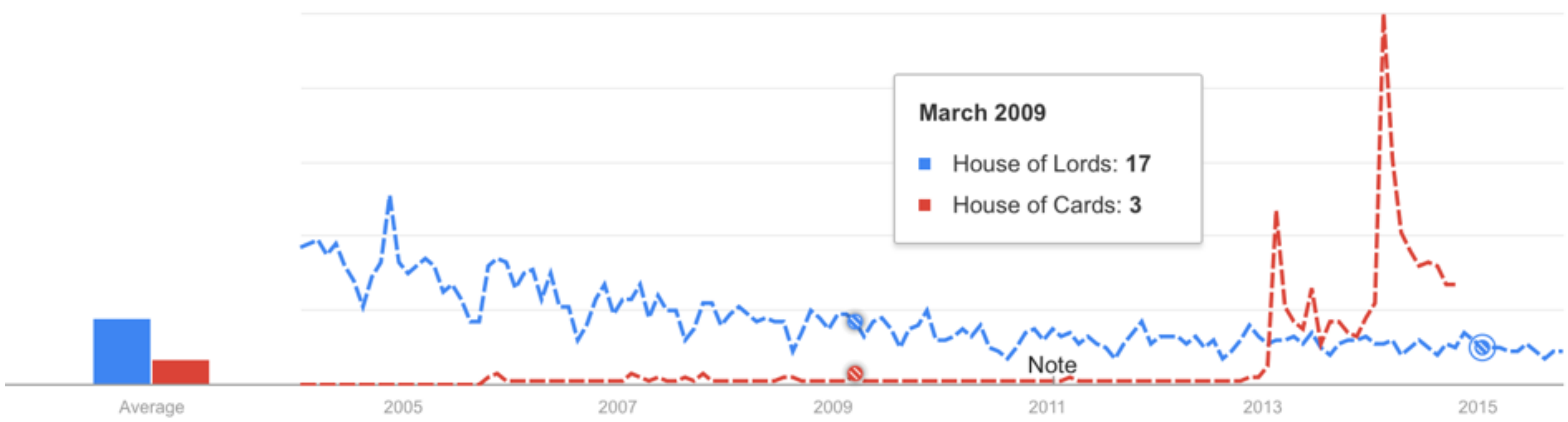
Foundations - 1

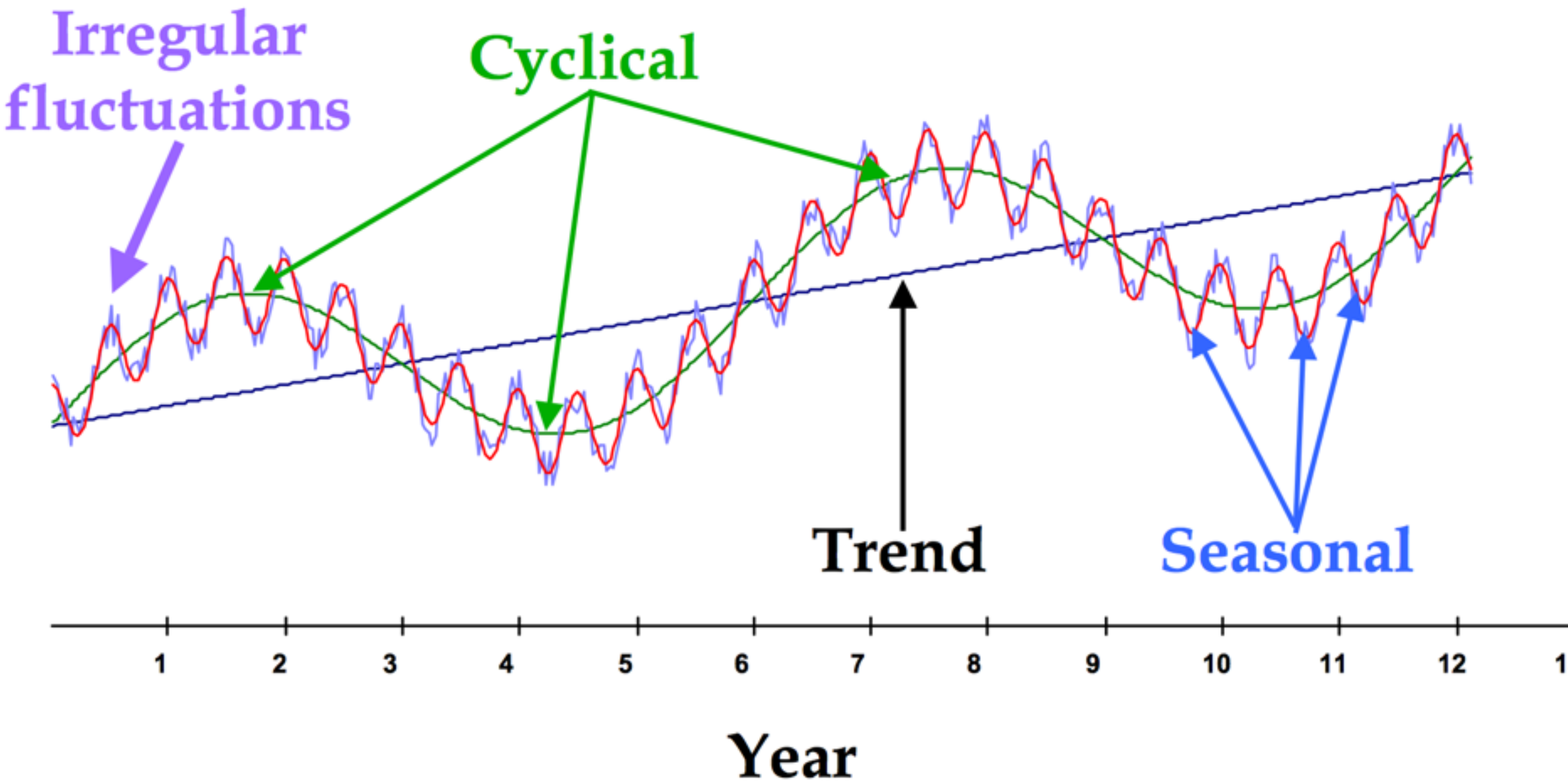
Time-series Analysis, Forecasting

Time Series

- An ordered sequence of values (data points) of variables at equally spaced time intervals



