Lec. 4 Heating and Cooling
and
Line Diagnostics for HII Regions

1. General Theory of Heating & Cooling
2. Line Cooling Function
3. Cooling of HII Regions
4. Diagnostics of HII Regions

References:

Spitzer, Ch. 6
Osterbrock & Ferland Ch. 3-6
Tielens, Chs. 3 & 4
McKee, ay216_2006_05_HIIThernal.pdf

The Heat Equation

The so far incorrect treatment of the temperature of HII regions requires a more general development of heating and cooling. This is done by including thermodynamics in the equations of hydrodynamics (or MHD) which express the *conservation laws* for fluids, in this case the ISM.

The **First Law of Thermodynamics** for fluids is

$$Tds = de - pd(1/\rho)$$

where *s* and *e* are the *entropy* and energy per unit mass, ρ is the *mass density* per unit volume, and *p* is the *pressure*; *e* is the sum over all constituents of kinetic, internal (excitation) and chemical (including ionization) energies:

$$e = e_{\rm kin} + e_{\rm ex} + e_{\rm chem}$$

For example, the chemical energy of an H^+ ion is 13.6eV and of an H_2 molecule is -4.48eV. We ignore the chemical energy and combine the first two terms into a combined thermal energy.

Heat Equation for an Ideal Gas

With two equations of state,

$$e = \frac{3}{2}\frac{kT}{m} \qquad p = \frac{2}{3}\rho e = nkT$$

and ρ=mn, n = number density and m = mass per particle, energy conservation is rewritten as the hydrodynamic **heat equation** (Spitzer Eq. 6-1)

$$\rho T \frac{ds}{dt} = n\frac{d}{dt}\left(\frac{3}{2}kT\right) - kT\frac{dn}{dt} = \Gamma - \Lambda$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \bullet \nabla$$

Γ = net energy **gain** per unit volume

Λ = net energy **loss** per unit volume

In steady state, $\Gamma - \Lambda = 0$. This condition determines T as a function of n.

Heat Equation for a Molecular Gas

We can generalize the treatment for point molecules to molecular gases by writing the energy as

$$e = \frac{1}{\gamma - 1} \frac{p}{\rho}$$

where γ is the ratio of specific heats (5/3 for point molecules, 7/5 for diatomic molecules, etc.), This form supposes thermal equilibrium, which may not apply to interstellar gases. For example, the most abundant interstellar molecule is H_2 , whose rotational levels have large excitation energies and are not fully thermalized.

By recognizing that $\boldsymbol{p/\rho = c^2 = kT/m}$, where c is the isothermal sound speed, the heat equation can be manipulated Into the form,

$$\frac{p}{\gamma - 1} \frac{d}{dt} \ln\left(\frac{p}{\rho^{\gamma}}\right) = \Gamma - \Lambda$$

Thus $\Gamma = \Lambda$ *requires adiabatic* changes*:* $p \propto \rho^{\gamma}$

Heating Processes

A heating process increases the thermal (mean kinetic) energy of the gas. This can only happen by particle collisions with some *non-thermal* component. We may think of this component *external* to the gas.

1. Photons - In ionizing or dissociating the gas, a suprathermal particle is produced, e.g., a photoelectron or a dissociated H atom, that heats the gas on collisions with ambient thermal particles. Usually the photons are in the UV to X-ray bands. Photoelectric heating may also be generated by photons absorbed by dust particles or large molecules such as PAHS.

2. **Cosmic rays** - High energy particles (> 1 MeV) slow down by ionizing and exciting H and He (and by direct collisions with ambient electrons). The ejected fast electrons heat the gas by collisions.

3. Dust-Gas Interaction - Collisions between gas and dust exchange energy, and heat the gas (cool) when the dust is warmer (cooler) than the gas.

Heating Mechanisms (cont'd)

4. Mechanical Heating - Dissipation in shocks and turbulence

5. Magnetic Heating - Reconnection and ambipolar-diffusion (ion-neutral collisions as fields slip through neutral gas.

Further Preliminary Comments on Heating:

- Photoelectric heating is the best understood. Heating of HII regions is the prototypical case, as discussed in Lecture 3.
- Cosmic ray heating is uncertain because of ignorance about the low-energy spectrum and MHD transport.
- Dust-Gas heating is reasonably well understood, although it depends on dust properties (abundance, size, surface).
- Mechanical heating is usually treated phenomenologically.
- Shock heating is affected by the detailed properties of shocks.

Heating processes will be discussed later in more detail, e.g., in Lec07; see also McKee's lecture, ay216_2006_05.

Cooling Processes

In contrast to the diversity (and uncertainty) of heating processes, cooling of interstellar gas is mainly by radiation, especially lines.

