

# Segmentation:

## Otsu's method

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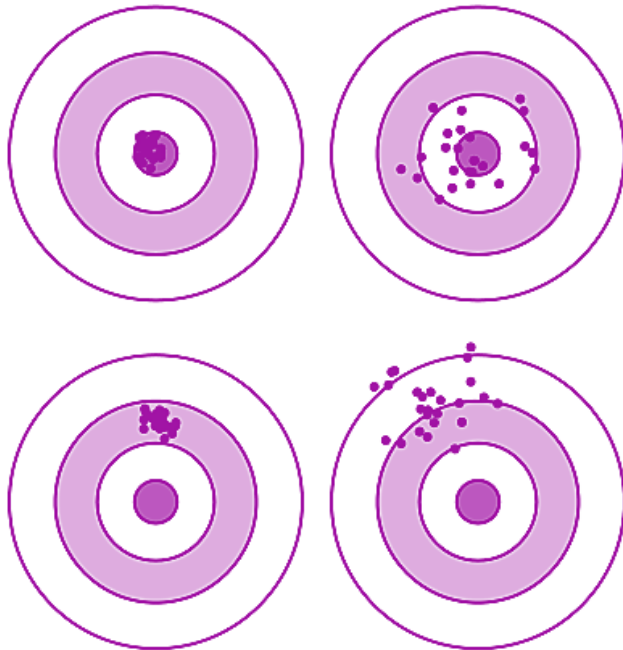
Dr. Tushar Sandhan

# Introduction

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## ■ Variance

- intraclass
- interclass

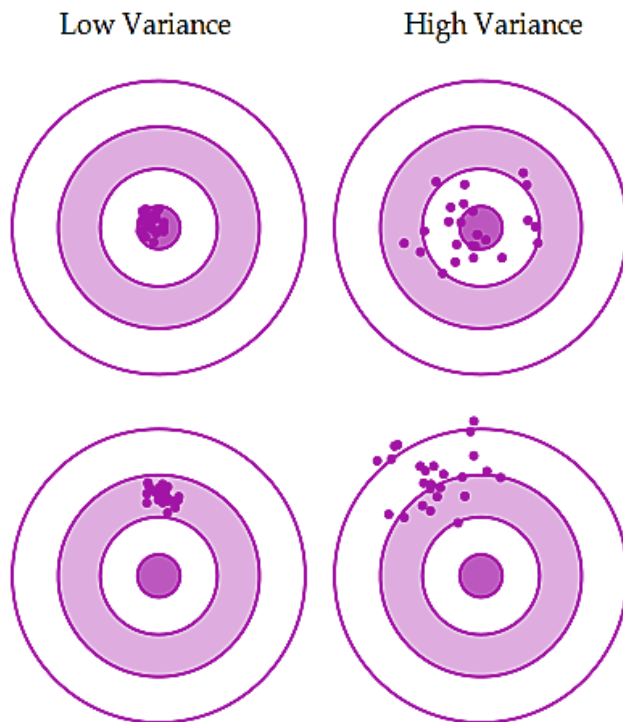


# Introduction

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## ■ Variance

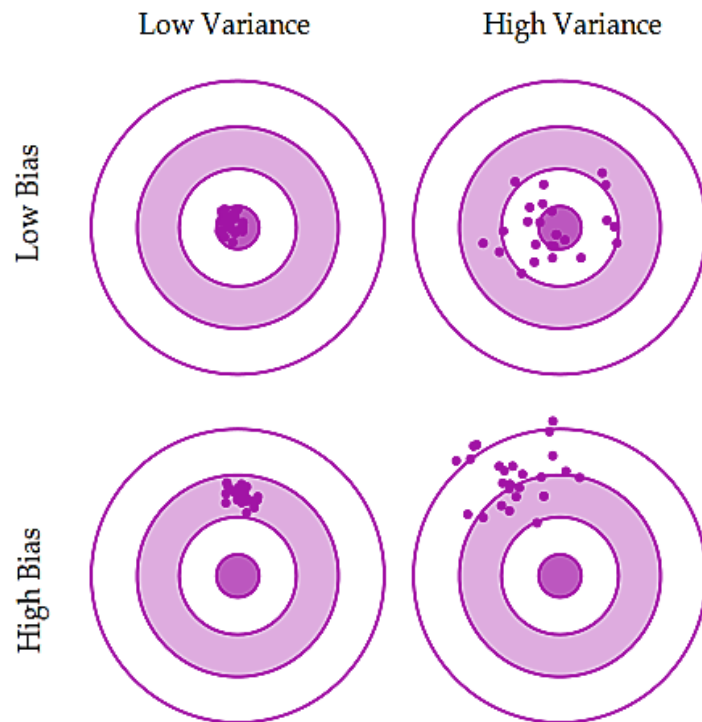
- intraclass
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# Introduction

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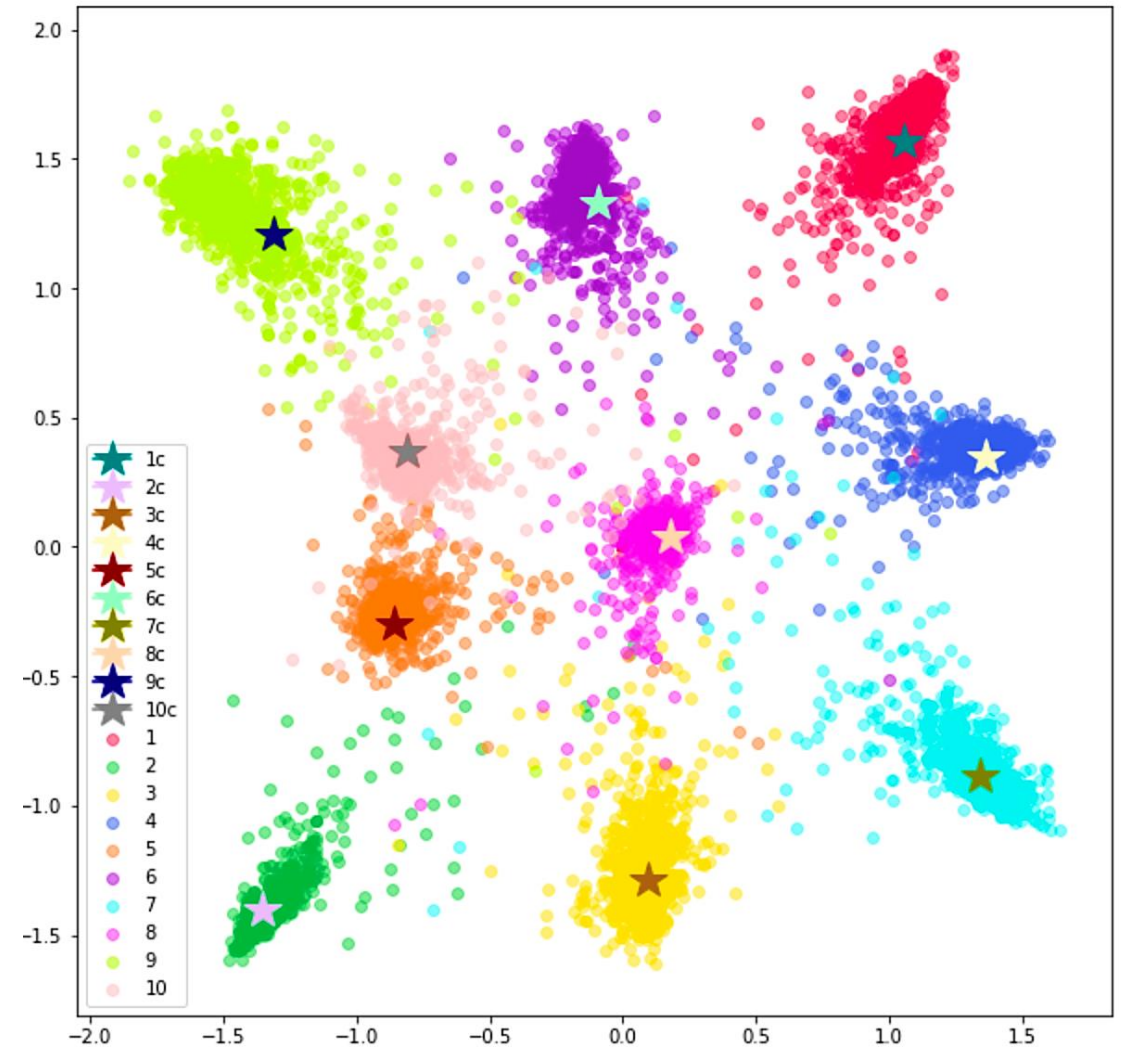
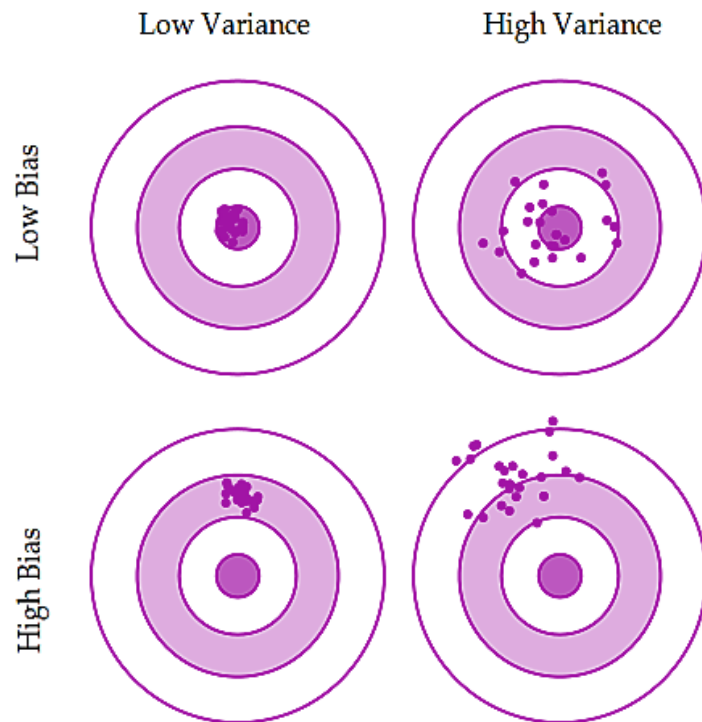
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# Introduction

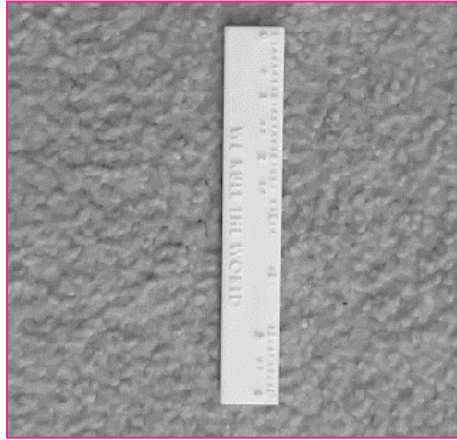
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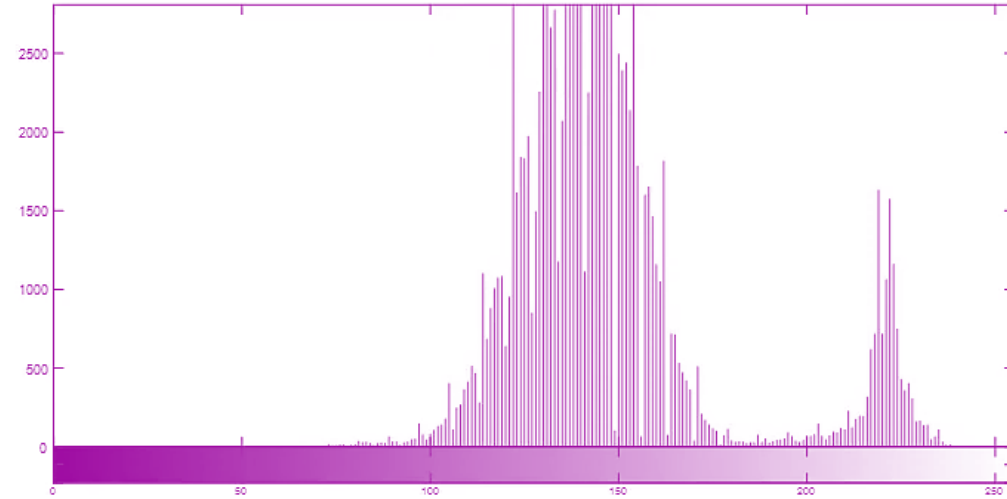
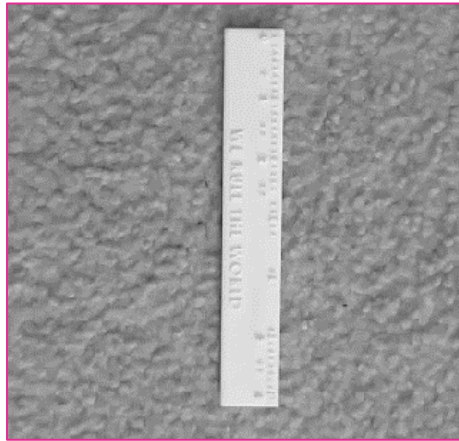
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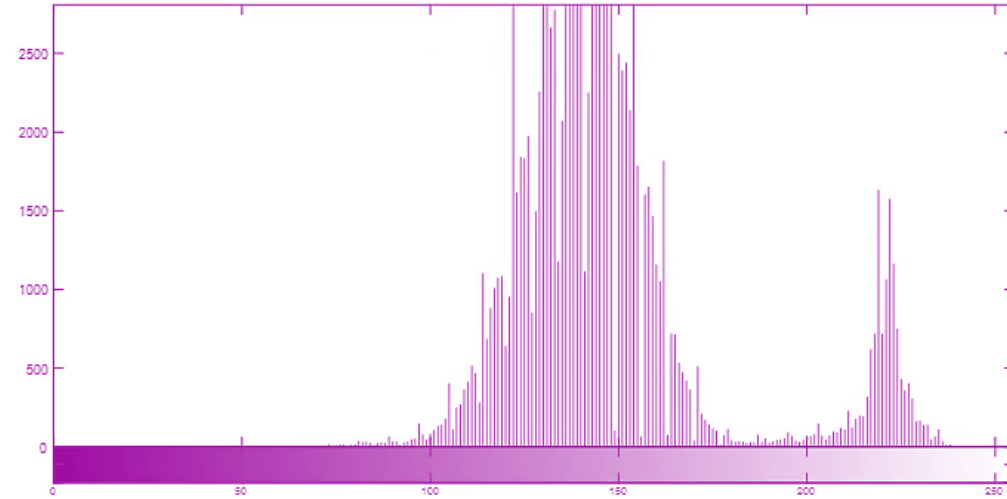
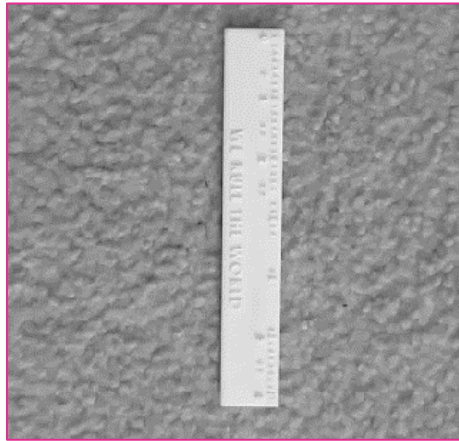
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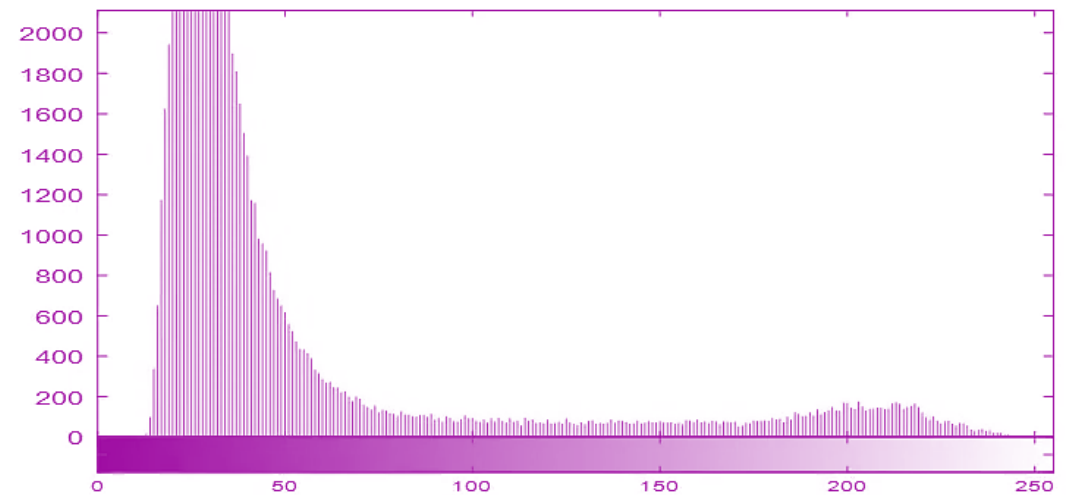
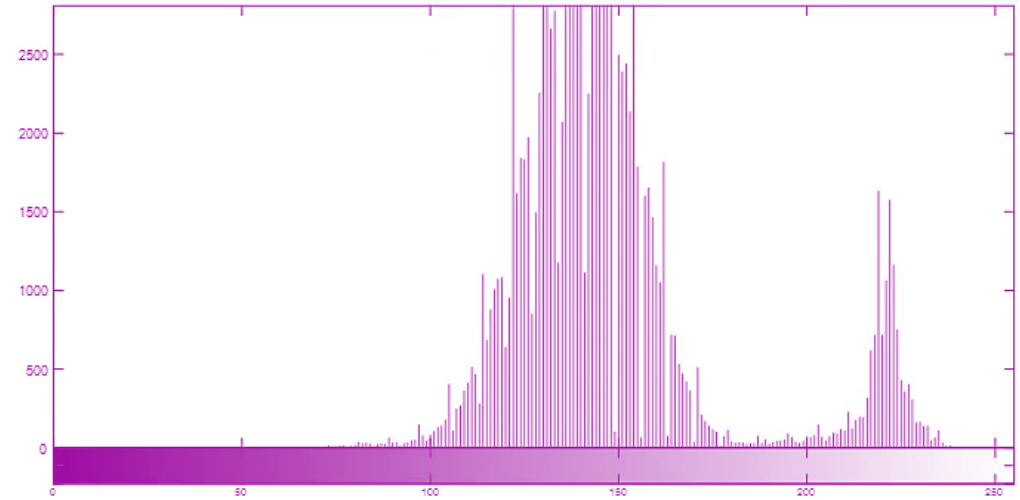
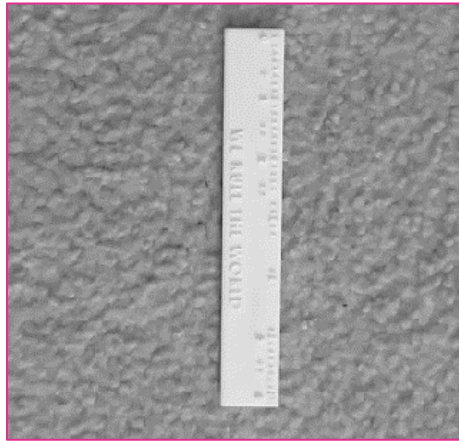
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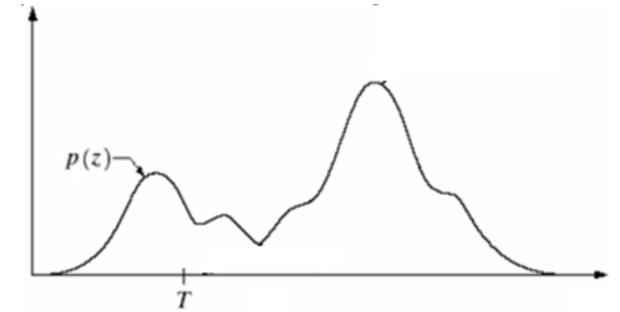
# Introduction



# Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches  $\forall T$  that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.

