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james r. graham

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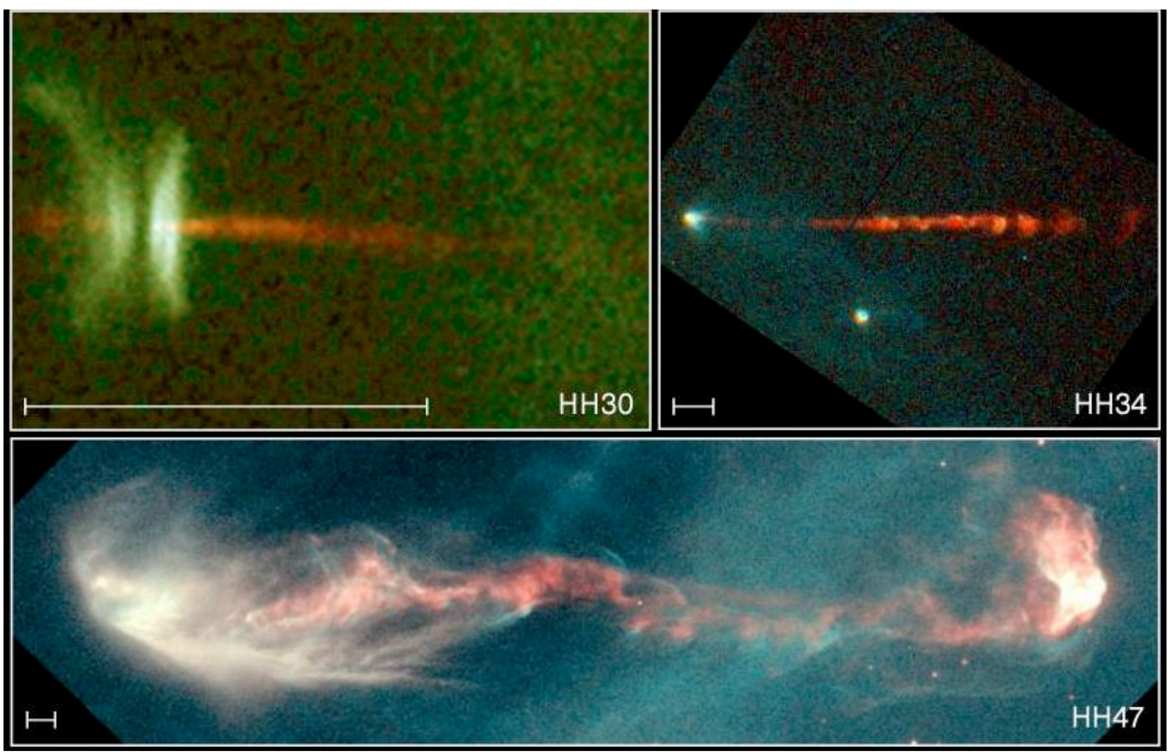
Interstellar Shocks, Supernovae & Bubbles

Violent events are ubiquitious in the ISM

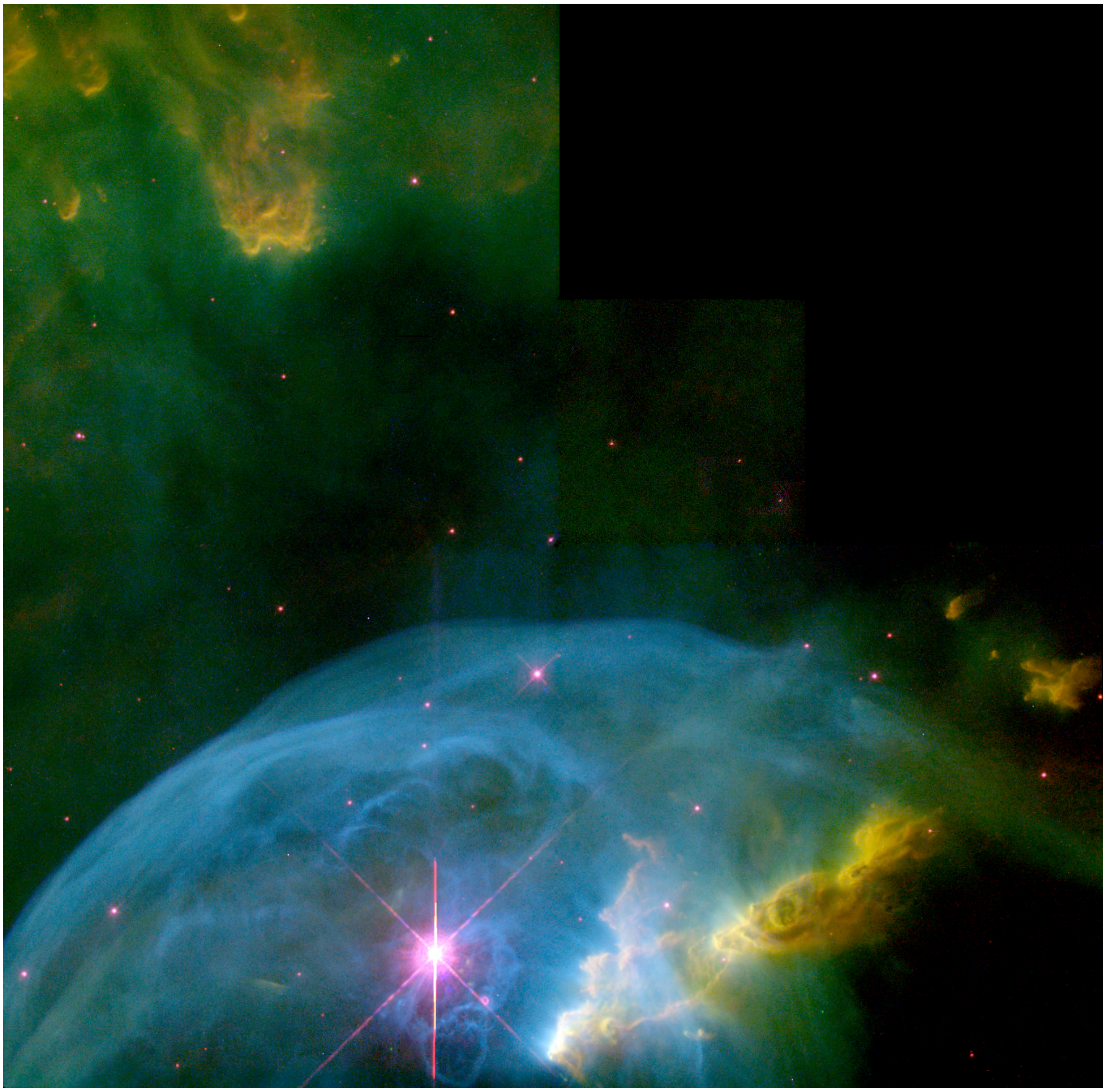
- Massive, young stars (OB stars)
  - Blow fast, energetic winds
  - Produce UV photons which ionize & heat their surroundings
- Stars explode as SN
  - Massive progenitors—core collapse SNII/SNIb/c
  - Low mass progenitor—SNIa
- These events create regions of overpressure in the ISM which drives expansion into lower pressure gas
  - Interstellar can typically cools efficiently
    - Sound speed is low and shocks are common
- Galaxies & even whole clusters collide with one another
  - AGN of individual galaxies spew out relativistic gas
- All of these events create shock waves, which are the major sources of ***hot gas***
- For reviews see
  - Ostriker & McKee (1988 Rev Mod Phys 60 1)
  - Bisnovatyi-Kogan & Silich (1995 Rev Mod Phys 67, 661

HST Images of Jets From Low Mass Stars

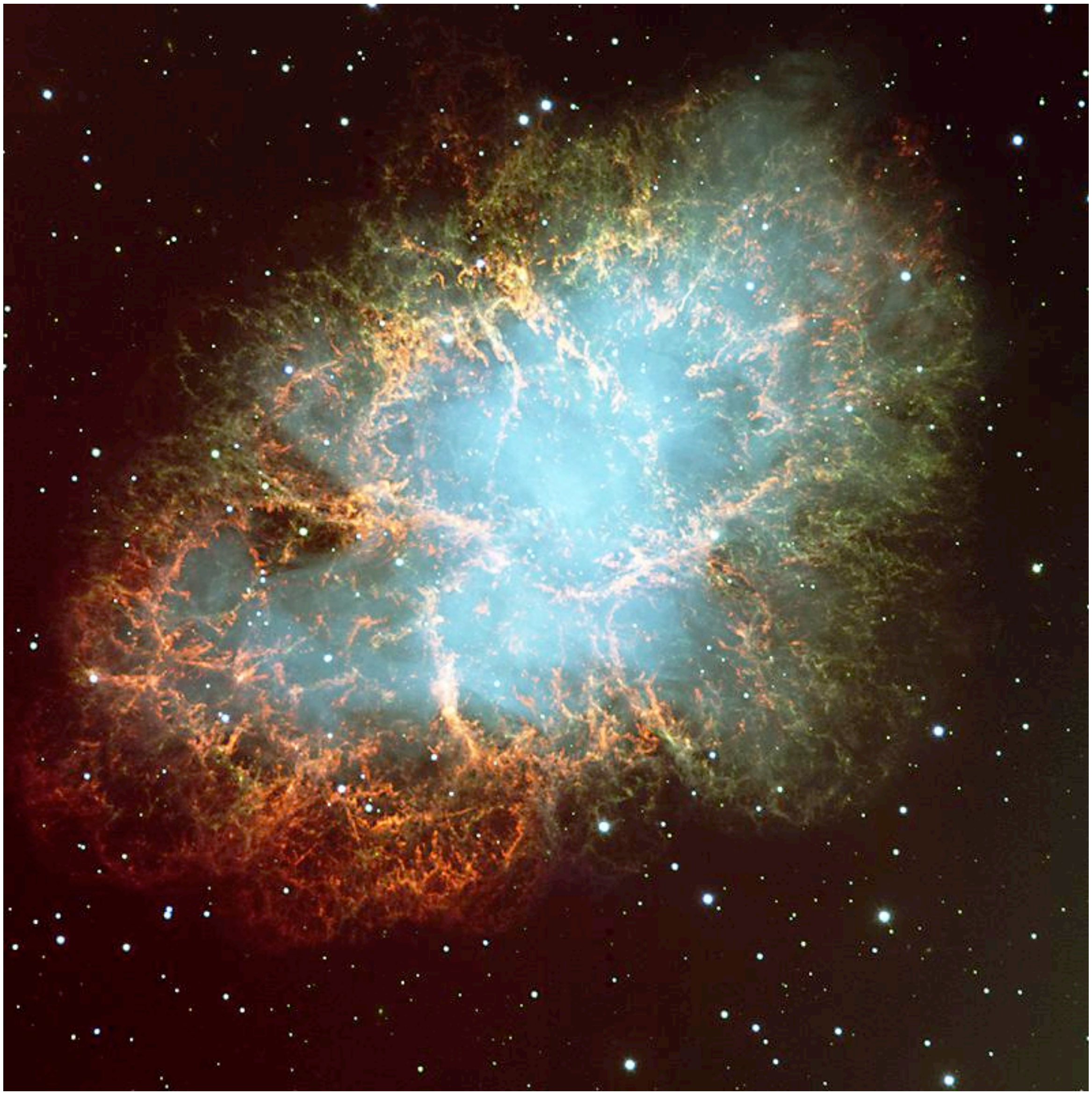


HII Region (Rosette Nebula NGC 2237)

Bubble Nebula (NGC 7635)

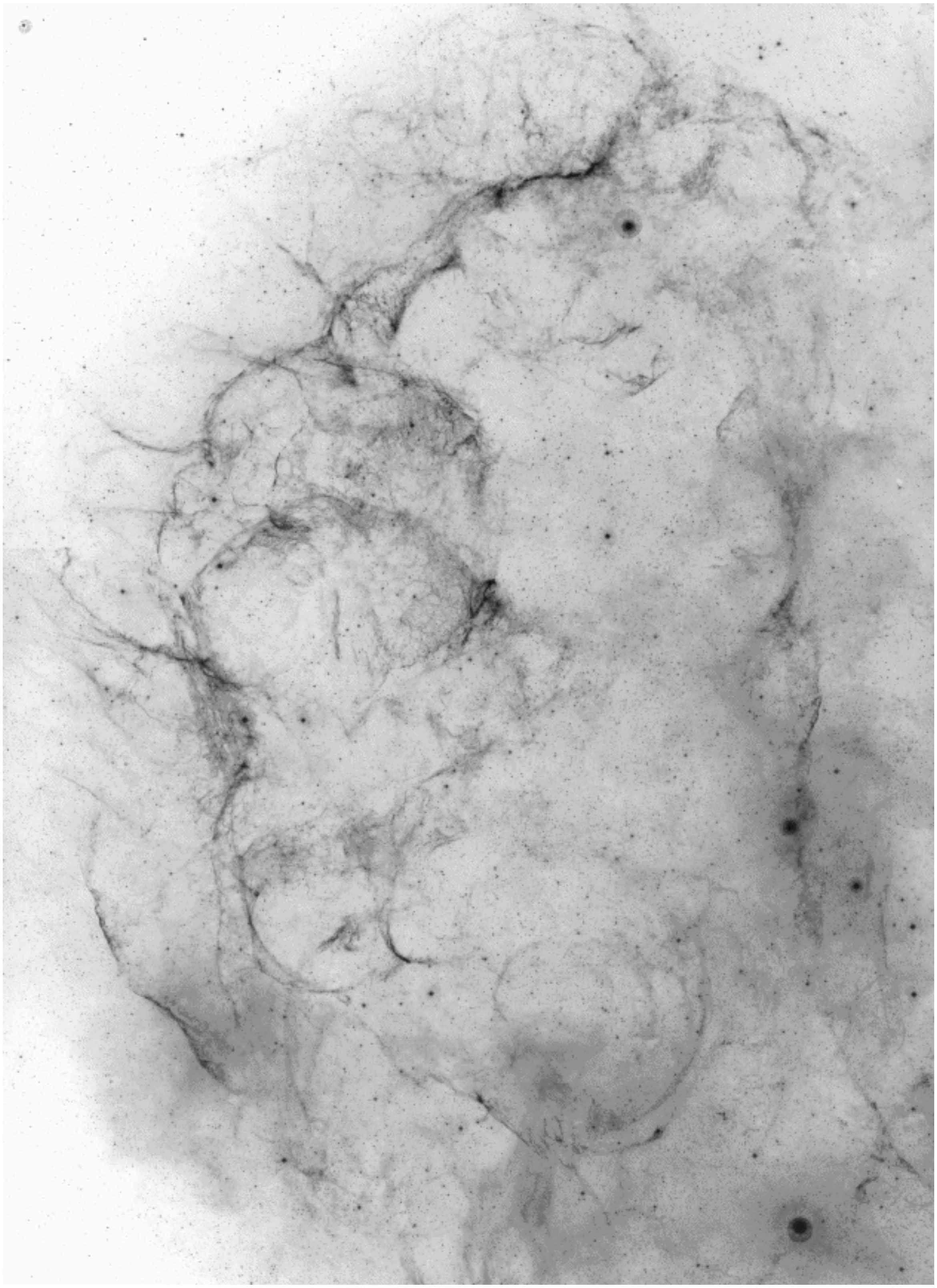


SN 1054 AD (Crab Supernova Remnant)

