INTERMODAL FIXED RADIUS DENSITY DEPENDENT COVERING SALESMAN PROBLEM IN UNCERTAIN ENVIRONMENT

Thesis is submitted for the degree of

Master of Technology

in

Operations Research

by

CHHATRA PAL SINGH

Roll no.: 22MA4105

Reg. No.: 22P10178

Under the Supervision of

DR. GOUTAM PANIGRAHI

(Assistant professor)

Department of Mathematics

NATIONAL INSTITUTE OF TECHNOLOGY

DURGAPUR, WB – 713209

May 15, 2024.

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this report entitled "**Intermodal fixed radius density dependent covering salesman problem in uncertain environment**" in fulfilment of the requirements for the award of the MASTER OF TECHNOLOGY in OPERATIONS RESEARCH and submitted in the Department of Mathematics, National Institute of Technology, Durgapur is an authentic record of my own work carried by me during July 2023 – November 2023, under the guidance of DR. GOUTAM PANIGRAHI, Assistant Professor Department of Mathematics, NIT Durgapur.

The matter presented in this report has not been submitted by me for the award of any other degree of this or any other Institute.

(CHHATRA PAL SINGH)

(22MA4105)

Department of Mathematics

NIT DURGAPUR, WB -713209

Date: **Place:** NIT, Durgapur

CERTIFICATE OF RECOMMENDATION

This is to certify that the report entitled "**Intermodal fixed radius density dependent covering salesman problem in uncertain environment**" submitted by M.Tech Student **CHHATRA PAL SINGH (22MA4105)** as per the requirements of Master of Technology program of National Institute of Technology, Durgapur is a record of bonafied work carried out by him under my supervision.

Dr. Goutam Panigrahi

(Assistant professor)

Department of Mathematics

National Institute of Technology, Durgapur - 713209

Date:

Place: NIT, Durgapur

CERTIFICATE OF APPROVAL

This is to certify that we have examined the thesis entitled "**Intermodal fixed radius density dependent covering salesman problem in uncertain environment"** and hereby accord our approval of it as a study carried out and presented in a manner required for its acceptance in partial fulfilment of the requirements for the award of the degree in **Master of Technology** in **Operations Research** for which it has been submitted. It is to be understood that by this approval the undersigned do not necessarily endorse or approve any statement made, opinion expressed or conclusion drawn therein but approve the thesis only for the purpose for which it is submitted.

DR. GOUTAM PANIGRAHI
(Supervisor)
NIT Durgapur

Head
Department of Mathematics
NIT DURGAPUR

Date:

Place: NIT, Durgapur

ACKNOWLEDGEMENT

I would like to express my gratitude to my supervisor, DR. GOUTAM PANIGRAHI, Assistant Professor, Department of Mathematics, for providing me with the opportunity to work under his guidance. His continuous support, mentorship, and constructive guidance have been instrumental in shaping my work. I am immensely grateful to him for his unwavering assistance and encouragement throughout the process. I am profoundly grateful to my dear family and friends, especially my parents, for their unconditional love, unwavering support, and endless encouragement. Their beliefs in my abilities and their constant motivation have been the driving force behind my academic pursuits. I am truly blessed to have them by my side.

CHHATRA PAL SINGH
(22MA4105)
Department of Mathematics
NIT DURGAPUR, WB -713209

Dedicated to my Parents

ABSTRACT

The Intermodal Fixed Radius Density Dependent Covering Salesman Problem poses a complex challenge in optimizing transportation logistics within an uncertain environment. This study addresses the critical need to develop efficient solutions for intermodal transportation systems where two modes of transportation are utilized and the density of demand points varies dynamically. In this report, we propose a novel approach that integrates concepts from covering salesman problems, intermodal transportation, and uncertainty modelling to tackle the **Intermodal Fixed Radius Density Dependent Covering Salesman Problem**. Our method considers the fixed radius constraint to ensure efficient coverage of demand points while optimizing the salesman's route across multiple transportation modes. Additionally, we incorporate uncertainty into the problem formulation, considering variations in demand, travel times, and transportation costs. We employ a hybrid optimization framework that combines genetic algorithms and optimization techniques to find near-optimal solutions under uncertainty. Through extensive experimentation and case studies, we demonstrate the effectiveness and robustness of our approach in real-world scenarios. The proposed methodology offers insights and decision support for transportation planners and logistics managers facing the challenges of intermodal transportation in uncertain environments, contributing to the advancement of transportation optimization research and practice.

LIST OF ABBREVIATIONS

Abbreviations	Description
CSP	Covering salesman Problem
TSP	Travelling Salesman Problem
GA	Genetic Algorithm
UR	Unimodal Road Transport
IR	Intermodal Road Transport
C_{UR}	Unimodal Road cost
C_{IM}	Intermodal Road cost
d_E	Euclidean Distance
d_M	Manhattan Distance
J	Jacobean matrix

TABLE OF CONTENTS

- **Total Distance calculation**
- **Concept of Break Even Distance**
- **Mathematical expression of drayage distance**
- **Concept of nodes within the Radius**
- **Calculating the cost of density nodes**
- **The Total Cost Formula**

4. Result and Discussion

5. Conclusions

6. Future Scope

7. Applications

8. References

1. INTRODUCTIONS

Covering Salesman Problem (CSP) is a generalization of Travelling Salesman Problem which was first introduced by Current and Schilling (Current, 1989). In CSP, unlike TSP, salesmen need not visit all the given nodes but a subset of nodes such that all other nodes be covered within a predetermined distance from the visited nodes. The CSP may be stated as follows: identify the minimum cost tour of a subset of n given cities such that every city not on the tour is within some predetermined covering distance standard, S, of a city that is on the tour. The CSP may be viewed as a generalization of the traveling salesman problem. A heuristic procedure for solving the CSP is presented and demonstrated with a sample problem.

In CSP, the objective is to find an optimal or near-optimal tour that minimizes the total cost or distance travelled while satisfying the covering constraint. Unlike TSP, where every city must be visited exactly once, CSP permits flexibility in node selection, focusing instead on ensuring coverage within the predetermined distance.

A heuristic approach is often employed to solve CSP due to its NP-hard nature, meaning that finding the exact optimal solution may be computationally intractable for large instances. Heuristic methods aim to provide reasonably good solutions within a reasonable amount of time by employing approximation techniques or rule-based strategies.

The essence of solving CSP lies in striking a balance between minimizing the tour cost and ensuring adequate coverage within the specified distance standard. This problem has applications in various domains such as transportation, logistics, telecommunications, and facility location, where optimizing routes while considering coverage constraints is essential for efficiency and cost-effectiveness.

Overall, CSP serves as a valuable generalization of the classic TSP, offering a more flexible framework for addressing real-world routing and coverage problems where strict node visitation requirements may not be practical.

Key Points: Covering salesman problem, genetic algorithm, C programming, travelling salesman problem, uncertainty Theory, fixed radius, intermodal, density dependent.

Objective:

The objective of studying the Covering Salesman Problem (CSP) is to develop efficient algorithms and methodologies for optimizing routing and coverage in scenarios where traditional Traveling Salesman Problem (TSP) formulations are inadequate.

Specifically, the objectives include:

Optimization of Routes: Develop algorithms to find near-optimal or optimal routes for salesmen to visit a subset of nodes while ensuring that all other nodes are covered within a predetermined distance standard.

Minimization of Costs: Minimize transportation costs associated with visiting nodes and ensuring coverage, taking into account factors such as distance travelled resource utilization, and operational expenses.

Maximization of Efficiency: Improve the efficiency of resource utilization by enabling flexible routing strategies that focus on covering critical nodes while minimizing unnecessary visits.

Practical Implementation: Develop practical solutions that can be applied in real-world scenarios across various domains such as transportation, logistics, telecommunications, and facility location.

Robustness to Constraints: Design algorithms that are robust to constraints such as time windows, capacity limits, geographic considerations, and uncertainty in demand or travel times.

Scalability: Develop scalable algorithms capable of handling large instances of the problem efficiently, considering the increasing complexity and size of real-world datasets.

Decision Support: Provide decision support for managers and planners by offering insights into optimal routing and coverage strategies, aiding in informed decision-making and resource allocation.

Advancement of Optimization Techniques: Contribute to the advancement of optimization techniques by tackling the additional complexity introduced by covering constraints, thereby addressing challenging problems in combinatorial optimization.

Overall, the objective is to address practical routing and coverage challenges by developing efficient algorithms and methodologies that balance optimization objectives, constraints, and real-world considerations, leading to improved efficiency, cost-effectiveness, and decision support capabilities.