$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

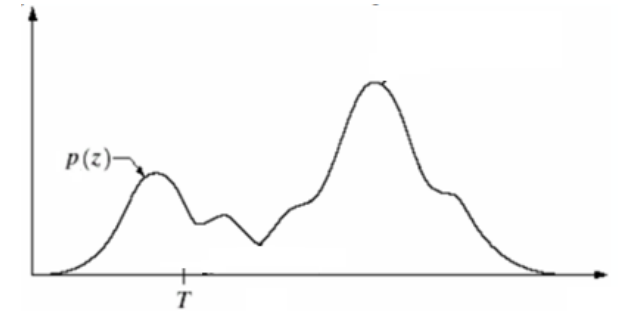


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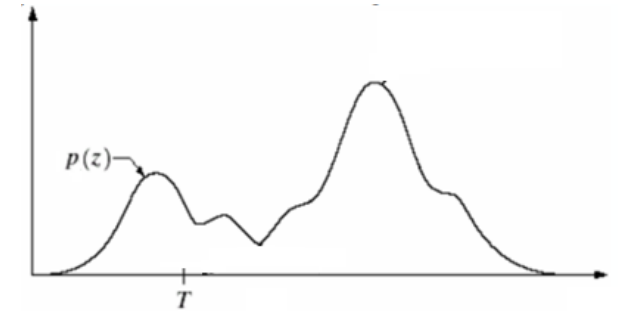
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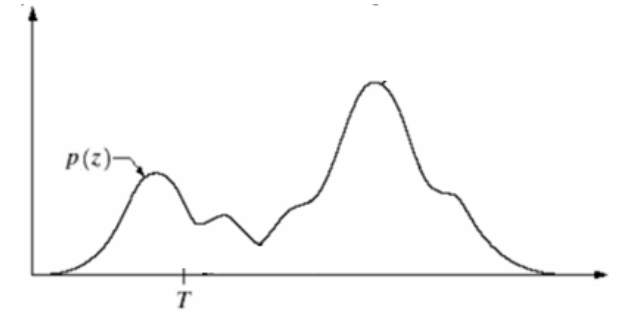
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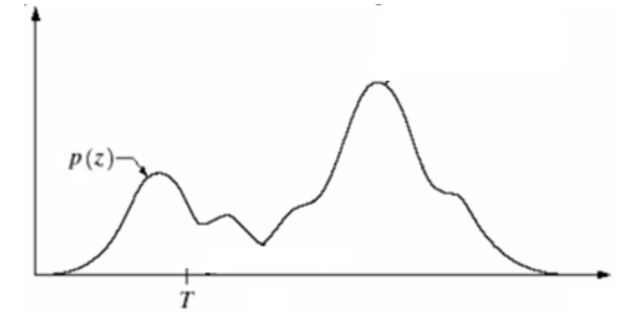
$$P_2(T) = \sum_{t=T}^{L-1} p(t)$$

$$\mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)}$$



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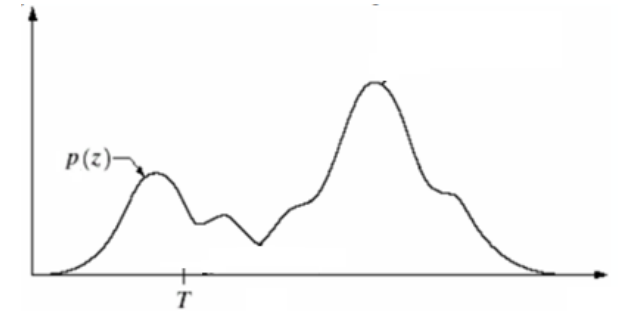
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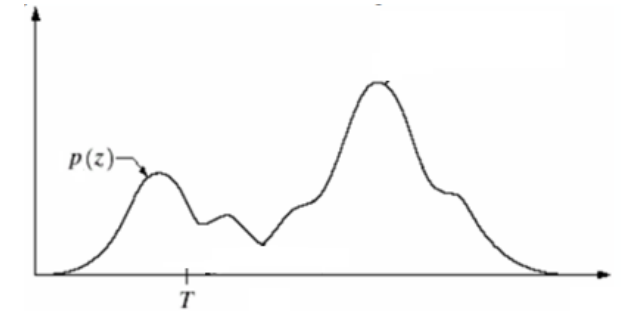
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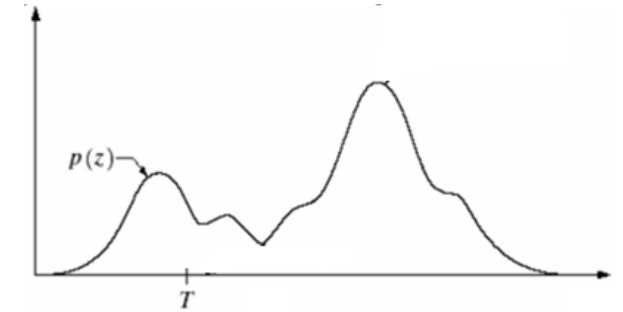
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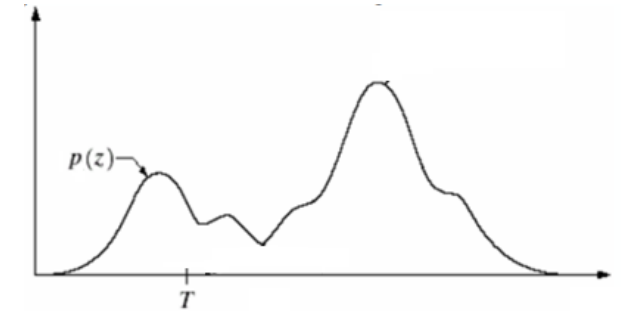
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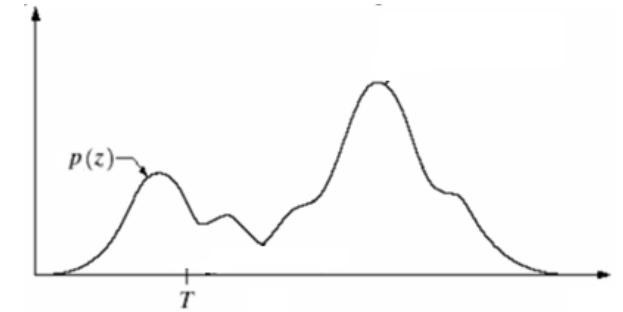
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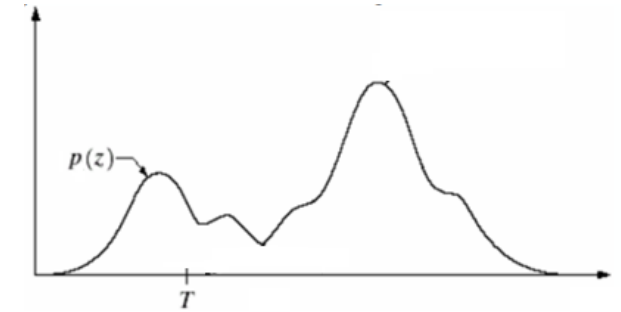
$$\sigma^2 = \sigma_e^2(T) + \sigma_r^2(T)$$

$$\sigma_1^2(T) = \sum_{t=0}^{T-1} \frac{(t - \mu_1(T))^2 \cdot p(t)}{P_1(T)}$$

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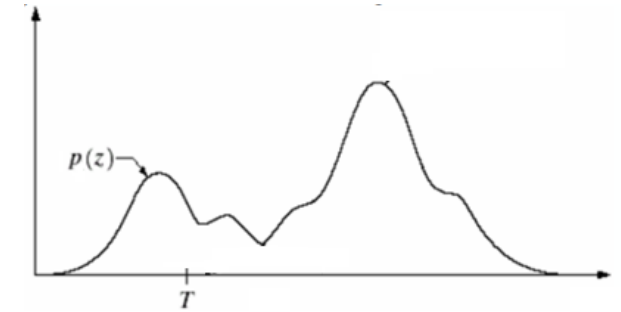
$$\sigma_e^2(T) = \sigma^2 - \sigma_r^2(T)$$

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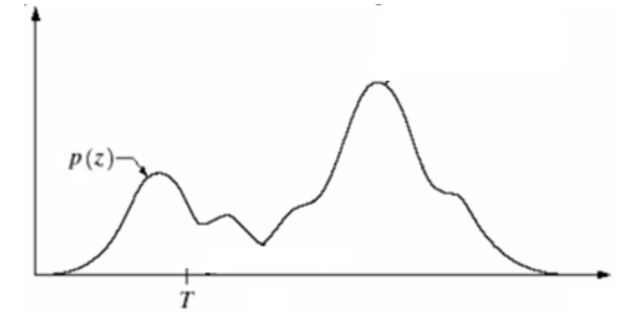
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$$\sigma_e^2(T) = P_1(T)P_2(T)(\mu_1(T) - \mu_2(T))^2$$

# Otsu's thresholding

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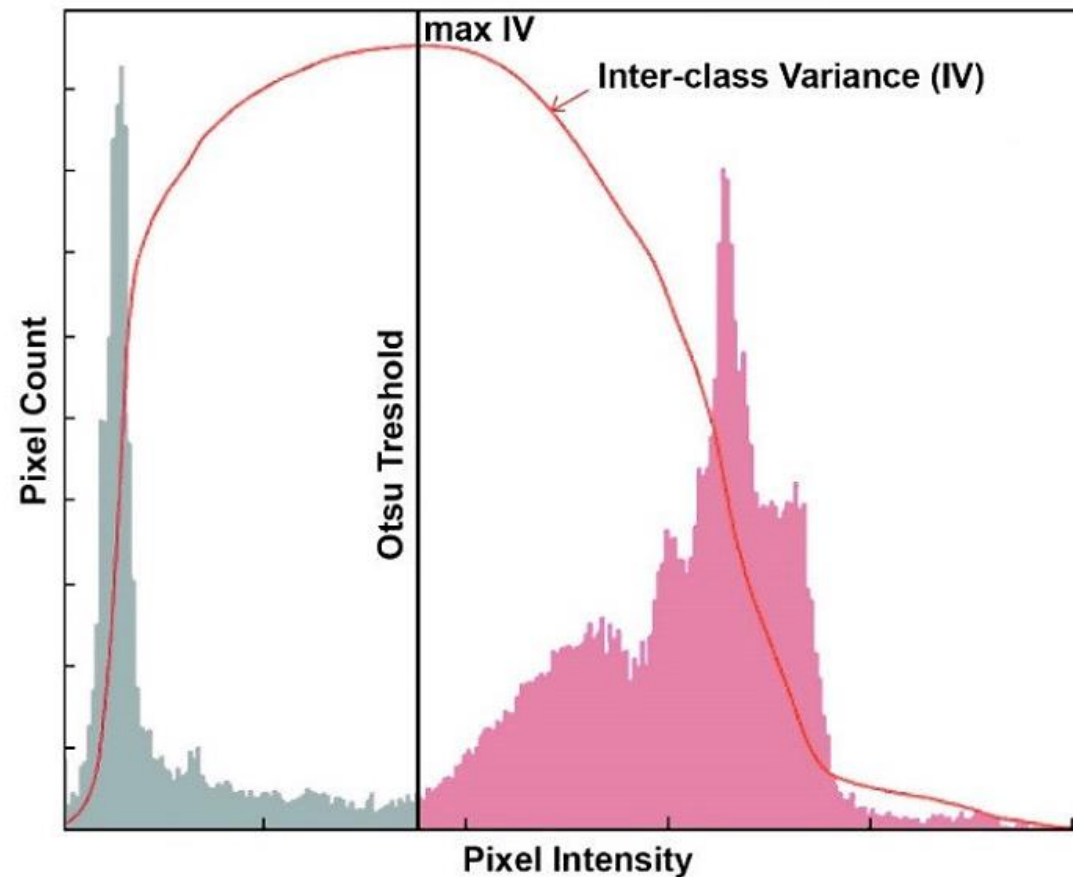
- 
1. Compute histogram and probabilities of each intensity level  $t$
  2. Set up initial  $P_i(0)$  and  $\mu_i(0)$
  3. Step through all possible thresholds  $T = 1, \dots, L$ 
    1. Update  $P_i(T)$  and  $\mu_i(T)$
    2. Compute  $\sigma_e^2(T)$
  4. Desired threshold  $T^*$  corresponds to the maximum  $\sigma_e^2(T)$
-

# Otsu's thresholding

- Variance variation

- inter-class var maximization

$$\sigma_e^2(T) = P_1(T)P_2(T)(\mu_1(T) - \mu_2(T))^2$$





# Otsu's thresholding

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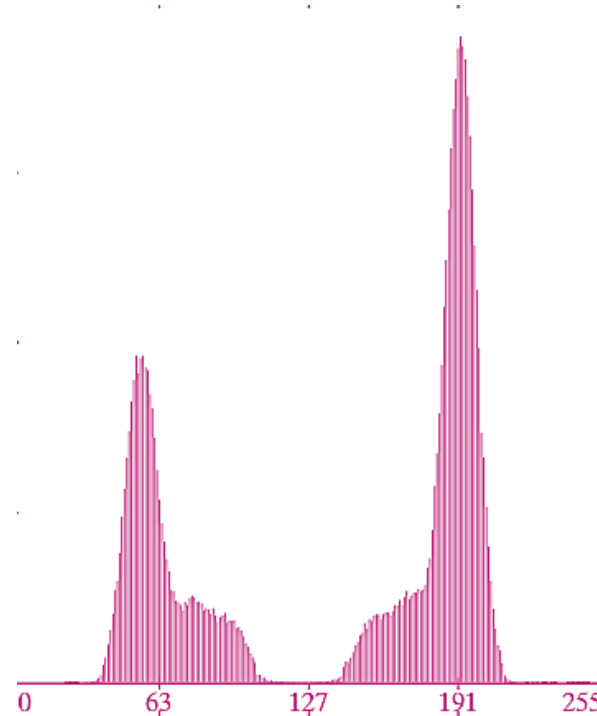
- Global: iterative adapting threshold:  $TH = 125$
- Global: Otsu's thresholding:  $TH = 125$



# Otsu's thresholding

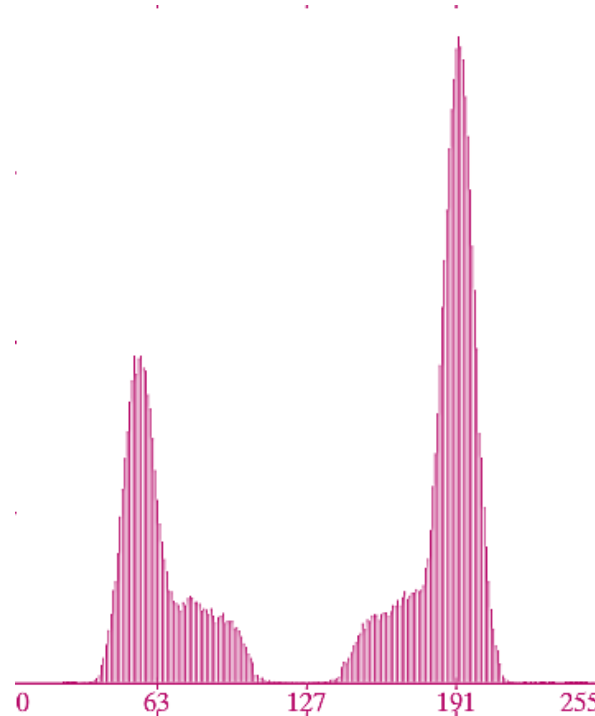
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# Otsu's thresholding

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- Global: Otsu's thresholding:  $TH = 125$



# Otsu's thresholding

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- Example

- microscopic image (polymer cells)

Input



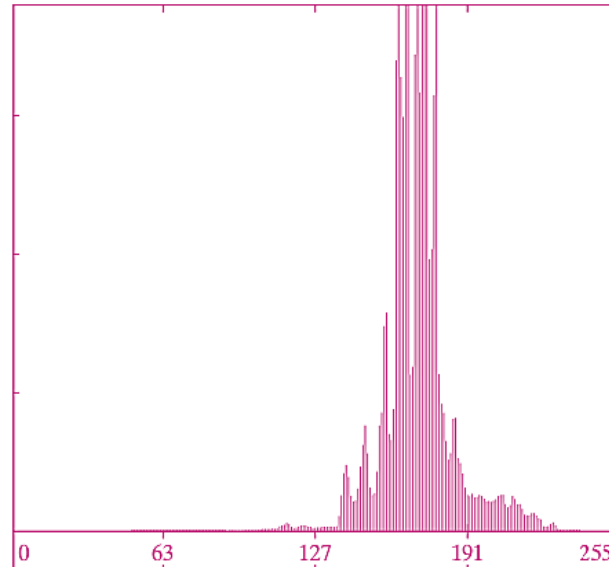
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# Otsu's thresholding

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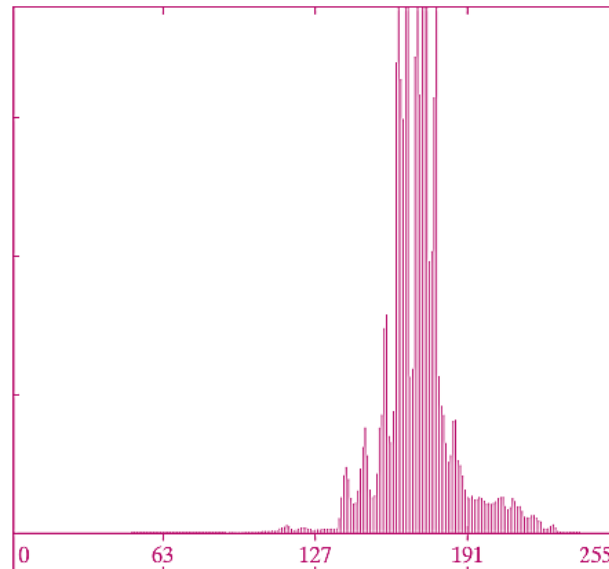
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Hist



# Otsu's thresholding

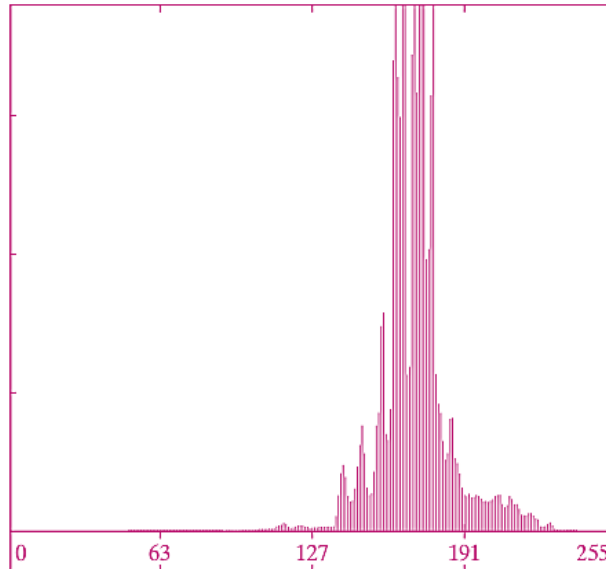
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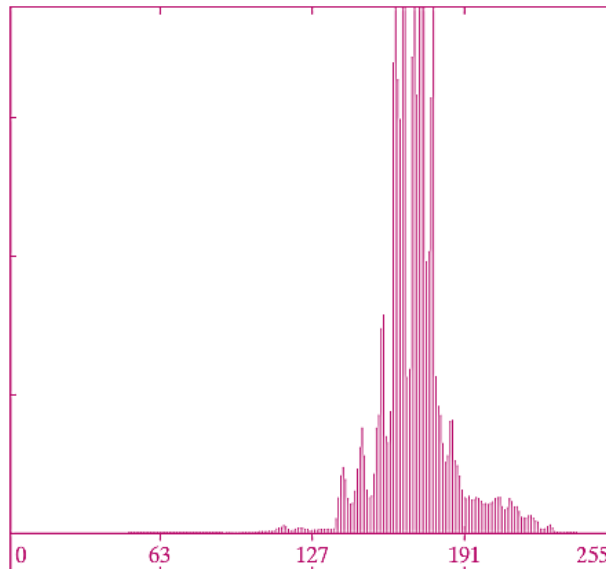
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Hist



Global: iterative adaptive TH





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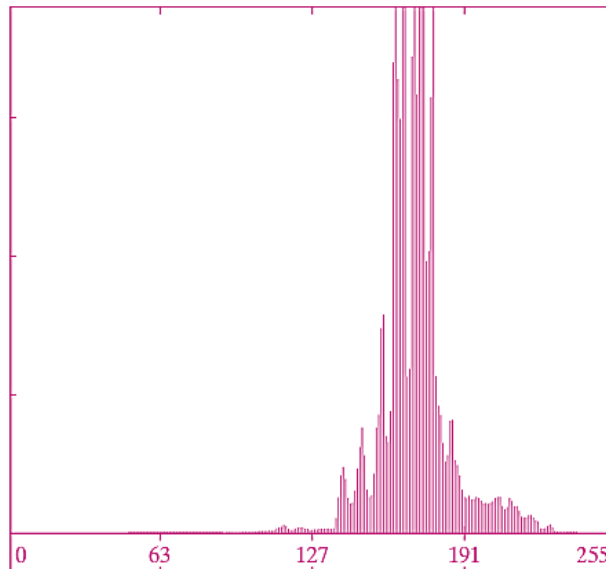
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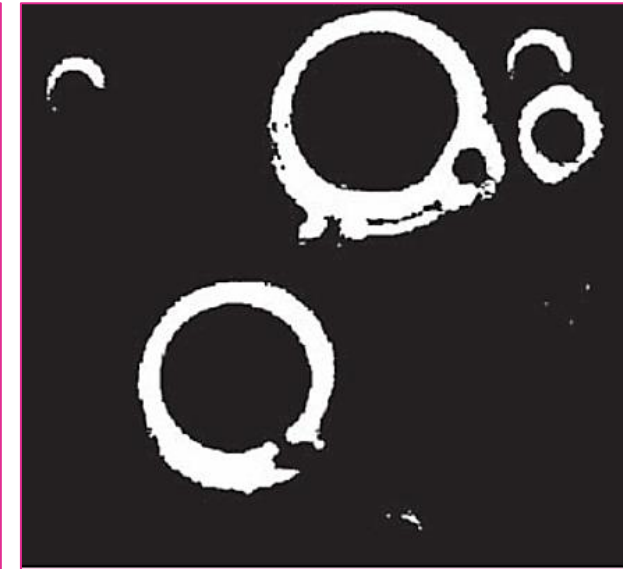
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Global: iterative adaptive TH



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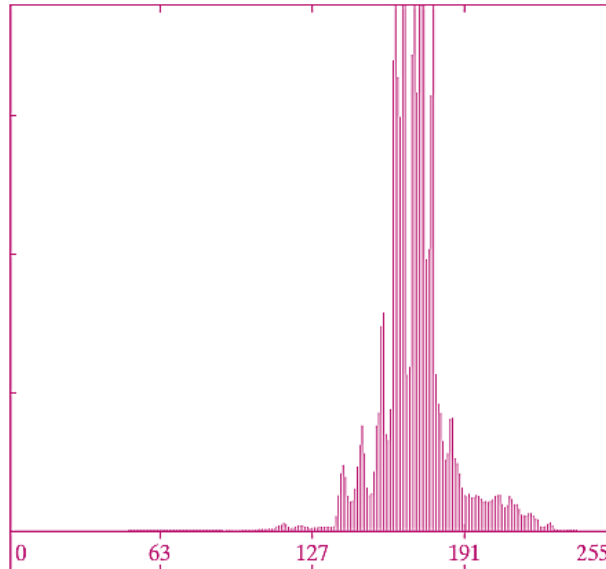
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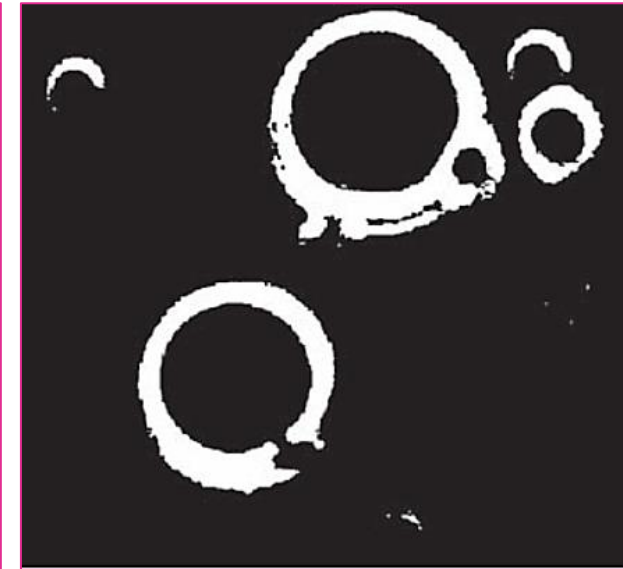
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Global: iterative adaptive TH



