Lecture 22 Stability of Molecular Clouds

1. Stability of Cloud Cores
2. Collapse and Fragmentation of Clouds
3. Applying the Virial Theorem

References
• Myers, "Physical Conditions in Molecular Clouds" in Origins of Stars & Planetary Systems eds. Lada & Kylafis
   http://www.cfa.harvard.edu/events/1999crete
• For the Virial Theorem: Shu II, Ch. 12 & Stahler, Appendix D

1. Stability of Molecular Cloud Cores

Summary of molecular cloud core properties (Lec. 21) relating to *virial equilibrium*:

1. Location of star formation
2. Elongated (aspect ratio ~ 2:1)
3. Internal dynamics dominated by thermal or by turbulent motion
4. Often in approximate virial equilibrium
5. Temperature: $T \sim 5 - 30$ K
6. Size: $R \sim 0.1$ pc
7. Ionization fraction: $x_e \sim 10^{-7}$
8. Size-line width relation*
   $$R \sim \sigma^{p}, \quad p = 0.5 \pm 0.2$$
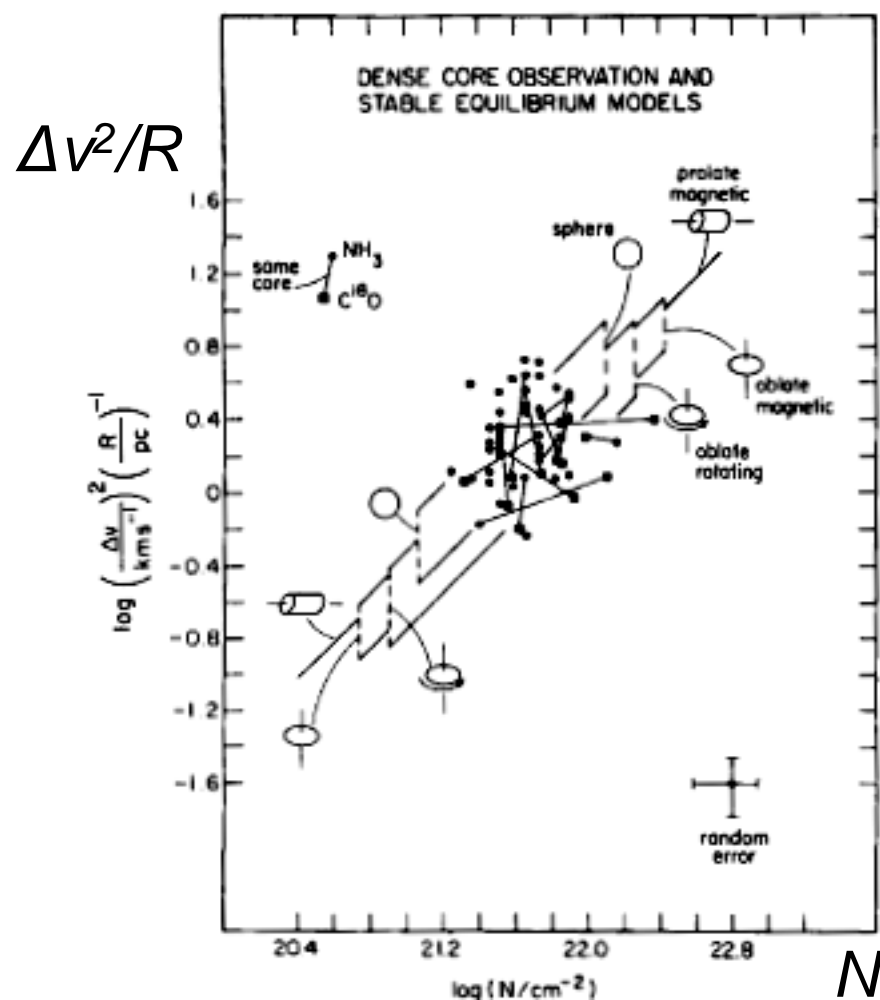9. Mass spectrum similar to GMCs.

* Does not apply always apply, e.g. the Pipe Nebula (Lada et al. 2008).

