$\nabla \cdot \vec{V} = 0$

Que $\vec{A} = 3y^2 z^2 \hat{i} + 4x^3 z^2 \hat{j} - 3x^2 y^2 \hat{k}$

is solenoidal

Soln $\nabla \cdot \vec{A} = 0$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(3y^2 z^2) + \frac{\partial}{\partial y}(4x^3 z^2) + \frac{\partial}{\partial z}(-3x^2 y^2)$$

$$= 0$$

$\Rightarrow \vec{A}$ Solenoidal.

Que Determine the constant b s.t.

$$\vec{A} = (bx + 4y^2 z)\hat{i} + (x^3 \sin z - 3y)\hat{j} - (e^x + 4\cos x^2 y)\hat{k}$$ is solenoidal

$b = 0$

Que If $\vec{A} = (ax^2 y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2 y^2)\hat{k}$ is

Solenoidal. Find $\vec{A}$

$a = -2$

Ques $\vec{A} = e^{xyz}(\hat{i}+\hat{j}+\hat{k})$

Find the curl at $(1,2,3)$

Soln $$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xyz} & e^{xyz} & e^{xyz} \end{vmatrix}$$

$$= i\left(\frac{\partial}{\partial y}(e^{xyz}) - \frac{\partial}{\partial z}(e^{xyz})\right)$$

$$+ j\left(\frac{\partial}{\partial z}e^{xyz} - \frac{\partial}{\partial x}e^{xyz}\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x}e^{xyz} - \frac{\partial}{\partial y}e^{xyz}\right)$$

$$= \left(e^{xyz}\cdot xz - e^{xyz}xy\right)\hat{i} + \left(xy\,e^{xyz} - yz\,e^{xyz}\right)$$

$$+ \left(e^{xyz}yz - e^{xyz}\cdot xz\right)\hat{k}$$

$$\nabla \times \vec{A}\Big|_{(1,2,3)}$$

$$= e^{6}\left[i(3-2) + j(2-6) + k(6-3)\right]$$

$$= e^{6}(i - 4j + 3\hat{k})$$ <u>Ans</u>

<u>Que</u> Find the curl (curl $\bar{A}$)

$$\vec{A} = x^2 y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$$

at $(1,0,2)$

$$\text{curl}(\text{curl}\,\vec{A}) = 4\hat{j}$$

<u>Que</u> Prove that

$$\vec{F} = 2xyz^2\,\hat{i} + \left[x^2 z^2 + z\cos(yz)\right]\hat{j} + \left(2x^2 yz + y\cos yz\right)\hat{k}$$

is a conservative vector field.

<u>Soln</u> For conservative vector field

$$\nabla \times \vec{F} = 0$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

$$\begin{vmatrix} & & \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz^2 & (x^2z^2 + z\cos yz) & (2x^2yz + y\cos yz) \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(2x^2yz + y\cos yz)\right]$$

$$+ \hat{j}\left[\frac{\partial}{\partial z}(2xyz^2) - \frac{\partial}{\partial x}(2x^2yz + y\cos yz)\right]$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}(x^2z^2 + z\cos yz) - \frac{\partial}{\partial y}(2xyz^2)\right]$$

$$= \hat{i}\left(\cancel{2x^2z} + \cancel{\cos yz} - \cancel{yz\sin yz} - \cancel{2x^2z} - \cancel{\cos yz} + \cancel{zy\sin yz}\right)$$

$$+ \hat{j}(4xyz - 4xyz) + \hat{k}(2xz^2$$

$-2xz^2)$

$= 0$

$\Rightarrow \vec{F}$ is Conservative vector Field

Que Determine the constants 'a' & 'b' s.t

$\text{Curl}\left((2xy+3yz)\hat{i} + \hat{j}(x^2+axz-4z^2) + (3xy+2byz)\hat{k}\right)$ is zero