1. Line Radiation - Thermal particle energy is expended in exciting atoms and molecules, whose re-radiation cools the gas if it escapes. This is the main topic of this lecture.

2. **Continuum Radiation** - One example already encountered in recombination cooling, where the recombination of the particles is accompanied by a continuum photon. Another example is free-free radiation or bremsstrahlung, which generates far-IR and radio continuum by HII regions, but plays a small role in cooling.

3. **Dust-Gas Collisional Energy Exchange**- This leads to gas cooling when the dust is cooler than the gas.

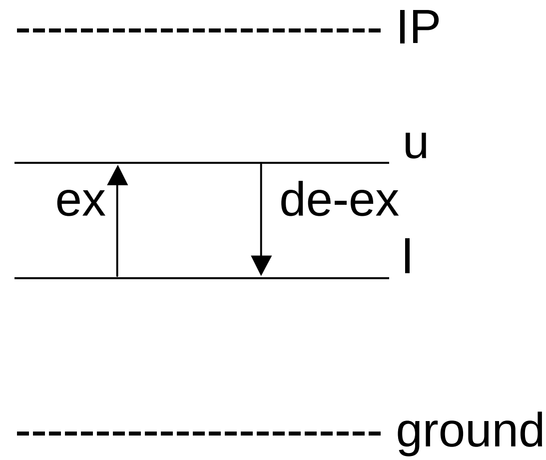
4. **Expansion Cooling** - This form of mechanical cooling occurs when the gas (cloud or wind) expands, in analogy to “adiabatic cooling”.

2. Line Cooling Function

$$c + A_l \leftrightarrow A_u + c'$$

$$E_{c'} = E_c - E_{ul}$$

NB Instead of *j* and *k* for lower and upper levels, we use *u* and *l*



“Atom“A

Λ is the net rate at which energy is lost in collisions. (For HII regions, electrons are the most important collision partners.) The collisions are described by upward and downward *rate coefficients* k_{lu} & k_{ul}, related by detailed balance.

$$k_{ul} = \langle v\sigma_{lu} \rangle \qquad k_{ul} = \langle v\sigma_{lu} \rangle$$

$$g_l k_{lu} = g_u k_{ul} e^{-E_{ul}/kT}$$

The net cooling per unit volume (Spitzer Eq. 6-5) is

$$\Lambda = \sum_{ul} n_c (n_l k_{lu} - n_u k_{ul}) E_{ul}$$

NB: If the levels are exactly in thermal equilibrium, Λ=0!

Compact Cooling Function

The levels are indeed *not* in thermal equilibrium because they are coupled to the radiation field by radiative decay.

$$\sum_l n_l n_c k_{lu} = \sum_l n_u n_c k_{ul} + \sum_l A_{ul} n_u$$

This equation holds locally, but the radiation emitted according to the second term may be absorbed before it escapes from the region of interest. To take account of this possibility, we introduce the *escape probability* β_{ul} by replacing A_{ul} by $\beta_{ul} A_{ul.}$. Now substituting The above population balance equation into the cooling function on the previous slide leads to

$$\Lambda = \sum_{ul} \beta_{ul} A_{ul} E_{ul} n_u$$

The escape probability depends on the geometry of the cloud, specifically on a mean optical depth form the source point to the cloud boundary. With this one complication, the above formula expresses the cooling in a compact form that resembles the optically thin emissivity of the transition.

The Escape Probability

Tielens Sec. 2.3.2 gives a practical introduction to the escape probability; see also Shu I Ch. 9 & Osterbrock/Ferland Sec.4.5 This topic will be also reviewed in Lecture 07.

To measure optical depth, use the absorption cross section at line center (c.f. Lec. 2, slides 11, 13, 16) ignoring stimulated emission:

$$s_\nu^{(lu)} = s_{lu}\varphi(\Delta\upsilon), \quad s_{lu} = \frac{\pi e^2}{m_e c} f_{lu}\,, \quad \varphi(0) = \frac{\lambda}{\pi^{1/2} b}$$

$$\frac{\pi e^2}{m_e c} = 0.0265 \text{ cm}^2\text{s}^{-1}, \quad b = 2^{1/2}\sigma$$

with σ the velocity (thermal plus turbulent) dispersion.
The absorption cross section at line center is then,