Global: Otsu's TH



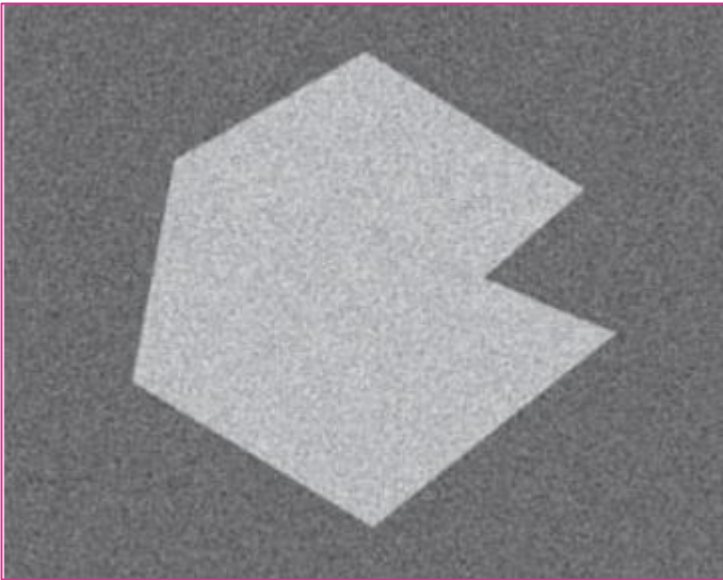
# Otsu's thresholding

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- Example

- noisy input as it is

Input



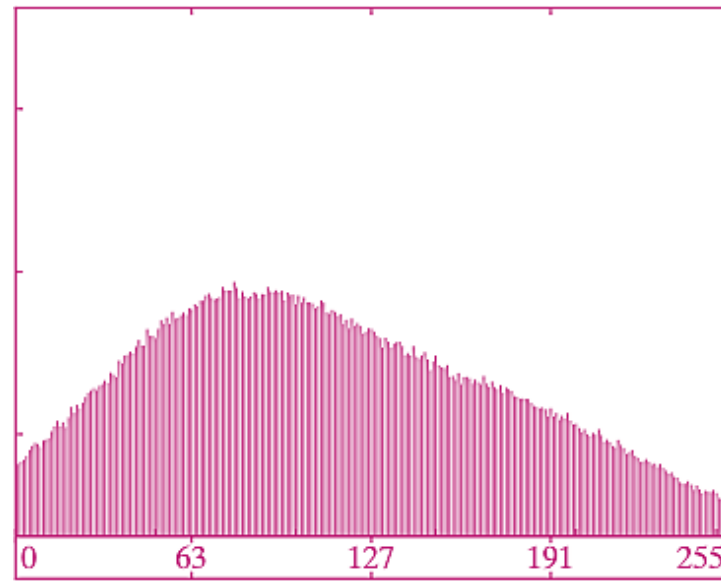
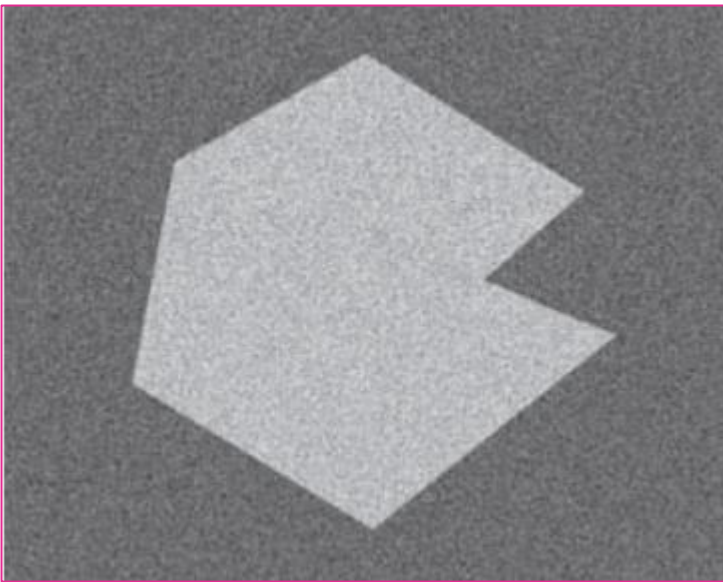
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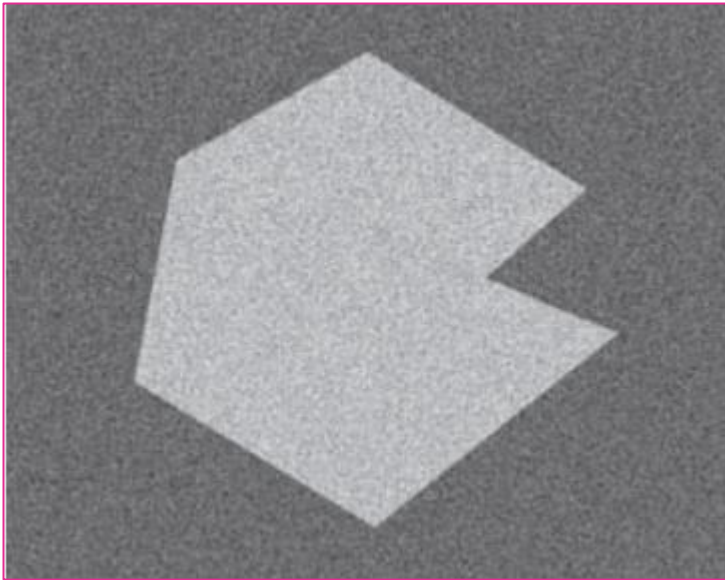
# Otsu's thresholding

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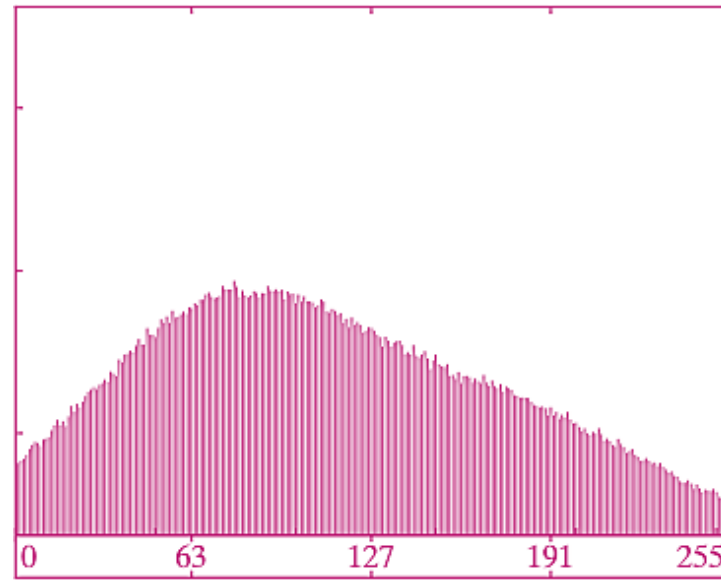
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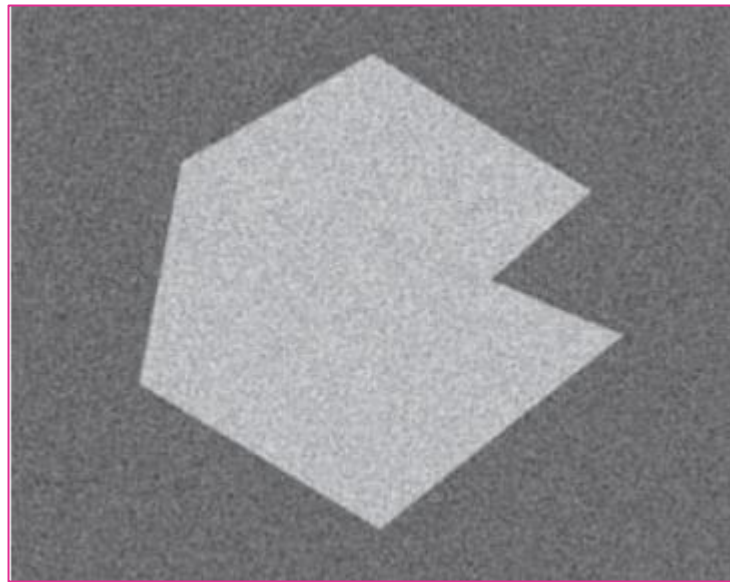


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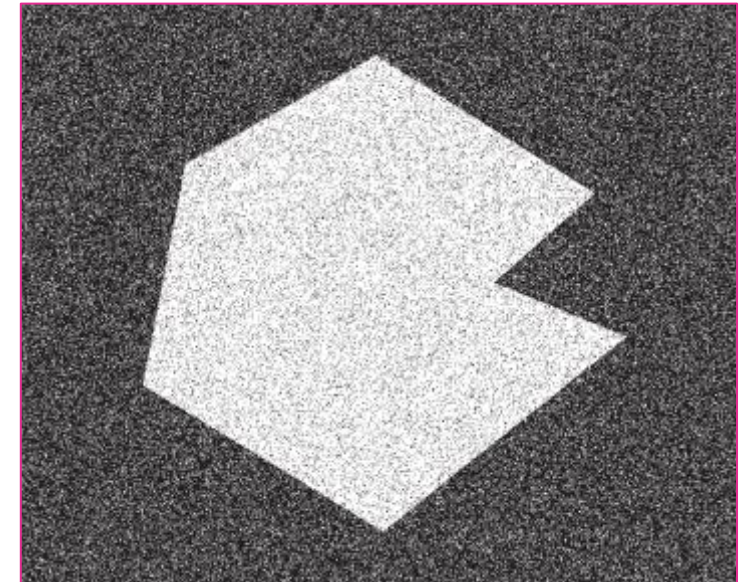
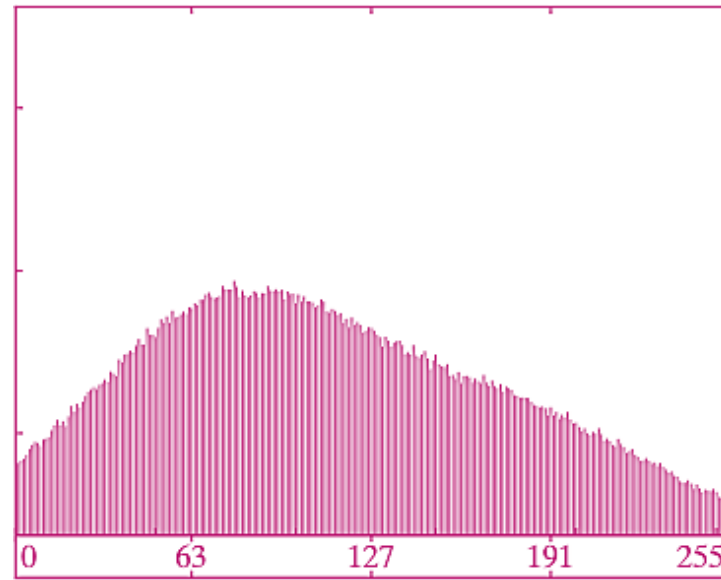
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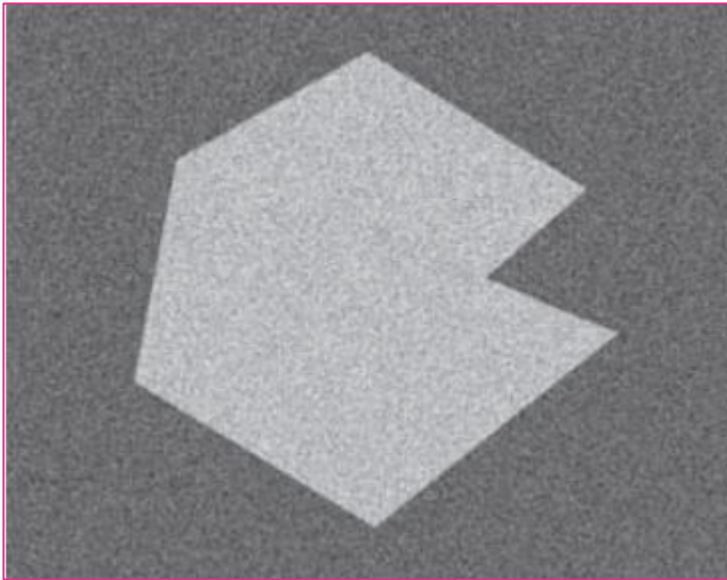


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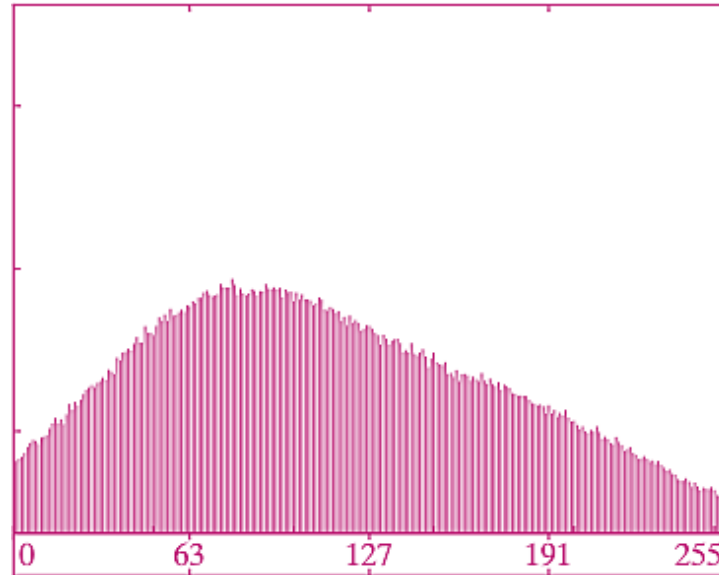
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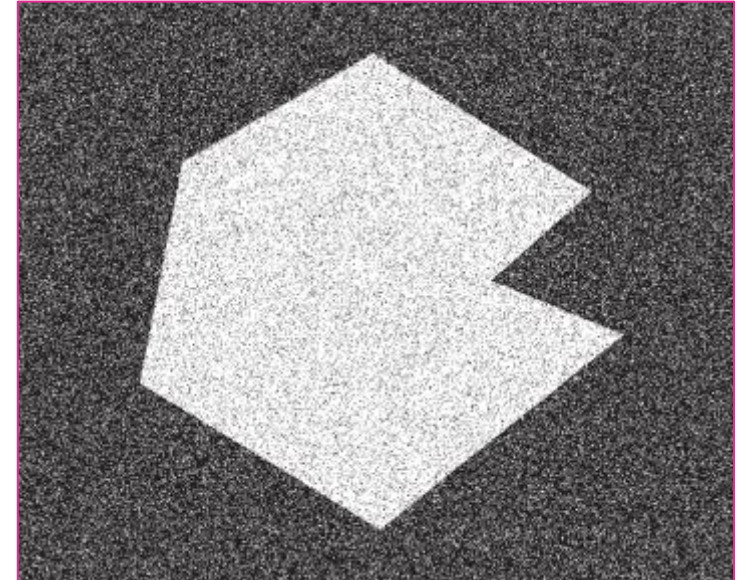
Input



Hist



Global: Otsu's TH



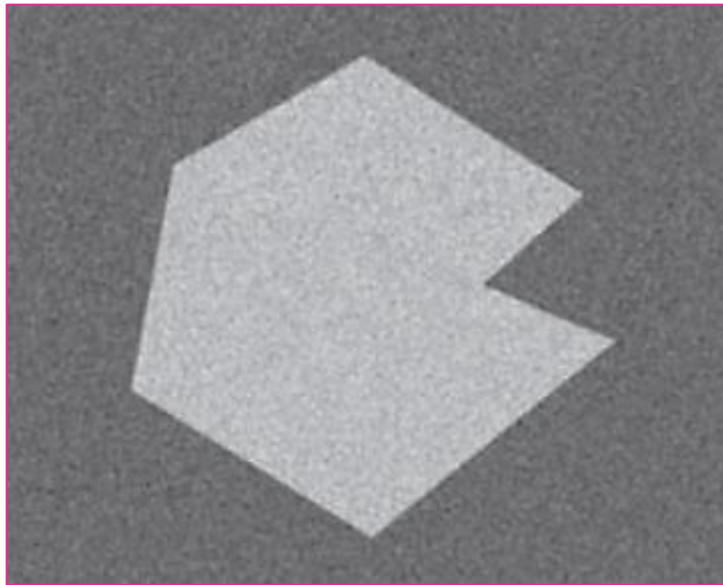
# Otsu's thresholding

---

- Example

- noisy input after minor smoothing

Input





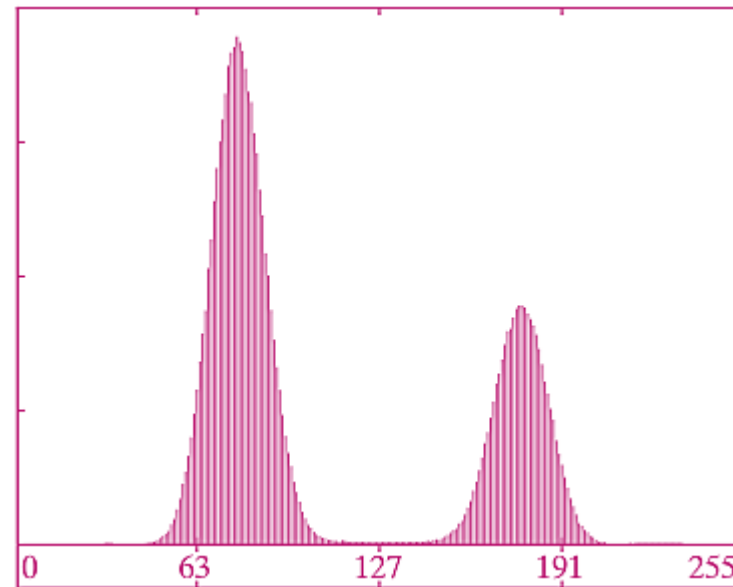
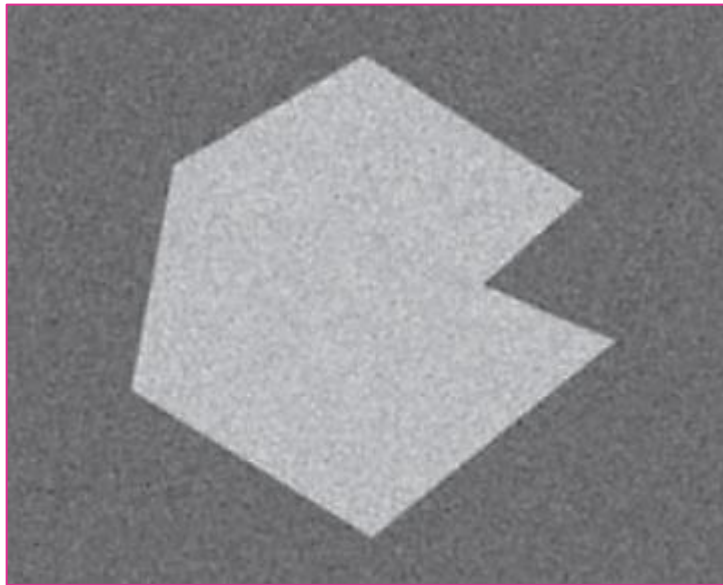
# Otsu's thresholding

