Theoretical Astrophysics
Exercise Sheet 3

HS 17
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Issued: 02.10.2017
Due: 09.10.2017

Exercise 1 [Collision rate of two fluids]
In the lecture the collision integral for outgoing particles was defined as

$$\left(\frac{\mathrm{D}f}{\mathrm{D}t}\right)_{out} = \int_{\mathbb{R}^3}\int_{4\pi} f_1\, f_2\, v\,\sigma\,\mathrm{d}\Omega\,\mathrm{d}^3u_2\,. \tag{1}$$

Now we define the collision rate of two fluids as

$$C_{ab} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3}\int_{4\pi} f_a\, f_b\, v\,\sigma\,\mathrm{d}\Omega\,\mathrm{d}^3u_b\,\mathrm{d}^3u_a\,, \tag{2}$$

where $\vec{v} = \vec{u}_a - \vec{u}_b$ is the relative velocity, and $v = |\vec{v}|$. Assume that the following holds true:

(i) The particles of both fluids are rigid elastic spheres with radius $d/2$ (see Exercise 2 of Exercise Sheet 2).

(ii) Both fluids follow a Maxwell-Boltzmann distribution and have the same temperature.

Show that the collision rate is given by

$$C_{ab} = n_a n_b d^2 \left(\frac{8\pi k_B T}{\widetilde{m}_{ab}}\right)^{\frac{1}{2}}\,, \tag{3}$$

where $\widetilde{m}_{ab} = \frac{m_a m_b}{m_a + m_b}$ is the reduced mass.

Exercise 2 [Heat equation]
The heat equation in a *stationary* and *uniform* medium is given by

$$\frac{\partial T}{\partial t} = \nu\Delta T(\vec{x},\, t)\,, \tag{4}$$

where $\nu = \frac{2\kappa}{3nk_B}$ is the *thermal diffusion coefficient*.

(a) Derive equation (4) using the energy conservation equation (derived in Exercise 3 of Exercise Sheet 2), the diffusion law $\vec{Q}(\vec{x},t) = -\kappa\vec{\nabla}T(\vec{x},t)$ for the heat flux, and the ideal gas relation $e = \frac{3}{2}nk_BT$.

(b) Using the Fourier transform, solve the heat equation for the initial condition of an infinite pulse at $t = 0$, $\vec{x} = 0$, and $T = 0$ everywhere else.

(c) Integrate the solution you found over the entire space (d^3x). What is your interpretation of the result?

Exercise 3 [Validity of the fluid approximation in astrophysics]
In astrophysics we often use the fluid approximation, treating the ensemble of particles as a continuous fluid. This is only valid when the mean free path is much smaller than the size of the astrophysical object under consideration.

(a) Calculate the mean free path λ for the cases listed below, and discuss whether the fluid approximation is valid or not. You can find the typical sizes on the Internet or in literature. Assume that if $T < 10^4\,\mathrm{K}$, the gas is neutral and the total collision cross section is given by $\sigma_{tot} = 10^{-15}\,\mathrm{cm}^2$. Otherwise, the gas is ionised and σ_{tot} is the Coulomb cross section,

$$\sigma_{tot} = \frac{e^4}{m^2 \sigma_v^4} \ln(\Lambda) \tag{5}$$

where e is the elementary charge, m is the mass of the considered species, σ_v is the velocity dispersion given by $\sigma_v^2 = \frac{k_B T}{m}$, and $\ln(\Lambda)$ is the Coulomb logarithm $\ln(\Lambda) \approx 20$.

(b) For each case, calculate the collision time τ_{coll}, the viscosity coefficient μ and the thermal conduction coefficient κ. Note that an ionized gas consists of electrons and protons. For the calculation of μ and κ, take the mass of the particle species which contributes dominantly to them.

Table 1: **Gas Properties**

	T[K]	n[H/cm^3]
Intracluster medium	10^8	10^{-6}
Intergalactic medium	10^5	10^{-4}
Interstellar medium	8000	10^{-1}
Giant molecular cloud	10	10^3
Proto planetary disk	10	10^{12}
Solar core	10^7	10^{26}