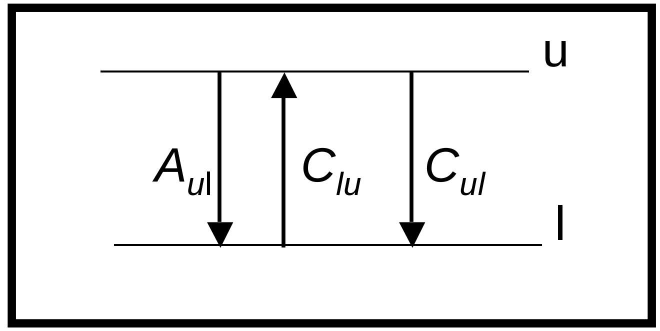
$$s_0 = 1.50\text{x}10^{-15} f_{lu}\left(\frac{\text{km/s}}{b}\right)(\lambda/\text{Å})\,\text{cm}^2$$

For the Lyα line, λ=1216 Å & f_{lu}=0.4162, so s_0=7.6x10^{-13}cm^2;
the line center becomes think for a H column of N(H) ~10^{12}cm^{-2}.

Model Two-Level System

This exactly solvable model (using escape probability) illustrates basic elements of radiative cooling and introduces the concept of critical density.

The steady balance equation is



$$(\beta_{ul} A_{ul} + C_{ul}) n_u = C_{lu}(n - n_u),$$

$$n = n_l + n_u \qquad C_{ul} = n_c k_{ul}, \text{ etc.}$$

The population of the upper level is then

$$\frac{n_u}{n} = \frac{C_{lu}}{\beta_{ul} A_{ul} + C_{ul} + C_{lu}} = \frac{\frac{g_u}{g_l} e^{-E_{ul}/kT}}{1 + \frac{g_u}{g_l} e^{-E_{ul}/kT} + \frac{\beta_{ul} A_{ul}}{n_c k_{ul}}}$$

after dividing the first form by C_{ul} and applying detailed balance

$$\frac{C_{lu}}{C_{ul}} = \frac{k_{lu}}{k_{ul}} = \frac{g_u}{g_l} e^{-E_{ul}/kT}$$

Two-Level Cooling Formula

The third term of the denominator of the population of the upper level suggests introducing the ***critical density***

$$n_{\rm crit} = \frac{\beta_{ul} A_{ul}}{k_{ul}}$$

so that

$$\frac{n_u}{n} = \frac{C_{lu}}{\beta_{ul} A_{ul} + C_{ul} + C_{lu}} = \frac{\frac{g_u}{g_l} e^{-E_{ul}/kT}}{1 + \frac{g_u}{g_l} e^{-E_{ul}/kT} + \frac{n_{crit}}{n_c}}$$

and the cooling rate is

$$\boxed{L_{ul} = \beta_{ul} A_{ul} n_u E_{ul} \; \frac{\frac{g_u}{g_l} e^{-E_{ul}/kT}}{1 + \frac{g_u}{g_l} e^{-E_{ul}/kT} + \frac{n_{crit}}{n_c}}}$$

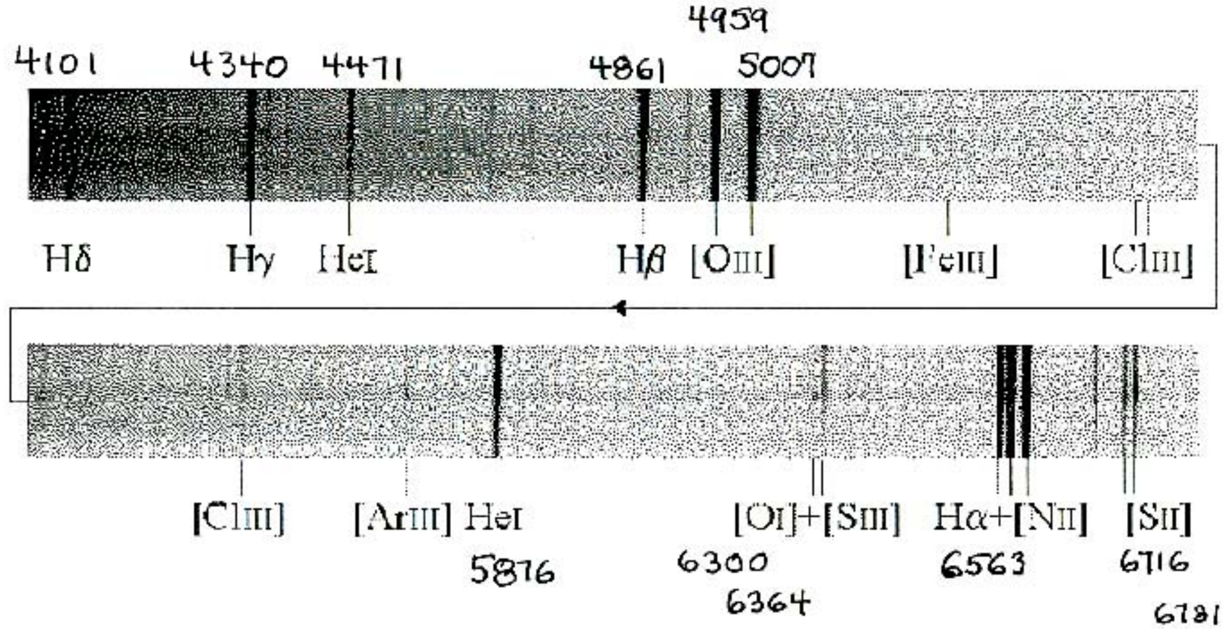
There are two simple limits:

$$n_c \ll n_{\rm crit} \;\text{(subthermal)}: \; L_{ul} = k_{lu} n_c n E_{ul}$$

$$n_c \gg n_{\rm crit} \;\text{(thermalized)}: \; L_{ul} = \beta_{ul} A_{ul} n E_{ul} \; \frac{g_u / g_l \; e^{-E_{ul}/kT}}{1 + g_u / g_l \; e^{-E_{ul}/kT}}$$

3. Cooling of HII Regions

Photoelectric heating balanced by recombination cooling in a pure hydrogen model predicted too high temperatures for HII regions.



Long slit optical spectrum of the Orion Bar. Notice the large strength of the OIII lines

The optical line emission of HII regions is dominated by the recombination lines of H & He and by the forbidden lines of heavy elements (even more so for SNRs and AGN). These lines are important for cooling. Thus, collisional excitation of heavy elements must be included in photoionization models.

Historical Note on Nebular Lines

The heavy element lines seen in HII and other photoionized regions have a long and important role in the history of physics and astronomy.

Helium – discovered in 1868 by Janssen in solar chromosphere (in eclipse) at 5816 Angstroms.

Identified as non-terrestrial by Lockyer & Franklund; later detected in minerals.

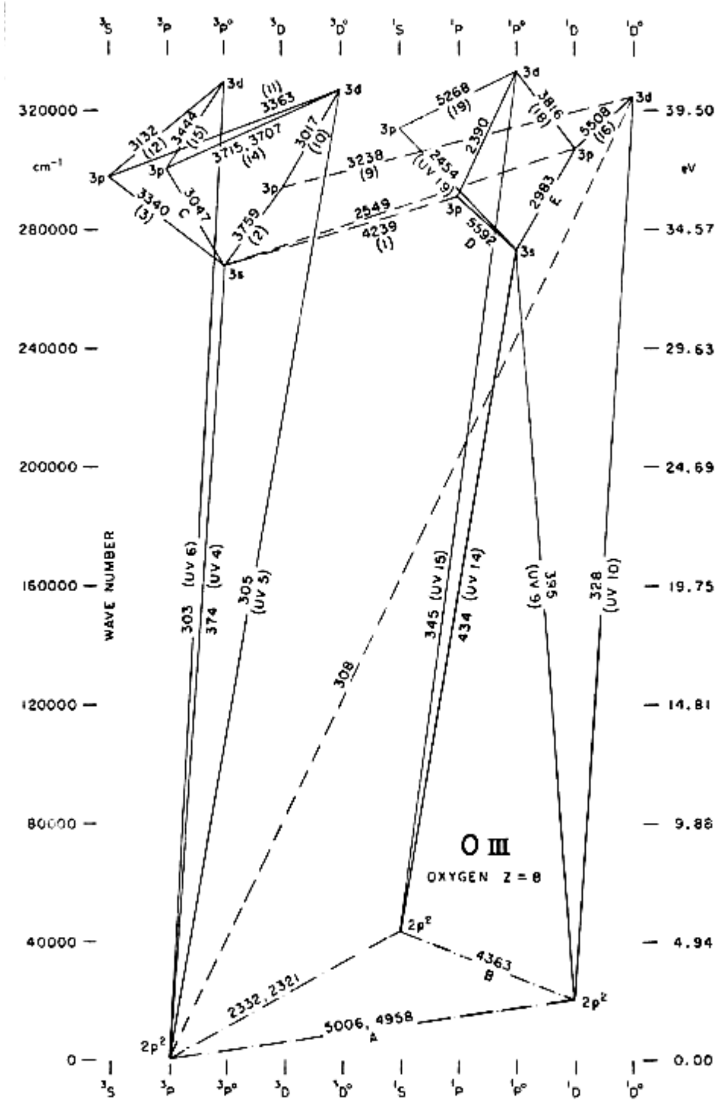
Significance: 40% of the visible mass had been missed (although the fact that most of it is hydrogen was unknown then).

