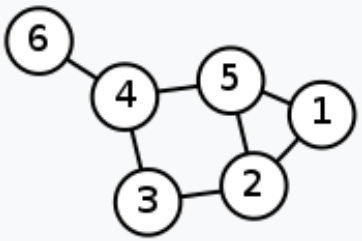
Spectral Algorithms

Spectral Algorithms

- classical method
- assign nodes to communities based on the eigenvectors of matrices, such as adjacency matrix

Laplacian Matrix

Labeled graph	Degree matrix	Adjacency matrix	Laplacian matrix
	$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$

Properties of Laplacian Matrix

$$L_{i,j} = \begin{cases} deg(v_i) & \text{if } i = j \\ -1 & \text{if } i \neq j \text{and} v_i \text{is adjacent to } v_j \\ 0 & otherwise \end{cases} \quad (1)$$

Properties of normalized Laplacian Matrix

$$L_{i,j} = \begin{cases} 1 & \text{if } i = j \text{and} deg(v_i) \neq 0 \\ -\frac{1}{\sqrt{deg(v_i)deg(v_j)}} & \text{if } i \neq j \text{and} v_i \text{is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Multi-level Graph Partitioning

Multi-level Graph Partitioning

- fast and high-quality graph partitioing
- shrink or coarsen the input graph
- partition this small graph
- project this partition back to the original graph

Step 1: Coarsening

- the goal is to produce a smaller graph that is similar to the original graph
- the strategy is to first construct a matching on the graph, where a matching is defined as a set of edges no two of which are incident on the same vertex
- for each edge in the matching, the vertices at the ends of the edge are collapsed together and are represented by a single node

Step 2: Initial Partitioning

- partitioning of the coarses graph is performed
- strategies such as spectral partitioning may be applied

Step 3: Uncoarsening

- partition on the current graph is used to initialize a partition on the finer graph
- this step is continued until we arrive at the original input graph