Image Enhancement:

Spatial domain

Dr. Tushar Sandhan

Introduction

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- Intensity transformations

- Distribution transformation

Spatial domain enhancements

- Transformations
  - intensity transformations
    - negatives
    - logs
    - power-law (gamma)
    - contrast stretching
    - level slicing
    - bit-plane slicing
  - distribution transformations
    - histogram equalization
- Spatial filtering
  - image filtering

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- Spatial filtering
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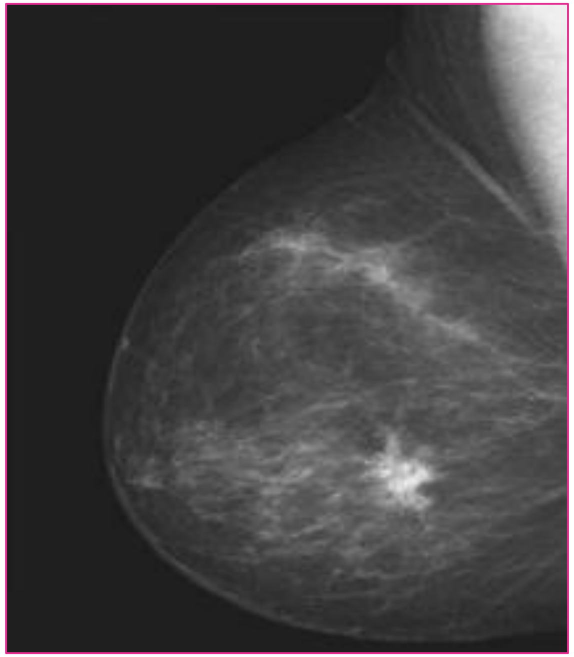

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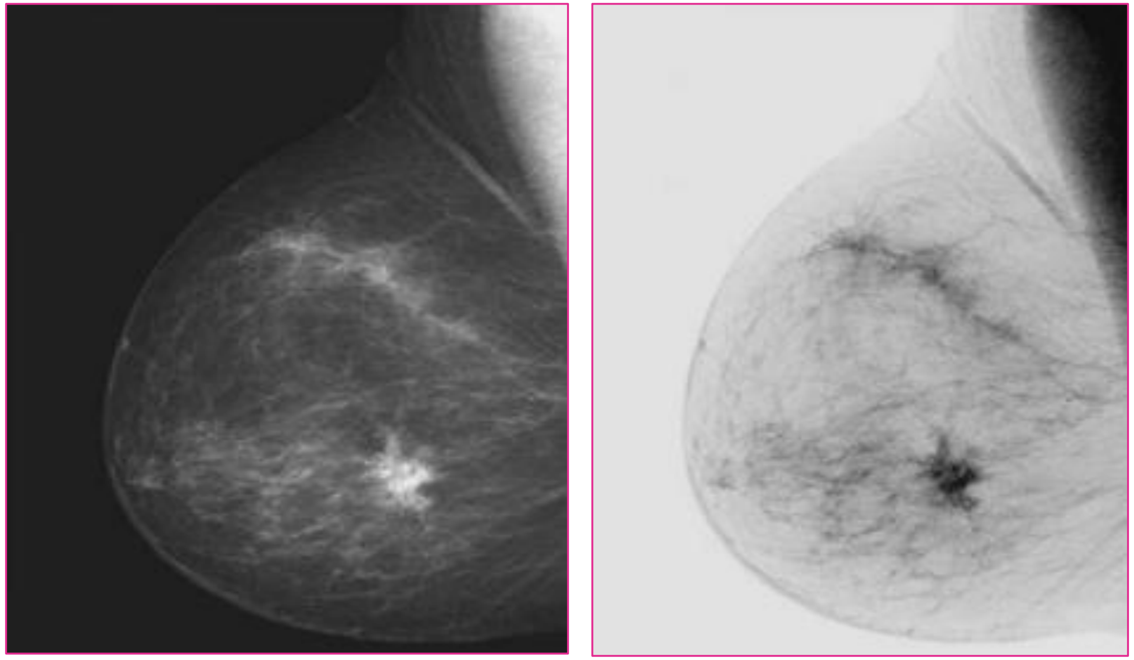

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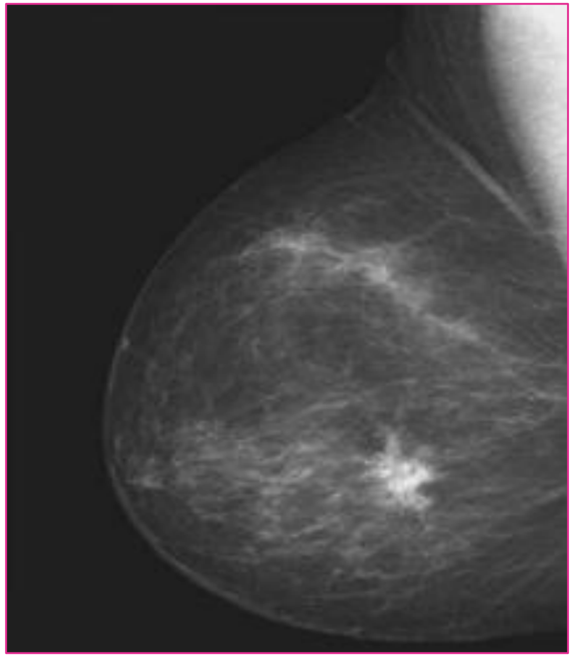
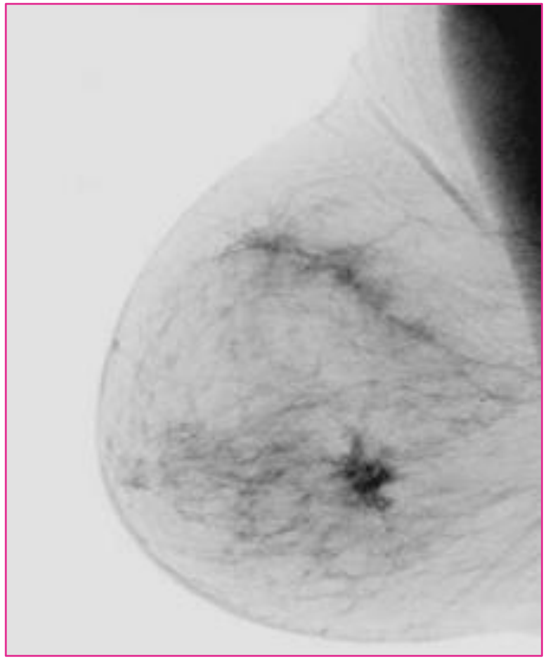

Logs

$$s = c \cdot log(1 + r)$$

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  - used to expand values of dark pixels
  - simultaneously compressing bright pixels
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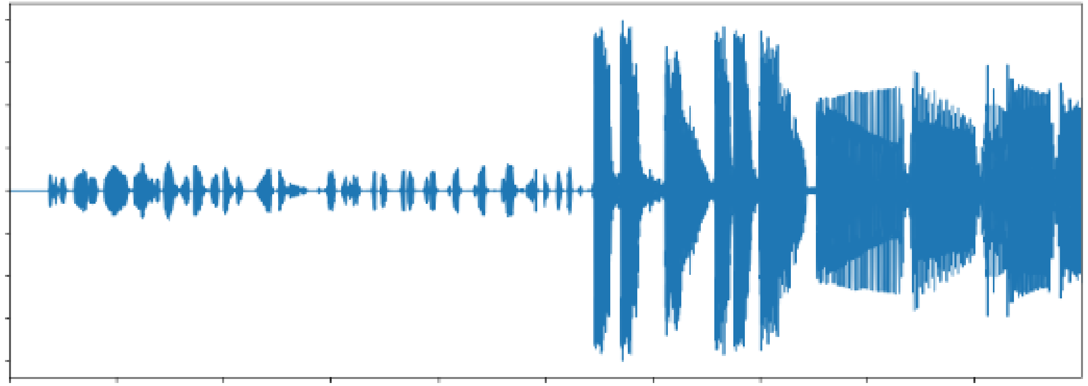

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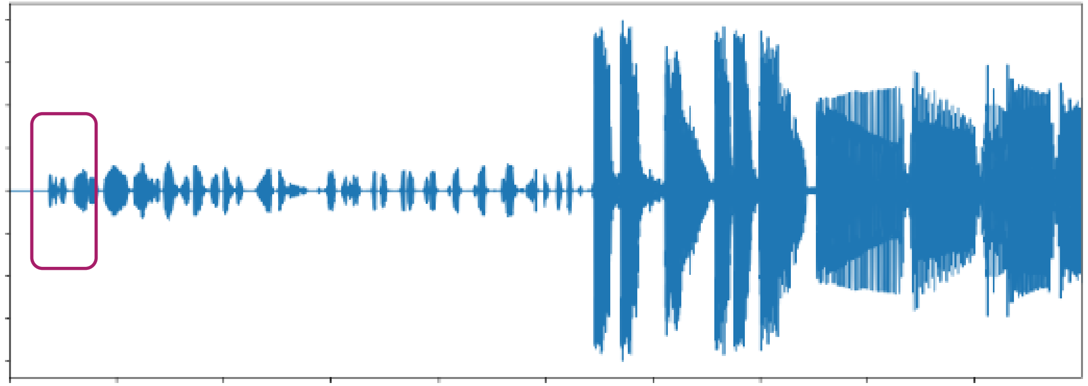

sandhan@iitk.ac.in

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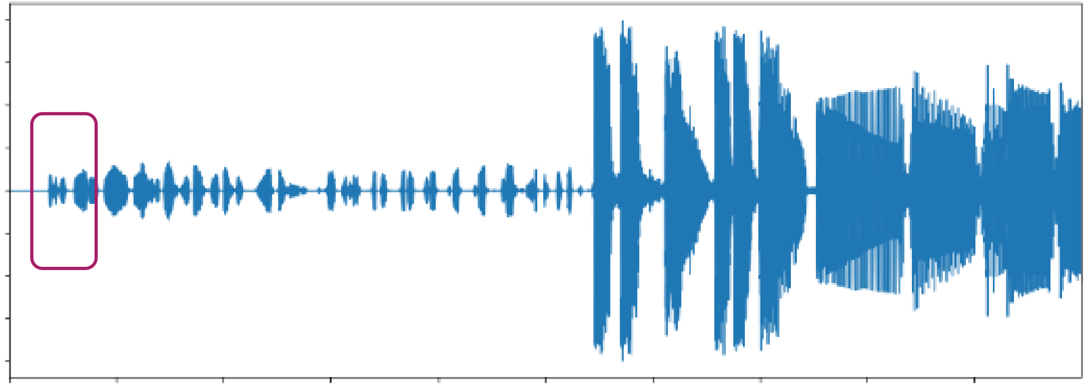

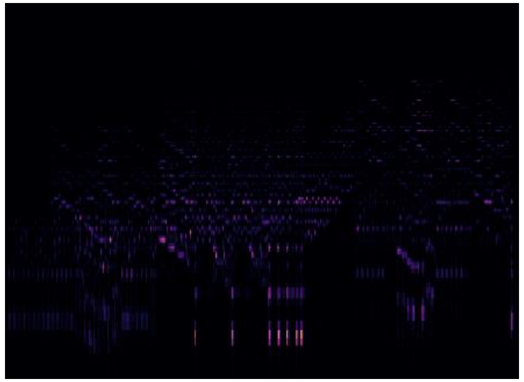

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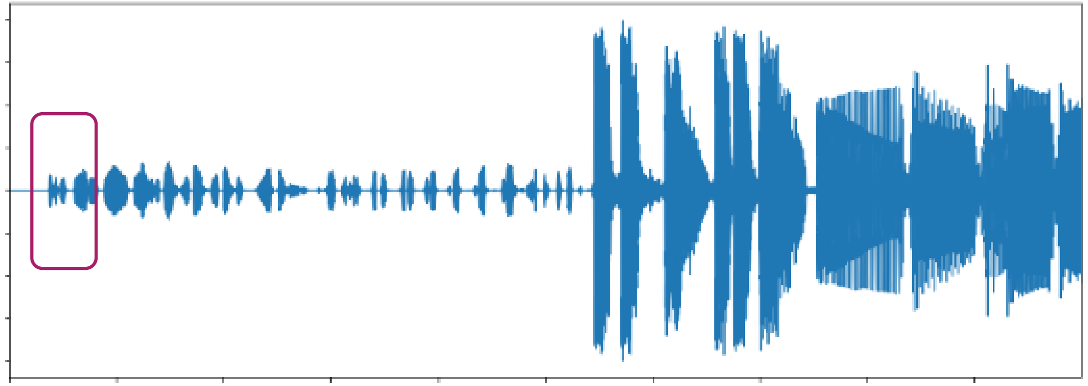

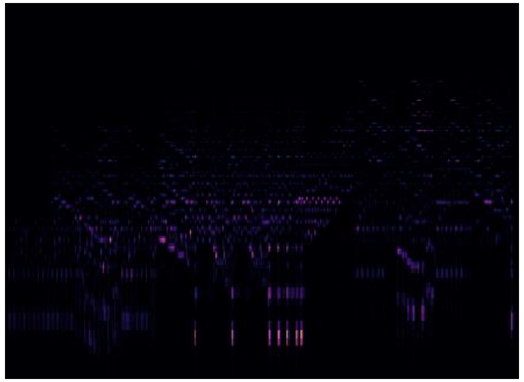

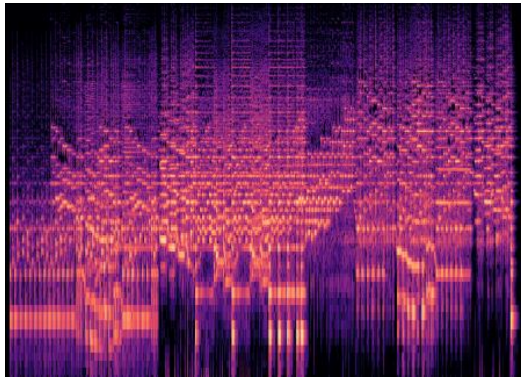

Gammas

$$\boxed{s = c \cdot r^{\gamma}}$$

- Power-law transformations
  - sensors respond according to power law
    - CMOS, scanners, printing, displays
    - CRT: intensity to voltage response as power function ($\gamma' = 1.8$~2.5)
  - gamma correction
    - device dependent γ
    - γ variation also varies the color ratios
    - correct color reproduction needs knowledge of γ
  - gamma injection
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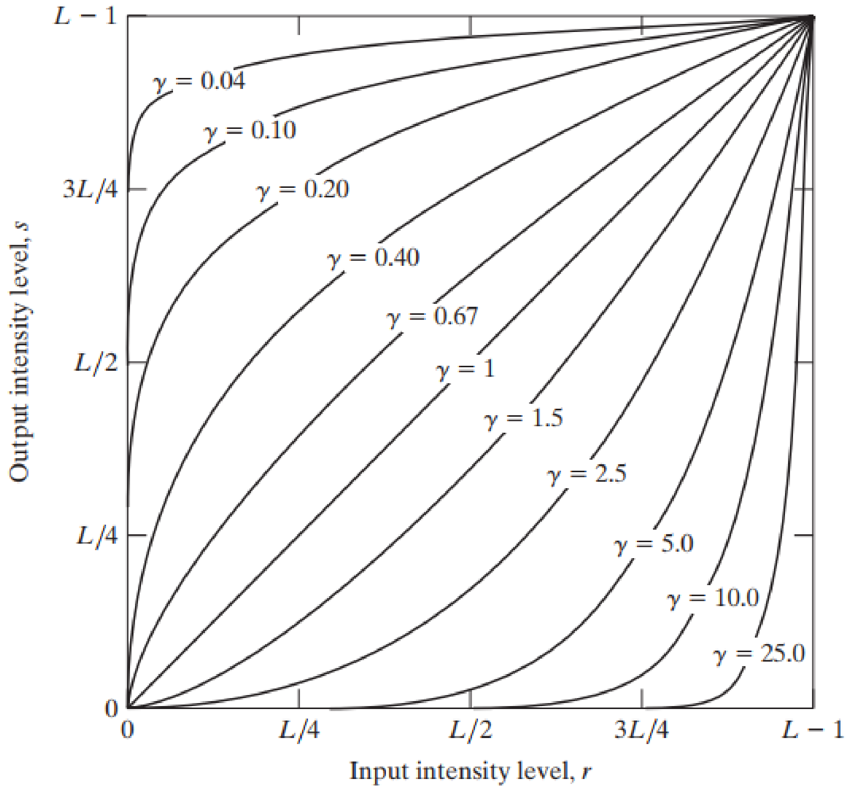

