Lec 3. Radiative Processes and HII Regions

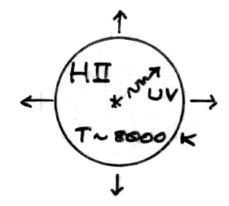
- 1. Photoionization
- 2. Recombination
- 3. Photoionization-Recombination Equilibrium
- 4. Heating & Cooling of HII Regions
- 5. Strömgren Theory (for Hydrogen)
- 6. The Role of Helium

References

Spitzer Secs. 5.1 & 6.1 Tielens Ch. 7 Dopita & Sutherland Ch.9 Osterbrock & Ferland, Ch. 3

Introductory Summary

The far ultraviolet radiation (FUV) from an O or B star Ionizes its immediate neighborhood and produces an HII region. Strömgren developed the theory in 1939 for a spherical model where the HII region slowly expands into uniform HI.



HII regions illustrate basic processes that operate in all photoionized regions of the ISM: super-Lyman radiation (λ < 916.6 Å - FUV or X-rays) photoionize sH:

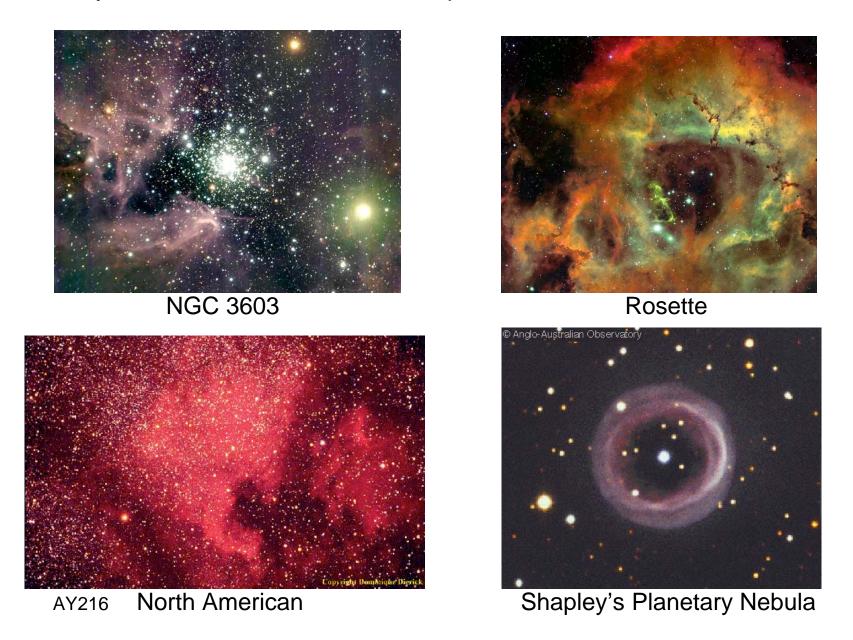
$$h\nu + H \rightarrow H^+ + e$$
.

The H+ ions recombine radiatively

$$H^+ + e \rightarrow H + hv$$
.

- The balance between these two reactions determines the ionization fraction.
- Any excess photon energy above the ionization potential (*IP* = 13.6 eV) is given to the ejected electron and is then equilibrated in collisions with ambient electrons, thereby heating the HII region.

Real HII regions are rarely spherical. Nonetheless, Strömgren's theory Illustrates the basic roles of photoionization and recombination.

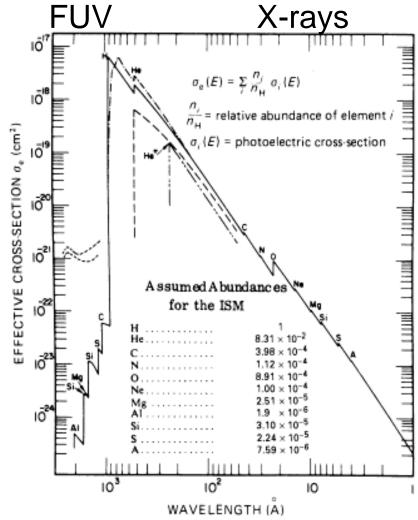


1. Photoionization

The ISM is opaque at 911 Å (the H Lyman edge) & partially transparent in the FUV and in the X-ray band above 1 keV.

