x1x3h4rj0

October 11, 2025

[1]:

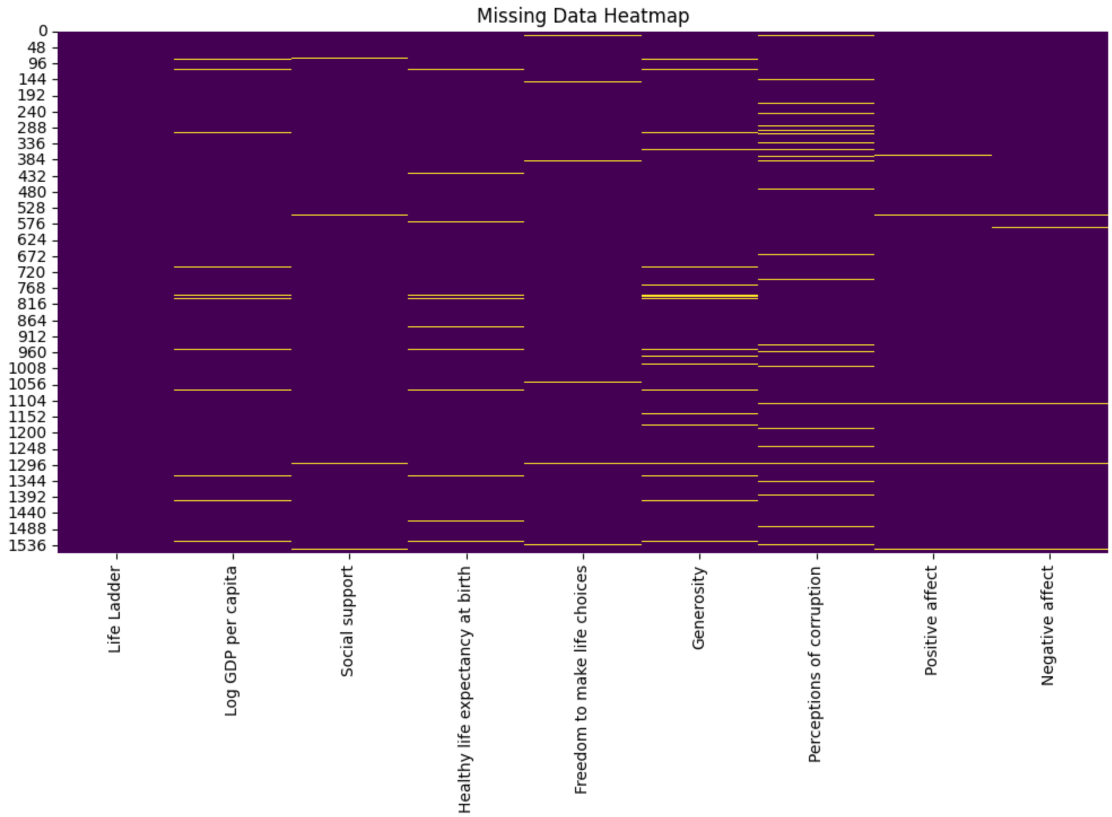
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import cross_val_score, KFold
from sklearn.linear_model import LinearRegression, SGDRegressor, Ridge, Lasso,␣
↪ElasticNet
from sklearn.preprocessing import StandardScaler,  PolynomialFeatures
from sklearn.metrics import mean_squared_error, r2_score, mean_absolute_error
import warnings
warnings.filterwarnings('ignore')


# Load your dataset
data = pd.read_csv('happiness_data_train.csv')
print(data.head())
print(data.info())
print(data.describe())


plt.figure(figsize=(16, 12))

# Plot missing data heatmap
plt.figure(figsize=(12, 6))
sns.heatmap(data.isnull(), cbar=False, cmap='viridis')
plt.title("Missing Data Heatmap")
plt.show()

# Dataset summary
num_rows, num_cols = data.shape
print(f"\nThe dataset contains {num_rows} rows and {num_cols} columns.\n")

# Identify categorical vs continuous features
categorical_features = data.select_dtypes(include=['object']).columns.tolist()
continuous_features = data.select_dtypes(include=['float64', 'int64']).columns.
↪tolist()
```

```python
print("Categorical features:", categorical_features)
print("Continuous features:", continuous_features)
```

```
   Life Ladder  Log GDP per capita  Social support  \
0        4.930               9.343           0.766
1        5.367              10.299           0.901
2        7.375              10.881           0.931
3        4.613               7.554           0.724
4        6.180              10.945             NaN

   Healthy life expectancy at birth  Freedom to make life choices  Generosity  \
0                             64.22                           NaN      -0.127
1                             67.48                         0.754      -0.201
2                             72.60                         0.942       0.077
3                             60.64                         0.702      -0.088
4                             65.80                           NaN         NaN

   Perceptions of corruption  Positive affect  Negative affect
0                      0.709            0.669            0.331
1                      0.726            0.702            0.199
2                      0.263            0.823            0.161
3                      0.768            0.566            0.195
4                        NaN              NaN              NaN
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1559 entries, 0 to 1558
Data columns (total 9 columns):
 #   Column                            Non-Null Count  Dtype
---  ------                            --------------  -----
 0   Life Ladder                       1559 non-null   float64
 1   Log GDP per capita                1533 non-null   float64
 2   Social support                    1549 non-null   float64
 3   Healthy life expectancy at birth  1522 non-null   float64
 4   Freedom to make life choices      1537 non-null   float64
 5   Generosity                        1488 non-null   float64
 6   Perceptions of corruption         1473 non-null   float64
 7   Positive affect                   1542 non-null   float64
 8   Negative affect                   1545 non-null   float64
dtypes: float64(9)
memory usage: 109.7 KB
None
       Life Ladder  Log GDP per capita  Social support  \
count  1559.000000         1533.000000     1549.000000
mean      5.469321            9.362995        0.812689
std       1.113280            1.160240        0.118762
min       2.375000            6.635000        0.290000
25%       4.649500            8.460000        0.753000
50%       5.374000            9.456000        0.835000
```

```
75%       6.272500           10.332000       0.905000
max       8.019000           11.648000       0.987000

       Healthy life expectancy at birth  Freedom to make life choices  \
count                       1522.000000                   1537.000000
mean                          63.331680                      0.741882
std                            7.478307                      0.142326
min                           32.300000                      0.260000
25%                           58.340000                      0.646000
50%                           65.180000                      0.763000
75%                           68.535000                      0.855000
max                           77.100000                      0.985000

        Generosity  Perceptions of corruption  Positive affect  \
count  1488.000000                1473.000000      1542.000000
mean      0.000343                   0.748248         0.708551
std       0.163385                   0.186708         0.108029
min      -0.335000                   0.035000         0.322000
25%      -0.113000                   0.691000         0.623000
50%      -0.025000                   0.804000         0.721000
75%       0.091000                   0.873000         0.798000
max       0.698000                   0.983000         0.944000

       Negative affect
count      1545.000000
mean          0.267644
std           0.084431
min           0.083000
25%           0.206000
50%           0.258000
75%           0.319000
max           0.705000
```

```
<Figure size 1600x1200 with 0 Axes>
```




```
The dataset contains 1559 rows and 9 columns.

Categorical features: []
Continuous features: ['Life Ladder', 'Log GDP per capita', 'Social support',
'Healthy life expectancy at birth', 'Freedom to make life choices',
'Generosity', 'Perceptions of corruption', 'Positive affect', 'Negative affect']
```

[2]:
```
print(data.isnull().sum())
target_col = 'Life Ladder'
X = data.drop(columns=[target_col])
y = data[target_col]
```

