Segmentation:

Regions & K-means

Dr. Tushar Sandhan

Introduction

- Goal of segmentation (multiclass)

Image credit: J. Jordan

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Region growing

- Partition the image I into m regions
  - every pix belong to some region
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  - all pix in a region, share similar property
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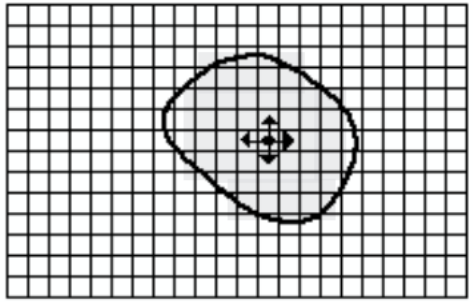

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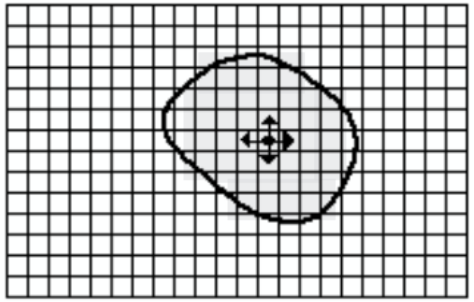
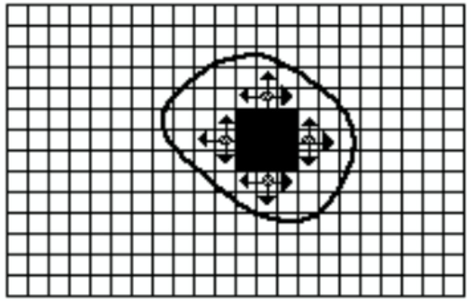




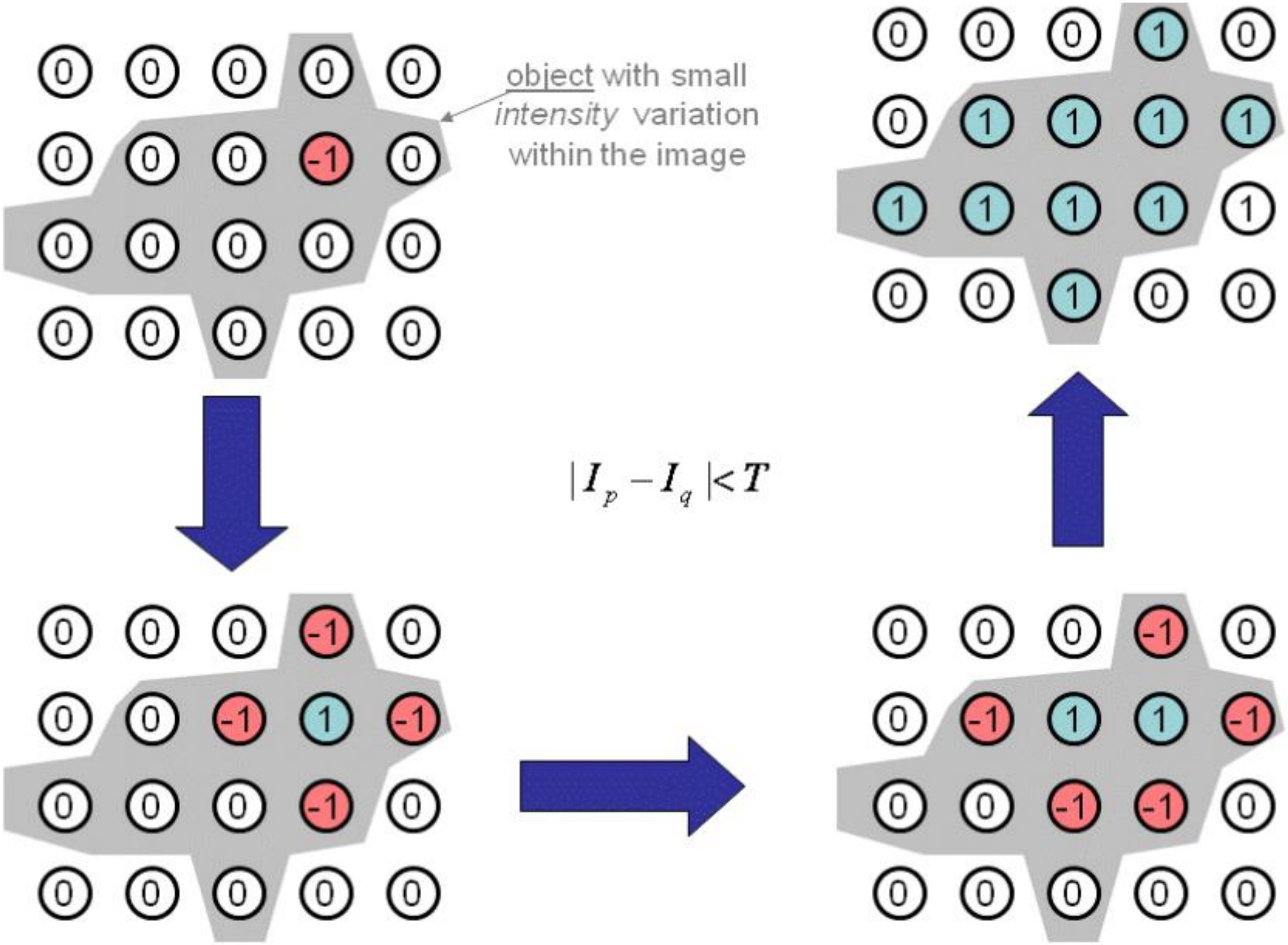
after few iterations

Region growing

- Partition and grow

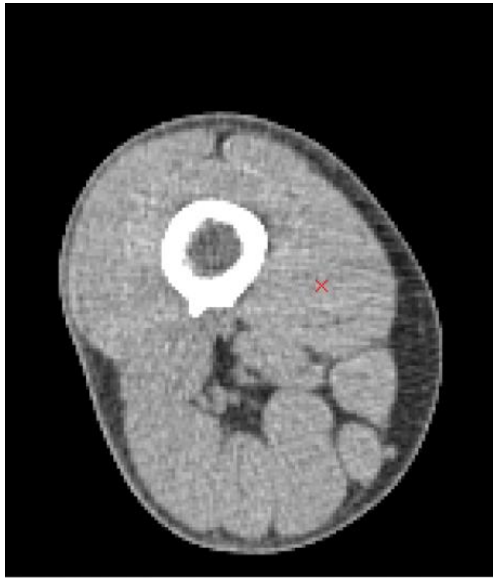
- Start with a seed point s_j for region R_j
- For every s_j
  - Initialize mean intensity of each region: $\mu_j = s_j$
  - Initialize region: $R_j = \{s_j\}$
- For each point p in R_j
  - Get its 4-connect neighborhood: $\mathcal{N}_i(p),\ i = 1, 2, 3, 4$
  - If $|\mathcal{N}_i(p) - \mu_j| < \tau$, $\mathcal{N}_i(p) \notin \mathcal{R}_k \quad j \neq k$
    - $\mathcal{R}_j \leftarrow \mathcal{R}_j \bigcup \mathcal{N}_i(p)$
    - update μ_j
  - Stop growing when no neighborhood pixel matches
- Move to the next seed point, until the whole image is partitioned.

