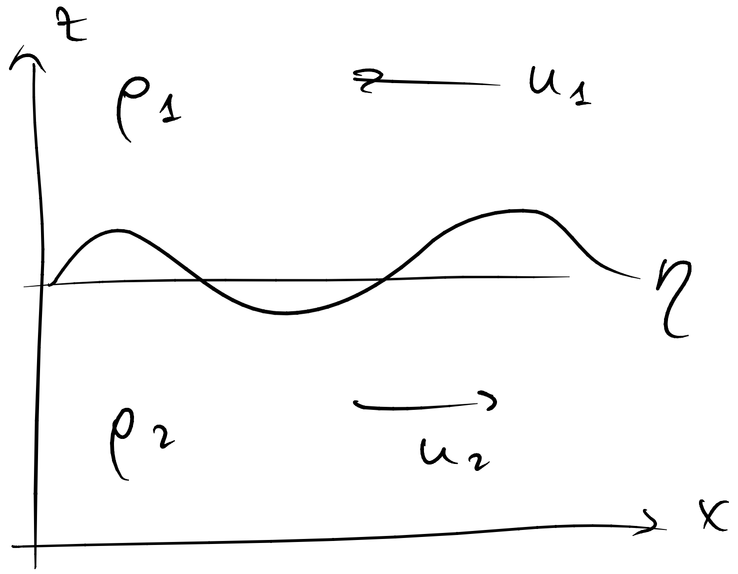
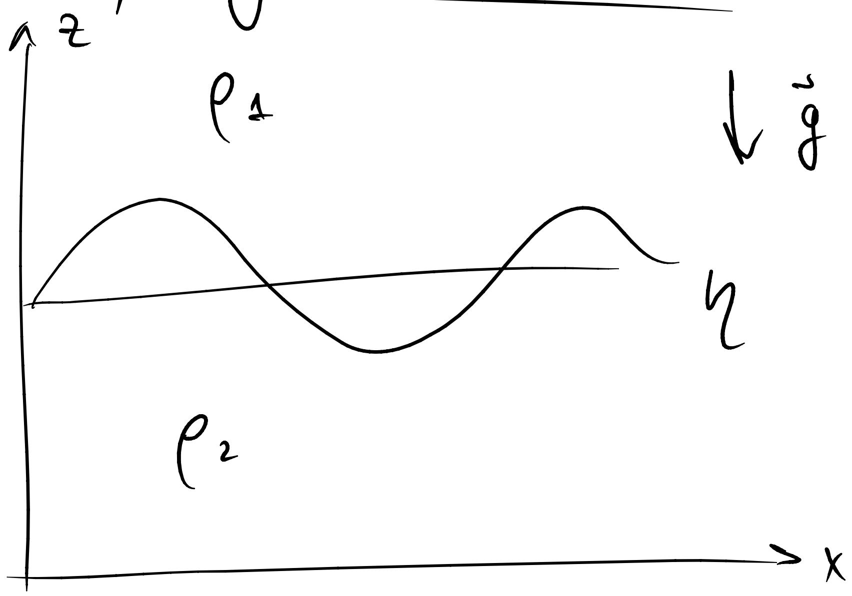
Th. Astro. - Turbulence

19 January 2016 12:32

Kelvin-Helmholtz and Rayleigh-Taylor inst.





- kinematic condition
- Euler equation (Bernoulli theorem) $\rightarrow$ equilibrium (pressure)

} linearize equation

↳ wave ansatz $\Rightarrow$ <u>dispersion relation</u>

- <u>Turbulence</u>:

- <u>Ergodic principle</u>: spatial / time averages equivalent to ensemble averages

↳ $\langle \frac{\partial \bar{v}}{\partial t} \rangle = \frac{\partial}{\partial t} \langle \bar{v} \rangle$ or $\langle \vec{\nabla} \cdot \vec{v} \rangle = \vec{\nabla} \cdot \langle \vec{v} \rangle$

$\langle \frac{\partial}{\partial t} \rangle = \frac{\partial}{\partial t} \langle v \rangle$ or $\langle \nabla \cdot v \rangle = \nabla \cdot \langle v \rangle$

- $\vec{v} = \vec{V} + \vec{v}'$ (true velocity = macrosc. vel. + turb. vel.)

