

(I) PDF Evolution: $M = 5, \bar{n}_H = 100 [H]/cc$

$$\text{pdf} = \frac{1}{\sqrt{2\pi\sigma_s^2}} \cdot \exp \left[-\frac{1}{2} \left(\frac{s - \bar{s}}{\sigma_s} \right)^2 \right]$$

$$\sigma_s = \sqrt{\log(1 + b^2 M^2)} ; \bar{s} = -\frac{\sigma_s^2}{2}$$

$$\varepsilon = 0.16 \quad \left(\frac{t^2}{E_H(\bar{n}_H)} = \varepsilon = (0.4)^2 \right)$$

$$z = \frac{[1 + 2\varepsilon \exp(s) - \sqrt{1 + 4\varepsilon \exp(s)}]}{2\varepsilon^2 \exp(s)}$$

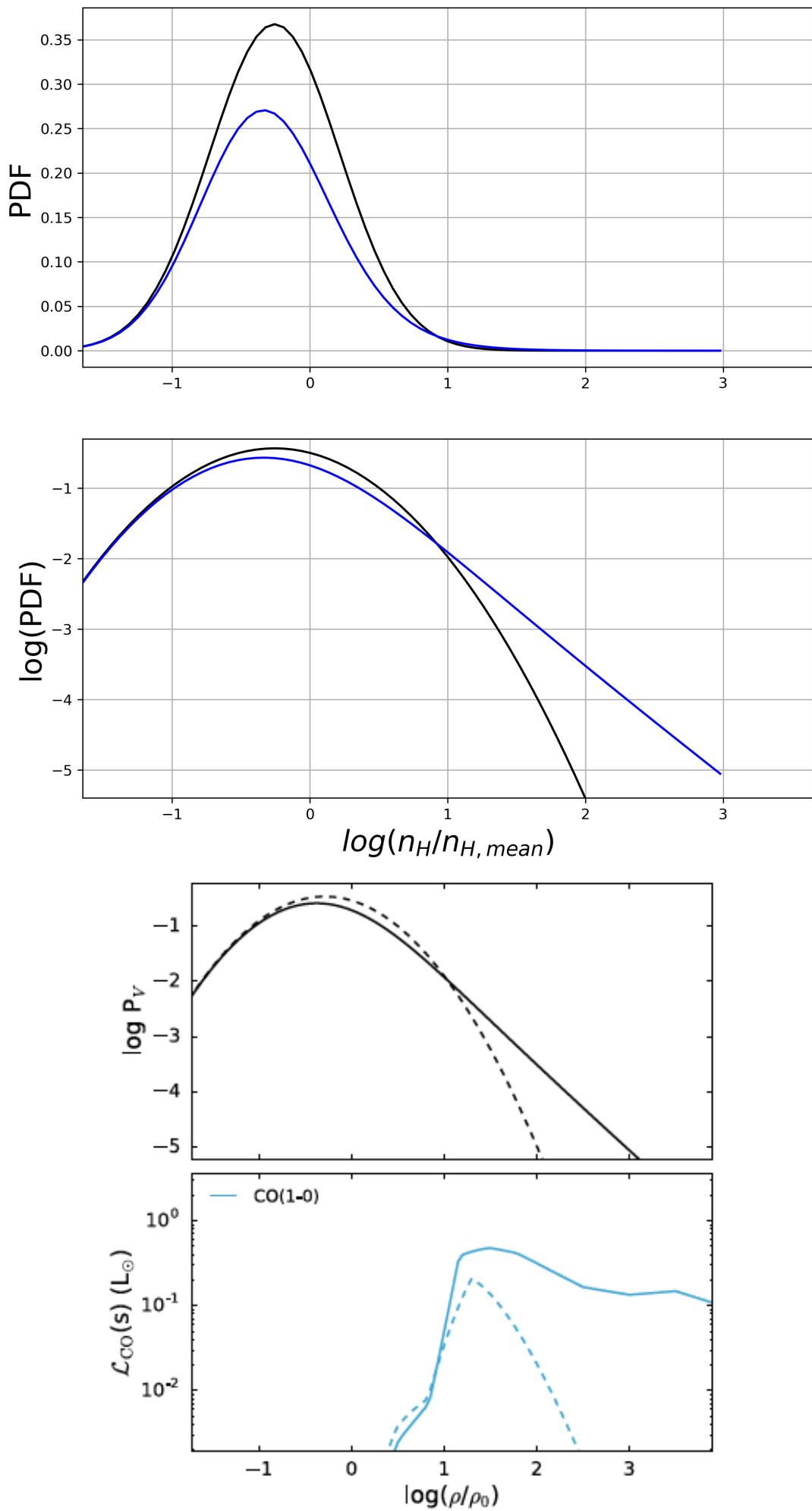
