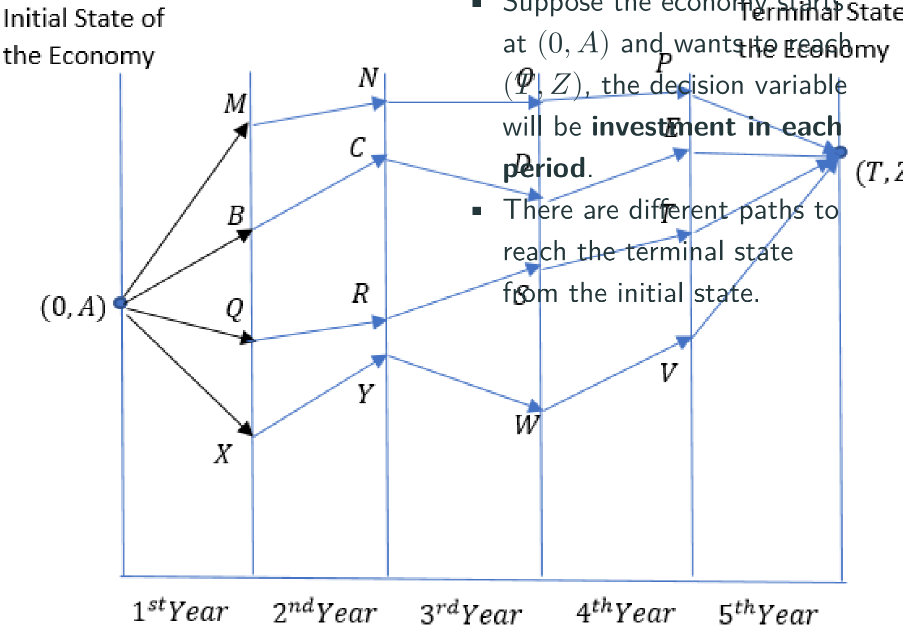
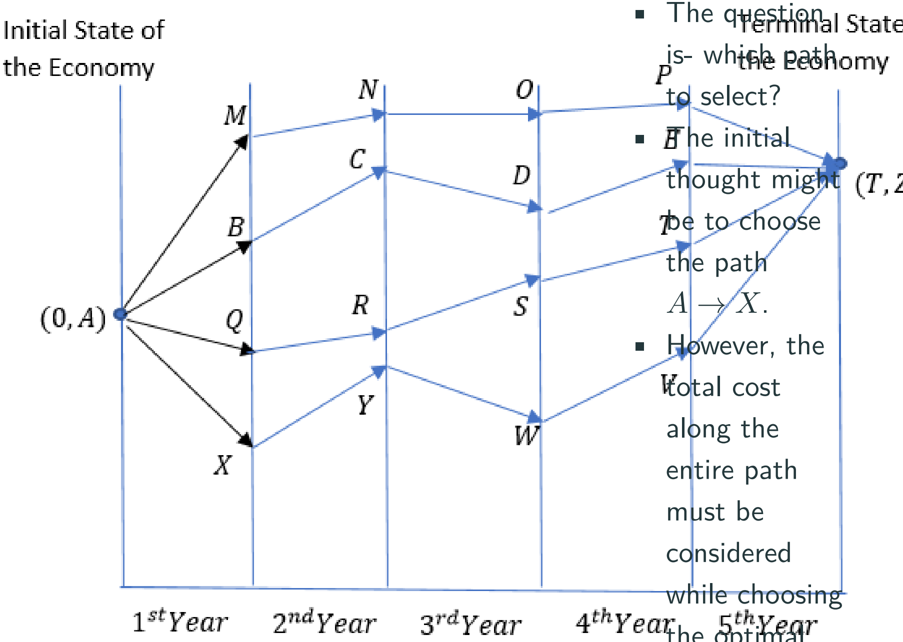
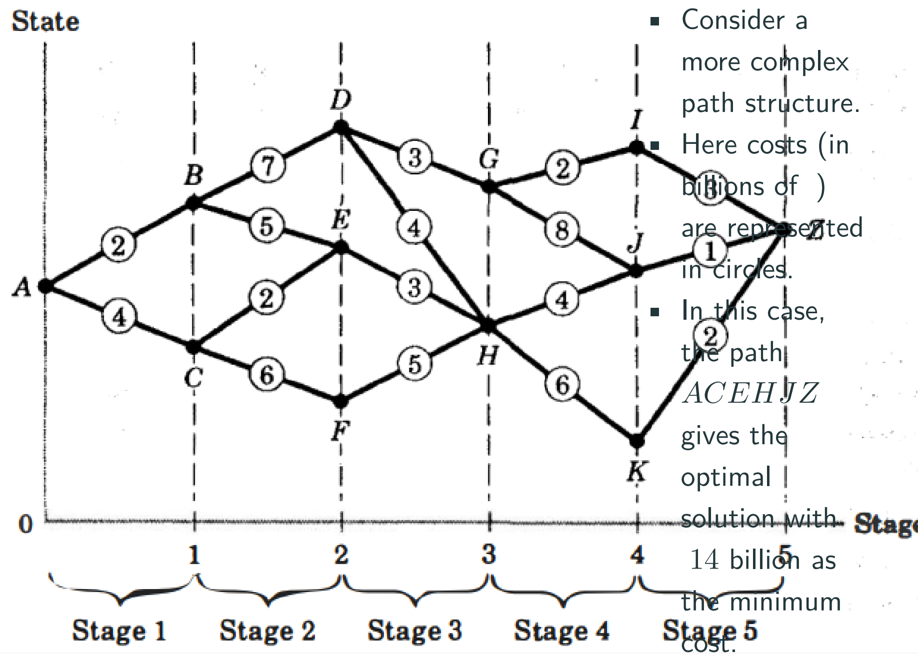
Untitled

