E1 213 Pattern Recognition and Neural Networks

Practice Problems: Set 1

1. Consider a 2-class problem with one dimensional feature space. Let the class conditional densities be: $f_0(x) = e^{-x},\ x > 0$, and $f_1(x) = 1/2a,\ x \in [-a,\ a],\ a > 0$. The prior probabilities are equal. Assume we are using 0–1 loss. Find the Bayes classifier. For the case when $a = 0.25$, find Bayes error.

2. Consider a 2-class PR problem with feature vectors in $\Re^2$. The class conditional density for class-I is uniform over $[1, 3] \times [1, 3]$ and that for class-II is uniform over $[2, 4] \times [2, 4]$. Suppose the prior probabilities are equal and we are using 0–1 loss. Consider line given by $x + y = 5$ in $\Re^2$. Is this a Bayes Classifier for this problem? Is Bayes Classifier unique for this problem? If not, can you specify two different Bayes classifiers? Suppose the class conditional densities are changed so that the density for class-I is still uniform over $[1, 3] \times [1, 3]$ but that for class-II is uniform over $[2, 5] \times [2, 5]$. Is the line $x + y = 5$ a Bayes classifier now? If not, specify a Bayes classifier now. Is the Bayes classifier unique now? For this case of class conditional densities, suppose that wrongly classifying a pattern into class-I is 10 times more expensive than wrongly classifying a pattern into class-II. Now, what would be a Bayes classifier?

3. Consider a 2-class problem with one dimensional feature vector and class conditional densities being normal. Specify a simple special case where Bayes classifier, Min-max classifier and Neyman-Pearson classifier would all be the same.