Lognormal PDF :

$$P_v(\rho) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{1}{2} \left(\frac{\ln(\rho/\rho_0) - \langle \ln(\rho/\rho_0) \rangle}{\sigma} \right)^2 \right]$$

ρ_0 = mean cloud density

σ = vel. dispersion

$\langle \ln(\rho/\rho_0) \rangle = -\frac{\sigma^2}{2}$

$\sigma^2 = \ln(1 + b\mathcal{M}^2)$

b = parametrises the KE injection mechanism
= 0.3 → in this paper

$\mathcal{M}$ = Mach no.

Time-evolution :

when self-gravity becomes important, it changes the lognormal distribution ~~to~~ at higher densities to a skewed function.

$$\rho = \rho_0 \left[1 - \left(\frac{t}{t_{ff}} \right)^2 \right]^{-2} \Rightarrow \left\{ \rho_0 = \rho \left[1 - \left(t/t_{ff} \right)^2 \right]^2 \right\}$$

(Giriclidis et al. 2014)

t_{ff} = free-fall time

$$= \sqrt{\frac{3\pi}{32\rho_0 G}}$$

then, evolution of PDF :

$$\left[P_v(\rho, t) = P_v(\rho_0, 0) \frac{d\rho_0}{d\rho} \right]$$

This is the reason why density PDF of a GMC is a function of two parameters :

① $\mathcal{M}$ → Mach no. → which affects the lognormal part of the distribution.

② ratio (t/t_{ff}) → that determines the point at which the PDF significantly deviates from the lognormal part of the distribution.

Note : Photoevaporation is neglected in this paper

$R_{GMC} \rightarrow$ Radius of the GMC

$$V_{tot} = \frac{4}{3}\pi R_{GMC}^3$$

mean density, $\rho_0 = n_0 \mu m_p$ (μ = mean molecular weight for H_2 & He = 1.4)

fixed Mach no. = $\mathcal{M}$

evolutionary time = t/t_{ff}

normalisation of volume-weighted density PDF:

$$V_{tot} = \int P_V \, d\rho \quad ; \text{ where } P_V \equiv P_V(\rho, \mathcal{M}, t/t_{ff})$$

if ρ_i = density element of each clump,

then, length scale of each clump, $$r_i = \left(V_{tot} \int_{\rho_i - \delta_i}^{\rho_i + \delta_i} P_V \, d\rho \right)^{1/3}$$

Column density of gas element, $$N_i(\rho_i) = \frac{\rho_i r_i}{\mu m_p}$$

then, CO emission per unit volume from each density element ρ_i:

$$l_{CO,J}(\rho_i) = \frac{1}{V_{tot}} \mathcal{E}_{CO,J} \cdot 4\pi r_i^2$$

where, $\mathcal{E}_{CO,J} \equiv \mathcal{E}_{CO,J}(n_i, N_i, Z, G_0)$ = emissivity for CO of the $J \rightarrow J-1$ transition.

n_i = no. density of each clump

Z = metallicity

G_0 = FUV flux $\simeq 1.6\times10^{-3}$ erg/cm²/sec (Habing, 1968)

Then, total CO emission from GMC:

$$\left[L_{CO,J}^{tot} = \int l_{CO,J} \, d\rho = \int \mathcal{L}_{CO,J}(s) \, ds \right]$$

where $s = \ln(\rho/\rho_0)$

$$\mathcal{L} = l_{CO,J} \cdot \left(\frac{P_V}{\rho}\right)$$

$\mathcal{E}_{CO,J}$ = first 9 rotational transitions of the CO molecule

→ CLOUDY version c13.03 (Ferland et al 2013)

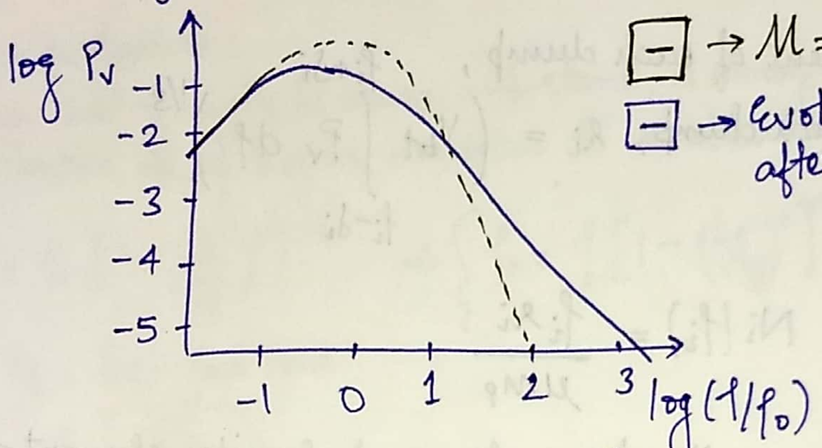
→ accounts for ~1000 reactions; including UMIST 2000 database (Le Teuff, Millar & Marwick, 2000). Also accounts for the dissociation of H_2 molecule, H_2 formation on the dust grain surface via gas-phase reactions, & also the primary & secondry cosmic ray ionisation processes.

