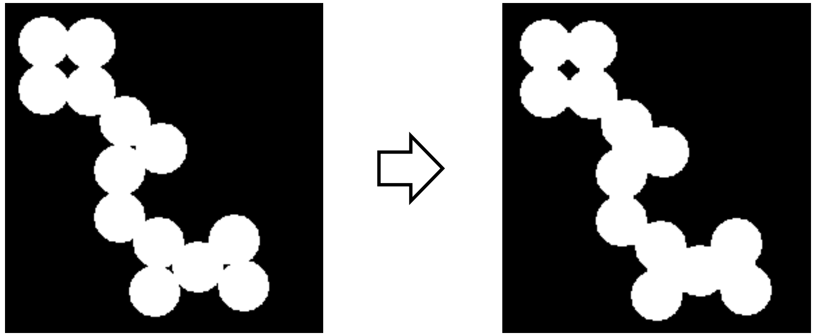
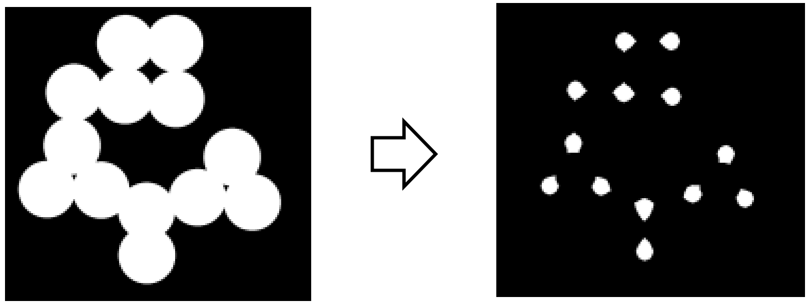
Morphological processing

Dr. Tushar Sandhan

Morphological processing

- Morphology
  - deals with shape
    - to adjust slight imperfections in shapes
  - operations carried out using operator
    - operator relies on structuring element
    - structuring element is the base shape (mould) which re-structures entire image
    - used for modifying, extracting shapes
  - processing is based on set-theoretic operations
    - mostly used in pre-processing or post-processing
    - mostly for binary images, but can be extended to grey scale (level sets)





Set operations

$A, B \in \mathbb{R}^2, \qquad w \in A, \qquad w = (x, y)$

- union

  $$A \cup B = \{w : w \in A \text{ or } w \in B\}$$

- intersection

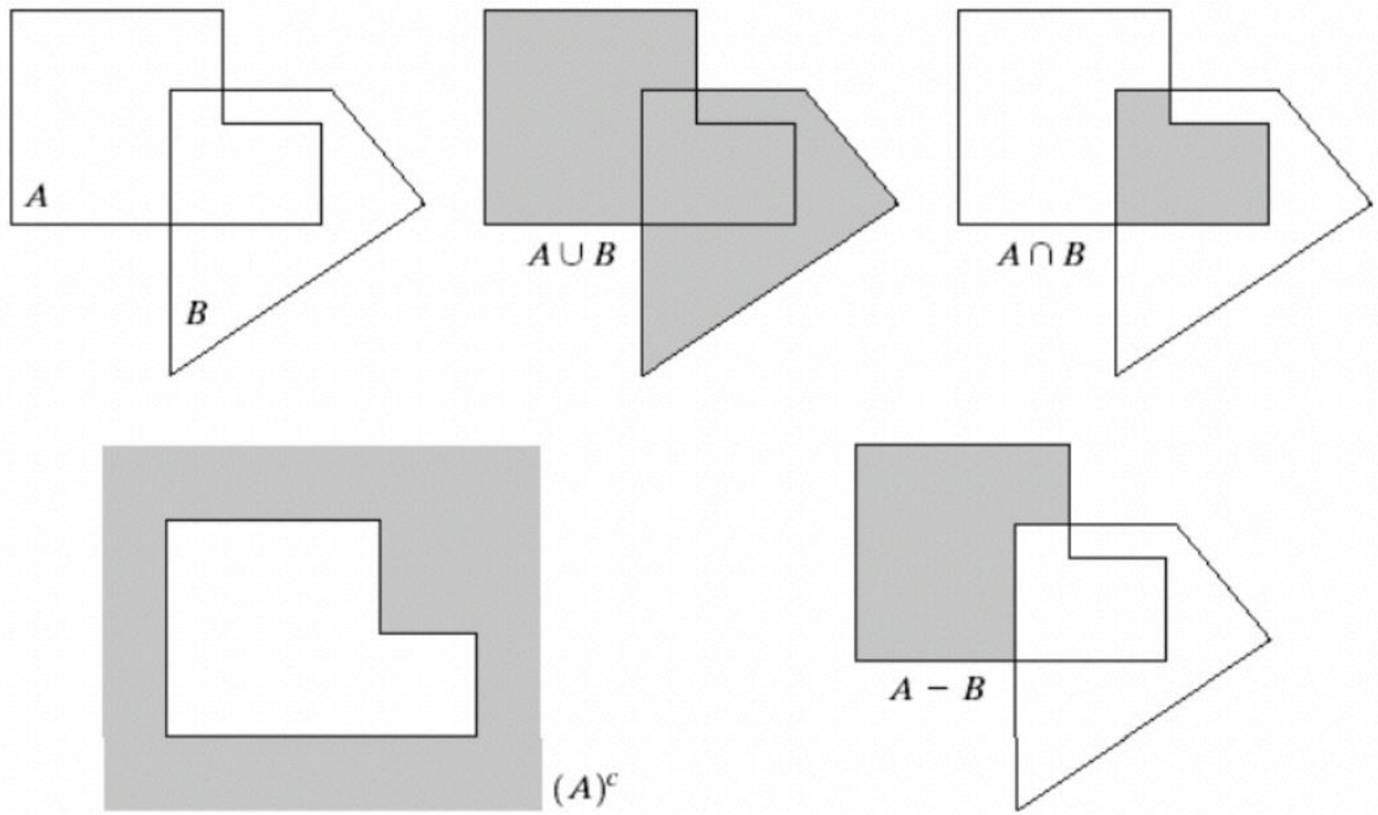
  $$A \cap B = \{w : w \in A \text{ and } w \in B\}$$

- complement

  $$A^c = \{w : w \notin A\}$$

- difference

  $$A \backslash B = \{w : w \in A, w \notin B\}$$



Set operations

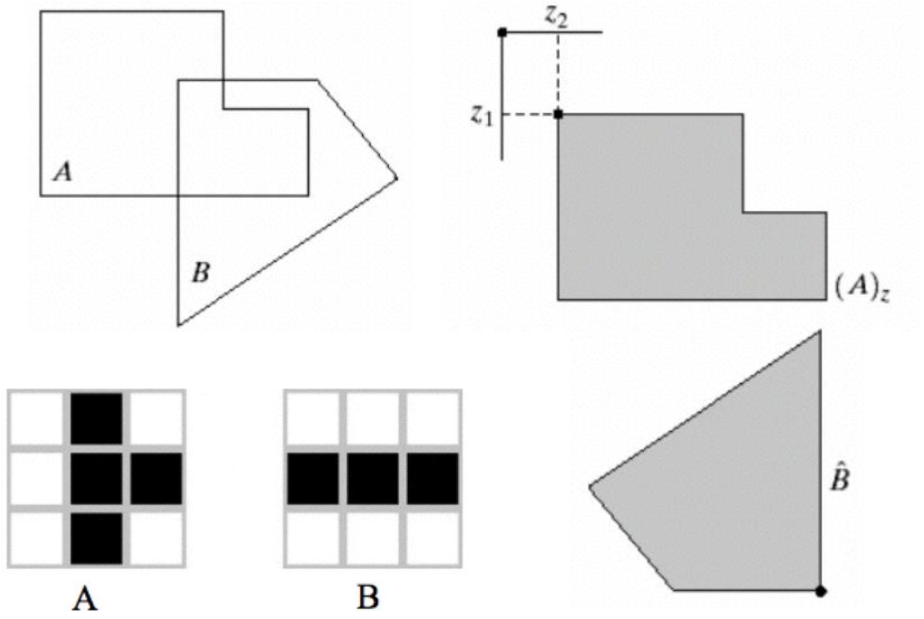
$A, B \in \mathbb{R}^2, \qquad w \in A, \qquad w = (x, y)$