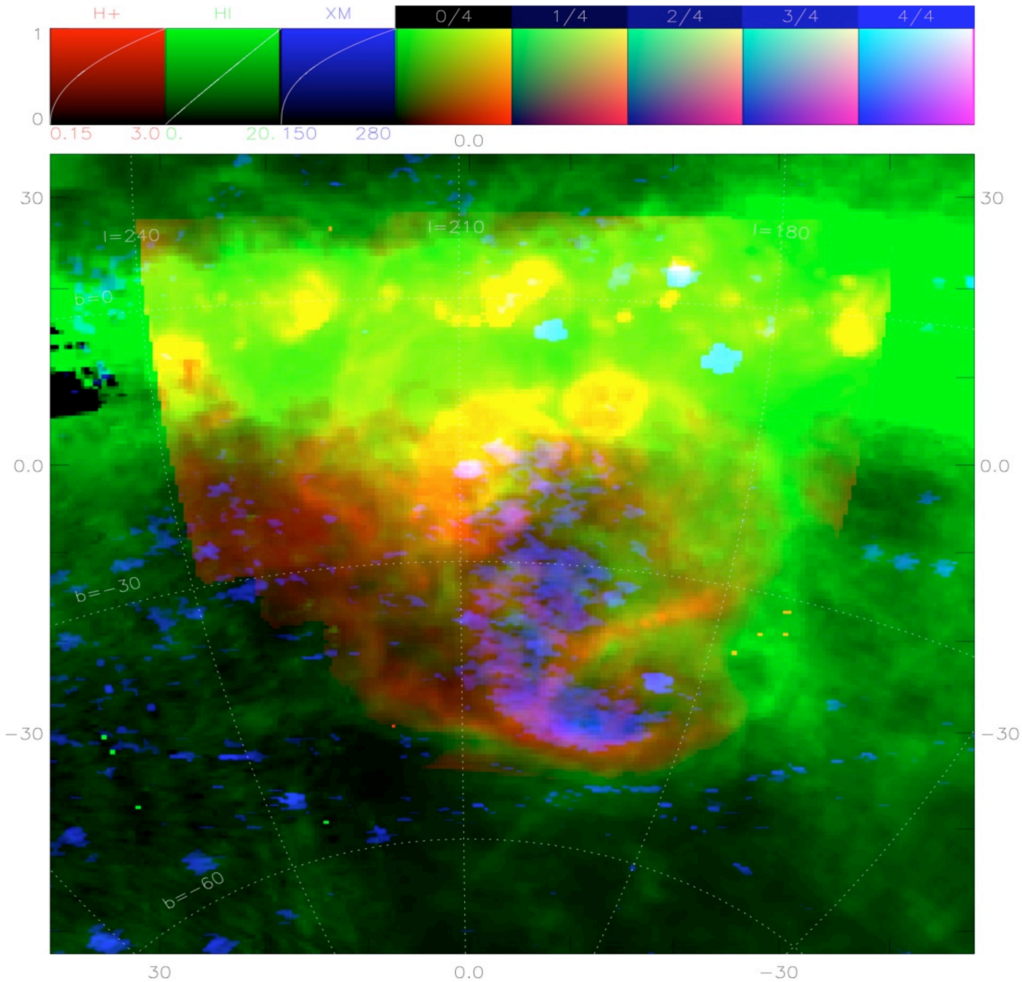


Cygnus Loop SNR

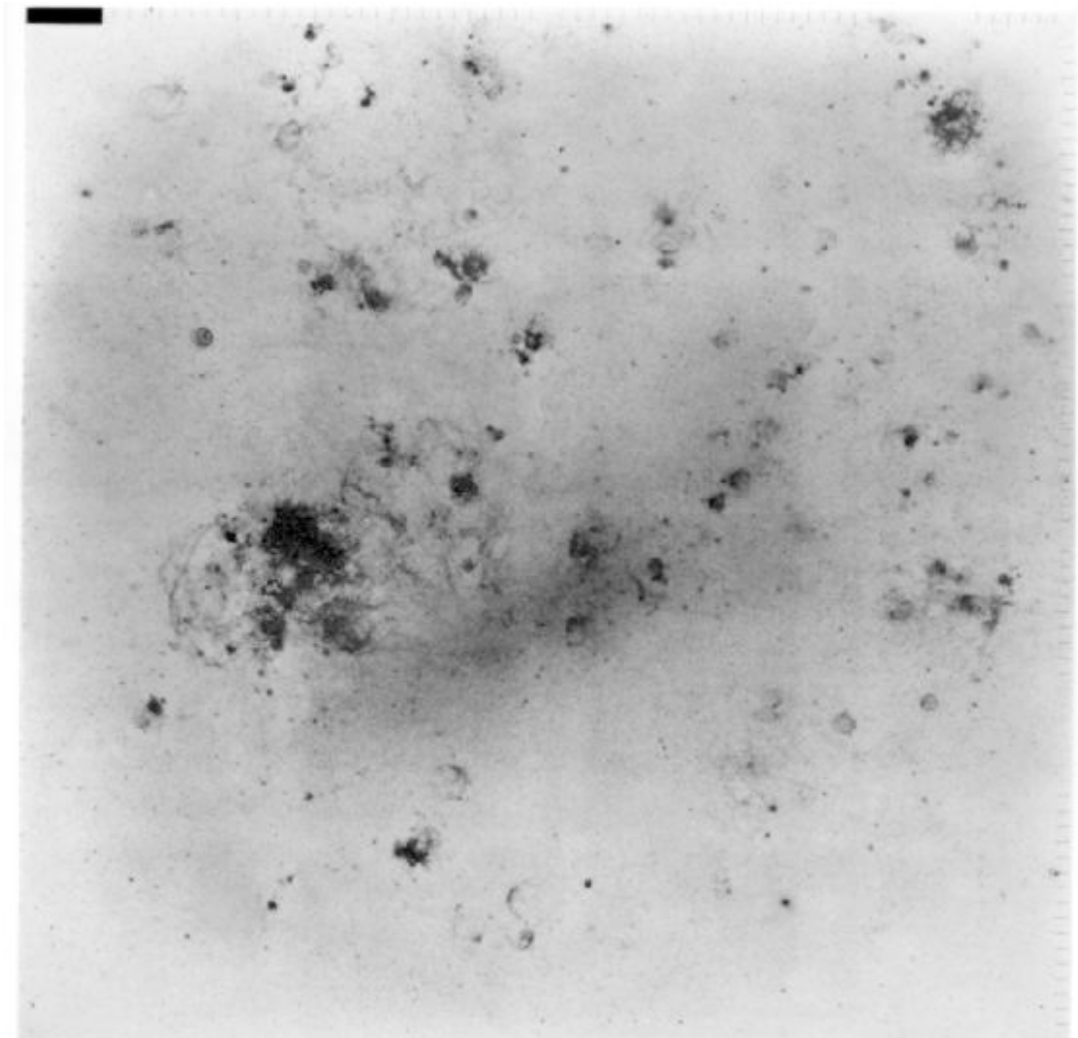
Vela SNR (H-alpha)



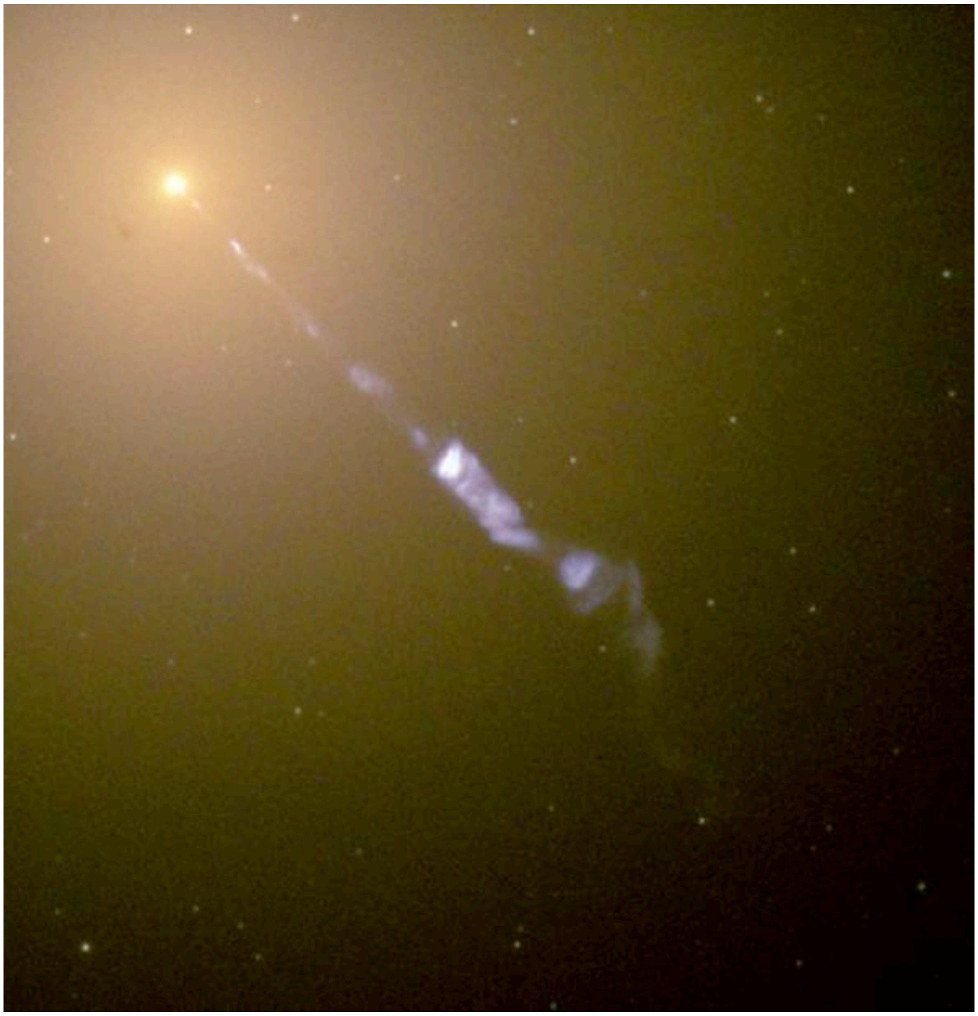
Eridanus Superbubble



LMC H-alpha Shells



M87 Jet



Subsections

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- Jump Shocks
- The Rankine-Hugoniot Jump Conditions
- Isothermal Shocks
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- The Hot Ionized Component
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Why Do Shocks Exist?

Consider reversible, isentropic ($\nabla s = 0$) ideal fluid flows, where $\mathbf{B} = 0$ and the radiation field can be ignored

- Such flows satisfy ***mass conservation***

  $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$

  stating that the rate of change of density is determined by the divergence of the mass flux

- For a 1-d flow

  $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

  Simplifying for a steady flow, i.e: $\partial / \partial t = 0$,

  $$\frac{d}{dx}(\rho v) = 0$$

- Momentum is governed by ***Euler's equation*** (*m* ***a*** = ***f***)

  $$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = \mathbf{F} - \nabla P,$$

  - Rate of change of momentum of material equals the forces acting upon this material.
    - External volume forces, ***F***, such as gravity
    - Internal surface forces resulting from pressure gradients
      - No viscosity
  - For 1-d flow and no external forces

    $$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} = -\frac{\partial P}{\partial x}.$$

  - For a steady flow $\partial / \partial t = 0$, and from continuity $\rho v =$ const., hence

$$\frac{d}{dx}\left(P+\rho v^2\right)=0$$

- To complete the description, we require the (adiabatic) ***equation of state,***

$$P=K\rho^\gamma$$

$K=K(s)$ is a constant, and $\gamma=c_p/c_V$

- $\gamma=5/3$: monatomic gas
- 7/5: diatomic molecular, e.g., H_2
- 4/3: polyatomic gas with many internal degrees of freedom (photon gas)
  - The equation of state replaces the energy equation

Propagation of disturbances: Small ampitude waves

Consider a small disturbance propagated in an equilibrium 1-d flow

- The fluid is initially at rest (v = 0)
  - Change in velocity is small as are changes in the other variables (ρ, P) compared with their initial values (ρ_0, P_0).
  - Neglect the second term on the LHS of the 1-d momentum equation
    - Product of two small quantities.
  - Neglect the second part of the the expansion of $d(\rho v)/dx$ in the continuity equation we get the ***linearized continuity equation***

$$\frac{\partial v}{\partial x}=-\frac{1}{\rho_0}\frac{\partial\rho}{\partial t}.$$

Using the equation of state we write the pressure change in terms of the density change,

$$dP=\left(\frac{\gamma P_0}{\rho_0}\right)d\rho.$$

and substitute in the ***linearized momentum equation***

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho_0}\left(\frac{\gamma P_0}{\rho_0}\right)\frac{\partial\rho}{\partial t}.$$

Differentiate the 1-d linearized continuity equation wrt to t and 1-d linearized momentum wrt x, and subtract

$$\frac{\partial^2 \rho}{\partial t^2} + \left(\frac{\gamma P_0}{\rho_0}\right)\frac{\partial^2 \rho}{\partial x^2} = 0,$$

to find the ***wave equation***. It implies that changes in density are propagated in the fluid at the sound speed, c_s;

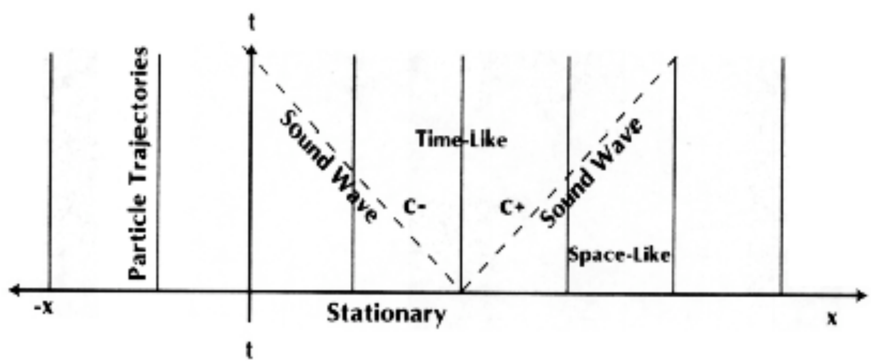
$$c_s^2 = \frac{\gamma P_0}{\rho_0}$$

This equation can be transformed by using $P = nkT_e$ and $\rho = \mu m_H n$.

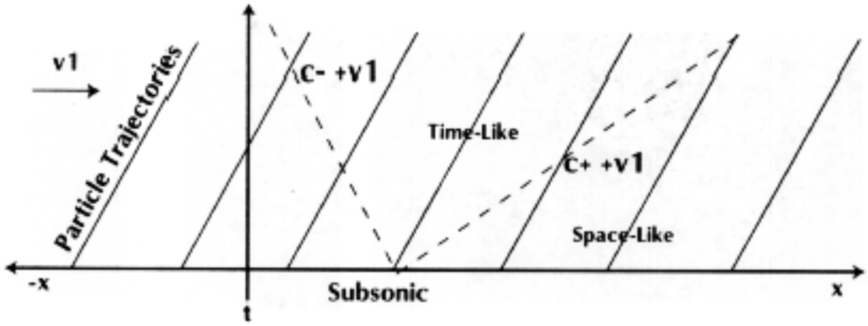
- If a change is made at one place and time, e.g., a sudden local change in P, the effects propagate relative to a given fluid element at c_s.

