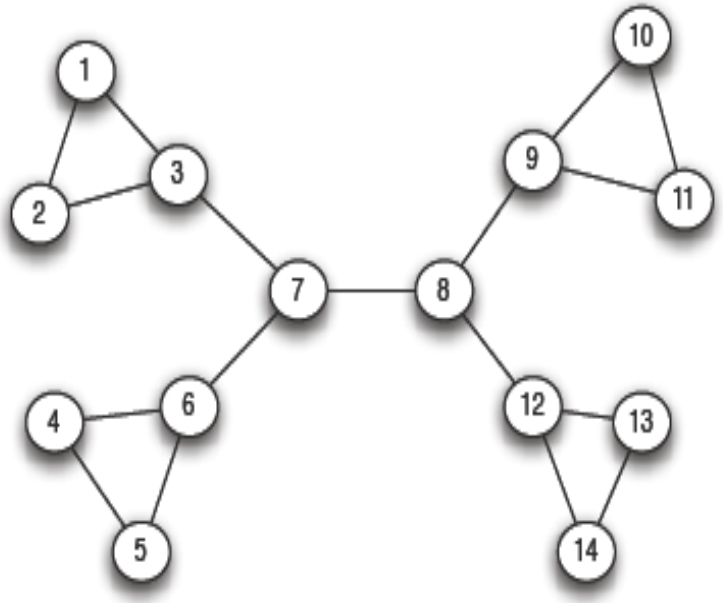
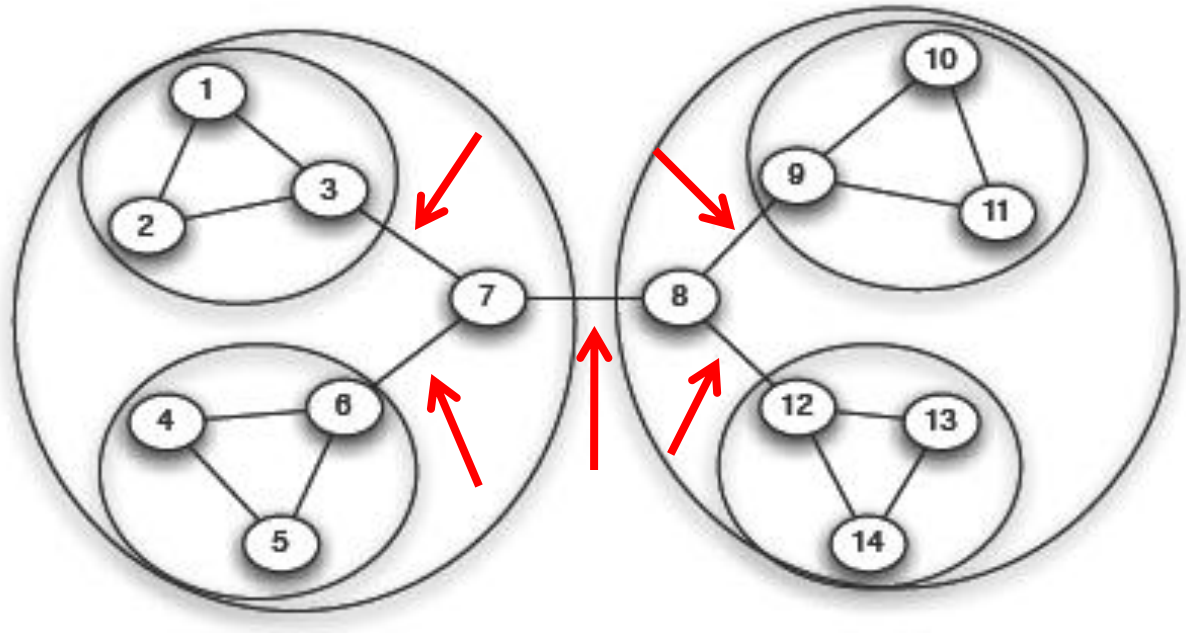
Edge Betweenness

- Betweenness of an edge: the total amount of flow it carries
  - counting flow between all pairs of nodes using this edge
- Ex:
  - Edge **7-8**: each pair of nodes between [1-7] and [8-14]; each pair with traffic = 1; total 7 x 7 = 49
  - Edge **3-7**: each pair of nodes between [1-3] and [4-14]; each pair with traffic = 1; total 3 x 11 = 33
  - Edge **1-3**: each pair of nodes between [1] and [3-14] (not node 2); each pair with traffic = 1; total 1 x 12 = 12
    - similar for edges 2-3, 4-6, 5-6, 9-10, 9-11, 12-13, and 12-14
  - Edge **1-2**: each pair of nodes between [1] and [2] (no other); each pair with traffic = 1; total 1 x 1 = 1
    - similar for edges 4-5, 10-11, and 13-14