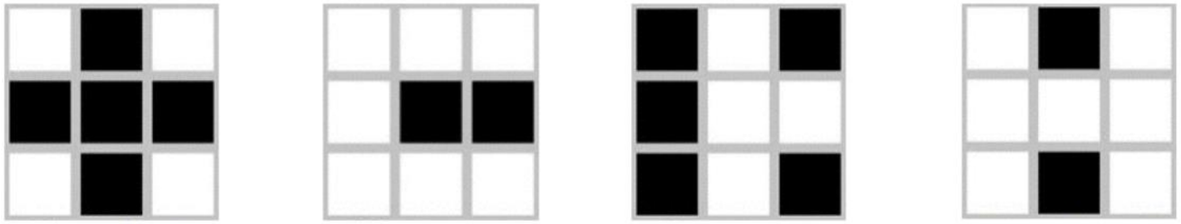
- translation

$$(A)_z = \{c : c = a + z, a \in A\}$$

- reflection

$$\hat{B} = \{w : w = -b, b \in B\}$$





$C = A \cup B$ $\qquad$ $C = A \cap B$ $\qquad$ $C = A^c$ $\qquad$ $C = A \backslash B$

Morphological processing

- Operations
  - set $A = I$ image
  - set $B = E$ element (structuring)
    - element can have any shape and size
    - symmetry is preferred but not necessary
    - filtering: element is translated over entire image

Morphological processing

- Operations
  - set $A = I$ image
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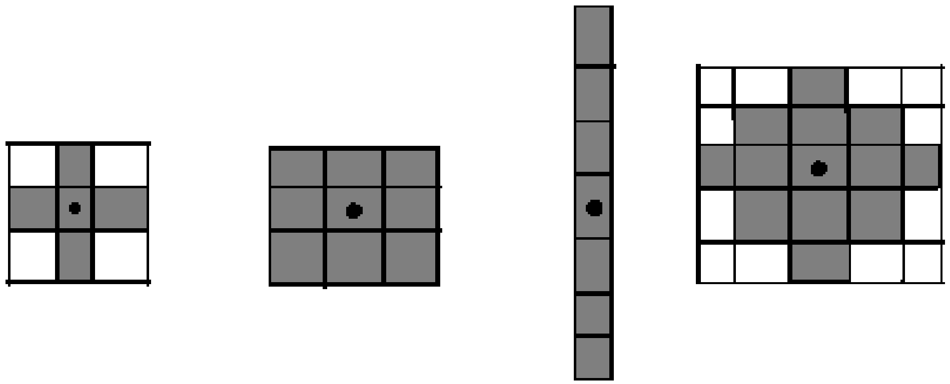
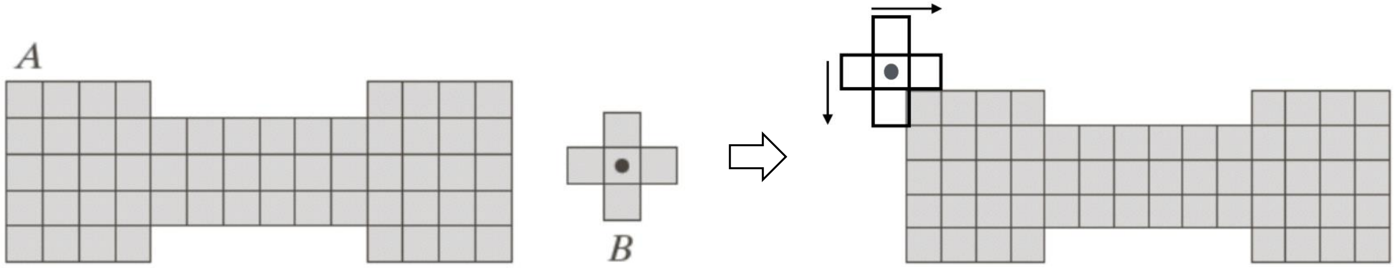
    - filtering: element is translated over entire image
    - E e.g.



Morphological processing

- Dilation
  - dilation of A by B
  - joining broken bridges

$$A \oplus B = \left\{ z \,\middle|\, \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$$



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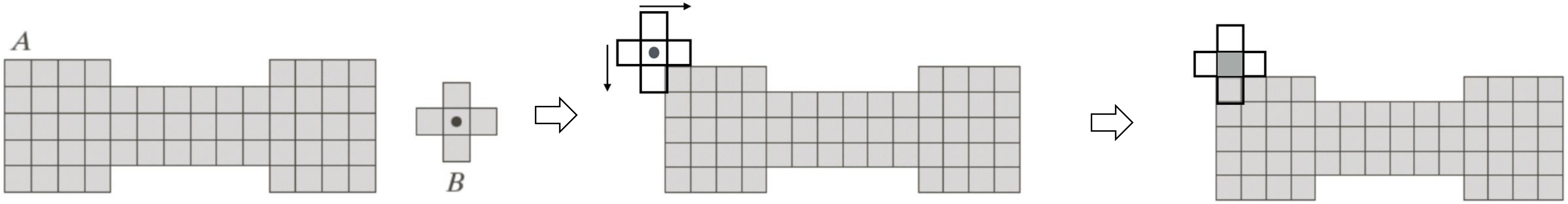

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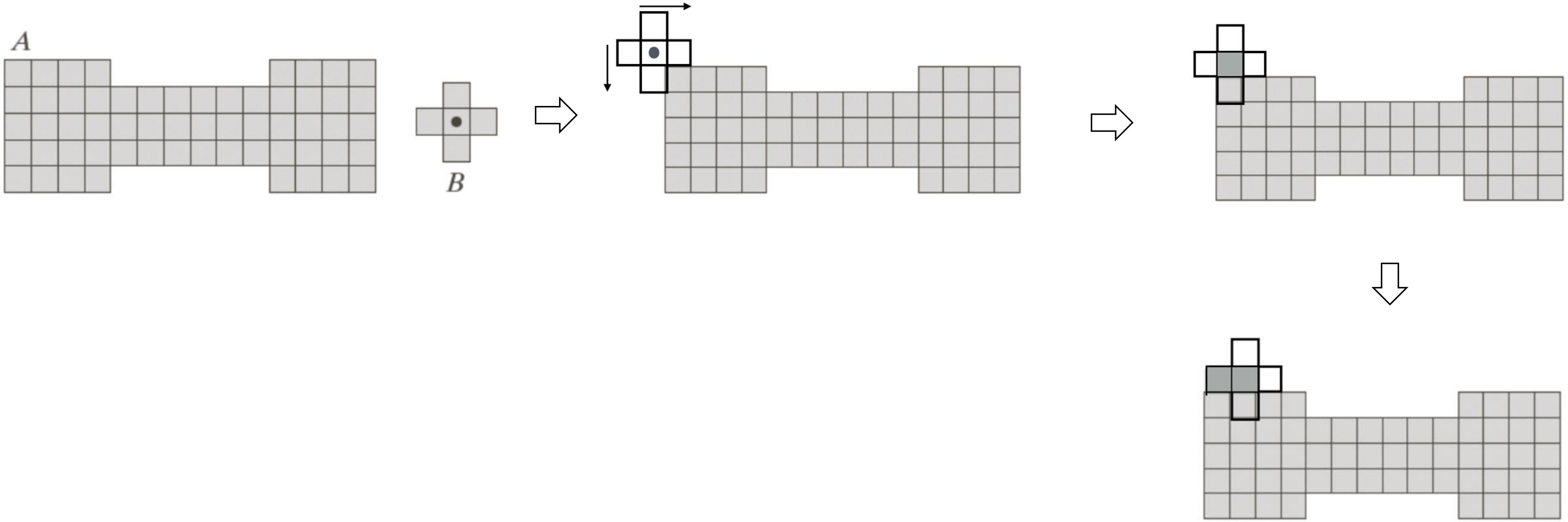