Supersonic and subsonic flow

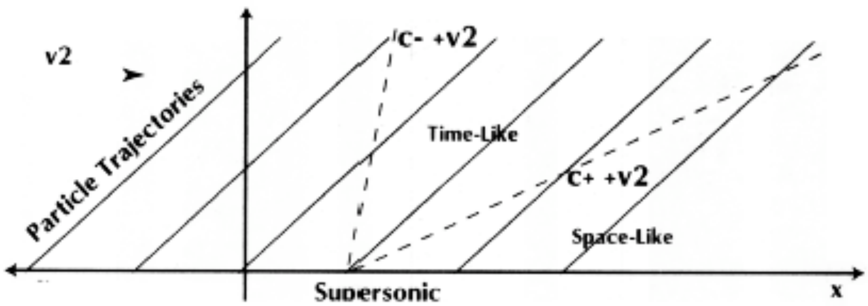
Space-Time diagrams show which points are connected causally through the propagation of sound waves



- Within a ***time-like*** regions the local fluid variables can change in response to the disturbance.
  - There is a space-like region within which no response can occur
  - In (x, y, t)-space, the time-like region is a cone
  - In (x, y, z, t)-space the time-like region is an (expanding) sphere.
- Motion of the medium tips the cone because the acoustic disturbance propagates relative to the fluid
  - For ***subsonic flow*** the sound wave eventually influences all x

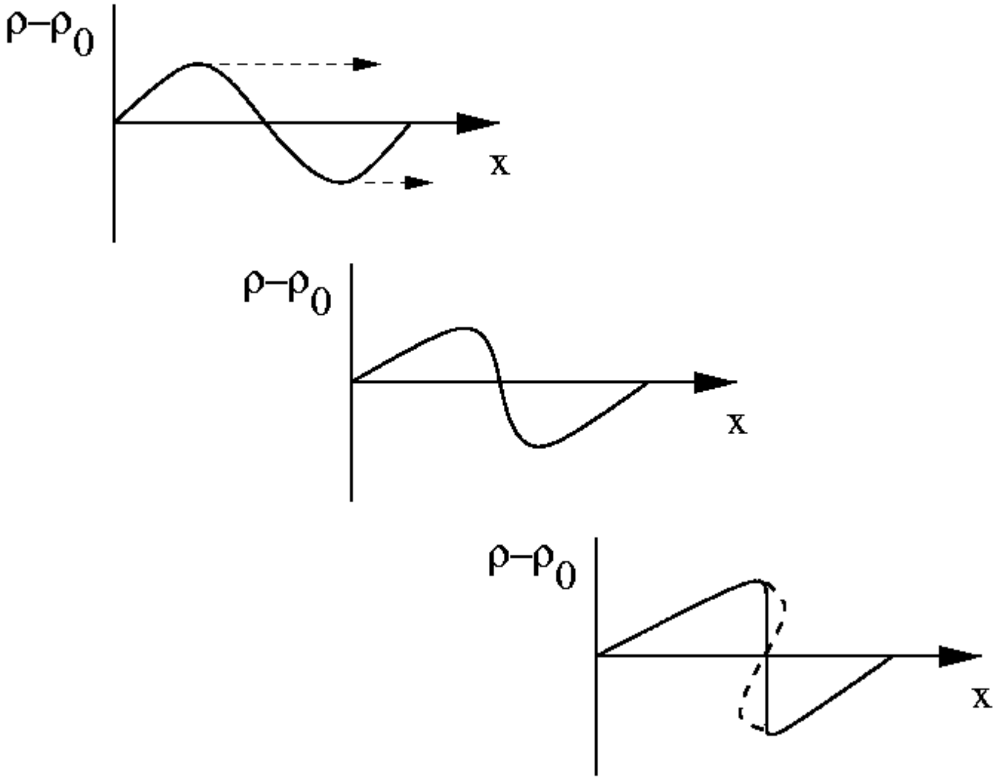


- If the velocity of the medium exceeds the speed of sound, information about the change can never be carried backward in the x-coordinate
  - ***Supersonic flow***



Finite amplitude disturbances

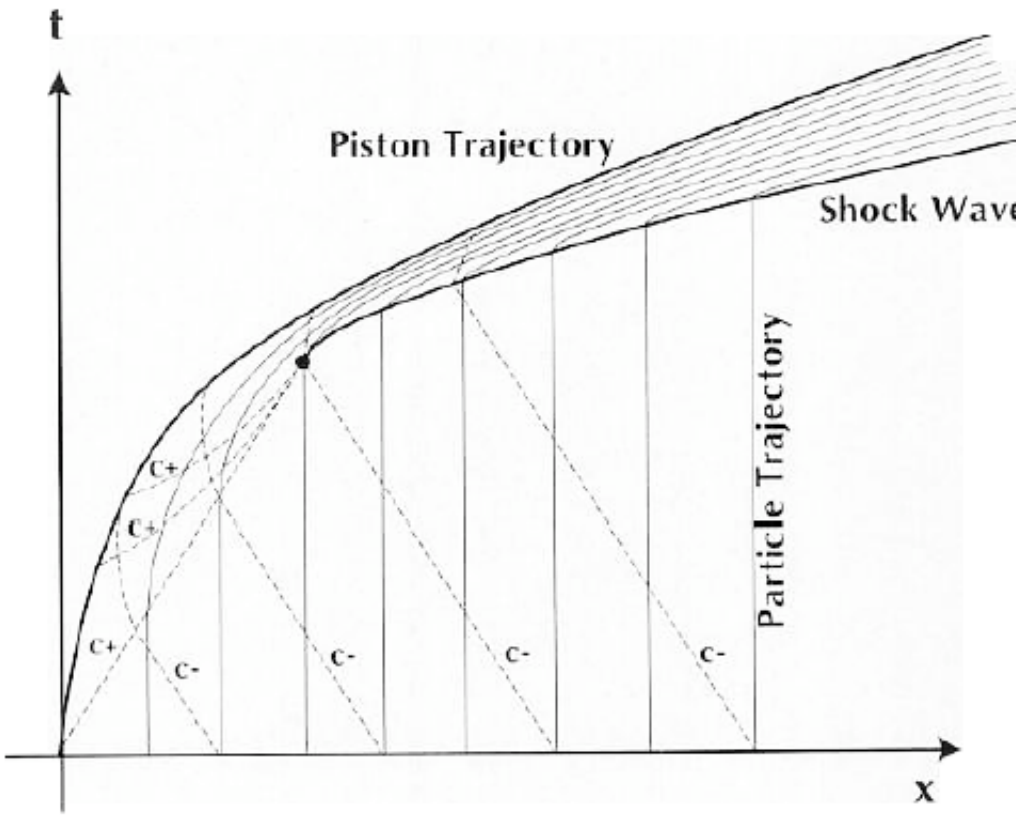
- Suppose that the change which is being propagated is an adiabatic compression
  - Locally the density increases from ρ_1 to ρ_2 ($\rho_2 > \rho_1$)
  - Locally the sound speed increases by a factor of $(\rho_2/\rho_1)^{(\gamma-1)/2}$ or $(\rho_2/\rho_1)^{1/3} > 1$ for $\gamma = 5/3$.
- The most compressed region tends to catch up with and to overrun the uncompressed region.
  - Since the fluid variables must be single valued a ***discontinuity may form.***



Accelerating piston

Suppose a piston accelerates smoothly from rest to some high velocity $> c_{s0}$ in a cylinder

- Initially, at low speeds the piston lags behind the sound waves
- Sound waves establish a smooth gradient of density ahead of the piston
  - The piston continues to accelerate, exceeding c_s
    - Forward propagating sound waves converge until they merge
  - At this point a discontinuity in the flow variables (a shock) develops
    - Information carried by the separate sound waves is lost



- Before merging information carried by the sound waves could (in principle) be used to reconstruct the details of the acceleration of the piston
  - Shock formation is ***irreversible*** corresponding $dS > 0$

- If the piston was withdrawn
  - Fluid variables remain continuous and particle trajectories diverge
    - Adiabatic cooling lowers the sound speed, and the forward and backward sound wave trajectories asymptotically approach the particle trajectories
  - If the piston is withdrawn too rapidly, the internal energy of the gas is depleted entirely
    - The maximum expansion velocity of the gas is (expansion into a vacuum)

$$\frac{2c_0}{(\gamma - 1)^{1/2}}$$

  - If the piston is withdrawn faster than this, a vacuum develops behind the piston
    - The disturbance which results is a rarefaction wave, but not a shock

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Jump Shocks

Ahead of an accelerating piston flow variables eventually change ***discontinuously***

- Good approximation for a plasma where ions & electrons are strongly coupled by Coulomb forces
  - The mean free path of any charged species is very short
  - Such shocks are referred to as ***jump-shocks (J-shocks)***
    - Distinguished from shocks in which some or all of the flow variable change in a more ***continuous*** manner (***C-shocks***)

Mass & momentum conservation

Across the shock discontinuity mass & momentum are conserved

- The momentum equation is easily modified to include $\mathbf{B}$
  - If the field is along the direction of motion, it plays no part in the hydrodynamic flow
    - Parallel $\mathbf{B}$ is unchanged by the flow and provides no pressure support to the flow
  - If $\mathbf{B}$ is transverse to the flow the field can be compressed or rarefied by the flow
- If the component of $\mathbf{B}$ transverse to the direction of flow is B, the momentum equation for 1-d, steady flow is

$$\frac{d}{dx}\left(P+\rho v^2+\frac{B^2}{8\pi}\right)=0.$$

  For ionized gas (high conductivity), the magnetic flux is frozen into the flow and we write an 1-d equation of magnetic flux conservation

$$\frac{d}{dx}\left(\frac{B}{\rho}\right)=0.$$

  or (from mass conservation)

$$\frac{d}{dx}(Bv)=0.$$

Energy conservation across the shock

- In the flow as a whole, energy may be lost or gained by the emission or absorption of radiation
  - The rate of change of energy of the gas within any volume must equal the net flux of energy:

$$\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2}+\rho\varepsilon+\rho\Phi\right)+\nabla\cdot\left[\rho\mathbf{v}\left(\frac{v^2}{2}+h+\Phi\right)\right]+\nabla\cdot\mathbf{F}=0.$$

  - The first term represents the rate of change of
    - KE
    - Internal energy
    - Gravitational potential energy
  - The second term is the divergence of the energy fluxes
    - $h=\varepsilon+P/\rho$is the specific enthalpy
  - The last term is the divergence of the radiative flux

$$\nabla\cdot\mathbf{F}=-4\pi\kappa J+n^2\Lambda$$

  - κ, is the net absorption coefficient of the gas
  - J is the mean intensity of the local radiation field
    - $-4\pi\kappa J$ represents the energy absorbed from the radiation field
  - Λ is the cooling function, and n is the total particle density
  - $n^2\Lambda$ represents the radiative losses from the test volume

For a 1-d, potential-free, steady flow, including a transverse magnetic field energy the ***energy equation*** is

$$\frac{d}{dx}\left[\rho v\left(\frac{v^2}{2}+h\right)+v\frac{B^2}{4\pi}\right]=4\pi\kappa J-n^2\Lambda.$$

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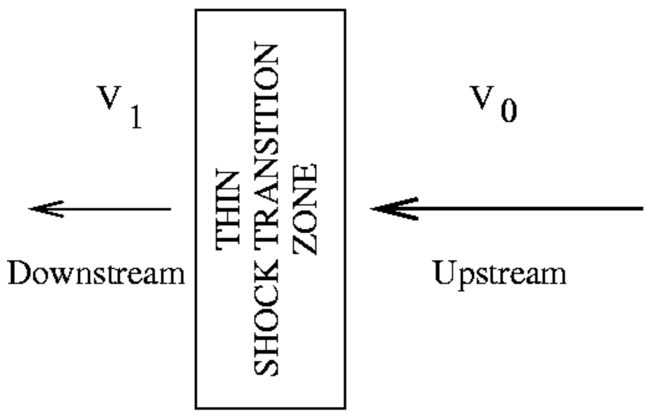
The Rankine-Hugoniot Jump Conditions

The equations of:

- Continuity
- Energy, momentum & magnetic flux conservation
- Equation of state

provide a complete description of all 1-d, potential-free, steady flows

- Integrated to provide relationships between any two points in the flow ***in the frame of the shock***



- Consider
  - An initial point in the flow with hydrodynamic variables v_0, P_0, ρ_0 and B_0
  - A later point in the flow where these have changed to v_1, P_1, ρ_1 and B_1

  Eliminate the specific enthaply, $h = \varepsilon + P/\rho = \gamma P/[(\gamma-1)\rho]$

  - Write the ***Rankine-Hugoniot jump conditions*** in terms of the difference in the fluid variables evaluated at the two points (0 and 1 respectively)

$$
\begin{aligned}
[\rho v]_0^1 &= 0 \\
[Bv]_0^1 &= 0 \\
\left[P + \rho v^2 + \frac{B^2}{8\pi}\right]_0^1 &= 0 \\
\left[\left(\frac{\rho v^2}{2} + \frac{\gamma P}{\gamma - 1} + \frac{B^2}{4\pi}\right) v + F\right]_0^1 &= 0
\end{aligned}
$$

- Provided
  - Initial conditions are specified
  - Radiative energy loss (gain) term, ***F*** is known
    - Derive the flow variables and the magnetic field at any other point in the flow
- Consider only ***steady flows***
  - For these conditions to apply, any discontinuities (shocks) must be ***stationary in the frame of reference in which the jump conditions are evaluated***
    - Otherwise we have to keep the time derivatives

Adiabatic* unmagnetized shocks

Consider for flows with $\mathbf{B} = \mathbf{0}$

- In a radiation-less transition (this is where the abominable nomenclature "adiabatic" comes from)
  - Appropriate to a shock in which the flow variables change suddenly
    - $[F]_0^1 = 0$.
  - The Rankine-Hugoniot jump conditions reduce to:

$$\begin{aligned} [\rho v]_0^1 &= 0 \\ [P + \rho v]_0^1 &= 0 \\ \left[\frac{v^2}{2} + \frac{\gamma}{\gamma - 1}\frac{P}{\rho}\right]_0^1 &= 0 \end{aligned}$$

From continuity $\rho_1 = \rho_0 v_0 / v_1$ so the momentum condition to be written as

$P_1 = P_0 + \rho_0 v_0 (v_0 - v_1)$

Substitute both results into the energy condition and collect terms

$$\frac{\gamma+1}{\gamma-1}v_1^2 - \frac{2\gamma}{\gamma-1}\left(\frac{P_0 + \rho_0 v_0^2}{\rho_0 v_0}\right) v_1 + \left(\frac{2\gamma}{\gamma-1}\frac{P_0}{\rho_0} + v_0^2\right) = 0.$$