Region growing



Region growing: CT scan

- Seed-1



stricter similarity

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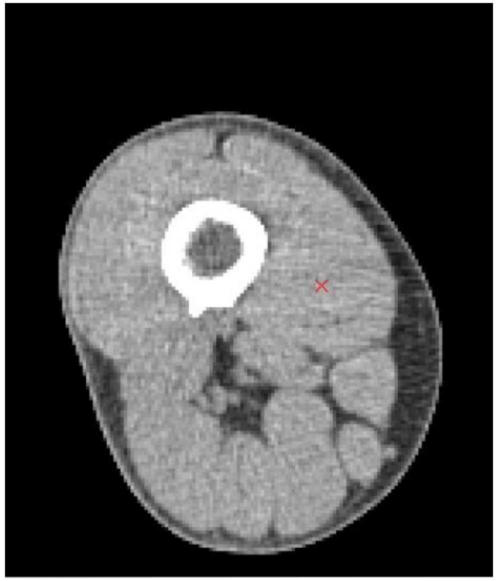

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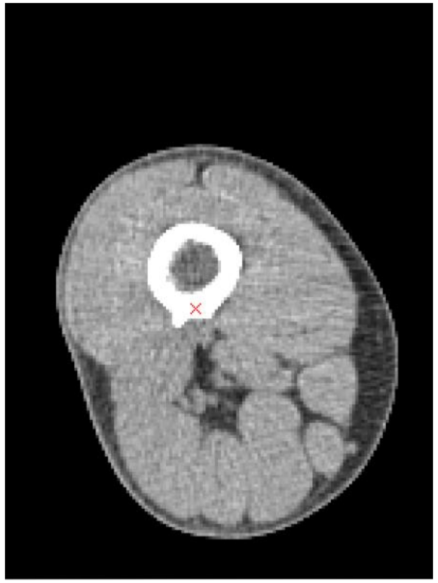
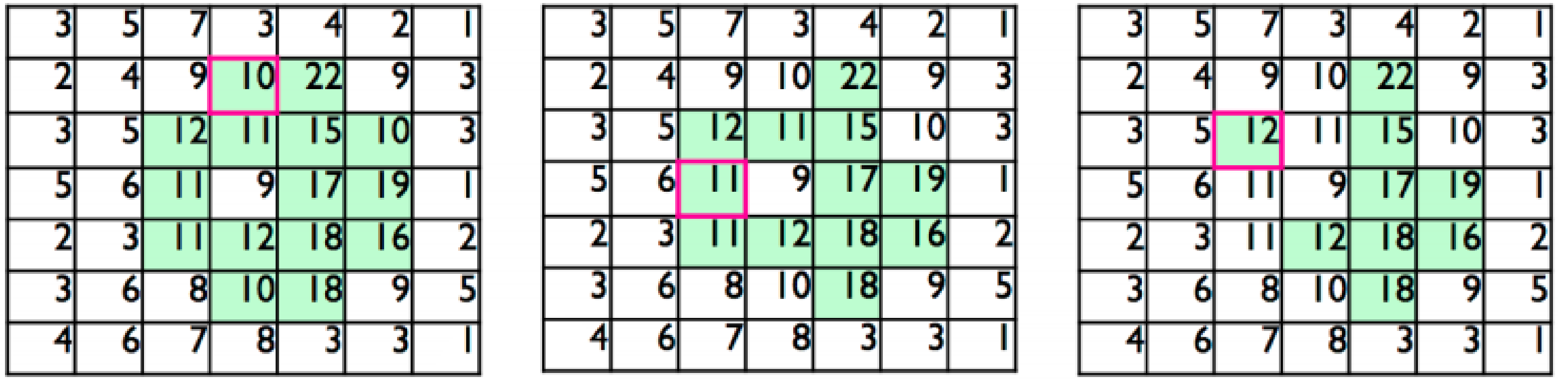
- Seed-2

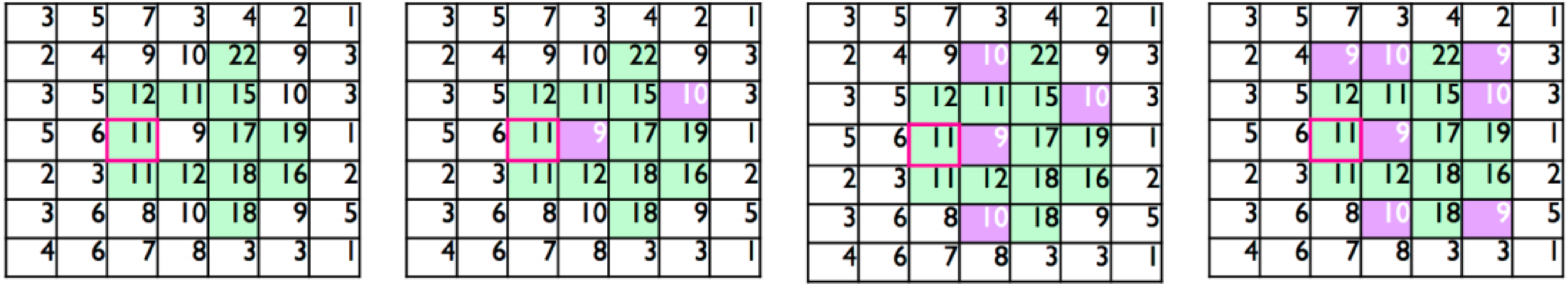


Comparative example

3	5	7	3	4	2	1
2	4	9	10	22	9	3
3	5	12	11	15	10	3
5	6	11	9	17	19	1
2	3	11	12	18	16	2
3	6	8	10	18	9	5
4	6	7	8	3	3	1



threshold $T \geq 10$ threshold $T \geq 11$ threshold $T \geq 12$



region growing with variance of 2 in respect to value 11 with reference to threshold $T \geq 11$