---

- Example

- noisy input after minor smoothing

Input



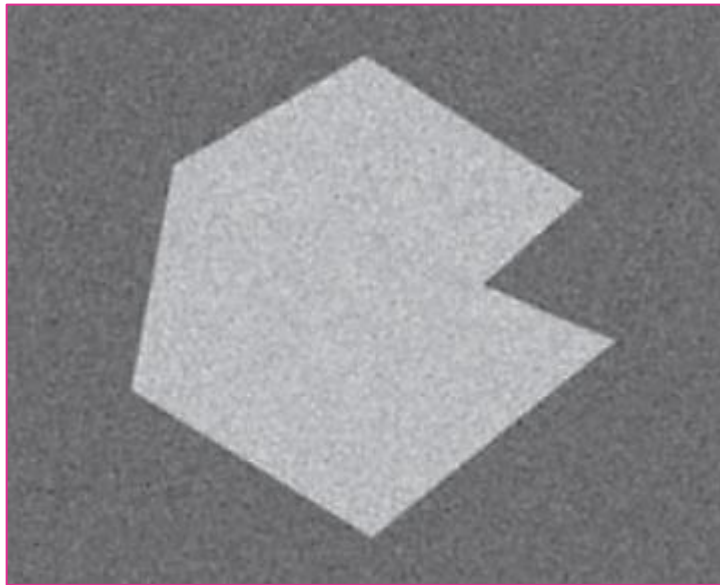
# Otsu's thresholding

---

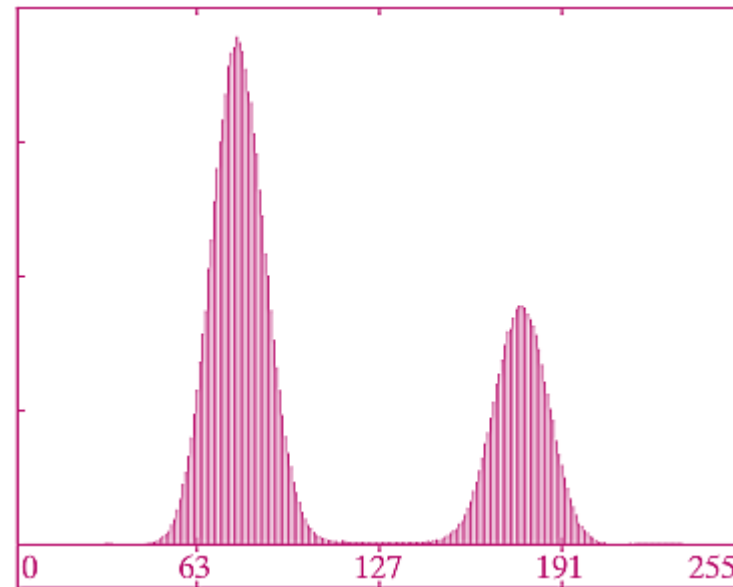
- Example

- noisy input after minor smoothing

Input



Hist

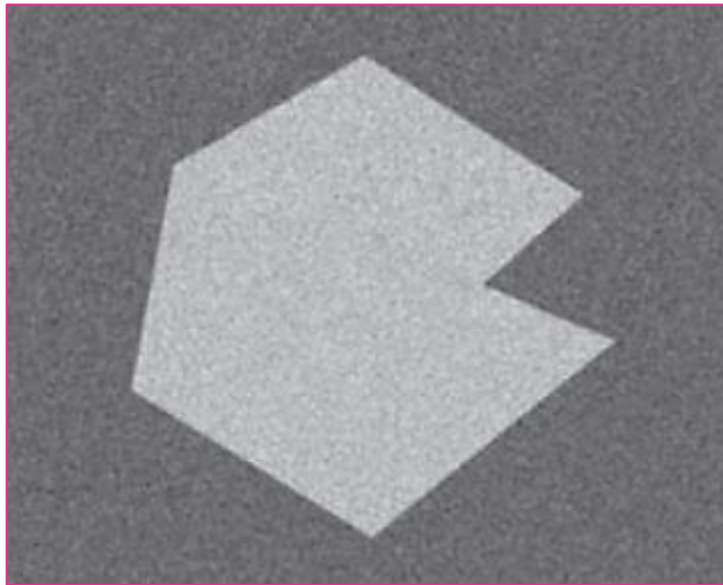


# Otsu's thresholding

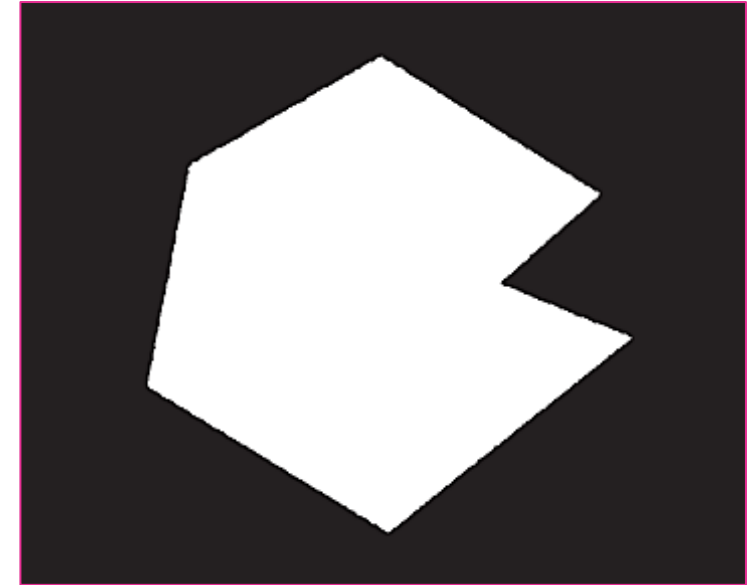
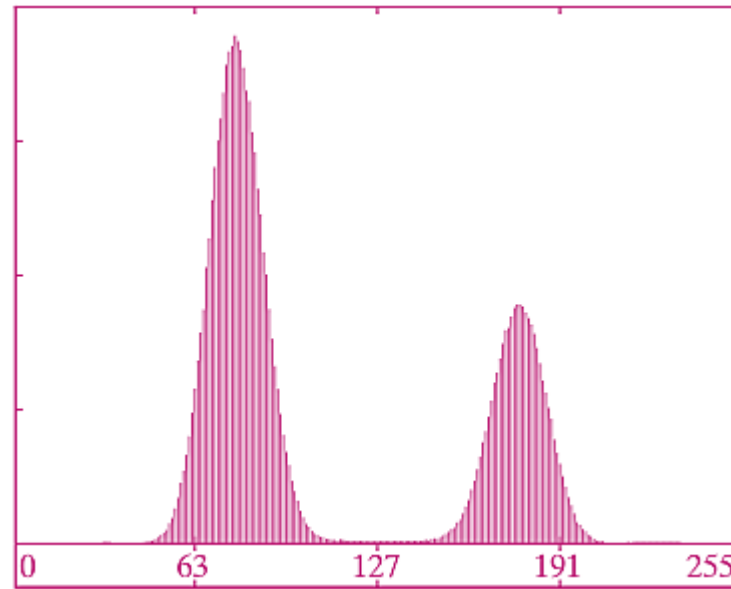
- Example

- noisy input after minor smoothing

Input



Hist

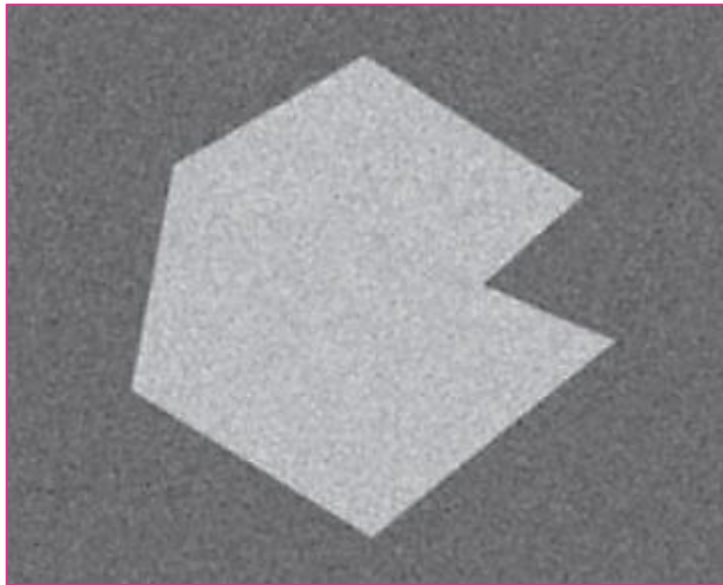


# Otsu's thresholding

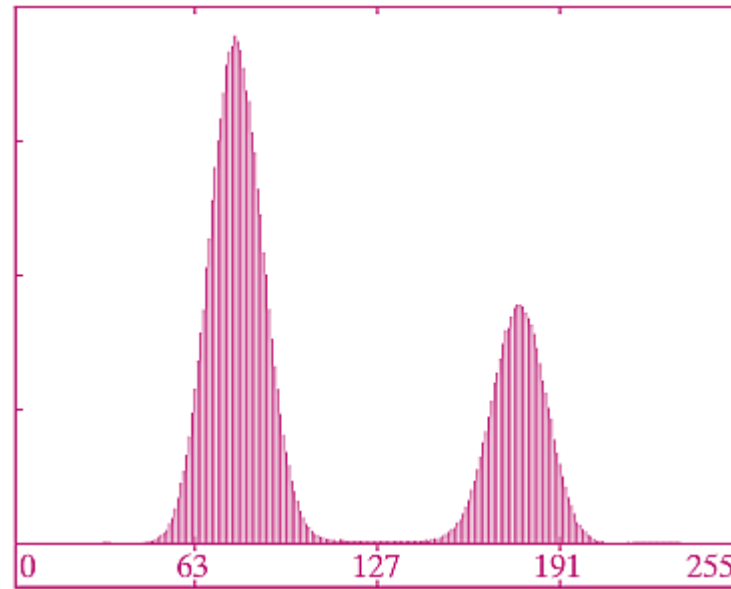
- Example

- noisy input after minor smoothing

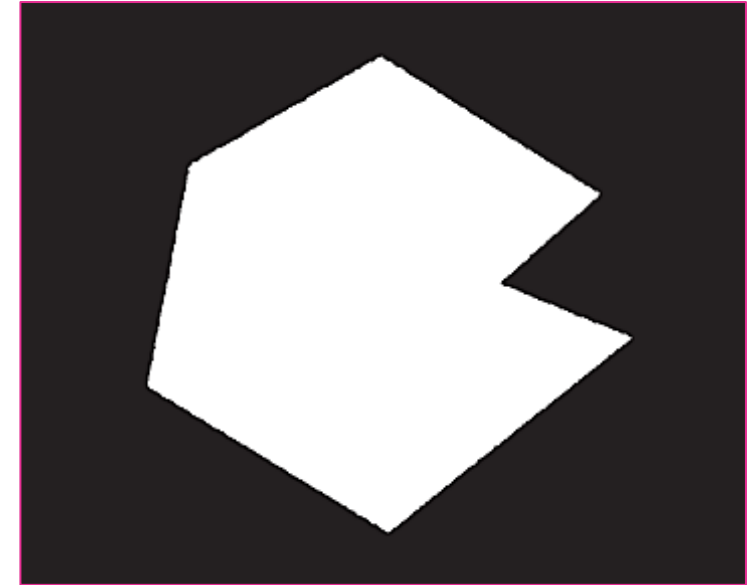
Input



Hist



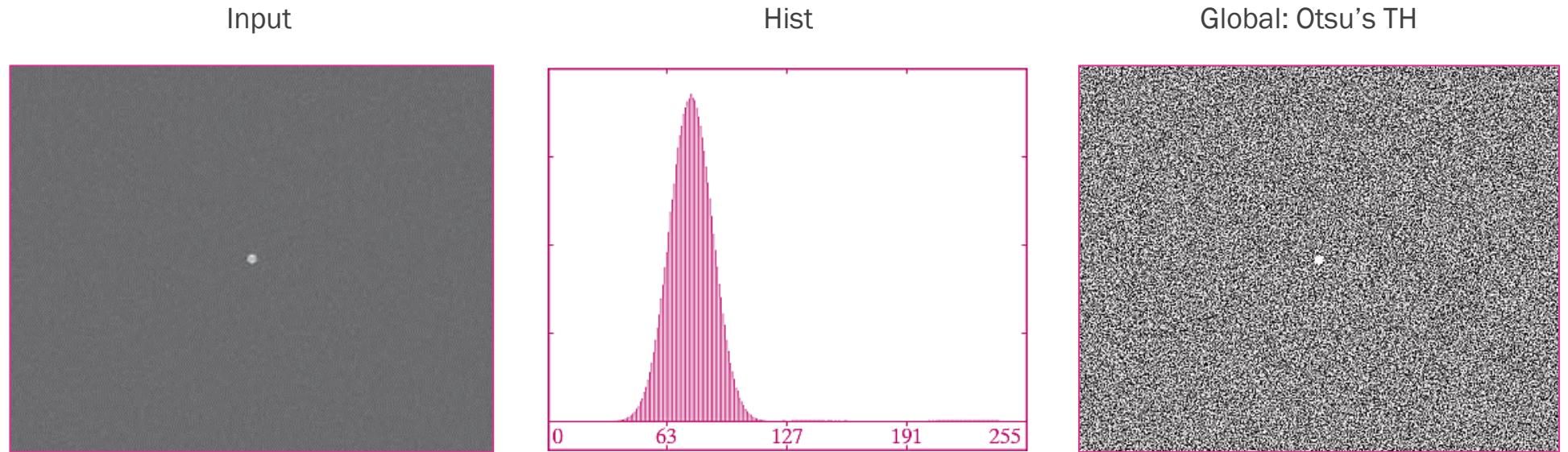
Global: Otsu's TH



# Otsu's thresholding

- Example

- small object's noisy image



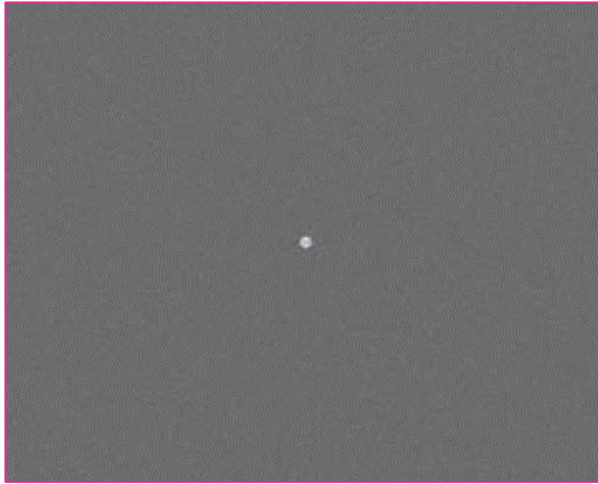


# Otsu's thresholding

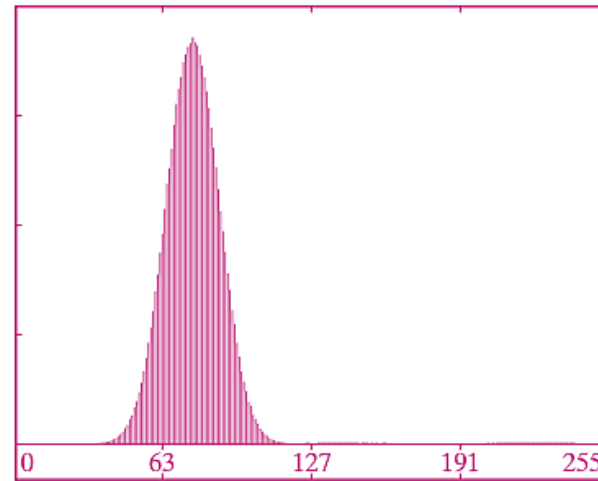
- Example

- small object's noisy image

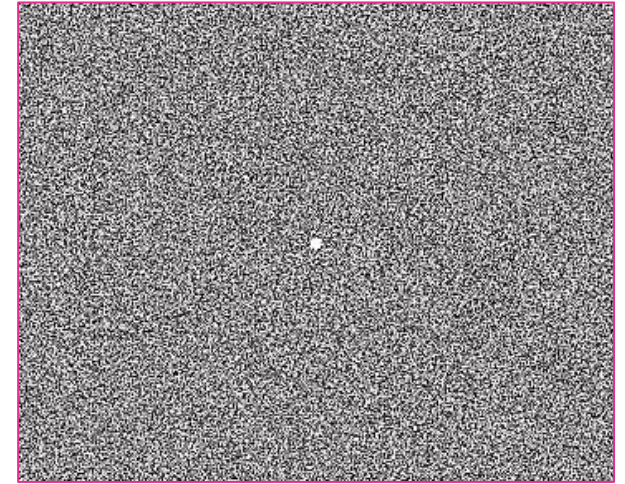
Input



Hist



Global: Otsu's TH



# Otsu's thresholding

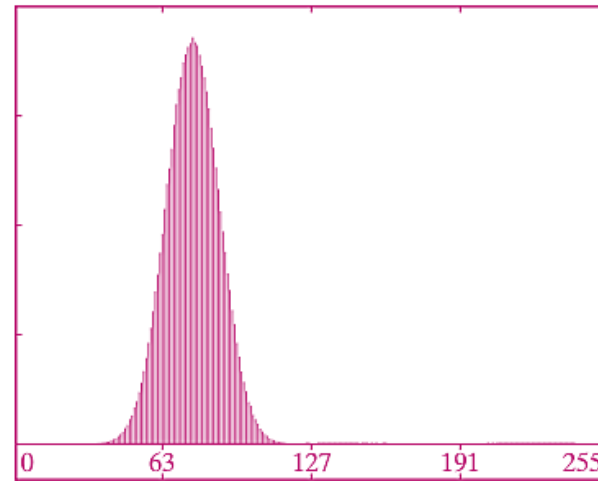
- Example

- small object's noisy image

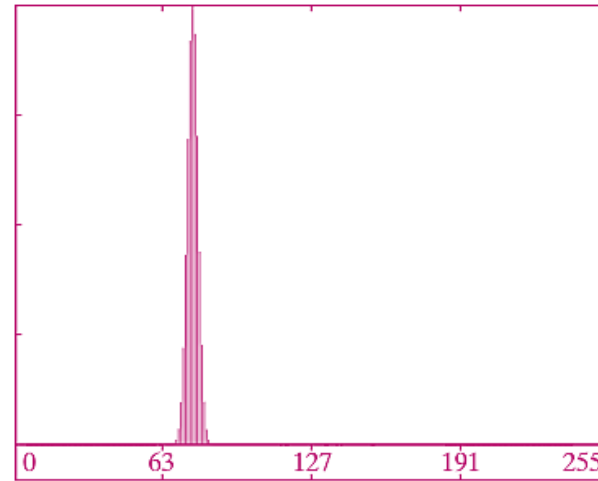
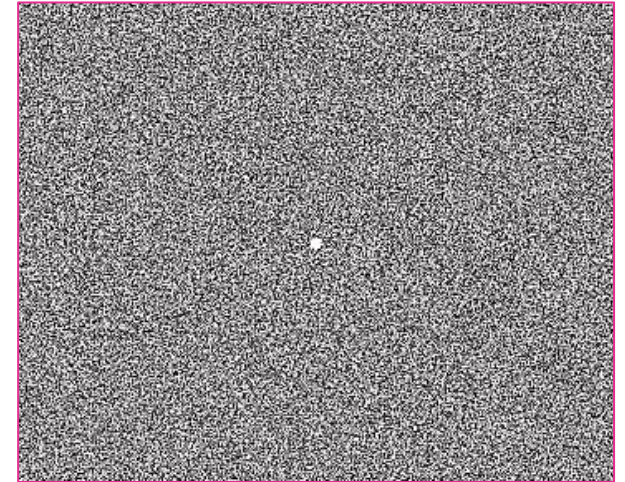
Input



