05/06/2019

$$I_{10} = \int j_{10}\, dz \quad [\text{erg cm}^{-2}\, \text{s}^{-1}]$$

$$I_\nu = \frac{I_{10}}{\Delta\nu} \qquad \Delta\nu = \nu_{10}\,\frac{\Delta v}{c}$$

$$\boxed{k_B T = \frac{c^2}{2\nu_{10}^2}\, I_\nu}$$

$$k_B T = \frac{c^2}{2\nu_{10}^2}\, \frac{I_{10}}{\Delta\nu}$$

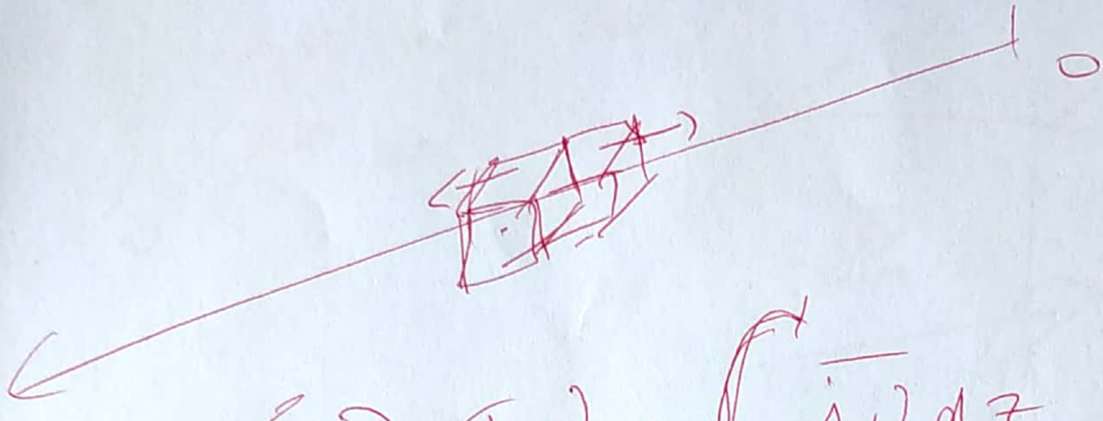
$$\left(k_B T \cdot \Delta v\right) = \boxed{\frac{c^2}{2\nu_{10}^2}\, \frac{I_{10}}{\nu_{10}}}\; c$$

K

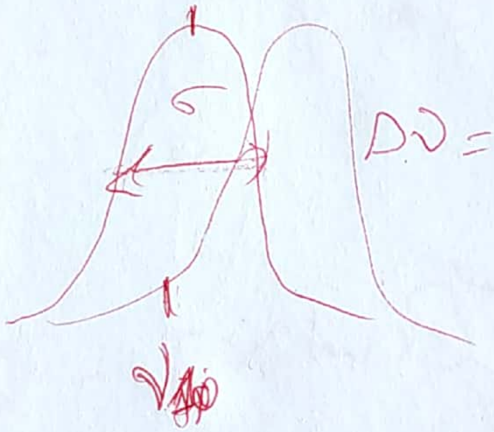
$c = 3 \times 10^{10}$ m/s

$\Delta v = 10^{7}$ m/s

$A = 10^{-4}$




$$(K)\ I_\nu = \int \overline{j_\nu}\, dz$$




$$\Delta\nu = \nu_{10}\frac{\sigma}{d} \quad [Hz]$$

$$\sigma \quad [km/s]$$

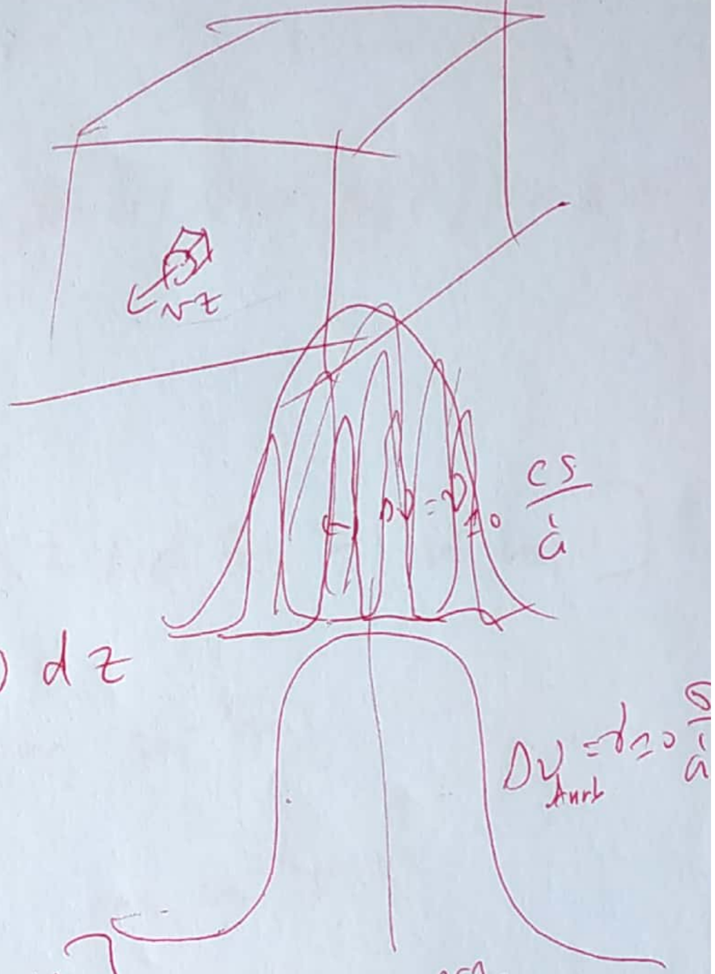
$$K \quad km.s^{-1}$$

$$\nu_{10} \rightarrow \nu_{10}' = \nu_{10} - \nu_{10}\frac{v_z}{d}$$

$$\left(I_\nu \Delta\sigma\right) = \int \overline{j_\nu}\,\sigma\, dz$$

$$\int_0^{+\infty} j_\nu \, \phi(\nu) \, d\nu$$

ergs




$$\cancel{\int_0^{+\infty} I_\nu \, d\Omega =}$$

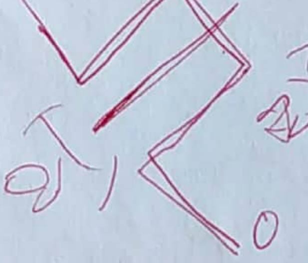
$$I_\nu = \int_0^L j_\nu \, dz$$

$$\int_0^{+\infty} I_\nu \, d\nu = [\nu \, I_\nu(\nu)]$$

$$= \int_0^L \int_0^{+\infty} j_\nu \, \phi(\nu) \, dz \, d\nu$$

$$I_\nu = \int_0^L j_{\nu_0} \phi(\nu) \, dz$$

erg/cm$^3$/s



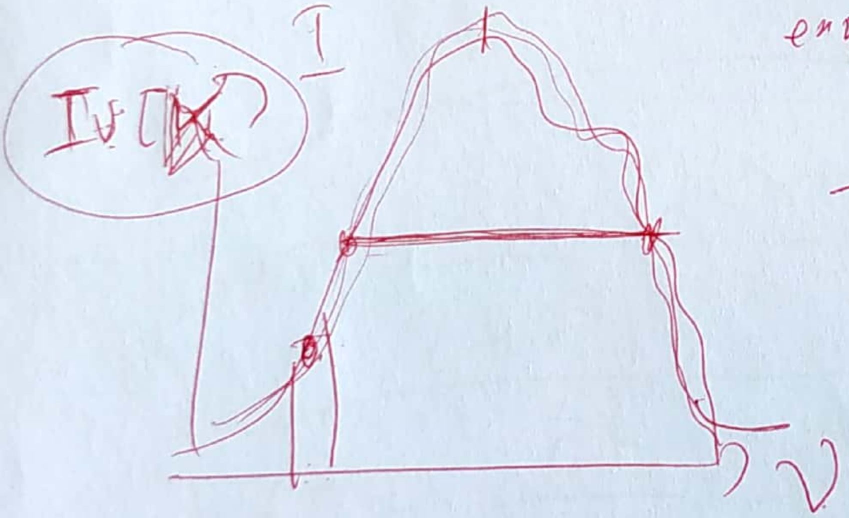
$$\int \phi(\nu)\, d\nu = 1$$

$$\int_0^{+\infty} I_\nu\, d\nu = \int_0^L \int_0^{+\infty} j_{10}(z)\, \cancel{\phi(\nu_z, \sigma, \nu)}\, d\nu\, dz$$

$$I = \int_0^L j_{10}(z)\, dz = [\mathrm{K\ km/s}]$$

erg cm$^{-3}$ s$^{-1}$

erg cm$^{-2}$ s$^{-1}$




$\Delta\nu = \nu_{10} \cdot \frac{\Delta v}{c}$

$$\frac{I}{\nu_{10} \cdot \frac{\Delta v}{c}} \rightarrow K \cdot \Delta v$$

$$I \cdot \frac{c}{\nu_{10}} = [\mathrm{K.\ km/s}]$$

$$\left(\int_0^L \overline{j_{10}(z)}\, dz\right) \cdot \frac{d}{\gamma_{10}}$$

$$\iint \mathcal{K}\, dx\, dy = K \text{ [illegible]}$$

$$\boxed{h_{BT} = \frac{a\nu}{2a\nu^3} c^2 I \nu}$$

$$h_{BT} = \frac{c^2}{2\nu^2} I \nu$$

$$h_{BT} = \frac{c^2}{2\nu_{10}^2} \frac{I}{\nu_{10} \frac{D\nu}{d}}$$

$$K' = h_{BT} \cdot D\nu = \frac{c^3}{2\nu_{10}^3} \frac{I}{\cancel{\nu_{10}}}$$

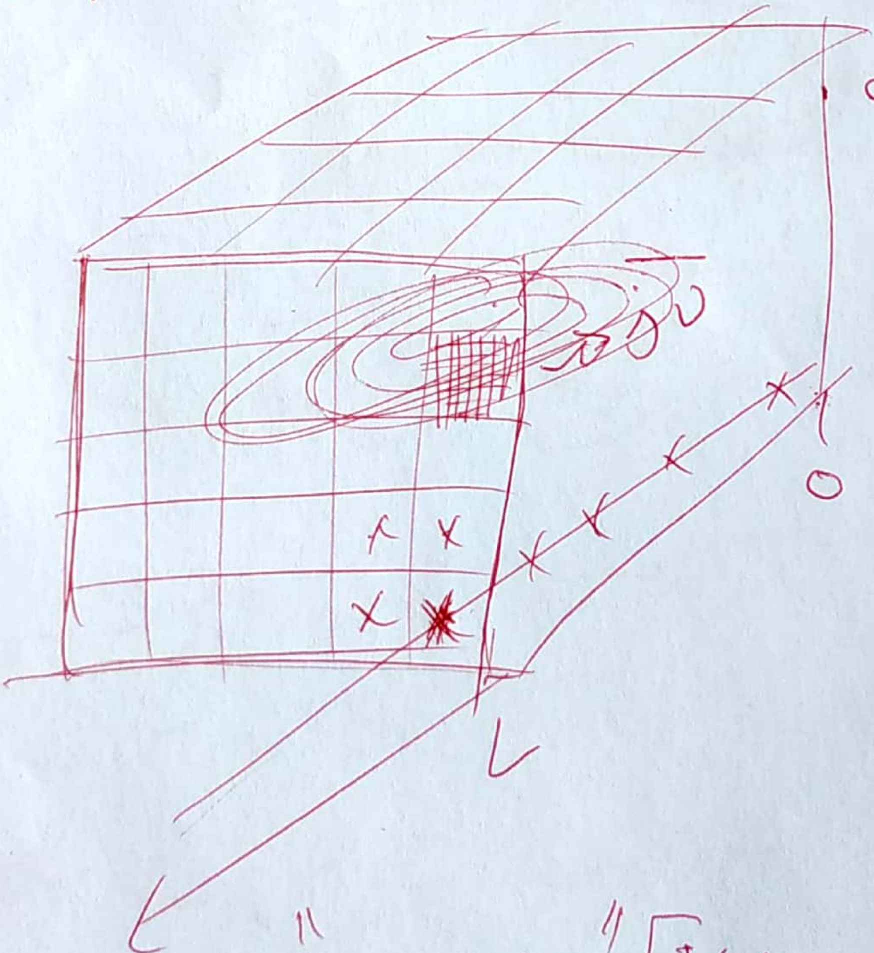
20|05|2019

03/06/19

$\overline{j_\nu}$ [erg cm$^{-3}$ s$^{-1}$ Hz$^{-1}$]

$$I_\nu = \int_0^L \overline{j_\nu}\, dz \quad [\text{erg cm}^{-2}\ \text{s}^{-1}\ \text{Hz}^{-1}]$$

$$= B_\nu(T) \quad K$$




z "$I_\nu \Delta v$" [K km s$^{-1}$]

