Since we changed our variables from n_H to s, I thought it'd be better to replot all the older graphs first.

So, first I plotted

1. s vs pdf
2. s vs log(pdf)
3. $\log(\lambda_J)$ vs s
4. s vs X_{H_2}

I have detailed the alogorithm & the verification also. So far everything seems as expected.

(I) $M = 5$

$$\sigma_s = \sqrt{\log(1 + 0.3^2 M^2)}$$

$$\bar{s} = -\frac{1}{2}\sigma_s^2$$

$$smin = -3\sigma_s + \bar{s}$$

$$smax = +3\sigma_s + \bar{s}$$

$$ds = (smax - smin)/1000$$

$$\bar{n}_H = 10{,}000$$

for (i → 0 to 1000)

$$s[i] = smin + i*ds$$

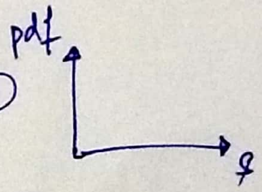
$$pdf = \frac{1}{\sqrt{2\pi\sigma_s^2}} \cdot \exp\left[-\frac{1}{2}\left(\frac{s-\bar{s}}{\sigma_s}\right)^2\right]$$

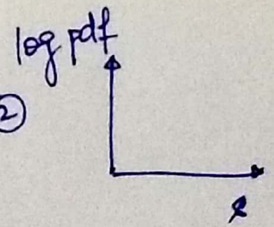
$$n_H = \bar{n}_H \cdot \exp(s)$$

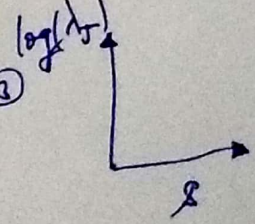
$$\lambda_J = \frac{\underbrace{\sqrt{\frac{K_b \cdot T_{mean}}{m_p}}}_{C_s}}{\sqrt{4\pi G\, n_H \cdot m_p}}$$

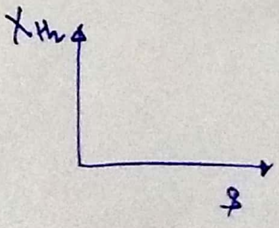
$$exp\_\tau = \exp(-\kappa * n_H * \lambda_J)$$

$$X_{H_2} = \left[\frac{1}{2 + \left(\frac{DC \cdot exp\_\tau \cdot rad\_out}{CC \cdot Z \cdot n_H}\right)}\right]$$

plot → ① 

② 

③ 

④ 

$T = 10K$, $K_b = 1.3806 \times 10^{-16}$ erg/K

$m_p = 1.6726 \times 10^{-24}$ g , $G = 6.674 \times 10^{-8}$ dyne cm^2/g^2

$$C_s = \sqrt{\frac{K_b T}{m_p}} = 28730.5 \text{ cm/s}$$

$n_H = \bar{n}_H \cdot \exp(s)$

$\bar{n}_H = 10{,}000$

$(s)_{min} = -3.8463038$

$(s)_{max} = 2.661134902$

$(n_H)_{min} = 213.585350$

$(n_H)_{max} = 143125.232046$

Analytical:

$$(\lambda_J)_{min} = \frac{C_s}{\sqrt{4\pi G m_p (n_H)_{max}}} = 6.41 \times 10^{16}$$

$$(\lambda_J)_{max} = \frac{C_s}{\sqrt{4\pi G m_p (n_H)_{min}}} = 1.659 \times 10^{18}$$