- Suppose the economy starts at $(0, A)$ and wants to reach (T, Z), the decision variable will be **investment in each period**.
- There are different paths to reach the terminal state from the initial state.

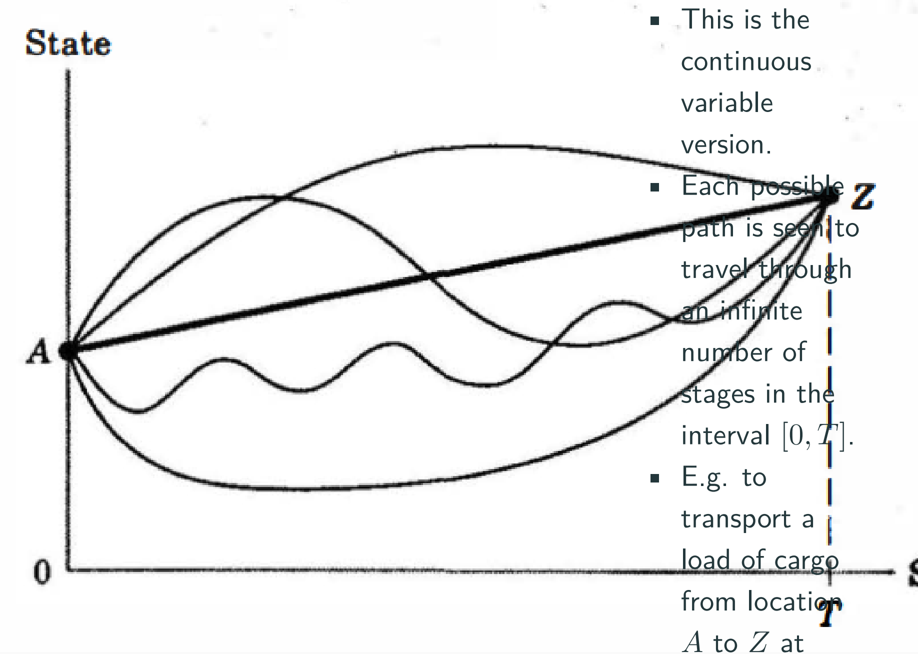




- The question is- which path to select?
- The initial thought might be to choose the path $A \to X$.
- However, the total cost along the entire path must be considered while choosing the optimal path.



- Consider a more complex path structure.
- Here costs (in billions of) are represented in circles.
- In this case, the path $ACEHJZ$ gives the optimal solution with 14 billion as the minimum cost.