Time Series Components

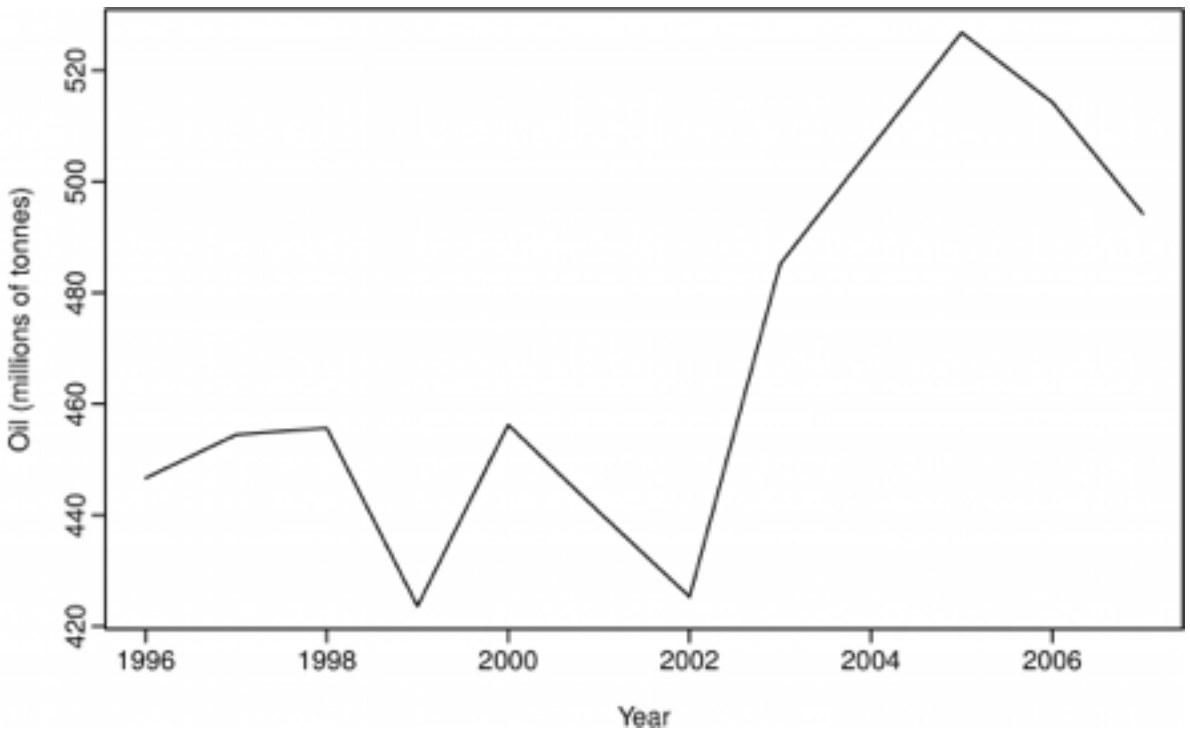


Forecasting

- Forecasting using time-series a classical task to predict the future values based on past observations
- Heavily used in many domains for forecasting:
  - Stock prices
  - Market adoption
  - Weather conditions
  - Sales of articles
  - Environmental carbon footprint

Forecasting

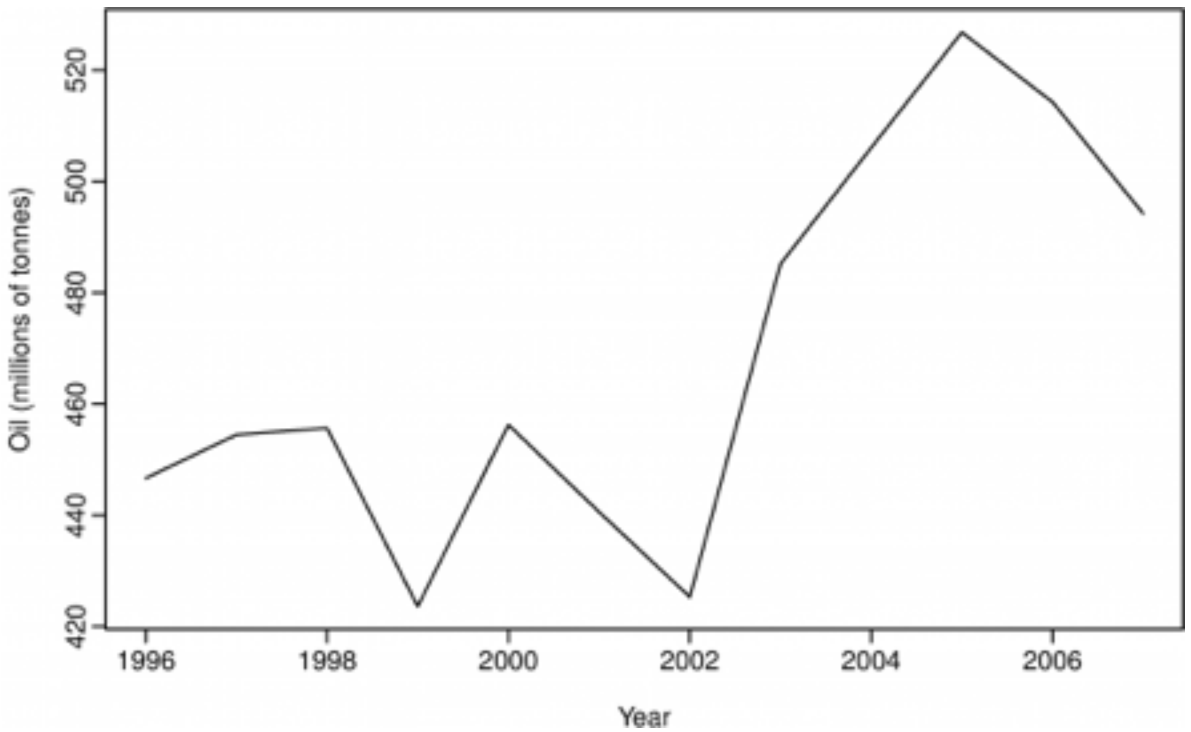
- Given a series of observations $\{y_T\}$ predict forecast the observation at $T+h$



