# Ranking-I

**Probabilistic Interpretation and Language Models**

# Ranking in IR

- Ranking documents important for information overload, quickly finding documents which are **“relevant”** for the query
- Interpretations and Modelling of relevance
    - Geometric Interpretation — Vector Space Similarity, Okapi BM25
    - Probabilistic interpretation — **Topic for today**

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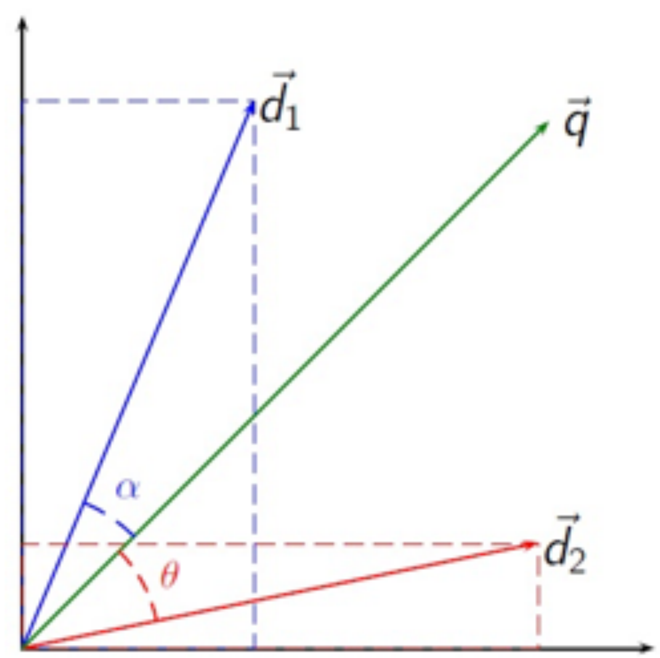
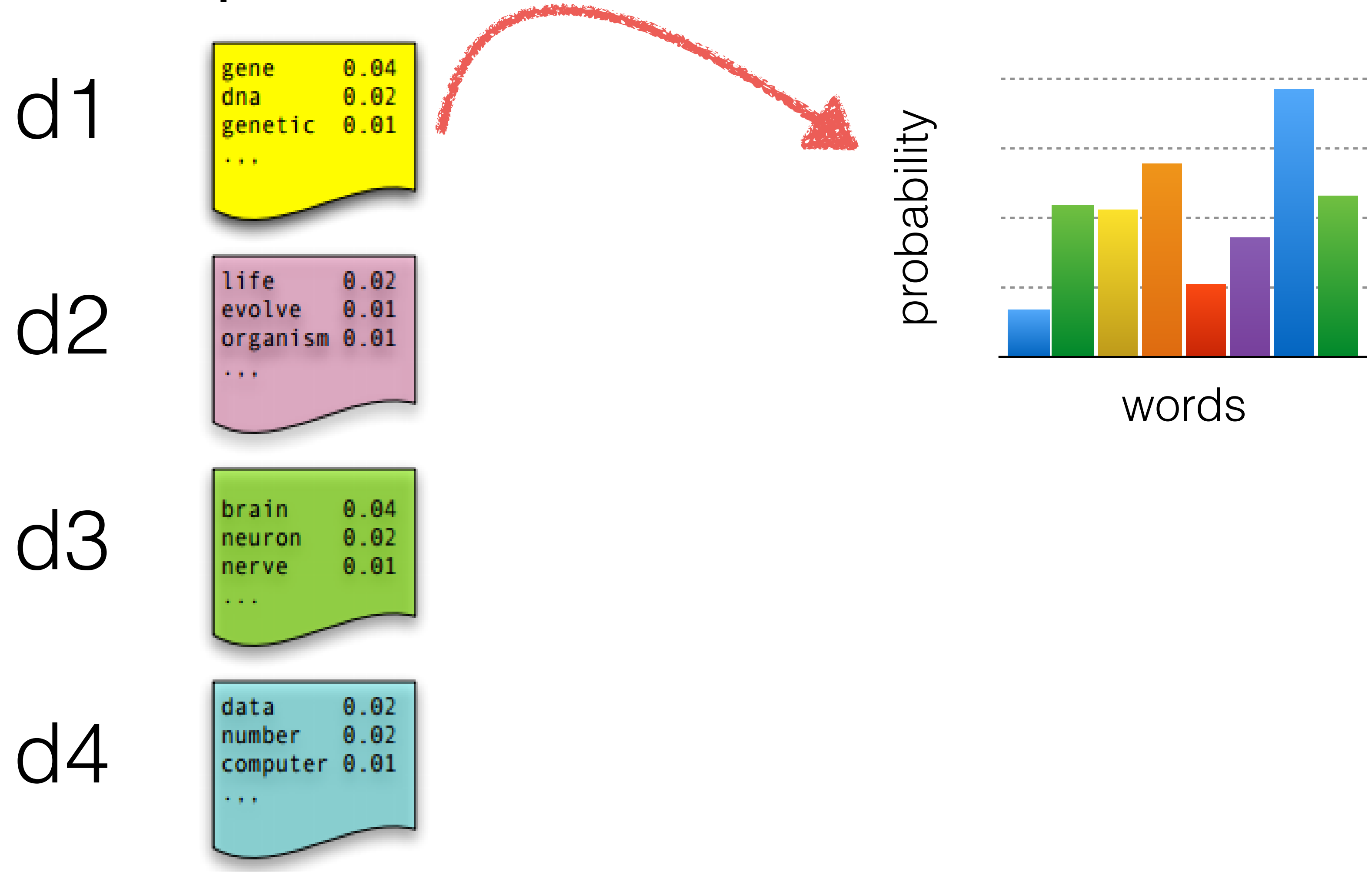



$$sim(d,q) = \frac{\sum_{i\in V} w_d(i).w_q(i)}{\sqrt{\sum_{i\in V} w_d(q_i)^2}\sqrt{\sum_{i\in V} w_q(i)^2}}$$

# Probabilistic Interpretation

- Each document is a probability distribution over words
- Each document is just a collection of words or a “bag of words”. Thus, the **order** of the words and the grammatical role of the words (subject, object, verbs, ...) are not considered in the model.
- Generative process



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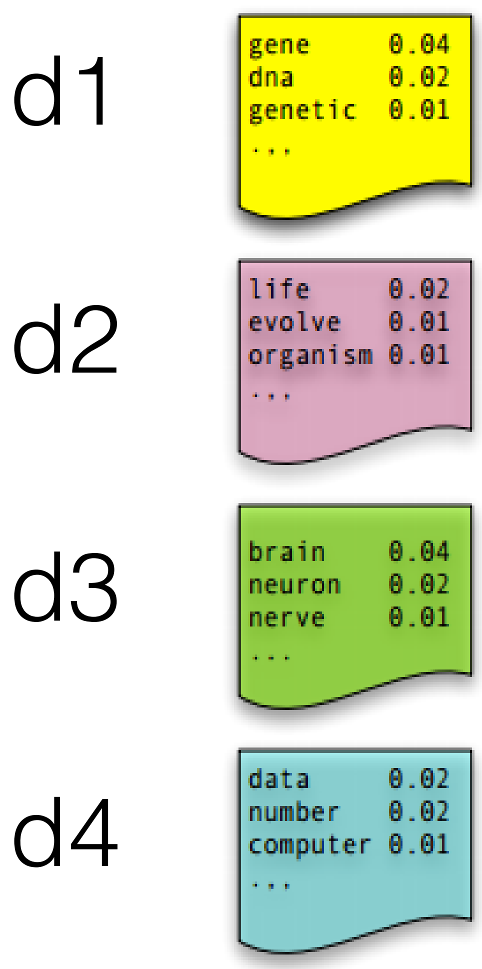
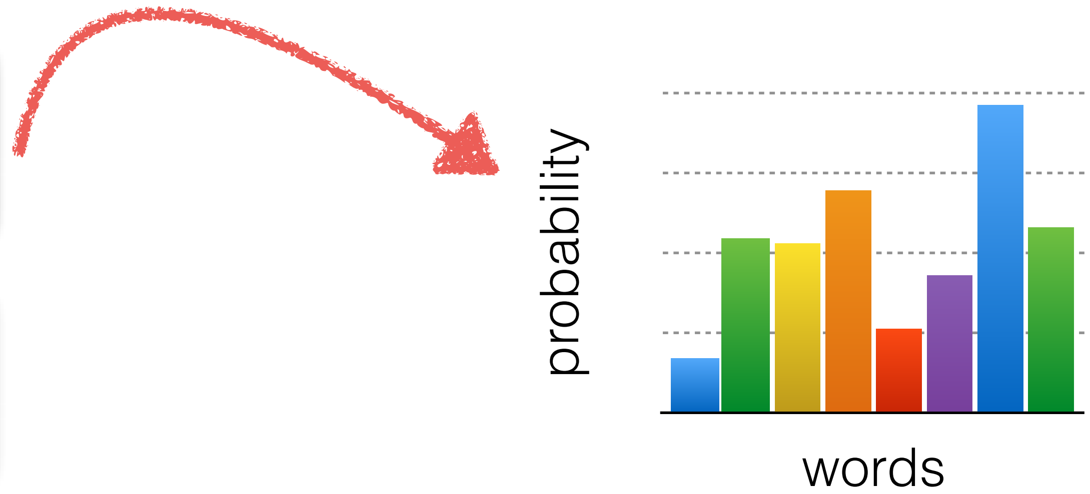