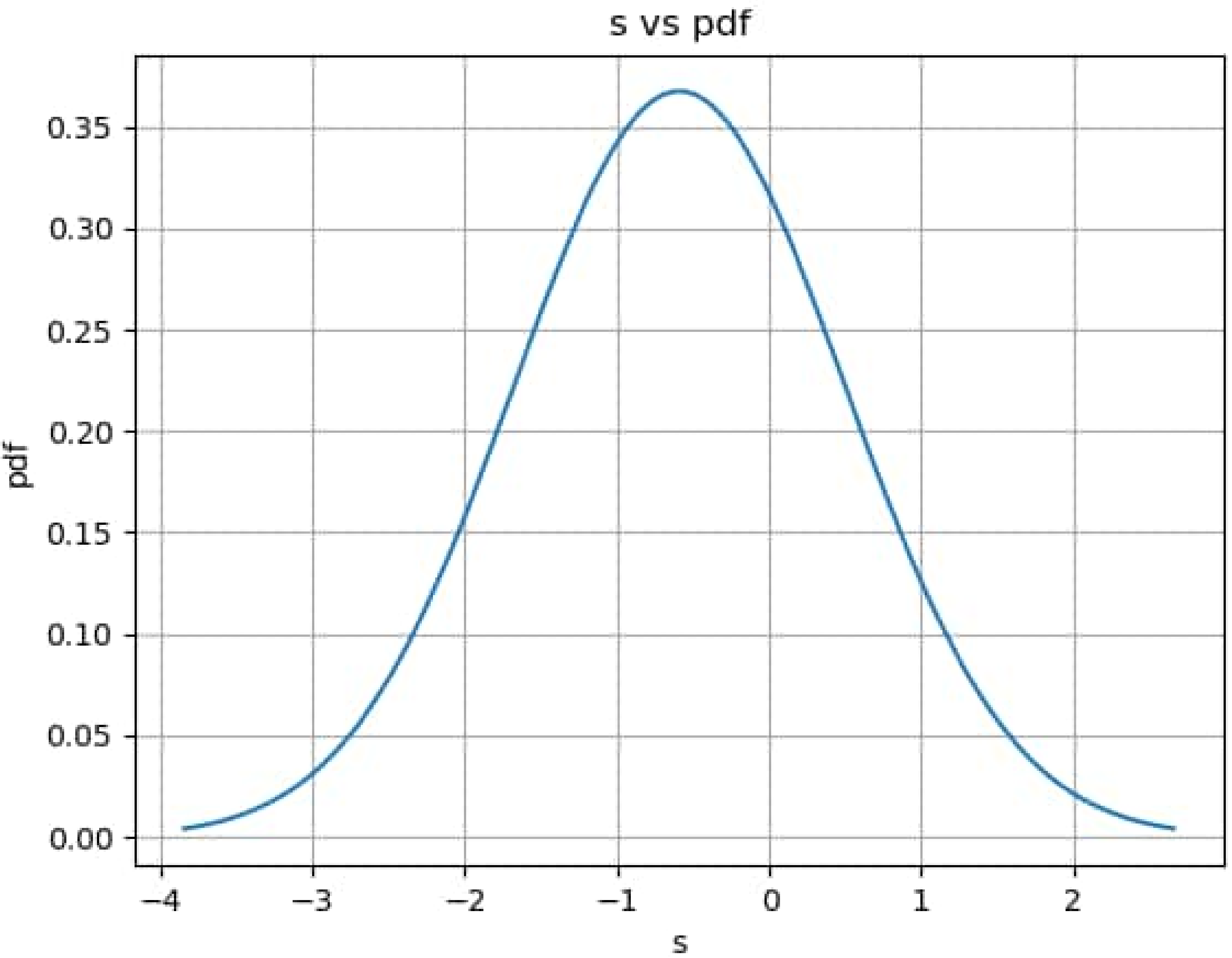
From code:

$(\lambda_J)_{min} = 6.4118 \times 10^{16}$

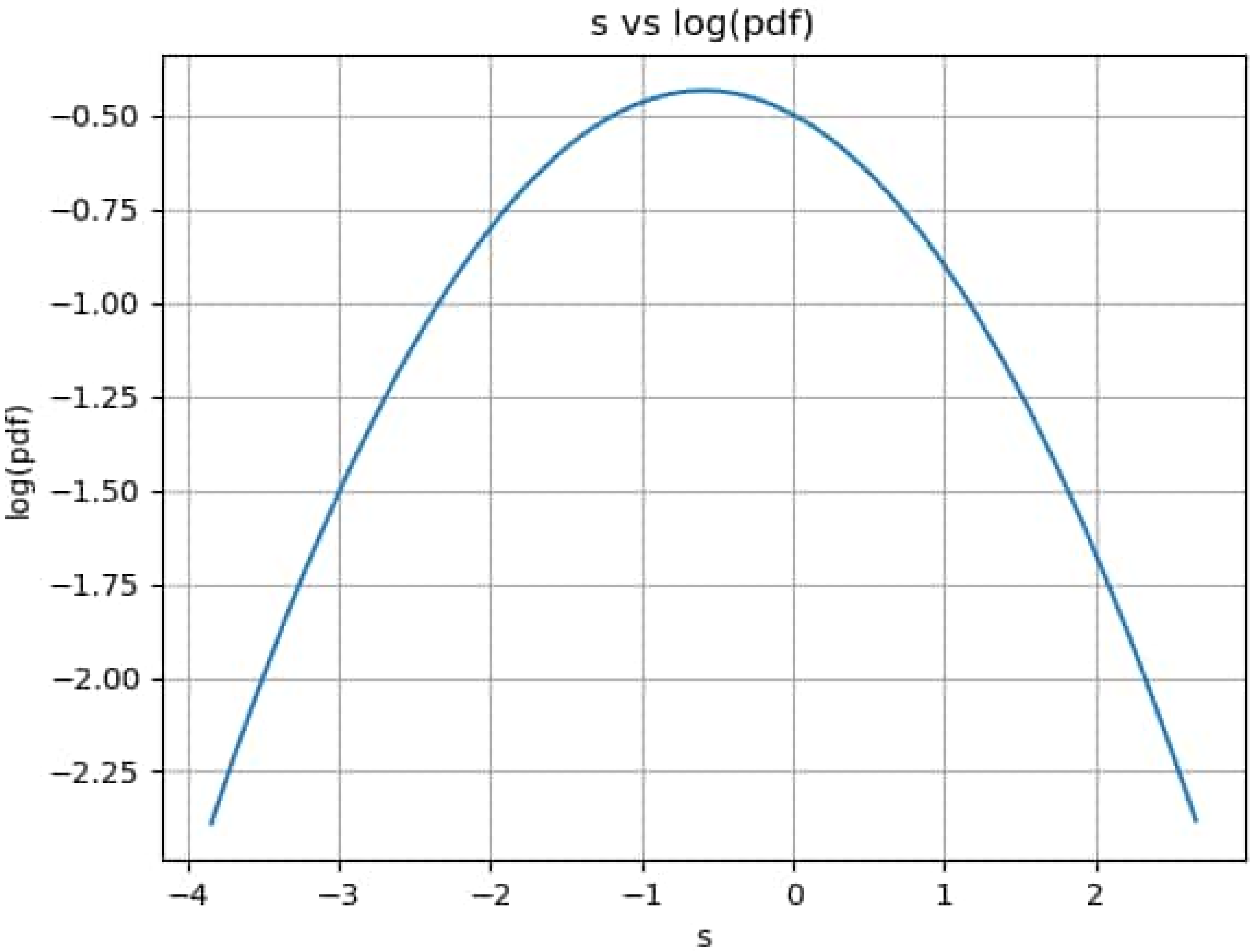
$(\lambda_J)_{max} = 1.6598 \times 10^{18}$