Ionizing photons come from

- 1. Massive young stars*
- 2. Hot white dwarfs
- 3. Planetary nebula stars
- 4. SNR shocks
- * Table 2.3 of Osterbrock & Ferland give $T(O5V) \approx 46,000 \text{ K}$ (BB peak at 3kT or 12 eV) & $3x10^{49}$ ionizing photons/s



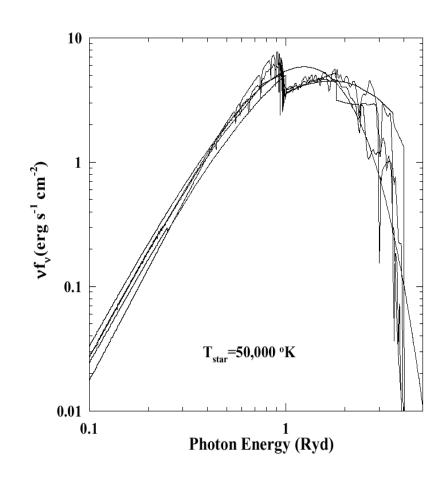
Absorption cross section per H nucleus (1-2000 Å, averaged over abundances)

Hot Stars as FUV Sources

Blackbody (smooth curve) is a poor approximation in the FUV and EUV.

State of the art model atmospheres disagree.

See Lejuene et al., A&AS 125, 229 1997 and NASA ADS: 1997yCat..41250229L, for an extensive library of stellar energy distributions.

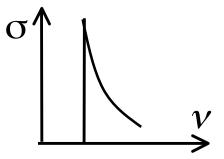


Note that the spectrum of this very hot star cuts off beyond 50 eV.

Photoionization of Hydrogenic Ions

Quantum theory predicts

$$\sigma_{v} = \frac{7.91 \times 10^{-18}}{Z^{2}} \left(\frac{v_{1}}{v}\right)^{3} g_{bf} \text{ cm}^{2}$$



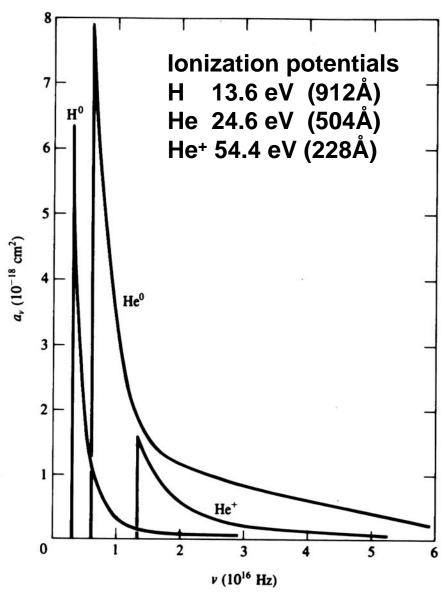
where $hv_1 = 13.6 Z^2$ eV and $g_{bf} \le 1$ is the QM Gaunt factor for bound-free transitions from n = 1. Compare this with Kramers' semi-classical formula:

$$\sigma_{v} = \sigma_{1} \left(\frac{v_{1}}{v}\right)^{3}$$
 with $\sigma_{1} = \frac{6.33 \times 10^{-18}}{Z^{2}} \text{ cm}^{2}$

Both give an *inverse-cube dependence on frequency*. The mean free path at 912 Å is very small:

$$l_{mfp}(v_1) = \frac{1}{n_{HI}\sigma_1} = \frac{1.58 \times 10^{17}}{n_{HI}} \text{ cm}^2$$

Photoionization Cross Section of He



- At high frequencies, the larger He cross section more than compensates for its lower abundance relative to H.
- Very hot stars are needed to ionize He⁺ (T_{*} > 50,000 K). He⁺⁺ does not occur in H II regions except in planetary nebulae, AGN and in fast shocks.
- Continuum radiation gets "harder" with increasing depth due to the rapid decrease of the cross section with *v*.