```
Life Ladder                          0
Log GDP per capita                  26
Social support                      10
Healthy life expectancy at birth    37
Freedom to make life choices        22
Generosity                          71
Perceptions of corruption           86
Positive affect                     17
Negative affect                     14
```

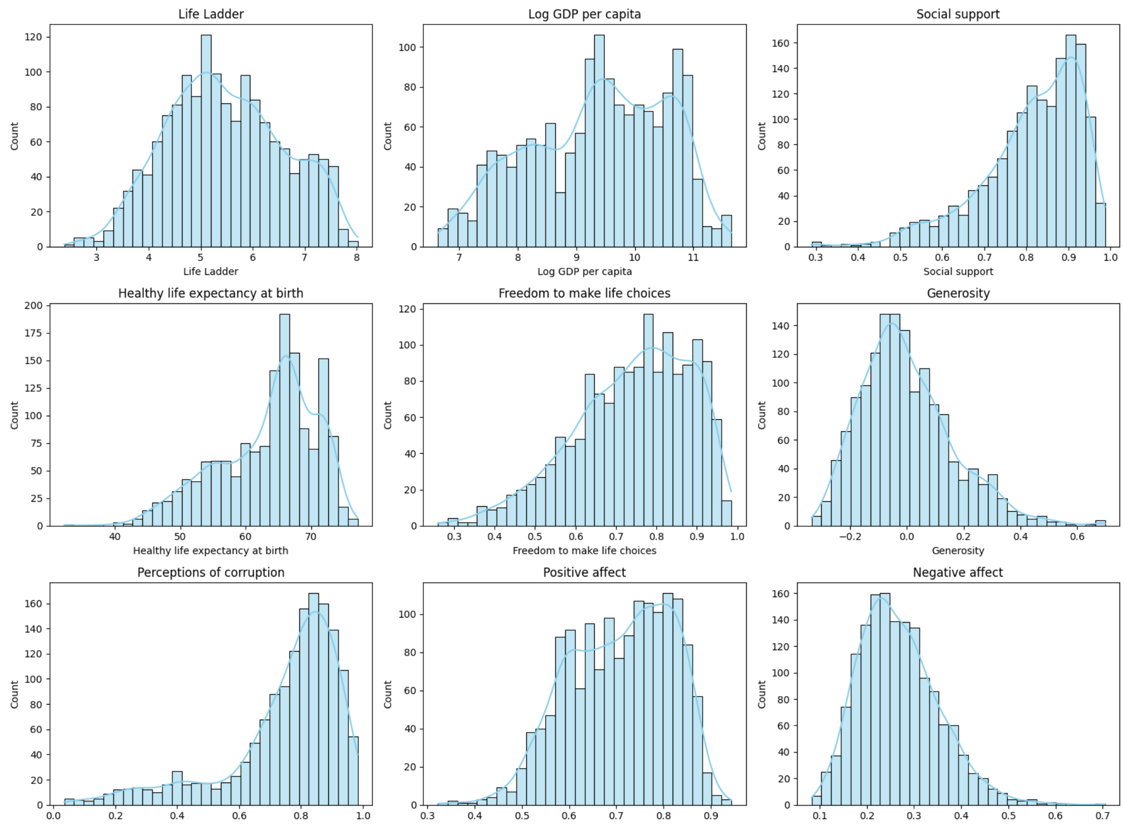
dtype: int64

0.1 Anwer to question 1:

1559 entries are present. All the features are continuous and no attribute is categorical. Some of the the features like perception of corruption has more missing values than others.

---

[3]:
```python
# Plot histograms for all continuous features
plt.figure(figsize=(16, 12))
for i, col in enumerate(data.columns, 1):
    plt.subplot(3, 3, i)
    sns.histplot(data[col], kde=True, bins=30, color='skyblue')
    plt.title(col)
plt.tight_layout()
plt.show()
```



0.2 Answer to B

Require Special Treatment for: Social support, Freedom to make life choices, Positive affect : strong ceiling effects. Nonlinear transformation required. Generosity : skewed with negatives; handle outliers or normalize. Perceptions of corruption : ceiling effect; may benefit from categorization (low vs. high corruption perception). ——————————————————————————-

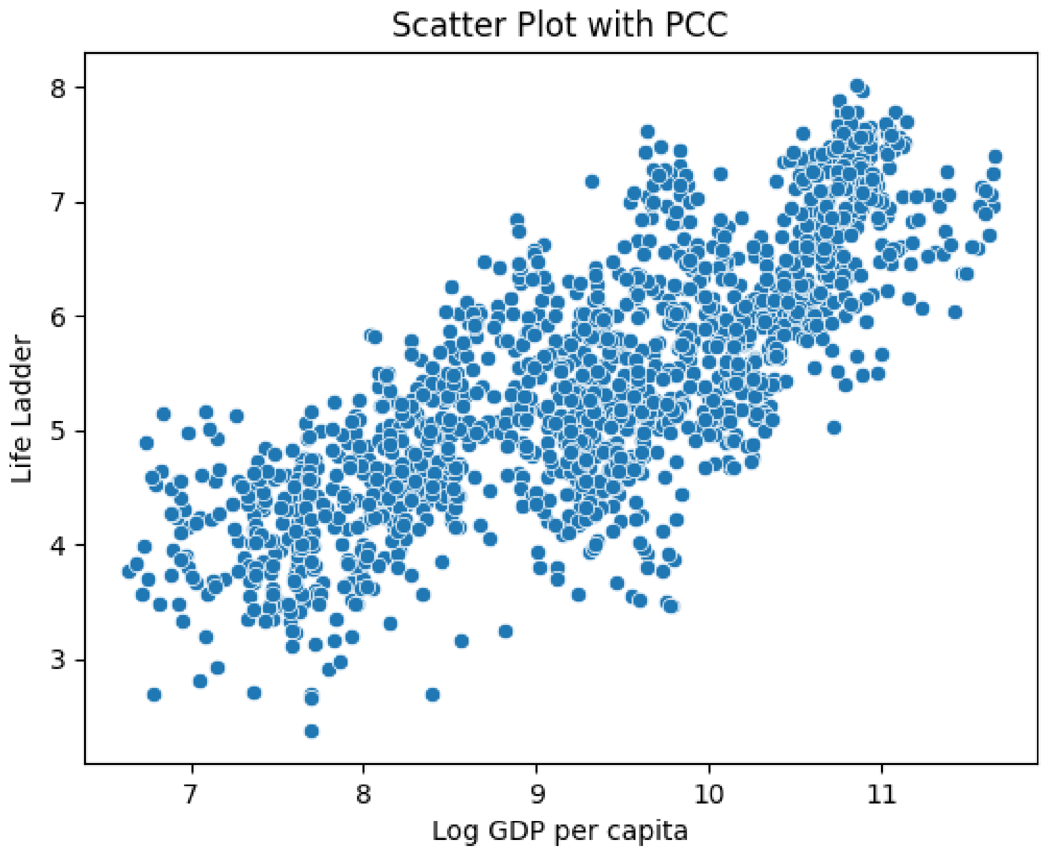
[4]:
```python
corr_matrix = data.corr(method='pearson')
print(corr_matrix)
```

```
                              Life Ladder  Log GDP per capita  \
Life Ladder                      1.000000            0.786712
Log GDP per capita               0.786712            1.000000
Social support                   0.703878            0.696893
Healthy life expectancy at birth    0.750773            0.850872
Freedom to make life choices     0.529819            0.359674
Generosity                       0.194970           -0.006162
Perceptions of corruption       -0.427562           -0.341493
Positive affect                  0.534640            0.296017
Negative affect                 -0.301821           -0.215581

                              Social support  \
Life Ladder                         0.703878
Log GDP per capita                  0.696893
Social support                      1.000000
Healthy life expectancy at birth       0.621460
Freedom to make life choices        0.420372
Generosity                          0.062367
Perceptions of corruption          -0.225764
Positive affect                     0.424982
Negative affect                    -0.403880

                              Healthy life expectancy at birth  \
Life Ladder                                           0.750773
Log GDP per capita                                    0.850872
Social support                                        0.621460
Healthy life expectancy at birth                         1.000000
Freedom to make life choices                          0.381830
Generosity                                            0.019077
Perceptions of corruption                            -0.322159
Positive affect                                       0.311821
Negative affect                                      -0.137307

                              Freedom to make life choices  Generosity  \
Life Ladder                                       0.529819    0.194970
Log GDP per capita                                0.359674   -0.006162
Social support                                    0.420372    0.062367
Healthy life expectancy at birth                     0.381830    0.019077
Freedom to make life choices                      1.000000    0.327079
Generosity                                        0.327079    1.000000
Perceptions of corruption                        -0.480826   -0.276108
Positive affect                                   0.614398    0.377166
```

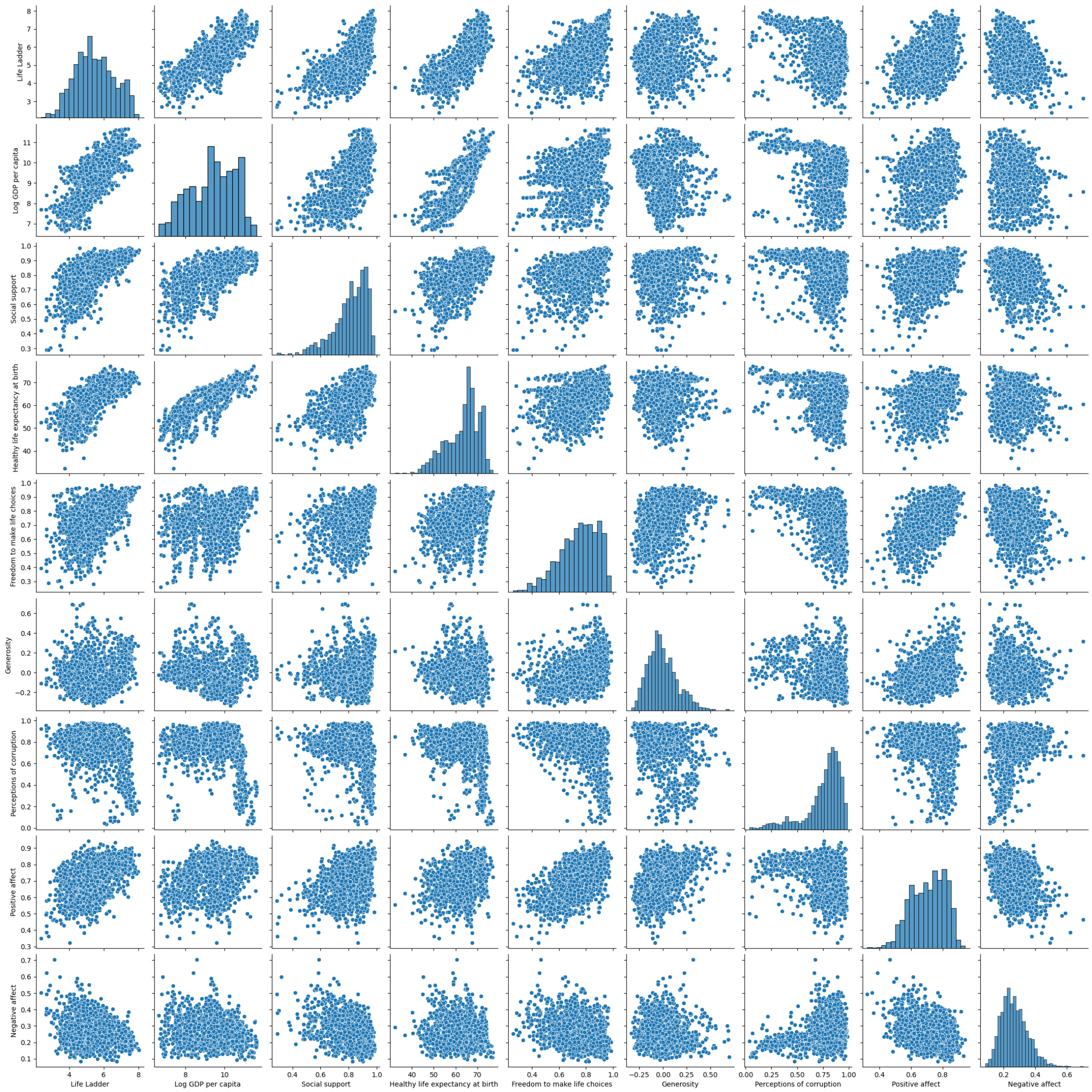
```
Negative affect                                 -0.272536   -0.106673

                                  Perceptions of corruption  Positive affect  \
Life Ladder                                       -0.427562         0.534640
Log GDP per capita                                -0.341493         0.296017
Social support                                    -0.225764         0.424982
Healthy life expectancy at birth                  -0.322159         0.311821
Freedom to make life choices                      -0.480826         0.614398
Generosity                                        -0.276108         0.377166
Perceptions of corruption                          1.000000        -0.300299
Positive affect                                   -0.300299         1.000000
Negative affect                                    0.277781        -0.357850