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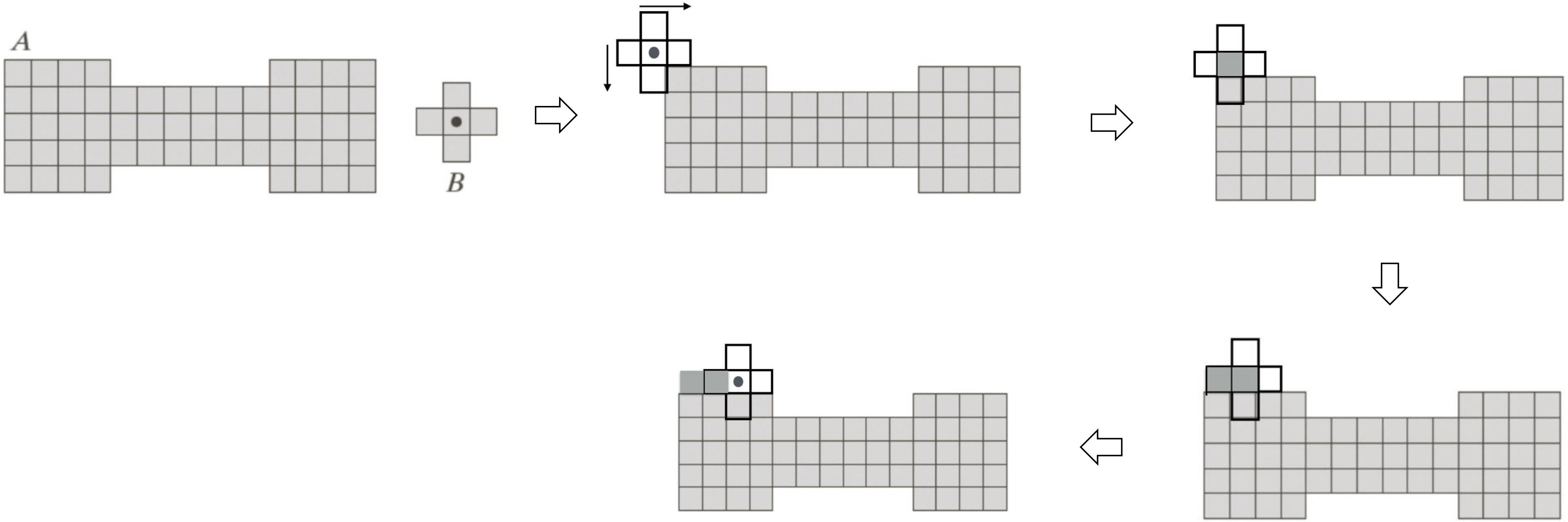

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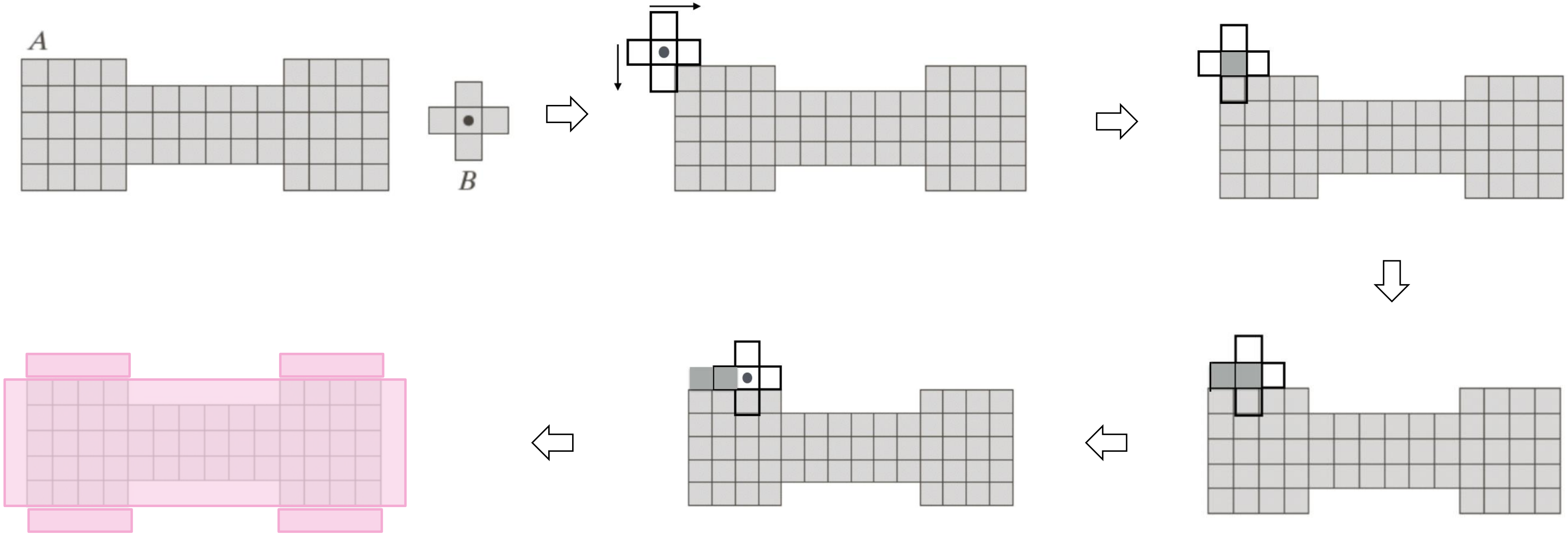
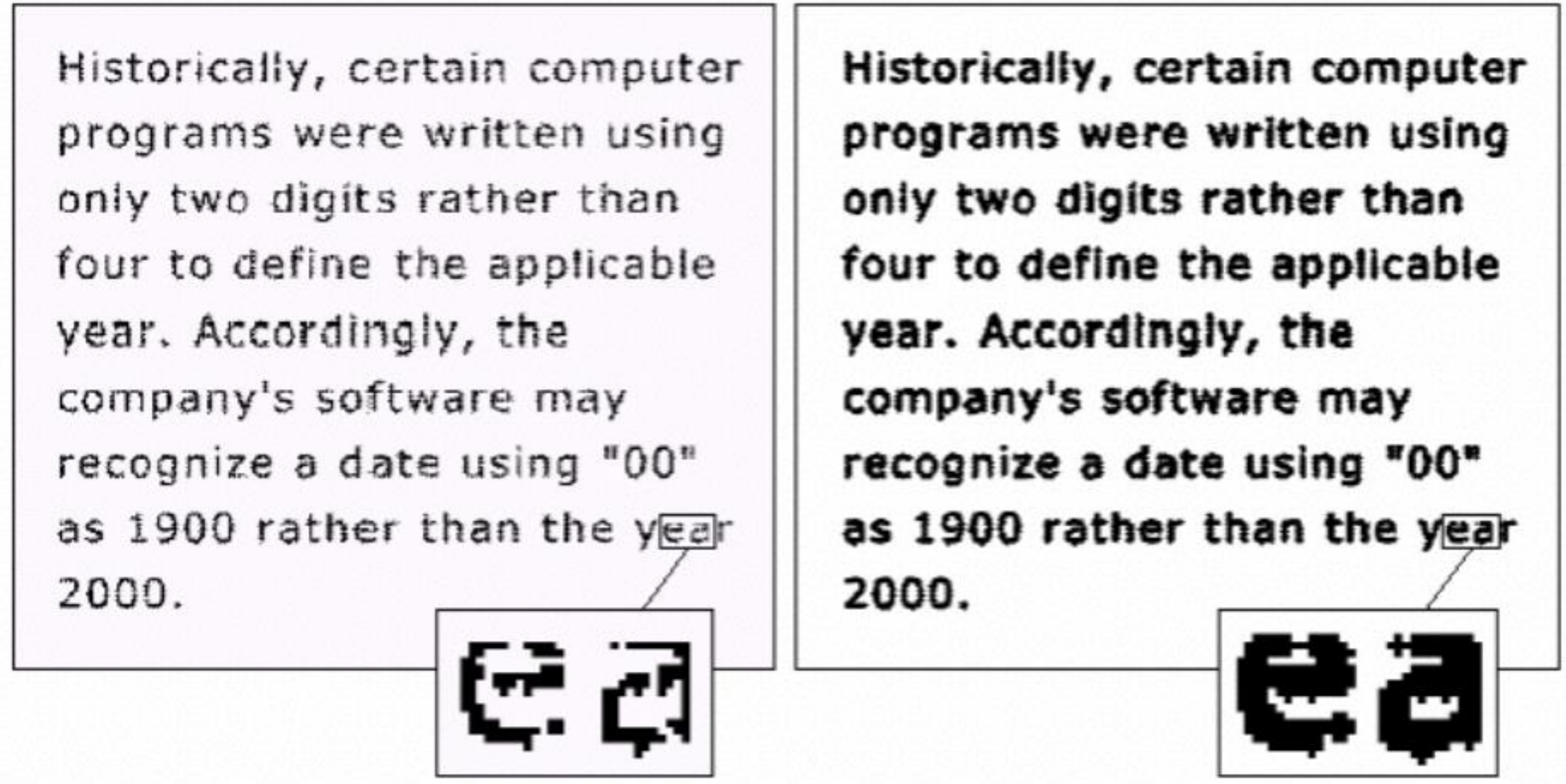


