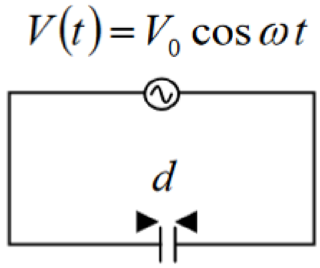
Unit I ELECTROMAGNETIC THEORY

Tutorial Sheet

Q1. An oscillating voltage $V(t) = V_o \cos(\omega t)$ is applied across a parallel plate capacitor having a plate separation d. Find the displacement current density through the capacitor.



Q2. An electric field $\vec{E}(r) = \alpha\hat{r} + \beta \sin\theta \cos\phi\,\hat{\phi}$ exists in space. What will be the total charge enclosed in a sphere of unit radius centered at origin?

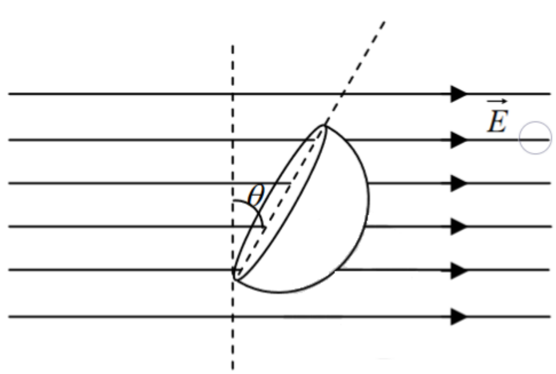
Q3. A small loop of wire of area $A = 0.01m^2, N = 40$ turns and resistance $R = 20\Omega$ is initially kept in a uniform magnetic field B in such a way that the field is normal to the loop. When it is pulled out of the magnetic field, a total charge of$Q = 2 \times 10^{-5}C$ flows through the coil. Then, calculate the magnitude of magnetic field B.

Q4. Find the electrostatic energy density corresponding to the electrostatic potential $V = 2x + 4y$ volts at some point (x, y).

Q5. Equipotential surface corresponding to a particular charge distribution are given by

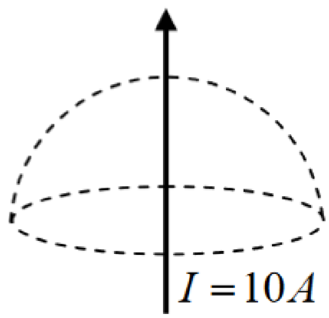
$4x^2 + (y-2)^2 + z^2 = V_i$, where the values of V_i are constants. Calculate the electric field at the origin.

Q6. A closed Gaussian surface consisting of a hemisphere and a circular disc of radius R, is placed in a uniform electric field $\vec{E}$, as shown in the figure. The circular disc makes an angle $\theta = 30^o$ with the vertical. Calculate the flux of the electric field vector coming out of the curved surface of the hemisphere.



Q7. A parallel-plate capacitor with plate area of $5cm^2$ and plate separation of $3mm$ has a voltage $50 \sin 10^3 t$ Volts applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_o$.

Q8. A current $I = 10A$ flows in an infinitely long wire along the axis of hemisphere (see figure). Find the value of $\int (\vec{\nabla} \times \vec{B}).\overrightarrow{ds}$ over the hemispherical surface.



Q9. The electric field of an electromagnetic wave is given by:

$$\vec{E} = (2\hat{k} - 3\hat{\jmath}) \times 10^{-3} \sin[10^7(x + 2y + 3z - \beta t)]$$

Calculate the value of the constant β.

Q10. Which one of the following is an impossible magnetic field $\vec{B}$?

(a) $\vec{B} = -2xy\hat{x} + yz^2\hat{y} + \left(2yz - \frac{z^3}{3}\right)\hat{z}$
(b) $\vec{B} = (xz + 4y)\hat{x} - yx^3\hat{y} + \left(x^3z - \frac{z^2}{2}\right)\hat{z}$
(c) $\vec{B} = -6xz\hat{x} + 3yz^2\hat{y}$

Q11. Given in free space, $\vec{E} = 20\cos(\omega t - 50x)\hat{y}$. Find $\vec{H}$

Q12. The electric field intensity of a spherical wave in free space is given by

$$\vec{E} = \frac{10}{r}\sin\theta\cos(\omega t - \beta r)\hat{\theta}$$

Find the corresponding magnetic field intensity $\vec{H}$.

Q13. In free space, $\vec{E}(z,t) = 50\cos(\omega t - \beta z)\hat{x}$ V/m. Find the average power crossing a circular area of radius 2.5m in the plane $z = constant$.