                                  Negative affect
Life Ladder                             -0.301821
Log GDP per capita                      -0.215581
Social support                          -0.403880
Healthy life expectancy at birth        -0.137307
Freedom to make life choices            -0.272536
Generosity                              -0.106673
Perceptions of corruption                0.277781
Positive affect                         -0.357850
Negative affect                          1.000000
```

0.3 Answer to Part C:

[5]:
```
sns.scatterplot(x='Log GDP per capita', y='Life Ladder', data=data)
plt.title("Scatter Plot with PCC")
plt.show()
```



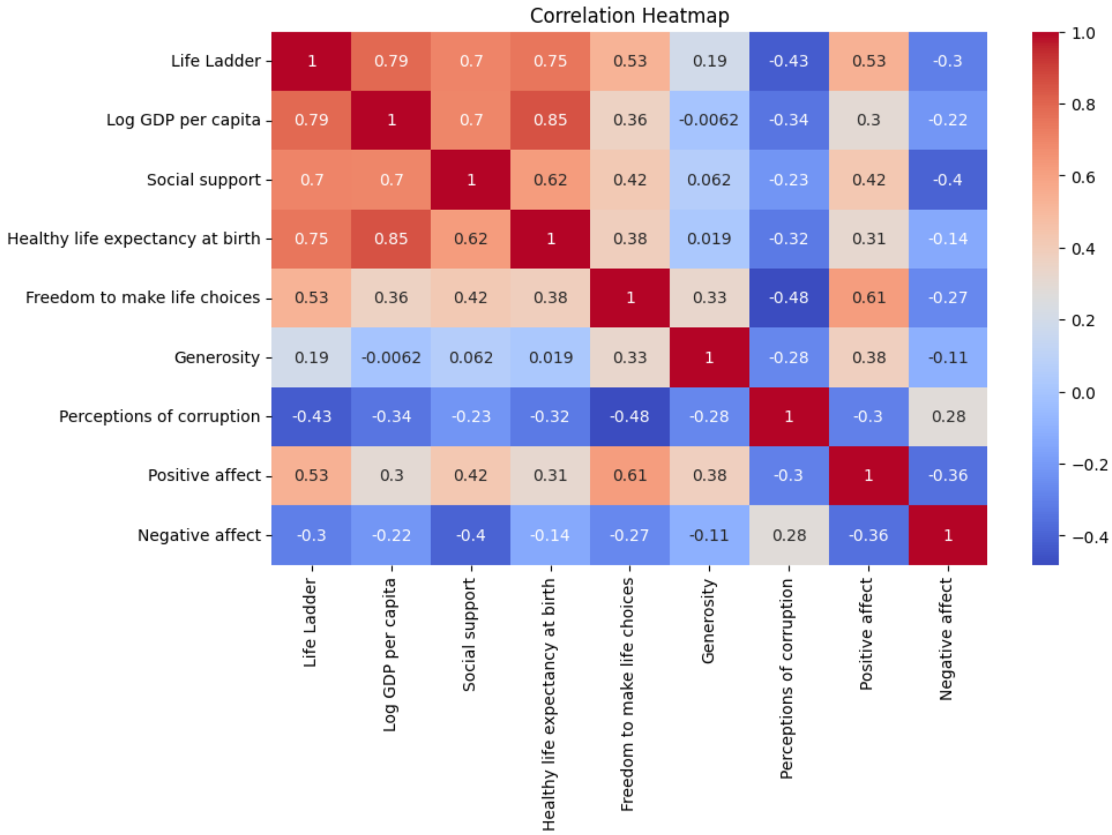
[6]:
```
sns.pairplot(data[['Life Ladder', 'Log GDP per capita', 'Social support',␣
  ↪'Healthy life expectancy at birth'    ,
'Freedom to make life choices',
'Generosity',
'Perceptions of corruption',
'Positive affect',
'Negative affect']])
plt.show()
```



Answer C: It can be said by looking at PCC and scatter plot that : Life Ladder is strongly related with Log GDP/Capita, social support, healthy life expectancy, freedom of make life choice. It is also weekly related with prerception of corruption, negative affect.

It is safe to say that cells corresponding to oranages and reds have strong correlation between the corresponding features. But Blue and grey cells have less correlation between the corresponding feature.

[7]:
```
plt.figure(figsize=(10,6))
sns.heatmap(corr_matrix, annot=True, cmap='coolwarm')
plt.title("Correlation Heatmap")
plt.show()
```



0.4 Part D starts here:

[8]:
```
RANDOM_SEED = 30
np.random.seed(RANDOM_SEED)
```

[9]:
```
target_col = 'Life Ladder'
X = data.drop(columns=[target_col])
y = data[target_col]
```

[10]:
```
valid_idx = ~y.isnull()
X = X[valid_idx]
y = y[valid_idx]    # drop null code, not necessary
```

[11]:
```
from sklearn.impute import SimpleImputer
imputer = SimpleImputer(strategy='median')
X_imputed = imputer.fit_transform(X)
X = pd.DataFrame(X_imputed, columns=X.columns)

print(f"\nAfter handling missing values:")
print(f"Dataset shape: {X.shape}")
```

```
print(f"Features used: {list(X.columns)}")
print(f"Target variable: {target_col}")
```

```
After handling missing values:
Dataset shape: (1559, 8)
Features used: ['Log GDP per capita', 'Social support', 'Healthy life expectancy
at birth', 'Freedom to make life choices', 'Generosity', 'Perceptions of
corruption', 'Positive affect', 'Negative affect']
Target variable: Life Ladder
```

[12]:
```
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)
```

[13]:
```
kfold = KFold(n_splits=4, shuffle=True, random_state=RANDOM_SEED)

print("\n" + "="*80)
print("PART 1: CLOSED-FORM SOLUTIONS")
print("="*80)
```

```
================================================================================
PART 1: CLOSED-FORM SOLUTIONS
================================================================================
```

[ ]:
```

```

[14]:
```
print("\n1. Linear Regression (Normal Equation/SVD)")
lr_model = LinearRegression()
cv_scores = cross_val_score(lr_model, X_scaled, y, cv=kfold,
                            scoring='neg_root_mean_squared_error')
cv_rmse = -cv_scores
print(f"   Cross-validation RMSE scores: {cv_rmse}")
print(f"   Mean CV RMSE: {cv_rmse.mean():.4f} (+/- {cv_rmse.std():.4f})")
```

```
1. Linear Regression (Normal Equation/SVD)
   Cross-validation RMSE scores: [0.50655203 0.54872898 0.56828078 0.57072328]
   Mean CV RMSE: 0.5486 (+/- 0.0257)
```

[15]:
```
lr_model.fit(X_scaled, y)
y_pred_lr = lr_model.predict(X_scaled)
train_rmse_lr = np.sqrt(mean_squared_error(y, y_pred_lr))
train_r2_lr = r2_score(y, y_pred_lr)
print(f"   Training RMSE: {train_rmse_lr:.4f}")
print(f"   Training R²: {train_r2_lr:.4f}")
```

```
print("\n" + "="*80)
print("PART 2: SGD WITHOUT REGULARIZATION")
print("="*80)
```

```
   Training RMSE: 0.5454
   Training R²: 0.7598

================================================================================
PART 2: SGD WITHOUT REGULARIZATION
================================================================================
```

[16]:
```
print("\n2. SGD Linear Regression (No Regularization)")
sgd_model = SGDRegressor(penalty=None, max_iter=1000, tol=1e-3,
                         random_state=RANDOM_SEED, learning_rate='invscaling',
                         eta0=0.01)
cv_scores_sgd = cross_val_score(sgd_model, X_scaled, y, cv=kfold,
                                scoring='neg_root_mean_squared_error')
cv_rmse_sgd = -cv_scores_sgd
print(f"   Cross-validation RMSE scores: {cv_rmse_sgd}")
print(f"   Mean CV RMSE: {cv_rmse_sgd.mean():.4f} (+/- {cv_rmse_sgd.std():.
 ↪4f})")

print("\n" + "="*80)
print("PART 3: REGULARIZATION EXPERIMENTS")
print("="*80)
```

```
2. SGD Linear Regression (No Regularization)
   Cross-validation RMSE scores: [0.50773048 0.54979493 0.56585417 0.57061505]
   Mean CV RMSE: 0.5485 (+/- 0.0248)

================================================================================
PART 3: REGULARIZATION EXPERIMENTS
================================================================================
```

[17]:
```
print("\n3. Ridge Regularization (L2)")
alpha_values = [0.01, 1.0, 100.0]
ridge_results = {}

for alpha in alpha_values:
    sgd_ridge = SGDRegressor(penalty='l2', alpha=alpha, max_iter=1000,
                             tol=1e-3, random_state=RANDOM_SEED,
                             learning_rate='invscaling', eta0=0.01)
    cv_scores = cross_val_score(sgd_ridge, X_scaled, y, cv=kfold,
                                scoring='neg_root_mean_squared_error')
    cv_rmse = -cv_scores
    ridge_results[alpha] = cv_rmse.mean()
```

```
    print(f"   Alpha={alpha:6.2f}: CV RMSE = {cv_rmse.mean():.4f} (+/- {cv_rmse.
↪std():.4f})")
```

```
3. Ridge Regularization (L2)
   Alpha=  0.01: CV RMSE = 0.5485 (+/- 0.0248)
   Alpha=  1.00: CV RMSE = 0.5963 (+/- 0.0290)
   Alpha=100.00: CV RMSE = 1.0874 (+/- 0.0143)
```

[18]:

```
print("\n4. Lasso Regularization (L1)")
lasso_results = {}

for alpha in alpha_values:
    sgd_lasso = SGDRegressor(penalty='l1', alpha=alpha, max_iter=1000,
                             tol=1e-3, random_state=RANDOM_SEED,
                             learning_rate='invscaling', eta0=0.01)
    cv_scores = cross_val_score(sgd_lasso, X_scaled, y, cv=kfold,
                                scoring='neg_root_mean_squared_error')
    cv_rmse = -cv_scores
    lasso_results[alpha] = cv_rmse.mean()
    print(f"   Alpha={alpha:6.2f}: CV RMSE = {cv_rmse.mean():.4f} (+/- {cv_rmse.
↪std():.4f})")
```

