Theoretical Astrophysics
Exercise Sheet 7

HS 17
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Exercise 1 [Protostars and accretion shocks]
Accretion shocks are shock fronts produced by accreting material at the surface of an object. An example of an accretion shock is the *standing* shock that forms at the surface of a protostar due to the *free-falling*, accreting material. This is illustrated in Figure 1.

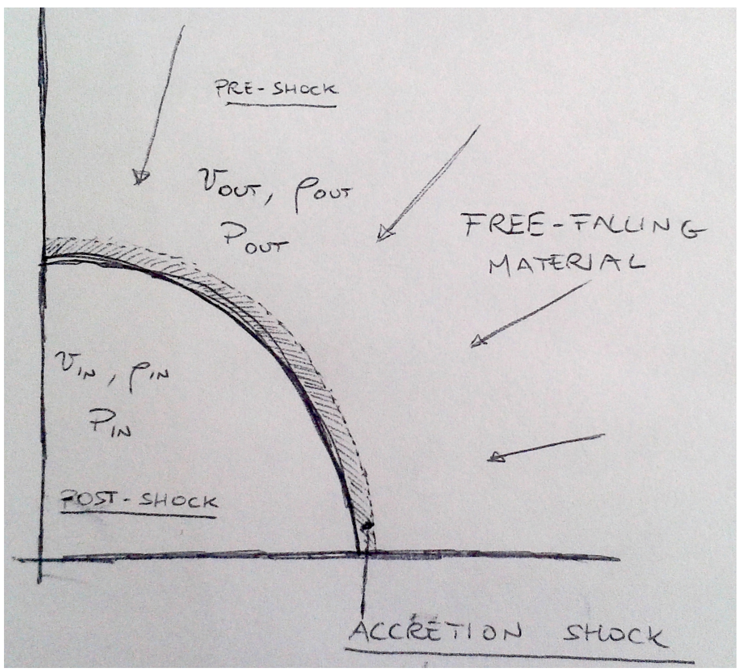

Figure 1: Graphical illustration of the accretion shock on to a protostar.

a) Accretion shocks are usually *strong* shocks (i.e. shocks with Mach number $\mathcal{M} \gg 1$). Show that the Rankine-Hugoniot jump conditions for a strong *adiabatic* shock propagating through a gas of neutral hydrogen reduce to:

$$\frac{\rho_{\rm in}}{\rho_{\rm out}} = \frac{v_{\rm out}}{v_{\rm in}} = \frac{\gamma+1}{\gamma-1}, \quad P_{\rm in} = \frac{2\rho_{\rm out}v_{\rm out}^2}{\gamma+1}, \quad T_{\rm in} = \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{m_p v_{\rm out}^2}{k_B}, \tag{1}$$

where the subscripts $_{\rm in}$ (post-shock) and $_{\rm out}$ (pre-shock) refer to Figure 1 *(Hint: In Exercise Sheet 1, we derived the relation $c_s^2(T) = \gamma P/\rho = \gamma k_B T/m$).*

Calculate the post-shock temperature assuming $v_{out} = 200 \mathrm{km\, s^{-1}}$. Can the post-shock gas remain neutral?

b) Let's now consider the spherical accretion shock of Figure 1. In steady-state, the continuity equation can be solved as $\dot{M} = 4\pi r^2 \rho v$. Use this result to calculate the post-shock properties $v_{\rm in}$, $\rho_{\rm in}$, $P_{\rm in}$, $T_{\rm in}$ of a *free-falling* gas just behind the accretion-shock front at the protostar radius R. Assume that the mass and the radius of the protostar are equal to those of the sun, $R = R_S$ and $M = M_S$, and that the accretion rate is given by $\dot{M} = 10^{-6} M_S/1\,\mathrm{year}$.

Exercise 2 [Blast waves]

a) Starting from the spherical Euler equations,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0 \tag{2}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0 \tag{3}$$

$$\frac{\partial e}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 e v\right) + \frac{P}{r^2}\frac{\partial}{\partial r}\left(v r^2\right) = 0 \tag{4}$$

derive the following differential equations for the post-shock values of a blast wave:

$$0 = -x\tilde{\rho}' + \frac{2}{(\gamma+1)x^2}\left(x^2\tilde{\rho}\tilde{v}\right)' \,, \tag{5}$$

$$0 = -3\tilde{v} - 2x\tilde{v}' + \frac{4}{\gamma+1}\tilde{v}\tilde{v}' + 2\frac{\gamma-1}{\gamma+1}\frac{\tilde{P}'}{\tilde{\rho}} \,, \tag{6}$$

$$0 = -3\tilde{P} - x\tilde{P}' + \frac{2}{x^2(\gamma+1)}\left(x^2\tilde{P}\tilde{v}\right)' + \frac{2\tilde{P}(\gamma-1)}{x^2(\gamma+1)}\left(\tilde{v}x^2\right)' \,, \tag{7}$$

where $\rho(x) = \rho_s\tilde{\rho}(x)$, $v(x) = v_s\tilde{v}(x)$, $P(x) = P_s\tilde{P}(x)$, a prime denotes a derivation by x, and:

$$x = \frac{r}{R(t)}\,, \quad R(t) = \left(\frac{E_0}{\rho_0}\right)^{1/5} t^{2/5}\,, \quad v_s = \frac{2}{\gamma+1}\dot{R}(t)\,, \quad \rho_s = \frac{\gamma+1}{\gamma-1}\rho_0\,, \tag{8}$$

$$P_s = \frac{2}{\gamma+1}\rho_0\dot{R}(t)^2\,. \tag{9}$$

b) Solve the system of equations (5)–(7) for $\gamma = 5/3$ by integrating numerically from x_s to 0. The inital values for the dimensionless variables are given by $\tilde{\rho}(x_s) = \tilde{v}(x_s) = \tilde{P}(x_s) = 1$. To find the value of x_s, first assume e.g. $x_s = 1.2$. If you obtain $v(0) > \epsilon$ for $\epsilon = 10^{-4}$, you should increase the value of x_s. If you obtain $v(0) < \epsilon$, you should decrease it. For which value of x_s do you obtain $|v(0)| \leq \epsilon$? For this value of x_s, plot your results for $\tilde{\rho} = \rho/\rho_s$, $\tilde{v} = v/v_s$ and $\tilde{P} = P/P_s$.