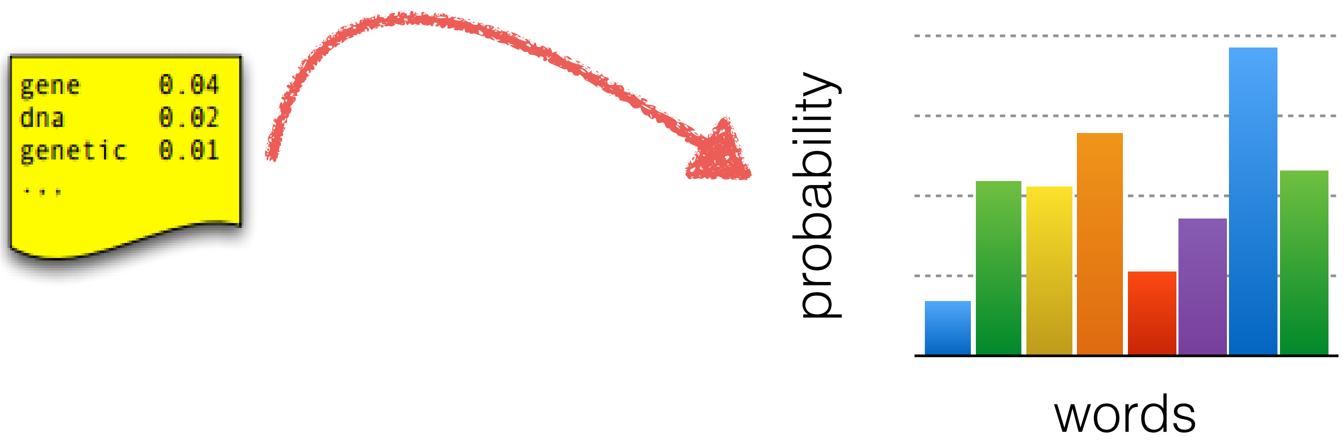




- How do we estimate the document distribution ?
- How do we rank documents given a query q ?
- How do we model topics ?

# Probabilistic Interpretation

- What is the probability of the word "dna" is generated from document d1 ?

d1



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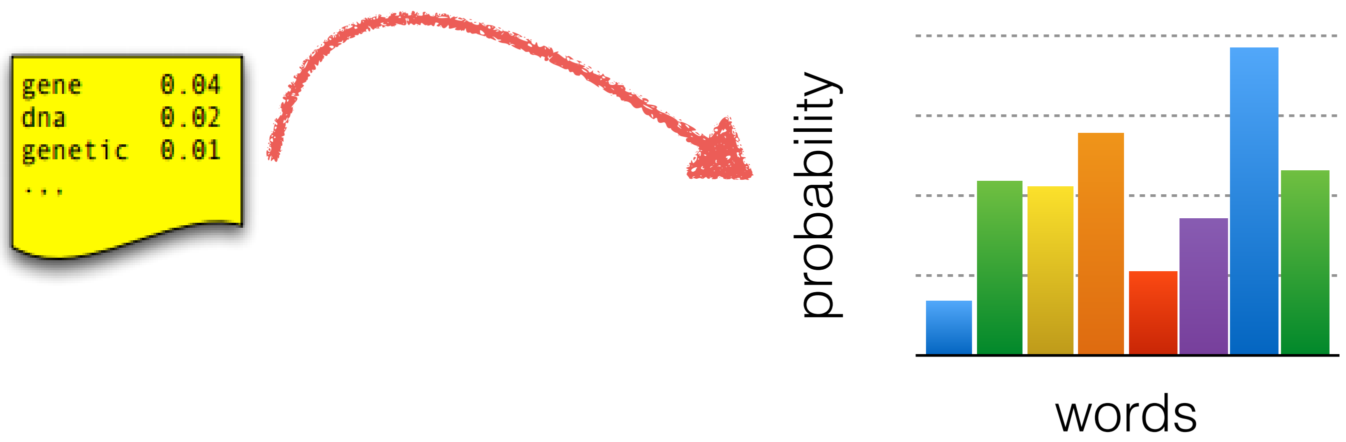


p (w | d)

- What is the probability of the words "dna" and "gene" being generated from document d1 ? (assume independence between words)

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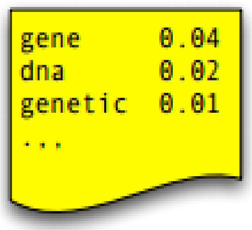


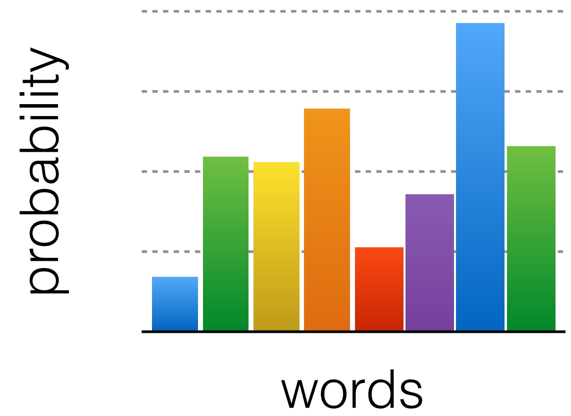


$p\ (w \mid d)$

- What is the probability of the words "dna" and "gene" being generated from document d1 ? (assume independence between words)

$$p\ (\text{“gene”}, \text{“dna”} \mid d) = p\ (\text{“gene”} \mid d).\ p\ (\text{“dna”} \mid d)$$

A is **conditionally** independent to C given B $\longrightarrow$ $P(A \mid B,C) = P(A \mid B)$

