Theoretical Astrophysics
Exercise Sheet 5

HS 17
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Exercise 1 [Accretion disk in the stationary case]

(a) The equation for mass conservation in cylindrical coordinates is

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{r} \cdot \frac{\partial}{\partial r}(\Sigma\, v_r\, r). \tag{1}$$

Show that, for a *Keplerian disk*, the expression for the radial velocity is

$$v_r = -\frac{3}{\Sigma r^{\frac{1}{2}}} \cdot \frac{\partial}{\partial r}(\Sigma\, \nu\, r^{\frac{1}{2}}). \tag{2}$$

And using equation the above two equations, show that the time derivative of the surface density Σ can be written as

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \cdot \frac{\partial}{\partial r}\left[r^{\frac{1}{2}} \frac{\partial}{\partial r}(\Sigma\, \nu\, r^{\frac{1}{2}})\right]. \tag{3}$$

(b) Find the solution $\Sigma(r)$ for the stationary case ($\frac{\partial \Sigma}{\partial t} = 0$), with the boundary condition $\Sigma(R_0) = 0$ (here R_0 is the inner boundary of the disk), as a function of $\dot{M}$, ν and R_0.

(c) The rate of viscous dissipation per unit area is

$$D(R) = \nu \Sigma \left(R \frac{\partial \Omega}{\partial R}\right)^2 \tag{4}$$

Show that

$$L_{\text{disk}} = \frac{1}{2} \frac{GM\dot{M}}{R_0}, \tag{5}$$

where L_{disk} is the total luminosity of the disk. This result indicates that half the gravitational energy is released in accreting the gas to radius R_0.

(d) For an optically-thick thin disk, show $T \propto r^{-3/4}$.

– please turn over –

Exercise 2 [Bondi Accretion]
Starting from the 2nd Bernoulli theorem and the general expression for a constant mass accretion rate

$$\frac{u^2}{2} + \Pi - \frac{GM}{r} = 0 \quad \text{with} \quad \Pi = \int_{\rho_\infty}^{\rho} \frac{dP}{\rho}, \tag{6}$$

$$4\pi r^2 \rho u = -\dot{M} = \text{const.} \tag{7}$$

(a) Calculate the sonic transition point and the corresponding dimensionless mass accretion rate

$$\lambda = \frac{\dot{M}}{\dot{M}_\mathrm{B}} \quad \text{with} \quad \dot{M}_\mathrm{B} = \frac{4\pi\rho_\infty G^2 M^2}{c_\infty^3}, \tag{8}$$

by following the same derivation as in the lecture for an *isothermal* gas ($P = c_\infty^2 \rho$). And like in the lecture, use the dimensionless variables $x = r/r_\mathrm{B}$ (here $r_\mathrm{B} = GM/c_\infty^2$ is called the *Bondi radius*), $v = |u|/c_\infty$ and $\alpha = \rho/\rho_\infty$.

(b) Now we want to consider the case of a polytropic gas with

$$P = P_\infty \left(\frac{\rho}{\rho_\infty}\right)^\gamma \quad \text{and} \quad c_\infty^2 = \gamma \frac{P_\infty}{\rho_\infty}. \tag{9}$$

Find the sonic transition point for the case $\gamma = 5/3$.

Exercise 3 [Parker's Solar Wind Solution]
Consider the solar wind as an isothermal and steady plasma. You can again start from the 2nd Bernoulli theorem, like in exercise 2, and get the expression for $u(r)$. But now we assume that the kinetic energy of the plasma vanishes at the origin. There is again a subsonic solution which accelerates until r_s and then decelerates again, and a solution with sonic transition at r_s, which accelerates until infinity (see Fig. 6.11 in F. Shu: Gas Dynamics). Compute now the sonic radius r_s and the sonic density ρ_s at the sonic transition $u = c_\mathrm{s}$, assuming a corona temperature of 2×10^6 K and a mass-loss rate of $\dot{M} = 2 \times 10^{-14}\ M_\odot\ \mathrm{yr}^{-1}$.