Q14. Find the skin depth δ at a frequency of 1.6 MHz in aluminum, where $\sigma = 38.2\ MS/m$ and $\mu_r = 1$.

Answers/Hints:

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Q.No: Displacement current $J_d = \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\varepsilon_0}{d} \frac{\partial V(t)}{\partial t}$

$$J_d = -\frac{\varepsilon_0 \omega V_0 \sin\omega t}{d}$$

Qne: 2 $Q_{enclosed} = \varepsilon_0 \oint \vec{E} \cdot \vec{da} = \varepsilon_0 \int \left(\alpha \hat{r} + \beta \sin\theta \cos\phi \,\hat{\phi}\right) \times \big(\underbrace{r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}}_{\vec{da}}\big)$

$= 4\pi \alpha \varepsilon_0 r^2$

$\vec{da}$ = differential normal surface area (we can choose this in any of 3 ways)

given : $r = 1$ $\quad\therefore\; Q_{encl} = 4\pi \alpha \varepsilon_0$

$$\vec{da} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r} = r \sin\theta \, dr \, d\phi \, \hat{\theta} = r \, dr \, d\theta \, \hat{\phi}$$

Ans: 3 Magnetic flux through the loop $\phi = NBA = 40 \times B \times 0.01$

Induced EMF $= \varepsilon = -\frac{d\phi}{dt}$

Induced current $i = \underbrace{-\frac{1}{R}\frac{d\phi}{dt}}_{\frac{V}{R}} = \underbrace{\frac{dQ}{dt}}_{\text{Rate of change of charge}}$

or

$$-\frac{1}{R} d\phi = dQ$$

$$\Rightarrow \frac{1}{20} \times 40 \times B \times 0.01 = 2 \times 10^{-5}$$

$$B = 1 \times 10^{-3} \text{ T}$$

Ans:4 $\vec{E} = -\vec{\nabla} V = -2\hat{x} - 4\hat{y}$

$\Rightarrow |\vec{E}| = \sqrt{4+16} = \sqrt{20}\ \frac{V}{m}$

$\therefore$ Electrostatic energy density $= \frac{1}{2}\varepsilon_0 |E|^2$

$= \frac{1}{2}\varepsilon_0 \times 20$

$= 10\,\varepsilon_0\ J/m^3$

Ans:5 $\vec{E} = -\vec{\nabla} V$

$= 8x\hat{x} + 2(y-2)\hat{y} + 2z\hat{z}$

$\vec{E}(0,0,0) = -4\hat{y}$

Ans:6 $\vec{E} = E\cos 30\,\hat{z} + E\sin 30\,\hat{x} = \frac{\sqrt{3}}{2}E\hat{z} + \frac{E}{2}\hat{x}$

$$\phi_E = \oint_S \vec{E}\cdot\vec{da} = \iint\left(\frac{\sqrt{3}}{2}E\hat{z} + \frac{1}{2}E\hat{x}\right)(r^2\sin\theta\, d\theta\, d\phi\, \hat{r})$$

$$\phi_E = R^2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\left(\frac{\sqrt{3}}{2}E\cos\theta + \frac{1}{2}E\sin\theta\cos\phi\right)\sin\theta\, d\theta\, d\phi$$

$$\left[\begin{aligned} \text{Note:}\quad \hat{x} &= \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi} \\ \hat{y} &= \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi} \\ \hat{z} &= \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta} \end{aligned}\right]$$

$$\phi_E = \frac{\sqrt{3}}{2}ER^2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\cos\theta\sin\theta\, d\theta\, d\phi + \frac{1}{2}ER^2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}(\sin^2\theta\cos\phi)\, d\theta\, d\phi$$

$$\phi_E = \frac{\sqrt{3}}{2}ER^2 \times 2\pi \times \frac{1}{2} + 0 = \frac{\sqrt{3}}{2}\pi R^2 E$$

or

$$\phi_E = \int \vec{E}\cdot\vec{da} = E\cos 30^\circ \times \pi R^2 = \frac{\sqrt{3}}{2}\pi R^2 E$$

Ans:7 $D = \varepsilon E = \varepsilon \frac{V}{d}$

$$J_d = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d}\frac{dV}{dt}$$

$$I_d = J_d \cdot \underset{\substack{\downarrow \\ \text{Area}}}{S} = \varepsilon \frac{S}{d}\frac{dV}{dt} = \text{same as conduction current}$$

$$\left(I_c = \frac{dQ}{dt} = S\frac{d\rho_v}{dt} = \frac{S\,dD}{dt} = \varepsilon S\frac{dE}{dt} = \frac{\varepsilon S}{d}\frac{dV}{dt} \right)$$

$$\therefore \quad I_d = 2 \times \frac{10^{-9}}{36\pi} \times \frac{5\times10^{-4}}{3\times10^{-3}} \times 10^{3} \times 50 \cos 10^{3} t$$

$$I_d = 147.4 \cos 10^3 t \quad nA$$

Ans:8 $\int(\vec{\nabla}\times\vec{B})\cdot\vec{dS} \overset{\text{stoke's theorem}}{=} \oint \vec{B}\cdot\vec{dl} = |B| \times 2\pi r$