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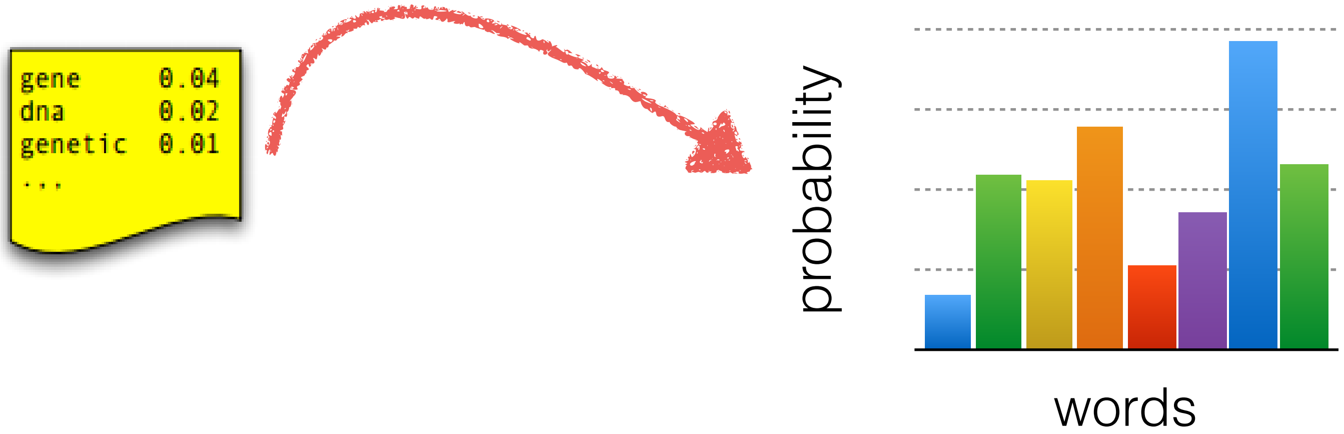


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A is **conditionally** independent to C given B $\longrightarrow$ $P(A \mid B,C) = P(A \mid B)$

- For a query q = w1,w2 the probability that it came from d is:

$$p\ (w1,w2 \mid d) = p\ (w1 \mid d).\ p\ (w2 \mid d)$$

# Parameter Estimation

Given **observed data** D

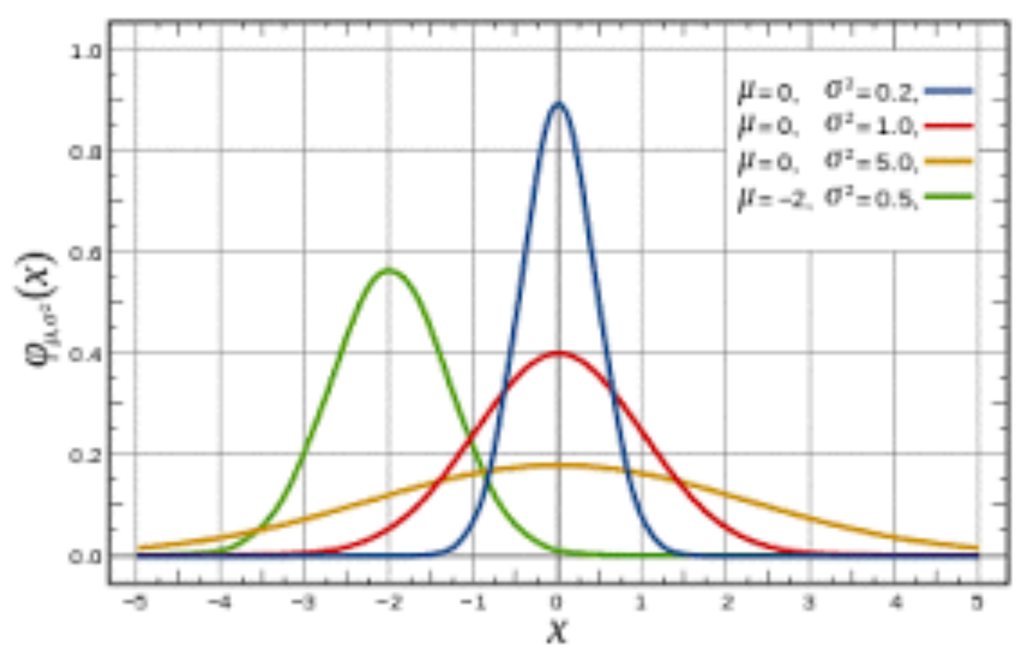
**Generative model:** Assume that you have a probability density func. p(x) with parameters $\theta$ which generates it

parameter estimation problem

What are the value of the parameters which best explain my data ?

Say you feel that D is generated from a Normal distribution

$$\theta = N(\mu, \sigma)$$



# Maximum Likelihood Estimator (MLE)

Say you feel that D is generated from a Normal distribution

$$\theta = N(\mu, \sigma)$$

The **likelihood** of observing this independent and identically distributed sample is

$$\mathcal{L}(\theta \mid x = (x_1, x_2, \ldots, x_n)) = p(x \mid \theta)$$

parameters to be estimated

observed data

MLE chooses $\hat{\theta}$ maximizes this likelihood of generating the observed data

# Maximum Likelihood Estimator (MLE)

- Desired probability distribution is the one that makes the observed data "most likely," which means that one must seek the value of the parameter vector $\hat{\theta}$ that maximizes the likelihood function $\mathcal{L}(\hat{\theta}|x)$

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- For computational convenience, the MLE estimate is obtained by maximizing the log-likelihood function $ln(\, \mathcal{L}(\hat{\theta}|x)\, )$

$$ln(\, \mathcal{L}(\hat{\theta}|x)\, ) = \sum_{x_i} ln \; p(x_i \,|\, \theta)$$

- If log-likelihood is differentiable find $\hat{\theta}$ partial derivatives

# Maximum Likelihood Estimator (MLE) - Gaussian

$$\mathcal{L}(\theta \mid x = (x_1, x_2, \ldots, x_n)) = \prod_{x_i} p(x_i \mid \theta) \longrightarrow ln(\,\mathcal{L}(\hat{\theta}|x)\,) = \sum_{x_i} ln\ p(x_i\,|\,\theta)$$

likelihood

log likelihood

- Find maxima by using partial derivatives i.e. $\dfrac{\partial\, ln\, \mathcal{L}(\theta \mid x)}{\partial\, \theta_i} = 0$

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- What are the estimated parameters of the gaussian ?

# Multinomial Distribution for Text

- Lets try to find a distribution for text, say a document
- You are given a document D with
  - tf ("game") = 5, tf ("of") = 15, tf ("thrones") = 5, tf ("arya") = 3, tf ("stark") = 4, ….
  - E = {all words in D} = {"game", "thrones", "stark",….}
- Multinomial distribution **best encodes** the generative process of text

term frequency of the word

multinomial distribution

$$\binom{n}{tf_1, tf_2, \ldots, tf_{|E|}} p(x_1)^{tf_1} \ldots . \; p(x_{|E|})^{tf_{|E|}}$$

prob. of the word occurrence

# Language Model for a Document

- What are the parameters to be estimated and how can we estimate them ?

total words in Doc

$$\binom{|D|}{tf_1, tf_2, \ldots, tf_{|V|}} p(x_1)^{tf_1} \ldots . \; p(x_{|V|})^{tf_{|V|}}$$

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- Now we have a Model or P (w|d) for each d

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- The MLE for each of the word prob. p(x) is the most natural estimate

$$p(x_i) \;=\; \frac{tf_1}{|D|}$$

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# Language Models in IR

- Unigram Model typically used in IR
  - $P(w1,w2,w3,w4, w5,..| d) = P(w1|d). P(w2|d)\ldots$
- Build a language model for each document D — $\{ P (w_i | D) \}$
- **Ranking**:
  - Query Likelihood: Given a query Q, find the relevance of D (rank acc to P (D|Q) )
  - KL-Divergence Model:
    - Build language model for the query P (w | Q)
    - Rank acc. to **KL (D || Q)** compares two distributions

# Ranking using Query Likelihood

- Query Likelihood: Given a query Q, find the relevance of D (rank acc to P (D|Q) )

$$P(D|Q) \; = \; \frac{P(Q \mid D).P(D)}{P(Q)}$$

$$P(D|Q) \; \propto \; P(Q \mid D).P(D)$$

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constant for all docs

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Authority of the docs. determined by Page Rank etc.

