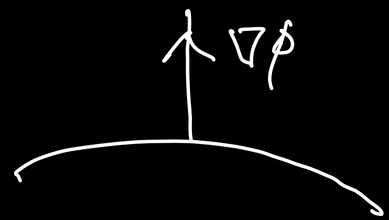
ϕ, $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

$$\vec{\nabla}\phi = i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

Gradient of $\phi \rightarrow$ Normal to the Surface



$\phi \rightarrow$ Scalar

$\phi = C$

$$d\phi = 0$$

$$\vec{\nabla}\phi \cdot d\vec{r} = 0$$

, $d\vec{r} \rightarrow$ tgt to the surface

$\Rightarrow \vec{\nabla}\phi \rightarrow \perp$ from tgt

$\Rightarrow \nabla\phi \rightarrow$ Normal to the surface.

<u>Directional Derivative</u>

let ϕ be a scalar pt func $\underline{\phi(x,y,z)}$

then $\frac{\partial \phi}{\partial x}$, $\frac{\partial \phi}{\partial y}$ & $\frac{\partial \phi}{\partial z}$ are the directional derivative of ϕ in the direction of the coord axes

If $\vec{V}(x,y,z)$ represents a vector pt func. the $\frac{\partial \bar{V}}{\partial x}, \frac{\partial \bar{V}}{\partial y}, \frac{\partial \bar{V}}{\partial z}$ represents directional derivatives of $\bar{V}$ in the direction of coord axes

<u>Note</u> i) The directional derivative of a scalar pt func ϕ in the direction of a line whose direction cosines are l, m, n

$$\boxed{= l\frac{\partial \phi}{\partial x} + m\frac{\partial \phi}{\partial y} + n\frac{\partial \phi}{\partial z}}$$

$$= \nabla \phi \cdot (l\hat{i} + m\hat{j} + n\hat{k})$$

$$= \left(i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{\partial y} + k\frac{\partial \phi}{\partial z}\right) \cdot (l\hat{i} + m\hat{j} + n\hat{k})$$

$$= l\frac{\partial \phi}{\partial x} + m\frac{\partial \phi}{\partial y} + n\frac{\partial \phi}{\partial z}$$

Direction der of ϕ in the direction of a line whose direc Cosines l, m, n

<u>Note 2</u> Directional derivative of a Scalar fun ϕ in the directon of a vector $\vec{a} \longrightarrow \nabla\phi \cdot \hat{a}$