Morphological processing

- Dilation
  - dilation of A by B
  - joining broken bridges
  - B: structuring element

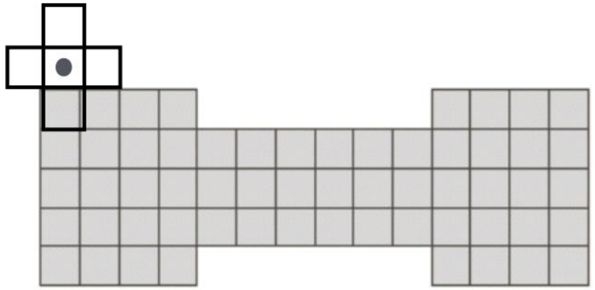
0	1	0
1	1	1
0	1	0



Morphological processing

- Erosion
  - erosion of A by B
  - peeling away layers

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



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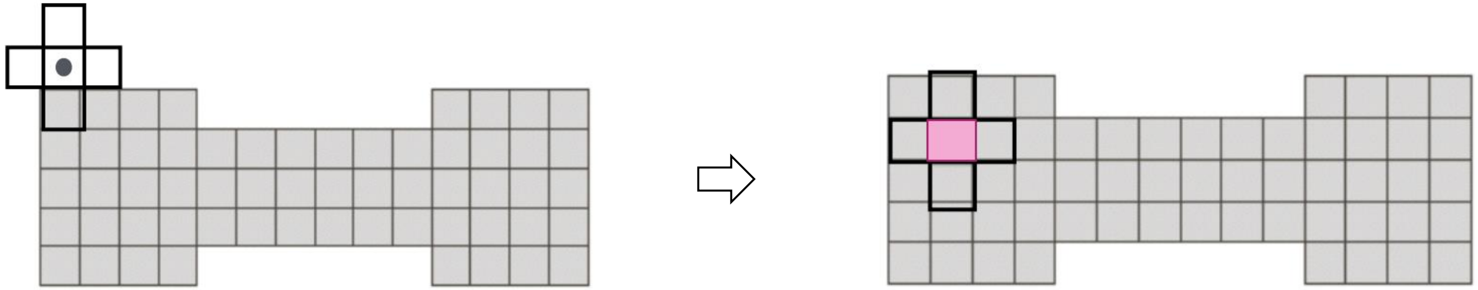

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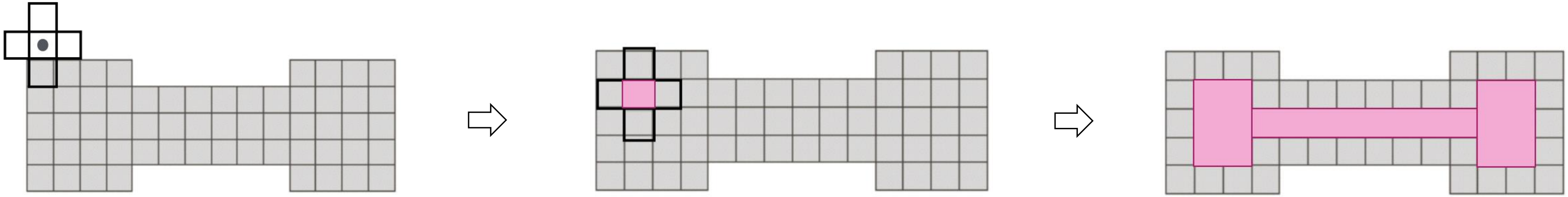
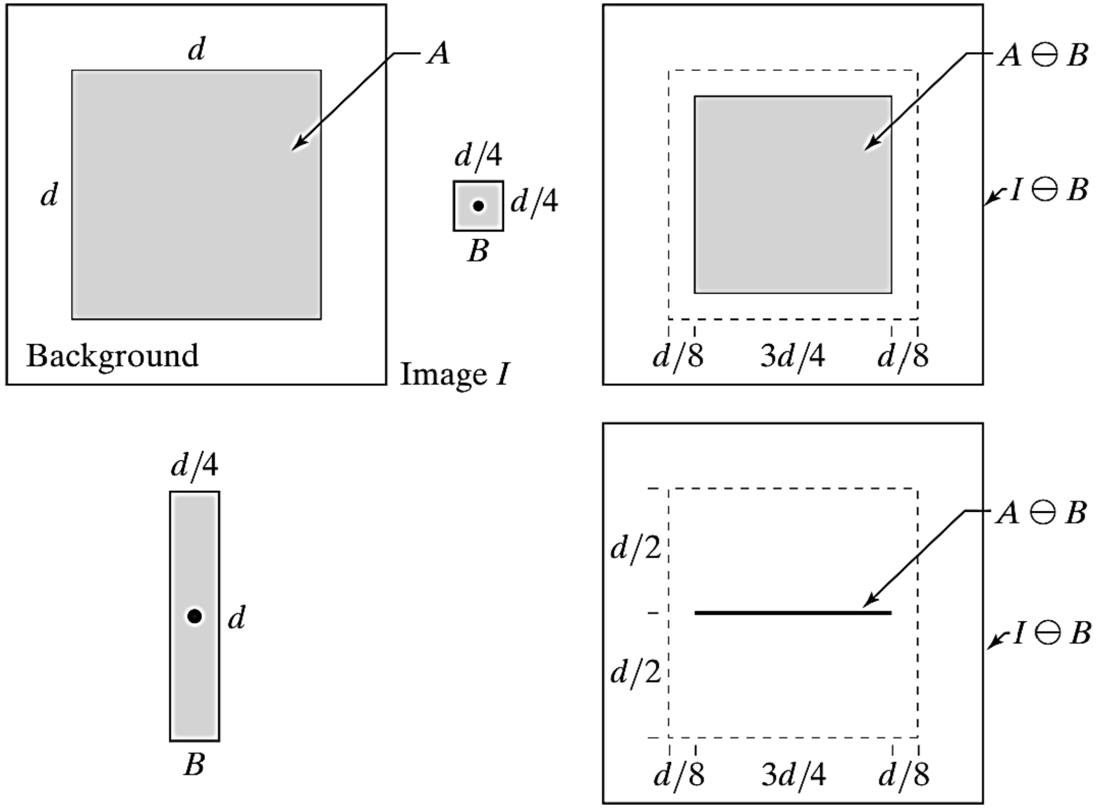


Morphological processing

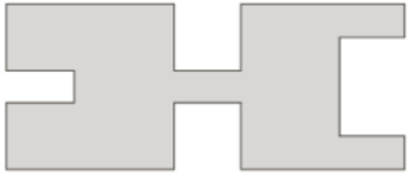
- Dilation and erosion
  - results vary significantly by changing the shape of B



Morphological processing

- Opening
  - dilate the eroded
  - i.e. fist erode then dilate
  - E is same

$$A \circ B = (A \ominus B) \oplus B$$



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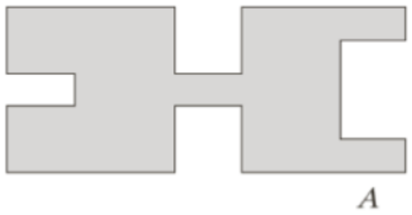