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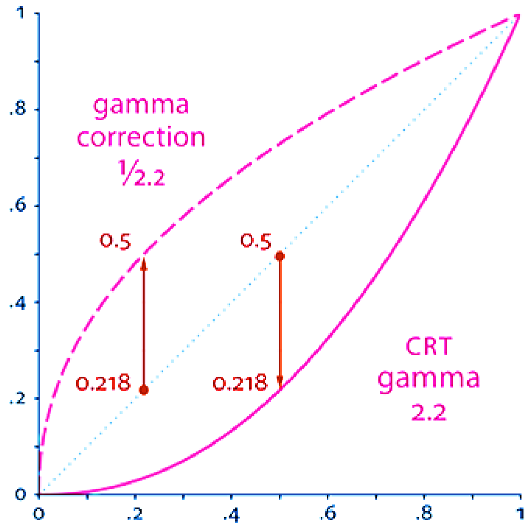

credit: blog.wolfire.com

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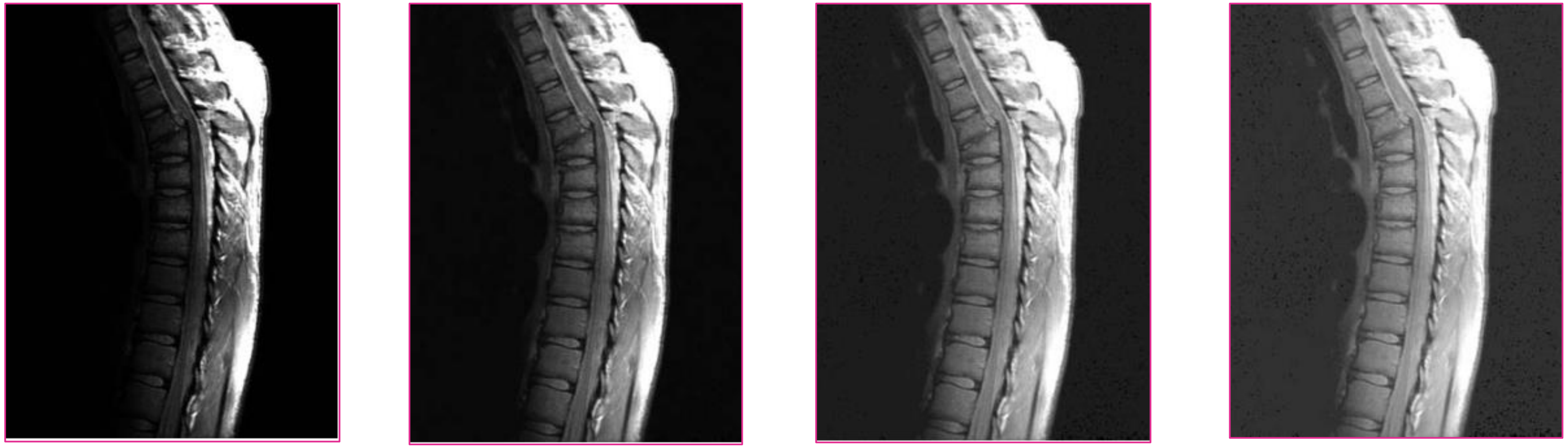

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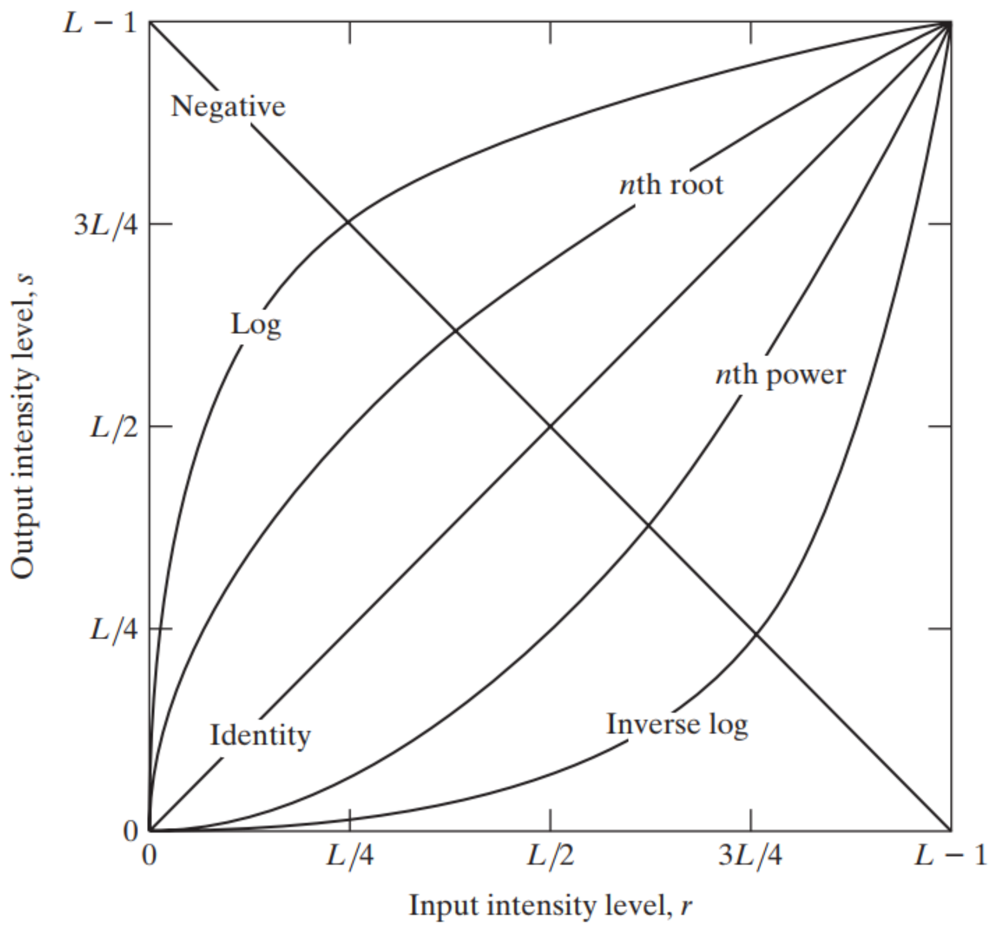
Transformations

- Compositions
  - piecewise combinations
  - piecewise linear
    - many T_i formulated with this
    - need more user input paras



Contrast stretching

- Contrast
  - low contrast images
    - due to poor illumination, low dynamic range sensors
    - wrong setting of lens aperture

  - full range stretching
    - $(r_1, s_1) = (r_{min}, 0)$
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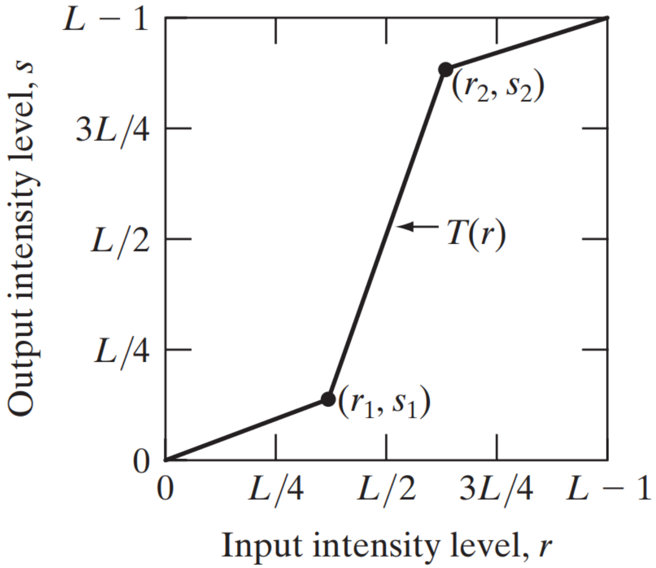

70~140

0~255

pixel ranges

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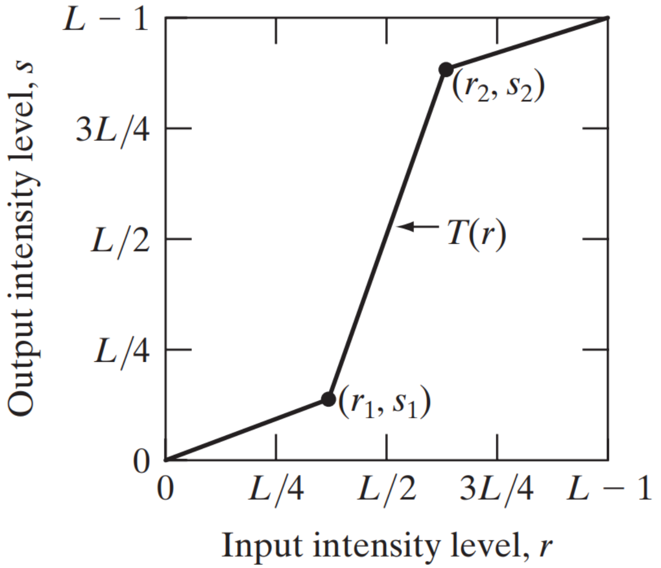

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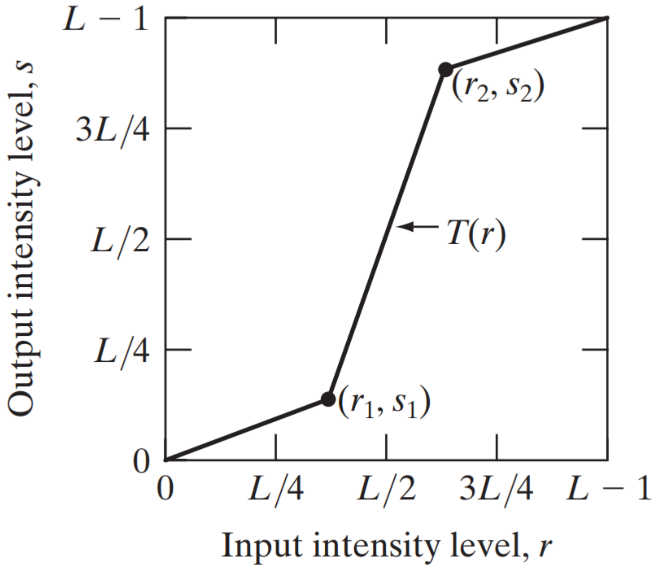

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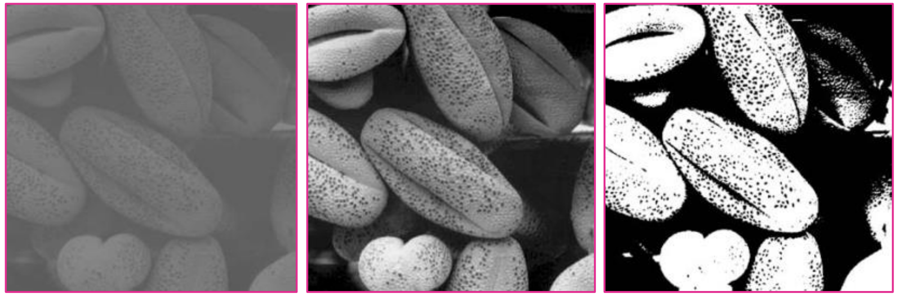

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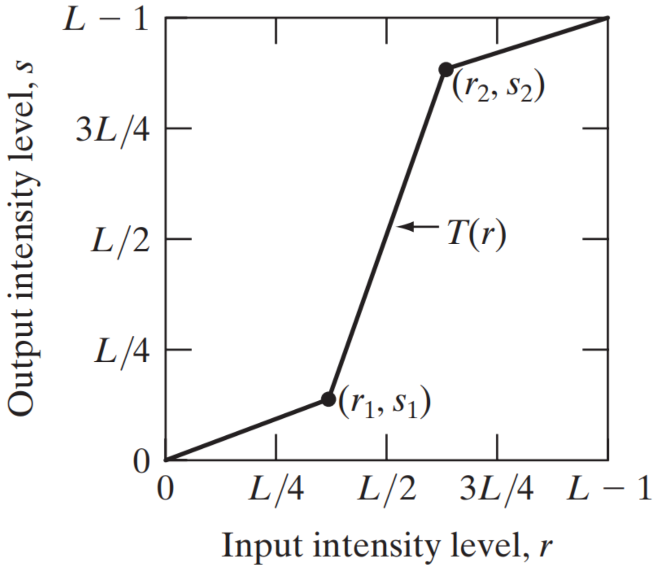

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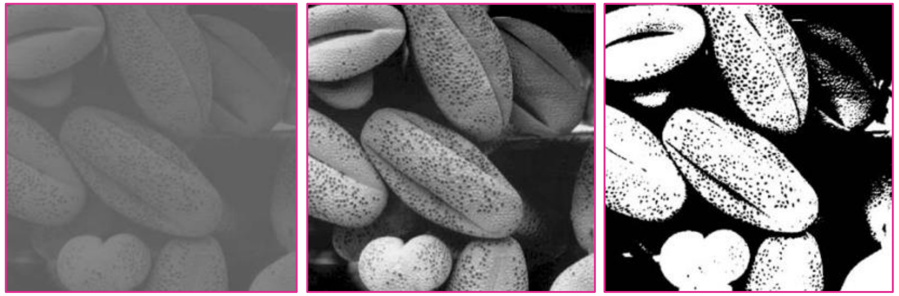
SEM image of pollen grains



Level slicing

- Intensity levels
  - local thresholding, stretching
  - enhancing only specific intensities
    - e.g. detecting water, wetland in sat. images

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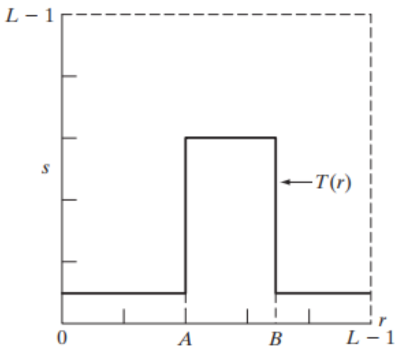