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Non-Thermal vs. Thermal Core Line Widths
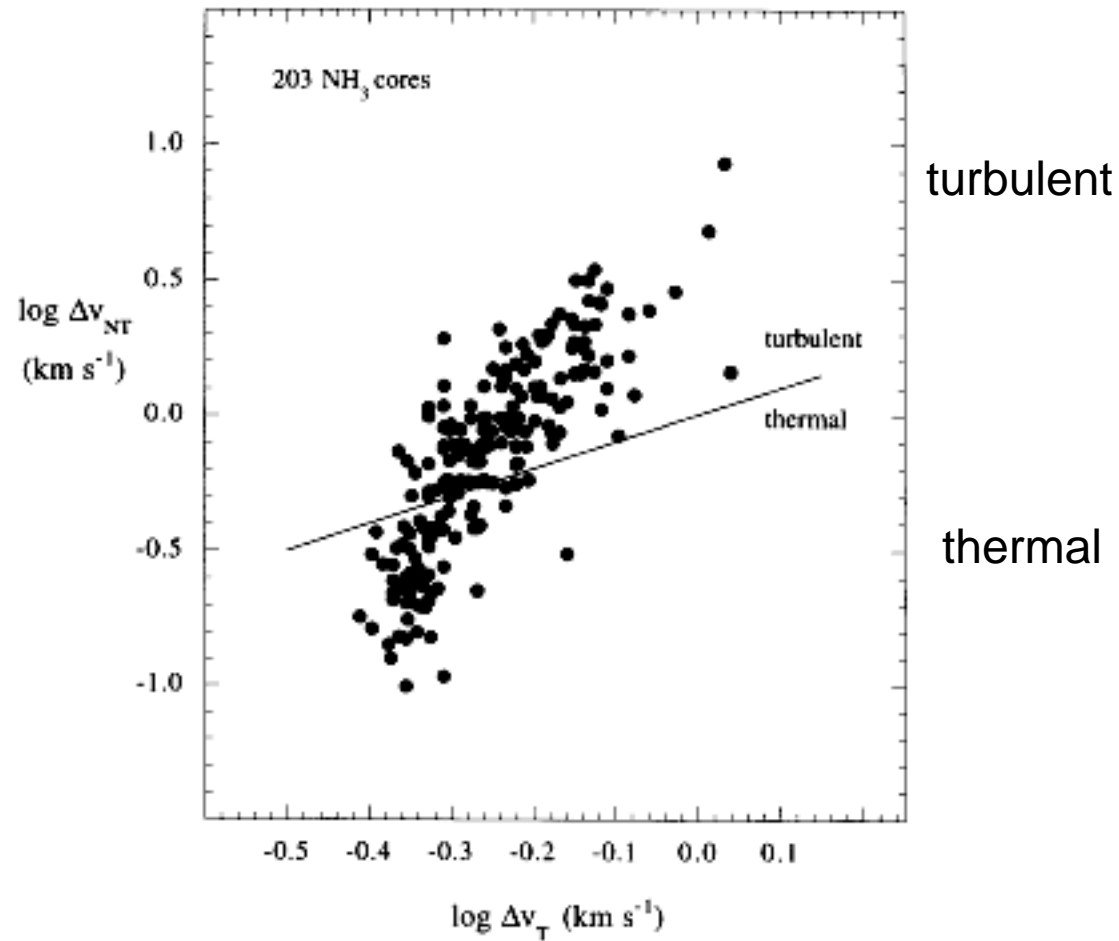
Turbulent cores are warmer than quiescent cores

Consistency of Cores and Virial Equilibrium

$\Delta v^2 / R$

DENSE CORE OBSERVATION AND STABLE EQUILIBRIUM MODELS

Gravitational energy per unit mass vs. Column Density

Models of virial equilibrium (funny symbols) easily fit the observations (solid circles [NH_3] and squares [$C^{18}O$]).

Recall that the linewidth-size relation implies that both the ordinate and abscissa are ~ constant. Myers' data plot illustrates this result, at least approximately for a diverse sample of cores.

