## 1D Wave Equation

$u_{tt} = u_{xx} \qquad for\ 0 \leq t \leq 4,\ 0 \leq x \leq 2$

### Boundary and initial contions

$u(x,0) = x(2-x) \qquad IC_1$
$u_t(x,0) = 0 \qquad IC_2$
$u(0,t) = u(2,t) = 0 \qquad BC$

### Analytical Solution

$$u(x,t) = \sum_{n=1}^{\inf} -\frac{8\pi n sin(\pi n) + 16 cos(\pi n) - 16}{\pi^3 n^3} cos\left(\frac{n\pi t}{2}\right) sin\left(\frac{n\pi x}{2}\right)$$

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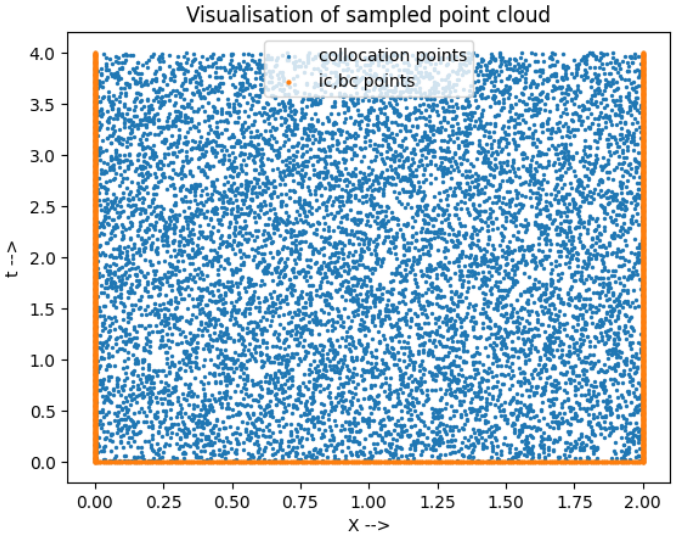
- ***Domain :***




N_f (collocation points) = 10000
N_u(points sampled from BC,IC) = 1000

Points are sampled using Latin hypercube sampling.

- ***Loss Function formulation:***

1) ICBC Loss = $\frac{1}{Nb}\sum_{i=0}^{Nb}(\hat{y} - y)^2 + \frac{1}{Ni}\sum_{i=0}^{Ni}\left(\frac{\partial u}{\partial t}\right)_{x=0}^2$

Where Nb = No. of boundary points , Ni = No. of initial condition points

2) PDE Loss = $\frac{1}{N}\sum_{i=0}^{N}\left(\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2}\right)^2$