Nebulium – discovered by Huggins in 1864 in nebulae at 5007, 4959 and 3726, 3729 Angstroms; as for He, ascribed to a new non-terrestrial element.

Identified in1927 by Bowen as OIII and OII.

Significance: highlighted the possibility of long-lived quantum states and focused attention on understanding *selection rules* in quantum mechanics.

Grotrian Diagram for OIII



NB Fine structure not shown

OIII ($1s^2 2s^2 2p^2$) has two 2p electrons (isoelectronic with NII and CI). The electron spins couple to a total spin *S* = 0,1. The two orbital ang. momenta couple to total *L* = 0,1,2. Of the 6 LS-coupling states, 1/2 satisfy the Pauli Exclusion Principle: $^1S_0$ $^1D_2$ ^3P_J (J =0 ,1,2), with different spatial wave functions and Coulomb energies.

$^1S_0$ -----------------------

4363

↓

$^1D_2$ ----------------------

4959, 5007

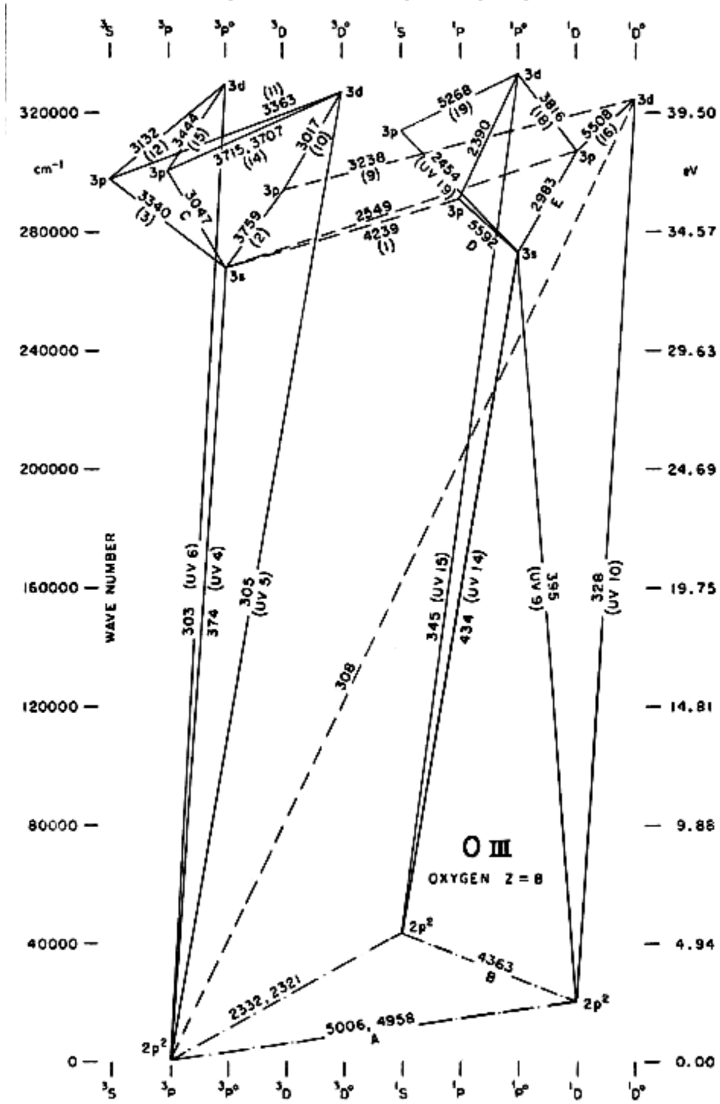
↓

^3P_J -----------------------

ground terms are called forbidden because hey are connected by magnetic dipole and electric quadrupole transitions.