For pure thermal support, $GM/R \sim kT/m$, $(m = 2.3\ m_H)$, or
$$R \approx 0.1\ (M / M_\odot)\ (10\ K\ /\ T)\ \text{pc.}$$

First Consideration of Core Stability

• Virial equilibrium seems to be an appropriate state from which cores proceed to make stars. However, because a significant fraction of cores have embedded protostars (observed in the IR by IRAS), they can't have been quiescent forever.

• Therefore core stability is an important issue, e.g., why are they stable (if they are), and how do they become de-stabilized and collapse under gravity to form stars.

• Although Myer's data presentation suggests that random motion (thermal and/or non-thermal) may stabilize cores against gravitational collapse, we also consider other possibilities, in particular rotation and magnetic fields.

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Effects of Rotation

Molecular cloud cores have *modest* velocity gradients,
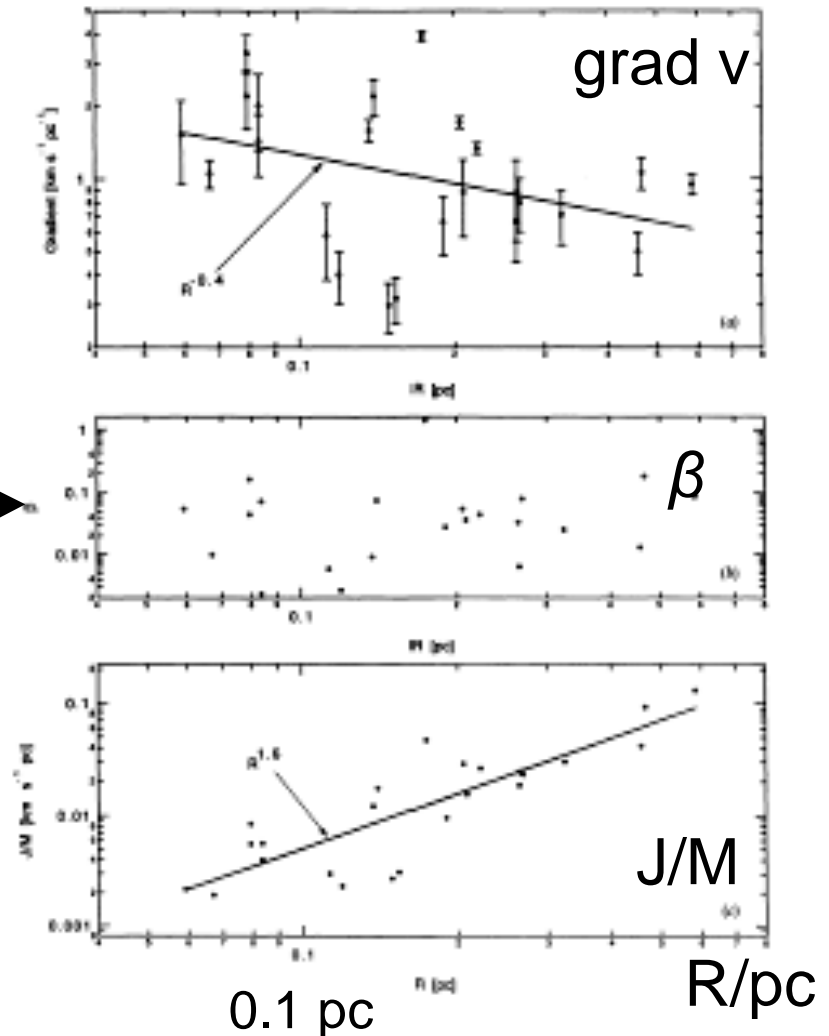$$0.4 - 3 \text{ km s}^{-1} \text{ pc}^{-1}$$
and angular speeds
$$\Omega \sim 10^{-14} - 10^{-13} \text{ rad s}^{-1.}$$

$$\beta = E_{rot} / E_{grav} \sim 0.02 \longrightarrow$$

Cores do not appear to be supported by rotation.