$$s' = \ln(z)$$

$$\text{pdf}' = q * \frac{1}{\sqrt{2\pi\sigma_s^2}} \cdot \exp \left[-\frac{1}{2} \left(\frac{s' - \bar{s}}{\sigma_s} \right)^2 \right]$$

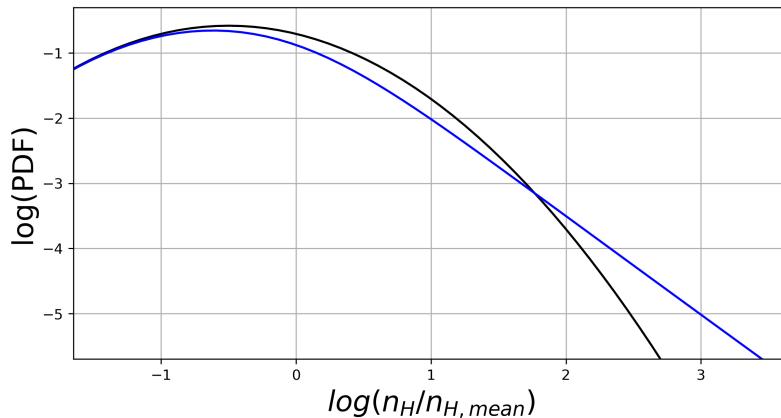
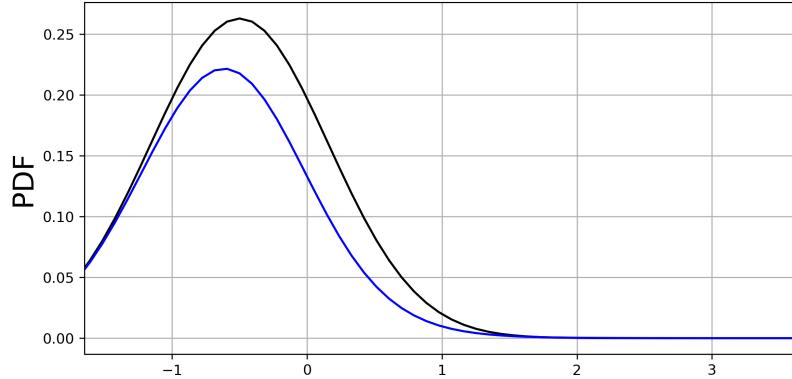
Plotting : $\begin{bmatrix} \log(n_H/\bar{n}_H) & \text{vs pdf \& log(pdf)} \\ \log(n_H/\bar{n}_H) & \text{vs pdf' \& log(pdf')} \end{bmatrix}$

$\begin{bmatrix} \log(n_H/\bar{n}_H) & \text{vs pdf' \& log(pdf')} \\ -\text{varying } \varepsilon \end{bmatrix}$