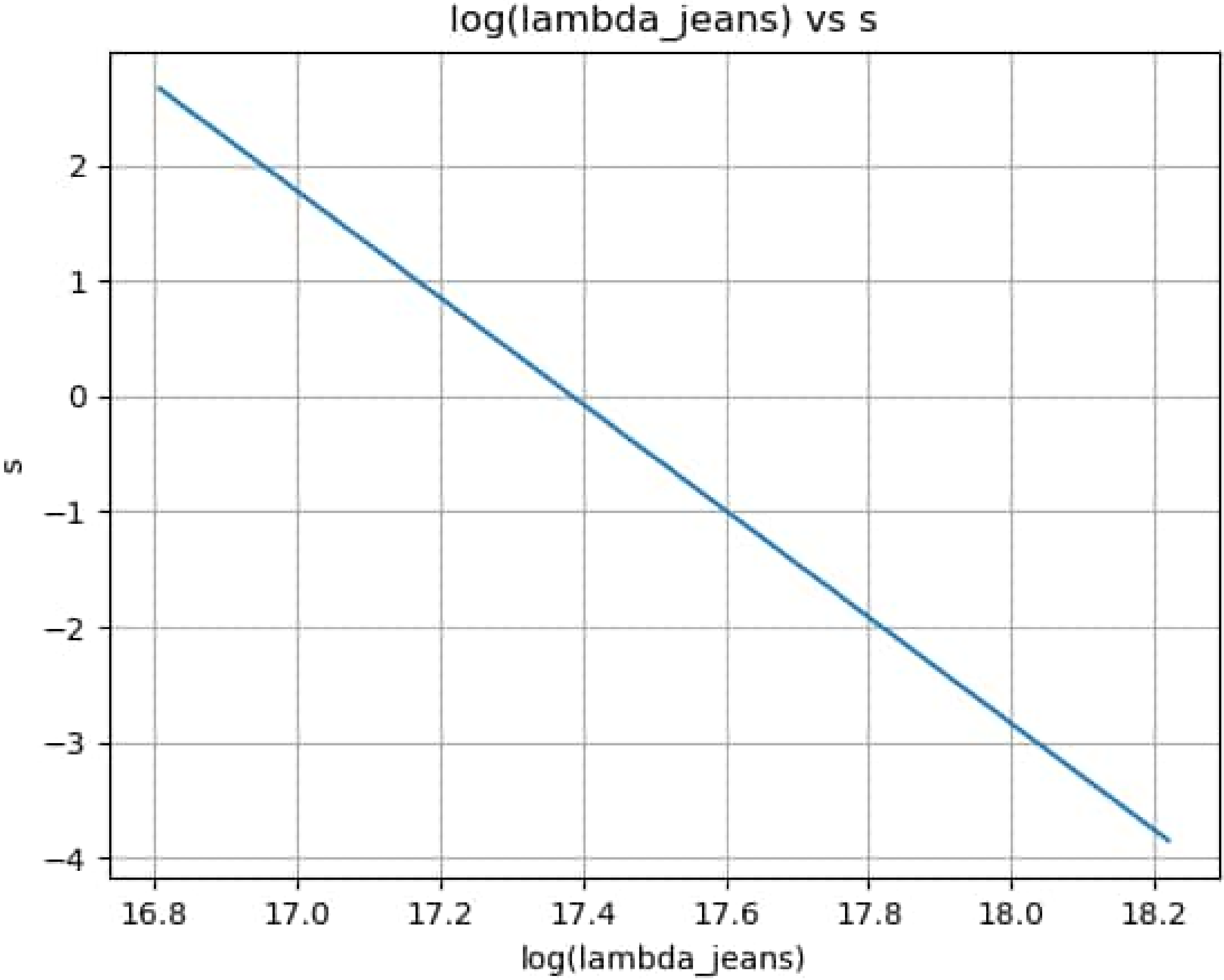
$\log(\lambda_{J,min}) = 16.81$

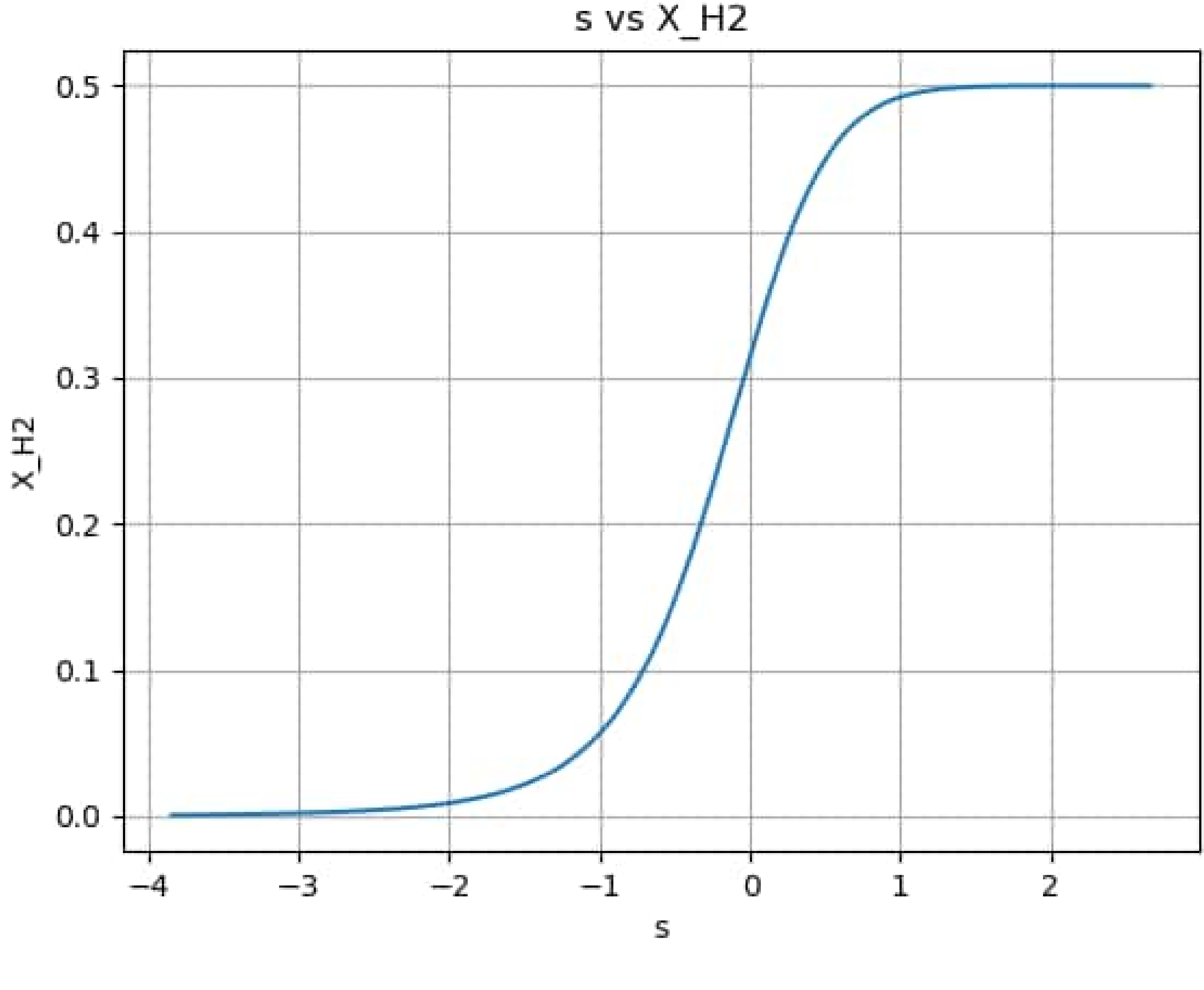
$\log(\lambda_{J,max}) = 18.22$



s vs pdf
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
pdf
−4
−3
−2
−1
0
1
2
s


s vs log(pdf)
log(pdf)
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−2.00
−2.25
−4
−3
−2
−1
0
1
2
s


log(lambda_jeans) vs s
s
2
1
0
−1
−2
−3
−4
16.8
17.0
17.2
17.4
17.6
17.8
18.0
18.2
log(lambda_jeans)


s vs X_H2
X_H2
s
0.0
0.1
0.2
0.3
0.4
0.5
−4
−3
−2
−1
0
1
2

Then, I tried to integrate to find $\int e^{s} P(s)\, ds$ & $\int e^{s} P(s)\, ds\ X_{H_2}$.

But, $tot\_\bar{n}_H = \int e^{s} P(s)\, ds \neq 1$

So, obviously $\bar{X}_{H_2} = \int e^{s} P(s)\, ds \cdot X_{H_2}$ wasn't as expected.

Just to see $\bar{X}_{H_2}$ → I plotted s vs $\bar{X}_{H_2}$.

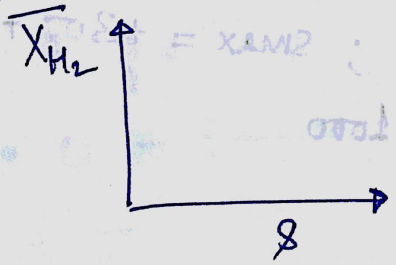
(II) for ($i \rightarrow 0$ to 1000)

$$tot\_\bar{n}_H = tot\_\bar{n}_H + [\exp(s) \cdot pdf \cdot ds]$$

$$\bar{X}_{H_2} = \bar{X}_{H_2} + [tot\_\bar{n}_H \cdot X_{H_2}]$$

plotting : (just to see the data pt.s)