NB For analytic fits for heavy atom photoionization cross sections, see Verner et al., ApJ, 465, 487 1996

Calculation of the Photoionization Rate

The photoionization rate for one H atom is

$$\zeta_{\pi} = \int_{v_1}^{\infty} dv \, \frac{4\pi J_{v}}{hv} \sigma_{v} = c \int_{v_1}^{\infty} dv \, n_{v} \sigma_{v}$$

where the *mean intensity* J_{v} , photon number, and energy density are derived from the specific intensity by

$$u_v = \frac{1}{c} \int d\Omega \, I_v \equiv \frac{4\pi}{c} J_v = n_v h v$$

The *total* photon number density,

$$n_{\pi} = \frac{1}{c} \int_{v_{1}}^{\infty} \frac{4\pi J_{v}}{hv} dv = \frac{4\pi J_{v_{1}}}{hc} I_{1},$$

where I_1 is the first inverse moment of J_{v} , defined by

$$I_n \equiv \int_{v_1}^{\infty} \frac{J_v}{J_{v_1}} \left(\frac{v_1}{v}\right)^n \frac{dv}{v_1}$$

Calculation of the Photoionization Rate (cont'd)

Because the cross section varies as v^3 , the ionization rate can also be expressed in terms of these moments.

$$\zeta_{\pi} = \int_{v_1}^{\infty} \frac{4\pi J_{v}}{hv} \sigma_{v} dv = \frac{4\pi J_{v_1}}{h} \int_{v_1}^{\infty} \frac{J_{v}}{J_{v_1}} \left(\frac{v_1}{v}\right) \sigma_1 \left(\frac{v_1}{v}\right)^3 \frac{dv}{v_1}$$

$$\zeta_{\pi} = \frac{4\pi J_{v_1}}{h} \sigma_1 I_4$$

Replacing the mean intensity by the photon number, this is

$$\zeta_{\pi} \approx n_{\pi} \sigma_{1} c \left(\frac{I_{4}}{I_{1}} \right)$$

The moments depend on the spectrum. A typical ratio appropriate for HII regions is $I_4/I_1 \approx \frac{1}{2}$.

Numerical Estimate of the Photoionization Rate

1 pc from an O5 star the H-ionizing photon density is

$$n_{\pi} = \frac{S_H}{4\pi R^2 c} = \frac{2.2 \times 10^{49} \,\text{s}^{-1}}{4\pi \cdot (3 \times 10^{18} \,\text{cm})^2 \cdot 3 \times 10^{10} \,\text{cms}^{-1}} = 6.1 \, \gamma \,\text{cm}^{-3}$$

$$\zeta_{\pi} = n_{\pi} \sigma_{\nu_1} c (I_4/I_1) = 6.1 \cdot 6.3 \times 10^{-18} \cdot 3 \times 10^8 \cdot \frac{1}{2} = 5.8 \times 10^{-7} \, s^{-1}$$

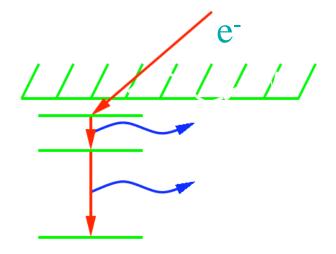
Ionization time t_π = 1/ζ_π = 1.7 x 10⁶ s ~ 20 days

NB In the 2nd line, the speed of light is given in m/s, whereas cgs units are used everywhere else, especially in the 1st line.