Given that observations have been made until time t

Forecasting

- Given a series of observations $\{y_T\}$ predict forecast the observation at T+h



$$\hat{y}_{t+h|t} = y_t$$

Given that observations have been made until time t

Forecasting

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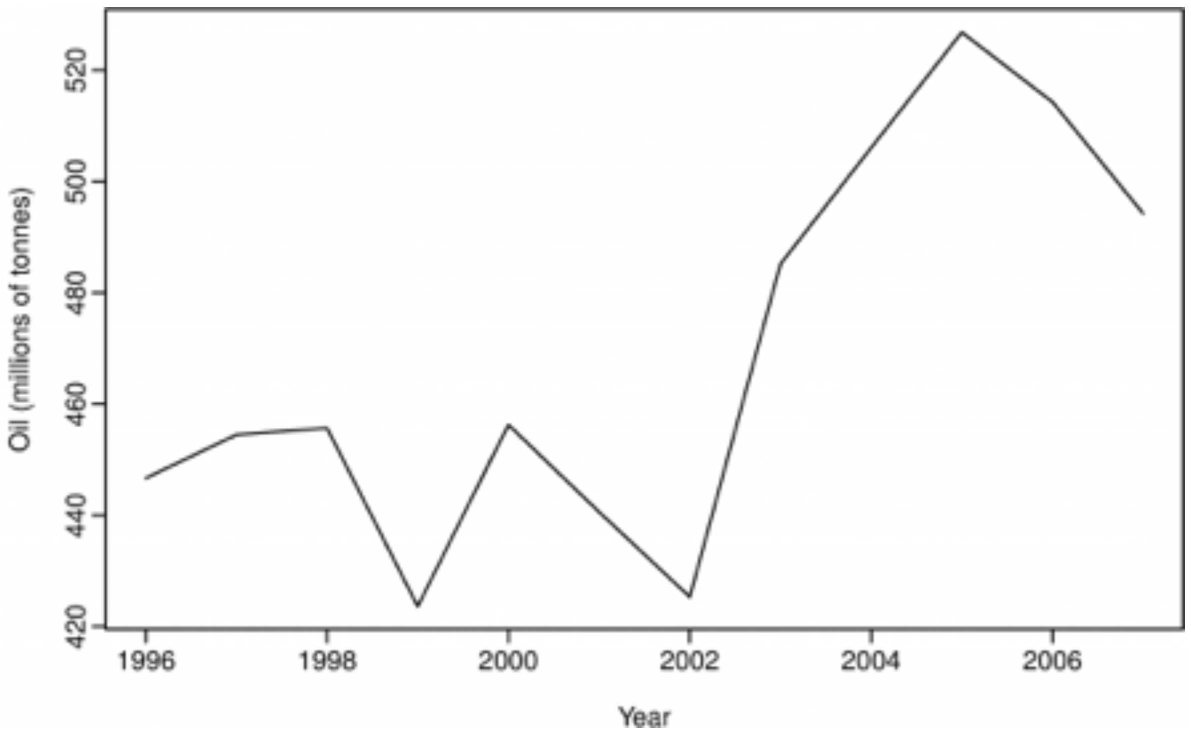

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Forecasting

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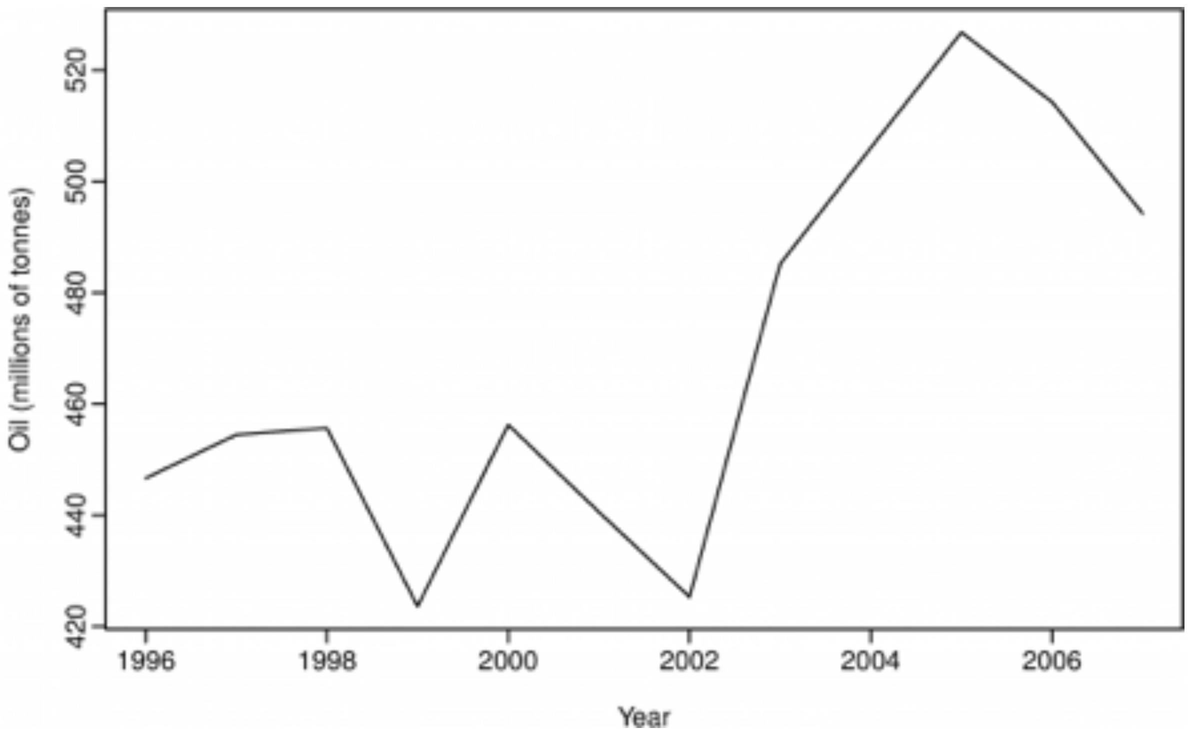


