Segmentation:
Mean-shift

Dr. Tushar Sandhan

Introduction

- Number of segments?

Introduction

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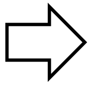
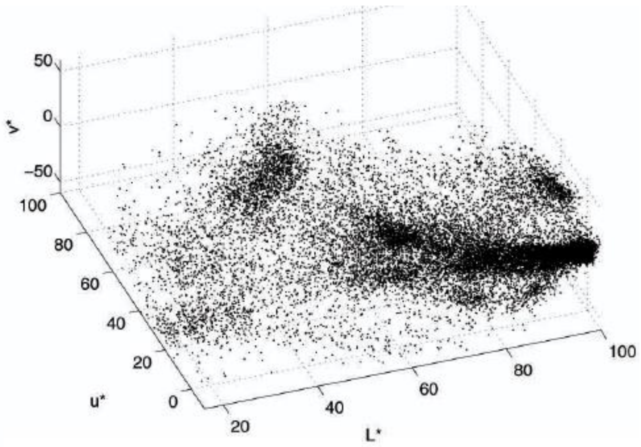
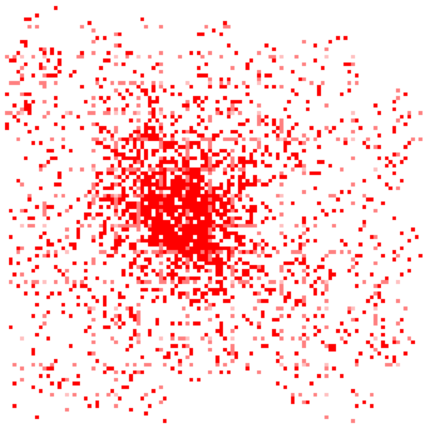
Mean-shift

- Mean-shift clustering
  - iterative steepest ascent method
  - seeks peaks of probability density in feature space
    - finds modes or local maxima
  - it tries to find all possible cluster centres
  - no need of initial guess of K clusters





Mean-shift



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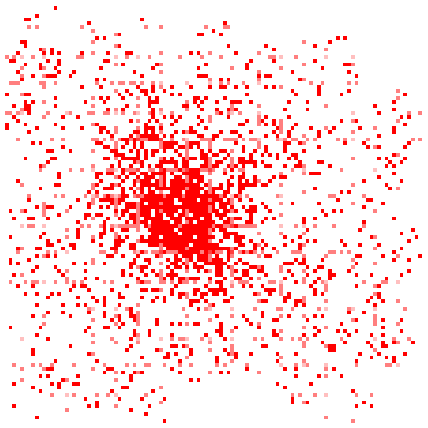

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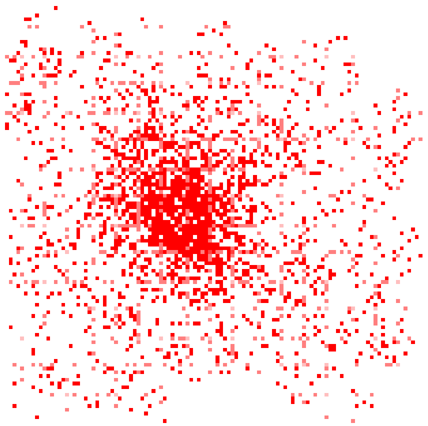
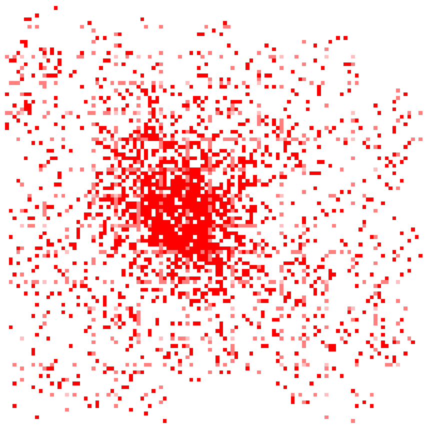


Non-parametric density estimation

Mean-shift



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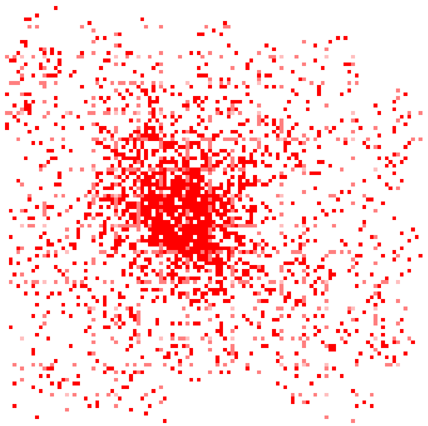

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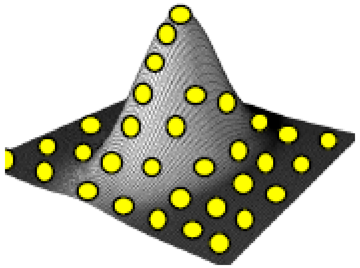

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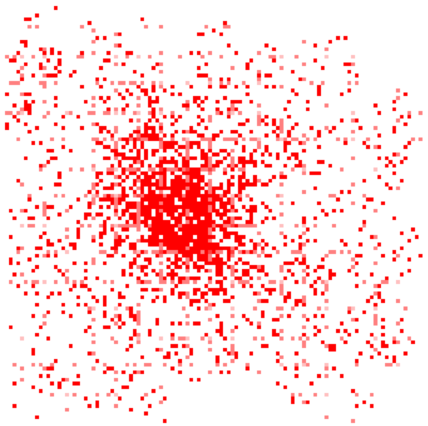

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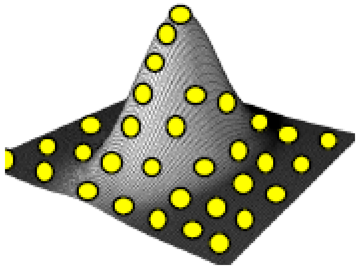
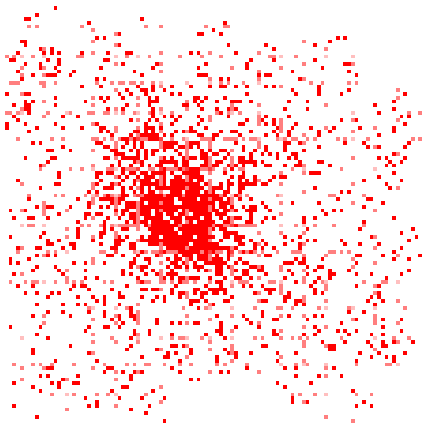


Non-parametric density gradient estimation

Mean-shift



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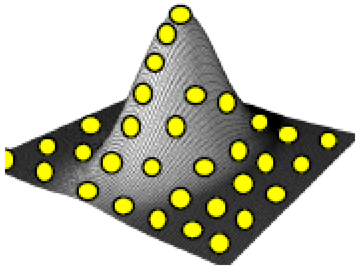

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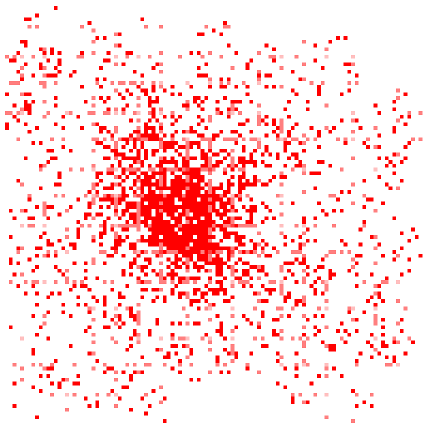

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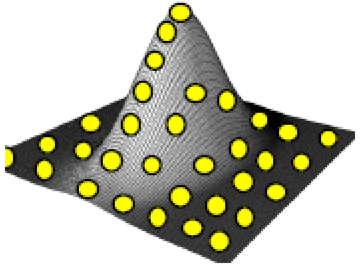

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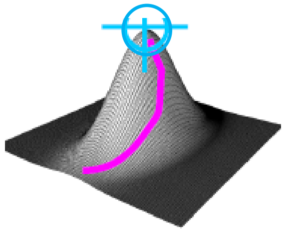