Region splitting & merging

- Split
  - sub-quadrants
    - e.g. 4 parts: quadregions
    - quadtree (having leaves as quadregions or quadimages)
  - continuous splitting
    - adjacent quadimages will be having identical properties

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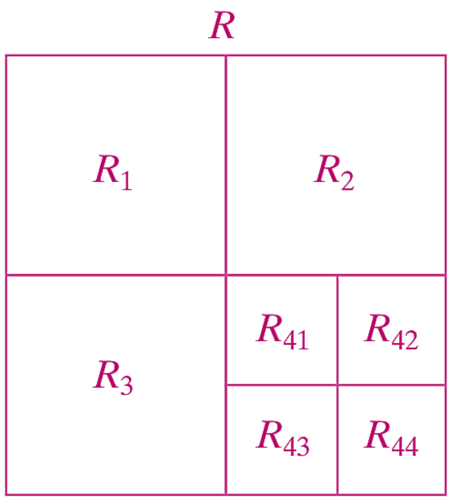

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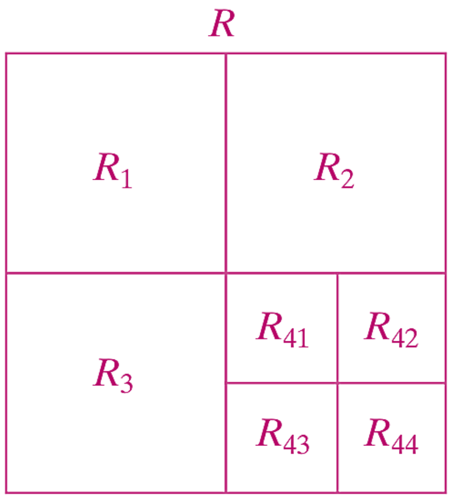

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    - merging begins when no further splitting is possible

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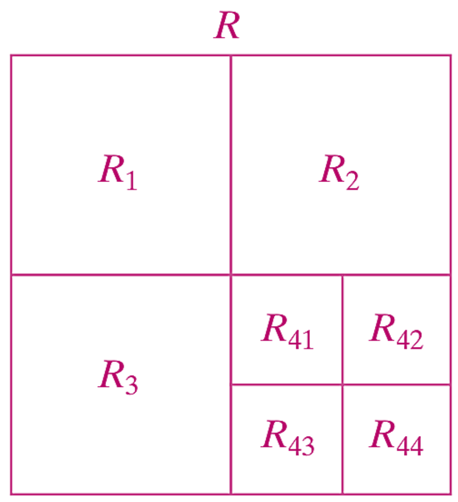

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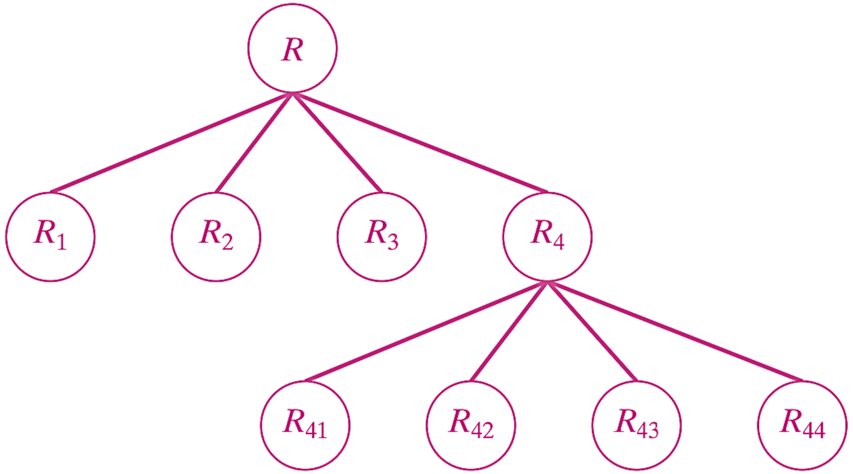
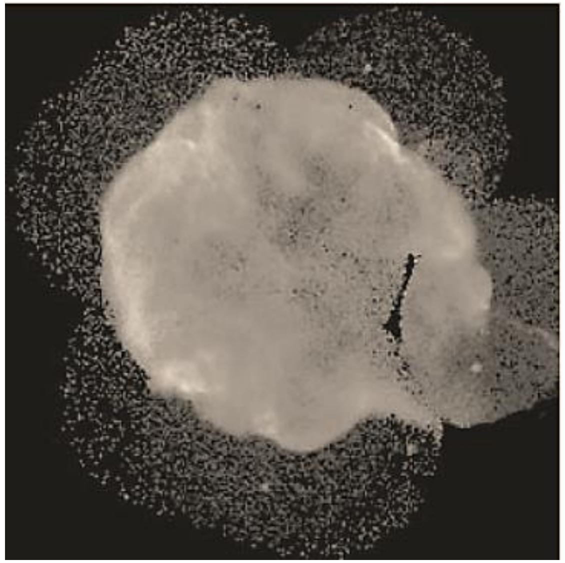


Region splitting & merging

- Segment the ring of supernova
  - quadimages size 32x32, 16x16 & 8x8
  - variance and mean of quadimages can be used as merging criterion

X-ray band image



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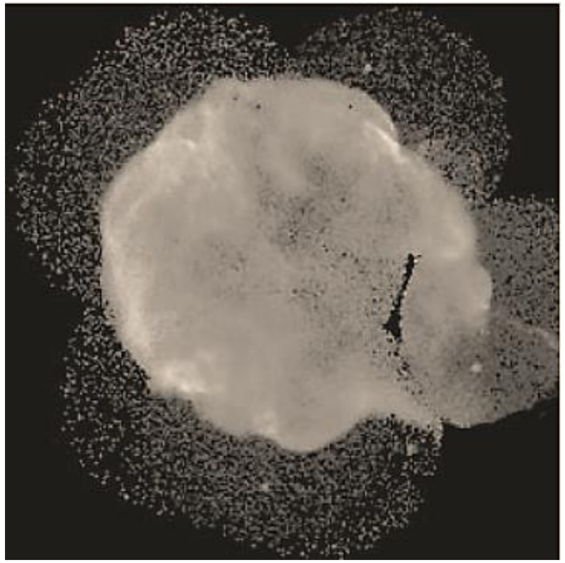