Eulers equation ↳ $$\boxed{\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V} + \vec{\nabla}\underbrace{(\vec{v}' \otimes \vec{v}')}_{\text{Reynolds tensor}} = -\frac{\vec{\nabla} P}{\rho_0}}$$

Reynolds tensor
(turbulent pressure tensor)

- stationary turbulent cascade:

flux through scales constant

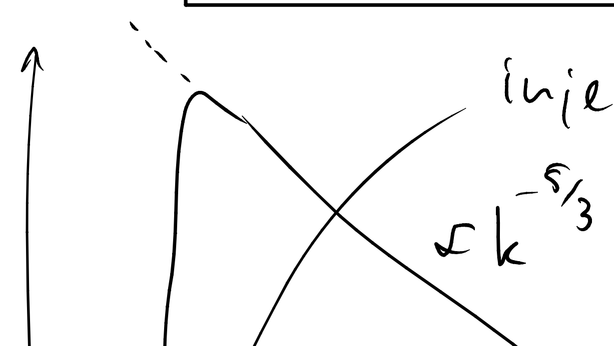
↳ conversion to smaller scales ~ bigger scales

Fourier transform of velocity field

$v = \sigma \cdot l$

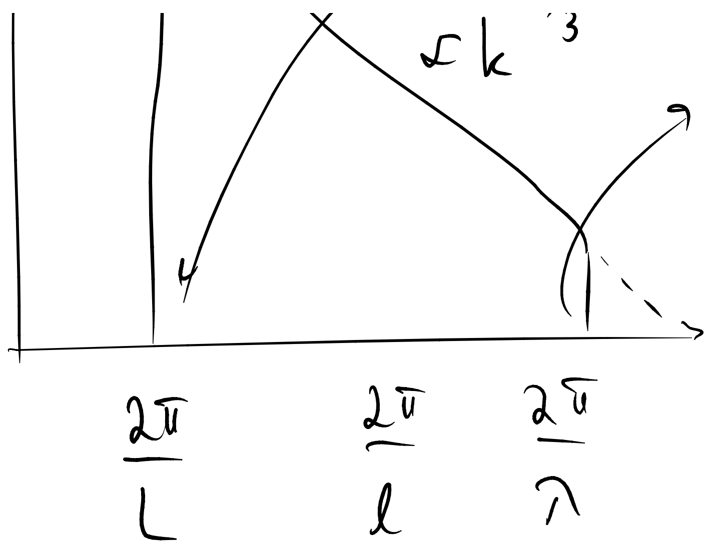
↳ $\boxed{E(k) = \Pi_0^{2/3} k^{-5/3}}$ $\boxed{\sigma(l) = \sqrt{2}\, \Pi_0^{1/3} l^{1/3}}$

(supersonic $\sigma \gg a$
turb. stress dominant)



injection (kinetic energy injection through instabilities)

dissipation (microscopic viscosity)

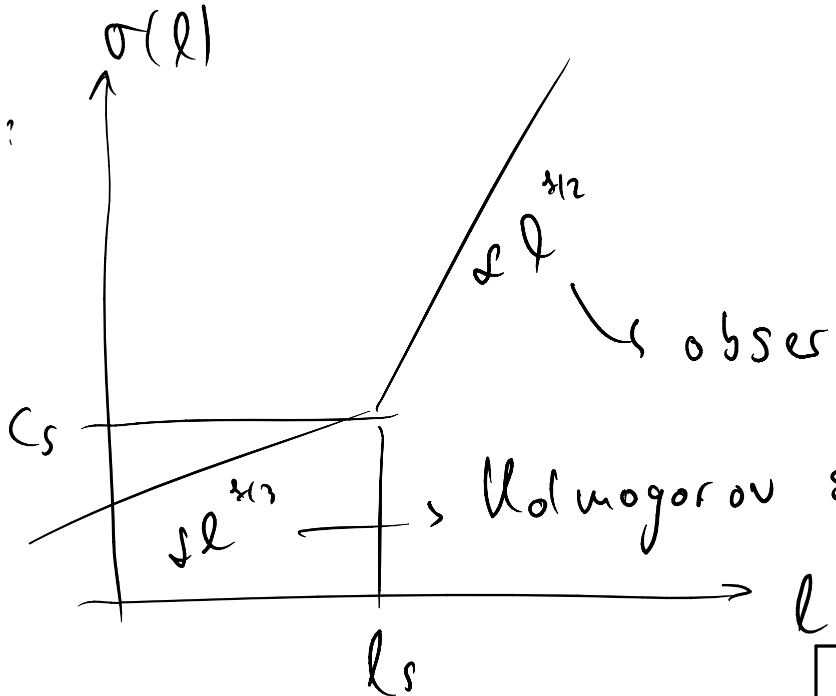


ν
dissipation (microscopic viscosity)

(truncated at λ and L using high-pass filter $W(kl)$)

· <u>ISM turbulence</u>:

$\rho(l)\, v(l) = \mu_0$



observed

Kolmogorov subsonic

Larsons sonic scale $\boxed{\sigma_l \sim 1\,\mathrm{km/s}\left(\frac{l}{1\,\mathrm{pc}}\right)^{1/2}}$