MATH 3283W Written Project

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Definition of Mathematical Induction.

Let $P(n)$ be a statement for all $n \in \mathbb{N}$. $P(n)$ is true if $P(1)$ is true, and $P(k)$ implies $P(k+1)$ for each $k \in \mathbb{N}$. We will use induction to prove the statement below.

Section 3.4 # 4.

Prove that $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$, for all $n \in \mathbb{N}$.

Let $P(n)$ be the statement $1^3 + 1^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$, for all $n \in \mathbb{N}$.

First, we will verify that $P(1)$ is true. On the right hand side, $P(1) = 1^3 = 1$. On the left hand side, $P(1) = \frac{1^2 * (1+1)^2}{4} = \frac{1*4}{4} = 1$. Thus, $P(1)$ is true.

Now, we will show that $P(k+1)$ follows from $P(k)$. Suppose $P(k)$ is true for all $k = n \in \mathbb{N}$. We have the following equalities:

$$\begin{aligned}
&= \frac{k^2(k+1)^2}{4} + (k+1)^3 \\
&= \frac{k^2(k+1)^2 + 4(k+1)^3}{4} \\
&= \frac{k^4 + 6k^3 + 13k^2 + 4}{4} \\
&= \frac{(k+1)^2(k+2)^2}{4} \\
&= \frac{(k+1)^2(k+1+1)^2}{4}.
\end{aligned}$$

Since $P(1)$ is true, and $P(k)$ implies $P(k+1)$, $P(n)$ is true for all natural numbers by mathematical induction.

Discussion.

It is easy to see that (3.4 #5) follows from this proof by the following equalities:

$$1^3 + 1^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} = \left(\frac{n(n+1)}{2}\right)^2 = (1^2 + 2^2 + \cdots + n^2)^2.$$

We can also see that the sum of n^k natural numbers always has a summation formula with a leading term of the form n^{k+1}. For example, the summation formula for naturals has a leading n^2 term, the summation for squares has a leading n^3 term, and the summation for cubes has a leading n^4 term. See the formulas below:

$$\begin{aligned} 1 + 2 + \cdots + n &= \frac{(n+1)(n+2)}{2} \\ 1^2 + 2^2 + \cdots + n^2 &= \frac{n(n+1)(2n+1)}{6} \\ 1^3 + 2^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4}. \end{aligned}$$