Image Filtering

Dr. Tushar Sandhan

Introduction

Input

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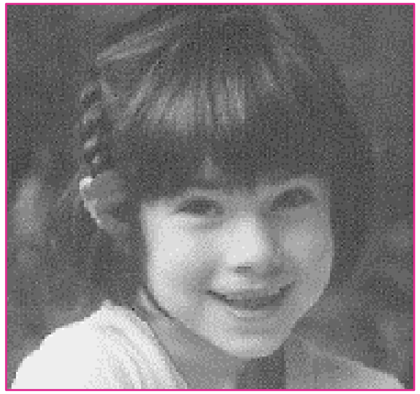
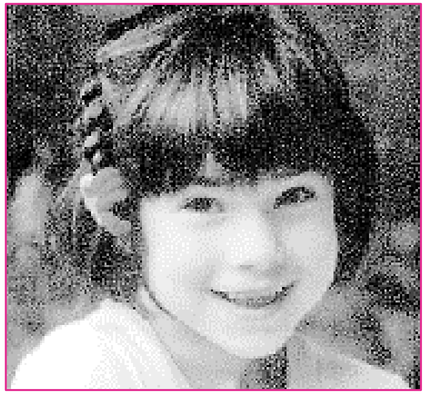
Input





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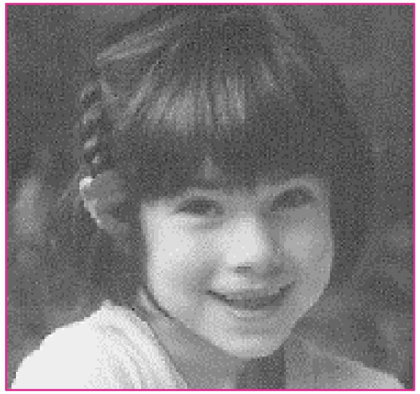

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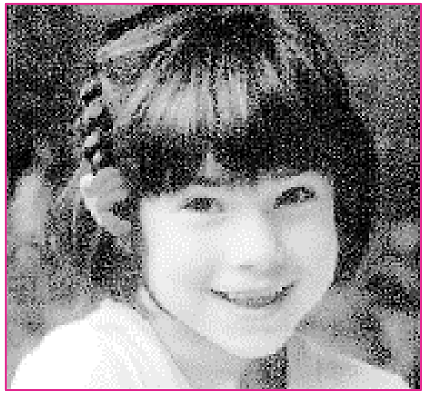

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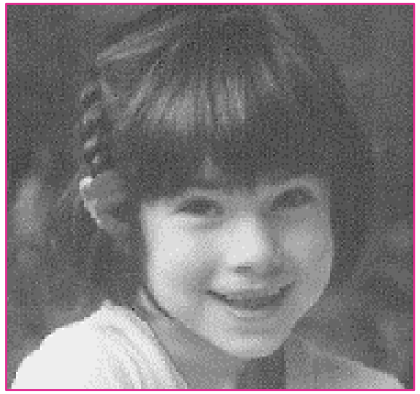

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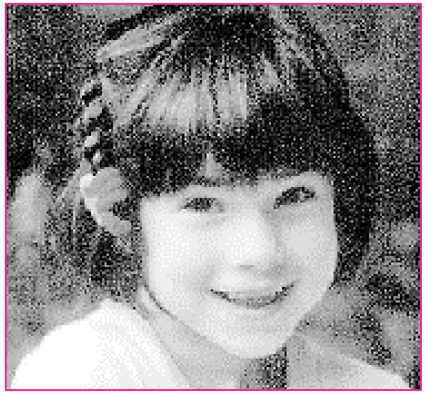

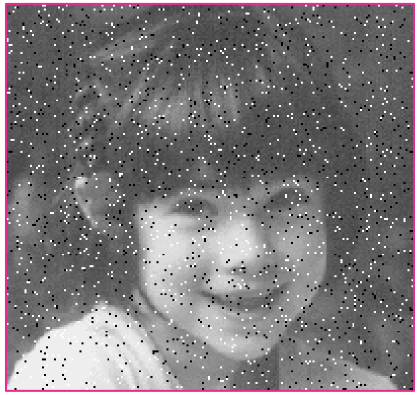

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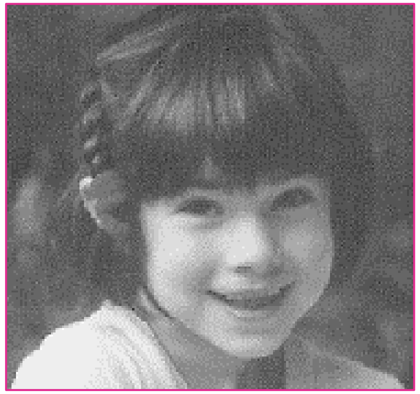

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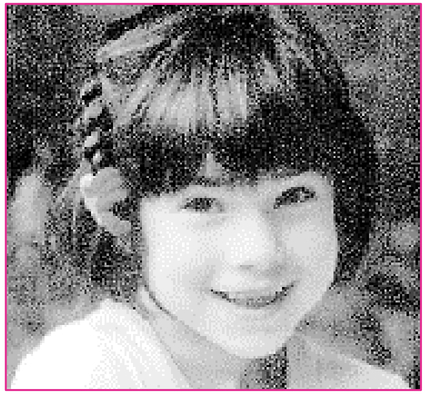

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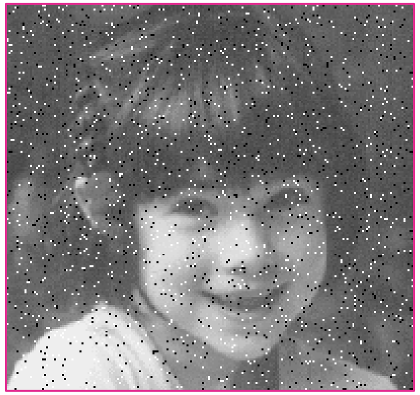

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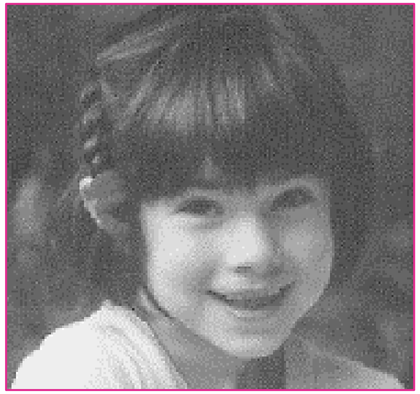

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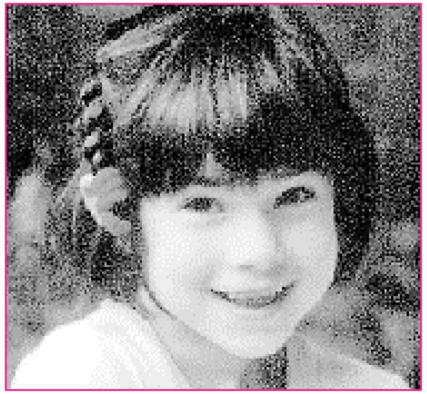

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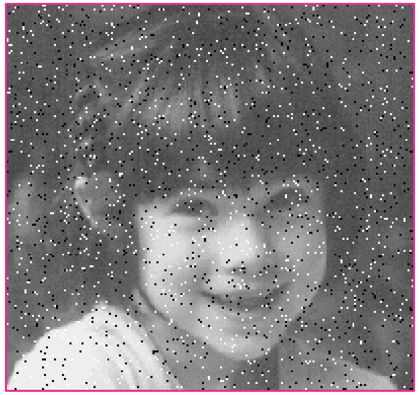

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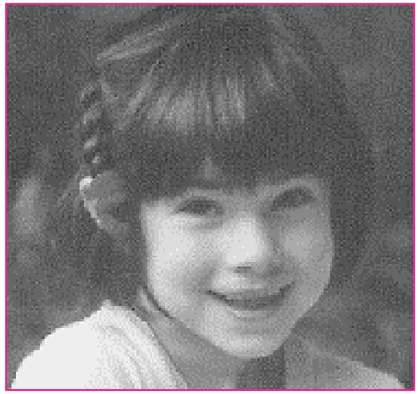

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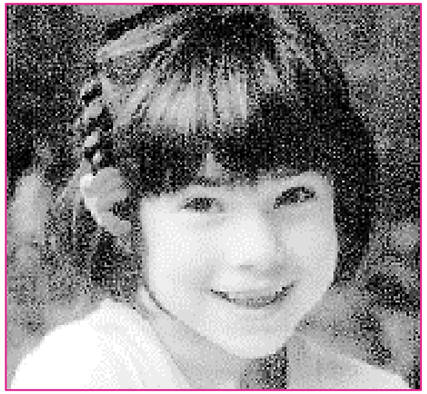

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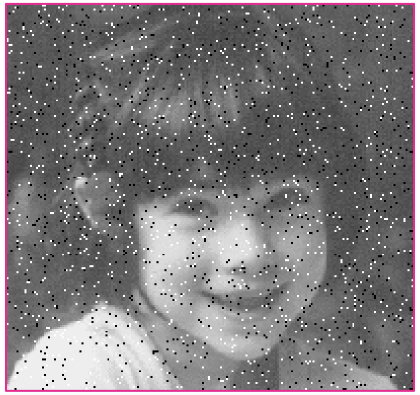

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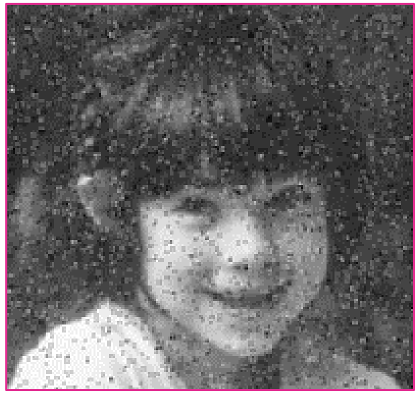

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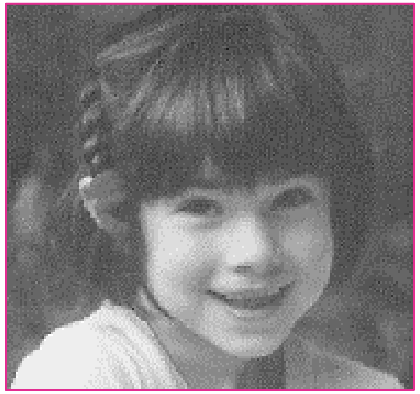

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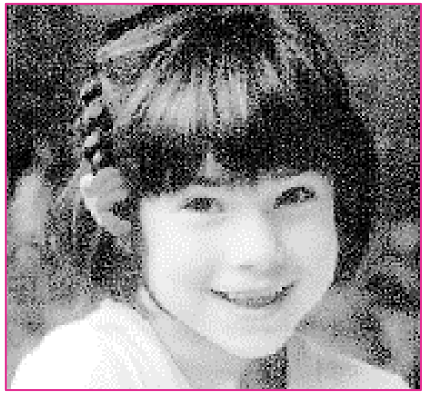

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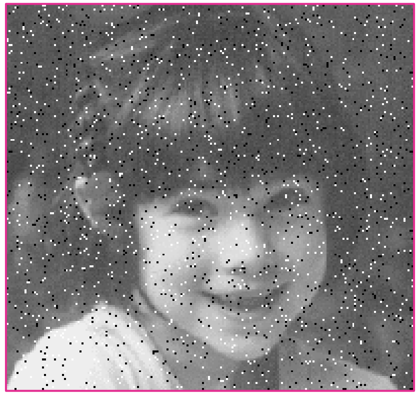

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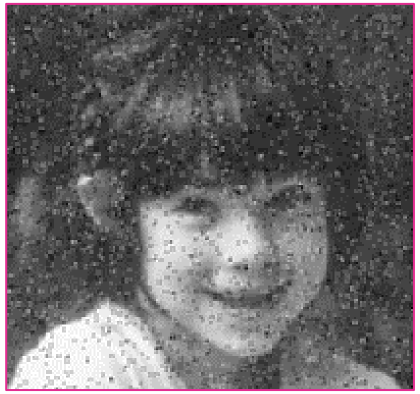

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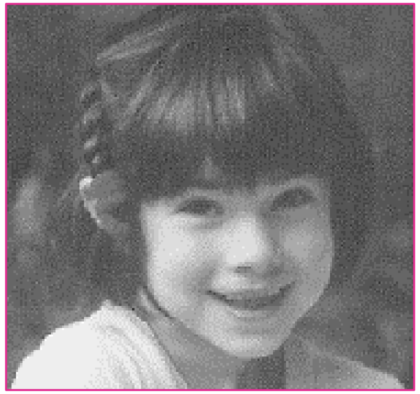

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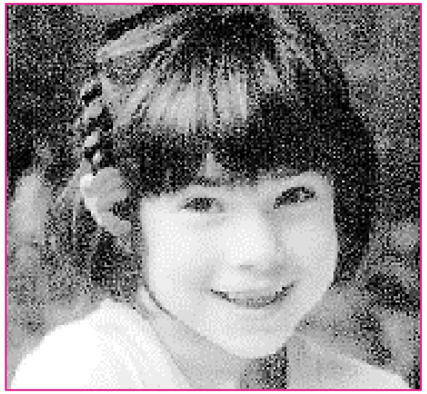

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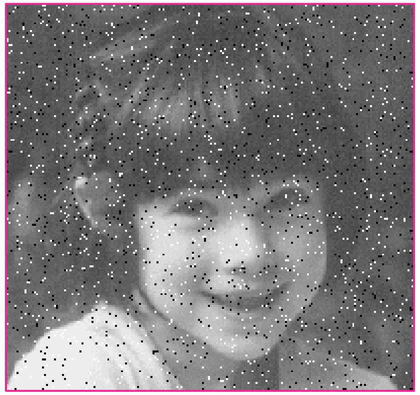

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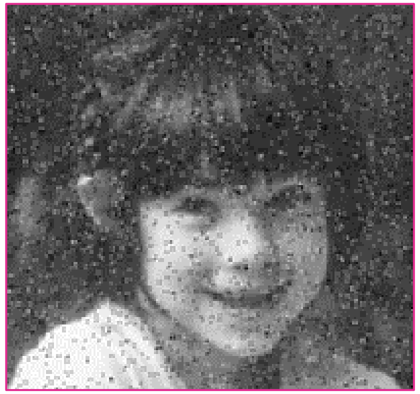

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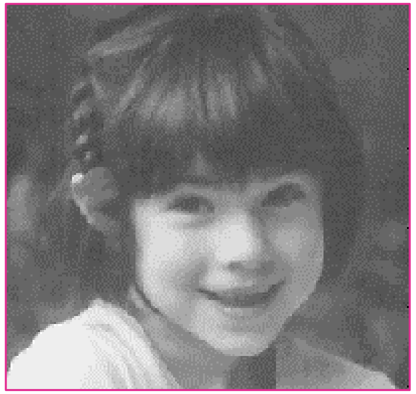

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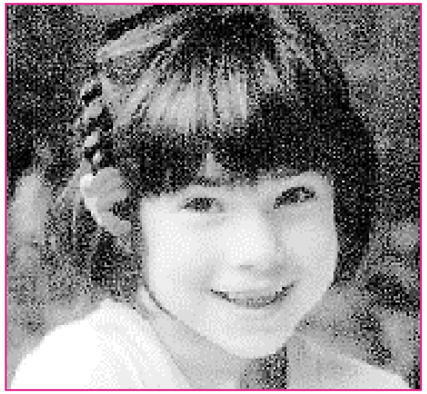

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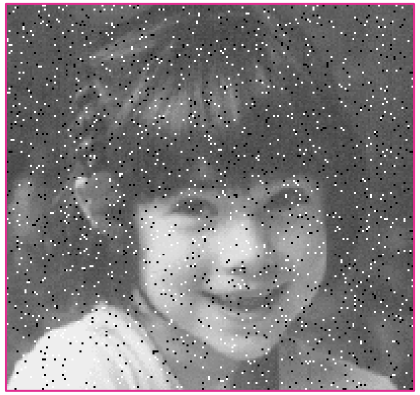

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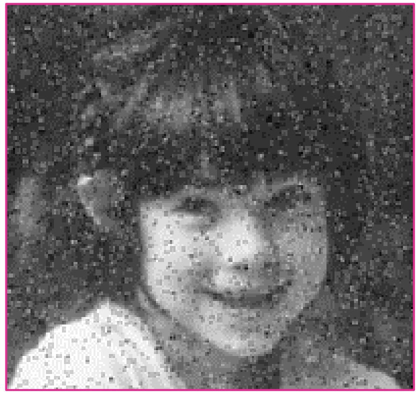

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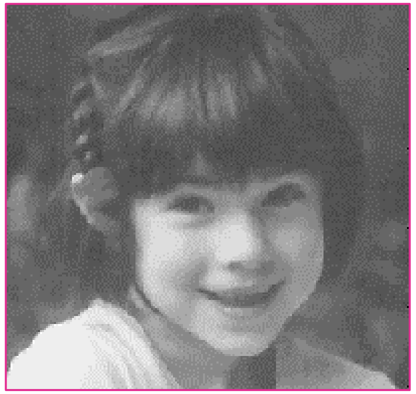

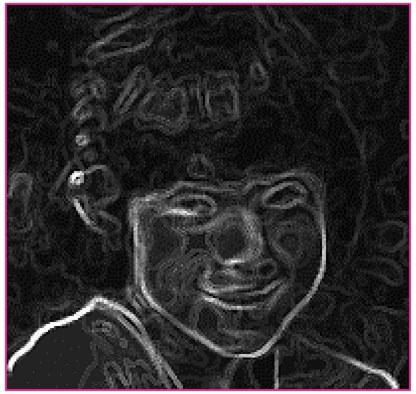

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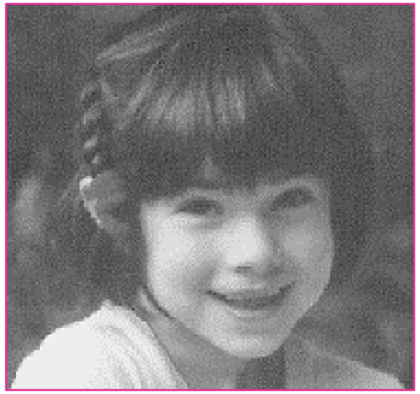

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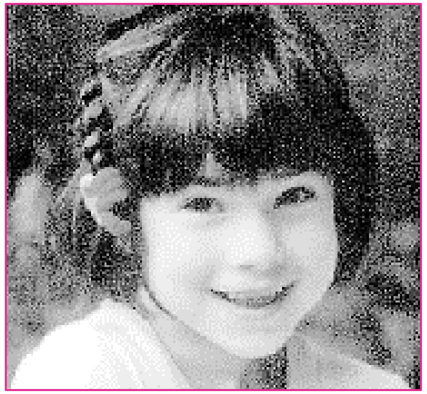

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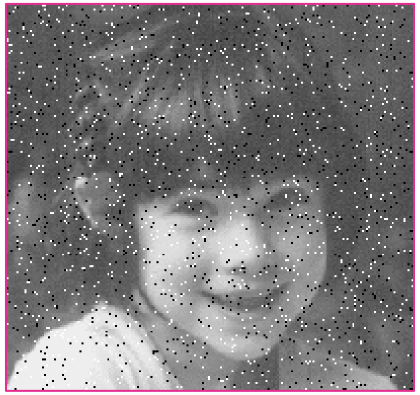

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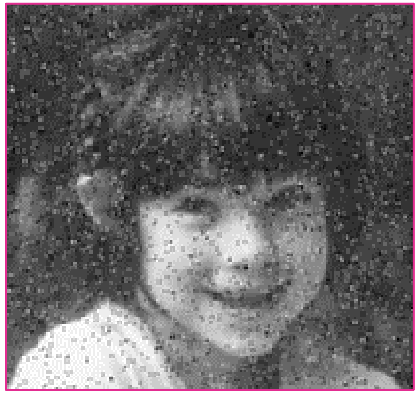

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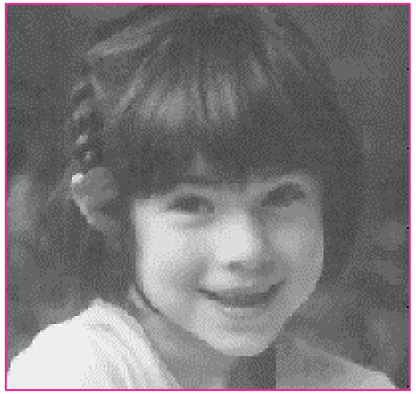

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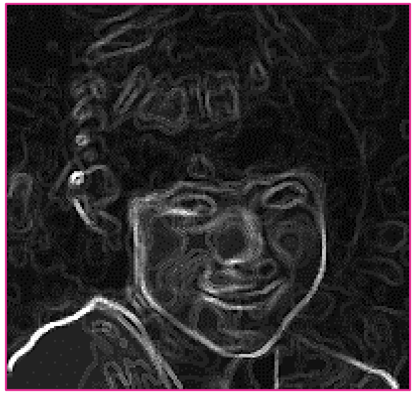

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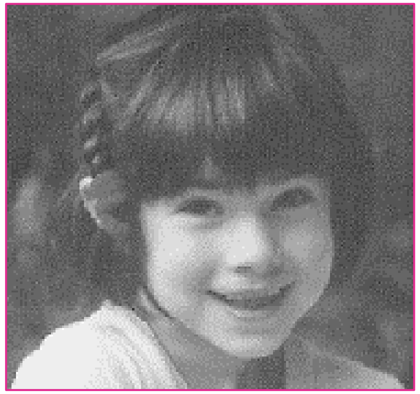

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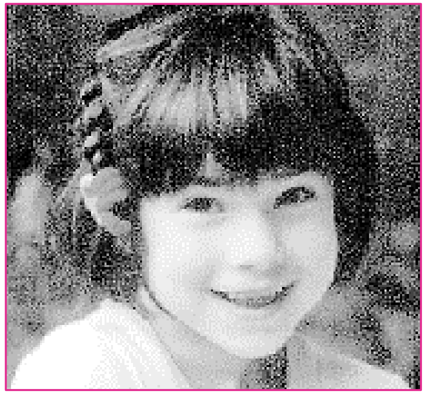