Given that observations have been made until time t

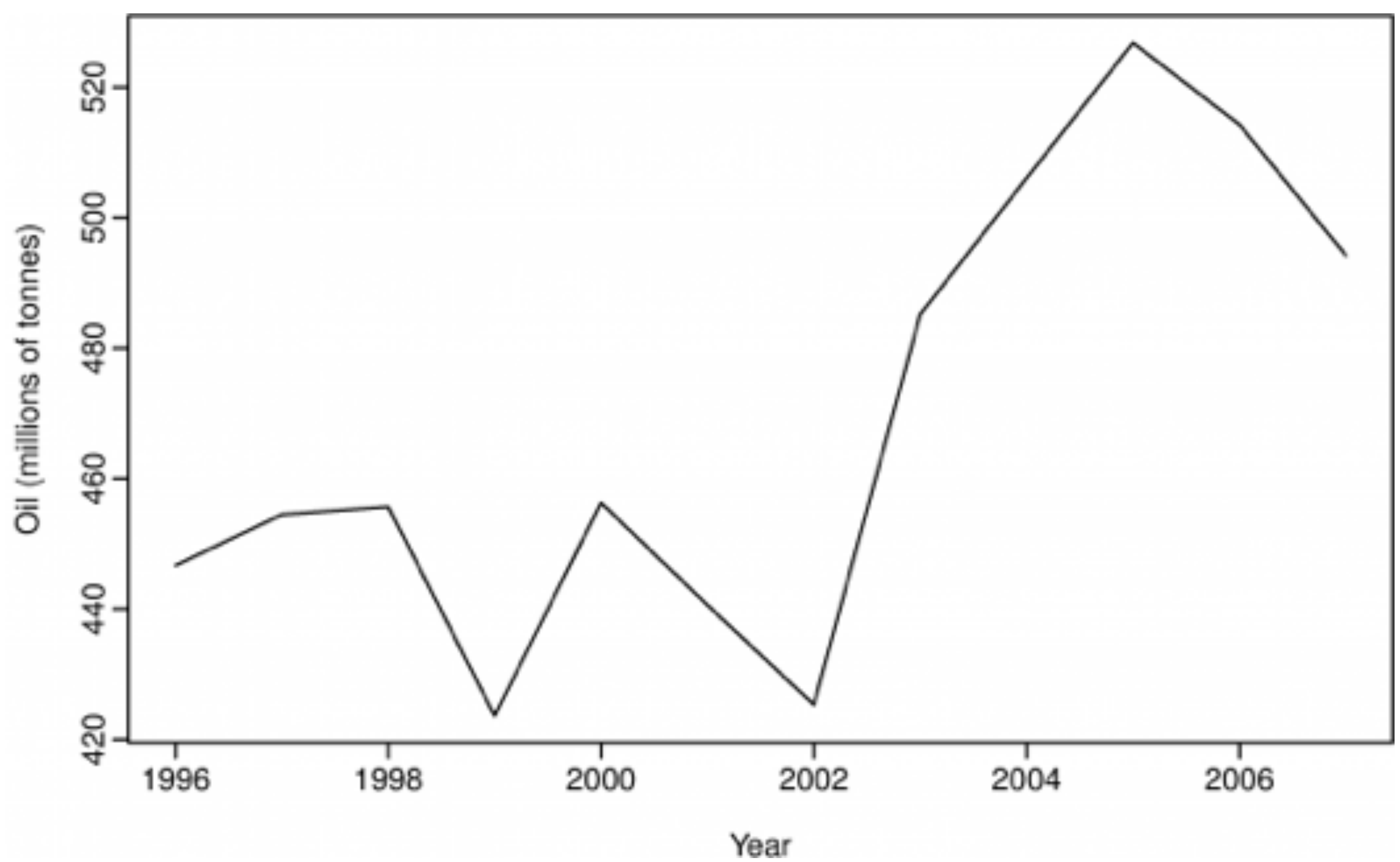
forecast for time t+h

$$\hat{y}_{t+h|t} = y_t$$

observed value at t

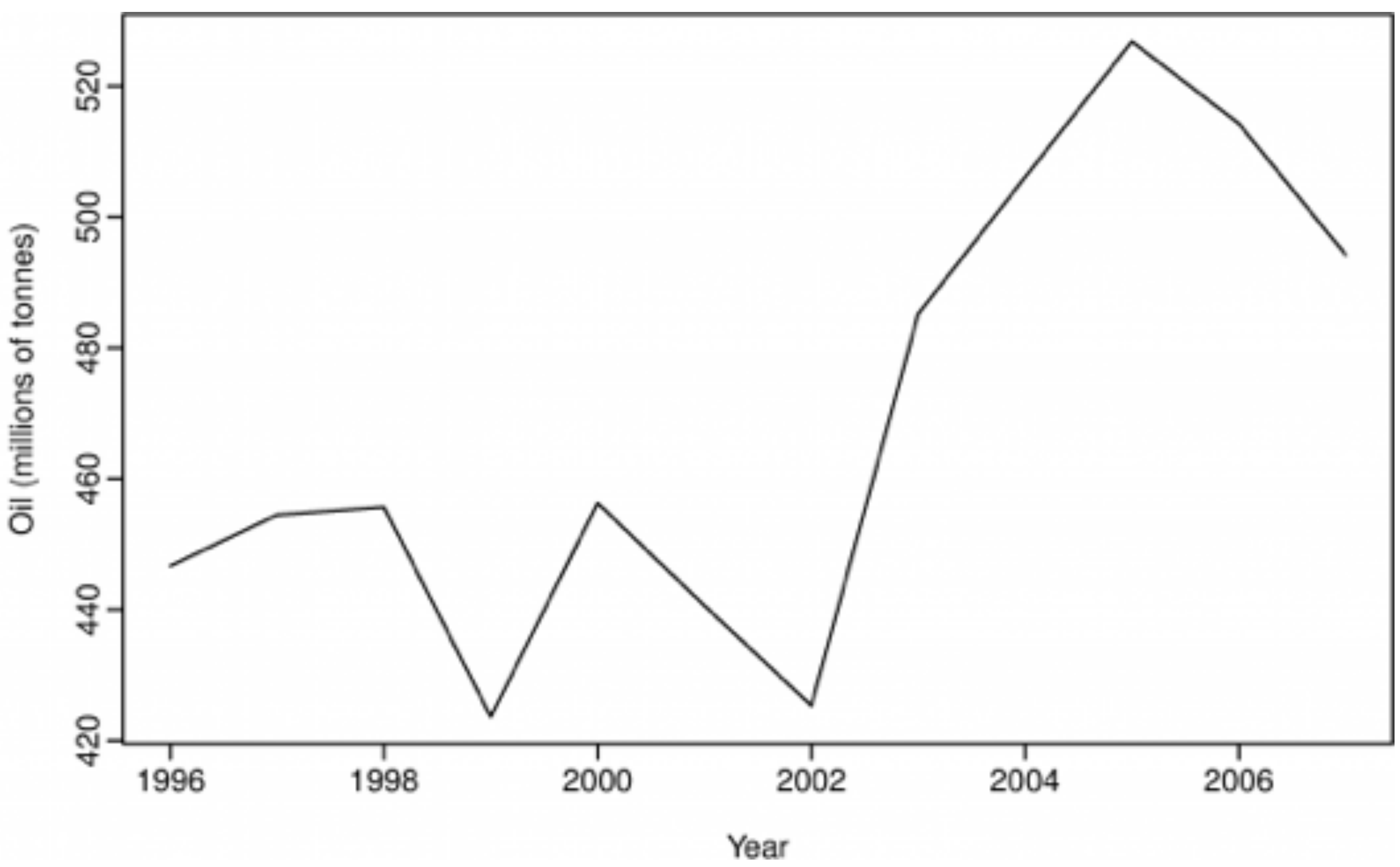
Forecasting

- What is the forecast for the next time point ?



Forecasting

- What is the forecast for the next time point ?
  - Simple Average
  - Moving Average
  - Linear Regression
  - Weighted Average



Idea : More weights to recent data

Simple Exponential Smoothing

$$\hat{y}_{t+1|t} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots ,$$

- Forecast for time t+1 given that t values have been observed
- Exponentially decreasing weights

Simple Exponential Smoothing

smoothing parameter in [0,1]

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smoothing
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- Forecast for time t+1 given that t values have been observed
- Exponentially decreasing weights

Observation	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$
y_T	0.2	0.4	0.6	0.8
y_{T-1}	0.16	0.24	0.24	0.16
y_{T-2}	0.128	0.144	0.096	0.032
y_{T-3}	0.1024	0.0864	0.0384	0.0064
y_{T-4}	$(0.2)(0.8)^4$	$(0.4)(0.6)^4$	$(0.6)(0.4)^4$	$(0.8)(0.2)^4$
y_{T-5}	$(0.2)(0.8)^5$	$(0.4)(0.6)^5$	$(0.6)(0.4)^5$	$(0.8)(0.2)^5$

SES: Component Form

$$\hat{y}_{t+1|t} = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots,$$

weighted average form

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

Component Form

Interpretation: SES is the linear combination of last observed value and last forecast value

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weighted average form

forecast for time t+1

forecast for time t

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

Component Form

Interpretation: SES is the linear combination of last observed value and last forecast value

Simple Exponential Smoothing

$$\hat{y}_{t+1|t} = \alpha y_t + (1 - \alpha)\hat{y}_{t|t-1}$$

$$\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$$ component form

$$\ell_{t-1} = \hat{y}_{t|t-1}$$

Simple Exponential Smoothing

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