Grotrian Diagrams for OIII and NII

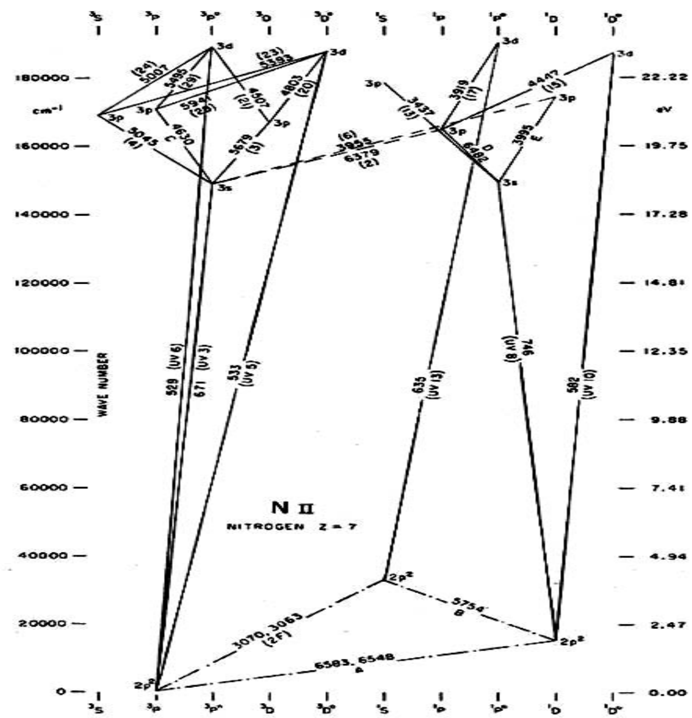
OIII IP=54.9 eV



$2p^2$ series: CI, NII, OIII, FIV, NeV

$3p^2$ series: SiI, PII, SIII, ClV, ArV

NII. IP=29.6eV



Critical Densities of Forbidden Transitions

The two-level model illustrates how the cooling depends on the density of the collision partner relative to the critical density,

$$n_{\text{crit}} = \frac{\beta_{ul} A_{ul}}{k_{ul}}$$

For HII regions, electrons do the excitation, and the collisional rate coefficients are given in standard form (Osterbrock/Ferland Eq. 3.20)

$$k_{ul} = \frac{8.629\text{x}10^{-6}}{T^{1/2}} \frac{\Omega_{ul}}{g_u},$$

where Ω_{ul} is the "collison strength" with rough magnitudes $O(1)$.

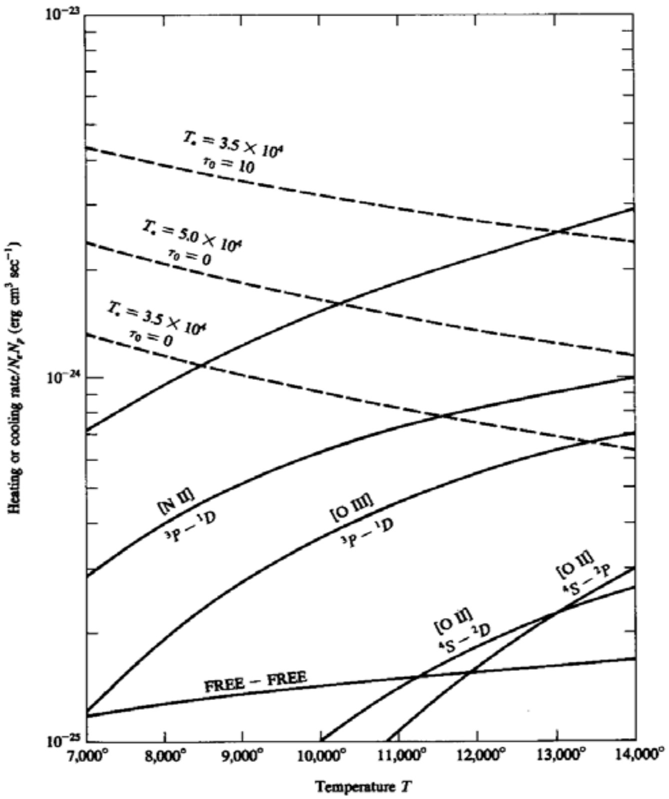
Typical values of k_{ul} are 10^{-7} $cm^3 s^{-1}$. Osterbrock & Ferland give tables of atomic properties of heavy elements. Table 3-15 gives a sampling of critical densities at 10,000K. For the $2p^2$ ions OIII & NII, we find

n_{crit}(NII: ^{1}D->^{3}P; 6500 Å) = $6.6 \text{x} 10^4 cm^{-3}$

n_{crit}(OIII: ^{1}D->^{3}P; 5000 Å) = $6.8 \text{x} 10^5 cm^{-3}$

These transitions will be sub-thermally excited in many HII regions. Atomic cross sections will be discussed further in Lecture 07.

Solution of the Temperature Problem for HII Regions



Heating (minus recombination cooling) and line cooling plotted vs.*T,* the former for stars with T_* = 35,000 & 50,000K (dashed lines). The solid lines are for line cooling with only a small contribution from free-free collisions. NII & OIII are the most Important. The unlabeled solid line is the total line cooling, and the solution where it crosses a dashed line is near8,000K

Figure 3.3 of Osterbrock (1988). In the new edition of Osterbrock& Ferland the figure is slightly different due to revised abundances and atomic parameters

Diagnostics of HII Regions

1. Measuring Temperature: The mere occurrence of optical forbidden lines suggests values of order 10^4 K.
NB: E = (12,400Å/λ) and 1eV corresponds to 11,605 K.