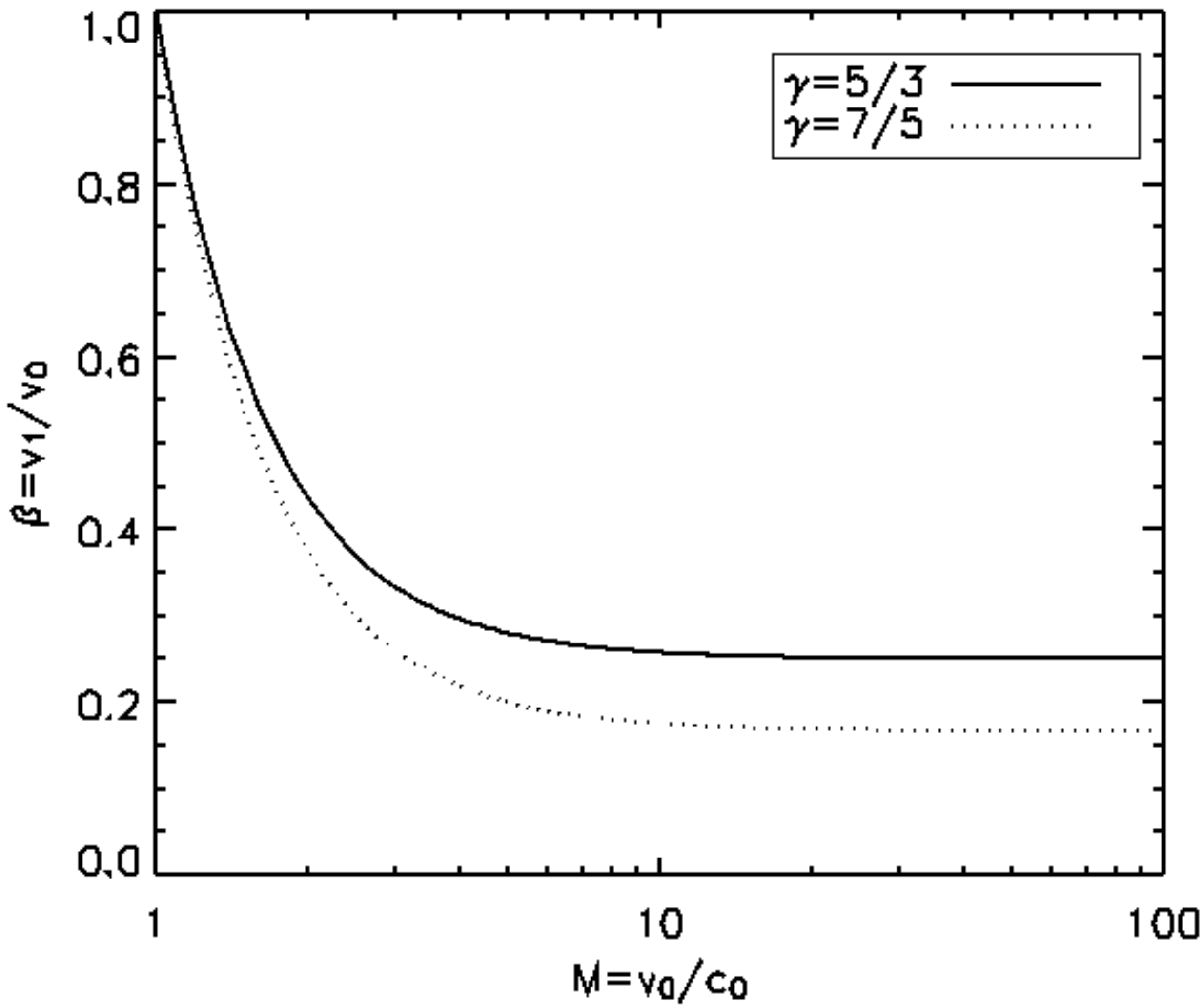
Eliminate P_0 in favor of $\boldsymbol{c_0}$, and dividing through by v_0^2 to make the dimensionless quadratic equation:

$$\frac{\gamma+1}{\gamma-1}\beta^2 - \frac{2}{\gamma-1}\left[\left(\frac{c_0}{v_0}\right)^2 + \gamma\right]\beta + \left[\frac{2}{\gamma-1}\left(\frac{c_0}{v_0}\right)^2 + 1\right] = 0.$$

where $\beta = (v_1/v_0)$.

- The ratio of the flow speed to the sound speed, v_0 / c_0 is defined as the ***Mach number*** of the flow, M, so

$$\frac{\gamma+1}{\gamma-1}\beta^2 - \frac{2}{\gamma-1}\left(M^{-2}+\gamma\right)\beta + \left(\frac{2M^{-2}}{\gamma-1}+1\right) = 0$$



For a fast enough flow, M^{-2} can be neglected, and in a ideal monatomic gas ($\gamma = 5/3$) we have

$$v_1 = v_0$$

(trivial solution) or

$$v_1 = \frac{v_0}{4}$$

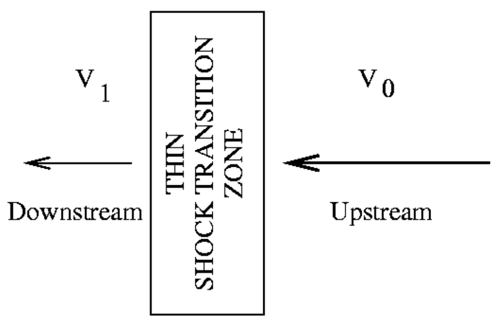
- The second, non-trivial solution is the asymptotic ***strong shock solution***
  - A ***monatomic gas is compressed by*** $\times 4$ in its passage through a strong shock

* An "adiabatic" shock is an oxymoron. Shocks are irreversible and increase the entropy across the front. Nonetheless, the term "adiabatic" shock is frequently used to refer to a shock, but describes the post-shock flow.

Strong shocks

The ***strong shock limit*** occurs when the ***gas pressure in the preshock gas is negligible*** compared with the ram pressure

- The shock is stationary in our frame of reference



  - Gas flows in at v_0 and out at v_1 such that $v_0 > v_1$
  - Consider the frame where the shock appears to advance at speed v_0 into stationary gas
    - v_0 is referred to as the ***shock velocity***, $v_0 = v_s$

The solution for the postshock flow variables in the strong shock limit ($\gamma = 5/3$) is

$$\begin{aligned} v_1 &= \frac{v_s}{4} \\ \rho_1 &= 4\rho_0 \\ P_1 &= \frac{3}{4}\rho_o v_s^2 \end{aligned}$$

- The bulk kinetic energy in the flow is decreased on passage through the shock
  - Energy is conserved (***F*** = 0)
    - The decrease in flow energy is balanced by an increase in thermal energy

- The specific internal energy of the postshock (monatomic) gas is

$$\varepsilon = \frac{3P_1}{2\rho_1} = \frac{9}{32}v_s^2$$

Interestingly, this is identical to the specific (rest frame) KE of the postshock gas

$$\frac{1}{2}(v_1 - v_0)^2 = \frac{1}{2}\left(\frac{3}{4}v_s\right)^2 = \frac{9}{32}v_s^2$$

A strong shock enforces equipartition between bulk kinetic energy and internal energy

- Writing the equation of state as $P/\rho = kT/\mu m_H$
  - The ***postshock temperature*** in the strong shock limit

$$\begin{aligned} T_1 &= \frac{3}{16k}\mu m_H v_s^2 \\ &\simeq 1.36 \times 10^5 v_{100}^2 : \text{ fully ionised} \\ &\simeq 2.90 \times 10^5 v_{100}^2 : \text{ neutal atomic plasma} \end{aligned}$$

The cases apply to ionized and atomic plasmas with solar composition, respectively, and shock velocity $v_s = 100 v_{100}$ km s^{-1}

  - Although the compression is limited (to a factor of four ($\gamma = 5/3$) the temperature and pressure increase without limit as the shock speed is raised

- As a result of the increased temperature of the postshock gas, the velocity of the postshock flow is subsonic
  - In a monatomic plasma $M_1 = 5^{-1/2} \simeq 0.447$
  - Because the flow is subsonic, its ram pressure can be neglected
  - Isobaric cooling ($P = const.$) is a good approximation of the subsequent flow
    - $\mathbf{B}$ does not influence the structure of the shock front
    - The magnetic pressure is increased at most by a factor of 16 across a strong shock

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Isothermal Shocks

When shock-heated gas radiates & cools, the full Rankine-Hugoniot conditions must be integrated to solve for the subsequent flow parameters.

- A limiting case occurs when the gas cools and returns to its original temperature
  - For example, when heating is established by external agencies
    - Initial & final temperatures are maintained as a balance between photoelectric heating & far-IR fine structure line cooling
- A fully radiative shock is an ***isothermal shock***
  - The sound speed in the post- and pre-shock gas is unchanged
  - An equation of state replaces energy conservation

$$\begin{aligned} [\rho v]_0^1 &= 0 \\ [P+\rho v^2]_0^1 &= 0 \\ [P/\rho]_0^1 &= 0 \end{aligned}$$

The equation of state for an isothermal gas is $P = P_0(\rho/\rho_0)$. Effectively, $\gamma = 1$ at the two control points in the preshock and postshock plasma

- The ***isothermal sound speed*** is $c_s^2 = P/\rho$.
  - In terms of the Mach number of the preshock flow yields the quadratic equation

$$\left(\frac{v_1}{v_s}\right)^2 - (M^{-2}+1)\left(\frac{v_1}{v_s}\right) + M^{-2} = 0$$

with trivial solution

$$v_1 = v_s$$

and significant solution

$$v_1 = v_s \ M^{-2}$$

  - From the shock solution and the equation of continuity the ***maximum compression*** in an isothermal shock is M^2

Magnetic field limited compression

The total energy radiated in the shock per unit area when $\mathbf{B}=0$ is

$$\dot{E}=\frac{\rho_0 v_s^3}{2}(1-M^{-2}).$$

- All the KE of the shock can be radiated away as M^2 tends to infinity
- Eventually, pressure due to the transverse component of the magnetic field, $\boldsymbol{B}^2/8\pi$, will dominate the gas pressure at some point in the postshock flow
    - Magnetic pressure increases as the square of the density
    - Gas pressure is limited by the ram pressure of the material entering the shock and decreases as gas cools radiatively

- At a certain point in the cooling plasma, the gas switches from being thermal pressure supported to being magnetic pressure supported

    - In postshock gas (0)
        - Magnetic field pressure dominates the gas pressure & the ram-pressure terms
    - In preshock gas (1)
        - Magnetic & thermal pressures in the preshock gas is negligible compared with the ram-pressure (fast shock limit)
    - From the momentum equation

$$\frac{B_1^2}{8\pi}=\rho_0 v_s^2.$$

for a frozen magnetic field (Bv=const.), equating the shock ram pressure to the postshock transverse magnetic pressure gives the ***maximum compression*** in magnetized shock

$$\frac{\rho_1}{\rho_0}=2^{1/2}M_A$$

M_A is the Alfven Mach number, the ratio of the shock velocity to the Alfven velocity,

$v_A^2=B^2/4\pi\rho$.

In fully ionized gas, the Alfven velocity in the preshock gas is likely to be of order of the sound speed, c_{II}, because turbulence in the ISM tends to wind up any preexisting field

- If $\mathbf{B}$ is intially dynamically negligible and and there is no significant magnetic pressure support then the turbulence is not damped by the emission of Alfven waves

- However, as the $\mathbf{B}$ increases (at a given density), the magnetic pressure and the magnetic energy both increase
- Eventually reach an equipartition value for $\mathbf{B}$, where the turbulence is dissipated through Alfven waves making the medium magnetically stiff
  - For a fully ionized plasma equipartition of thermal and magnetic energy densities gives

$$\frac{B^2}{4\pi} \sim \frac{3}{2} nkT_0$$

where n is the total particle density. This is effectively the same as the condition $v_A \simeq c_{II}$, which gives,

$$\frac{B^2}{4\pi} \sim \gamma P = \gamma nkT_0$$

If these conditions are assumed to hold then the transverse field is related to the preshock density by

$$\frac{B_0^2}{4\pi} \sim n_0 kT_0.$$

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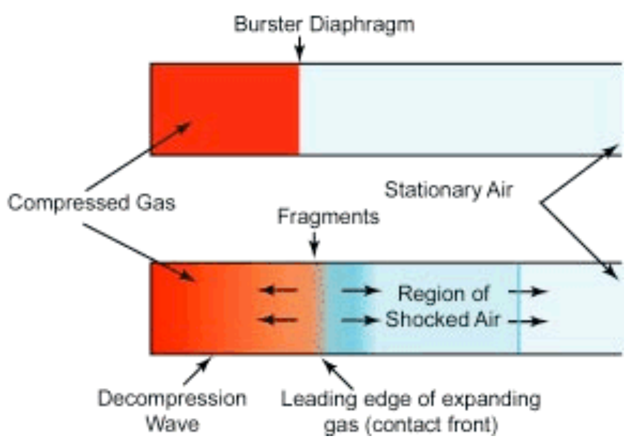
Interstellar Shock Drivers

Insterstellar shock are driven by "pistons"

- These arise in the ISM in:
  - Energetic outflows
    - From newly born low-mass stars (T Tauri stars)
    - Nuclei of active galaxies (outflows may be relativistic)
  - Radiatively driven stellar winds from in hot stars
    - OB stars or the central stars of planetary nebulae
  - Explosions which deposit a large quantity of energetic plasma at a single point in space and time
    - Supernovae
    - Gamma ray bursts

All outflows eventually interact with the surrounding medium, either pre-existing CSM, ISM or IGM

- A simple way to think of such flows is to consider high-pressure gas expanding supersonically down a pipe into a low pressure ambient medium (shock tube analogy)
    - Constant cross section tube
      - The flow simulates a jet
    - Constant opening angle
      - The flow simulates a sector of a spherical outflow
  - Suppose at some point in the tube a diaphragm prevents the compressed from filling the low pressure medium that initially exists in the remainder of the tube

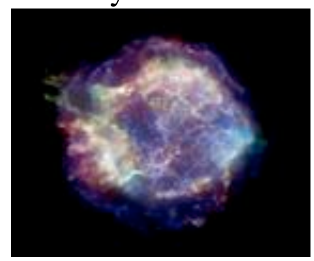


    - This atmospheric gas simulates the ISM or circumstellar medium.
    - As the expanding high-pressure gas reaches the diaphragm is removed

[Shock Tube with x10 pressured differential (calculated using http://rainman.astro.uiuc.edu/ddr/)]

- High pressure supersonically expanding gas pushes against the stationary gas it has encountered
  - Drives a shock into the stationary (CSM/ISM/IGM) gas
  - Energy needed to heat and accelerate the stationary gas comes from the kinetic energy of the expanding medium--this gas is abruptly slowed
  - This occurs through another shock that is transmitted into the expanding gas (which represents the stellar ejecta or the jet flow)

- Imagine the shock from the point of view of an outside stationary observer
  - Although propagating into the out flowing gas, initially we would probably see this (inner) shock moving outward down the tube, as long as the outflow velocity in the flow exceeds the (inward) shock velocity



  - The outer shock is called the blast wave, and the inner shock is called the reverse shock
  - The velocities of these two shocks are not in general the same and the temperatures of the medium shocked by the blast wave and the expanding medium shocked by the reverse shock are not the same
    - However, the pressures of these two shocked gases must be the same
    - Between the two gases, where the diaphragm was initially located, we must therefore have a jump in temperature accompanied by a jump in density such that the product nT is constant
    - The gas on both sides of the discontinuity have the same velocity
    - Such a discontinuity of density and temperature (at constant pressure) is called a contact discontinuity

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Supernovæ

Supernova explosions deposit kinetic energy ~ 10^{51} ergs into the ISM

- For massive stars, (M > 8 M_o) the explosion is driven by the outward pressure exerted by escaping neutrinos produced in the collapse of the stellar core to a neutron star or a black hole
- For low-mass stars (< 8 M_o initially, and $\simeq 1.4 M_\odot$ at the time of the explosion), the explosion results from explosive thermonuclear burning under initially electron-degenerate conditions
  - Much of the star being burnt to nuclear statistical equilibrium, which is dominated by Fe-peak elements
- In the initial fireball stage, the ejected material cools by adiabatic expansion following emergence of the shock at the stellar surface
  - The bright optical display is driven by the diffusion of radiation
    - Radioactive decay of ^{56}Ni is a significant contributor
  - As the stored radioactive energy diffuses away, the fireball cools and recombines until finally it becomes optically thin to the escape of radiation
  - At this time, it has become clumpy through the action of thermal instabilities and incomplete mixing, and, because the internal pressure is nearly equalised
    - Expansion is approximately homologously ($v \propto r$)