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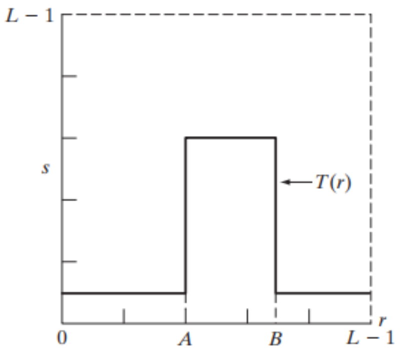

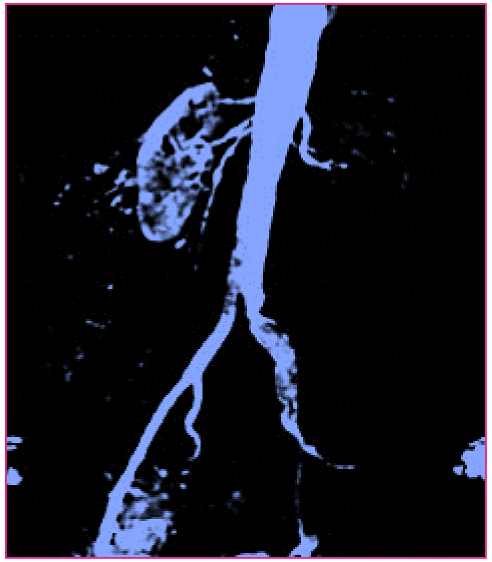

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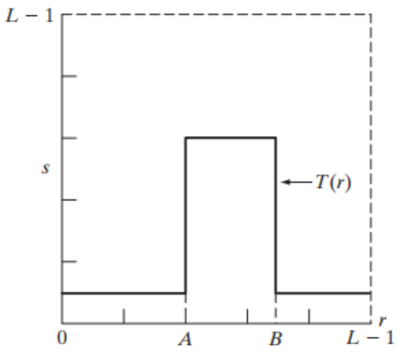

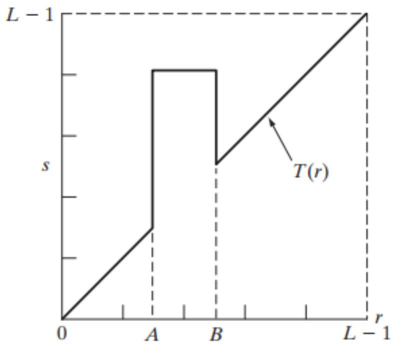

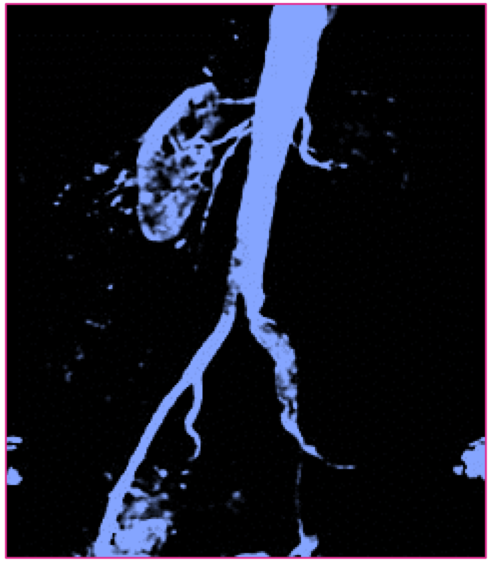

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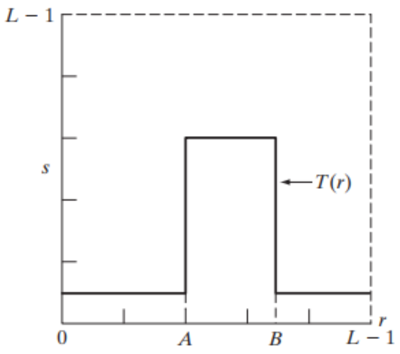
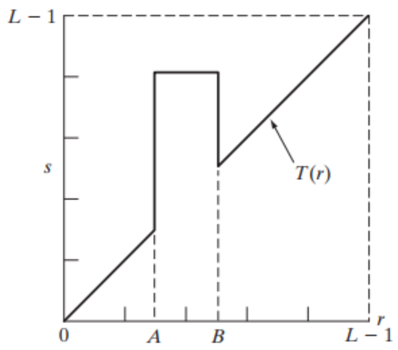
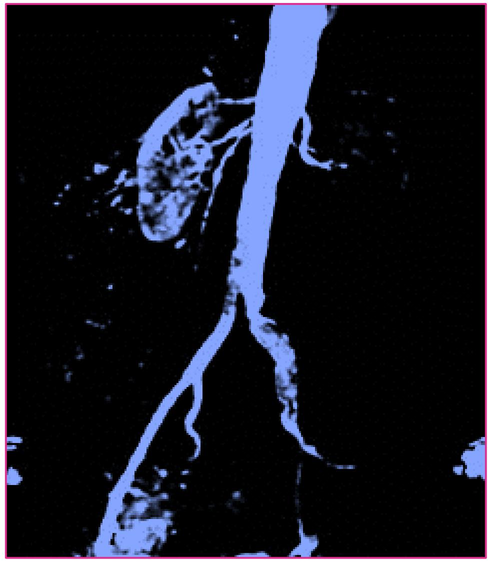
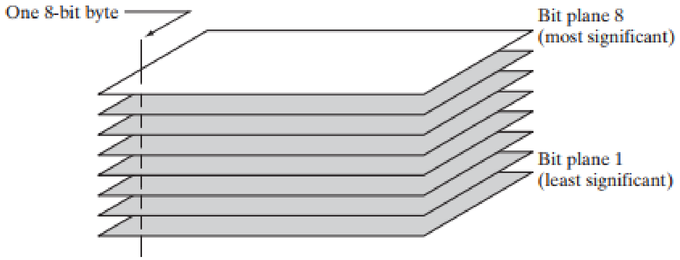






Bitplane slicing

- Bitplanes
  - contribution of each bit for total image appearance
  - gives clue for a compression



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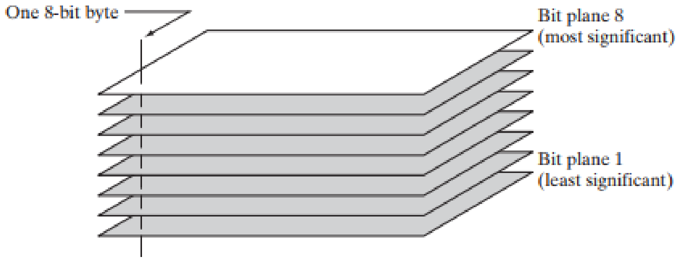

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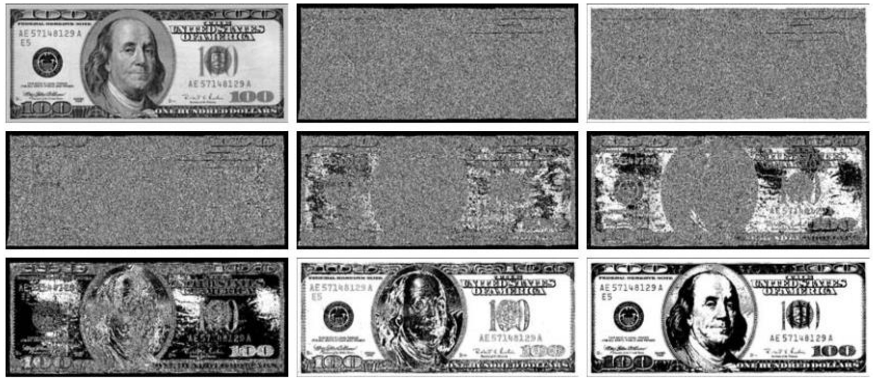

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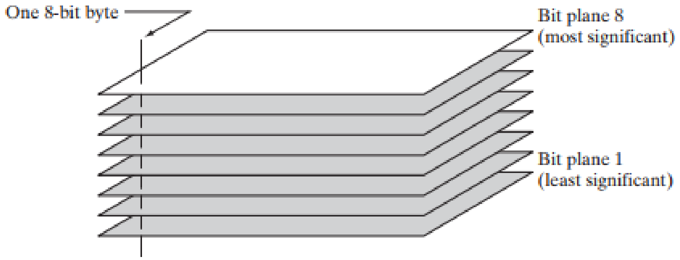

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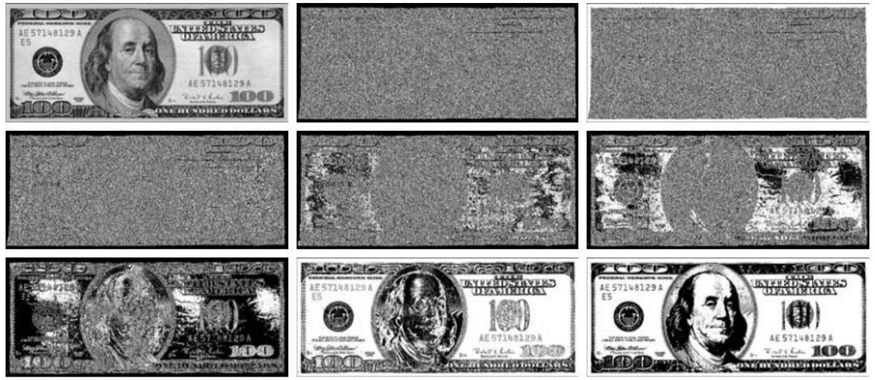

- reconstruction

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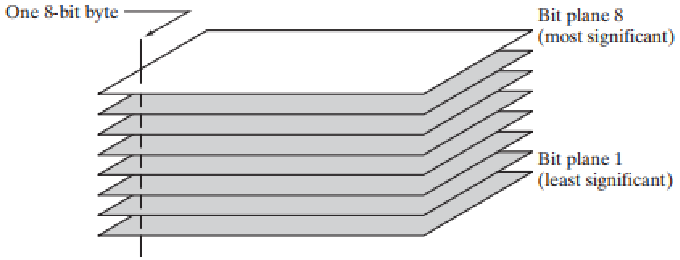

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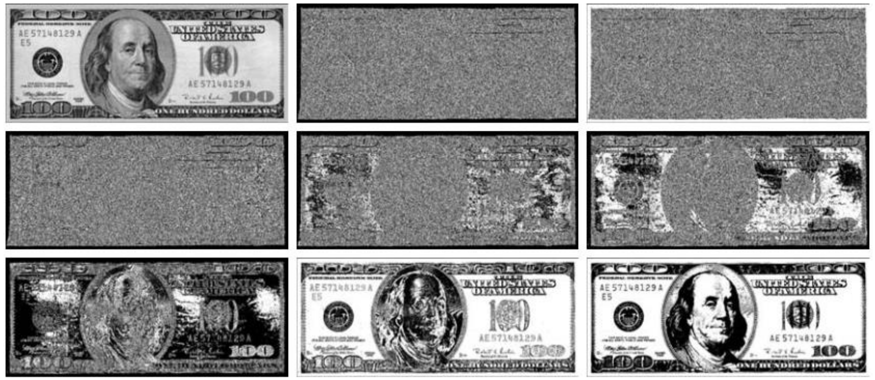

- reconstruction

- bitplanes (8+7)

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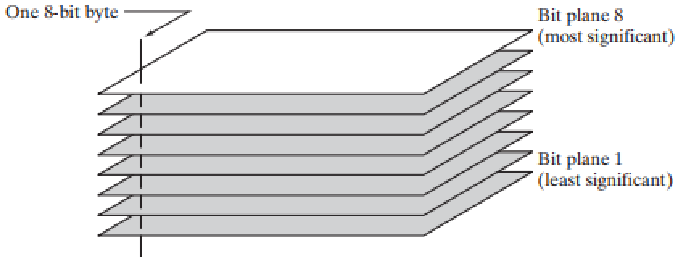

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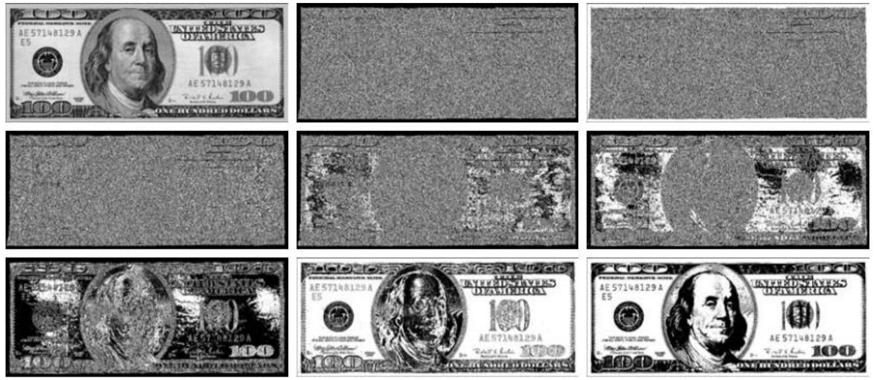

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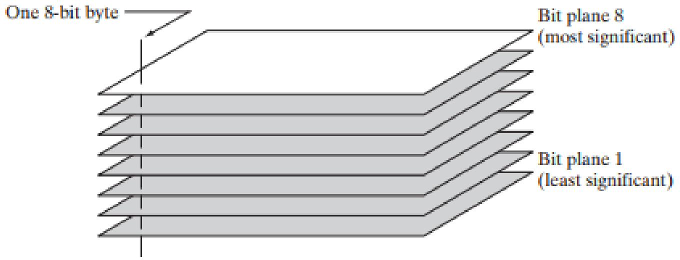

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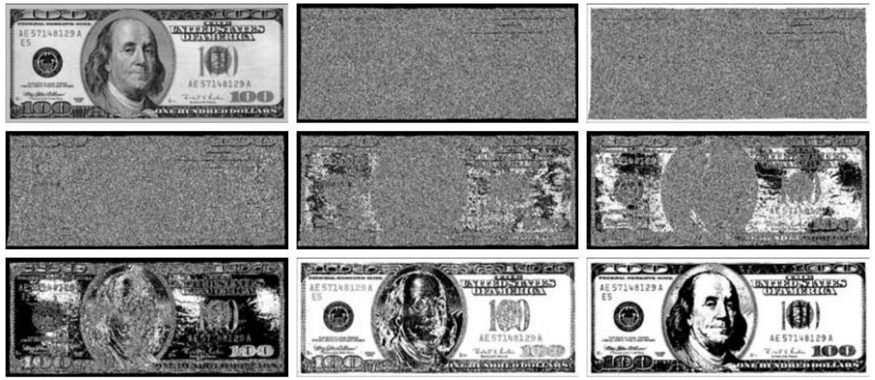

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- bitplanes (8+7+6+5)

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  - distribution transformations
    - histogram equalization

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- Spatial filtering
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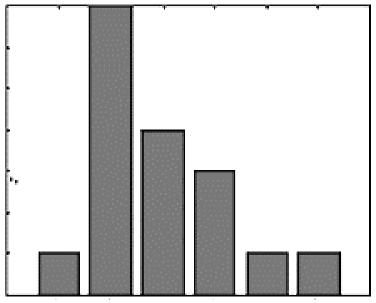
Histograms