2. Radiative Recombination

$$H^+ + e \rightarrow H + hv$$

Radiative recombination is the inverse of photoionization. Milne calculated the cross section from Kramer's cross section using *detailed balance* (Spitzer Eq. 5-9, Rybicki & Lightman Sec.10.5)



The cross section for capture to level *n* is

$$\sigma_{fb}(w) = 2n^2 \left(\frac{hv}{m_e cw}\right)^2 \sigma_{bf}(v) = \frac{hv_1}{m_e c^2} \frac{hv_1}{\frac{1}{2}m_e w^2} \frac{v_1}{v} \frac{\sigma_1}{n^3}$$

It depends on electron speed as w^2 . The *rate-coefficient*, $\alpha_n = \langle w\sigma(w)\rangle_{th}$, is small and decreases as 1/T, e.g., the *total* rate coefficient (next slide), summed over n at 10,000K is $\alpha(10^4\text{K}) \sim 4\text{x}10^{-13}\,\text{cm}^3\,\text{s}^{-1}$.

Rate Coefficient for Radiative Recombination

The rate coefficient $\alpha^{(n)}$ (units, cm³s⁻¹) gives the rate of direct recombinations per unit volume to level n; n_i is the ion density (in this case H⁺),

$$n_e n_i \langle \sigma_n(w) w \rangle = n_e n_i \alpha_n$$

Summing the rates for all levels $m \ge n$, gives the total recombination rate coefficient to level n:

$$\alpha^{(n)} = \sum_{m=n}^{\infty} \alpha_m = 2.06 \times 10^{-11} Z^2 T^{-1/2} \phi_n(\beta) \text{ cm}^3 \text{s}^{-1}$$

 $\phi_n(hv/kT)$ is a slowly varying function of T, introduced and tabulated by Spitzer (p. 107). The values for n = 1 & 2 at 8000K are 2.09 & 1.34, respectively, so that $\alpha^{(1)}=5x10^{-13}$ and $\alpha^{(2)}=3x10^{-13}$ cm³s⁻¹

On The Spot Approximation for H

The rate coefficient $\alpha^{(1)}$ includes recombination to the ground state, but that process produces another ionizing photon that is easily absorbed locally at high density, as if the recombination had not occurred.

In this on the spot approximation, the effective recombination rate omits recombination to the ground state

$$\alpha^{(2)} = \sum_{m=2}^{\infty} \alpha_m \approx 2.60 \times 10^{-13} Z^2 T_4^{-0.8} \text{cm}^3 \text{s}^{-1}$$

The corresponding recombination time is

$$t_{rec} = \frac{1}{n_e \alpha^{(2)}} \approx 3.85 \times 10^{12} n_e^{-1} T_4^{0.8} \text{ sec}$$

Compared with the ionization time (slide 10), $\tau_{rec} >> \tau_{\pi}$ in most cases, and the gas around hot stars is highly ionized.

3. Photoionization Equilibrium for H

The ISM is not in LTE

The Saha Equation (chemical equilibrium) is not valid.

However, the time scales for dynamical changes for gas and star are much longer than t_{rec} and t_{ion} . Thus the ionization fraction is in a *quasi-steady-state* called *photoionization equilibrium* where *the* rate of ionization ζ_{π} out of ionization state i-1 = rate of recombination into state i:

$$\zeta_\pi n_{i-1} = \alpha \ n_e n_i \qquad \qquad \alpha = \alpha^{(1)} \ \text{or} \ \alpha^{(2)}, \ \text{depending}$$

$$n_i / n_{i-1} = \zeta_\pi / \alpha n_e = t_{rec} / t_\pi \qquad \text{on optical depth}$$

For H, assume $\alpha = \alpha^{(2)}$ and $n_{\pi} = n_{\pi} (hv > 13.6 \text{ eV})$

$$\frac{n_{\text{H}^+}}{n_{\text{H}^0}} = \frac{\xi_{\pi}}{\alpha^{(2)}n_{\text{e}}} = \frac{n_{\pi}c\sigma_1 I_4/I_1}{\alpha^{(2)}x_{\text{e}}n_{\text{H}}}$$