Motivation:

The motivations for studying the Covering Salesman Problem (CSP) are rooted in addressing real-world logistics and routing challenges where traditional Traveling Salesman Problem (TSP) formulations may not suffice. Some key motivations include:

Efficiency in Resource Utilization: CSP allows for more efficient resource utilization by enabling salesmen to visit only a subset of nodes while still ensuring that all nodes are covered within a predetermined distance. This flexibility can lead to more optimal routes and reduced resource wastage.

Practicality in Real-World Scenarios: In many real-world scenarios, it may not be feasible or necessary for salesmen to visit every single node. CSP reflects the practicality of such situations where partial coverage is acceptable as long as it meets certain predefined criteria.

Cost Reduction: By optimizing routes to cover a subset of nodes within a specified distance standard, CSP can help in minimizing transportation costs. This is particularly relevant in scenarios where minimizing distance travelled or resource usage directly translates into cost savings.

Flexibility in Route Planning: CSP offers greater flexibility in route planning compared to TSP, allowing for adaptive routing strategies that can be tailored to specific requirements or constraints such as time windows, capacity limits, or geographic considerations.

Applications in Various Domains: CSP has applications across a wide range of domains including transportation, logistics, telecommunications, facility location, and urban planning. By providing a generalized framework for routing and coverage optimization, CSP addresses critical challenges in these domains and contributes to improved efficiency and performance.

Overall, the motivations for studying CSP stem from the need to address practical routing and coverage challenges in various application domains while considering factors such as efficiency, cost-effectiveness, and adaptability to real-world constraints.

2. LITERATURE REVIEW

Here we are giving some literature review for the project report.

OPTIMIZATION TECHNIQUES

Optimization, also known as mathematical programming, the collection of mathematical principles and methods used for solving quantitative problems in many disciplines, including physics, biology, engineering, economics, and business. The subject grew from a realization that quantitative problems in manifestly different disciplines have important mathematical elements in common. Because of this commonality, many problems can be formulated and solved by using the unified set of ideas and methods that make up the field of optimization. The historical term mathematical programming, broadly synonymous with optimization, was coined in the 1940s before programming became equated with computer programming. Mathematical programming includes the study of the mathematical structure of optimization problems, the invention of methods for solving these problems, the study of the mathematical properties of these methods, and the implementation of these methods on computers. Faster computers have greatly expanded the size and complexity of optimization problems that can be solved. The development of optimization techniques has paralleled advances not only in computer science but also in operations research, numerical analysis, game theory, mathematical economics, control theory, and combinatorics.

Optimization problems typically have three fundamental elements. The first is a single numerical quantity, or objective function, that is to be maximized or minimized. The objective may be the expected return on a stock portfolio, a company's production costs or profits, the time of arrival of a vehicle at a specified destination, or the vote share of a political candidate. The second element is a collection of variables, which are quantities whose values can be manipulated in order to optimize the objective. Examples include the quantities of stock to be bought or sold, the amounts of various resources to be allocated to different production activities, the route to be followed by a vehicle through a traffic network, or the policies to be advocated by a candidate. The third element of an optimization problem is a set of constraints, which are restrictions on the values that the variables can take. For instance, a manufacturing process cannot require more resources than are available, nor can it employ less than zero resources. Within this broad framework, optimization problems can have different mathematical properties. Problems in which the variables are continuous quantities (as in the resource allocation example) require a different approach from problems in which the variables are discrete or combinatorial quantities (as in the selection of a vehicle route from among a predefined set of possibilities).

COMBINATORIAL OPTIMIZATION

Combinatorial optimization is a subfield of mathematical optimization. The aim of combinatorial optimization is to find an optimal object from a finite set of objects those candidate objects are called feasible solutions while the optimal one is called an optimal solution that consists of finding an optimal object from a finite set of objects, where the set of feasible solutions is discrete or can be reduced to a discrete set. Combinatorial optimization is a powerful field that deals with finding the best solution from a finite set of possibilities. It has wide-ranging applications in various industries, from logistics and scheduling to finance and telecommunications.

It is the process of finding the best arrangement or solution from a set of possibilities. It plays a crucial role in various fields such as computer science, operations research, and logistics. This section provides an overview of combinatorial optimization, highlighting its definition, importance, and applications. Combinatorial optimization refers to the process of finding the best solution from a finite set of possible solutions for a specific problem. This field has various applications, including logistics, scheduling, network design, and more.

UNCERTAINTY THEORY

Introduction:-

Uncertainty theory is a branch of mathematics based on normality, monotonicity, self-duality, countable subadditivity, and product measure axioms.

Uncertainty refers to a lack of certainty or predictability about the outcome of a situation or the future state of events. It encompasses various factors such as incomplete information, variability, randomness, and ambiguity that can affect decision-making processes and outcomes.

In many real-world scenarios, uncertainty is inherent due to factors such as:

Incomplete Information: Lack of complete knowledge about the state of the system, including missing data or unreliable information.

Variability: Natural fluctuations or variations in parameters, conditions, or external factors that influence the outcome.

Randomness: Unpredictable events or stochastic processes that introduce randomness into the system.

Ambiguity: Unclear or vague information that makes it difficult to determine the correct interpretation or course of action.

Four axioms of uncertainty:

Axiom 1. (Normality Axiom) $\boldsymbol{\mathcal{M}}\{\boldsymbol{\Gamma}\} = \mathbf{1}$ $for\ the\ universal\ set\ \boldsymbol{\Gamma}.$

Axiom 2. (Self-Duality Axiom) $\boldsymbol{\mathcal{M}}\{\wedge\} + \boldsymbol{\mathcal{M}}\{\wedge^c\} = \mathbf{1}$ for any event $\wedge$.

Axiom 3. (Countable Subadditivity Axiom) For every countable sequence of events $\wedge_1$, $\wedge_2$, $\wedge_3$,… , we have

$$\boldsymbol{\mathcal{M}}\{\cup_{i=1}^{\infty} \wedge_i\} \leq \sum_{i=1}^{\infty} \boldsymbol{\mathcal{M}}\{\wedge_i\} .$$

Axiom 4. (Product Measure Axiom) Let $(\boldsymbol{\Gamma_k}, \boldsymbol{\mathcal{L}_k}, \boldsymbol{\mathcal{M}_k})$ be uncertainty spaces for $k = 1,2,3, \dots, n$. Then the product uncertain measure is an uncertain measure on the product σ-algebra satisfying

$$\boldsymbol{\mathcal{M}}\left\{\prod_{i=1}^{n} \wedge_i\right\} = \min_{1\le i\le n} \boldsymbol{\mathcal{M}}\{\wedge_i\}$$

There are different types of uncertainty,

1. **Linear Uncertainty**
2. **Zigzag Uncertainty**
3. **Normal Uncertainty**
4. **Lognormal Uncertainty**

In this report I have used Linear Uncertainty, So I am explaining here only Linear Uncertainty that is defines as,

Linear Uncertainty Distribution: An uncertain variable δ is called linear if it has a linear uncertainty distribution.

$$\boldsymbol{\phi(x)} = \begin{Bmatrix} \mathbf{0}, & \boldsymbol{if\ x \le 0} \\ \boldsymbol{(x-a)/(b-a)}, & \boldsymbol{if\ a \le x \le b} \\ \mathbf{1}, & \boldsymbol{if\ x \ge b} \end{Bmatrix}$$

denoted by $\mathcal{L}(\mathrm{a},\mathrm{b})$ where a and b are real numbers with $a \le b$

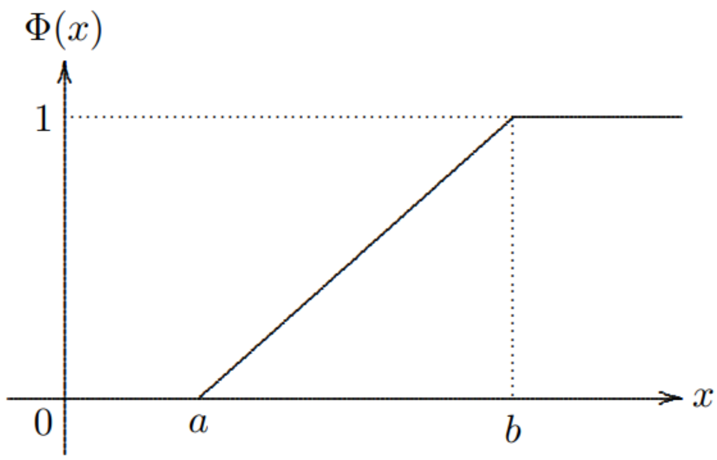


Figure: Linear Uncertainty Distribution

Now we have to define the expected value of linear uncertainty distribution:

If a and b are two real no. and $\mathcal{L}(a,b)$ be the linear uncertainty distribution then the expected value of this linear uncertainty distribution is given by:

$$E[\mathcal{L}(a,b)] = \frac{a+b}{2}$$

This expected value of the linear uncertainty distribution is used to find the cost of ring nodes.

