E1 213 Pattern Recognition and Neural Networks

Practice Problems: Set 2

1. Let $x_1, \cdots, x_n$ be *iid* data drawn according to exponential density with parameter λ. Derive the ML estimate for λ. (The exponential density is given by $f(x) = \lambda e^{-\lambda x},\ x > 0$).

2. Suppose X is uniformly distributed over $[0,\ \theta]$, with $\theta > 0$ being the unknown parameter. (The uniform density is given by $f(x) = 1/\theta$, if $0 \leq x \leq \theta$ and $f(x) = 0$ otherwise). Suppose we have three *iid* samples, $1.75, 0.5, 2.2$. What is the value of the likelihood function $L(\theta|\mathcal{D})$ for (i). $\theta = 10$, (ii). $\theta = 1.9$? Now consider the general case where we represent the three *iid* samples as x_1, x_2, x_3. Plot the likelihood function (that is, plot $L(\theta|\mathcal{D})$ versus θ). Now, consider the case where we have n *iid* samples, what is the ML estimate for θ.

3. Suppose you have n samples from a normal density with mean μ and variance 1. You estimated the mean using the sample mean. Then you discover that your friend had m samples from the same density and has estimated the mean using sample mean. How should you combine your estimates to get a better estimate.

4. We know that sample mean is an unbiased estimater of the expectation (or population mean). We also know that the variance of the sample mean estimater goes down as $1/n$ where n is the number of samples. Suppose we replicate each sample thus doubling the sample size. Will the variance (and hence the mean square error) decrease by half? Explain.