```
4. Lasso Regularization (L1)
   Alpha=  0.01: CV RMSE = 0.5487 (+/- 0.0251)
   Alpha=  1.00: CV RMSE = 1.1133 (+/- 0.0082)
   Alpha=100.00: CV RMSE = 1.1133 (+/- 0.0082)
```

[19]:

```
print("\n5. Elastic Net Regularization (L1 + L2, l1_ratio=0.5)")
elasticnet_results = {}

for alpha in alpha_values:
    sgd_elastic = SGDRegressor(penalty='elasticnet', alpha=alpha, l1_ratio=0.5,
                               max_iter=1000, tol=1e-3,␣
↪random_state=RANDOM_SEED,
                               learning_rate='invscaling', eta0=0.01)
    cv_scores = cross_val_score(sgd_elastic, X_scaled, y, cv=kfold,
                                scoring='neg_root_mean_squared_error')
    cv_rmse = -cv_scores
    elasticnet_results[alpha] = cv_rmse.mean()
    print(f"   Alpha={alpha:6.2f}: CV RMSE = {cv_rmse.mean():.4f} (+/- {cv_rmse.
↪std():.4f})")
```

```
5. Elastic Net Regularization (L1 + L2, l1_ratio=0.5)
   Alpha=  0.01: CV RMSE = 0.5486 (+/- 0.0250)
   Alpha=  1.00: CV RMSE = 0.8782 (+/- 0.0058)
```

```
Alpha=100.00: CV RMSE = 1.1133 (+/- 0.0082)
```

[20]:
```
print("\n" + "="*80)
print("PART 4: HYPERPARAMETER EXPLORATION - LEARNING RATE")
print("="*80)
```

```
================================================================================
PART 4: HYPERPARAMETER EXPLORATION - LEARNING RATE
================================================================================
```

[21]:
```
print("\n6. Impact of Learning Rate (with Ridge, alpha=1.0)")
learning_rates = [0.001, 0.01, 0.1]
lr_results = {}

for lr in learning_rates:
    sgd_lr = SGDRegressor(penalty='l2', alpha=1.0, max_iter=1000, tol=1e-3,
                          random_state=RANDOM_SEED, learning_rate='constant',
                          eta0=lr)
    cv_scores = cross_val_score(sgd_lr, X_scaled, y, cv=kfold,
                                scoring='neg_root_mean_squared_error')
    cv_rmse = -cv_scores
    lr_results[lr] = cv_rmse.mean()
    print(f"   Learning Rate={lr:.4f}: CV RMSE = {cv_rmse.mean():.4f} (+/-␣
↪{cv_rmse.std():.4f})")

print("\n   Impact: Lower learning rates converge slowly but more stably.")
print("           Higher learning rates converge faster but may overshoot or␣
↪diverge.")

print("\n7. Impact of Learning Rate Schedule (with Ridge, alpha=1.0, eta0=0.
↪01)")
schedules = ['constant', 'optimal', 'invscaling', 'adaptive']
schedule_results = {}

for schedule in schedules:
    sgd_schedule = SGDRegressor(penalty='l2', alpha=1.0, max_iter=1000,␣
↪tol=1e-3,
                                random_state=RANDOM_SEED,␣
↪learning_rate=schedule,
                                eta0=0.01)
    cv_scores = cross_val_score(sgd_schedule, X_scaled, y, cv=kfold,
                                scoring='neg_root_mean_squared_error')
    cv_rmse = -cv_scores
    schedule_results[schedule] = cv_rmse.mean()
    print(f"   Schedule={schedule:12s}: CV RMSE = {cv_rmse.mean():.4f} (+/-␣
↪{cv_rmse.std():.4f})")
```

```
print("\n   Impact: Different schedules affect convergence speed and final␣
↪performance.")
print("           'optimal' and 'invscaling' typically work well for most␣
↪problems.")
```

```
6. Impact of Learning Rate (with Ridge, alpha=1.0)
   Learning Rate=0.0010: CV RMSE = 0.6003 (+/- 0.0228)
   Learning Rate=0.0100: CV RMSE = 0.6140 (+/- 0.0434)
   Learning Rate=0.1000: CV RMSE = 0.7090 (+/- 0.1281)

   Impact: Lower learning rates converge slowly but more stably.
           Higher learning rates converge faster but may overshoot or diverge.

7. Impact of Learning Rate Schedule (with Ridge, alpha=1.0, eta0=0.01)
   Schedule=constant    : CV RMSE = 0.6140 (+/- 0.0434)
   Schedule=optimal     : CV RMSE = 0.5986 (+/- 0.0241)
   Schedule=invscaling  : CV RMSE = 0.5963 (+/- 0.0290)
   Schedule=adaptive    : CV RMSE = 0.5988 (+/- 0.0244)

   Impact: Different schedules affect convergence speed and final performance.
           'optimal' and 'invscaling' typically work well for most problems.
```

[22]:
```
print("\n" + "="*80)
print("PART 5: SUMMARY AND BEST MODEL SELECTION")
print("="*80)
```

```
================================================================================
PART 5: SUMMARY AND BEST MODEL SELECTION
================================================================================
```

[23]:
```
all_results = {
    'Linear Regression': cv_rmse.mean(),
    'SGD (no reg)': cv_rmse_sgd.mean(),
}

for alpha in alpha_values:
    all_results[f'Ridge ( ={alpha})'] = ridge_results[alpha]
    all_results[f'Lasso ( ={alpha})'] = lasso_results[alpha]
    all_results[f'ElasticNet ( ={alpha})'] = elasticnet_results[alpha]

print("\nAll Model Performance (CV RMSE):")
for model_name, rmse in sorted(all_results.items(), key=lambda x: x[1]):
    print(f"   {model_name:30s}: {rmse:.4f}")
```

```
All Model Performance (CV RMSE):
   Ridge ( =0.01)                : 0.5485
   SGD (no reg)                  : 0.5485
   ElasticNet ( =0.01)           : 0.5486
   Lasso ( =0.01)                : 0.5487
   Ridge ( =1.0)                 : 0.5963
   Linear Regression             : 0.5988
   ElasticNet ( =1.0)            : 0.8782
   Ridge ( =100.0)               : 1.0874
   Lasso ( =100.0)               : 1.1133
   ElasticNet ( =100.0)          : 1.1133
   Lasso ( =1.0)                 : 1.1133
```

[24]:

```
best_model_name = min(all_results, key=all_results.get)
best_rmse = all_results[best_model_name]
print(f"\nBest Model: {best_model_name} with CV RMSE = {best_rmse:.4f}")

# Determine best hyperparameters
if 'Ridge' in best_model_name:
    best_penalty = 'l2'
    best_alpha = float(best_model_name.split(' =')[1].rstrip(')'))
elif 'Lasso' in best_model_name:
    best_penalty = 'l1'
    best_alpha = float(best_model_name.split(' =')[1].rstrip(')'))
elif 'ElasticNet' in best_model_name:
    best_penalty = 'elasticnet'
    best_alpha = float(best_model_name.split(' =')[1].rstrip(')'))
else:
    best_penalty = None
    best_alpha = 0.0001

print("\n" + "="*80)
print("PART 6: TRAINING BEST MODEL WITH EPOCH-BY-EPOCH TRACKING")
print("="*80)

# Train best model with epoch tracking
print(f"\nTraining best model: {best_model_name}")
print("Tracking training and validation loss by epoch...\n")

# Split data for validation tracking
from sklearn.model_selection import train_test_split
X_train, X_val, y_train, y_val = train_test_split(
    X_scaled, y, test_size=0.2, random_state=RANDOM_SEED
)
```

```
Best Model: Ridge ( =0.01) with CV RMSE = 0.5485
```

```
================================================================================
PART 6: TRAINING BEST MODEL WITH EPOCH-BY-EPOCH TRACKING
================================================================================

Training best model: Ridge ( =0.01)
Tracking training and validation loss by epoch…
```

[25]:
```python
if best_penalty == 'elasticnet':
    best_sgd_model = SGDRegressor(
        penalty=best_penalty, alpha=best_alpha, l1_ratio=0.5,
        max_iter=1, warm_start=True, random_state=RANDOM_SEED,
        learning_rate='invscaling', eta0=0.01, tol=None
    )
else:
    best_sgd_model = SGDRegressor(
        penalty=best_penalty, alpha=best_alpha,
        max_iter=1, warm_start=True, random_state=RANDOM_SEED,
        learning_rate='invscaling', eta0=0.01, tol=None
    )
```

[26]:
```python
n_epochs = 100
train_losses = []
val_losses = []

for epoch in range(n_epochs):
    best_sgd_model.fit(X_train, y_train)

    # Compute training and validation RMSE
    y_train_pred = best_sgd_model.predict(X_train)
    y_val_pred = best_sgd_model.predict(X_val)

    train_rmse = np.sqrt(mean_squared_error(y_train, y_train_pred))
    val_rmse = np.sqrt(mean_squared_error(y_val, y_val_pred))

    train_losses.append(train_rmse)
    val_losses.append(val_rmse)

    if (epoch + 1) % 10 == 0:
        print(f"Epoch {epoch+1:3d}: Train RMSE = {train_rmse:.4f}, Val RMSE = {val_rmse:.4f}")
```

```
Epoch  10: Train RMSE = 0.5549, Val RMSE = 0.5065
Epoch  20: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch  30: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch  40: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch  50: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch  60: Train RMSE = 0.5549, Val RMSE = 0.5066
```

```
Epoch  70: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch  80: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch  90: Train RMSE = 0.5549, Val RMSE = 0.5066
Epoch 100: Train RMSE = 0.5549, Val RMSE = 0.5066
```

[27]:
```
print("\n\nTraining final model on entire training dataset...")
if best_penalty == 'elasticnet':
    final_model = SGDRegressor(
        penalty=best_penalty, alpha=best_alpha, l1_ratio=0.5,
        max_iter=100, random_state=RANDOM_SEED,
        learning_rate='invscaling', eta0=0.01, tol=1e-3
    )
else:
    final_model = SGDRegressor(
        penalty=best_penalty, alpha=best_alpha,
        max_iter=100, random_state=RANDOM_SEED,
        learning_rate='invscaling', eta0=0.01, tol=1e-3
    )

final_model.fit(X_scaled, y)
y_pred_final = final_model.predict(X_scaled)
final_rmse = np.sqrt(mean_squared_error(y, y_pred_final))
final_r2 = r2_score(y, y_pred_final)
final_mae = mean_absolute_error(y, y_pred_final)
```