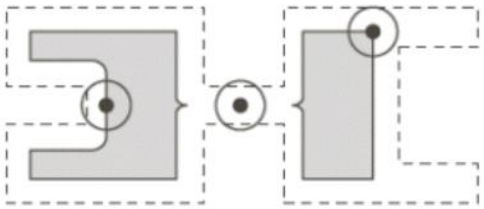

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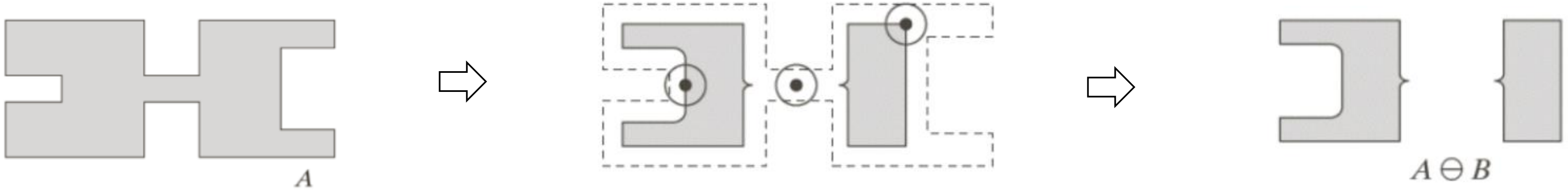

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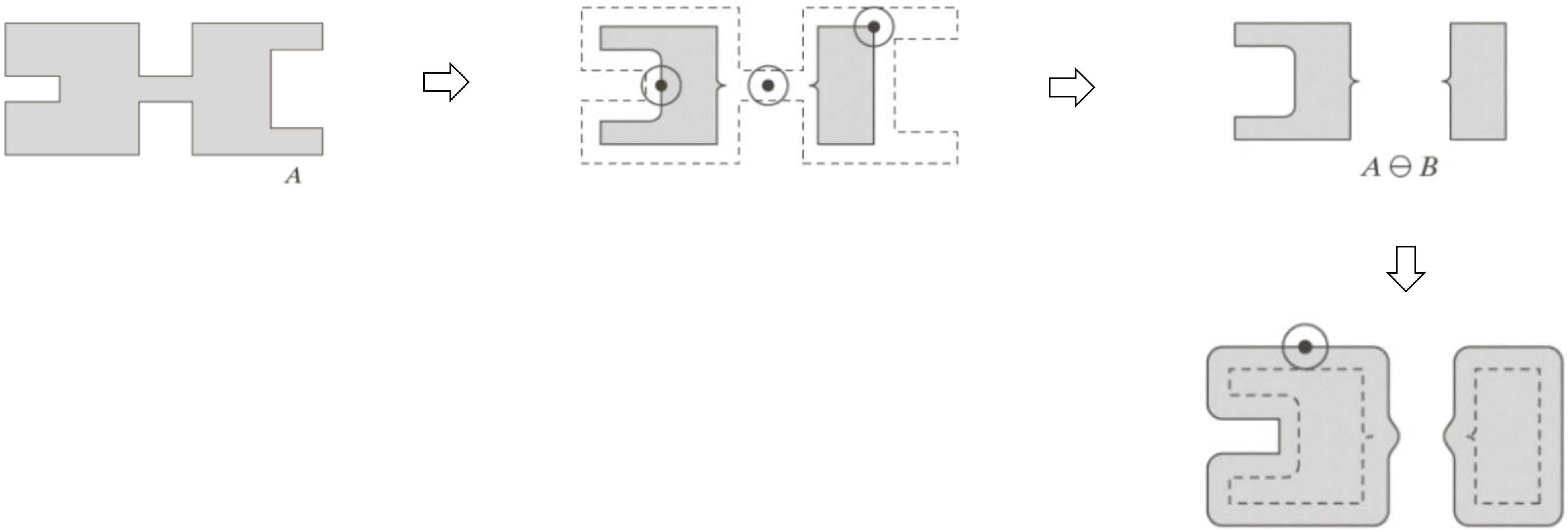

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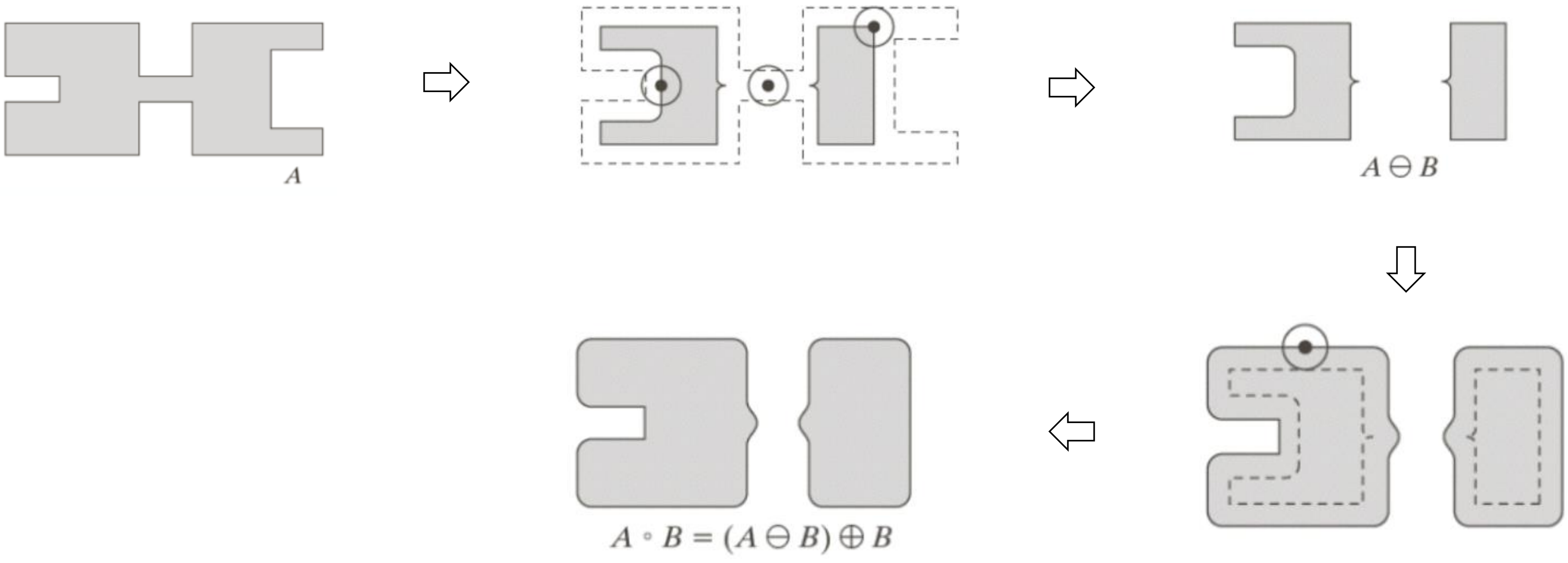
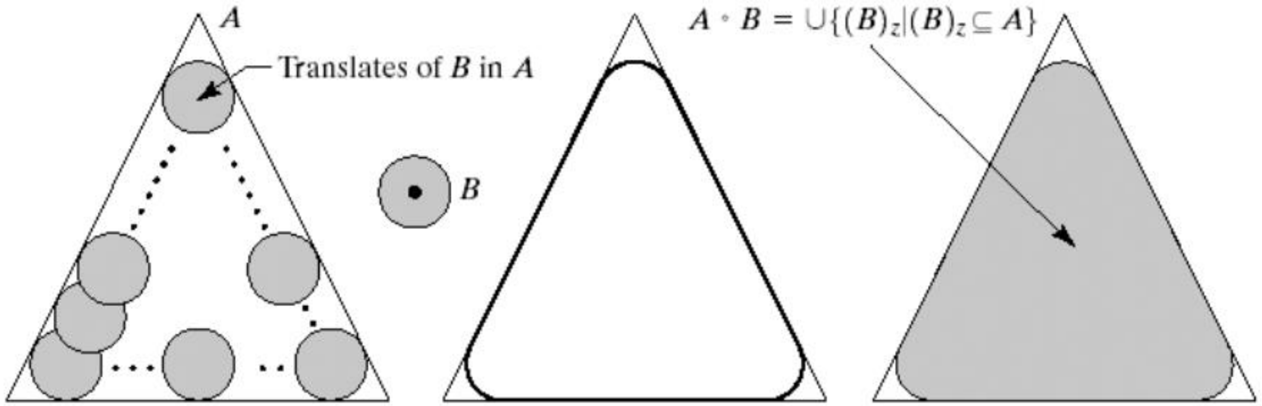