Hist



Global: Otsu's TH

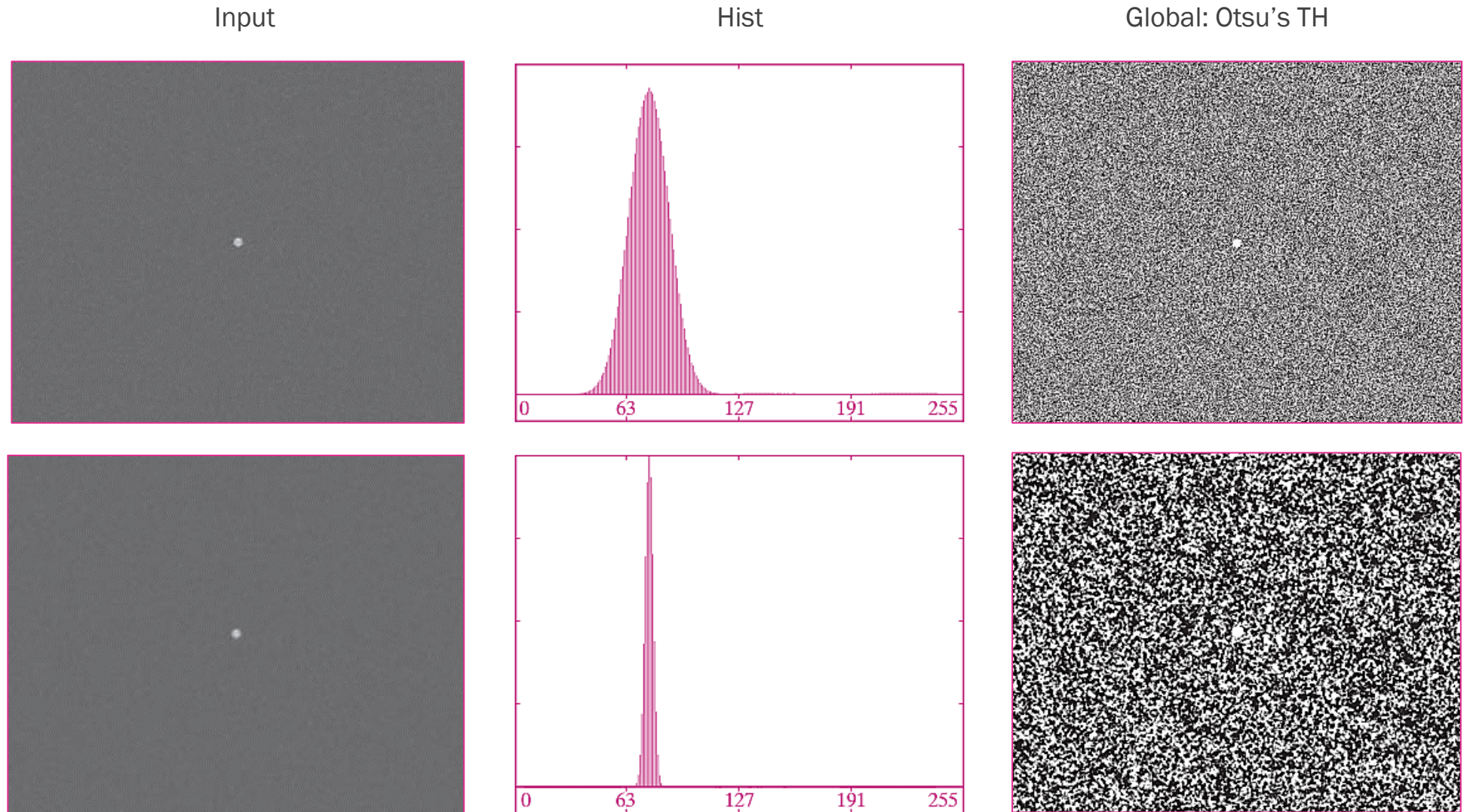




# Otsu's thresholding

- Example

- small object's noisy image

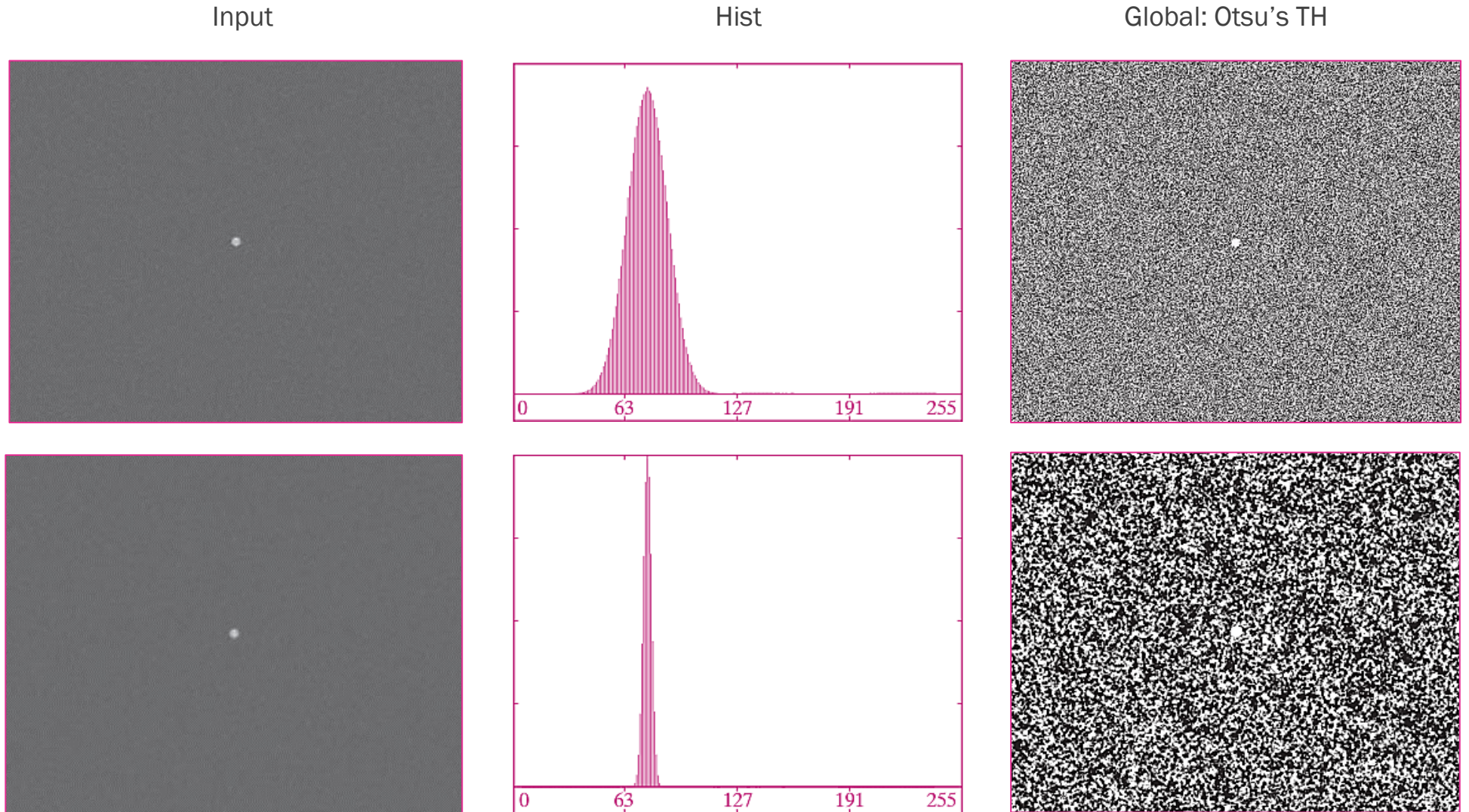




# Otsu's thresholding

## ■ Example

- small object's noisy image
- smoothing degrades the performance

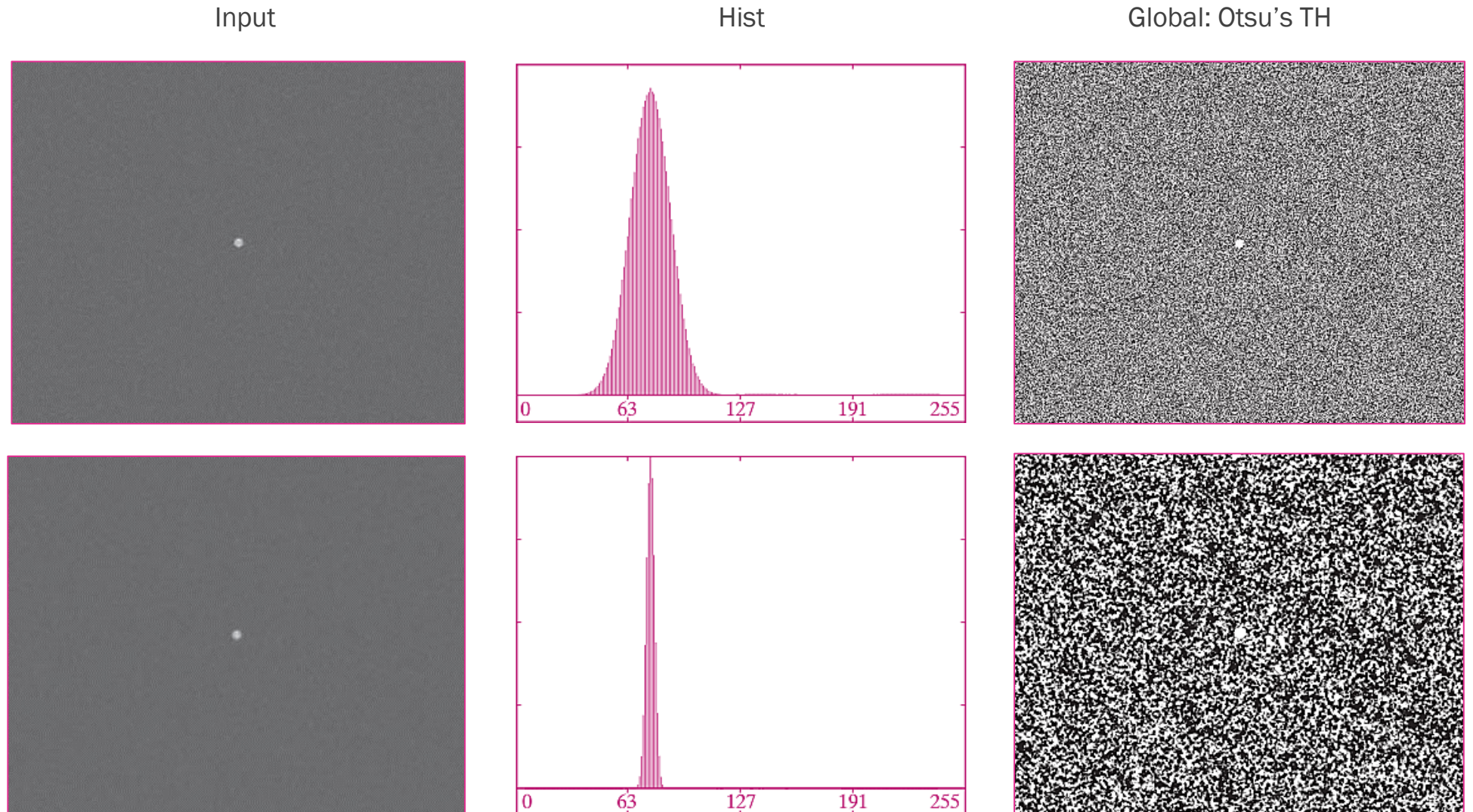




# Otsu's thresholding

## ■ Example

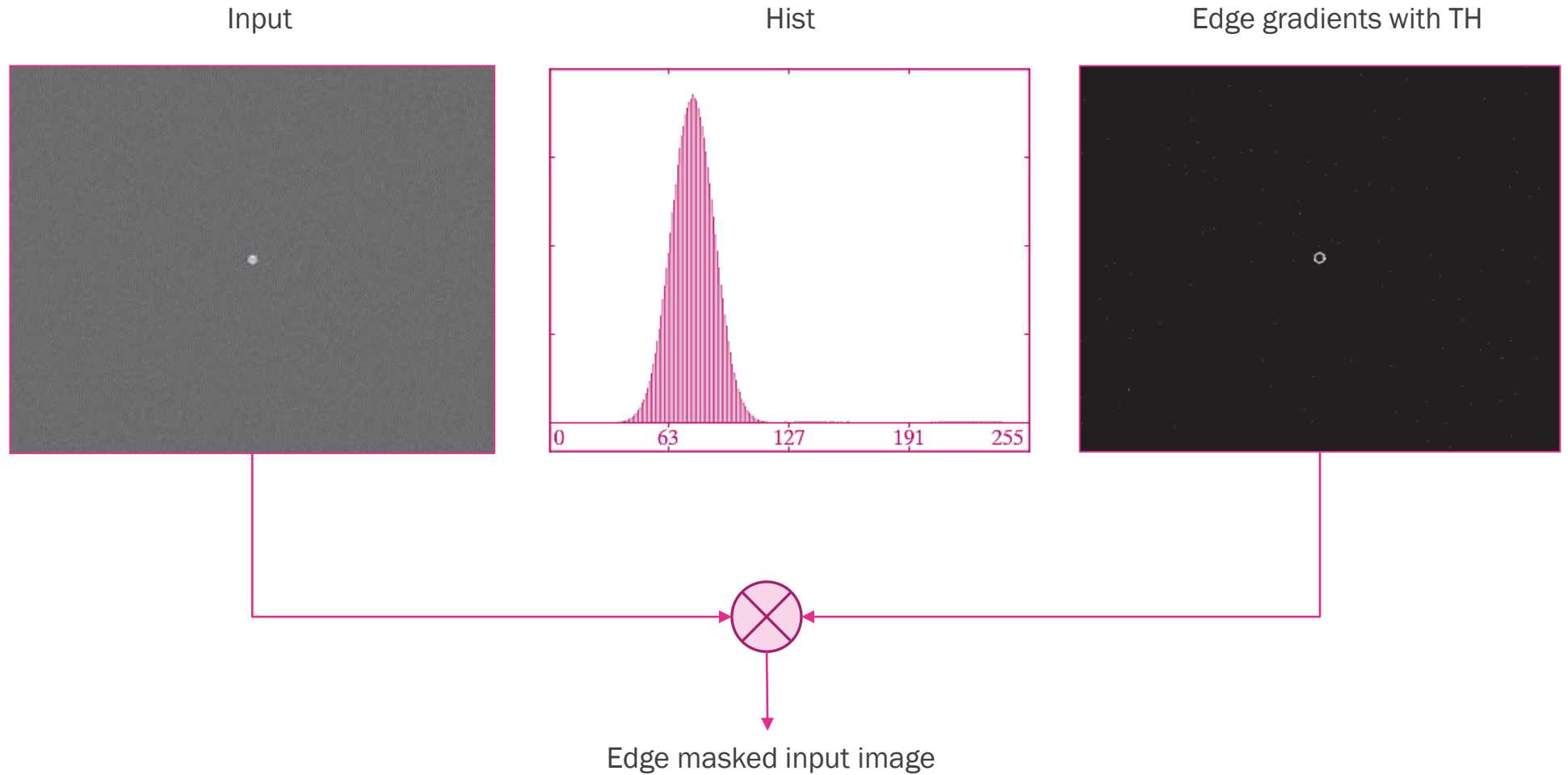
- small object's noisy image
- smoothing degrades the performance
- what caused the problem?
- how to solve the problem?



# Otsu's thresholding

## ■ Example

- small object's noisy image
- edge masks

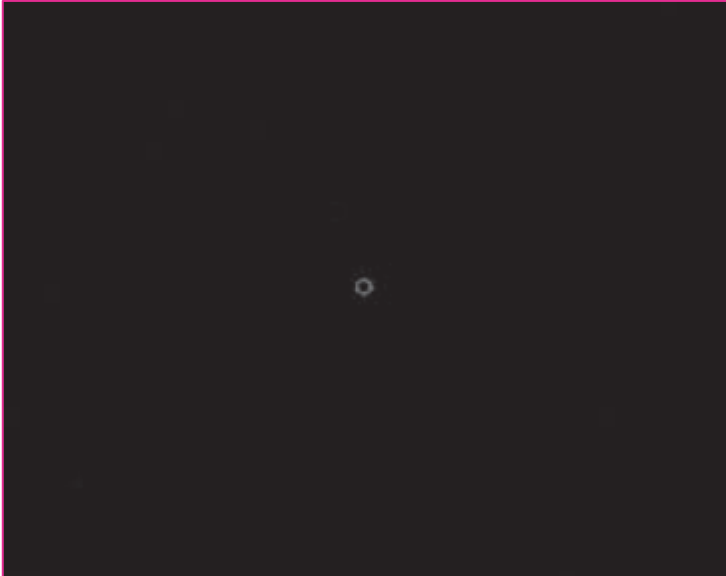


# Otsu's thresholding

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- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

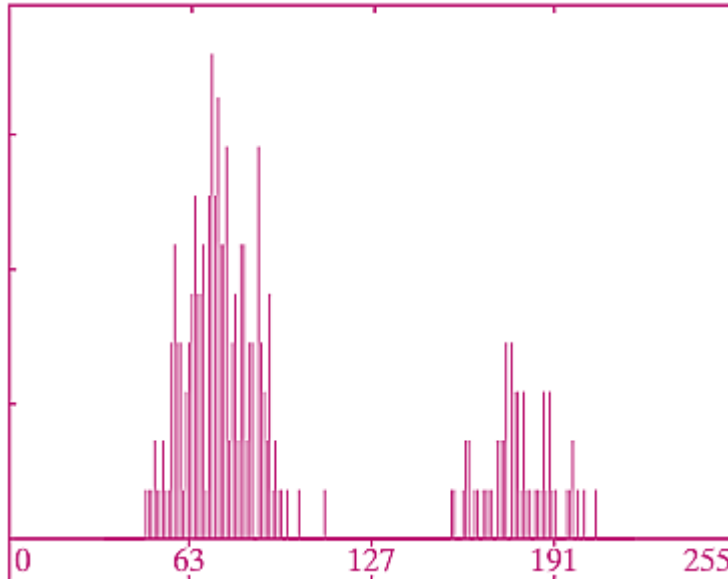
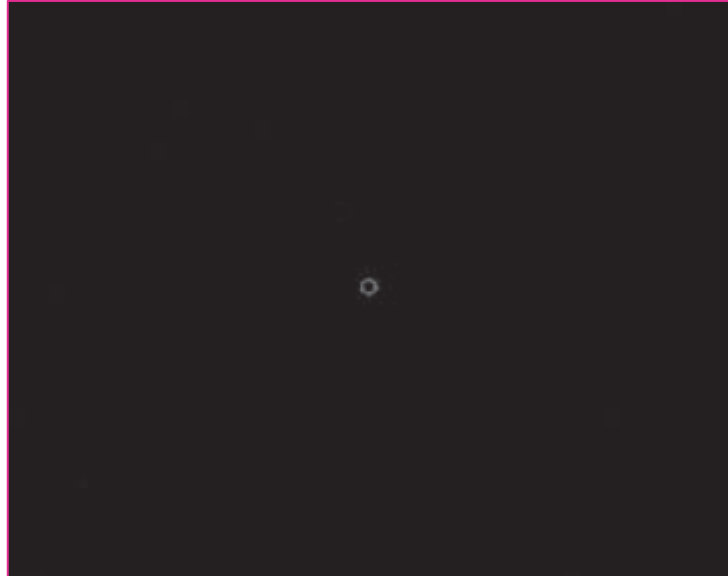
Edge masked input image



# Otsu's thresholding

- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

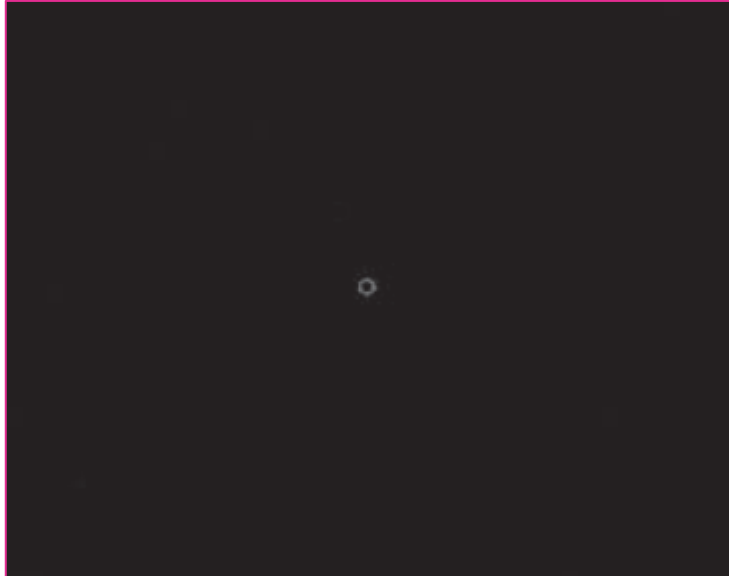
Edge masked input image



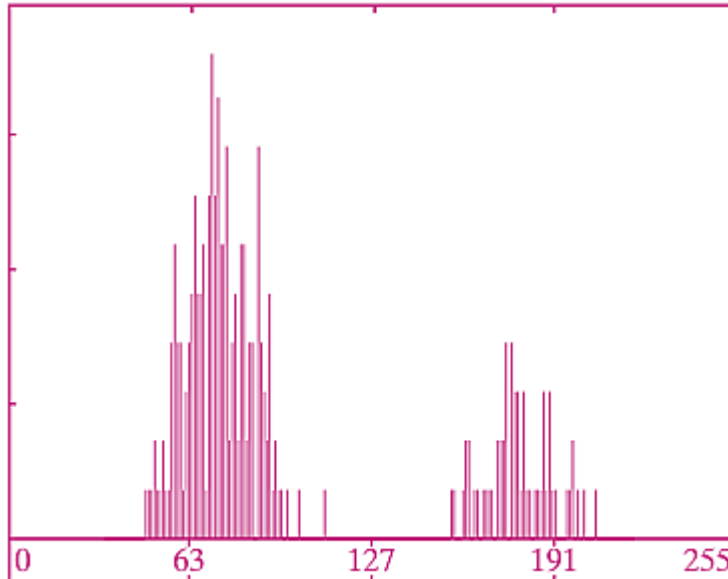
# Otsu's thresholding

- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

Edge masked input image



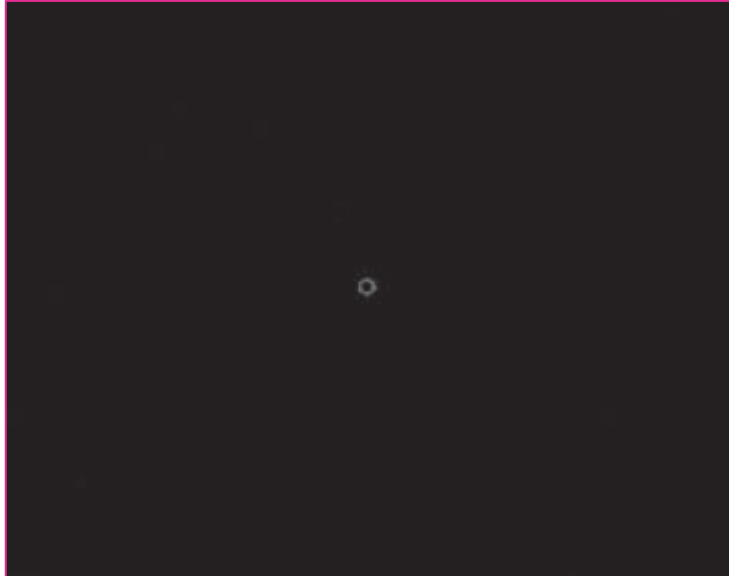
Hist



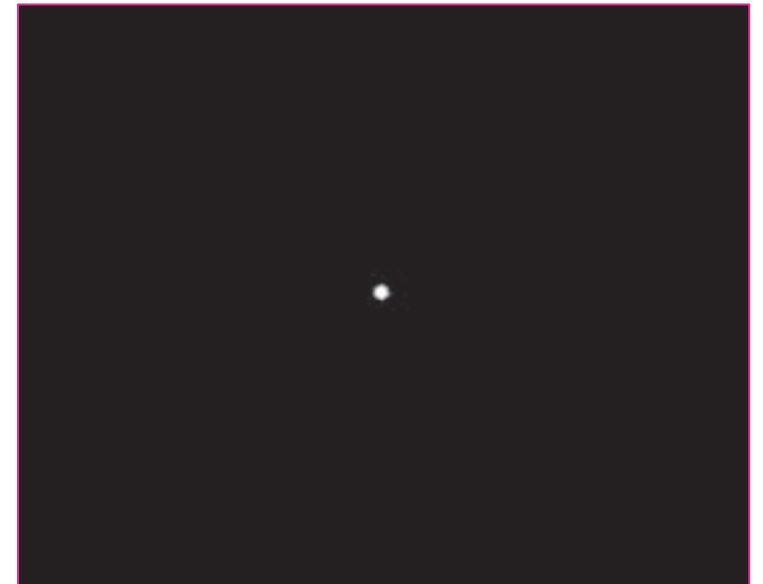
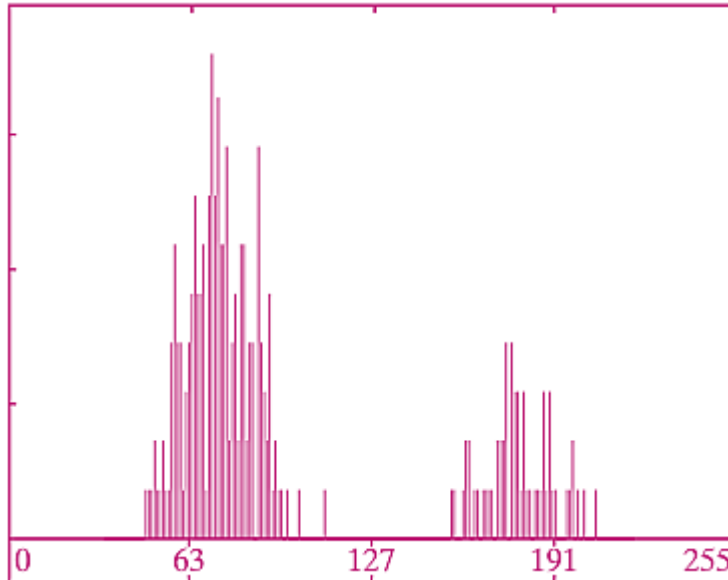
# Otsu's thresholding

- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

Edge masked input image



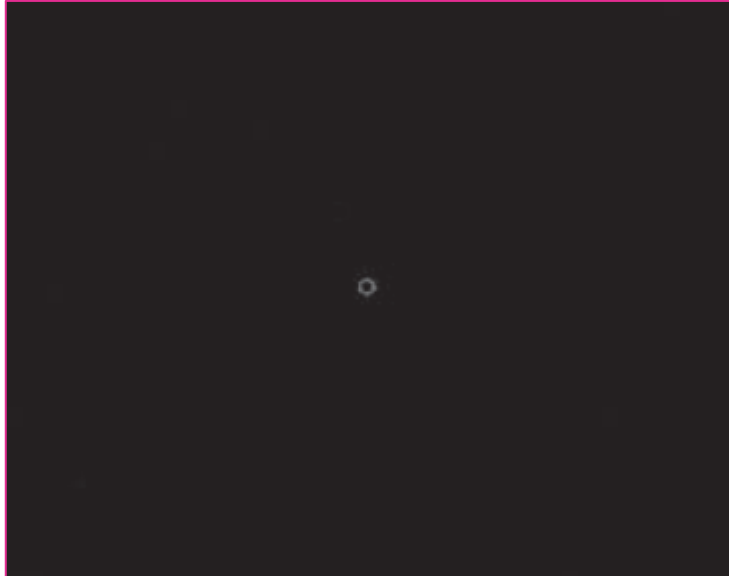
Hist



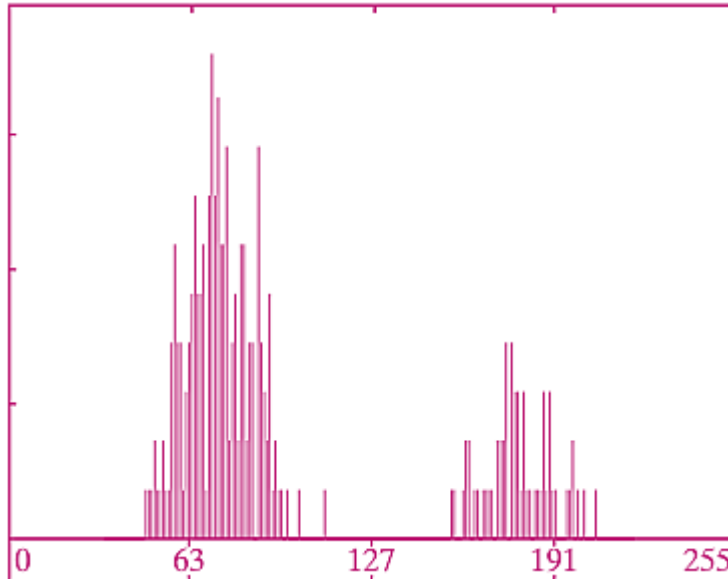
# Otsu's thresholding

- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

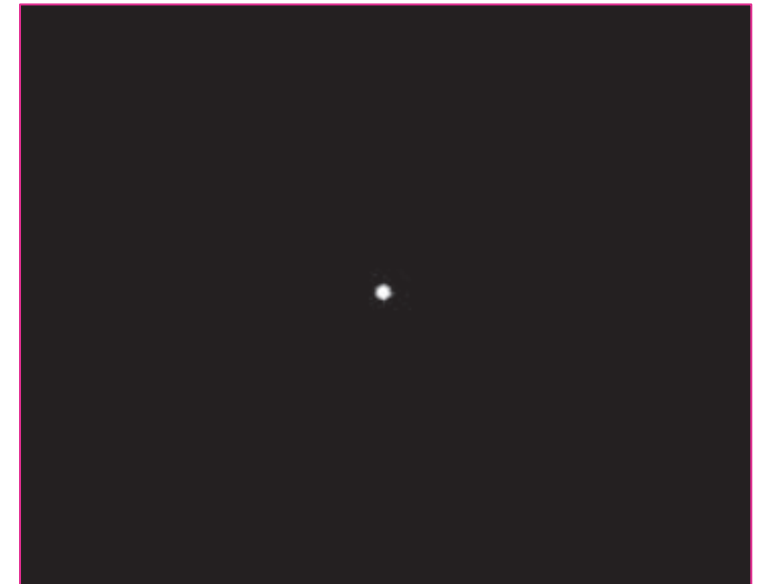
Edge masked input image



Hist



Global: Otsu's TH





# Improving Otsu for underwater imaging

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# Improving Otsu for underwater imaging

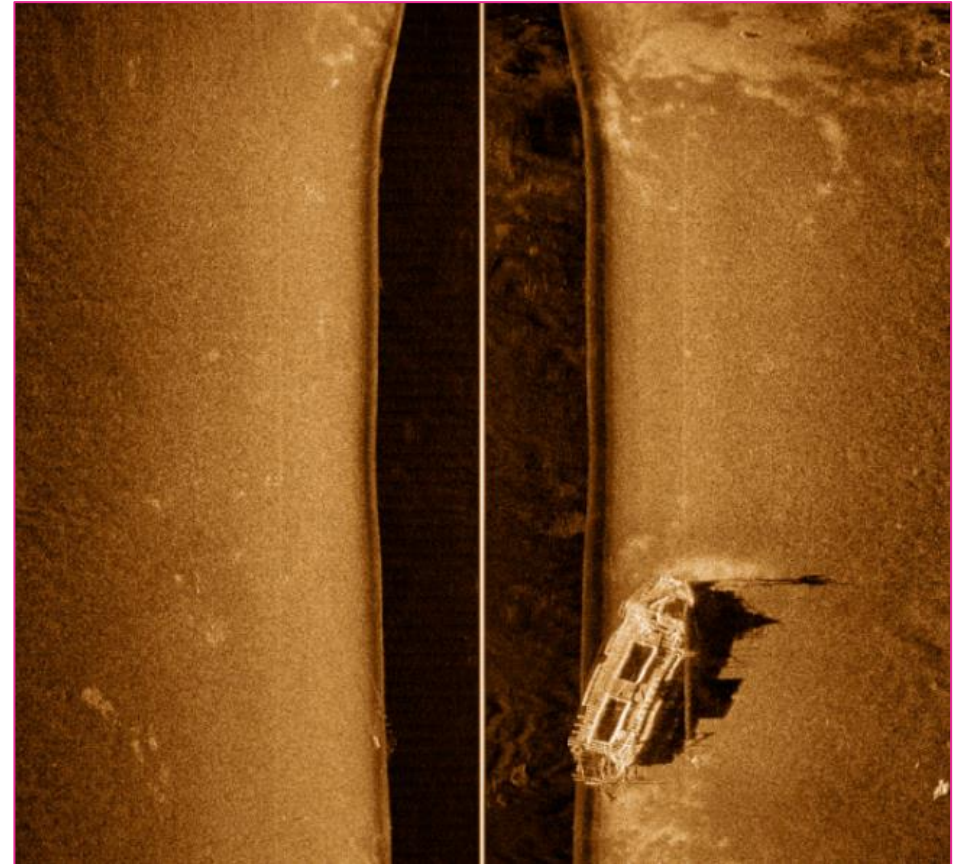
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- SONAR
  - Sound Navigation And Ranging

# Improving Otsu for underwater imaging

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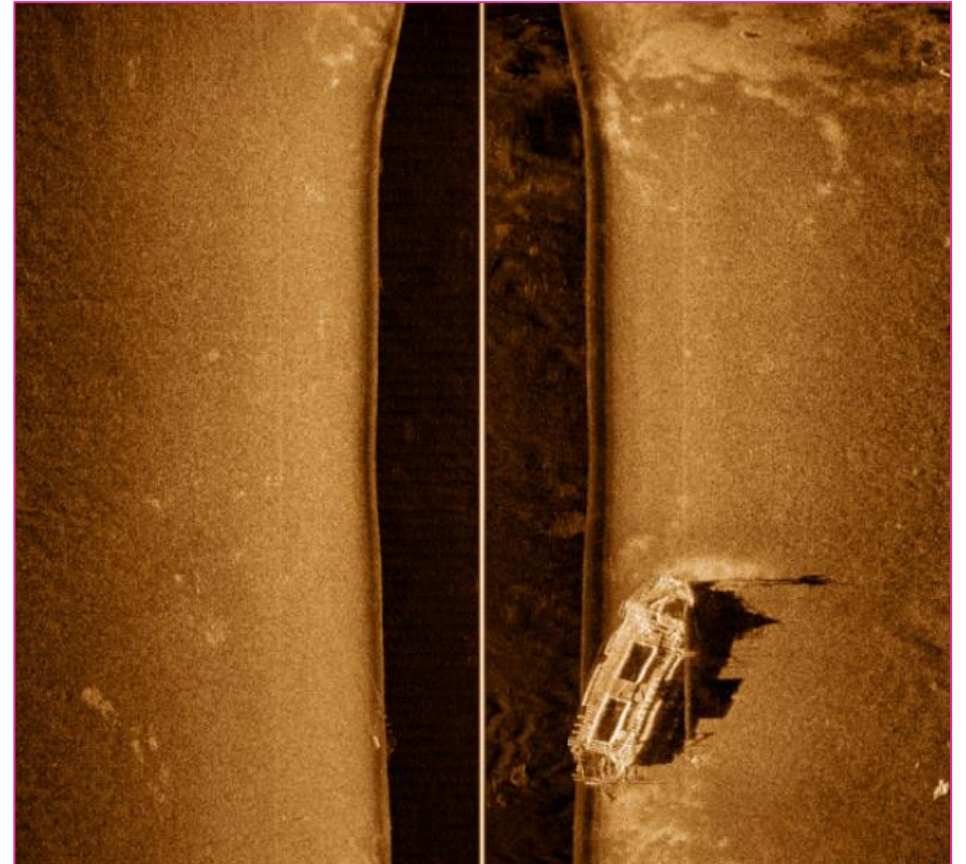
- SONAR
  - Sound Navigation And Ranging



# Improving Otsu for underwater imaging

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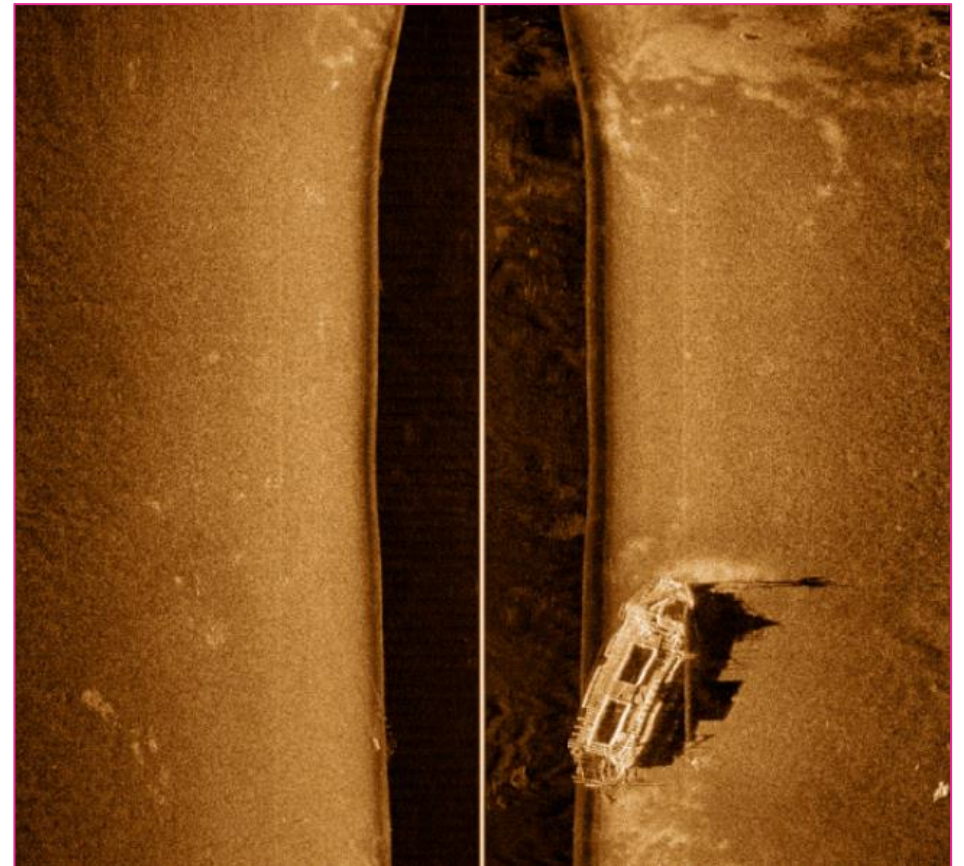
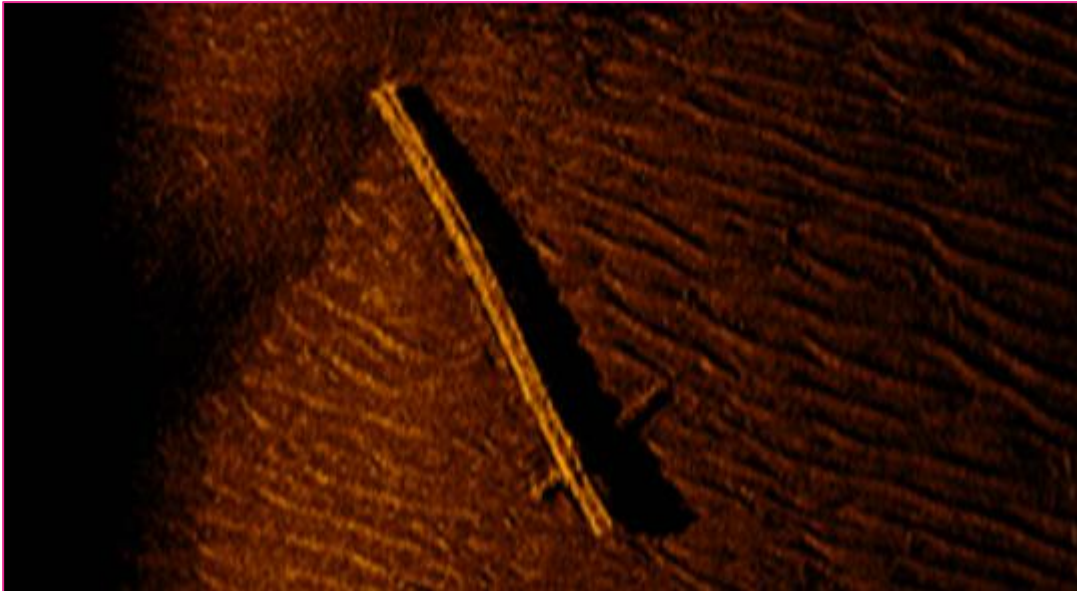
- SONAR
  - Sound Navigation And Ranging



Stockholm sea image

# Improving Otsu for underwater imaging

- SONAR
  - Sound Navigation And Ranging

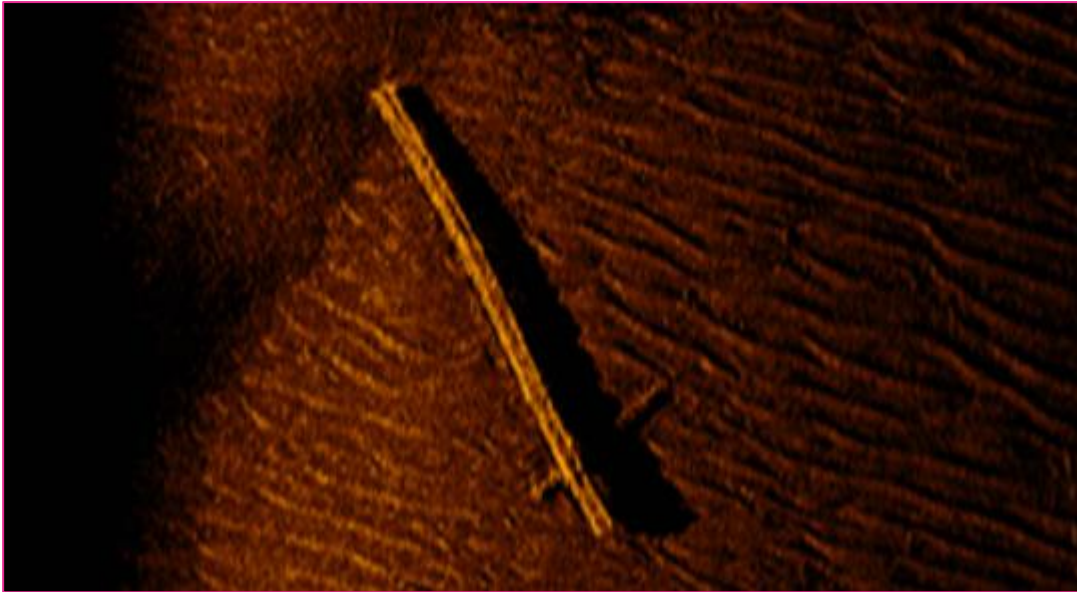


Stockholm sea image

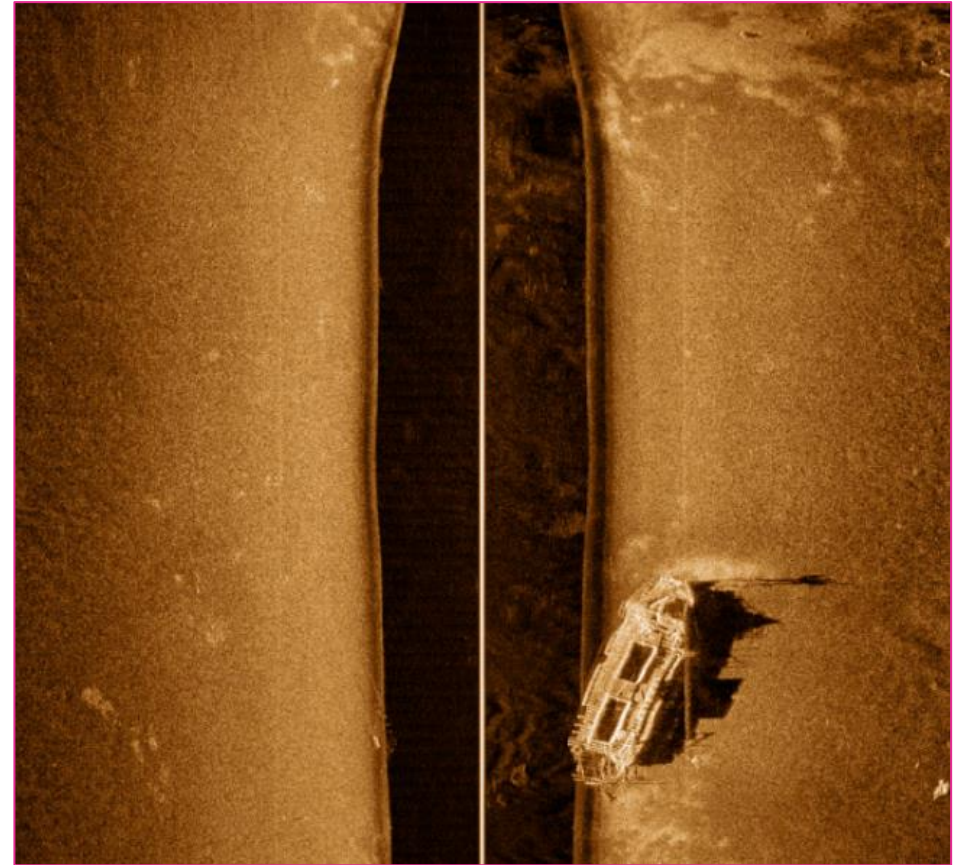


# Improving Otsu for underwater imaging

- SONAR
  - Sound Navigation And Ranging



Underwater plank (by ECA group company)



Stockholm sea image

# Improving Otsu for underwater imaging

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# Improving Otsu for underwater imaging

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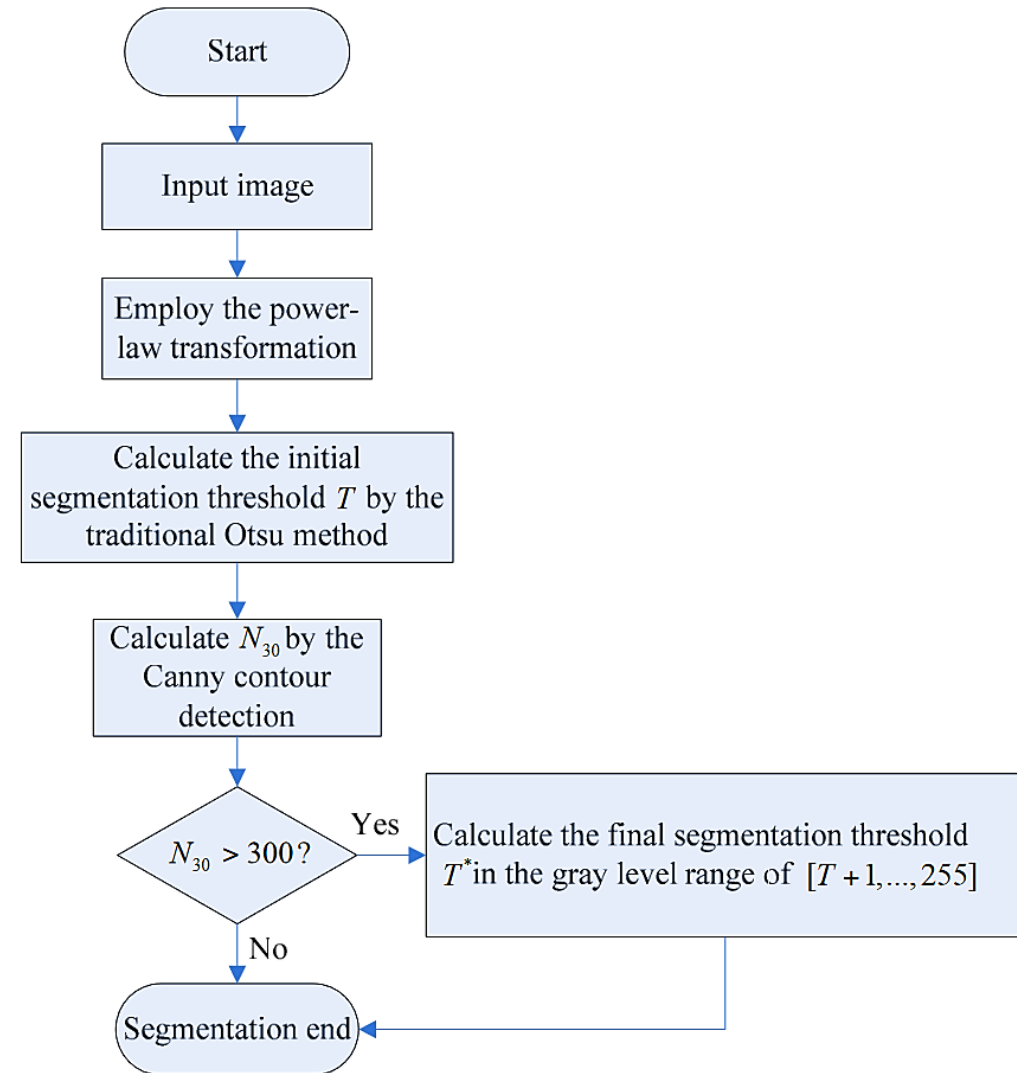
- SONAR
  - Sound Navigation And Ranging