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$$P(D|Q) = \frac{P(Q \mid D).P(D)}{P(Q)}$$

constant for all docs

$$P(D|Q) \propto P(Q \mid D).P(D)$$

Assuming all documents are equally likely

$$P(D|Q) \propto P(Q \mid D)$$

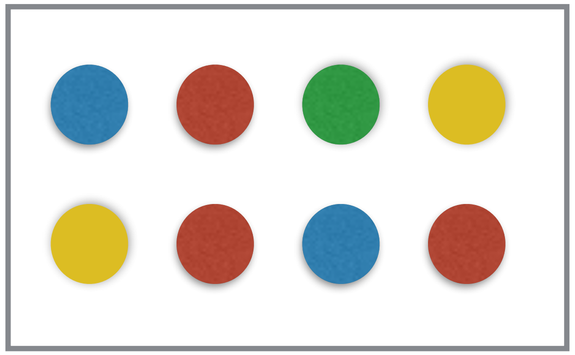
$$P(D|Q) \propto \prod_{w_i \in Q} P(w_i|D)$$

unigram language model

# Language Model for a Document

- Unigram Language Model provides a probabilistic model for representing text in Information retrieval

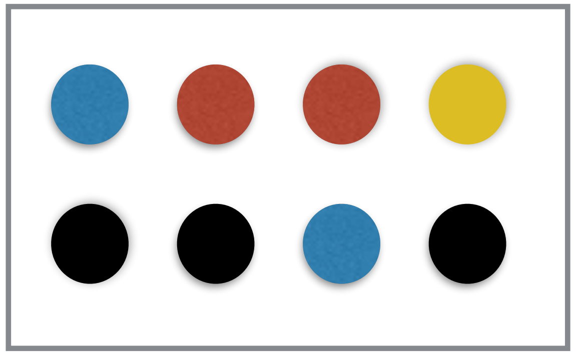
$D_1$



$p(\bullet \mid D_1) = 1/4$ $p(\bullet \mid D_1) = 1/4$

$p(\bullet \mid D_1) = 1/8$ $p(\bullet \mid D_1) = 3/8$

$D_2$



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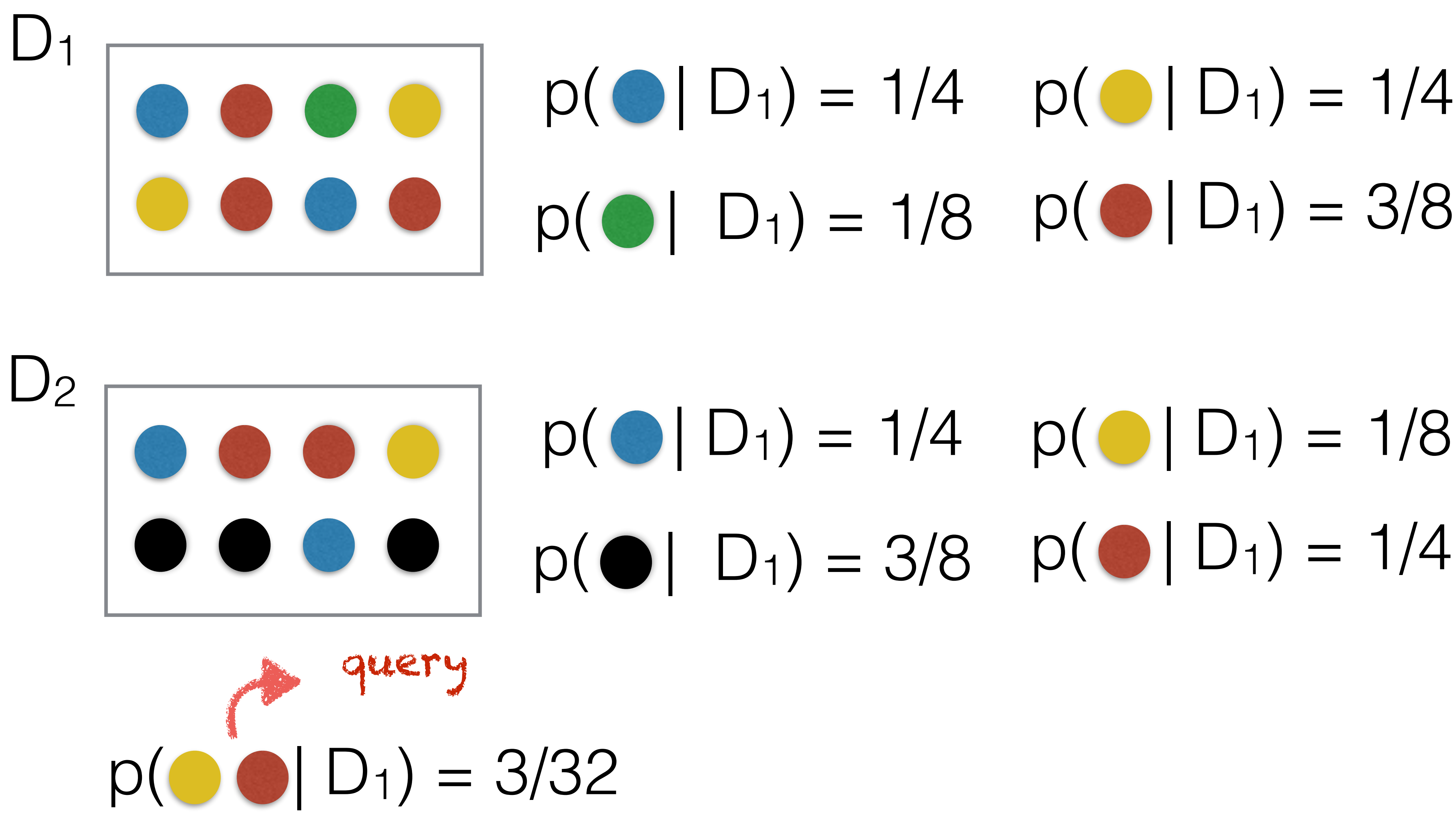

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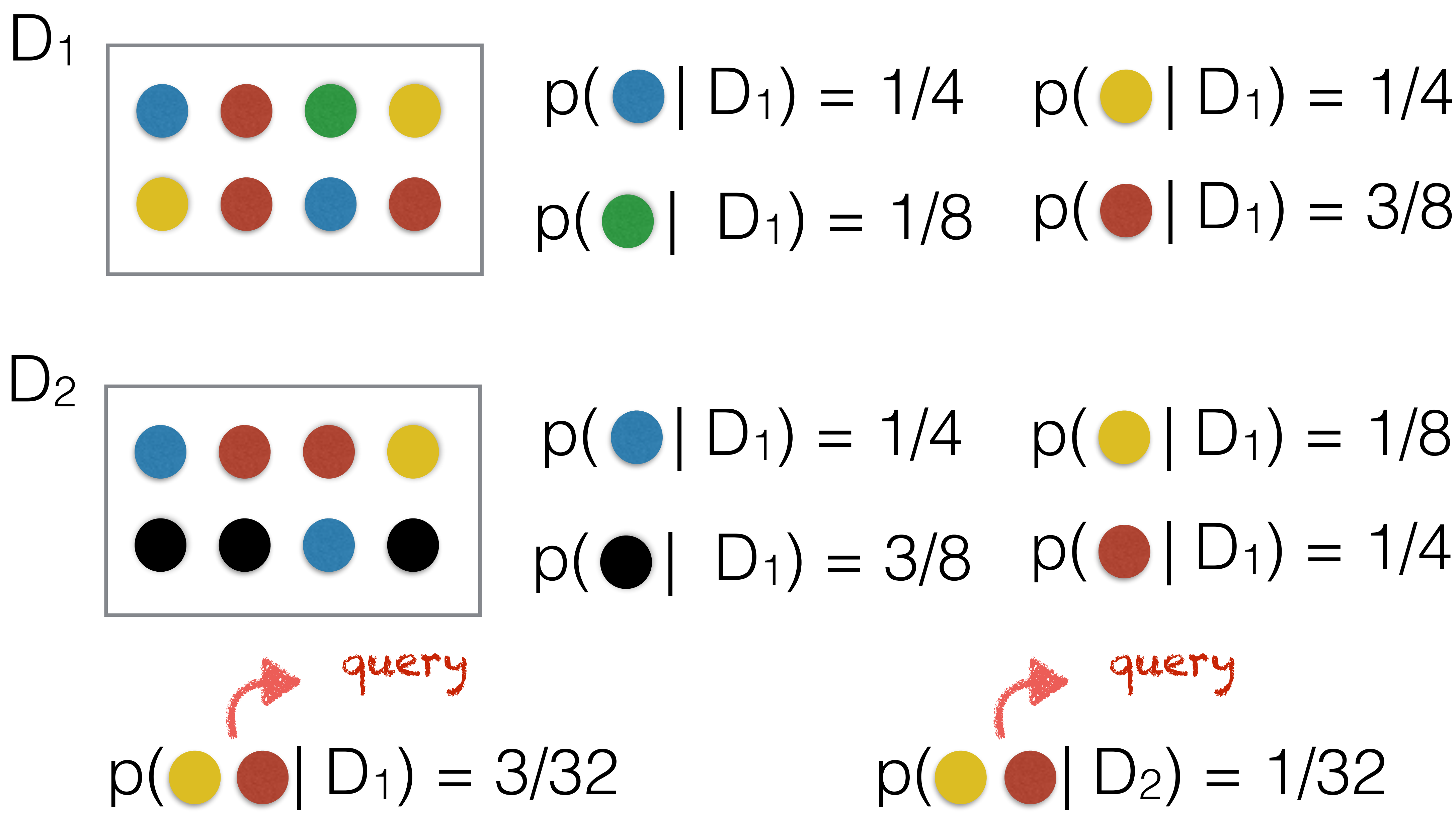