NB Not all of the observed gradients correspond to the overall rotation of the core

grad v

β

J/M

0.1 pc

R/pc

Goodman et al. ApJ 406 528 1993

Magnetic Pressure

- Magnetic fields are difficult to measure in cloud cores with the Zeeman effect
- Example of dark cloud B1:

  $|\mathbf{B}| \cos \theta \approx -19 \pm 4$ µG

  $B^2/8\pi \sim 3 \times 10^5$ K cm^{-3}

- This value of $B^2/8\pi$ is much greater than the thermal pressure of a 10 K core with n ~ 10^3 cm^{-3}, but cores may well have higher densities. (OH probes $n_H \approx 10^3$ cm^{-3})
- Magnetic fields are potentially important.

Crutcher 1993 ApJ 407 175
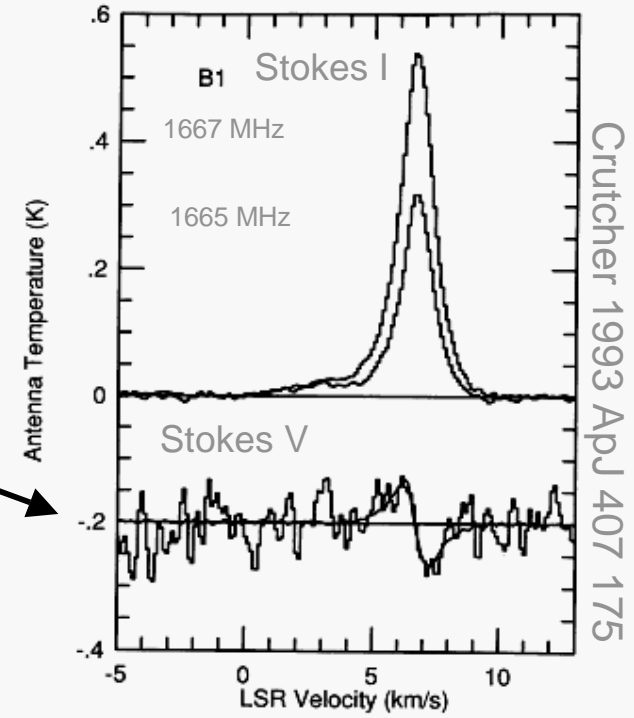
B1 - First dark cloud OH Zeeman measurement
Recall Lecture 9 and 21 cm measurements of the HI Zeemaneffect.

2. Collapse and Fragmentation

How do cores condense out of the lower density regions of GMCs?

- The conventional wisdom is local **gravitational instabilities** in a globally stable GMC via the classical **Jeans instability**.
- Consider a uniform isothermal gas in hydrostatic equilibrium with gravity balanced by the pressure gradient. Now suppose a small spherical region of size r is perturbed

$$\rho_0 \rightarrow X\rho_0 \quad \text{with} \quad X > 1$$

The over-density $X\rho_0$ generates an outward pressure force per unit mass:

Simple Stability Criterion

$$F_p \sim |\nabla p| / \rho_0 \sim Xc^2 / r$$

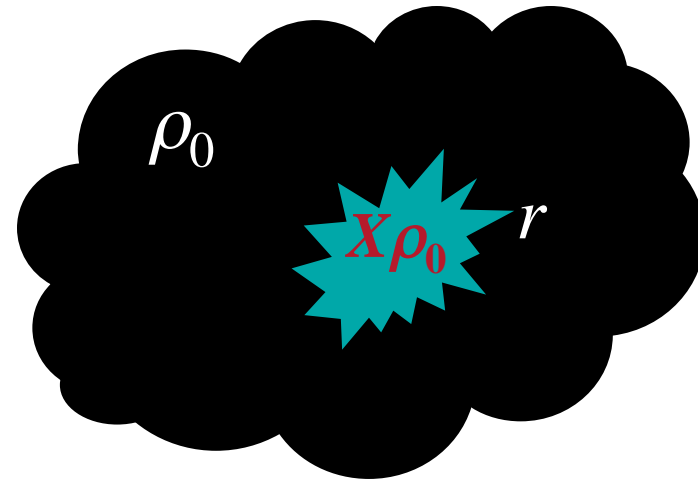The increased density leads to an inward gravtitational force per unit mass