Ionization Parameter

The H⁺/H ratio depends on the ratio of photon density (strength of the radiation field) to the particle density. This fact is a general property of external energy sources and is recognized by the *ionization parameter* $U = n_{\pi} / n_{H}$. The H⁺/H ratio can now be rewritten as

$$\frac{n_{H^{+}}}{n_{H^{0}}} = \frac{U}{U_{H}} \qquad U_{H} = \frac{\alpha^{(2)} x_{e}}{\sigma_{v_{1}} c (I_{4}/I_{1})} = 1.37 \times 10^{-6} \frac{x_{e}}{T_{4}^{0.7} (I_{4}/I_{1})}$$

A typical value in an HII region is $U \sim 10^{-2.5} >> U_{\rm H}$ Since $x_{\rm e} = n_{\rm e} / n_{\rm H} \le 1.2$ (<10% He), hydrogen in HII regions is fully ionized.

Similar equations apply to all elements using appropriate values of n_{π} and U_{Z} (replacing $U_{H)}$.

4. Temperature of Photoionized Gas

This section follows closely the treatment of the heating and cooling of HII regions given in Spitzer Sec. 6.1

The one important heating mechanism involves the dissipation of the excess energy of the photoelectrons (generated by the absorption of stellar UV photons) in Coulomb collisions with ambient electrons:

$$E_e = hv - hv_1 \sim kT$$

The mean energy of the photoelectrons is

$$\overline{E}_{2} = \frac{\int_{v_{1}}^{\infty} h(v - v_{1}) \sigma_{v} \frac{4\pi J_{v}}{hv} dv}{\int_{v_{1}}^{\infty} \sigma_{v} \frac{4\pi J_{v}}{hv} dv}$$

where J_{ν} is the mean intensity of the radiation field.

Magnitude of the Photoelectric Heating

Spitzer Sec. 6.1, expresses the mean photoelectron energy in terms of the stellar effective temperature

$$\psi = \overline{E}_2 / kT_*$$

With $\zeta_{\pi} n_{H}$ the photoionization rate per unit volume, the volumetric heating rate is

$$\Gamma = \xi_{\pi} n_{\rm H} \ \psi \ kT^*$$

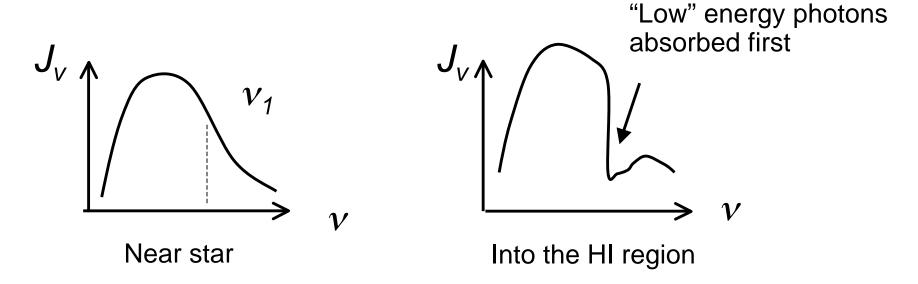
Spitzer, Table 6.1

$T_{\rm e}(K)$	4000	8000	16,000	32,000	64,000
ψ_0	0.977	0.959	0.922	0.864	0.775
$\langle \psi_0 \rangle$	1.051	1.101	1.199	1.380	1.655

 ψ_0 - value near star, $\langle \psi_0 \rangle$ is averaged over an HII region; the 1st decreases and the 2nd increases with T*, as does the ratio.

Hardening of the Ionizing Radiation

The spectrum hardens because of v⁻³ absorption close to the star.



The mean photoelectron energy increases as the softer photons get preferentially absorbed. This is why the entries of the previous table, and especially the ratio $\langle \psi_0 \rangle / |\psi_0 \rangle$, increase with stellar effective temperature.

Recombination Cooling

Radiation is the main way that interstellar matter cools.