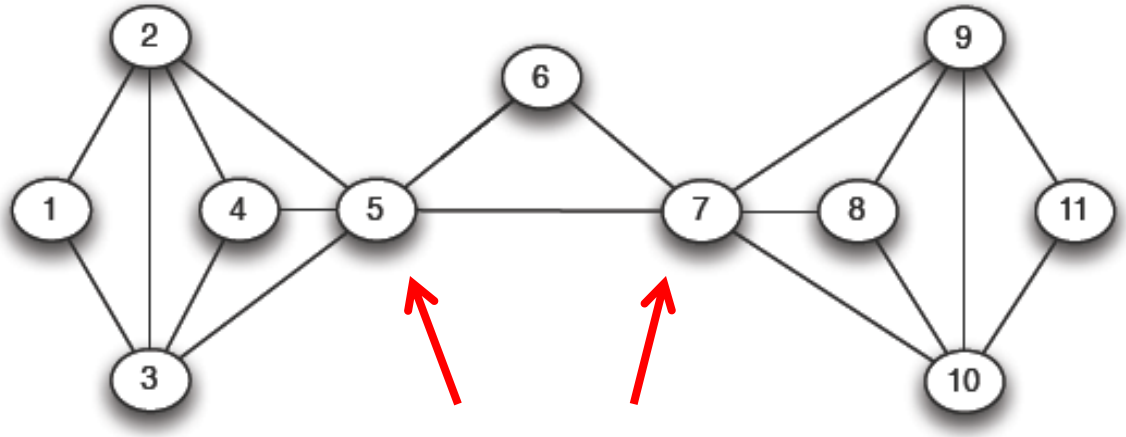
Betweenness for Partitioning

- Divisive: remove edges with high betweenness



Betweenness of Nodes

- Betweenness of a node: total amount of flow that it carries, when a unit of flow between each pair of nodes is divided up evenly over shortest paths (same as for edges)
  - nodes of high betweenness occupy critical roles in the network ("gatekeepers")



Girvan-Newman Partitioning Alg.

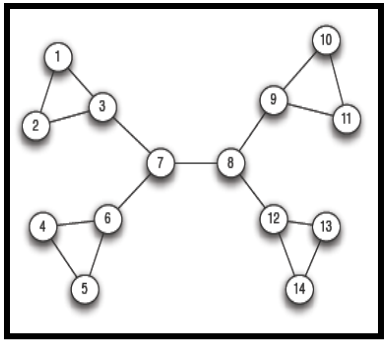
Successively Deleting Edges of High Betweenness

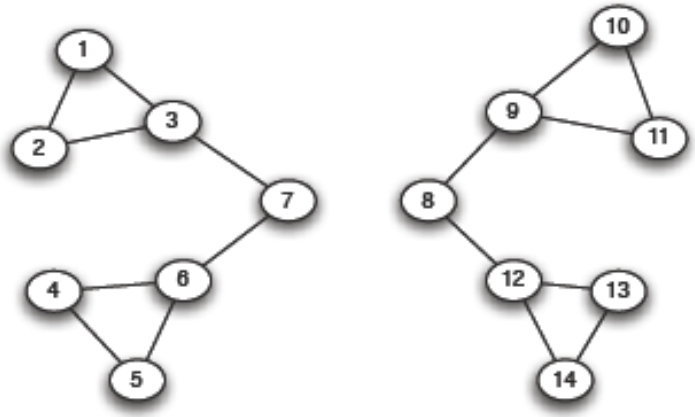
(1) Find the edge of highest betweenness — or multiple edges of highest betweenness, if there is a tie — and remove these edges from the graph. This may cause the graph to separate into multiple components. If so, this is the first level of regions in the partitioning of the graph.

(2) Now recalculate all betweennesses, and again remove the edge or edges of highest betweenness. This may break some of the existing components into smaller components; if so, these are regions nested within the larger regions.

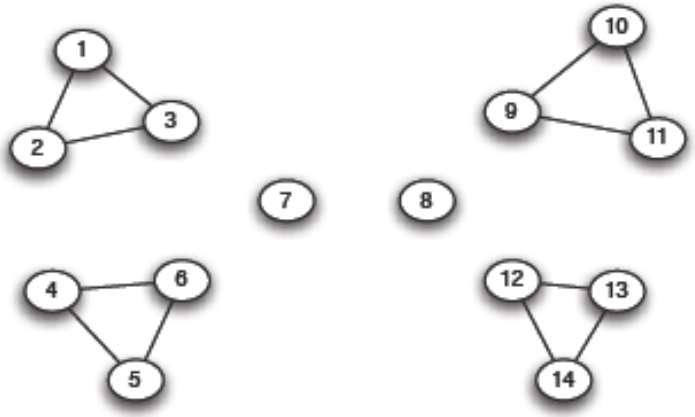
(...) Proceed in this way as long as edges remain in graph, in each step recalculating all betweennesses and removing the edge or edges of highest betweenness.