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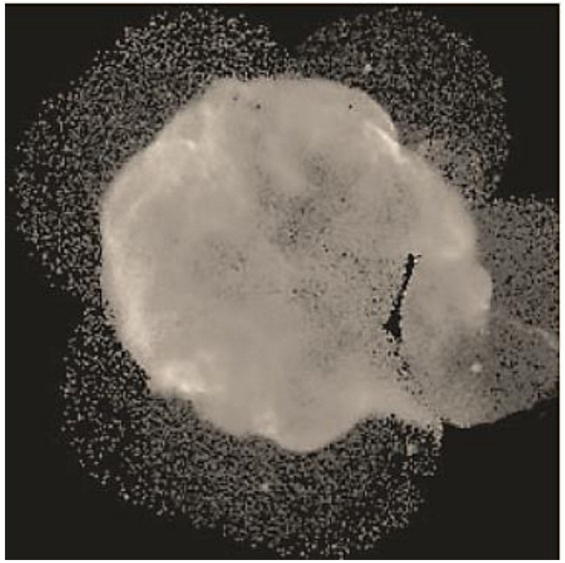

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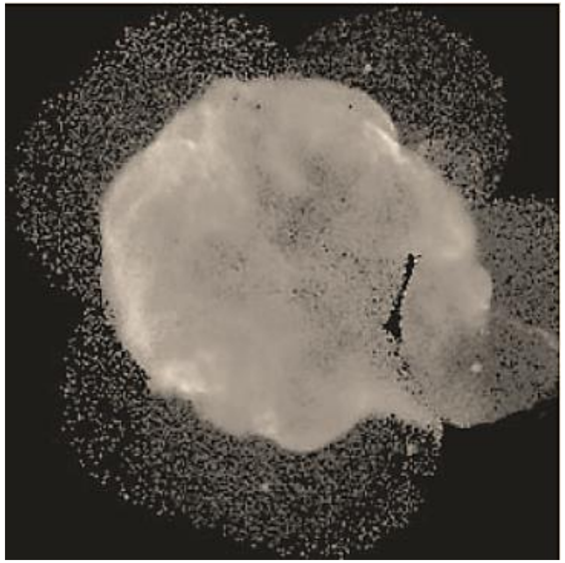

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Clustering

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- Organizing data into multiple (#clusters) classes s.t.:
  - intra-class variance is low (high similarity)
  - inter-class variance is high (low similarity)
- Unsupervised learning paradigm
  - finding class labels directly from data
  - training data labels are not available
- What are similarity measures:
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    - e.g. euclidian, cosine
  - density
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K-Means

- K-means clustering
  - unsupervised learning method: requires data but not labels
  - useful for pattern recognition, when we don't know what to look for
  - detects united patterns e.g. groups of text topics, regions of images
  - pros: simple iterative
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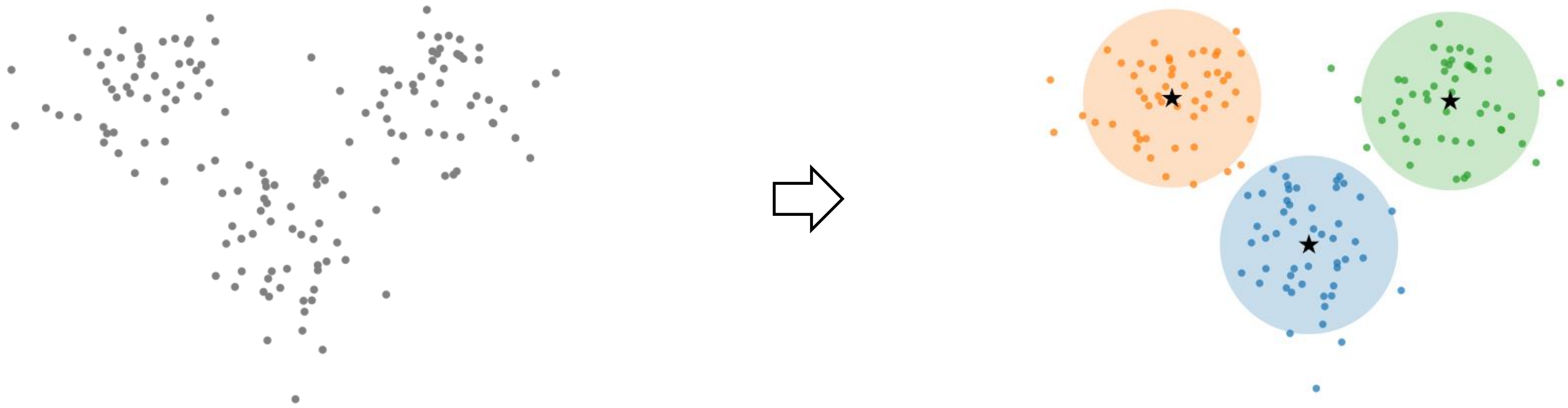

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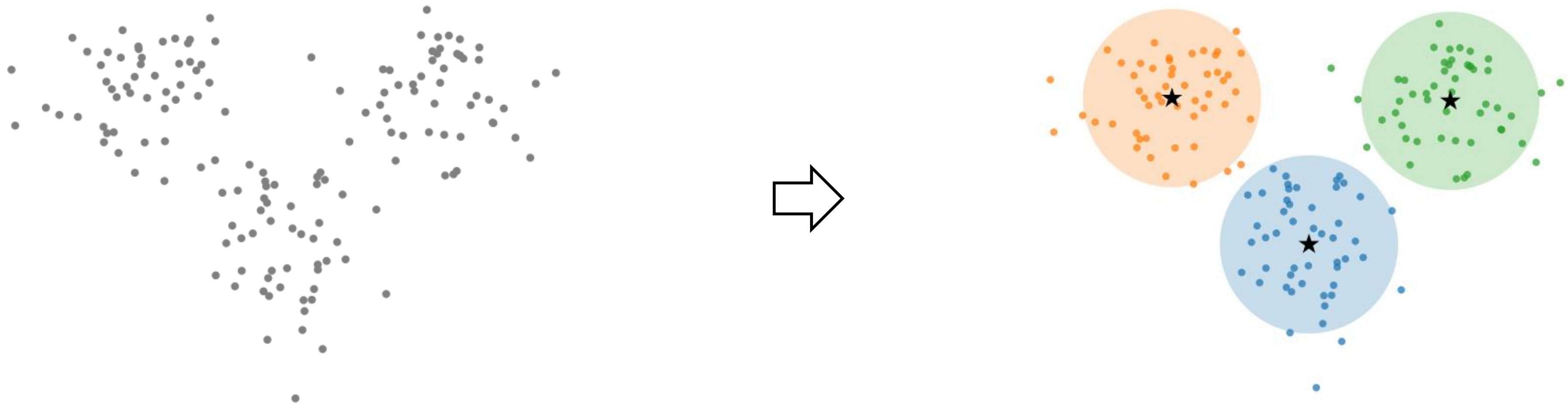