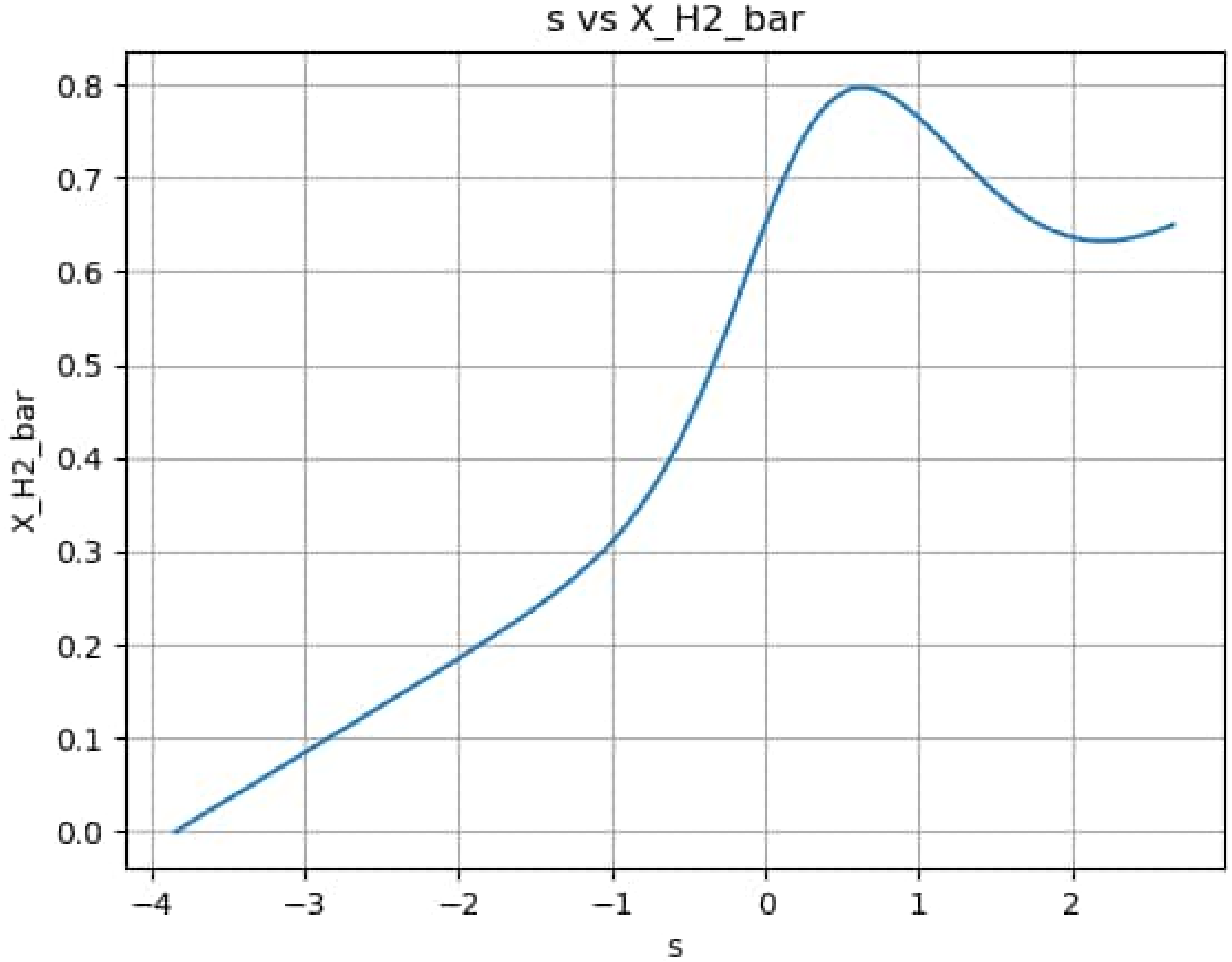
①



$\Rightarrow tot\_\bar{n}_H \neq 1$

$(tot\_\bar{n}_H)_{min} = 5.68 \times 10^{-4}$

$(tot\_\bar{n}_H)_{max} = 2.17$



s vs X_H2_bar
X_H2_bar
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−4
−3
−2
−1
0
1
2
s

I skipped this problem for the time being, because I wanted to plot $\overline{X}_{H_2}$. So, I assumed that tot_$\overline{n}_H$ = 0.999686855 so that I can see whether the problem was only in tot_$\overline{n}_H$. And plotted s vs $\overline{X}_{H_2}$, and it was close to what was expected. So the problem is that tot_$\overline{n}_H \neq 1$.

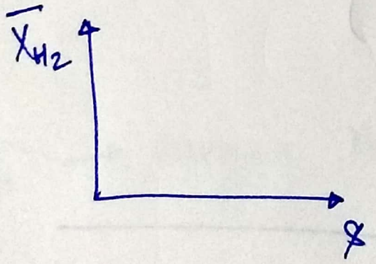
0.999686855 → value I got when n_H was the variable (last week).

