Indira Gandhi Delhi Technical University For Women

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PROBABILITY AND STATISTICS (BAS-103)

TUTORIAL SHEET -1

(Concepts of Probability)

Q1. Show that $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

Q2. Show that the probability that exactly one of the events E or F occurs is equal to

$$P(E) + P(F) - 2P(E \cap F).$$

[Hint: Consider $P((E \cap \bar{F}) \cup (\bar{E} \cap F))$]

Q3. A bin contains 5 defective (that immediately fail when put in use), 10 partially defective (that fail after a couple of hours of use), and 25 acceptable transistors. A transistor is chosen at random from the bin and put to use. If it does not immediately fail, what is the probability it is acceptable? [Ans. 5/7]

Q4. A laboratory blood test is 99 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false-positive" result for 1 percent of the healthy persons tested (that is, if a healthy person is tested, then, with probability 0.01, the test result will imply he or she has the disease). If 0.5 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive? [Ans. 0.3322]

Q5. Of three cards, one is painted red on both sides; one is painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?

[Ans. 2/3]

Q6. You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability 0.8; with water, it will die with probability 0.15. You are 90 percent certain that your neighbor will remember to water the plant. What is the probability that the plant will be alive when you return? Also, if it is dead, what is the probability your neighbor forgot to water it? [Ans. 0.785 and 0.372]

Q7. If E and F are independent, then prove that E and F^C are independent.

Q8. A maintenance firm has gathered the following information regarding the failure mechanisms for air conditioning systems:

		evidence of gas leaks	
		yes	no
evidence of	yes	55	17
electrical failure	no	32	3

The units without evidence of gas leaks or electrical failure showed other types of failure. If this is a representative sample of AC failure, find the probability:

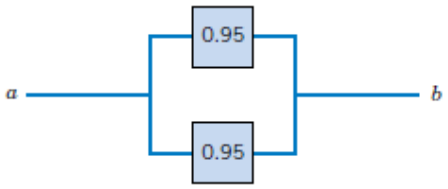
(a) That failure involves a gas leak.

(b) That there is evidence of electrical failure given that there was a gas leak.

(c) That there is evidence of a gas leak given that there is evidence of electrical failure.

[Ans. (a) 0.813 (b) 0.632 (c) 0.764]

Q9. The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?



[Ans. 0.9975]

Q10. Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

(a) What is the probability that a product attains a good review?

(b) If a new design attains a good review, what is the probability that it will be a highly successful product?

(c) If a product does not attain a good review, what is the probability that it will be a highly successful product? [Ans. (a) 0.615 (b) 0.618 (c) 0.052]