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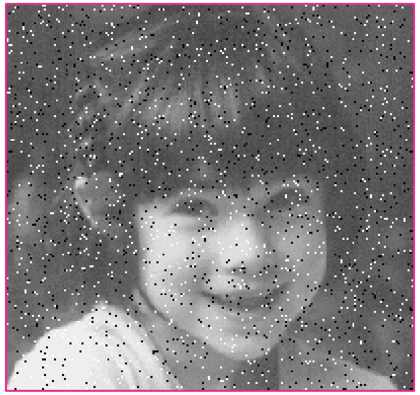

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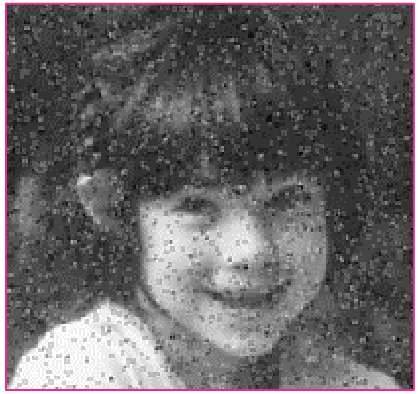

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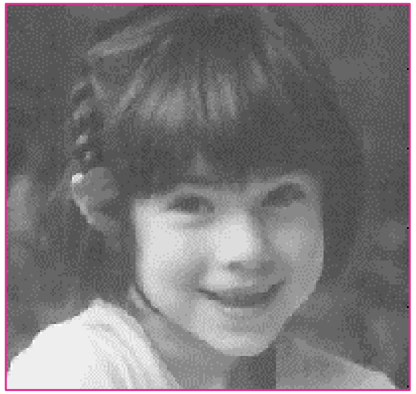

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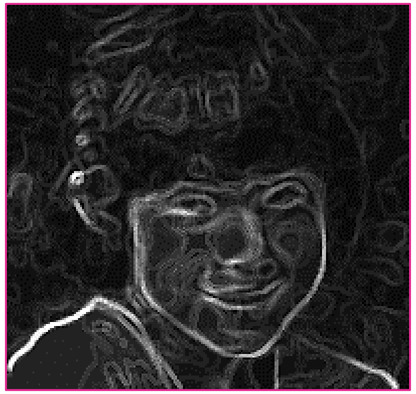

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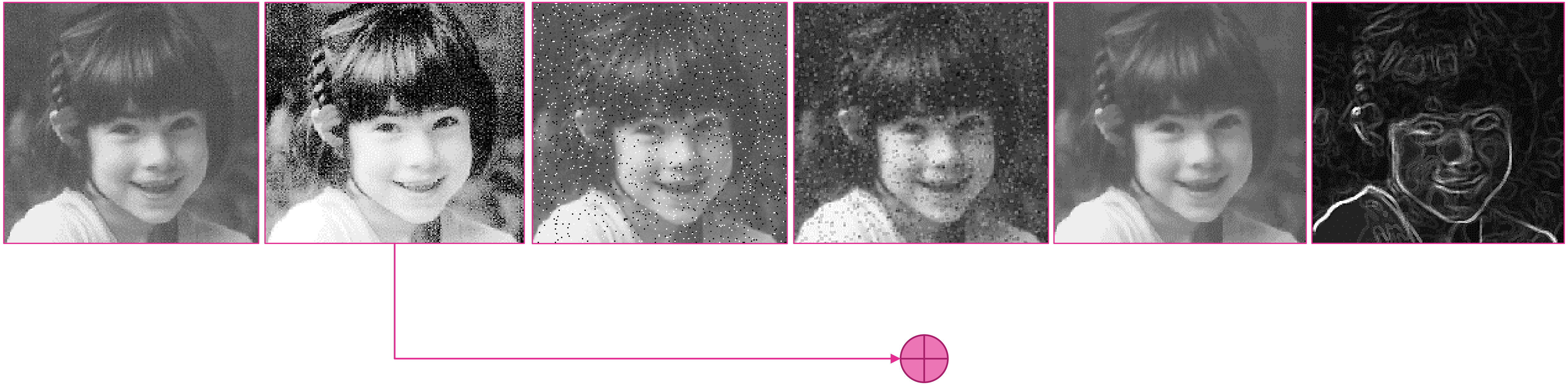

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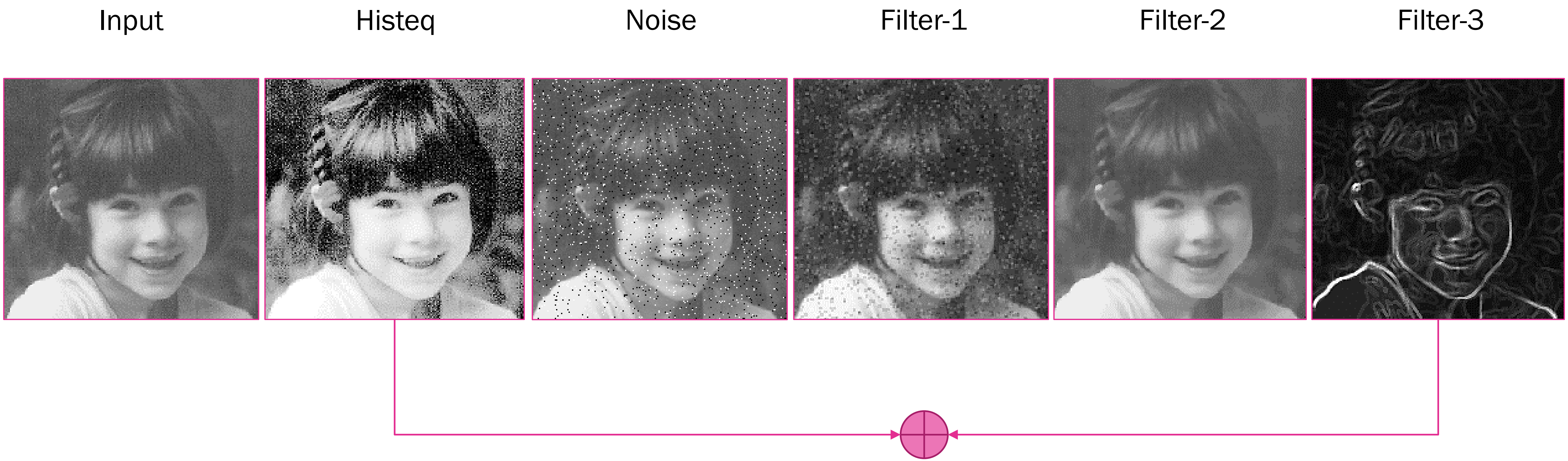

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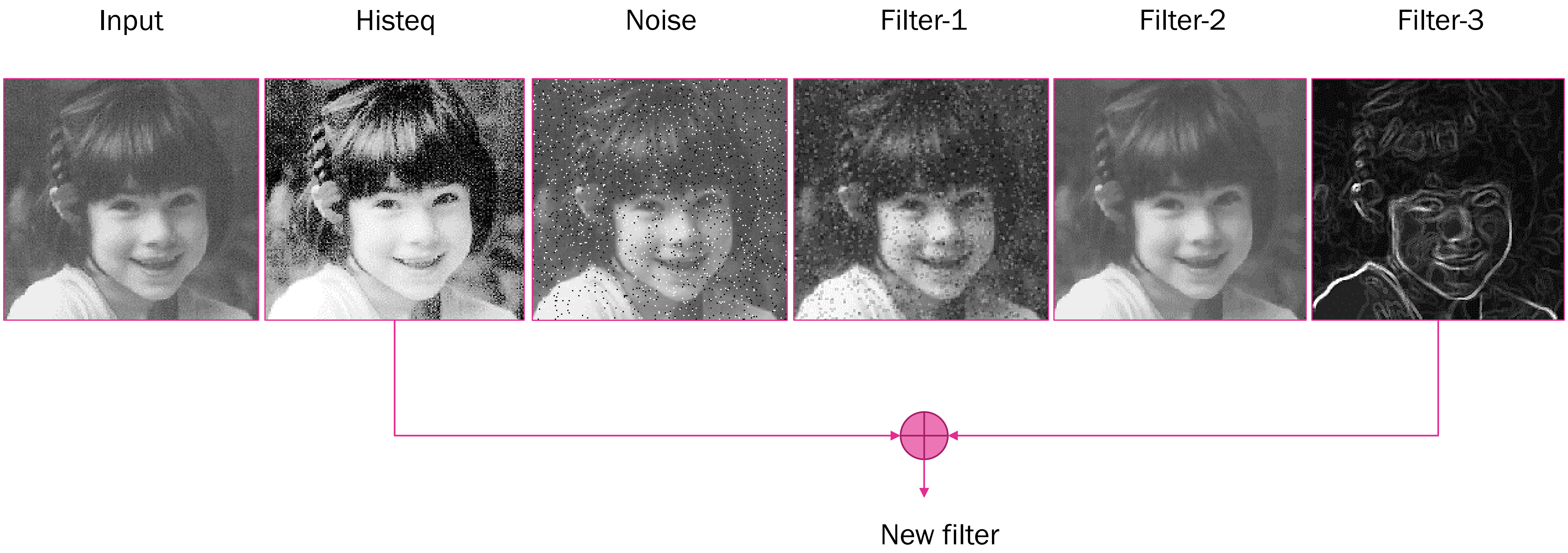

Input

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Noise

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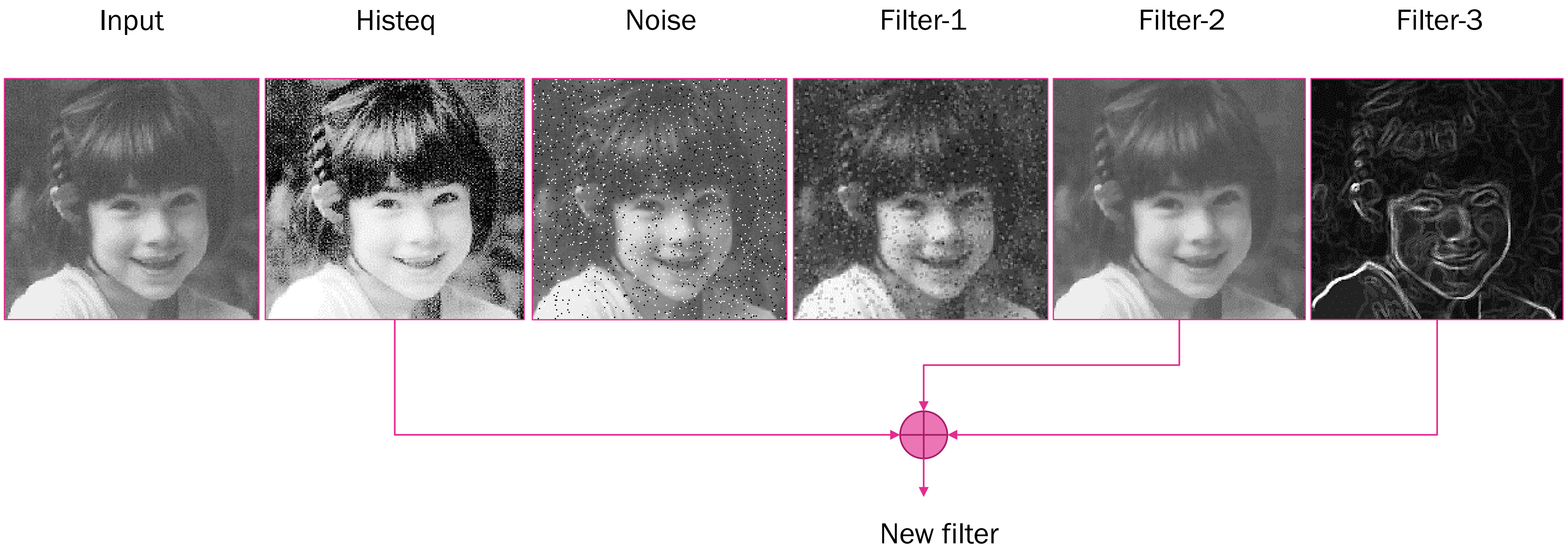
Filter-3

New filter

Introduction



Input

Histeq

Noise

Filter-1

Filter-2

Filter-3

New filter

Linearity

- Operations
  - linear
    - additivity

$$\mathcal{T}[a \cdot f_1(x,y) + b \cdot f_2(x,y)] = \mathcal{T}[a \cdot f_1\ (x,y)] + \mathcal{T}[b \cdot f_2\ (x,y)]$$

    - homogeneity

$$\mathcal{T}[a \cdot f_1(x,y)] = a \cdot \mathcal{T}[f_1(x,y)]$$

  - non-linear
    - not satisfying above

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- Examples
  - linear
    - negatives
  - non-linear
    - gammas

Correlation & Convolution

- Correlation
  - measures similarity between the two signals
  - windowed signal (kernel) is not reversed
  - sliding vectors dot product
  - orthogonal signals are uncorrelated

Correlation & Convolution

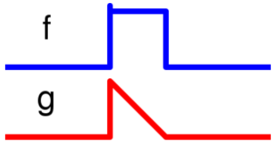
- Correlation
  - measures similarity between the two signals
  - windowed signal (kernel) is not reversed
  - sliding vectors dot product
  - orthogonal signals are uncorrelated

- Convolution
  - measure the effect of one signal on the another
  - windowed signal (kernel) is reversed
    - for symmetric kernels convolution = correlation

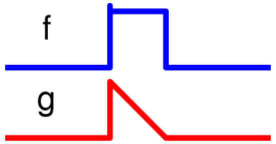
‘✩, ⊛, ✲, ✺, ✶, ✷, ✯, ✭, ✬, ☆, ✡, ✰, ✱, ✻, ✪’

Correlation & Convolution

- Correlation



- Convolution



credit: wikipedia

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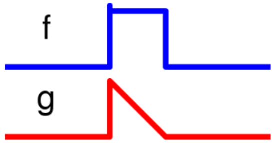
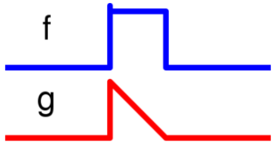


$$R(x) = f(x) \star g(x)$$

$$R(x) = \int_{-\infty}^{\infty} f(z)g(x+z)dz$$

- Convolution



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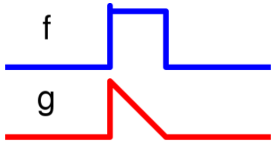

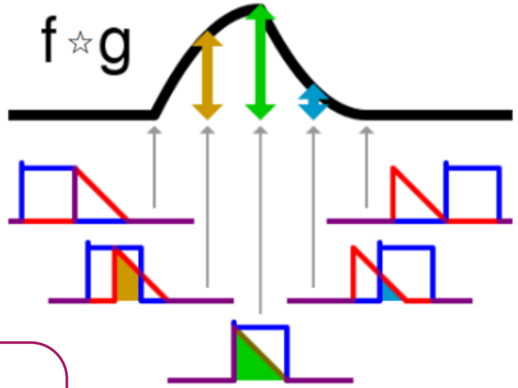

$$R(x) = f(x) \star g(x)$$

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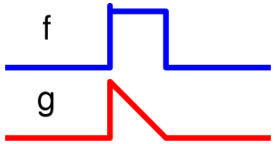
- Convolution



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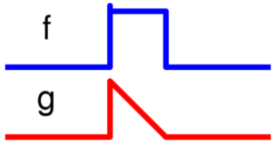
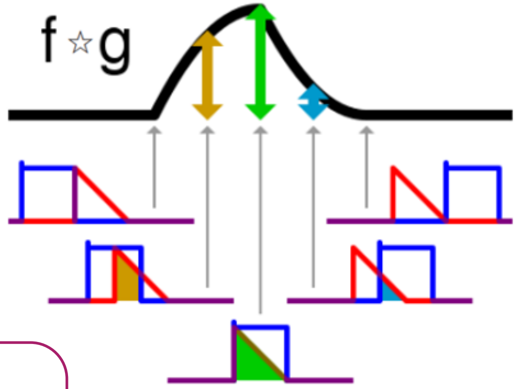
Correlation & Convolution

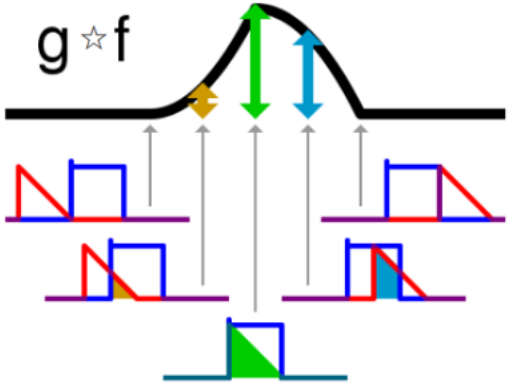
- Correlation





$$R(x) = f(x) \star g(x)$$

$$R(x) = \int_{-\infty}^{\infty} f(z)g(x+z)dz$$



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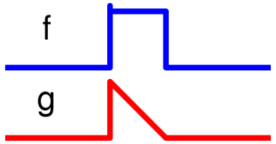

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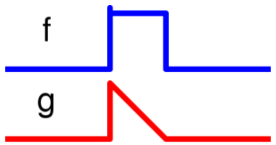
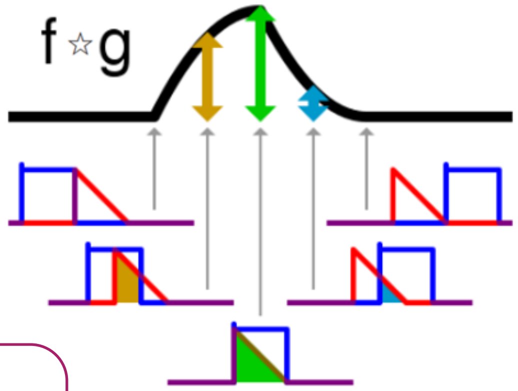
- Correlation

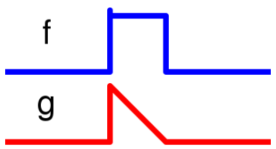




$$R(x) = f(x) \star g(x)$$

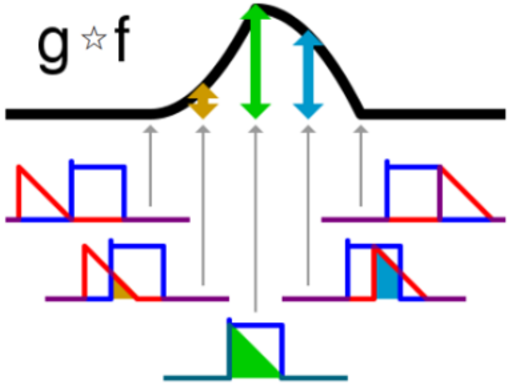
$$R(x) = \int_{-\infty}^{\infty} f(z) g(x+z) dz$$

- Convolution



$$G(x) = f(x) \star g(x)$$

$$G(x) = \int_{-\infty}^{\infty} f(z) g(x-z) dz$$



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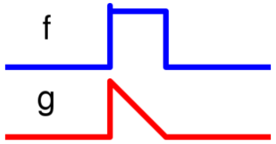
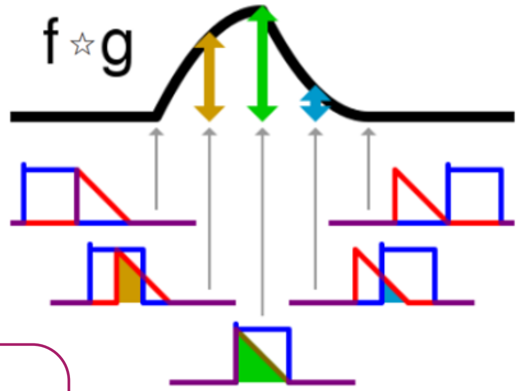
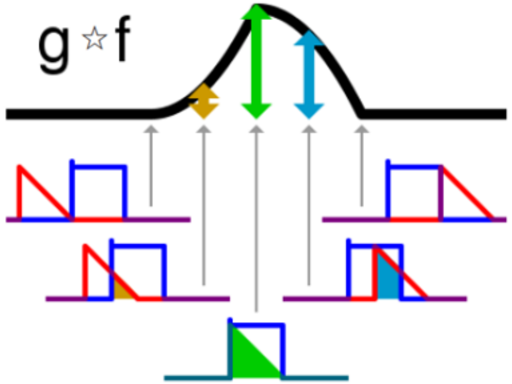
- Correlation



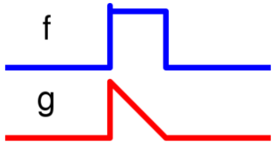


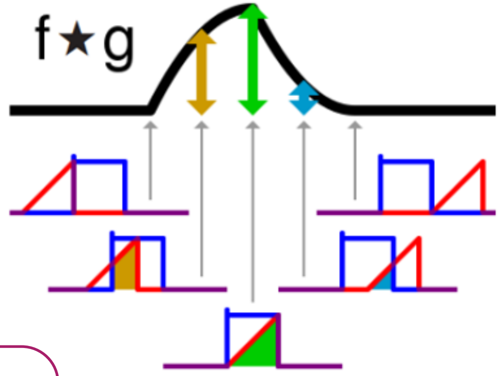
$$R(x) = f(x) \mathbin{\text{☆}} g(x)$$

$$R(x) = \int_{-\infty}^{\infty} f(z)g(x+z)dz$$



- Convolution





$$G(x) = f(x) \star g(x)$$

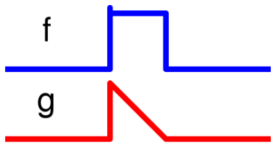
$$G(x) = \int_{-\infty}^{\infty} f(z)g(x-z)dz$$

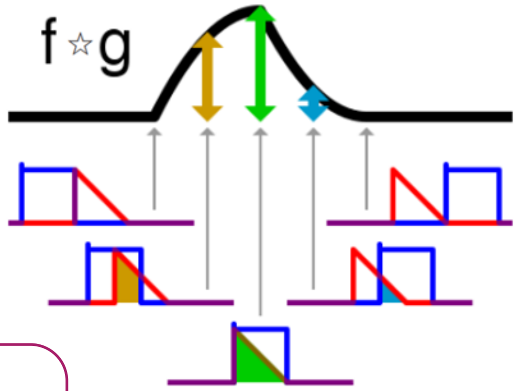
credit: wikipedia

‘✯, ⊛, ✲, ✺, ✶, ✳, ✯, ✬, ✪, ☆, ✡, ✰, ✱, ❋, ✪’