GENETIC ALGORITHM

Introduction:

A genetic algorithm (GA) is a search and optimization algorithm inspired by the process of natural selection and genetics. It is developed by John Holland in 1960s, Gas are the part of the broader class of evolutionary algorithms and are used to find approximate solution to optimization and search problems. These problems often used to find the best combination of parameters or solution in a large solution space.

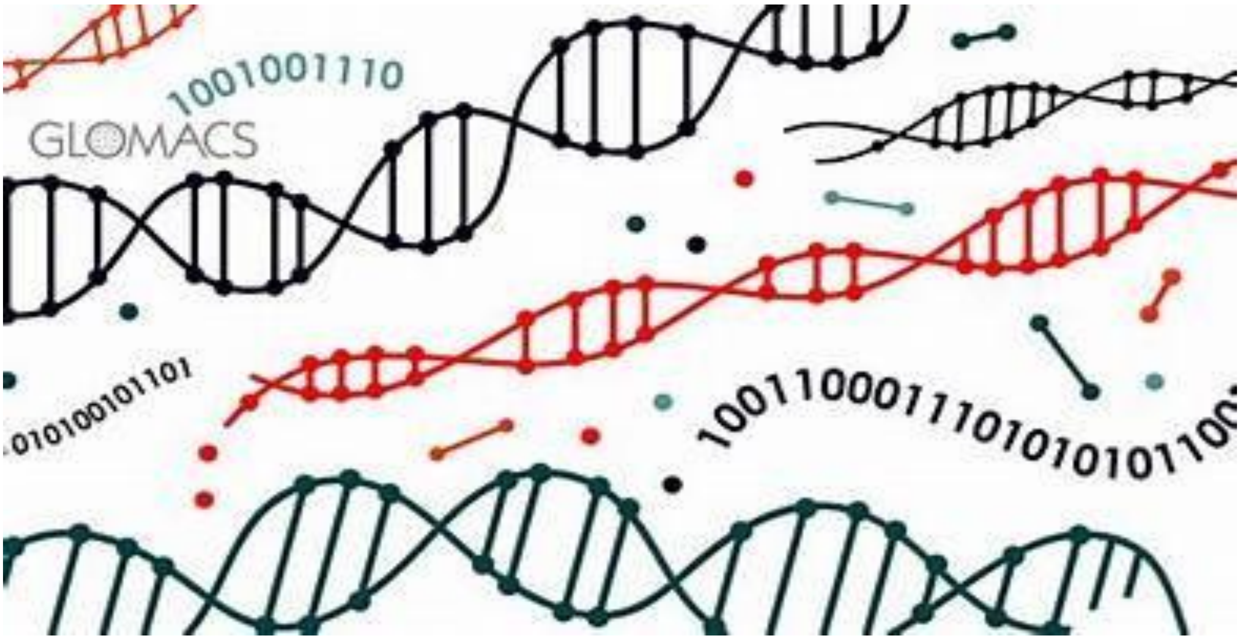


Figure – 2

Genetic algorithms are optimization techniques inspired by the principles of natural selection and genetics. They are commonly used to find optimal or near-optimal solutions to complex problems where traditional algorithms may be impractical or ineffective.

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions.

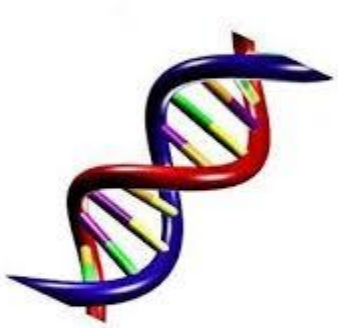
At each step, the genetic algorithm selects individuals from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. You can apply the genetic algorithm to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly non-linear. The genetic algorithm can address problems of mixed integer programming, where some components are restricted to be integer-valued.

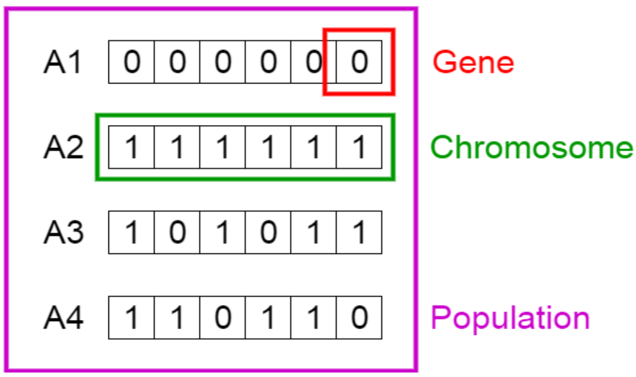
Genetic algorithms are particularly useful for optimization problems with large search spaces, non-linear or discontinuous fitness landscapes, and complex constraints. They have been successfully applied in various fields, including engineering, finance, bioinformatics, and machine learning.

Key Concepts of GAs:-

Here are some key concepts of genetic algorithms

1. **Gene**: An individual is characterized by a set of parameters (variables) known as Genes.





2. **String**: Genes are joined into a string to form a Chromosome (solution). In a genetic algorithm, the set of genes of an individual is represented using a string, in terms of an alphabet.

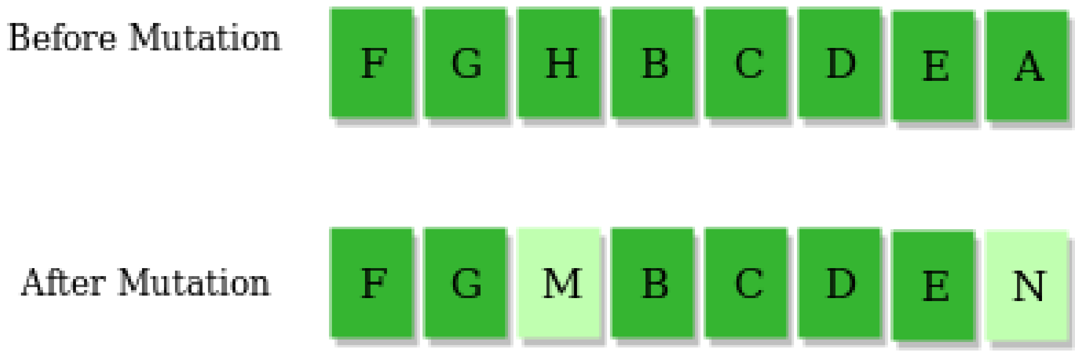
3. **Chromosome:** A representation of a solution in the population, often as a string of values or a set of parameters.

4. **Population:** A group of potential solutions to the optimization problem is known as population.

5. **Fitness Function:** A function that assigns a fitness value to each solution based on how well it solves the optimization problem.

6. **Crossover (Recombination):** The process of combining genetic material from two parent solutions to create new offspring solutions.

7. **Mutation:** Random changes made to some solutions in the population to introduce diversity.



Overview of GAs:-

Here's an overview of how genetic algorithms work:

1. **Introductions:** A population of potential solutions is generated randomly. Each solution represents a possible candidate for the optimization problem.

2. **Evaluation:** The fitness of each solution in the population is evaluated using a predefined fitness function. This function measures how well a solution solves the optimization problem.

3. **Selection:** Solutions are selected to form a new generation, with a higher probability of selection for solutions that have higher fitness values. This mimics the process of natural selection, where fitter individuals are more likely to survive and reproduce.

4. **Crossover (Recombination):** Pairs of selected solutions (parents) are combined to create new solutions (offspring) through crossover or recombination. This involves exchanging or combining parts of the parent solutions to create new solutions.

5. **Mutation:** Some solutions in the new generation undergo random changes or mutations. This introduces diversity into the population and helps explore different regions of the solution space.

6. **Replacement:** The new generation, consisting of both the offspring from crossover and mutated solutions, replaces the old generation.

7. **Termination:** The algorithm continues iterating through generations until a stopping criterion is met. This could be a fixed number of generations, reaching a satisfactory solution, or other specified conditions.

8. **Solution Extraction:** Once the algorithm terminates, the best solution or a set of good solutions found during the optimization process is extracted as the output.

