16-720B Computer Vision
Homework 4
3D Reconstruction

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Submitted on: 11/14/2018
Late days used: 2

Q1.1)

Fundamental matrix $\rightarrow F$

Point 1 $\rightarrow x_1$

Point 2 on other camera $\rightarrow x_2$

F is a 3X3 matrix:

$$F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$$

we know

$$x_1^T = [0 \quad 0 \quad 1]$$

$$x_2^T = [0 \quad 0 \quad 1]$$

Using relation $\rightarrow$ $x_2^T F x = 0$

$$\underset{(1\times3)}{[0 \quad 0 \quad 1]} \underset{(3\times3)}{\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}} \underset{(3\times1)}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} = \underset{(1\times1)}{0}$$

simplifying

$$[F_{31} \quad F_{32} \quad F_{33}] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \Rightarrow \boxed{F_{33} = 0}$$

Q1.2) Let's assume:

translational vector $t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$

since, translational vector is $\parallel$ to x axis t_2 & $t_3 = 0$

$$t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix}$$

Rotational Matrix $\Rightarrow$ $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

since, it is pure translational, hence rotational matrix $= I$

Cross product matrix of translation is given by $\Rightarrow t_x$

$$t_x = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

Essential Matrix $\Rightarrow E$

$$E = t_x R$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

Now, we know that

$$l_2 = E x_1$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix}$$

Epipolar line $\rightarrow l_2$

$$x_2^T l_2 = 0$$

Equation of l_2

$$\hookrightarrow \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix} = 0$$

$$\Rightarrow \boxed{\begin{array}{l} L_2 \\ -t_1 y_2 + t_1 y_1 = 0 \end{array}}$$

From above equation of line, we can see that L_2 is parallel to x axis.

Q1.3) Let X be the 3D point in the world frame

$x_i \rightarrow$ points in the image

Pinhole camera equation:

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = k(R_i \mid t_i) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Take 2 Points

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = k\left(R_1 \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} + t_1\right)$$

and

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = k\left(R_2 \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} + t_2\right)$$

Rewriting above equation.

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = R_2^{-1}\left(\left(k^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}\right) - t_2\right)$$

Substituting above equations in the point equation of first frame:

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K \left(R_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_1 \right)$$

$$= K \left(R_1 \left(R_2^{-1} \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) \right) + t_1 \right)$$

$$= K R_1 R_2^{-1} K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - K R_1 R_2^{-1} t_2 + K R_1 t_1$$

We can tell that t_{rel} and R_{rel} :

$$\boxed{\begin{aligned} t_{relation} &= -K R_1 R_2^{-1} t_2 + K R_1 t_1 \\ R_{relation} &= K R_1 R_2^{-1} K^{-1} \end{aligned}}$$

and the fundamental matrix F and essential matrix E are given by :

$$\boxed{E = t_{relation\times} \; R_{relation}}$$

[$t_{relation\times}$ is the cross product matrix of translational matrix]

$$\boxed{F = K^{-1} E K = K^{-1} t_{rel\times} R_{rel} K}$$

$$\boxed{F = K^{-1} t_{rel\times} R_{rel} K}$$

Q 1.4)

Fundamental matrix $\rightarrow F$

Real world coordinates of Image $\rightarrow W$

Real world coordinates of reflection Image $\rightarrow W'$

Corresponding ~~f~~ image point $\rightarrow w$

Corresponding reflection point $\rightarrow w'$

Pinhole camera equation in homogenous form

$\hookrightarrow$ $\lambda_1 w = KW$

and

$\lambda_2 w' = KW'$

where, K is intrinsic Matrix of camera $\rightarrow W = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

Since there is only a reflection between W and W',

we can infer that the ~~R~~ rotation matrix is an identity matrix, i.e. $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

there exists only a translation, which can be described as $T = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$

Now, we know:

$$w' = RW + T$$

Simplifying further:

$$\lambda_2 K^{-1} w'^{-1} = \lambda_1 R K^{-1} w + T$$

$$\lambda_2 K^{-1} \tilde{T} w'^{-1} = \lambda_1 R K^{-1} \tilde{T} w + T\tilde{T}$$

$\tilde{T} \rightarrow$ Taking cross product with T

such that $T\tilde{T} = 0$

$$\therefore \quad \lambda_2 K^{-1} \tilde{T} w^{-1} = \lambda_1 R K^{-1} \tilde{T} w$$