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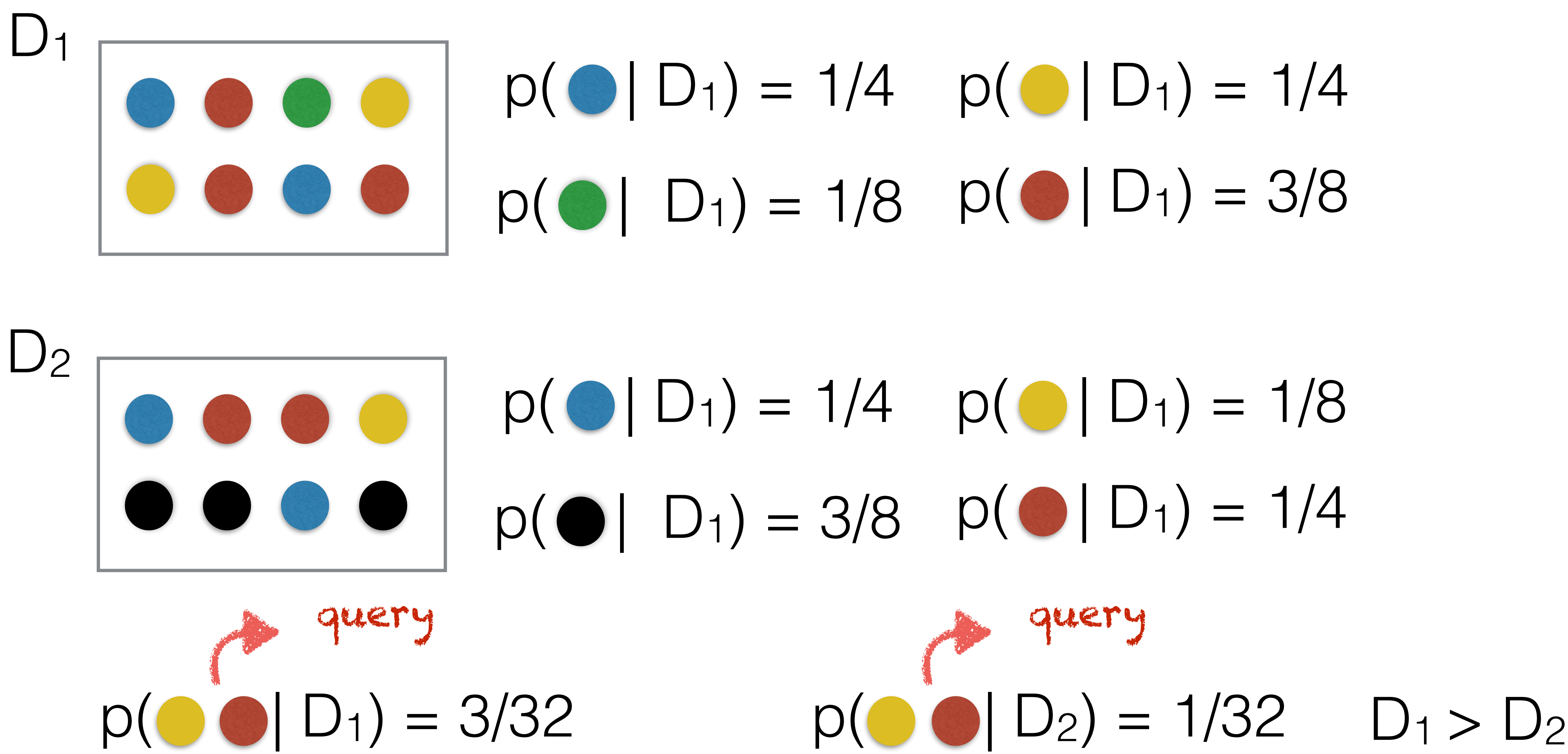

# Example - Language Model + MLE

- Unigram Language Model provides a probabilistic model for representing text

| Model $M_1$ | | Model $M_2$ | |
|---|---|---|---|
| the | 0.2 | the | 0.15 |
| a | 0.1 | a | 0.12 |
| frog | 0.01 | frog | 0.0002 |
| toad | 0.01 | toad | 0.0001 |
| said | 0.03 | said | 0.03 |
| likes | 0.02 | likes | 0.04 |
| that | 0.04 | that | 0.04 |
| dog | 0.005 | dog | 0.01 |
| cat | 0.003 | cat | 0.015 |
| monkey | 0.001 | monkey | 0.002 |
| ... | ... | ... | ... |

| s | frog | said | that | toad | likes | that | dog |
|---|---|---|---|---|---|---|---|
| $M_1$ | 0.01 | 0.03 | 0.04 | 0.01 | 0.02 | 0.04 | 0.005 |
| $M_2$ | 0.0002 | 0.03 | 0.04 | 0.0001 | 0.04 | 0.04 | 0.01 |

$P(s|M_1) = 0.000000000000048$
$P(s|M_2) = 0.000000000000000384$

- The MLE for each of the word prob. p(x) is the most natural estimate

$$p(x_i) = \frac{tf_1}{|D|}$$

# Zero Probability Problem

- What if some of the queried terms are absent in the document ?
- MLE based estimation results in a zero probability for query generation

| Model $M_1$ | | Model $M_2$ | |
|---|---|---|---|
| the | 0.2 | the | 0.15 |
| a | 0.1 | a | 0.12 |
| frog | 0.01 | frog | 0.0002 |
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| dog | 0.005 | dog | 0.01 |
| cat | 0.003 | cat | 0.015 |
| monkey | 0.001 | monkey | 0.002 |
| ... | ... | ... | ... |

- P("frog", "ape" | $M_1$) = 0.01x **0**
- P("frog", "ape" | $M_2$) = 0.0002x **0**

- Need to smooth the probability estimates for terms to avoid zero probabilities
- Take the prob. mass from each term and redistribute among missing terms