Evolutionary stages of supernova remnants

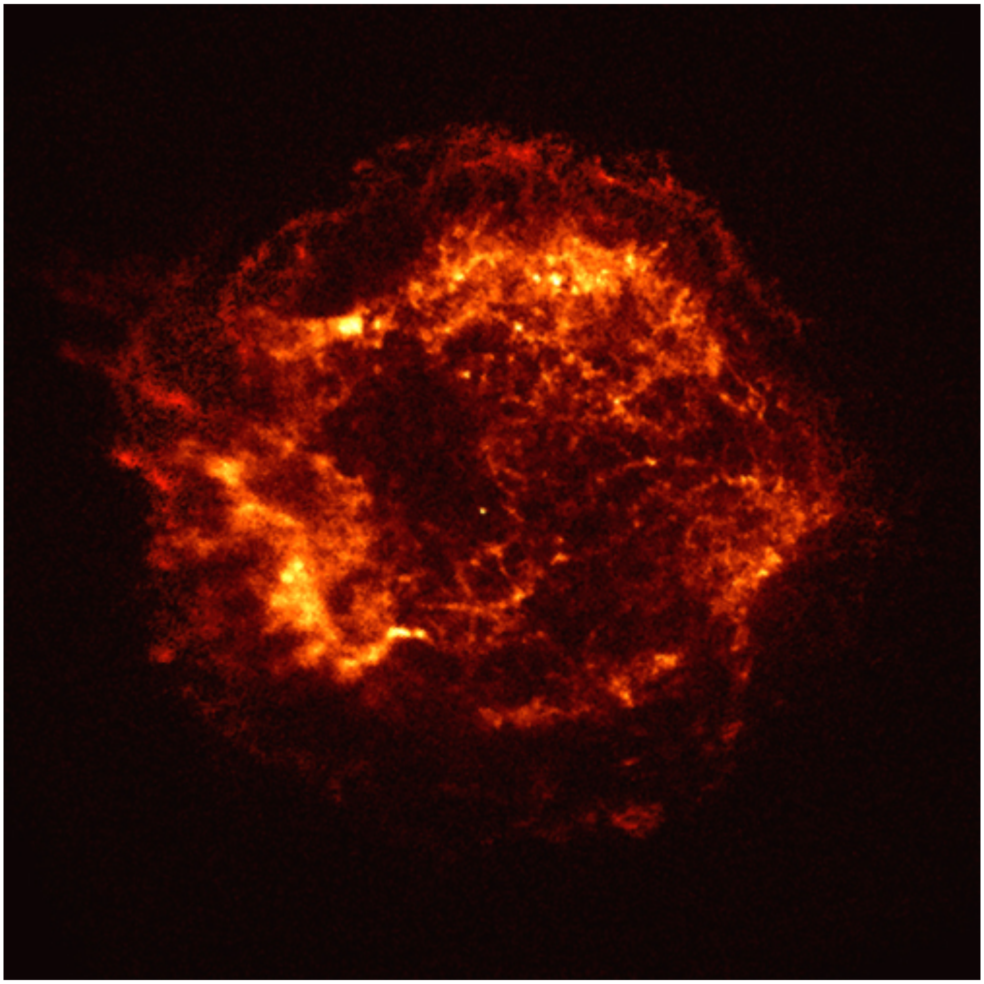
1) Interaction with the circumstellar/interstellar enviroment

- When ejecta first interact with the surrounding ISM/CSM with density ρ_0, at radius R, they initially drive a shock at a velocity v_s determined by the fastest moving ejecta with density $\rho(R,t)$ and with velocity $v(R,t)$;

$$\rho_0 v_s^2 = \rho(R,t) v^2(R,t).$$

- Thereafter the blast wave is slowed down as ISM/CSM is swept into a shell
  - A reverse shock propagates back into the freely expanding ejecta
    - Converts KE of expansion into thermal energy
  - Hot shocked ejecta & the hot shocked ISM/CSM are separated by a contact discontinuity
    - The gas on either side of the contact discontinuity has different temperatures and

densities, although the thermal pressures are the same
- If the shocked stellar ejecta are cooler and denser than the swept-up ISM/CSM, the contact discontinuity is Rayleigh-Taylor unstable, since the velocity of expansion is slowing with time
- This configuration produces mixing across the contact discontinuity



Chandra X-ray image of the Cassiopeia A supernova remnant. Two shock waves are visible: a fast outer shock and a slower inner shock. The inner shock is believed to be due to the collision of the ejecta from the supernova explosion with a circumstellar shell of material. Fluid instabilities, such as the Rayleigh-Taylor instability, have caused the ejected to become clumped.

Swept-up gas

Consider the case when the ejecta are not clumpy and the interstellar gas has a density ρ_0

- When the mass of swept up ISM/CSM, M_{su} is $\gg$ than the mass ejected, M_{ej}, the reverse shock has swept back down to the explosion center, and all of the ejecta have been shock heated.
  - This marks the start of ***Sedov-Taylor*** phase of evolution.

    The swept up mass is

$$M_{su} = \frac{4}{3}\pi R^3 \rho_0 = 0.145\ n_0 R_{pc}^3\ M_{\odot}.$$

so that $M_{su} = M_{ej}$ when

$$R = 1.9\ \left(\frac{M_{ej}}{n_0}\right)^{1/3}\ \text{pc}$$

- Massive SN progenitors can create low density wind bubbles--cavities with $n_0 \simeq 10^{-2} - 10^{-3}$ cm $^{-3}$.
  - In this case the SN ejecta interact with CSM first
  - The radius is larger than for typical interstellar values and values of $\simeq 10$ pc may be expected, especially for SN with massive progenitors.

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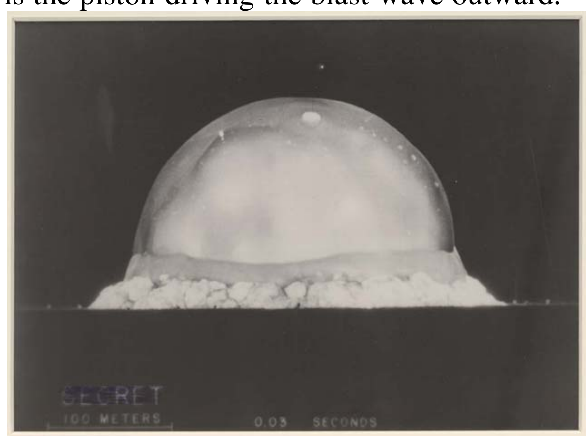
2) Sedov-Taylor expansion

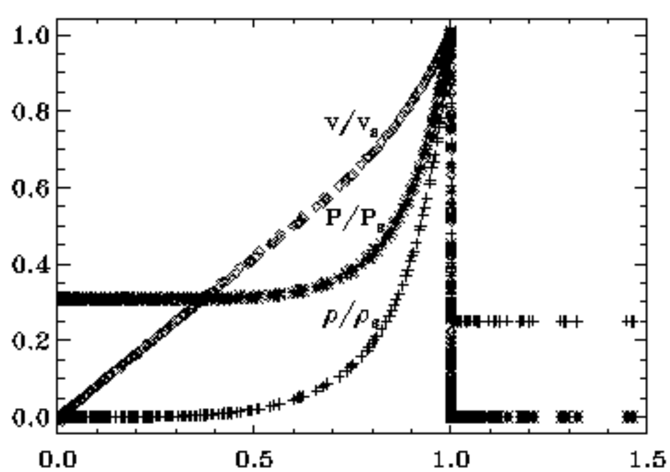
In the Sedov-Taylor solution the expanding bubble of hot gas is the piston driving the blast wave outward.



- The specific internal (thermal) and kinetic energies behind a strong radiationless blast-wave shock at radius R

$$\varepsilon = \frac{1}{2}v^2 = \frac{9}{32}\dot{R}^2$$

  are equal where $v_s = dR/dt = \dot{R}$.
  - This is true so long as the shock is strong (***M*** >> 1)
- Since the blast wave is decelerating, the specific internal energy of the gas varies with radius within the bubble of hot gas
  - The thermodynamic and flow variables, vary in a self-similar way with respect to the dimensionless radial space variable r/R
  - The total energy in the bubble of hot gas, E_0, equal to the energy injected by the supernova (neglect radiative losses)



$$E_0 = \phi \frac{4}{3}\pi R^3 \rho_0\ \dot{R}^2,$$

where $\phi \simeq 1$ is a dimensionless parameter, which accounts for the distribution of specific internal energy within the bubble. This is the equation of motion of the bubble.

When $t \rightarrow 0$, $R \rightarrow 0$, so that this equation has the solution,

R/R.

$$R = \left(\frac{25}{3\pi\phi}\right)^{1/5} \left(\frac{E_0}{\rho_0}\right)^{1/5} t^{2/5}$$

the blast-wave velocity in the Sedov- Taylor phase is therefore,

$$v_s = \frac{2}{5} \left(\frac{25}{3\pi\phi}\right)^{1/5} \left(\frac{E_0}{\rho_0}\right)^{1/5} t^{-3/5}$$

Inserting numerical values, with time in years,

$$R = 0.314 \ \left(\frac{E_{51}}{n_0}\right)^{1/5} t_y^{2/5} \ \mathrm{pc},$$

$$v_s = 1.23 \times 10^5 \ \left(\frac{E_{51}}{n_0}\right)^{1/5} t_y^{-3/5} \ \mathrm{km\ s^{-1}},$$

$$T_s = 2.09 \times 10^{11} \ \left(\frac{E_{51}}{n_0}\right)^{2/5} t_y^{-6/5} \ \mathrm{K},$$

- The form of these equations can be found by dimensional analysis
  - R at a given time is determined by the only two parameters available in the problem, E_0 and ρ_0.
  - E_0 has dimensions ML ^{2} T $^{-2}$ and ρ_0 ML $^{-3}$
    - It is evident that $R \sim (E_0/n_0)^{1/5}\ t^{2/5}$

Radiative Losses

- The Sedov-Taylor phase terminates when radiative losses behind the blast wave become important
  - A thin shell of swept-up cooling gas forms near the outer boundary
  - This phase sets in when the cooling timescale of the shocked plasma at the blast wave

$$\tau_{cool} = \frac{(3/2)(n_e + n)kT}{n^2\Lambda}$$

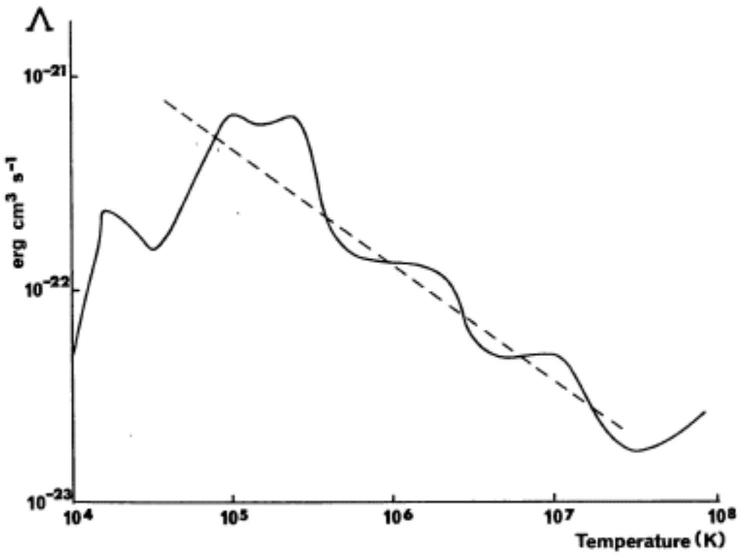
is shorter than the dynamical expansion time

$$\tau_{exp} = \frac{R}{v_s} = \frac{5t}{2}$$

The SNR luminosity, primarily radiated in soft X-rays, is

$$\begin{aligned} L_X &\propto n_0^2 R^3 \Lambda(T_s) \\ &\propto n_0^2 R^3 T^{-1/2} \\ &\propto R^{9/2} \end{aligned}$$

where the last steps follows from the Kahn (1976 AA 50 145) cooling approximation, $\Lambda \simeq 1.6 \times 10^{-19} T^{-1/2}$ ergs s^{-1} cm^3 so that $T^{-1/2} \propto v_s^{-1} \propto R^{3/2}$.



The atomic cooling function for hot gas in coronal ionization equilibrium (Raymond Cox & Smith 1976 ApJ 204 290) and Kahn's approximation (dashed line)

- The onset of cooling is a steep function of the remnant size (see later for numerical values)

---

3) Pressure driven snowlplough (PDS)

- When cooling is important PdV work done by the hot interior gas on the ISM is subsequently radiated away, decreasing the energy of the hot bubble
  - The energy equation is

$$\dot{E} = -4\pi R^2 P \dot{R},$$

and the adiabatic equation of state gives

$$E = \frac{4}{3}\pi R^3 \frac{P}{\gamma - 1}.$$

where E is the instantaneous energy content of the bubble.

  - The equations of mass and momentum conservation for a thin shell are

$$M = \frac{4}{3}\pi R^3 \rho_0$$

and

$$\frac{d(M\dot{R})}{dt} = 4\pi R^2 P.$$

This gives the intermediate evolution of the shell, $R \propto t^{2/7}$ (McKee & Ostriker 1977 ApJ 218 148).

- Fits to numerical hydrodynamic calculations that include cooling (Cioffi et al. 1988 ApJ 334 252) show that the onset of the PDS phase occurs at

$$t_{pds} = 1.33 \times 10^4 \ \frac{E_{51}^{3/14}}{n_0^{4/7}} \ \mathrm{yr}.$$

- A version of the PDS that provides continuity with the Sedov-Taylor expansion is given by Cioffi et al.

$$R = R_{pds}\left[\frac{4}{3}\frac{t}{t_{pds}} - \frac{1}{3}\right]^{3/10}$$

where

$$R_{pds} = 14.0 \ E_{51}^{2/9} n_0^{-3/7} \ \mathrm{pc}.$$

- This is valid until $\simeq 35 \ t_{pds}$ and a dense shell forms at $t_{sf} \simeq 2.72 \ t_{pds}$.
    - Cioffi's solution allows for cooling of interior gas, which is neglected in the analytic PDS.

---

4) Momentum conserving coasting & merging with the ISM

In the final phase, the stored thermal energy has been entirely radiated away, and we can neglect the pressure

- Only the momentum of the dense shell keeps the remnant expanding
    - If momentum of the SNR is deposited in swept-up material, momentum conservation is expressed

$$M_0 v_0 = \frac{4\pi}{3} R^3 \rho_0 \dot{R},$$

where the product of M_0, the mass of the ejecta and the mean ejecta velocity v_0, represents the initial momentum

- The ***momentum conserving snowplough*** phase of evolution was first considered by Oort (1946 BAN 10 187)
- This equation has the solution,

$$R = \left(\frac{3M_0 v_0}{\pi\rho_0}\right)^{1/4} t^{1/4}.$$

- Towards the end of this phase, the expansion velocity becomes sonic or sub-sonic with respect to the sound speed or the magnetosonic speed
  - Remaining KE of the aging supernova remnant is dissipated through turbulent cascade

- Merger of the SNR with the ISM occurs when the rate of expansion is comparable to the sound speed in the abmient gas, or the velocity dispersion of clouds.
  - Returning to Cioffi et al.'s PDS solution for late times this equality is expressed at

$$v_{merge} \simeq v_{pds}\left(\frac{4}{3}\frac{t_{merge}}{t_{pds}}\right)^{-7/10} \simeq \beta c_0$$

β is a constant $\mathcal{O}(1)$ and c_0 is the isothermal sound speed.

  - Thus

$$\begin{aligned} t_{merge} &= \frac{3}{4}\left(\frac{v_{pds}}{\beta c_0}\right)^{10/7} t_{pds} \\ &= 153\left(\frac{E_{51}^{1/14} n_0^{1/7}}{\beta c_{06}}\right)^{10/7} t_{pds} \\ &= 2.04\times 10^6\ E_{51}^{31/98} n_0^{-18/49} (\beta c_{06})^{-10/7}\ \mathrm{yr} \end{aligned}$$

The isothermal sound speed, $c_0 = \sqrt{P_0/\rho_0}$, where P_0 is the total pressure, including any turbulent contribution.