K-Means

- **Input:** $x^{(1)}, x^{(2)}, ..., x^{(n)}$
- **Output:** Set of clusters $C_1, C_2, ...C_k$
- **Initialization:** Randomly pick ***k*** centroids $z^{(1)}, z^{(2)}, \ldots, z^{(k)}$
- **Itereate** until convergence or up to iterations T
  - **Assignment:** Assign each point to its closest centroid

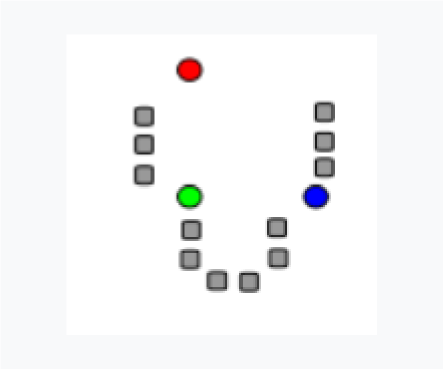
    for each $j = 1, .., k$

    $C_j = \{i | \text{s.t. } x^{(i)} \text{ is closest to } z^{(j)}\}$

  - **Update:** Recompute centroids with newly assigned points

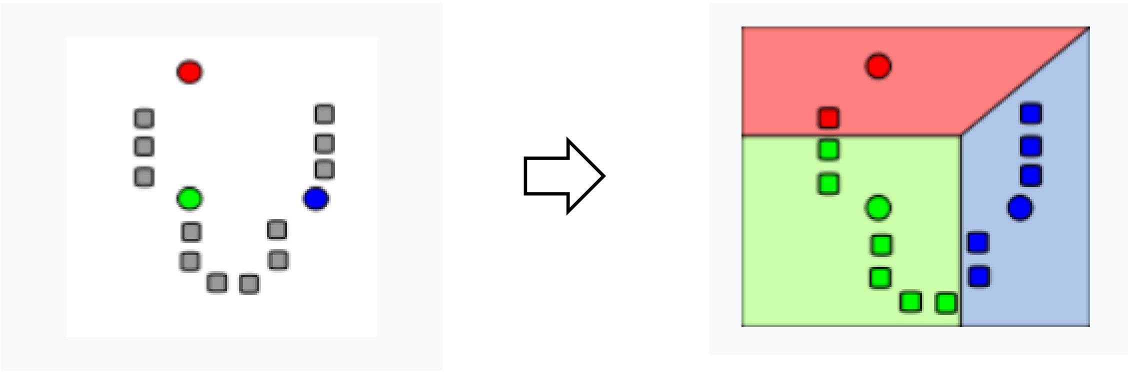
$$z^{(j)} = \frac{1}{|C_j|} \sum_{i \in C_j} x^{(i)}$$

K-Means



Courtesy: wiki

K-Means



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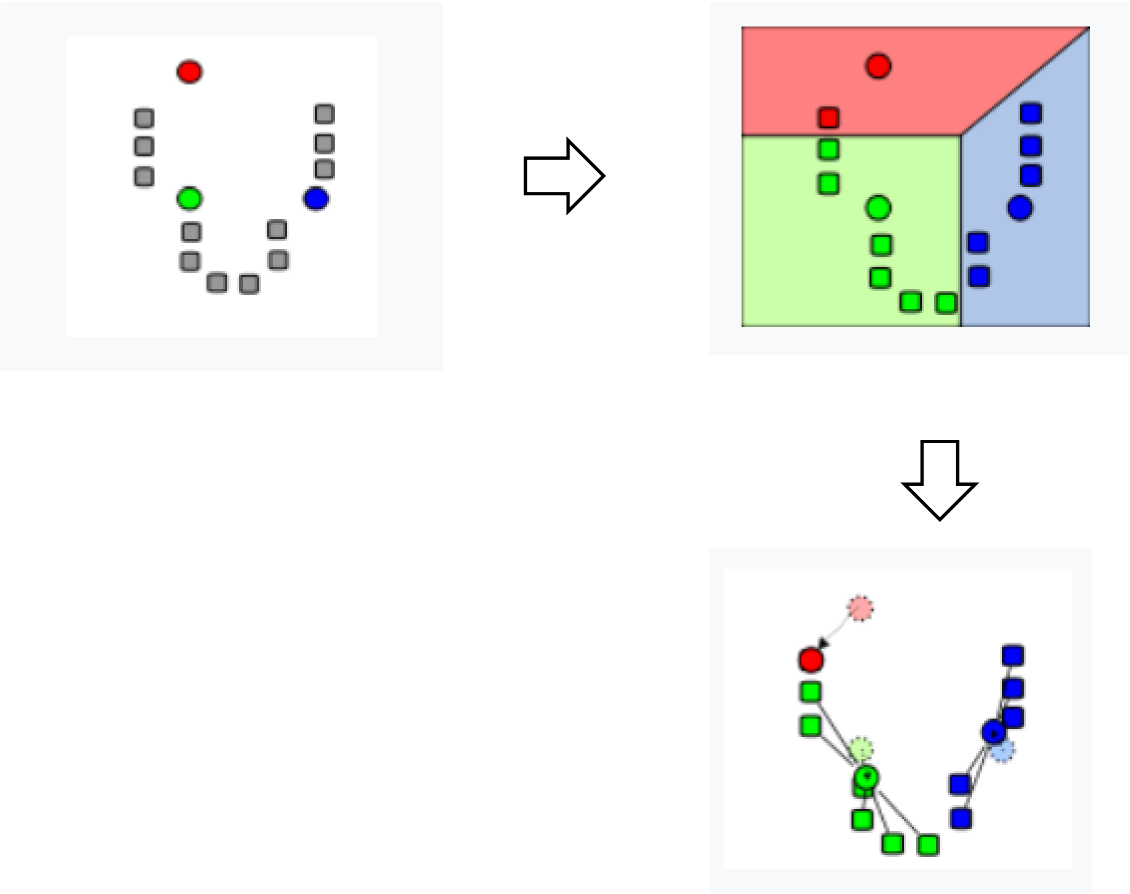