# Smoothing Methods

- **Jelinek-Mercer Smoothing :** Linear combination of document and corpus statistics to estimate term probabilities

$$P(Q|D) = \prod_{w_i \in Q} \overbrace{\lambda . P(w_i|D)}^{\text{doc. contrib.}} + \overbrace{(1-\lambda) . P(w_i|C)}^{\text{corpus contrib.}}$$

- Collection frequency: fraction of occurence of term in the entire collection
- Document frequency: fraction of document occurence of term in the entire collection

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collection freq. or
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param. regulates contribution

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# Smoothing Methods

- Smoothing with **Dirichlet Prior:**

term freq
of word in
Doc

$$P(Q|D) = \prod_{w_i \in Q} \frac{tf(w_i;D) + \mu . P(w_i|C)}{|D| + \mu}$$

- Takes the corpus distribution as a prior to estimating the prob. for terms
- works well for short queries

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# Topic Models

## Vocabulary Mismatch Problem

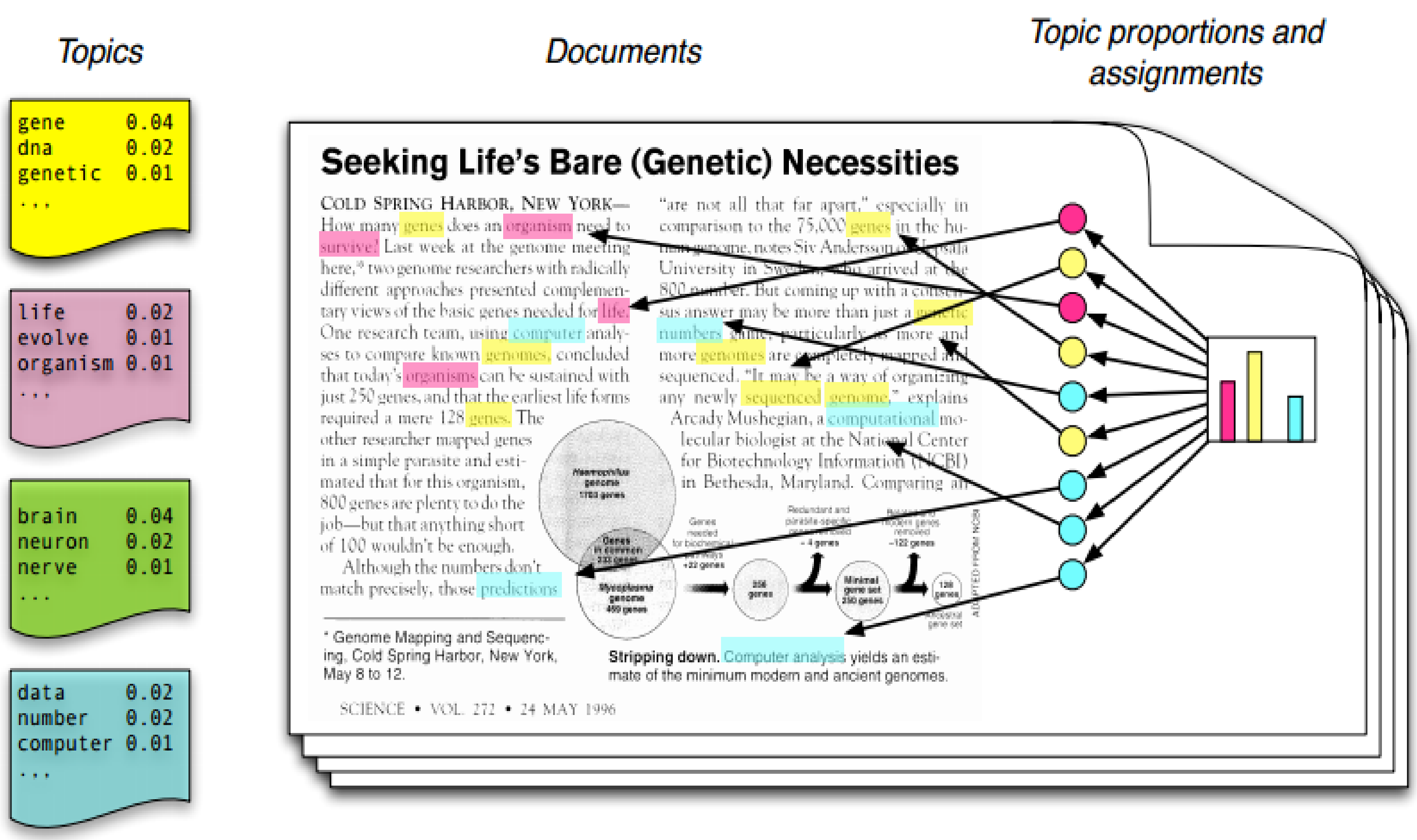
- One concept can be represented by several different words!
- Two documents might not contain similar terms (for instance due to writing styles) but refer to a single concept.
- Queries can contain words not present in a document and still be very relevant to that document!

## Topic model (Probabilistic Latent Semantic Indexing — PLSI)

- Given a set documents D
- A set of topics, classes, concepts {z1, z2,….,zk}
- A set of words {w1, w2,…}
- How do we find topics (word distributions) and documents (topic distribution) ?

# Generative Process

The generative process:

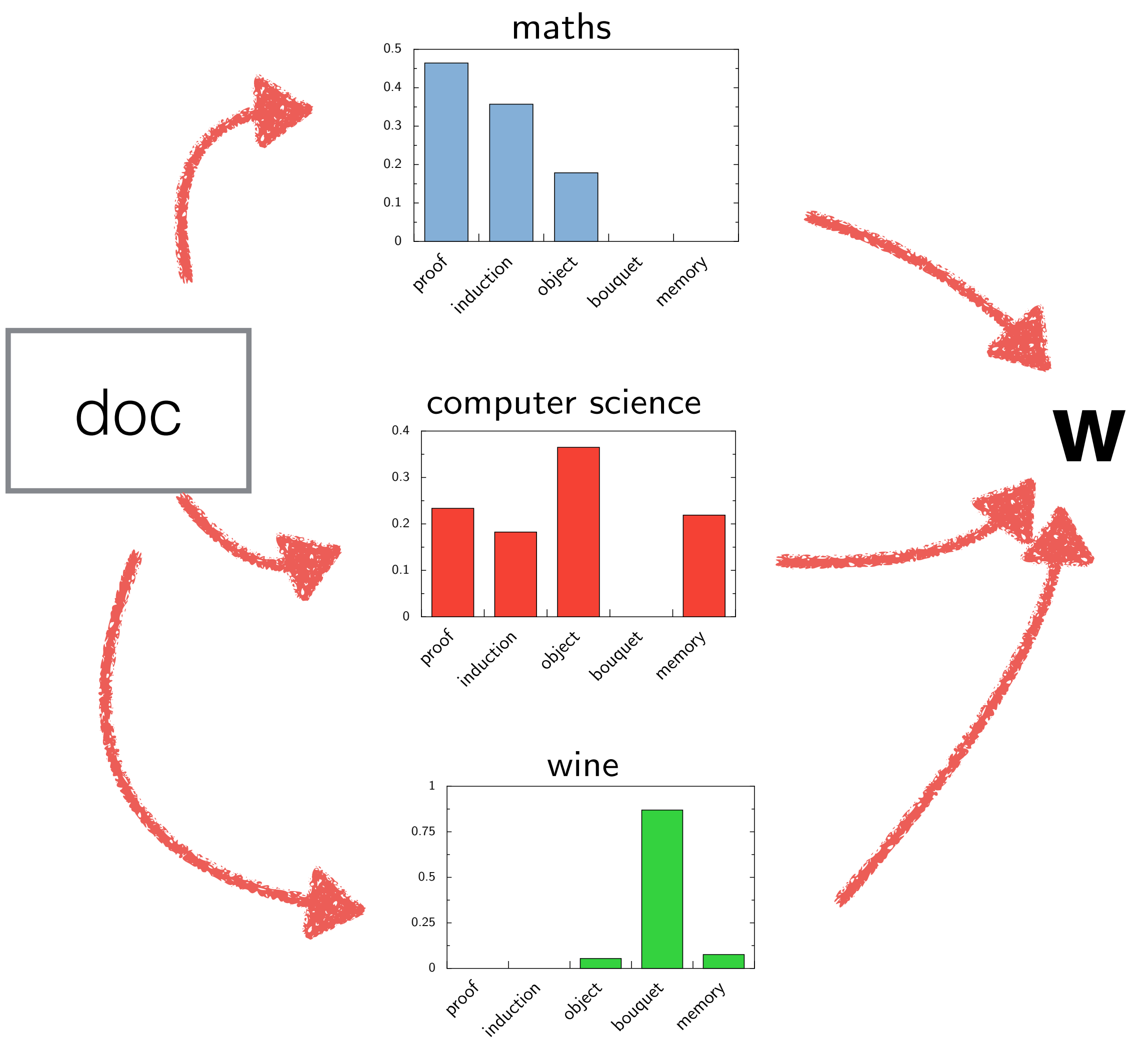



- Each **document** is a mixture of corpus-wide topics
- Each **concept** is distribution over words
- Each **word** is drawn from one of these topics
- We only observe the words within the documents and the other structures are **hidden variables**.

# PLSI representation

What is the probability of seeing a word w **and** a document d ?

$$p(d,w) = p\,(w|d).p(d)$$



# PLSI representation

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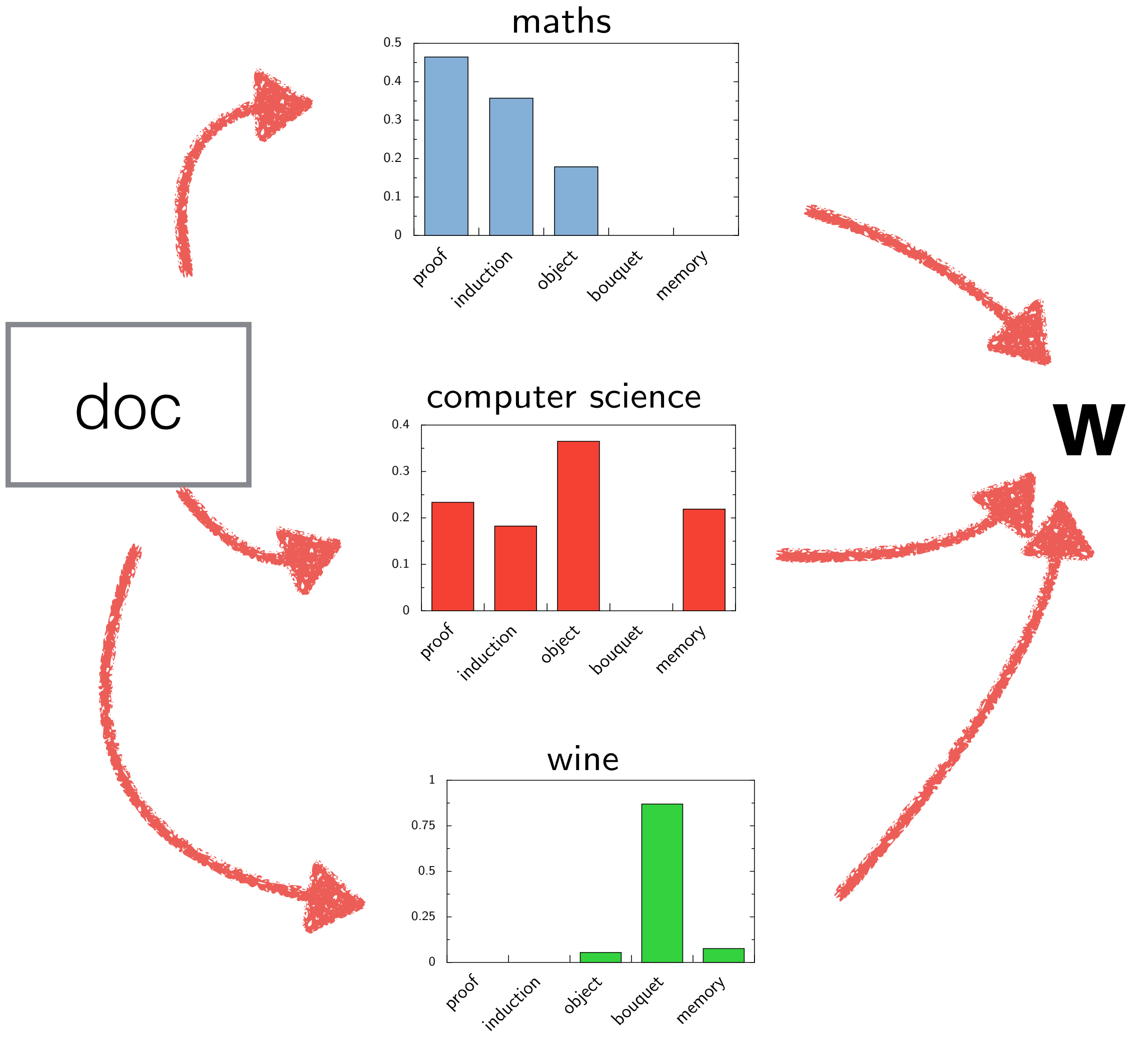
$$p(d,w) = p\,(w|d).p(d)$$

- Select a document with probability $P(d)$
- Pick a latent class $z$ with probability $P(z|d;\theta)$
- Generate a word $w$ with probability $P(w|z;\pi)$



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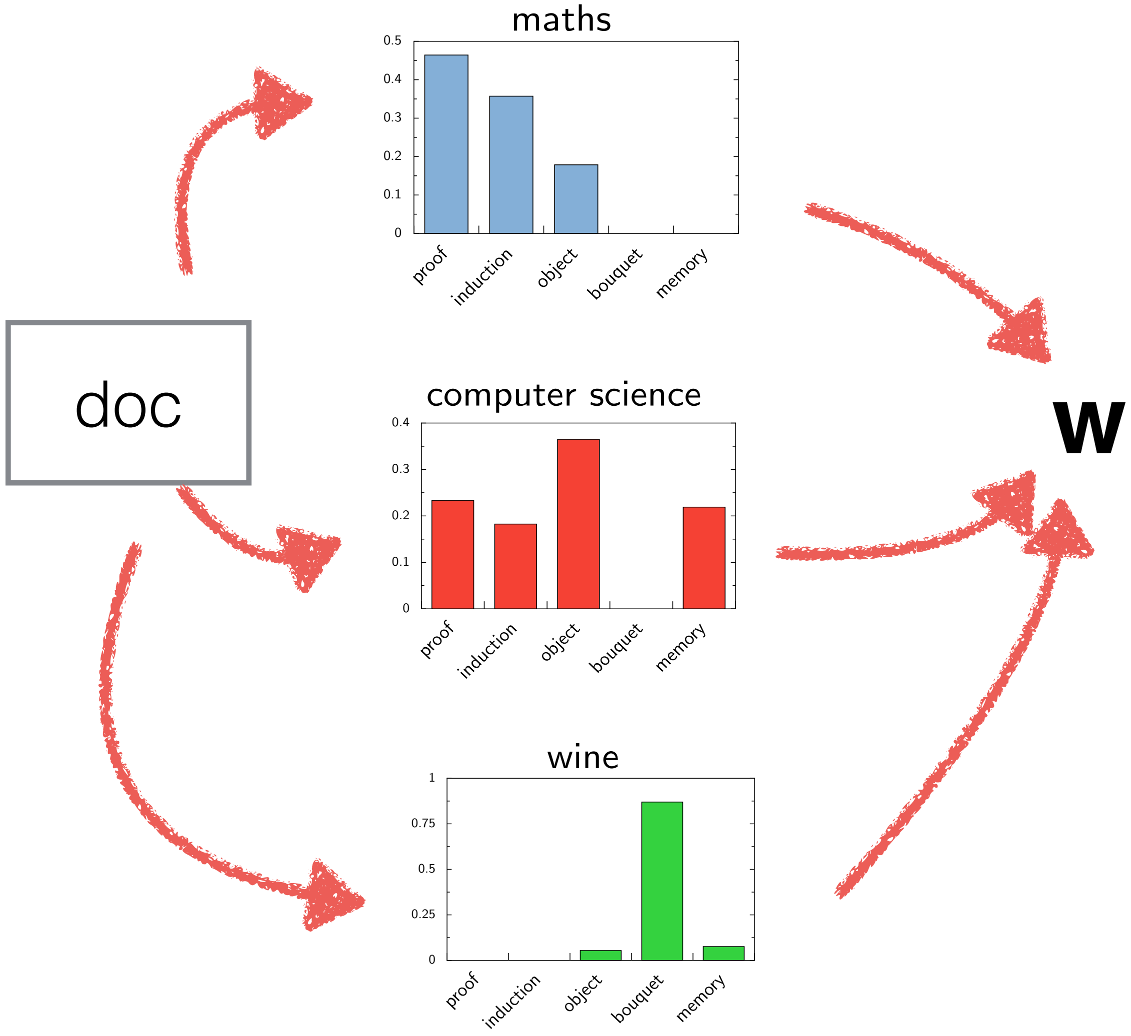
- Select a document with probability $P(d)$
- Pick a latent class $z$ with probability $P(z|d;\theta)$
- Generate a word $w$ with probability $P(w|z;\pi)$