$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r = \mu_0 I = 10\mu_0$$

Ans:9 $\vec{E} = (2\hat{k} - 3\hat{j}) \times 10^{-3} \sin\left[10^7 (x + 2y + 3z - \beta t)\right]$

$\vec{k} = 10^7(\hat{x} + 2\hat{y} + 3\hat{z})$

$|\vec{k}| = 10^7\sqrt{1+4+9} = 10^7\sqrt{14}$

$\omega = 10^7\beta$

$c = \frac{\omega}{|\vec{k}|} = \frac{10^7\beta}{10^7\sqrt{14}}$

or $\beta = \sqrt{14}\,c$

$\beta = 3\times10^8 \times \sqrt{14}$

Ans: 10 calculate $\vec{\nabla}\cdot\vec{B}$, if $\vec{\nabla}\cdot\vec{B}=0 \Rightarrow$ possible magnetic field

only (c) $\vec{\nabla}\cdot\vec{B} = -6z + 3z^2 \neq 0$

$\therefore$ (c) not possible

Ans 11.

$$\vec{E} = 20\cos(\omega t - \underbrace{50x}_{\vec{k}\cdot\vec{r}})\,\hat{y}\,.$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0}\left(\frac{K}{\omega}\left(\hat{k}\times\vec{E}\right)\right)$$

Here, $\hat{k} = \hat{x}$ and $\vec{E}$ along $\hat{y}$ $\quad\therefore\ \hat{x}\times\hat{y} = \hat{z}$

$\vec{k} = 50\hat{x}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0}\times\frac{K}{\omega}\times 20\cos(\omega t - 50x)\,\hat{z}$$

also $\quad \frac{1}{\mu_0\varepsilon_0} = c^2 = \left(\frac{\omega}{K}\right)^2 \quad \Rightarrow \frac{1}{\mu_0} = \varepsilon_0\left(\frac{\omega}{K}\right)^2$

$$\Rightarrow \vec{H} = \cancel{\frac{1}{\mu_0}}\,\frac{\cancel{K}}{\omega}\times\varepsilon_0\left(\frac{\omega}{K}\right)^2\times 20\cos(\omega t - 50x)\,\hat{z}$$

or $\quad \vec{H} = \frac{20}{K}\,\varepsilon_0\omega\cos(\omega t - 50x)\,\hat{z}$

Here $K = 50$

$$\vec{H} = \frac{20}{50}\,\varepsilon_0\omega\cos(\omega t - 50x)\,\hat{z}$$

$$= 0.4\,\omega\varepsilon_0\cos(\omega t - 50x)\,\hat{z}$$

Ans 12: $\frac{10\beta}{\omega \mu_0 r} \sin\theta \cos(\omega t - \beta r)\, \hat{\phi} \quad \frac{A}{m}$

Ans 13. $\bar{E}(z,t) = 50 \cos(\omega t - \beta z)\, \hat{x}$

$\bar{H}(z,t) = \frac{5}{12\pi} \cos(\omega t - \beta z)$

[Here note: $|B| = \frac{|E|}{c}$

$\therefore \quad |H| = \frac{|E|}{\mu_0 c} = \frac{|E|}{\mu_0} \times \sqrt{\mu_0 \varepsilon_0} = |E| \times \sqrt{\frac{\varepsilon_0}{\mu_0}}$

$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ = Impedance of free space

$\eta_0 = 12\pi$]

Avg Poynting vector $= \frac{1}{2} \frac{\vec{E} \times \vec{B}}{\mu_0} = \vec{E} \times \vec{H}$

$= \frac{1}{2}\, 50 \times \frac{5}{12\pi} \quad \hat{z} \quad \frac{W}{m^2}$

=

flow is normal to the area, so

$P_{avg} = \frac{1}{2} \times 50 \times \frac{5}{12\pi} \times (2.5)^2 = 65.1\ W$

Ans 14. $\sigma = 38.2\ MS/m$

$\nu = 1.6\ MHz$

$\mu_r = 1$

$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.44 \times 10^{-5}\ m = 64.4\ \mu m$