Now, taking the dot product with W' on both sides

$$(\lambda_2 K^{-1} w')^T (\lambda_2 K^{-1} \tilde{T} w') = (\lambda_2 K^{-1} w')^T (\lambda_1 R K^{-1} \tilde{T} w)$$

$$= (\lambda_2)^2 K^{-1^T} \tilde{T} w'^T K^{-1} \tilde{T} w' = \lambda_2 \lambda_1 K^{-1^T} w'^T R K^{-1} \tilde{T} w$$

Now, $K^{-1^T} \tilde{T} a'^T K^{-1} \tilde{T} a' = 0$

since volume $= 0$

$$\lambda_2 \lambda_1 K^{-1^T} w'^T R K^{-1} \tilde{T} w = 0$$

$$K^{-1^T} w'^T R K^{-1} \tilde{T} w = 0$$

$$w'^T (K^{-T} R K^{-1} \tilde{T}) w = 0$$

$$w'^T F w = 0$$

where

$$\boxed{F = K^{-T} R K^{-1} \tilde{T}}$$

Since $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and K won't affect skew-symmetricity of T matrix.

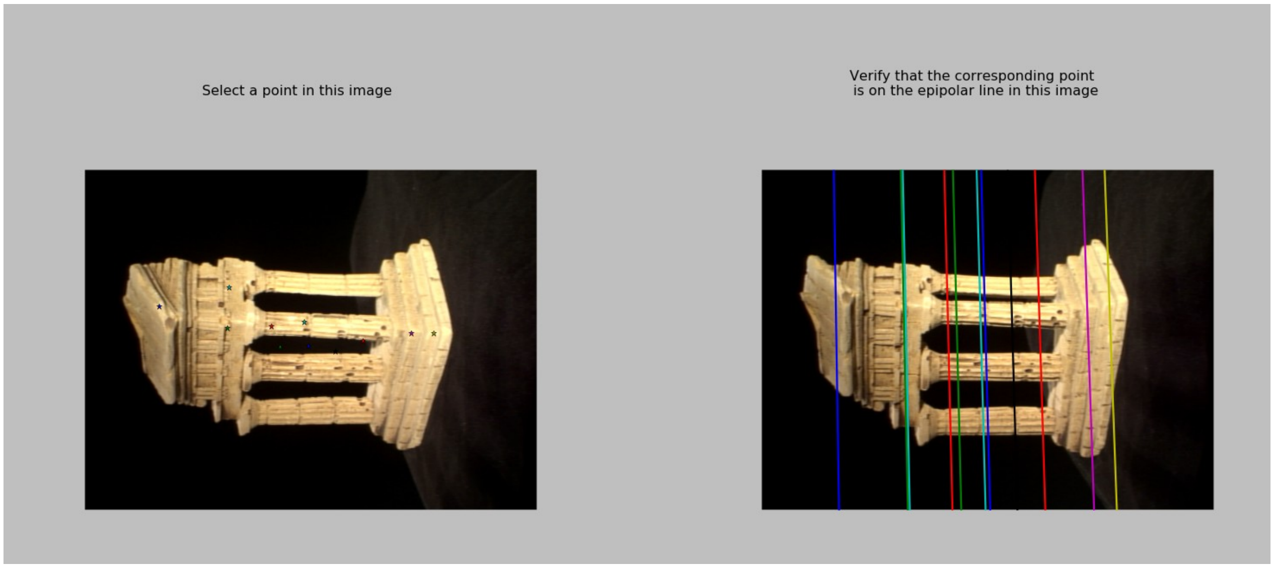
$$\tilde{T} = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

$$\boxed{-\tilde{T}^T = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \tilde{T}}$$

this shows that $\tilde{T}$ is a skew symmetric matrix
and hence, F is a skew symmetric matrix
hence fundamental matrix => skew symmetric

Q2.1)

8 point algorithm is used.
Following are the results:



Following is the fundamental matrix:

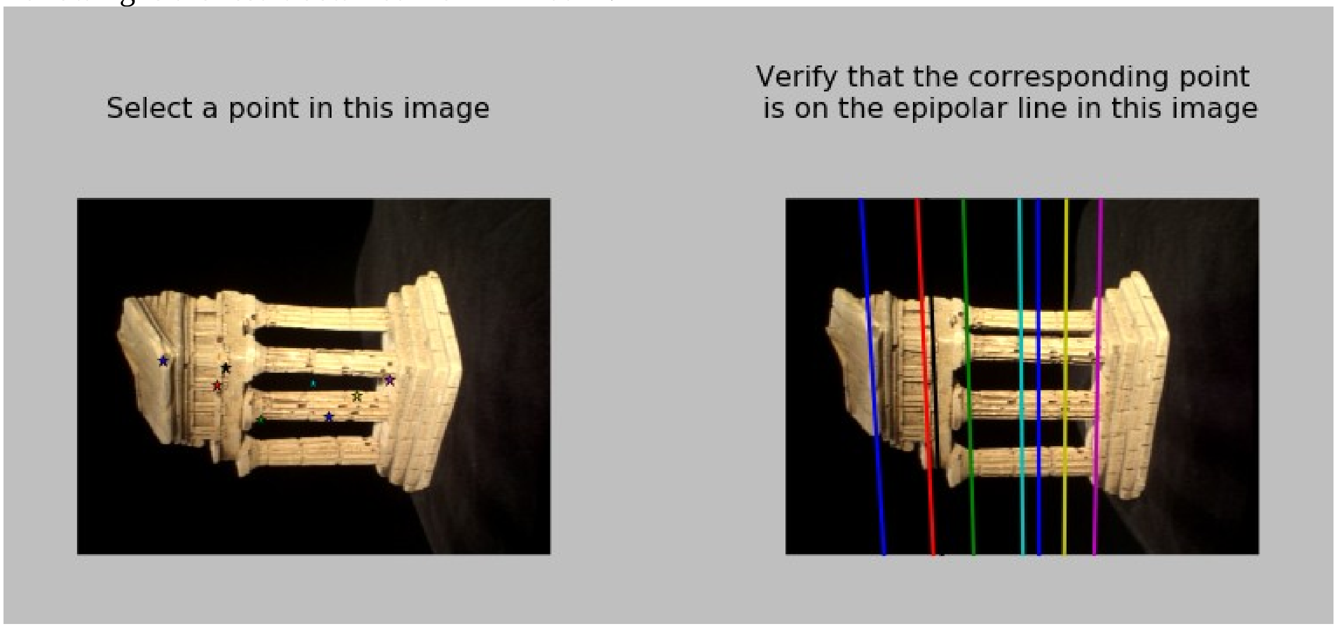
```
[[ 9.78833288e-10 -1.32135929e-07  1.12585666e-03]
 [-5.73843315e-08  2.96800276e-09 -1.17611996e-05]
 [-1.08269003e-03  3.04846703e-05 -4.47032655e-03]]
```

Results are saved in:
q2_1.npz

Q2.2)

7 point algorithm is used:

Following is the result obtained from 1st matrix:



F7 algorithm gave these 3 Fundumental matrices:

```
[array([[-7.80472372e-09,  6.72331321e-07, -1.54912851e-03],
       [-3.75353995e-07, -1.68423649e-08,  1.37436692e-04],
       [ 1.47600530e-03, -1.73021383e-04,  1.15477639e-02]]), array([[-7.985
27136e-08,  1.08728153e-06, -3.00951555e-04],
       [-1.01238913e-06,  7.66689515e-08,  2.32050293e-04],
       [ 3.21742003e-04, -2.85887631e-04,  3.47256046e-03]]), array([[-8.700
61744e-08,  1.12848088e-06, -1.77023255e-04],
       [-1.07563872e-06,  8.59534511e-08,  2.41444235e-04],
       [ 2.07138150e-04, -2.97093833e-04,  2.67079415e-03]])]
```

First of this matrix gave good result and has been used for above representation.

F7 matrix and points have been saved in below file:
q2_2.npz

Q3.1)

Essential matrix was computed using F8 (eight point fundamental matrix)

```
[[ 2.26268685e-03 -3.06552495e-01  1.66260633e+00]
 [-1.33130407e-01  6.91061098e-03 -4.33003420e-02]
 [-1.66721070e+00 -1.33210351e-02 -6.72186431e-04]]
```

Q3.2)

Let $pt1_{i1}$ and $pt1_{i2}$ represent x and y coordinates of the i^{th} point from set of points 1.