- distribution of discrete intensities
  - distribution is also discrete

4	1	3	2
3	1	1	1
0	1	5	2
1	1	2	2



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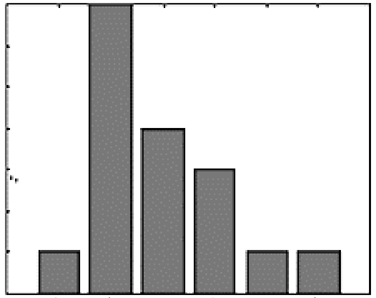

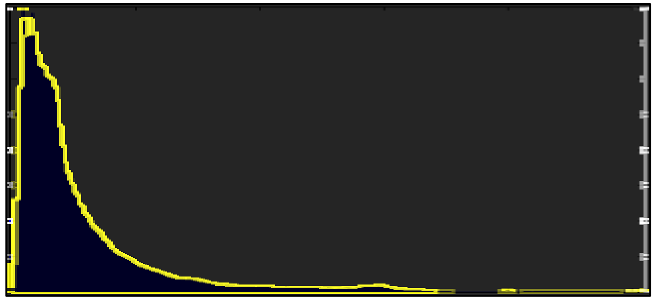

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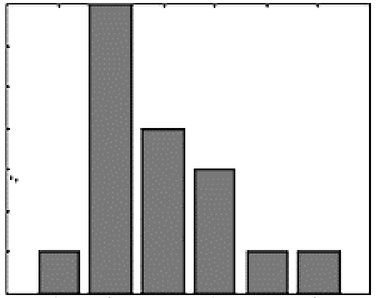

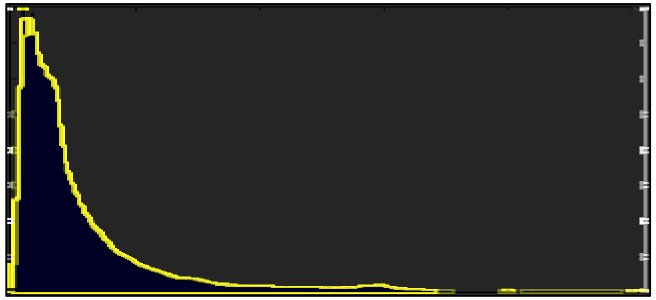

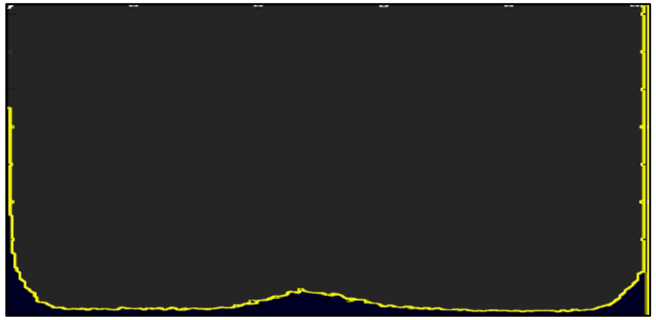

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- Color images

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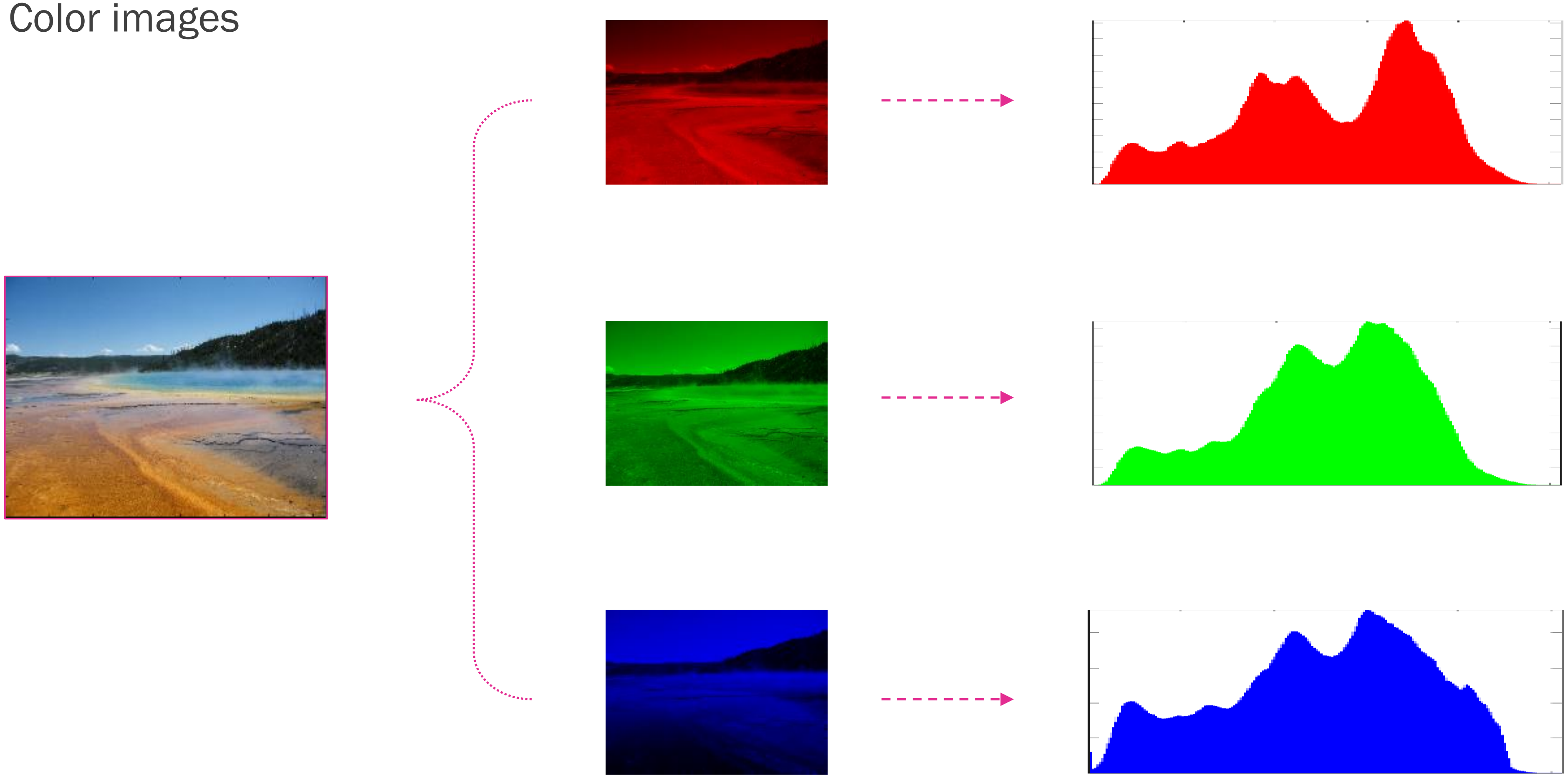
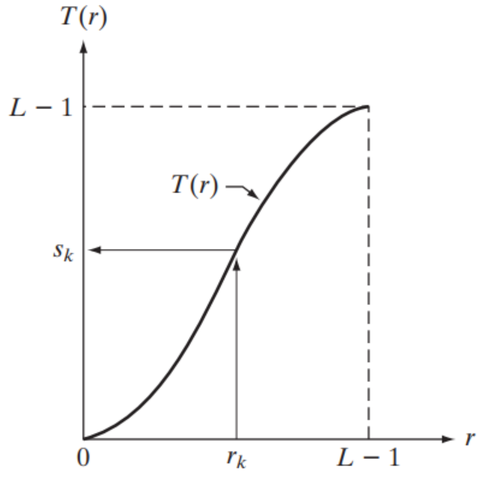


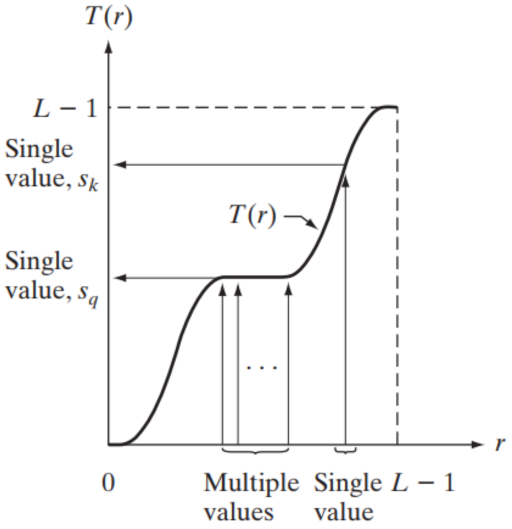
Histogram equalization

- Assume
  - $T(r)$ is monotonic $\uparrow$
  - bounded $0 \le T(r) \le L-1$
  - variable equivalence
    - to cover all notations

$$s = T(r)$$

$$0 \le r \le L - 1$$





Histogram equalization

- Assume
  - $T(r)$ is monotonic ↑
  - bounded $0 \leq T(r) \leq L-1$
  - variable equivalence
    - to cover all notations

$$Y = T(X)$$

$$s = T(r)$$

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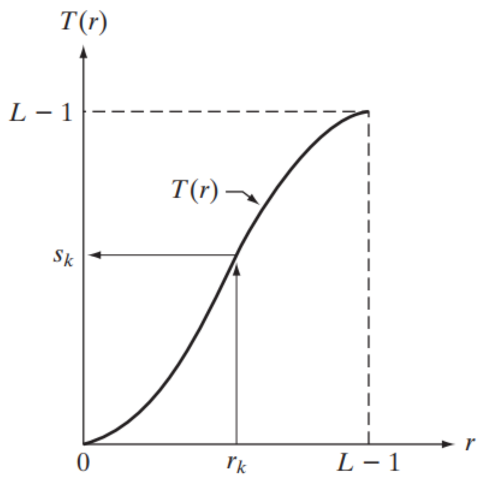

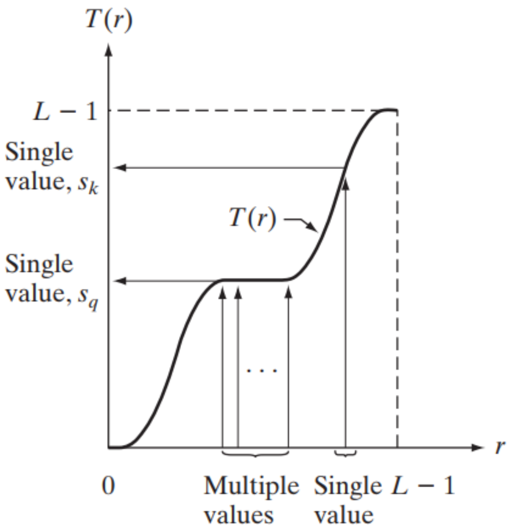

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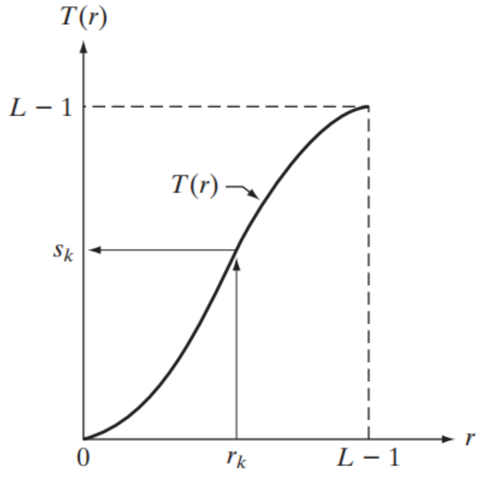

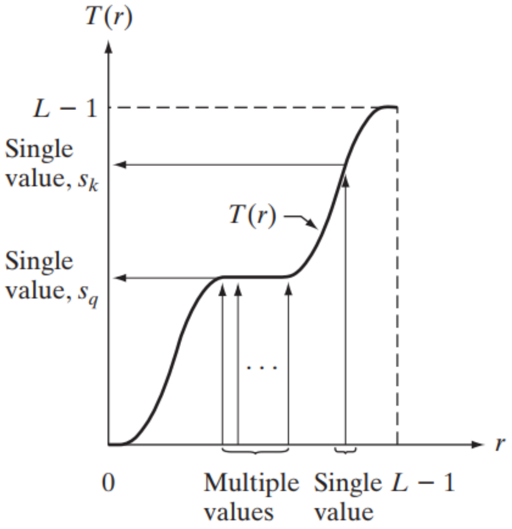

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$$p_s(s) \qquad p_r(r)$$

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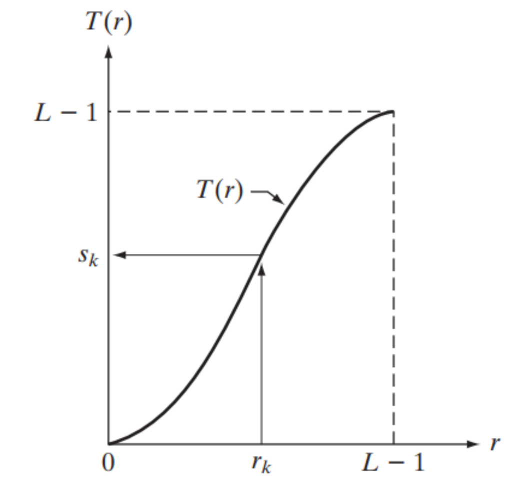

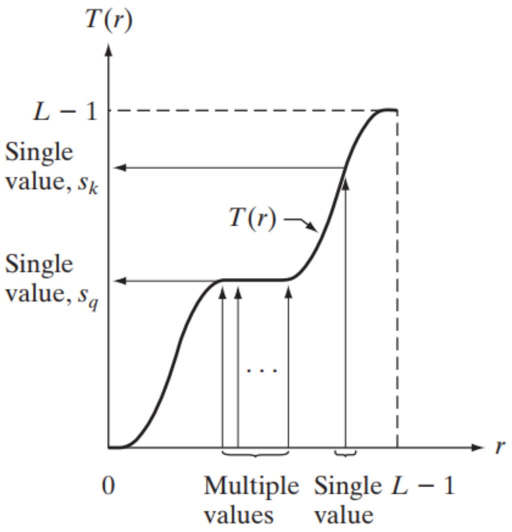

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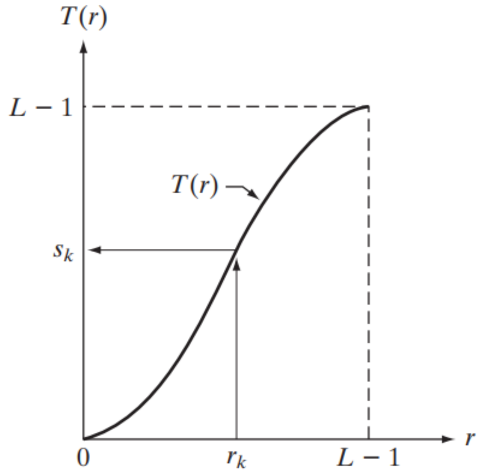