Morphological processing

- Opening
  - useful for removing
    - small objects
    - connections
    - protrusions

$$A \circ B = (A \ominus B) \oplus B$$



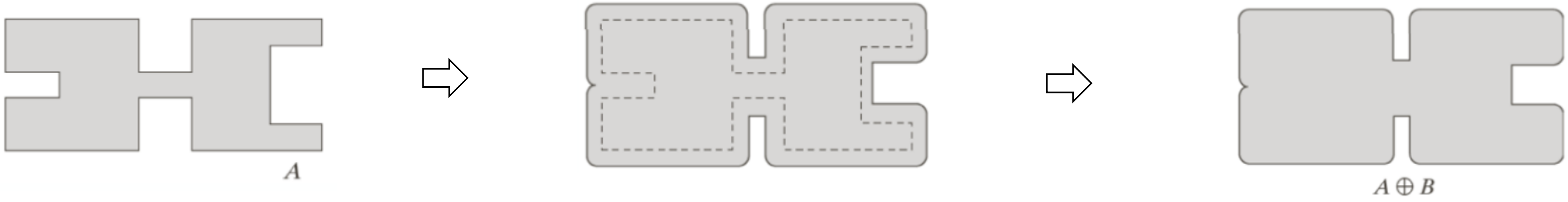
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Morphological processing

- Closing
  - erode the dilated
  - i.e. fist dilate then erode
  - E is same

$$A \cdot B = (A \oplus B) \ominus B$$



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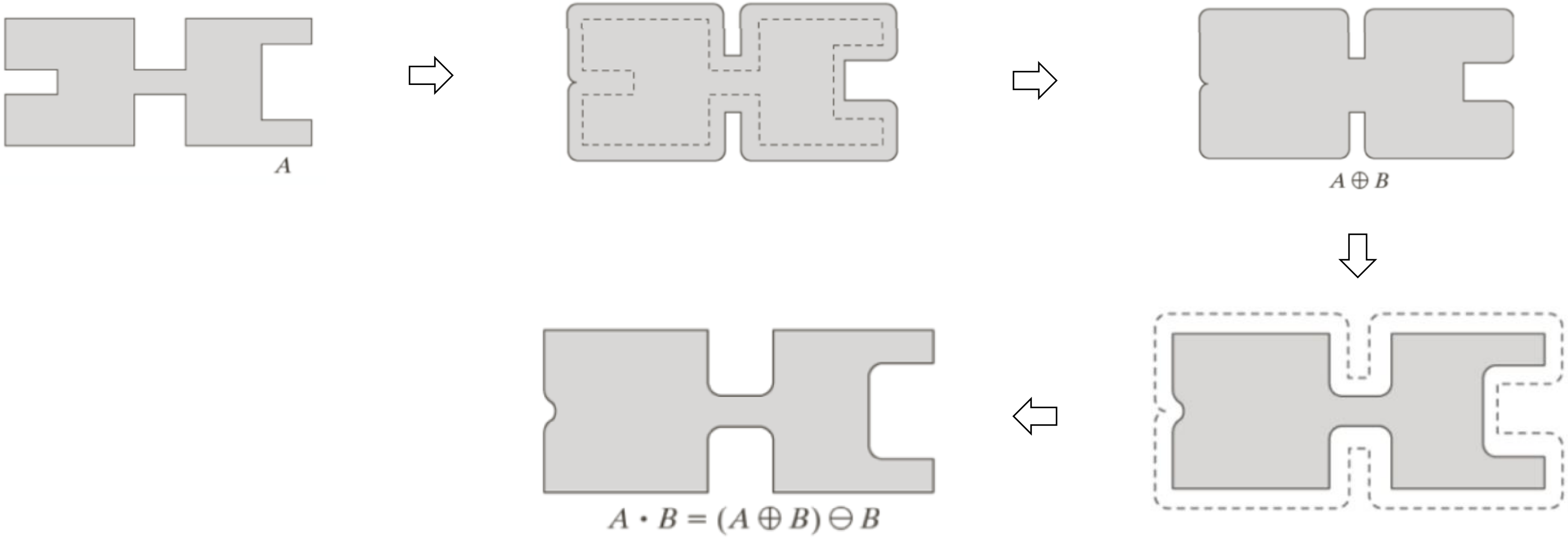


