E1 213 Pattern Recognition and Neural Networks

Practice Problems: Set 4

1. Given two sets of points in $\Re^d$, show that they are linearly separable if and only if their convex hulls do not intersect.

2. Consider the following 'guess-the-number' game. The teacher picks some number, c^*, from $[0,\ 1]$. The learner is given a set of examples of the form $\{(x_i, y_i), i = 1, \cdots, n\}$ where $x_i \in [0,\ 1]$ and $y_i = 1$ if $x_i \leq c^*$ and $y_i = 0$ if $x_i > c^*$. Suggest a PAC learning algorithm for this problem. First formulate the problem in the PAC framework by specifying what are $\mathcal{X}, \mathcal{Y}$ and $\mathcal{C}$ here. Then specify an algorithm which, for any n, takes n examples as input and outputs a number in $[0,\ 1]$. Then show that this is a PAC learning algorithm.

3. Let $\mathcal{X} = \Re^2$, $\mathcal{Y} = \{0,\ 1\}$. Let c^r denote the circular disc of radius r with center at origin. That is, $c^r = \{(x, y) \in \Re^2 \ : \ x^2 + y^2 \leq r^2\}$. Let $\mathcal{C}$ be family of all such concentric circles with centre at origin. Show that this concept class is PAC learnable.

4. A monomial over Boolean variables is a conjunction of literals. A literal is a variable or its compliment. For example, $x_1,\ x_2x_3,\ \bar{x}_1x_2x_3$ are all monomials over three Boolean variables. Here, $\bar{x}_1$ denotes the literal which is complement of x_1. Consider a 2-class problem with n Boolean features. Suppose we know that all patterns can be correctly classified by some monomial. (That is, the correct monomial would have value 1 on all feature vectors of C_0 and would have value 0 on all feature vectors from C_1). We want to learn the monomial given some examples. Consider a learning algorithm for this as given below. We start with the monomial $x_1\bar{x}_1x_2\bar{x}_2 \cdots x_n\bar{x}_n$. (Note that this monomial classifies all patterns as C_1). The algorithm is an incremental algorithm processing one example in each iteration. At each iteration we modify the current monomial as follows. If the next example is from C_1 we do nothing. If the next example is from C_0, then, for each $i, 1 \leq i \leq n$, if the example has value 1 for i^{th} feature, then we delete the literal $\bar{x}_i$ (if present) from the current monomial; if the example has value 0 for i^{th} feature, then we delete the literal x_i (if present) from the current monomial. For $n = 3$, take some example target monomial and show how the algorithm works if you take each of the 8 possible feature vectors one

by one. Next, show that this is a PAC-learning algorithm. That is, show that given any ϵ and δ, we can find n such that after n random examples, the probability that the error of the classifier learnt by the algorithm is greater than ϵ is less than δ.