

Maxwell equations in differential form

Gauss's law for electricity

$$\oint \mathbf{D} \cdot d\mathbf{S} = q_{\text{enclosed}}$$

$$\oint \mathbf{D} \cdot d\mathbf{S} = \int \rho dV$$

$$\oint \mathbf{A} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{A} dV \text{ Divergence theorem}$$

$$\nabla \cdot \mathbf{D} = \rho$$

Gauss's law for Magnetism

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\int \nabla \cdot \mathbf{B} dV = 0 \text{ Using Divergence theorem}$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law

$$\varepsilon = -\frac{d\phi_B}{dt}$$

(here ϕ_B denotes the Magnetic flux)

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \int_S \nabla \times \mathbf{E} \cdot d\mathbf{S}$$

$$\text{Stokes theorem } \oint \mathbf{A} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{A} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampere's Law

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

Maxwell's addition to Maxwell Law

$$\oint \mathbf{H} \cdot d\mathbf{l} = I + \frac{d\phi_E}{dt}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

$$\int_S \nabla \times \mathbf{H} \cdot d\mathbf{S} = \int_S \mathbf{J} \cdot d\mathbf{S} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S} \quad \text{Applying stokes theorem}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$$