$$\nabla\phi \cdot \hat{a} = \frac{\nabla\phi \cdot \vec{a}}{|a|}$$

$$\nabla\phi \cdot \hat{a} = |\nabla\phi| \, |\hat{a}| \cos\theta$$

$\theta \rightarrow$ Angle between $\nabla\phi$ & $\hat{a}$

$$\vec{\nabla}\phi \cdot \hat{a} = |\nabla\phi| \cos\theta$$

This value will be max

when $\theta = 0$

$\Rightarrow$ when both the vectors

are in the same direction

OR

$\Rightarrow$ when $\vec{a}$ is in the the

direction of $\nabla\phi$

$\Rightarrow$ Max direction of the

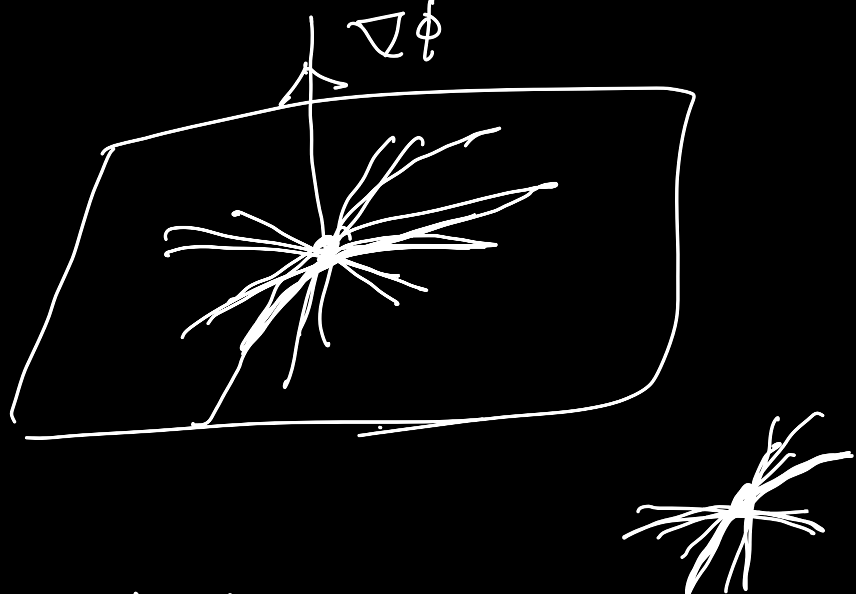
directional derivation of ϕ

with any vector $\vec{a}$ is in

the direction of $\vec{\nabla}\phi$ or

Normal to the surface

Equation of tgt plane with the help of Gradient of $\phi = c$



$\nabla\phi$ normal to surface at pt $P_0(x_0, y_0, z_0)$

Let (x, y, z) be any pt on the tgt the the vector

$$(x-x_0)\,i + (y-y_0)\,j + (z-z_0)\,k$$

is a vector in the tgt plane

$$\Rightarrow \nabla\phi(P_0) \cdot \left[(x-x_0)i+(y-y_0)j + (z-z_0)k\right] = 0$$

$$\Rightarrow \frac{\partial\phi}{\partial x}(P_0)(x-x_0) + \frac{\partial\phi}{\partial y}(P_0)(y-y_0) + \frac{\partial\phi}{\partial z}(P_0)(z-z_0) = 0$$

This represents eqn of the tgt plane at P_0

OR

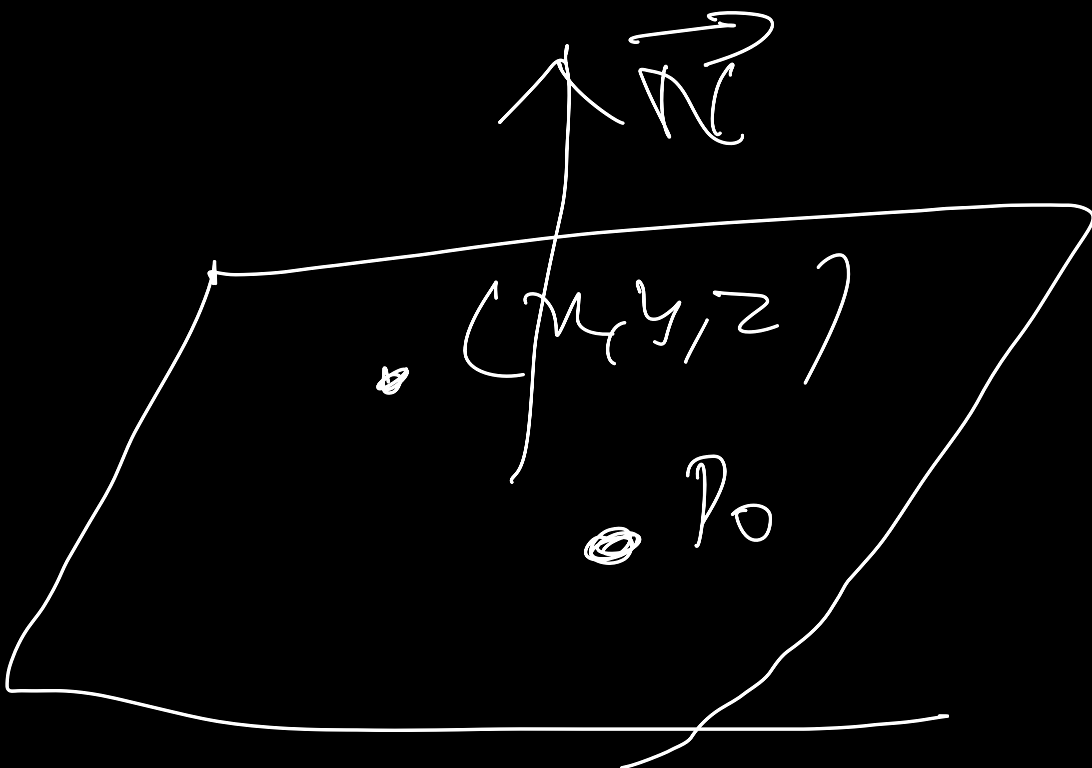
Eqn of the tgt plane passing through a pt r_0