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is the level (or the smoothed value) of the series at time t

Simple Exponential Smoothing

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$$\ell_{t-1} = \hat{y}_{t|t-1}$$

is the level (or the smoothed value) of the series at time t

$$\begin{aligned}\ell_t &= \ell_{t-1} + \alpha(y_t - \ell_{t-1}) \\ &= \ell_{t-1} + \alpha e_t\end{aligned}$$

error correction form

Simple Exponential Smoothing

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

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$$\begin{aligned} \ell_t &= \ell_{t-1} + \alpha(y_t - \ell_{t-1}) \\ &= \ell_{t-1} + \alpha e_t \end{aligned}$$

error correction form

Interpretation: SES is sum of last observed value and smoothed error from the last measurement

Simple Exponential Smoothing

$$\hat{y}_{t+1|t} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots ,$$

Weighted Average form

$$\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$$

component form

$$\ell_t = \ell_{t-1} + \alpha(y_t - \ell_{t-1})$$

error correction form

SES gives us the forecast for level

Simple Exponential Smoothing

in action

Query vol. for "hannover"

1	23	NA	23
2	40	23	
3	25	26.4	
4	27	26.12	
5	32	26.296	
6	48	27.437	
7	33	31.549	
8	37	31.840	
9	37	32.872	
10	50	33.697	

since no prior info. exists

$$\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$$ component form

Simple Exponential Smoothing

in action

Query vol. for "hannover"

1	23	NA	23
2	40	23	(.2)(40)+(.8)(23)=26.4
3	25	26.4	(.2)(25)+(.8)(26.4)=26.12
4	27	26.12	(.2)(27)+(.8)(26.12)=26.296
5	32	26.296	(.2)(32)+(.8)(26.296)=27.437
6	48	27.437	(.2)(48)+(.8)(27.437)=31.549
7	33	31.549	(.2)(48)+(.8)(31.549)=31.840
8	37	31.840	(.2)(33)+(.8)(31.840)=32.872
9	37	32.872	(.2)(37)+(.8)(32.872)=33.697
10	50	33.697	(.2)(50)+(.8)(33.697)=36.958

since no prior info. exists

$$\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$$ component form

Choice of Weights

- To estimate the smoothing parameter, forecasts are computed for equal to .1, .2, .3, …, .9 and the sum of squared forecast error is computed for each