$L_{CO}$ "$I_\nu \Delta v \Delta x^2$" [K km s$^{-1}$ pc$^2$]

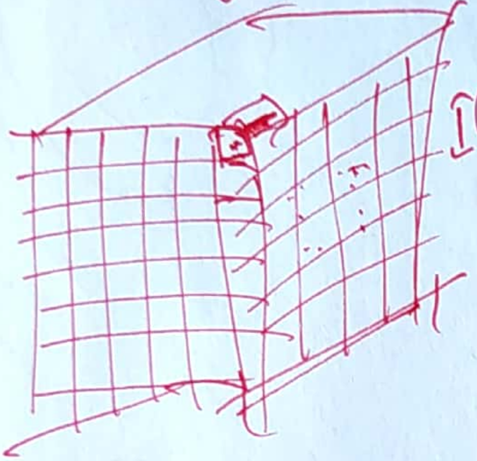
20|05|2019

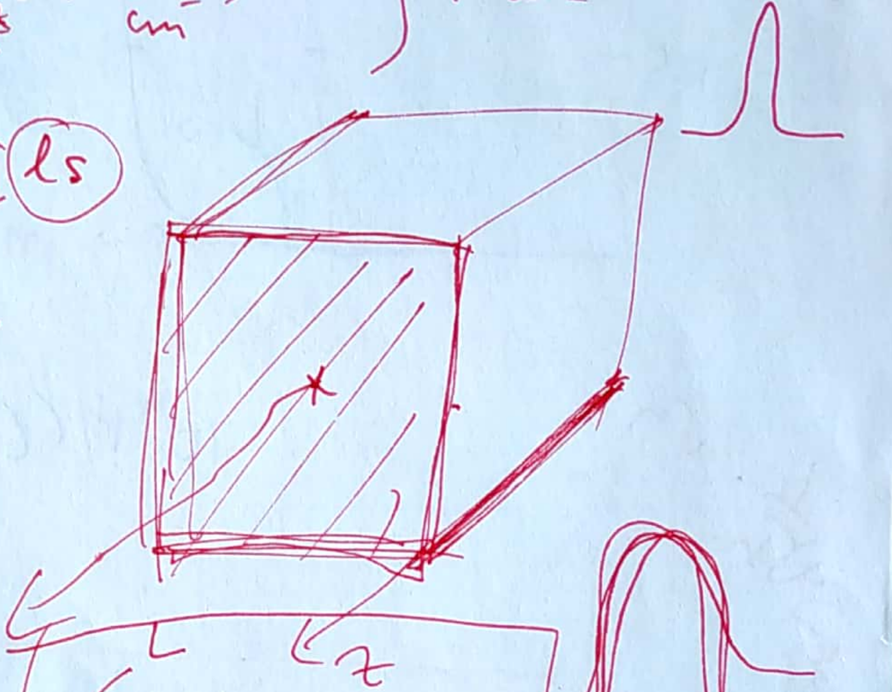
29|05|19

$$V = N_s . V_s$$

$$\int P \, ds = 1$$

erg $s^{-1}$ $Hz^{-1}$ $cm^{-3}$




$$\boxed{\int_0 \beta j_\nu \frac{dz}{\Delta\Omega_{beam}} = I_{\nu_i}}$$

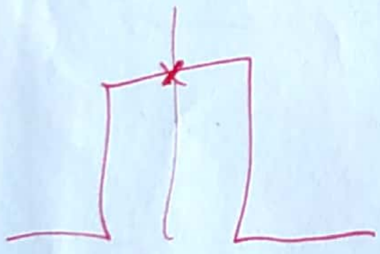
ds.

erg $cm^{-2}$ $s^{-1}$ $Hz^{-1}$ ⟶ K

|||

$$I_{\nu_i} = B_{\nu_i}(T) \longrightarrow T.$$

$$T = B_\nu^{-1}(I_\nu)$$

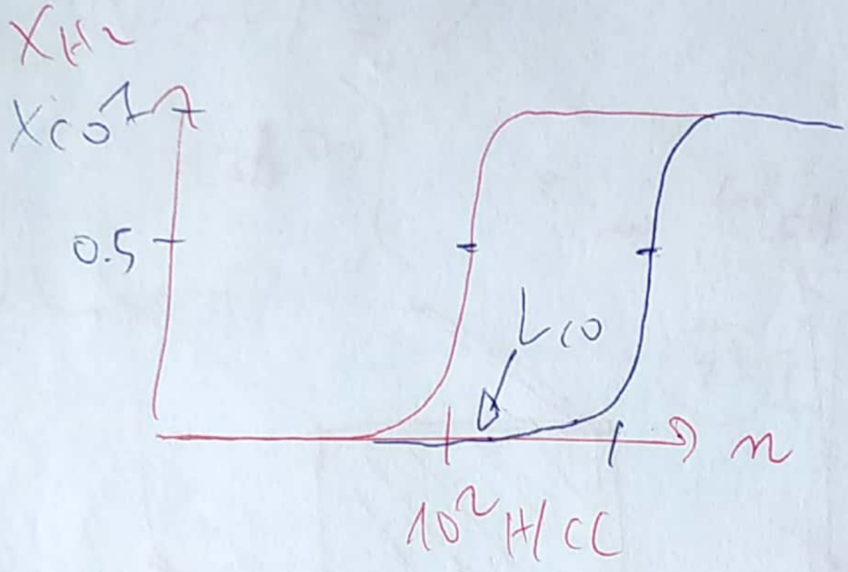


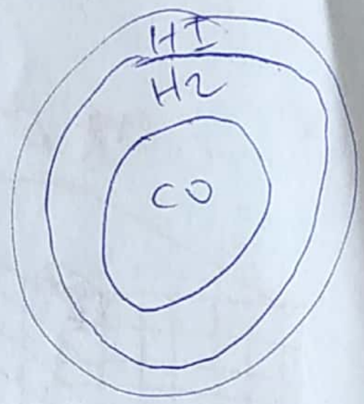

$$K \, pc^2 \, km/s = erg \, s^{-1} = L_{CO}$$

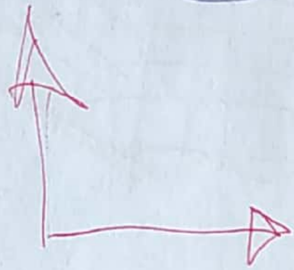
$$\overline{\beta j_\nu} = \int_{cell} \beta j_\nu \frac{dv}{P ds}$$

$$\int P \, dv = 1$$

23/05/2019



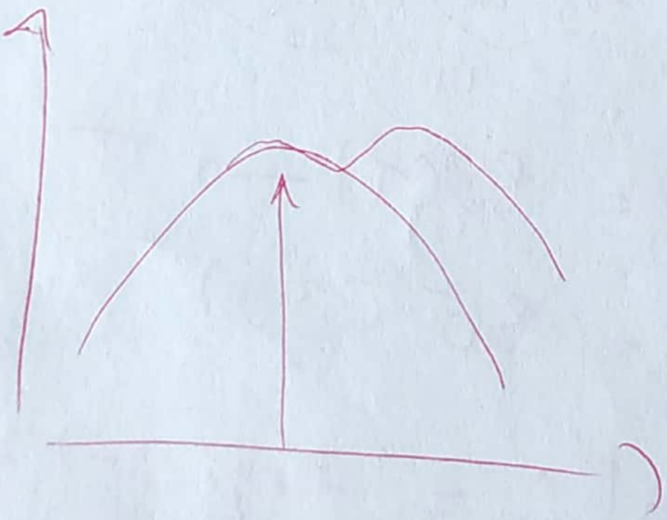



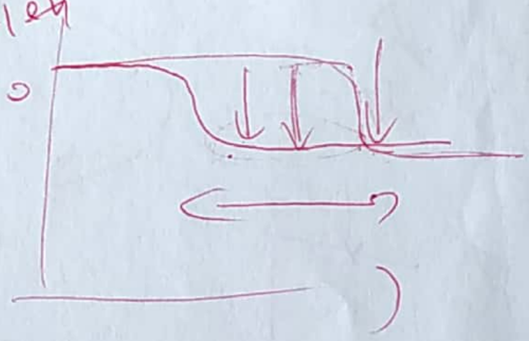



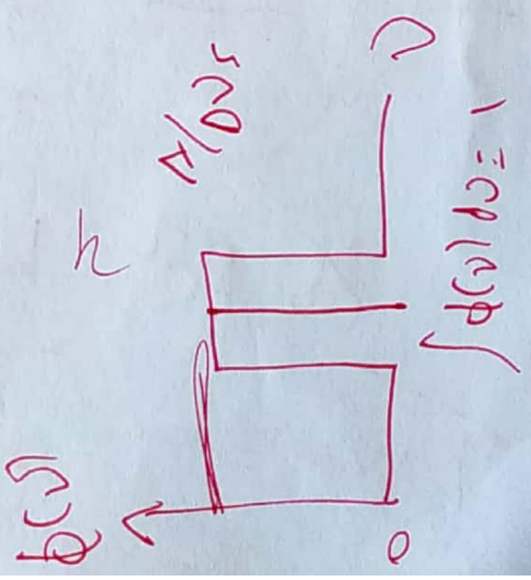
$$\boxed{M_{H_2} = \alpha_{CO} L_{CO}}$$

$\Delta \nu_{Ant} = \frac{6 \pi k_B \nu_{ij}}{c}$ · $\frac{T}{m}$

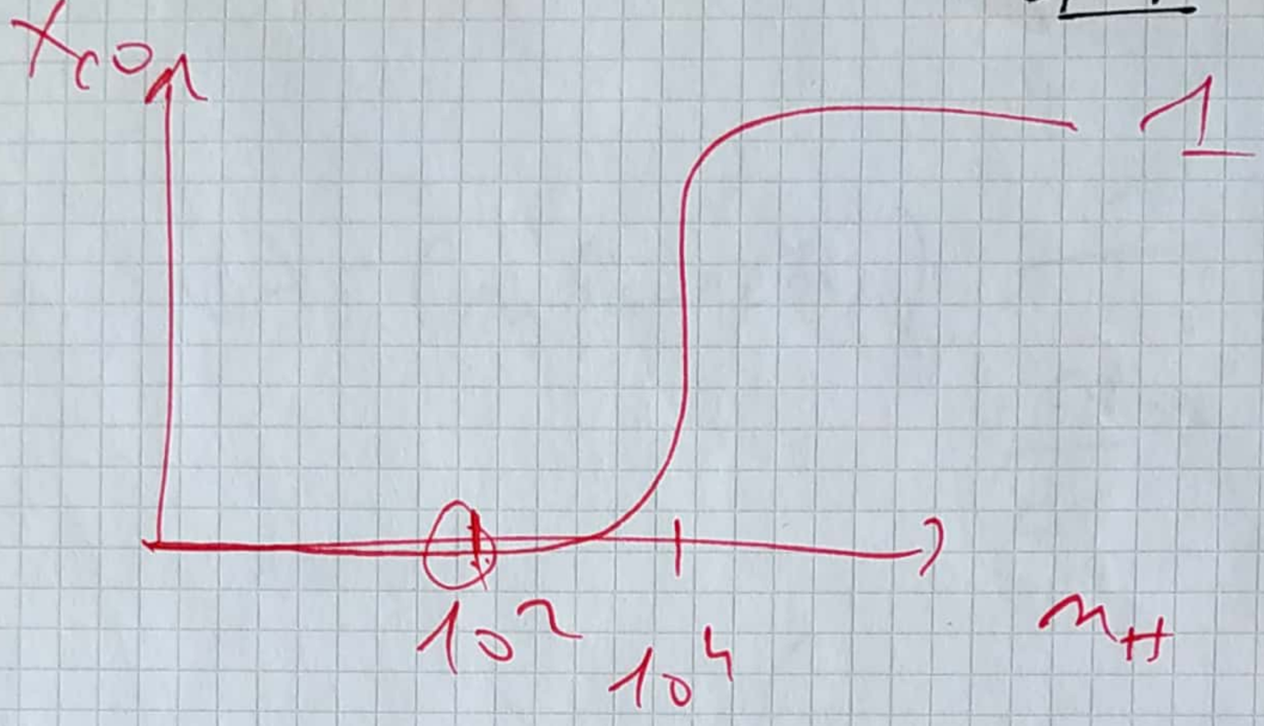
$\Delta \nu_s = \frac{c_s}{c} \nu_{ij}$



29/05/19 (1)




