Theoretical Astrophysics
Exercise Sheet 12

FS 17

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Exercise 1 [Galactic Relaxation]
The relaxation time for a galaxy or, more generally, a stellar system is defined as

$$t_{\text{relax}} = n_{\text{relax}} t_{\text{orbital}} \tag{1}$$

where

$$n_{\text{relax}} \approx \frac{N}{8 \ln \Lambda} \tag{2}$$

with N being the number of particles of the system (i.e. stars in the stellar system). The orbital time is given by

$$t_{\text{orbital}} = R/v \tag{3}$$

The term $\ln \Lambda$ is called the Coulomb logarithm and Λ is approximately given by

$$\Lambda \approx \frac{Rv^2}{Gm} \tag{4}$$

where m represents the typical mass of a star in the system.

(a) The speed v has not been specified any further. Knowing that it must represent a typical speed in the system, what speed could you use as a rough estimate of it? Here, you are not asked for a number, but for a conceptual answer.

(b) Calculate the relaxation time for the following stellar systems assuming $m = 1\ M_\odot$

(i) a galaxy like the Milky Way with $N \approx 10^{11}$, $v \approx 200$ km/s and $R \approx 10$ kpc

(ii) a typical globular cluster with $N \approx 10^5$, $v \approx 5$ km/s and $R \approx 5$ pc

(iii) a typical open cluster with $N \approx 100$, $v \approx 0.5$ km/s and $R \approx 5$ pc

(c) Given a lifetime for the galaxy, globular cluster and open cluster of respectively 10 Gyr, 10 Gyr and 100 Myr, conclude which one of these systems is collisionless.

Exercise 2 [[Power-law density model]
Many galaxies have luminosity profiles that approximate a power-law over a large range in radius. Consider the structure of a system in which the mass density drops off as some power of the radius:

$$\rho(r) = \rho_0 \big(\frac{r_0}{r}\big)^\alpha \tag{5}$$

(a) Show that the mass inside a radius r, $M(r)$ is given by

$$M(r) = \frac{4\pi\rho_0 r_0^\alpha}{3-\alpha} r^{3-\alpha} \tag{6}$$

(Hint: you need to consider two cases depending on the value of α)

(b) Give an expression for the circular velocity squared $v_c^2(r)$ by using the virial theorem.

(c) Compute the potential difference between radius r and the reference radius r_0 in terms of the circular velocity at these radii. (Hint: mind that there are different cases for different values of α)

(d) Based on the equality between kinetic and potential energy, find an expression for the escape velocity and compute it in terms of the circular velocity $v_c(r)$ and the parameter α (assume the constraint that $\alpha > 2$).

(e) Discuss what happens to the quantities calculated above when varying the value of the parameter α. What constraints do you find for α such that the above quantities physically make sense for a galaxy?