```
Training final model on entire training dataset…
```

[28]:
```
print(f"\nFinal Model Performance on Full Training Set:")
print(f"   RMSE: {final_rmse:.4f}")
print(f"   R²:   {final_r2:.4f}")
print(f"   MAE:  {final_mae:.4f}")

print("\n" + "="*80)
print("PART 7: VISUALIZATIONS")
print("="*80)
```

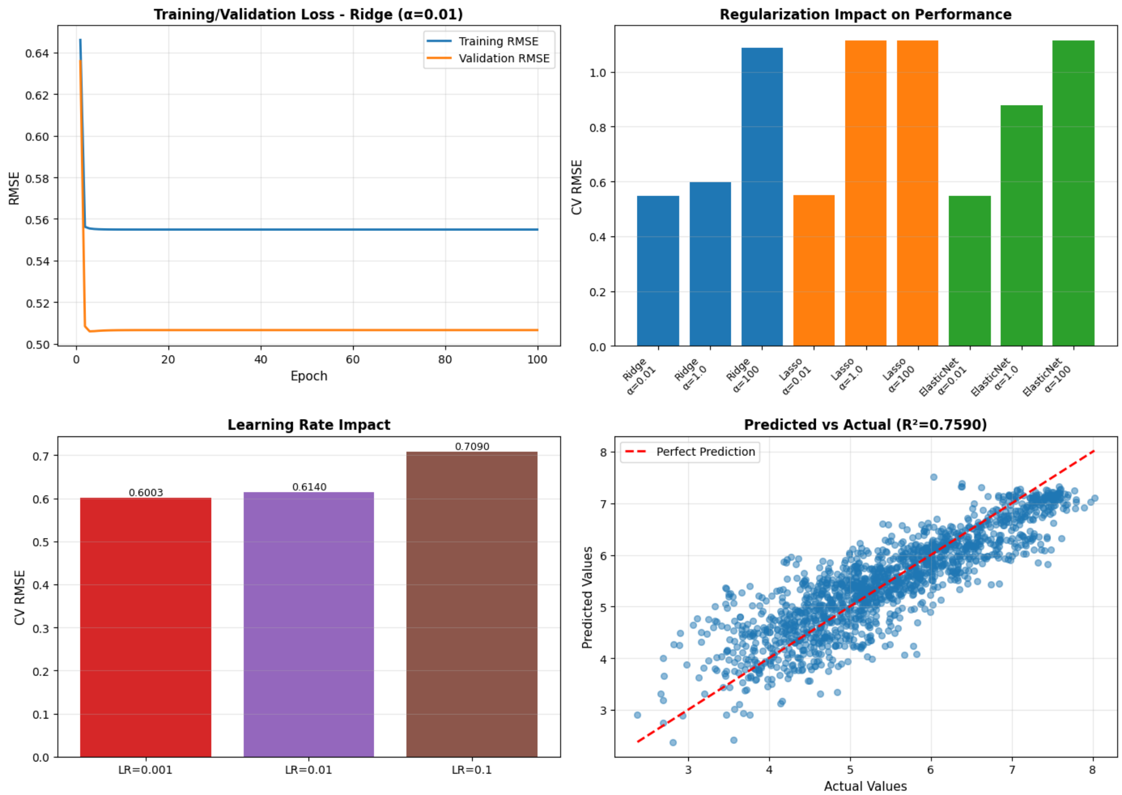
```
Final Model Performance on Full Training Set:
   RMSE: 0.5463
   R²:   0.7590
   MAE:  0.4238

================================================================================
PART 7: VISUALIZATIONS
================================================================================
```

[29]:

```
fig, axes = plt.subplots(2, 2, figsize=(14, 10))

# Plot 1: Training and Validation Loss over Epochs
ax1 = axes[0, 0]
ax1.plot(range(1, n_epochs + 1), train_losses, label='Training RMSE', linewidth=2)
ax1.plot(range(1, n_epochs + 1), val_losses, label='Validation RMSE', linewidth=2)
ax1.set_xlabel('Epoch', fontsize=11)
ax1.set_ylabel('RMSE', fontsize=11)
ax1.set_title(f'Training/Validation Loss - {best_model_name}', fontsize=12, fontweight='bold')
ax1.legend()
ax1.grid(True, alpha=0.3)

# Plot 2: Regularization Impact Comparison
ax2 = axes[0, 1]
reg_comparison = {
    'Ridge\n =0.01': ridge_results[0.01],
    'Ridge\n =1.0': ridge_results[1.0],
    'Ridge\n =100': ridge_results[100.0],
    'Lasso\n =0.01': lasso_results[0.01],
    'Lasso\n =1.0': lasso_results[1.0],
    'Lasso\n =100': lasso_results[100.0],
    'ElasticNet\n =0.01': elasticnet_results[0.01],
    'ElasticNet\n =1.0': elasticnet_results[1.0],
    'ElasticNet\n =100': elasticnet_results[100.0],
}
colors = ['#1f77b4']*3 + ['#ff7f0e']*3 + ['#2ca02c']*3
bars = ax2.bar(range(len(reg_comparison)), list(reg_comparison.values()), color=colors)
ax2.set_xticks(range(len(reg_comparison)))
ax2.set_xticklabels(list(reg_comparison.keys()), rotation=45, ha='right', fontsize=9)
ax2.set_ylabel('CV RMSE', fontsize=11)
ax2.set_title('Regularization Impact on Performance', fontsize=12, fontweight='bold')
ax2.grid(True, alpha=0.3, axis='y')

# Plot 3: Learning Rate Impact
ax3 = axes[1, 0]
lr_names = [f'LR={lr}' for lr in learning_rates]
bars = ax3.bar(lr_names, [lr_results[lr] for lr in learning_rates],
               color=['#d62728', '#9467bd', '#8c564b'])
ax3.set_ylabel('CV RMSE', fontsize=11)
ax3.set_title('Learning Rate Impact', fontsize=12, fontweight='bold')
ax3.grid(True, alpha=0.3, axis='y')
```

```
for i, (lr, rmse) in enumerate(lr_results.items()):
    ax3.text(i, rmse, f'{rmse:.4f}', ha='center', va='bottom', fontsize=9)

# Plot 4: Predicted vs Actual
ax4 = axes[1, 1]
ax4.scatter(y, y_pred_final, alpha=0.5, s=30)
min_val = min(y.min(), y_pred_final.min())
max_val = max(y.max(), y_pred_final.max())
ax4.plot([min_val, max_val], [min_val, max_val], 'r--', linewidth=2,␣
 ↪label='Perfect Prediction')
ax4.set_xlabel('Actual Values', fontsize=11)
ax4.set_ylabel('Predicted Values', fontsize=11)
ax4.set_title(f'Predicted vs Actual (R²={final_r2:.4f})', fontsize=12,␣
 ↪fontweight='bold')
ax4.legend()
ax4.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig('linear_regression_analysis.png', dpi=300, bbox_inches='tight')
print("\nVisualization saved as 'linear_regression_analysis.png'")
plt.show()

print(f"\nBest configuration: {best_model_name} with RMSE = {best_rmse:.4f}")
```

```
Visualization saved as 'linear_regression_analysis.png'
```



Best configuration: Ridge (=0.01) with RMSE = 0.5485

0.5 Solution of E starts here.

[34]:
```
degrees = [1, 2, 3, 4, 5]
degree_results = {}

print("\nTesting polynomial degrees [1, 2, 3, 4, 5]...")
print("-" * 80)

for degree in degrees:
    print(f"\nDegree {degree}:")

    # Create polynomial features
    poly = PolynomialFeatures(degree=degree, include_bias=False)
    X_poly_train = poly.fit_transform(X_train)
    X_poly_val = poly.transform(X_val)

    # Standardize
    scaler = StandardScaler()
    X_poly_train_scaled = scaler.fit_transform(X_poly_train)
```

```
    X_poly_val_scaled = scaler.transform(X_poly_val)

    print(f"  Number of polynomial features: {X_poly_train_scaled.shape[1]}")

    # Train SGD model
    sgd_model = SGDRegressor(
        penalty='l2', alpha=0.01, max_iter=1000, tol=1e-3,
        random_state=RANDOM_SEED, learning_rate='invscaling', eta0=0.01
    )
    sgd_model.fit(X_poly_train_scaled, y_train)

    # Predictions
    y_train_pred = sgd_model.predict(X_poly_train_scaled)
    y_val_pred = sgd_model.predict(X_poly_val_scaled)

    # Metrics
    train_rmse = np.sqrt(mean_squared_error(y_train, y_train_pred))
    val_rmse = np.sqrt(mean_squared_error(y_val, y_val_pred))
    train_r2 = r2_score(y_train, y_train_pred)
    val_r2 = r2_score(y_val, y_val_pred)

    degree_results[degree] = {
        'train_rmse': train_rmse,
        'val_rmse': val_rmse,
        'train_r2': train_r2,
        'val_r2': val_r2,
        'n_features': X_poly_train_scaled.shape[1]
    }

    print(f"  Train RMSE: {train_rmse:.4f} | Val RMSE: {val_rmse:.4f}")
    print(f"  Train R²:   {train_r2:.4f} | Val R²:   {val_r2:.4f}")
    print(f"  Gap (Val-Train RMSE): {val_rmse - train_rmse:.4f}")

print("\n" + "="*80)
print("3. OVERFITTING/UNDERFITTING ANALYSIS")
print("="*80)

print("\nAnalyzing model fit based on train-validation gap:\n")
for degree, metrics in degree_results.items():
    gap = metrics['val_rmse'] - metrics['train_rmse']
    gap_pct = (gap / metrics['train_rmse']) * 100

    print(f"Degree {degree}:")
    print(f"  Train RMSE: {metrics['train_rmse']:.4f}")
    print(f"  Val RMSE:   {metrics['val_rmse']:.4f}")
    print(f"  Gap:        {gap:.4f} ({gap_pct:.1f}%)")
```

