Problem 4 - Ay 216 Interstellar Medium, Spring 2008

The CO Molecule

Although CO is the second most abundant interstellar molecule, observationally it is the most useful. It is therefore important to understand some of the basic facts about its spectrum and its emissivity.

1. Energy Levels and Transitions

The rotational energy levels of a rigid rotator are $E = J(J+1)B$, where J is the angular momentum quantum number and B is the rotational constant.

a) Make a table of the rotational energies of ^{12}CO and ^{13}CO in their ground vibrational and electronic states. The appropriate rotational constants (in wavenumbers) are $B(^{12}\mathrm{CO})$ = 1.9225290 cm^{-1} and $B(^{13}\mathrm{CO})$ = 1.8379720 cm^{-1}. Compute the energies of the first 10 J-levels and the wavelengths and frequencies of all of the dipole permitted transitions between 100 GHz and 1 GHz using the rigid-rotor model. Tabulate the results and sketch the rotational energy level diagrams, labeling the energies in temperature units, i.e., E/k.

b) Compare your energy levels against the NIST molecular data base

`http://physics.nist.gov/PhysRefData/MolSpec/Diatomic/index.html`

What is the origin of the disagreement?

c) Which of these lines can be observed from the ground (at zero redshift). Answer the question by making a plot of the vertical transmission through the earth's atmosphere between 100 GHz and 1 THz. What are the primary factors determining atmospheric transmission, and explain why is the ALMA mm telescope being built in the Atacama desert?

d) The Einstein A-value for the J=1-0 transition of ^{12}CO is $A(1,0) = 7.18 \times 10^{-8}$ s^{-1}; for the other transitions it is given approximately by,

$$A(J, J-1) = \frac{3J^4}{(2J+1)} A(1,0).$$

The rate coefficient for collisional deexcitation by H$_2$ can be crudley expressed as,

$$k(J', J) = 1.0 \times 10^{-11}\, T^{0.5}\ \mathrm{cm^3 s^{-1}},$$

where $|J' - J| = 1$ or 2. Add the critical density for each transition to your previous table of wavelengths and frequencies. Try to locate accurate collisional rate coefficients to compare with the above approximation.

2. Radiative Transfer for the CO Lines

a) Write down the steady-state rate equations describing the population of the rotational levels of CO. Treat the radiation field by assuming that some fraction $\varepsilon \leq 1$ of the photons, produced following spontaneous decay, escape from the cloud containing the emitting molecules. In this approximation, the radiative term,

$$n_k A_{kj} + (n_k B_{kj} - n_j B_{ij}) J_\nu,$$

can be replaced by $n_k \varepsilon A_{kj}$.

The level population is now a function of three parameters: T, $n(\mathrm{H_2})$ (the collision partner density), and the escape probability ε.

Write the population rate equations in matrix form and solve for the level population. Check that the number of equations and the number of unknowns is sufficient to find a solution.

b) Plot the fractional level populations, $n(J = 0)/n$, $n(J = 1)/n$ etc. (where n is the total density of CO), as functions of temperature ($5 < T/\mathrm{K} < 200$) and $\mathrm{H_2}$ density ($10^3 < n(\mathrm{H_2})/\mathrm{cm^{-3}} < 10^5$) for $\varepsilon = 0.01$, 0.1, and 1. Include the thermal equilibrium fractional population in each plot. You may make 1-d graphs (e.g., holding density constant and varying the temperature) or use contour plots.

c) Explain how radiation trapping modifies the level populations and suggests a new definition of critical density.

d) Compute the optical depth at line center for the CO J=1-0 and CO J=2-1 transitions for a typical giant molecular cloud with $A_V = 10$ mag. and a CO/$\mathrm{H_2}$ abundance ratio of 10^{-4}. Assume a Doppler broadened line with a FWHM width of 3 km s^{-1}.

f) What is the relation between ε and line optical depth?