For star formation :

STARBURST99 → used to calculate the spectral energy distribution (SED) of the radiation field by assuming an initial mass function in the range 1–100 $M_\odot$, stellar age 10 Myr, $Z_* = 1Z_\odot, 0.2Z_\odot, 0.05Z_\odot$, $G_0 = 10^0 - 10^{4.5}$

(Leitherer 1999)

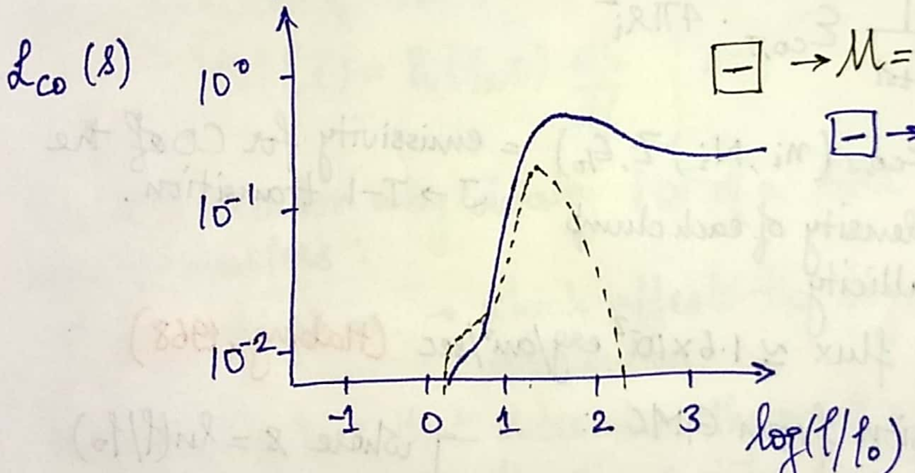
By adopting ~~these~~ this procedure :



→ $\mathcal{M} = 5$, initial lognormal PDF

→ Evolved PDF + tail after $t/t_{ff} = 0.4$

& resulting CO (1–0) luminosities :



→ $\mathcal{M} = 5$, initial lognormal PDF

→ Evolved PDF + tail after $t/t_{ff} = 0.4$

$t/t_{ff} = 0.4$ → this maximises the mass in the tail, highlighting the effect of the tail on the CO emission. (97% of the total mass of GMC)

Cloud properties → $n_0 = 100\ cm^{-3}$, $\rho_0 = 2.3 \times 10^{-22}\ g/cm^3$

$R_{GMC} = 15\ pc$, $\log(Z/Z_0) = 0$

$G_0 = 2$

[Larson 1981, Solomon 1987, Bolatto 2008] → $\boxed{\text{Larson Laws}}$

GMCs are in approximate virial equilibrium & obey these scaling relations. These form the link between size (R), velocity dispersion (σ) & CO luminosity (L'_{CO})

$$\left\{ \underset{(\text{in } L_\odot)}{L^*_{CO} = L'_{CO}} \times 3\times10^{-11}\,\nu_r^3 \; ; \; \nu_r = \text{rest frequency of the line expressed in GHz.} \right\}$$

(Carilli & Walter 2013)

$$\sigma \sim 0.7\, R^{0.5} \text{ km/s}$$
$$L'_{CO} \sim 130\,\sigma^5 \text{ K km/s pc}^2$$
$$\therefore \left\{ L'_{CO} \simeq 25\, R^{2.5} \text{ K pc}^2 \text{ km/s} \right\}$$

→ Assumption: molecular gas is dominating the mass enclosed in the cloud radius & therefore the virial mass (M_{vir}) is a good measure of the H_2 traced by CO.

virial equilibrium implies: $\sigma^2 \simeq \frac{GM_{vir}}{R}$

$$\therefore \left[M_{vir} \simeq 39\, L'^{\,0.81}_{CO}\, M_\odot \right]$$

For agreement with observations, they set:

M_{GMC}, R_{GMC}, P_v at $t/t_{ff} = 0.1$ → when 50% of the mass is in the tail, & the other 50% in the lognormal part.

for M:

$$M_{c_s} = \sqrt{\frac{GM_{GMC}}{R_{GMC}}} \quad , \quad c_s \simeq 0.3 \text{ km/s}$$

CO-to-H_2 conversion factor :

$$\alpha_{CO} \equiv \left(\frac{M_{H_2}}{L'_{CO}}\right) M_\odot \ (K\,km/s\,pc^2)^{-1}$$

M_{H_2} = molecular gas mass.

↳ for the CO(1-0) line luminosity

For M_{H_2}, or N_i (column density), there are 3 methods :

① assume GMCs are in virial equilibrium & derive H_2 mass from the CO line width → used in this paper (Larson 1981, Solomon 1987)

② assume a constant dust-to-gas ratio & use the dust continuum emission, possibly combined to H1 measurements, to infer N_{i,H_2} (Leroy 2011, Pineda 2008, Magdis 2011, Sandstrom 2013)

③ through γ-ray emission induced by cosmic ray interactions with H_2 molecules. (Padovani, Galli & Glassgold 2009)

~~[illegible]~~ Observationally, α_{CO} is determined by combining independent measurements of H_2 gass mass with the detection of the CO(1-0) line.

~~[illegible]~~