THE MINIMUM COST DIET PROBLEM

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1. Introduction

The idea of this project is to solve the problem of finding the minimum cost diet satisfying a set of "Recommended Daily Allowances" (RDAs) for a particular set of nutrients and possible kind of food.

1.1. **History.**

This problem was first "solved" by Stigler 1945 for a set of foods and prices and RDA requirements (see Dantzig 1990 for an entertaining discussion of what "solved" meant in that context). Times have changed: the variety of different kinds of food, food prices, and RDA requirements are all quite different from what they were for our grandparents.

1.2. **Dietary Guidelines.**

A compilation of dietary guidelines are provided at `https://health.gov/sites/default/files/2019-09/2015-2020_Dietary_Guidelines.pdf` (See especially Appendix 7); these provide recommended levels of 31 different nutrients by age and sex.

2. The Model

Stigler's insight was that the minimum cost diet (MCD) problem was most naturally posed as a linear program. If there are n different kinds of food one can buy, represent quantities consumed of these as a vector x. Each kind of food has a corresponding price that the consumer takes as given, call this vector of prices p. Then the cost of a consumer's diet is $p'x$, where the prime indicates the inner or dot product of the two vectors.

Each unit of a given kind of food is assumed to provide a set of nutrients, of the sort often reported on food labels. If there are m nutrients, then let A be a matrix with m rows and n columns describing the nutritional content of a single unit of each of kind of food.

There are various sources of recommendations regarding nutrition. These can take the form of either equalities or inequalities. For example, it is recommended that females in their twenties consume 2000

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kilo-calories, 46 grams of protein, and 28 grams of fiber per day, but less than 23 grams of sodium. Then using the matrix notation above, we can write these constraints as something like

$$Ax \geq \underline{b}$$

where $\underline{b}$ is a vector of recommendations about *minimum* amounts of different nutrients. Similarly, if there are some things we want to make sure we eat *less* of (e.g., mercury, sodium, calories), that can be written as a set of linear inequalities

$$Ax \leq \bar{b},$$

where $\bar{b}$ is a vector of recommendations about *maximum* amounts of different nutrients. Note that this constraint can *also* be expressed as a greater than constraint by multiplying both sides by -1.

Note that if some some nutrients we want exact equalities these can be specified by specifying both a greater-than and a less-than constraint.

Putting this all together, the linear program to compute the minimum cost diet looks like

$$\min_x p'x$$

such that

$$\begin{bmatrix} A \\ -A \end{bmatrix} x \geq \begin{bmatrix} \underline{b} \\ -\bar{b} \end{bmatrix}.$$

Solving problems like this by hand is generally difficult for more than a few variables and constraints, but numerical software has been developed that can handle large (millions) of both variables and constraints. There are a variety of different implementations; the one we'll use in our prototype is part of the `scipy.optimize` package.

3. References

Dantzig, George B. 1990. The diet problem. *Interfaces* 20 (4): 43–47.

Stigler, George. 1945. The cost of subsistence. *The Journal of Farm Economics* 27:303–314.