Applied Mathematics-I, BAS-201

Applied Mathematics-I, BAS101
Tutorial Sheet -6
Infinite Series

Q1. Shown that the following series are convergent:

i. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$

ii. $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$

iii. $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\cdots$

iv. $\frac{\log 2}{2^2}-\frac{\log 3}{3^2}+\frac{\log 4}{4^2}-\cdots$

Q2.Prove that the following series are absolutely convergent:

i. $1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots$

ii. $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$

iii. $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$

Q3.Show that $\lim_{n\to\infty}\frac{n^r}{x^n}=0$,if $x>0$

Q4.Show that the following series are conditionally convergent:

i. $\sum\frac{(-1)^{(n+1)}}{\sqrt{n}}$

ii. $\sum\frac{(-1)\hat{}(n+1)}{3n-2}$

Q5.Show that the series $\sum\frac{(-1)^{(n+1)}}{n^p}$ is absolutely convergent for $p>0$

But conditionally convergent for $0<p\le 1$.

Q6.Show that the following series are absolutely convergent:

i. $\sum(-1)^{(n-1)}\left\{\frac{1}{n^2}+\frac{1}{(n+1)^2}\right\}$

ii. $\sum(-1)^{(n-1)}\left\{\frac{1}{n^{5/2}}+\frac{1}{(n+1)^{5/2}}\right\}$

iii. $\sum(-1)^n\ \frac{n+2}{2^n+5}$

Q7.Show that the series $\sum\left(\frac{1}{n}+\frac{(-1)^{(n-1)}}{\sqrt{n}}\right)$ is divergent.

Q8.Show that the series $1-\frac{1}{3.4}+\frac{1}{5.4^2}-\frac{1}{7.4^3}+\cdots$ converges.

Q9.Use Cauchy's Integral Test to show that the following series converge:

i. $\sum_{n=0}^{\infty}\left(\frac{1+n}{1+n^2}\right)^2$

ii. $\sum_{n=2}^{\infty}\frac{1}{\sqrt{x}}\log\frac{n+1}{n-1}$

Q10.Show that the following series are absolutely convergent:

i. $\sum\frac{\sin n\alpha}{n^2}$

ii. $\sum(-1)^{(n+1)}\frac{n^3}{2^n}$

iii. $\frac{1}{2}-\frac{1}{2}.\frac{1}{2^2}+\frac{1}{3}.\frac{1}{2^3}-\frac{1}{4}.\frac{1}{2^4}+\cdots$

Q11.Show that the following series are conditionally convergent:

i. $\sum_{n=1}^{\infty}\frac{(-1)^{(n+1)}}{\log(n+1)}$

ii. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n-\log n}$

Q12.Establish the divergence of the series $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$

Q13.Show that if $\sum a_n^2$ and$\sum b_n^2$ are convergent infinite series ,then $\sum a_n b_n$ is an absolutely Convergent series.

Q14.Show that if the series $\sum a_n$ is absolutely convergent, then the series $\sum\frac{n+1}{n}a_n$ is also absolutely convergent.