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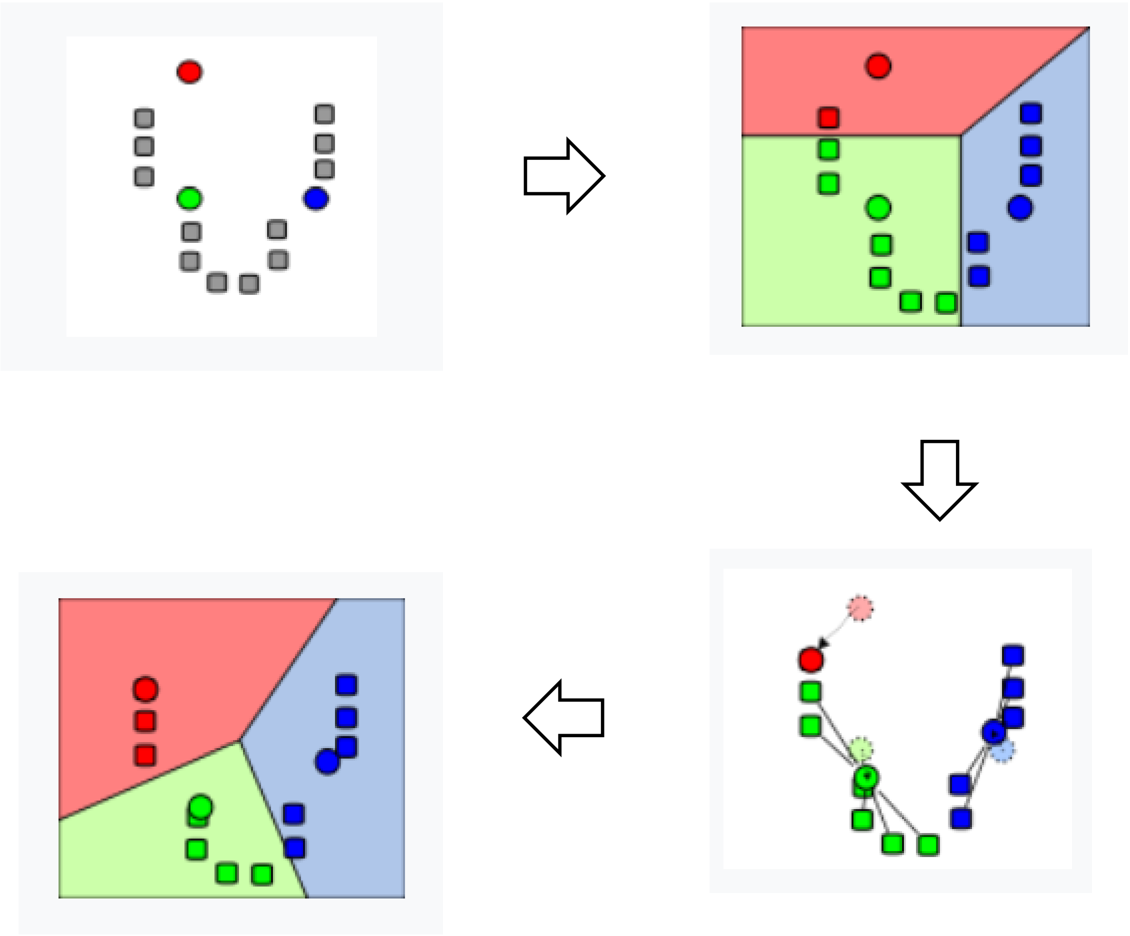

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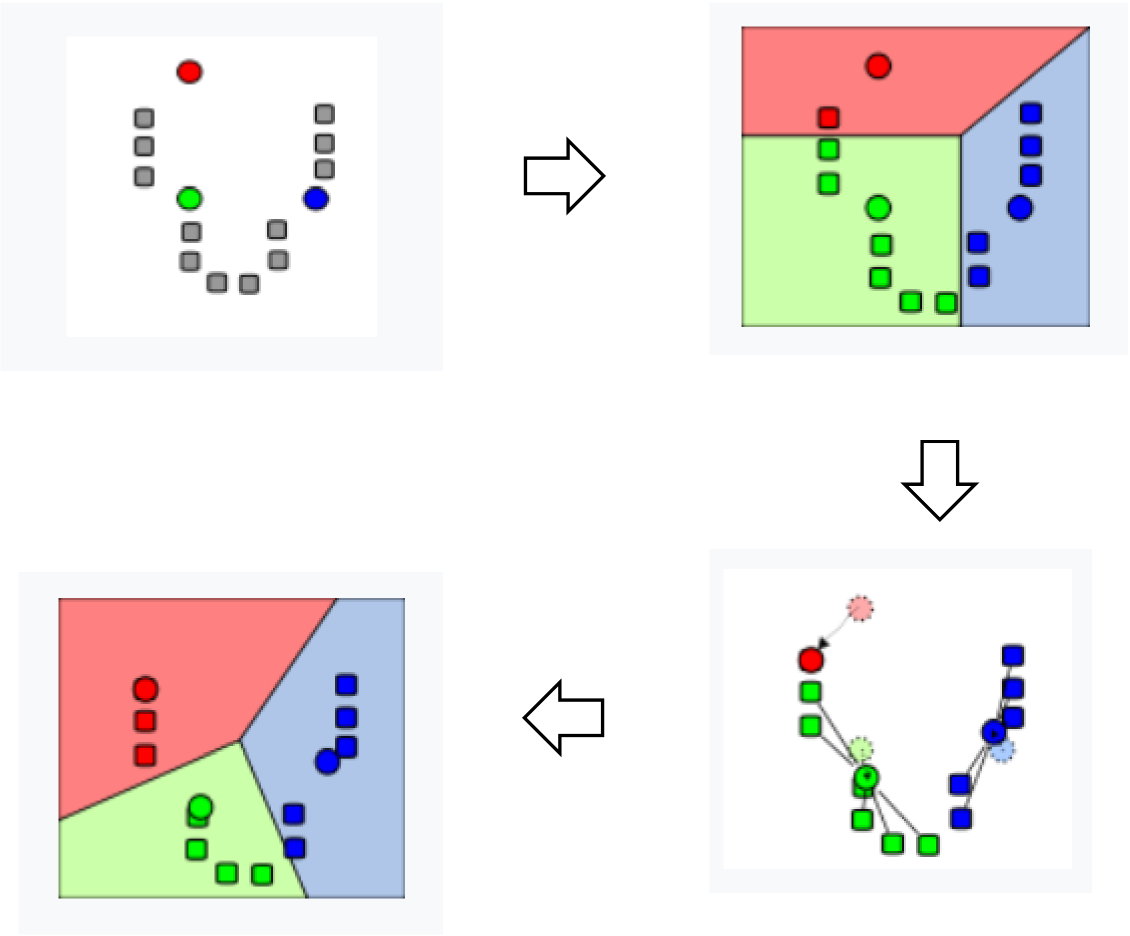