Correlation & Convolution

- Correlation





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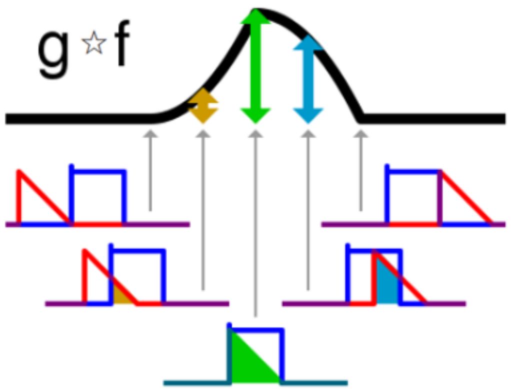
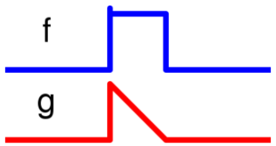
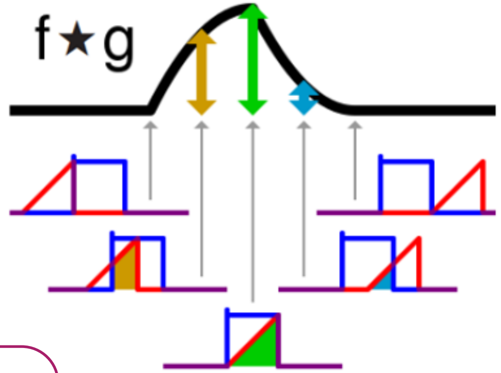


- Convolution





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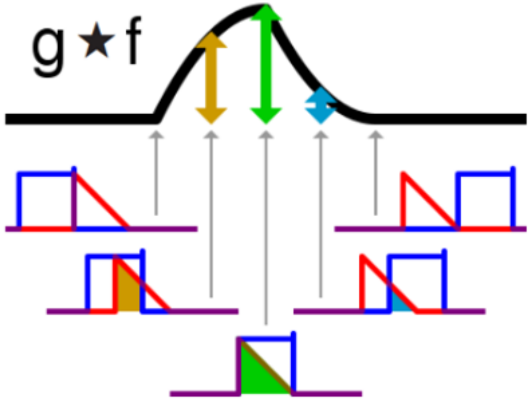

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Correlation & Convolution

- 2D correlation
  - cross-correlation
  - filtering algos internally use it
    - w need to be appropriately reflected before filtering

$$(w \star f)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

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- 2D convolution
  - $w \;\rightarrow m \times n$
  - $a = \frac{m-1}{2}, b = \frac{n-1}{2}$
    - a, b are assumed to be odd integers
    - note the kernels do not depend on (x, y)

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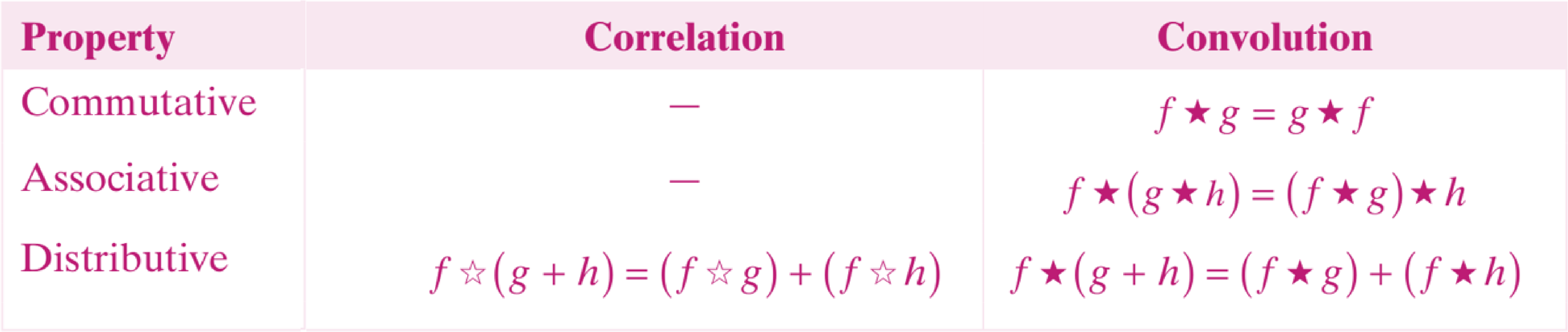

Property	**Correlation**	**Convolution**
Commutative	—	$f \bigstar g = g \bigstar f$
Associative	—	$f \bigstar (g \bigstar h) = (f \bigstar g) \bigstar h$
Distributive	$f \mathbin{\text{☆}} (g+h) = (f \mathbin{\text{☆}} g) + (f \mathbin{\text{☆}} h)$	$f \bigstar (g+h) = (f \bigstar g) + (f \bigstar h)$

Filtering

- Image filtering
  - spatial filtering
  - convolving a kernel with an image
  - filtering: $g(x, y) = (w \star f)(x, y)$

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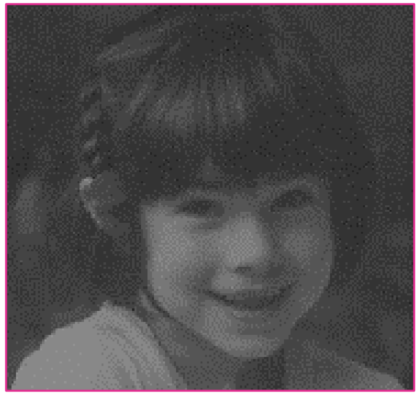
Input

Filter-1



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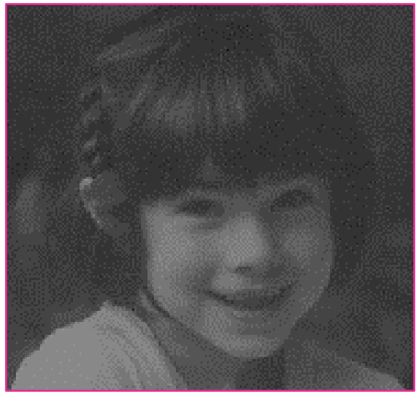

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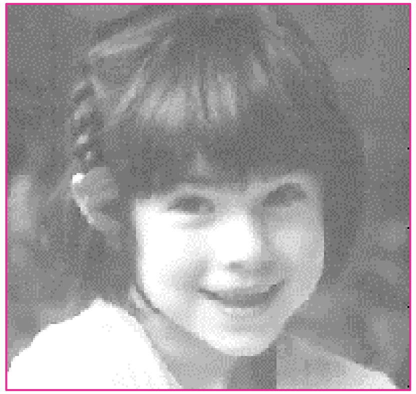
Filter-2



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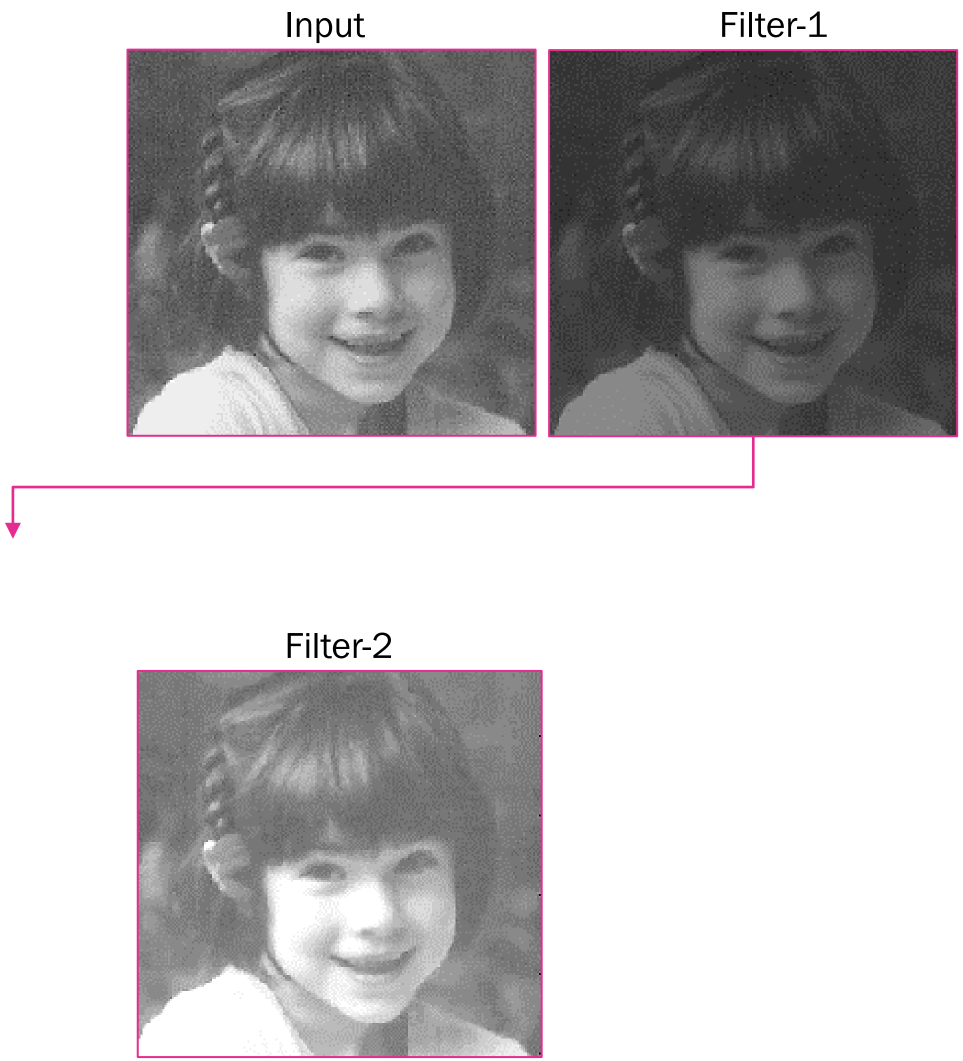

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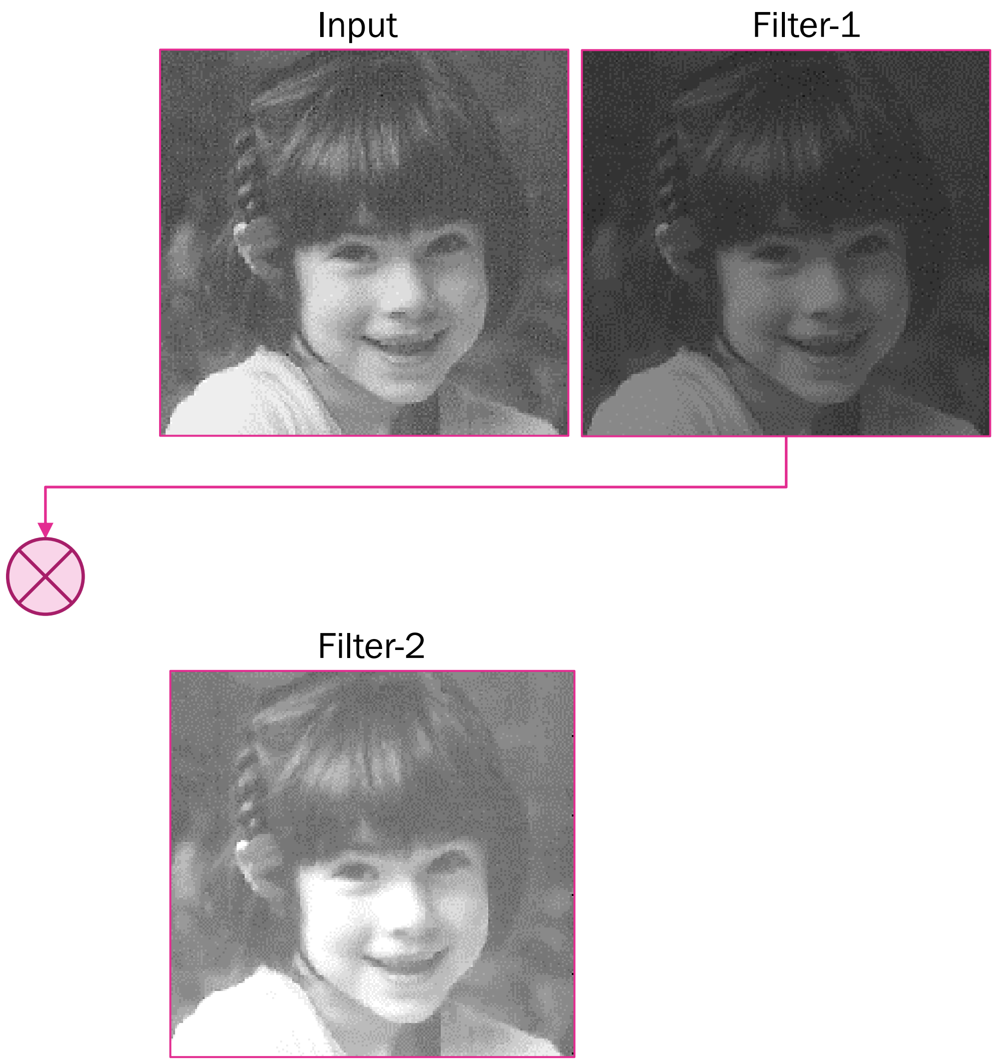

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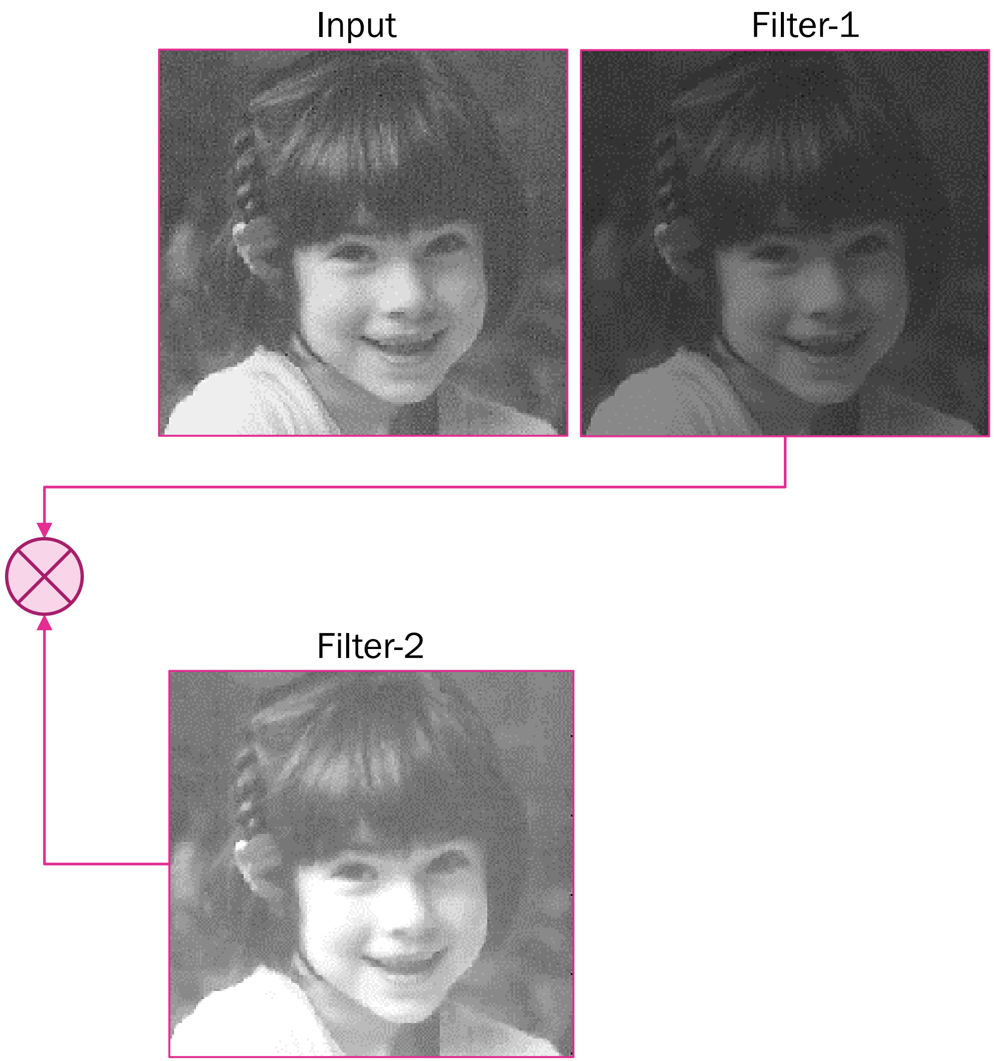

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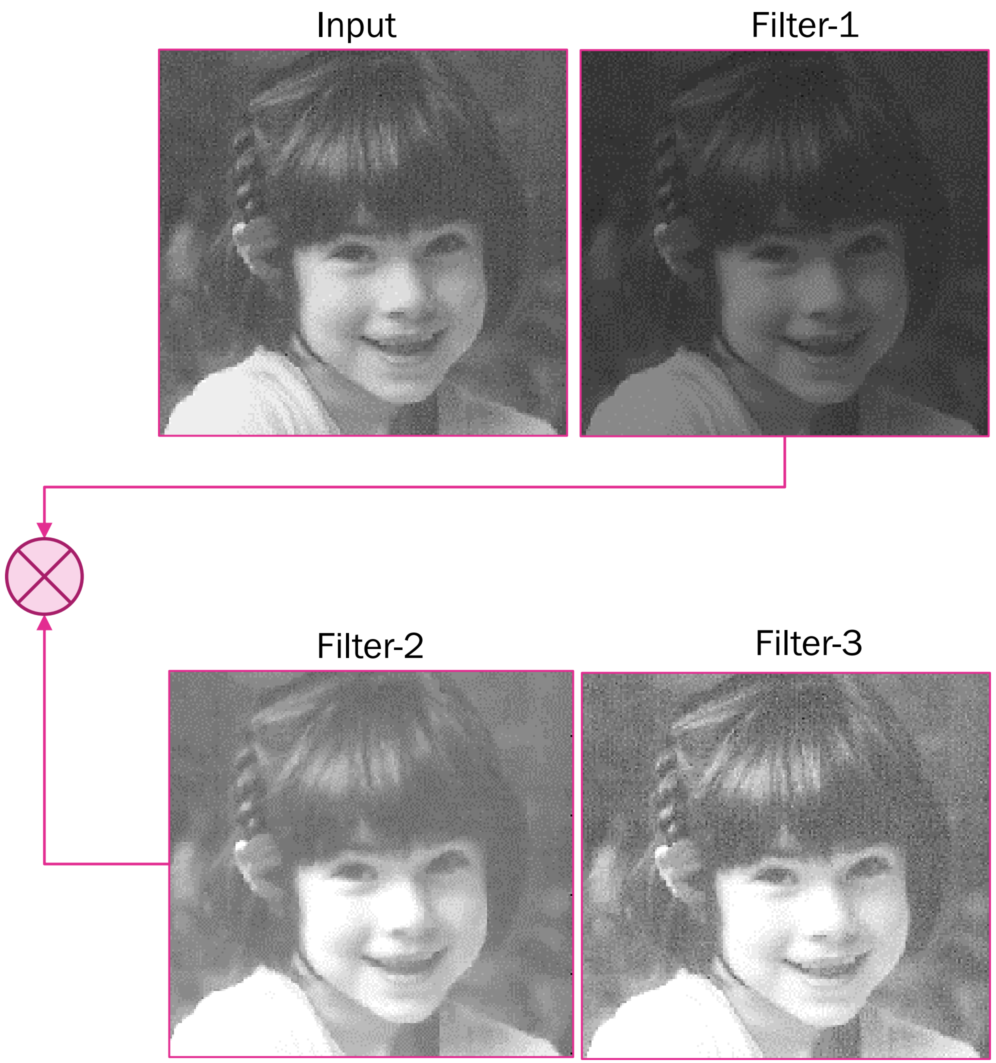

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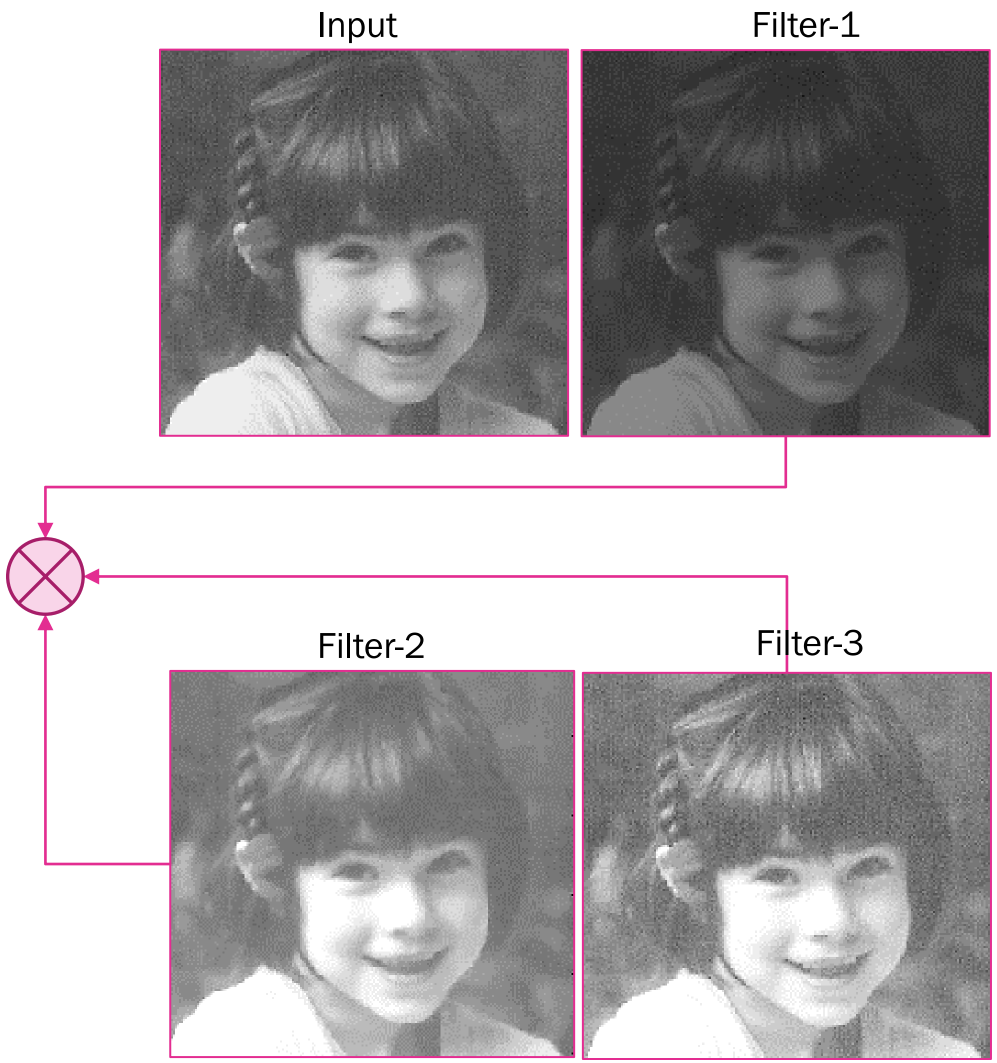