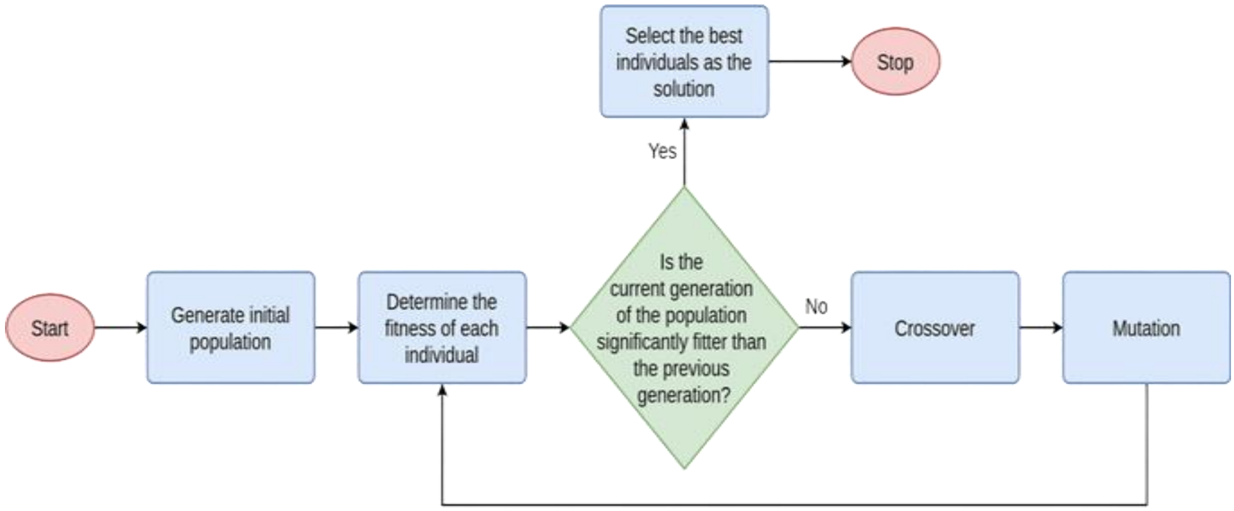


Figure – 3

Genetic algorithms are versatile and have been successfully applied to various optimization problems, including scheduling, engineering design, financial modelling, and machine learning. They are particularly useful in complex and high-dimensional search spaces where traditional optimization methods might struggle.

TRAVELING SALESMAN PROBLEM

The objective of the Traveling Salesman Problem (TSP) is to find a tour that starts at one city, visits each city exactly once, and returns to the starting city while minimizing the total distance travelled.

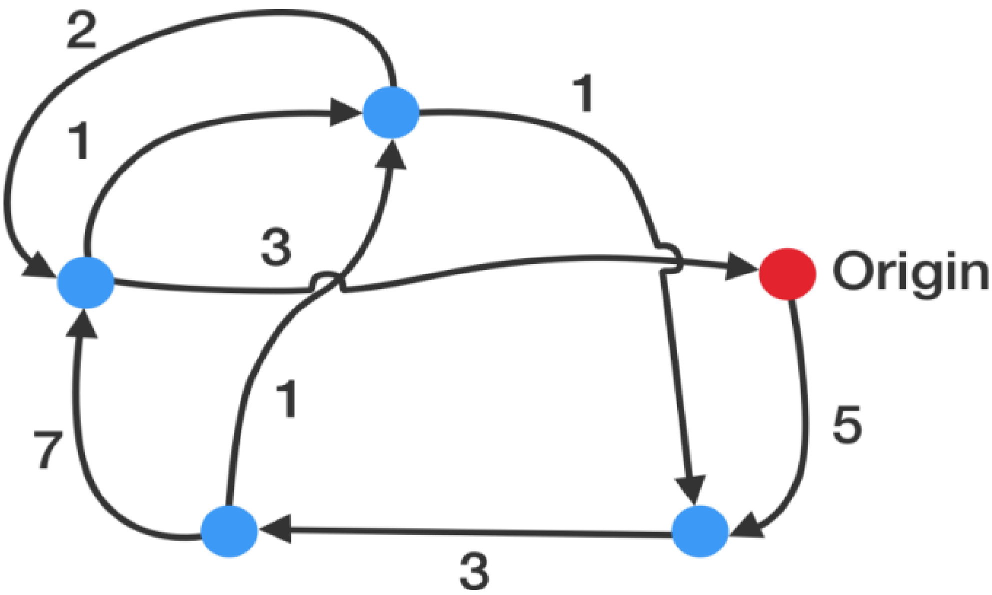


Figure – 4

In this figure the salesman start his journey from origin go to 5 unit distance and visit first node and in this way by visiting all the nodes exactly ones and then the salesman return to the origin

In TSP, the goal is to find the shortest possible tour that visits a given set of cities exactly once and returns to the starting city. There are various approaches to find the solution to the travelling salesman problem: naive approach, greedy approach, dynamic programming approach, etc.

The travelling salesman problem is a graph computational problem where the salesman needs to visit all cities (represented using nodes in a graph) in a list just once and the distances (represented using edges in the graph) between all these cities are known. The solution that is needed to be found for this problem is the shortest possible route in which the salesman visits all the cities and returns to the origin city.

The given below is the real world example of TSP in which the salesman visit some cities exactly once and return to origin city.

Mathematical Formulations:

If c_{ij} is the cost of going from city i to j and $x_{ij} = 1$, if the salesman goes directly from city i to city j and zero otherwise,

Then the problem is to find x_{ij} which is defined by

$$\boldsymbol{minimize\ Z = \sum_{i=1}^{n} . \sum_{j=1}^{n} c_{ij} x_{ij}}$$

Subject to,

$$\boldsymbol{\sum_{j=1}^{n} x_{ij} = 1}$$

$$\boldsymbol{\sum_{i=1}^{n} x_{ij} = 1}$$

And

$$\boldsymbol{x_{ij} = 0\ or\ 1;\ \ i = 1, 2, 3, \dots, n\ ,}$$

With the two additional constraints that no city is to be visited twice before the tour of all the cities is completed and that going from city i to directly i is not permitted which means $\boldsymbol{c_{ii}} = \infty$.

COVERING SALESMAN PROBLEM

INTRODUCTION:

The covering salesman problem which was first introduced by Current and Schilling (Current 1989). The objective of Covering Salesman Problem is not only to find a tour that visits all cities but also to ensure that every city is visited at least once. This means that in addition to minimizing the total travel distance, there is a requirement to cover all cities. The Covering Salesman Problem is a variant of the Traveling Salesman Problem that adds an extra constraint to the problem and is used in scenarios where complete coverage of cities is essential.

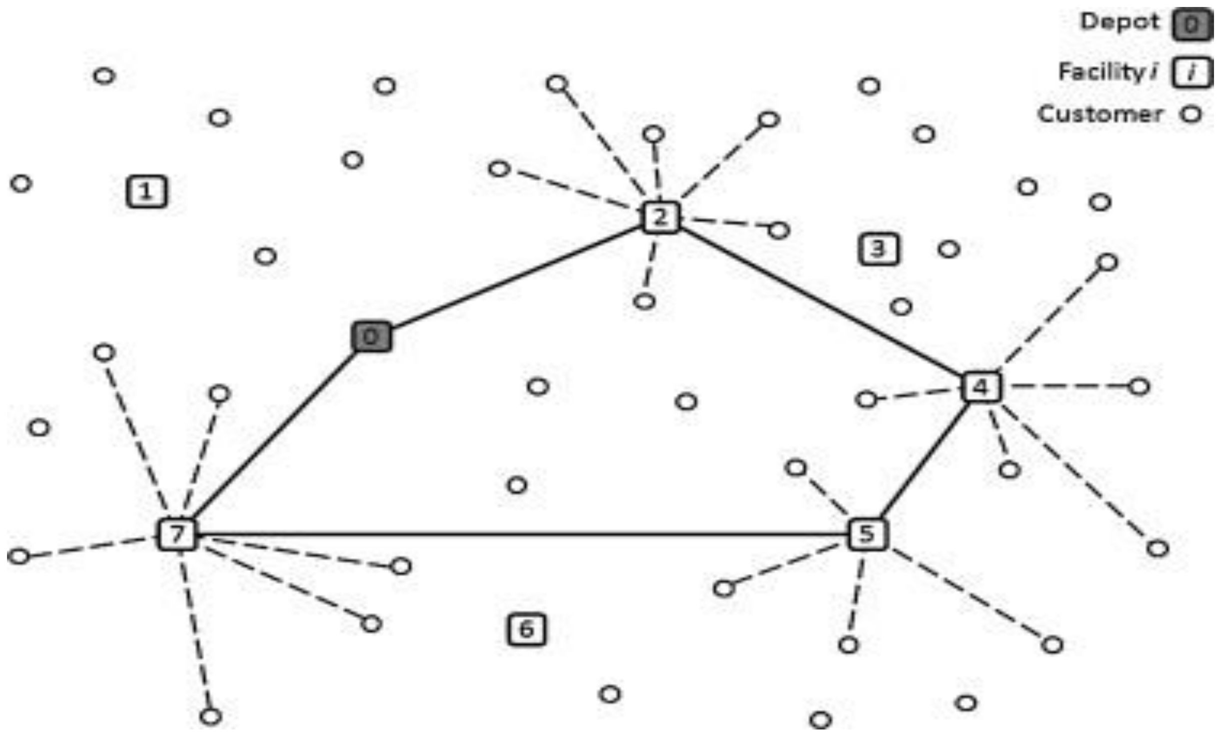


Figure – 6(a)

Covering salesman problem (CSP) is an extension of the classic traveling salesman problem (TSP). Unlike in the case of TSP, not all the cities need to be visited in CSP. However, each unvisited city needs to be within the coverage radius of at least one city in the tour. Formally, the problem is defined on an undirected complete weighted graph $G = (V, E)$ with a set of vertices $V = \{1,2,3, \ldots, n\}$ and a set of edges $E = \{ (i, j): i, j \in V, i \neq j \}$.