$\log(n_H/n_{H, mean})$ vs PDF & log(PDF): Mach=5

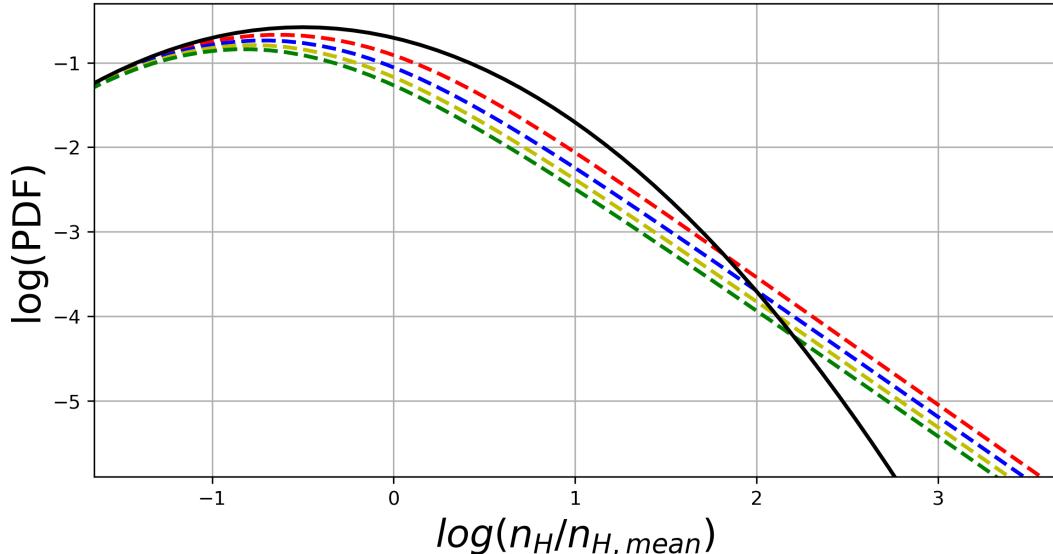
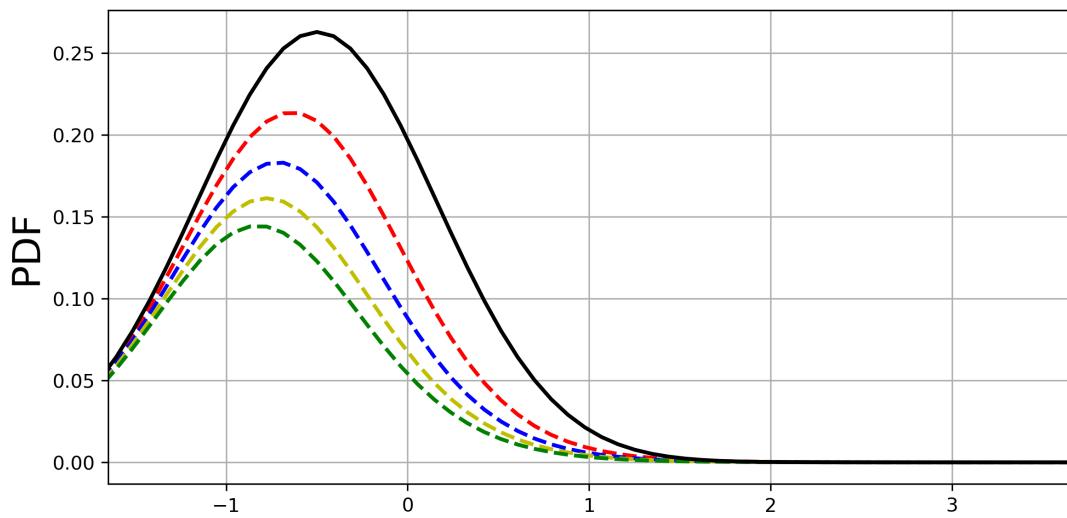


$\log(n_H/n_{H, \text{mean}})$ vs PDF & log(PDF): Mach=10



$\log(n_H/n_{H, \text{mean}})$ vs PDF & log(PDF): varying $t/t_{ff, \text{mean}}$, Mach = 10

— $t/t_{ff, \text{mean}} = 0.0$ - - - $t/t_{ff, \text{mean}} = 0.2$ - · - $t/t_{ff, \text{mean}} = 0.4$ - - . $t/t_{ff, \text{mean}} = 0.6$ - - - $t/t_{ff, \text{mean}} = 0.8$



② Updating the corrected RT model :

$$\text{eV-to-erg} = 1.602 \times 10^{-12}$$

$$\alpha_{\text{co}} = \frac{1}{2K_B} \left(\frac{C}{J} \right)^3 \times 1e^{-5} \times (3.24 \times 10^{-19})^2 \times M_{\odot}$$

[erg to $K \text{ km s}^{-1} \text{ pc}^2 \text{ M}_{\odot}^{-1}$ conversion]

$$l_{\text{co}} = h_{\text{ev}} \cdot J_{10} \cdot X_1 \cdot A_{10} \cdot 1e^{-4} \cdot n_{\text{H}}$$

$$\times \text{eV-to-erg} / M_{\odot}$$

[erg $s^{-1} g^{-1}$]

$$l'_{\text{co}} = \frac{h_{\text{ev}} J_{10} \times A_{10} \cdot n_{\text{co}} \times \text{eV-to-erg}}{M_{\odot}}$$

