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Paper-1

Convex MPC :-

* Abstract :- MPC $\longrightarrow$ to determine GRFs $\longrightarrow$ torque controlled quadruped robot.

simplified robot dynamics $\rightarrow$ to convert/formulate problem as convex optimization.

what does this mean?

$\Rightarrow$ With the simplified model, GRF planning problems are formulated for prediction horizons of up to 0.5 seconds, and are solved to optimality in under 1ms at a rate of 20-30 Hz.

* Swing leg controller :-

$$\tau_i = J_i^T \left[K_p ({}_B p_{i,ref} - {}_B p_i) + K_d ({}_B v_{i,ref} - {}_B v_i) \right] + \tau_{i,ff}$$

* $J_i \in \mathbb{R}^{3\times3}$; foot Jacobian
* $K_p, K_d \in \mathbb{R}^{3\times3}$; diag. +ve definit proportional & derivative gain matrices.
* ${}_B p_i, {}_B v_i \in \mathbb{R}^3$; foot position & velocity in body frame. (of ith foot).
* ${}_B p_{i,ref}, {}_B v_{i,ref} \in \mathbb{R}^3$; ref. for pos & vel.
* $\tau_{i,ff} \in \mathbb{R}^3$ is a feedforward torque

$$\tau_{i,ff} = J^T \Lambda_i ({}_B a_{i,ref} - \dot{J}_i \dot{q}_i) + C_i \dot{q}_i + G_i$$

i??

$\Lambda_i \in \mathbb{R}^{3\times3}$ space inertia matrix.

Manipulator Dynamics ??

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★ Desired foot location calculation :-

$$p^{des} = p^{ref} + v^{com} \Delta t/2.$$

Δt - time the foot will spend on the ground.

p^{ref} - location on the ground beneath the hip of the robot.

↳ world frame

★ Ground force control :-

$$\tau_i = J_i^T R_i^T f_i$$

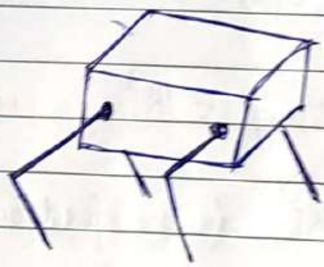
R is the rotation mat. which transforms from body to world coordinates

$J \in \mathbb{R}^{3 \times 3}$ is foot Jacobian.

f_i is the vector of forces calc. from MPC.

★ Simplified Robot Dynamics :-

* SRB model :-



$$\Rightarrow \ddot{p} = \frac{\sum_{i=1}^{n} f_i}{m} - g$$

$$\Rightarrow \frac{d}{dt}(I\omega) = \sum_{i=1}^{n} (r_i \times f_i)$$

$$\Rightarrow \dot{R} = \hat{\omega} R$$

} Non-linear.

For every GRF; $f_i \in \mathbb{R}^3$, the vector from the centre of mass to

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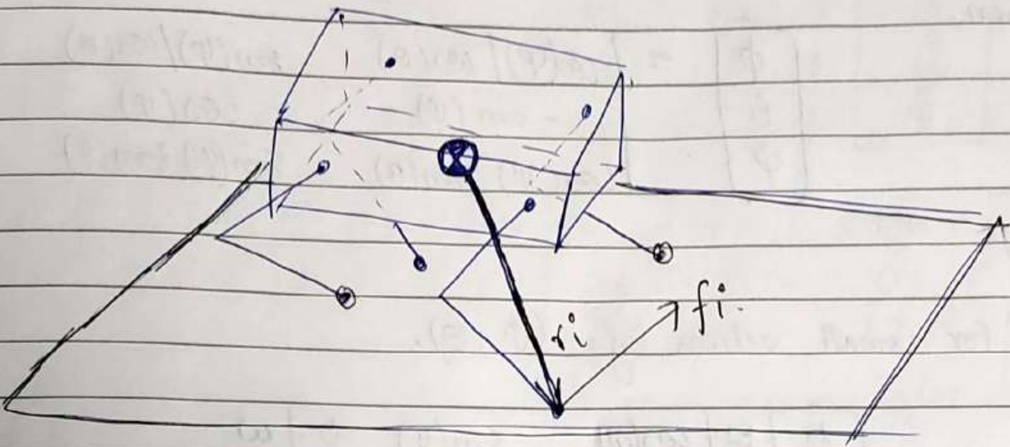
to the pt. where the force is applied is $r_i \in \mathbb{R}^3$.