```
print("\nKEY FINDINGS:")
print("-" * 80)
best_val_degree = min(degree_results.keys(), key=lambda d:␣
↪degree_results[d]['val_rmse'])
print(f"• Best validation RMSE at degree {best_val_degree}:␣
↪{degree_results[best_val_degree]['val_rmse']:.4f}")
print(f"• Degree 1 (linear) baseline: {degree_results[1]['val_rmse']:.4f}")
print(f"• As degree increases, training error decreases but validation may␣
↪increase")
print(f"• High-degree polynomials risk overfitting with {X.shape[1]} original␣
↪features")

print("\n" + "="*80)
print("4. REGULARIZATION EXPERIMENTS WITH POLYNOMIAL FEATURES")
print("="*80)

# Use best polynomial degree for regularization experiments
print(f"\nUsing polynomial degree {best_val_degree} for detailed experiments...
↪")
print("-" * 80)

poly = PolynomialFeatures(degree=best_val_degree, include_bias=False)
X_poly_train = poly.fit_transform(X_train)
X_poly_val = poly.transform(X_val)

scaler = StandardScaler()
X_poly_train_scaled = scaler.fit_transform(X_poly_train)
X_poly_val_scaled = scaler.transform(X_poly_val)

# Test different regularization types and strengths
regularization_experiments = []
alpha_values = [0.0001, 0.001, 0.01, 0.1, 1.0, 10.0]

print("\n4.1 Ridge Regularization (L2)")
for alpha in alpha_values:
    sgd = SGDRegressor(penalty='l2', alpha=alpha, max_iter=1000, tol=1e-3,
                       random_state=RANDOM_SEED, learning_rate='invscaling',␣
↪eta0=0.01)
    sgd.fit(X_poly_train_scaled, y_train)

    train_rmse = np.sqrt(mean_squared_error(y_train, sgd.
↪predict(X_poly_train_scaled)))
    val_rmse = np.sqrt(mean_squared_error(y_val, sgd.
↪predict(X_poly_val_scaled)))
```

```
    regularization_experiments.append({
        'type': 'Ridge',
        'alpha': alpha,
        'train_rmse': train_rmse,
        'val_rmse': val_rmse
    })
    print(f"   ={alpha:7.4f}: Train={train_rmse:.4f}, Val={val_rmse:.4f}, Gap={val_rmse-train_rmse:.4f}")

print("\n4.2 Lasso Regularization (L1)")
for alpha in alpha_values:
    sgd = SGDRegressor(penalty='l1', alpha=alpha, max_iter=1000, tol=1e-3,
                       random_state=RANDOM_SEED, learning_rate='invscaling', eta0=0.01)
    sgd.fit(X_poly_train_scaled, y_train)

    train_rmse = np.sqrt(mean_squared_error(y_train, sgd.predict(X_poly_train_scaled)))
    val_rmse = np.sqrt(mean_squared_error(y_val, sgd.predict(X_poly_val_scaled)))

    regularization_experiments.append({
        'type': 'Lasso',
        'alpha': alpha,
        'train_rmse': train_rmse,
        'val_rmse': val_rmse
    })
    print(f"   ={alpha:7.4f}: Train={train_rmse:.4f}, Val={val_rmse:.4f}, Gap={val_rmse-train_rmse:.4f}")

print("\n4.3 Elastic Net Regularization (L1 + L2)")
for alpha in alpha_values:
    sgd = SGDRegressor(penalty='elasticnet', alpha=alpha, l1_ratio=0.5,
                       max_iter=1000, tol=1e-3, random_state=RANDOM_SEED,
                       learning_rate='invscaling', eta0=0.01)
    sgd.fit(X_poly_train_scaled, y_train)

    train_rmse = np.sqrt(mean_squared_error(y_train, sgd.predict(X_poly_train_scaled)))
    val_rmse = np.sqrt(mean_squared_error(y_val, sgd.predict(X_poly_val_scaled)))

    regularization_experiments.append({
        'type': 'ElasticNet',
        'alpha': alpha,
        'train_rmse': train_rmse,
```

```
        'val_rmse': val_rmse
    })
    print(f"   ={alpha:7.4f}: Train={train_rmse:.4f}, Val={val_rmse:.4f}, Gap={val_rmse-train_rmse:.4f}")

print("\n" + "="*80)
print("5. LEARNING RATE EXPLORATION")
print("="*80)

learning_rates = [0.001, 0.005, 0.01, 0.05, 0.1, 0.5]
lr_results = []

print(f"\nTesting learning rates with Ridge ( =0.01), degree={best_val_degree}:")
print("-" * 80)

for lr in learning_rates:
    sgd = SGDRegressor(penalty='l2', alpha=0.01, max_iter=1000, tol=1e-3,
                       random_state=RANDOM_SEED, learning_rate='constant', eta0=lr)
    sgd.fit(X_poly_train_scaled, y_train)

    train_rmse = np.sqrt(mean_squared_error(y_train, sgd.predict(X_poly_train_scaled)))
    val_rmse = np.sqrt(mean_squared_error(y_val, sgd.predict(X_poly_val_scaled)))

    lr_results.append({
        'lr': lr,
        'train_rmse': train_rmse,
        'val_rmse': val_rmse
    })
    print(f"  LR={lr:.4f}: Train={train_rmse:.4f}, Val={val_rmse:.4f}")

print("\n" + "="*80)
print("6. BEST MODEL SELECTION")
print("="*80)

# Find best configuration
best_config = min(regularization_experiments, key=lambda x: x['val_rmse'])
print(f"\nBest configuration based on validation RMSE:")
print(f"  Regularization: {best_config['type']}")
print(f"  Alpha: {best_config['alpha']}")
print(f"  Polynomial Degree: {best_val_degree}")
print(f"  Validation RMSE: {best_config['val_rmse']:.4f}")
print(f"  Training RMSE: {best_config['train_rmse']:.4f}")
```

```python
print("\n" + "="*80)
print("7. TRAINING BEST MODEL WITH EPOCH TRACKING")
print("="*80)

print("\nTraining best model with epoch-by-epoch tracking...")

# Prepare polynomial features for best degree
poly_best = PolynomialFeatures(degree=best_val_degree, include_bias=False)
X_poly_train = poly_best.fit_transform(X_train)
X_poly_val = poly_best.transform(X_val)

scaler_best = StandardScaler()
X_poly_train_scaled = scaler_best.fit_transform(X_poly_train)
X_poly_val_scaled = scaler_best.transform(X_poly_val)

# Initialize best model
if best_config['type'] == 'Ridge':
    best_model = SGDRegressor(penalty='l2', alpha=best_config['alpha'],
                              max_iter=1, warm_start=True, tol=None,
                              random_state=RANDOM_SEED,
 ↪learning_rate='invscaling', eta0=0.01)
elif best_config['type'] == 'Lasso':
    best_model = SGDRegressor(penalty='l1', alpha=best_config['alpha'],
                              max_iter=1, warm_start=True, tol=None,
                              random_state=RANDOM_SEED,
 ↪learning_rate='invscaling', eta0=0.01)
else:  # ElasticNet
    best_model = SGDRegressor(penalty='elasticnet', alpha=best_config['alpha'],
 ↪l1_ratio=0.5,
                              max_iter=1, warm_start=True, tol=None,
                              random_state=RANDOM_SEED,
 ↪learning_rate='invscaling', eta0=0.01)

# Track losses over epochs
n_epochs = 150
train_losses_epoch = []
val_losses_epoch = []

for epoch in range(n_epochs):
    best_model.fit(X_poly_train_scaled, y_train)

    y_train_pred = best_model.predict(X_poly_train_scaled)
    y_val_pred = best_model.predict(X_poly_val_scaled)

    train_rmse = np.sqrt(mean_squared_error(y_train, y_train_pred))
    val_rmse = np.sqrt(mean_squared_error(y_val, y_val_pred))
```

```
    train_losses_epoch.append(train_rmse)
    val_losses_epoch.append(val_rmse)

    if (epoch + 1) % 30 == 0:
        print(f"Epoch {epoch+1:3d}: Train RMSE={train_rmse:.4f}, Val␣
↪RMSE={val_rmse:.4f}")

print("\n" + "="*80)
print("8. FINAL MODEL TRAINING ON FULL TRAINING SET")
print("="*80)

# Train on full training data with best hyperparameters
print("\nTraining final model on entire training dataset...")

# Prepare full training data
X_full = X  # All training data
y_full = y

poly_final = PolynomialFeatures(degree=best_val_degree, include_bias=False)
X_poly_full = poly_final.fit_transform(X_full)

scaler_final = StandardScaler()
X_poly_full_scaled = scaler_final.fit_transform(X_poly_full)

# Train final model
if best_config['type'] == 'Ridge':
    final_model = SGDRegressor(penalty='l2', alpha=best_config['alpha'],
                               max_iter=150, tol=1e-3, random_state=RANDOM_SEED,
                               learning_rate='invscaling', eta0=0.01)
elif best_config['type'] == 'Lasso':
    final_model = SGDRegressor(penalty='l1', alpha=best_config['alpha'],
                               max_iter=150, tol=1e-3, random_state=RANDOM_SEED,
                               learning_rate='invscaling', eta0=0.01)
else:  # ElasticNet
    final_model = SGDRegressor(penalty='elasticnet',␣
↪alpha=best_config['alpha'], l1_ratio=0.5,
                               max_iter=150, tol=1e-3, random_state=RANDOM_SEED,
                               learning_rate='invscaling', eta0=0.01)

final_model.fit(X_poly_full_scaled, y_full)

# Evaluate on training set
y_pred_train_final = final_model.predict(X_poly_full_scaled)
final_train_rmse = np.sqrt(mean_squared_error(y_full, y_pred_train_final))
final_train_r2 = r2_score(y_full, y_pred_train_final)
final_train_mae = mean_absolute_error(y_full, y_pred_train_final)
```

```python
print(f"\nFinal Model Performance on Full Training Set:")
print(f"  RMSE: {final_train_rmse:.4f}")
print(f"  R²:   {final_train_r2:.4f}")
print(f"  MAE:  {final_train_mae:.4f}")

print("\n" + "="*80)
print("9. TEST SET PREDICTIONS")
print("="*80)