- The smallest Mean Square Error (MSE) is chosen for use in producing the future forecasts.

$$\frac{1}{n}\sum_{t=1}^{n}(\hat{y}_t - y_t)^2$$

error $e_t = \hat{y}_t - y_t$

Goodness of Forecast

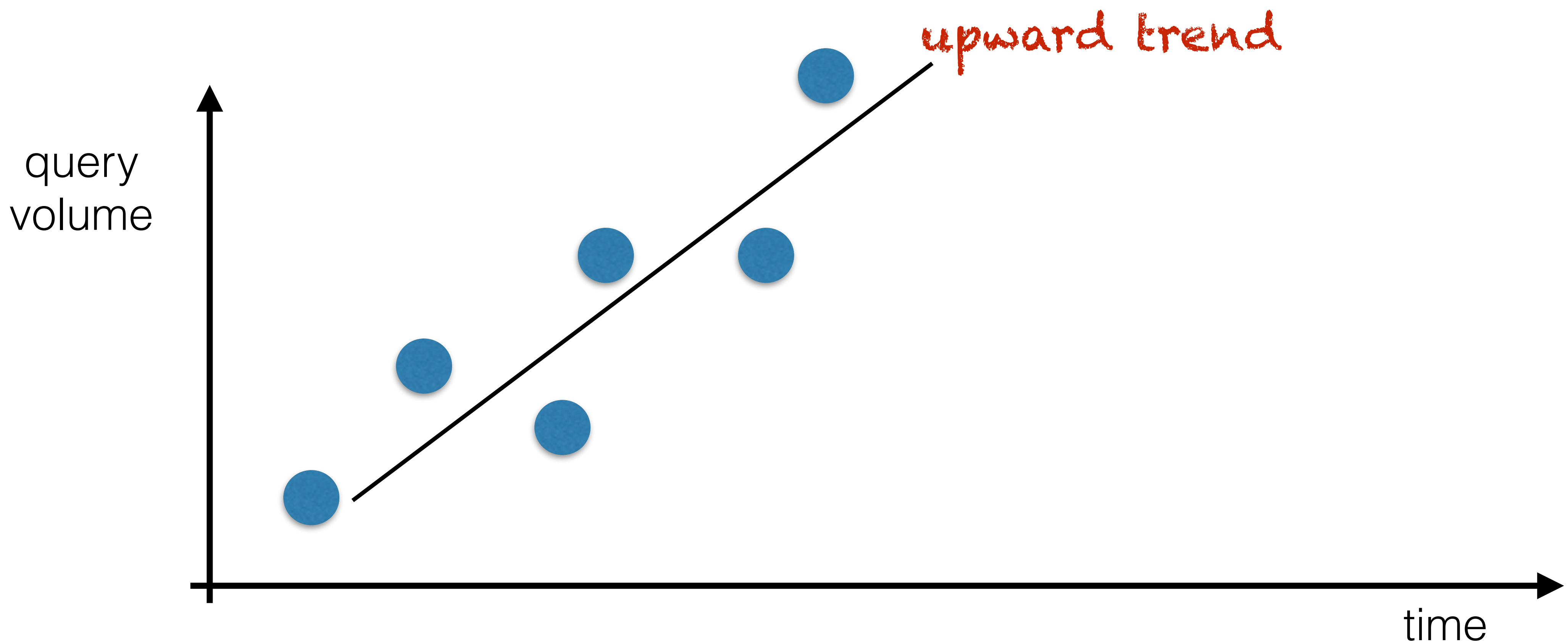
- Root Mean Squared Error (RMSE) : $\sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$

- Mean Absolute Error (MAE): $\frac{1}{n}\sum_{t=1}^{n} |e_t|$

- Mean Absolute Percentage Error (MAPE): $\frac{100\%}{n}\sum_{t=1}^{n} \left|\frac{e_t}{y_t}\right|$

Holt's Model

Double Exponential Smoothing



SES responds slow to trend data

$$\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$$

simple exponential smoothing

Holt's model explicitly models **Trend** alongwith **Level**

Holt's Model

Double Exponential Smoothing

$$\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} \boxed{+ b_{t-1}})$$

Level Equation

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$$

Trend Equation

Adds a growth factor (or trend factor) to the smoothing equation as a way of adjusting for the trend.

Holt's Model

Double Exponential Smoothing

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Level Equation

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Trend Equation

Adds a growth factor (or trend factor) to the smoothing equation as a way of adjusting for the trend.

$$\hat{y}_{t+h|t} = \ell_t \boxed{+ hb_t}$$

Forecast Equation

Choice of Weights

- To weights α and β can be selected subjectively or by minimizing a measure of forecast error such as RMSE

- Large weights result in more rapid changes in the component. Small weights result in less rapid changes.