- Typically $\beta \simeq 1$, except for a strongly magnetized medium where $\beta = \sqrt{2}$, so that βc_0 is the Alfven speed.

- The lifetime of a SNR is relatively brief compared to the lifetimes of typical massive progenitors.

Summary of idealized SNR evolution

For a uniform (!) medium

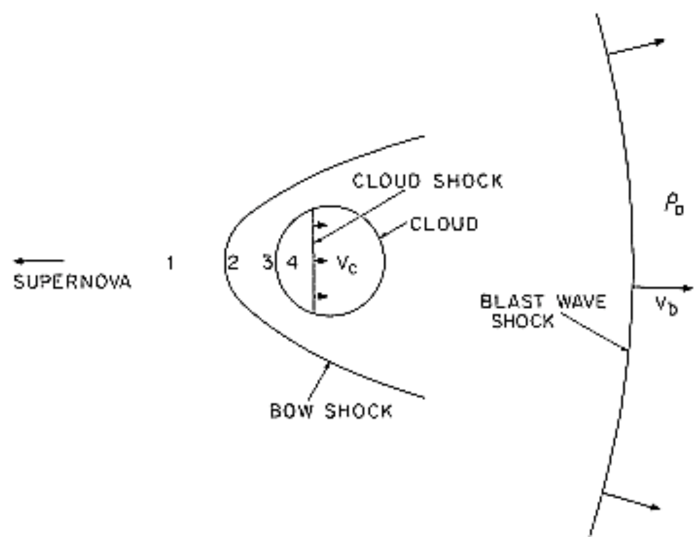
- The SN expansion velocity is constantly slowed by its interaction with the surrounding medium
  - Initially, $R \propto t$ (free expansion), but later $R \propto t^{2/5}$ (Sedov), then $R \propto t^{2/7}$ (PDS), and finally $R \propto t^{1/4}$ (snow plough)
    - The middle two phases correspond to the phases where energetic radiative or partially

radiative shock waves are seen
  - At first bright in the X-rays, but later at optical wavelengths as the stored internal energy is drained away in doing PdV work on the interstellar gas

- If the ejecta or the interstellar medium is cloudy, the evolution is more complex
  - The onset of the Sedov-Taylor phase is blurred, as first the intercloud medium is thermalized, and then the clouds are thermalized over an extended period of time
  - In a cloudy interstellar medium consisting of small clouds, the blast wave sweeps over and compresses the denser regions which then evaporate slowly into the shock-heated low-density postshock medium through the processes of turbulent shredding, mixing, and thermal conduction (?) (Cowie & McKee 1977 ApJ 211 135; McKee & Cowie 1977 ApJ 215 213)
  - When clouds have sizes comparable to the remnant, then the interaction is dominated by hydrodynamic, not diffusive effects (Graham et al. 1996)

Real supernova remnants

The real ISM is inhomogeneous and the encounter of a SN blast wave with a cloud drives a shock into the cloud



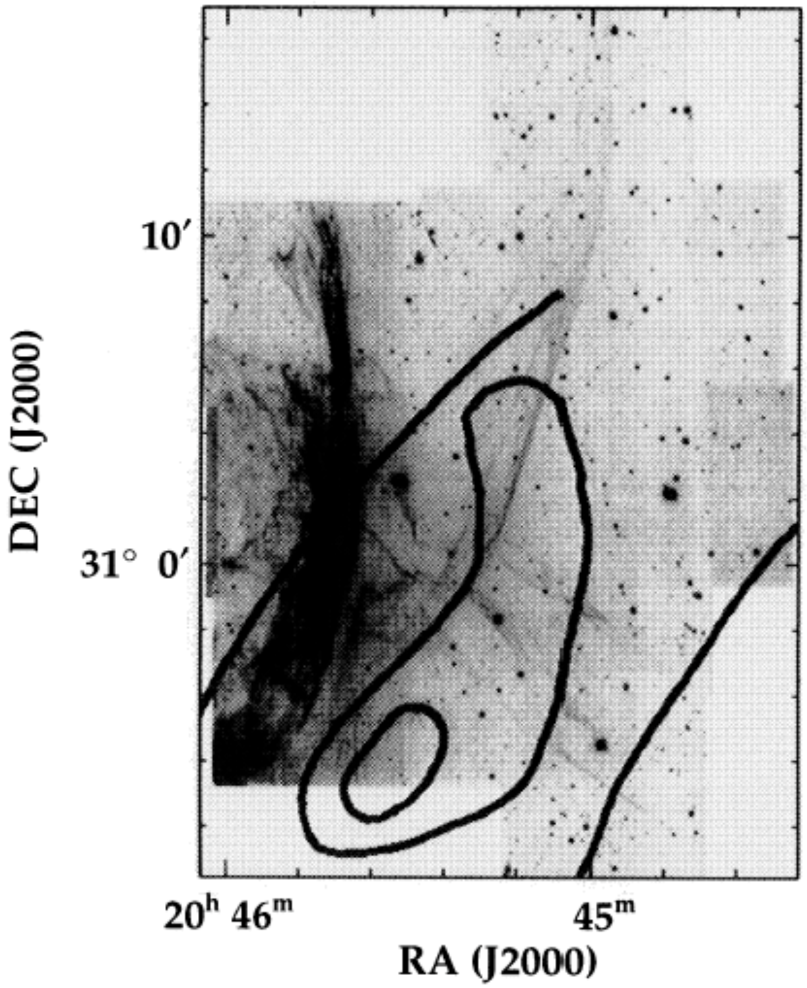
- Initially the pressure at the cloud surface is very high, and the rapid deceleration of blast wave by the cloud produces pressures up to $(3\gamma - 1)/(\gamma - 1) = 6$ times the blast wave pressure (Landau & Lifshitz and Spitzer 1982 ApJ 262 315).
  - For a small cloud engulfed by the blast wave the pressure relaxes so that there is approximate pressure balance behind the shock driven into the cloud and the blast wave

$$\rho_c v_c^2 \simeq \rho_0 v_s^2$$

- The blast wave propagates through the low density intercloud medium since the shock is faster in low density gas,
  - The cloud is compressed & accelerated
  - Velocity shear drives hydrodynamic instabilites which destroy a cloud of radius, R_c , on time scales of the order of the cloud crossing time

$$t_{cross} = \frac{R_c}{v_c}$$

  See Klein et al. (1994 ApJ 420 213) for purely hydrodynamic calculation or Maclow et al. (1994 ApJ 433 757) for MHD results which show that the presence of $\mathbf{B}$ tends to supress instabilities.



H-alpha image and CO 1-0 contours at the western edge of the Cygnus Loop (Levenson et al. 1996 ApJ 468 323)

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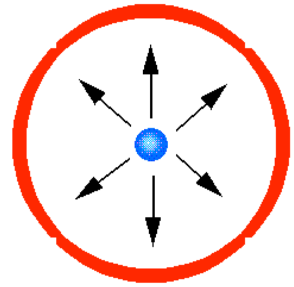
Stellar Wind Bubbles

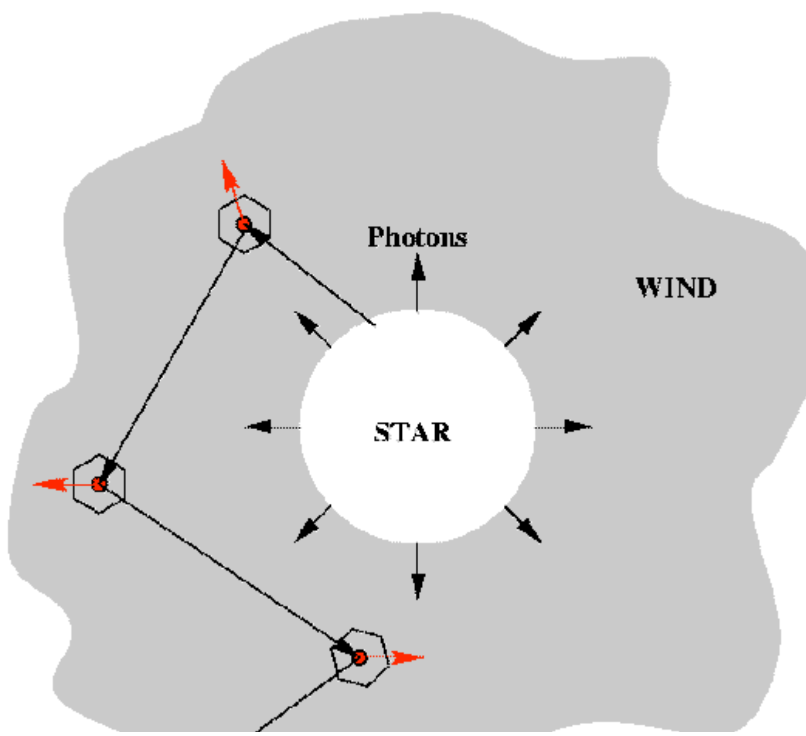


Hot stars produce fast winds driven by radiation pressure

- Physics of such winds is relatively well understood (e.g., Kudritzki & Puls AARA 2000 38 613)
- Radiation from the star is scattered by the atmosphere by UV resonance lines
  - Momentum carried in the radiation field, $L*/c$, is transferred to the atmosphere, to produce a wind
- If v_w is the terminal velocity attained by the wind, and $\dot{M}_w$ its mass flux (typically $\sim 10^{-5} M_\odot$ yr $^{-1}$ for massive stars and $\sim 10^{-8} M_\odot$ yr $^{-1}$ for the central stars of planetary nebulae), then

$$\dot{M}_w v_w = \frac{\eta L_*}{c}$$

η, which may be greater than unity for an optically thick envelope, accounts for the fact that each photon may be scattered many times before it escapes, enhancing the total amount of momentum that can be deposited.



- The wind cannot carry away more energy than is produced by the star:

$$\frac{1}{2}\dot{M}_w v_w^2 < L_*$$

Thus,

$$\eta < 2\frac{c}{v_w}.$$

Radiatively driven wind theory shows that $\eta \simeq 3-4$

- In radiatively driven winds, the outflow velocity is a factor $1 \tilde{<} \epsilon \tilde{<} 3$ times the

escape velocity at the base of the outflowPoCH

$$v_w = \epsilon \left(\frac{GM_*}{r_*} \right)^{1/2} \simeq 1000 - 4000 \mathrm{km\,s}^{-1}.$$

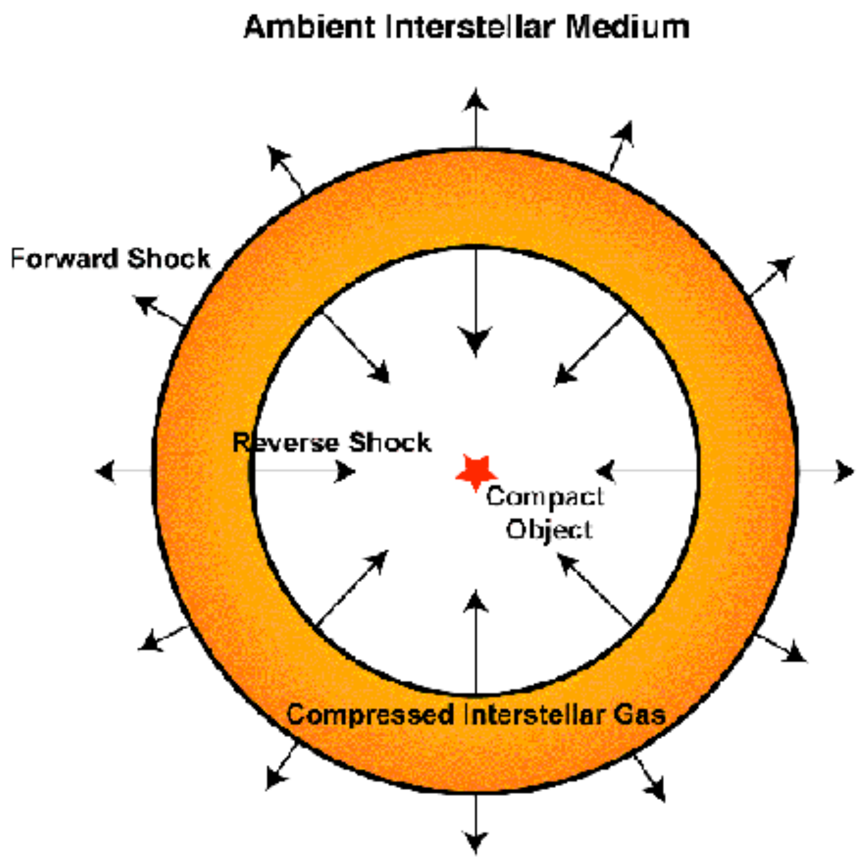
The wind expands freely until it is slowed by the collision with the surrounding ISM. At this point it passes through an shock, at an inner radius R_{in}, analogous to the reverse shock in a young SNR.

The wind bulk KE is "thermalized" by the shock and feeds into a thick, hot shell of gas.

- Hot gas is the "piston" that inflates the stellar wind bubble
  - The pressure, P, throughout the region between the inner shock and the outer shock is ~ constant, since the hot gas has a sound speed of order 500 km s $^{-1}$,
    - The sound-crossing timescale in the hot gas << the dynamical expansion timescale of the bubble, which has an expansion velocity in the range $\simeq 20 - 100$ kms $^{-1}$.
  - Pressure in the hot plasma is given by the rate of change of momentum per unit area of the stellar wind across the inner shock:

$$4\pi R_{in}^2 P = \frac{3}{4} \dot{M}_w v_w$$

The low expansion speed of the bubble ensures that the outer shock is radiative and isothermal at the temperature ($\simeq 10,000$ K) of the preshock gas, which is ionized by the central star.



- Assume that the shocked ISM is swept up into a thin shell at the outer radius of the bubble
  - The equation of conservation of momentum here is

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_0 \dot{R}\right) = 4\pi R^2 P,$$

or,

$$\frac{P}{\rho_0} = \dot{R}^2 + \frac{1}{3}\ddot{R}R.$$

If the hot gas occupies a constant fraction

$$\phi = 1 - \left(\frac{R_{in}}{R}\right)^3$$

of the total volume, and the energy input by the stellar wind,

$$\dot{E}_w = \frac{\dot{M}_w v_w}{2}$$

is the sum of the rate of change of thermal energy in the hot gas and the rate of PdV work done on interstellar gas, the energy conservation equation is,

$$\dot{E}_w = \frac{d}{dt}\left(\frac{3P}{2}\frac{4\pi\phi}{3}R^3\right) + P\frac{d}{dt}\left(\frac{4\pi}{3}R^3\right)$$

- Eliminate pressure to give the equation of motion of the shell.
  - Assume a power law , $R \propto t^\beta$ with the correct boundary conditions, $R \rightarrow 0$ $r \rightarrow 0$, then it follows by substituting for R and its derivatives with respect to time in the equation of motion that $\beta = 3/5$

$$R = \left[\frac{125}{\pi(70\phi + 84)}\right]^{1/5}\left(\frac{\dot{E}_w}{\rho_0}\right)^{1/5} t^{3/5}$$

(Castor et al. ApJL, 1975, 200, 107; Weaver et al. 1977 ApJ 218 377). This form can also be established through dimensional arguments, similarly to the Sedov solution for supernova remnants.

