Problem 2 - Ay 216 Interstellar Medium, Spring 2008

Forbidden Line Diagnostics

The forbidden transitions within the ground state configurations of abundant heavy ions provide diagnostics for H II regions and the warm ionized interstellar medium (WIM) of galaxies. They are used to determine the temperature, electron density, and abundances of these regions.

1.) Draw energy level diagrams for the first three ions of oxygen (O I, O II, & O III) that show the levels of the ground state configuration, usually the first five or six. Label the levels with integers (i = 1, 2, 3, ...), standard spectroscopic notation ($^{2S+1}L_J$) and the energy above the ground state (in cm^{-1} and eV). Draw the downward radiative transitions and label them with their wavelengths in microns and A-values in s^{-1}.

2.) A simplified framework for calculating forbidden line emission from H II regions assumes that the levels are populated by electronic collisions and depopulated by electronic collisions and by spontaneous radiative decay (neglecting stimulated emission). The tables at the end of the problem set give the pertinent atomic data for O II. Review the treatment of the two-level atom in the lecture notes (alos Tielens §2.3) and calculate the critical densities for the transitions out of the i = 2, 3, 4 and 5 levels of this ion.

3.) Approximate the O II ion as a *three-level* system (i = 1, 2, and 3). What is the justification for this approximation? Consider the limiting cases $n_e \ll n_{cr}$ and $n_e \gg n_{cr}$, and find the population ratios n_2/n_1 and n_3/n_1 and the line intensity ratio I_{3-1}/I_{2-1}. For $n_e \gg n_{cr}$, assume that local thermodynamic equilibrium holds and that the electrons are described by a Maxwell-Boltzman distribution with $T_e = 8000$ K.

4.) Show that the lines in part (3) are optically thin for an H II region produced by an O5 star, assumed to be a Strömgren sphere with uniform properties: $n_{\mathrm{H}} = 10^4\,\mathrm{cm}^{-3}$ and $T = 8,000\,\mathrm{K}$. Take the worst case scenario where the photons escape from a location close to the star. Why is this calculation necessary to validate the assumptions of (3)?

5.) Numerically calculate the populations of the five lowest levels of O II. Start by writing a general equation describing the population and depopulation of the levels by collisions and by spontaneous decay. Use the language of linear algebra to describe the resultant system of simultaneous equations. Choose a programming environment, such as MATLAB, IDL, or even EXCEL, to set up the numerical solution and obtain the populations n_j as a function of n_e and T_e. Of course you can develop your own algorithm if you prefer. In any case, be sure to check that your mathematical description of the problem is well-posed before you calculate. Check your results by showing how the level populations approach the correct low and high density limits established in (3) at a fixed temperature. Make a contour diagram that displays I_{3-1}/I_{2-1} for temperatures and densities typical of H II regions.

You will need access to relevant atomic data. A standard reference had been the 1993 article by C. Mendoza (IAU Symposium 102, "Planetary Nebulae"). The O II data below is from that source. Although it is now out of date, it is still useful for getting a quick overview of the facts. The newer textbooks have some useful tables:

Appendix B of Dopita & Sutherland

Ch. 3 of Osterbrock & Ferland

Table 2.6 of Tielens

The standard reference for energy levels and A-values is the NIST database:
http://physics.nist.gov/PhysRefData/ASD/. As always, check for consistency[1]!

Atomic Data for O II

Energies (erg)

Level	1	2	3	4	5
1	0.0	5.32589e-12	5.32976e-12	8.03878e-12	8.03900e-12
2		0.0	3.87000e-15	2.71289e-12	2.71311e-12
3			0.0	2.70902e-12	2.70924e-12
4				0.0	2.20000e-16
5					0.0

A values (inverse seconds)

Level	1	2	3	4	5
1	0	3.82e-05	1.65e-04	5.64e-02	2.32e-02
2		0	1.20e-07	1.17e-01	6.15e-02
3			0	6.14e-02	1.02e-01
4				0	0
5					0

Collision strengths (symmetric)

Level	1	2	3	4	5
1	0	8.01e-01	5.34e-01	2.70e-01	1.35e-01
2	..	0	1.17e+00	7.30e-01	2.95e-01
3	..	..	0	4.08e-01	2.75e-01
4	..	..	..	0	2.87e-01
5	..	..	..	..	0

[1]One AY216 instructor discovered that all NIST wavelengths between 1 and 5 μm were incorrect due to an error in the refractive index of air.