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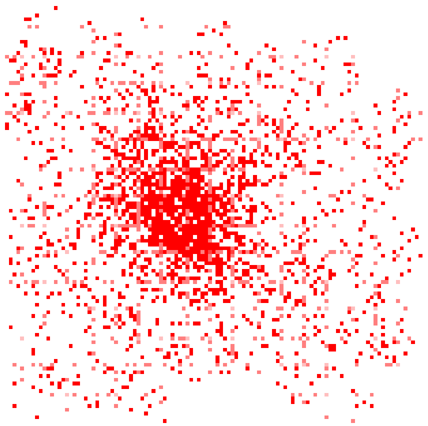

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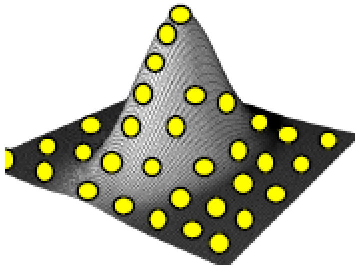
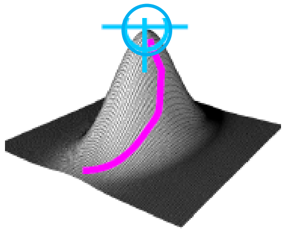


Non-parametric density gradient estimation

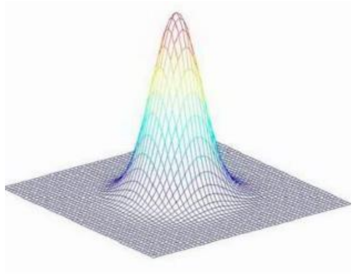
(Mean shift)

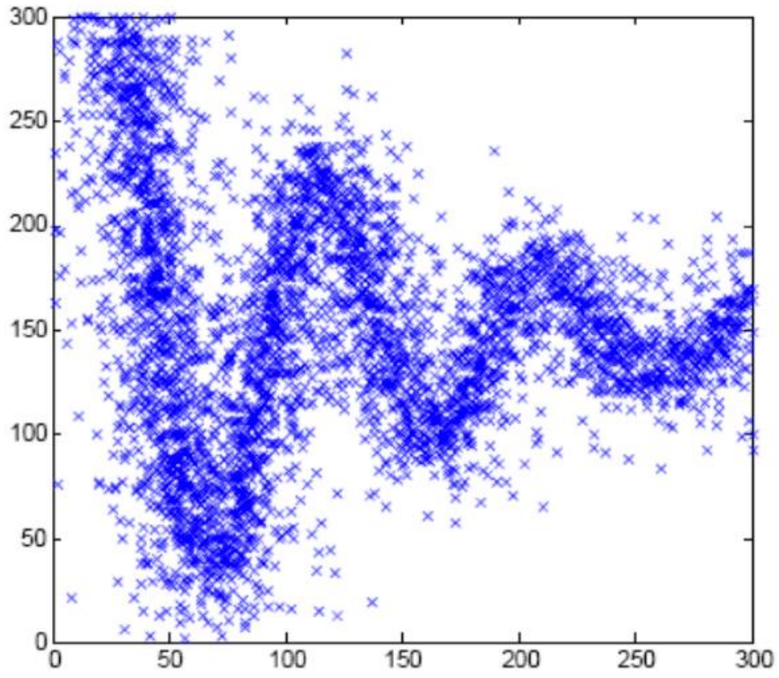


Mean-shift

- Density estimation
  - find the underlying distribution that generates the given data

- Non-parametric density estimation
  - use data points to define distribution
  - put a small probability mass around each data-point via kernel
  - e.g. Gaussian kernel





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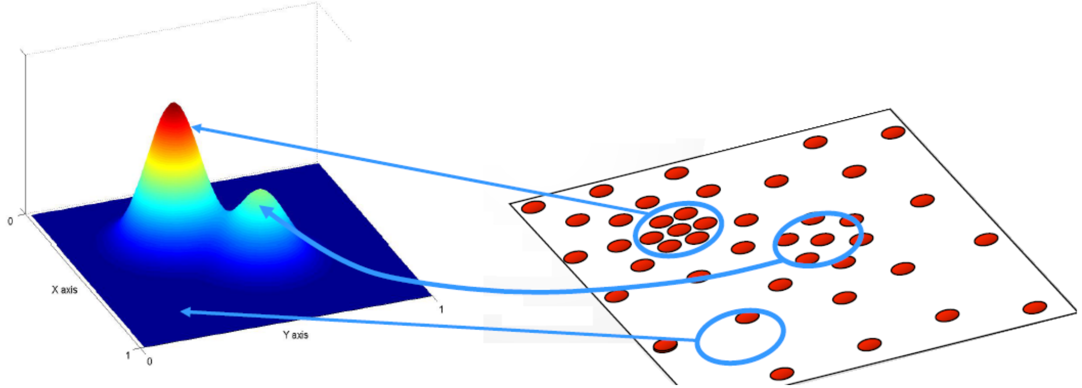
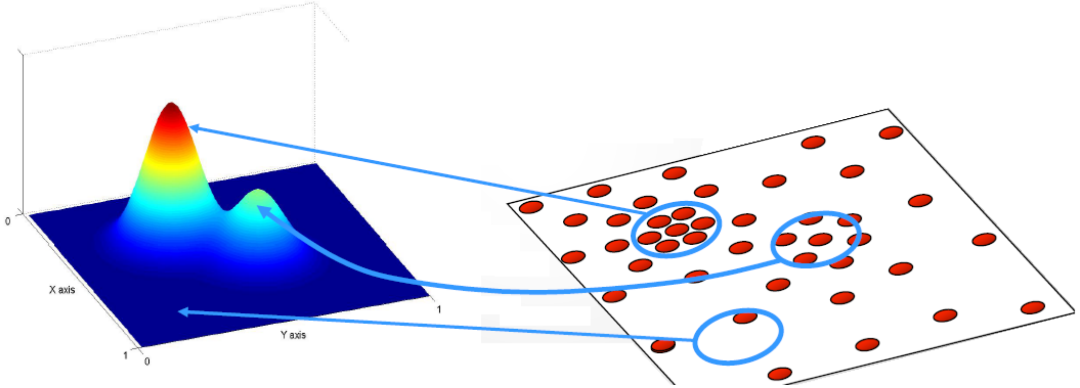


Data point density is similar to PDF value

Courtesy: T. Tappen

Mean-shift



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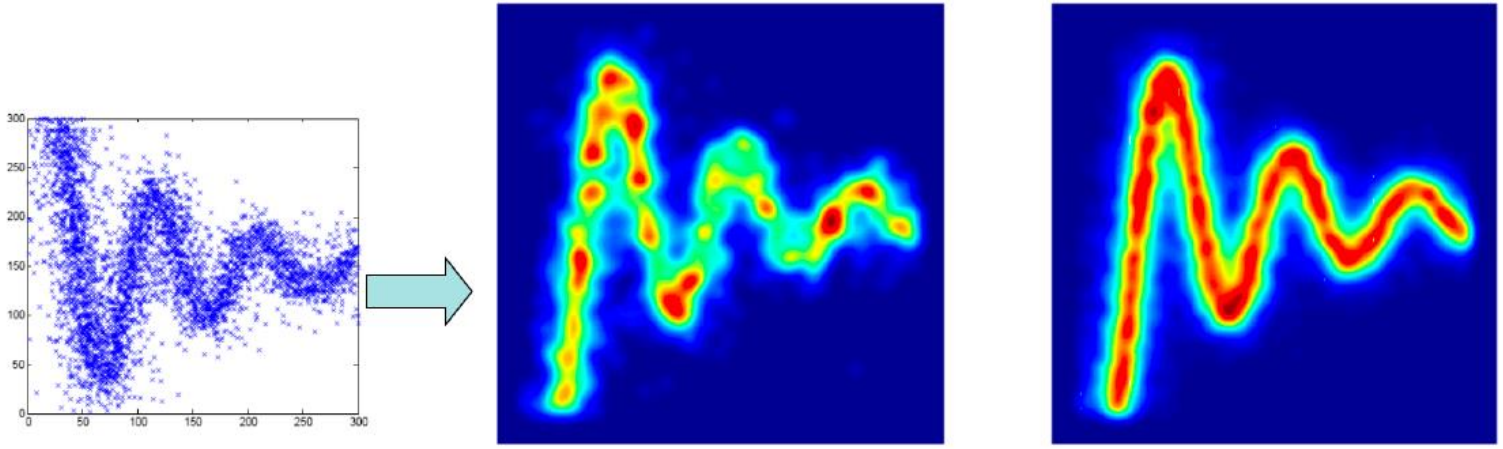