$$F_G \sim GM / r^2 \sim GX\rho_0 r.$$

Gravity wins out if

$$r^2 > \frac{c^2}{G\rho_0} \quad \text{or} \quad r > \frac{c}{\sqrt{G\rho_0}}$$

The right hand side is essentially the Jeans length:

$$\lambda_J = c\sqrt{\frac{\pi}{G\rho_0}}$$

ρ_0

$X\rho_0$ r

The Jeans Length & Mass

From the Jeans length

$$\lambda_J = c\sqrt{\frac{\pi}{G\rho_0}} = 0.189 \text{ pc} \left(T/10\text{K}\right)^{1/2} \left(n(\text{H}_2)/10^4\,\text{cm}^{-3}\right)^{-1/2}$$

we get a corresponding Jeans mass

$$M_J = \lambda_J^3 \rho_0 = \left(\frac{\pi c^2}{G}\right)^{3/2} \rho_0^{-1/2} \propto \frac{c^3}{\sqrt{\rho_0}}$$

whose numerical value is

$$M_J = 4.79 M_{\text{sun}} \left(T/10\text{K}\right)^{3/2} \left(n(\text{H}_2)/10^4\,\text{cm}^{-3}\right)^{-1/2}$$

Length scales > λ_J or masses > M_J are unstable against gravitational collapse.

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Relevance of the Jeans Analysis

Many assumptions have been made implicitly in the above analysis that ignore the basic fact that **molecular clouds are not quiescent but are turbulent** on large scales.

Turbulence is believed to provide an effective pressure support, and its supersonic motions must be dissipated by shocks before the collapse can proceed. Turbulent pressure is usually mocked up by replacing the sound velocity *c,* e.g., in the Jeans formulae by

$$c^2 = \frac{kT}{m} + \sigma_{turb}^2$$

As the gas condenses, the Jeans mass decreases, which suggests **fragmentation** (Hoyle 1953 ApJ 118 513), i.e., the collapse cascades into smaller and smaller masses.

Does Fragmentation Occur?

- The dispersion relation for a perturbation $\delta\rho \sim e^{i(\omega t - kx)}$ in the Jeans problem is
$$\omega^2 = c^2 k^2 - 4\pi G\rho_0$$

or

$$\omega^2 = c^2 (k^2 - k_J^2), \quad k_J^2 = 4\pi G\rho_0 / c^2$$

- $k^2 - k_J^2 < 0$ makes ω imaginary:
  - Exponential growth occurs for $k < k_J$
  - Growth rate $-i\omega$ increases monotonically with *decreasing k,* which implies that
    - the longest wavelength perturbations (largest mass) grow the fastest
    - the fast collapse on the largest scales suggests that ***fragmentation is unlikely***

    (Larson 1985 MNRAS 214 379)

Swindled by Jeans?

Consider an alternate model, a thin sheet of surface density $\sum$.
The dispersion relation is

$$\omega^2 = c^2 k^2 - 2\pi \, G \sum |k|$$

or

$$\omega^2 = c^2 \left(k^2 - k_c \, |k| \right)$$

Exponential growth occurs for

$$k < k_c = 2\pi \, G \sum / c^2$$

with growth rate

$$-i \, \omega = \left(2\pi \, G \sum k - c^2 k^2 \right)^{1/2}$$

which is maximum at $k_f = k_c/2 = \pi \, G \sum / c^2$
and gives a preferred mass,

$$M_f \sim (2\pi / k_f)^2 \sum = 4 \, c^4 / G^2 \sum$$

This is a different result than the previous Jeans analysis for an isotropic 3-d medium. The maximum growth rate now occurs on an intermediate scale that is smaller than predicted by Jeans, possibly favoring fragmentation.