Morphological processing

- Closing
  - useful for filling
    - small holes
    - gaps

$$A \cdot B = (A \oplus B) \ominus B$$

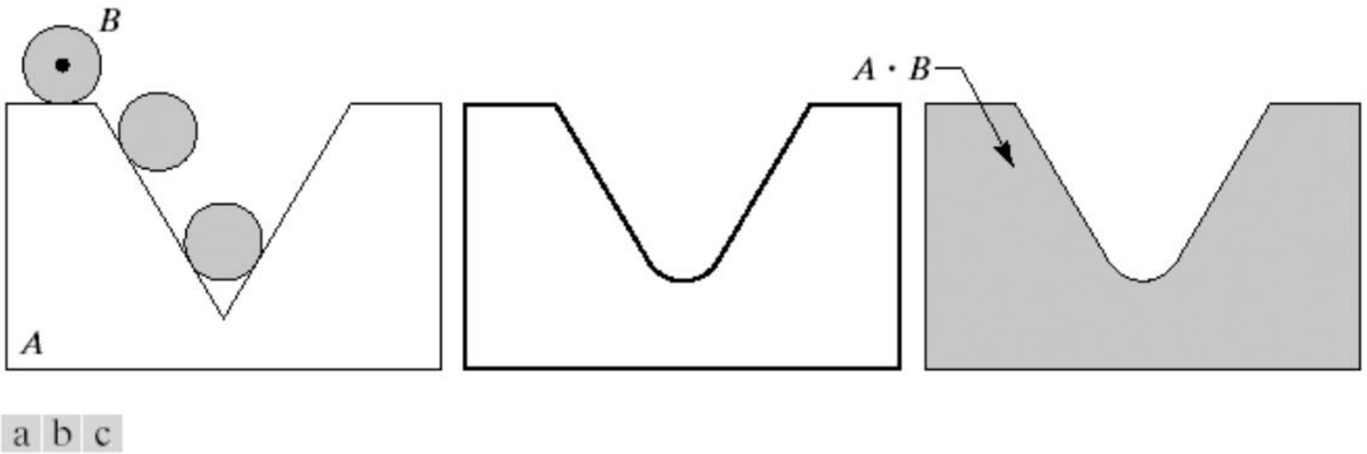


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Morphological processing

- Duality
  - opening and closing are dual to each other

$$(A \circ B)^c = A^c \bullet \hat{B}$$

$$(A \bullet B)^c = A^c \circ \hat{B}$$

  - dilation and erosion are dual to each other

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

- Idempotency

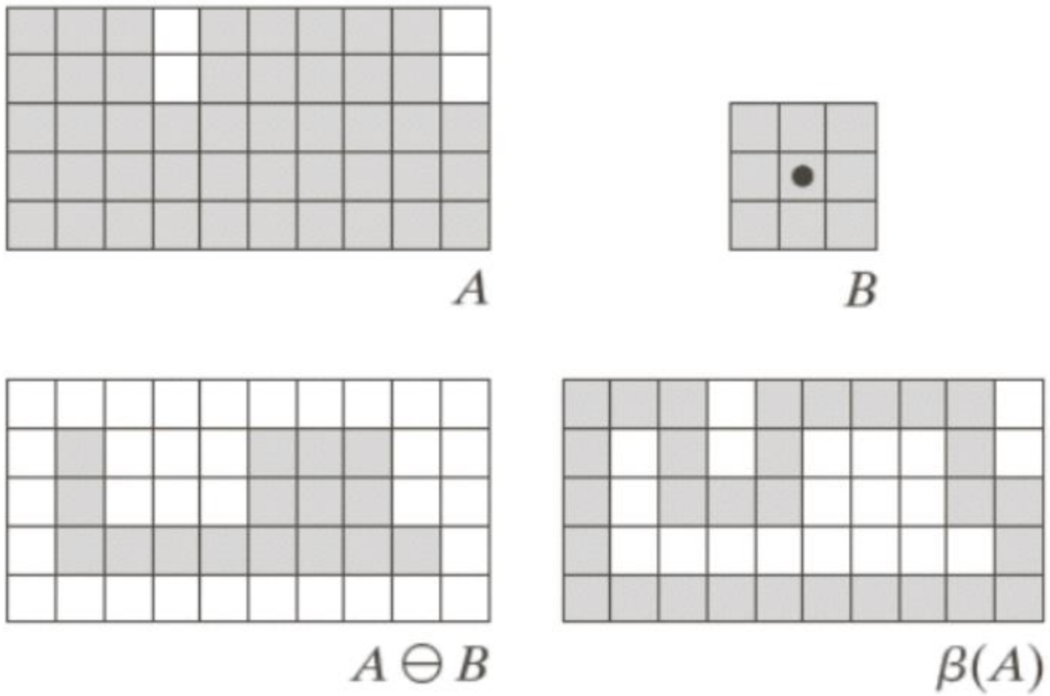
$$A \circ B \circ B = A \circ B$$

$$A \bullet B \bullet B = A \bullet B$$

Morphological processing

- Boundary
  - extracting boundary from a region

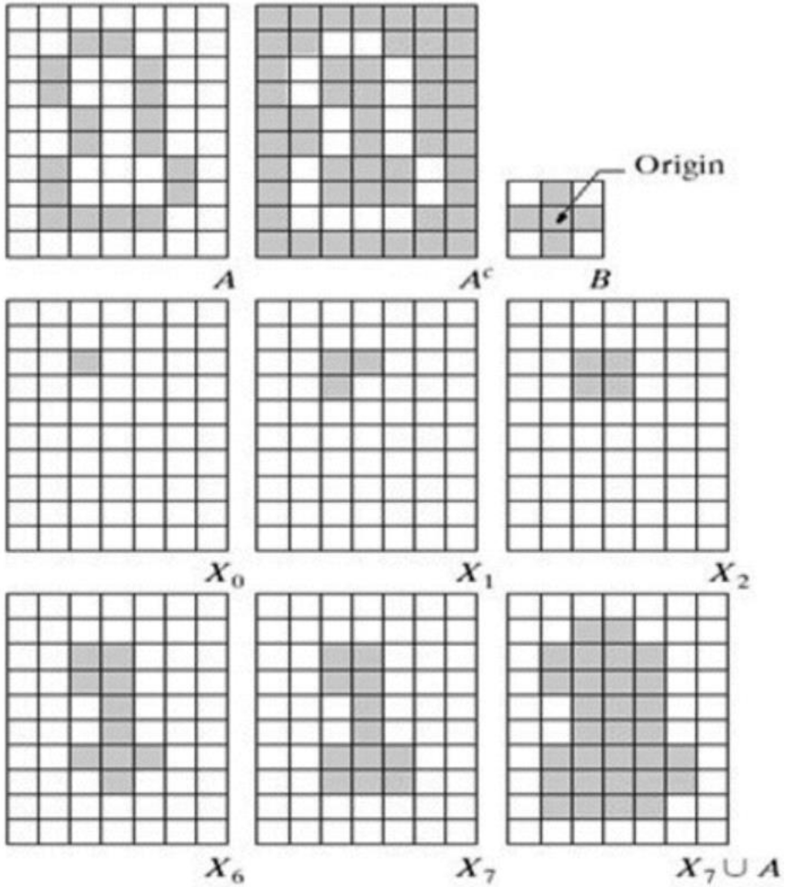
$$\beta(A) = A - (A \ominus B)$$



Morphological processing

- Region filling

$$X_k = (X_{k-1} \oplus B) \cap A^c$$



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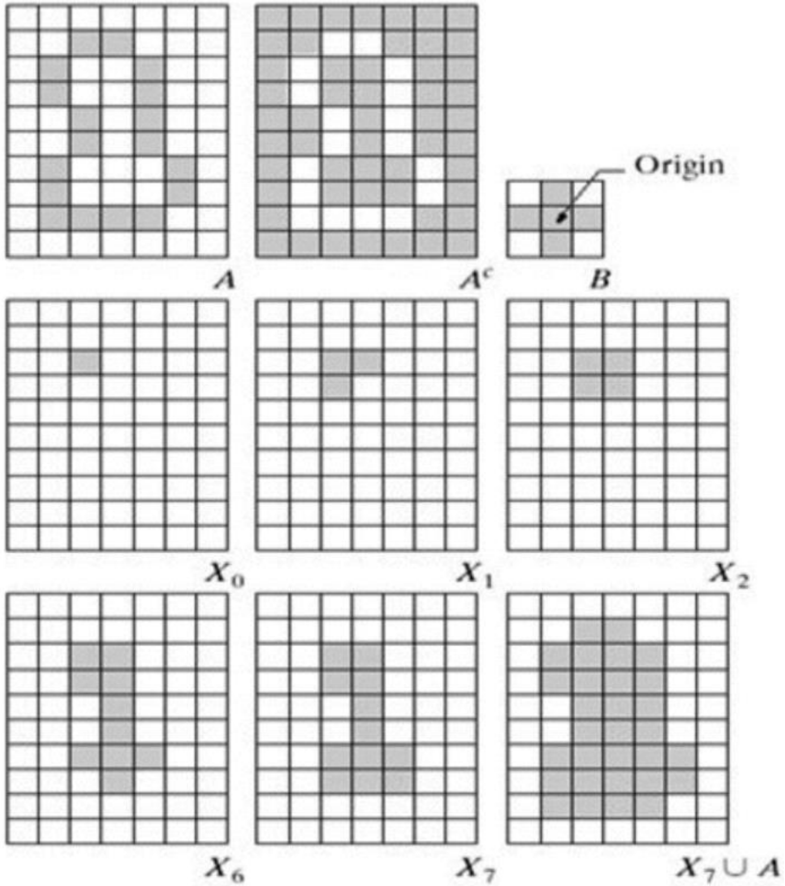
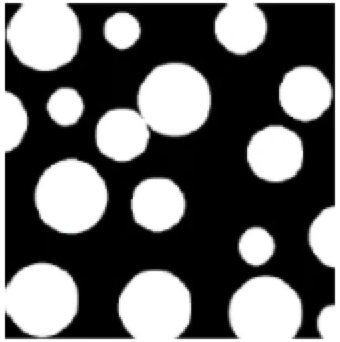
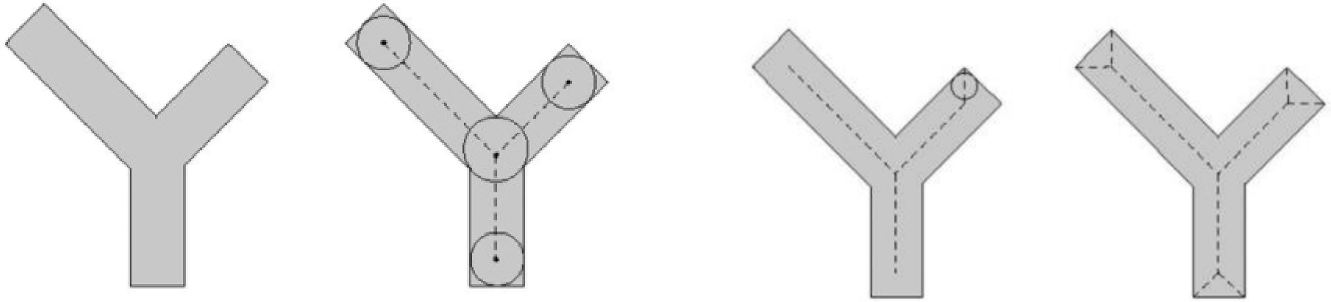


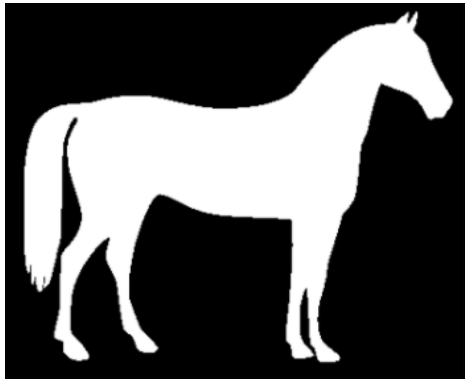


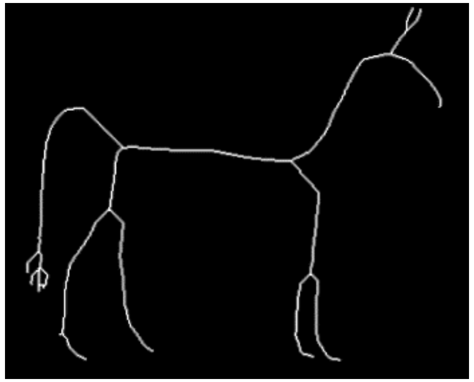
Morphological processing

- Skeleton
  - a set consists of the centres of max enclosing discs
  - repeatedly run adjusted erosions

Erosions	Openings	Set differences
A	$A \circ B$	$A - (A \circ B)$
$A \ominus B$	$(A \ominus B) \circ B$	$(A \ominus B) - ((A \ominus B) \circ B)$
$A \ominus 2B$	$(A \ominus 2B) \circ B$	$(A \ominus 2B) - ((A \ominus 2B) \circ B)$
$A \ominus 3B$	$(A \ominus 3B) \circ B$	$(A \ominus 3B) - ((A \ominus 3B) \circ B)$
$\vdots$	$\vdots$	$\vdots$
$A \ominus kB$	$(A \ominus kB) \circ B$	$(A \ominus kB) - ((A \ominus kB) \circ B)$

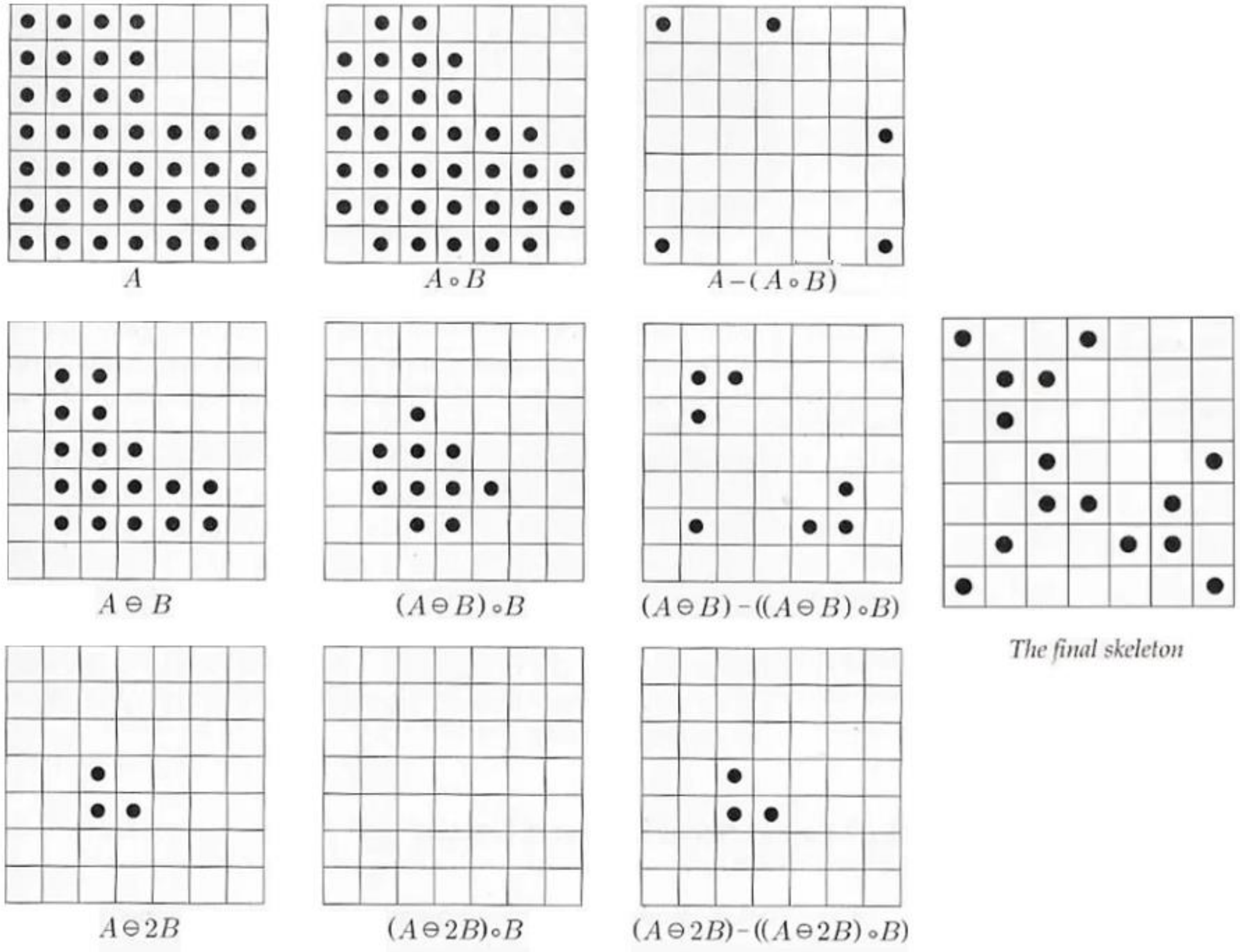






Morphological processing

- Skeleton



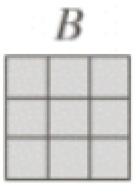
Morphological processing

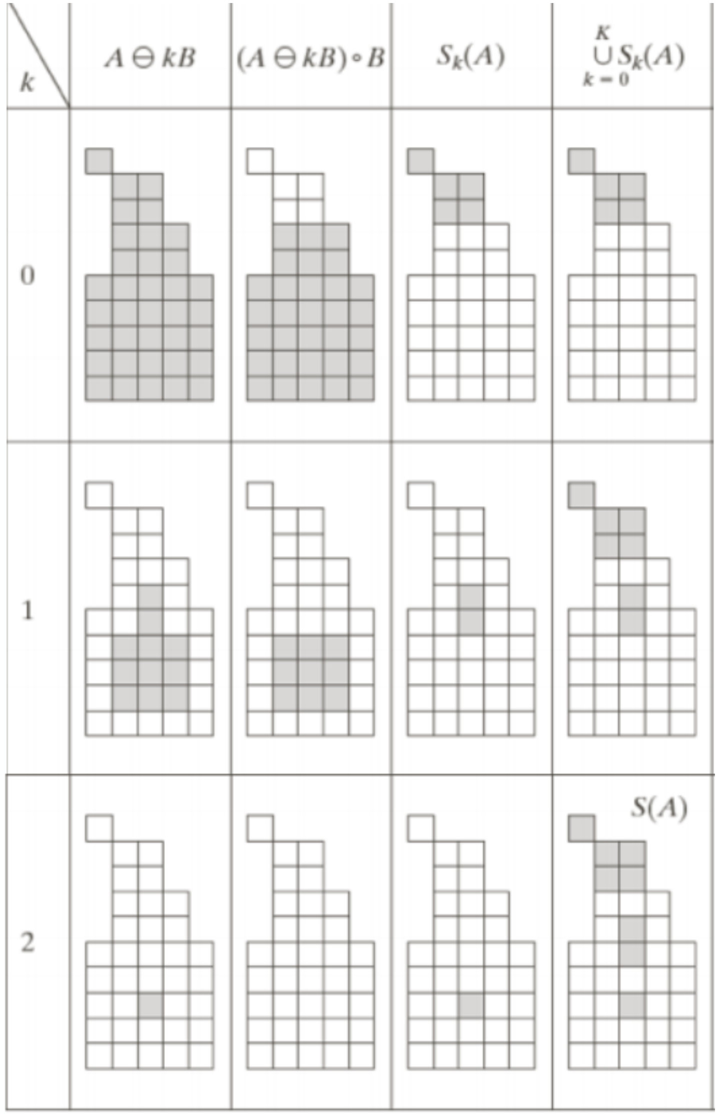
- Skeleton

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ \boldsymbol{B}$$

$$K = max\{k|(A \ominus kB) \neq \emptyset\}$$





Morphological processing

- Fingerprint image processing

$A \ominus B$

$(A \ominus B) \oplus B$

$(A \circ B) \oplus B$

B

1	1	1
1	1	1
1	1	1

$$((A \circ B) \oplus B) \ominus \mathrm{B}$$
$$= (A \circ B) \cdot B$$

Conclusion

- Dilation
- Erosion
- Opening
- Closing

How will the closing by look like?