(a) 2000 Samples

(b) 20000 Samples

Courtesy: T. Tappen

Mean-shift

- Kernal density estimation
  - find the underlying distribution
  - that generates the given data
  - in non-parametric way

$$\hat{f}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{x}_i)$$

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points $\mathbf{x}_i$, $i = 1, \ldots, n$ in the d-dimensional space R^d

kernel $K(\mathbf{x})$

symmetric positive definite $d \times d$ bandwidth matrix $\mathbf{H}$

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$$\hat{f}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{x}_i)$$

d-variate kernel $K(\mathbf{x})$ is a bounded function compact support satisfying

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$$\int_{R^d} K(\mathbf{x})\mathrm{d}\mathbf{x} = 1 \qquad \lim_{\|\mathbf{x}\|\to\infty} \|\mathbf{x}\|^d K(\mathbf{x}) = 0$$

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where c_K is a constant

points $\mathbf{x}_i$, $i = 1, \ldots, n$ in the d-dimensional space R^d

kernel $K(\mathbf{x})$

symmetric positive definite $d \times d$ bandwidth matrix $\mathbf{H}$

Mean-shift

- Kernal density estimation
  - multivariate kernel can be generated from a symmetric univariate $K_1(x)$ in two different ways:

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$K^S(\mathbf{x})$ from rotating $K_1(x)$ in R^d

$$K^S(\mathbf{x}) = a_{k,d} K_1(\|\mathbf{x}\|)$$

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$$a_{k,d}^{-1} = \int_{R^d} K_1(\|\mathbf{x}\|)\mathrm{d}\mathbf{x}$$

Mean-shift

- Kernal density estimation
  - radial symmetric kernels $K^S(\boldsymbol{x})$
  - profile of a kernel $k(x)$

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$$K^S(\mathbf{x}) \text{ from rotating } K_1(x) \text{ in } R^d$$

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Mean-shift

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$$K(\mathbf{x}) = c_{k,d} k(\|\mathbf{x}\|^2)$$

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$K^S(\mathbf{x})$ from rotating $K_1(x)$ in R^d

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it suffices to define the function $k(x)$

$K^S(\mathbf{x})$ from rotating $K_1(x)$ in R^d

$$K^S(\mathbf{x}) = a_{k,d} K_1(\|\mathbf{x}\|)$$

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it suffices to define the function $k(x)$

$K^S(\mathbf{x})$ from rotating $K_1(x)$ in R^d

constant $c_{k,d}$, which makes $K(\mathbf{x})$ integrate to one

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Mean-shift

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$$K_E(\mathbf{x}) = \begin{cases} \frac{1}{2}c_d^{-1}(d+2)(1 - \|\mathbf{x}\|^2) & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise}, \end{cases}$$

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$$K(\mathbf{x}) = c_{k,d} k(\|\mathbf{x}\|^2)$$

$$\hat{f}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{x}_i)$$

$$K_{\mathbf{H}}(\mathbf{x}) = | \mathbf{H} |^{-1/2} K(\mathbf{H}^{-1/2}\mathbf{x})$$

$$k_E(x) = \begin{cases} 1 - x & 0 \le x \le 1 \\ 0 & x > 1, \end{cases}$$

$$\int_{R^d} K(\mathbf{x}) \mathrm{d}\mathbf{x} = 1 \qquad \lim_{\|\mathbf{x}\| \to \infty} \|\mathbf{x}\|^d K(\mathbf{x}) = 0$$

$$\int_{R^d} \mathbf{x} K(\mathbf{x}) \mathrm{d}\mathbf{x} = 0 \qquad \int_{R^d} \mathbf{x}\mathbf{x}^\top K(\mathbf{x}) \mathrm{d}\mathbf{x} = c_K \mathbf{I},$$

$$K_E(\mathbf{x}) = \begin{cases} \frac{1}{2} c_d^{-1}(d+2)(1 - \|\mathbf{x}\|^2) & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise}, \end{cases}$$

c_d is the volume of the unit d-dimensional sphere

$K^S(\mathbf{x})$ from rotating $K_1(x)$ in R^d

$$K^S(\mathbf{x}) = a_{k,d} K_1(\|\mathbf{x}\|)$$

$$a_{k,d}^{-1} = \int_{R^d} K_1(\|\mathbf{x}\|) \mathrm{d}\mathbf{x}$$

Mean-shift

- Kernal density estimation
  - radial symmetric kernels $K^S(\boldsymbol{x})$
  - profile of a kernel $k(x)$
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  - non-differentiable at boundary

Mean-shift

- Kernal density estimation
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  - profile of a kernel $k(x)$
  - exponential profile

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Mean-shift

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  - radial symmetric kernels $K^S(\boldsymbol{x})$
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$$k_N(x) = \exp\left(-\frac{1}{2}x\right) \qquad x \geq 0$$

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Mean-shift

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$$k_N(x) = \exp\left(-\frac{1}{2}x\right) \qquad x \geq 0$$

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$$K_N(\mathbf{x}) = (2\pi)^{-d/2}\exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right)$$

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  - final density estimator with this kernel

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Mean-shift

$$x_1 = x_0 + \eta \nabla f(x_0)$$

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$c_{k,d}$ is volume of unit sphere $\in R^d$

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Mean-shift

$x_1 = x_0 + \eta \nabla f(x_0)$

Define a function of a kernel $g(x) = -k'(x)$

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Define a function of a kernel $g(x) = -k'(x)$

$$\begin{aligned} &\hat{\nabla} f_{h,K}(\mathbf{x}) \\ &= \frac{2c_{k,d}}{nh^{d+2}} \sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{x}) g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right) \\ &= \frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)\right] \left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right] \end{aligned}$$

Mean-shift

$$x_1 = x_0 + \eta \nabla f(x_0)$$

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$$= \frac{2c_{k,d}}{nh^{d+2}} \left[\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)\right] \left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]$$

$$\boxed{\mathbf{m}_{h,G}(\mathbf{x}) = \frac{\sum_{i=1}^{n} \mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}}$$

Mean-shift

$x_1 = x_0 + \eta \nabla f(x_0)$

$$\hat{f}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right)$$

$$\hat{f}_{h,K}(\mathbf{x}) = \frac{c_{k,d}}{nh^d} \sum_{i=1}^{n} k\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)$$

$c_{k,d}$ is volume of unit sphere $\in R^d$

at the modes: $\nabla f(\mathbf{x}) = \mathbf{0}$

$$\hat{\nabla} f_{h,K}(\mathbf{x}) \equiv \nabla \hat{f}_{h,K}(\mathbf{x}) = \frac{2c_{k,d}}{nh^{d+2}} \sum_{i=1}^{n} (\mathbf{x}-\mathbf{x}_i) k'\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)$$