Let C be an $n \times n$ distance matrix where c_{ij} $(i, j = 1,2 \ldots, n, i \neq j)$ represents the travel cost between two corresponding vertices i and j. Each vertex $i \in V$ is associated with a coverage radius r_i .

A vertex j is covered by vertex i if it is within the coverage radius r_i of i. The aim of the problem is to seek for a Hamiltonian cycle of the lowest cost across a subset of vertices $M \subset V$, such that every vertex $i \in V$ must be either visited or covered by the tour.

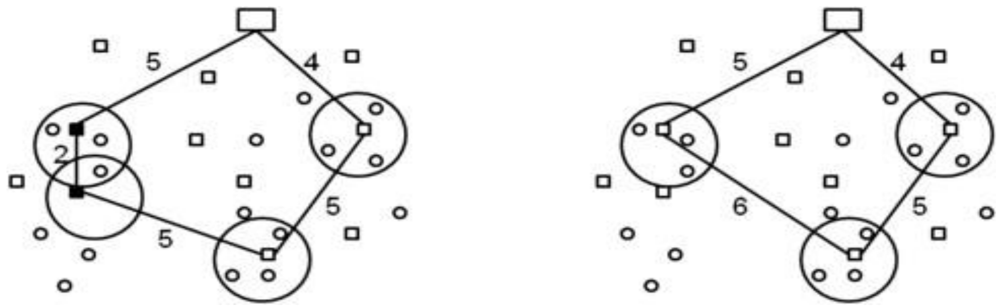
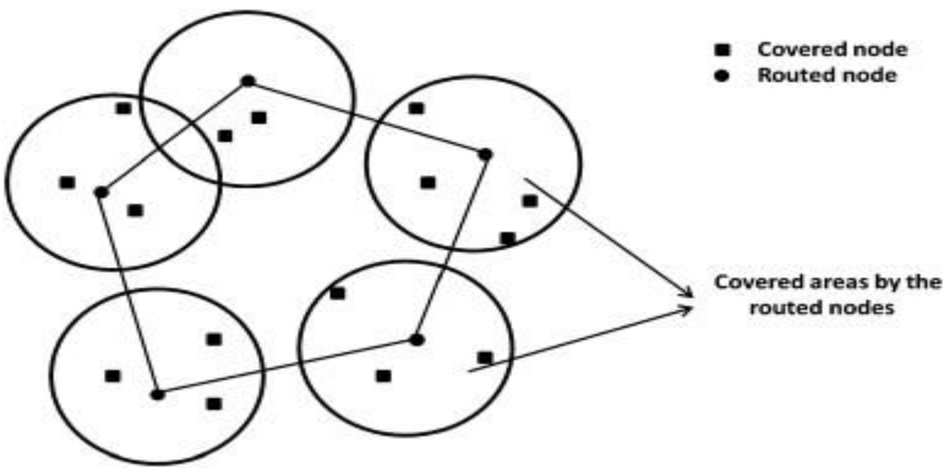


Figure – 6(b)

One notices that CSP reduces to TSP for the case where every vertex has a coverage radius of 0.

Given that CSP is a combination of subset selection and permutation optimization, its complexity is higher than in the general TSP case classifying it as an NP-hard problem (A problem is classified as NP-Hard when an algorithm for solving it can be translated to solve any NP problem and A problem is called NP (Non deterministic polynomial) if its solution can be guessed and verified in polynomial time; nondeterministic means that no particular rule is followed to make the guess. If a problem is NP and all other NP problems are polynomial-time reducible to it, the problem is NP-complete).



Mathematical Formulation of CSP:

Given a complete graph $\boldsymbol{G = (N, A)}$, minimize the total tour cost such that a salesman starts from an initial node, visits a subset $\boldsymbol{N' \subset N}$ of nodes in which no node is visited more than once and comes back to the initial node, so that each of the nodes not on tour are in a predetermined distance from at least one of the visited nodes.

The mathematical formulation of this problem may be stated as:

$$\boldsymbol{Minimize \quad Z = \sum_{i=1}^{|N|} . \sum_{j=1}^{|N|} c_{ij} x_{ij}} \qquad (1)$$

Subject to:

$$\boldsymbol{\sum_{i=1}^{n} . \sum_{j \in D_l} x_{ij} \geq 1, \ \ \forall\, l \in N} \qquad (2)$$

$$\boldsymbol{\sum_{i=1}^{|N|} x_{ik} = \sum_{j=1}^{|N|} x_{kj} = 0 \ or \ 1, \qquad \forall\, k \in N} \qquad (3)$$

$$\boldsymbol{x_{ij} \in \{0, 1\}} \qquad (4)$$

$$\boldsymbol{\sum_{i \in S} . \sum_{j \in N} x_{ij} \leq \ |S| - 1, \ \ \forall S \subset N' \subset N, \qquad 2 \leq |S| \leq |N'| - 2} \qquad (5)$$

Where ***N'*** is the set of visiting nodes, $\boldsymbol{C_{ij}}$ is the cost from the node ***i*** to the node ***j***.

And,

\`

$$\boldsymbol{x_{ij} = \begin{cases} 1, & if \ \exists \ a \ path \ from \ the \ node \ i \ to \ the \ node \ j. \\ 0, & Otherwise \end{cases}}$$

$$\boldsymbol{D_l = \{j\text{: } d_{ij} \leq \triangle_j\}.}$$

$$\boldsymbol{d_{ij} = shortest \ distance \ between \ i \ and \ j.}$$

Where The Equations indicate:

i.e. equation -

(1) Minimizes the total tour cost.

(2) Ensures every node of the graph is either visited or is covered by the visited nodes.

(3) Indicates that for a vertex, in degree and out degree are the same.

(4) Implies the binary character of x and

(5) is the sub tour elimination constraint,

i.e., if a node is on tour, it is visited not more than once (only the initial node is visited twice).

The above equation (1) - (5) can be rewritten as follows:

Let $\boldsymbol{N} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \ldots, \boldsymbol{x_{|N|}}\}$ be the set of nodes.

Determine a complete tour $(x_{a_1}, x_{a_2}, x_{a_3}, \ldots, x_{a_m})$, $m \leq |N|$

$$\boldsymbol{To\ minimize}$$

$$\sum_{i=1}^{m-1} \boldsymbol{c}(\boldsymbol{x}_{\boldsymbol{a}_i}, \boldsymbol{x}_{\boldsymbol{a}_{i+1}}) + \boldsymbol{c}(\boldsymbol{x}_{\boldsymbol{a}_m}, \boldsymbol{x}_{\boldsymbol{a}_1}); \qquad (6)$$

Such that,

$\boldsymbol{x_i} \boldsymbol{\epsilon}\, \overline{B}\ \boldsymbol{\epsilon}\ (\boldsymbol{x_{a_1}}, \Delta_{\boldsymbol{a_i}})$, $\forall\, x_i$ ϵ N and for some i; (7)

Where, $\boldsymbol{a_i} \boldsymbol{\epsilon} \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, |\boldsymbol{N}|\}$

And $\boldsymbol{a_i} \neq \boldsymbol{a_j}$, for $\boldsymbol{i} \neq \boldsymbol{j}$, $\boldsymbol{c}(\boldsymbol{i}, \boldsymbol{j}) = \boldsymbol{c_{ij}}$,

$\overline{\boldsymbol{B}}(\boldsymbol{a}, \boldsymbol{r})$ is a closed disc with *centre* ***a*** and *radius* ***r***,

$\boldsymbol{\Delta}_{\boldsymbol{j}}$ = maximum covering distance at node ***j***.