$$\boxed{n_{CO} = X_{CO} \times 10^{-4}\, n_H . \frac{T}{T_0}}$$

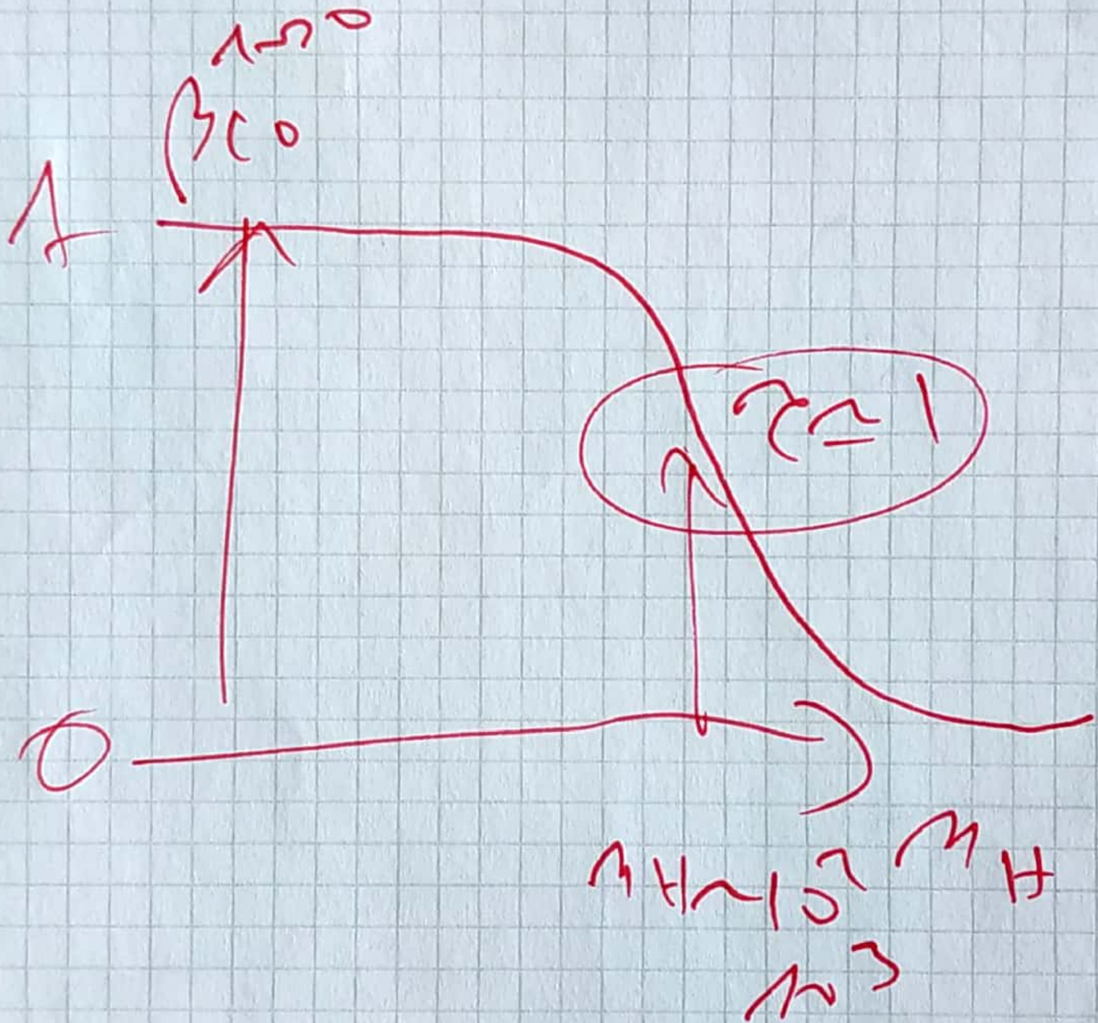
$n_{CO} \simeq 1\ cm^{-3} \Rightarrow n_H = 10^{5}$ $\frac{H}{cc}$

$X_{CO} \simeq 10^{-2}$

$n_{CO} \simeq 10^{-2} \times 10^{-4} \times 10^{2}$

$\simeq 10^{-4}$

29/05/18 (2)




Scanned by CamScanner

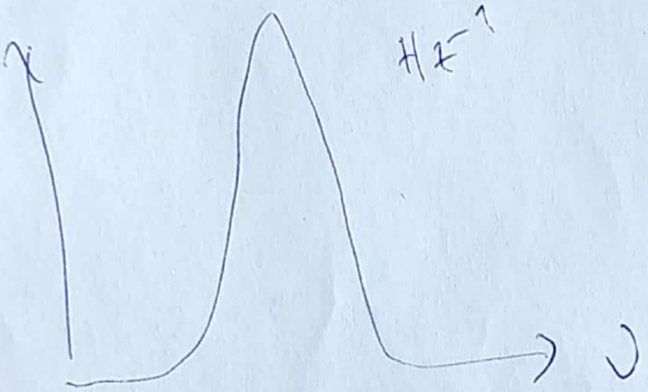
$\boxed{20|05|2019}$

$$\tau_\nu = \frac{h\nu_{ij}}{4\pi} N (n_j B_{ji} - n_i B_{ij}) \frac{\lambda_J}{c_s \nu_{ij}}$$

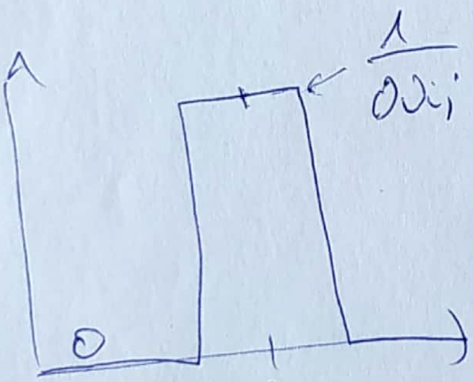
$\cancel{1.064}$

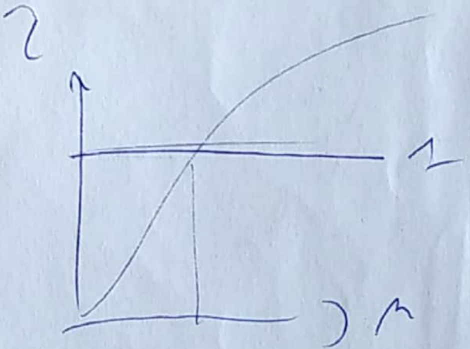
$\phi(\nu)$

$$\tau_\nu = \frac{h\nu_{ij}}{4\pi} N \lambda_J (n_j B_{ji} - n_i B_{ij}) \frac{1}{\frac{c_s}{\lambda} \nu_{ij} = \Delta\nu_{ij}}$$




$$\int \phi_\nu \, d\nu = 1$$




$$\boxed{\tau_\nu = \frac{h\nu_{ij}}{4\pi} N \lambda_J (n_j B_{ji} - n_i B_{ij}) \frac{1}{\Delta\nu_{ij}}}$$

$$\boxed{cm^2\ erg^{-1}\ Hz = B_{ij}}$$

$m^2\ eV^{-1}\ s^{-1} \rightarrow cm^2\ erg^{-1}\ s^{-1}$

erg/cm^3/sec/Hz

jν = ? Time

jν =

$\int dx$

erg/cm^2/sec/Hz = BνCT(1 − $\boxed{4}$

ν_{obs}

ℓos

$\int d\Omega$

↓

K. km/sec

L_{CO} = K. km/s pc^2

N_{H2}

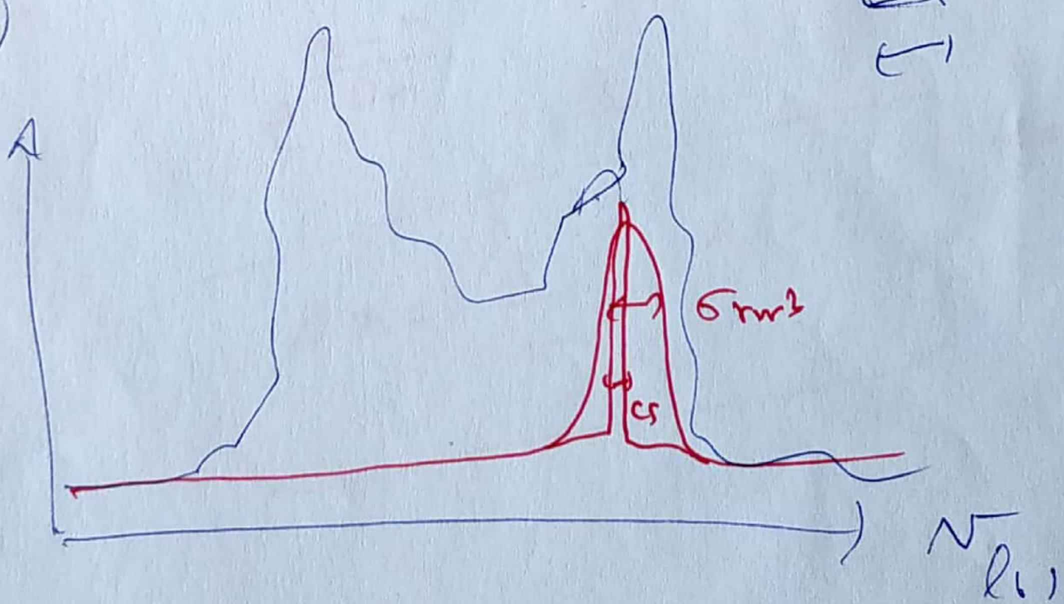
$\int dx^2$

↓

K km/sec pc^2

|||

erg/sec

23/05/2019

$$\frac{\frac{1}{2}\, e^{r(x-c)}}{1+\frac{1}{2}\left(e^{r(x-c)}-1\right)} = \frac{e^{r(x-c)}}{1+e^{r(x-c)}}$$

$$\boxed{f(x) = \frac{1}{e^{r(c-x)}+1}}$$

$$f(n) = \frac{1}{e^{\frac{n_c-n}{\Delta n}}+1}$$

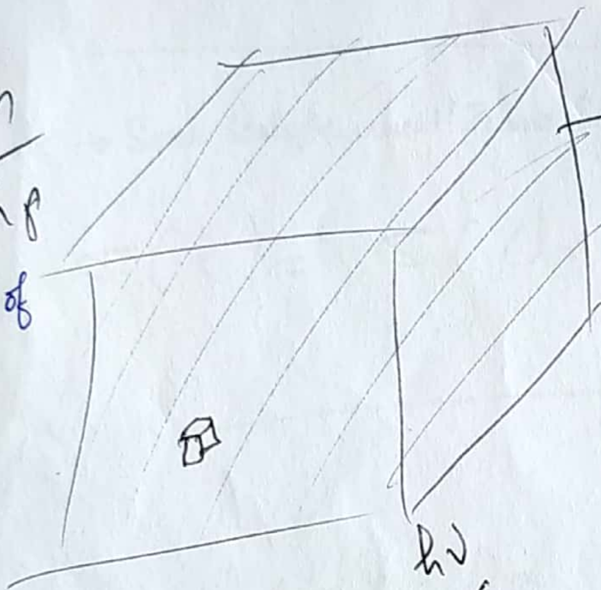
$$\boxed{\Delta n = \frac{1}{r} \qquad n_c = c}$$

