Model – III (M/M/1) : (K/FIFO)

Formulae:

1) Server Utilization $\rho = \dfrac{\lambda}{\mu}$

2) $P_0 = \dfrac{1-\rho}{1-\rho^{k+1}}$

3) $\lambda' = \mu(1-P_0)$ (effective arrival rate)

4) $L_s = \dfrac{\rho}{1-\rho} - \dfrac{(k+1)\rho^{k+1}}{1-\rho^{k+1}}$

5) $L_q = L_s - \dfrac{\lambda'}{\mu}$

6) $W_s = \dfrac{L_s}{\lambda'}$

7) $W_q = \dfrac{L_q}{\lambda'}$

8) $P[\text{a customer turned away}] = P_k = \rho^k P_0$

Problem 4.11:

The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4/hr (exponential service time) (N/D 2014)

(i) What percentage of time is the barber idle?
(ii) What fraction of the potential customers are turned away?
(iii) What is the expected number of customers waiting for a hair-cut?
(iv) How much time can a customer expect to spend in the barber shop?

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate:

$$\lambda = 5 \text{ per hr} = \frac{5}{60} \text{ per min} = \frac{1}{12} \text{ per min}$$

Service Rate:

$$\mu = 4 \text{ per hr} = \frac{4}{60} \text{ per min} = \frac{1}{15} \text{ per min}$$

$k = 5$ (System Capacity)

Server Utilization:

$$\rho = \frac{\lambda}{\mu} = \frac{1/12}{1/15} = \frac{15}{12} = \frac{5}{4} = 1.25$$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-1.25}{1-(1.25)^6} = \frac{-0.25}{-2.815} = 0.089$$

$$\lambda' = \mu(1-P_0) = \frac{1}{15}(1-0.089) = 0.061$$

i)

$$P(\text{the barber idle}) = P_0 = 0.089$$

$\therefore$ The percentage value of barber idle $= 0.089 \times 100$

$$= 8.9 \simeq 9\%$$

ii) Fraction of the potential customers turned away:

$$P(\text{a custom turnes away}) = \rho^k P_0$$

$$= (1.25)^5 \times 0.089 = 0.27$$

iii) Expected number of customers waiting for a hair-cut:

We have to find L_q.

$$L_s = \frac{\rho}{1-\rho} - \frac{(k+1)\,\rho^{k+1}}{1-\rho^{k+1}}$$

$$= \frac{1.25}{1-1.25} - \frac{6(1.25)^6}{1-(1.25)^6}$$

$$= -5+8.132$$

$$L_s = 3.132$$

$$\therefore L_q = L_s - \frac{\lambda'}{\mu} = 3.132 - \frac{0.061}{1/15}$$

$$= 3.132-0.915$$

$$L_q = 2.217$$

iv) Customer waiting time in the shop:

$$W_s = \frac{Ls}{\lambda'} = \frac{3.132}{0.061} = 51.34 \text{ mins}$$

Problem 4.12:

Patients arrive at a clinic according to Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour. (1) Find the effective arrival rate at the clinic (2) What is the probability that an arriving patient will not wait? (3) What is the expected waiting time until a patient is discharged from the clinic?

(N/D 2015),(A/M 2018)

Solution:

The given problem is "Model-III (M/M/1) : (k/FIFO)".

Arrival Rate:

$$\lambda = 30 \text{ per hr} = \frac{30}{60} \text{ per min} = \frac{1}{2} \text{ per min}$$

Service Rate:

$$\mu = 20 \text{ per hr} = \frac{20}{60} \text{ per min} = \frac{1}{3} \text{ per min}$$

$$k = 14+1 = 15 \quad \text{(14 waiting and 1 getting service)}$$

Server Utilization:

$$\rho = \frac{\lambda}{\mu} = \frac{1/2}{1/3} = \frac{3}{2} = 1.5$$

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}} = \frac{1-1.5}{1-(1.5)^{16}} = 0.00076 \simeq 0.001$$

$$\lambda' = \mu\left(1-P_0\right) = \frac{1}{3}\left(1-0.001\right) = 0.333$$

(1) Effective Arrival:

$$\lambda' = 0.333 \text{ per min}$$

(2) Probability that an arriving patient will not wait:

$$P\left(\text{a patient will not wait}\right) = P_0 = 0.001$$

(3) Expected waiting time of a customer in the clinic:

We have to find W_s.