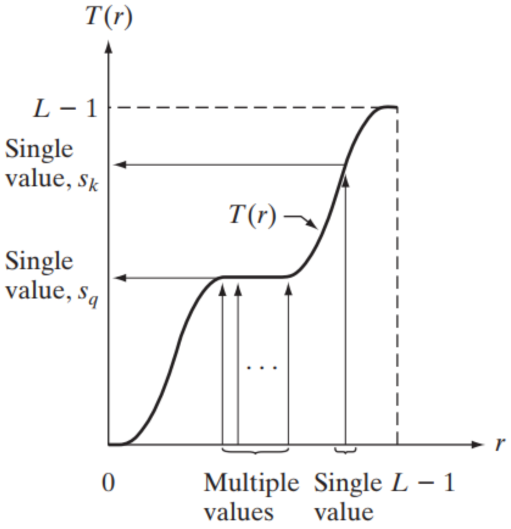

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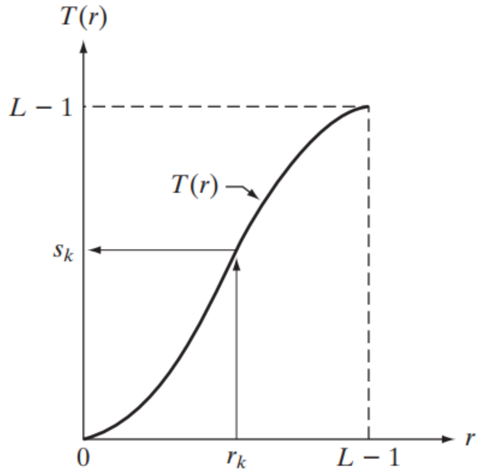
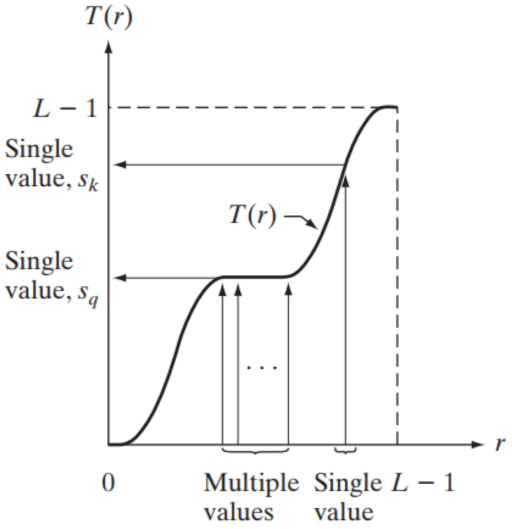
$T(r)$ is cts & differentiable





Histogram equalization

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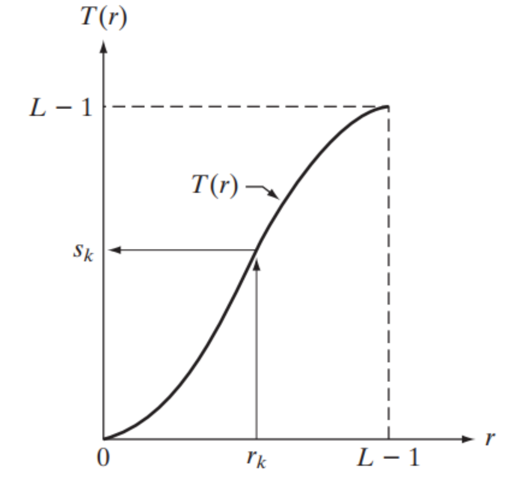
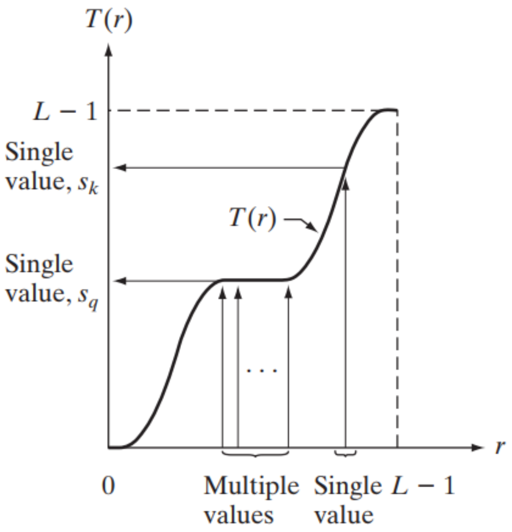


- cumulative function satisfies above properties for $T(r)$



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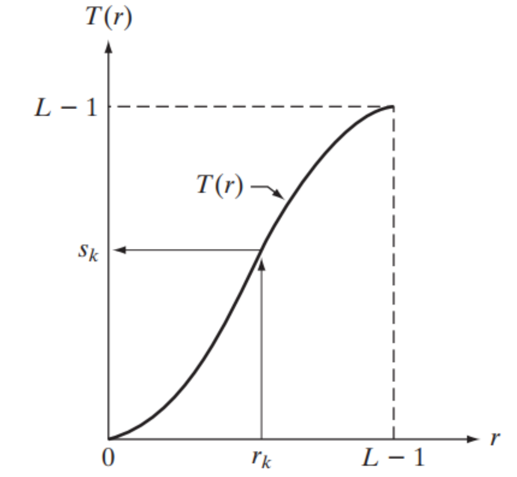
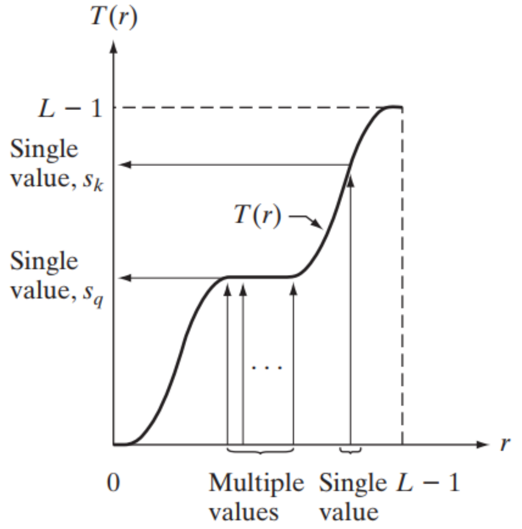


- cumulative function satisfies above properties for $T(r)$

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$Y = T(X) = (L-1)\int_0^X p_X(x)dx$$



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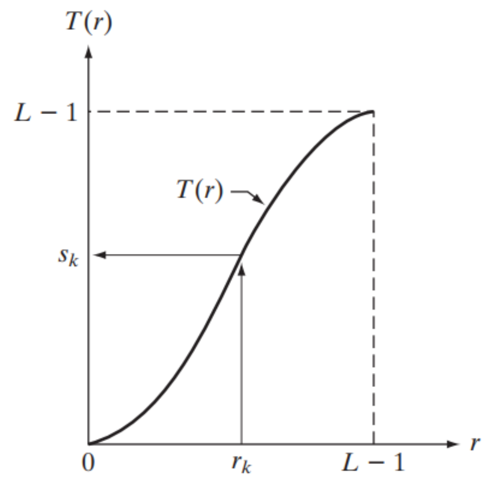

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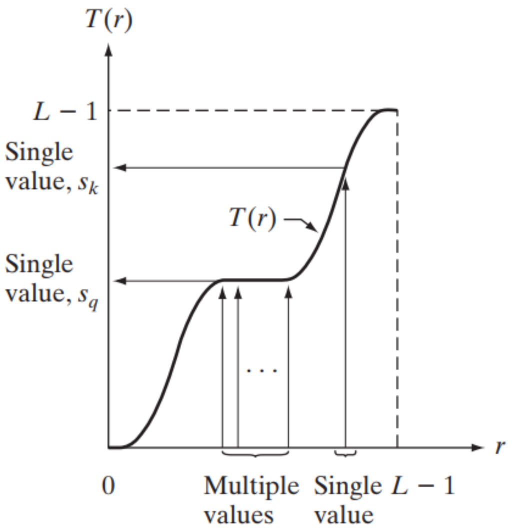
$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$Y = T(X) = (L-1)\int_0^X p_X(x)dx$$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$



Histogram equalization

$$Y = T(X) = (L-1)\int_0^X p_X(x)dx$$

$$Y = T(X)$$

$$p_Y(y) \qquad p_X(x)$$

$T(X)$ is cts & differentiable

What is $p_Y(y)$?

Histogram equalization

$$Y = T(X) = (L-1)\int_0^X p_X(x)dx$$

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$T(X)$ is cts & differentiable

What is $p_Y(y)$?

$$\int_0^y p_Y(z)dz = \text{ probability that } 0 \le Y \le y$$

Histogram equalization

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Histogram equalization

$$Y = T(X) = (L-1)\int_0^X p_X(x)dx$$

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Histogram equalization

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$p_Y(y)$ $p_X(x)$

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$$\frac{d}{dy}\left(\int_0^y p_Y(z)dz\right)$$

Histogram equalization

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$$\frac{d}{dy}\left(\int_0^y p_Y(z)dz\right) \quad = p_X(T^{-1}(y))\frac{d}{dy}(T^{-1}(y))$$

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What is $p_Y(y)$?

$$p_Y(y) \qquad = p_X(T^{-1}(y))\frac{d}{dy}(T^{-1}(y))$$

$$= \frac{1}{L-1} \cdot \frac{dT}{dx}\Big|_{x=T^{-1}(y)}\frac{d}{dy}(T^{-1}(y))$$

Histogram equalization

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Histogram equalization

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$$= \frac{1}{L-1}$$

Histogram equalization

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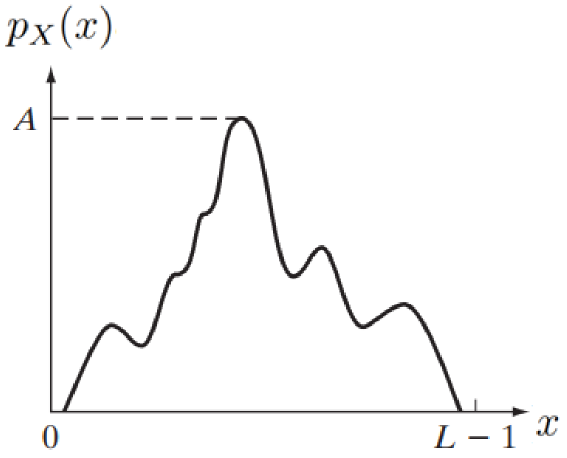

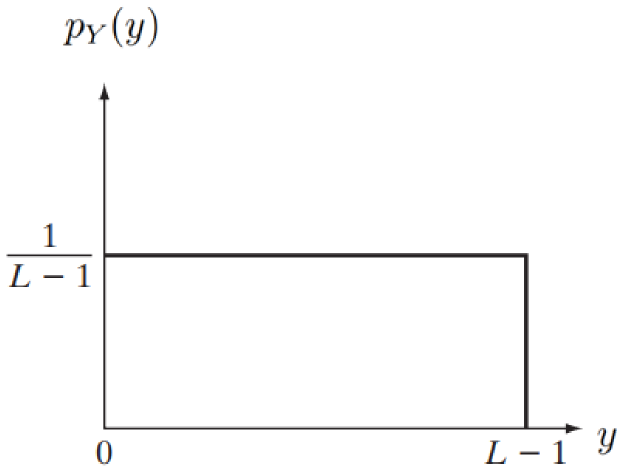

Histogram equalization

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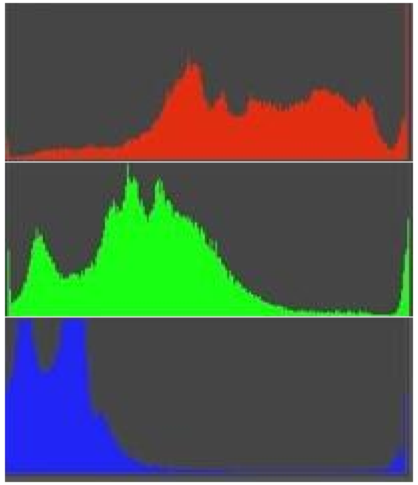

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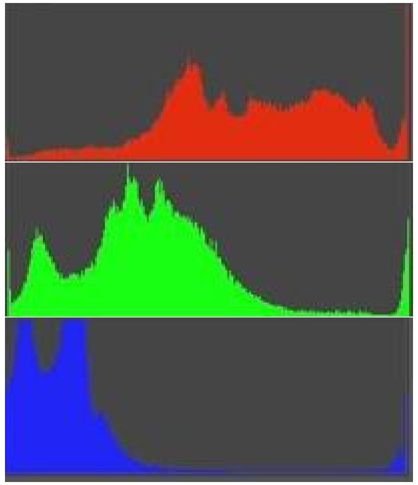

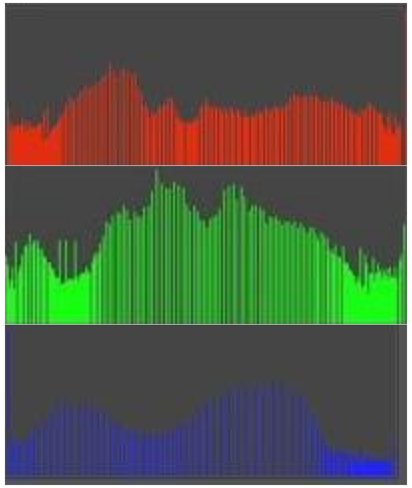

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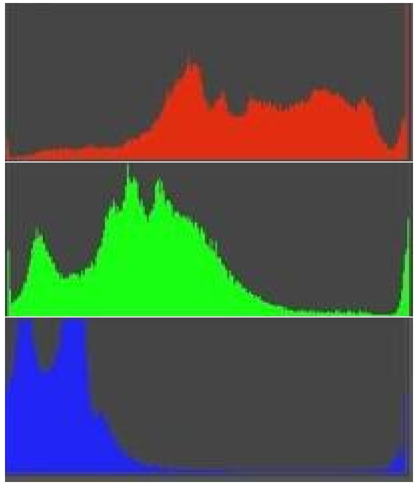