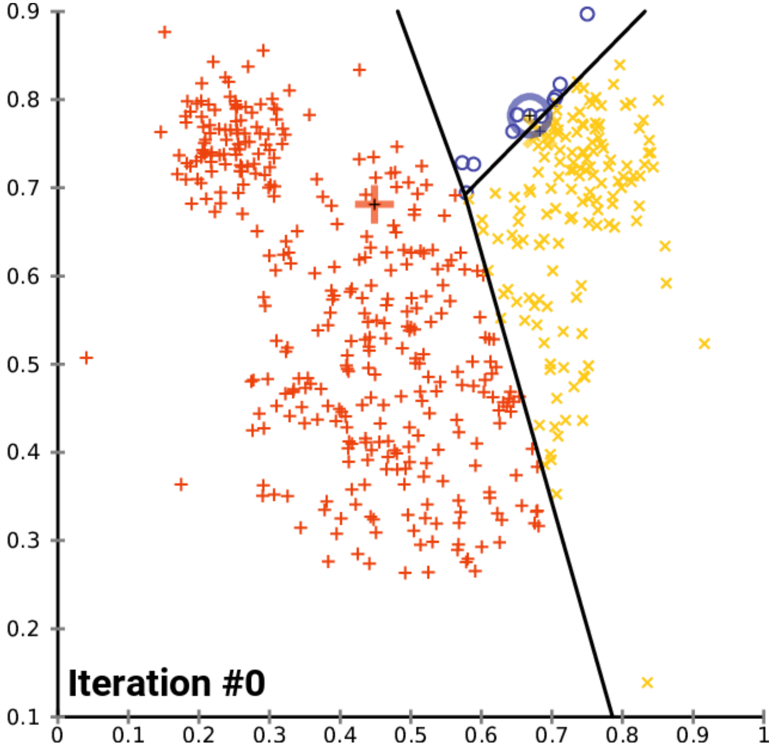

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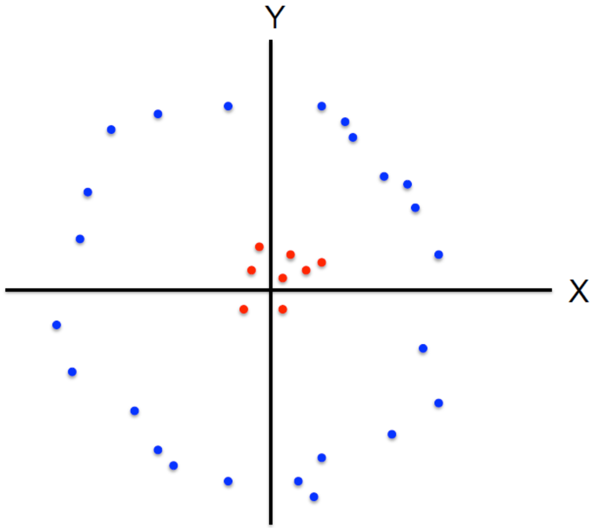

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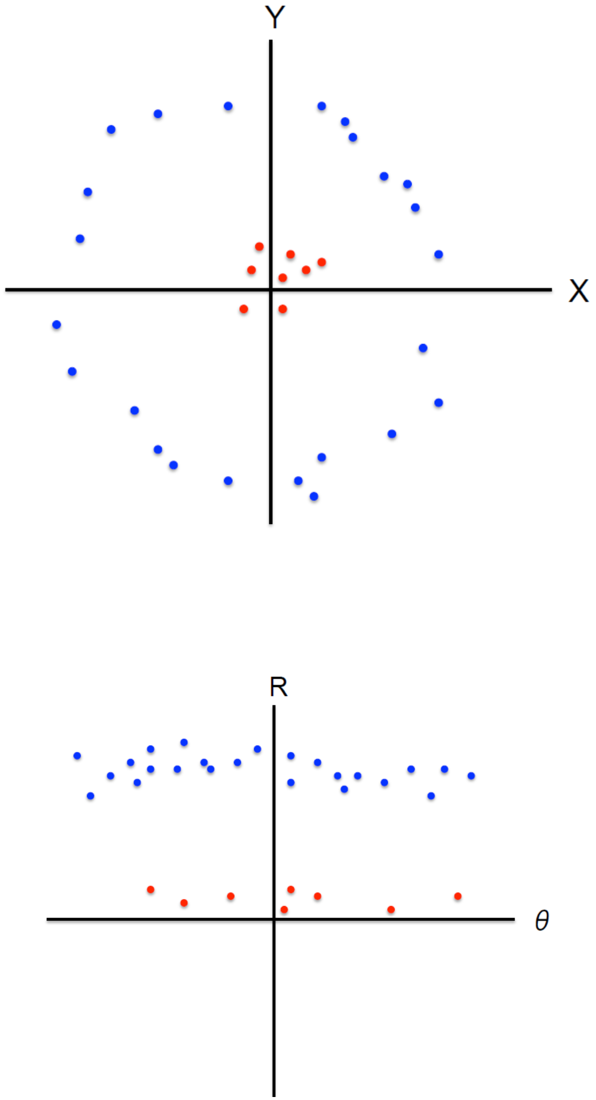

K-Means

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  - update: fix C, optimize for z

$$J(z) = \min_{z^{(1)},\ldots,z^{(k)}} \sum_{j=1}^{k} \sum_{i \in C_j} \|x^{(i)} - z^{(j)}\|^2$$