$$\hat{P}_{LSA}(w|d) = \sum_{z \in Z} P(w|z;\theta)P(z|d;\pi)$$

$$\hat{P}_{LSA}(d,w) = P(d)\sum P(w|z)P(z|d) = \sum P(d|z)P(z)P(w|z)$$



# MLE Formulation

Find all the parameters such that the probability of observing the corpus is maximized.

Likelihood function to be maximized: $L = \prod_{i=1}^{N} \prod_{j=1}^{M} P(d_i, w_j)^{n(d_i, w_j)}$

$$\log L = \sum_{i=1}^{N} \sum_{j=1}^{M} n(d_i, w_j) \log P(d_i, w_j) = \sum_{i=1}^{N} \sum_{j=1}^{M} n(d_i, w_j) \log \sum_{k=1}^{K} P(d_i) P(z_k|d_i) P(w_j|z_k)$$

$$= \sum_{i=1}^{N} n(d_i) \left[ \log P(d_i) + \sum_{j=1}^{M} \frac{n(d_i, w_j)}{n(d_i)} \log \left[ \underbrace{\sum_{k=1}^{K} P(z_k|d_i) P(w_j|z_k)}_{\text{Only this is the difficult part}} \right] \right]$$

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Document

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$$= \sum_{i=1}^{N} n(d_i) \left[ \log P(d_i) + \sum_{j=1}^{M} \frac{n(d_i, w_j)}{n(d_i)} \log \underline{\left[ \sum_{k=1}^{K} P(z_k | d_i) P(w_j | z_k) \right]} \right]$$

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corpus/
collection

Document

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Only this is the
difficult part

# Expectation Maximisation

$$\log L = \sum_{i=1}^{N} n(d_i) \left[ \log P(d_i) + \sum_{j=1}^{M} \frac{n(d_i, w_j)}{n(d_i)} \log \left[ \underbrace{\sum_{k=1}^{K} P(z_k|d_i) P(w_j|z_k)}_{\text{marginal prob. w/ sum}} \right] \right]$$

Estimated directly from data:

- $P(d_i)$: uniform or related to popularity of the document $d_i$
- $n(d_i)$: number of words in $d_i$
- $n(d_i, w_j)$: count of word $w_j$ in $d_i$

- Use Expectation Maximisation (EM Algorithm) procedure when there are **latent variables**
- **Issue:** we have a marginal probability which is difficult to maximize analytically (mainly because of the sum)

# Expectation Maximisation

EM for our problem (repeat until convergence):

1. **E-step:** Calculate posterior probabilities for latent variables given the observations and current estimates
2. **M-step:** Update parameters using the posterior probabilities in E-step to increase $\log L$

$$\log L = \sum_{i=1}^{N} n(d_i) \left[ \log P(d_i) + \sum_{j=1}^{M} \frac{n(d_i, w_j)}{n(d_i)} \log \left[ \sum_{k=1}^{K} P(z_k|d_i) P(w_j|z_k) \right] \right]$$

1. **E-step:** Calculating posterior probabilities using the current estimates

$$P(z_k|d_i, w_j) = \frac{P(w_j, z_k|d_i)}{P(w_j|d_i)} = \frac{P(w_j|z_k, d_i) P(z_k|d_i)}{\sum_{i=1}^{K} P(w_j|z_i, d_i) P(z_i|d_i)}$$

2. **M-step:** Maximizing $\log L$ having the posterior probability

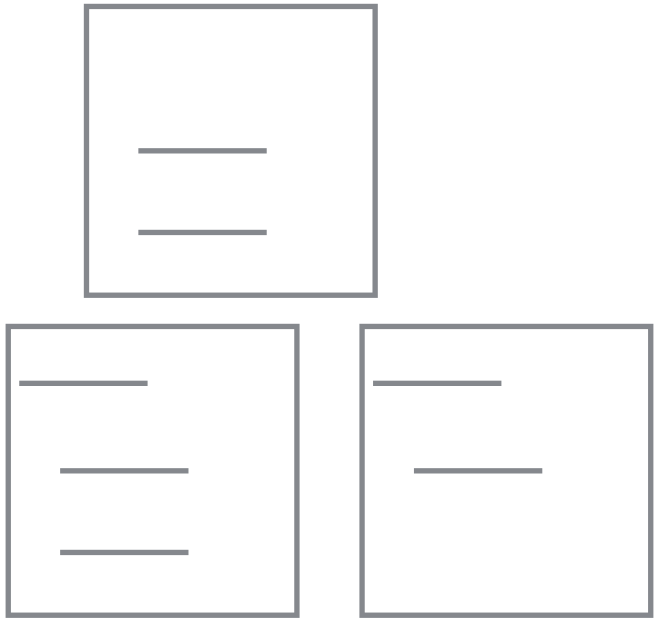
$$P(w_j|z_k) = \frac{\sum_{i=1}^{N} n(d_i, w_j) P(z_k|d_i, w_j)}{\sum_{m=1}^{M} \sum_{i=1}^{N} n(d_i, w_m) P(z_k|d_i, w_m)} \qquad P(z_k|d_i) = \frac{\sum_{j=1}^{M} n(d_i, w_j) P(z_k|d_i, w_j)}{n(d_i)}$$

# References and Further Readings


- Information retrieval: (http://www.ir.uwaterloo.ca/book/)
    - Stefan Büttcher, Google Inc. , Charles L. A. Clarke, Univ. of Waterloo, Gordon V. Cormack, Univ. of Waterloo
- Foundations of Information retrieval: Manning, Schutze, Raghavan
    - http://nlp.stanford.edu/IR-book/pdf/12lmodel.pdf
- Hofmann, Thomas. "Probabilistic latent semantic indexing." Proceedings of the 22nd annual international ACM SIGIR conference on Research and development in information retrieval. ACM, 1999.

# Temporal Ranking

- Queries with temporal expressions
  - fifa world cup in 1998
- Documents also mention temporal expressions
  - “Clinton was the president in 1990s”
  - “Interstellar was slated for release in july 2014”



$$P(Q|D) \;=\; P(Q_{text}|D_{text}).P(Q_{time}|D_{time})$$

- Assume time mentions are independent of text
- Assume temporal expressions are independent of each other

How do we compute $P(Q_{time}|D_{time})$