This is the mathematical understanding of covering salesman problem.

Difference between and TSP and CSP:

TSP	CSP
Objective: The main goal of the TSP is to find the shortest possible route that visits a given set of cities and returns to the original city (salesman's home city). The objective is to minimize the total distance or cost of the tour. **Constraint:** Each city must be visited exactly once, and the tour must form a closed loop.	**Objective:** The CSP is concerned with finding the shortest possible route that covers a set of given locations, rather than visiting each location exactly once. The objective is to minimize the total distance or cost of the tour while ensuring that every location is covered. **Constraint:** There is a requirement to cover a specified set of locations in a predetermined distance, but there may be flexibility in how many times each location is visited

3. PROJECT WORK

Project Title:

"

INTERMODAL FIXED RADIUS DENSITY DEPENDENT COVERING SALESMAN PROBLEM IN UNCERTAIN ENVIRONMENT.

"

• Summarize the problem statement:

The Intermodal Fixed Radius Density Dependent Covering Salesman Problem poses a complex challenge in optimizing transportation logistics within an uncertain environment. This study addresses the critical need to develop efficient solutions for intermodal transportation systems where two modes of transportation are utilized and the density of demand points varies dynamically.

Intermodal transportations, entails the seamless integration of different modes of transport, from trucks to trains to ships. Each mode brings its own set of advantages and constraints, requiring careful coordination to optimize efficiency.

The notion of a fixed radius introduces a spatial constraint, compelling decision-makers to operate within a defined geographical area. Within this boundary, they must devise strategies to cover all relevant locations effectively.

Density dependency adds another layer of complexity, acknowledging that certain areas might require more attention due to higher population density or increased demand. As such, the solution must adapt to these varying levels of activity and resource allocation.

The essence of the salesman problem emerges as the need to find the most efficient route to navigate through the designated area, visiting key locations and ultimately returning to the starting point. However, unlike traditional scenarios, this puzzle unfolds in an uncertain environment, where conditions are subject to change unpredictably.

Thus, tackling this challenge involves not only optimizing routes and schedules but also devising robust strategies to cope with unforeseen disruptions. It demands a blend of advanced mathematical algorithms, real-time data analytics, and strategic foresight.

In essence, the Intermodal Fixed Radius Density Dependent Covering Salesman Problem in an Uncertain Environment encapsulates the multifaceted nature of modern logistics and transportation planning. It represents a formidable challenge yet also an opportunity for innovation and efficiency improvement in the increasingly dynamic landscape of supply chain management.

• Intermodal transportation:

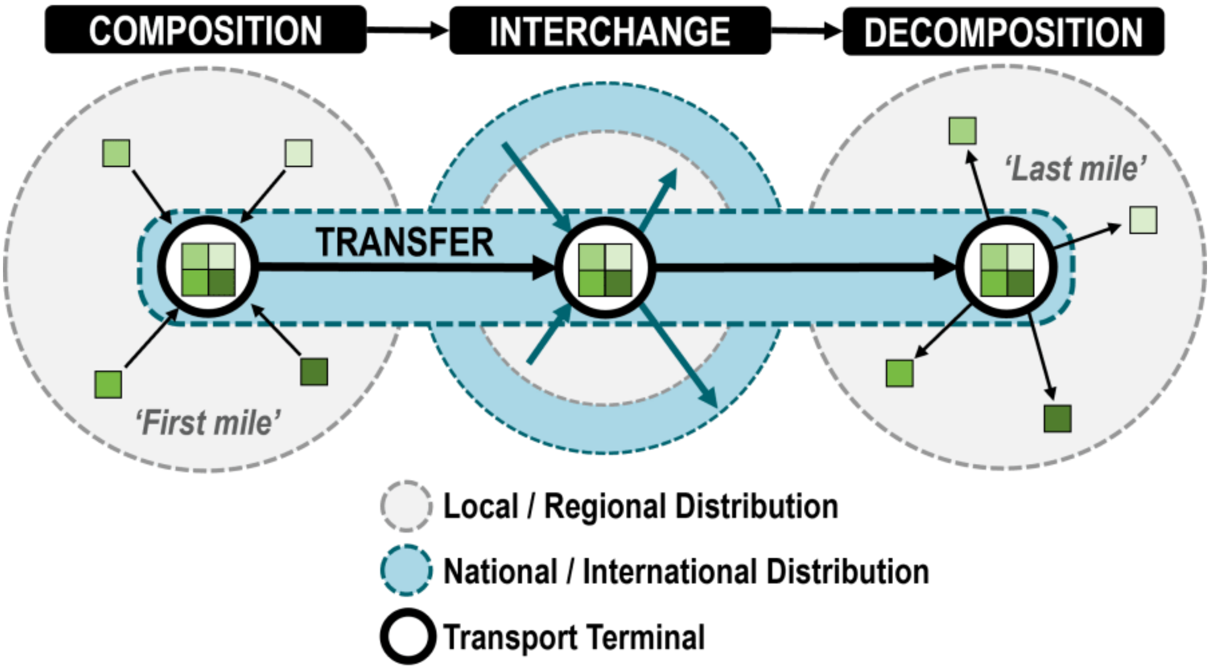
Intermodal transportation means moving large-sized goods in the same steel-based containers through two or more modes of transport. It's a typical way of moving goods in modern times. Intermodal transfer may involve truck, rail, ship, and then truck again. Basically, instead of shifting goods from one vehicle to the next in their journey, intermodal transport handles these special standardized containers instead. This process brings many benefits, such as increased safety for the goods and faster delivery. Because different categories of transports used for distribution from any nodes on ring, the transports are rail, road etc.

This mode of transportation system dates back to 18th-century Britain. The British used it to move coal stored in containers over their canal network. But it wasn't until the 1960s that intermodal become the preferred choice for sea transport.

Logistics companies and international organizations made efforts to integrate the other modes of transport through intermodalism. Later, containerization and standardizing international container sizes made intermodal transport even more lucrative by enabling easy handling between modal systems. Currently, intermodal transport is the dominant mode for the global supply chain and logistics.

How Intermodal Transport Work:

At the start of the intermodal transport process, an empty truck arrives at the consignee point. The shipper or transporter in charge loads the goods onto the container at the back of the truck. Most likely, nobody will handle these goods from this point until they reach the destination.
The truck travels to a rail yard through the road network. Here, the logistics company transfers the containers onto a train. That train might transfer the container to a railroad station in the destination city, or it might go to a port for shipping.



The shipping company moves these containers through the maritime network to the destination port. People unload the container and transfer it to another truck. Or they may put it on another train for more inland transport and then to the truck. This truck may deliver the container to the destination modal station. The trucks used to transfer these goods are called drayage, a service that only certain companies provide.

What happens next?
The logistics company now removes the goods from the container. Now, the container is empty and ready for a new load. Finally, the logistics company carries out the last-mile delivery to the end user.

Why do companies use intermodal transport?
It allows a logistics company to benefit from the advantages of each means of transport. Using rail service is often cheaper for inland transport. Shipping allows faster international travel over long distances. And trucks are essential for the last mile of local pickups and deliveries.

- ## Density dependent:

In this case the nodes covered by any node on ring are uniformly distributed within a circle of fixed radius. We are considering the total network as a supply chain.
The nodes on ring supplying more than two nodes are considered as high density node.
The nodes on ring supplying less than two nodes are considered as low density nodes.

In the context of the Covering Salesman Problem (CSP), the concept of density dependence plays a crucial role in optimizing the efficiency of node coverage within a network, particularly when considering a supply chain scenario. Density dependence refers to the distribution of covered nodes relative to each other within a fixed radius, typically represented as a circle.

In this scenario, the network is envisioned as a supply chain where nodes represent critical points of distribution or service provision. Each node on the ring is tasked with covering surrounding nodes within its designated radius. The distribution of nodes covered by any given node is assumed to be uniform within the circle.

Nodes that supply more than two neighbouring nodes are classified as high-density nodes, indicating their significance as central hubs within the network. Conversely, nodes that supply less than two neighbouring nodes are categorized as low-density nodes, suggesting their peripheral or less pivotal role within the network structure.

Optimizing the CSP within this framework involves strategically assigning tasks to nodes based on their density classification. High-density nodes may be prioritized for covering neighbouring nodes due to their capacity to efficiently serve multiple connections, thereby maximizing coverage with minimal redundancy. On the other hand, low-density nodes may require additional support or coordination to ensure adequate coverage of surrounding areas.