[erg $s^{-1} g^{-1}$]

$$\bar{l}_{\text{co}} = l'_{\text{co}} \times \text{pdf} \times ds$$

[erg $s^{-1} g^{-1}$]

$$l_{\text{co}, \text{SI}} = l_{\text{co}} \times \alpha_{\text{co}}$$

[$K \text{ km s}^{-1} \text{ pc}^2 \text{ M}_{\odot}^{-1}$]

~~$$l'_{\text{co}, \text{SI}} = l'_{\text{co}} \times \alpha_{\text{co}}$$~~

($K \text{ km s}^{-1} \text{ pc}^2 \text{ M}_{\odot}^{-1}$)

$$\bar{l}_{\text{co}, \text{SI}} = \bar{l}_{\text{co}} \times \alpha_{\text{co}}$$

[$K \text{ km s}^{-1} \text{ pc}^2 \text{ M}_{\odot}^{-1}$]

③ Updated RT model + PDF evolution :

$$[l_{\text{co}}, l'_{\text{co}}, \bar{l}_{\text{co}}] \rightarrow (\text{erg } s^{-1} g^{-1})$$

$$\rightarrow [K \text{ km s}^{-1} \text{ pc}^2 \text{ M}_{\odot}^{-1}]$$

Here,

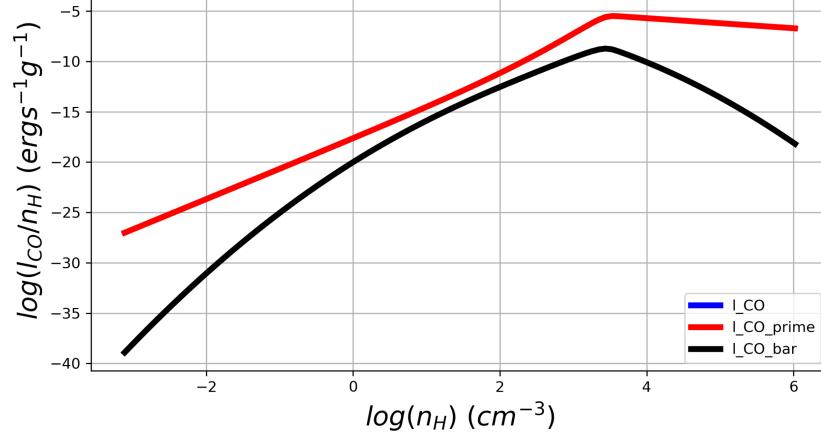
$$\bar{l}_{\text{co}} = l'_{\text{co}} \times \text{pdf}' \times ds$$

Plotting : $\begin{cases} \log(n_{\text{H}}) \text{ vs } \log(l_{\text{co}}/n_{\text{H}}) \\ \log(n_{\text{H}}) \text{ vs } \log(l'_{\text{co}}/n_{\text{H}}) \\ \log(n_{\text{H}}) \text{ vs } \log(\bar{l}_{\text{co}}/n_{\text{H}}) \end{cases}$