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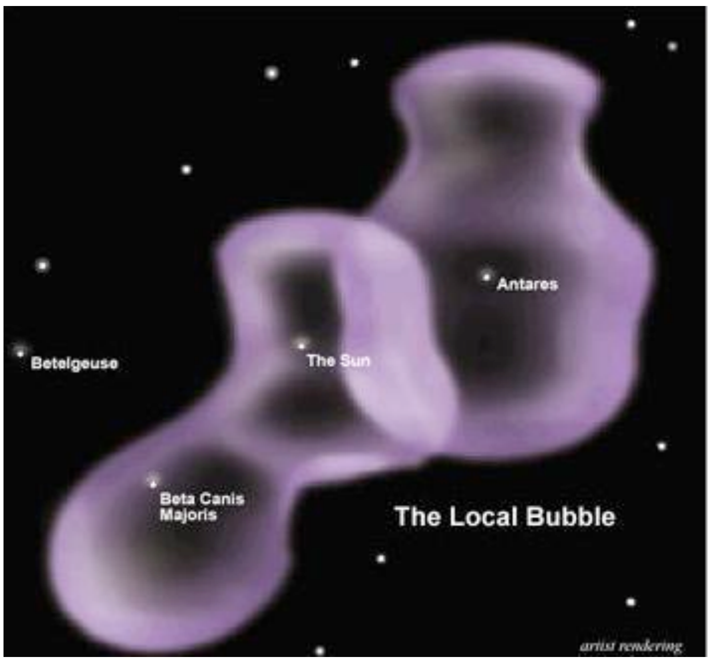
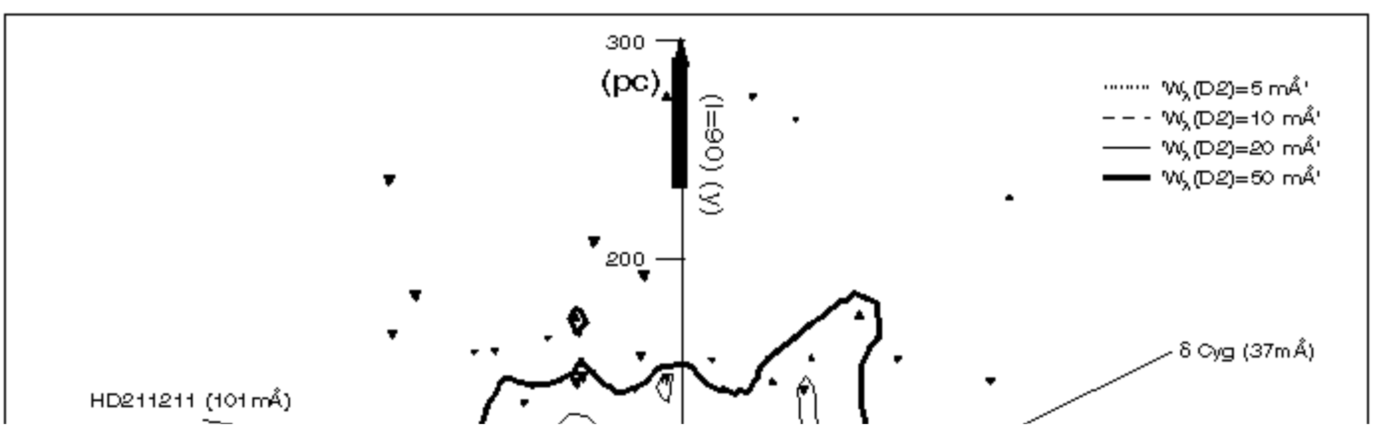
The Hot Ionized Medium (HIM)

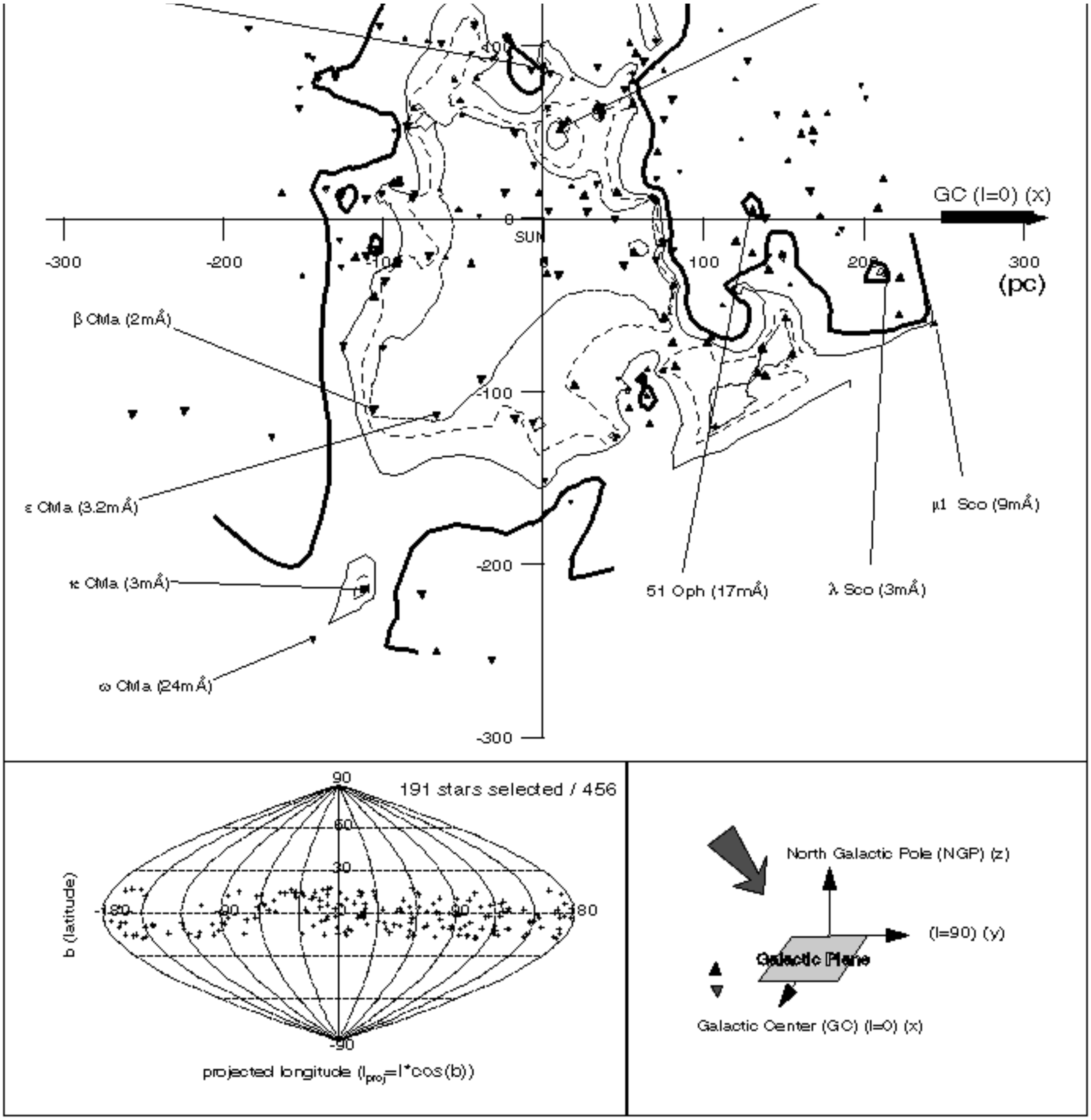
The hot phase of the ISM is thermally unstable—cooling time get shorter as the temperature drops

- The HIM exists because its density is so low & its cooling timescale is longer than the timescale over which it is reheated by supernova shocks
  - The prototype for the HIM is the local hot (d ~ 100 pc) bubble with $T \simeq 10^6$ K, and $n_H \sim 5 \times 10^{-3}$ cm $^{-3}$.
  - Schematically:



NaI EW (Sfeir et al. 1999 AA 346 785)





- SNR preferentially expand in the low-density phase
    - Low density gas leads to a high shock velocity
    - The mean interval between arrival of supernova remnant shocks is about 5×10^5 yr, locally
        - The hot medium sweeps over clouds in the cooler phases, shredding them, and heating the remnants by thermal conduction (?)

Entropy of the Hot Ionized Medium

We can use entropy arguments to trace the origin of the HIM

From the first law $TdS = dE + PdV$. For a monatomic, perfect gas, up to an addative constant the specific entropy is

$$\begin{aligned} s &= \frac{k}{\mu} \log\left(\frac{T^{3/2}}{n}\right) \\ &= \frac{k}{\mu} \log s_* \, . \end{aligned}$$

The quantity $s_* = T^{3/2}/n$, is proportional to the phase space per particle ($s_* \propto v^3 x^3$), is a measure of the specific entropy.

ISM Entropy

Phase	n	T	s_*	$t_{\rm cool}$
	(cm $^{-3}$)	(K)	(cm ^{3} K $^{3/2}$)	
CNM	30	100	30	...
WNM	0.4	8000	2×10^6	...
HIM	3×10^{-3}	5×10^6	1.2×10^{11}	7.6 Myr

- Shock heating increases the entropy of gas to a post-shock value of

$$\begin{aligned} s_* &= \frac{1}{4n_0}\left(\frac{3}{16}\frac{\mu v_s^2}{k}\right)^{3/2} \\ &= 1.28 \times 10^{10} \, \frac{v_{s8}^3}{n_0} \;\; \mathrm{cm^3 \; K^{3/2}} \end{aligned}$$

  Fast shocks in the WNM are a likely source of the HIM

Cooling time

- For a collisionally ionized gas the radiative cooling is approximated by a power law (Kahn 1976)

$$\Lambda = 1.6 \times 10^{-19} \; T^{-1/2} \; \mathrm{erg \; s^{-1} \; cm^3}$$

  for $10^5 < T/K < 10^{7.5}$

If s_H is the entropy per H atom, then the cooling is described by the First Law

$$nT\,\frac{ds_H}{dt} = -n^2\Lambda$$

where

$$s_H = x_t k \log(s_*)$$

and x_t is the (assumed constant) total number of particles per H.

Thus

$$x_t P \frac{d\log(s_*)}{dt} = -n^2\Lambda$$

For our simple cooling law, $\Lambda = \Lambda_0 T^{-1/2}$

$$x_t nkT\frac{d\log(s_*)}{dt} = -n^2\frac{\Lambda_0}{T^{1/2}}$$

or

$$\frac{x_t kT^{3/2}}{n}\frac{d\log(s_*)}{dt} = -\Lambda_0$$

The factor in front of the time derivative involves s_* so

$$\frac{ds_*}{dt} = -\frac{\Lambda_0}{x_t k}$$

which can be integrated

$$s_* = s_{*0} - \frac{\Lambda_0}{x_t k}t,$$

and the gas cools from $T \gg 10^5$ K down to 10^5 K in

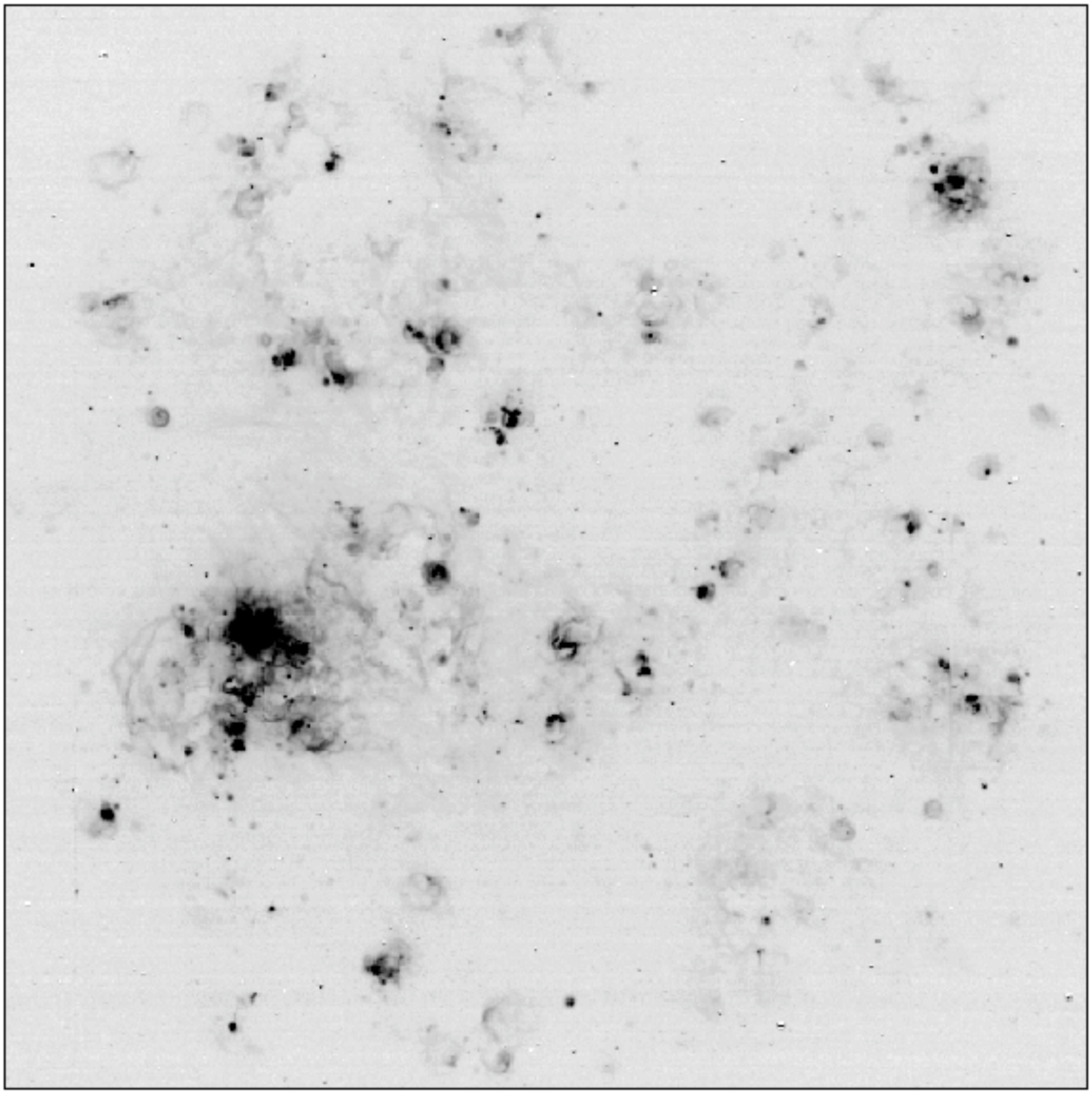
$$\begin{aligned} t_{cool} &= \frac{x_t k s_*}{\Lambda_0} \\ &= 6.3\times 10^5 s_{*10}\ \mathrm{yr} \end{aligned}$$

- Compare with the interval between the arrival of SN shocks of 5×10^5 yr

---

Porosity

- A key parameter is the ***porosity,*** Q, which is the filling factor of this medium
    - $Q \ll 1$, regions of hot plasma remain isolated from one another, like the holes in Swiss cheese
    - $Q \tilde{>} 0.5$, hot bubbles connect, and the cooler phases are compressed into a ramified network of blobs and filaments.
        - This appears to be the case which most closely corresponds to what we see in our solar neighborhood, or in the Magellanic Clouds.
    - $Q \rightarrow 1$, the cooler phases are confined to isolated clouds
        - This is the situation most likely to apply in the hot Galactic halo