# Improving Otsu for underwater imaging

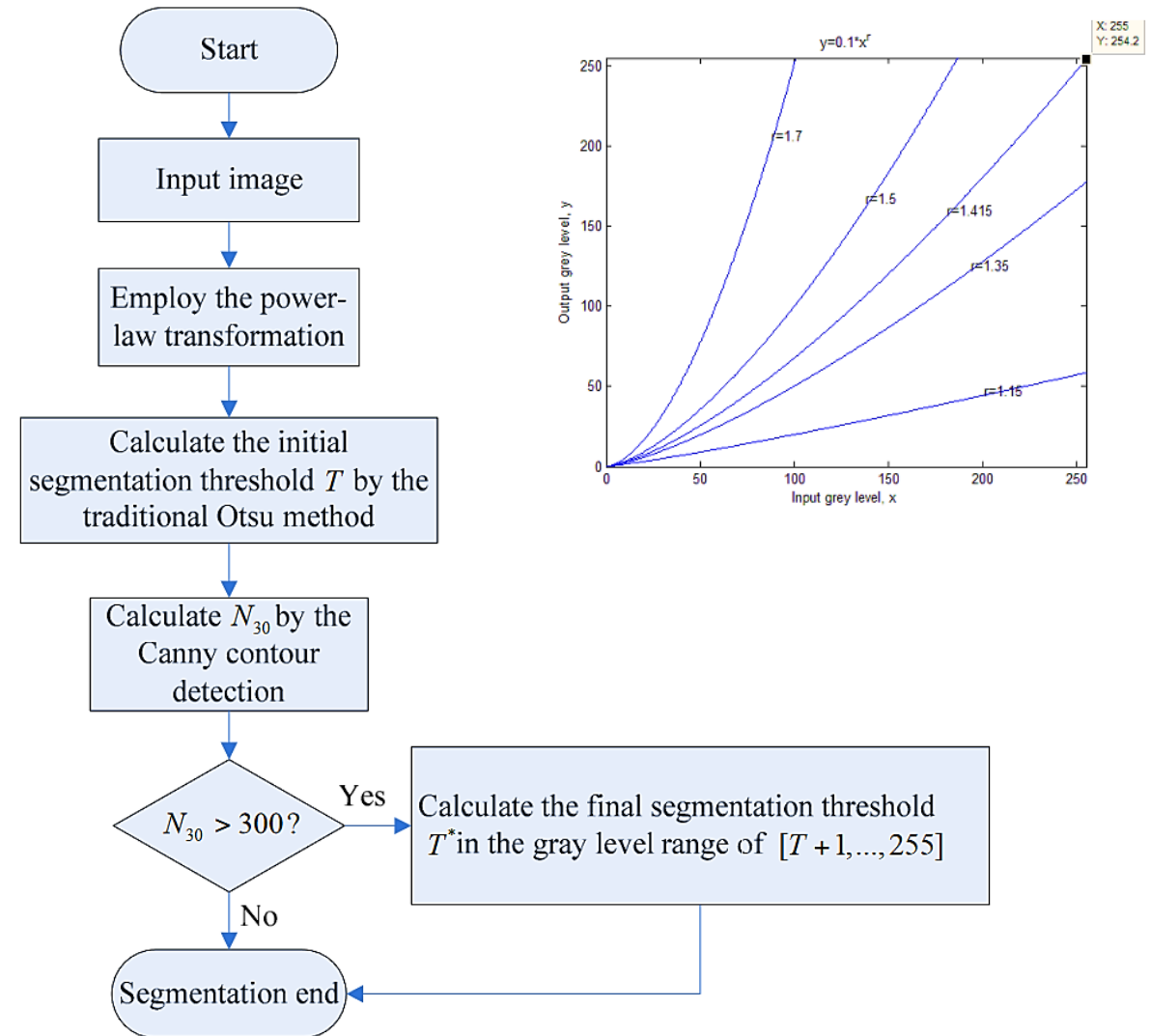
- SONAR

- Sound Navigation And Ranging



# Improving Otsu for underwater imaging

- SONAR
  - Sound Navigation And Ranging

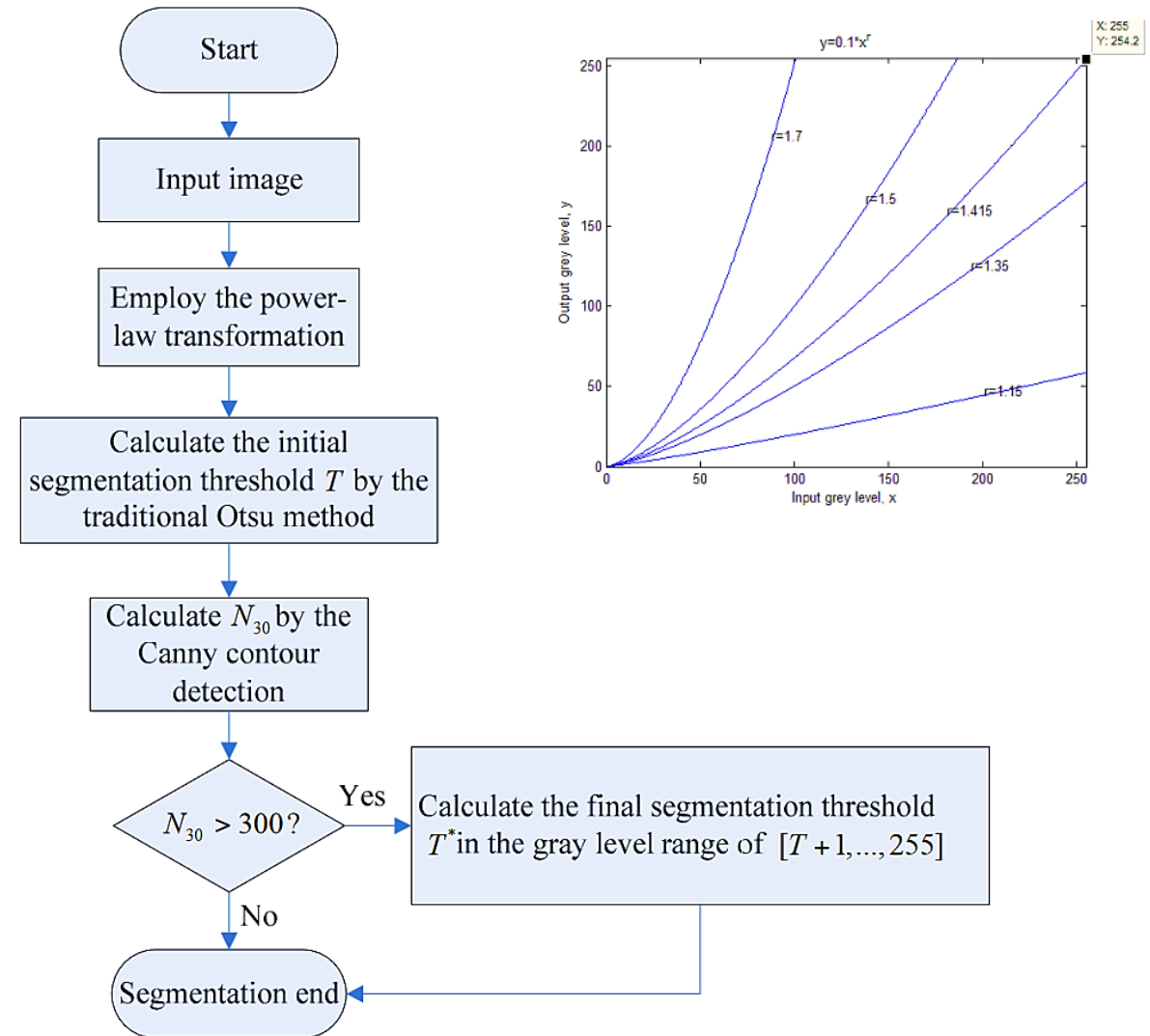


# Improving Otsu for underwater imaging

## ■ SONAR

### ○ Sound Navigation And Ranging

$$T^* = \arg \left\{ \max_{T+1 \leq i \leq 255} \left\{ \sigma_B^2(i) \right\} \right\} = \arg \left\{ \min_{T+1 \leq i \leq 255} \left\{ \sigma_W^2(i) \right\} \right\}$$



# Improving Otsu for underwater imaging

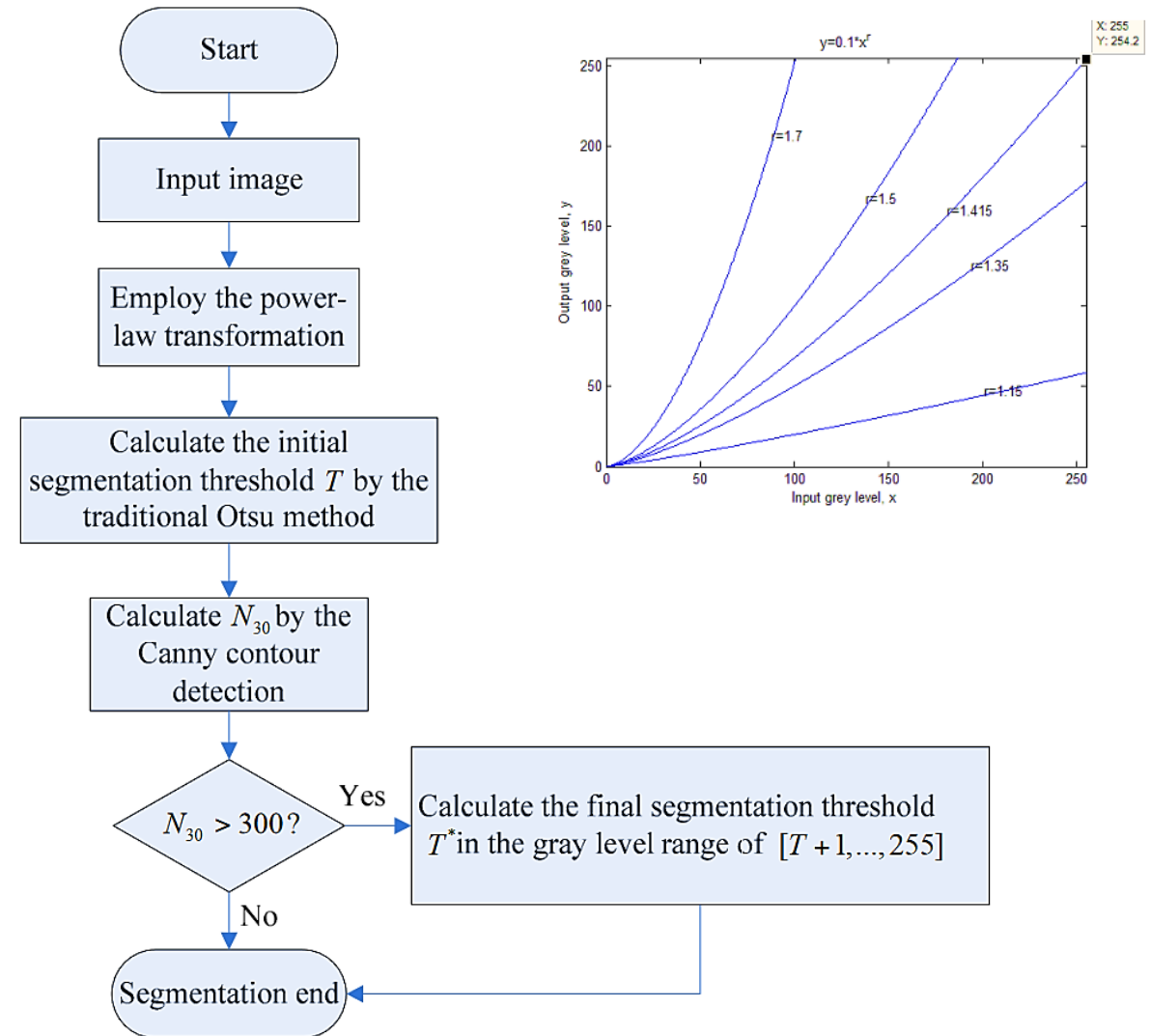
## ■ SONAR

### ○ Sound Navigation And Ranging

$$T^* = \arg \left\{ \max_{T+1 \leq i \leq 255} \left\{ \sigma_B^2(i) \right\} \right\} = \arg \left\{ \min_{T+1 \leq i \leq 255} \left\{ \sigma_W^2(i) \right\} \right\}$$

between-class variance  $\sigma_B^2$

within-class variance  $\sigma_W^2$



# Improving Otsu for underwater imaging

## ■ SONAR

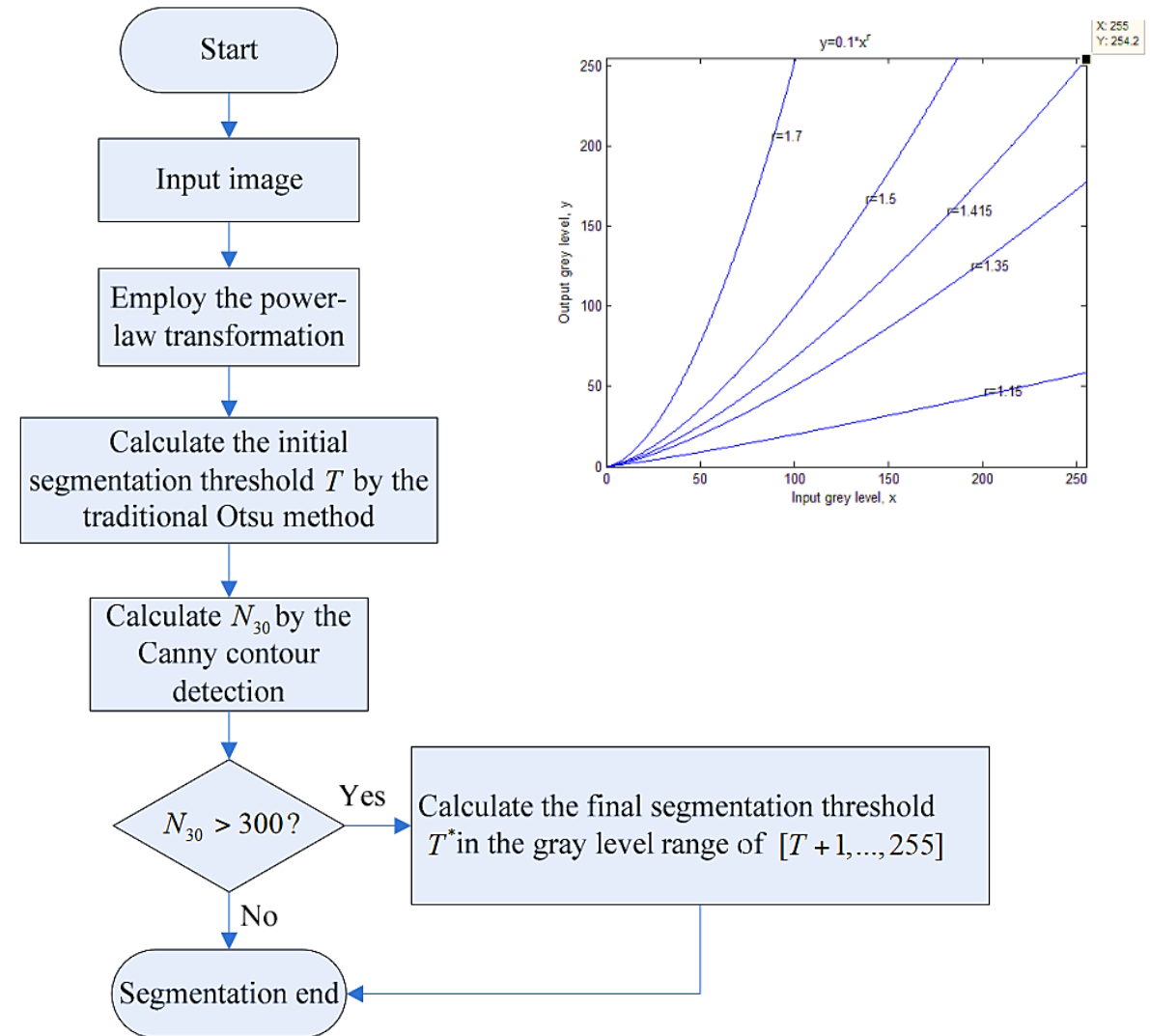
### ○ Sound Navigation And Ranging

$$T^* = \arg \left\{ \max_{T+1 \leq i \leq 255} \left\{ \sigma_B^2(i) \right\} \right\} = \arg \left\{ \min_{T+1 \leq i \leq 255} \left\{ \sigma_W^2(i) \right\} \right\}$$

between-class variance  $\sigma_B^2$

within-class variance  $\sigma_W^2$

- area size of the background spots is < 30 pixels
- $N_{30}$ : # contours to be found with an area of 30 pixels



# Improving Otsu for underwater imaging

## ■ SONAR

### ○ Sound Navigation And Ranging

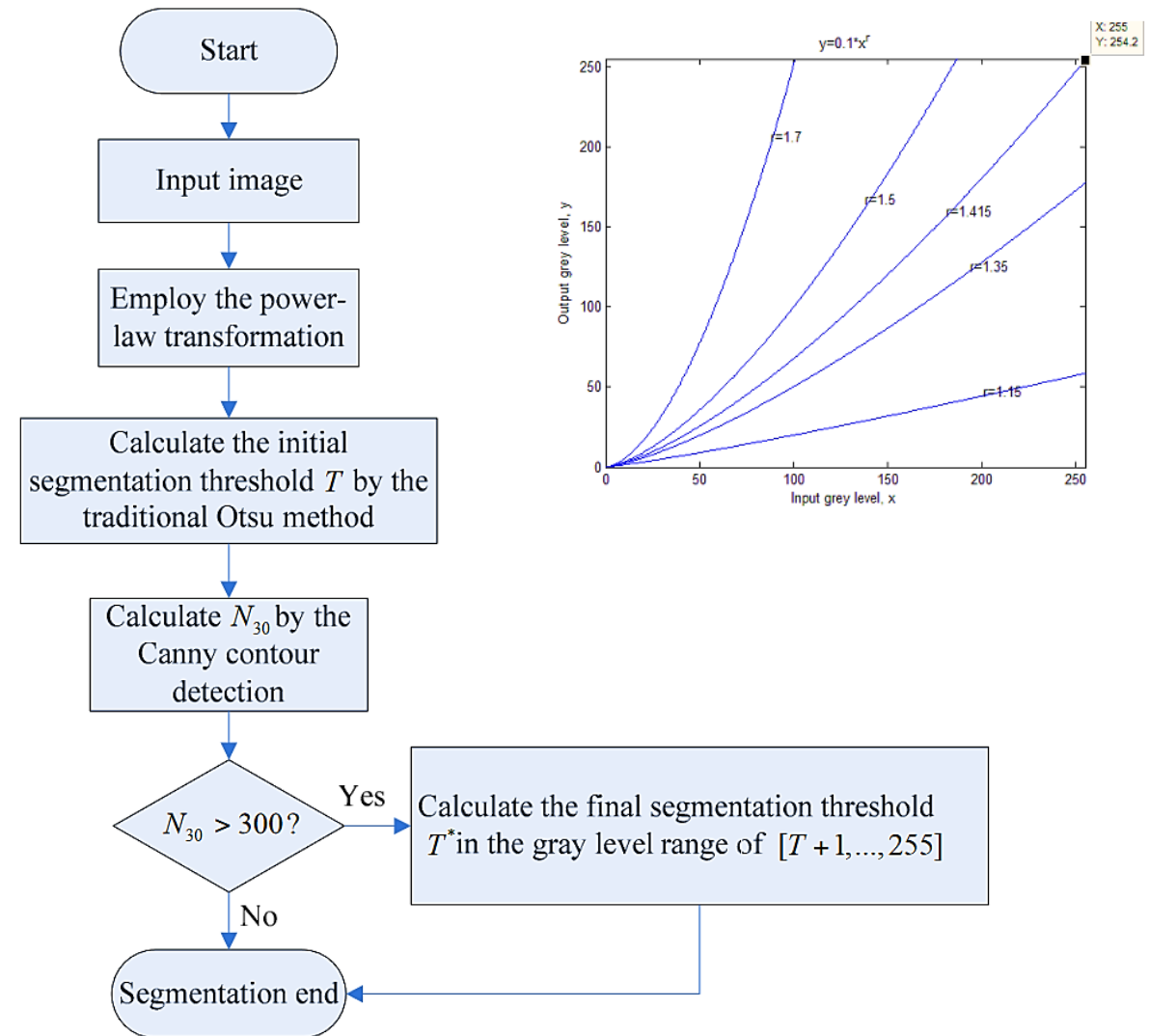
$$T^* = \arg \left\{ \max_{T+1 \leq i \leq 255} \left\{ \sigma_B^2(i) \right\} \right\} = \arg \left\{ \min_{T+1 \leq i \leq 255} \left\{ \sigma_W^2(i) \right\} \right\}$$

between-class variance  $\sigma_B^2$

within-class variance  $\sigma_W^2$

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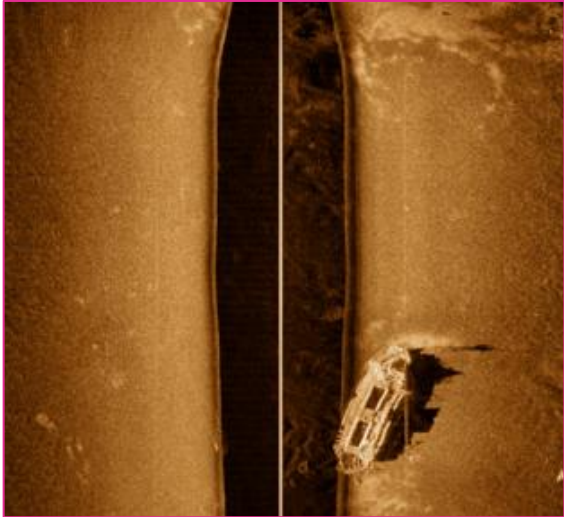
X. Yuan et al. "An Improved Otsu Threshold Segmentation Method for Underwater SLAM Navigation", Sensors, 2016