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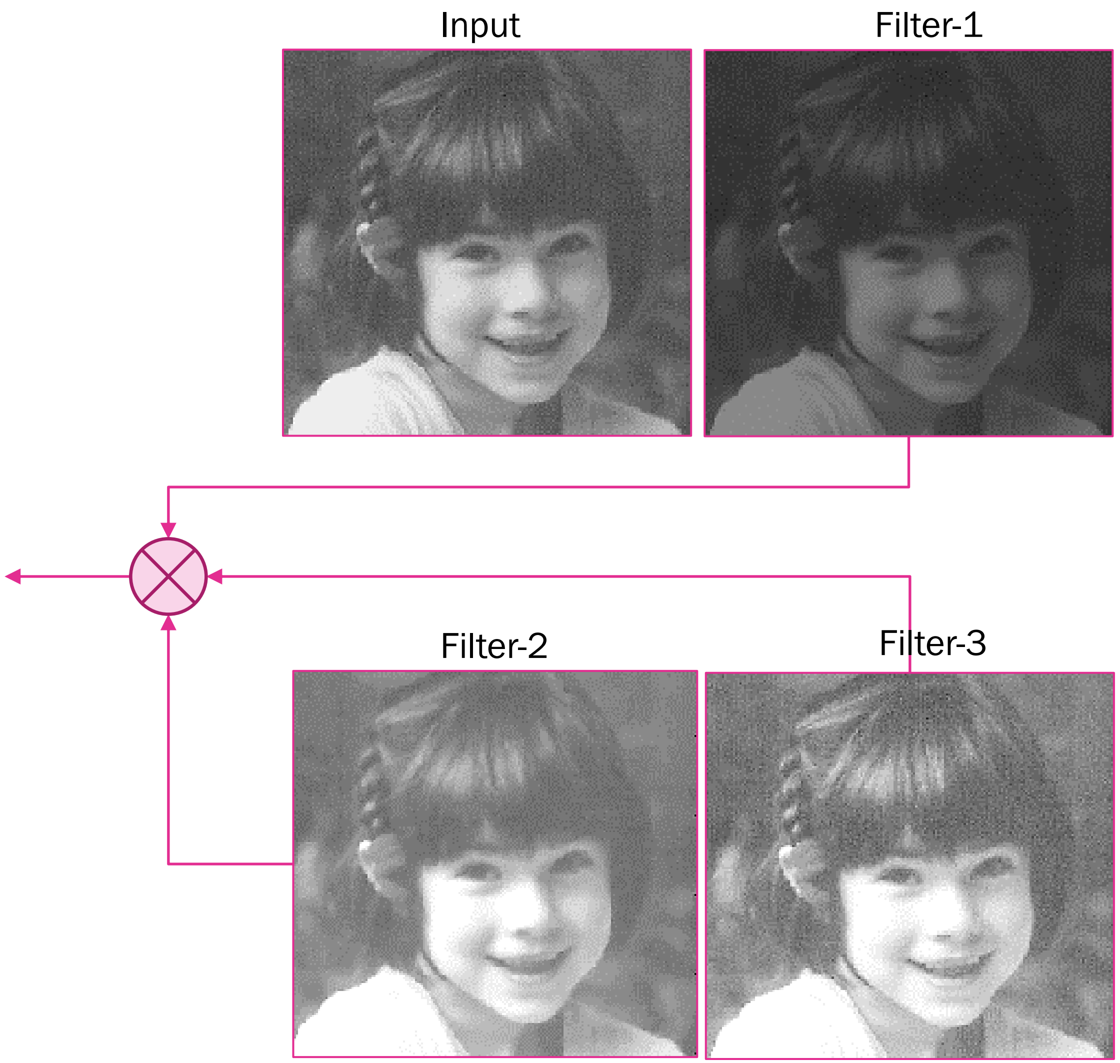

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$$g(x,y) = w_3 \star w_2 \star w_1 \star f(x,y)$$

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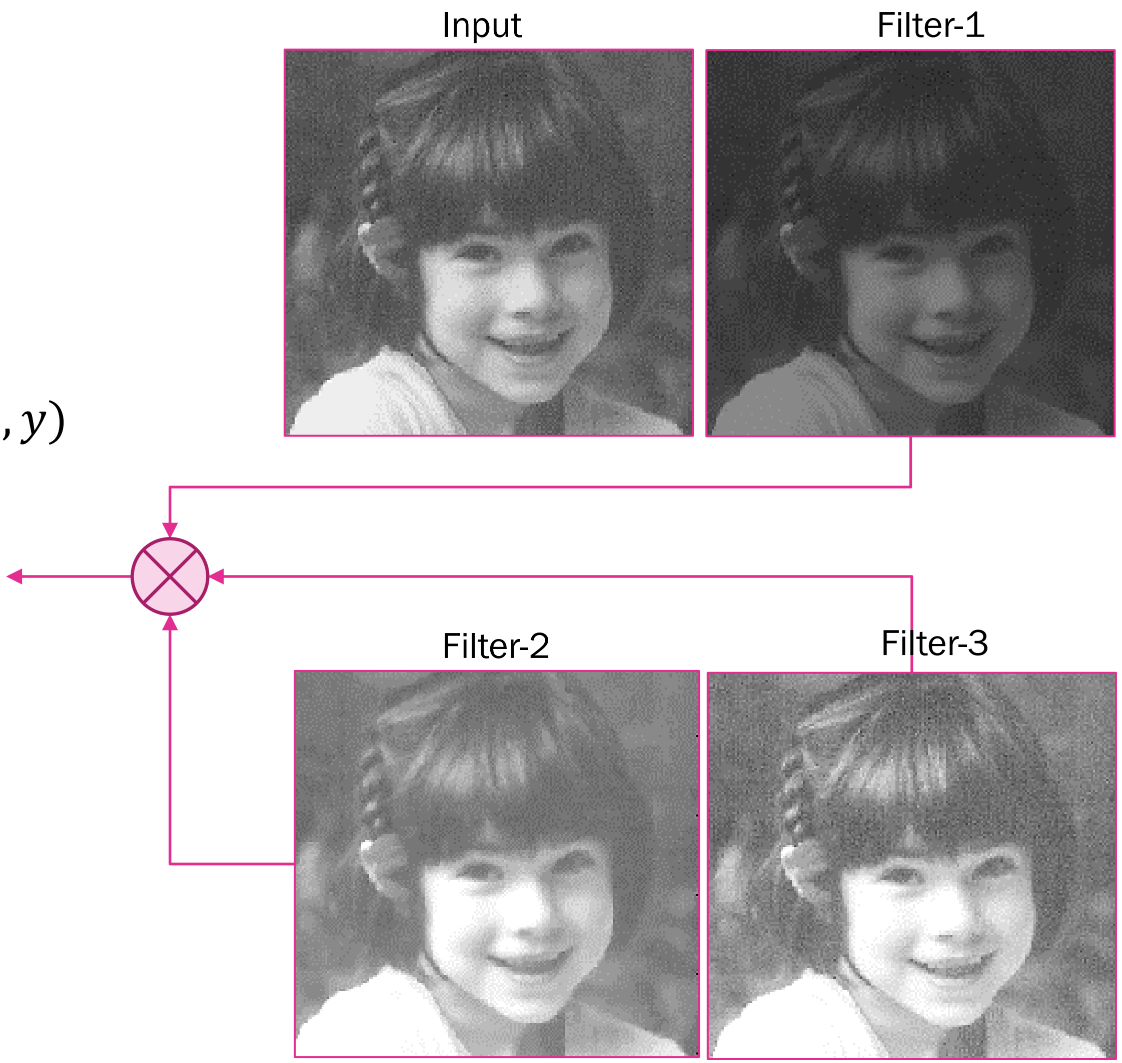

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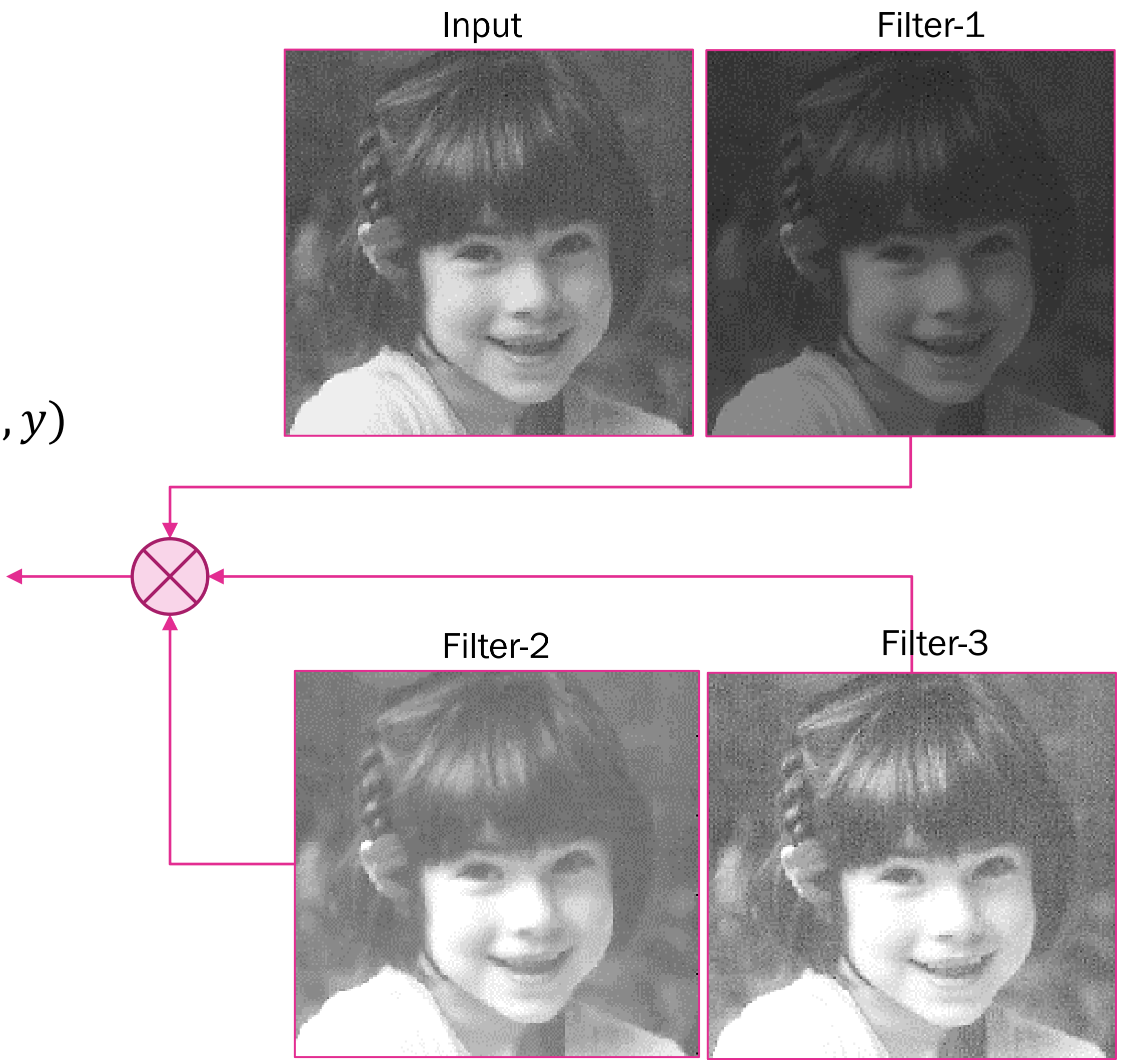

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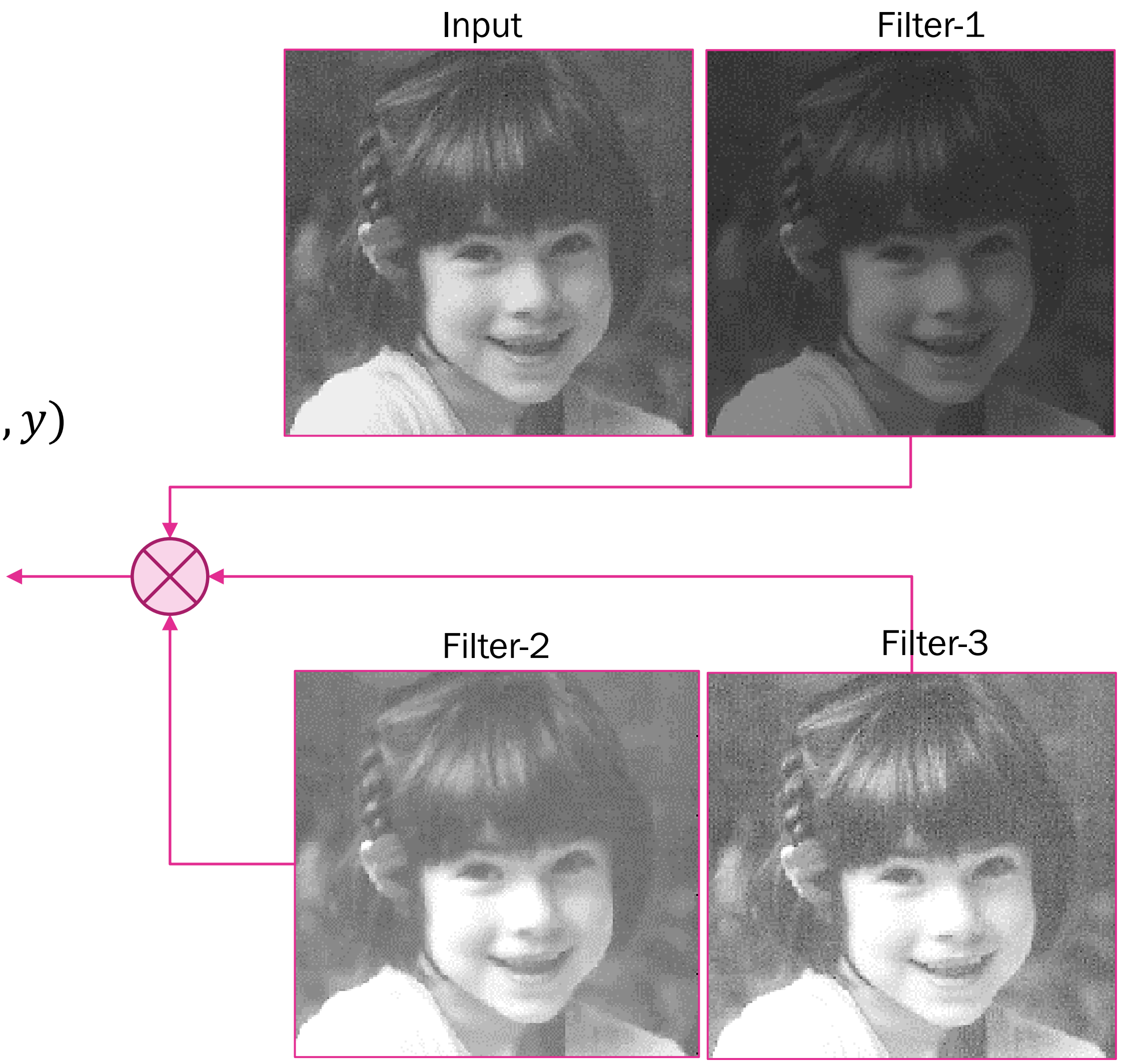
$$(w_1 \star w_2 \star w_3) \star f(x, y)$$



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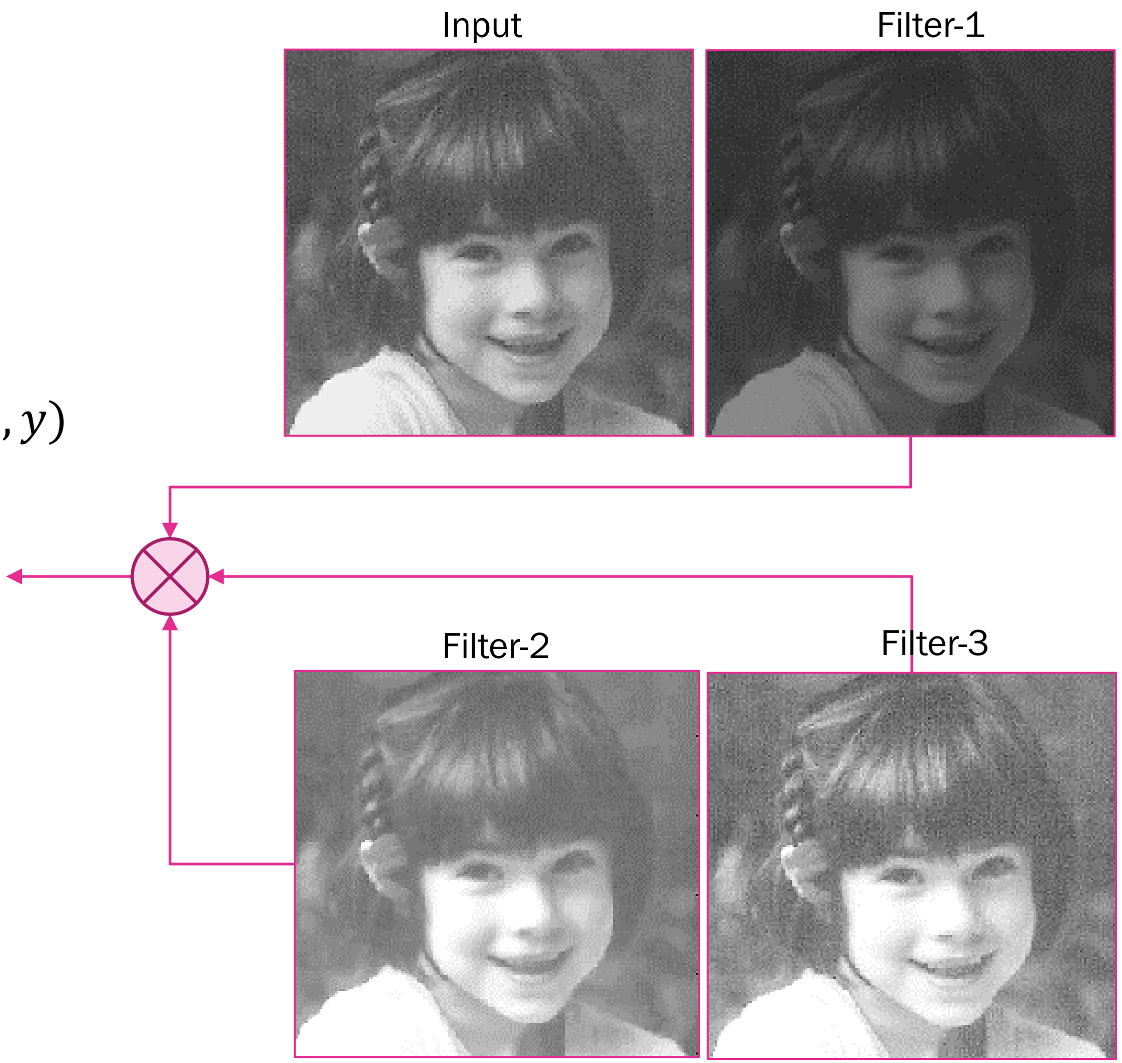
$$g(x,y) = w \star f(x,y)$$

$$w = w_1 \star w_2 \star w_3$$



Filtering

- Filter
  - kernel, mask, window, template
  - $w(i,j)$ or $k(i,j) \;\; \forall\, i,j \in N_K,\;\; K$- kernel size
    - K : determine neighbourhood of operation
    - $w(i,j)$: filter coefficients – determine nature of the filter