Soln Curl $\vec{A} = 0$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+3yz & (x^2+axz-4z^2) & (3xy+2byz) \end{vmatrix} = 0$$

$$i\left[\frac{\partial}{\partial y}(3xy+2byz)-\frac{\partial}{\partial z}(x^2+axz-4z^2)\right]$$

$$+\hat{j}\left[\frac{\partial}{\partial z}(2xy+3yz)-\frac{\partial}{\partial x}(3xy+2byz)\right]$$

$$+\hat{k}\left[\frac{\partial}{\partial x}(x^2+axz-4z^2)-\frac{\partial}{\partial y}(2xy+3yz)\right]$$

$$=0$$

$$i\left[3x+2bz-ax+8z\right]$$

$$+j\left[3y-3y\right]+k\left[2x+az-2x-3z\right]$$

$$=0$$

$$\Rightarrow \left[(3-a)x+2z(b+4)\right]i$$

$+ (0)\hat{j} + z(a-3)\hat{k} = 0$

$a = 3$

$b = -4$

Que Show that

$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$

is both Solenoidal & irrotational

$\nabla \cdot \vec{F} = 0$, $\nabla \times \vec{F} = 0$

Que Find the directional derivative of the div. $\vec{F}$, $\vec{F} = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$

at (2, 1, 2) in the direction of the outer normal to the sphere

$x^2+y^2+z^2=9$

<u>Ans</u> $\vec{F} = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$

$$\nabla\cdot\vec{F} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(xy^2) + \frac{\partial}{\partial z}(z^2)$$

$$\nabla\cdot\vec{F} = y + 2xy + 2z$$

$$\nabla(\nabla\cdot\vec{F}) = \text{Grad}(\text{Div}\,\vec{F})$$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(y+2xy+2z)$$

$$= 2y\hat{i} + (1+2x)\hat{j} + 2\hat{k}$$

$$\nabla(\nabla\cdot\vec{F})\Big|_{(2,1,2)} = 2i + 5\hat{j} + 2\hat{k}$$

Normal to the sphere $= \vec{a}$

$$= \nabla \cdot (x^2+y^2+z^2)$$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)$$

$$= 2(x\hat{i} + y\hat{j} + z\hat{k})$$

Normal at $(2,1,2)$

$$= 2(2i+j+2k) = \vec{a}$$

Directional derivative in the direction of outward normal to the sphere

$$= (2i+5j+2\hat{k})\cdot\hat{a}$$

$$= (2\hat{i}+5\hat{j}+2\hat{k})\cdot(4i+2j+4k)$$

$$\sqrt{16+4+16}$$

$$= \frac{1}{6}(8+10+8)$$

$$= \frac{13}{3}$$

$$\vec{V} = \nabla \cdot \phi$$

If $\vec{V}$ is a conservative vector field

the $\exists$ a scalar potential ϕ

st $\quad \vec{V} = \nabla \cdot \phi$

OR

When $\vec{V}$ is irrotational

$$\nabla \times \vec{V} = 0$$

Ques (a) A vector field V is irrotational if curl $\vec{V} = 0$. Find the constants a, b, c s.t

$$\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$ is irrotational

sol:

Curl $V = 0$, $a = 4$, $b = 2$, $c = -1$

$$\vec{V} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)\hat{k}$$

b) Show that $\vec{V}$ can be

expressed as the gradient on a scalar func

let $\vec{V} = \nabla \phi$

$(x+2y+4z)i + (2x-3y-z)j + (4x-y+2z)\hat{k}$

$= \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} \hat{k}$

$\frac{\partial \phi}{\partial x} = x+2y+4z$ — (1)

$\frac{\partial \phi}{\partial y} = 2x-3y-z$ — (2)

$\frac{\partial \phi}{} = 4x-y+2z$ — (3)

$-\partial z$

Integrate (i) partially wrt x treating y & z as const

$$\phi = \frac{x^2}{2} + 2yx + 4zx + f(yz)$$

$$\phi = 2xy - \frac{3y^2}{2} - zy + g(xz)$$

$$\phi = 4xz - yz + z^2 + h(xy)$$

$$\phi = \frac{x^2}{2} + 2yx + 4zx - yz$$
$$- \frac{3y^2}{2} + z^2$$

$$\vec{V} = \nabla \cdot \phi$$

$$\boxed{\text{Curl } \vec{V} = 0}$$

Conservative