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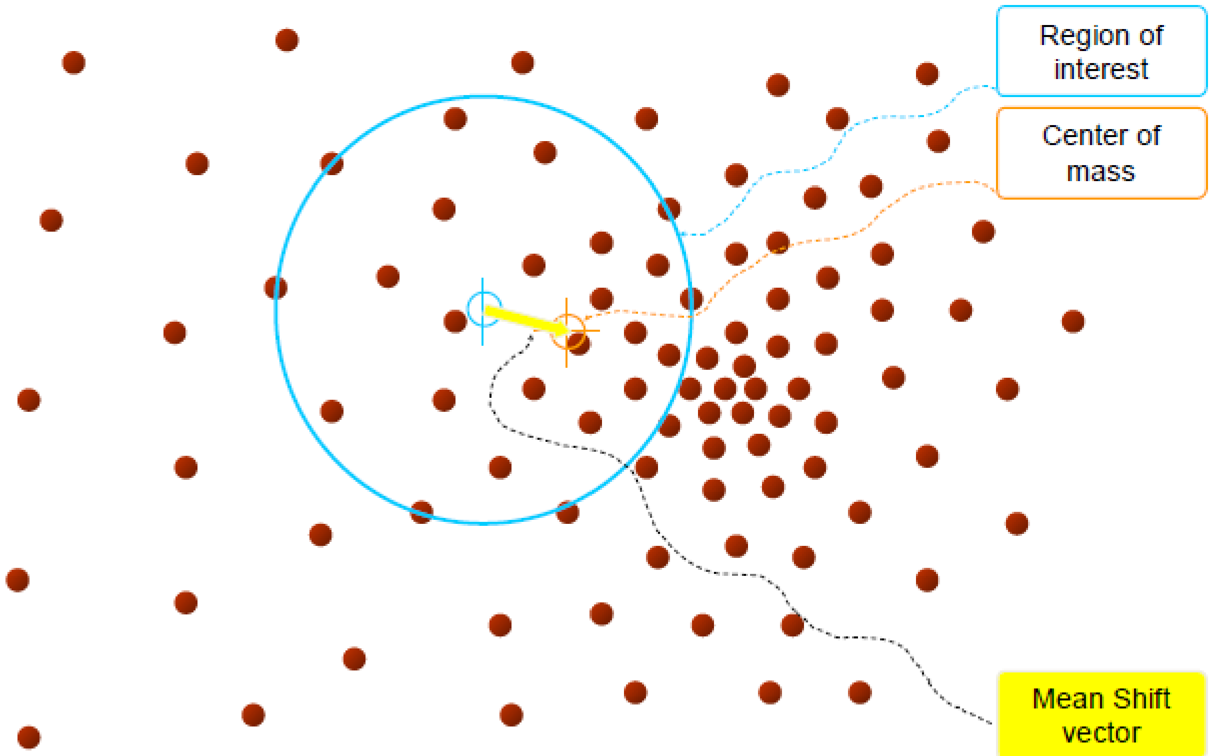
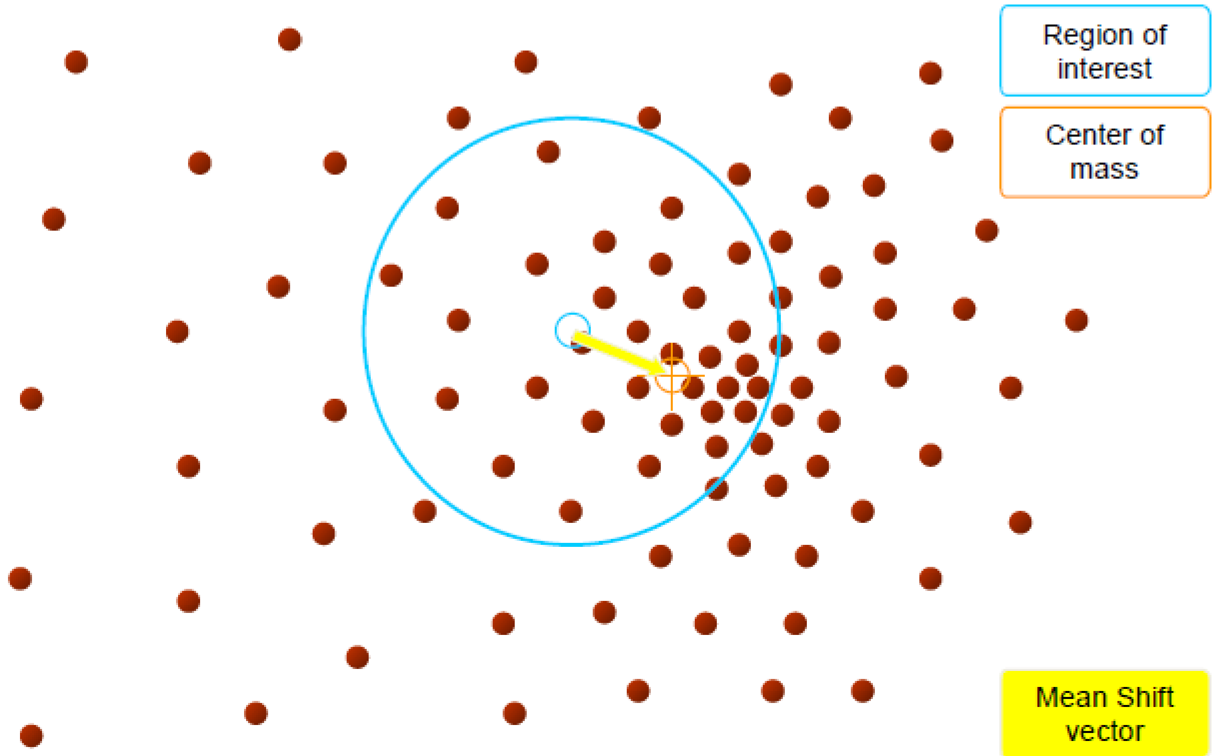
$$\hat{\nabla} f_{h,K}(\mathbf{x}) = \hat{f}_{h,G}(\mathbf{x}) \frac{2c_{k,d}}{h^2 c_{g,d}} \mathbf{m}_{h,G}(\mathbf{x}) \qquad \mathbf{m}_{h,G}(\mathbf{x}) = \frac{1}{2} h^2 c \frac{\hat{\nabla} f_{h,K}(\mathbf{x})}{\hat{f}_{h,G}(\mathbf{x})}$$