$n = x$

# → Tine's model for $H_2$ and CO formation

$n_H = \frac{f}{m_p}$

↳ total no. of Hydrogen atoms



→ $N_{LW}$ [$cm^{-3}$]

↳ photon density in the Lyman-Werner band. Usually expressed in units of:

Habing flux

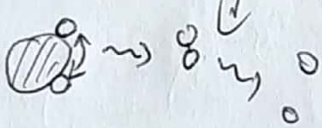
which corresponds to:

$N_{Hab} = 1$

$G_0$ (ISRF)

- Inter-Stellar Radiation Field
- Corresponds to the Solar neighbourhood
- Usually the $H_2$ destruction rate is given in these units





~~$dN_{H_2}$~~

$$\underbrace{\frac{dn_{H_2}}{dt}}_{\text{no. of } H_2 \text{ molecules per unit time}} = \overbrace{\alpha\, n_H\, Z\, n_{HI}}^{\text{creation rate}} - \overbrace{\sigma c\, n_{H_2}\, n_{LW}}^{\text{destruction rate}}$$

- $\alpha$ → coefficient
- $n_H$ → $= \frac{1}{m_p}$
- $Z$ → metallicity or dust fraction
- $\sigma$ → cross section
- $c$ → speed of light
- $n_{H_2}$ → no. density of $H_2$ molecules
- $n_{LW}$ → no. density of Lyman-Werner photons (in Habing unit) (that reach the H atoms in box)

$$\boxed{n_{LW} = (n_0)\, e^{-\tau}}$$

- $\tau$ → optical depth
- $(n_0)$ ↳ avg. no. of LW photons in the whole box

$$\tau = \sigma_{H_2} n_{H_2} Z$$

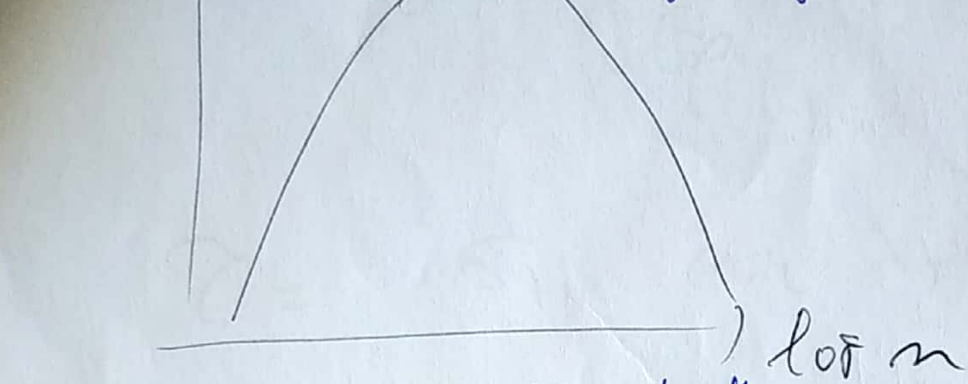
$\sigma_{H_2}$ ← cross-section of the $H_2$ absorption coefficient

$n_{H_2} Z$ → column density

$$\tau = \sigma_{H_2} N_{H_2}$$

$\sigma_{H_2}$ [$cm^2$], $N_{H_2}$ [$cm^{-2}$]

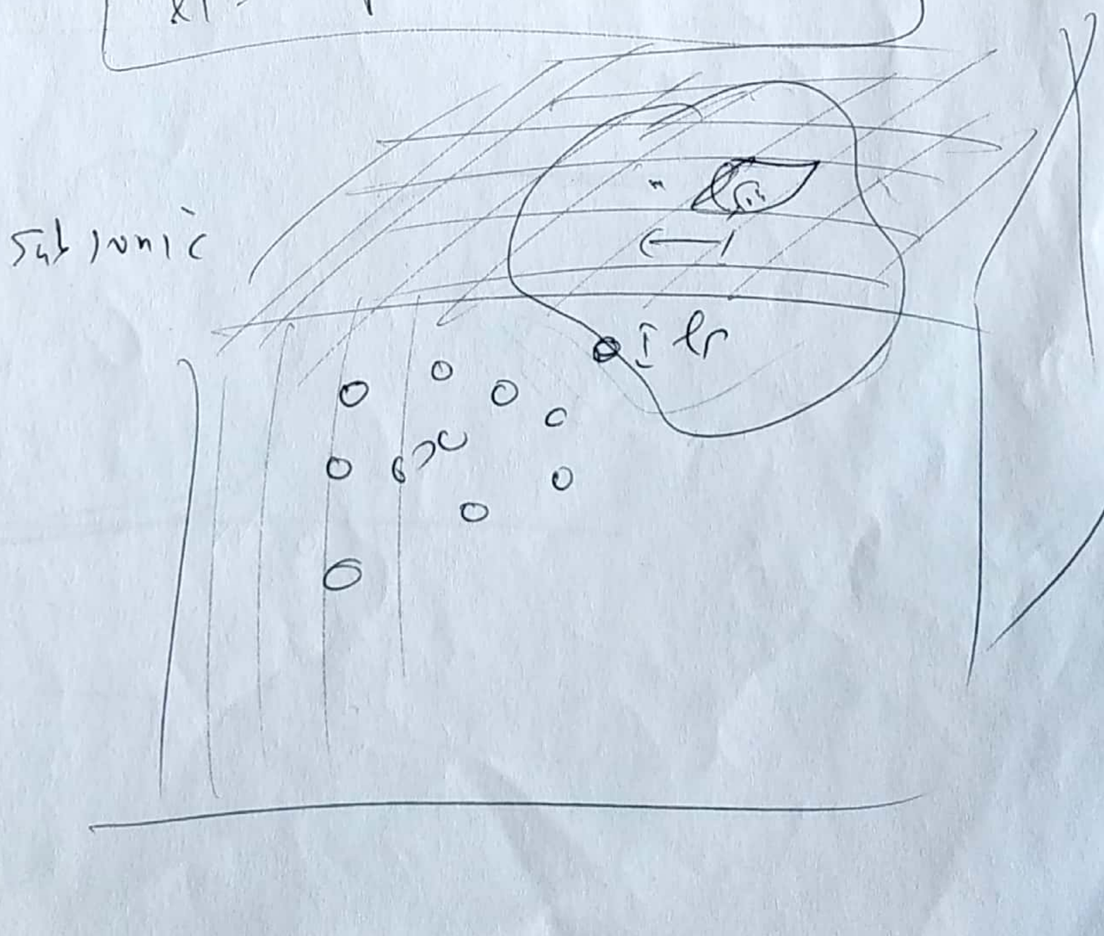
Important to realise that this PDF is the volume fraction occupied by elements of density $n$.




Length scale → Sonic Scale/'thermal' Jeans Length

$$\sigma(l) = \sigma(\Delta x)\left(\frac{l}{\Delta x}\right)^{0.5} = c_s$$

$$\boxed{\begin{array}{l} l_s = \Delta x \left(\dfrac{c_s}{\sigma(\Delta x)}\right)^2 \\ l_s = 0.1 \text{ pc} \end{array}}$$



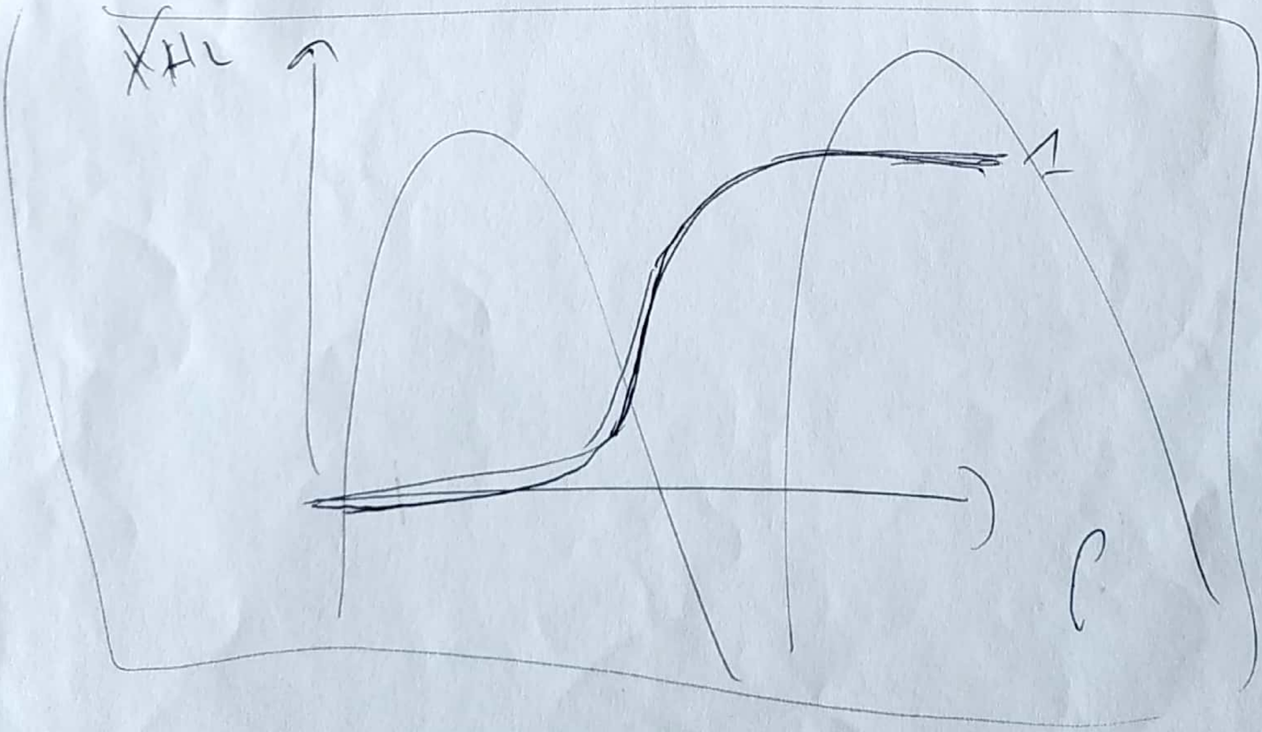
(2)

$$\rho \longrightarrow \lambda_J = \frac{a}{\sqrt{G\rho}}$$

$$\tau = \sigma_{HL}\, n_{HL}\, \lambda_J$$

$$N_H = n_H \cdot \tau = \underbrace{n_H \lambda_J}_{\propto \sqrt{\rho}}$$

$X_{HL}$



$$\overline{\rho_{H_2}} = \int x_{H_2} \rho P(\rho) d\rho = \int x_{H_2} \bar{\rho} e^{s} P(\rho) d\rho$$

