Th. Astro. – Atomic radiative processes

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3 Categories:

free-free | bound-free | bound-bound

- **Thompson scattering**: (free-free)

$I_\nu = cE^2 \longrightarrow j_{rad} = cE_{rad}^2 \, dA\,dt$

$\longrightarrow j_{rad} = \frac{e^4}{m_e^2 c^4}(cE^2)$ with $\boxed{\sigma_T = \frac{e^4}{m_e^2 c^4}}$

$\longrightarrow \boxed{j_\nu = n_e \sigma_T I_\nu}$ $\quad \hookrightarrow r_T^2 \propto \sigma_T$

if $h\nu \ll m_e c^2$: $\boxed{j_\nu = n_e \sigma_T B_\nu}$

- **Line radiation**: (bound-bound)

$\boxed{}$ $f_r \quad p^2 \quad e^2$

Bohr radius: $\boxed{r_0 = \frac{h^2}{me^2}}$ from $\frac{p^2}{2m} = \frac{e^2}{r}$ and Heisenberg ($p \cdot r = h$)

for hydrogen: $\boxed{E_n = \frac{E_0}{n^2} = \frac{-e^2}{n^2 r_B} \simeq \frac{-13.6\,eV}{n^2}}$

· Kirchhofs law relates emission and absorption:

$j_\nu = \alpha_\nu B_\nu$ → thermal radiation

↳ spontaneous emission / absorption / stimulated emission

A_{21} (sec^{-1}) $\qquad B_{12}\bar{J}$ (sec^{-1}) $\qquad B_{12}\bar{J}$

$+ n_2 C_{12} \qquad + C_{21} n_2$

↳ $\boxed{n_1 B_{12} \bar{J} = n_2 A_{21} + n_2 B_{22} \bar{J}}$

$\boxed{}$

$$\Rightarrow \boxed{\bar{J} = \frac{A_{21}/B_{21}}{\left(\frac{n_1}{n_2}\right)\left(\frac{B_{12}}{B_{21}}\right) - 1}}$$

↳ Boltzmann relation: $\boxed{\frac{n_1}{n_2} = \frac{g_1}{g_2} \exp\left(-\frac{\Delta E_{21}}{k_B T}\right)}$

at LTE: $\bar{J} = B_\nu \Rightarrow A_{21} = \frac{2h\nu^3}{c^2} B_{21}$

in general: $\frac{C_{21}}{C_{12}} = \frac{n_1}{n_2}$

$$\boxed{n_\nu = \frac{4\pi}{c} \frac{J_\nu}{h\nu}}$$

- oscillator strength:

$$B_{21} = \frac{4\pi}{h\nu_{21}} \sigma_T \underbrace{\Gamma_\alpha}_{\frac{f_{21} c}{r_T}}$$

(drag coefficient; sec^{-1})

$$\Rightarrow \boxed{B_{21} = \frac{4\pi}{h\nu} r_T c f_{21}}$$

oscillation strength $f_{21} < 1$

$\Rightarrow \boxed{B_{21} = \frac{\;}{h\nu_{21}} \cdots F_{21}} \qquad F_{21} < 1$

- **Bound-free radiation** $\quad H^0 \rightleftharpoons H^+ + e^-$

- ionization: 1) collisional ioniz. α_{ion} / 2) photo ioniz. Γ_{ion}

- recombination: 3) spontaneous recomb. β_{rec} / 4) stimulated recomb. δ_{rec} / 5) dielectronic recomb. $\delta_{diel.}$

- Rate equation:

$$\boxed{\frac{dn_{H^0}}{dt} = n_{H^+}n_{e^-}\beta_{rec} + n_{H^+}n_{e^-}n_\nu \delta_{rec} + n_{H^+}n_{e^-}^2\delta_{diel.} - n_{H^0}n_{e^-}\alpha_{ion} - n_{H^0}n_\nu \Gamma_{ion}}$$

- no radiation $(n_\nu = 0)$ + $n_e > n_{crit} \left(= \frac{\beta_{rec}}{\delta_{diel}}\right)$ + LTE:

$$\frac{n_{H^+}n_{e^-}}{n_{H^0}} = \frac{\alpha_{ion}}{\delta_{diel}} = \frac{g_{H^++e^-}}{g_{H^0}} \exp\left(-\frac{\chi_I + \frac{1}{2}m_e v^2}{k_B T}\right)$$

$\left(\;\; {}^3 \;\; 2g^+ \;\; \chi_I \right.$

$$\Longrightarrow \boxed{\frac{n_{H^+} n_{e^-}}{n_{H^0}} = \left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} \frac{2g^+}{g^0} e^{-\frac{\chi_I}{k_B T}}}$$ Saha relation