Mean-shift

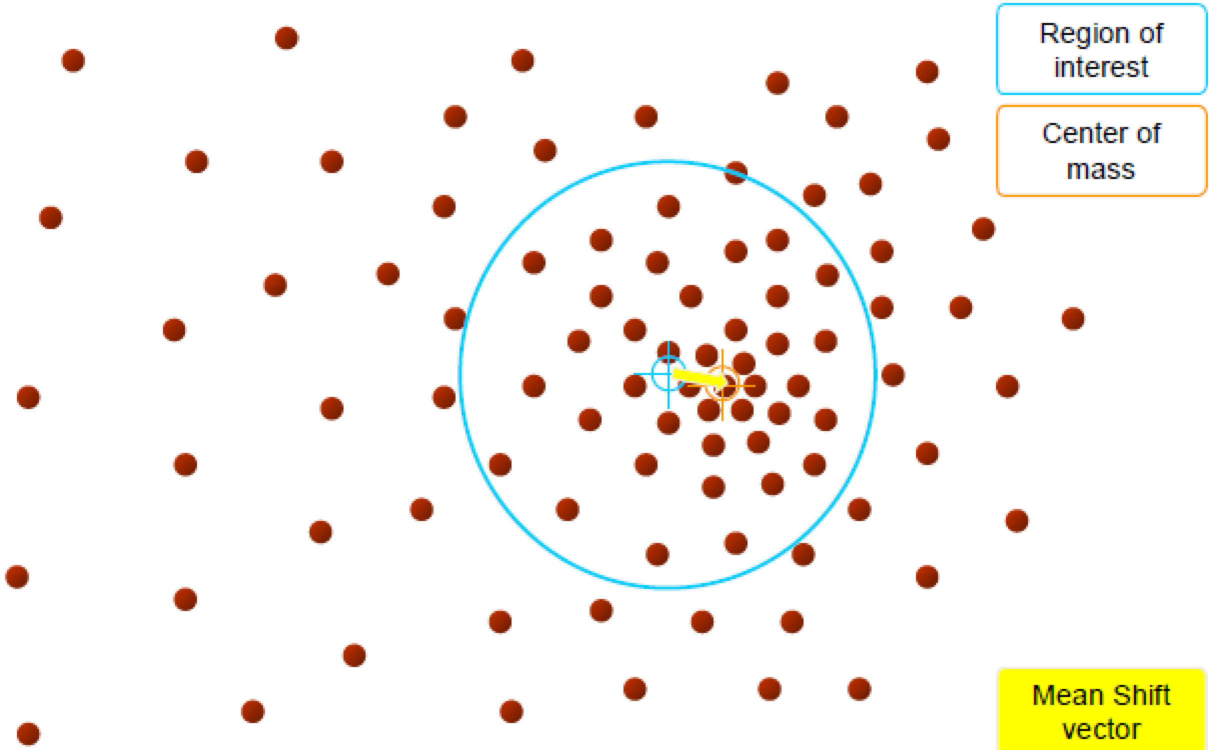


Courtesy: Y. Ukrainitz

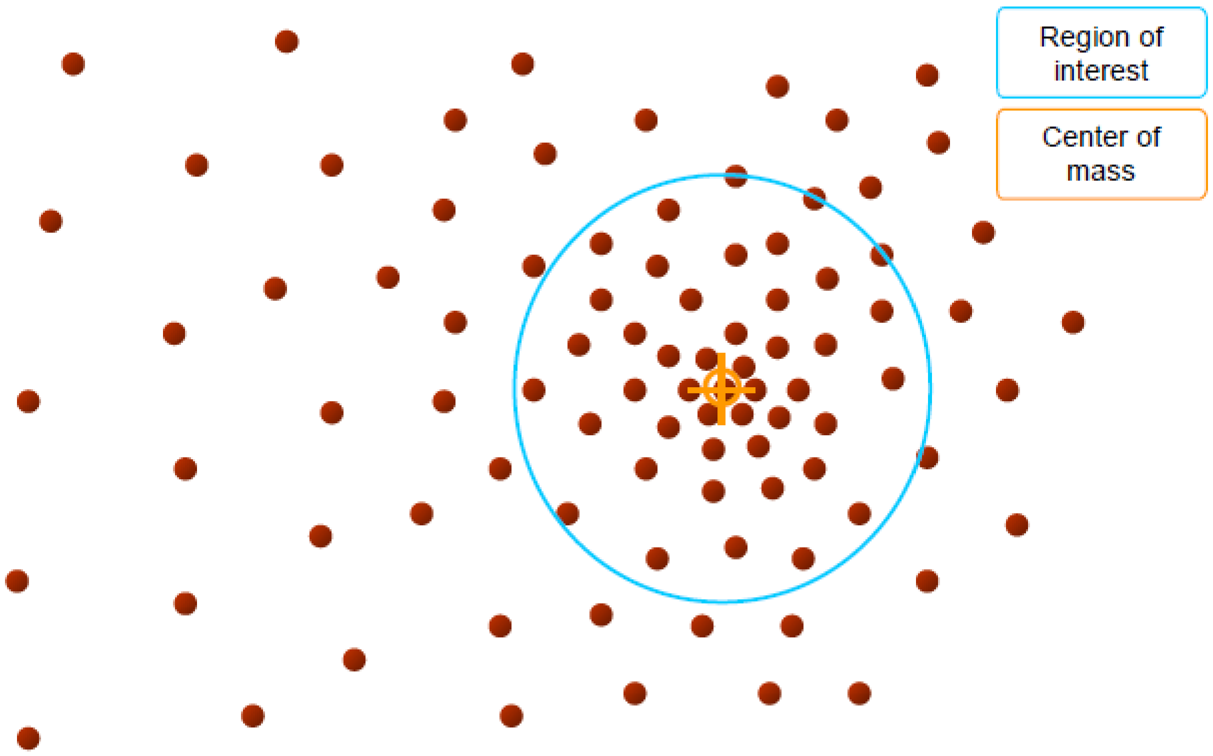
Mean-shift



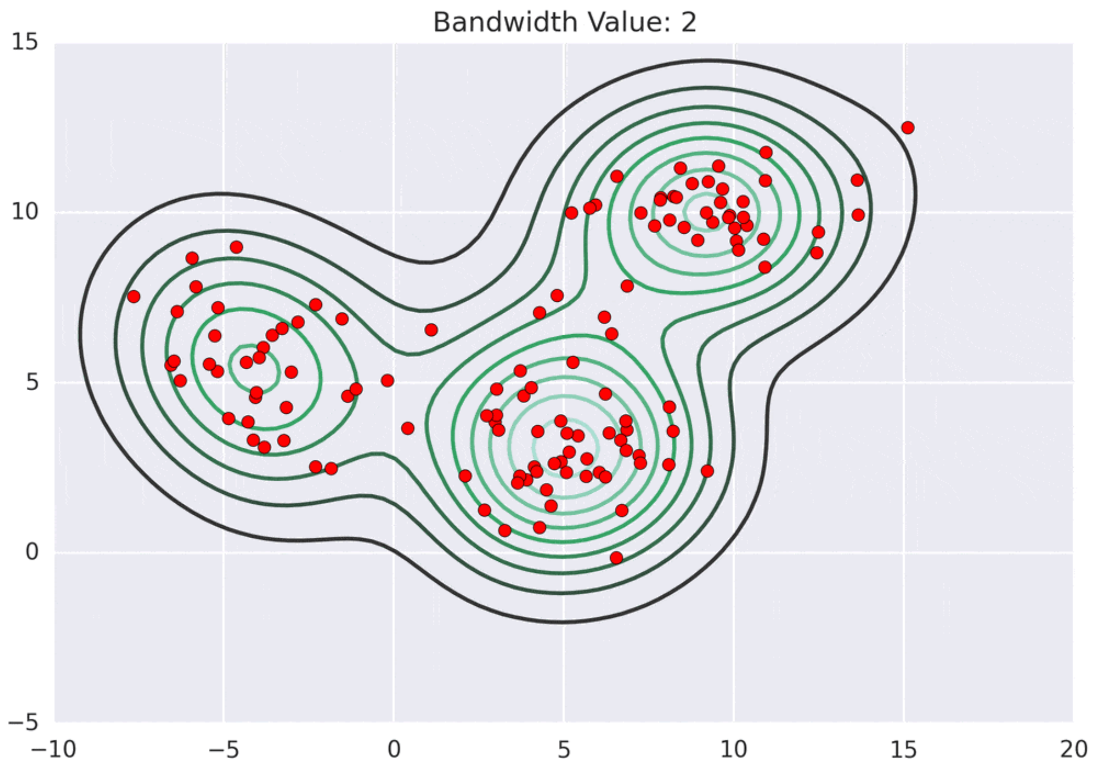
Mean-shift



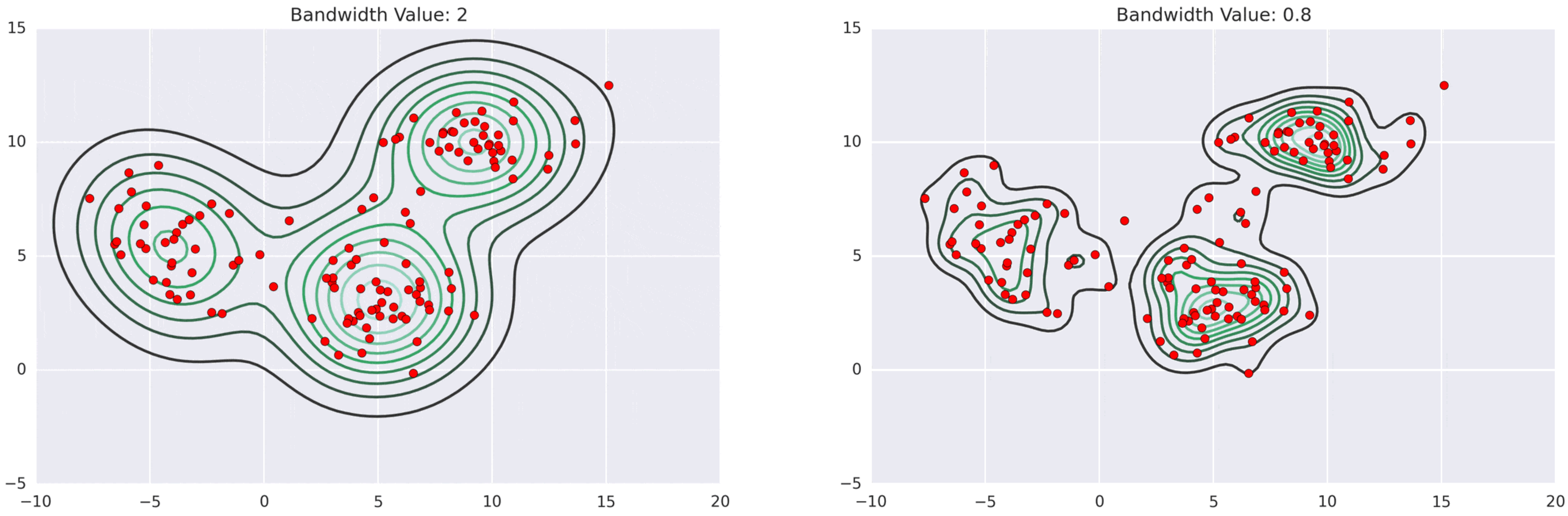
Mean-shift



Mean-shift



Mean-shift


Bandwidth Value: 2
15
10
5
0
−5
−10
−5
0
5
10
15
20
Bandwidth Value: 0.8
15
10
5
0
−5
−10
−5
0
5
10
15
20