In HII regions, radiation from recombination provides a minimum amount of cooling: each recombination drains thermal energy $\frac{1}{2} m_e w^2$ from the gas. The total cooling rate per ion is

$$\frac{1}{2}m\sum_{j=k}^{\infty}\left\langle w^{3}\sigma_{j}\right\rangle$$

The recombination cross section varies as the inverse square of the speed w. Therefore the rate of cooling by recombination is determined by the thermal average of $w^3xw^2=w$, or $T^{1/2}$. This is borne out roughly by the exact calculations described by Spitzer:

Recombination Cooling Rate

The volumetric cooling, neglecting recombinations to the ground state (on-the-spot rate approximation) is

$$\Lambda_{\rm rec} = \alpha^{(2)} n_{\rm e} n({\rm H}^+) k T(\frac{\chi^{(2)}}{\phi^{(2)}})$$

T/K	4000	8000	16,000	64,000
$\chi^{(2)}/\phi^{(2)}$	0.73	0.69	0.63	0.51

 ϕ = recombination function (Spitzer Table 5-2, p. 106 χ = energy gain function - which come from Spitzer's calculation of the recombination cooling (Eq. 6-8, p. 135) Roughly 2/3 kT of electron thermal energy is lost in each recombination in an HII region.

Preliminary Thermal Balance for Pure H

$$\Gamma_{ep} = \frac{2.07 \times 10^{-11} n_e n_p}{T^{1/2}} \left\{ \overline{E}_2 \phi_1(\beta) - kT \chi_1(\beta) \right\} \frac{\text{erg}}{\text{cm}^3 \text{s}}, \quad (6-9)$$

NB Spitzer's formula for photo-electric heating subtracts recombination cooling. HII regions, where the on-the-spot approximation applies, require the use of $\varphi^{(2)}$ and $\chi^{(2)}$.

For recombination cooling to balance with photoelectric heating requires $\Gamma_{\rm ep}$ = 0, or

$$T/T^* = (\varphi/\chi) \langle \psi \rangle$$

Both φ/χ and $\langle \psi \rangle$ increase with T, so $\varphi/\chi \rangle \langle \psi \rangle$ is typically ~2-3. Thus the predicted temperature is much greater than observed, e.g., for a B0.5 star with $T^* = 32,000$, T would be ~ 100,000K. **There must be other coolants** - not surprising since many emission lines are observed in HII regions.

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5. The Strömgren Sphere

Real HII regions are inhomogeneous. Their properties are determined by the local *ionization parameter*. Modeling HII regions requires a good calculation of the stellar FUV radiation field (usually Monte Carlo). Here we avoid all such complications by using Strömgren's simple but very useful theory of a *uniform spherical region* of radius $R_{\rm st}$.

 $R_{\rm st}$ is determined by equating the *total* rates of ionization and recombination inside a sphere of radius $R_{\rm s}$:

$$S_{H} = \frac{4\pi}{3} R_{St}^{3} n_{e}^{2} \alpha^{(2)}$$

Here $S_H = 10^{49} \,\text{s}^{-1} \, S_{H,49}$ is the rate at which the central star produces photons that ionize H.

The Strömgren Radius

$$R_{St} = \left(\frac{3}{4\pi} \frac{S_H}{x_e n^2 \alpha^{(2)}}\right)^{1/3} \approx 61.7 \left(\frac{S_{H49}}{n^2}\right)^{1/3} \text{pc}$$

The numerical value assumes *T*=7000 K.

As mentioned above, HII regions are neither uniform nor spherical. Rather, the dynamics determine n, e,g., expansion into the non-uniform surrounding ISM.



Numerical parameters are given in the next slide from Osterbrock's book.