Similarly, for set of points 2

$pt2_{i1}$ and $pt2_{i2}$ → x and y coordinates

Let $C1_{i:}$ represent the i^{th} row of C_1 and let C_{2i} represent the i^{th} row of C_2

where both $C1_{i:}$ and $C_{2i:}$ is a vector of length 4.

$$A_i = \begin{bmatrix} pt1_{i1}\, C1_{3:} - C1_{1:} \\ pt1_{i2}\, C1_{3:} - C1_{2:} \\ pt2_{i1}\, C2_{3:} - C2_{1:} \\ pt2_{i2}\, C2_{3:} - C2_{2:} \end{bmatrix}$$

Q3.3)

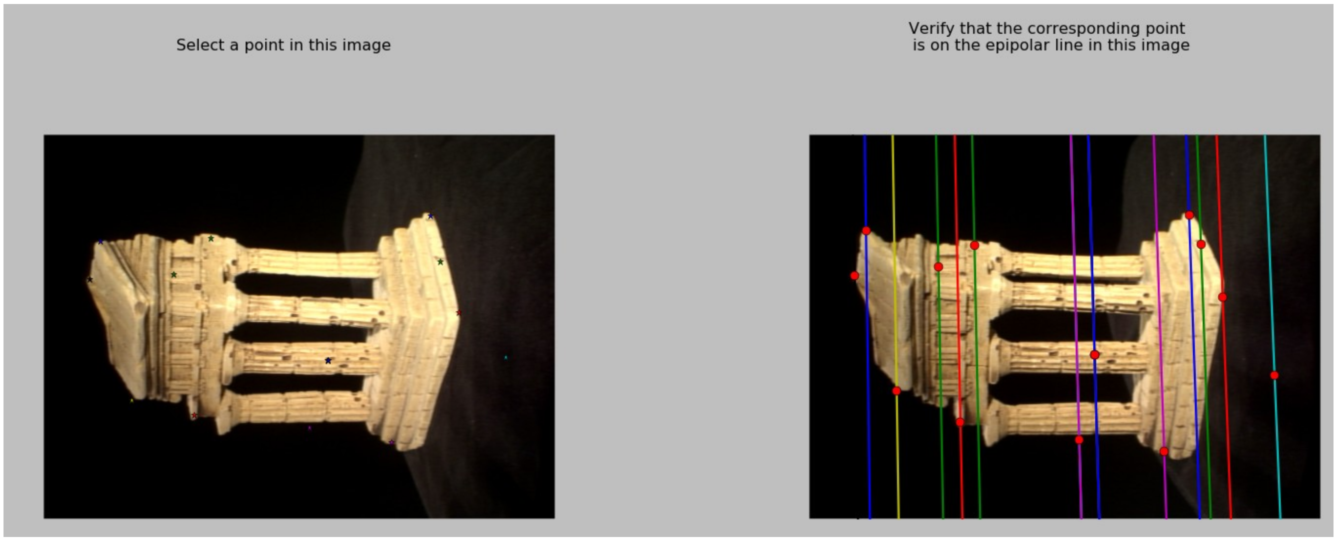
Best matrix using find M2 is:

```
[[ 0.99942701  0.03331428  0.0059843  -0.02601138]
 [-0.03372743  0.96531375  0.25890503 -1.        ]
 [ 0.00284851 -0.25895852  0.96588424  0.07981688]]
```

Result has been stored in q3 3.npz

Q4.1)

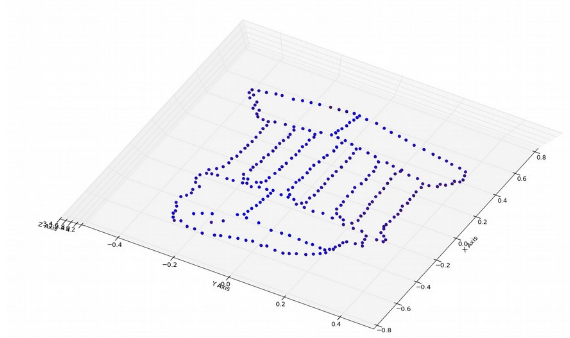
Corresponding points using Epipolar correspondence have been found out.
This has been made using 8 point algorithm