Preferred Length & Mass

- Apply the thin sheet model to the Taurus dark cloud.
  - $T = 10$ K, $c = 0.19$ km/s
    - $\lambda_c \sim 0.05$ pc
  - A(V) $\approx$ 5 mag. or $\sum \approx 0.032$ g cm^{-2}
    - $\lambda_f \sim 0.1$ pc, $M_f \sim 2$ M$_\odot$
    - $t_f = \lambda_f /(\pi\, c) \sim 2 \times 10^5$ yr
- More realistic analyses tend to show that, when thermal pressure provides the dominant support against gravity, there is a minimum length & mass scale which can grow
  - If the cloud is non-uniform there is a preferred scale
    - Typically $\sim$ few times the characteristic length of the background, e.g., the scale height, $H = c^2 /\pi G\sum$, for a gaseous equilibrium sheet (Larson 1985 MNRAS 214 379)

3.The Virial Theorem and Stability

A. <u>General Considerations</u>

> The derivation of virial theorem starts with the equation of motion for the *macroscopic* velocity of the fluid after averaging over the random thermal velocities. The mathematics is given by Shu II Ch. 24 and Stahler Appendix D.

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla}P - \rho\vec{\nabla}\phi + \vec{\nabla}\cdot\vec{\vec{T}}$$

$$T_{ij} = B_i B_j / 4\pi - B^2\delta_{ij} / 8\pi$$

$$\vec{\nabla}\cdot\vec{\vec{T}} = -\frac{1}{8\pi}\nabla B^2 + \frac{1}{4\pi}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}$$

The perspective of the Virial Theorem is that these equations contain too much information for quick comprehension. Hence, one takes moments, specifically the first moment:

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Virial Theorem: LHS Analysis

Take the dot product with **r** and integrate the equation of motion over a finite volume, and analyze the right and left hand sides separately.

The LHS side yields two terms since:

$$\int_{V} \rho \, \ddot{\vec{r}} \cdot \vec{r} \, dV = \frac{1}{2} \frac{D^2}{Dt^2} \int_{V} (\vec{r} \cdot \vec{r}) \rho \, dV - \int_{V} (\vec{v} \cdot \vec{v}) \rho \, dV$$

and

$$\frac{D^2}{Dt^2}(\vec{r} \cdot \vec{r}) = 2\left(\dot{\vec{r}} \cdot \dot{\vec{r}} + \vec{r} \cdot \ddot{\vec{r}} \right)$$

and finally

$$\frac{1}{2} \frac{D^2}{Dt^2} \left(\int_{V} r^2 \rho \, dV \right) - \int_{V} v^2 \rho \, dV = \frac{1}{2} \frac{D^2 I}{Dt^2} - 2E_K$$

V

The 1st term vanishes for a static cloud.
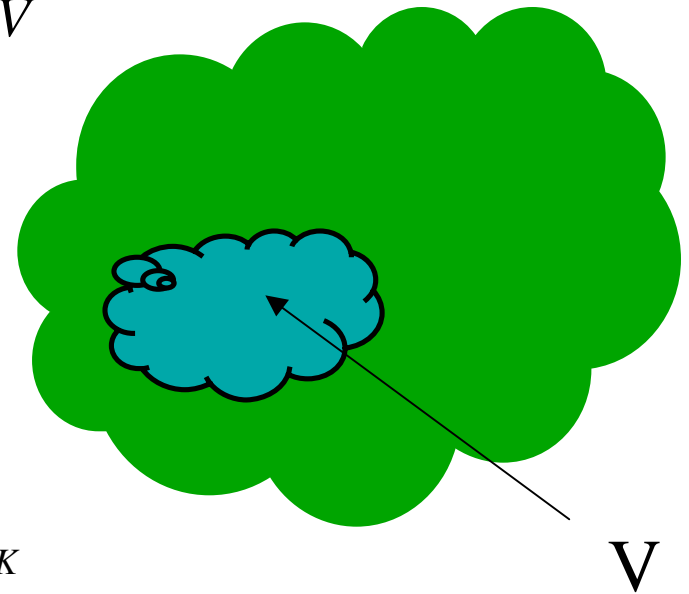
Virial Theorem: RHS Analysis

- The RHS yields volume and surface terms

$$\int_V 3P\,dV + \int_V \frac{B^2}{8\pi}\,dV - \int_V \left(\vec{r}\cdot\vec{\nabla}\phi\right)dm$$

$$-\int_S \left(P + \frac{B^2}{8\pi}\right)\vec{r}\cdot d\vec{S} + \frac{1}{4\pi}\int_S \left(\vec{r}\cdot\vec{B}\right)\left(\vec{B}\cdot d\vec{S}\right)$$

- The 3rd term becomes the gravitational energy.
- The 1st and 4th terms measure the difference between the external and mean internal pressures.
- The other terms are transformed into magnetic pressure and tension.