Properties of Strömgren Spheres

$$R_{St} \approx 61.7 \left(\frac{S_{H49}}{n^2}\right)^{1/3} \text{pc}$$

Spectral type	M_v	$T_*({}^{\circ}K)$	$egin{aligned} \operatorname{Log}\ Q(H^0)\ (ext{photons/sec}) \end{aligned}$	$\log N_e N_p r_1^3$ $(N \text{ in cm}^{-3};$ $r_1 \text{ in pc})$	$r_1 ext{ (pc)}$ $(N_e = N_p)$ $= 1 ext{ cm}^{-3}$
O5	- 5.6	48,000	49.67	6.07	108
O6	-5.5	40,000	49.23	5.63	74
07	-5.4	35,000	48.84	5.24	56
O8	-5.2	33,500	48.60	5.00	51
O9	-4.8	32,000	48.24	4.64	34
O9.5	-4.6	31,000	47.95	4.35	29
B0	-4.4	30,000	47.67	4.07	23
B0.5	-4.2	26,200	46.83	3.23	12

NOTE: $T = 7500^{\circ}$ K assumed for calculating α_B .

Characteristics of Strmgren Spheres

1. Ionization parameter and radial column density

Consider a location just inside R_{st} where $x_e = 1$; ignore attenuation of the spectrum, and apply ionization equilibrium:

$$U_{St} = \frac{n_{\pi}}{n} = \frac{S}{4\pi R_{St}^{2} cn} = \frac{(4\pi/3)R_{St}^{3} n^{2} \alpha^{(2)}}{4\pi R_{St}^{2} cn} = \frac{\alpha^{(2)}}{3c} nR_{St}$$

$$nR_{St} = n \left(\frac{3}{4\pi} \frac{S}{n^{2} \alpha^{(2)}}\right)^{1/3} = \left(\frac{3}{4\pi} \frac{nS}{\alpha^{(2)}}\right)^{1/3} = \left(\frac{3}{4\pi \alpha^{(2)}}\right)^{1/3} (nS)^{1/3}$$

and finally

$$U = \frac{\alpha^{(2)}}{3c} \left(\frac{3}{4\pi\alpha^{(2)}}\right)^{1/3} (nS)^{1/3}$$

Both $U_{\rm st}$ and $nR_{\rm st}$ are proportional to (nS)^{1/3}.

2. Numerical values of $U_{\rm st}$ and $nR_{\rm st}$

Substitute the numerical values, $\alpha = 3.65 \times 10^{-13} \text{ cm}^3 \text{s}^{-1}$, $n = 100 \text{ cm}^{-3} n_2$ and $S = 10^{49} S_{49} \text{ s}^{-1}$

$$U_{st} = 3.49 \times 10^{-3} (n_2 S_{49})^{1/3} = 10^{-2.46} (n_2 S_{49})^{1/3},$$

$$nR_{st} = 8.6 \times 10^{20} (n_2 S_{49})^{1/3} \text{ cm}^2$$

The radial column density nR_{st} is related to the "average column density:

$$\overline{N} = \frac{\frac{4}{3}\pi nR_{St}^3}{\pi R_{St}^2} = \frac{4}{3}nR_{St} \approx 1.15 \times 10^{21} (S_{49}n_2)^{1/3} \text{ cm}^{-2}$$

As n increases for fixed S, the column increases as $n^{1/3}$: small dense HII regions can have large columns, as in ultra compact (UC) HII regions.

3. The H+/H Ratio

Start from previous results for the ionization parameter

$$\frac{n_{H^{+}}}{n_{H^{0}}} = \frac{U}{U_{H}} \qquad U_{H} \equiv \frac{\alpha^{(2)} x_{e}}{\sigma_{v_{1}} c (I_{4}/I_{1})} \qquad U_{St} = \frac{\alpha^{(2)}}{3c} n R_{St}$$

to find

$$\frac{n_{H^+}}{n_{H^0}} = \frac{1}{3}\sigma_{\nu_1} nR_{St}(I_4/I_1) = \frac{1}{4}\sigma_{\nu_1} \overline{N}(I_4/I_1)$$

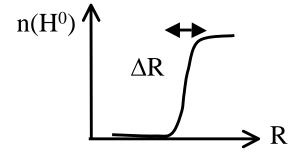
This expresses the H+/H ratio in terms of the optical depth at the Lyman edge. Recalling $\sigma_1 = 6.33 \times 10^{-18} \, \text{cm}^2$.

$$\tau_{\nu_1} = \sigma_{\nu_1} \overline{N} = (6.33 \times 10^{-18}) (1.15 \times 10^{21}) (S_{49} n_2)^{1/3}$$
$$= 7280 (S_{49} n_2)^{1/3}$$

The H+/H ratio is about 1/8 of this value, or about 900.