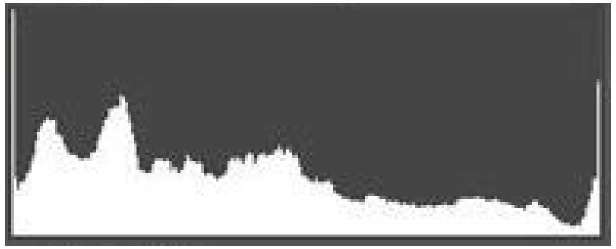

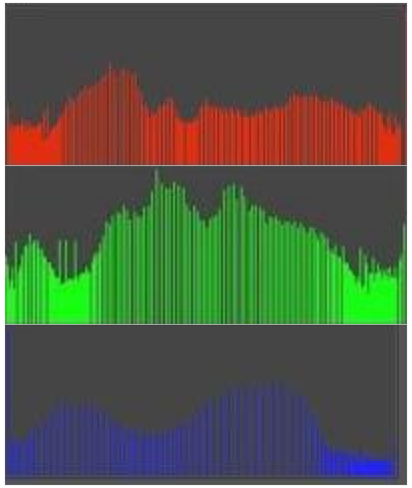

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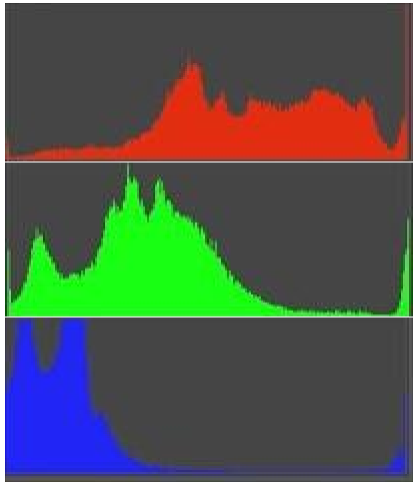
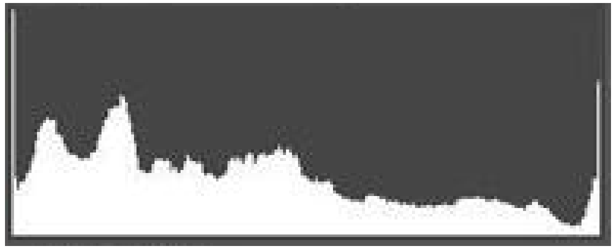
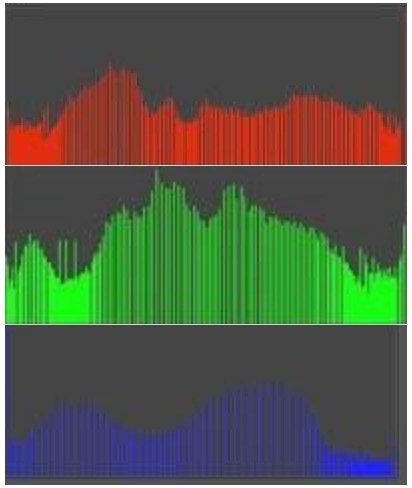
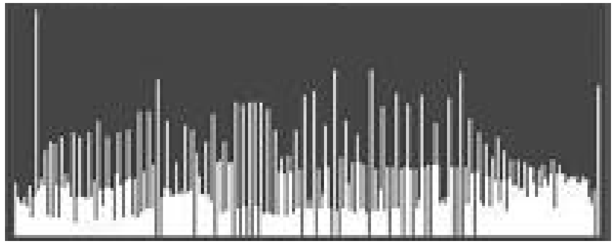
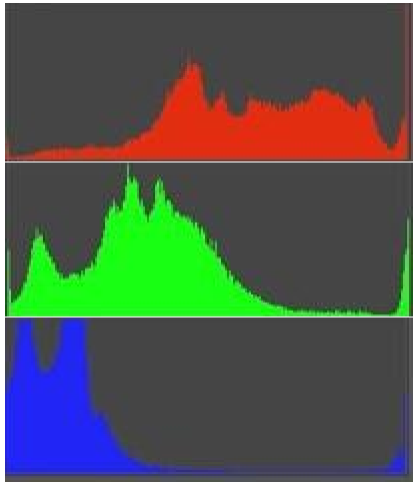


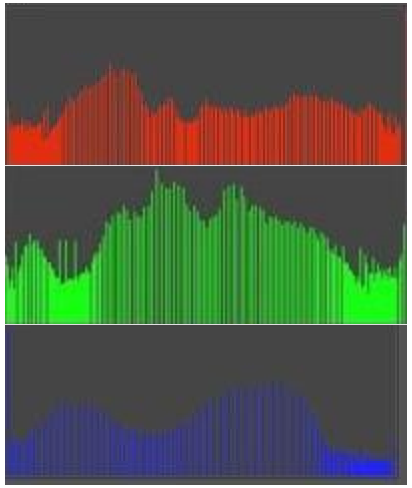
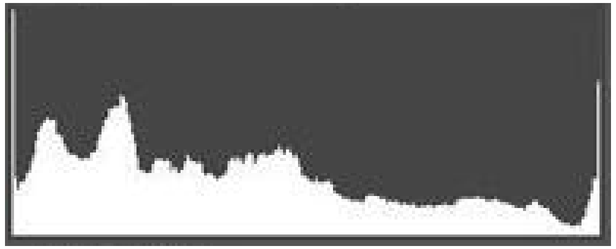


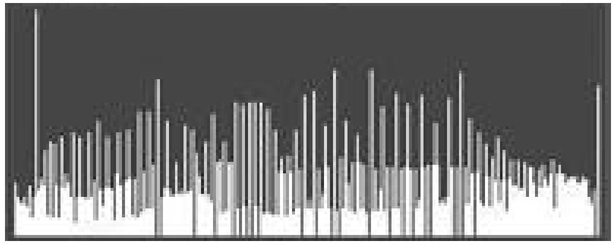




Histogram equalization



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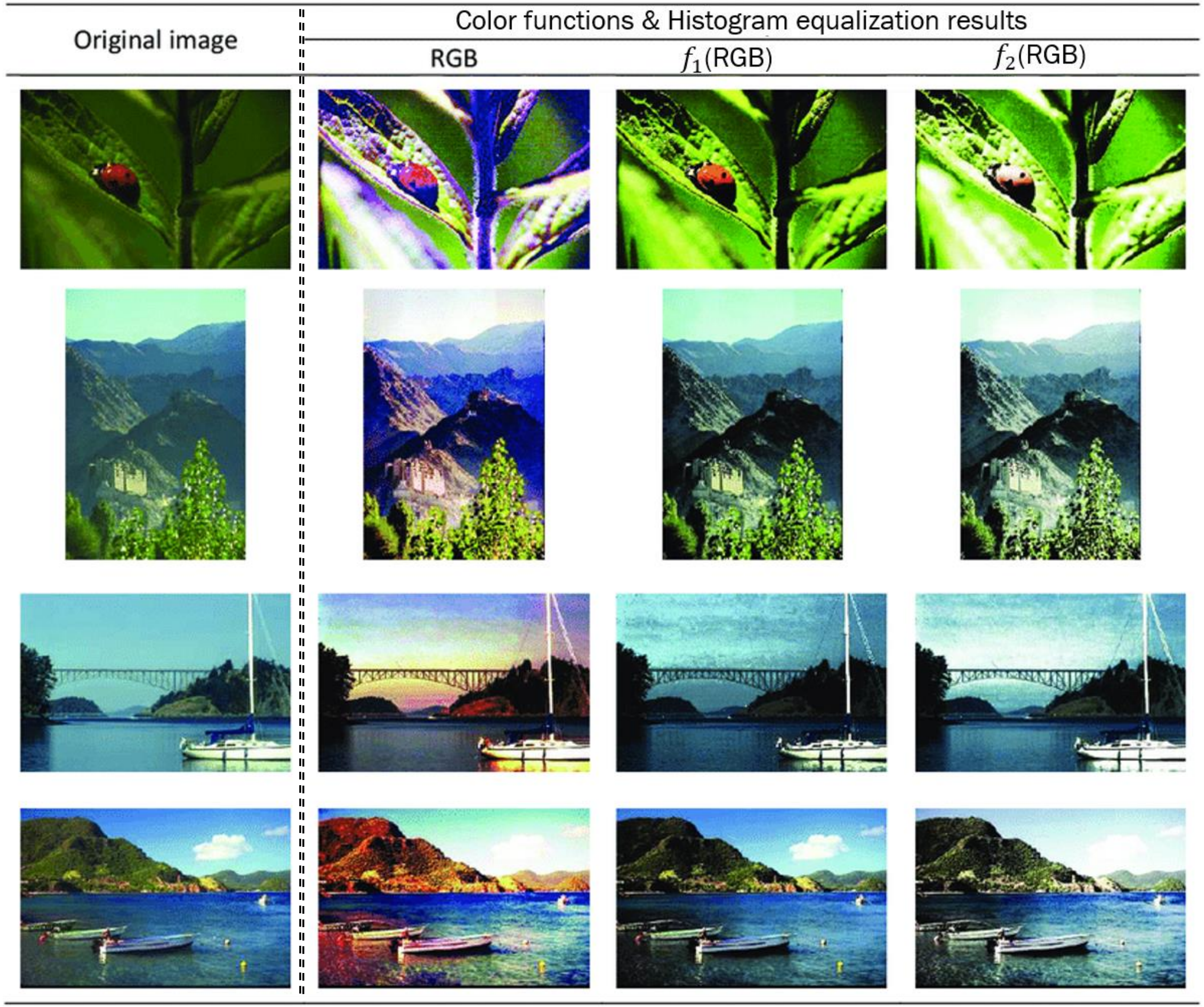
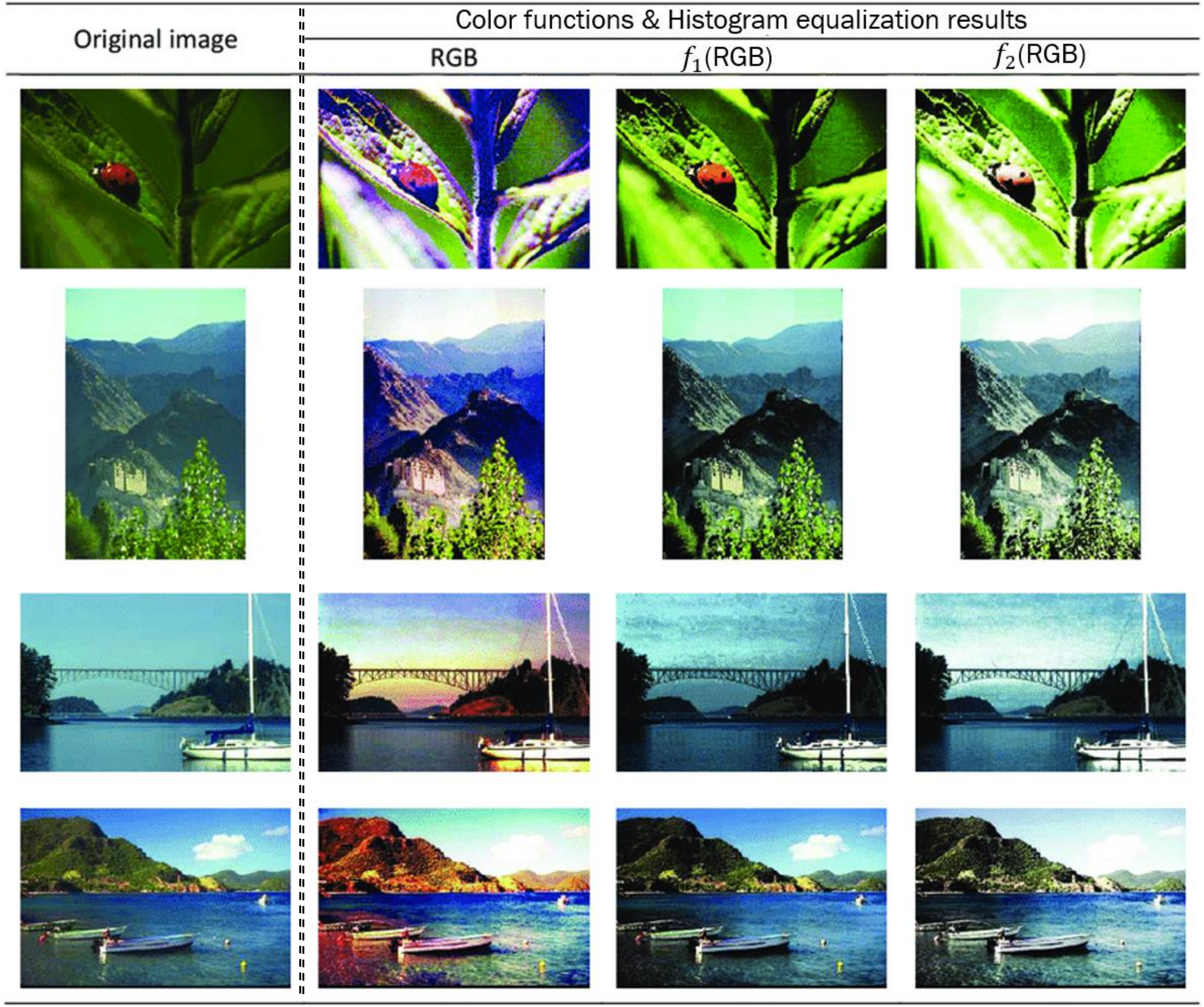
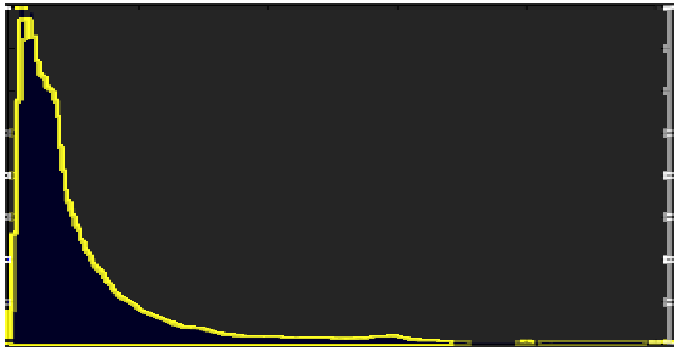


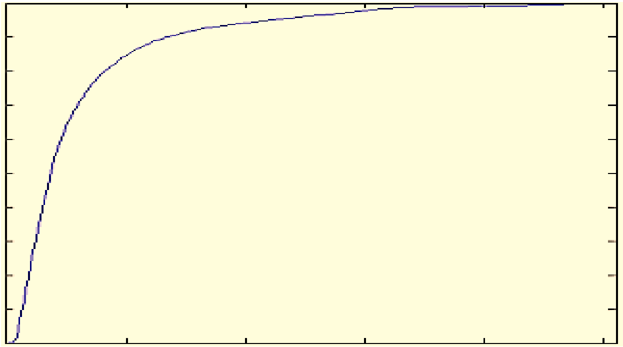
Histogram equalization

- Color conversions
  - colors can be mapped with certain functions
  - mapped images then histogram equalized