# Improving Otsu for underwater imaging

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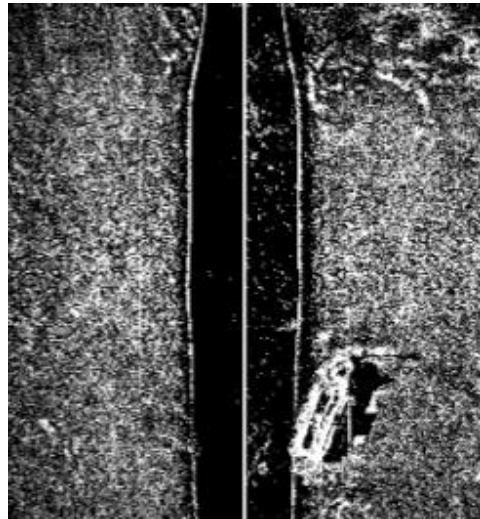
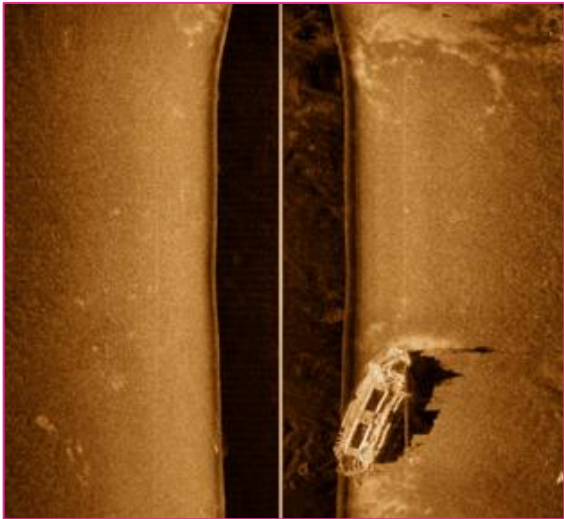
Input



# Improving Otsu for underwater imaging

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Input

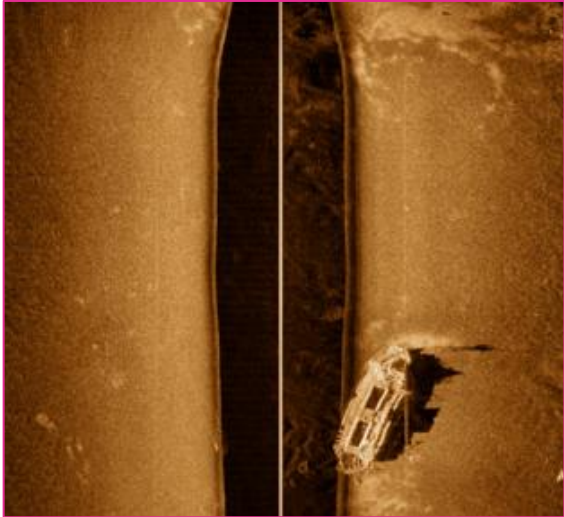




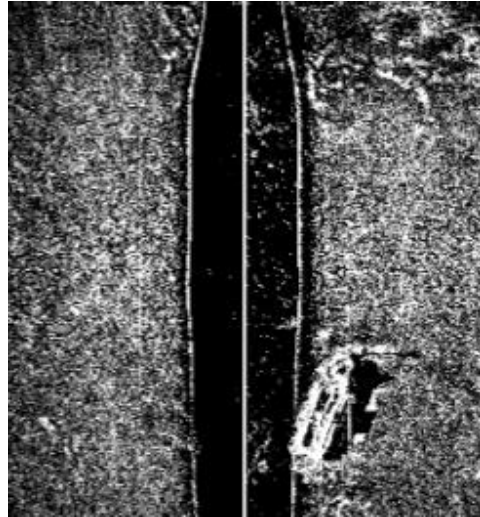
# Improving Otsu for underwater imaging

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Input



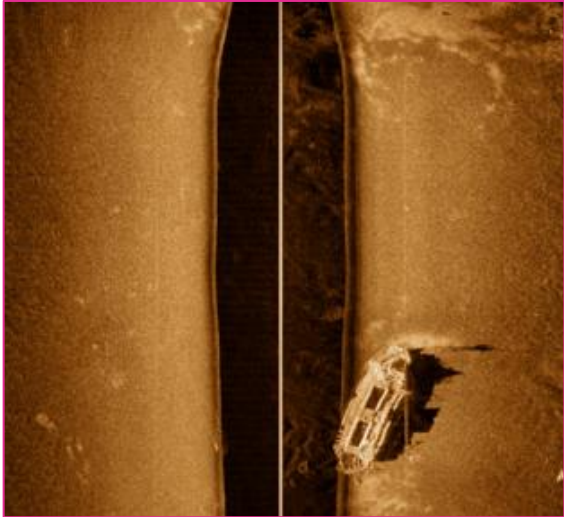
Local TH



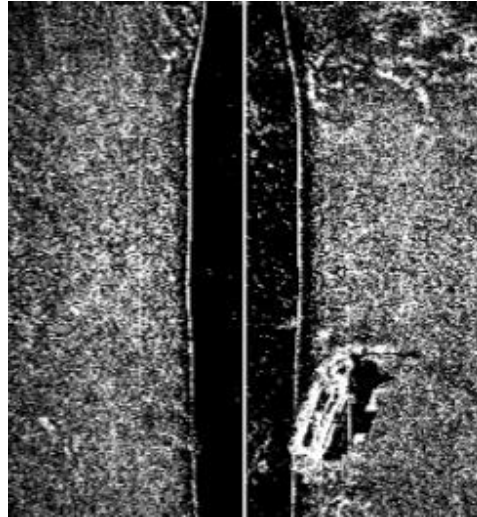
# Improving Otsu for underwater imaging

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Input



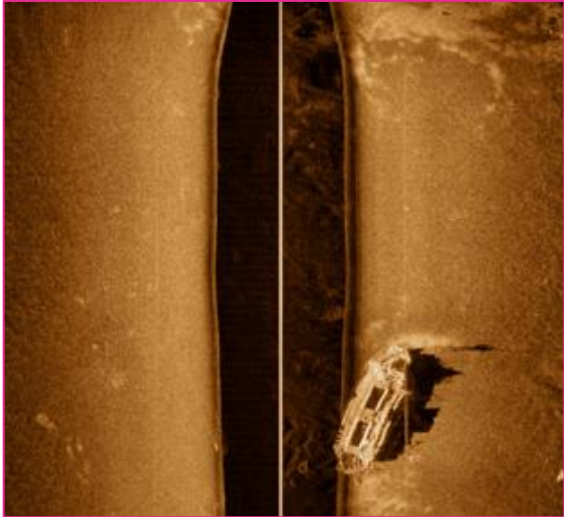
Local TH



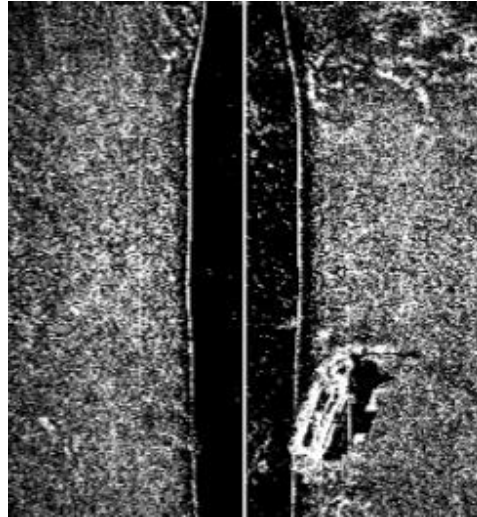
# Improving Otsu for underwater imaging

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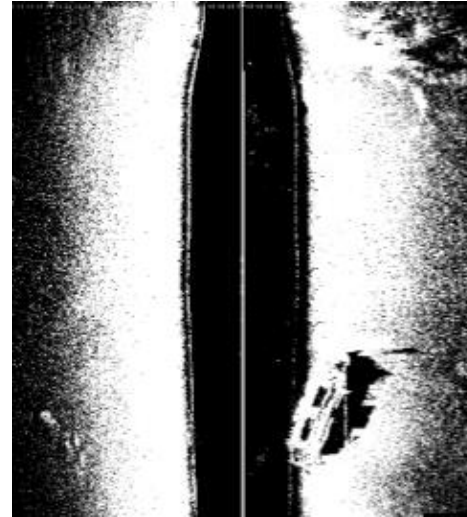
Input



Local TH



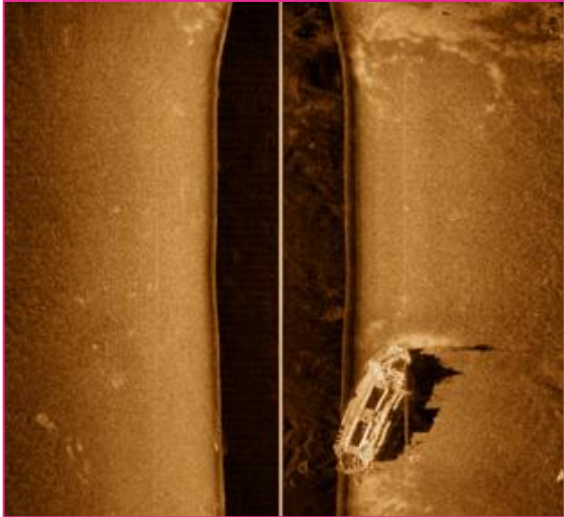
Otsu



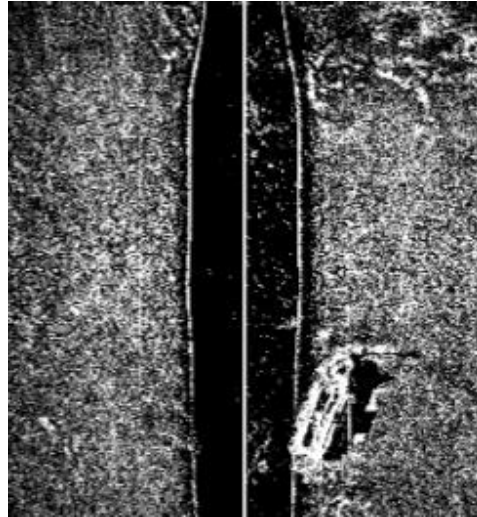
# Improving Otsu for underwater imaging

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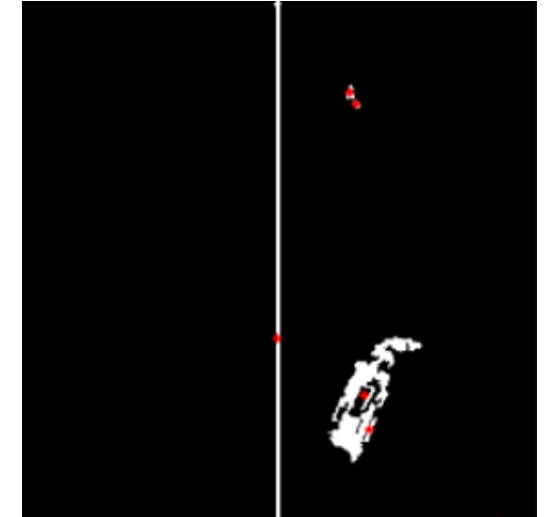
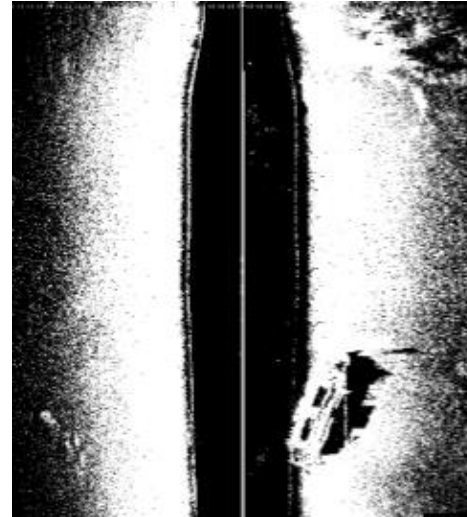
Input



Local TH



Otsu

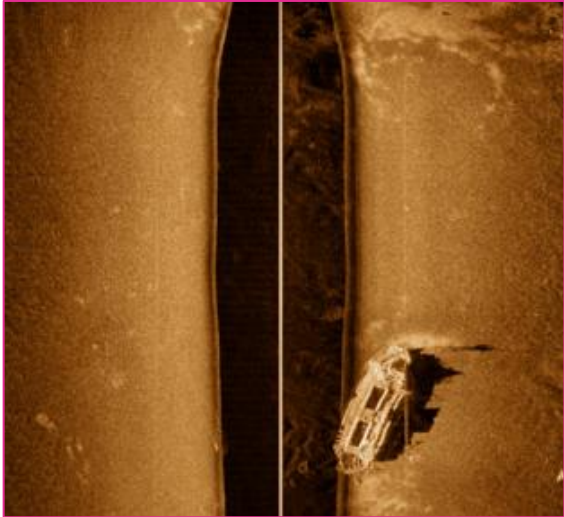




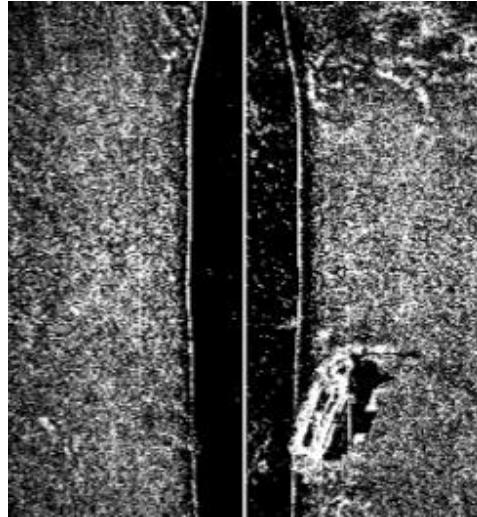
# Improving Otsu for underwater imaging

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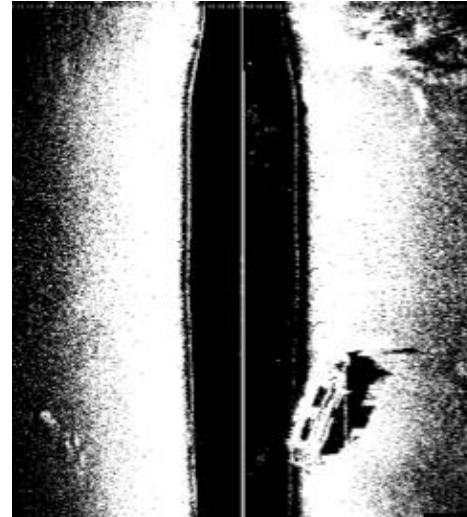
Input



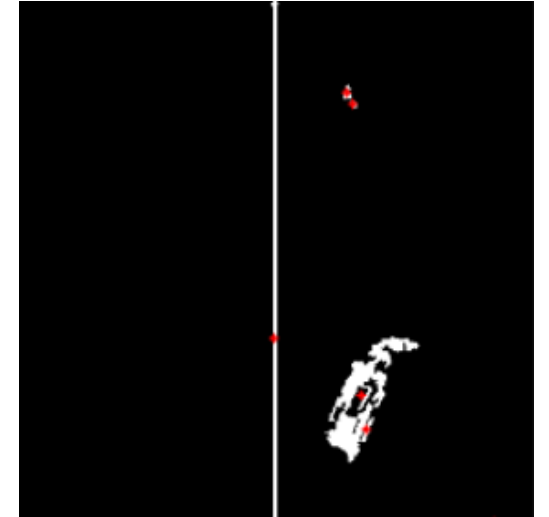
Local TH



Otsu

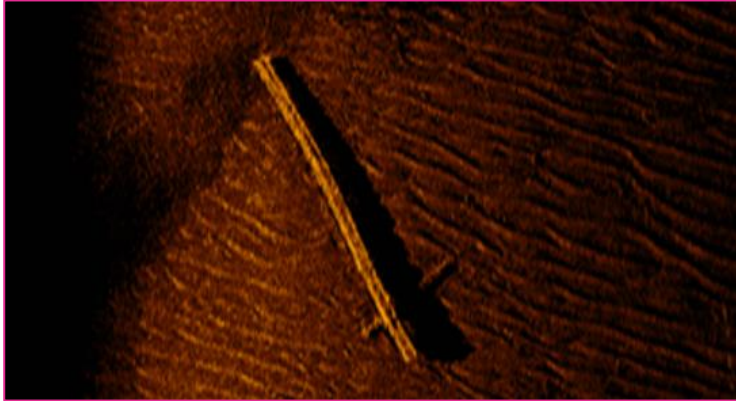


Underwater Otsu



# Improving Otsu for underwater imaging

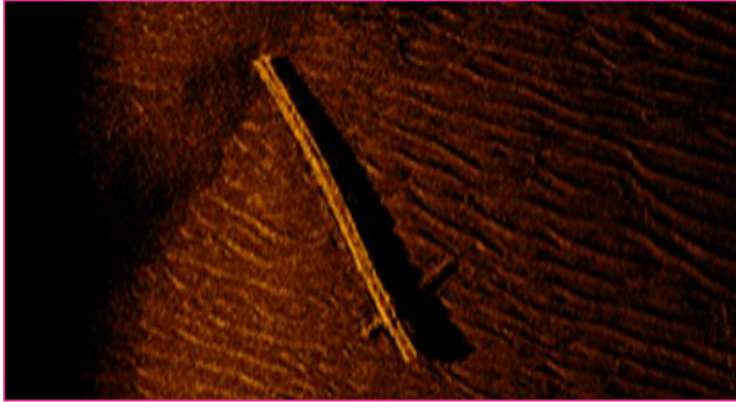
---



# Improving Otsu for underwater imaging

---

Input

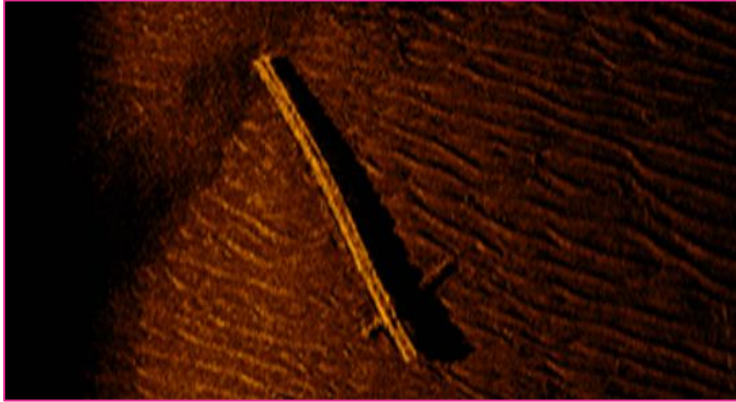




# Improving Otsu for underwater imaging

---

Input



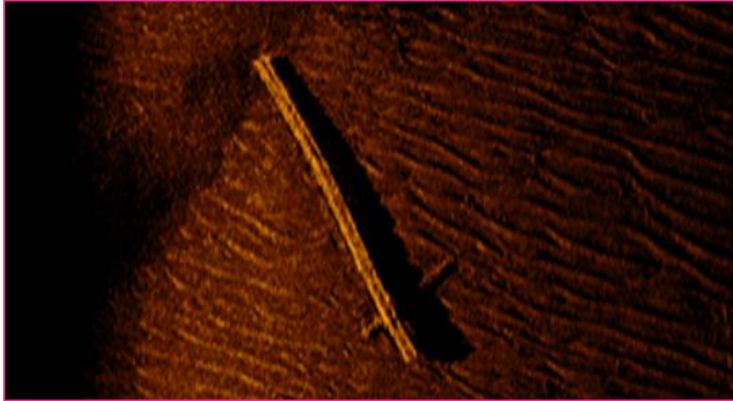
Local TH



# Improving Otsu for underwater imaging

---

Input



Local TH



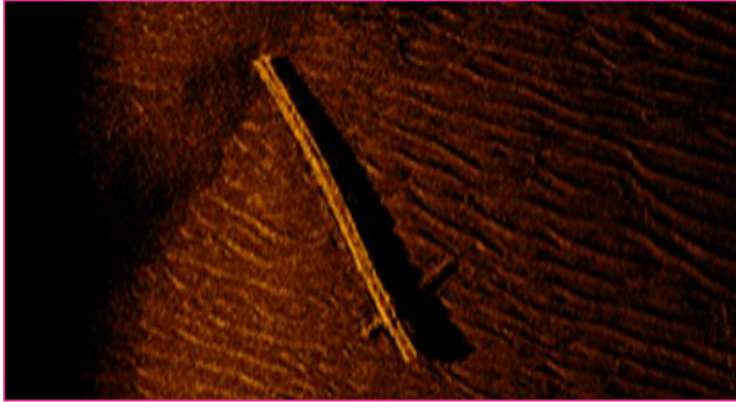
Otsu



# Improving Otsu for underwater imaging

---

Input



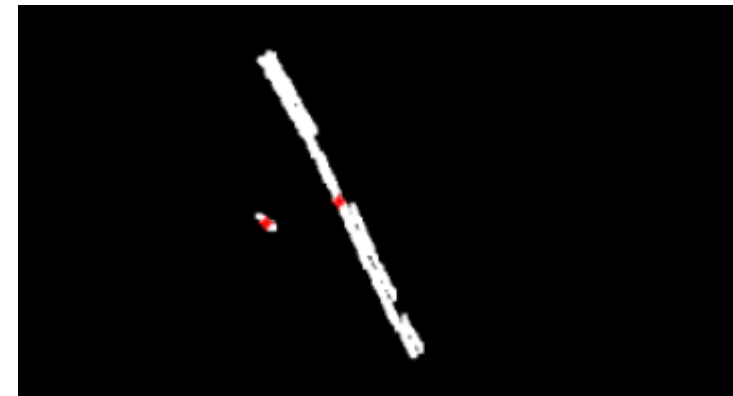
Local TH



Otsu



Underwater Otsu



# Conclusion

- Segmentation via thresholding (Otsu)

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❑ Global optimal

❑ Global Otsu's method

- Input image histogram processing
- Noise handled via smoothing
- Small object issues handled via edge masks

# Conclusion

- Segmentation via thresholding (Otsu)

- ❑ Global optimal
- ❑ Global Otsu's method
  - Input image histogram processing
  - Noise handled via smoothing
  - Small object issues handled via edge masks

## A Threshold Selection Method from Gray-Level Histograms

NOBUYUKI OTSU

**Abstract**—A nonparametric and unsupervised method of automatic threshold selection for picture segmentation is presented. An optimal threshold is selected by the discriminant criterion, namely, so as to maximize the separability of the resultant classes in gray levels. The procedure is very simple, utilizing only the zeroth- and the first-order cumulative moments of the gray-level histogram. It is straightforward to extend the method to multithreshold problems. Several experimental results are also presented to support the validity of the method.

### I. INTRODUCTION

It is important in picture processing to select an adequate threshold of gray level for extracting objects from their background. A



# Conclusion

- Segmentation via thresholding (Otsu)

❑ Global optimal

❑ Global Otsu's method

- Input image histogram processing
- Noise handled via smoothing
- Small object issues handled via edge masks

Threshold the Otsu's paper via Otsu's method:

