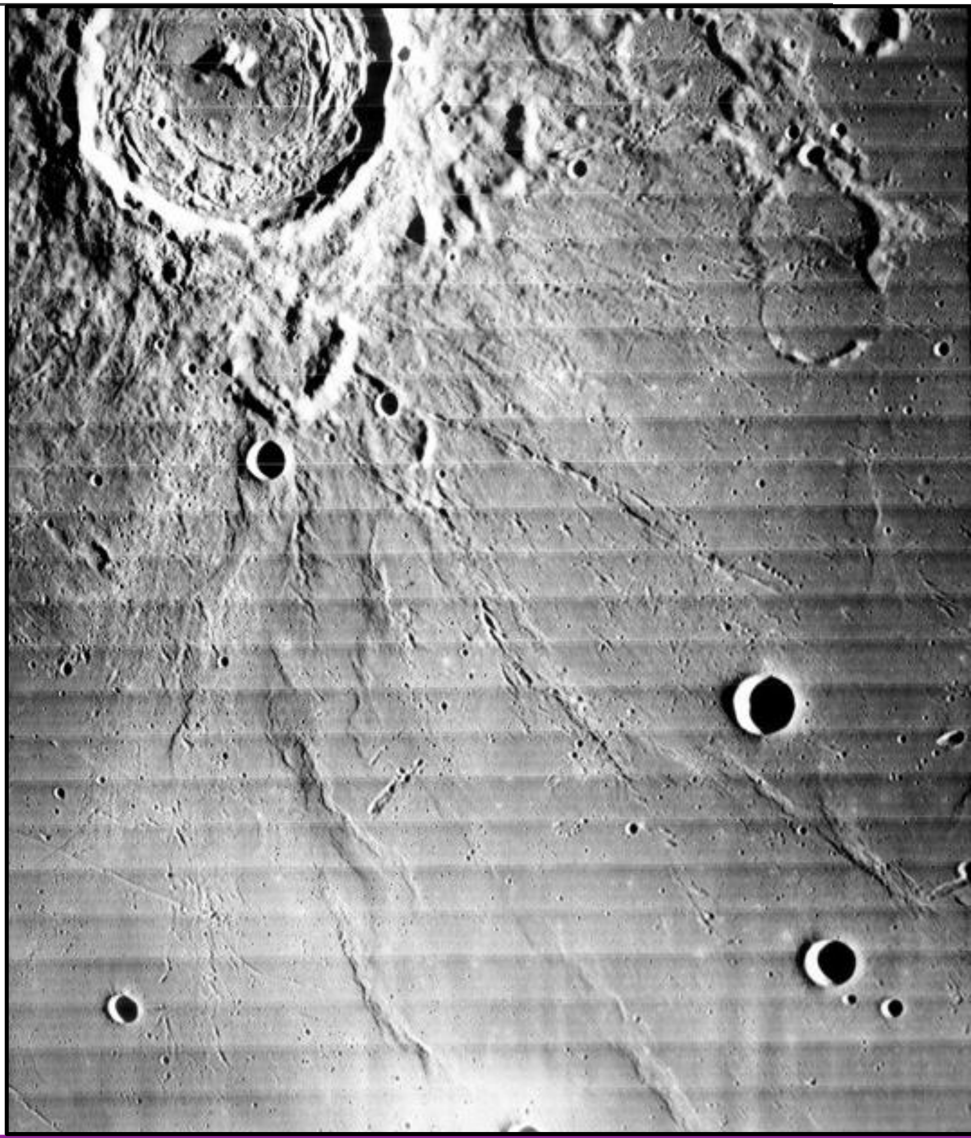
Enhancement & Denoising:
Frequency domain filtering

Dr. Tushar Sandhan

Introduction



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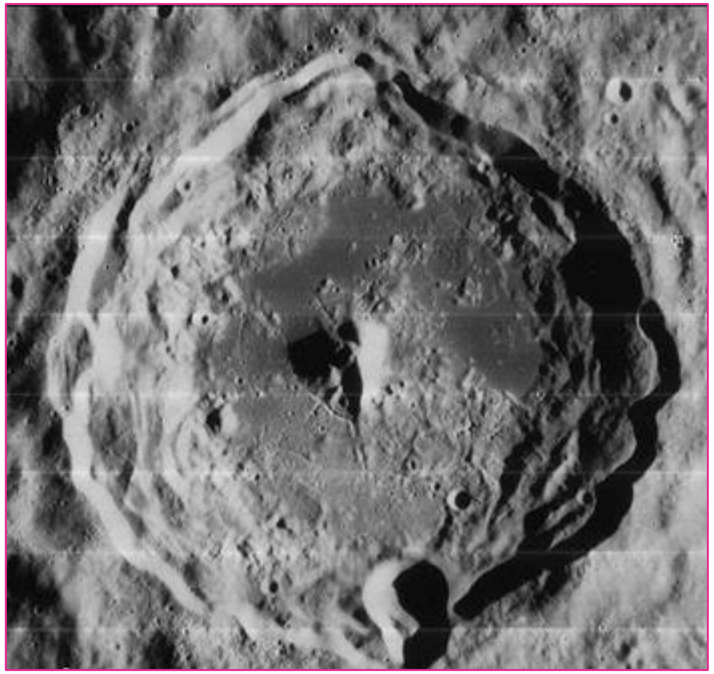

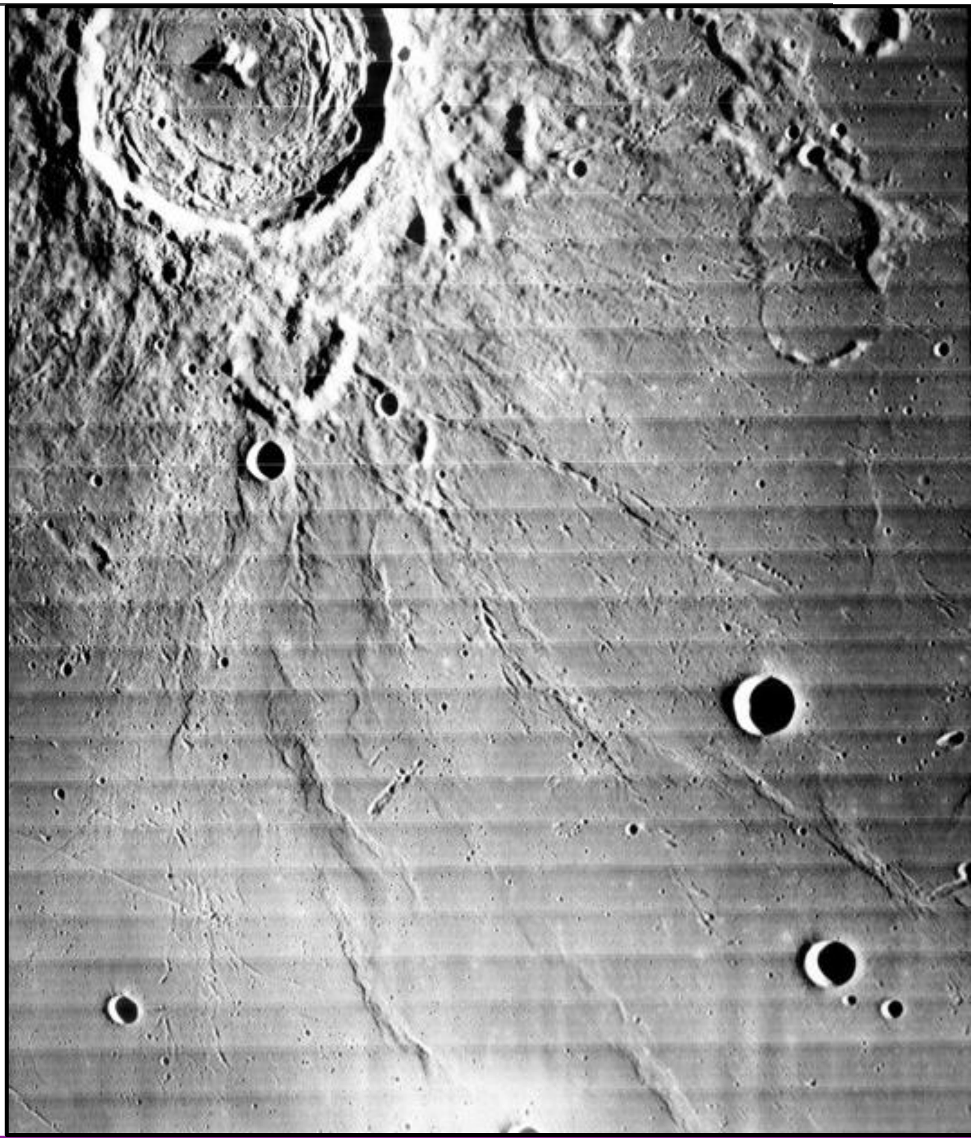

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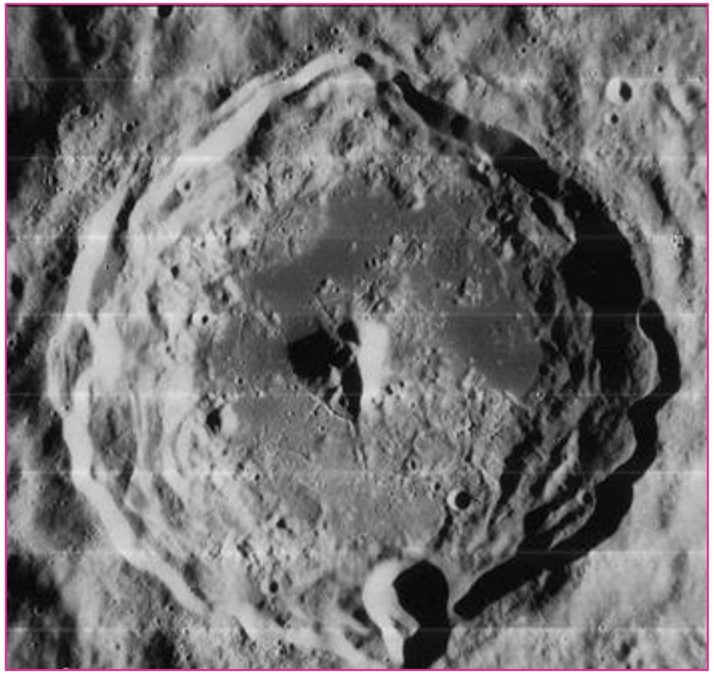

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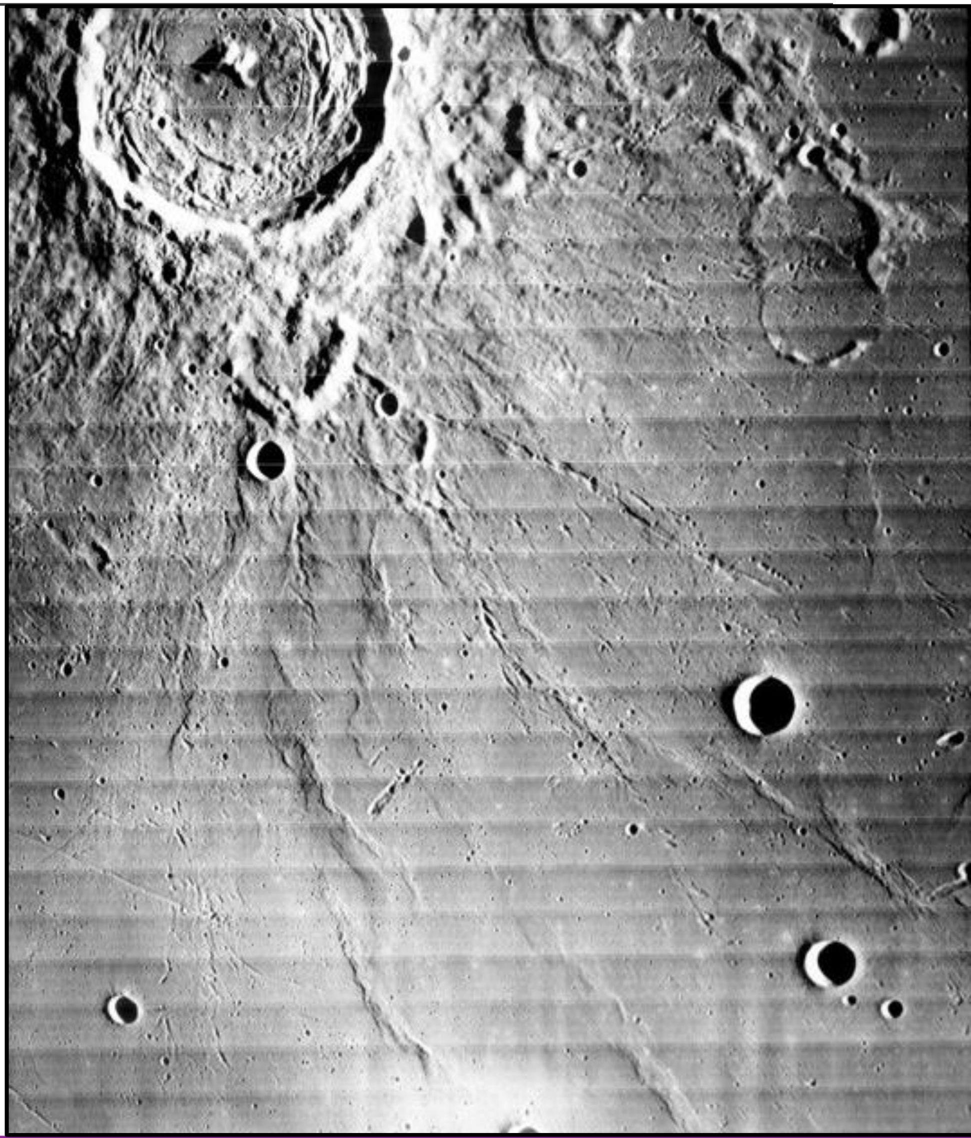

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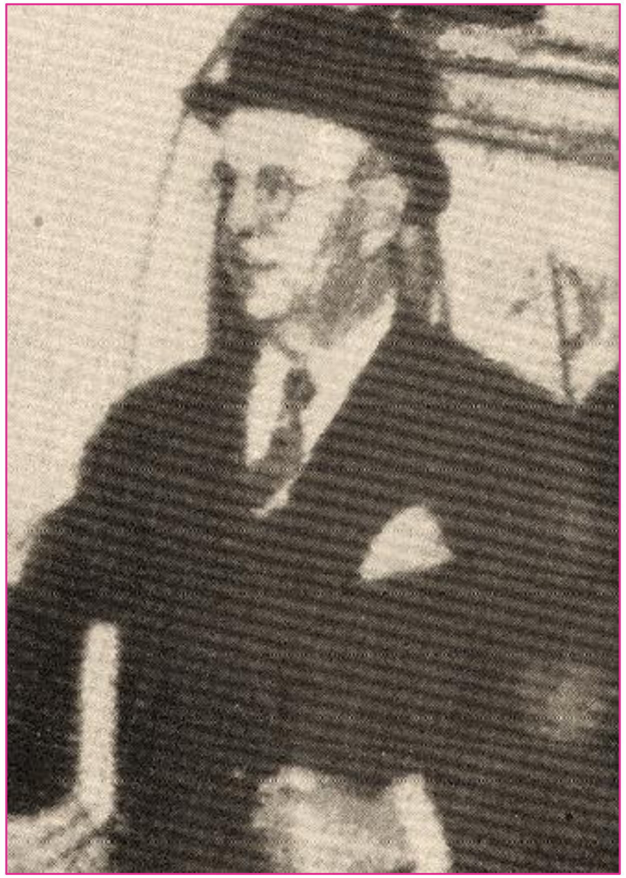

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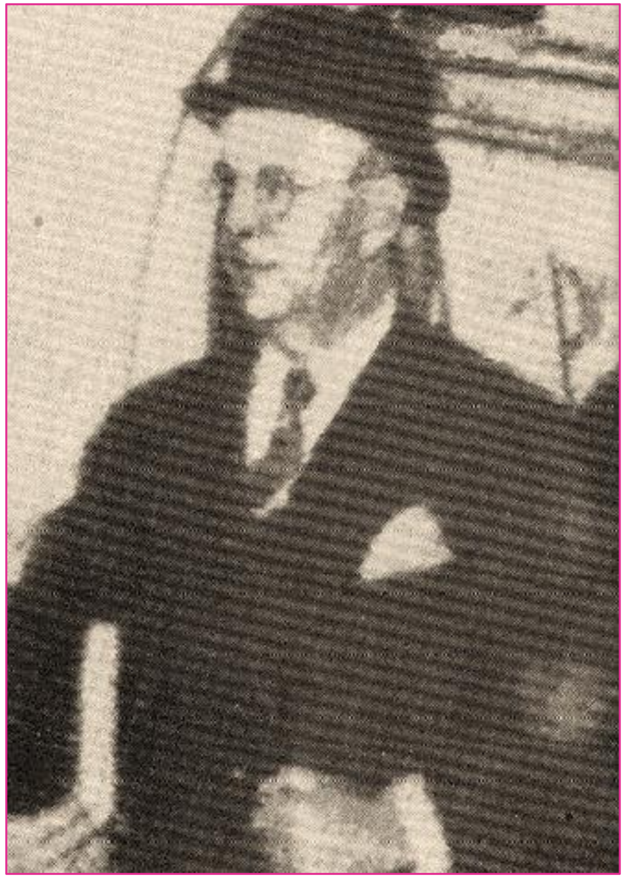

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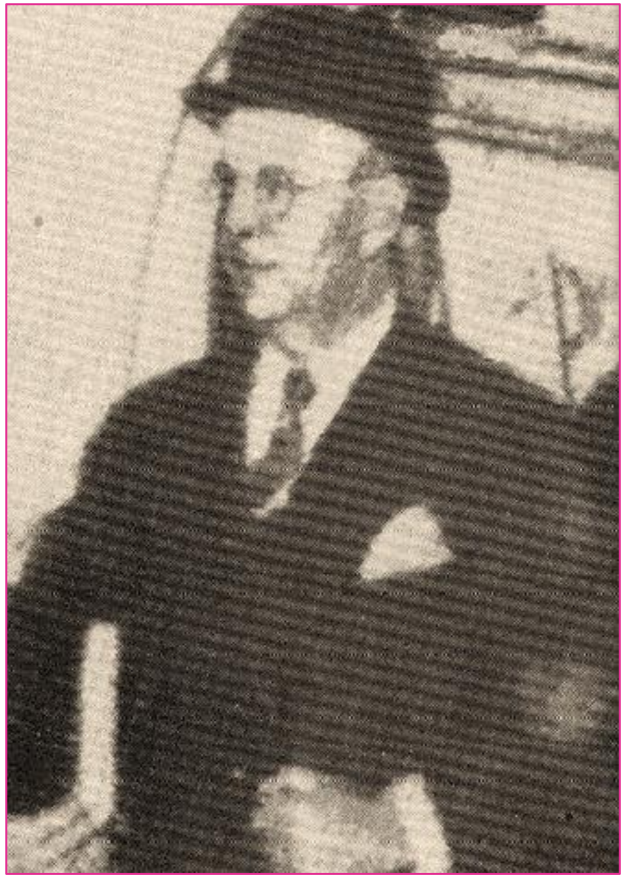

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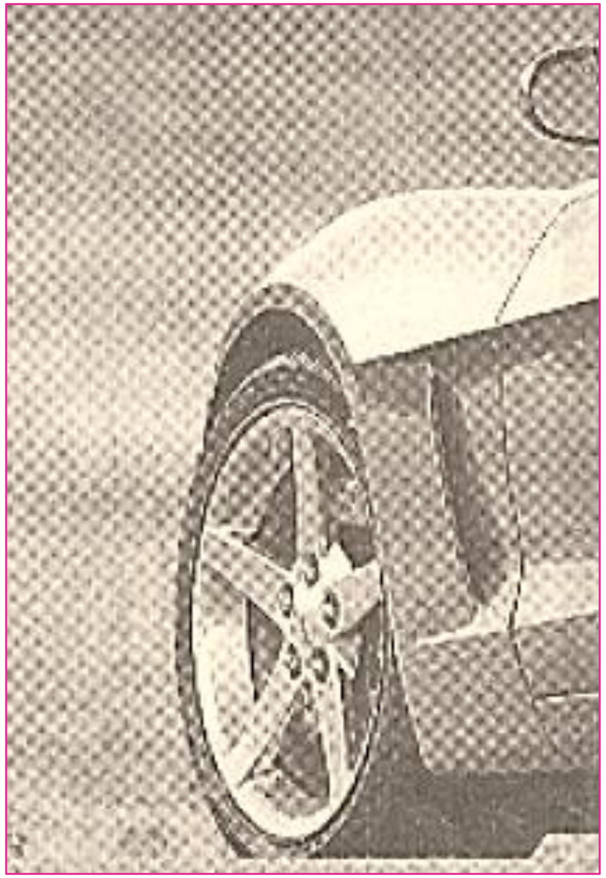

Fourier Translation & Scaling

- 2D Fourier Transform

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$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

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$$FT[f(x - x_0, y - y_0)] = F(u, v) \cdot \exp[-j2\pi(ux_0 + vy_0)/N]$$

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$$FT[f(ax, by)] = \frac{1}{ab} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

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- Spatial filtering to frequency filtering

$$f \star g$$

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Fast Fourier transform (FFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi \frac{nk}{N}} \qquad 0 \le k < N$$

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$$t = [0 : \ 0.01 : 10]$$
$$x(t) = 10\sin(t) + 10\cos(t)$$
$$X(\omega) = FFT(\, x(t)\,)$$

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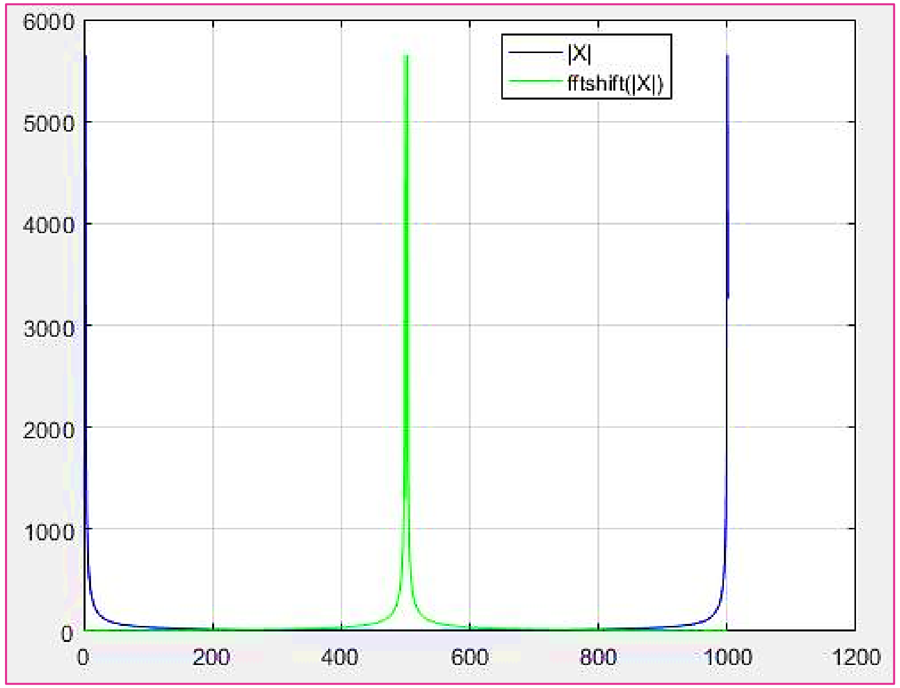

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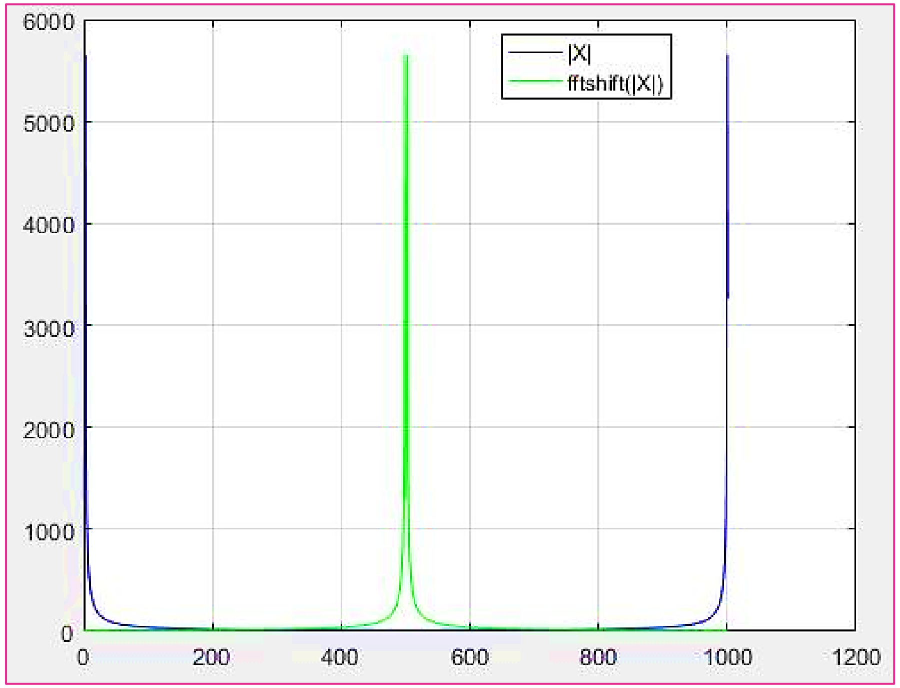
- fftshift: visualize FFT within $[-\frac{F_S}{2}, \frac{F_S}{2}]$ instead of $[0\ \ F_S]$



2D Fourier Transform

- Repositioning the quadrants

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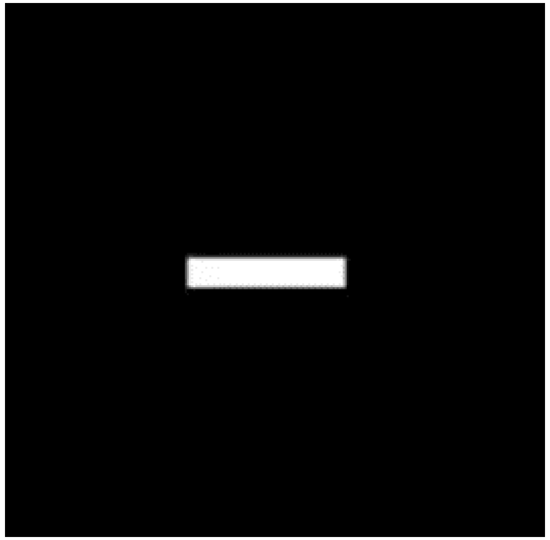
$f(x,y)$



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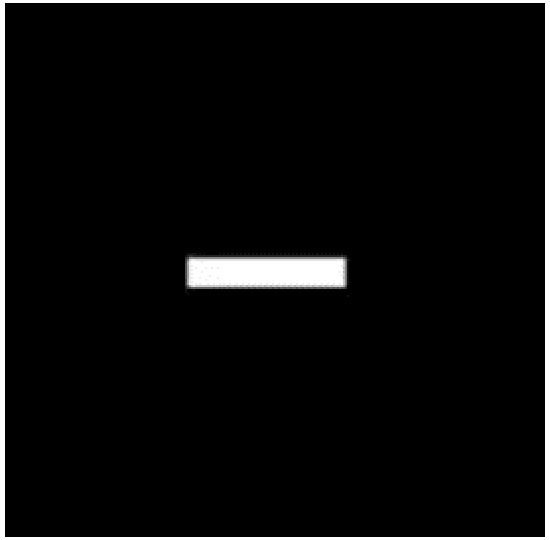

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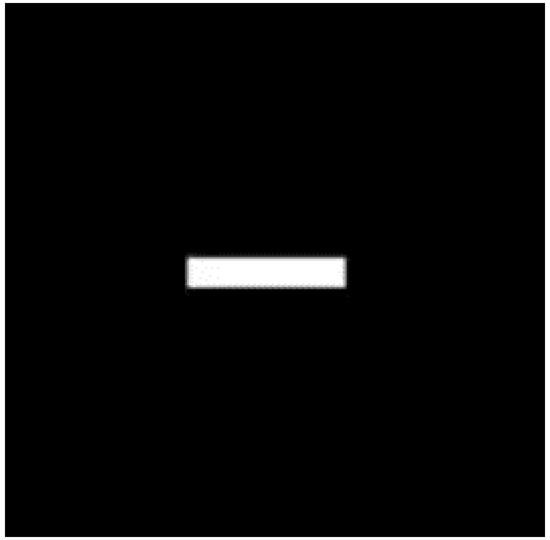

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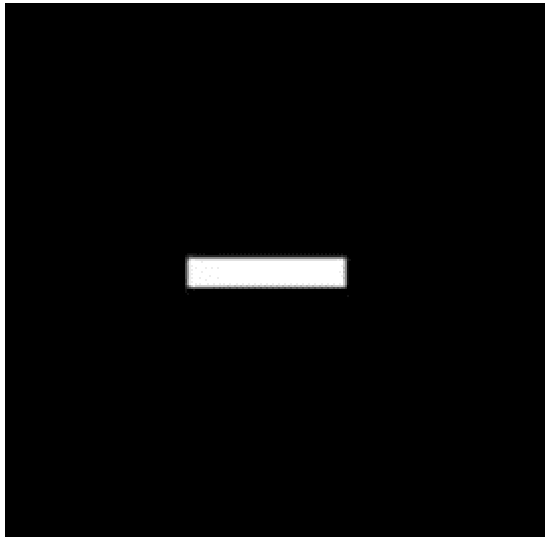

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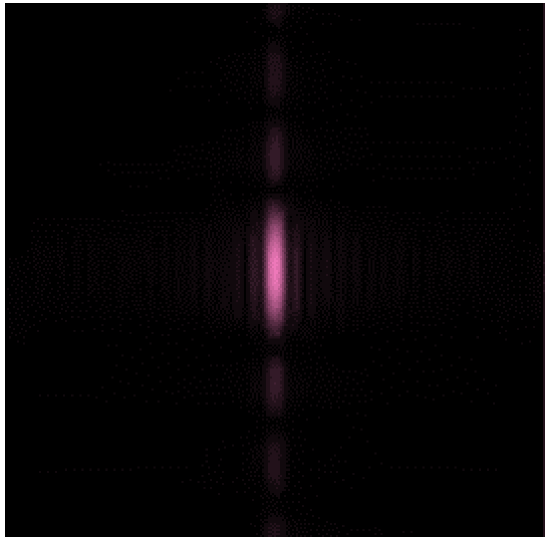

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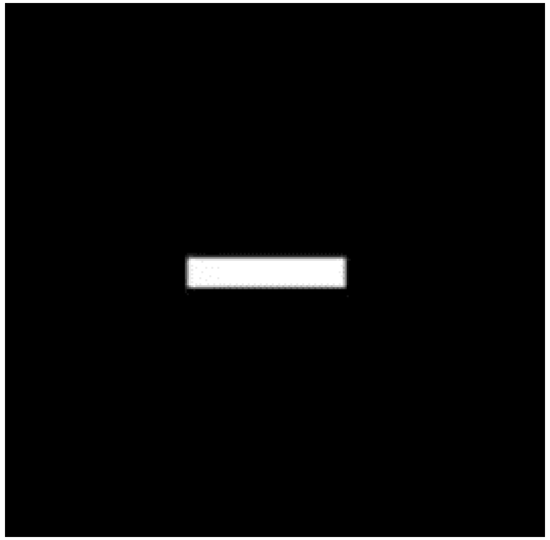
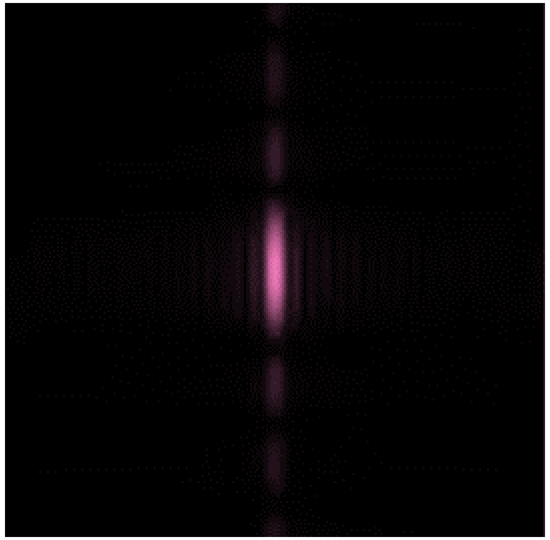


$|F(u, v)|$

$Shift(|F(u, v)|)$



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- FT as image & intensity transformations

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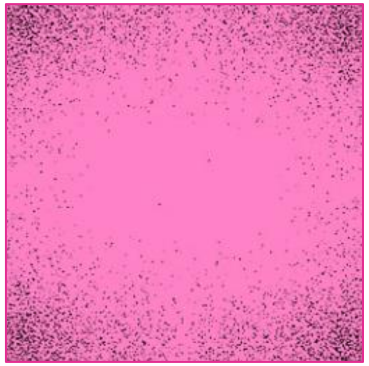

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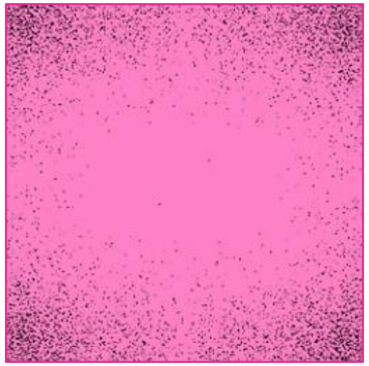

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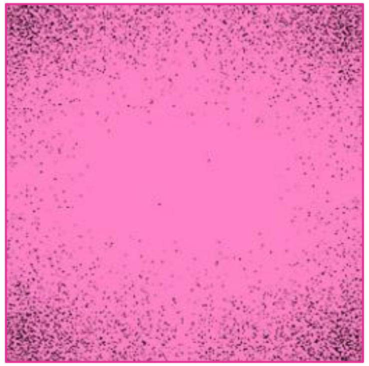

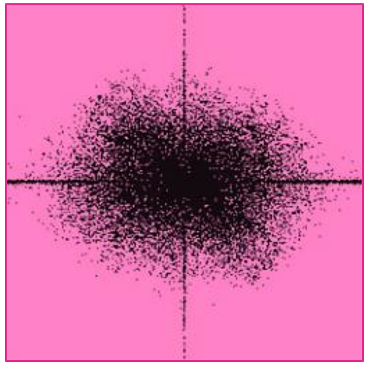

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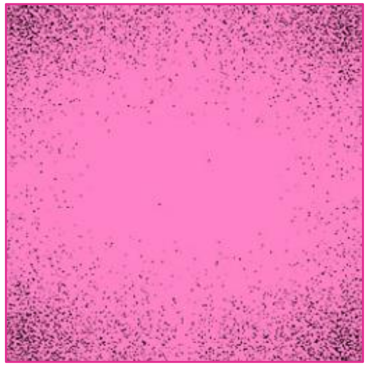

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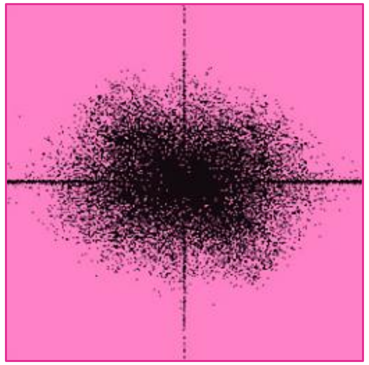

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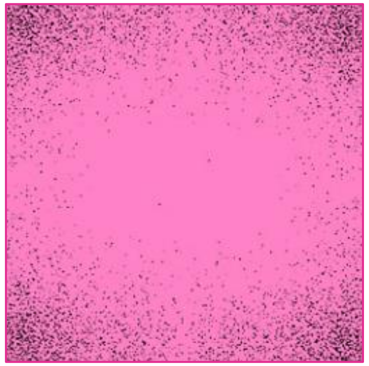

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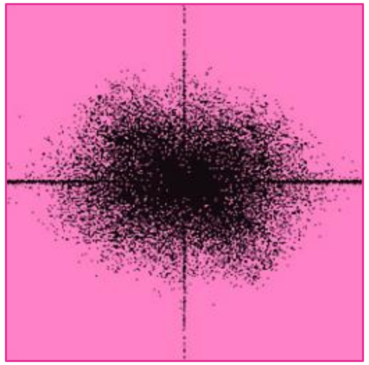

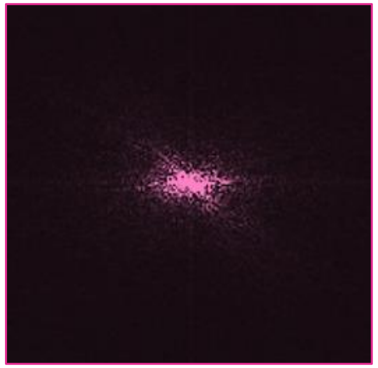

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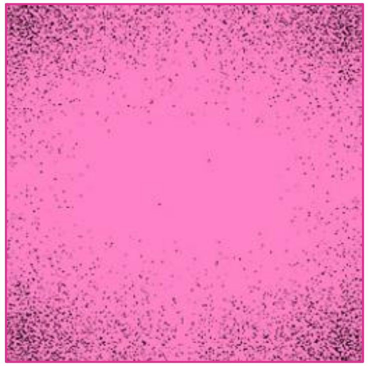

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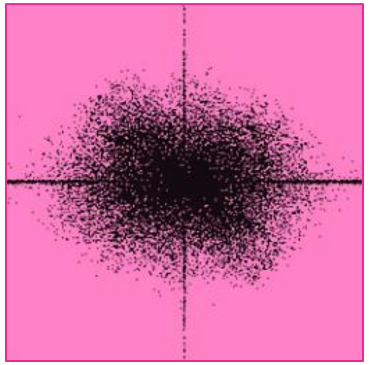

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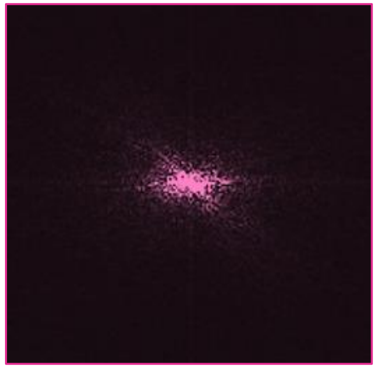

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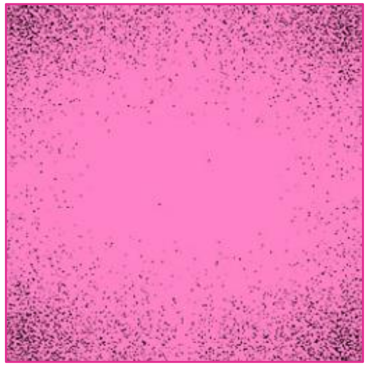

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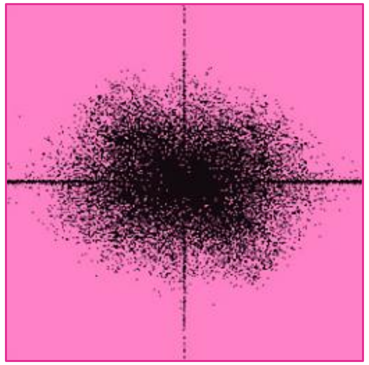

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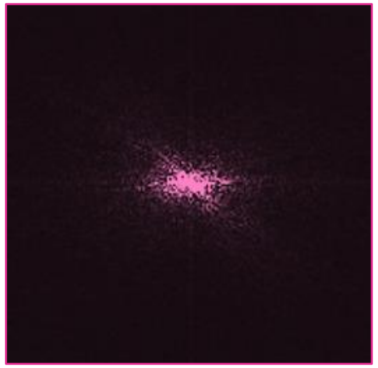

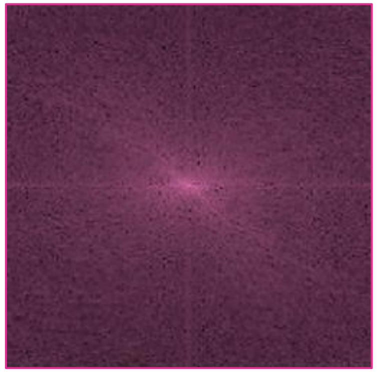

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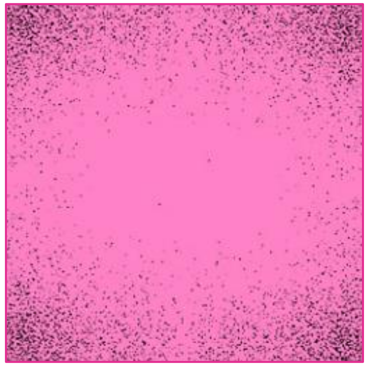

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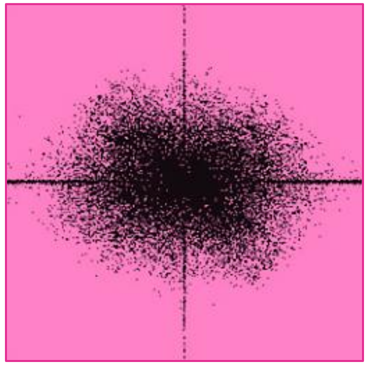

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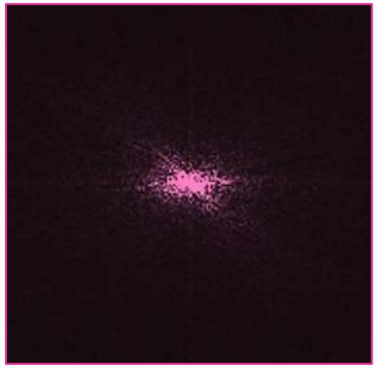

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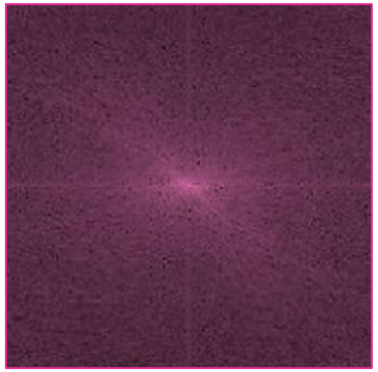

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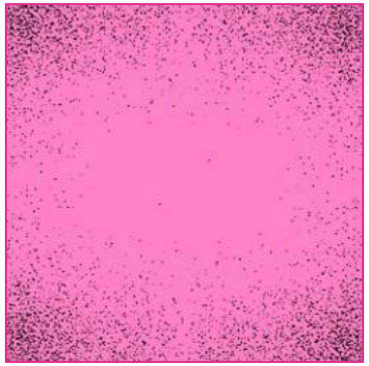

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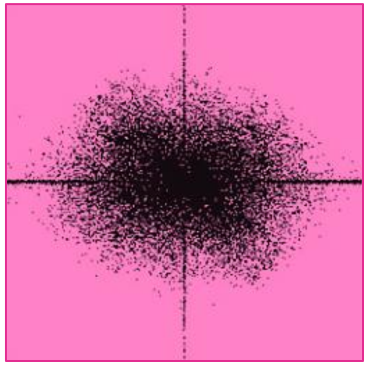
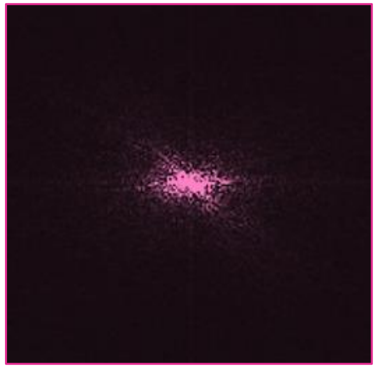
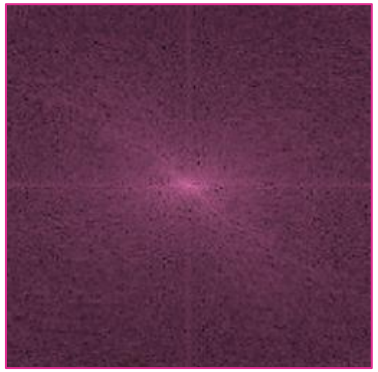
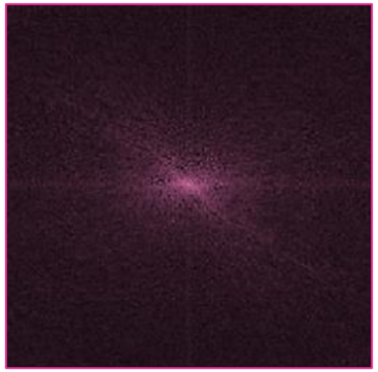


Scaling



Log

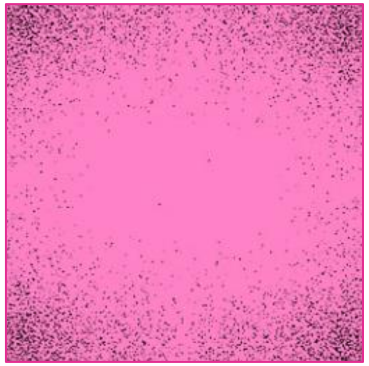




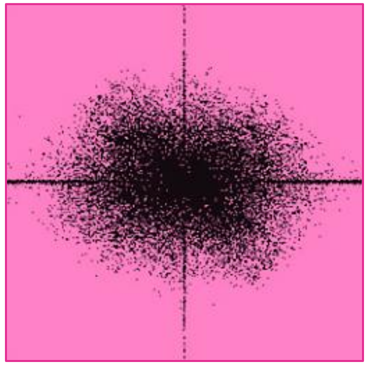
2D Fourier Transform

- FT as image & intensity transformations

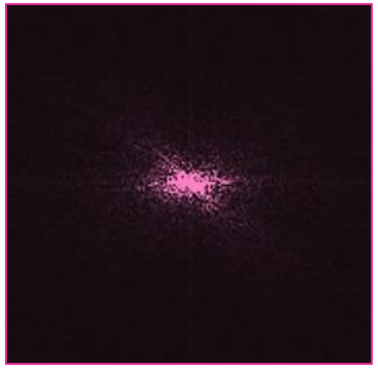
FT



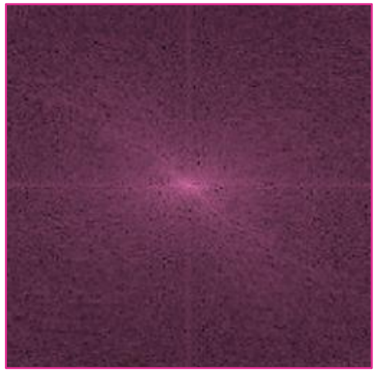
Thresholding



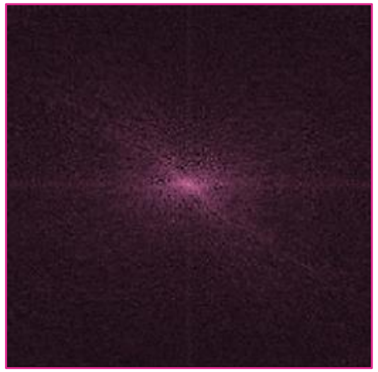
Scaling



Log



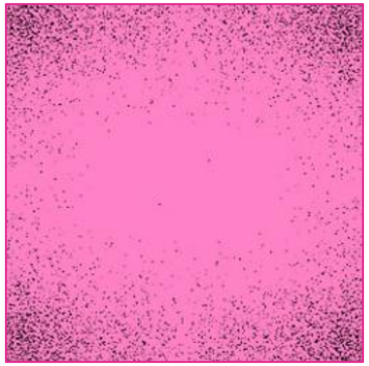
Log+Scaling



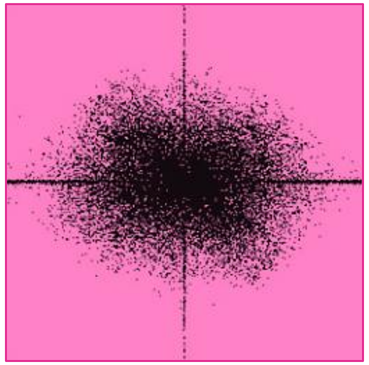
2D Fourier Transform

- FT as image & intensity transformations

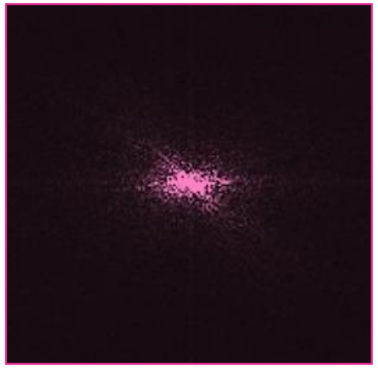
FT



Thresholding



Scaling



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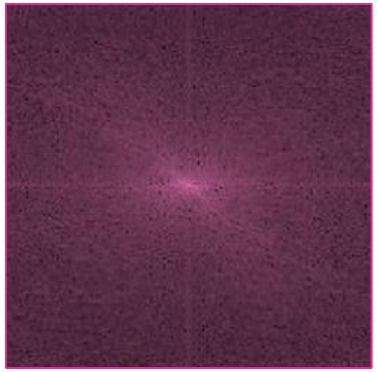

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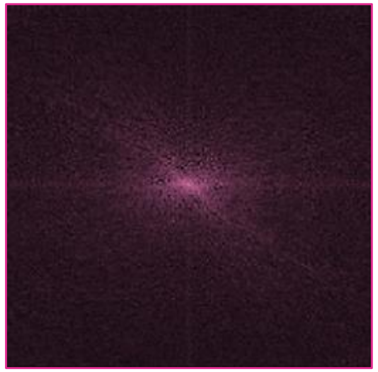
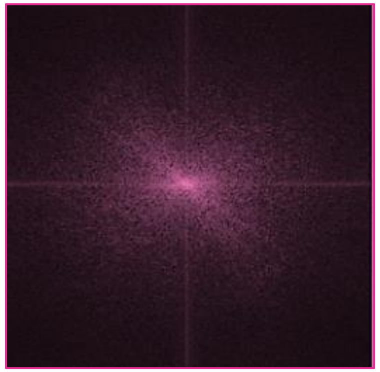
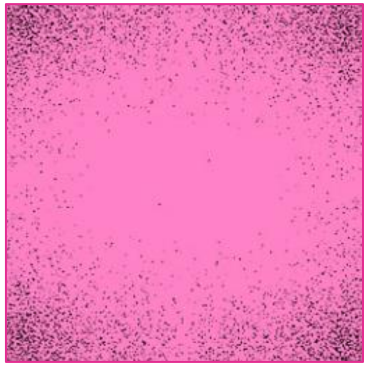




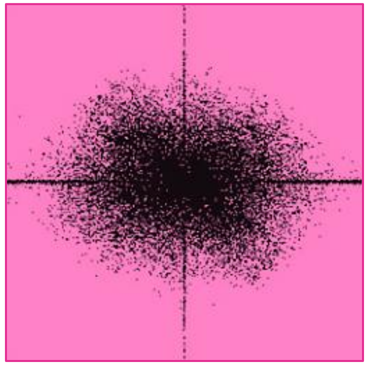
2D Fourier Transform

- FT as image & intensity transformations

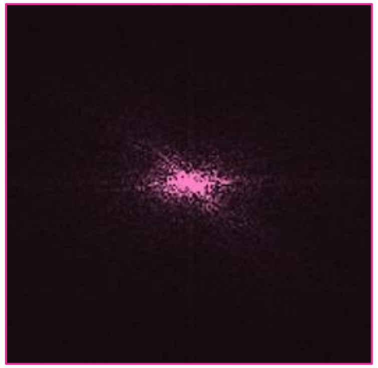
FT



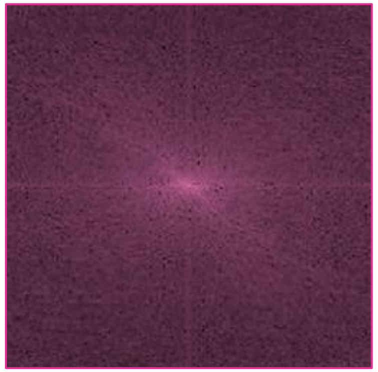
Thresholding



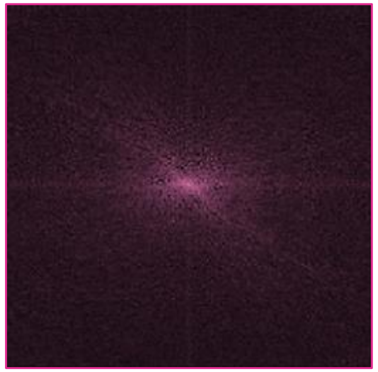
Scaling



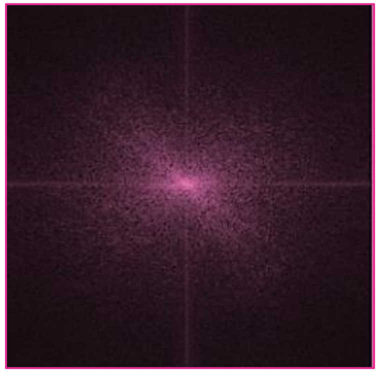
Log



Log+Scaling

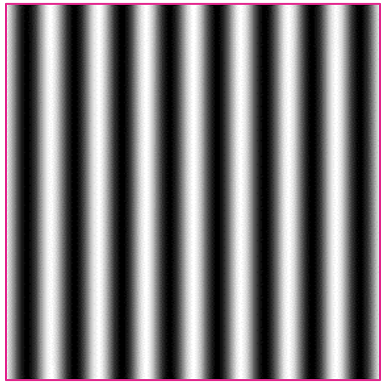


Log+Scaling+Histeq



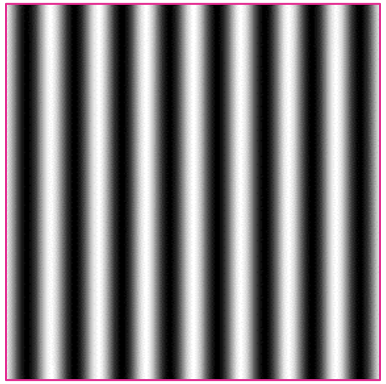
2D Fourier Transform

$f_1(x, y)$



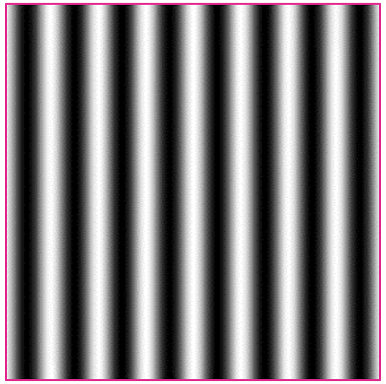
2D Fourier Transform

$f_1(x, y)$



2D Fourier Transform

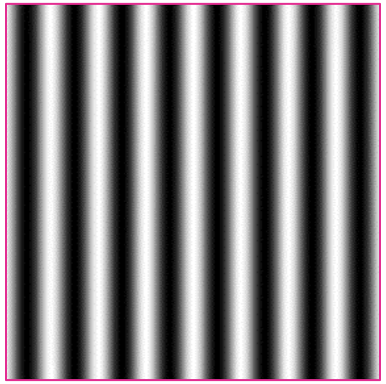
$f_1(x, y)$



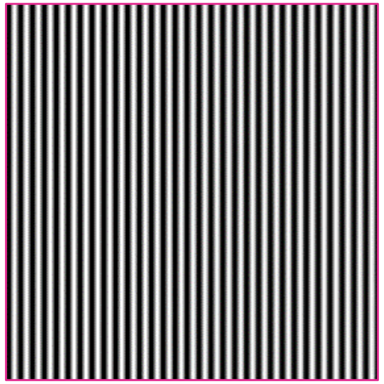
$\log(|F_1(u, v)|)$

2D Fourier Transform

$f_1(x, y)$

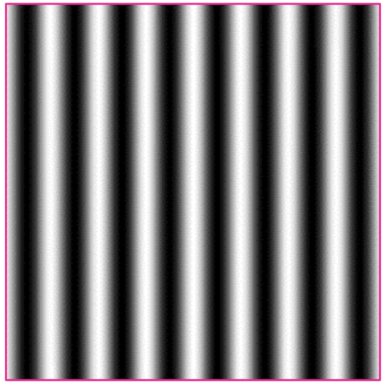


$\log(|F_1(u, v)|)$



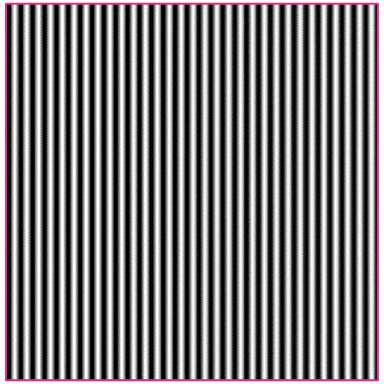
2D Fourier Transform

$f_1(x, y)$



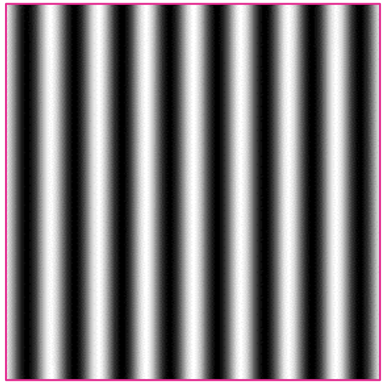
$\log(|F_1(u, v)|)$

$f_2(x, y)$



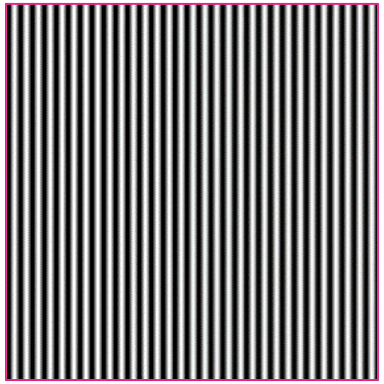
2D Fourier Transform

$f_1(x,y)$



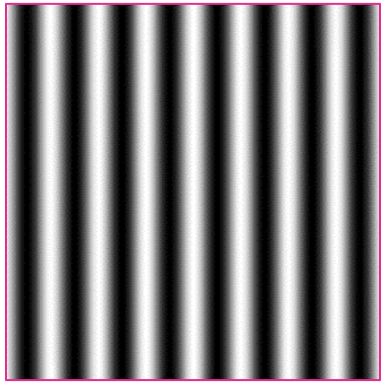
$\log(|F_1(u,v)|)$

$f_2(x,y)$



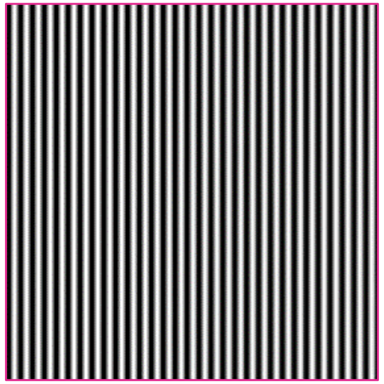
2D Fourier Transform

$f_1(x, y)$



$\log(|F_1(u, v)|)$

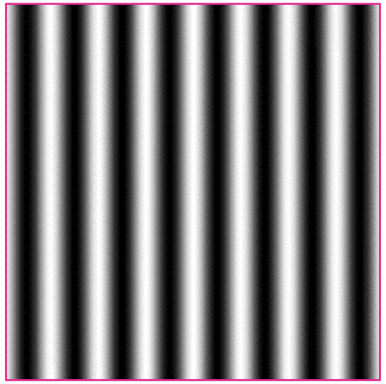
$f_2(x, y)$



$\log(|F_2(u, v)|)$

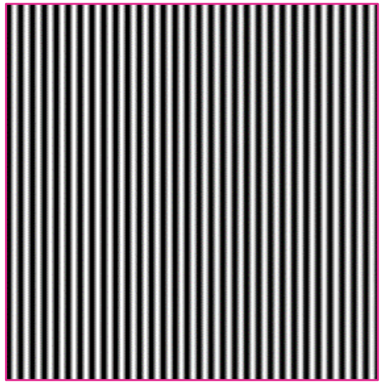
2D Fourier Transform

$f_1(x, y)$



$\log(|F_1(u, v)|)$

$f_2(x, y)$

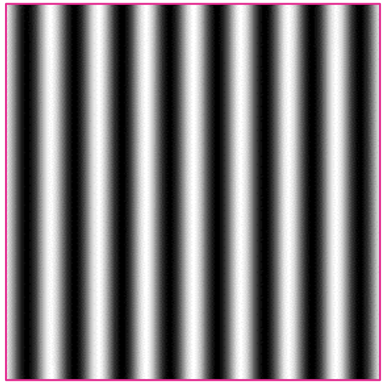


$\log(|F_2(u, v)|)$

$f_1(x, y) + f_2(x, y)$

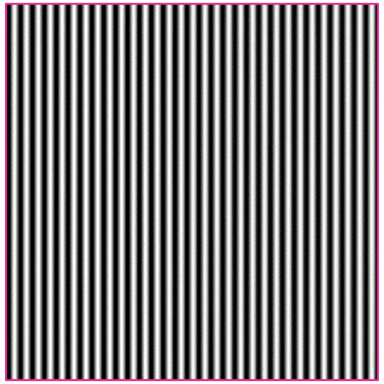
2D Fourier Transform

$f_1(x, y)$



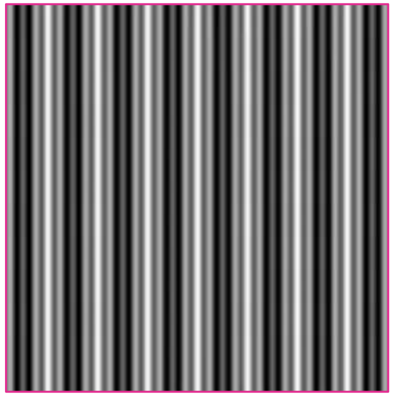
$\log(|F_1(u, v)|)$

$f_2(x, y)$



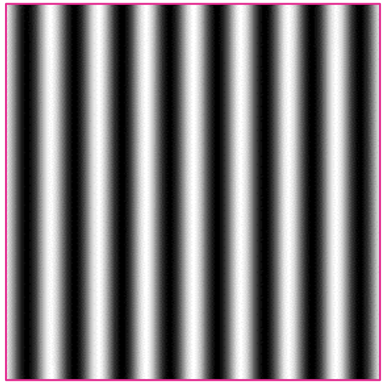
$\log(|F_2(u, v)|)$

$f_1(x, y) + f_2(x, y)$



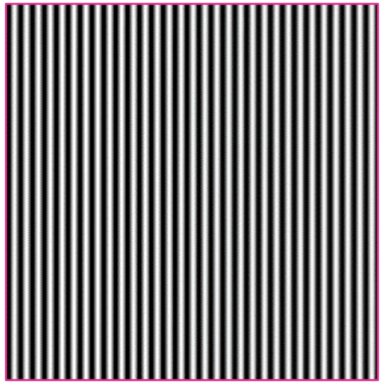
2D Fourier Transform

$f_1(x, y)$



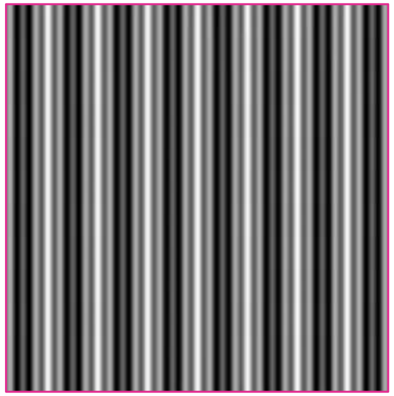
$\log(|F_1(u, v)|)$

$f_2(x, y)$



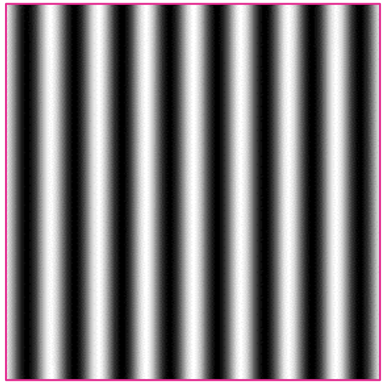
$\log(|F_2(u, v)|)$

$f_1(x, y) + f_2(x, y)$



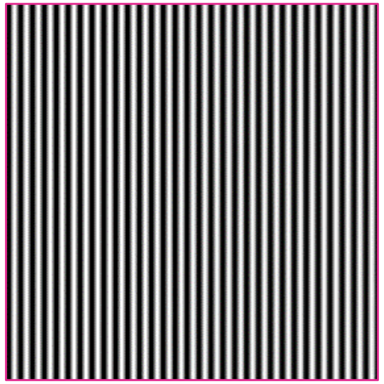
2D Fourier Transform

$f_1(x, y)$



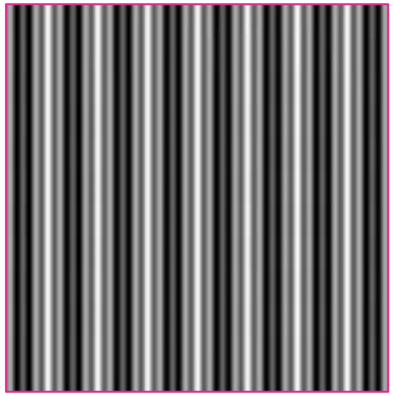
$\log(|F_1(u, v)|)$

$f_2(x, y)$



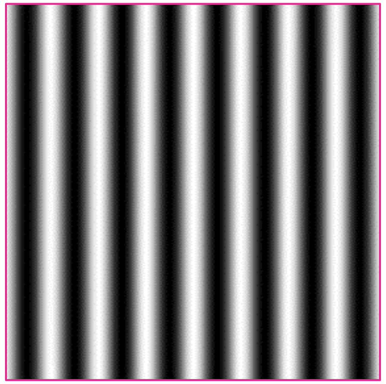
$\log(|F_2(u, v)|)$

$f_1(x, y) + f_2(x, y)$



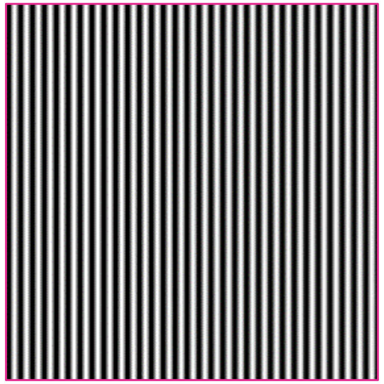
2D Fourier Transform

$f_1(x, y)$



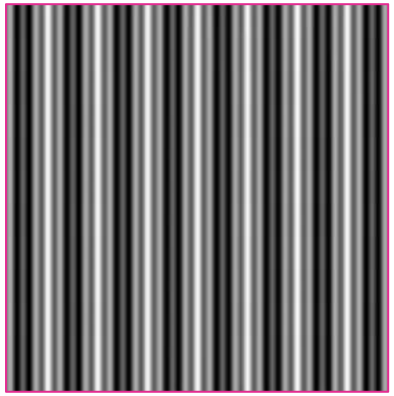
$\log(|F_1(u, v)|)$

$f_2(x, y)$



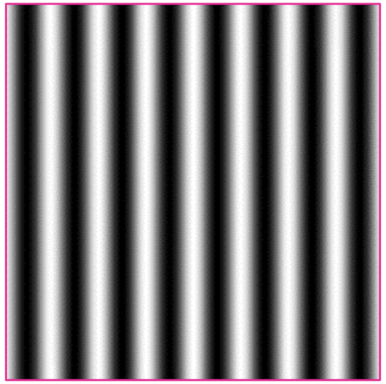
$\log(|F_2(u, v)|)$

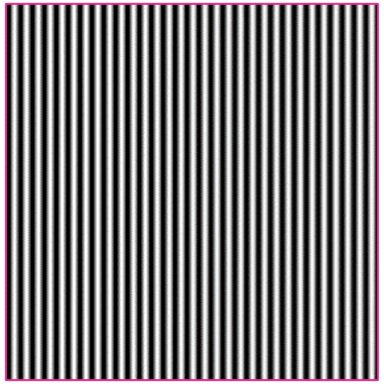
$f_1(x, y) + f_2(x, y)$

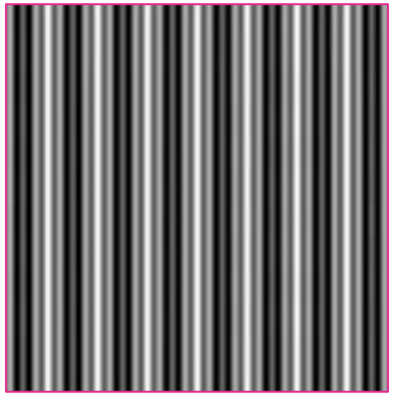


$\log(|F_1(u, v) + F_2(u, v)|)$

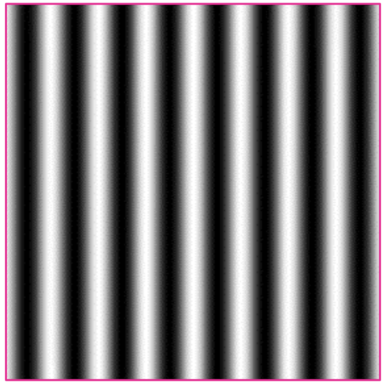
2D Fourier Transform

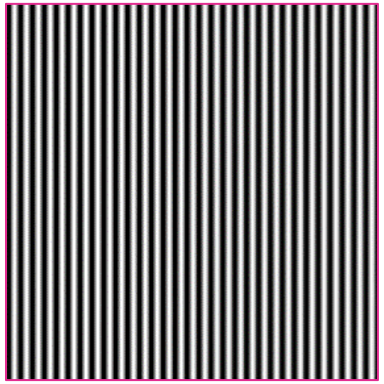


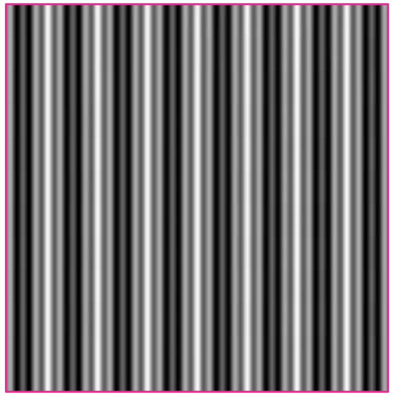


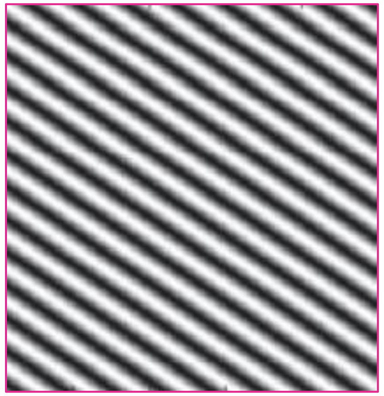



2D Fourier Transform

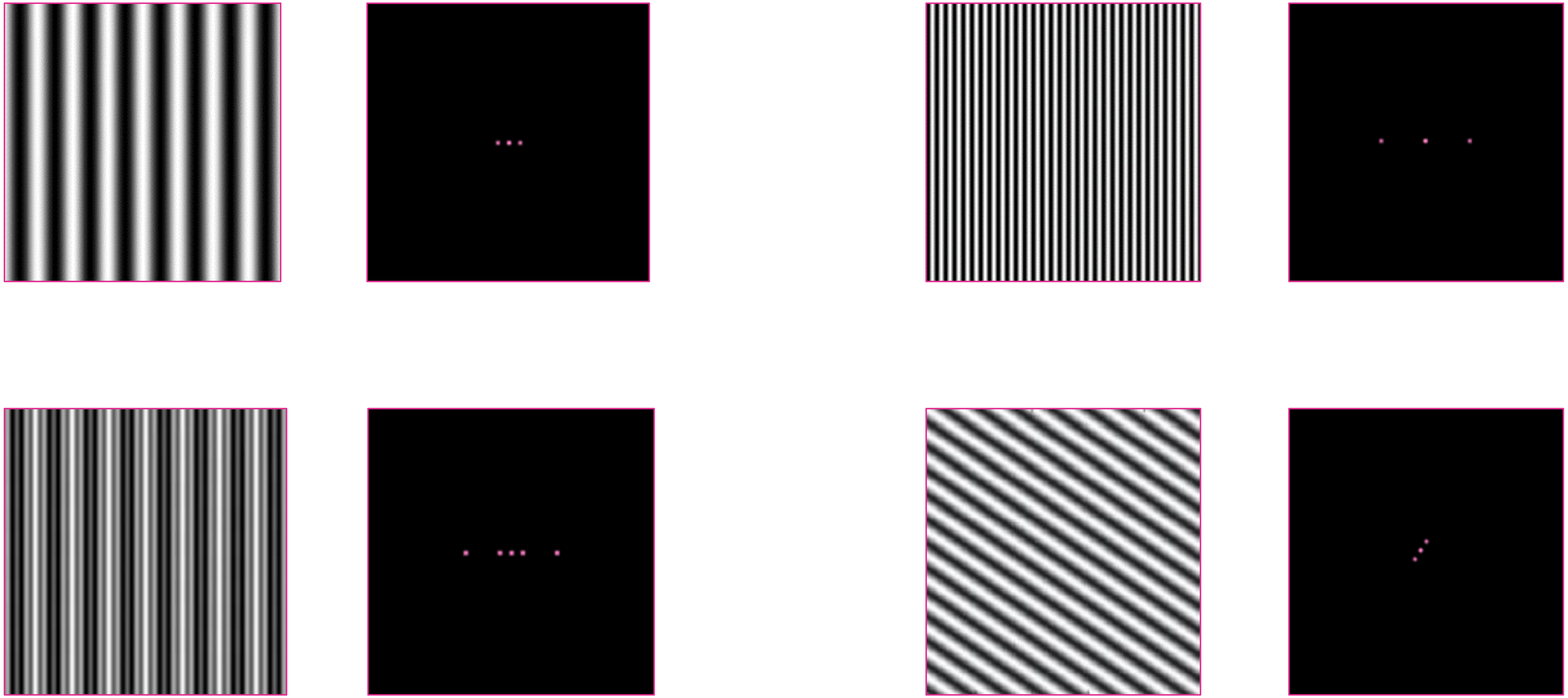




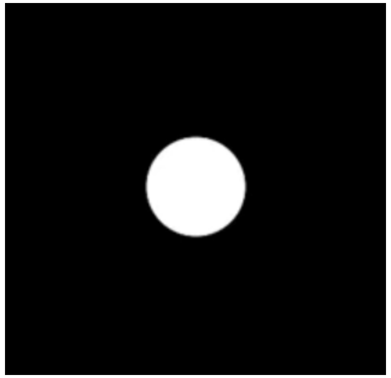




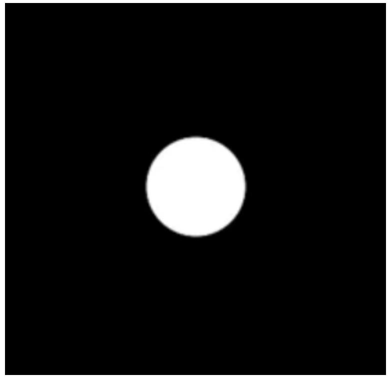
2D Fourier Transform

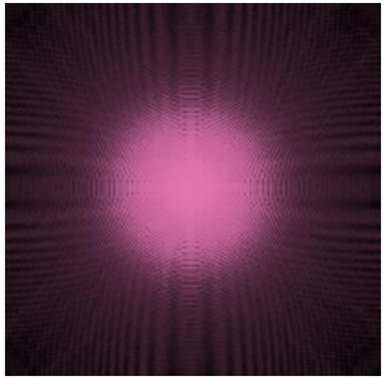


2D Fourier Transform

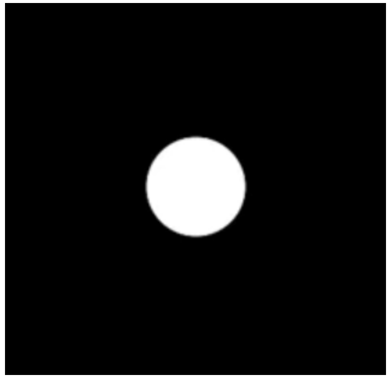


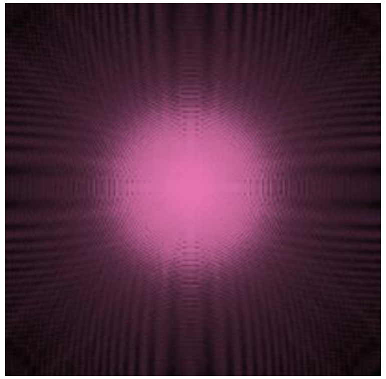
2D Fourier Transform

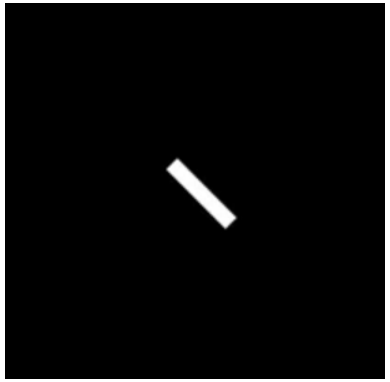




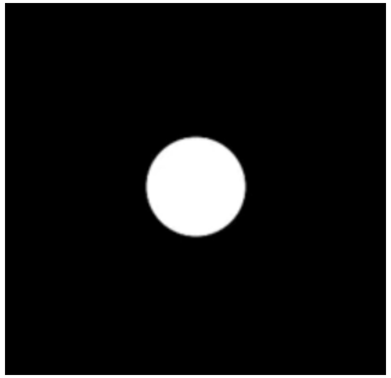
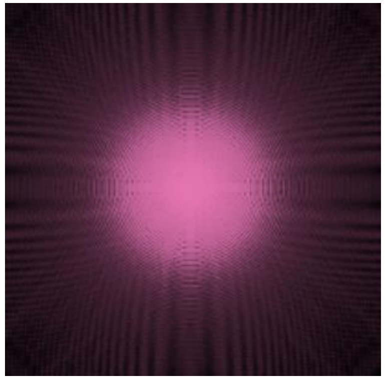
2D Fourier Transform

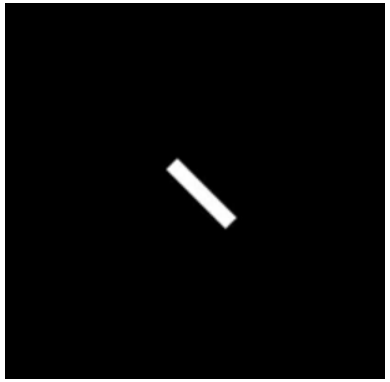
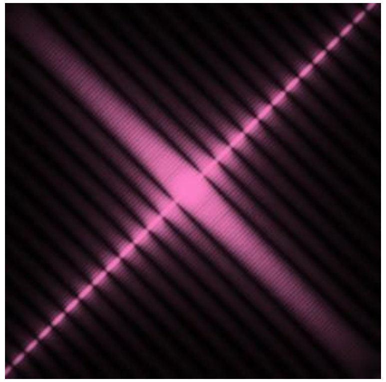




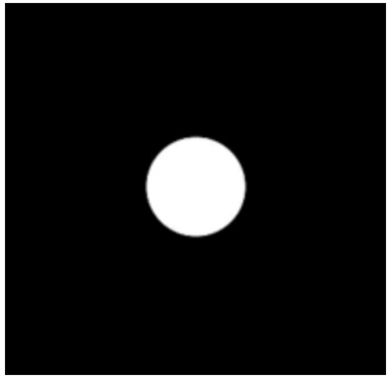
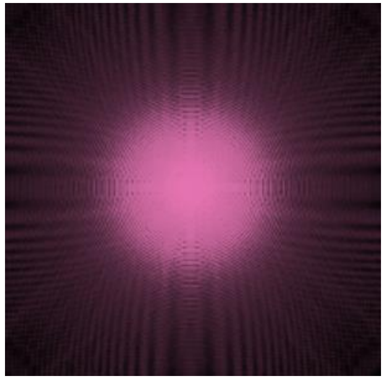
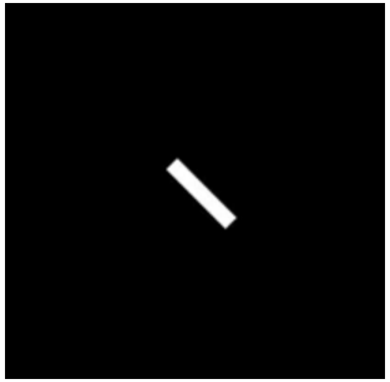
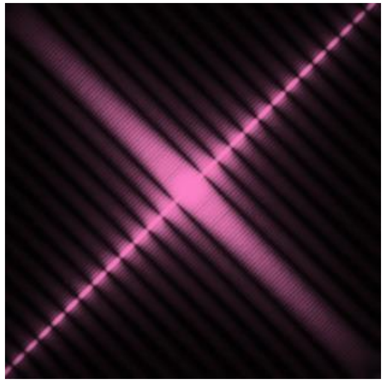
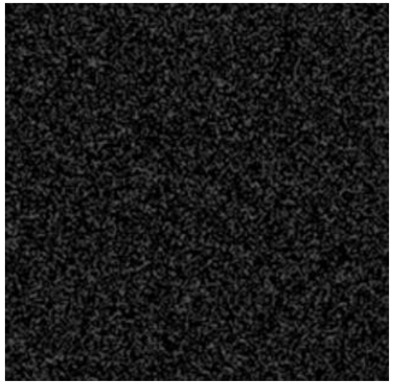


2D Fourier Transform

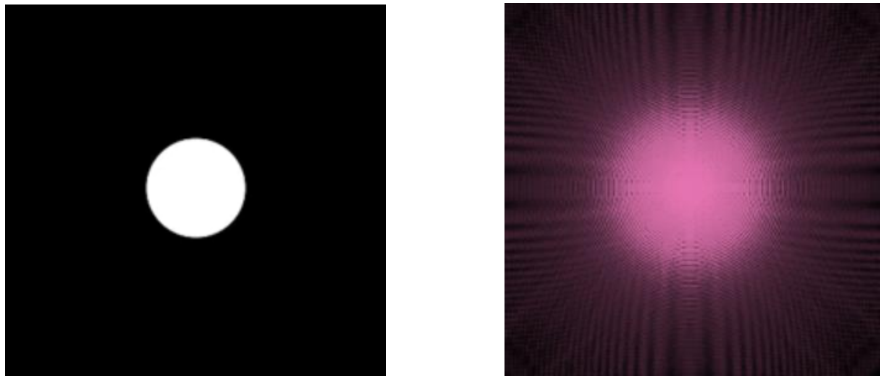
 

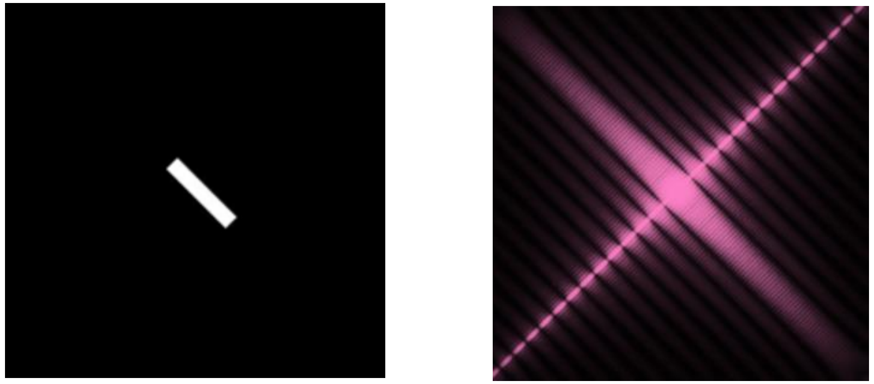
 

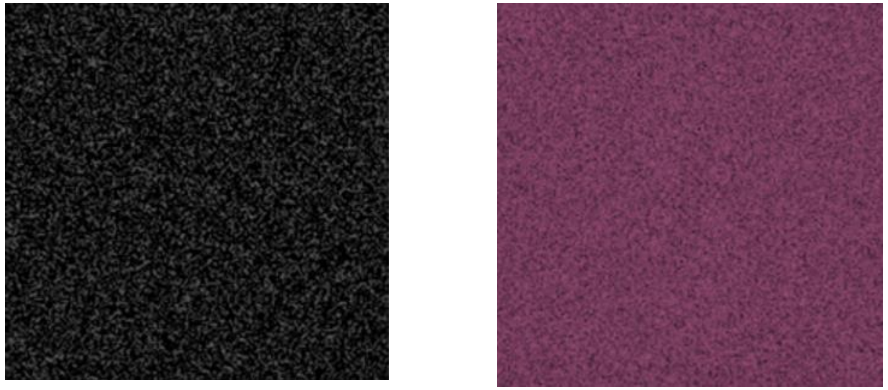
2D Fourier Transform

2D Fourier Transform







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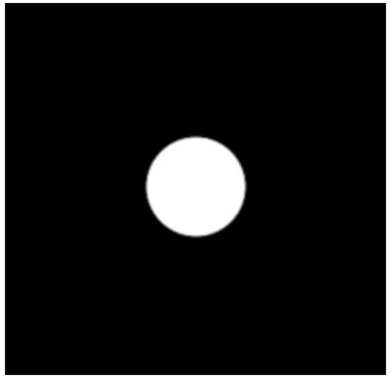
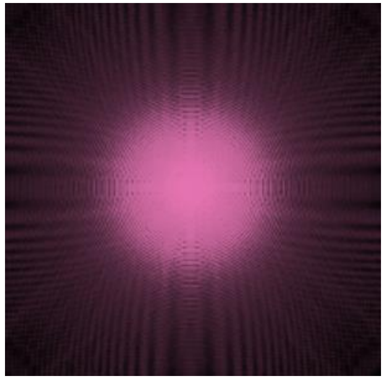
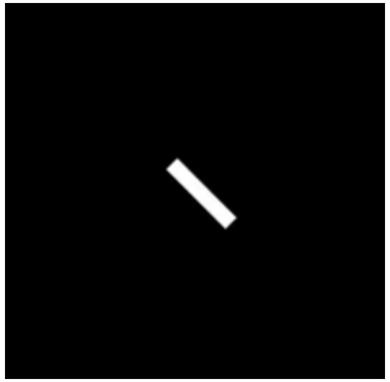
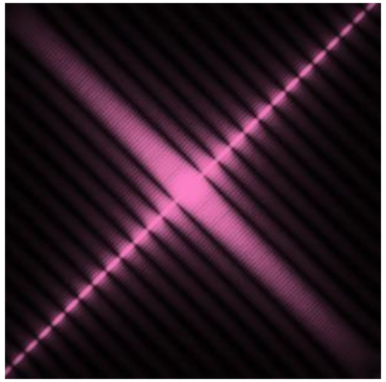
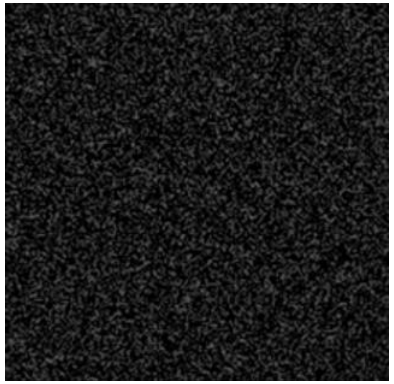
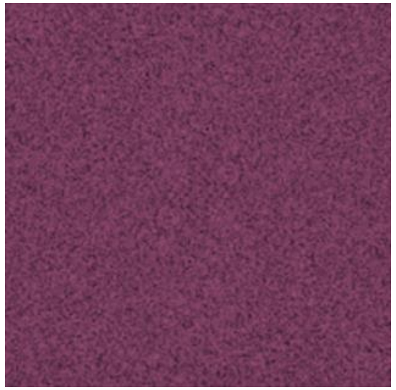


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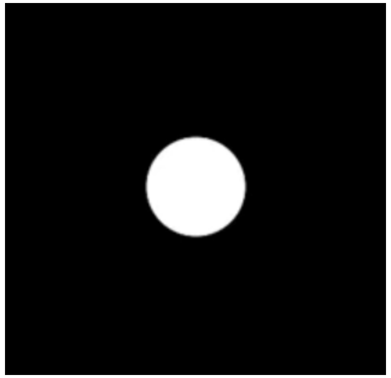
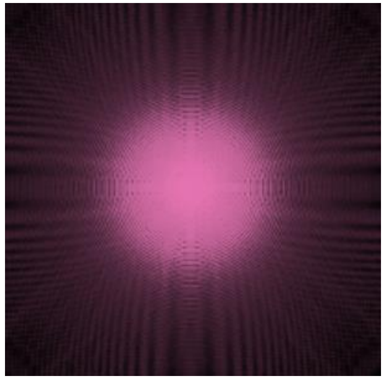
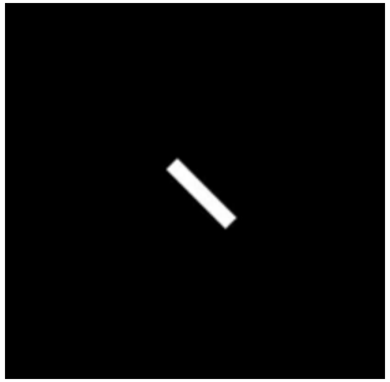
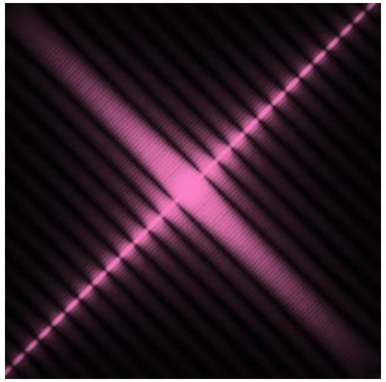
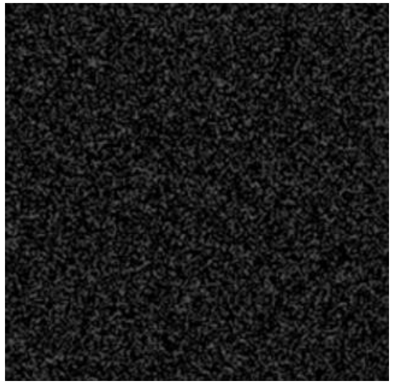
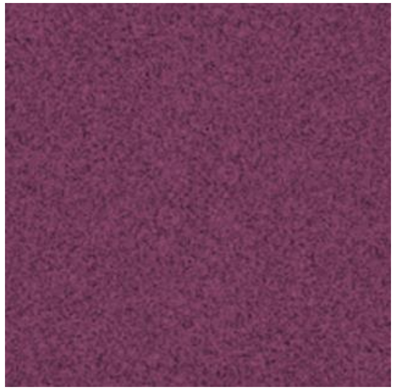

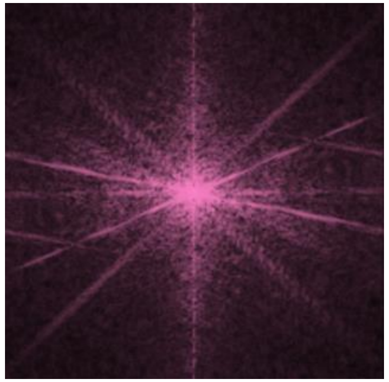

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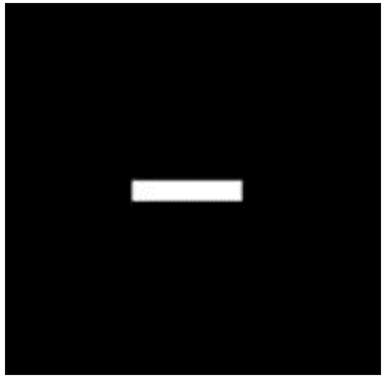

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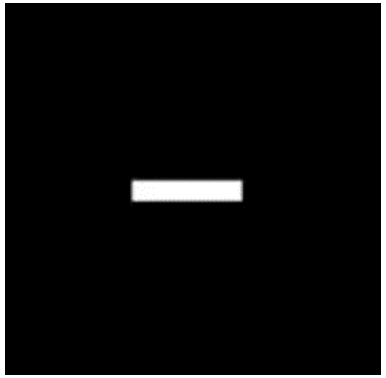

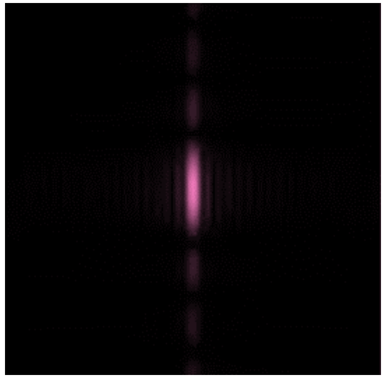

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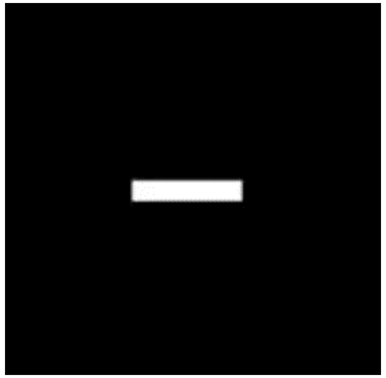
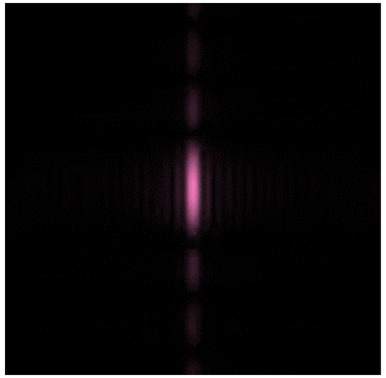
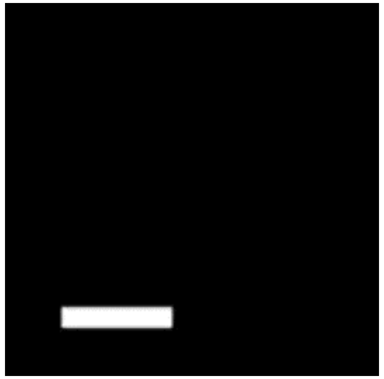
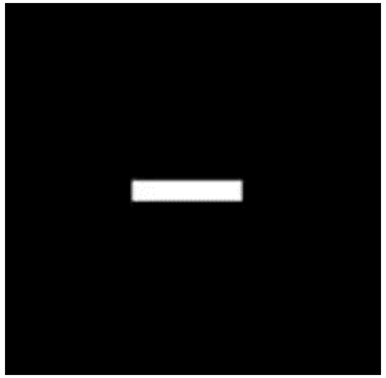


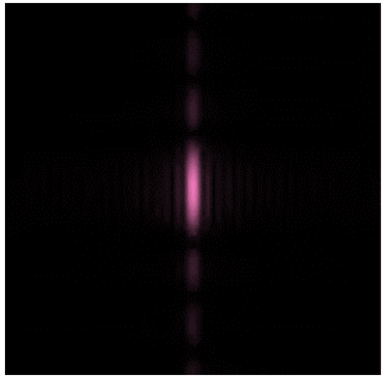


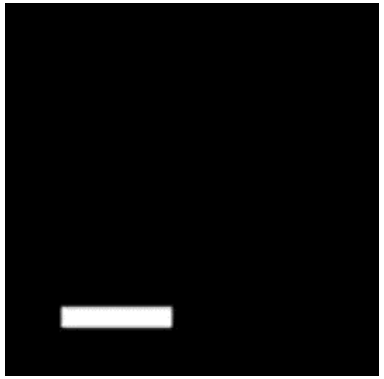


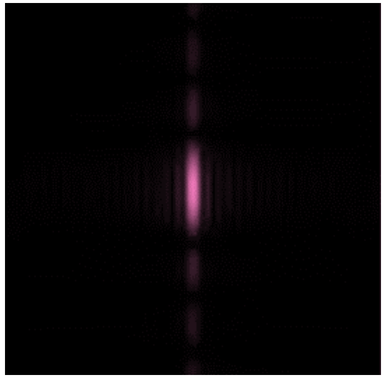
2D Fourier Transform

2D Fourier Transform









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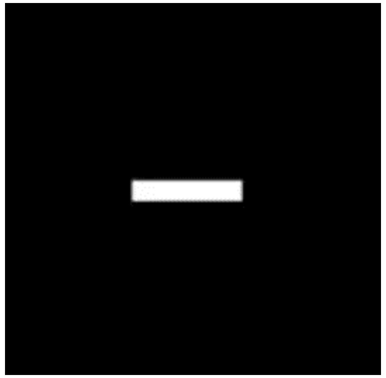

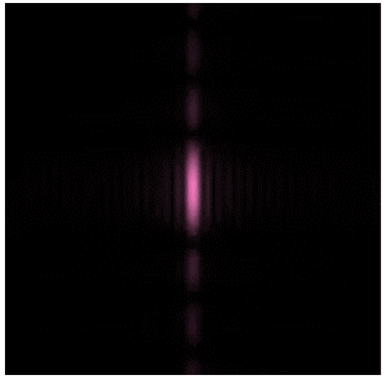

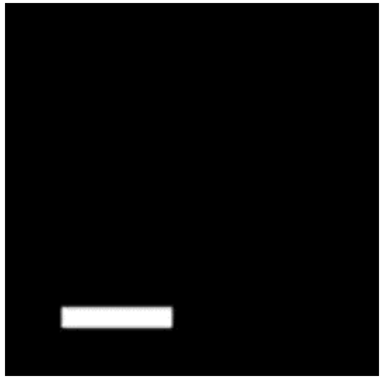

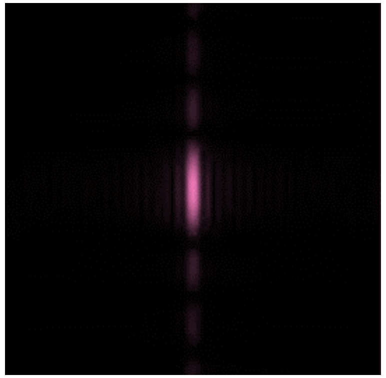

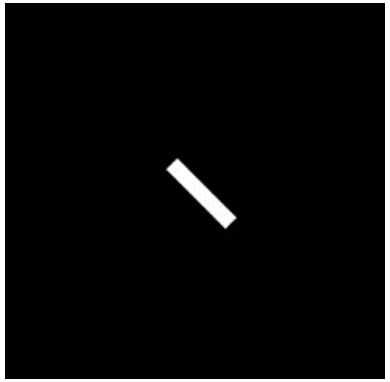

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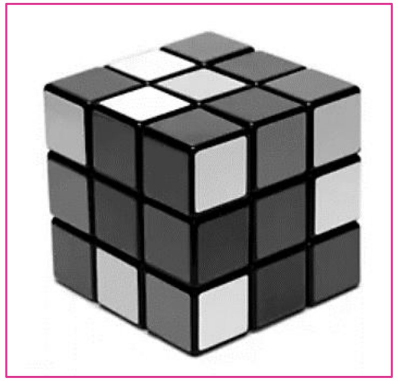

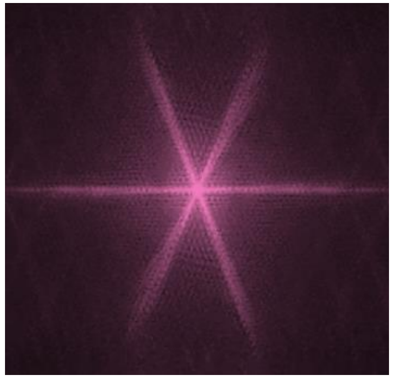

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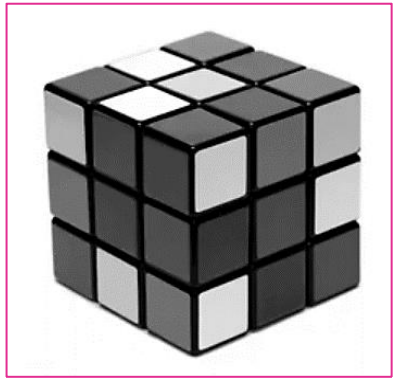

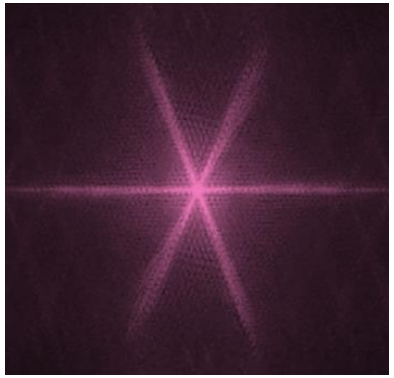

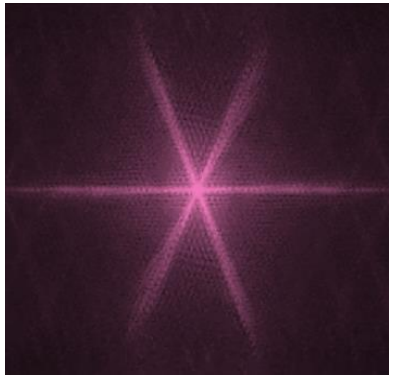

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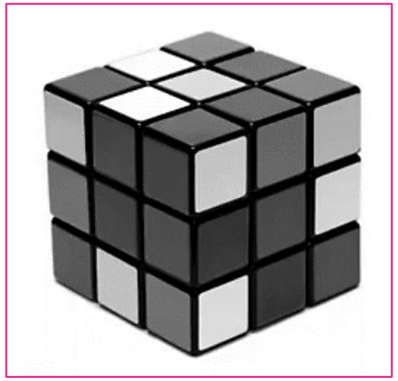

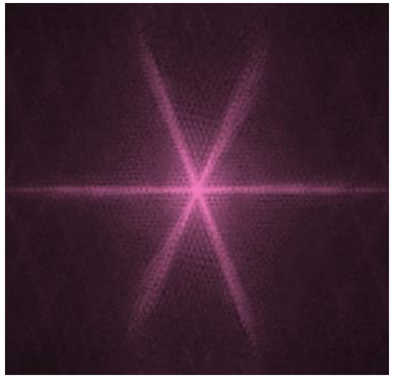

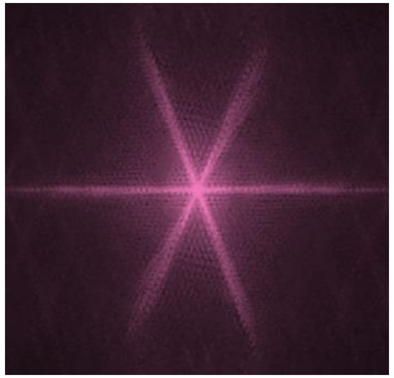

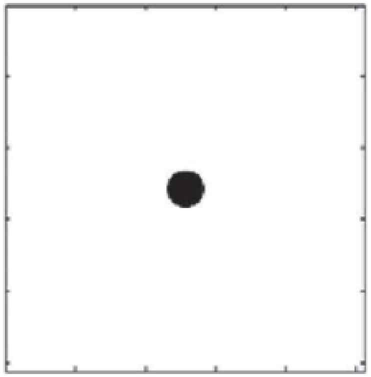

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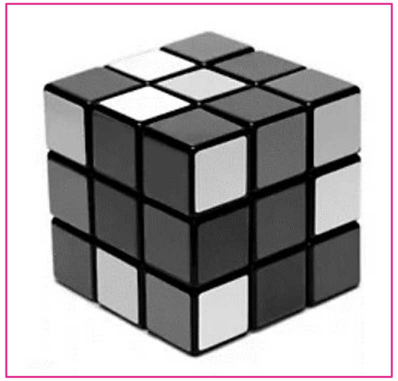

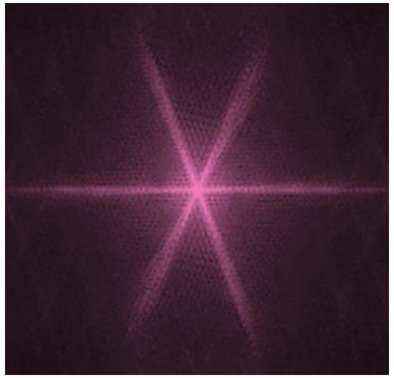

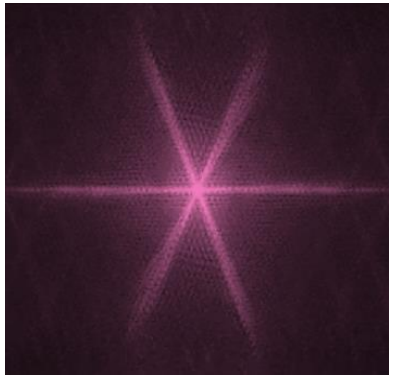

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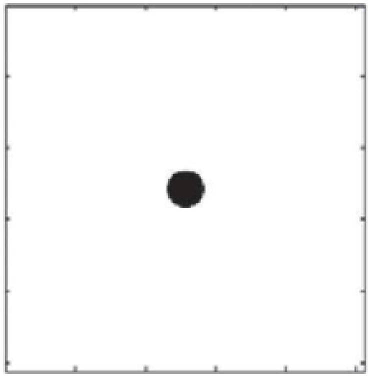

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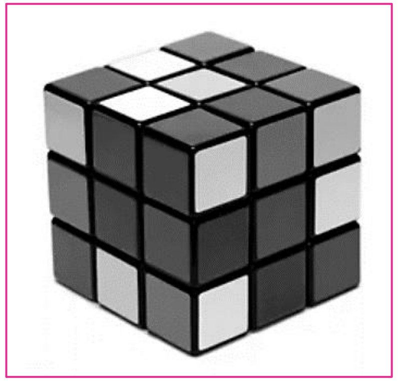