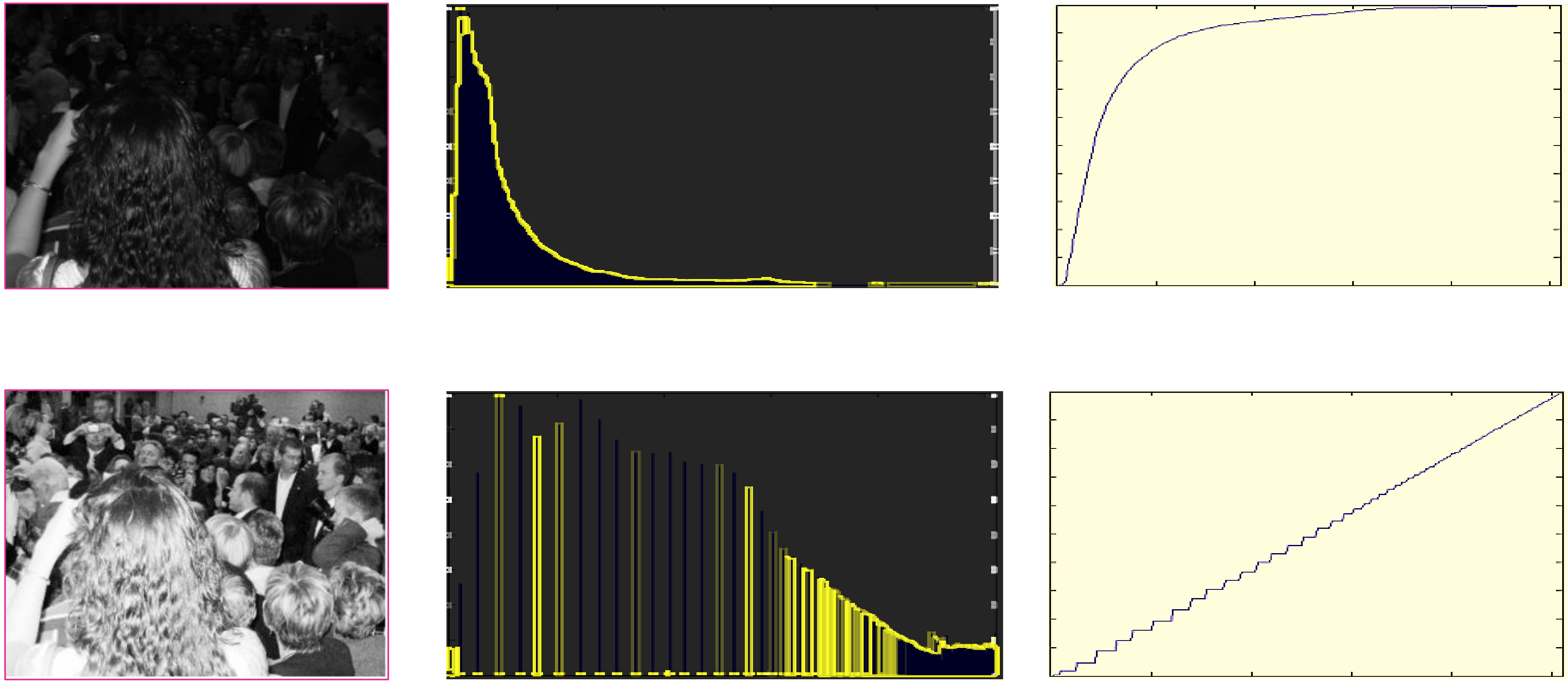


Histogram equalization



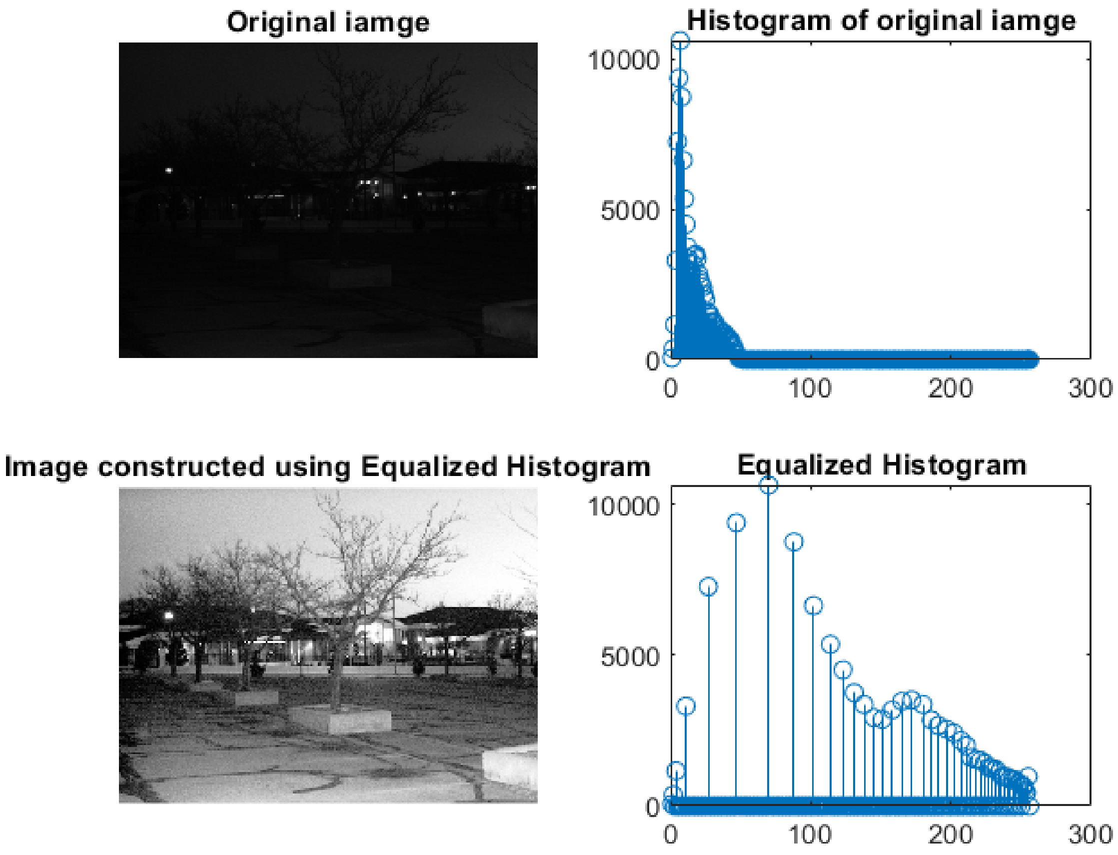


Histogram equalization



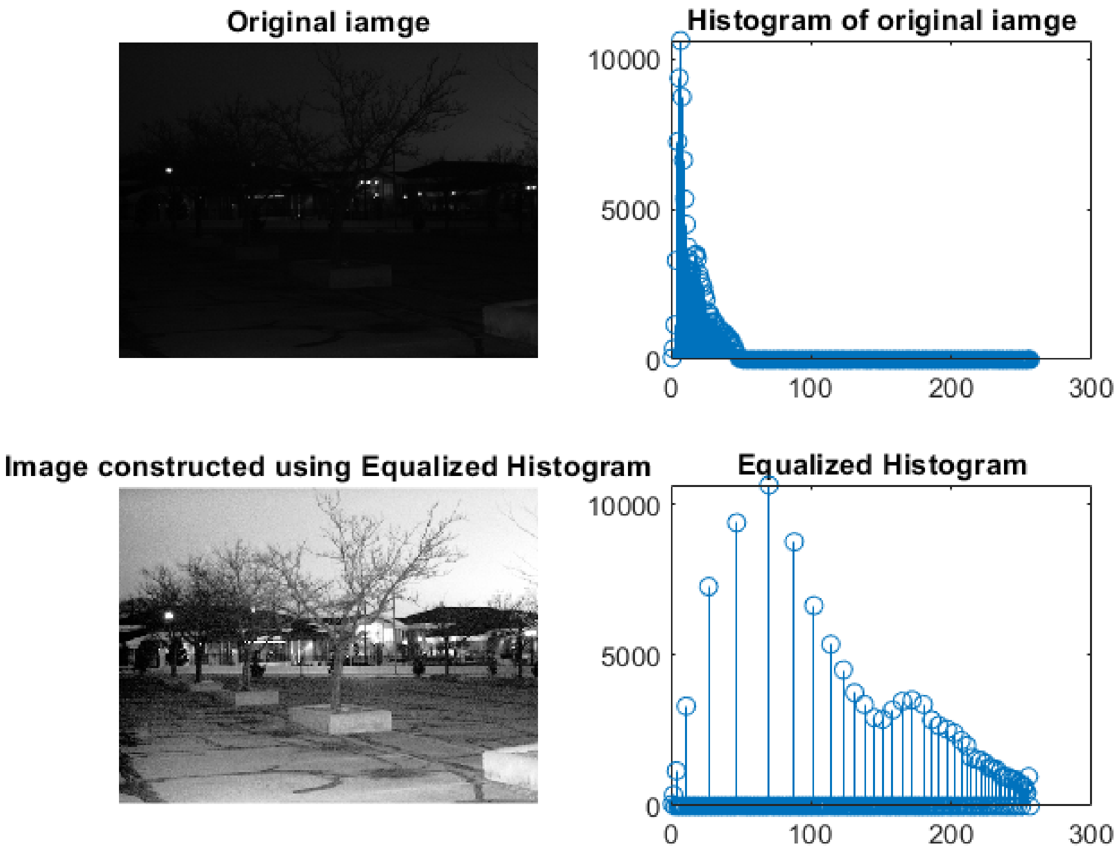
Histogram equalization

- Global



Histogram equalization

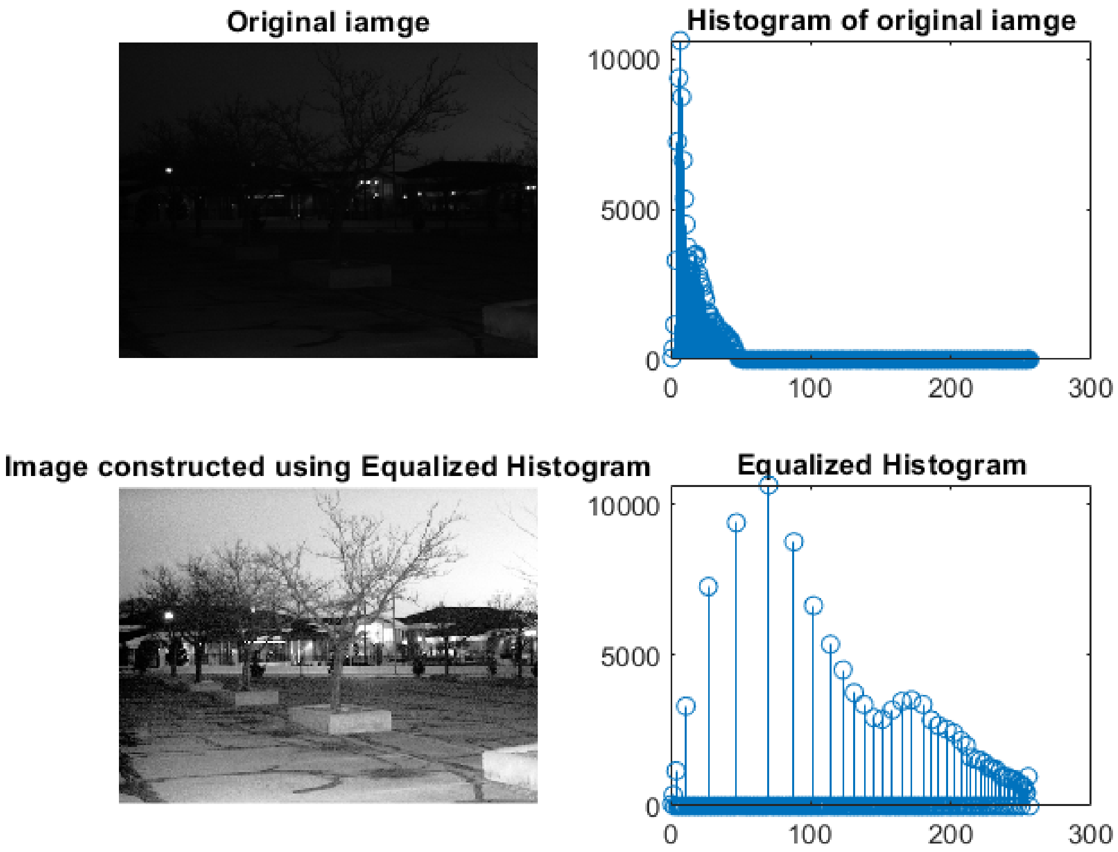
- Global



- Local

Histogram equalization

- Global



- Local

Histogram equalization

- Local

Ref: wikipedia

Histogram equalization

- Local

- Bilinear interpolation

Ref: wikipedia

Histogram equalization

- Local

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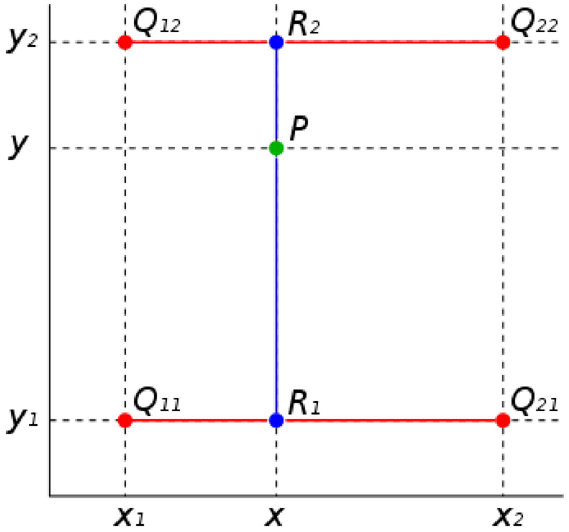


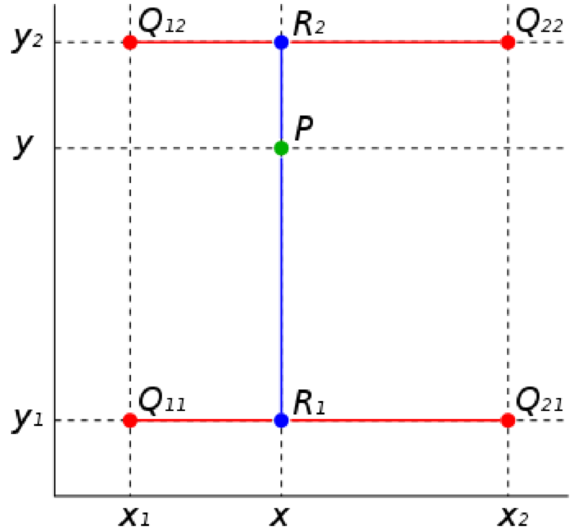
Ref: wikipedia

Histogram equalization

- Local

- Bilinear interpolation

$$f(x, y_1) = \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$
$$f(x, y_2) = \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}).$$



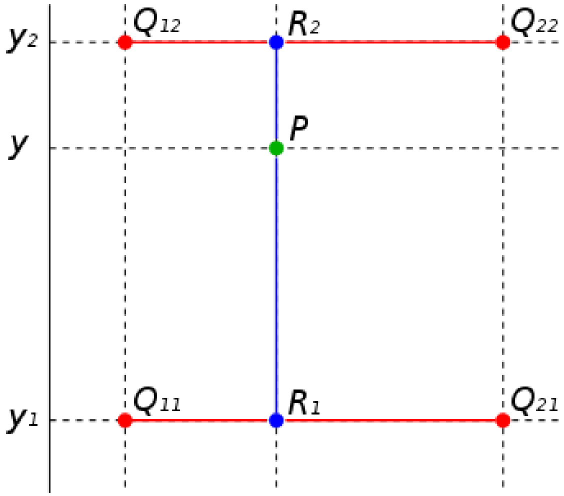
Ref: wikipedia

Histogram equalization

- Local

- Bilinear interpolation

$$f(x, y_1) = \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}),$$
$$f(x, y_2) = \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}).$$



$$\begin{aligned}
f(x,y) &= \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \\
&= \frac{y_2 - y}{y_2 - y_1} \left(\frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}) \right) + \frac{y - y_1}{y_2 - y_1} \left(\frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}) \right) \\
&= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \big(f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1) \big) \\
&= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \begin{bmatrix} x_2 - x & x - x_1 \end{bmatrix} \begin{bmatrix} f(Q_{11}) & f(Q_{12}) \\ f(Q_{21}) & f(Q_{22}) \end{bmatrix} \begin{bmatrix} y_2 - y \\ y - y_1 \end{bmatrix}.
\end{aligned}$$