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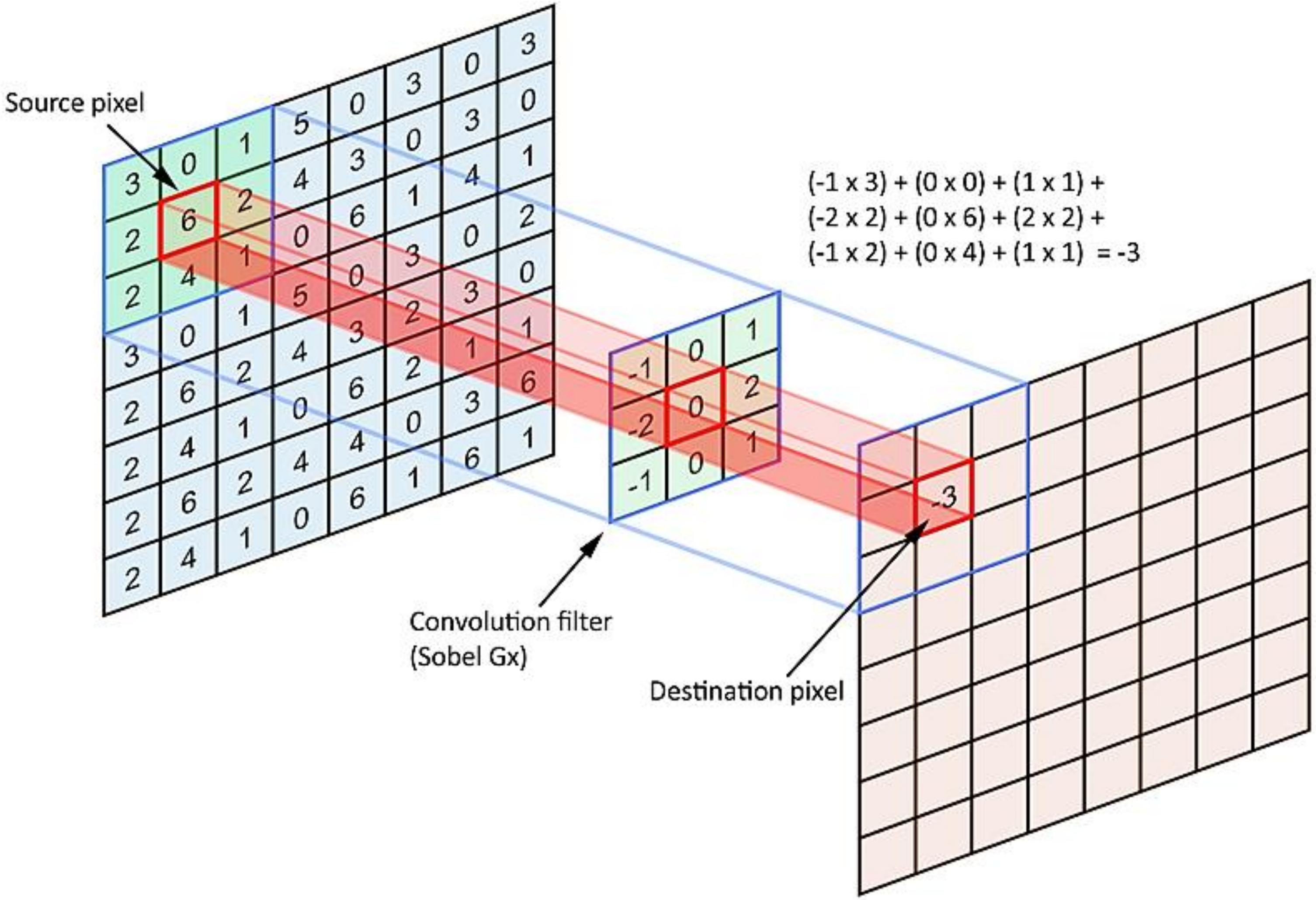
- Nature of a filter
    - neighbour interactions
        - filter coefficients define severity of interaction
    - smoothing
    - sharpening
    - noise handling capacity

Filtering



3	0	1	5	0	3	0	3
2	6	2	4	3	0	3	0
2	4	1	0	6	1	4	1
3	0	1	5	0	3	0	2
2	6	2	4	3	2	3	0
2	4	1	0	6	2	1	1
2	6	2	4	4	0	3	6
2	4	1	0	6	1	6	1

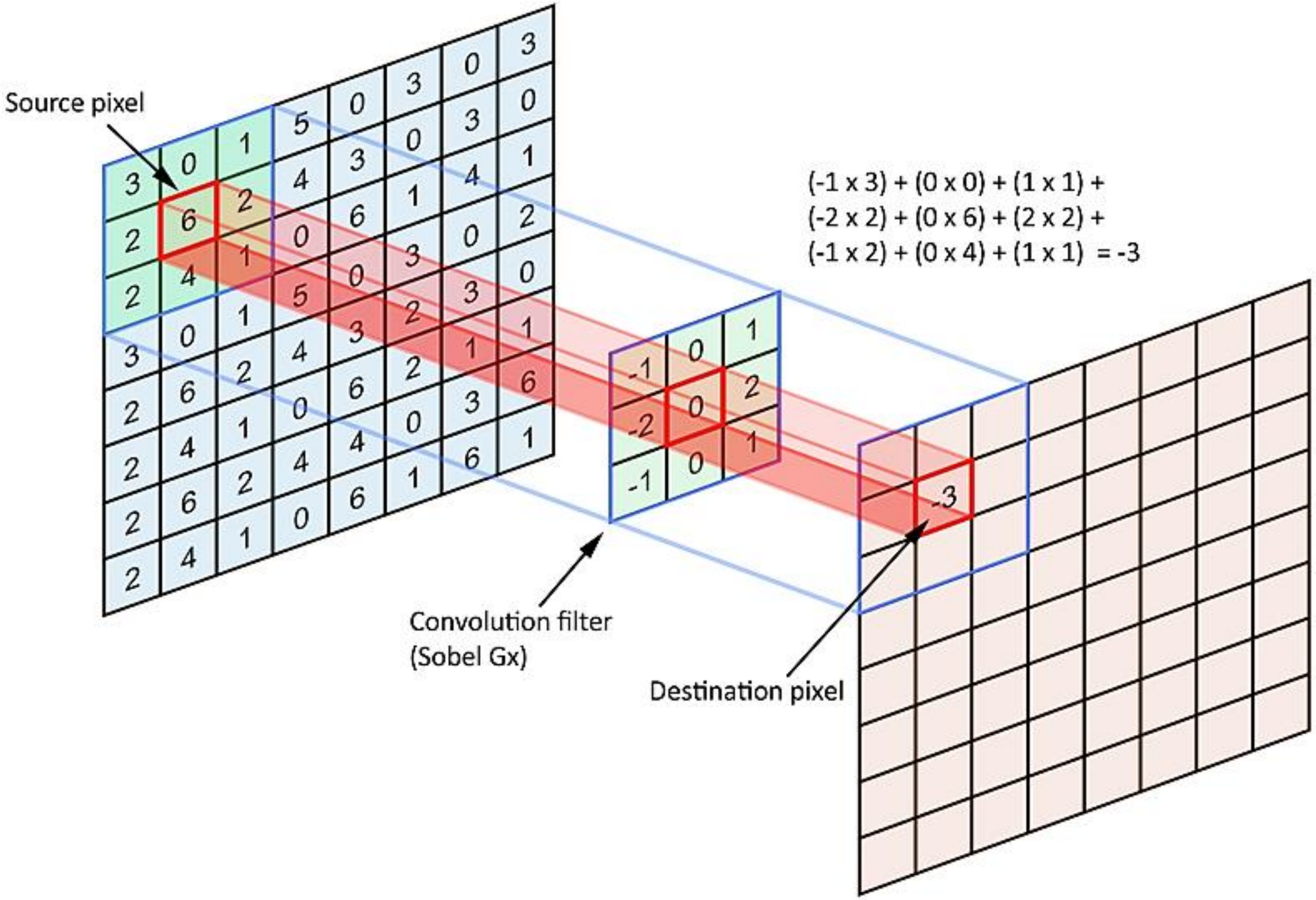
-1	0	1
-2	0	2
-1	0	1

-3

(-1 x 3) + (0 x 0) + (1 x 1) +
(-2 x 2) + (0 x 6) + (2 x 2) +
(-1 x 2) + (0 x 4) + (1 x 1) = -3

Filtering

- Paddings
  - zero
  - mirror
  - replicate



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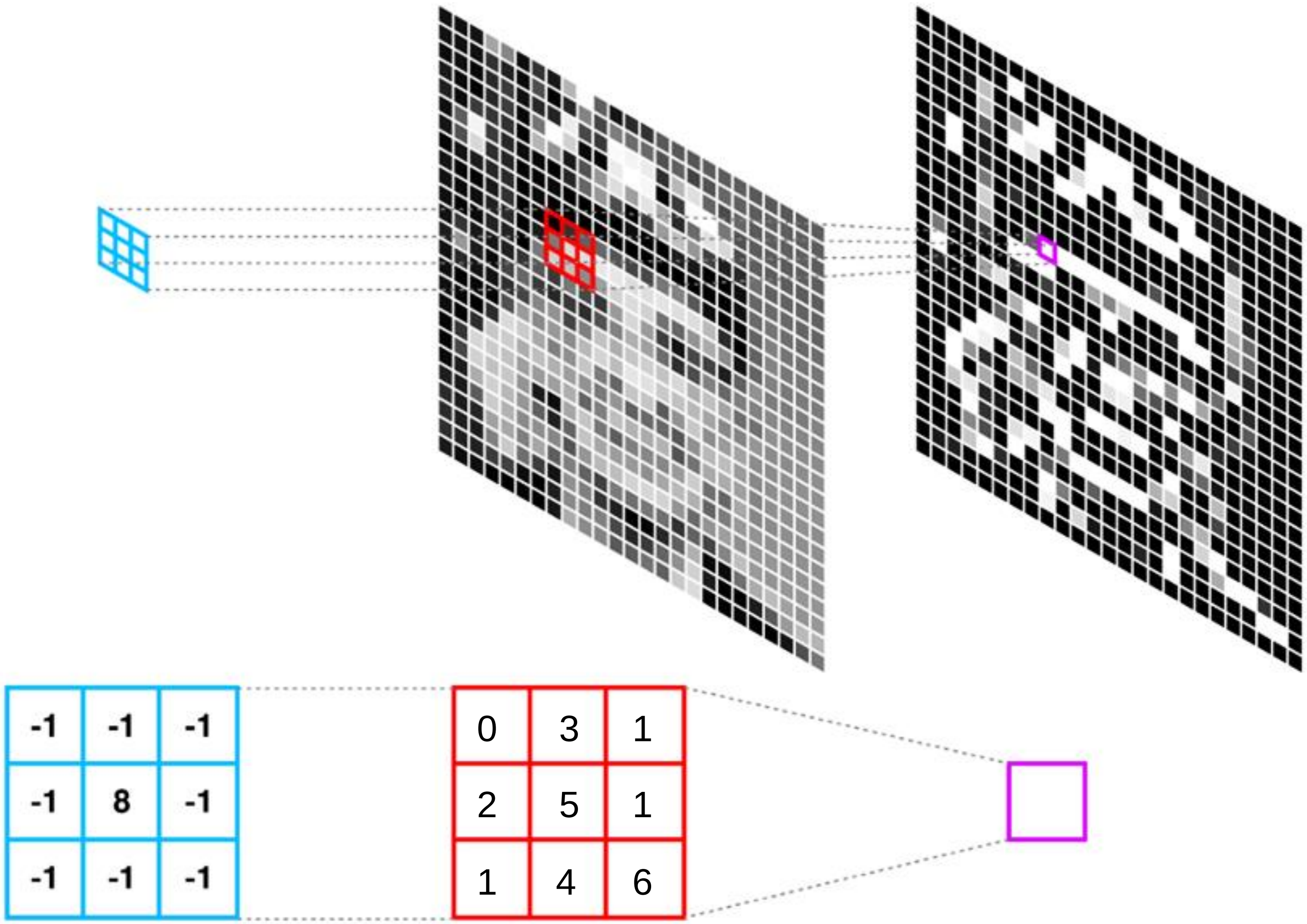
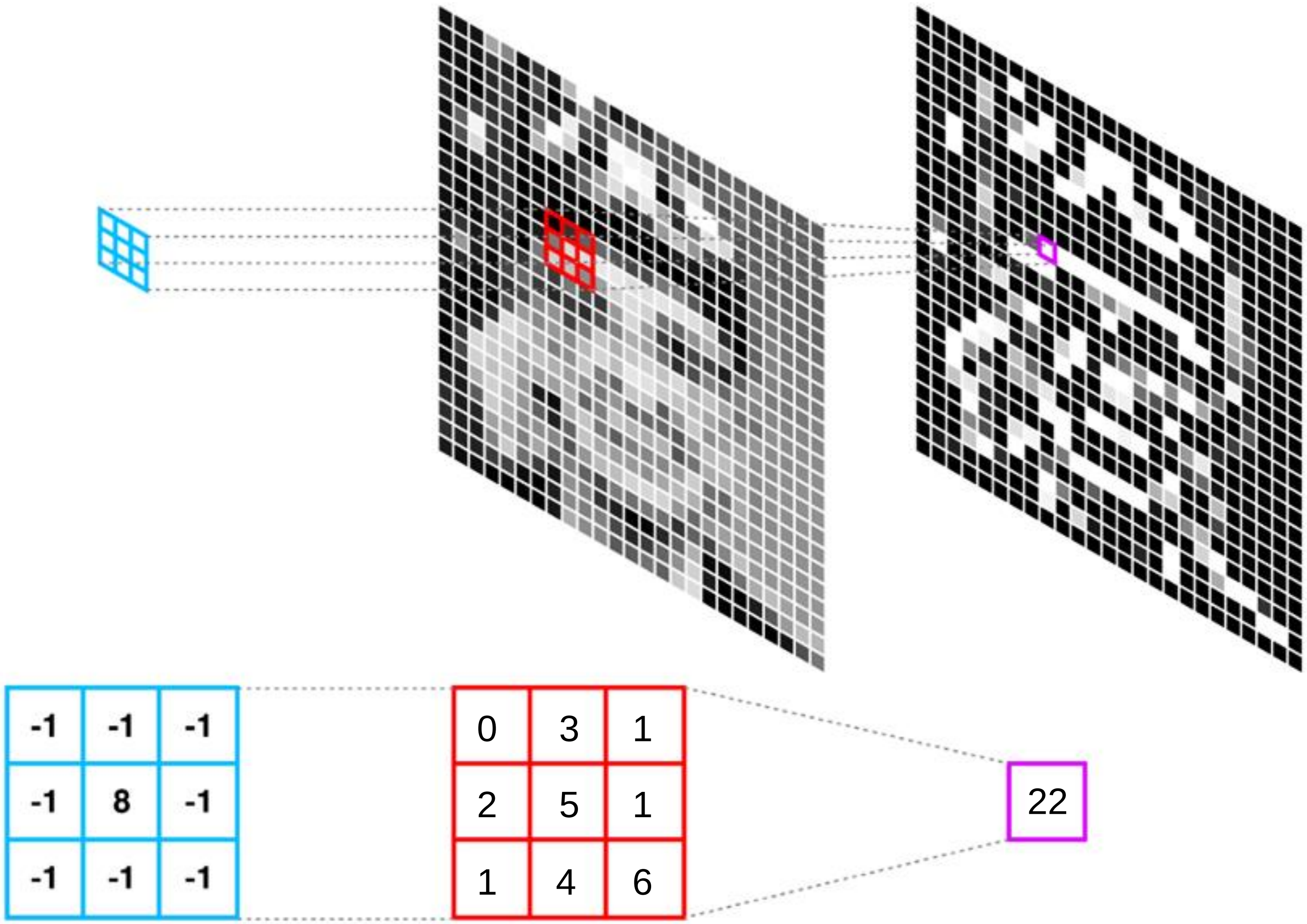


-1	-1	-1
-1	8	-1
-1	-1	-1

0	3	1
2	5	1
1	4	6

Filtering



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-1	-1	-1

0	3	1
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1	4	6

22

Filtering

- Separable kernels
  - a kernel in a matrix form can be represented as outer product of two vectors
  - $w = uv^T$
    - $u \in m \times 1$
    - $v \in n \times 1$
    - sq. kernels $w = uu^T,\ w \in m \times m$

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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    - outer product of vectors is same as their 2D conv

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$$\mathbf{c}\,\mathbf{r}^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = w$$

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    - advantage factor $= \dfrac{mn}{m+n}$

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Filtering

- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Filtering

- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

- Use cases
  - random noise reduction
    - reducing sharp transitions in intensity
    - favours blurring along perpendicular directions
  - reduce aliasing
    - smoothing prior to resampling
  - reduce quantization noise
    - reduce false contours of intensities
  - essential in composite filtering
    - multistage filters

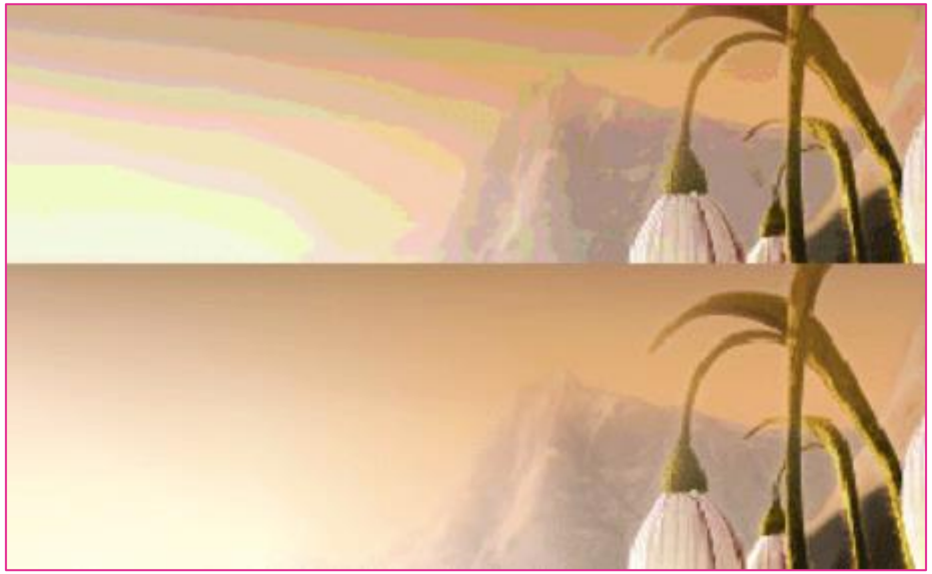
$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Filtering

- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

- Use cases
  - random noise reduction
    - reducing sharp transitions in intensity
    - favours blurring along perpendicular directions
  - reduce aliasing
    - smoothing prior to resampling
  - reduce quantization noise
    - reduce false contours of intensities
  - essential in composite filtering
    - multistage filters

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$



Filtering

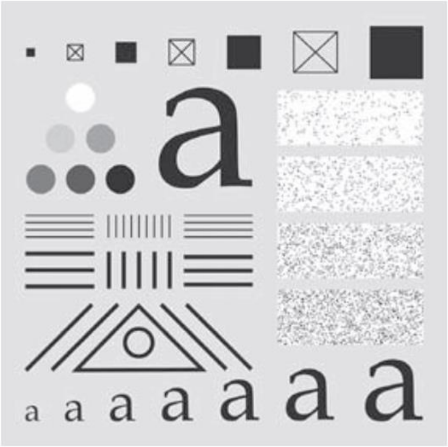
- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Filtering

- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

input

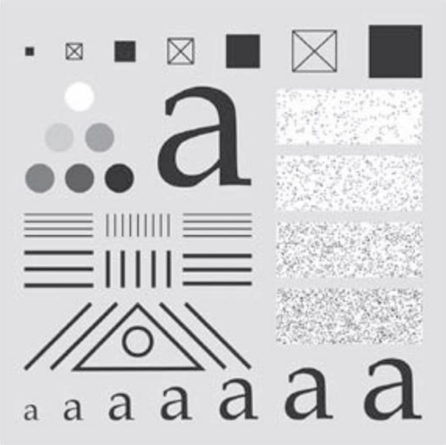


$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Filtering

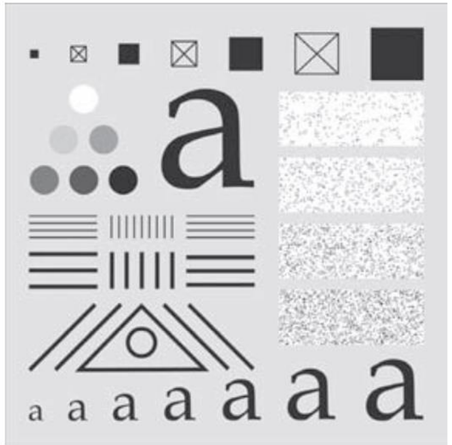
- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

input



$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

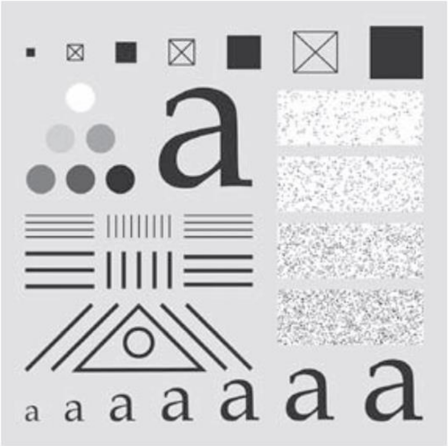
m=3



Filtering

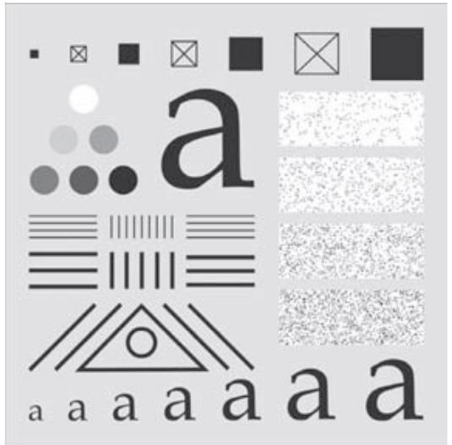
- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

input

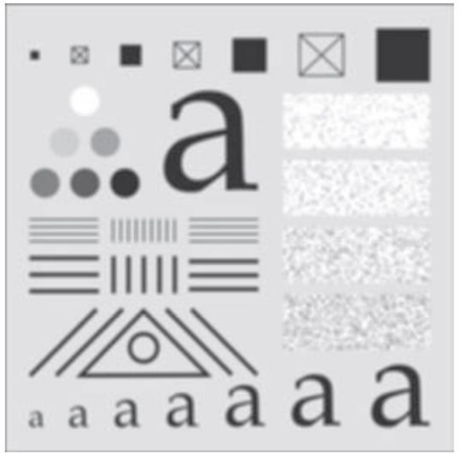


$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1\\ \hline 1 & 1 & 1\\ \hline 1 & 1 & 1\\ \hline\end{array}$$

