$$\text{But } L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{1.5}{1-1.5} - \frac{16(1.5)^{16}}{1-(1.5)^{16}}$$

$$= -3 + 16.02 = 13.02$$

$$\therefore W_s = \frac{L_s}{\lambda'} = \frac{13.02}{0.333} = 39 \text{ mins}$$

Problem 4.13:

Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number of trains in the system, given that the inter arrival time and service time are following exponential distribution.

(M/J 2012)

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate: $\lambda = 15 \text{ mins} = \dfrac{1}{15}$ per min

Service Rate: $\mu = 33 \text{ mins} = \dfrac{1}{33}$ per min

$k = 5$ (System Capacity)

Server Utilization: $\rho = \dfrac{\lambda}{\mu} = \dfrac{1/5}{1/33} = \dfrac{33}{15} = \dfrac{11}{5} = 2.2$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-2.2}{1-(2.2)^6} = 0.011$$

$$\lambda' = \mu(1-P_0) = \frac{1}{33}(1-0.011) = 0.03$$

i) **Probability that the yard is empty:**

$$P(\text{yard is empty}) = P_0 = 0.011$$

ii) Average number of trains in the system:

$$L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{2.2}{1-2.2} - \frac{6(2.2)^6}{1-(2.2)^6} = -1.833 + 6.053 = 4.22$$

Problem 4.14:

Customers arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space is front of window, including that for the serviced car can accommodate a maximum of three cars. Others cars can wait outside this space. (A/M 2011)

(1) What is the probability that an arriving customer can drive directly to the space in front of the window?

(2) How long is an arriving customer expected to wait before being served?

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate: $\lambda = 10$ per hr $= \dfrac{10}{60}$ per min $= \dfrac{1}{6}$ per min

Service Rate: $\mu = 5 \text{ mins} = \frac{1}{5}$ per min

$k = 3$ (System Capacity)

Server Utilization: $\rho = \frac{\lambda}{\mu} = \frac{1/6}{1/5} = \frac{5}{6} = 0.83$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-0.83}{1-(0.83)^4} = 0.3236$$

$$\lambda' = \mu(1-P_0) = \frac{1}{5}(1-0.3236) = 0.1353$$

(1) Probability that an arriving customer can drive directly to the space in front of the window:

$$P_0 = 0.3236$$

(2) Expected waiting time of a customer in the queue:

We have to find W_q.

$$L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{0.83}{1-0.83} - \frac{4(0.83)^4}{1-(0.83)^4} = 4.8824 - 3.6131 = 1.2693$$

$$L_q = L_s - \frac{\lambda'}{\mu} = 1.2693 - \left(\frac{0.1353}{1/5}\right) = 0.5928$$

$$W_q = \frac{L_q}{\lambda'} = \frac{0.5928}{0.1353} = 4.3814$$

Problem 4.15:

Consider a single server queueing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units in the system is two. Find the steady state probability distribution of the number of calling units in the system and the expected number of calling units in the system. (N/D 2013),(N/D 2017)

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate:

$$\lambda = 3 \text{ per hr} = \frac{3}{60} \text{ per min} = \frac{1}{20} \text{ per min}$$

Service Rate:

$$\mu = 0.25 \text{ hrs} = \frac{1}{0.25} \text{ per hr}$$

$$= \frac{1}{0.25 \times 60} \text{ per min} = \frac{1}{15} \text{ per min}$$

$k = 2$ (System Capacity)

Server Utilization:

$$\rho = \frac{\lambda}{\mu} = \frac{1/20}{1/15} = \frac{15}{20} = \frac{3}{4} = 0.75$$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-0.75}{1-(0.75)^3} = 0.433$$

$$\lambda' = \mu(1-P_0) = \frac{1}{15}(1-0.433) = 0.038$$