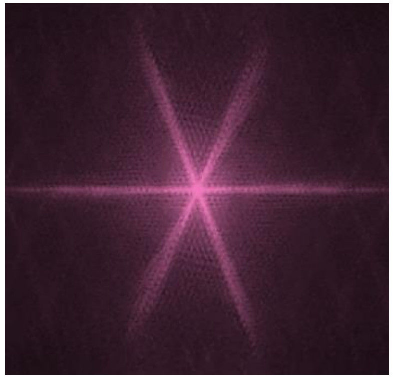

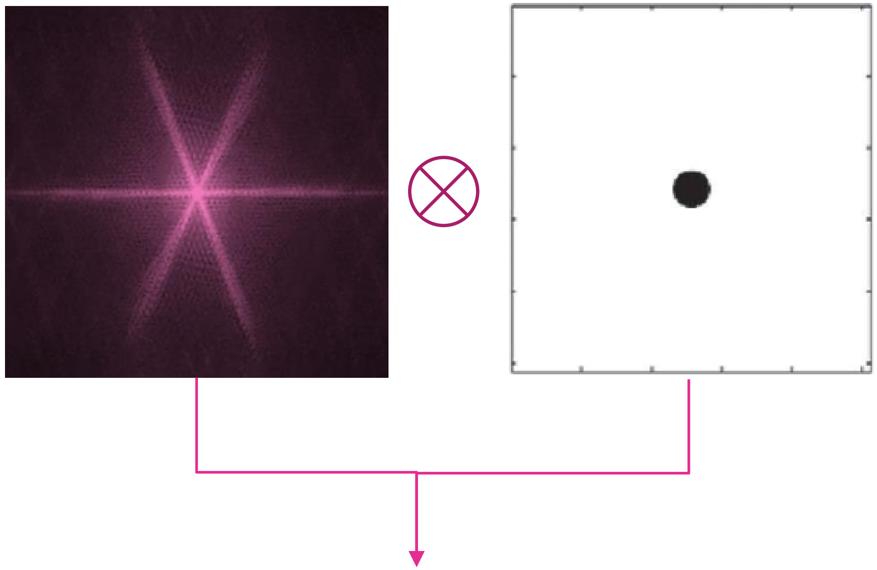

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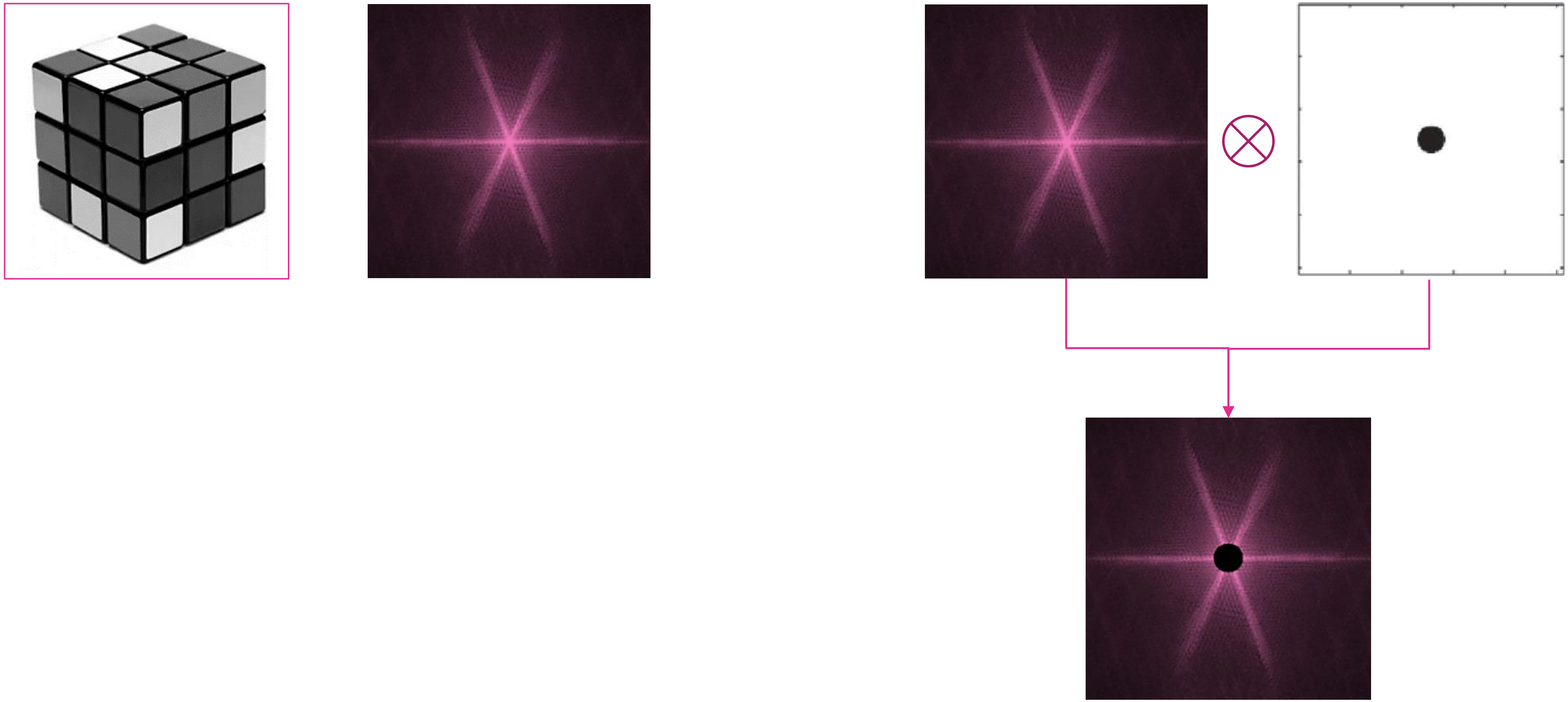

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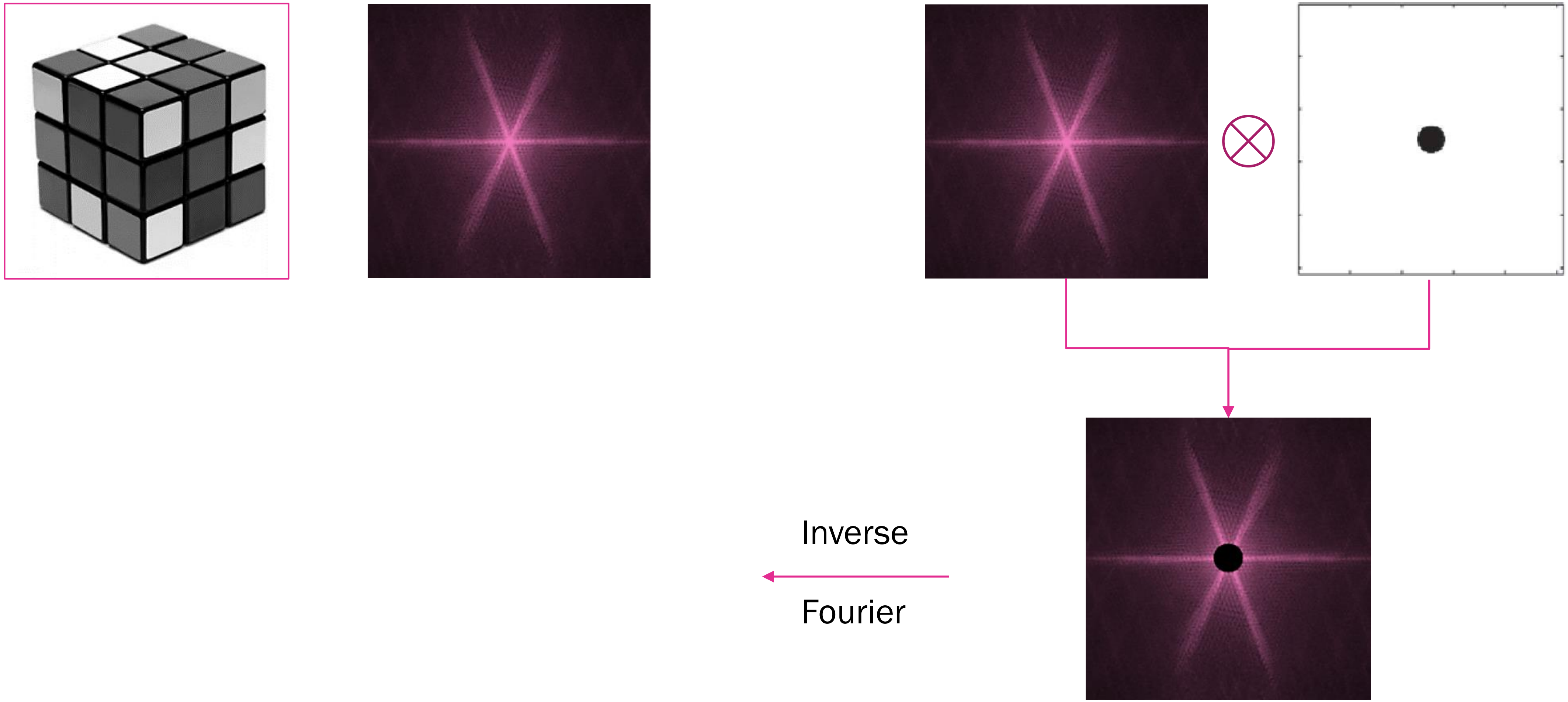

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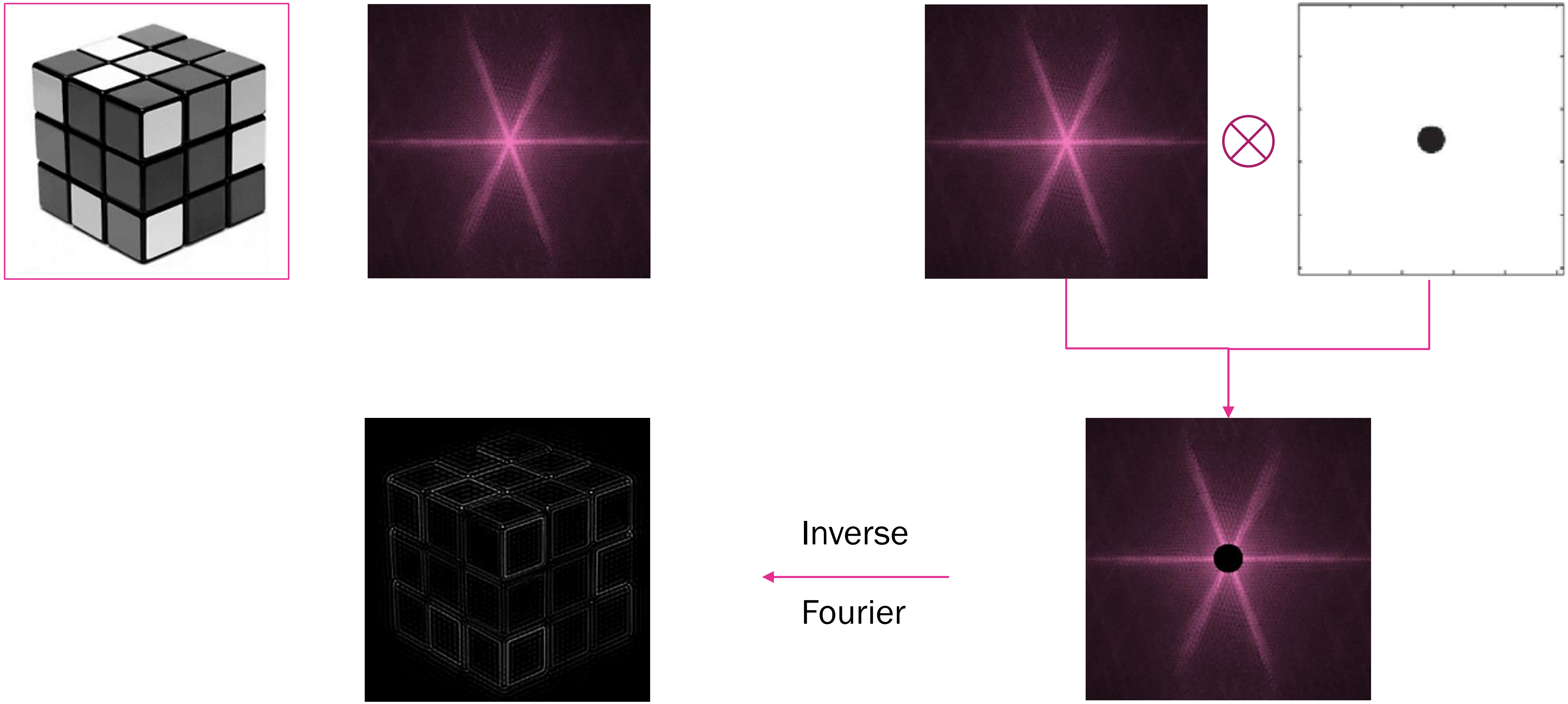

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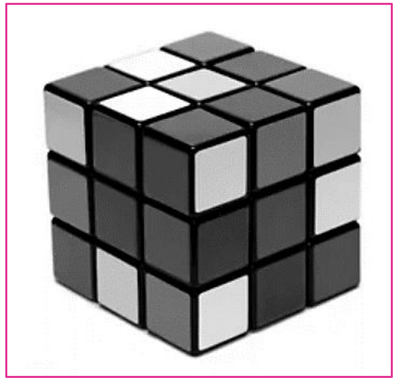

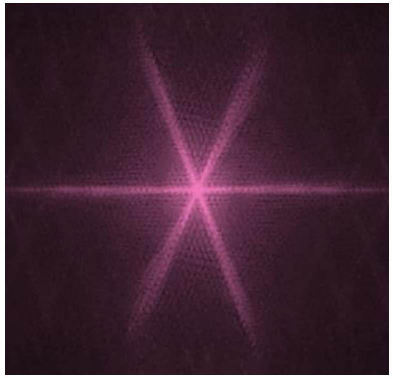

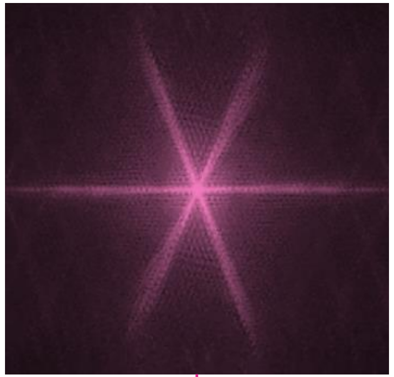

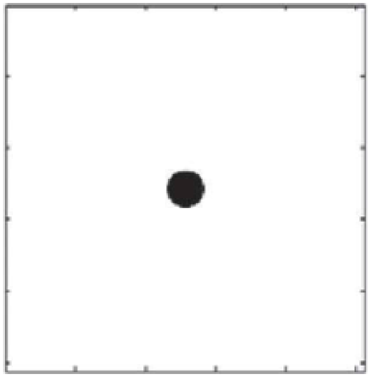

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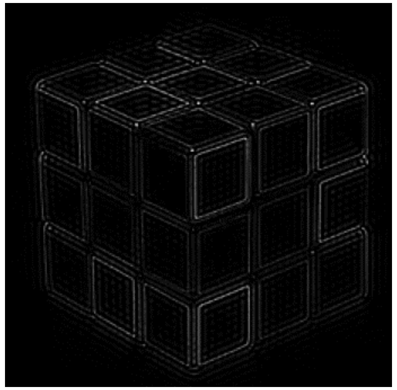
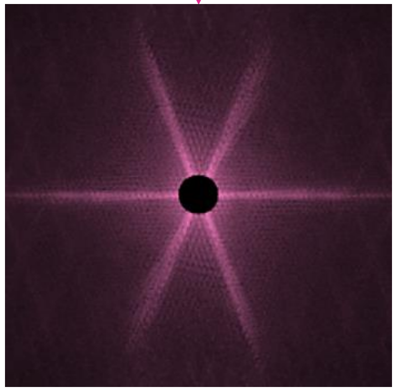


Inverse

Fourier



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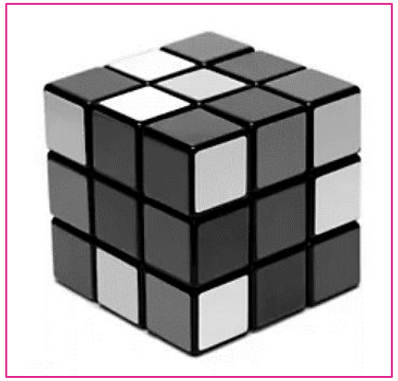

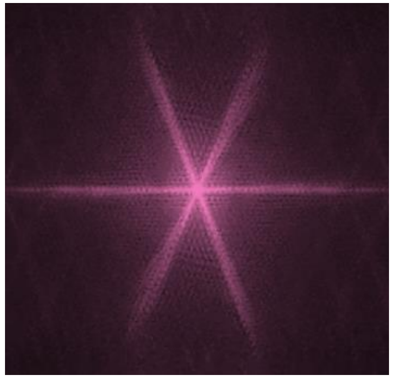

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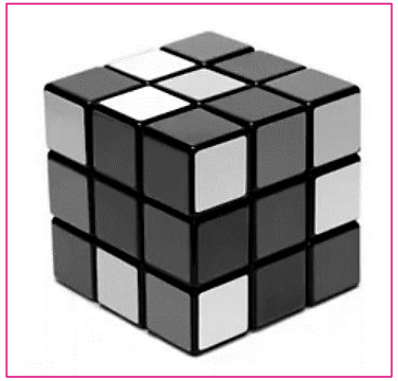

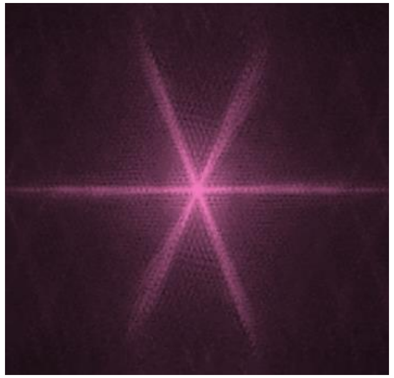

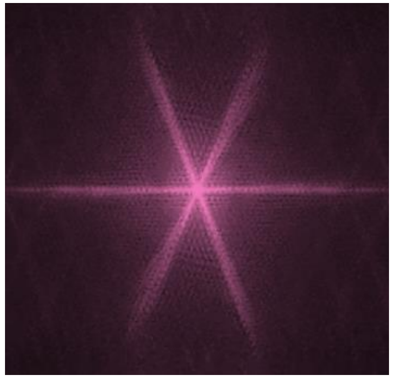

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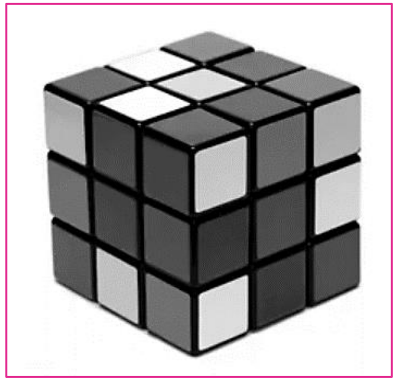

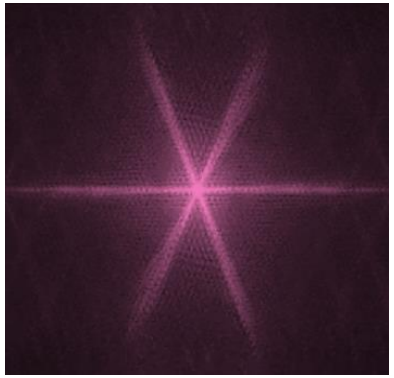

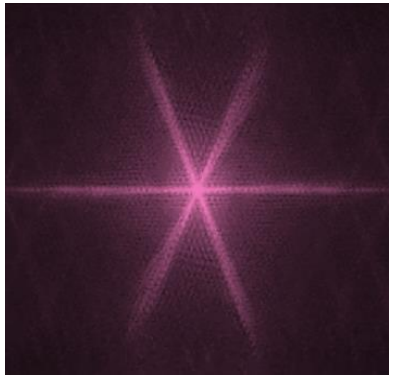

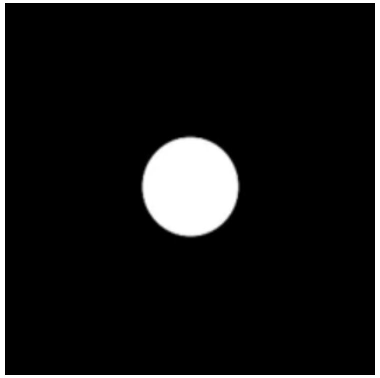

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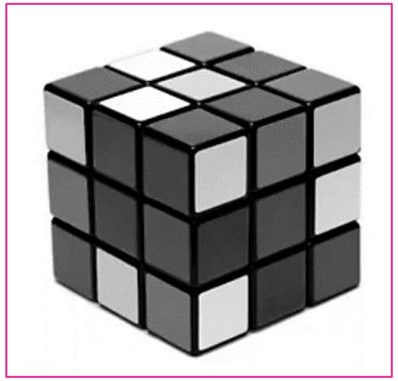

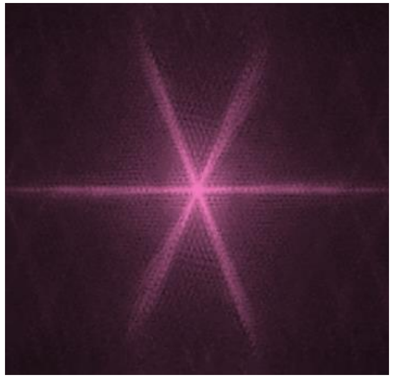

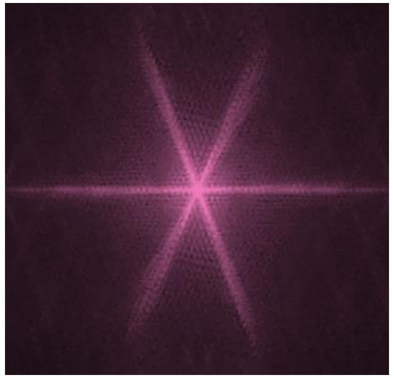

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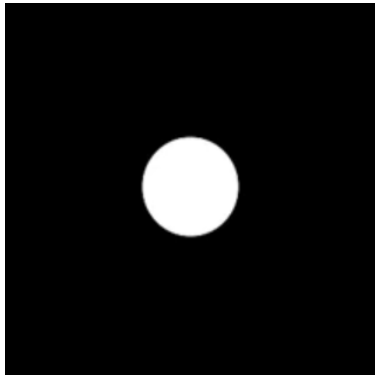

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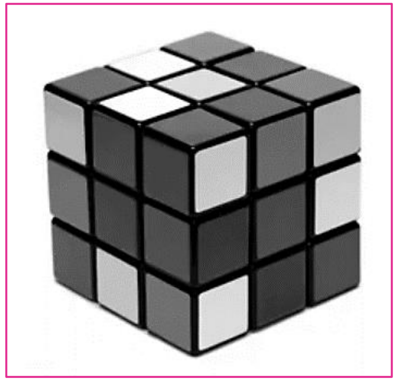

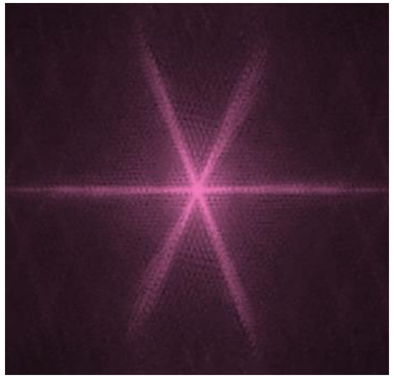

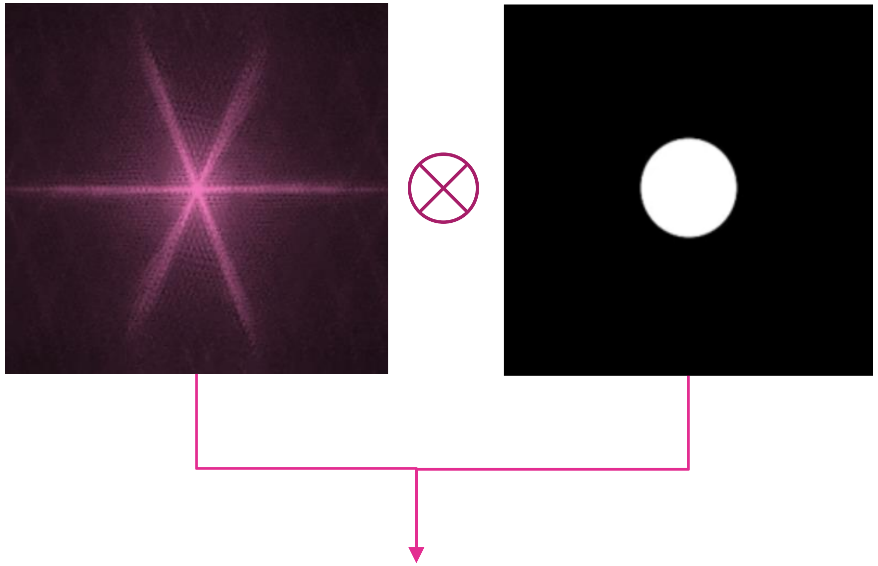

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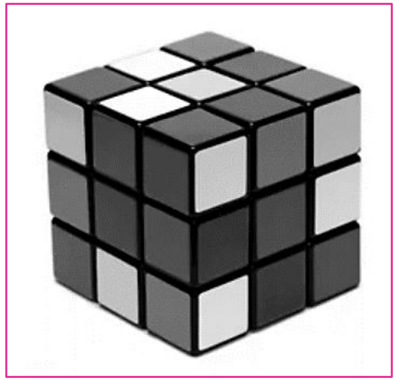

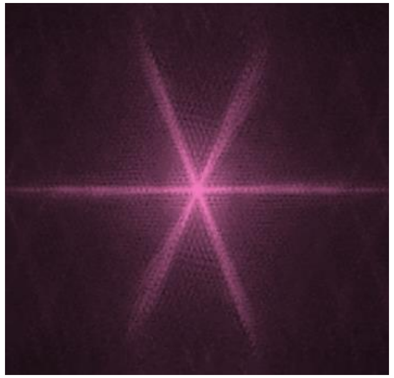

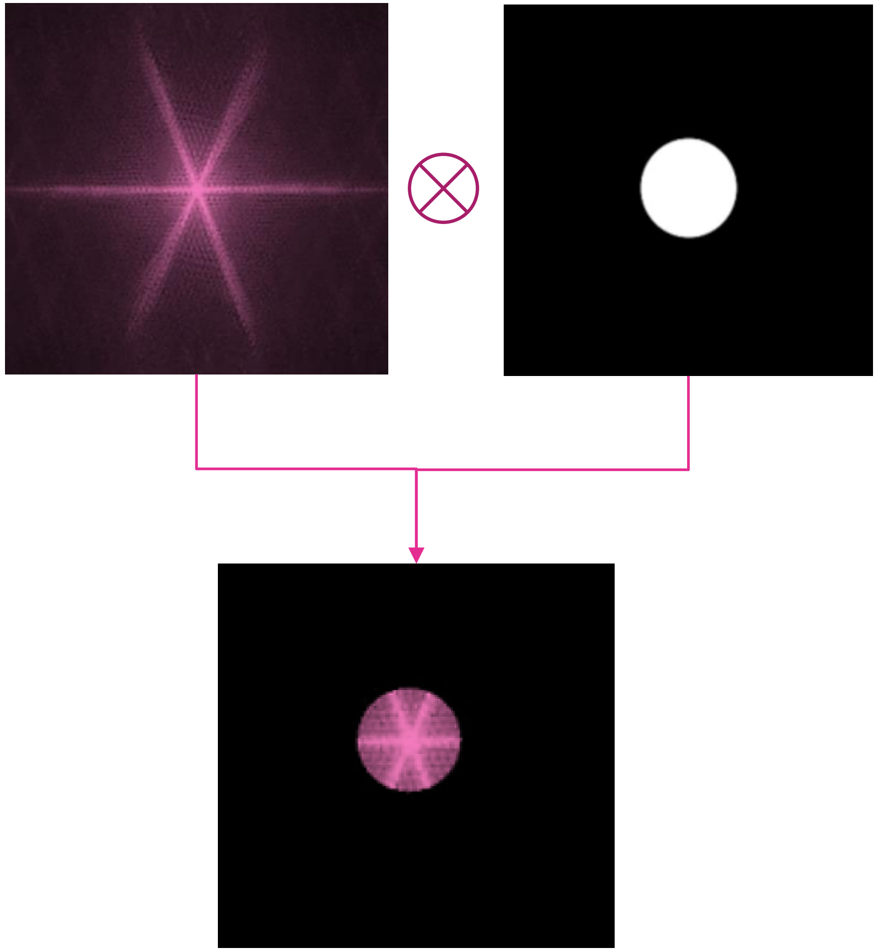

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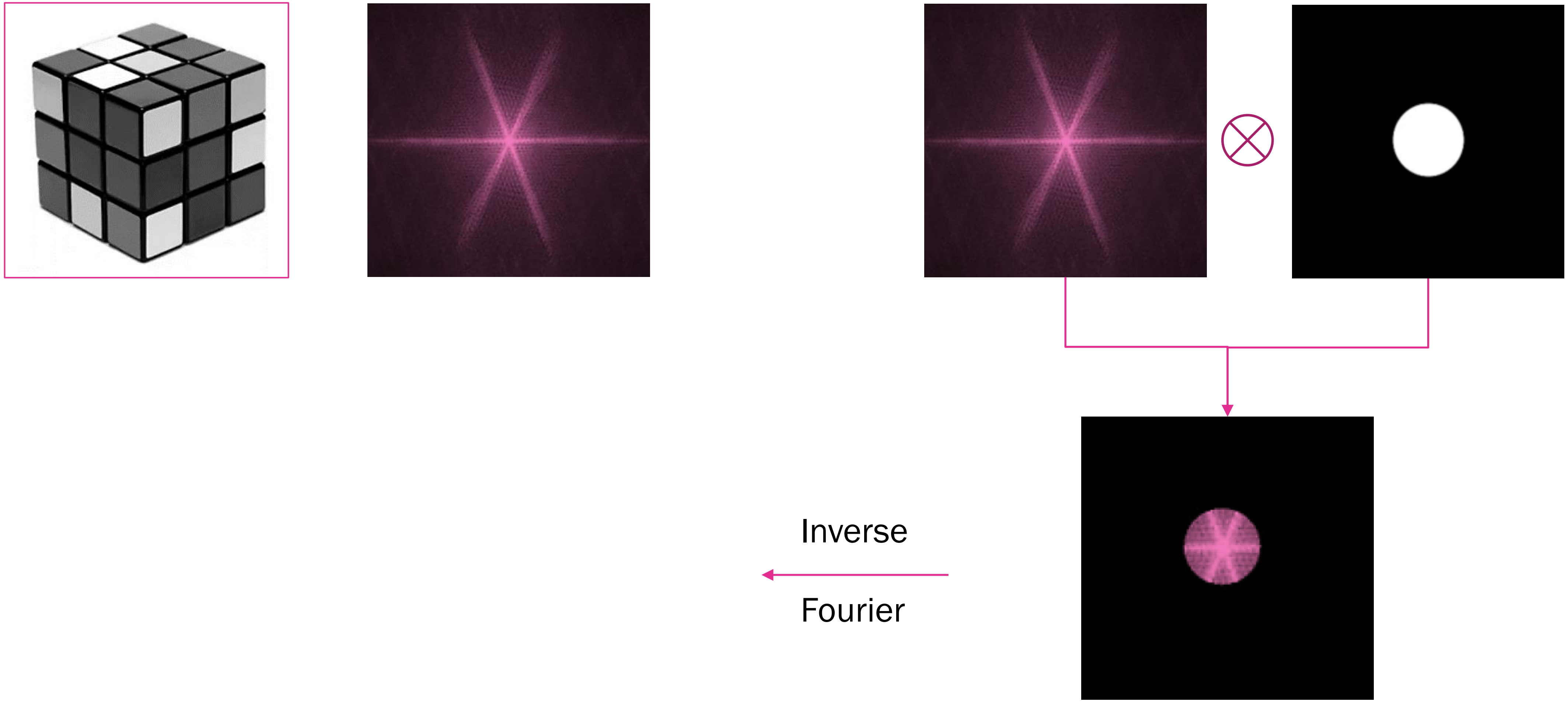

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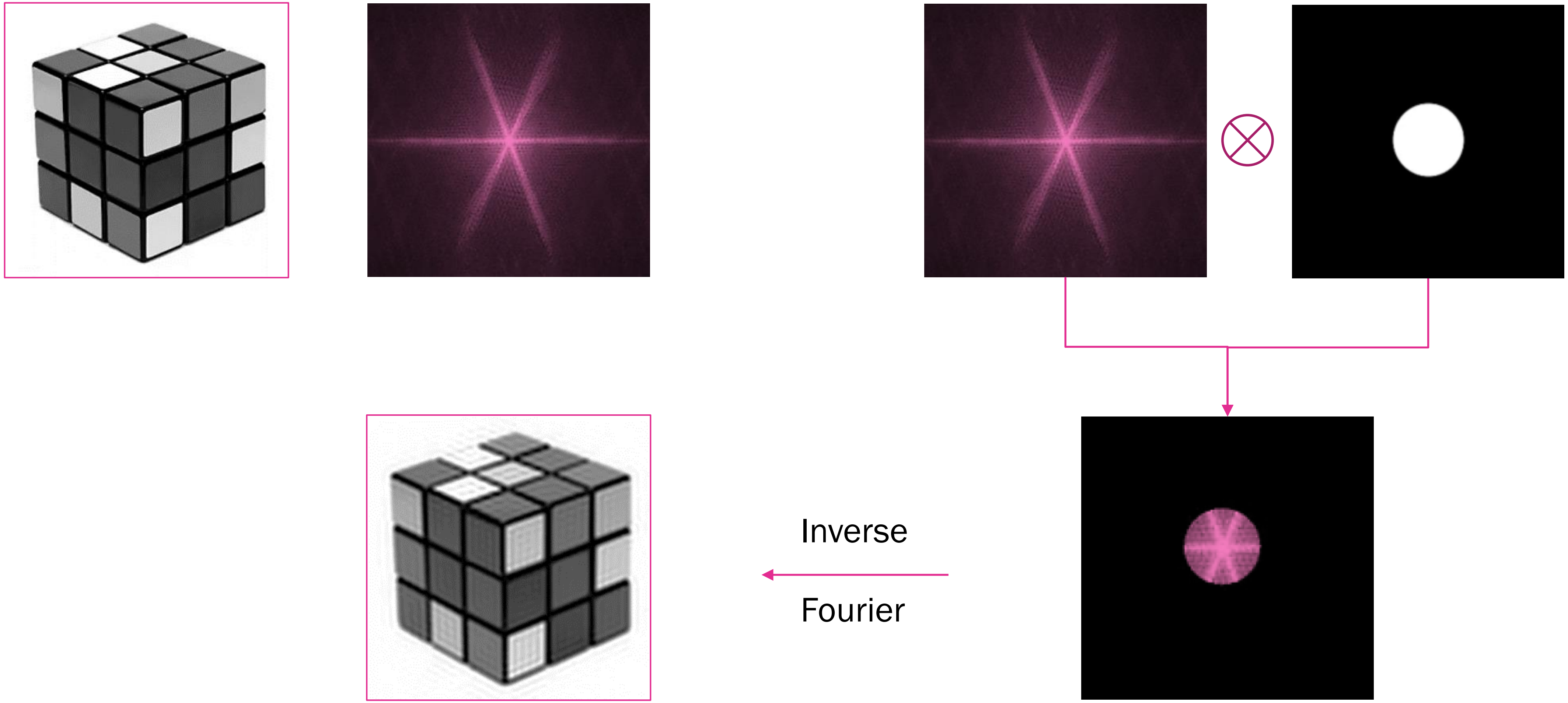
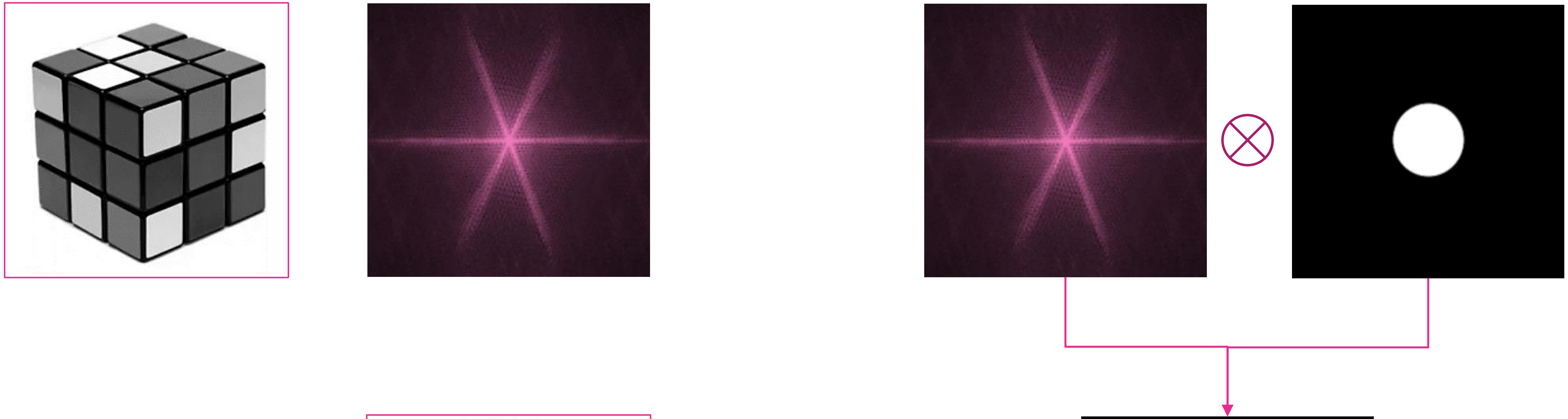
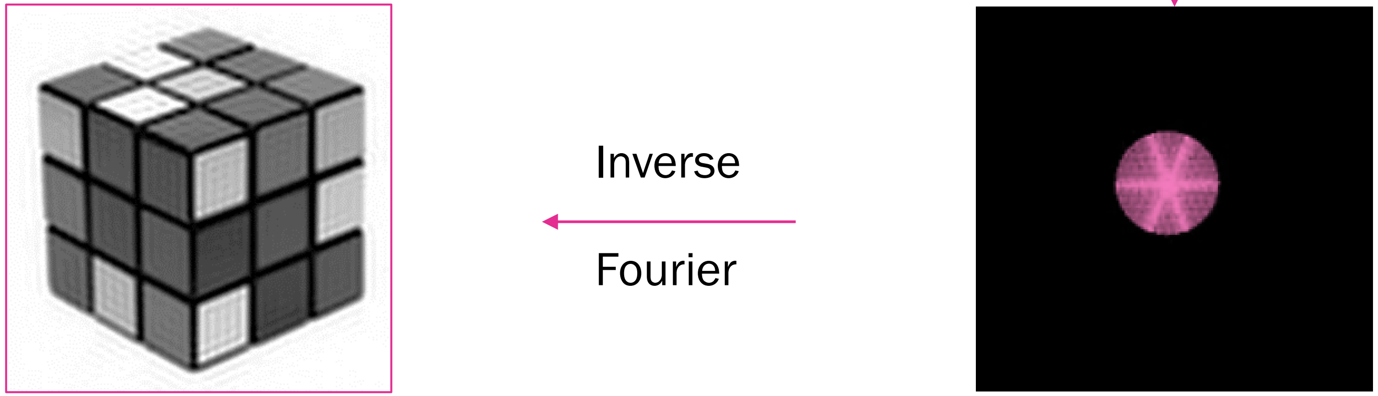