m=3



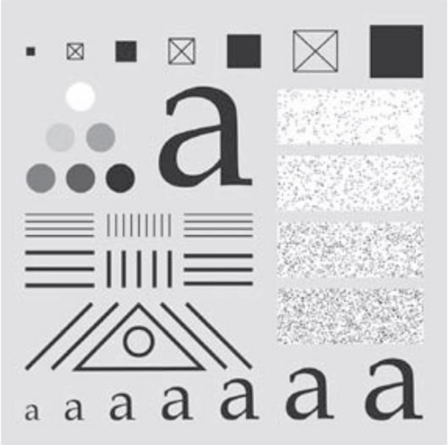
m=11



Filtering

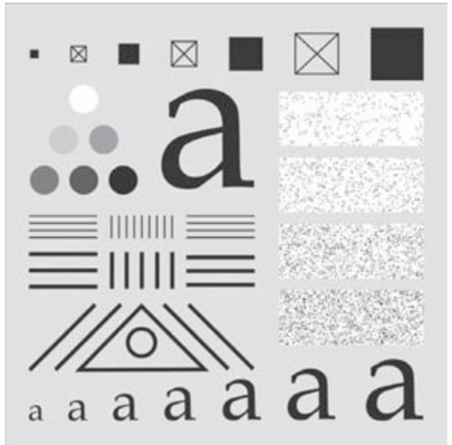
- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

input

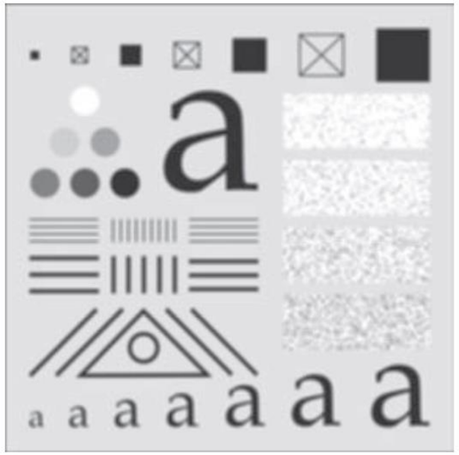


$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$

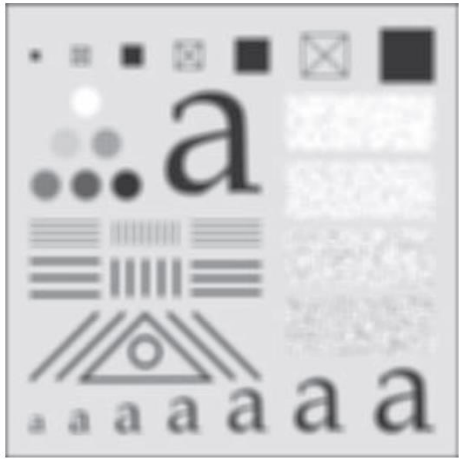
m=3



m=11



m=21



Filtering

- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

$$w(s,t) = G(s,t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$

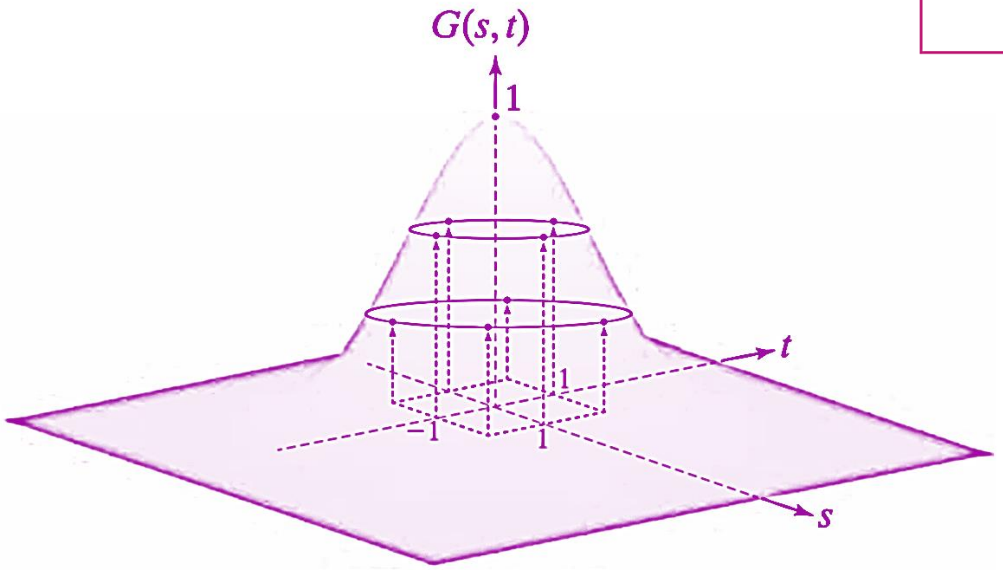
$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

Filtering

- Gaussian filter
    - smoothing filter
    - defocused lens approximators
    - isotropic
        - response is independent of orientation
        - circularly symmetric

$$w(s,t) = G(s,t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$

$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$



Filtering

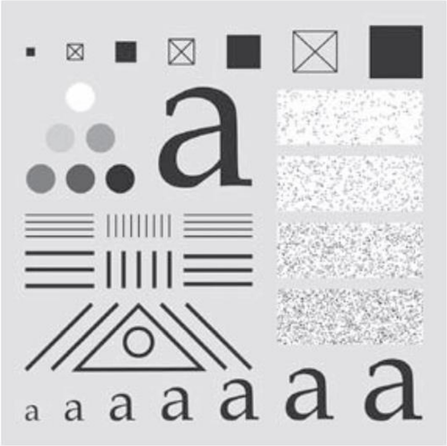
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

Filtering

- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input

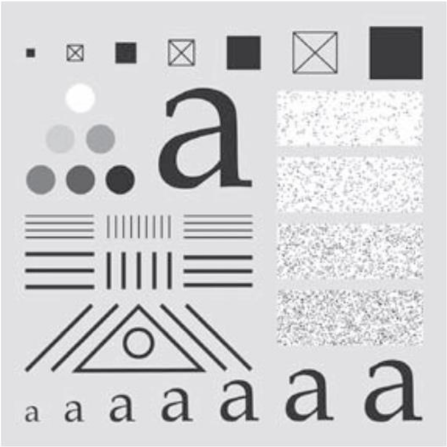


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

Filtering

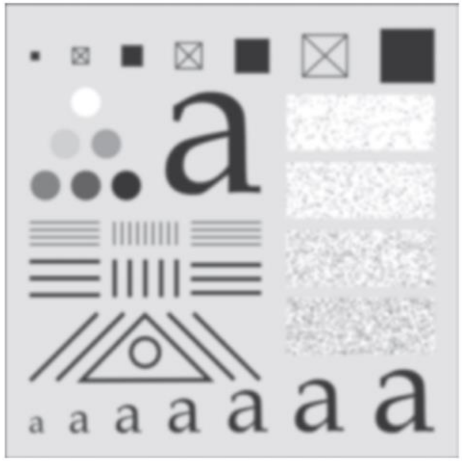
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input



$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

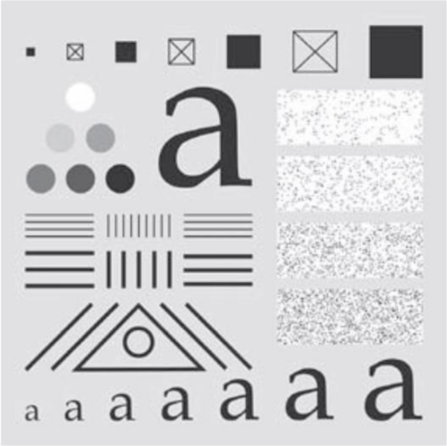
m=21 σ=3.5 Gauss



Filtering

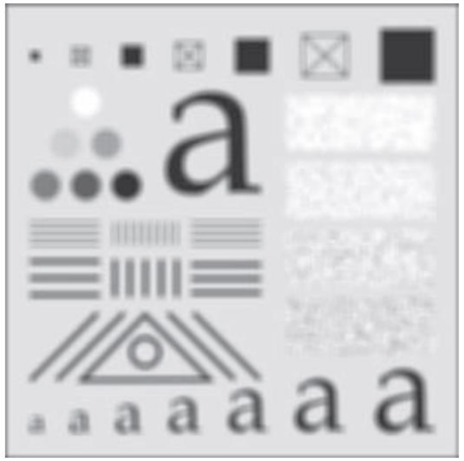
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input

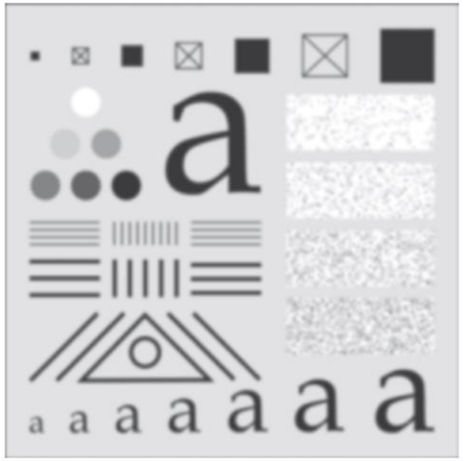


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

m=21 box



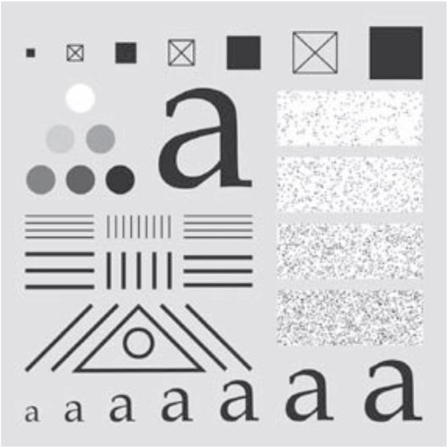
m=21 σ=3.5 Gauss



Filtering

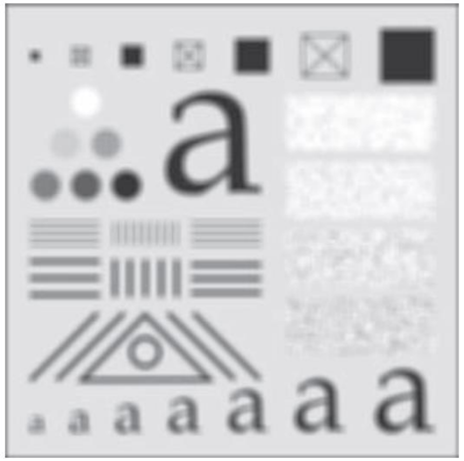
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input

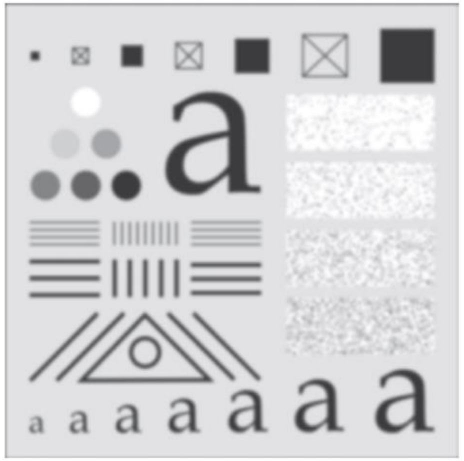


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

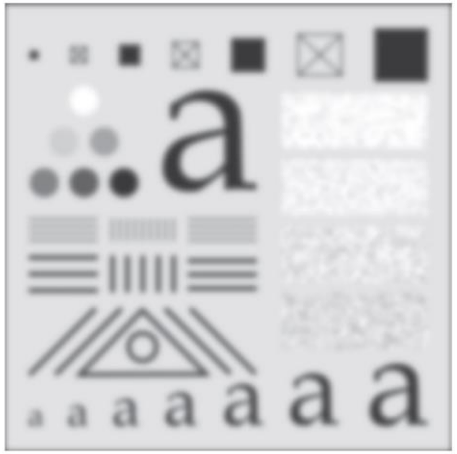
m=21 box



m=21 σ=3.5 Gauss



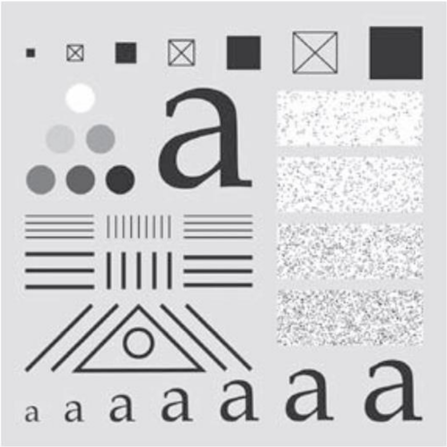
m=43 σ=7 Gauss



Filtering

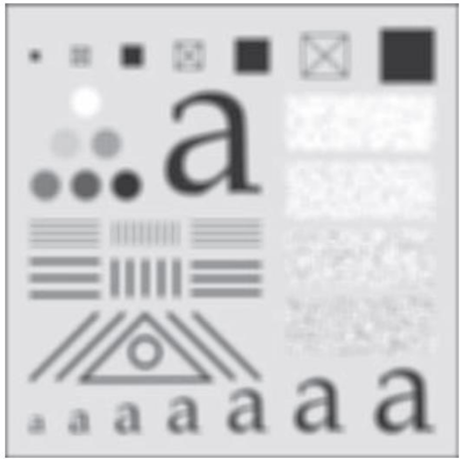
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input

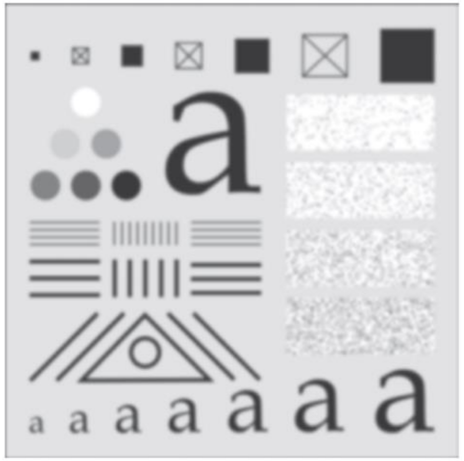


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

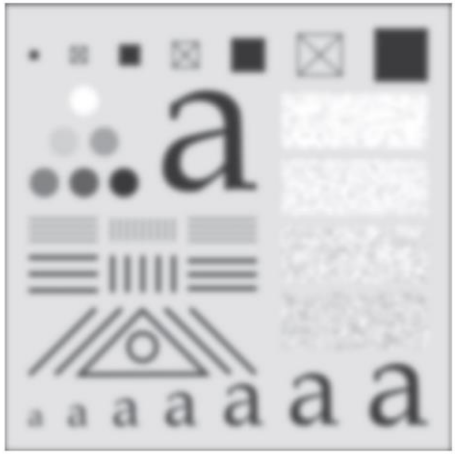
m=21 box



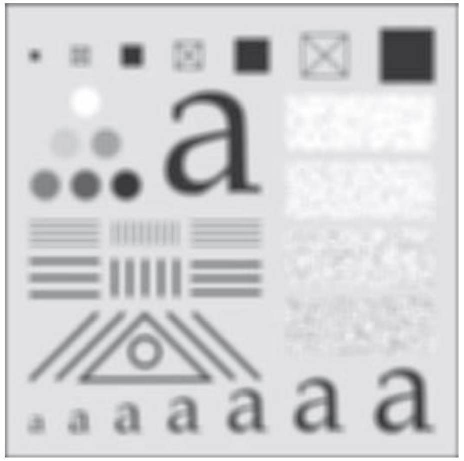
m=21 σ=3.5 Gauss



m=43 σ=7 Gauss



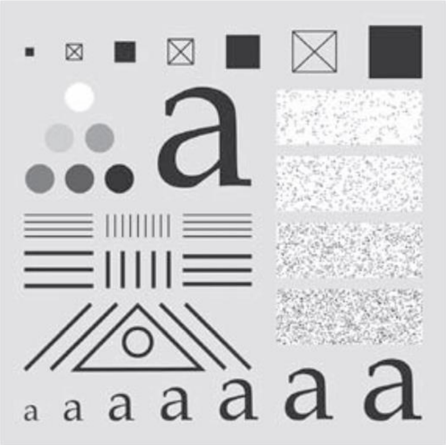
m=21 box



Filtering

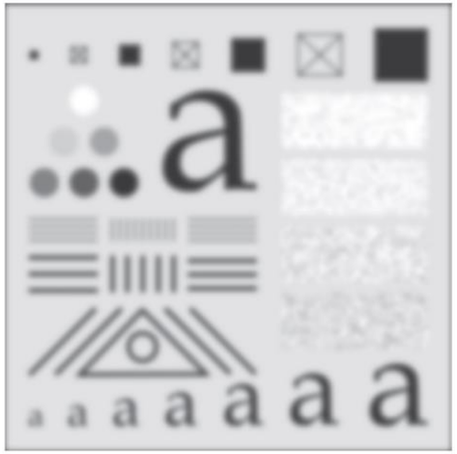
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input

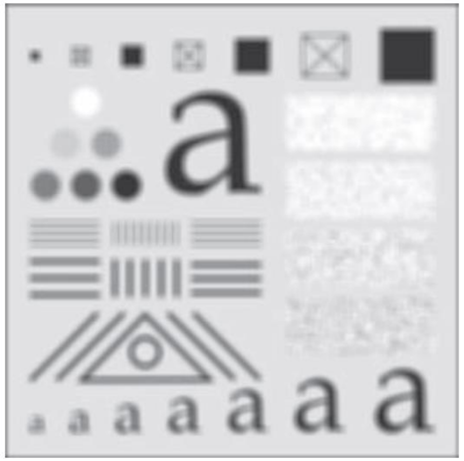


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

m=43 σ=7 Gauss



m=21 box



Filtering

- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
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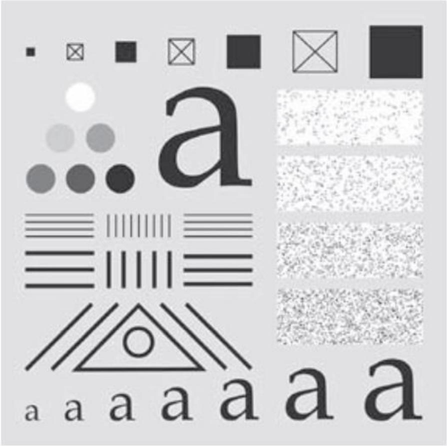
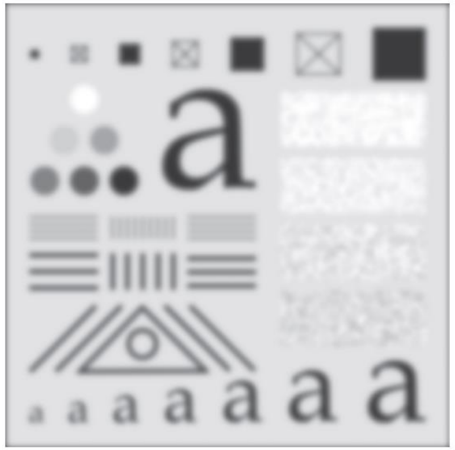
input

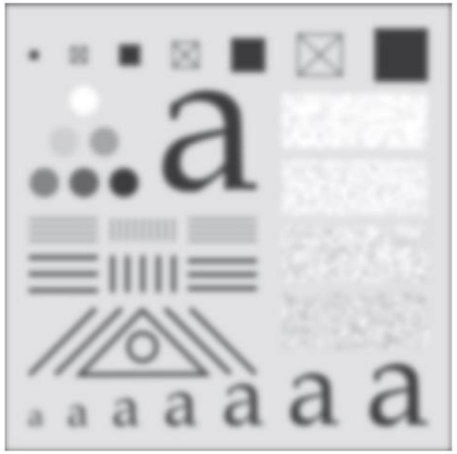


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

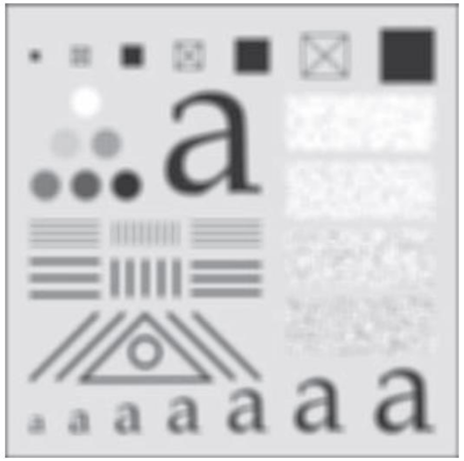
m=85 σ=7 Gauss



m=43 σ=7 Gauss



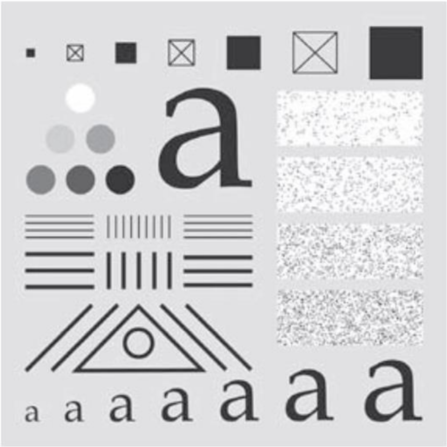
m=21 box



Filtering

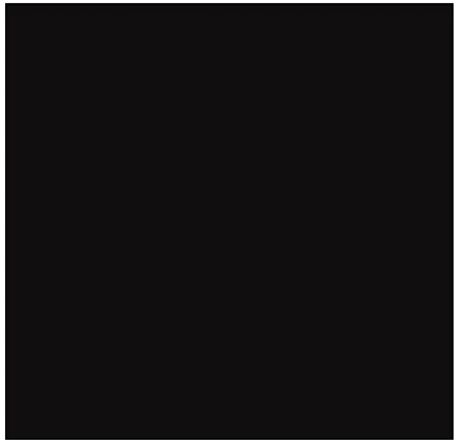
- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

input

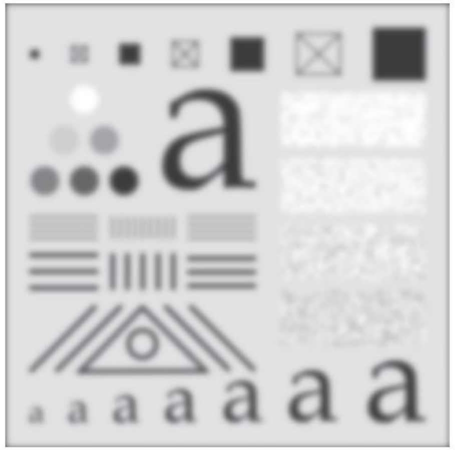


$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|}\hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