Example 1

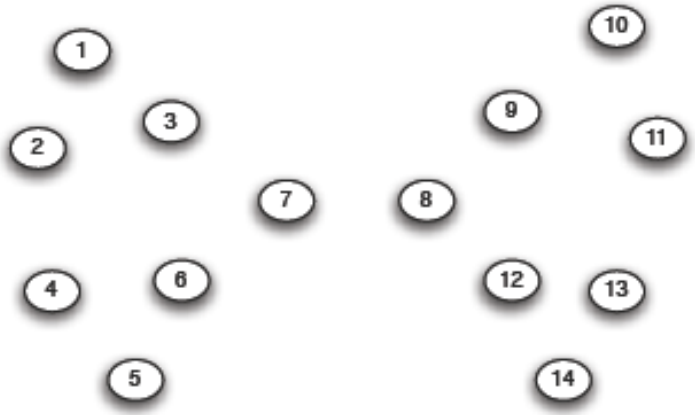




(a) *Step 1*

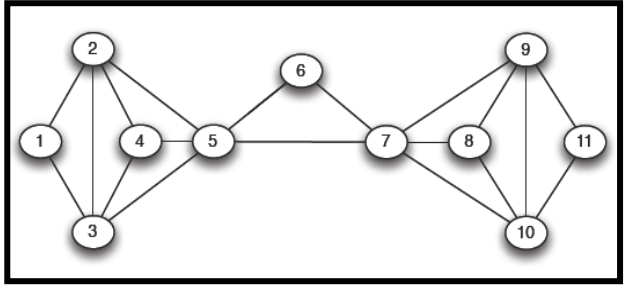


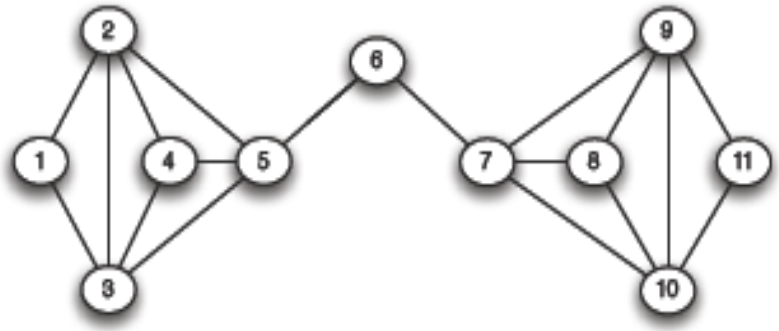
(b) *Step 2*



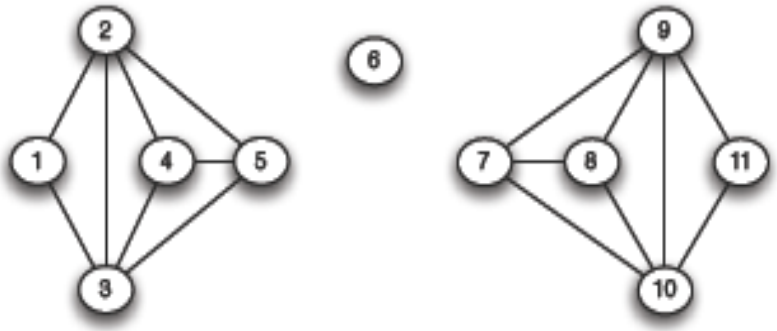
(c) *Step 3*

Example 2

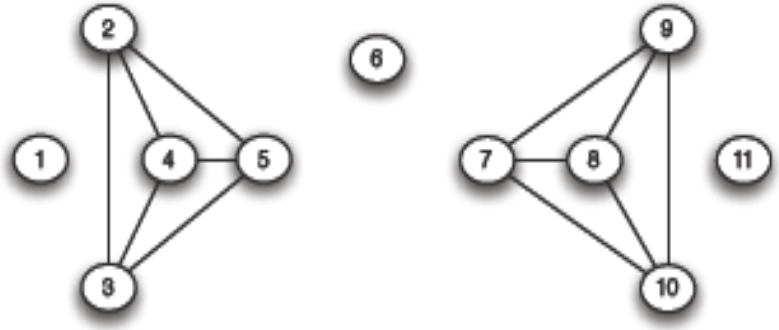




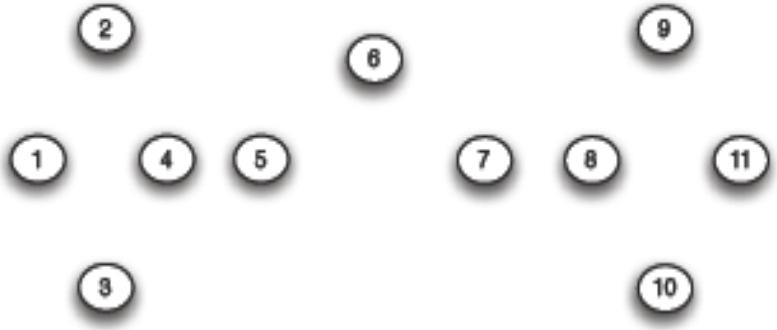
(a) *Step 1*



(b) *Step 2*



(c) *Step 3*



(d) *Step 4*