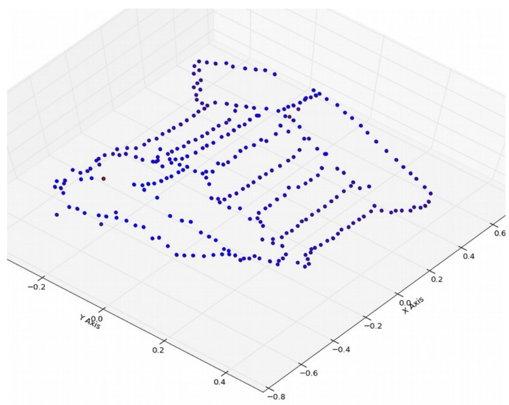


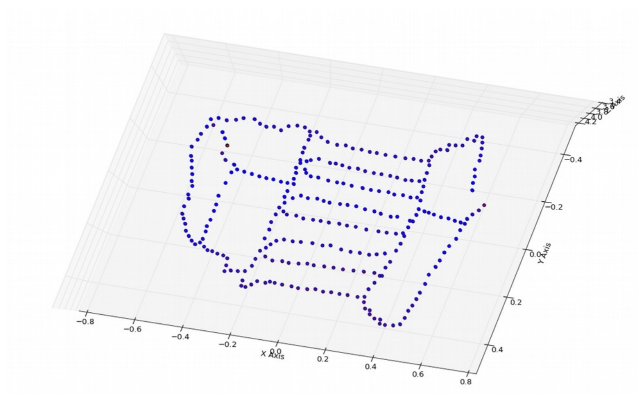
File has been saved with the name:

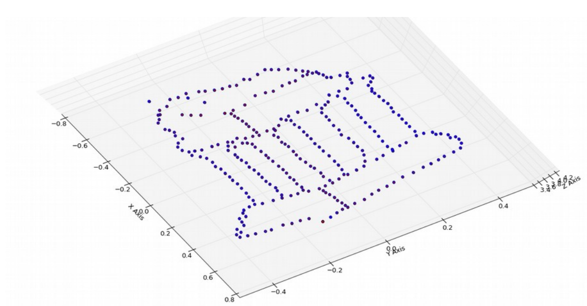
q4 1.npz
It includes F8 matrix and points used to create 8 point algorithm (As said on piazza post)

Q4.2)
Point cloud Visualization
This has been made in visualize.py
Following are the different views in :









Results are saved in
q4_2.npz

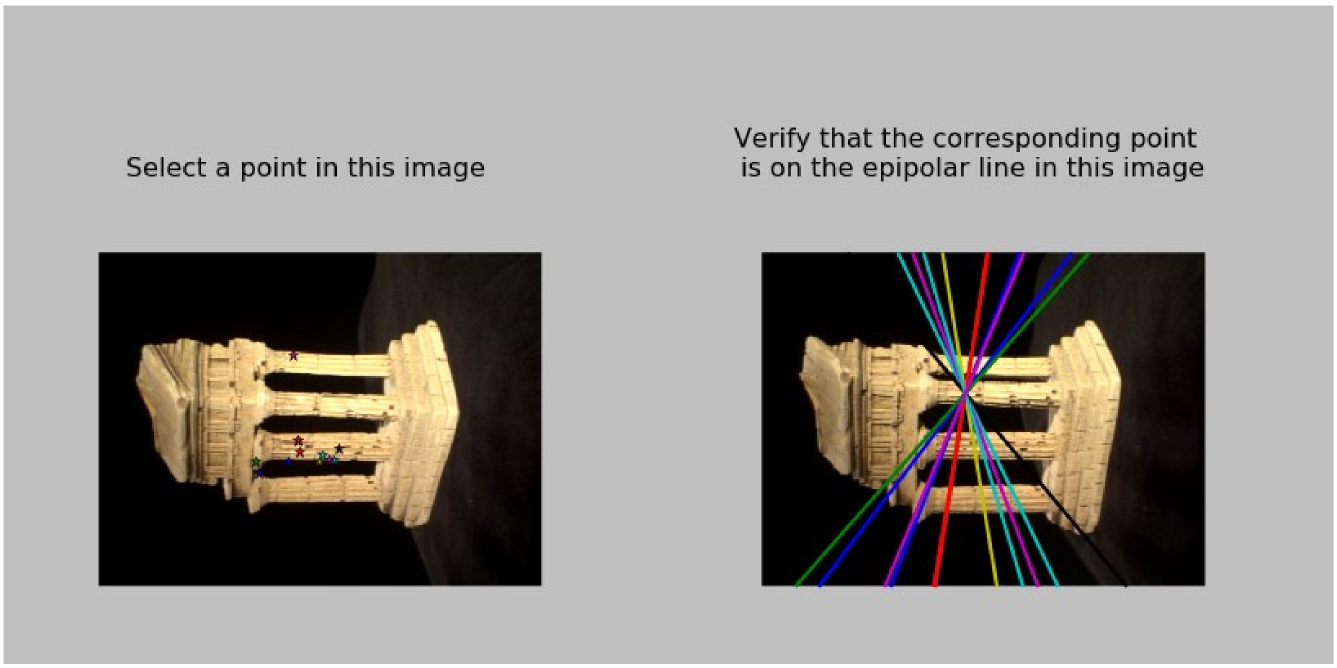
Q5.1)