More precisely, observing transitions of the same ion from different upper levels measures T, e.g., using the ratio of intensities of the 5007/4959 and 4363 lines of OIII.

$^1S_0$ -----------------------

4363

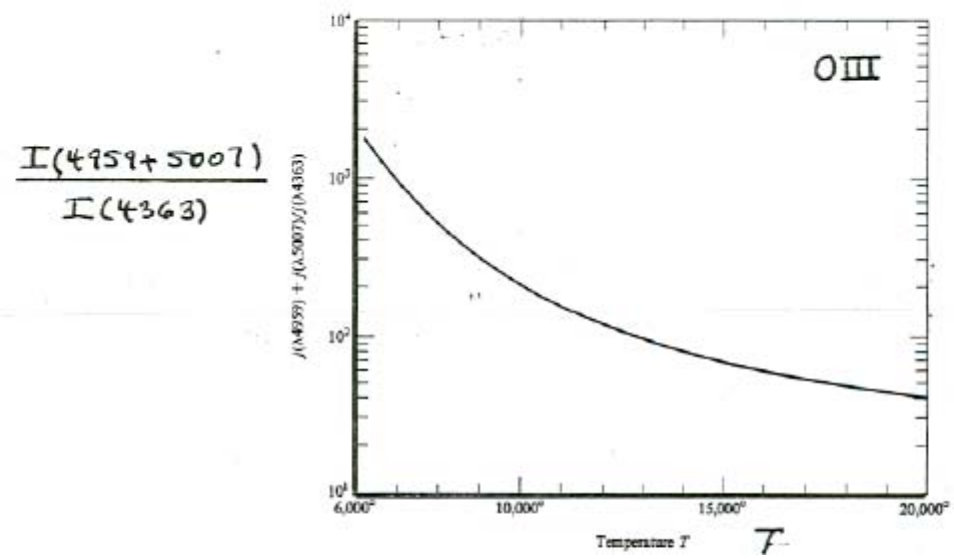
↓

$^1D_2$ -----------------------

4959, 5007

↓

^3P_J -----------------------



The ratio changes from 2000 too 400 as T varies from 6,000 to 20,000 K

Diagnostics of HII Regions (cont'd)

2. Measuring Electron Density.

The critical densities vary from transition to transition because *A*-values and rate coefficients do, e.g., in the case of doublets leading to the ground state the variation arises from statistical weights.

SII has strong red lines that arise from the first excited fine-structure doublet. The critical densities are ~ 10^4 cm s^{-1}.

At low densities, the intensities are determined by the collision strengths, at high densities by the *A*-values. The net effect is a line-intensity ratio that varies significantly with density.

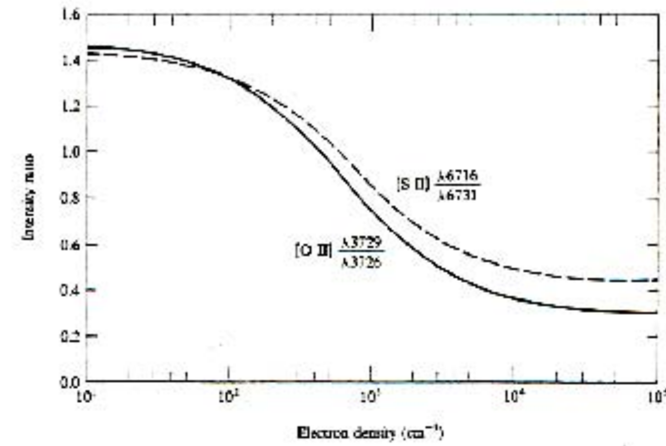
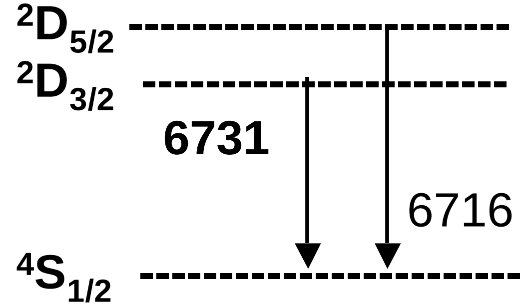


FIGURE 5.3
Calculated variation of [O II] (*solid line*) and [S II] (*dashed line*) intensity ratios as function of N_e at $T = 10,000°$ K. At other temperatures the plotted curves are very nearly correct if the horizontal scale is taken to be $N_e(10^4/T)^{1/2}$.