By leveraging density dependence metrics, such as the classification of nodes as high or low density, CSP algorithms can effectively allocate resources and tasks within the network to optimize coverage and minimize overall costs or inefficiencies. This approach enables the design of more robust and efficient supply chain networks by strategically leveraging the inherent density distribution of nodes within the system.

We assume the high dense nodes in a circular zone with uniform distribution follows Euclidean distance pattern and nodes in low density region follows Manhattan distance pattern.

• Concept of Drayage Distance:

Drayage refers to short distance movements in part of the supply chain. The concept of intermodal drayage means the transportation of cargo by truck, sea or through rail.

In the context of logistics and transportation, the concept of drayage distance refers to the distance traveled by a vehicle or carrier for the pickup and delivery of goods within a localized area, typically between ports, terminals, warehouses, or distribution centers. Drayage distance plays a crucial role in supply chain management, particularly in scenarios involving intermodal transportation where goods need to be transferred between different modes of transportation, such as ships, trains, trucks, or airplanes.

The term "drayage" originally referred to the movement of goods over short distances using horse-drawn carts, but it has evolved to encompass modern-day trucking operations. Drayage distance is influenced by factors such as the location of pickup and delivery points, traffic conditions, road infrastructure, and regulatory requirements.

Efficient management of drayage distance is essential for optimizing transportation costs, reducing transit times, and enhancing overall supply chain efficiency. Strategies to minimize drayage distance may include route optimization, consolidation of shipments, coordination of pickup and delivery schedules, and the use of technology such as GPS tracking and real-time monitoring systems.

In the context of the Covering Salesman Problem (CSP), drayage distance could be a factor considered when determining the optimal sequence of node visits to minimize overall transportation costs and travel time. By incorporating drayage distance into the CSP framework, logistics managers can make informed decisions about route planning, vehicle allocation, and resource utilization to streamline operations and improve customer service levels.

Drayage distance in a circular market area is categorized into two types:

i) Euclidean Distance
ii) Manhattan Distance

In high density region the drayage distance is considered as Euclidian Distance and in low density region, the drayage distance is assumed to be Manhattan distance.

In our report, Nodes on circle connected with two or more nodes is considered as nodes within high density.

When dealing with high-density regions where nodes are densely interconnected, Euclidean Distance is utilized as the measure of drayage distance. Euclidean Distance represents the straight-line distance between two points in space and is suitable for scenarios where the movement between nodes can be relatively unrestricted, such as in densely populated areas with multiple connections.

Conversely, if the number of nodes around a node on ring is less than two, it is low density node i.e. in low-density regions where nodes are sparsely connected, Manhattan Distance is employed as the measure of drayage distance. Manhattan Distance calculates the distance between two points by summing the lengths of the horizontal and vertical segments between them, mimicking the movement along city blocks. This distance metric is particularly suitable for areas with limited direct connections between nodes, where travel paths may need to follow specific routes due to logistical constraints.

In the research context outlined, nodes on the circular market area that are connected with three or more nodes are identified as high-density nodes. These nodes are likely central hubs within the network, serving multiple connections and facilitating efficient movement of goods or services. By categorizing drayage distance based on the density of nodes and selecting appropriate distance metrics (Euclidean or Manhattan), logistics managers can tailor their transportation strategies to the specific characteristics of each region within the market area, optimizing efficiency and reducing costs.

In the proposed solution for the covering salesman problem (CSP), the distribution of nodes around a central node on a ring follows a circular pattern. The solution utilizes a genetic algorithm to determine the nodes on the ring and those covered by them within a predefined fixed radius.

Once the nodes on the ring and their covered nodes are identified, they are categorized into two types based on the number of nodes they cover:

i) High dense nodes: Nodes on the ring that cover three or more nodes are classified as high dense nodes. These nodes represent areas of concentrated activity or connectivity within the network.

ii) Low dense nodes: Nodes on the ring that cover fewer than three nodes are categorized as low dense nodes. These nodes indicate areas with less activity or connectivity compared to high dense nodes.

In the solution, different distance patterns are applied depending on whether a node is categorized as high dense or low dense:

For high dense nodes: Euclidean distance pattern is followed. Euclidean distance calculates the straight-line distance between two points, which is suitable for areas with dense connections and relatively unrestricted movement.

For low dense nodes: Manhattan distance pattern is followed. Manhattan distance calculates the distance between two points by summing the lengths of the horizontal and vertical segments between them, mimicking movement along city blocks. This pattern is more appropriate for areas with sparse connections and limited direct paths between nodes.

By categorizing nodes based on their density and applying appropriate distance patterns, the solution optimizes the efficiency of the CSP by tailoring the transportation strategy to the specific characteristics of each area within the network.

- ## Total Distance calculation:

According to the dense nodes we have to find the total distance that is calculated as:

$$\begin{aligned}&\boldsymbol{The\ Total\ distance}\\&=\ \boldsymbol{Distance\ of\ Ring\ node}\\&+\ \boldsymbol{Sum\ of\ the\ nodes\ covered\ by\ nodes\ on\ ring\ in\ high\ dense\ area}\\&+\ \boldsymbol{Sum\ of\ the\ nodes\ covered\ by\ nodes\ on\ ring\ in\ low\ dense\ area.}\end{aligned}$$

The objective of this distance calculation is a **strategic decision** because the vehicle type plays a vital role if more number of nodes around a depot is present then Euclidean pattern is followed, because there are different options of roads are available but in low dense area Manhattan distance pattern is followed.

Indeed, the strategic decision to choose between Euclidean and Manhattan distance patterns based on the density of nodes around a depot is crucial in optimizing transportation efficiency in the covering salesman problem (CSP).

In areas with a high density of nodes around the depot, where numerous options for transportation routes are available, employing the Euclidean distance pattern makes sense. Euclidean distance calculates the shortest straight-line distance between two points, which is advantageous in areas with multiple road options or relatively uncongested traffic conditions. This approach allows vehicles to navigate efficiently through densely populated areas, potentially reducing travel time and costs.

Conversely, in low-density areas where nodes are sparsely distributed and direct connections between them are limited, the Manhattan distance pattern is more appropriate. Manhattan distance, which considers the sum of horizontal and vertical distances between points, aligns well with the structured grid-like layout often found in less densely populated regions. By following the Manhattan distance pattern, vehicles can navigate through city blocks and follow specific routes more effectively, optimizing travel paths despite the limited direct connections between nodes.

The choice of distance calculation method based on node density reflects a strategic approach to route planning and resource allocation. By adapting to the characteristics of different regions within the network, such as node density, the transportation strategy can be tailored to maximize efficiency and minimize costs. This strategic decision-making contributes to the overall optimization of the CSP solution, ensuring that vehicles are routed in a manner that takes full advantage of the available road networks and minimizes unnecessary detours or delays.

The travel plan for two transport options depends on the distance of two market area.

- ## Concept of Break Even Distance:

In the context of the Covering Salesman Problem (CSP), the concept of Break Even Distance refers to the critical point at which the cost or distance incurred by a salesperson or vehicle to cover additional nodes is balanced by the benefits or gains obtained from covering those nodes. In other words, it represents the distance threshold at which the marginal benefits of covering additional nodes equal the marginal costs incurred in doing so.

Break Even Distance is a key consideration in optimizing the route planning and resource allocation in the CSP. It helps decision-makers determine the optimal stopping point for the salesperson or vehicle, considering factors such as transportation costs, time constraints, and the value of covering additional nodes.

Once the Break Even Distance is reached, covering additional nodes beyond this point may result in diminishing returns or increased costs that outweigh the benefits. Therefore, it is essential to identify this threshold to ensure that resources are allocated efficiently and that the route plan is economically viable.

Strategies for determining the Break Even Distance in the CSP may involve mathematical modelling, optimization algorithms, or heuristic approaches that take into account various factors such as node distribution, transportation costs, and the value associated with covering additional nodes. By identifying the Break Even Distance, decision-makers can make informed decisions about route planning, resource allocation, and overall strategy to optimize the efficiency and cost-effectiveness of the CSP solution.

In transportation, intermodal transport can be considered as an alternative of unimodal transportation.

There is a break even distance under road cost equal to the intermodal transport cost

$$\boldsymbol{C_{UR} = C_{IM}}$$

Then splitting the cost into distance dependent variable cost and distance independent variable cost

$$\boldsymbol{FC_{UR} + VC_{UR}\,BE = FC_{IM} + VC_{IM}BE}$$

Where

1. $\boldsymbol{FC_{UR}}$ is Unimodal fright.
2. $\boldsymbol{VC_{UR}BE}$ is Break Even distance for unimodal fright.
3. $\boldsymbol{FC_{IM}}$ is Intermodal fright.
4. $\boldsymbol{VC_{IM}BE}$ is Break Even distance for intermodal freight.