Frequency Filtering



- LPF



Frequency Filtering

Input

Frequency Filtering

Input

Frequency Filtering

Input

LPF

Frequency Filtering

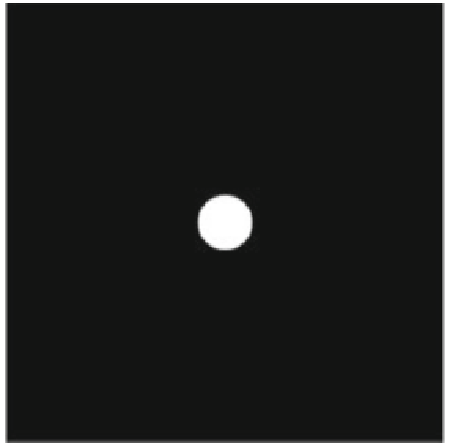
Input

LPF

Frequency Filtering

Input

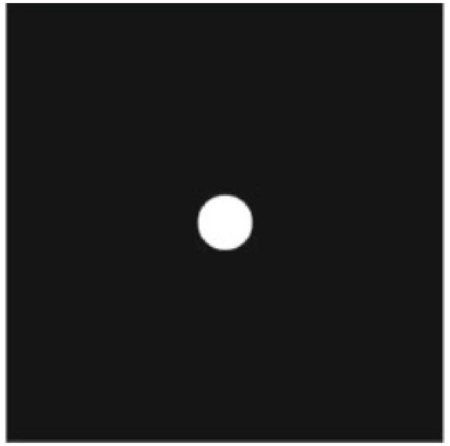
LPF



Frequency Filtering

Input

LPF

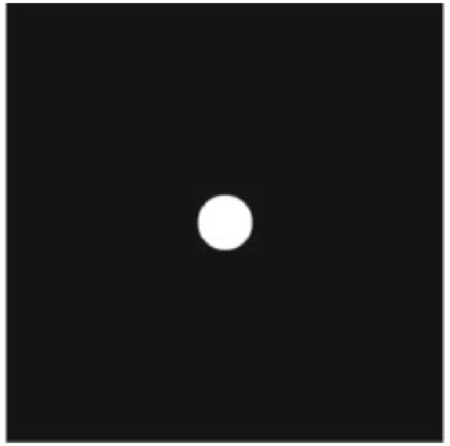


HPF

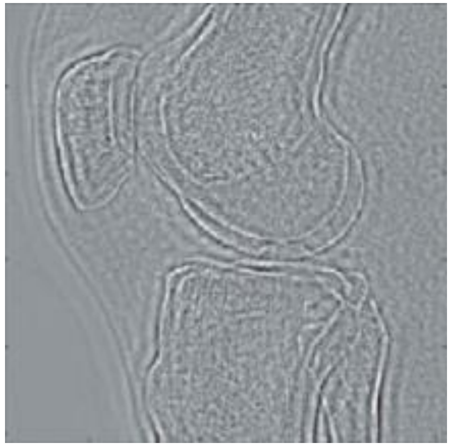
Frequency Filtering

Input

LPF

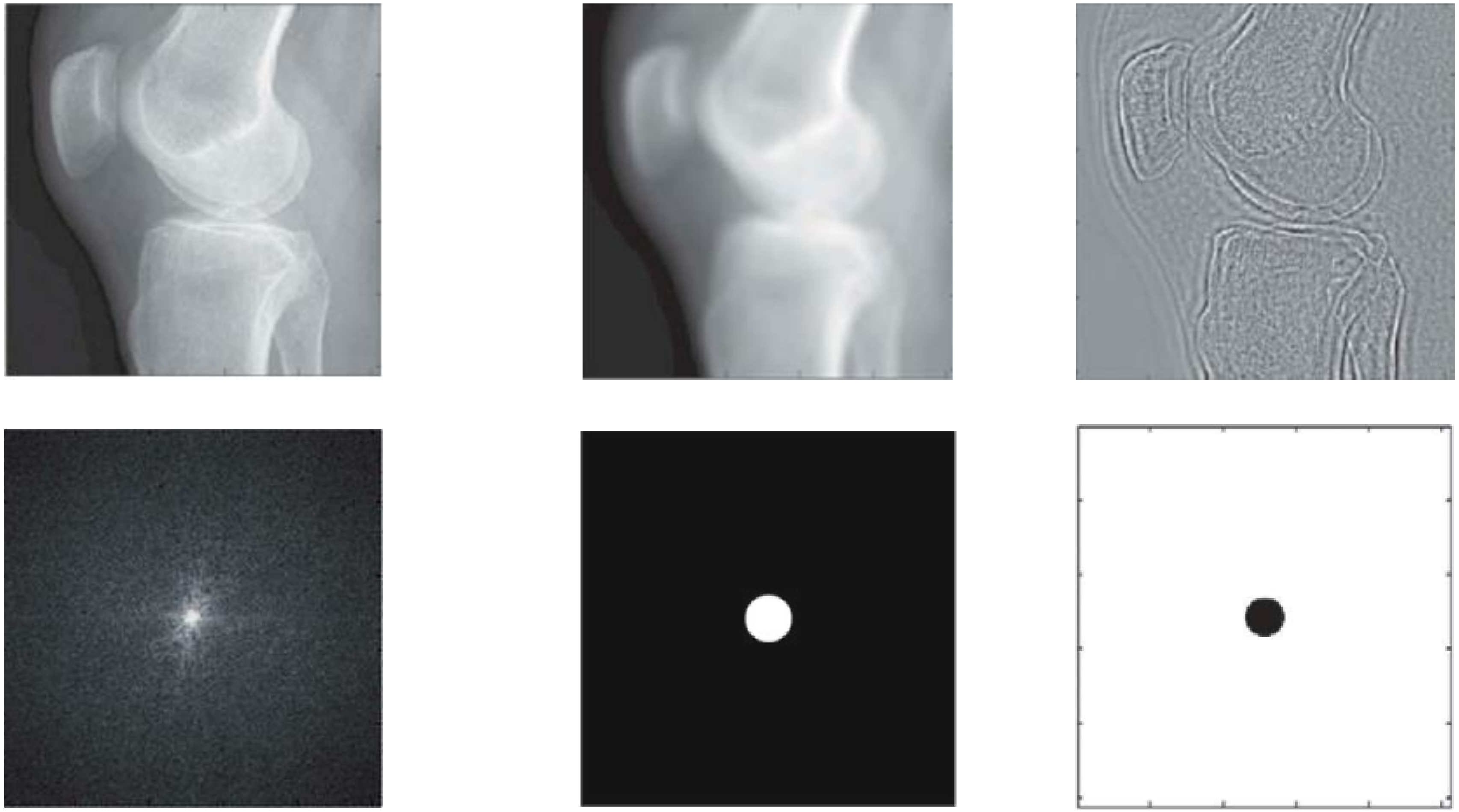


HPF



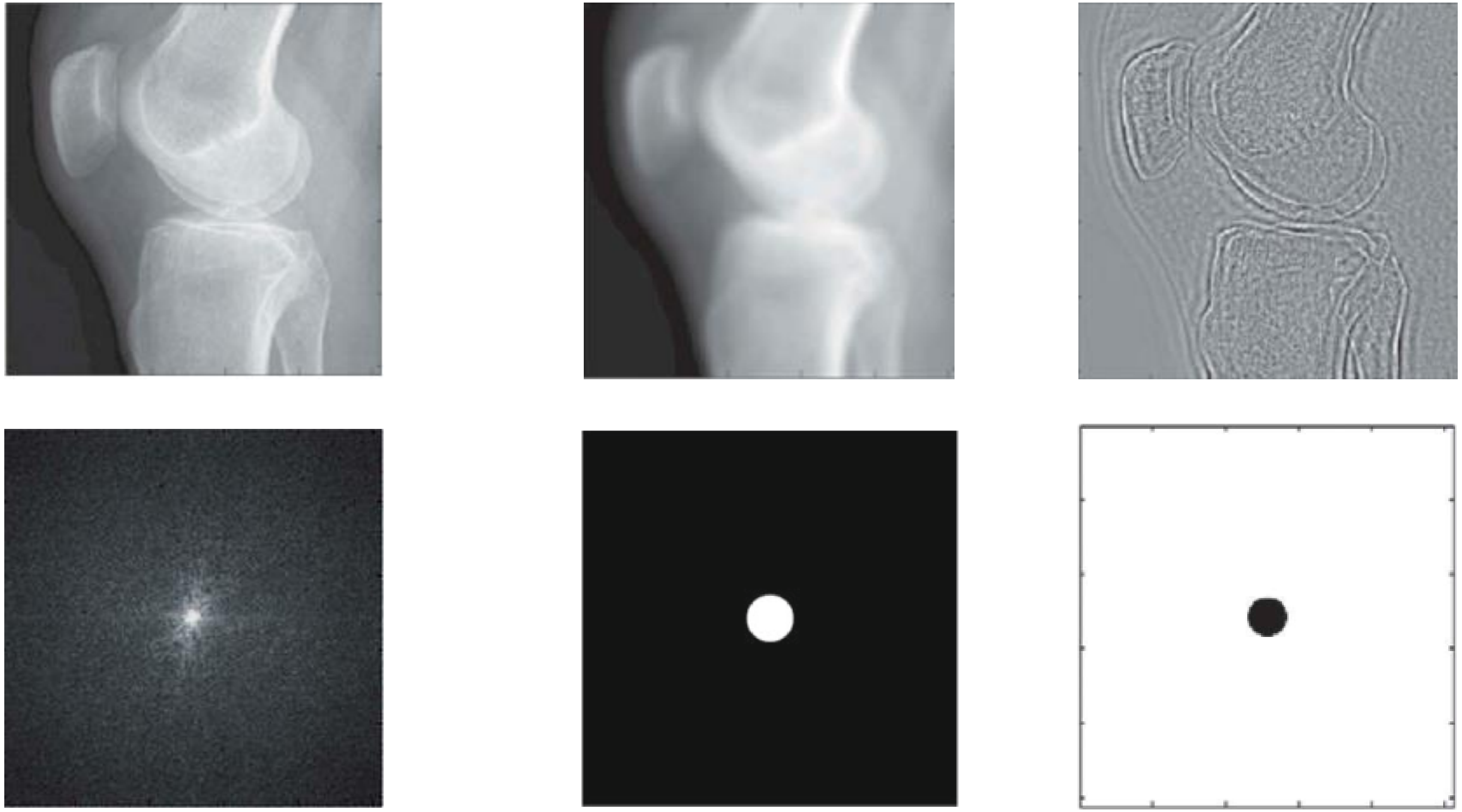
Frequency Filtering

Input LPF HPF



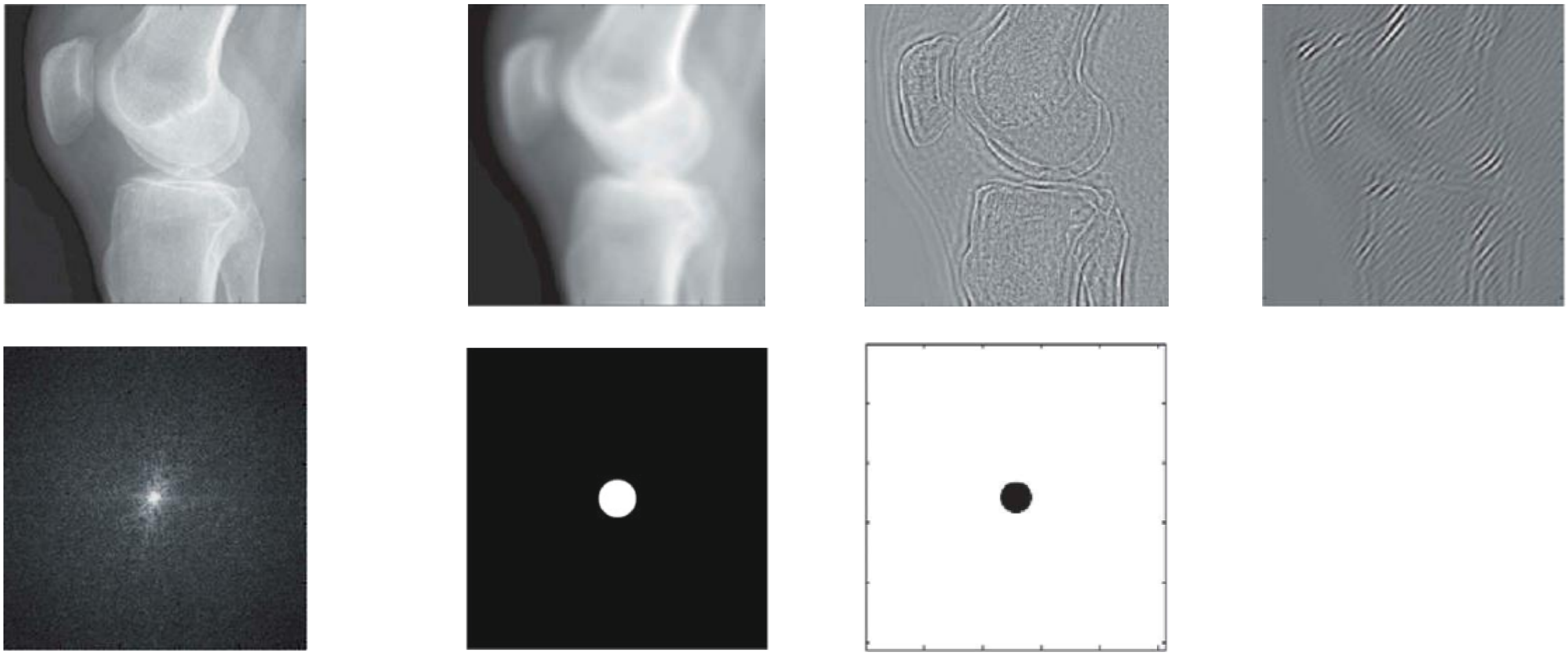
Frequency Filtering

Input LPF HPF BPF



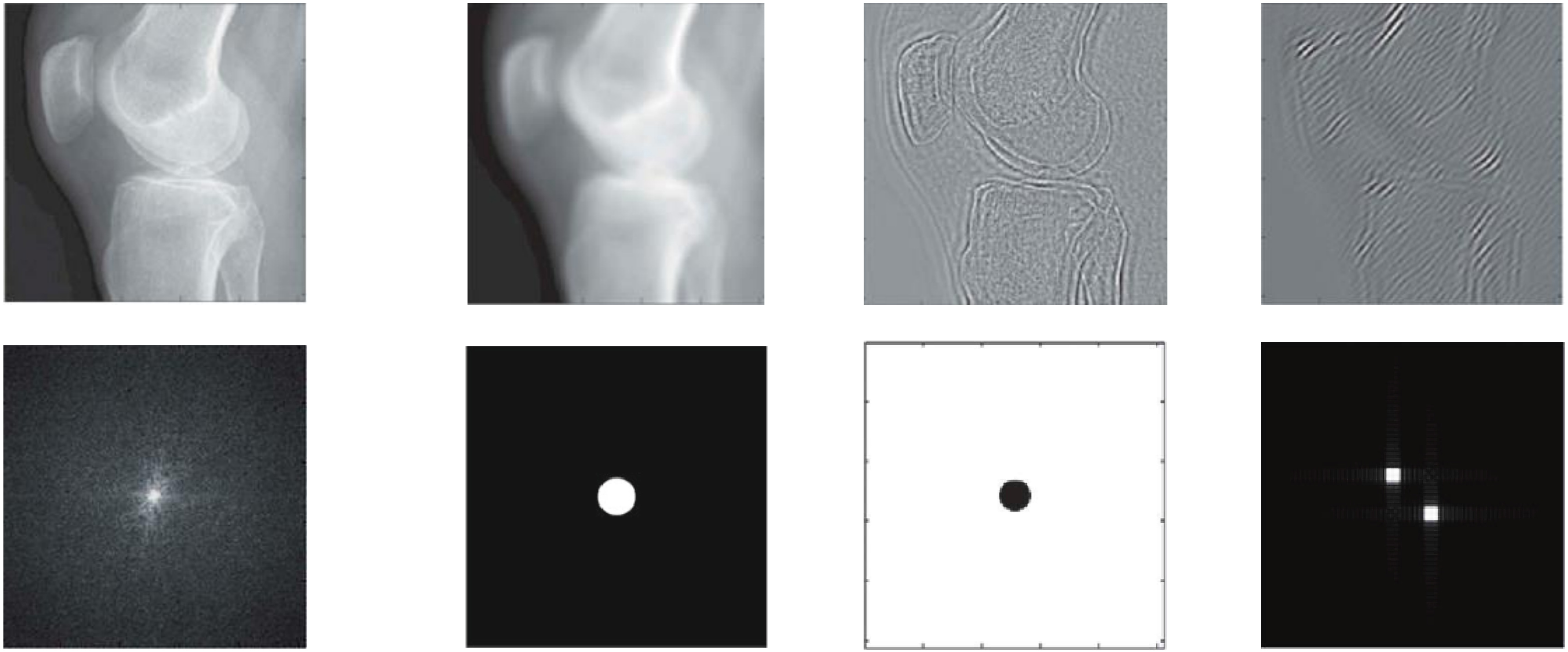
Frequency Filtering

Input LPF HPF BPF



Frequency Filtering

Input LPF HPF BPF

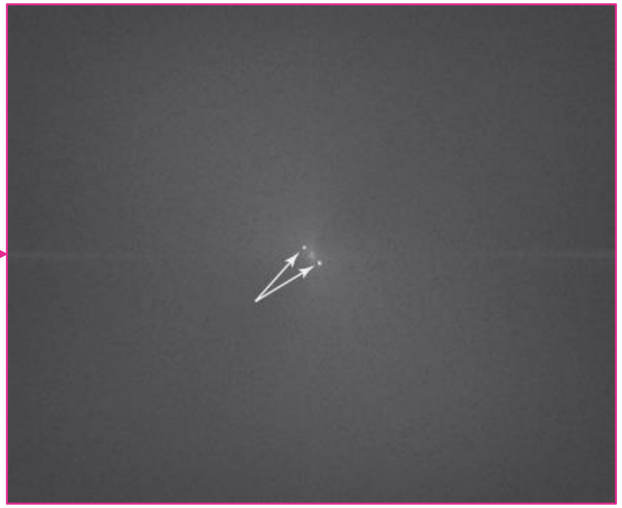


Frequency Filtering

Input

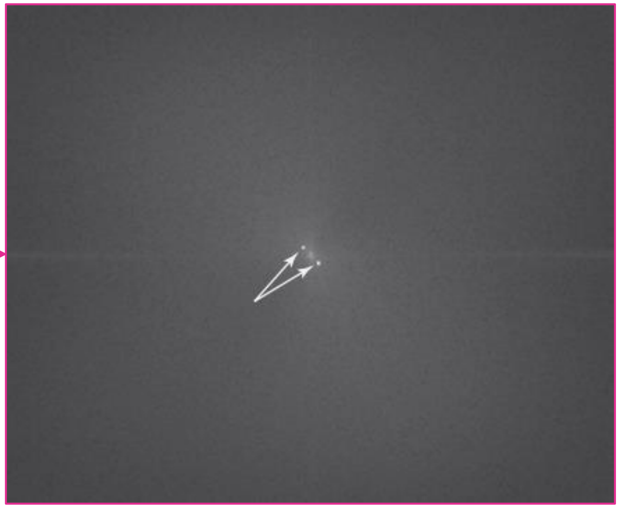
Frequency Filtering

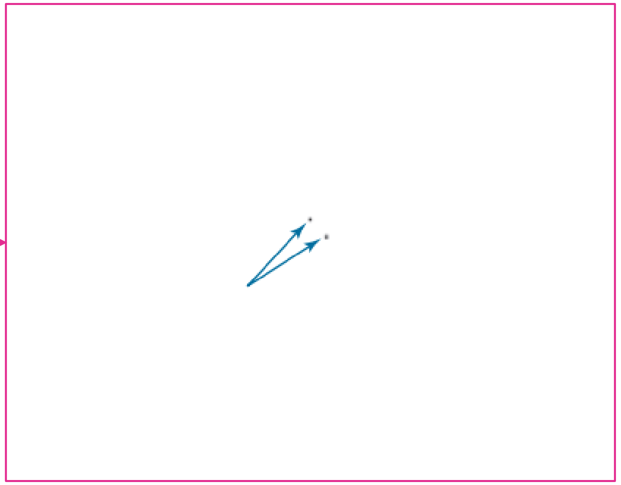
Input



Frequency Filtering

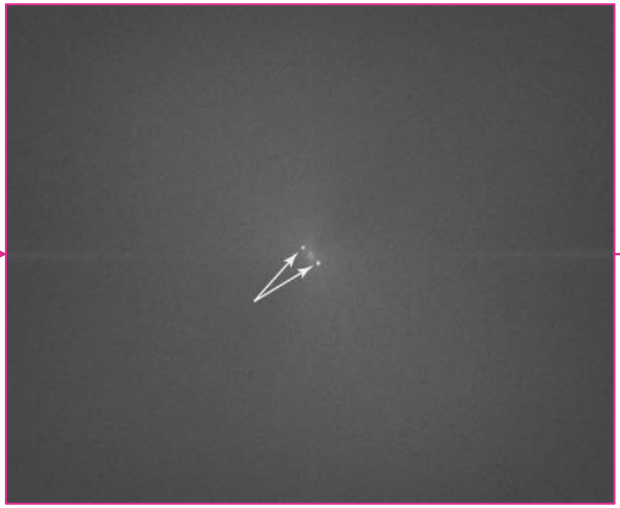
Input

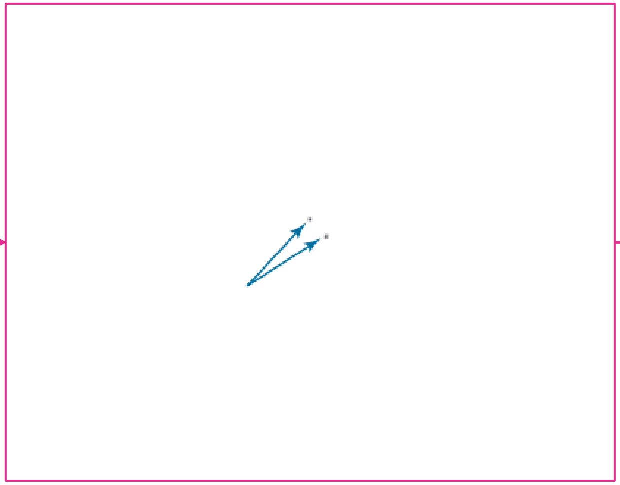




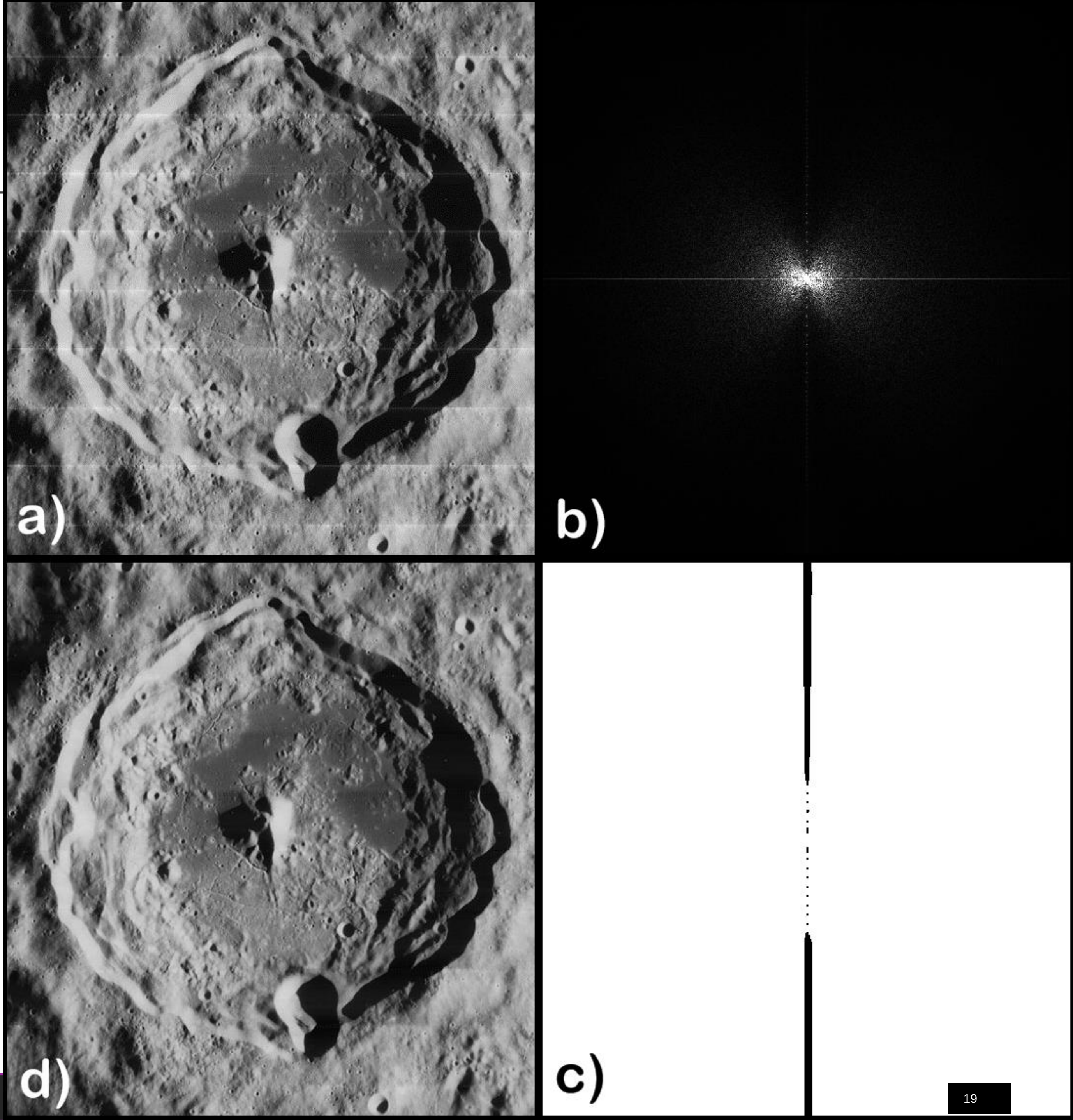
Frequency Filtering

Input



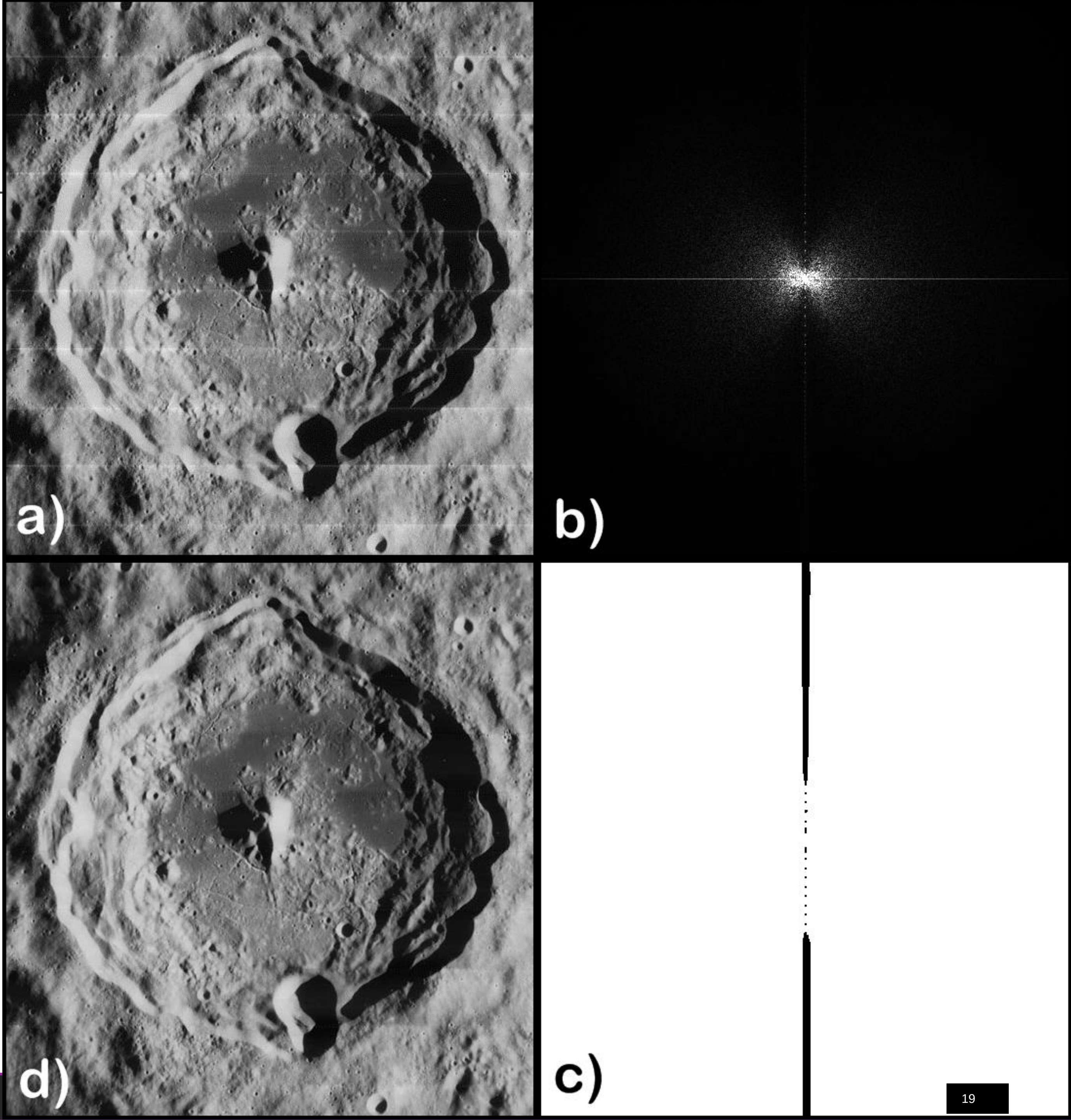


Frequency Filtering



Frequency Filtering

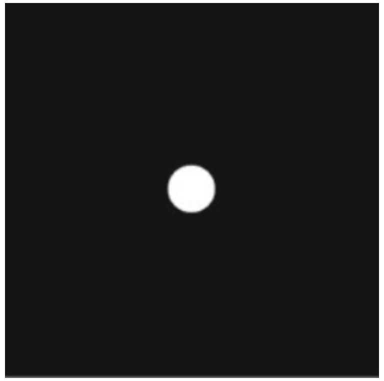
- (a) Input image
- (b) Freq representation
- (c) 2D Mask
- (d) Freq filtered image
  - Inverse Fourier transform after getting dot product between (b) and (c) image