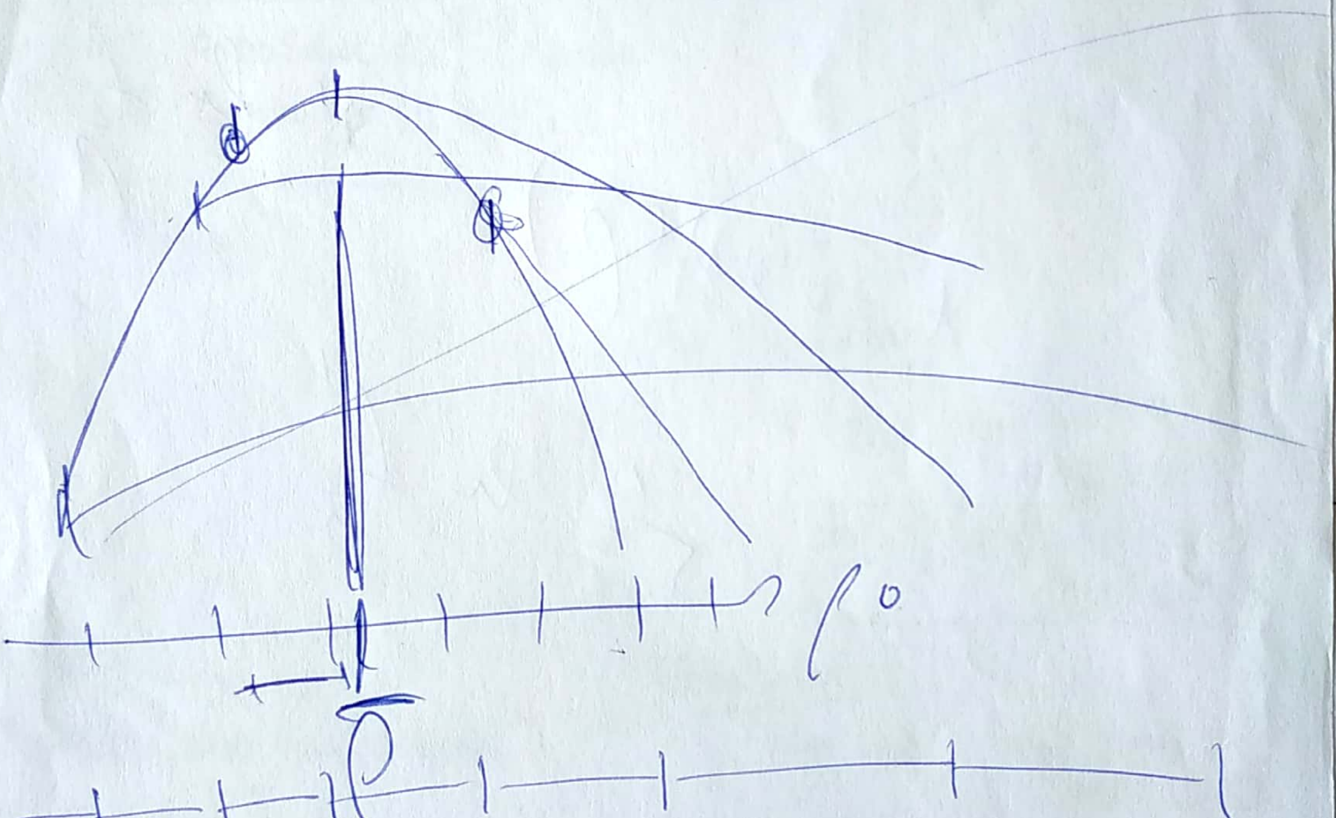
$$\bar{\rho} = \int \rho P(\rho) d\rho = \bar{\rho}$$

$$\overline{x_{H_2}} = \frac{\overline{\rho_{H_2}}}{\bar{\rho}} = \int x_{H_2} e^{s} P(s) ds$$

$$\rho = \bar{\rho} e^{s}$$

13/12/18

$$\rho = \rho^0 \frac{1}{\left(1-\left(\frac{v}{v_{eff}(\rho)}\right)^{n}\right)^2}$$




$v_{eff}(\bar{\rho})$

$$r'(t) = r \frac{1}{\left(1-\left(\frac{t}{A(r)}\right)^2\right)^2}$$

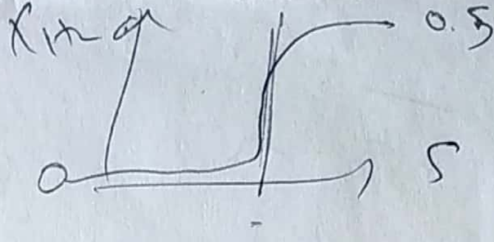
$$\delta = \ln \frac{r'}{r}$$

$$t = 0.4\, A(\bar{r})$$

$$r' = \ln \frac{r'}{r}$$

$(\overline{m_H}, \mu)$

27/11/18




$$m_H(s) = \overline{m_H}\, e^{s}$$

$$P(s) = \frac{1}{\sqrt{2\pi \sigma_s^2}}\, e^{-\frac{1}{2}\frac{(s-\bar{s})^2}{\sigma_s^2}}$$

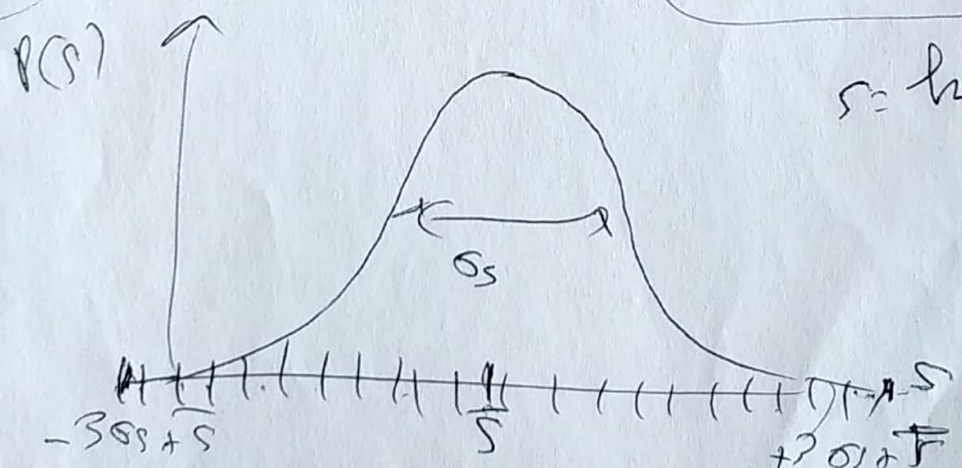
$$\bar{s} = -\frac{1}{2}\sigma_s^2$$

$$\sigma_s = \sqrt{\ln(1 + e^2 \mu^2)}$$

$$\int_{-\infty}^{+\infty} e^{s} P(s)\, ds = 1$$

$$\int_{s_{min}}^{s_{max}} P(s)\, ds = 1$$

$$s = \ln \frac{\rho}{\bar{\rho}} = \ln \frac{m_H}{\overline{m_H}}$$

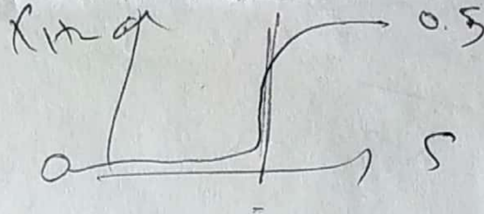



$$s = [-3\sigma_s + \bar{s}\,;\ +3\sigma_s + \bar{s}]$$

$$\overline{X_{H_2}} = \int_{s_{min}}^{s_{max}} X_{H_2}(s)\, e^{s} P(s)\, ds$$

$X_{H_2}(m_H)$

$( \overline{m_H} , \mu )$




27/11/18

$$m_H(s) = \overline{m_H}\, e^{s}$$

$$P(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}}\, e^{-\frac{1}{2}\frac{(s-\bar{s})^2}{\sigma_s^2}}$$

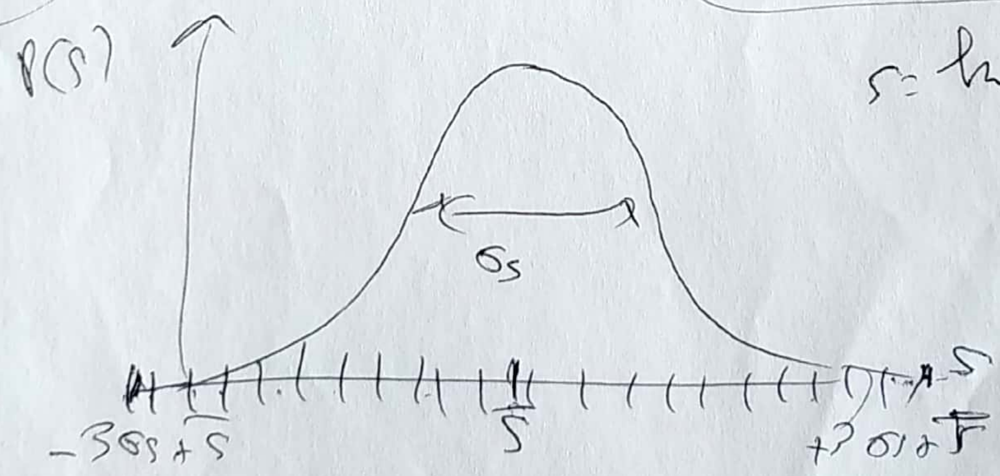
$$\bar{s} = -\frac{1}{2}\sigma_s^2$$

$$\sigma_s = \sqrt{\ln(1 + e\mu^2)}$$

$$\int_{-\infty}^{+\infty} e^{s} P(s)\, ds = 1$$

$$\int_{s_{min}}^{s_{max}} P(s)\, ds = 1$$

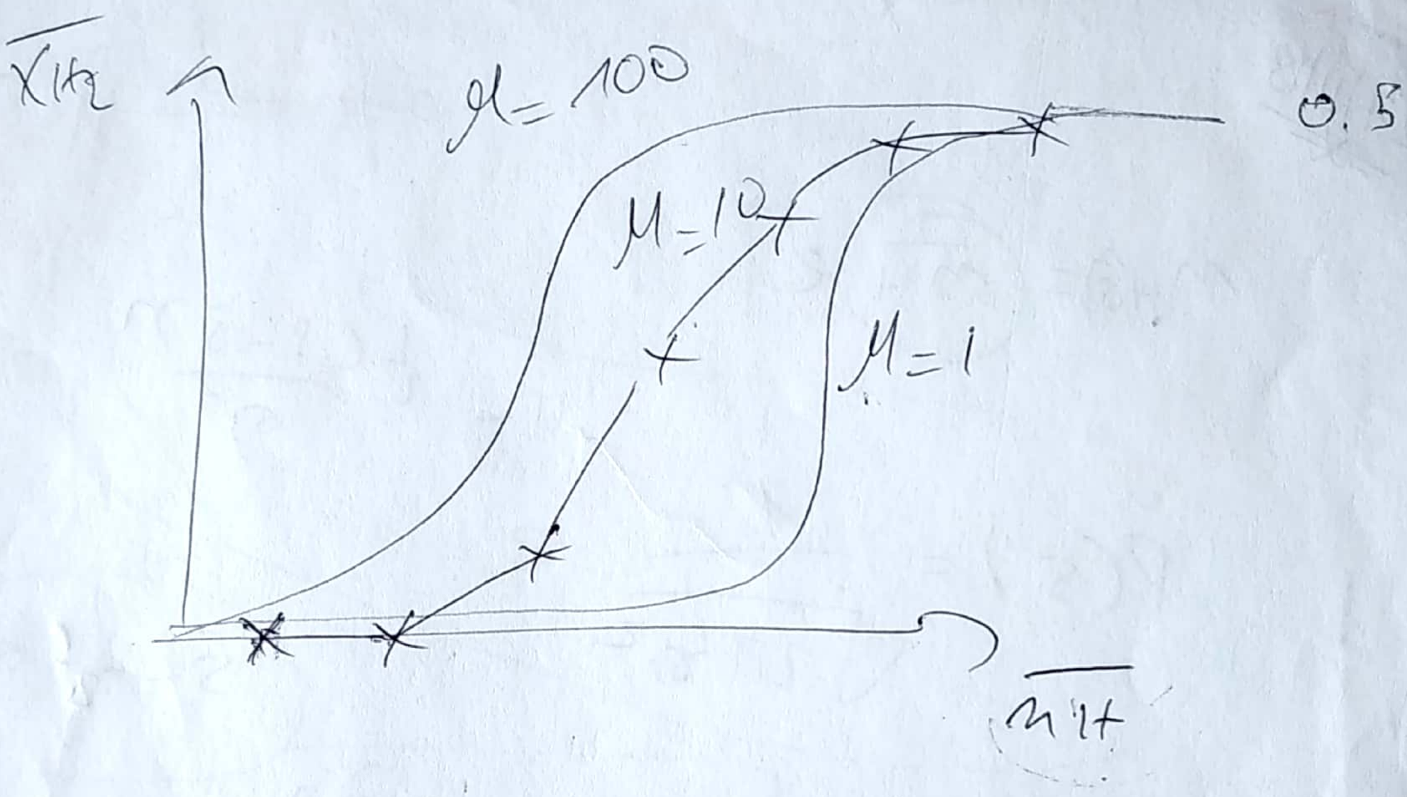
$$s = \ln \frac{\rho}{\bar{\rho}} = \ln \frac{m_H}{\overline{m_H}}$$



