i) Steady state probability distribution of the number of calling units:

$$P_n = \rho^n P_0 = (0.75)^n \times 0.433$$

ii) Expected number of calling units in the system:

We have to L_s.

$$L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\,\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{0.75}{1-0.75} - \frac{3(0.75)^3}{1-(0.75)^3} = 3 - 2.189 = 0.81$$

Problem 4.16:

A one-person barber shop has 6 chairs to accommodate people waiting for a haircut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the average rate of 3 per hour and spend an average of 15 min in the barber's chair.

a) What is the probability that a customer can get directly into the barber's chair upon arrival?
b) What is the expected number of customers waiting for a haircut?
c) How much time can a customer expect to spend in the barber shop?
d) What fraction of potential customers are turned away?

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate:

$$\lambda = 3 \text{ per hr} = \frac{3}{60} \text{ per min} = \frac{1}{20} \text{ per min}$$

Service Rate:

$$\mu = 15 \text{ min} = \frac{1}{15} \text{ per min}$$

$$k = 6+1 = 7 \quad \text{(6 waiting and 1 getting service)}$$

Server Utilization:

$$\rho = \frac{\lambda}{\mu} = \frac{1/20}{1/15} = \frac{15}{20} = \frac{3}{4} = 0.75$$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-0.75}{1-(0.75)^8} = 0.278$$

$$\lambda' = \mu(1-P_0) = \frac{1}{15}(1-0.278) = 0.048$$

a) **Probability that a customer can get directly into the barber's chair:**

$$P_0 = 0.278$$

b) **Expected number of customers waiting for haircut:**

We have to find L_q.

$$L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{0.75}{1-0.75} - \frac{8(0.75)^8}{1-(0.75)^8} = 3-0.89 = 2.11$$

$$\therefore L_q = L_s - \frac{\lambda'}{\mu} = 2.11 - \frac{0.048}{1/15} = 2.11 - 0.72 = 1.39$$

c) Expected waiting time of a customer in the shop:

$$W_s = \frac{L_s}{\lambda'} = \frac{2.11}{0.048} = 43.95 \simeq 44 \text{ mins}$$

d) Fraction of potential customers turned away:

$$P[\text{a customer turnes away}] = \rho^k P_0$$

$$= (0.75)^7 \times 0.278 = 0.037$$

Problem 4.17:

A car park contains 5 cars. The arrival of cars is Poisson at a mean rate 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 2 min. How many cars are in the car park on an average and what is the probability of a newly arriving customer finding the car park full and leaving to park his car elsewhere.

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate:

$$\lambda = 10 \text{ per hr} = \frac{10}{60} \text{ per min} = \frac{1}{6} \text{ per min}$$

Service Rate:

$$\mu = 2 \text{ min} = \frac{1}{2} \text{ per min}$$

$$k = 5 \quad \text{(System Capacity)}$$

Server Utilization:

$$\rho = \frac{\lambda}{\mu} = \frac{1/6}{1/2} = \frac{2}{6} = \frac{1}{3} = 0.33$$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-0.33}{1-(0.33)^6} = 0.668$$

$$\lambda' = \mu(1-P_0) = \frac{1}{2}(1-0.668) = 0.166$$

i) Average number of cars in the park:

$$L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{0.33}{1-0.33} - \frac{6(0.33)^6}{1-(0.33)^6} = 0.4925 - 0.0078 = 0.4847$$

ii) A customer turned away:

$$P[\text{a customer turnes away}] = \rho^k P_0$$

$$= (0.33)^5 \times 0.668 = 0.0027$$