The Virial Theorem

$$\frac{D^2 I}{Dt^2} = 2E_K + 3\left(\langle P \rangle - P_{ext}\right) V + W + M$$

where

$$M = \int dV \frac{B^2}{8\pi} + \frac{1}{4\pi} \int d\vec{S} \cdot \vec{B}(\vec{B} \cdot \vec{r}) - \frac{1}{8\pi} \int d\vec{S} \cdot \vec{r}\, B^2$$

For a static cloud, we can solve for the external pressure

$$P_{ext}\, V - \langle P \rangle\, V = \frac{2}{3} E_K + \frac{1}{3} W + \frac{1}{3} M$$

and then insert approximate expressions in the RHS.

P_{ext} becomes a function of the global variables of the volume V and of the gas inside. If there is no rotation, $E_K = 0$.

Simplest Application of the Virial Theorem

Consider a spherical, isothermal, unmagnetized and non-turbulent cloud of mass M and radius R in equilibrium :

$$4\pi R^3 P_{ext} = 3Mc^2 - \frac{3}{5}\frac{GM^2}{R}$$

where we have used $P = \rho c^2$. If we now also ignore gravity, we check that the external and internal pressures are the same. In the presence of gravity, the second term serves to lower the external pressure needed to confine the gas Inside volume V.

Next, divide by $4\pi R^3$ and plot the RHS vs. R
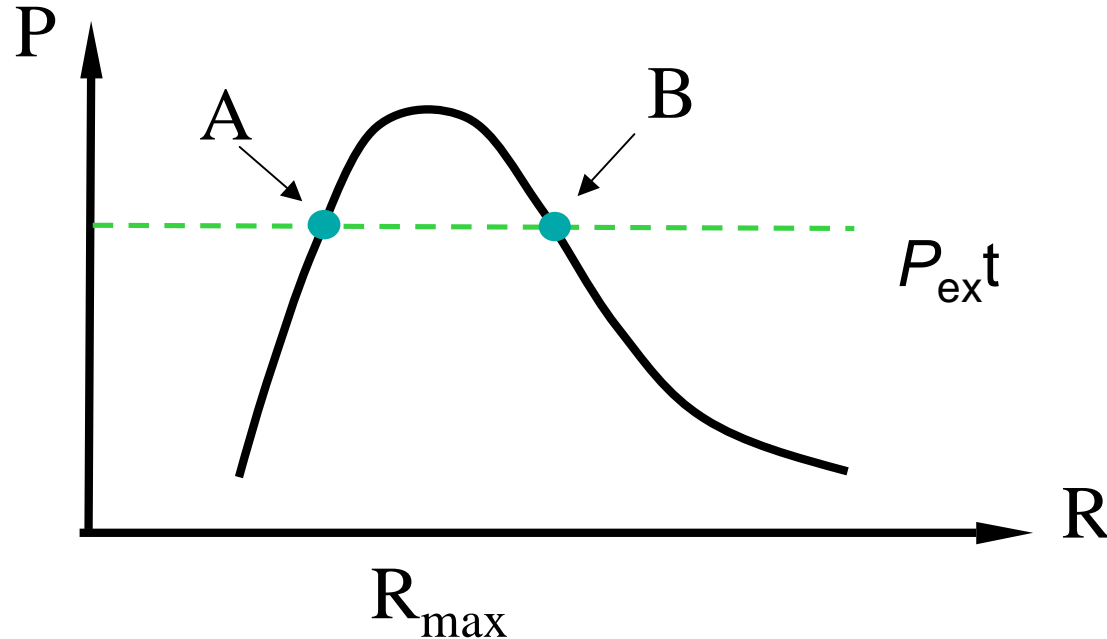
Simplest Application of the Virial Theorem