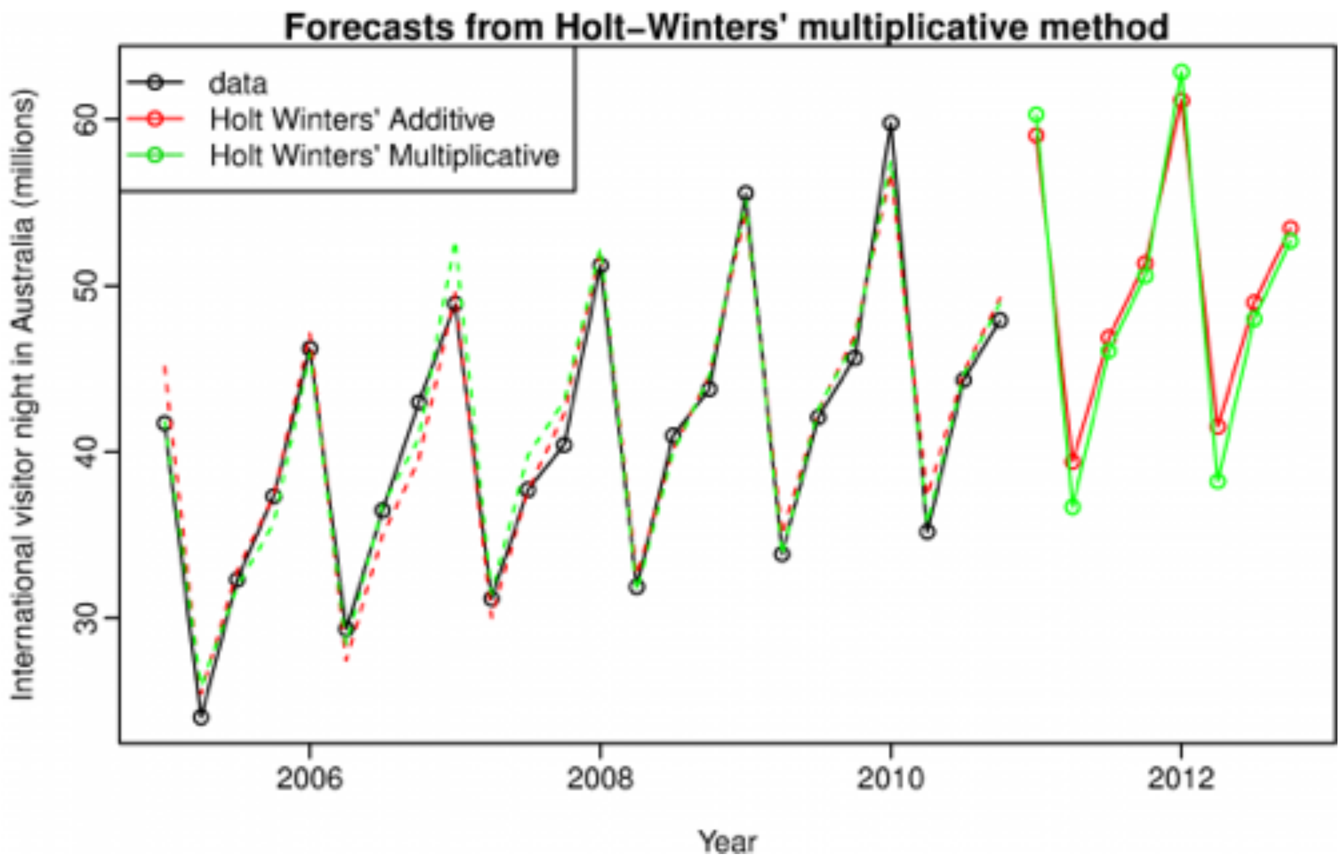
Holt-Winters Model

Triple Exponential Smoothing

- Winter's exponential smoothing model is the second extension of the basic Exponential smoothing model.
- It is used for data that exhibit both trend and seasonality.
- It is a three parameter model that is an extension of Holt's method.
- An additional equation adjusts the model for the seasonal component.

Holt-Winters Model

Triple Exponential Smoothing



$$s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m},$$

Holt-Winters Model

Triple Exponential Smoothing

$$s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m},$$

Seasonality Equation

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$$

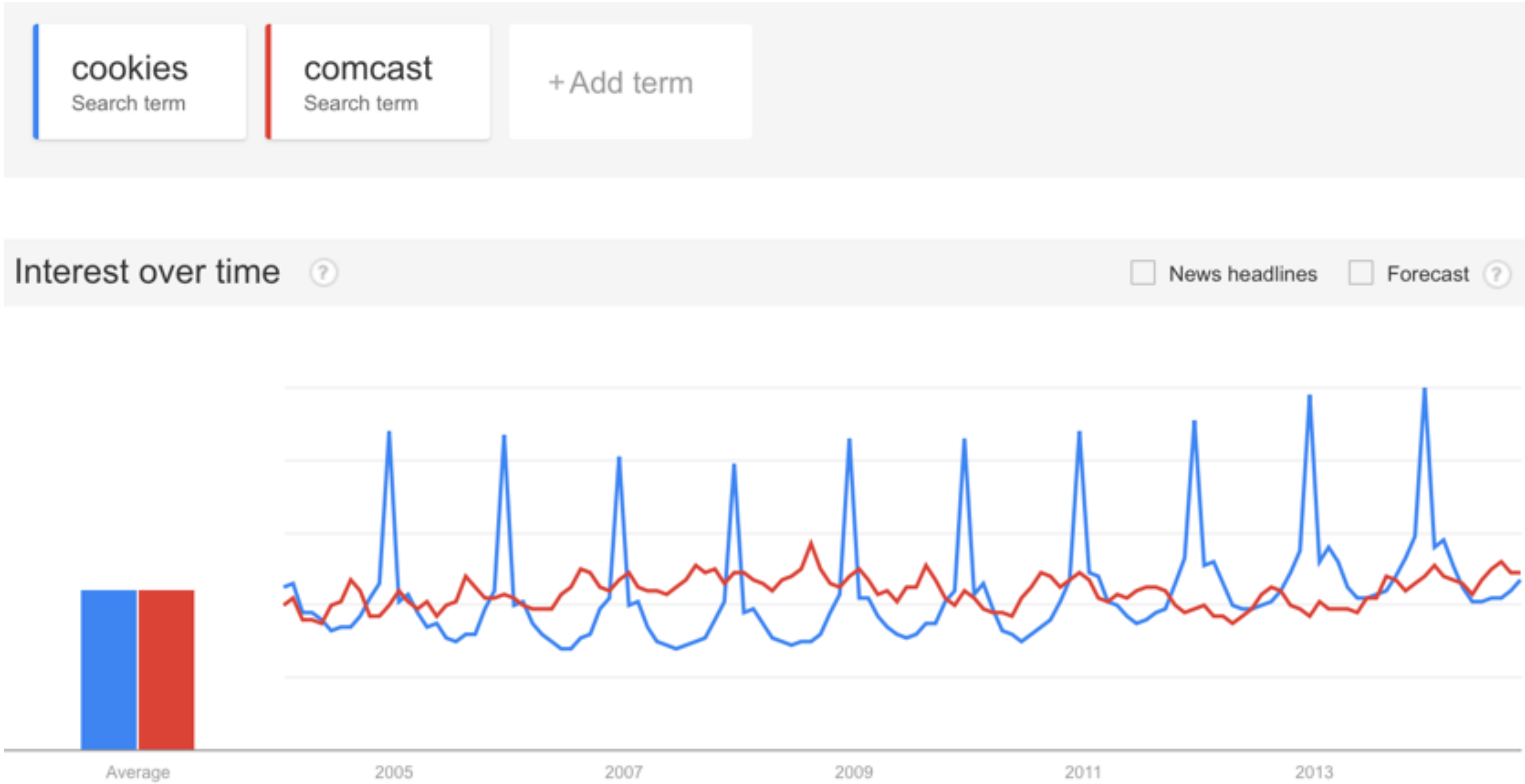
Trend Equation

$$\ell_t = \alpha(y_t \boxed{- s_{t-m}}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$$