Ref: wikipedia

Histogram equalization: CLAHE

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- AHE
  - Adaptive hist eq

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- CL
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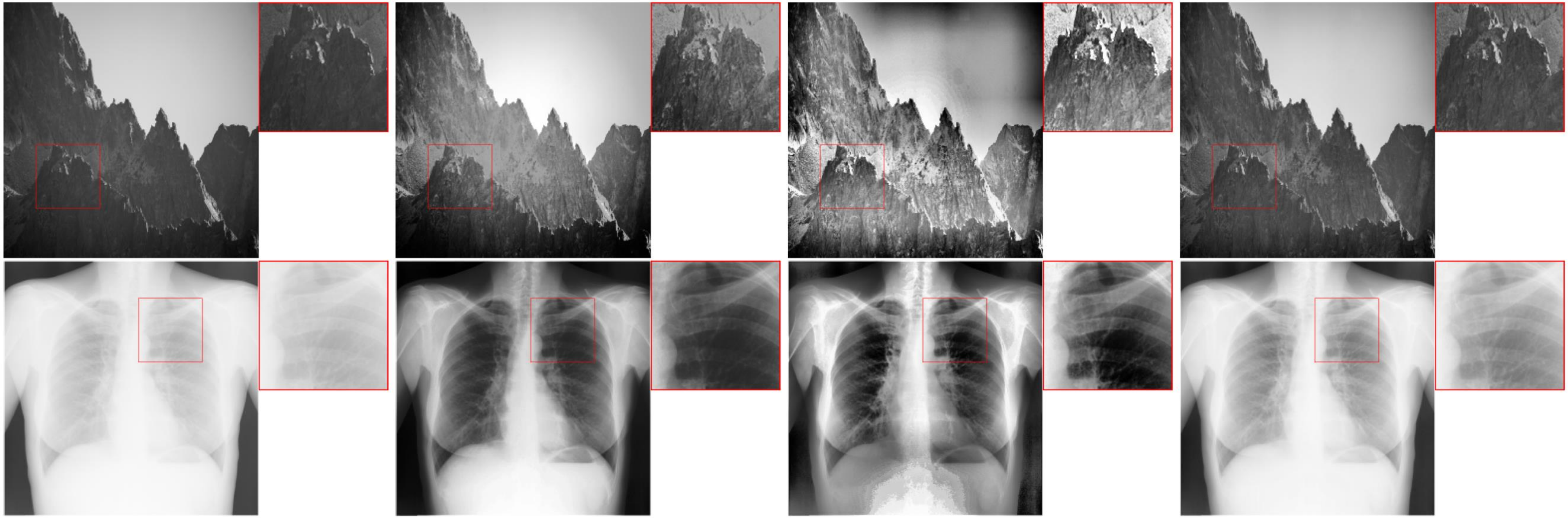

Histogram equalization: CLAHE

- AHE
  - Adaptive hist eq
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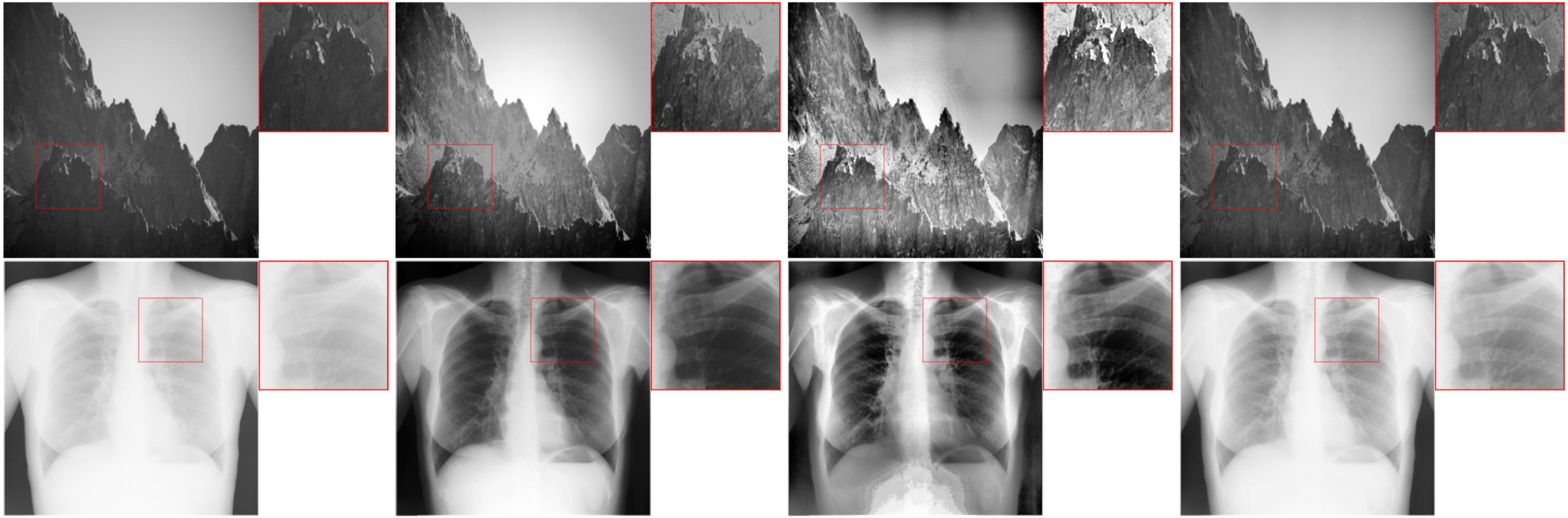
Input



Histogram equalization: CLAHE

- AHE
  - Adaptive hist eq
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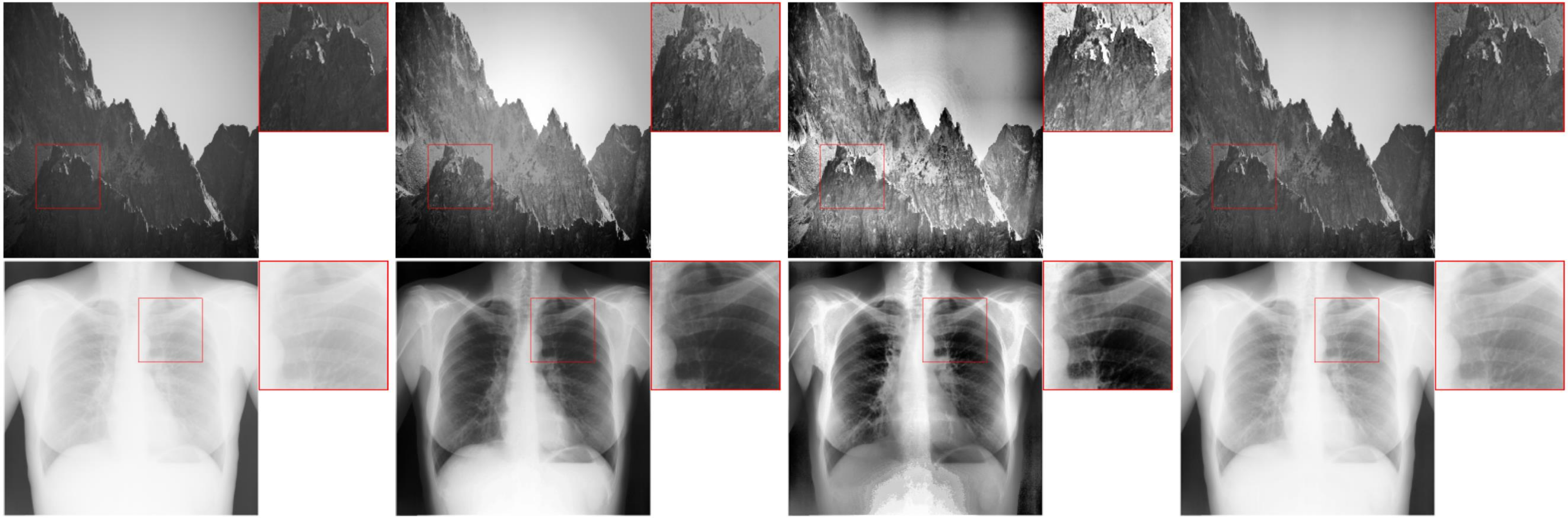
  - Bilinear

Input GHE



Histogram equalization: CLAHE

- AHE
  - Adaptive hist eq
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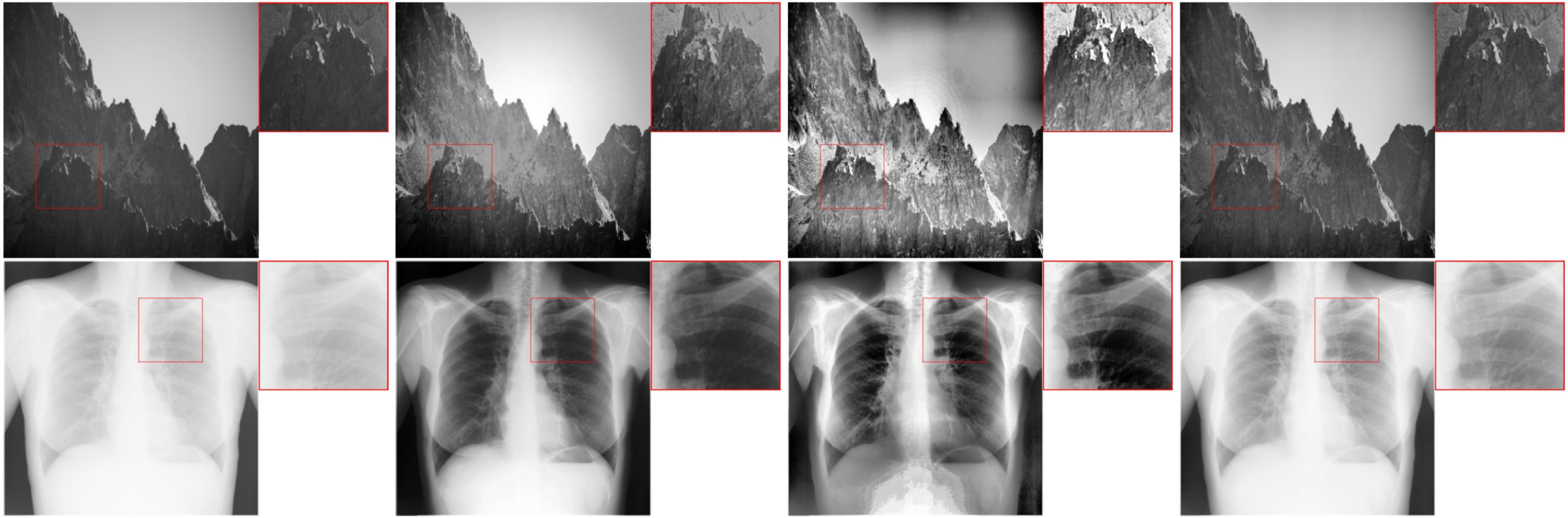
Input GHE AHE



Histogram equalization: CLAHE

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  - Adaptive hist eq
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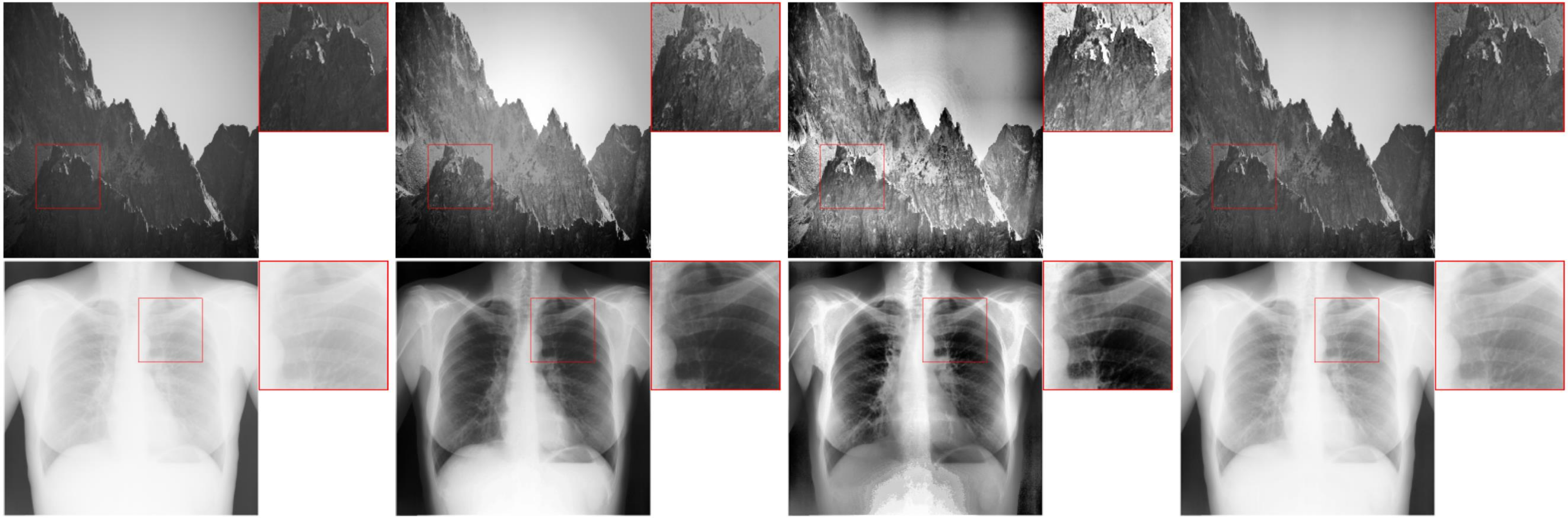
  - Bilinear

Input GHE AHE CLAHE



Conclusion

- Intensity transforms
- Distribution transforms

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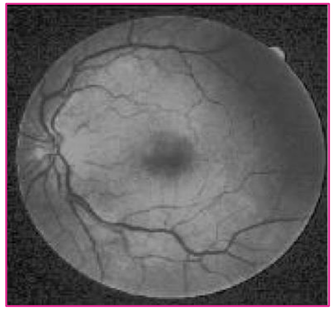
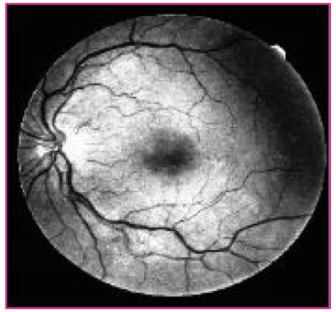
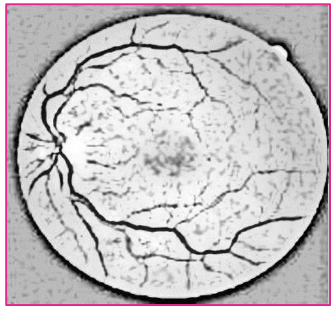
- Intensity transforms
- Distribution transforms







Conclusion

- Intensity transforms
- Distribution transforms

❑ Intensity transformations

- negatives
- logs
- power-law (gamma)
- contrast stretching
- level slicing
- bit-plane slicing

❑ Distribution transformations

- Histogram equalization