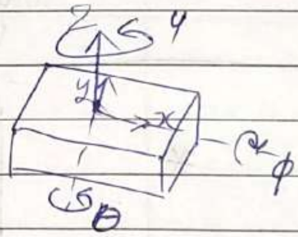


where $p \in \mathbb{R}^3$ is robot's position.
m is robot's mass.
g is acc. of gravity.
$I \in \mathbb{R}^3$; is robot's inertia tensor.
$\omega \in \mathbb{R}^3$; is the robot's angular velocity.
R , is the rotation matrix; Body $\rightarrow$ World.

★ Approximated angular velocity dynamics :-

Robot's orientation is expressed as a vector of Z-Y-X Euler angles $\Theta = [\phi \ \theta \ \psi]^T$

ϕ - roll
θ - pitch
ψ - Yaw



$$R = R_z(\psi) R_y(\theta) R_x(\phi)$$

* Angular velocity in world coordinate frame.

$$\omega = \begin{bmatrix} \cos(\theta)\cdot\cos(\psi) & -\sin(\psi) & 0 \\ \cos(\theta)\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

* If robot is not pointed vertically (ie $\cos(\theta) \neq 0$

then.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos(\psi)/\cos(\theta) & \sin(\psi)/\cos(\theta) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ \cos(\psi)\tan(\theta) & \sin(\psi)\tan(\theta) & 1 \end{bmatrix} \omega$$

* For small values of (ϕ, θ).

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \approx \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \omega$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \approx R_z(\psi) \cdot \omega$$

Note:- Order of Euler angle rotations are imp.
* For alternate seq of rot. approx. will be inaccurate.

Why??

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* $$\boxed{\frac{d}{dt}(I\omega) = I\dot{\omega} + \omega \times (I\omega) \approx I\dot{\omega}}$$

discards the effects of precession and nutation of the rotating body.

The inertia tensor in World Coordinate frame is given by :-

$$I = R_B I R^T$$

$$\Rightarrow \boxed{\hat{I} = [R_z(\psi)][{}_B I][R_z(\psi)^T]}$$

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* Simplified Robot Dynamics:-

$$\frac{d}{dt}\begin{bmatrix}\hat{\theta}\\ \hat{p}\\ \hat{\omega}\\ \hat{\dot{p}}\end{bmatrix} = \begin{bmatrix}0_3 & 0_3 & R_z(\psi) & 0_3\\ 0_3 & 0_3 & 0_3 & I_3\\ 0_3 & 0_3 & 0_3 & 0_3\\ 0_3 & 0_3 & 0_3 & 0_3\end{bmatrix}\begin{bmatrix}\hat{\theta}\\ \hat{p}\\ \hat{\omega}\\ \hat{\dot{p}}\end{bmatrix}$$

$$+\begin{bmatrix}0_3 & \cdots & 0_3\\ 0_3 & \cdots & 0_3\\ \hat{I}^{-1}[\hat{r}_1] & \cdots & \hat{I}\hat{r}_n\\ I_3/m & \cdots & I_3/m\end{bmatrix}\begin{bmatrix}f_1\\ f_2\\ \vdots\\ f_n\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ g\end{bmatrix}$$

above eq^n can be re written with "g" as additional state to get state-space form. — ??

$$\dot{x}(t) = A_c(\psi)\,x(t) + B_c(r_1, \ldots r_n, \psi)\,u(t)$$

$$A_c \in \mathbb{R}^{13\times 13}, \quad B_c \in \mathbb{R}^{13\times 3n}.$$

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