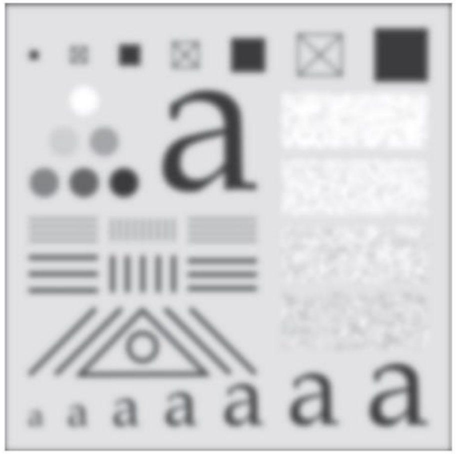
difference $m_{85} - m_{43}$



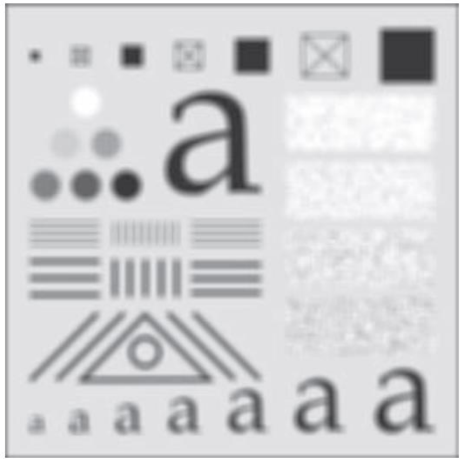
m=85 σ=7 Gauss



m=43 σ=7 Gauss

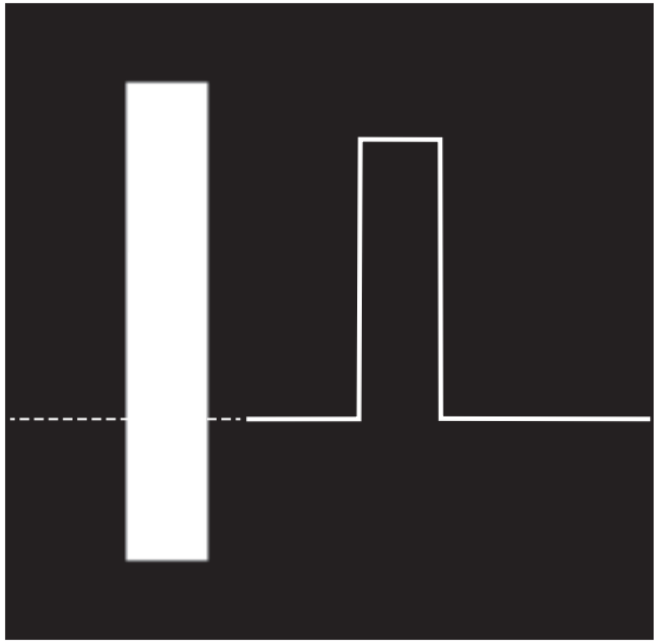


m=21 box



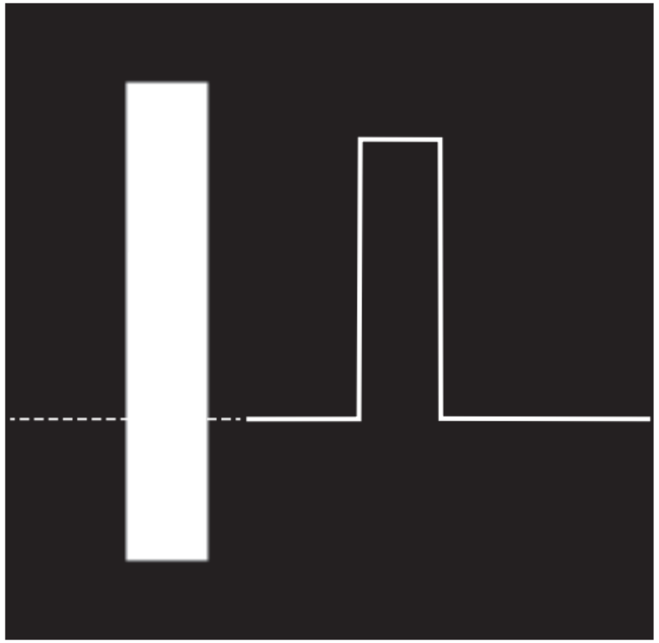
Filtering

- Box vs Gaussian
  - blur profile
  - blurred rects having same shape

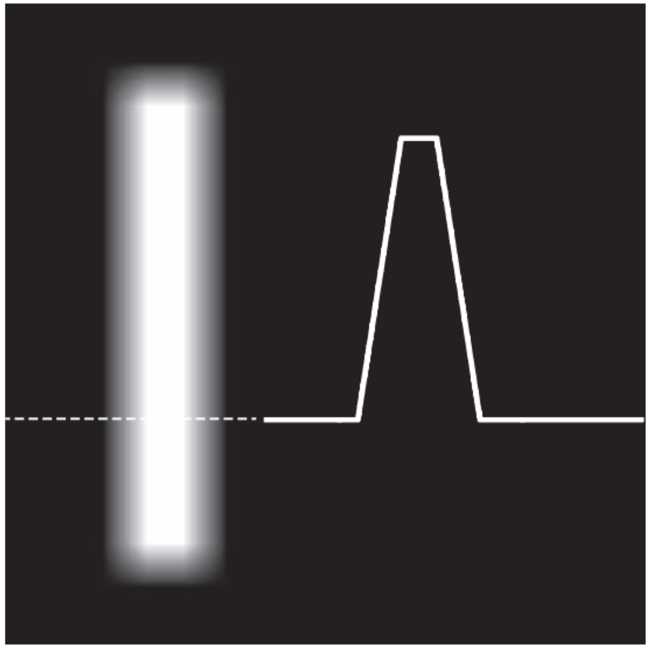


Filtering

- Box vs Gaussian
  - blur profile
  - blurred rects having same shape

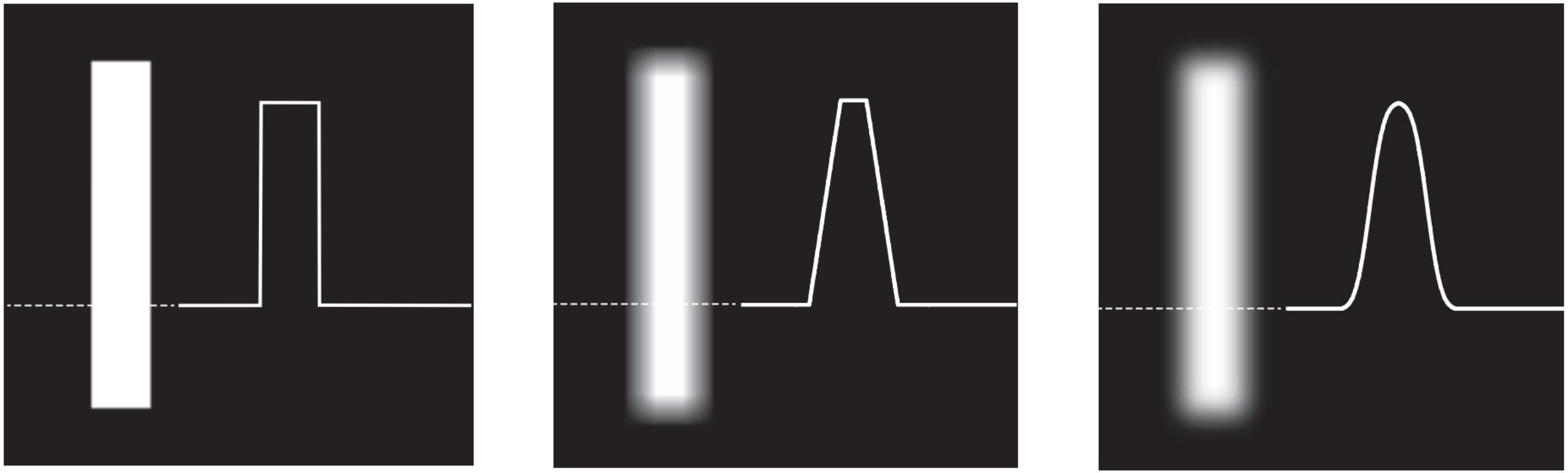


m=71 box



Filtering

- Box vs Gaussian
  - blur profile
  - blurred rects having same shape



m=71 box

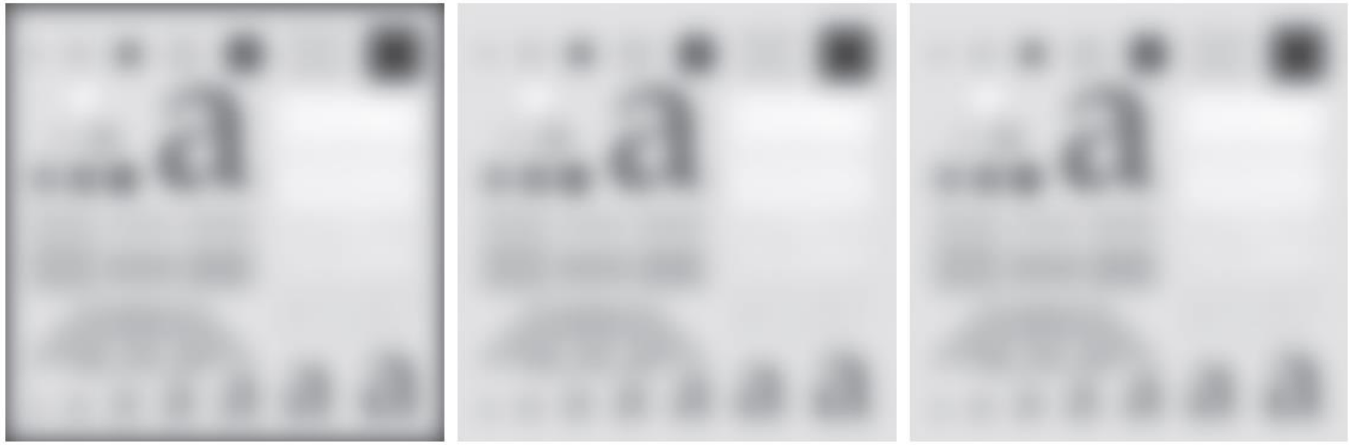
m=151 σ=25 Gauss

Filtering

- Padding effects

m=187 σ=31 Gauss

image 1024x1024



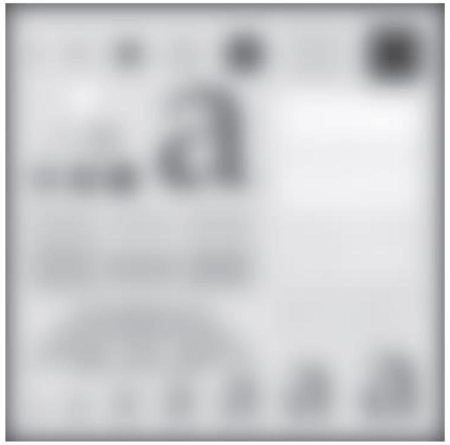
Filtering

- Padding effects

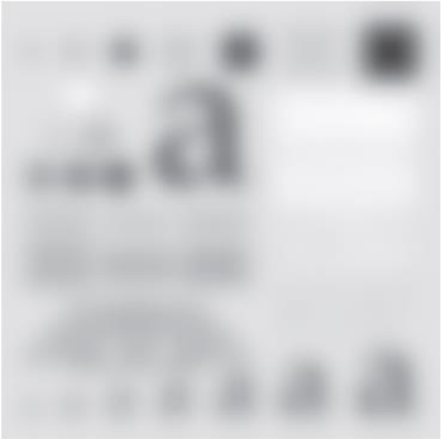
m=187 σ=31 Gauss

image 1024x1024

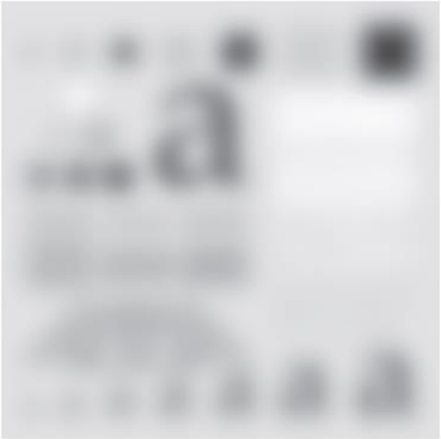
zero



mirror



replicate



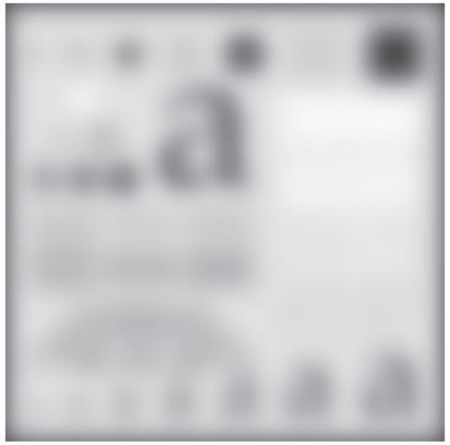
Filtering

- Padding effects

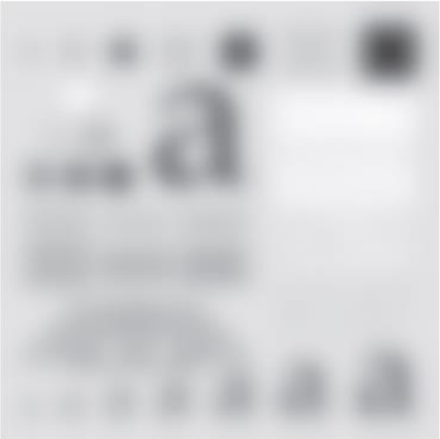
m=187 σ=31 Gauss

image 1024x1024

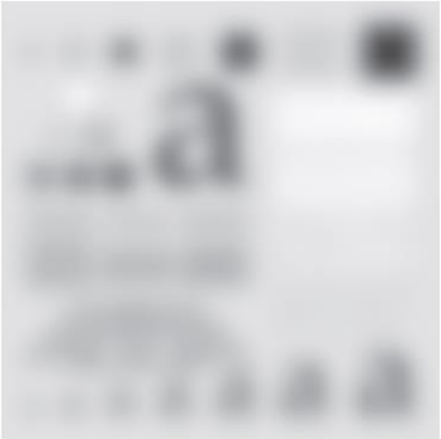
zero



mirror



replicate



- Relative size effect

m=187 σ=31 Gauss

image 4096x4096

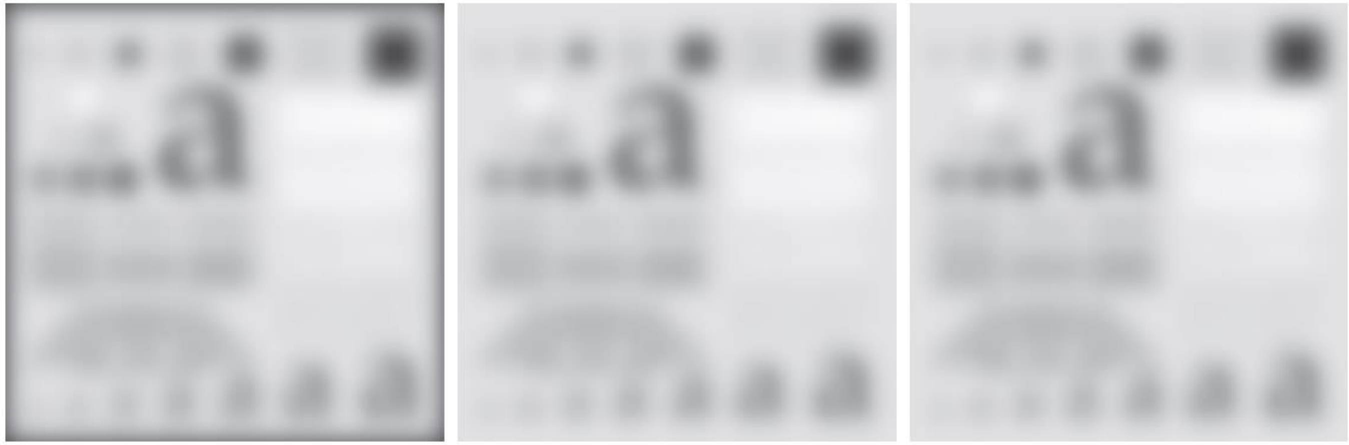
Filtering

- Padding effects

m=187 σ=31 Gauss

image 1024x1024

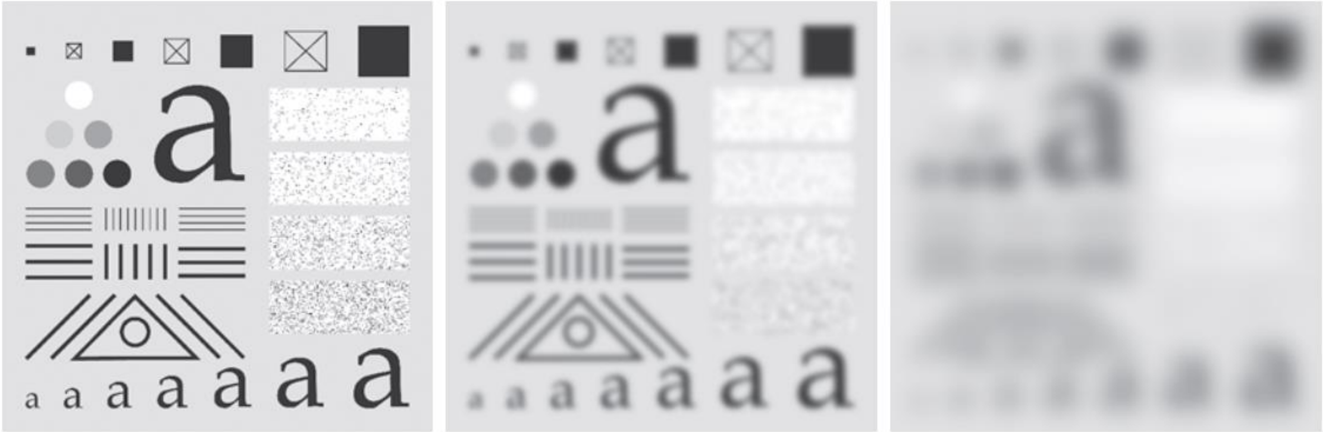
zero mirror replicate



- Relative size effect

m=187 σ=31 Gauss

image 4096x4096



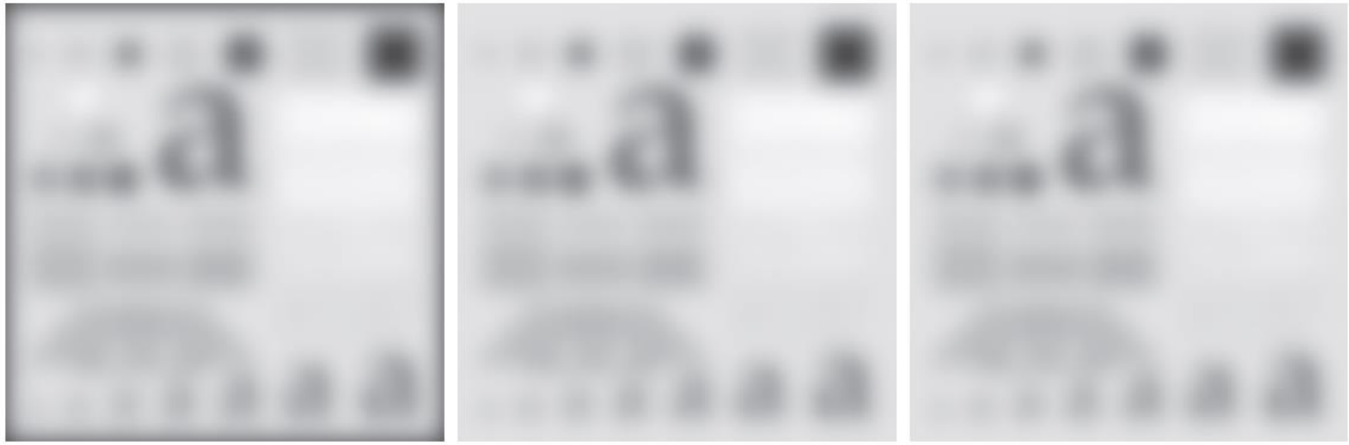
Filtering

- Padding effects

m=187 σ=31 Gauss

image 1024x1024

zero mirror replicate

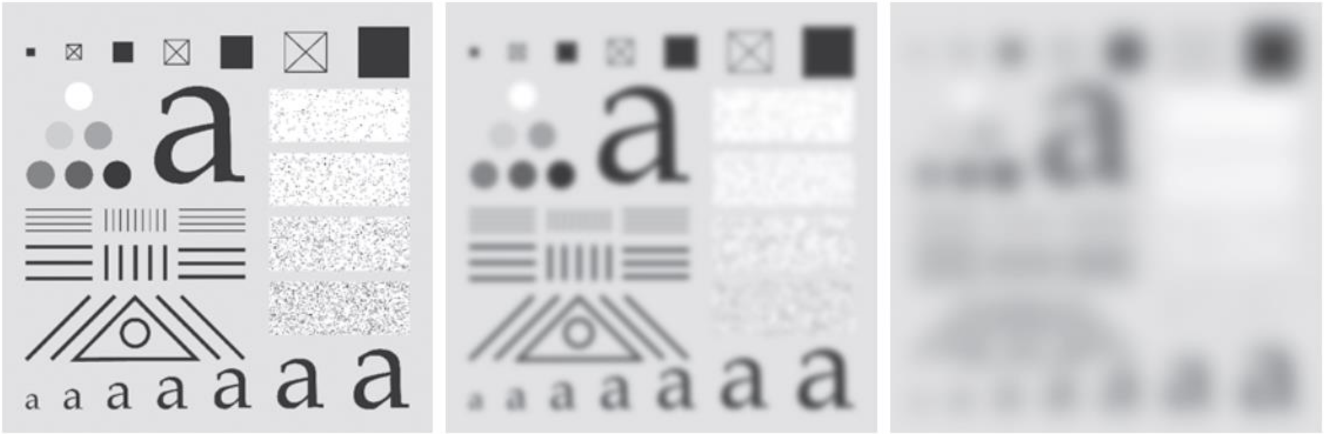


- Relative size effect

m=187 σ=31 Gauss

image 4096x4096