RANSAC function was used for this question.

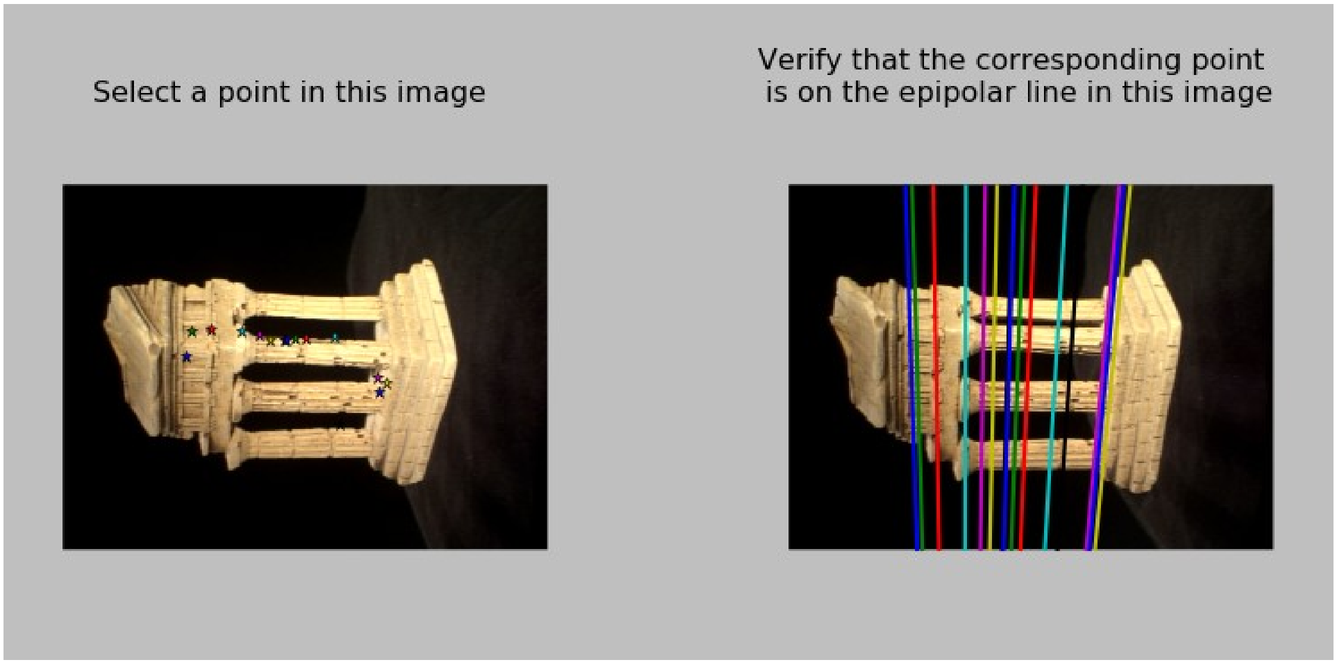
Following tasks has been done in this algorithm:

1) Select 7 random points → Compute 7 point Fundamental matrix
2) Epipolar constraints → x2.t F x2 = 0 is used for each point
3) Error matrix has to be small, as ideally it has to be 0
4) Parameters: Iterations → 250, tolerance → 0.001
5) Roughly 100 inliers have been returned

Below figure shows output from eight point algorithm from some_corresp_noisy.npz



Below figure shows output when F is computed through ransac from some_corresp.npz



Q5.3)
Bundle adjustment

Initial reprojection error:
Final reprojection error:

<< Code is almost complete → Just few minor fixes and plotting could be done >>

→ Request you to grant partial marks.