$$(r-r_0)\cdot\vec{N} = 0$$

$$\vec{r} = xi + yj + zk$$

$$\vec{r_0} = x_0 i + y_0 j + z_0 k$$

$$(\vec{r} - \vec{r_0}) \cdot \vec{N} = 0$$



$P_0(x_0, y_0, z_0)$

$$(x-x_0)\hat{i}+(y-y_0)\hat{j}$$

$$+(z-z_0)k$$

$$= \vec{dr}$$

$$\vec{dr}\cdot\vec{N}=0$$

Que $2xz^2-3xy-4x=7$

at $(1,-1,2)$

Find the eqn of the tgt plane

$$\vec{N}=\vec{\nabla\phi}\Big|$$

$(1, -1, 2)$

$$\nabla\phi = (2z^2 - 3y - 4)\hat{i} - 3x\hat{j} + 4zx\hat{k}$$

$$\nabla\phi\Big|_{(1,-1,2)} = \vec{N} = 7\hat{i} - 3\hat{j} + 8\hat{k}$$

Equn of the tgt plane

$$(\vec{r} - \vec{r}_0) \cdot \vec{N} = 0$$

$$\left[(xi + yj + zk) - (i - j + 2k)\right] \cdot [7i - 3j + 8k] = 0$$

$$7(x-1) - 3(y+1) + 8(z-2) = 0$$

Eqn of the tgt plane.

Que Find the directional derivative of $\phi = xy^2 + yz^2$ at $(2,-1,1)$ in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\nabla\phi\Big|_{(2,-1,1)} \cdot \hat{a}$$

$$\nabla\phi = i\frac{\partial}{\partial x}(xy^2 + yz^2) + j\frac{\partial}{\partial y}(xy^2 + yz^2) + k\frac{\partial}{\partial z}(xy^2 + yz^2)$$

$$= \hat{i}\,y^2 + \hat{j}(2xy + z^2) + \hat{k}(2yz)$$

$$\nabla\phi\Big|_{(2,-1,1)} = i - 3\hat{j} - 2\hat{k}$$

$$\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|a|} = \frac{i + 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}}$$

$$\nabla\vec{\phi}\cdot\hat{a} = (i-3j-2k)\cdot\left(\frac{i+2j+2k}{3}\right)$$

$$= \frac{1-6-4}{3} = -3$$

Que $\phi = xy^2 + yz^2 + zx^2$ along the tgt to the curve $x=t,\ y=t^2,\ z=t^3$ at the pt $(1,1,1)$

Soln Tangent to the Curve

$$\vec{T} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\hat{i}+y\hat{j}+z\hat{k})$$

$$= \frac{d}{dt}(t\hat{i}+t^2\hat{j}+t^3\hat{k})$$

$$\vec{T} = i + 2t\hat{j} + 3t^2\hat{k}$$

At $(1,1,1)$, $t=1$

$$\vec{a} = \vec{T} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\phi = xy^2 + yz^2 + zx^2$$

$$\nabla\phi = i\frac{\partial}{\partial x}(xy^2+yz^2+zx^2) + j\frac{\partial}{\partial y}(xy^2+yz^2+zx^2) + k(xy^2+yz^2+zx^2)$$

$$= i(y^2+2xz) + j(2xy+z^2) + k(2yz+x^2)$$

$$\nabla\phi\big|_{(1,1,1)} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Directional derivative of ϕ in the direction of $\vec{T} = \vec{a}$

$$= \nabla\phi \cdot \hat{T} = \frac{\nabla\phi \cdot \vec{T}}{|T|}$$

$$= \frac{(3i+3j+3k)\cdot(i+2j+3k)}{\sqrt{1+4+9}}$$

$$= \frac{18}{\sqrt{14}}$$