SII red lines

Combined Diagnostics: OIII Optical and Far Infrared Lines Applied to Planetary Nebulae

Osterbrock (1988), Fig. 5.6

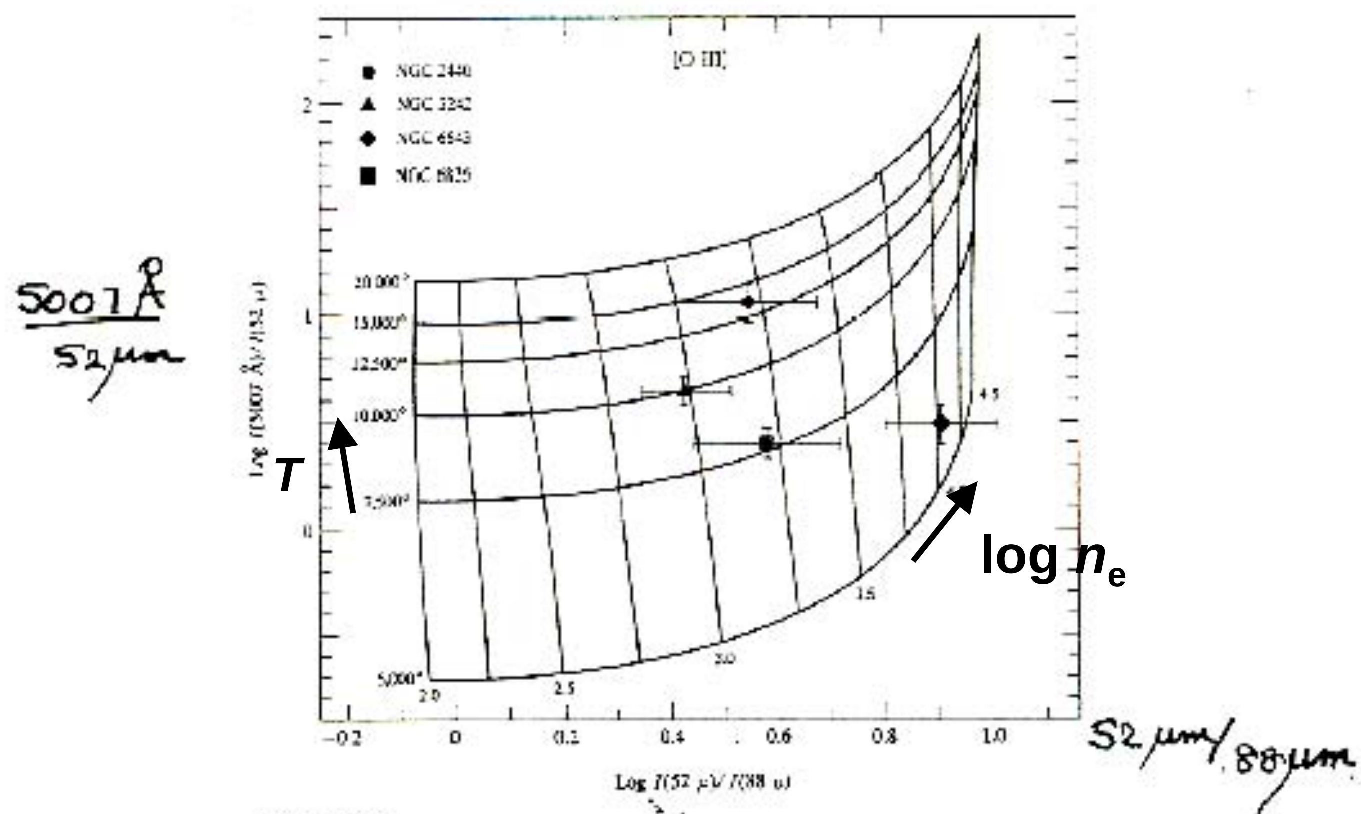


FIGURE 5.6
Calculated variation of [O III] forbidden-line relative-intensity ratios as functions of T (5000° to 20,000° K) and N_e. Observed planetary-nebula ratios plotted with indication of probable errors.

Summary

We have discussed the processes that produce HII regions around young, massive stars and that generate diagnostic emission lines that can be used to measure T and n_e.

The forbidden lines of heavy atoms & ions are excited by electronic collisions (in contrast to the recombination lines) so that, even in this simplest example of a photoionized nebulae, collisional phenomena play a role in so-called photoionization equilibrium.

Similar methods apply to the study of planetary nebulae and quasar clouds, although additional processes have to be considered, e.g., extreme FUV and X-ray ionization.

Many things are missing from this discussion, e.g., dynamic effects, especially the fact that massive stars have powerful outflows, as well as the role of dust. For further details, see the book by Ferland & Osterbrock.