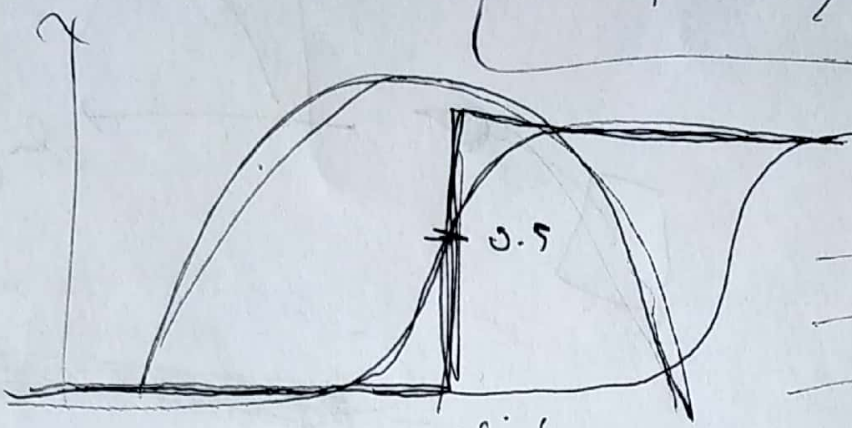

$$s = [-3\sigma_s + \bar{s} ; +3\sigma_s + \bar{s}]$$

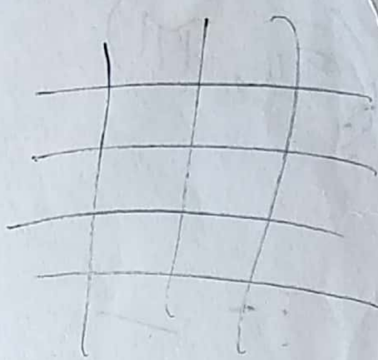
$$\overline{X_{H_2}} = \int_{s_{min}}^{s_{max}} X_{H_2}(s)\, e^{s} P(s)\, ds$$

$X_{H_2}(m_H)$

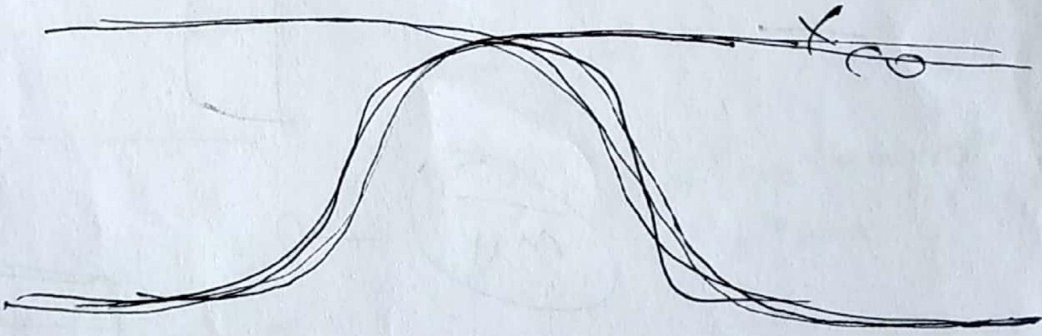

$\overline{X_{H_2}}$
$M = 100$
$M = 10$
$M = 1$
0.5
$\overline{n_H}$

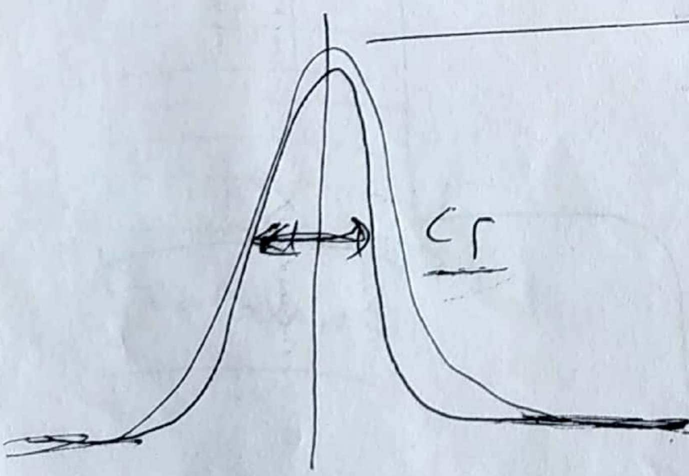
K / 8r / km/s

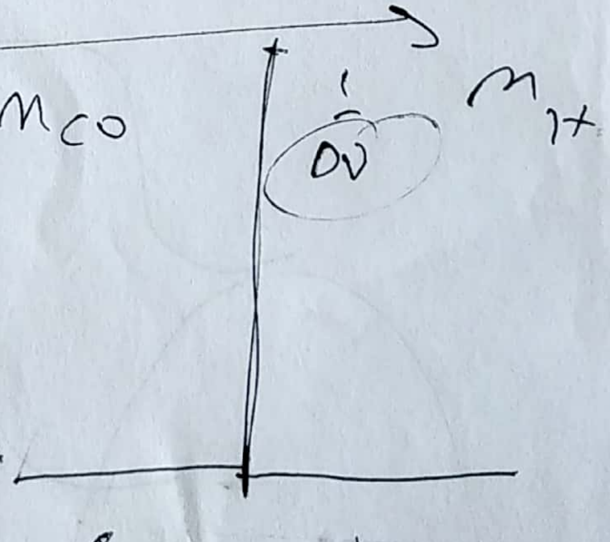


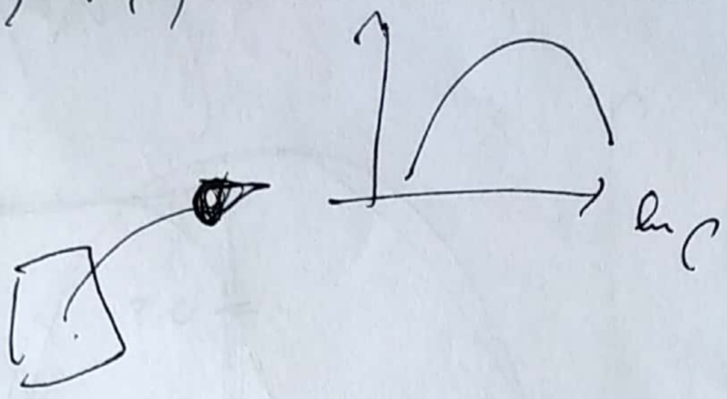
$$\overline{X_{H_2}} = \frac{\int_{\rho_{lim}}^{\rho_{lim'}} f \rho \, g \, dv}{\overline{\rho}_{bar}}$$



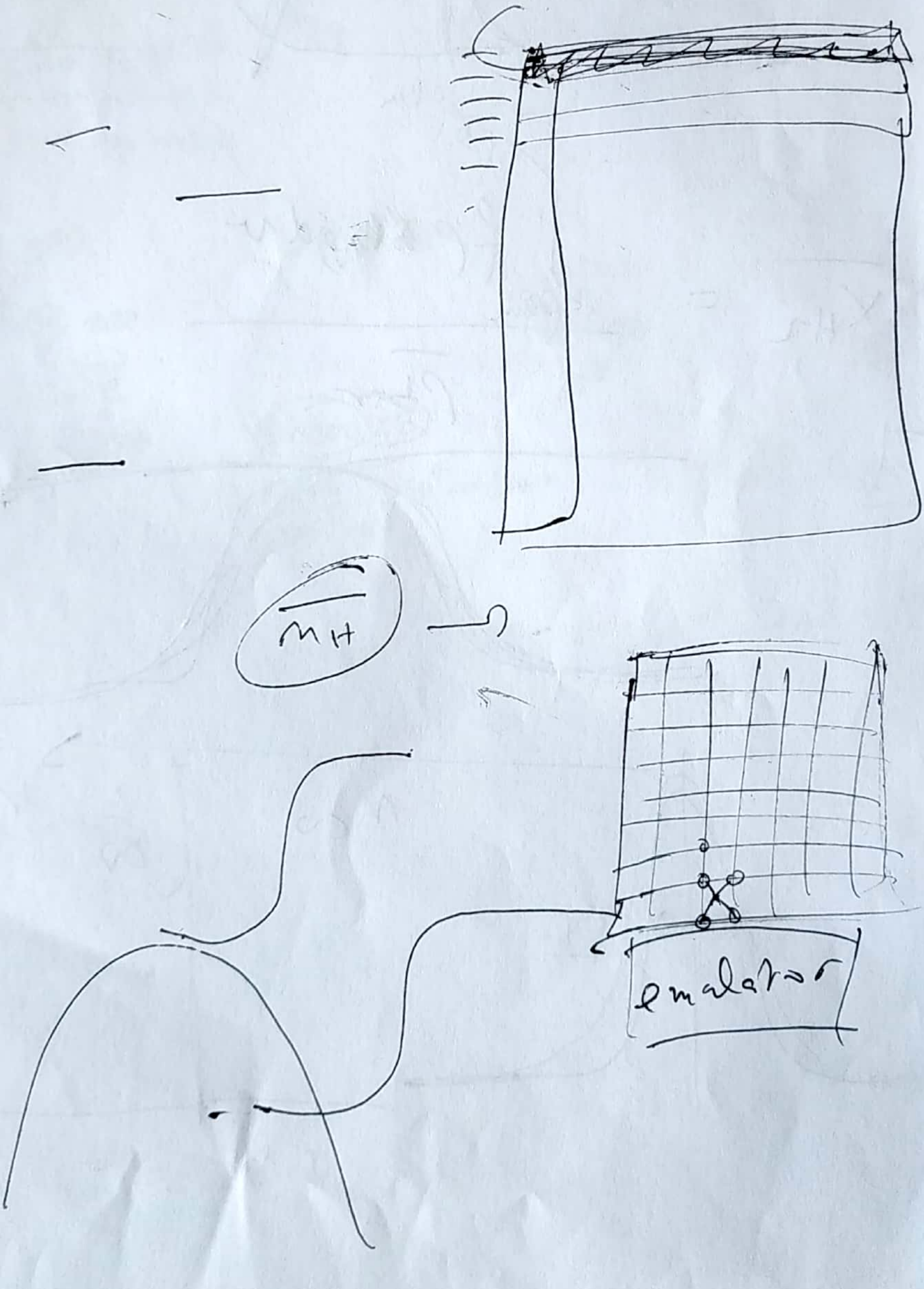



X_ith_lar( $\bar{m}_H$, M)




( $\bar{m}_H$, M)