- if $n_e < n_{crit}$: 2) $\simeq$ 3) + $\cancel{4)}$ with radiation field

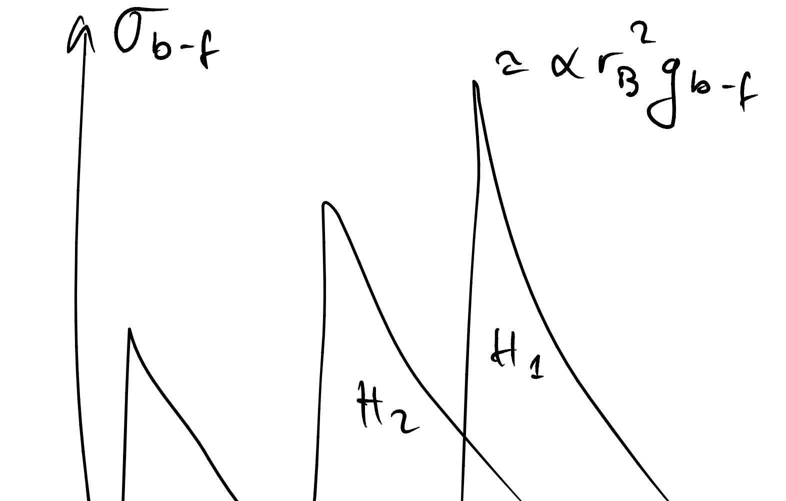
  1) $\simeq$ 3) w/o radiation field

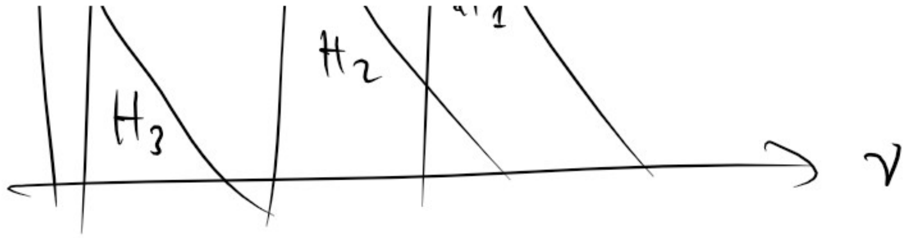
  1) + 2) $\simeq$ 3) coronal equilibrium

- Einstein-Milne relations : 2) = 3) + 4)

detailled balance : $n_{H^0} h\nu \sigma_\nu \, d\nu \, c = n_{H^+} n_{e^-} \underbrace{\sigma_{rec}}_{F(\nu)} v dv + n_{H^+} n_{e^-} \underbrace{\beta_{rec} \, v \, \frac{n_\nu}{n_0}}_{B_\nu G(\nu)} d\nu$

$$\hookrightarrow \boxed{F(\nu) = \frac{2h\nu^3}{c^2} G(\nu)}$$ same as line emission





· photon occupation number:

$$B_\nu = \frac{h^4 \nu^3}{c^2} f_\nu \; ; \; N_\nu = \frac{2}{e^{\frac{h\nu}{k_B T}} - 1} \; ; \; f_\nu = \frac{2\left(e^{\frac{h\nu}{k_B T}} - 1\right)}{h^3}$$

↳ $n_{H^+} n_{e^-} F(\nu) \left[1 + N_\nu \right]$ → Bose enhancement factor

for ionization: $\left[1 - N_e \right]$ → Fermi suppression factor

· Bremstrahlung (free-free): $P = \frac{8\pi}{3} \frac{e^2}{c^3} (a(t))^2$

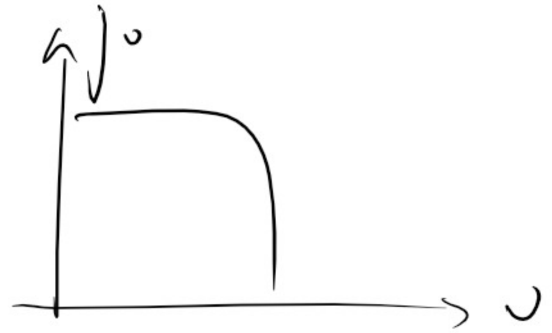
$$\boxed{\frac{dE}{d\omega \, dV \, dt} = \frac{16 e^6}{3 c^3 m_e^2 v} n_e n_i Z^2 \underbrace{\ln\left(\frac{b_{max}}{b_{min}}\right)}_{\overline{g_{ff}}}}$$

thermal Bremsstrahlung:

$$j^\nu = \frac{1}{4\pi} 6.8 \cdot 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-\frac{h\nu}{k_B T}} \overline{g_{ff}}$$

$\uparrow j^\nu$

$\int^{\infty}$



$$\to \Lambda_{brems}(T) = \int_0^\infty 4\pi j_\nu \, d\nu$$

- **Summary of atomic radiative processes:**