(III) setting $tot\_\bar{n}_H$ = 0.9996868549

for ($i \rightarrow 0$ to 1000)

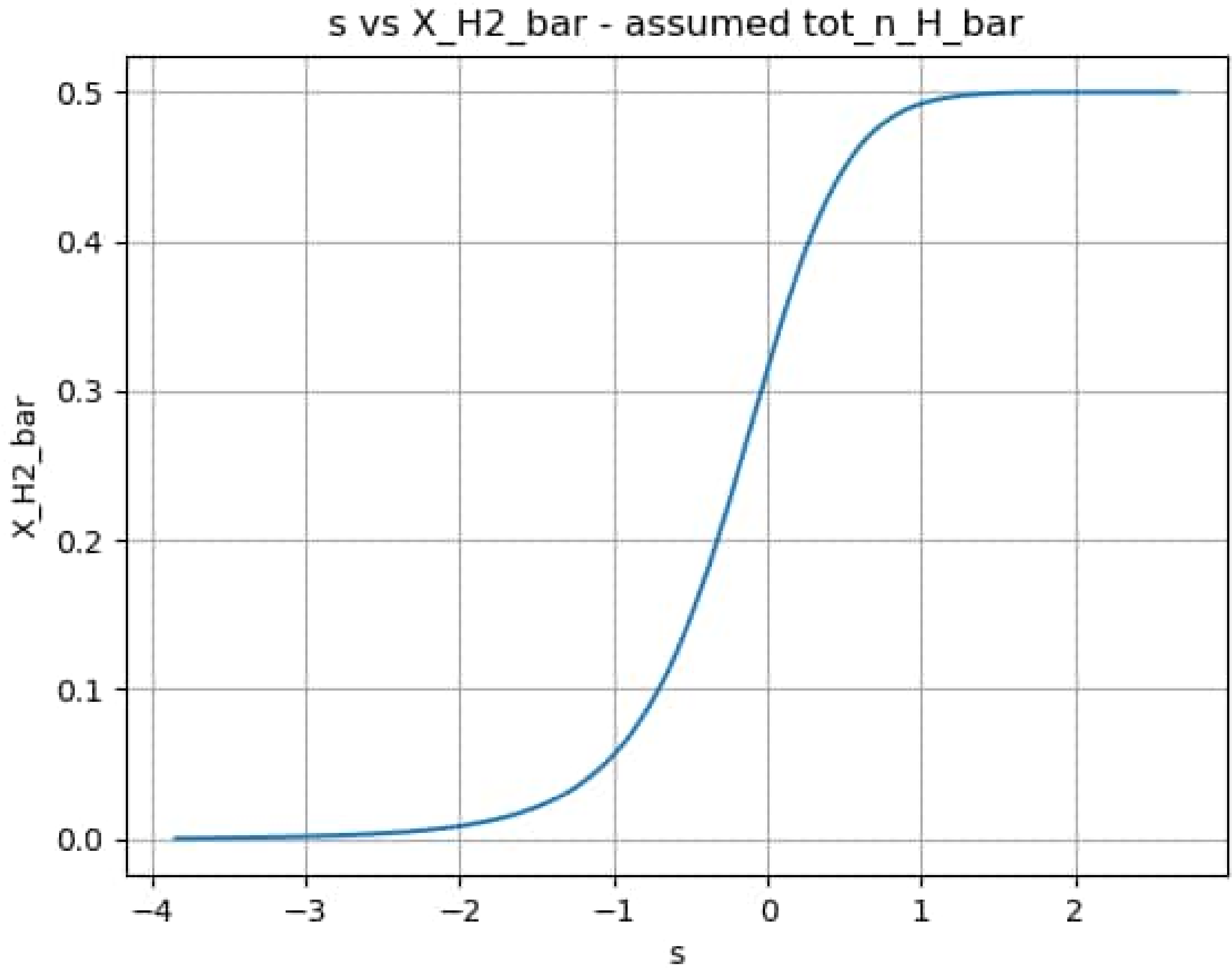
$$\left[\begin{array}{l} tot\_\bar{n}_H = tot\_\bar{n}_H + [\exp(x) \cdot pdf \cdot dx] \\ \bar{X}_{H_2} = \bar{X}_{H_2} + [0.9996868549 \cdot X_{H_2}] \end{array}\right.$$

plotting:

① 

⇒ almost as expected

⇒ problem in $tot\_\bar{n}_H$


s vs X_H2_bar - assumed tot_n_H_bar
X_H2_bar
0.0
0.1
0.2
0.3
0.4
0.5
−4
−3
−2
−1
0
1
2
s

For my next step, I assumed $tot\_\bar{n}_H = 0.999686855$

Now I took mach no. M in the form of an array.

I varied ~~this~~ M so that σ_s, $\bar{s}$, smin, smax & ds would vary, so that I could plot $\bar{s}$ vs $\overline{X}_{H_2}$. Each point is a different value of $\overline{X}_{H_2}$ for different $\bar{s}$.

(IV) $M\_array = [1, 5, 10, 15, 20, 50, 100]$

for (m $\to$ 0 to len(M_array))

$M = M\_array[m]$

$\sigma_s = \sqrt{\log(1 + 0.3^2 M^2)}$

$\bar{s} = -\frac{1}{2}\sigma_s^2$

$smin = -3\sigma_s + \bar{s}$; $smax = +3\sigma_s + \bar{s}$

$ds = (smax - smin)/1000$

$\bar{n}_H = 10{,}000$

for (i $\to$ 0 to 1000)