→ Lognormal PDF

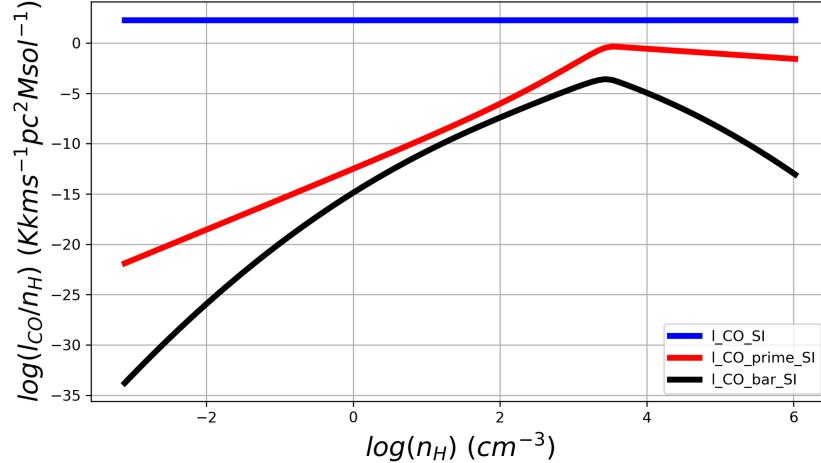
→ PDF Evolution

$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (ergs}^{-1}g^{-1}\text{)}$

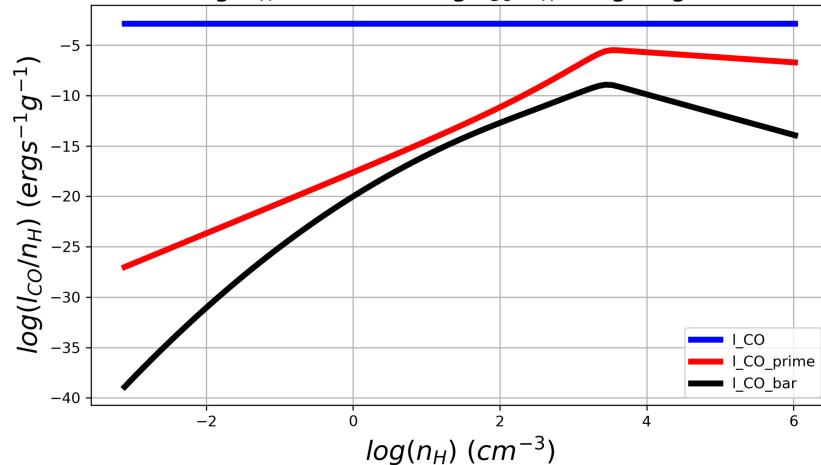


Yesterday's plots
(Without PDF evolution)

$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (Kkms}^{-1}pc^2Msol^{-1}\text{)}$

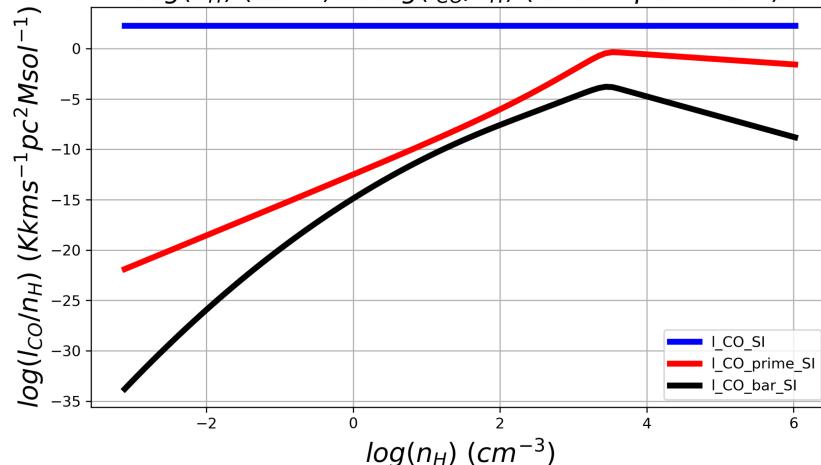


$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (ergs}^{-1}g^{-1}\text{)}$



With PDF Evolution

$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (Kkms}^{-1}pc^2Msol^{-1}\text{)}$



(IV) Using Δv instead of $c_{s,co}$:

$$\Delta x = \approx 1 \text{ pc}$$

$$\sigma_{turb} = 1 \text{ km/s}$$

$$\Delta v = \sigma_{turb} \left(\frac{\lambda_J}{\Delta x} \right)^{1/2}$$

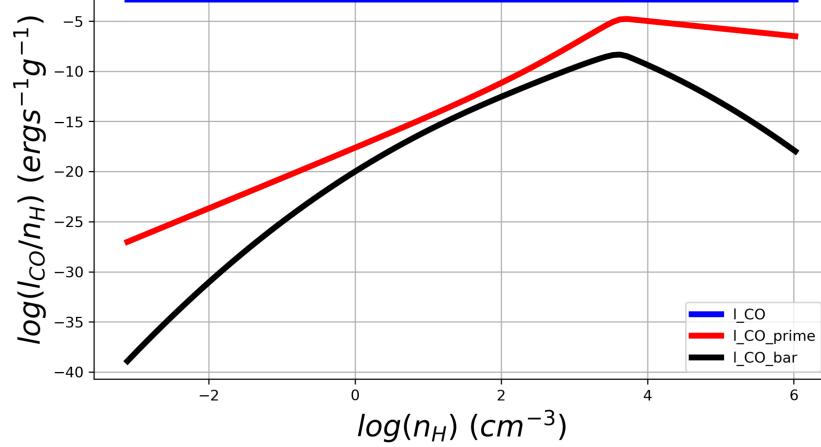
$$\therefore \Delta J = \left(\frac{\Delta v}{c} \right) J \quad [\text{Hz}]$$