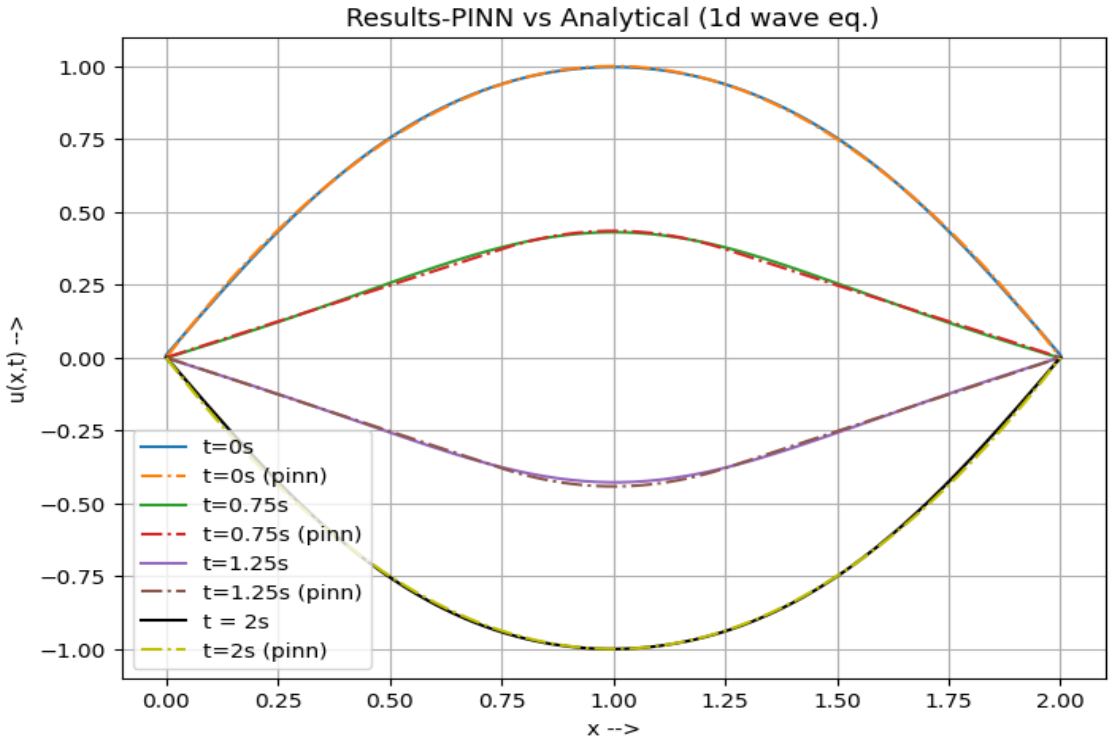
Where N = N_f + N_u

- ***Activation function used:*** tanh()

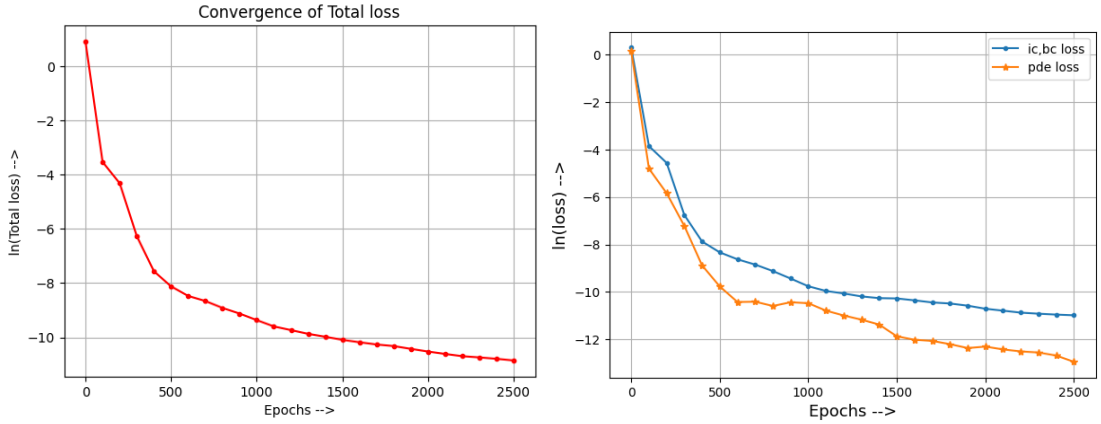
- ***RESULTS:***

| Layers | Neurons | Residual loss | Epochs |
|---|---|---|---|
| 4 | 20 | 8.995484e-05 | 2500 |
| 4 | 30 | 3.890816e-05 | 2500 |
| 4 | 40 | 3.214692e-05 | 2500 |
| 4 | 50 | 2.465390e-05 | 2500 |
| 4 | 60 | 1.937172e-05 | 2500 |