Frequency-Spatial Filtering

Input $f(x, y)$

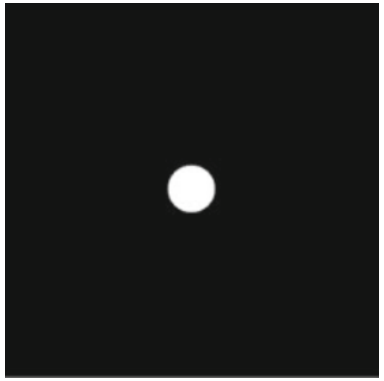
W_1

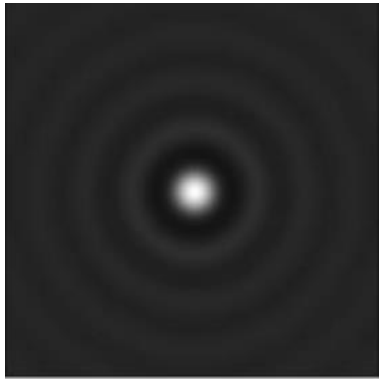


Frequency-Spatial Filtering

Input $f(x, y)$

W_1

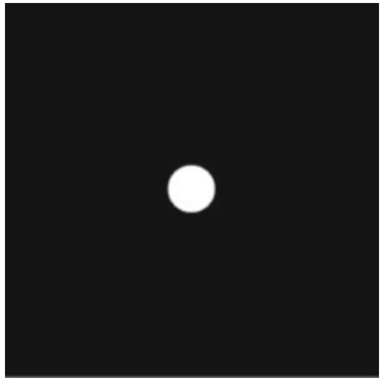




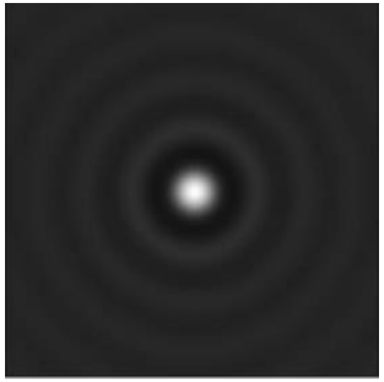
Frequency-Spatial Filtering

Input $f(x, y)$

W_1



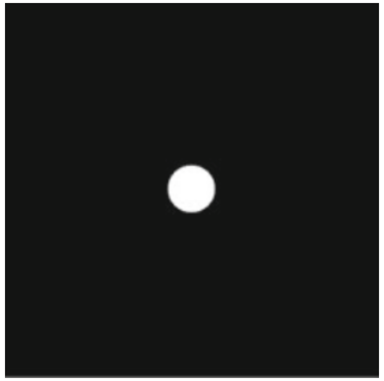
w_1



Frequency-Spatial Filtering

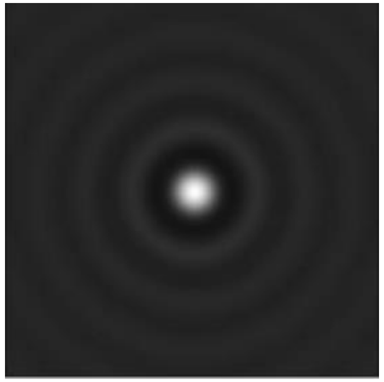
Input $f(x, y)$

W_1



W_2

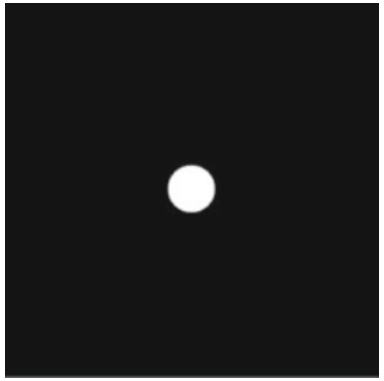
w_1



Frequency-Spatial Filtering

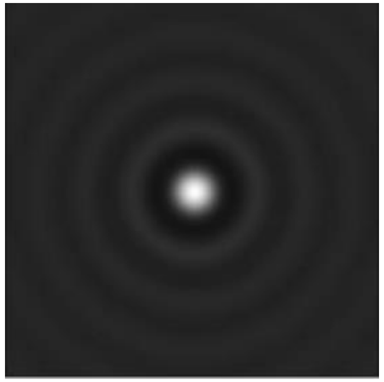
Input $f(x, y)$

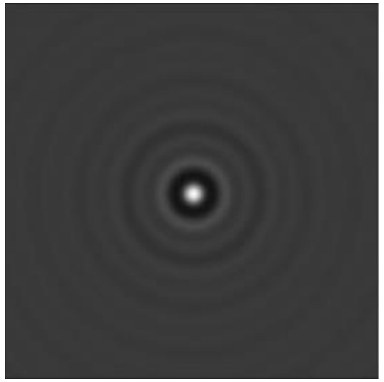
W_1



W_2

w_1

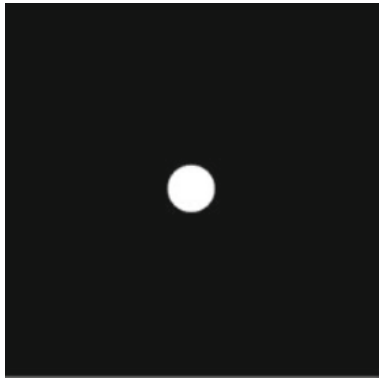




Frequency-Spatial Filtering

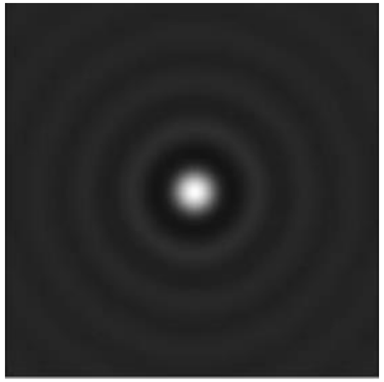
Input $f(x, y)$

W_1

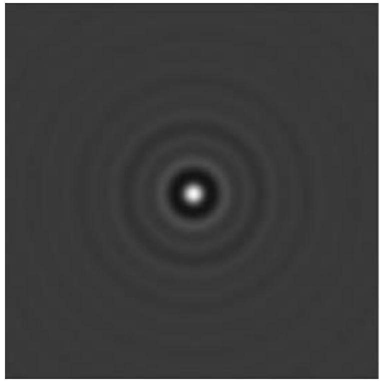


W_2

w_1



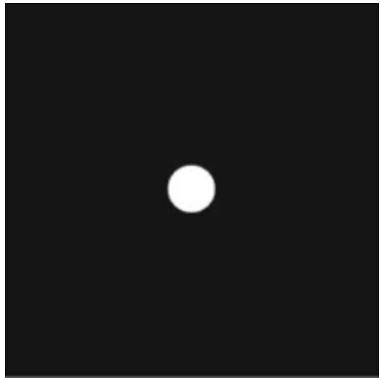
w_2



Frequency-Spatial Filtering

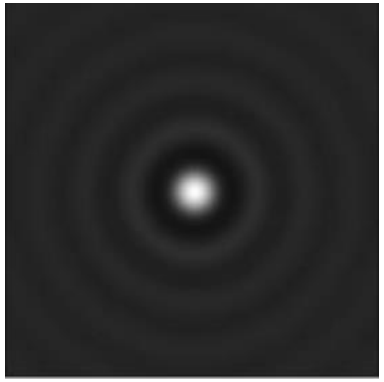
Input $f(x,y)$

W_1

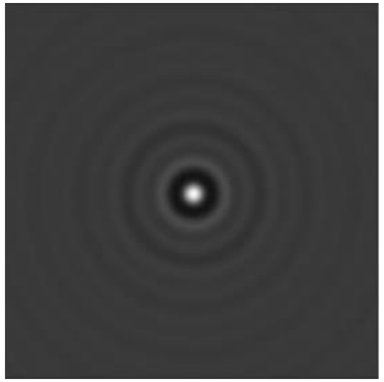


W_2

w_1



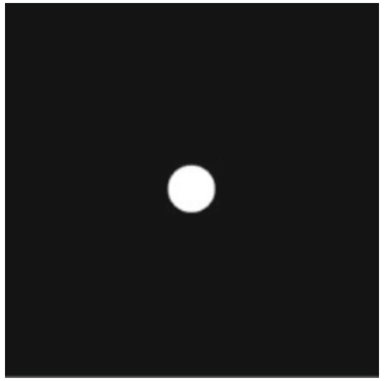
w_2



Frequency-Spatial Filtering

Input $f(x, y)$

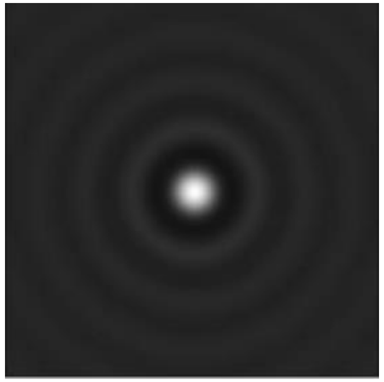
W_1



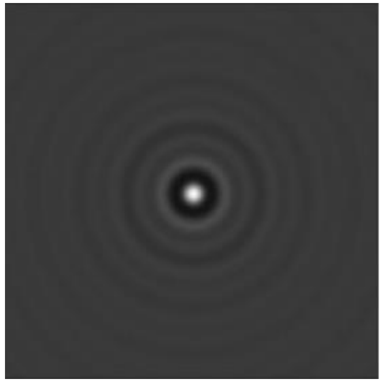
W_2

$w_1 \star f$

w_1



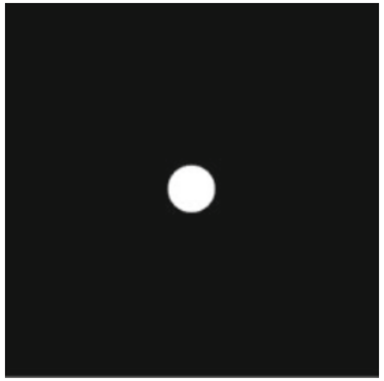
w_2



Frequency-Spatial Filtering

Input $f(x, y)$

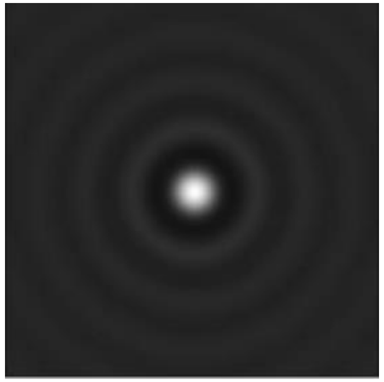
W_1



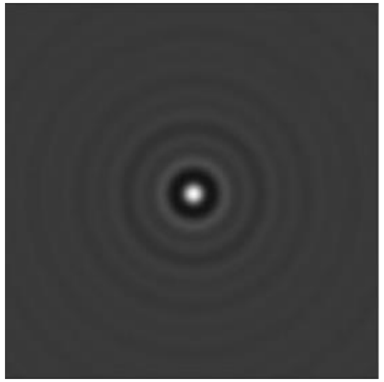
W_2

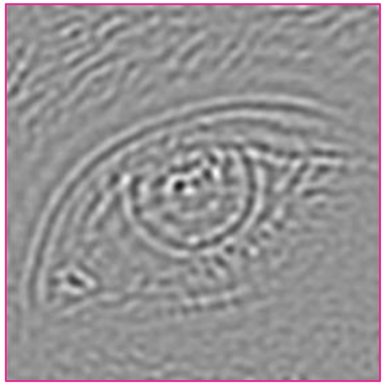
$w_1 \star f$

w_1



w_2

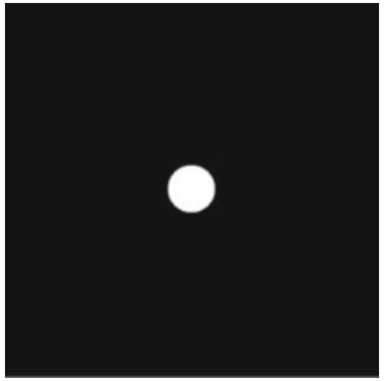




Frequency-Spatial Filtering

Input $f(x, y)$

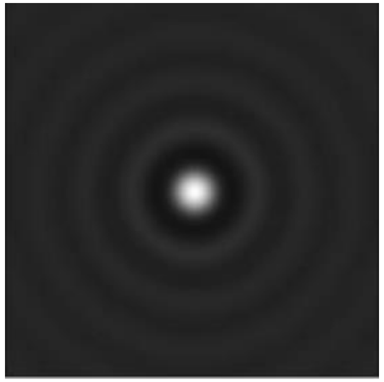
W_1



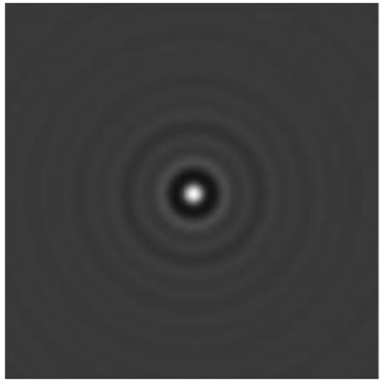
W_2

$w_1 \star f$

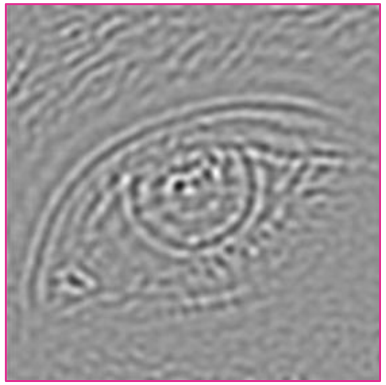
w_1



w_2



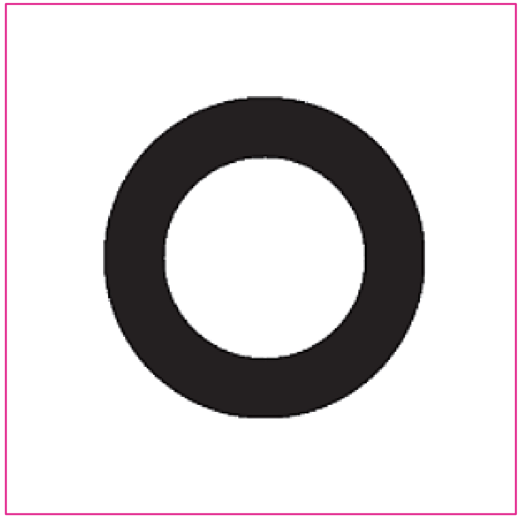
$w_2 \star f$



Frequency-Spatial Filtering

- Prior smoothing to reduce ripple effects

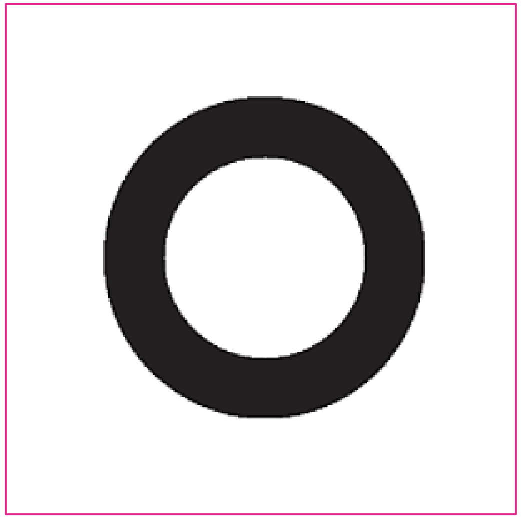
Ideal Mask



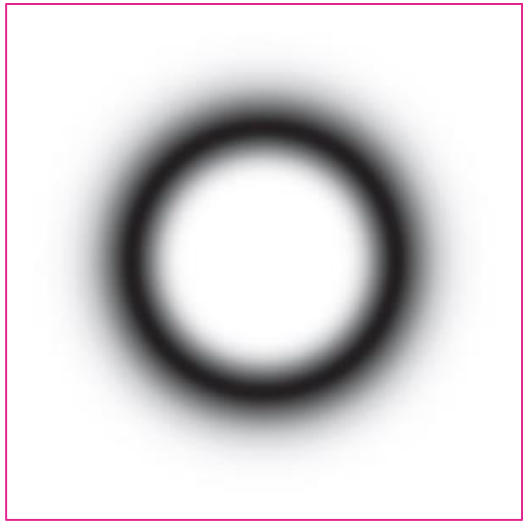
Frequency-Spatial Filtering

- Prior smoothing to reduce ripple effects

Ideal Mask



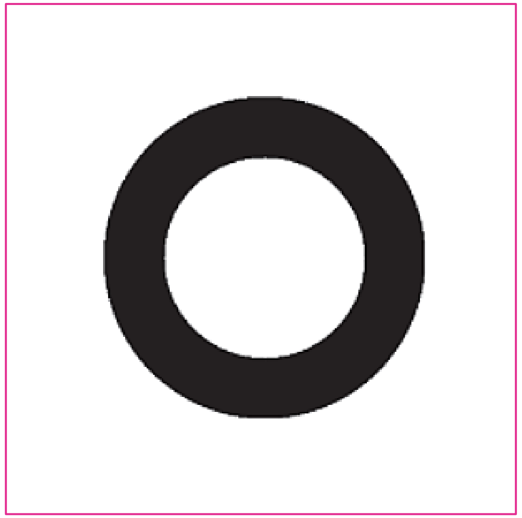
Smooth Mask-1



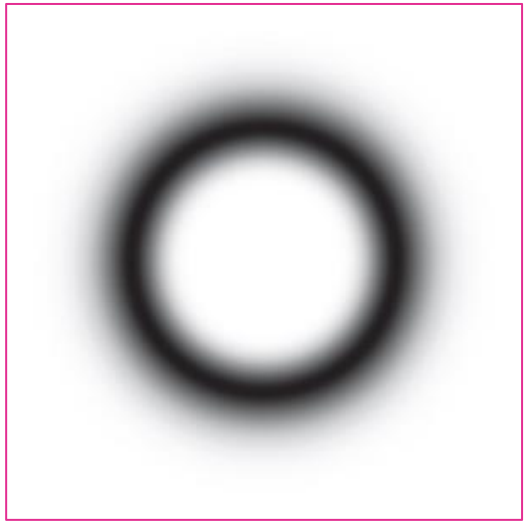
Frequency-Spatial Filtering

- Prior smoothing to reduce ripple effects

Ideal Mask



Smooth Mask-1



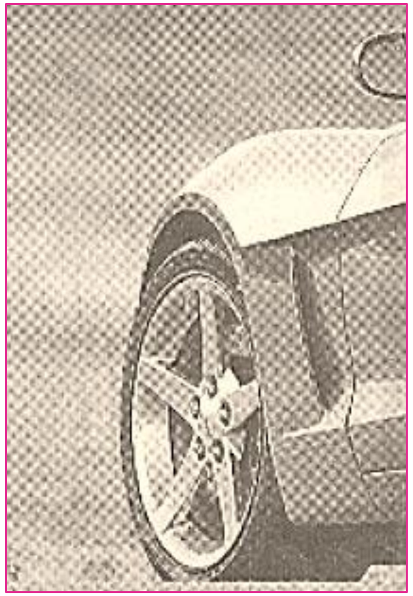
Smooth Mask-2

Frequency-Spatial Filtering

Forget me, but
don’t forget my car!

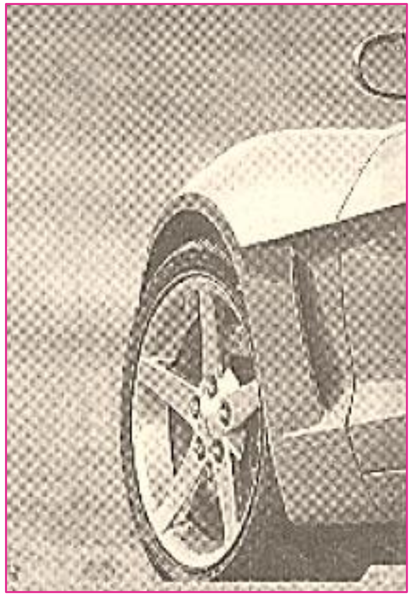
Frequency-Spatial Filtering

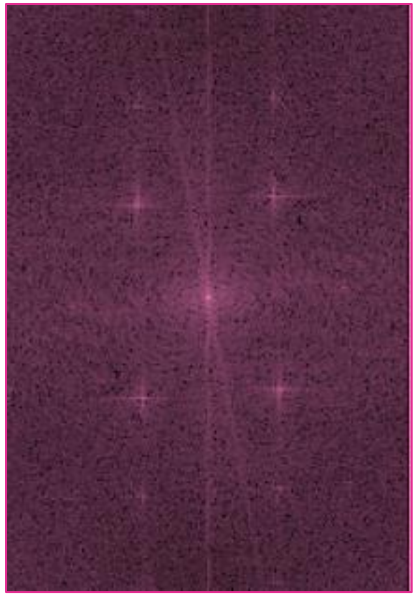
Forget me, but
don’t forget my car!



Frequency-Spatial Filtering

Forget me, but
don't forget my car!





Frequency-Spatial Filtering

Forget me, but
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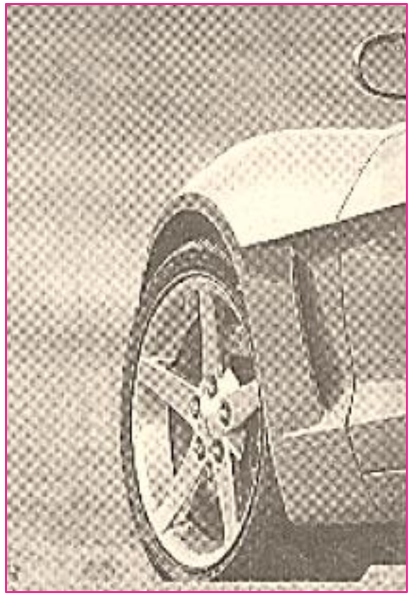
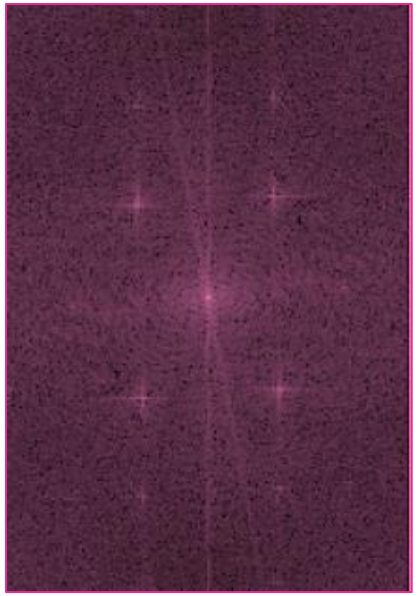
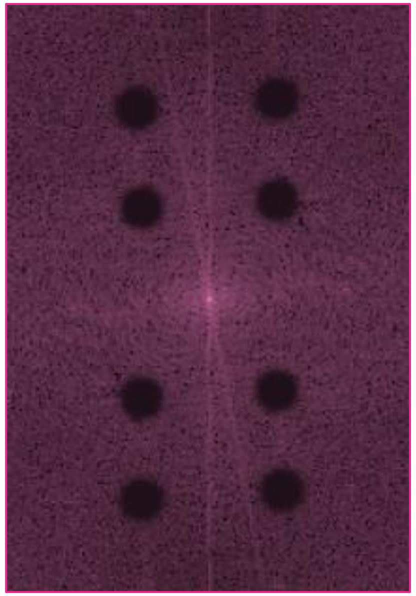

Frequency-Spatial Filtering

Forget me, but
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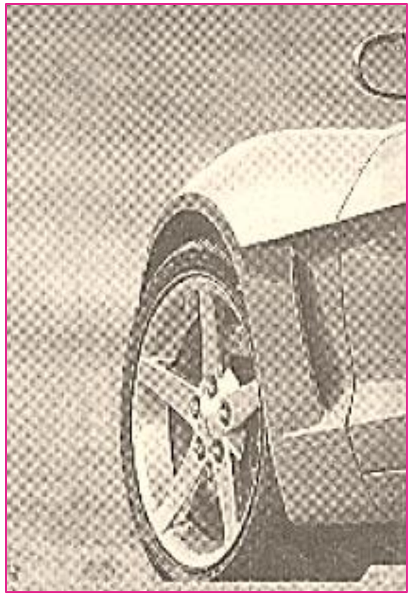
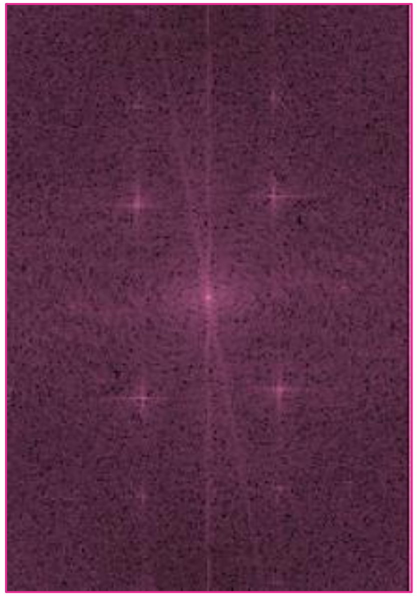
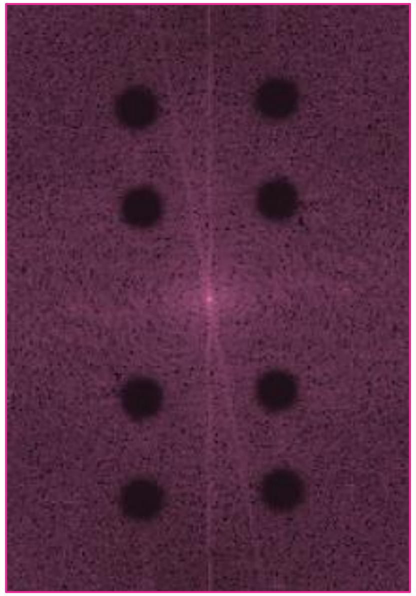
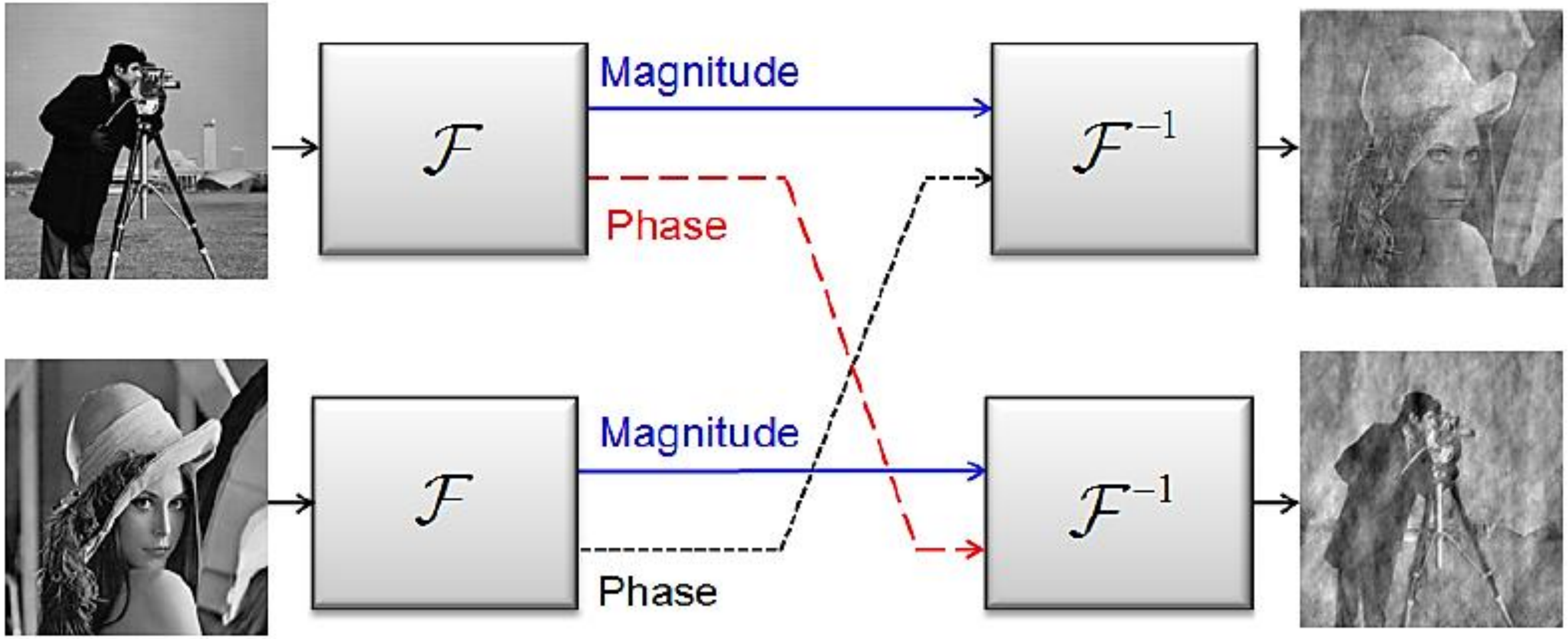





Importance of Fourier Phase

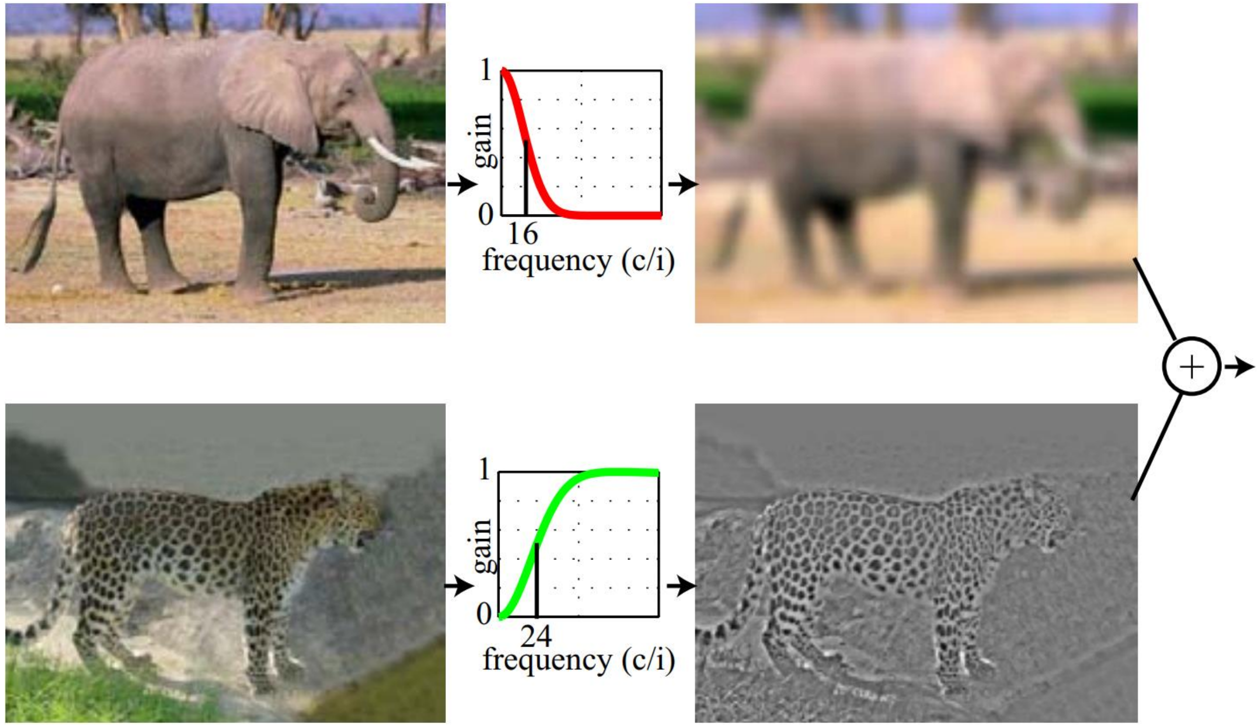


Credit: Y. Shechtman et al. 2014

Frequency Filtering & HVS

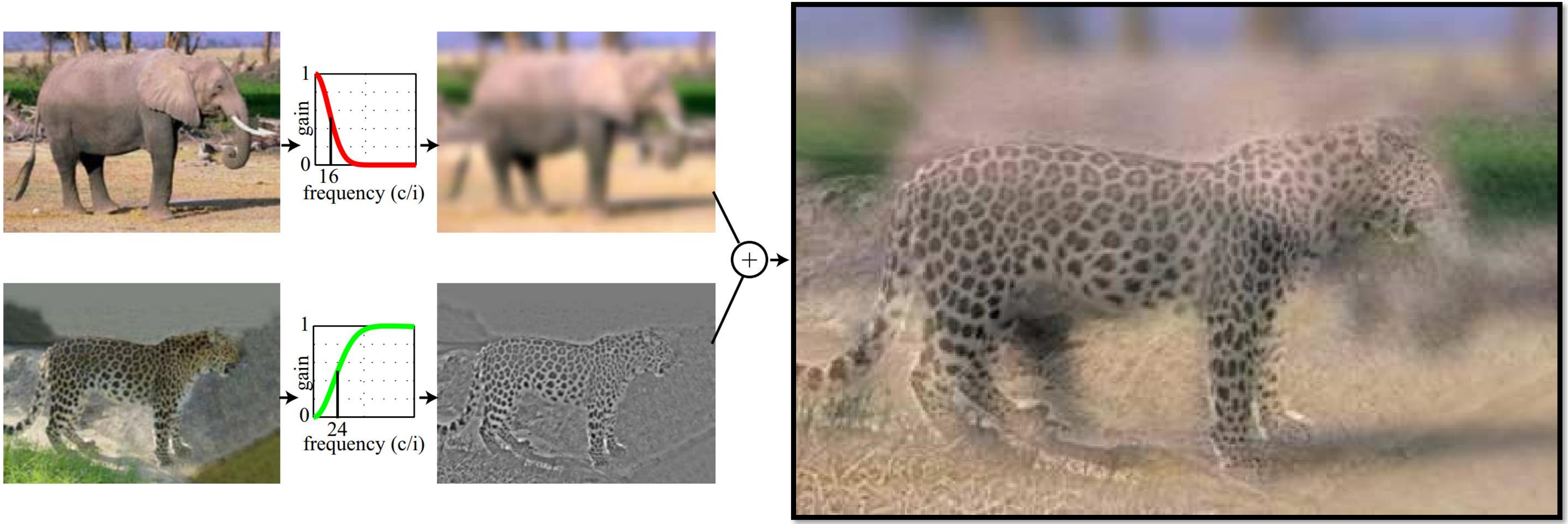
credit: A. Oliva

Frequency Filtering & HVS



credit: A. Oliva

Frequency Filtering & HVS



credit: A. Oliva

Conclusion

- 2D FT properties & images
- Frequency filtering

Conclusion

- 2D FT properties & images
- Frequency filtering

❑ 2D Fourier Transform

- Properties
- Convolution theorem
- 2D FT images

❑ Frequency filtering

- Filtering in FT domain
- Freq-spatial filtering
- Freq-mixing

Conclusion

- 2D FT properties & images
- Frequency filtering

- 2D Fourier Transform
  - Properties
  - Convolution theorem
  - 2D FT images
- Frequency filtering
  - Filtering in FT domain
  - Freq-spatial filtering
  - Freq-mixing