4. Transition from H+ to H

How thick is the region in which x(H) goes from 0 to 1?
Roughly the distance for an ionizing photon to be absorbed:



$$\tau_{v_1} = \Delta R n_{H^0} \sigma_{v_1} = 1$$

Neglect hardening of the spectrum and define the transition where n(H) = 0.5 n,

$$\frac{\Delta R}{R_{St}} = \frac{1}{\frac{1}{2}n_{H^0}\sigma_{v_1}R_{St}} = \frac{1}{\frac{3}{8}\sigma_{v_1}\overline{N}} = \frac{2}{3}\frac{U_H}{U_{St}}\frac{I_4}{I_1} \approx 3.5 \times 10^{-4} \left(S_{49}n_2\right)^{-1/3}$$

This property, as well as the three previous ones, all depend on the Strömgren parameter $(Sn)^{1/3}$.

5. The Role of He in HII Regions

1. High IP H - 13.6 eV (912Å)

He - 24.6 eV (504Å)

He+ - 54.4 eV (228Å)

Very hot stars are needed to ionize He⁺ (T* > 50,000 K). O stars won't do, so their HII regions have no He⁺⁺. Need planetary nebula stars or AGN.

- 2. The radiation that ionizes He also ionizes H.
- 3. He recombination radiation photoionizes H.
- 4. He has a dual set of energy levels arising from the two total spin states: S=0 (singlet) and S=1 (triplet). See JRG 2006 Lec-03 on HII regions for further discussion.
- 5. The He threshold photoionization cross section is larger than that for H, largely compensating for its smaller abundance

Ionization of He by OB Stars

Example 1: B0 star, $T_{\text{eff}} \approx 30,000 \text{K}$

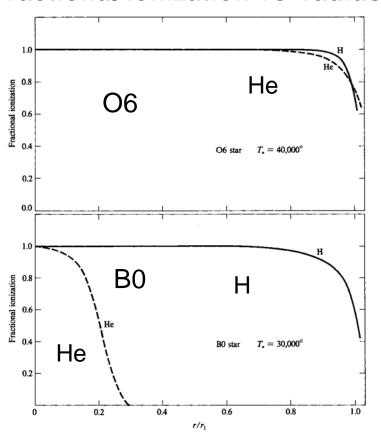
- Spectrum peaks at ~13.6 eV
 - Many photons in 13.6 24.6 eV range
 - Few photons with hv > 24.6 eV
- Two Strömgren spheres
 - Small central He+ zone surrounded by large H+ region

Example 2: O6 star, $T_{\text{eff}} \approx 40,000 \text{ K}$

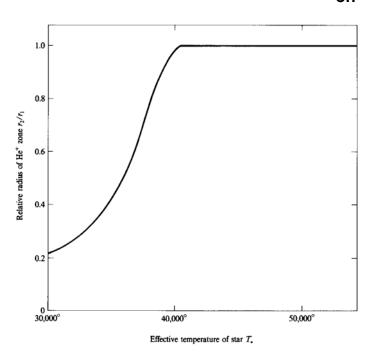
- Spectrum peaks beyond 24.6 eV
 - Lots of photons with hv > 24.6 eV
- Single Strömgren sphere
 - H+ and He+ zones coincide

He+ Zones in Model H II Regions

Fractional ionization vs. radius



He+/H+ radius ratio vs. $T_{\rm eff}$



Osterbrock Figures 2.4 & 2.5

For an O6 star, the abundant supply of He-ionizing photons keeps both H and He ionized, whereas the smaller number generated by a BO star are absorbed close to the star.