Theoretical Astrophysics
Exercise Sheet 2

HS 17
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Exercise 1 [Rutherford's Formula]

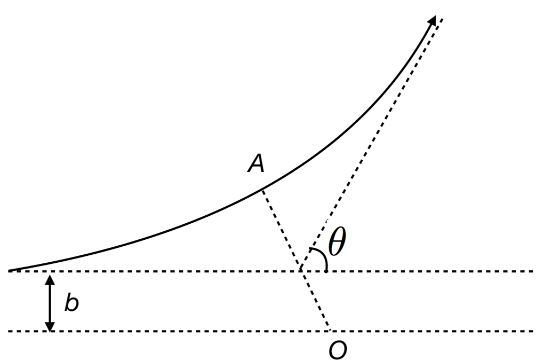

Figure 1: Scattering of a charged particle in a Coulomb field

Consider a charged particle being scattered in a Coulomb potential field ($\Phi = er^{-1}$). (a) Show that the distance of the closest approach $r_{\rm min}$ (OA in Figure 1) can be found by solving

$$r_{\rm min} = \frac{2e}{mv^2(1-c^2)}, \tag{1}$$

where $c = b/r_{\rm min}$. m and v in the equation represent the mass and initial speed of the charged particle, respectively. Solve for $r_{\rm min}$ using the above equation.

Hint: Use spherical coordinates to simplify your calculation. The formula you use is similar to that used for finding the Keplerian orbits.

(b) The *differential cross section parameter* is defined as

$$\sigma(\theta) = b|\partial b/\partial\theta|/\sin\theta. \tag{2}$$

For the Coulomb potential field $\Phi = er^{-1}$, show that

$$\sigma(\theta) = (\frac{e}{2mv^2})^2 \frac{1}{\sin^4(\theta/2)}. \tag{3}$$

Exercise 2 [Collision between rigid bodies]

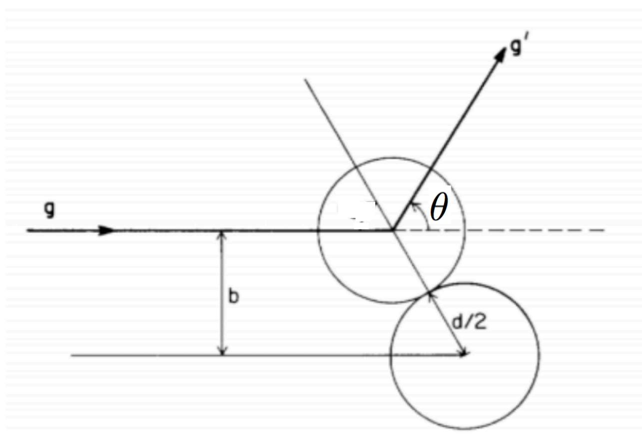

Figure 2: Two rigid elastic spheres colliding

Starting from the geometrical model where interacting particles are rigid elastic spheres with radius $d/2$ (see figure above) derive the *differential cross section parameter*

$$\sigma = \frac{1}{4}d^2 \tag{4}$$

and show that the total cross section $\sigma_T = \int d\Omega\, \sigma$ is

$$\sigma_T = \pi d^2. \tag{5}$$

Hint: You can start from equation (2).

Exercise 3 [Energy conservation equation]
During the lecture, you derived the conservation equations for mass and momentum by taking the first and second moment of the Boltzmann equation. Now take the third moment $< \frac{1}{2}mu^2 >$ to obtain the energy conservation equation:

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot (E\vec{v} + \vec{v} \cdot \overline{\overline{P}} + \vec{Q}) = \rho \vec{a} \cdot \vec{v} \tag{6}$$

with the total energy, internal energy, pressure tensor and heat flux defined as

$$E \equiv \rho\epsilon + \frac{1}{2}\rho|\vec{v}|^2 \tag{7}$$

$$\rho\epsilon \equiv \int \frac{1}{2}mw^2 f d^3u \tag{8}$$

$$P_{ij} \equiv \int mw_i w_j f d^3u \tag{9}$$

$$Q_i \equiv \int \frac{1}{2}mw_j^2 w_i d^3u \tag{10}$$

The particle velocity is split into two parts $\vec{u} = \vec{v} + \vec{w}$ with $\vec{v} =< \vec{u} >$ and $\vec{w}$ the random thermal velocity.