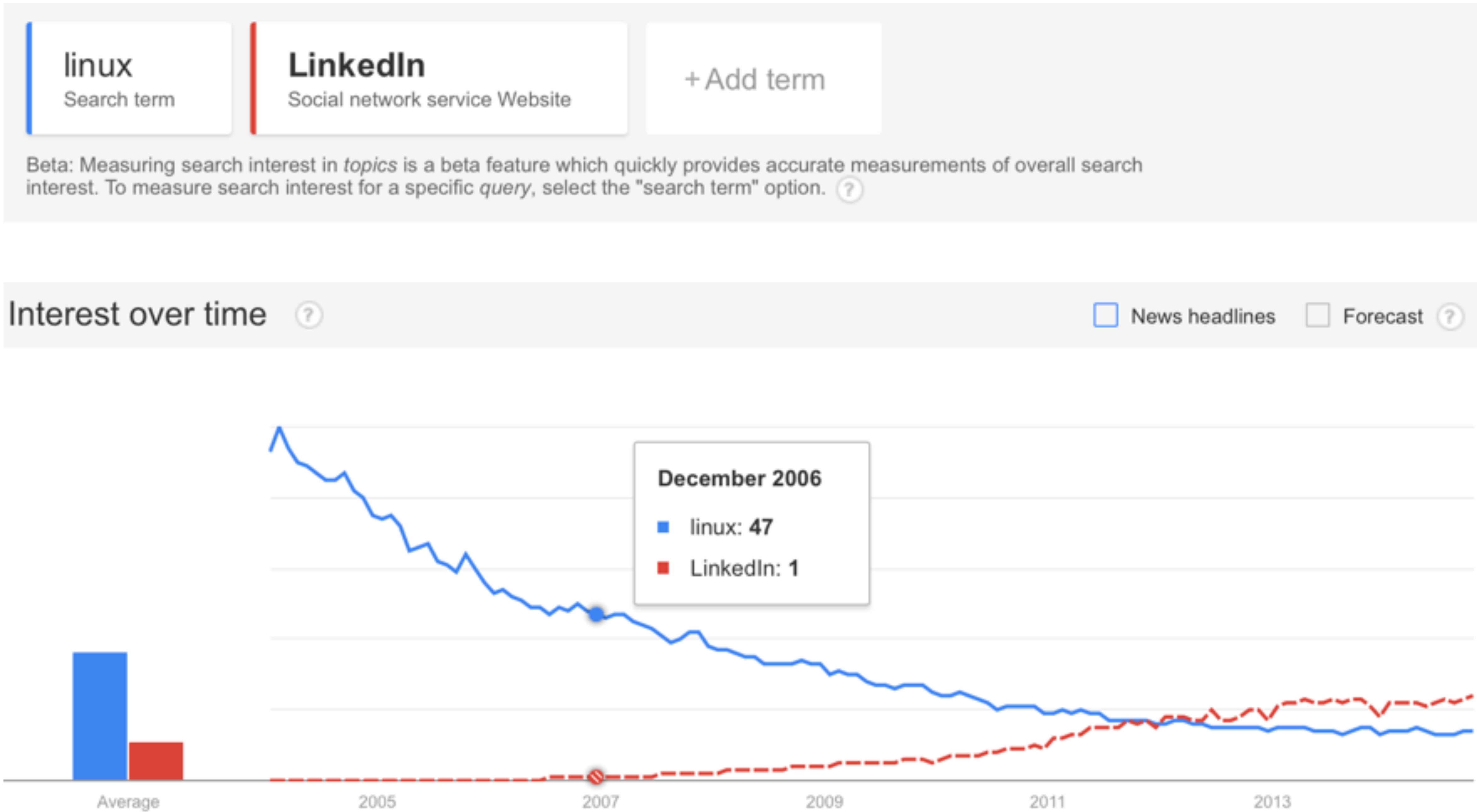
Level Equation

$$\hat{y}_{t+1|t} = \ell_t + hb_t \boxed{+ s_{t-m+h_m^+}}$$

Which smoothing model to use ?



Which smoothing model to use ?



Summary

- Forecasting of time-series and their applications in IR applications
- Components of a Time series - Level, Trend and Seasonality
- Exponential Smoothing Methods vs Averaging Methods
- Level oriented Methods - Simple Exponential Smoothing
- Adding Trend - Double Exponential Smoothing
- Adding Seasonality - Triple Exponential Smoothing

References

- **Forecasting: principles and practice.** (Chapter 7)
  - https://www.otexts.org/fpp/
- **Introduction to Time Series and Forecasting.**
  - http://www.masys.url.tw/Download/2002-Brockwell-Introduction%20Time%20Series%20and%20Forecasting.pdf

Projects

http://www.l3s.de/~anand/tir14/projects.html

- Each project has a mentor who you should co-ordinate with
- Mentor provides you a concrete task definition and data
- Each mentor tracks your progress and guides you
- Drop an email to the mentor to fix an appointment
- Bitbucket vs SVN..Vote next week

Projects

http://www.l3s.de/~anand/tir14/projects.html

- **Temporal and Phrase-based Indexing** - Avishek
- **Temporal Retrieval Models** - Jaspreet
- **Temporal Query Enrichments** - Avishek
- **Crawling for Temporal Collections** - Gerhard
- **Temporal Extraction** - Helge