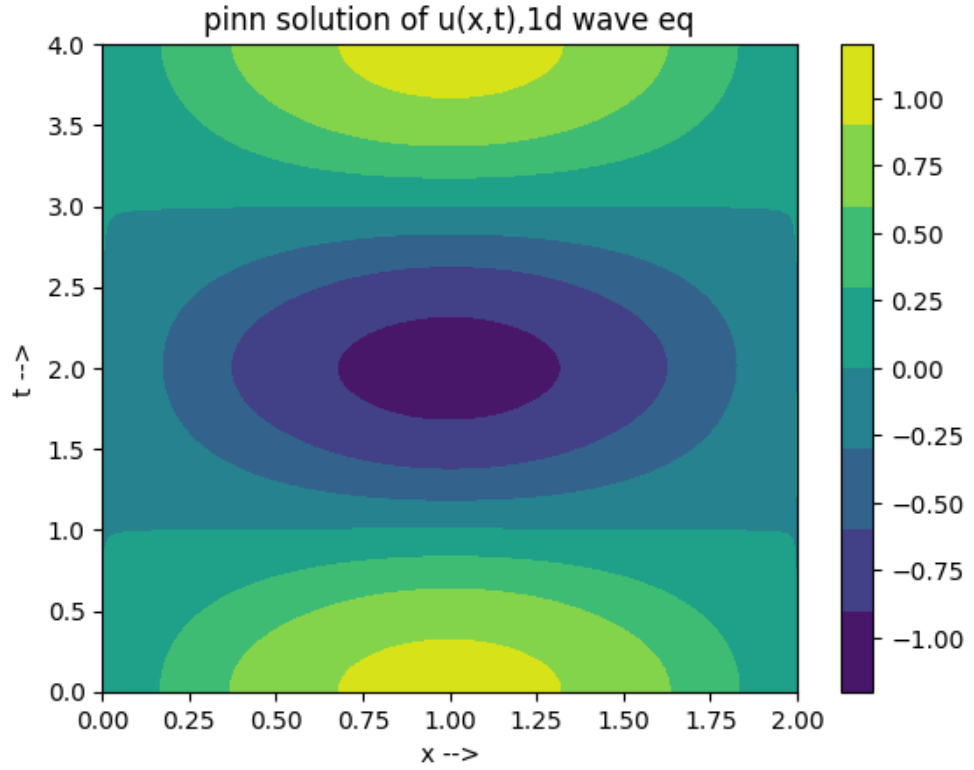
➢ **Comparison of PINNS results from analytical results at particular Time:**



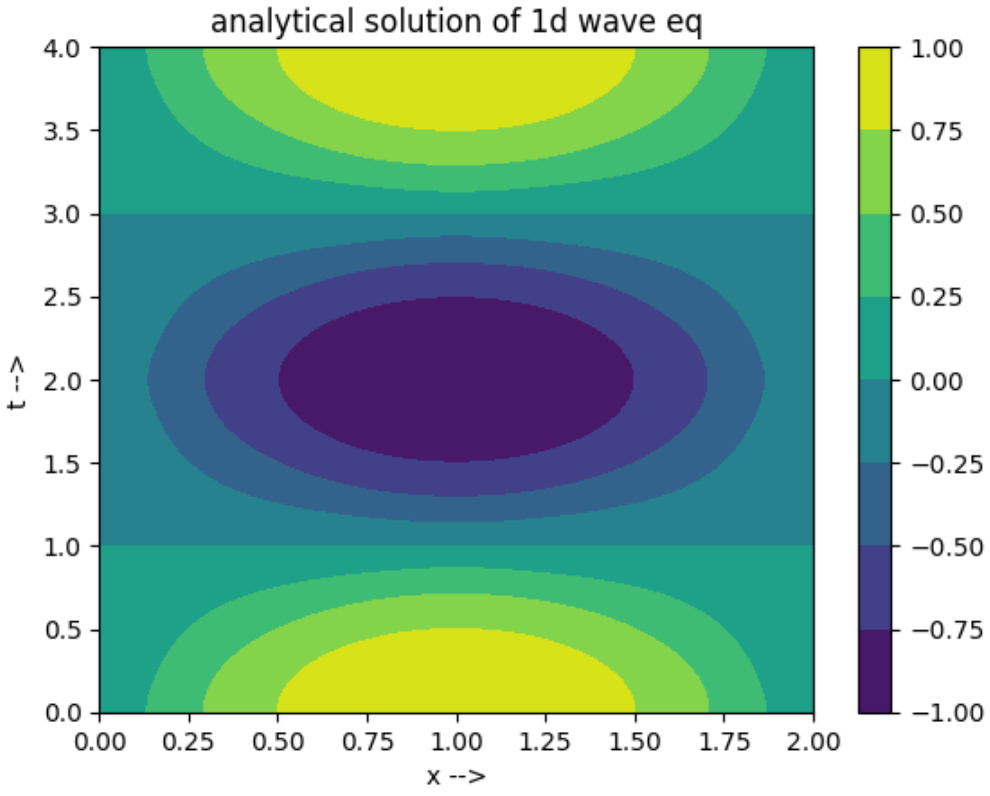

➢ **Loss convergence**: Loss is plotted in logarithmic scale.



➢ **Contour plot (PINN solution):**




➢ **Contour plot (Analytical solution):**



➢ **Error between Analytical solution and PINN solution:**