17/12/2018

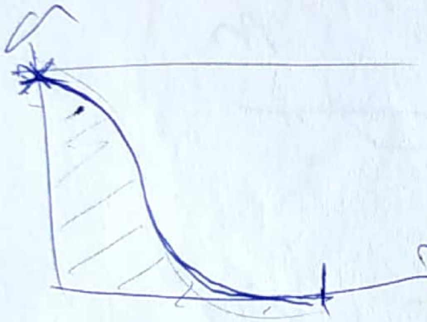
$H_2$ self shielding

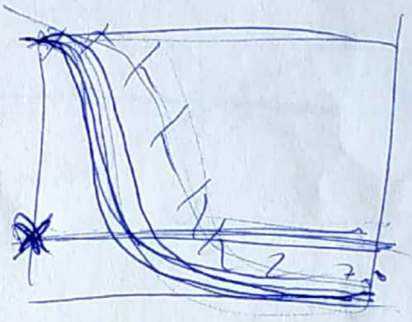

$\lambda_J(\rho)$

$\boxed{\text{Draine Formula}}$

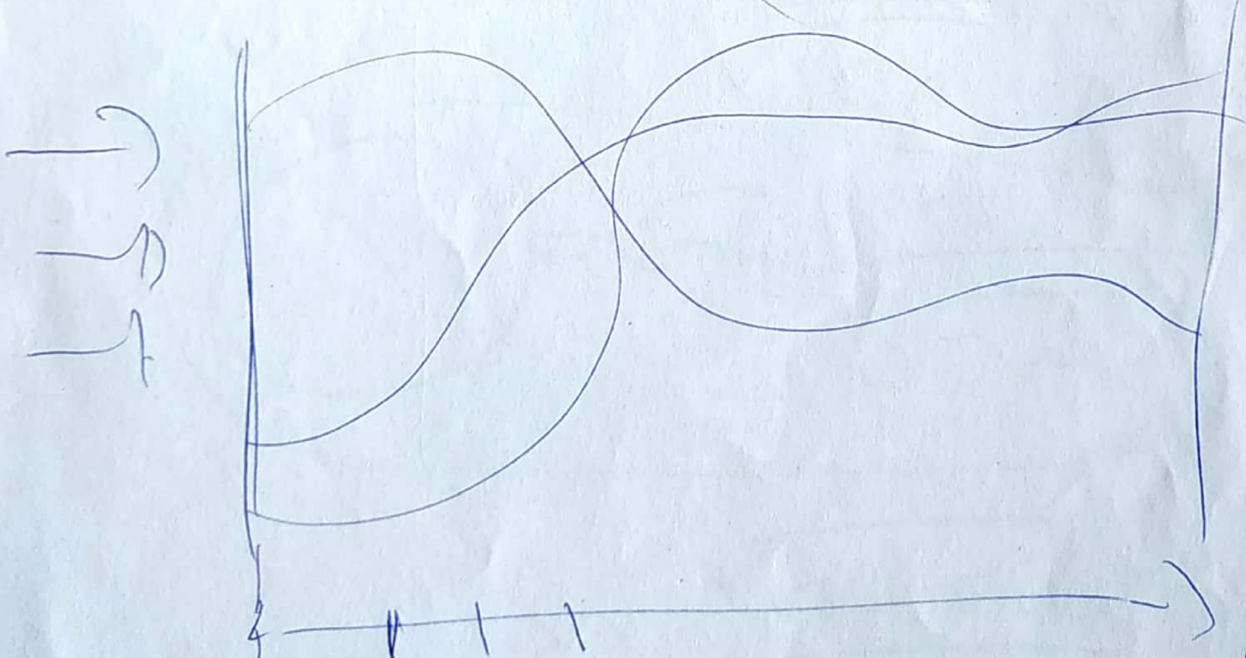
$$G = G_0 e^{-\tau} e^{-\tau_{H_2}} (1 - X_{H_2})$$

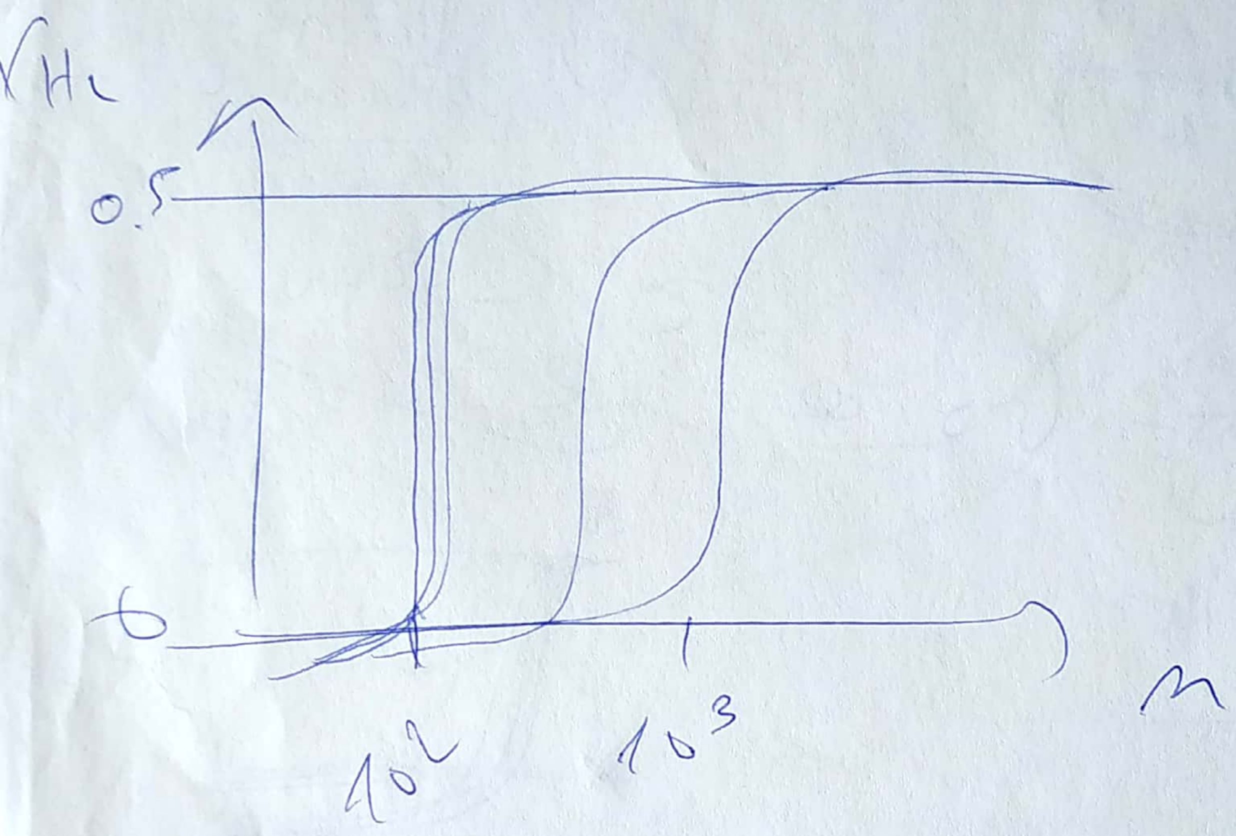
$X_{H_2}$   $X_{H_2} =$



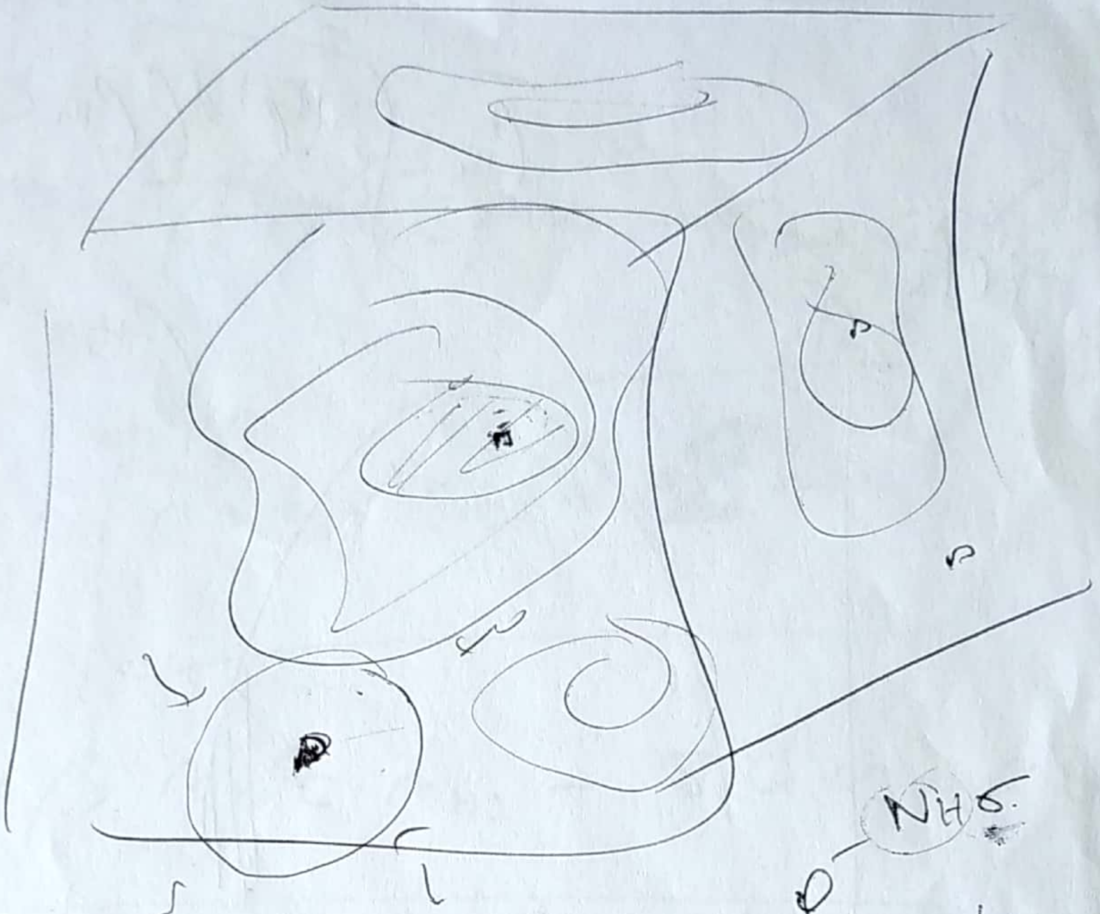
$\lambda_J \quad (n, T, G_0)$




rHe
0.5
0
$10^2$
$10^3$
n

05/11/18

(1)