Mean-shift

- Algorithm

  - transform image into feature space
  - initialize window at each feature point
  - for each window
    - compute mean shift vector $m(x)$
    - move density estimation window by $m(x)$
    - repeat till convergence
  - merge windows that end up near same peak

Mean-shift

- Pros
  - automatically finds various number of modes
  - no need of initial guessing of cluster centres
  - does not assume spherical clusters
  - robust to outliers
  - just a single para (window size w)

- Cons
  - bandwidth or windows size is an imp. Para
    - slight change in w, translates varied output
  - computationally expensive
    - complexity: $O(n^2T)$
  - not scalable with dimensionality

Mean-shift

input

Mean-shift

input

Mean-shift

input

mean shift segmentation

Mean-shift

input

mean shift segmentation

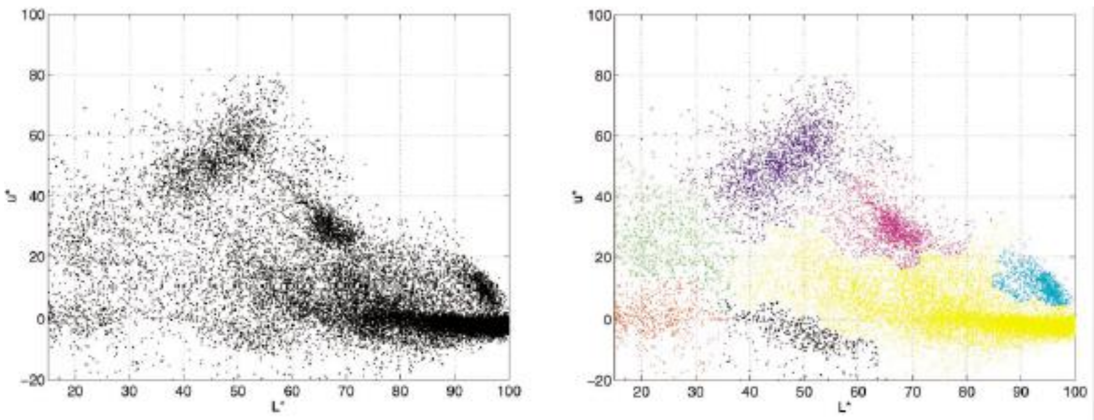
Mean-shift

input

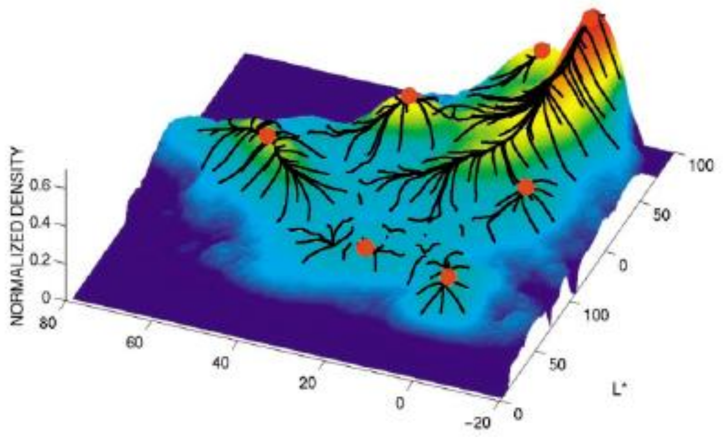
mean shift segmentation

mean shift region boundaries

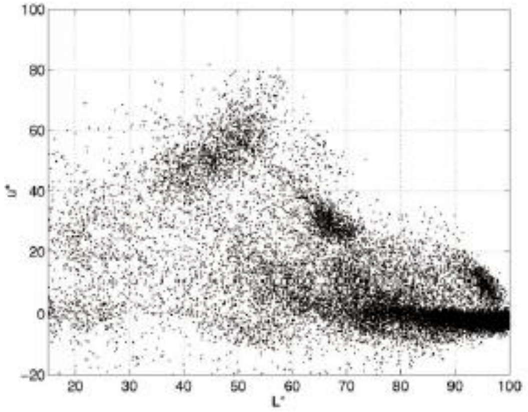
Mean-shift



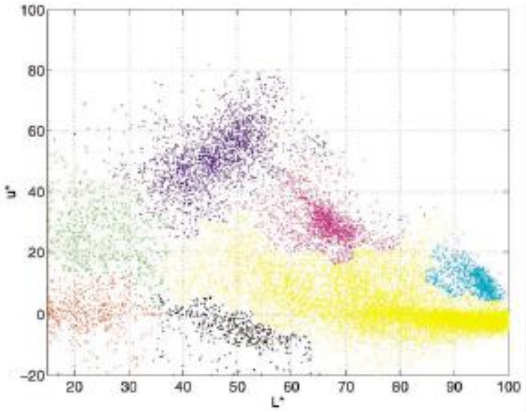
(a) (b)



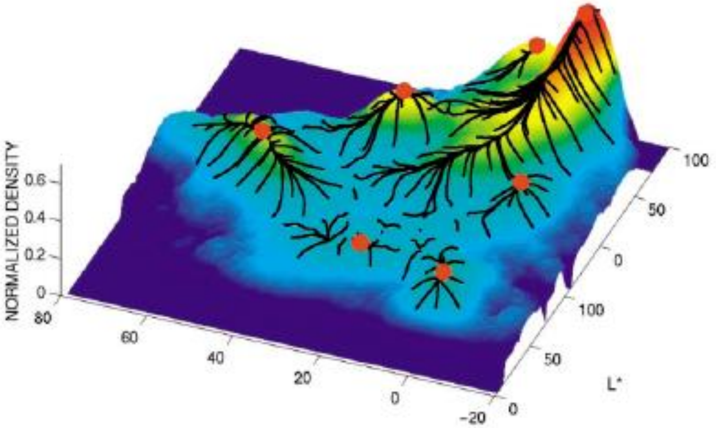
Mean-shift



(a)

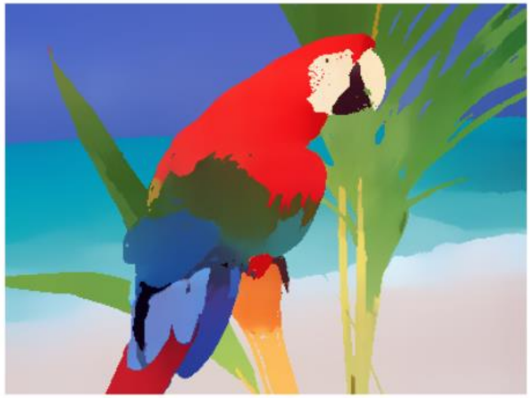


(b)