$$\therefore z = \frac{\ln(\lambda_J) \lambda_J n_{co} (x_0 B_{01} - x_1 B_{10})}{4\pi(\Delta J)}$$

plotting : $\begin{bmatrix} \log(n_H) \text{ vs } \log(l_{co}/n_H) \\ \log(n_H) \text{ vs } \log(l'_{co}/n_H) \\ \log(n_H) \text{ vs } \log(\bar{l}_{co}/n_H) \end{bmatrix} \rightarrow \text{Lognormal PDF}$

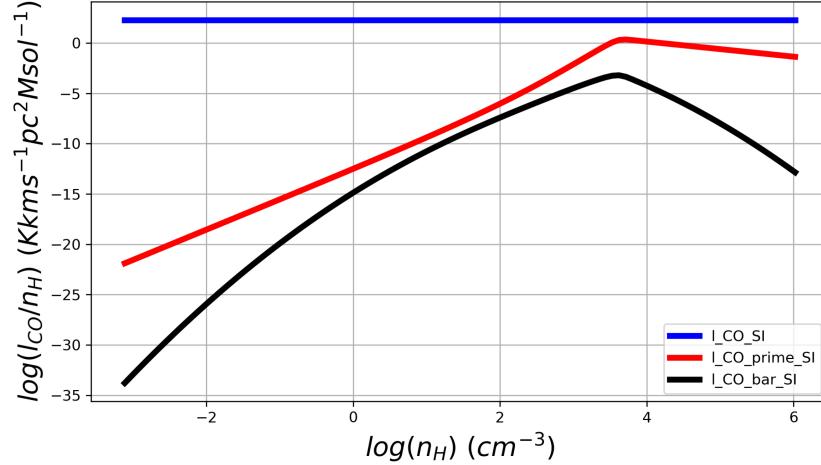
$\rightarrow \text{PDF Evolution.}$

$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (ergs}^{-1}g^{-1}\text{)}$

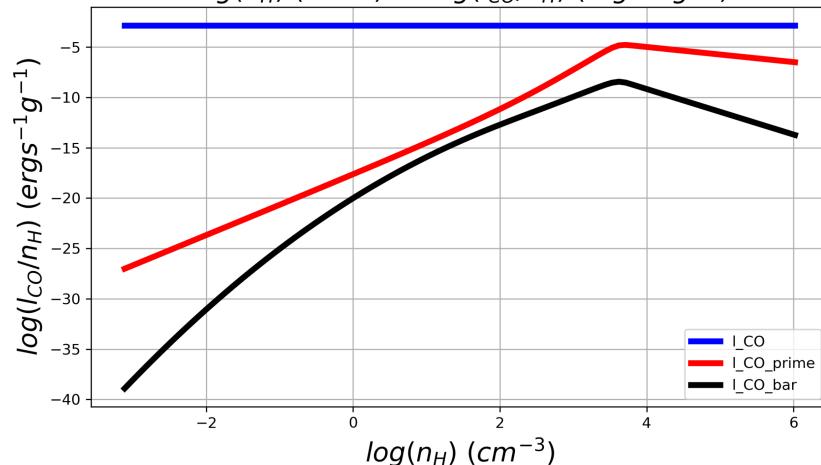


Without PDF Evolution

$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (Kkms}^{-1}pc^2Msol^{-1}\text{)}$



$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (ergs}^{-1}g^{-1}\text{)}$



With PDF Evolution

$\log(n_H) \text{ (cm}^{-3}\text{)} \text{ vs } \log(I_{CO}/n_H) \text{ (Kkms}^{-1}pc^2Msol^{-1}\text{)}$