$$z^{(j)} = \frac{1}{|C_j|} \sum_{i \in C_j} x^{(i)}$$

$$\frac{\delta J(z)}{\delta z^{(j)}} \rightarrow 0$$

K-Means

input

Courtesy: D. Sontag

K-Means

input

$K = 2$

Courtesy: D. Sontag

K-Means

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K = 2

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Courtesy: D. Sontag

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K = 3

K = 10

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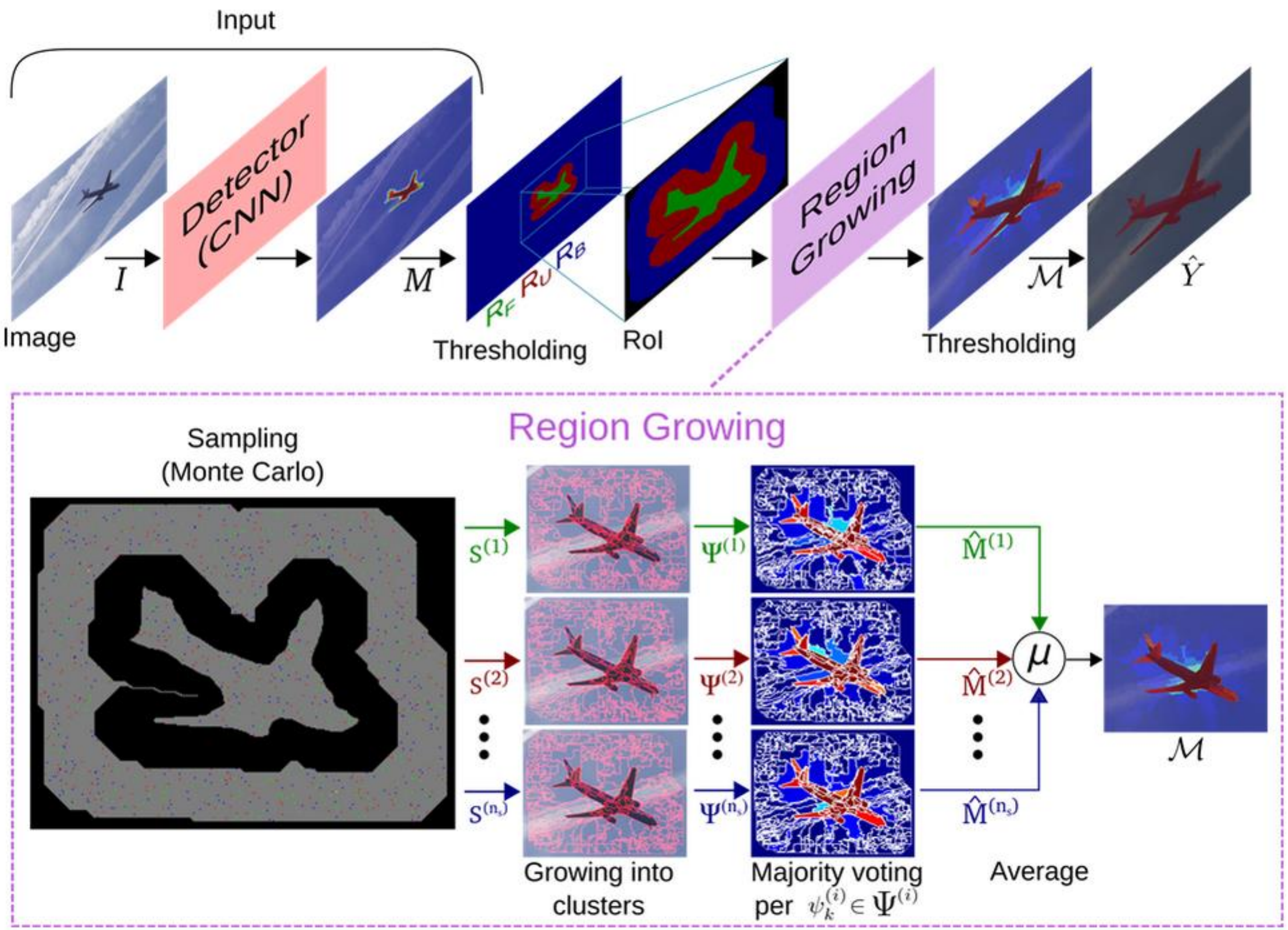
Region growing in feature space

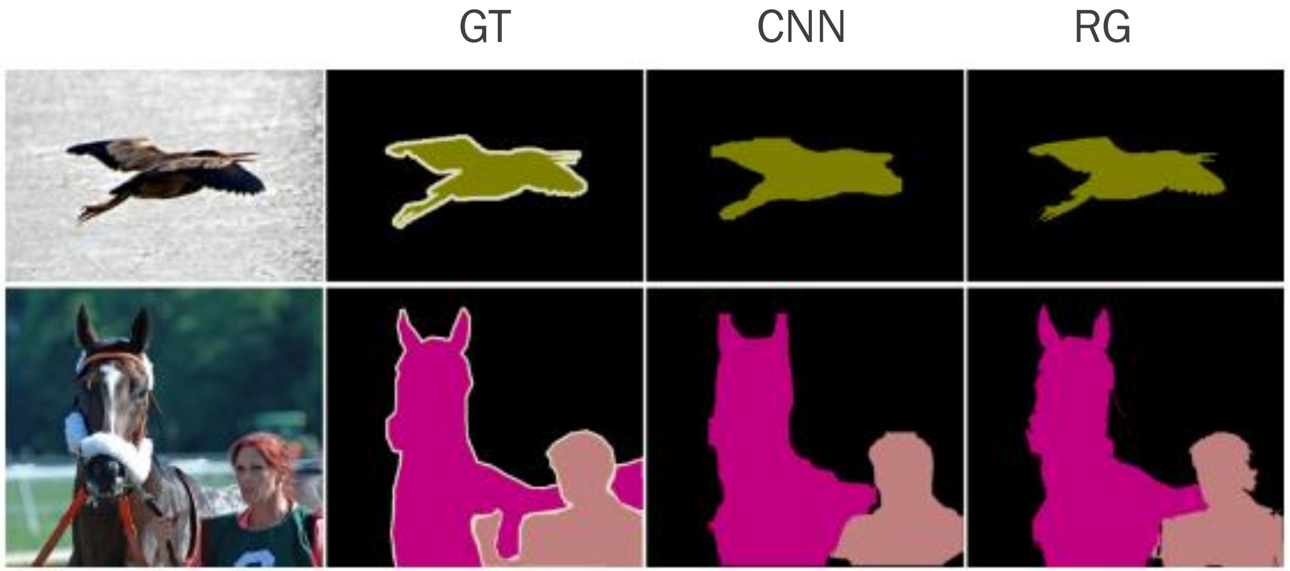


$$R_F = \{p_j | M(p_j) > \tau_F\},$$
$$R_U = \{p_j | \tau_B < M(p_j) < \tau_F\},$$
$$R_B = \{p_j | M(p_j) < \tau_B\},$$

Image credit: PA Dias

Conclusion

- Regions
- Clustering



Conclusion

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- Clustering

- ❑ Region growing
- ❑ Region splitting & merging
- ❑ Clustering
  - K-means clustering