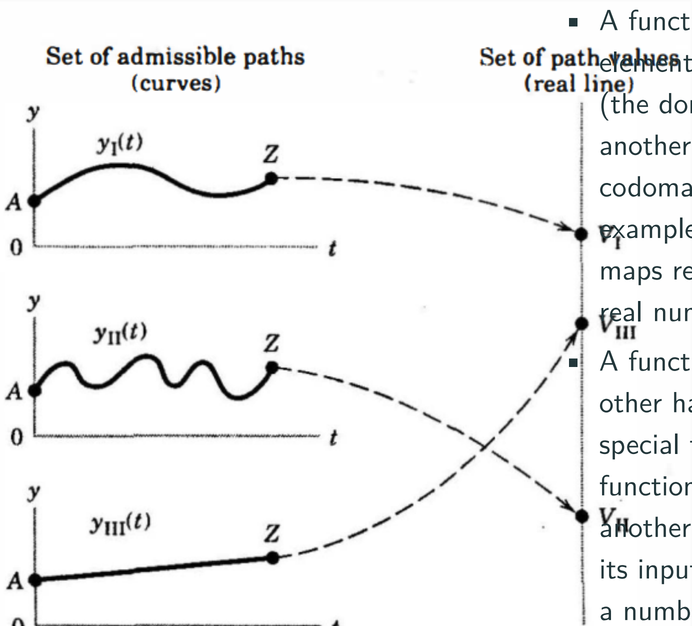


- This is the continuous variable version.
- Each possible path is seen to travel through an infinite number of stages in the interval $[0, T]$.
- E.g. to transport a load of cargo from location A to Z at minimum

Important elements of DO

1. In DO, we have initial state $[0, A]$ and terminal state $[T, Z]$.
2. There are different paths to achieve the terminal state.
3. There should be a **decision variable**. In our example, it's investment.
4. We should have an **objective functional** which we are trying to optimize.

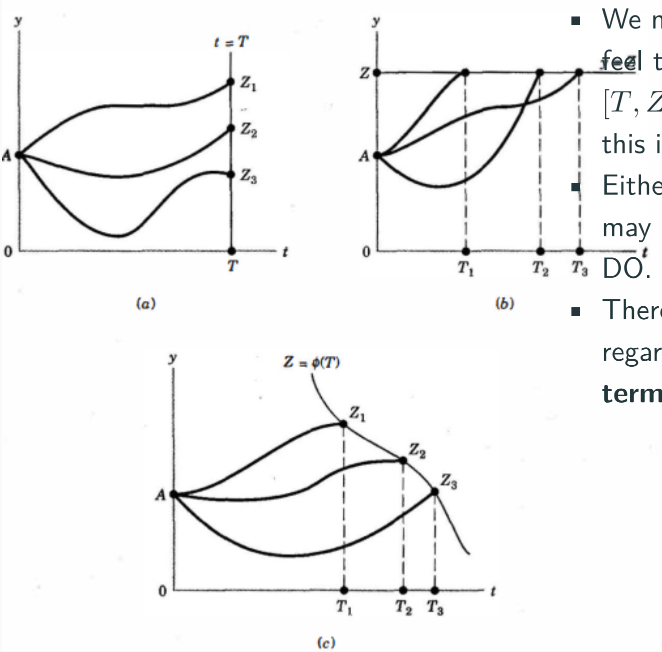
Objective function vs objective functional



- A function maps elements from one set (the domain) to another set (the codomain). For example, $f(x) = x^2$ maps real numbers to real numbers.
- A functional, on the other hand, is a special type of function that takes another function as its input and returns a number (or more

- Examples:-
  1. The definite integral is a functional:
  - Input: A function $f(x)$
  - Output: A single number representing the area under $f(x)$
  - Example: $\int_0^1 f(x)dx$ takes any function f and returns its integral from 0 to 1
  2. The maximum value functional:
  - Input: A function $f(x)$ defined on an interval $[a, b]$
  - Output: The maximum value of $f(x)$ on that interval
  - Example: $max f(x) : x \in [0, 1]$ takes a function and returns its highest value
  3. The norm of a function is a functional:
  - Input: A function $f(x)$
  - Output: A non-negative real number measuring the "size" of the function
  - Example: $L_2 norm : ||f|| = \sqrt{}(\int |f(x)|^2 dx)$

- Functionals are particularly important when finding the shortest path between two points on a surface, we're actually minimizing a functional that takes a path (which is a function) as input and returns its length as output.
- A key distinction is that functions operate on points (numbers, vectors, etc.), while functionals operate on entire functions. This makes functionals particularly useful in:
  - Optimization problems where we're looking for optimal functions rather than optimal points



- We might apparently feel that $[0, A]$ and $[T, Z]$ are fixed. But this is not the case.
- Either T or Z or both may be variable in DO.
- There are alternatives regarding the **terminal situation**.

- Let us assume we have an asset/resource stock from which we want to derive 2 types of benefits:
    1. **Flow benefit**: the value assumed during the use period of the resource.
    2. **Scrap value**: the value derived from a resource after it becomes obsolete. E.g.a car sold after 20 years or more as scrap.
- Our objective is to maximize the **total benefit (flow + scrap value)** from the resource.
- Let us denote

  V : flow benefit

  F : the scrap value