$s[i] = smin + i * ds$

$n_H[i] = \bar{n}_H \cdot \exp(s[i])$

$X_{H_2}[i] =$ ———

$pdf[i] =$ ———

$tot\_\bar{n}_H[i] = \exp(s[i]) \cdot pdf[i] \cdot ds$

$\bar{X}_{H_2}[i] = 0.9996868549 \cdot X_{H_2}[i]$

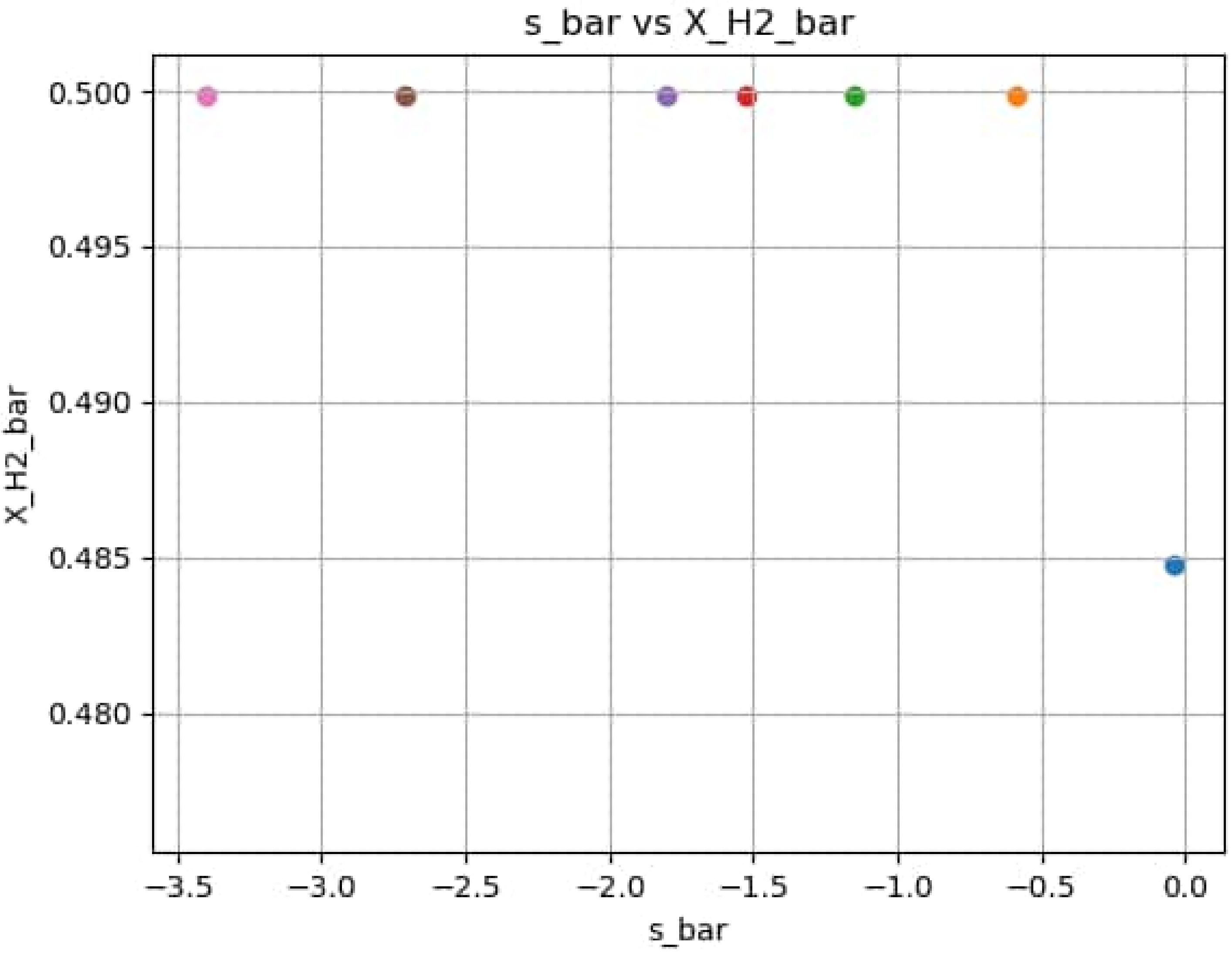
plot ($\bar{s}$ & $\bar{X}_{H_2}$)

saving the plot

$tot\_\bar{n}_H \neq 1$

varying M $\to$ varying $\bar{s}$

$\hookrightarrow$ then plotting $\bar{s}$ vs $\bar{X}_{H_2}$



s_bar vs X_H2_bar
0.500
0.495
0.490
0.485
0.480
X_H2_bar
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
s_bar

Now, again, I assumed $tot\_\bar{n}_H = 0.999686855$

This time, I fixed $M = 5$, & varied $\bar{n}_H$.

Varying $\bar{n}_H$ would change n_H (because $n_H = \bar{n}_H \cdot \exp(s)$),

which will change X_{H_2} & $\bar{X}_{H_2}$.

Then I plotted $\bar{n}_H$ vs $\bar{X}_{H_2}$.

This plot is very similar to the plot of s vs X_{H_2}.

Each pt. is a different value for X_{H_2} as $\bar{n}_H$ is varied.