$e^{-NH\sigma}$

$N_H = n_H \cdot e$

$= n_H \frac{c_s}{\sqrt{G\rho}}$

$e^{-n\sigma}$

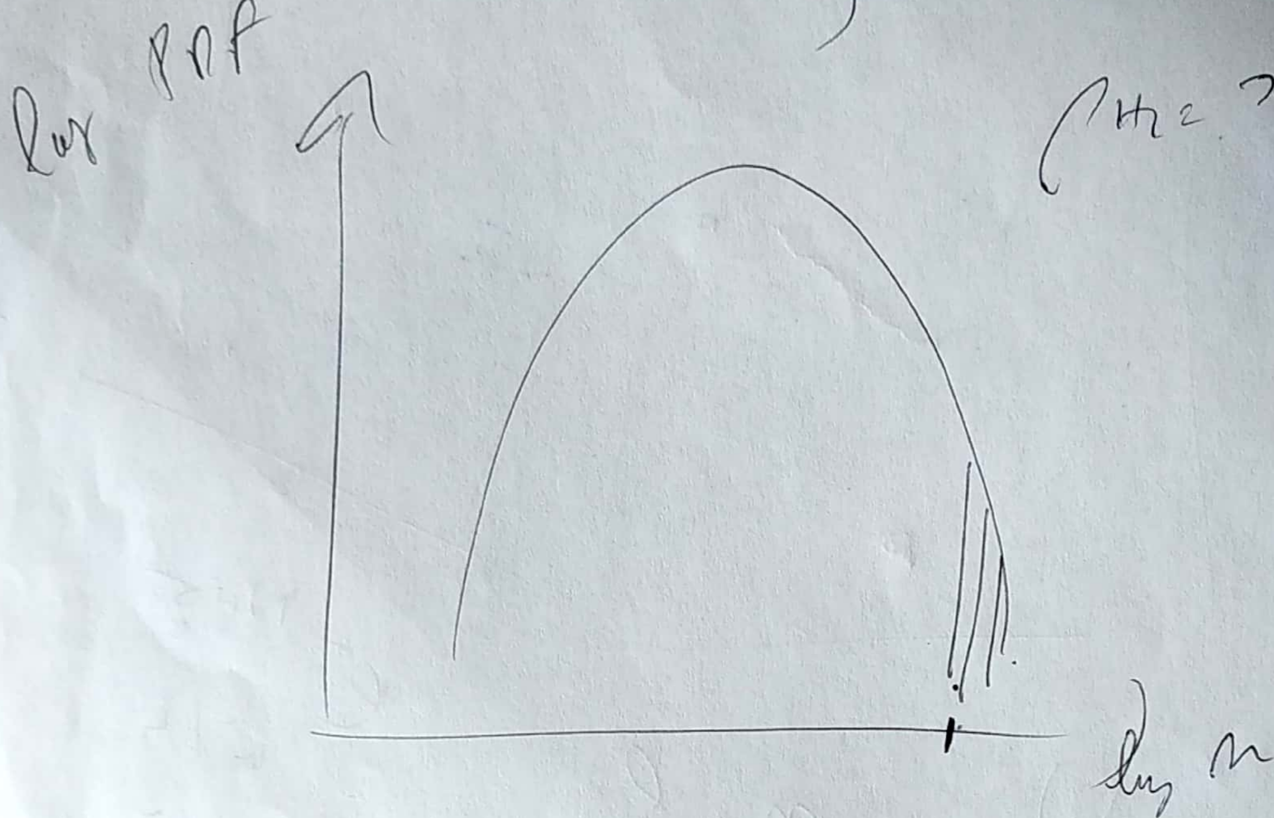
$e^{-\rho k}$

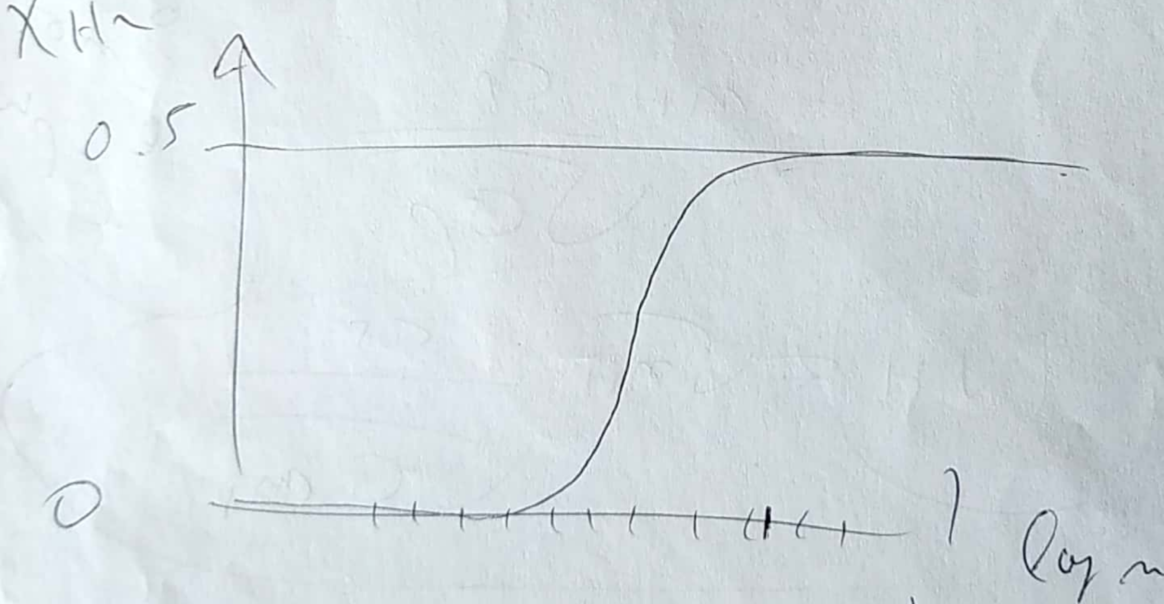
$N_H \propto \sqrt{n_H} \frac{c_s}{\sqrt{G m_p}}$

$\lambda = \frac{\sqrt{c_s^2 + \sigma_T^2}}{\sqrt{G\rho}}$

$$\bar{\rho}_{H_2} = \int \boxed{P(\rho)\, d\rho}\; \rho_{H_2}(\rho) \quad \tilde{\rho}$$

$$\rho_{H_2} = x_{H_2} \cdot \rho$$




$$\bar{\rho}_{H_2}(\bar{\rho}, \mathcal{M}, G)$$

$G_0 = 1$

$$\frac{d n_{H_2}}{dt} = \beta\, \cancel{n_H}\, \tau\, n_{HI} - \sigma_{dust}\, G\, \cancel{n_{H_2}}\, n_{LW}$$

$$\boxed{n_H = n_{HI} + 2 n_{H_2}}$$

$$0 = \beta\, n_H\, \tau\, n_{HI} - \sigma\, G\, n_{H_2}\, n_{LW}$$

$$\boxed{n_{HI} = \frac{\sigma G\, n_{LW}}{\beta \tau\, n_H}\, n_{H_2}}$$

$$n_H = \left(\frac{\sigma G\, n_{LW}}{\beta \tau\, n_H} + 2\right) n_{H_2}.$$

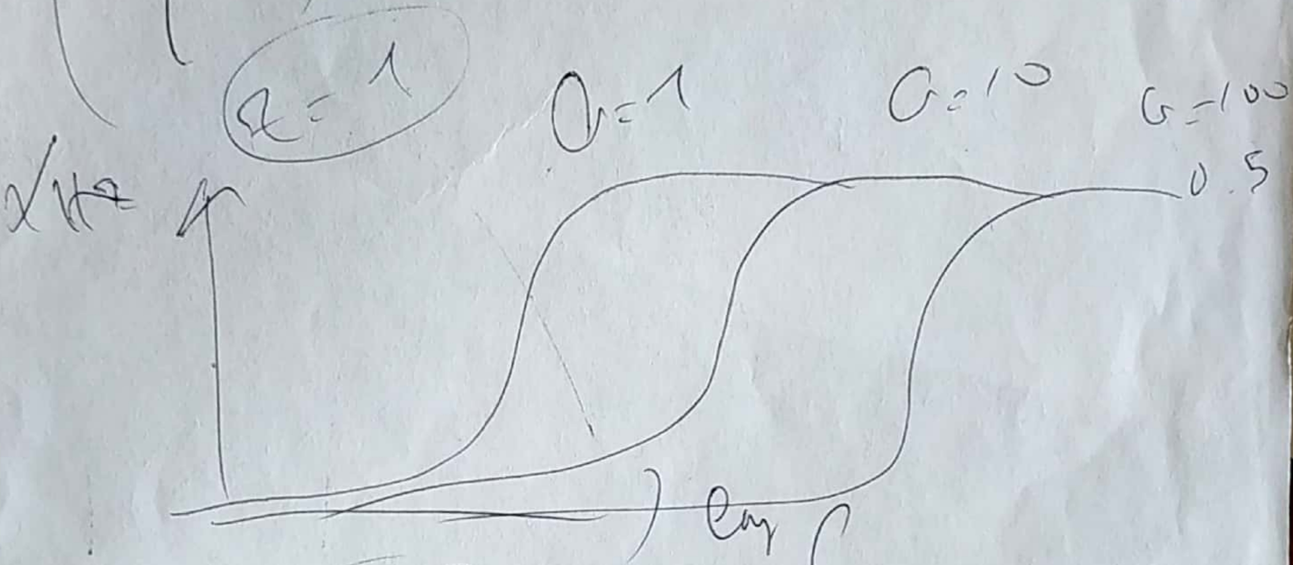
$$\boxed{x_{H_2} = \frac{n_{H_2}}{n_H} = \frac{1}{2 + \frac{\sigma G\, n_{LW}}{\beta \tau\, n_H}}}$$

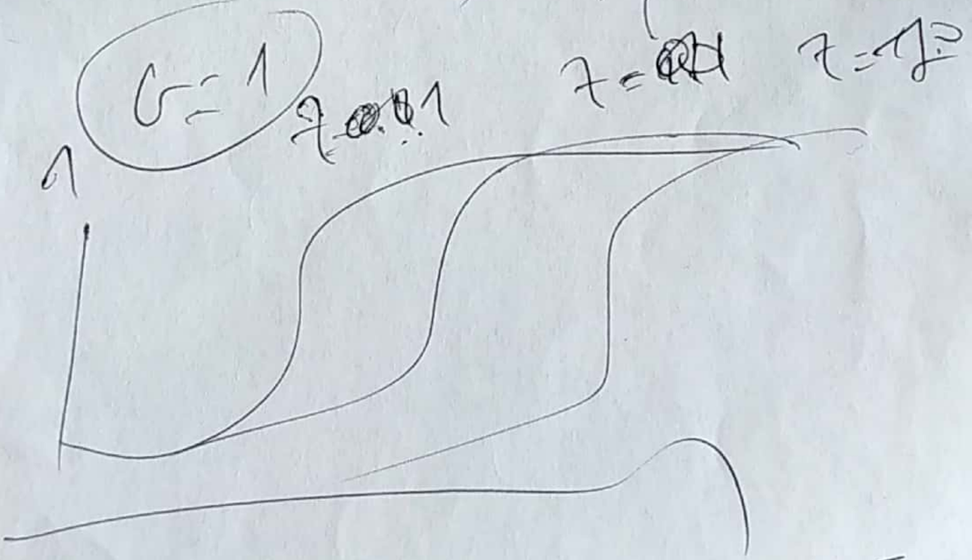
$g/cm^3 \quad cm^2/g \quad cm$

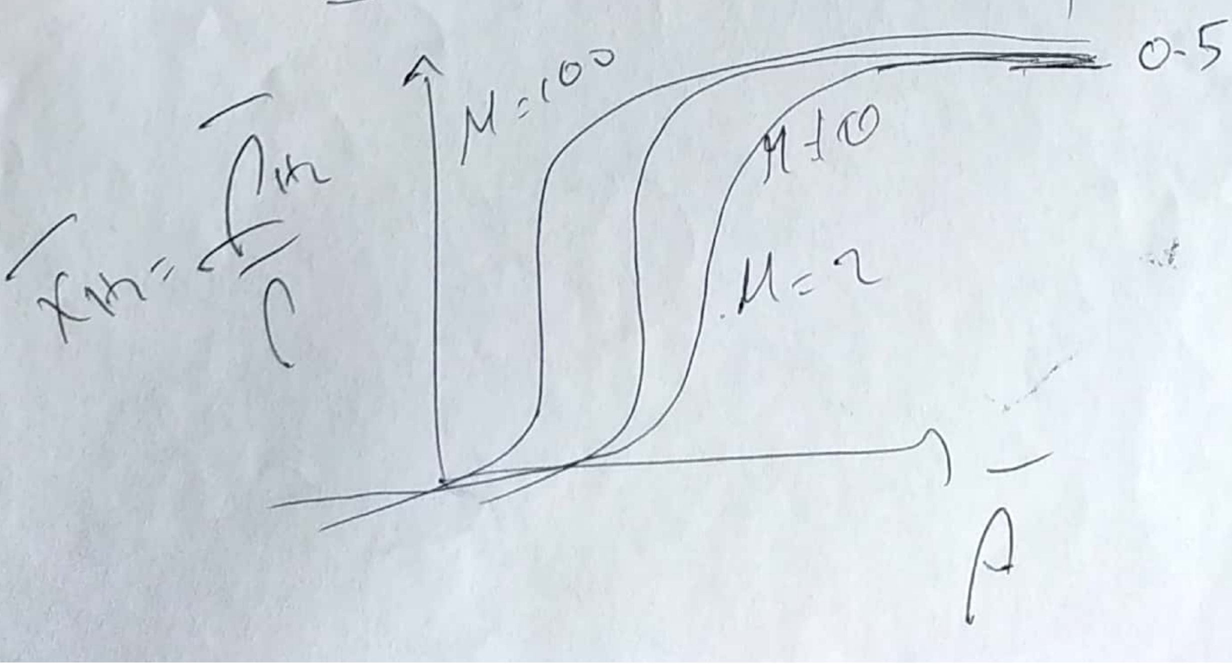
$$\tau = \rho \cdot \kappa_{dust}\, l$$

$$n_{LW} = (n_{LW})_{outside}\, e^{-\rho \kappa_{dust} l}$$

$$(\bar{\rho}, M, G, z)$$


z = 1
G = 1
G = 10
G = 100
0.5
$x_{H_2}$
$\rho_{eq}$


G = 1
z = 0.1
z = 1
z = 10


M = 100
M = 10
M = 2
0.5
$\bar{x}_{H_2} = \frac{\bar{\rho}_{H_2}}{\bar{\rho}}$
$\bar{\rho}$