m=745 σ=124 Gauss



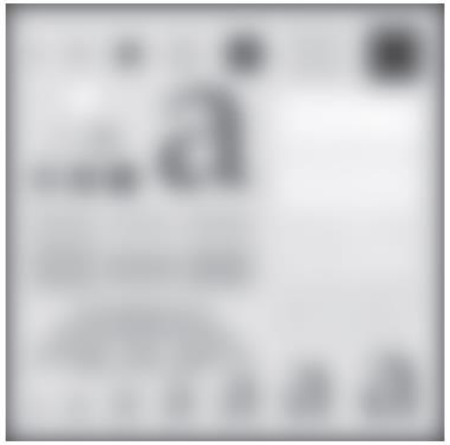
Filtering

- Padding effects

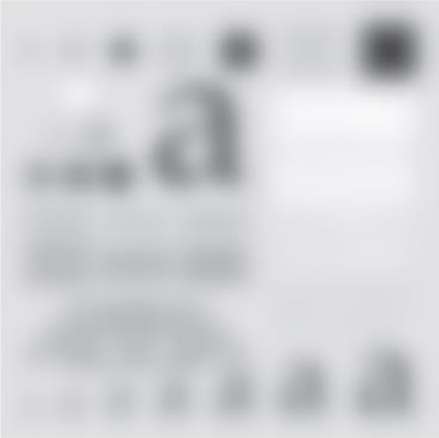
m=187 σ=31 Gauss

image 1024x1024

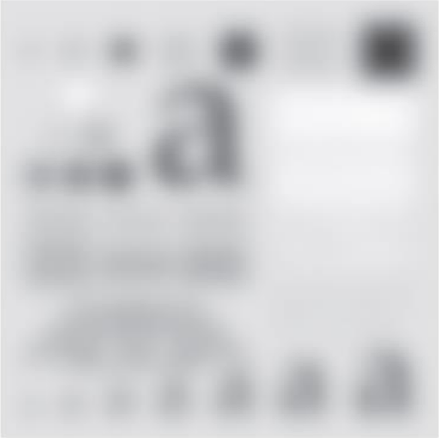
zero



mirror



replicate



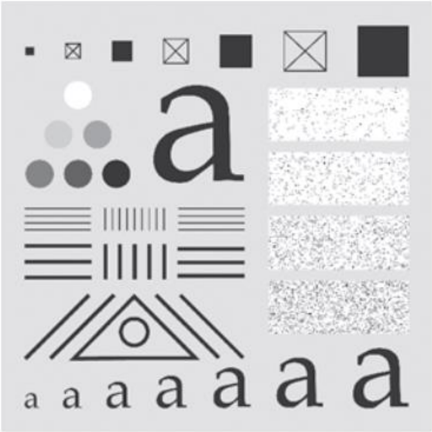
- Relative size effect

m=187 σ=31 Gauss

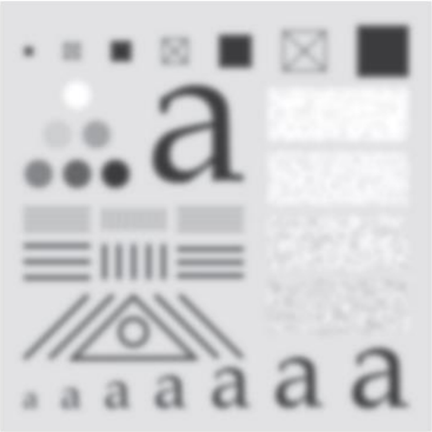
image 4096x4096

m=745 σ=124 Gauss

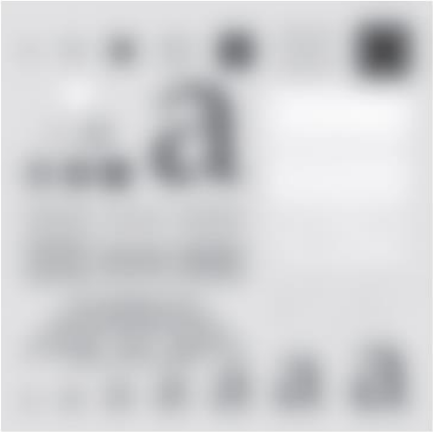
input



m=187 σ=31



m=745 σ=124

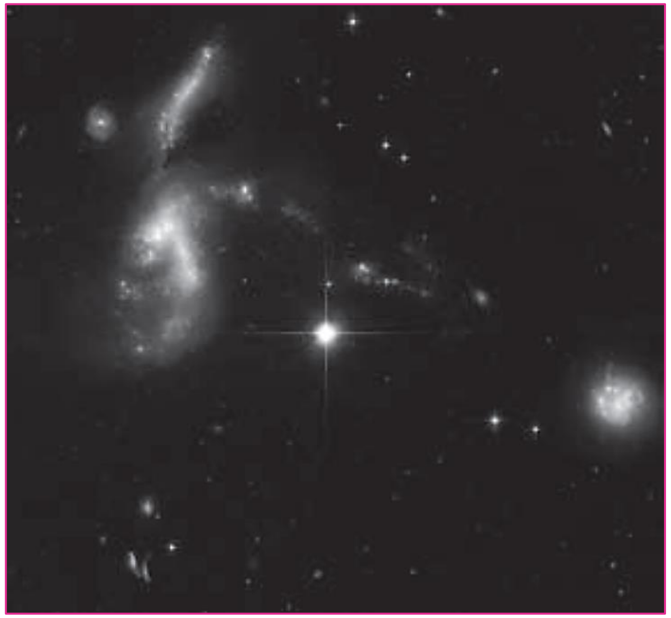


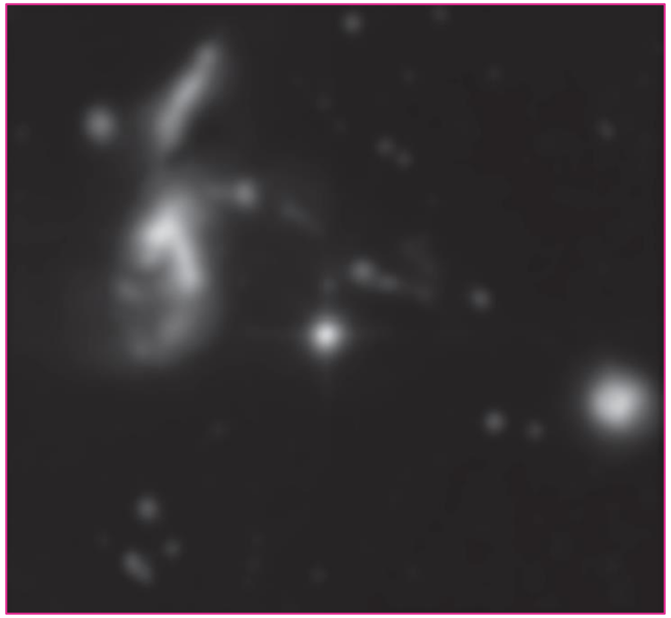
Filtering

- Relevant region extraction

Filtering

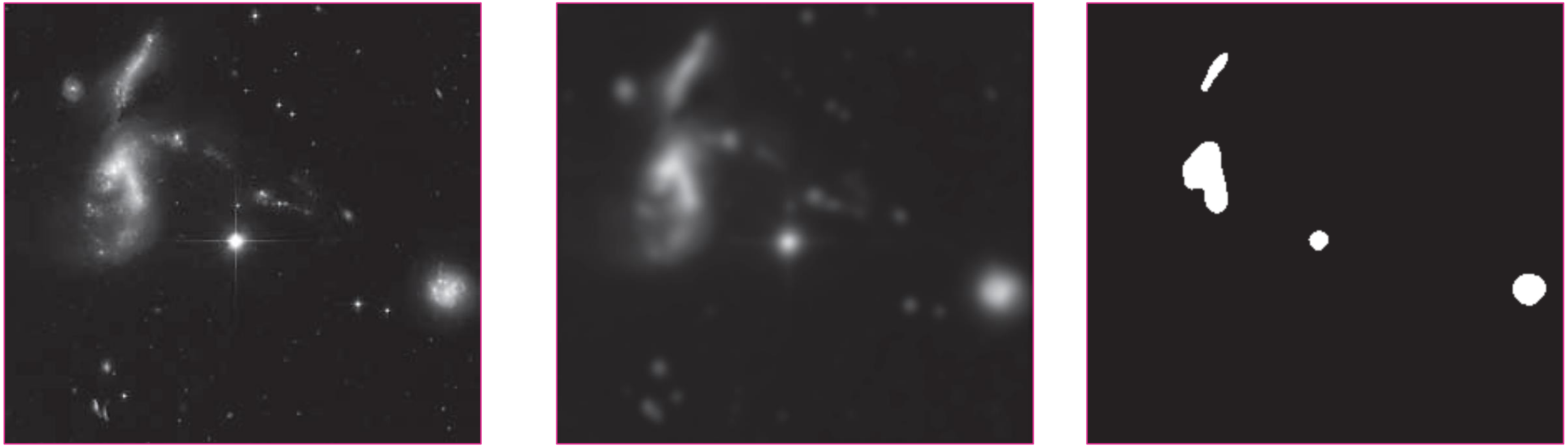
- Relevant region extraction





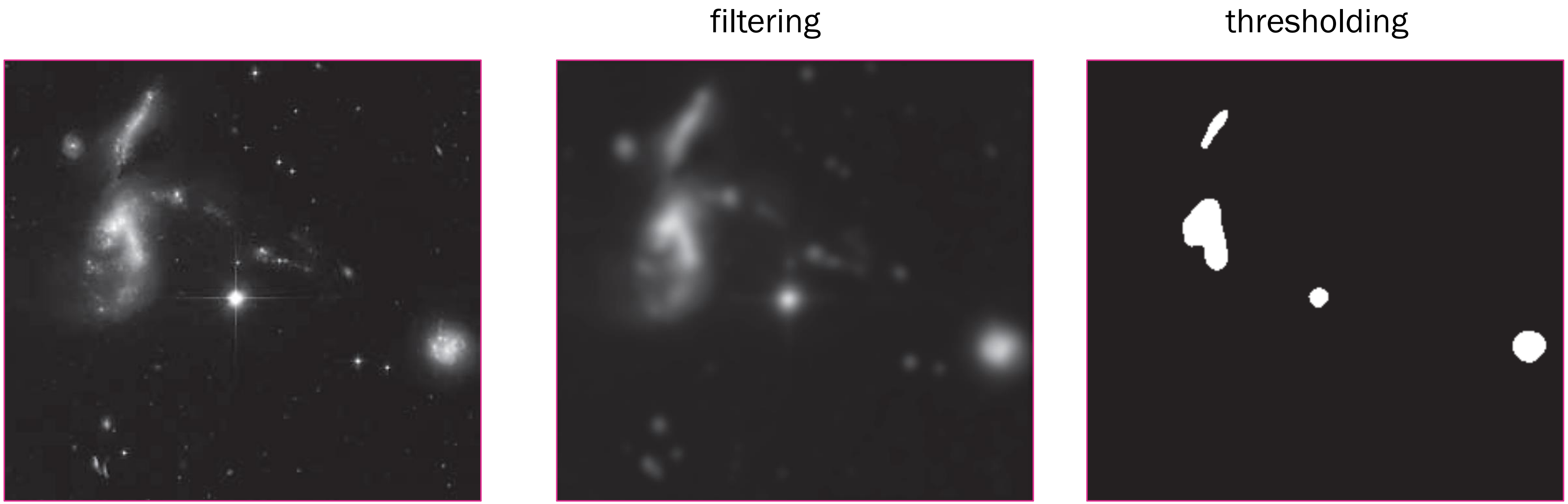
Filtering

- Relevant region extraction



Filtering

- Relevant region extraction

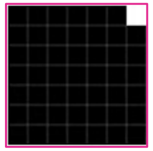


filtering

thresholding

Filtering

- Shifting

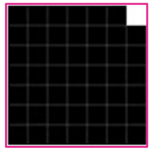


Filtering

- Shifting

filter

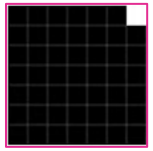
output



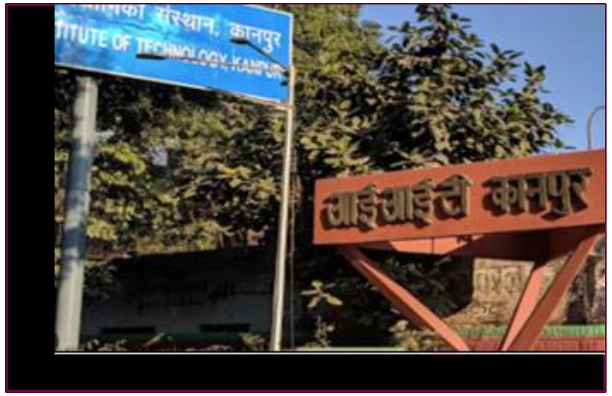
Filtering

- Shifting

filter

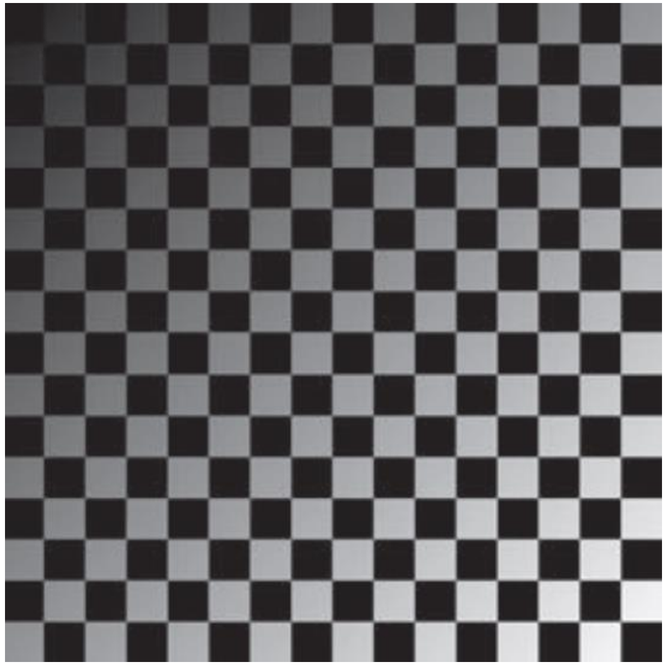


output



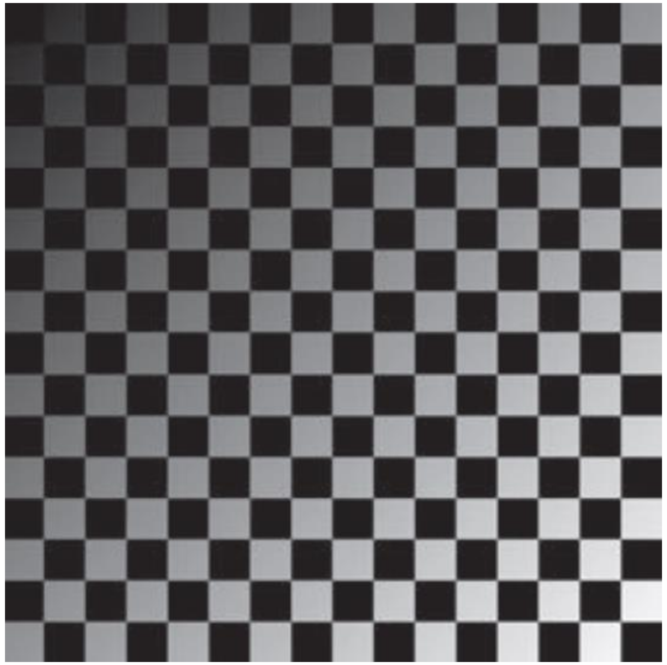
Filtering

- Shading correction



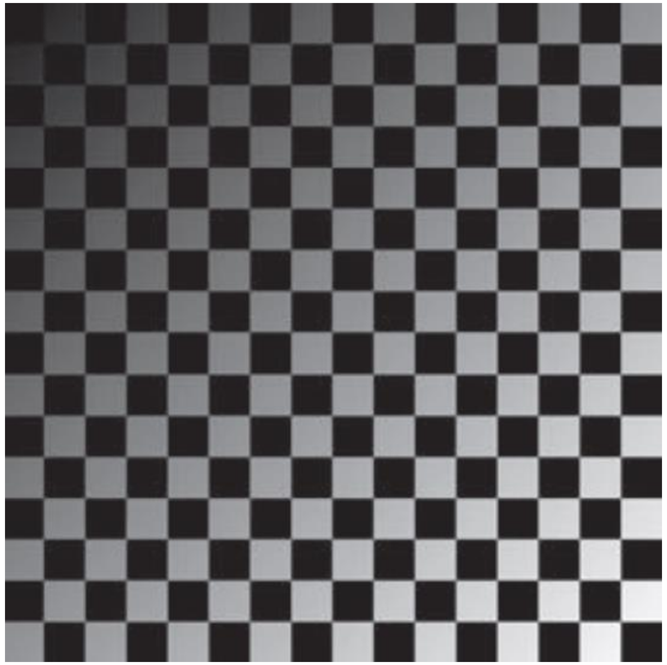
Filtering

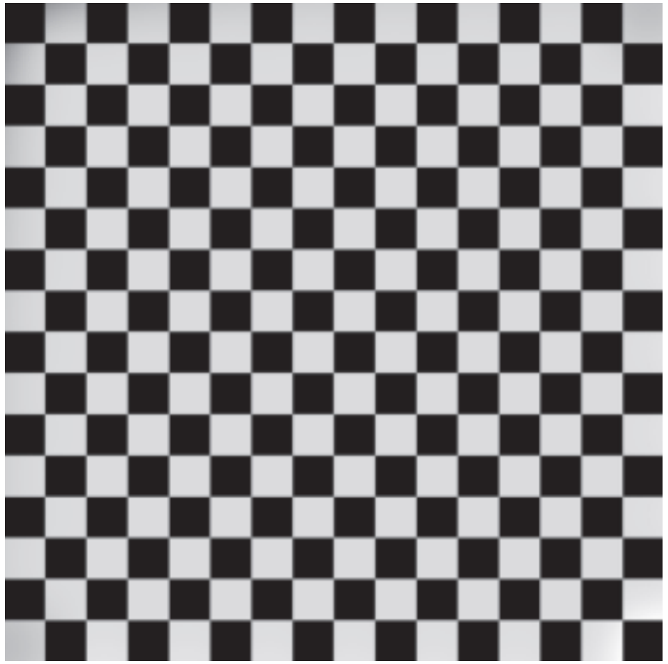
- Shading correction



Filtering

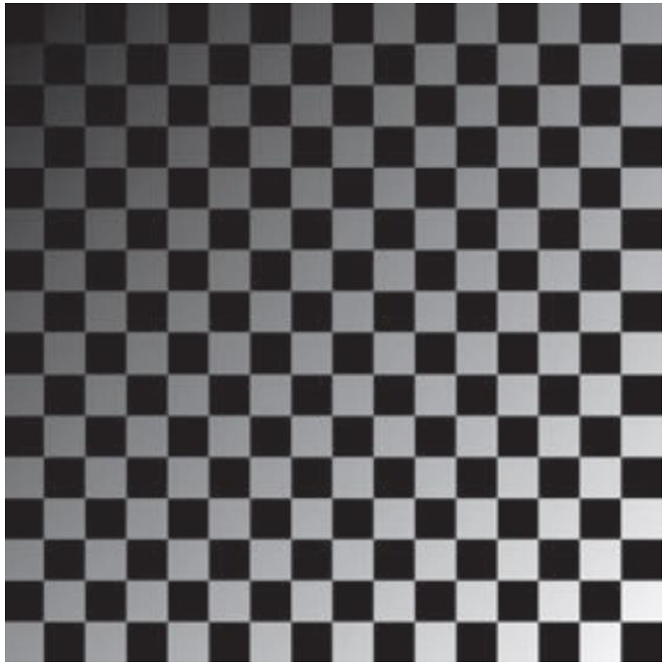
- Shading correction





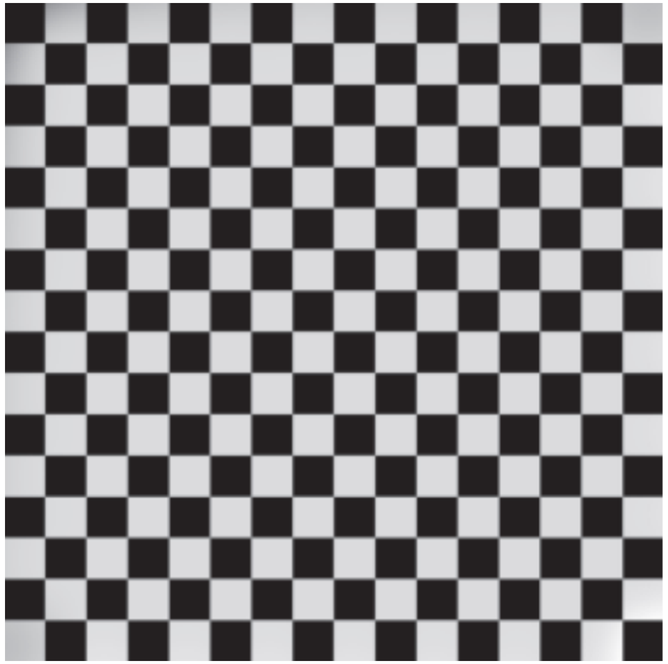
Filtering

- Shading correction



filtering

scaling



Conclusion

- Filtering
  - Separable kernels
  - Correlation Vs Convolution
  - Filter properties
  - Smoothing filters

Filtering

credit: S. Bidun'20