Interstellar bubbles in the Large Magellanic Cloud

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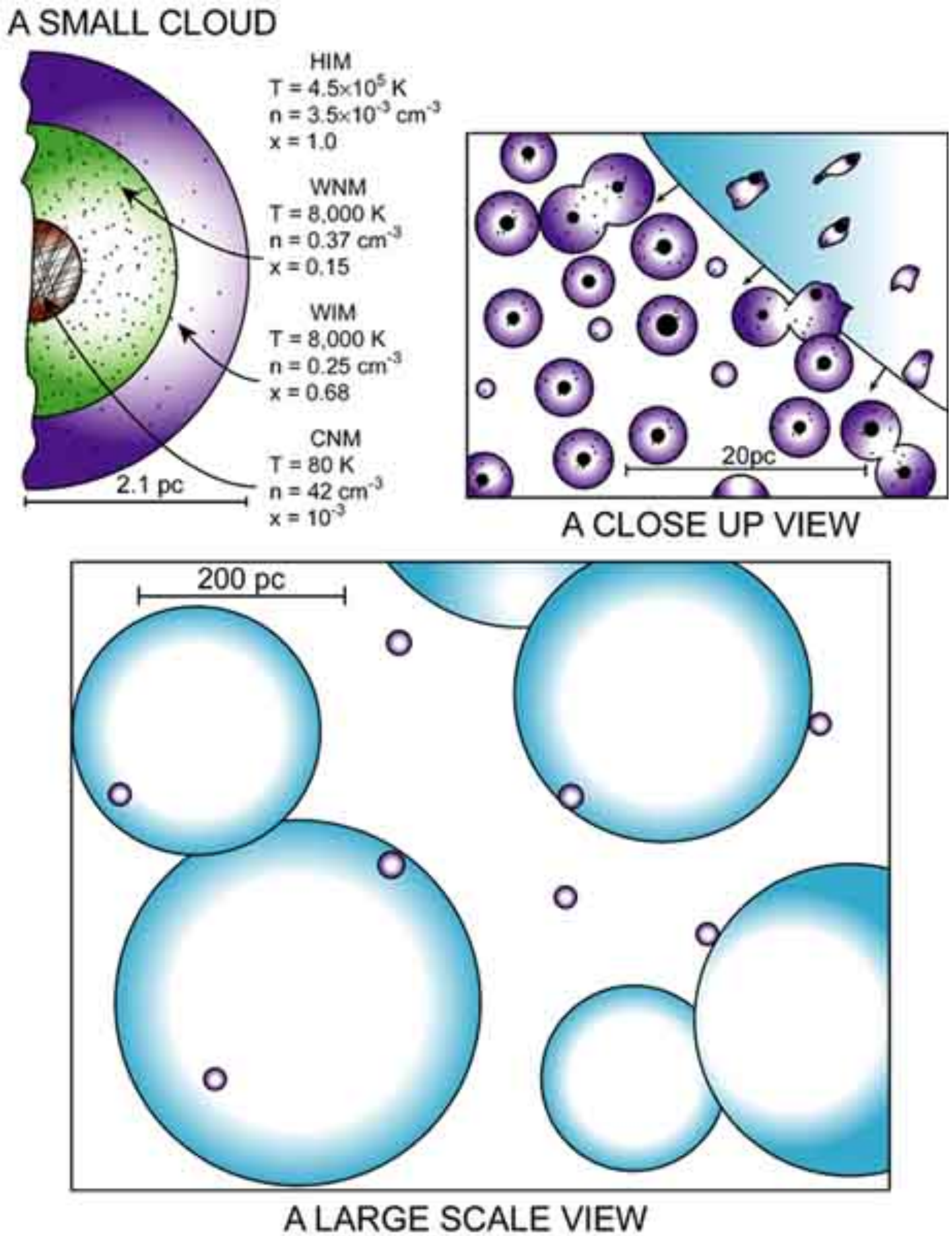
The Three Phase Model

Basic idea of the McKee & Ostriker (1977 ApJ 218 148) three phase model of the ISM is that the hot phase is pervasive & the ISM is dominated by individual SN explosions



- HIM fills the interior of SN remnants and powers their blast waves, which sweep up the gas & magnetic field inside the bubble and pile them into the shell
  - Shocked gas cools & recombines, forming the Cold Neutral Medium (CNM)
  - Soft X-rays produced by immediately adjacent HIM penetrate the halos of CNM clouds. Outermost more ionized portions are the Warm Ionized Medium (WIM), and more interior regions are Warm Neutral Medium (WNM)
- The sound speed in the HIM is high (> 100 km s^{-1})
  - The HIM communicates the background pressure to the other phases of the Galactic medium
  - This communication may occur via the halo, as local regions of over pressure are vented through galactic chimneys into the extended hot halo of the Galaxy

The MO model is impressive in its physical development & internal consistency.

- Makes many predictions & stands as the accepted paradigm against which most discussions and interpretations are compared
  - McCray & Snow (1979 AARA 17 213) present details & basic physics behind all aspects of the MO model
  - A modern perspective is given by Heiles astro-ph/0010047
  - A critical review is in Cox 2005 ARAA 43 337

The filling factor of the hot gas (Cox & Smith 1974 ApJ 189 L105) determines the ***porosity*** of the ISM due to SNR

- Cox & Smith showed that the porosity could be significant and that tunnels form in the ISM when adjacent SNR overlap

  Let $dQ(t)$ be the probability that a given point is inside a SNR of age $t \rightarrow t + dt$:

  $$dQ = SV(t)dt,$$

  S is the SN rate/unit volume and V is the volume of a SNR.

  If the effective disk area is 530 kpc ^{2} with thickness as 300 pc, then

  $$S \simeq 1.4 \times 10^{-13} \text{ pc}^{-3} \text{ yr}^{-1}.$$

  The expected # of SNRs encompassing a given point, all younger than t is

  $$Q(t) = S \int_0^t V(t')\ dt'$$

  where the integrand is the SNR 4-volume

  If SNRs expand as t^η for $t < t_m$ then

  $$\begin{aligned} Q(t) &= S\frac{4}{3}\pi R_m^3 \int_0^{t_m} \left(\frac{t}{t_m}\right)^{3\eta} dt \\ &= \frac{SV_m t_m}{3\eta + 1} \end{aligned}$$

  To find the total Q_{SNR} we can use Cioffi et al. (1988) for R_m and t_m,

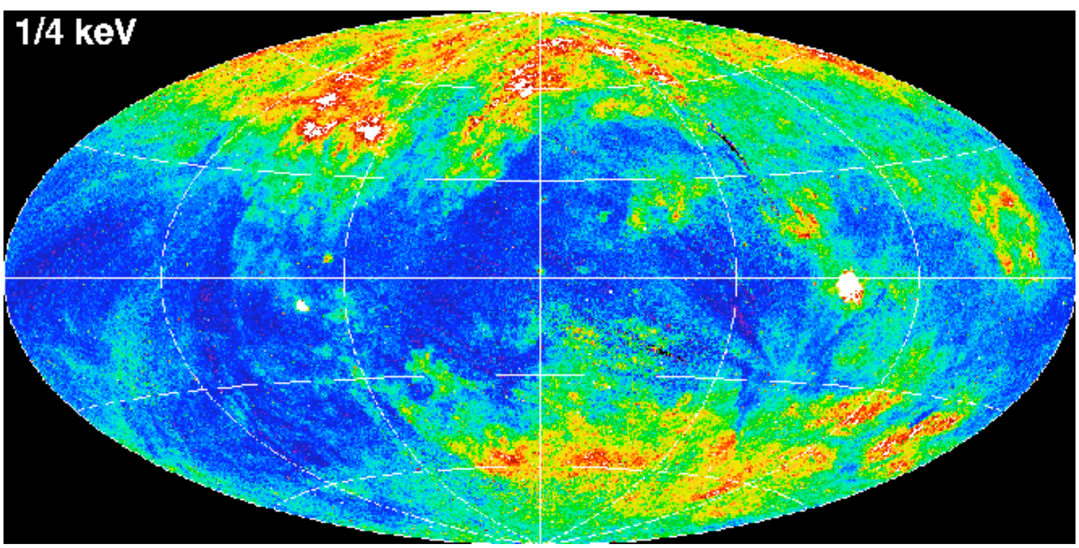
  $$Q_{SNR} = 0.78 \frac{S_{-13} E_{51}^{1.26}}{n_0^{0.14} P_{04}^{1.36}}$$

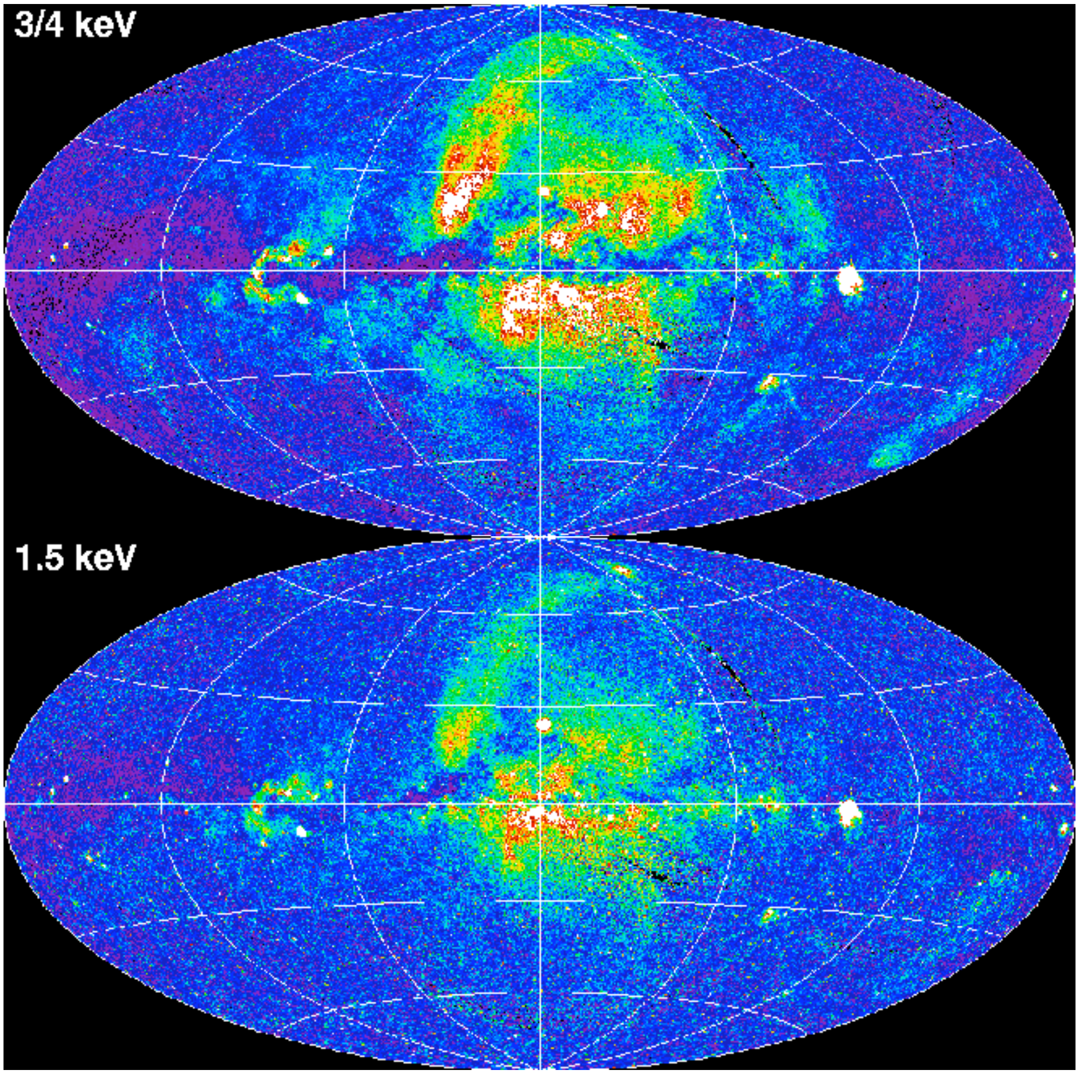
- McKee & Ostriker show that for a standard inter-cloud medium of FGH with $P_0 = 3000$ K cm $^{-3}$,
    - $Q_{SNR} > 1$
        - SNR overlap & we must consider a model in which hot gas is pervasive
            - ***A two phase FGH ISM is unstable in the presence of supernovae***
        - 21 cm observations (Heiles 2001 ApJL 551L 105) shows a significant fraction of WNM (> 47%) exists in the thermally unstable region between 500 and 5000 K
    - Slavin & Cox (1993 ApJ 417 187) include a magnetic field with $B = 5\mu$ G, adopt $S_{-13} = 0.4$, do not include SN in large association, which make superbubbles, $E_{51} = 0.75$ and $P_{04} = 0.9$, which leads to $Q_{SNR} = 0.18$.

- Theoretical models are very approximate and the value of Q must be resolved by observations.
    - Current estimates imply $Q = 0.4 \pm 0.2$.
        - There are big differences in the topology depending on whether $Q = 0.2$ or $Q = 0.6$!

Evidence for hot gas

- MO works well for the HIM:
    - The HIM is produced by SN shocks and resides in SN remnants and superbubbles.
        - Mapped via soft X-ray emission.
        - Cooler remnants can be observed with 0.25 keV X rays if the intervening HI column density is low enough ($N_{HI} < 0.6 \times 10^{20}$ cm^{-2}). This is tiny!
        - Even the thinnest cloud obscures 0.25 keV emission, so we can only map nearby structures, of which there are two prominent ones: the ***North Polar Spur*** and the ***Eridanus Superbubble*** (Snowden et al. 1997 ApJ 485 125).





- Gas having $T \lesssim 0.7 \times 10^6$ K must exist, because the HIM cools down as it ages. But such cool gas cannot be traced with X-rays
  - UV absorption lines of the He$^+$-like ions OVI, NVI, and CIV trace HIM at $T \lesssim 3 \times 10^5$ K and below.
    - Such lines provide velocity resolution and column density information (e.g. Shelton & Cox 1994 ApJ 434 599)
  - These lines can also be seen weakly in emission, which provides the EM instead of column density, so a comparison of absorption and emission provides n_e
    - OVI emission reveals gas at $T \simeq 3 \times 10^5$ K and implies high, $P/k \simeq 40{,}000$ cm^{-3} K (Dixon et al. 1996 ApJ 465 288).
    - CIV reveals $T \simeq 1 \times 10^5$ K and implies much lower pressures, $P/k \simeq 2000$ cm^{-3} K (Martin & Bowyer 1990 ApJ 350 242)
    - Data are poor and these results are provisional
      - SPEAR will make much better measurements, e.g., Welsh et al. 2007 A&A

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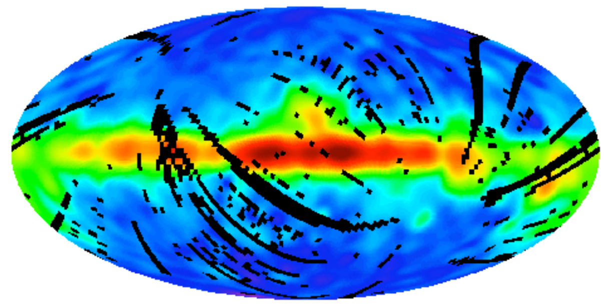
- The MO model does not cover everything
  - The MO model predicted the existence of the WIM as the envelopes of WNM clouds
    - Does not provide a consistent picture for the interpretation of the WIM via H-alpha EM and pulasr DM
  - MO is based on individual SN
    - Consider clustering of massive stars that produces superbubbles
  - CNM structures need to be relatively large across the line of sight compared to along the line of sight
    - Suggests morphologies are sheetlike---CNM is supposed to be formed by shocks
    - MO assume spherical clouds
  - The magnetic field and cosmic rays play important roles in the overall dynamics of the ISM
    - Their pressures are larger than the typically adopted thermal pressures of the gas components
    - The magnetic field links the different gas phases with each other, and also with the cosmic rays, so that components cannot act independently
    - Analysis of the dynamics and equilibria must include the whole ensemble, and this leads to new modes of behavior such as magnetically-linked clouds

COBE FIRAS 158 μm C^+ Line Intensity



COBE FIRAS 205 μm N^+ Line Intensity