First I varied $\bar{n}_H$ as : $[10, 10^2, 10^3, 10^4, 10^5, 10^6]$

But then I noticed there were big gaps in the curve, so I added more data-points : $[10, 10^2, 250, 500, 750, 10^3, 2500, 5000, 10^4, 10^5, 10^6]$

(V) ~~$\overline{n}_H$ (array) = [10, 100, 1000, ..., ...]~~

$\overline{n}_H$-array = $[10, 10^2, {}^{250, 500, 750}\ 10^3, 10^4, {}_{2500, 5000}\ 10^5, 10^6]$

for (m → 0 to len($\overline{n}_H$-array))

$M = 5$

σ_s, $\overline{s}$, smin, smax, ds → fixed

$\overline{n}_H$ = $\overline{n}_H$-array[m]

for (i → 0 to 1000)

s[i] = —

$n_H[i] = \overline{n}_H \cdot \exp(s[i])$

$X_{H_2}[i]$ = —

pdf [i] = —

tot_$\overline{n}_H$ [i] = —

$\overline{X}_{H_2}[i] = 0.99 - \ \cdot X_{H_2}[i]$

plot ($\log \overline{n}_H$) & $\overline{X}_{H_2}$)

saving the plot

---

tot_$\overline{n}_H \neq 1$

varying $\overline{n}_H$ → varying n_H → varying X_{H_2}

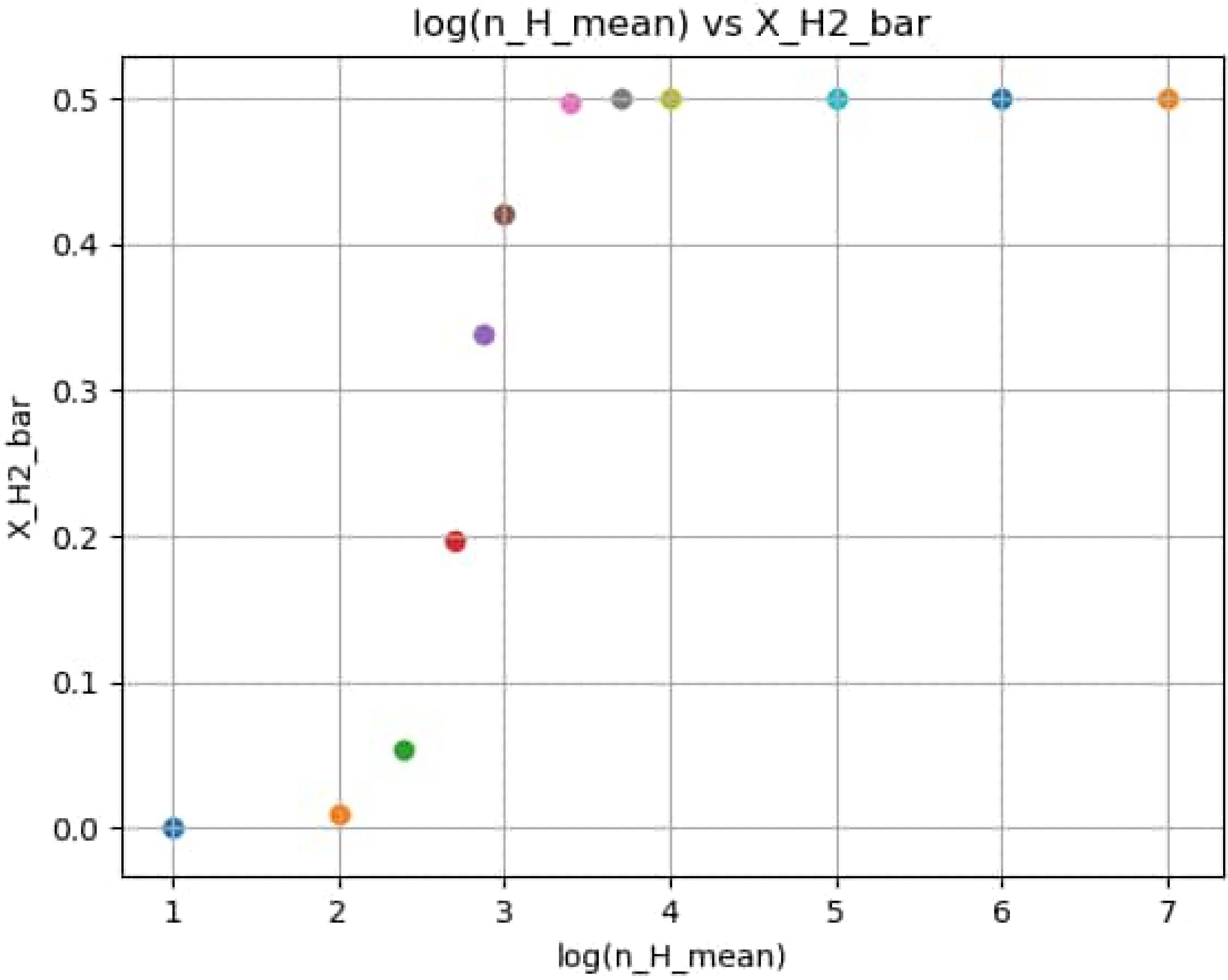
↓

varying $\overline{X}_{H_2}$

↓

then plotting

$\log(\overline{n}_H)$ vs $\overline{X}_{H_2}$


log(n_H_mean) vs X_H2_bar
X_H2_bar
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
5
6
7
log(n_H_mean)

If the plot of $\overline{n}_H$ vs $\overline{X}_{H_2}$ is correct, then the only problem is that tot_$\overline{n}_H \neq 1$.