# Load test data
try:
    test_data = pd.read_csv('happiness_data_test.csv')
    print(f"\nTest dataset loaded successfully!")
    print(f"Test dataset shape: {test_data.shape}")

    # Check if test has target column (for evaluation) or not
    has_test_target = target_col in test_data.columns

    if has_test_target:
        X_test = test_data.drop(columns=[target_col])
        y_test = test_data[target_col]
        print(f"Test set has target variable for evaluation")
    else:
        X_test = test_data
        y_test = None
        print(f"Test set does not have target variable (blind test)")

    # Impute missing values in test set using same imputer
    X_test_imputed = imputer.transform(X_test)
    X_test = pd.DataFrame(X_test_imputed, columns=X_test.columns)

    # Apply polynomial features and scaling
    X_test_poly = poly_final.transform(X_test)
    X_test_poly_scaled = scaler_final.transform(X_test_poly)

    # Make predictions
    y_test_pred = final_model.predict(X_test_poly_scaled)

    print(f"\nPredictions generated for {len(y_test_pred)} test samples")
    print(f"Prediction statistics:")
    print(f"  Mean: {y_test_pred.mean():.4f}")
    print(f"  Std:  {y_test_pred.std():.4f}")
    print(f"  Min:  {y_test_pred.min():.4f}")
    print(f"  Max:  {y_test_pred.max():.4f}")

    # If test has labels, calculate test RMSE
    if has_test_target and y_test is not None:
```

```
        # Remove any NaN values in test target
        valid_test_idx = ~y_test.isnull()
        y_test_clean = y_test[valid_test_idx]
        y_test_pred_clean = y_test_pred[valid_test_idx]

        test_rmse = np.sqrt(mean_squared_error(y_test_clean, y_test_pred_clean))
        test_r2 = r2_score(y_test_clean, y_test_pred_clean)
        test_mae = mean_absolute_error(y_test_clean, y_test_pred_clean)

        print(f"\n{'='*80}")
        print("TEST SET PERFORMANCE (OFFICIAL SCORE)")
        print(f"{'='*80}")
        print(f"  TEST RMSE: {test_rmse:.4f}  ")
        print(f"  TEST R²:   {test_r2:.4f}")
        print(f"  TEST MAE:  {test_mae:.4f}")
        print(f"{'='*80}")

    # Save predictions
    predictions_df = pd.DataFrame({
        'Predicted_Life_Ladder': y_test_pred
    })
    predictions_df.to_csv('test_predictions.csv', index=False)

except FileNotFoundError:
    print("\n  Warning: 'happiness_data_test.csv' not found")
    print("Please ensure the test file is in the same directory")

print("\n" + "="*80)
print("10. VISUALIZATIONS")
print("="*80)

# Create comprehensive visualizations
fig = plt.figure(figsize=(16, 12))
gs = fig.add_gridspec(3, 3, hspace=0.3, wspace=0.3)

# Plot 1: Training and Validation Loss over Epochs
ax1 = fig.add_subplot(gs[0, :2])
ax1.plot(range(1, n_epochs + 1), train_losses_epoch, label='Training RMSE',␣
↪linewidth=2, color='#1f77b4')
ax1.plot(range(1, n_epochs + 1), val_losses_epoch, label='Validation RMSE',␣
↪linewidth=2, color='#ff7f0e')
ax1.set_xlabel('Epoch', fontsize=11)
ax1.set_ylabel('RMSE', fontsize=11)
ax1.set_title(f'Training/Validation Loss - {best_config["type"]}␣
↪( ={best_config["alpha"]}, degree={best_val_degree})',
              fontsize=12, fontweight='bold')
ax1.legend(fontsize=10)
```

```
ax1.grid(True, alpha=0.3)

# Plot 2: Polynomial Degree Impact
ax2 = fig.add_subplot(gs[0, 2])
degrees_list = list(degree_results.keys())
train_rmses = [degree_results[d]['train_rmse'] for d in degrees_list]
val_rmses = [degree_results[d]['val_rmse'] for d in degrees_list]
ax2.plot(degrees_list, train_rmses, 'o-', label='Train RMSE', linewidth=2,␣
 ↪markersize=8)
ax2.plot(degrees_list, val_rmses, 's-', label='Val RMSE', linewidth=2,␣
 ↪markersize=8)
ax2.set_xlabel('Polynomial Degree', fontsize=11)
ax2.set_ylabel('RMSE', fontsize=11)
ax2.set_title('Polynomial Degree Impact', fontsize=12, fontweight='bold')
ax2.legend(fontsize=9)
ax2.grid(True, alpha=0.3)
ax2.set_xticks(degrees_list)

# Plot 3: Overfitting Analysis (Gap)
ax3 = fig.add_subplot(gs[1, 0])
gaps = [degree_results[d]['val_rmse'] - degree_results[d]['train_rmse'] for d␣
 ↪in degrees_list]
colors_gap = ['green' if g < 0.05 else 'orange' if g < 0.15 else 'red' for g in␣
 ↪gaps]
bars = ax3.bar(degrees_list, gaps, color=colors_gap, alpha=0.7)
ax3.axhline(y=0, color='black', linestyle='--', linewidth=1)
ax3.set_xlabel('Polynomial Degree', fontsize=11)
ax3.set_ylabel('Val RMSE - Train RMSE', fontsize=11)
ax3.set_title('Overfitting Gap Analysis', fontsize=12, fontweight='bold')
ax3.grid(True, alpha=0.3, axis='y')
ax3.set_xticks(degrees_list)

# Plot 4: Regularization Comparison (Ridge)
ax4 = fig.add_subplot(gs[1, 1])
ridge_exps = [e for e in regularization_experiments if e['type'] == 'Ridge']
alphas_ridge = [e['alpha'] for e in ridge_exps]
val_rmses_ridge = [e['val_rmse'] for e in ridge_exps]
ax4.semilogx(alphas_ridge, val_rmses_ridge, 'o-', linewidth=2, markersize=8,␣
 ↪color='#1f77b4')
ax4.set_xlabel('Alpha (log scale)', fontsize=11)
ax4.set_ylabel('Validation RMSE', fontsize=11)
ax4.set_title('Ridge Regularization Strength', fontsize=12, fontweight='bold')
ax4.grid(True, alpha=0.3)

# Plot 5: Regularization Comparison (Lasso)
ax5 = fig.add_subplot(gs[1, 2])
lasso_exps = [e for e in regularization_experiments if e['type'] == 'Lasso']
```

```
alphas_lasso = [e['alpha'] for e in lasso_exps]
val_rmses_lasso = [e['val_rmse'] for e in lasso_exps]
ax5.semilogx(alphas_lasso, val_rmses_lasso, 's-', linewidth=2, markersize=8, color='#ff7f0e')
ax5.set_xlabel('Alpha (log scale)', fontsize=11)
ax5.set_ylabel('Validation RMSE', fontsize=11)
ax5.set_title('Lasso Regularization Strength', fontsize=12, fontweight='bold')
ax5.grid(True, alpha=0.3)

# Plot 6: Learning Rate Impact
ax6 = fig.add_subplot(gs[2, 0])
lrs = [r['lr'] for r in lr_results]
val_rmses_lr = [r['val_rmse'] for r in lr_results]
ax6.plot(lrs, val_rmses_lr, 'D-', linewidth=2, markersize=8, color='#2ca02c')
ax6.set_xlabel('Learning Rate', fontsize=11)
ax6.set_ylabel('Validation RMSE', fontsize=11)
ax6.set_title('Learning Rate Impact', fontsize=12, fontweight='bold')
ax6.grid(True, alpha=0.3)

# Plot 7: Predicted vs Actual (Training)
ax7 = fig.add_subplot(gs[2, 1])
ax7.scatter(y_full, y_pred_train_final, alpha=0.5, s=30, color='#1f77b4')
min_val = min(y_full.min(), y_pred_train_final.min())
max_val = max(y_full.max(), y_pred_train_final.max())
ax7.plot([min_val, max_val], [min_val, max_val], 'r--', linewidth=2, label='Perfect Prediction')
ax7.set_xlabel('Actual Life Ladder', fontsize=11)
ax7.set_ylabel('Predicted Life Ladder', fontsize=11)
ax7.set_title(f'Training: Predicted vs Actual (R²={final_train_r2:.4f})', fontsize=12, fontweight='bold')
ax7.legend(fontsize=9)
ax7.grid(True, alpha=0.3)

# Plot 8: All Regularization Types Comparison
ax8 = fig.add_subplot(gs[2, 2])
ridge_best = min([e for e in regularization_experiments if e['type'] == 'Ridge'], key=lambda x: x['val_rmse'])
lasso_best = min([e for e in regularization_experiments if e['type'] == 'Lasso'], key=lambda x: x['val_rmse'])
elastic_best = min([e for e in regularization_experiments if e['type'] == 'ElasticNet'], key=lambda x: x['val_rmse'])

