Theoretical Astrophysics
Exercise Sheet 8

HS 17
Prof. Romain Teyssier

http://www.ics.uzh.ch/
To be corrected by: Lichen Liang
Office: Y11 F 72, e-mail: lliang@physik.uzh.ch

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Exercise 1 [Convective flux in the Sun]

The convective heat transport rate per unit area is

$$Q_{\text{conv}} \sim C\rho v \Delta T \tag{1}$$

where C is the heat capacity, v is the velocity of the buoyant element, and ΔT is the temperature differential between the element and the surroundings to which it finally imparts this temperature.

(a) Show that the acceleration of the element due to buoyancy is

$$\ddot{r} = \frac{GM(r)}{r^2}\frac{\Delta T}{T}. \tag{2}$$

Hint: $\Delta P = \Delta(\rho T) = 0$, i.e. the pressure of the element and the environment always equal.

(b) Assuming that the *mixing length* (i.e. the length over which an element is transported before it diffusively mixes with the surroundings) is one-tenth of the solar radius, we get the velocity v is of the order of $(\ddot{r}R_\odot/10)^{1/2}$. You can now use this result and estimate the temperature gradients over which the convective flux exceeds radiative heat transfer, i.e.

$$4\pi R_\odot^2 Q_{\text{conv}} > L_\odot. \tag{3}$$

Hint: Use $T \sim T_c$ (T_c can be found from your previous exercise sheet), and $C = 2 \times 10^8$ erg g^{-1} K^{-1}.

(c) Compare your result from (b) to the actually temperature gradient in the Sun, $\sim T_c/R_\odot$. What is the difference by orders of magnitude?

Exercise 2 [Komolgorov turbulence]
The turbulence σ of the giant molecular cloud (GMC) can be calculated through

$$\frac{1}{2}\sigma^2 = \int_{1/l}^{\infty} dk\, E(k) \tag{4}$$

where $E(K)$ is the turbulence power spectra and $l = 100$ pc is the typipcal size of the GMC.

(a)

$$E(k) = E_1 \cdot k^{-5/3} \text{ (subsonic)} \tag{5}$$

$$E(k) = E_2 \cdot k^{-2} \text{ (supersonic)} \tag{6}$$

If two regimes are separated by $l_s = 0.1$ pc, where $\sigma(l_s) = c_s = 0.2$ km s^{-1} (corresponding to $T_{\mathrm{GMC}} = 10$ K), can you calculate the normalization factor E_1 and E_2?

(b) Find σ for the GMC, assuming $l = 100$ pc.

Exercise 3 [Rayleigh-Taylor instability]
Derive the dispersion relation $\omega(k)$ for the Rayleigh-Taylor instability in the presence of surface tension. You need to consider that there exists tension T normal to the interface which is described by the Laplace law

$$T = \sigma\frac{1}{R}, \tag{7}$$

where σ is a constant and R is the curvature radius.