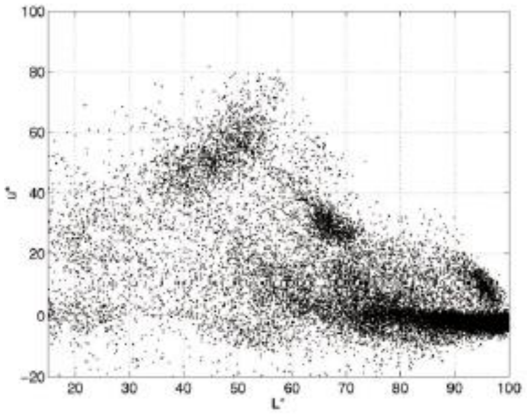


input image

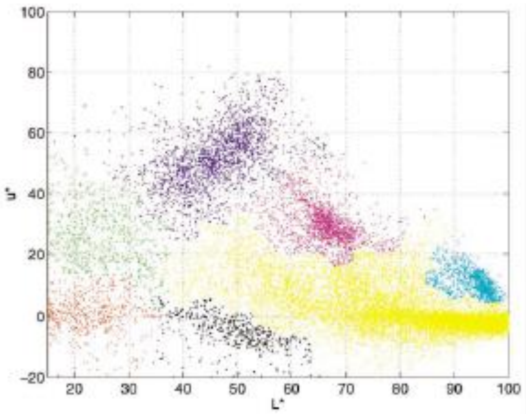
output image



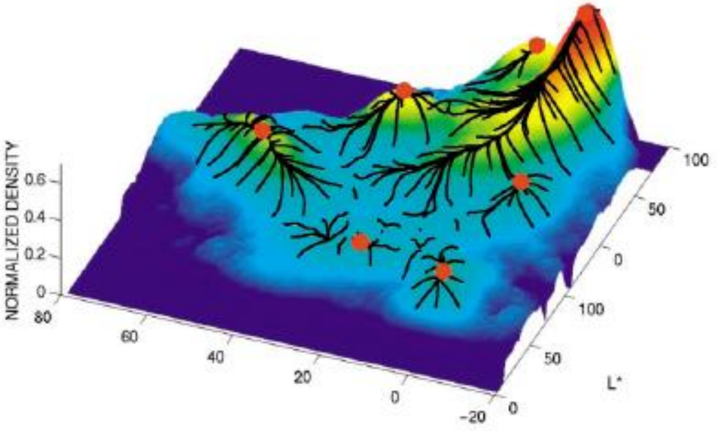
Mean-shift

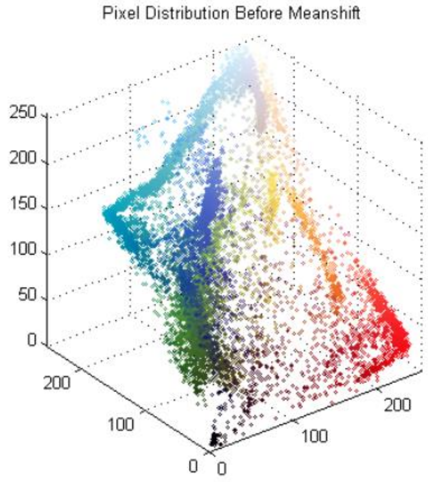


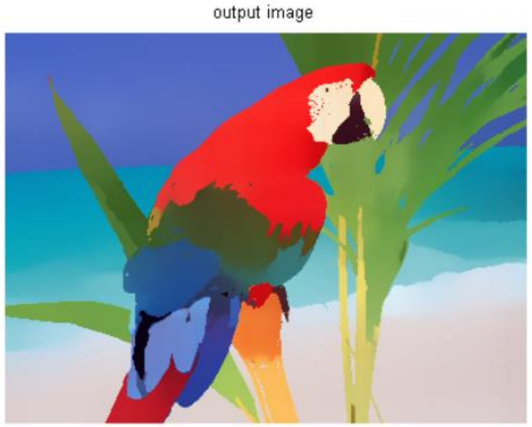
(a)

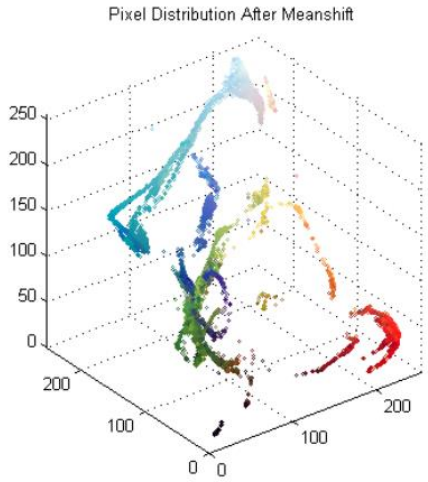


(b)









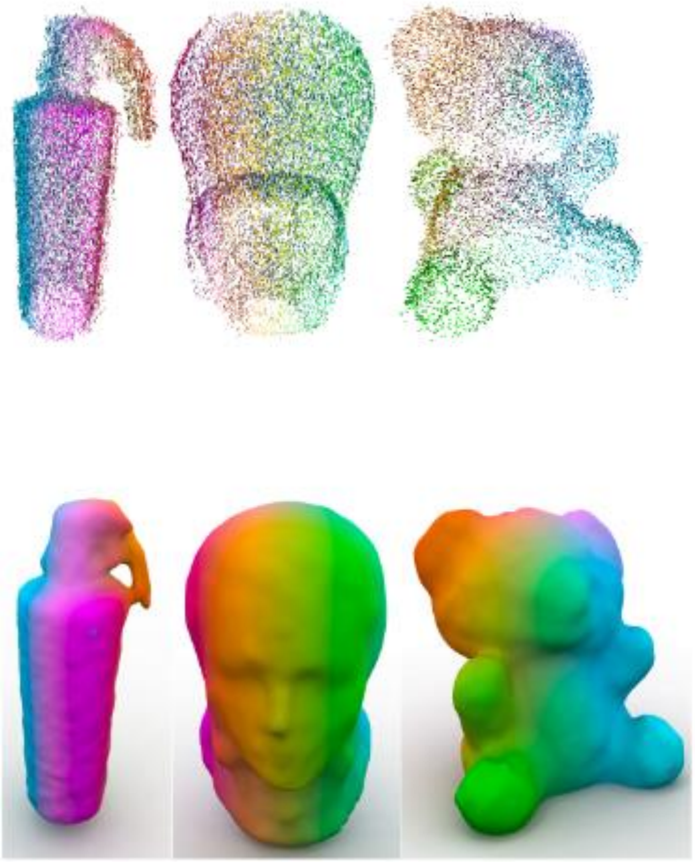
Mean-shift

- Surface reconstruction

- Image denoising

Mean-shift

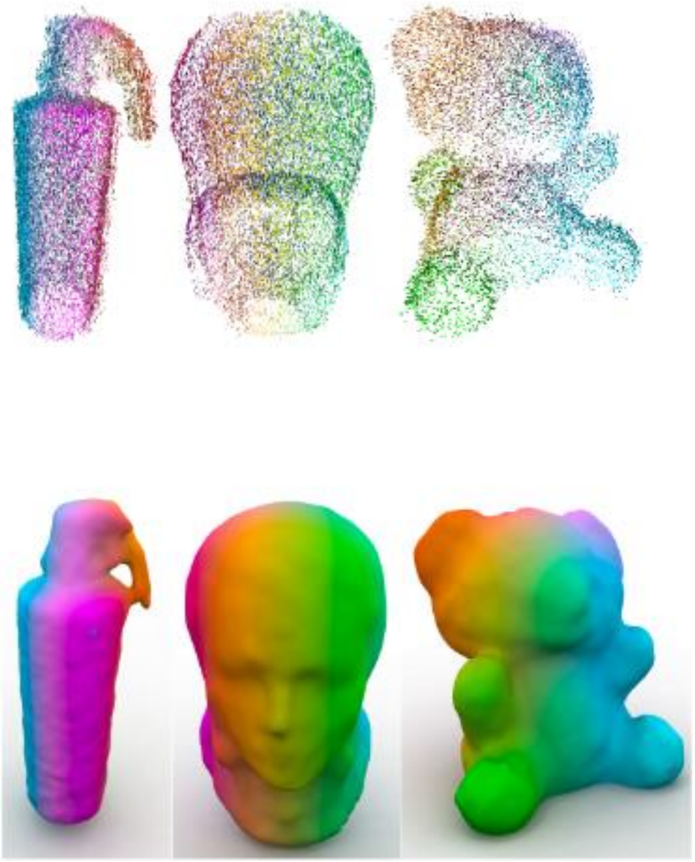
- Surface reconstruction



- Image denoising

Mean-shift

- Surface reconstruction



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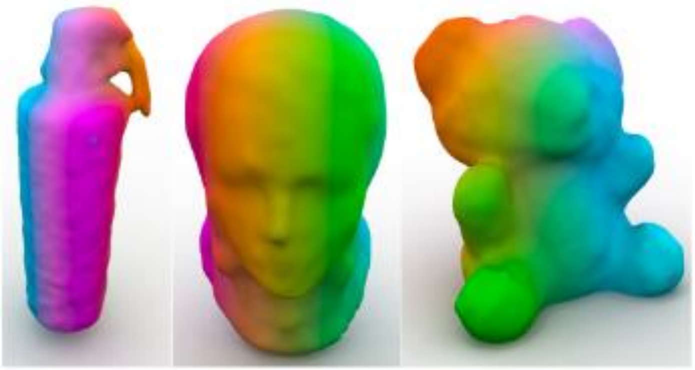