$$P_{\text{ext}} = \frac{3c^2 M}{4\pi}\frac{1}{R^3} - \frac{3GM^2}{20\pi}\frac{1}{R^4}$$
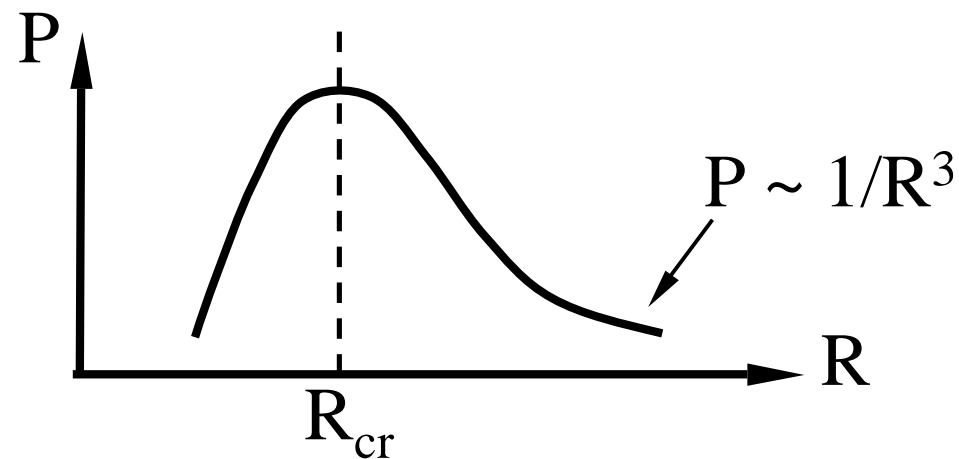
- There is a minimum radius below which this model cloud cannot support itself against gravity.
- States along the left segment, where *P* increases with *R,* are unstable.

Stable and Unstable Virial Equlibria

For a given external pressure, there are two equilibria, but only one is stable.

Squeeze the cloud at *B (decrease R)*
- – Requires more pressure to confine it: re-expands (stable)

Squeeze the cloud at *A (decrease R)*
- – Requires less pressure to confine it: contracts (unstable)

The Critical Pressure

Differentiating the external pressure for fixed M and c

$$P_{\text{ext}} = \frac{3c^2 M}{4\pi} \frac{1}{R^3} - \frac{3GM^2}{20\pi} \frac{1}{R^4}$$

gives the critical radius,

$$R_{\text{cr}} \approx \frac{GM}{c^2}$$

The corresponding density and mass are

$$\rho_{\text{cr}} \approx \frac{c^6}{M^3} \qquad M_{\text{cr}} \approx \frac{c^3}{\sqrt{\rho_{\text{cr}}}}$$

This last result is essentially the same as gotten from the Jeans analysis.

Fragmentation vs. Stability
Reconciliation of Jeans and Virial Analysis

- Both methods yield a similar characteristic "Jeans" mass
  - The difference is in the initial conditions
    - Cloud scenario assumes a self-gravitating cloud core in hydrostatic equilibrium
      - The cloud may or may not be close to the critical condition for collapse
    - In the gravitational fragmentation picture there is no "cloud"
      - It cannot be distinguished from the background **until it has started to collapse**

  Both perspectives may be applicable in different parts of GMCs
    - There are pressure confined clumps in GMCs
      - Not strongly self-gravitating
    - Cores make stars
      - Must be self-gravitating
      - If fragmentation produces cores these must be collapsing and it is too late to apply virial equilibrium

Fragmentation vs. Stability

- A relevant question is how quickly cores form (relative to the dynamical time scale)

    – NH_3 cores seem to be close to virial equilibrium but they could be contracting (or expanding) slowly

- Statistics of cores with and without stars should reveal their ages

    – If cores are stable, there should be many more cores without stars than with young stars

    – Comparison of NH_3 cores and *IRAS* sources suggests ~ 1/4 of cores have young stellar objects (Wood et al. ApJS 95 457 1994), but Jijina et al. give a large fraction (The corresponding life time is < 1 Myr (T Onishi et al. 1996 ApJ 465 815)

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