reg_types = ['Ridge', 'Lasso', 'ElasticNet']
best_rmses = [ridge_best['val_rmse'], lasso_best['val_rmse'], elastic_best['val_rmse']]
colors_reg = ['#1f77b4', '#ff7f0e', '#2ca02c']
```

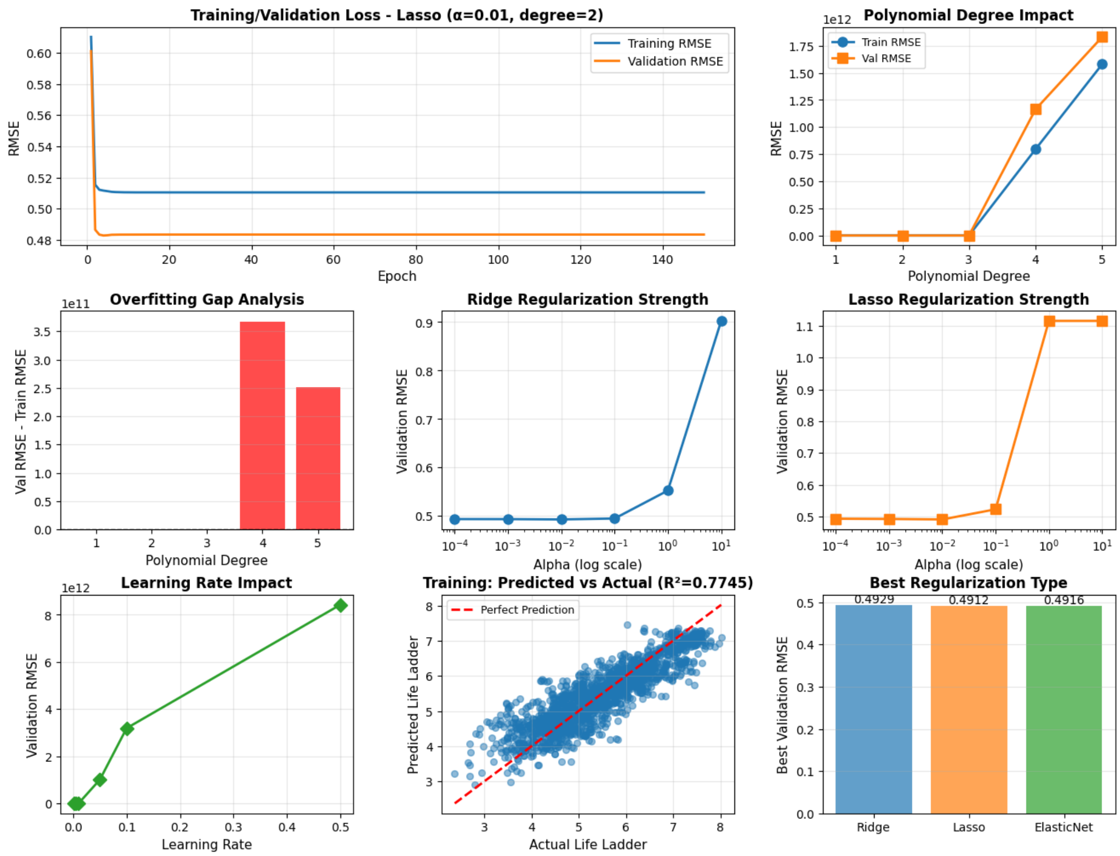
```python
bars = ax8.bar(reg_types, best_rmses, color=colors_reg, alpha=0.7)
ax8.set_ylabel('Best Validation RMSE', fontsize=11)
ax8.set_title('Best Regularization Type', fontsize=12, fontweight='bold')
ax8.grid(True, alpha=0.3, axis='y')
for i, (bar, rmse) in enumerate(zip(bars, best_rmses)):
    ax8.text(i, rmse, f'{rmse:.4f}', ha='center', va='bottom', fontsize=10)

plt.savefig('polynomial_regression_analysis.png', dpi=300, bbox_inches='tight')
plt.show()
```

```
Testing polynomial degrees [1, 2, 3, 4, 5]…
--------------------------------------------------------------------------------

Degree 1:
  Number of polynomial features: 8
  Train RMSE: 0.5560 | Val RMSE: 0.5091
  Train R²:   0.7502 | Val R²:   0.7911
  Gap (Val-Train RMSE): -0.0468

Degree 2:
  Number of polynomial features: 44
  Train RMSE: 0.4999 | Val RMSE: 0.4929
  Train R²:   0.7981 | Val R²:   0.8042
  Gap (Val-Train RMSE): -0.0069

Degree 3:
  Number of polynomial features: 164
  Train RMSE: 827.6986 | Val RMSE: 734.9670
  Train R²:   -553695.2339 | Val R²:   -435337.3450
  Gap (Val-Train RMSE): -92.7316

Degree 4:
  Number of polynomial features: 494
  Train RMSE: 796024722480.5966 | Val RMSE: 1163661749190.4272
  Train R²:   -512129974479686029279232.0000 | Val R²:
-1091301415382701412712448.0000
  Gap (Val-Train RMSE): 367637026709.8307

Degree 5:
  Number of polynomial features: 1286
  Train RMSE: 1579392186488.5076 | Val RMSE: 1831057506969.8611
  Train R²:   -2016076715002242172715008.0000 | Val R²:
-2702061124223802428882944.0000
  Gap (Val-Train RMSE): 251665320481.3535

================================================================================
```

```
3. OVERFITTING/UNDERFITTING ANALYSIS
================================================================================

Analyzing model fit based on train-validation gap:

Degree 1:
  Train RMSE: 0.5560
  Val RMSE:   0.5091
  Gap:        -0.0468 (-8.4%)
Degree 2:
  Train RMSE: 0.4999
  Val RMSE:   0.4929
  Gap:        -0.0069 (-1.4%)
Degree 3:
  Train RMSE: 827.6986
  Val RMSE:   734.9670
  Gap:        -92.7316 (-11.2%)
Degree 4:
  Train RMSE: 796024722480.5966
  Val RMSE:   1163661749190.4272
  Gap:        367637026709.8307 (46.2%)
Degree 5:
  Train RMSE: 1579392186488.5076
  Val RMSE:   1831057506969.8611
  Gap:        251665320481.3535 (15.9%)

KEY FINDINGS:
--------------------------------------------------------------------------------
• Best validation RMSE at degree 2: 0.4929
• Degree 1 (linear) baseline: 0.5091
• As degree increases, training error decreases but validation may increase
• High-degree polynomials risk overfitting with 8 original features

================================================================================
4. REGULARIZATION EXPERIMENTS WITH POLYNOMIAL FEATURES
================================================================================

Using polynomial degree 2 for detailed experiments…
--------------------------------------------------------------------------------

4.1 Ridge Regularization (L2)
   = 0.0001: Train=0.4996, Val=0.4936, Gap=-0.0060
   = 0.0010: Train=0.4996, Val=0.4936, Gap=-0.0061
   = 0.0100: Train=0.4999, Val=0.4929, Gap=-0.0069
   = 0.1000: Train=0.5075, Val=0.4948, Gap=-0.0127
   = 1.0000: Train=0.5729, Val=0.5526, Gap=-0.0203
   =10.0000: Train=0.8947, Val=0.9024, Gap=0.0077
```

```
4.2 Lasso Regularization (L1)
   = 0.0001: Train=0.4996, Val=0.4934, Gap=-0.0063
   = 0.0010: Train=0.4999, Val=0.4926, Gap=-0.0073
   = 0.0100: Train=0.5092, Val=0.4912, Gap=-0.0180
   = 0.1000: Train=0.5605, Val=0.5230, Gap=-0.0375
   = 1.0000: Train=1.1123, Val=1.1154, Gap=0.0031
   =10.0000: Train=1.1123, Val=1.1154, Gap=0.0031

4.3 Elastic Net Regularization (L1 + L2)
   = 0.0001: Train=0.4996, Val=0.4935, Gap=-0.0061
   = 0.0010: Train=0.4998, Val=0.4929, Gap=-0.0069
   = 0.0100: Train=0.5046, Val=0.4916, Gap=-0.0130
   = 0.1000: Train=0.5334, Val=0.5047, Gap=-0.0287
   = 1.0000: Train=0.8782, Val=0.8636, Gap=-0.0145
   =10.0000: Train=1.1123, Val=1.1154, Gap=0.0031

================================================================================
5. LEARNING RATE EXPLORATION
================================================================================

Testing learning rates with Ridge ( =0.01), degree=2:
--------------------------------------------------------------------------------
  LR=0.0010: Train=0.5005, Val=0.4924
  LR=0.0050: Train=0.5411, Val=0.5186
  LR=0.0100: Train=0.6116, Val=0.6490
  LR=0.0500: Train=1047459978293.1356, Val=1009846654679.7688
  LR=0.1000: Train=3389089237091.6392, Val=3188476514442.4971
  LR=0.5000: Train=8050744554355.9785, Val=8422481883575.1543

================================================================================
6. BEST MODEL SELECTION
================================================================================

Best configuration based on validation RMSE:
  Regularization: Lasso
  Alpha: 0.01
  Polynomial Degree: 2
  Validation RMSE: 0.4912
  Training RMSE: 0.5092

================================================================================
7. TRAINING BEST MODEL WITH EPOCH TRACKING
================================================================================

Training best model with epoch-by-epoch tracking…
Epoch  30: Train RMSE=0.5105, Val RMSE=0.4835
Epoch  60: Train RMSE=0.5105, Val RMSE=0.4835
Epoch  90: Train RMSE=0.5105, Val RMSE=0.4835
```

```
Epoch 120: Train RMSE=0.5105, Val RMSE=0.4835
Epoch 150: Train RMSE=0.5105, Val RMSE=0.4835

================================================================================
8. FINAL MODEL TRAINING ON FULL TRAINING SET
================================================================================

Training final model on entire training dataset…

Final Model Performance on Full Training Set:
  RMSE: 0.5284
  R²:   0.7745
  MAE:  0.4113

================================================================================
9. TEST SET PREDICTIONS
================================================================================

Test dataset loaded successfully!
Test dataset shape: (334, 9)
Test set has target variable for evaluation

Predictions generated for 334 test samples
Prediction statistics:
  Mean: 5.4649
  Std:  0.9335
  Min:  3.4131
  Max:  7.2703

================================================================================
TEST SET PERFORMANCE (OFFICIAL SCORE)
================================================================================
  TEST RMSE: 0.5400
  TEST R²:   0.7867
  TEST MAE:  0.4224
================================================================================

================================================================================
10. VISUALIZATIONS
================================================================================
```


Training/Validation Loss - Lasso (α=0.01, degree=2)
Training RMSE
Validation RMSE
RMSE
Epoch
Polynomial Degree Impact
1e12
Train RMSE
Val RMSE
RMSE
Polynomial Degree
Overfitting Gap Analysis
1e11
Val RMSE - Train RMSE
Polynomial Degree
Ridge Regularization Strength
Validation RMSE
Alpha (log scale)
Lasso Regularization Strength
Validation RMSE
Alpha (log scale)
Learning Rate Impact
1e12
Validation RMSE
Learning Rate
Training: Predicted vs Actual (R²=0.7745)
Perfect Prediction
Predicted Life Ladder
Actual Life Ladder
Best Regularization Type
0.4929
0.4912
0.4916
Best Validation RMSE
Ridge
Lasso
ElasticNet

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