- Image denoising

Mean-shift

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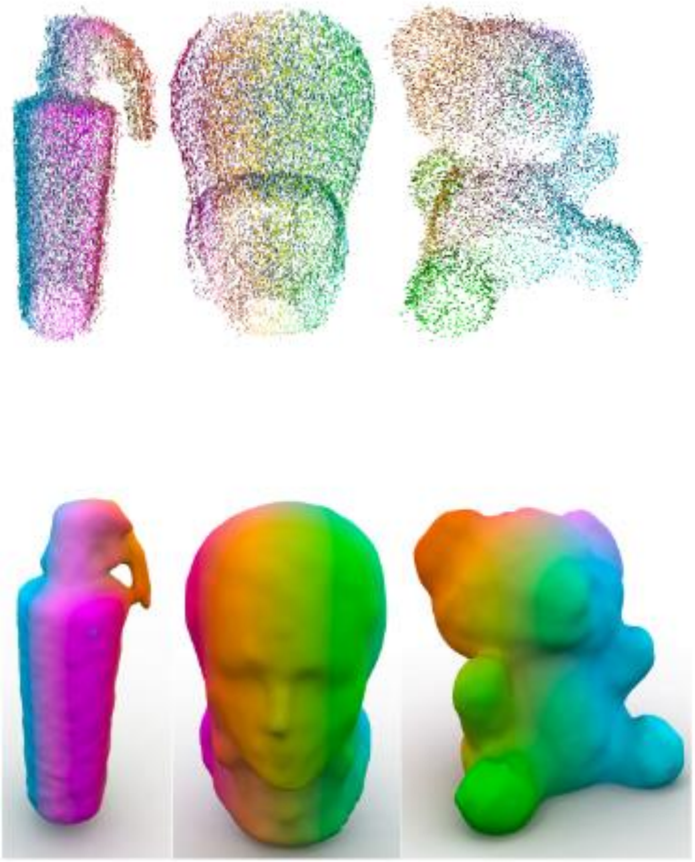

- Image denoising

Mean-shift

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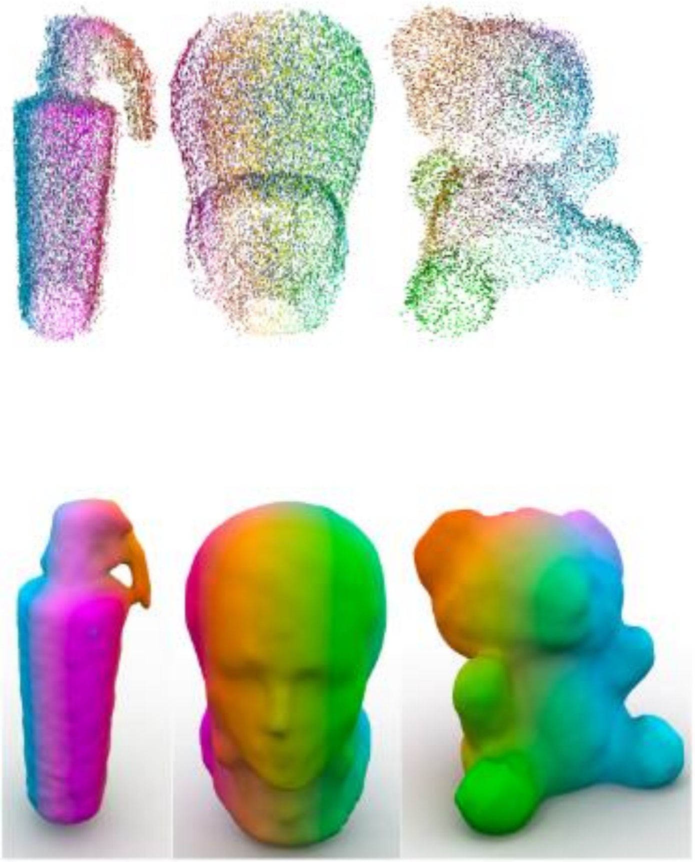
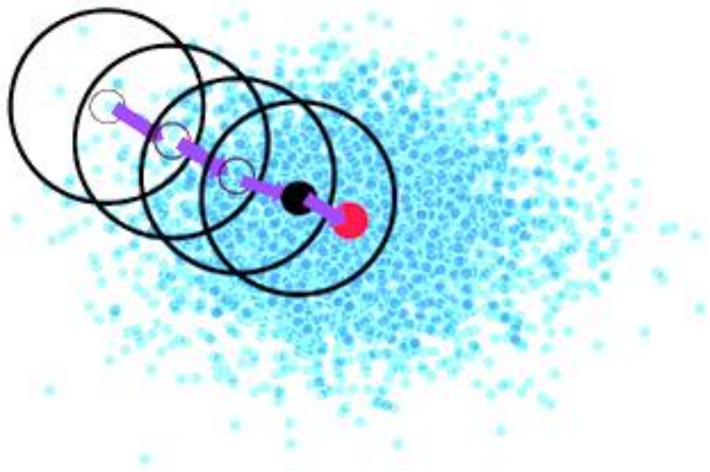


- Image denoising

Conclusion

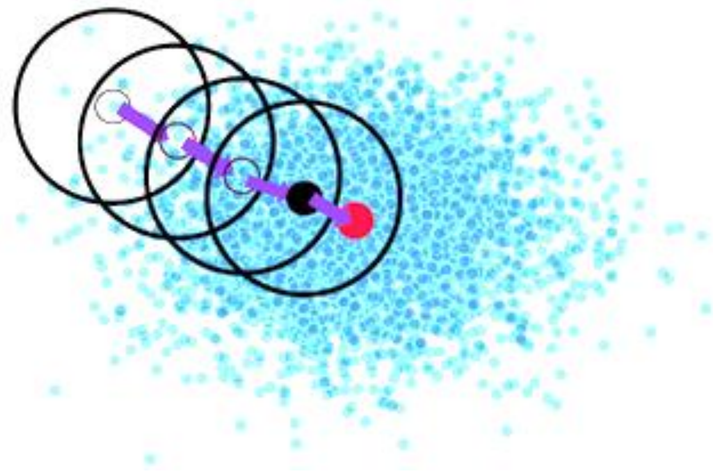
- Mean-shift



Conclusion

- Mean-shift

- Mean-shift
  - Number of cluster specification is not needed
  - Mode seeking algorithm
  - Computationally expensive



Conclusion

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❑ Mean-shift

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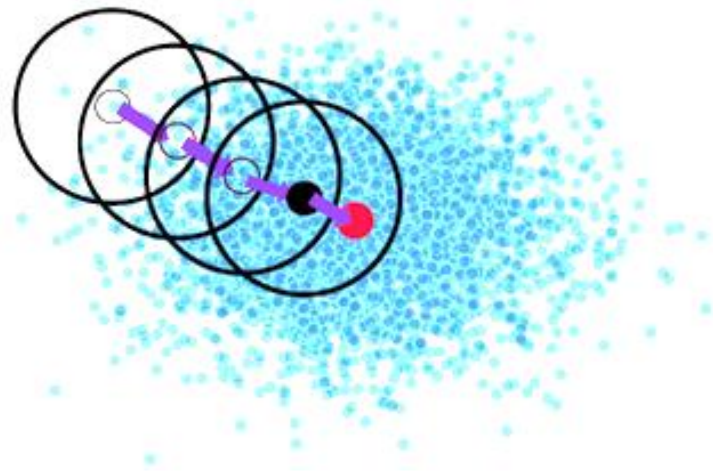

❑ Mean-shift also can do

- surface reconstruction
- Image filtering