Here $\boldsymbol{FC_{UR}}$ and $\boldsymbol{FC_{IM}}$ are fixed and $\boldsymbol{VC_{UR}}$ and $\boldsymbol{VC_{IM}}$ variable costs of a unimodal road and intermodal freight transport, respectively, and **BE** is the break-even distance.

In fright transport we mainly transport of goods using rail, road, air, any medium.

The concept of intermodal drayage means the transportation of cargo by truck, sea or rail rather than just in one mode of transportation

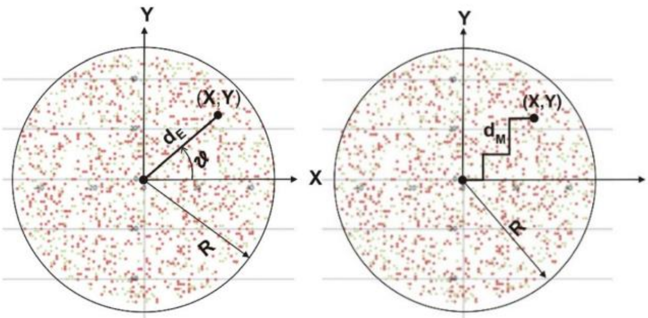
- **<u>Mathematical expression of drayage distance:</u>**

It is the transportation of shipping containers by truck to its final destination. Drayage is often part of a longer overall move, such as from a ship to a warehouse. Some research defines it specifically as a truck pickup from or delivery to a seaport, border point, inland port, or intermodel terminal with both the trip origin and destination in the same urban area. Port drayage is the term used when describing short hauls from ports and other areas to nearby locations. It can also refer to the movement of goods within large buildings such as convention centers. Drayage is a key aspect of the transfer of shipments to and from other means of transportation. The term drayage is also used for the fee paid for such services**.**

How mathematically we can express drayage distance in a circular market area. The shape is circular.

The shortest Euclidean distance is reasonable solution to use as a drayage distance in high density area.

The Manhattan distance is considered as in low density area.



In the first circle d_E represents the Euclidean distance and in second circle d_M represents the Manhattan distance.

Drayage distance depends on the shape of the market area, the terminal's location and the distribution of shippers and consignees in the market area. In this research, the shape of the market area is assumed to be a circle, the intermodal terminal is assumed to lie in the centred of the market area, and all shippers and consignees are assumed to be uniformly and randomly distributed in the origin and destination market areas. Drayage distances are calculated as both Euclidean and Manhattan distances. They are presented in Fig. 1 and calculated below,

The Nodes on circle connected with two or more nodes is considered as high density notes.

If less than two nodes then it is considered as low density nodes.

Calculating average Euclidean drayage distance

If $(\boldsymbol{X}, \boldsymbol{Y})$ is a random point in the unit disc then Euclidean distance d_E is given by,

$$\boldsymbol{d_E = \sqrt{X^2 + Y^2}}$$

The random variable $g(\boldsymbol{x}, \boldsymbol{y})$

$$\boldsymbol{g(x, y) = \sqrt{x^2 + y^2}}$$

And the joint probability density function,

$$\boldsymbol{f(x, y) = \frac{1}{\pi} : \ x^2 + y^2 \leq 1}$$

The Expected distance $\boldsymbol{E[d_E]}$ in the centre of the unit disc is given by,

$$\boldsymbol{E[d_E] = E\left[\sqrt{X^2 + Y^2}\right] = \frac{1}{\pi} \iint_{x^2+y^2 \leq 1} \sqrt{x^2 + y^2} \ dx\, dy}$$

By changing x and y in polar coordinates $\boldsymbol{d_E \ and \ \theta}$

$$\boldsymbol{x = d_E \cos\theta \ \ and \ \ y = \ d_E \sin\theta}$$

Then the Jacobian is given by,

$$J = \begin{pmatrix} \frac{\partial x}{\partial d_E} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial d_E} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} cos\,\theta & -d_E\,sin\,\theta \\ sin\,\theta & d_E\,cos\,\theta \end{pmatrix}$$

The determinant of the Jacobian matrix is,

$$|J| = \cos\theta\ \ d_E \cos\theta - (-d_E \sin\theta)\sin\theta$$

$$= \ \cos\theta\ \ d_E \cos\theta + \sin\theta\, d_E \sin\theta$$

$$|J| = \ \ d_E(cos^2\theta + sin^2\theta) = \ d_E$$

And,

$$dx\,dy = d_E\,dd_E\,d\theta$$

Which by integration into range

$$0 \le d_E \le 1\ and\ 0 \le \theta \le 2\pi$$

Then,

$$E[d_E] = \frac{1}{\pi}\int_0^{2\pi} d\theta\ \int_0^1 {d_E}^2\,dd_E$$

$$= \frac{1}{\pi}\int_0^{2\pi} d\theta\ \left\{\frac{{d_E}^3}{3}\right\}_0^1$$

$$= \ \frac{1}{\pi}\int_0^{2\pi} d\theta\left\{\frac{1^2}{3} - \frac{0^2}{3}\right\}$$

$$= \frac{1}{3\pi}\int_0^{2\pi} d\theta$$

$$= \ \frac{1}{3\pi}[\theta]_0^{2\pi}$$

$$= \frac{1}{3\pi}[2\pi - 0] = \frac{2\pi}{3\pi} = \frac{2}{3}$$

Hence,

$$E[d_E] = \frac{2}{3}$$

The expected distance$E[d_E]$, denoted as the average drayage distance $\overline{d_E}$ in the circle with radius R, is given by

$$E[d_E] = \frac{2}{3}R = 0.67R = \overline{d_E}$$

<u>Now calculating the average Manhattan drayage distance:</u>

This can be calculated similarly as the average Euclidean distance,

If (X, Y) is a random point in the unit disc then Euclidean distance d_M is given by,

$$d_M = |X| + |Y|$$

The random variable $g(x, y)$

$$g(x, y) = x + y$$

And the joint probability density function

$$f(x, y) = \frac{1}{\pi} : x^2 + y^2 \leq 1$$

The expected Manhattan distance $E[d_M]$ to the center of the unit disc can be expressed by the equation

$$E[d_M] = E[X + Y] = \frac{1}{\pi}\iint_{x^2+y^2\leq 1}^{.} (|x| + |y|)\ dx\, dy$$

By changing x and y in polar coordinates $\boldsymbol{d_M}$ ***and*** $\boldsymbol{\theta}$

$$\boldsymbol{x = d_M \cos\theta \ \ and \ \ y = \ d_M \sin\theta}$$

Then the Jacobian is given by,

$$\boldsymbol{J} = \begin{pmatrix} \frac{\partial x}{\partial d_M} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial d_M} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{cos\,\theta} & \boldsymbol{-d_M\,sin\,\theta} \\ \boldsymbol{sin\,\theta} & \boldsymbol{d_M\,cos\,\theta} \end{pmatrix}$$

The determinant of the Jacobian matrix is,

$$|\boldsymbol{J}| = \cos\theta \ \ d_M \cos\theta - (-d_M \sin\theta) \sin\theta$$
$$= \ \cos\theta \ \ d_M \cos\theta + \sin\theta \, d_M \sin\theta$$

$$|\boldsymbol{J}| = \ \ d_M(cos^2\,\theta + sin^2\,\theta) = \ d_M$$

Using the same calculation as for the Euclidean distance, the expected Manhattan distance $\boldsymbol{E[d_M]}$ is given by

$$\boldsymbol{E[d_M]} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 (|d_M \cos\theta| + |d_M \sin\theta|) \ \ d_M dd_M d\theta$$

Let's first consider one quadrant, say from θ=0 to θ=π/2.

For this quadrant:

$$\boldsymbol{E[d_M]} = \frac{1}{\pi}\int_0^{\pi/2}\int_0^1 (d_M \cos\theta + d_M \sin\theta) \ \ d_M dd_M d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\int_0^1 {d_M}^2 \cos\theta \, dd_M d\theta + {d_M}^2 \sin\theta \ \ dd_M d\theta$$

$$= \frac{1}{\pi}\left(\frac{{d_M}^3}{3}\right)_0^1 \int_0^{\pi/2} (cos\theta + sin\theta)\, d\theta$$

$$= \frac{1}{\pi}\left(\frac{1^3}{3} - 0\right)[sin\theta - cos\theta]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{3\pi}\left[\left\{sin\left(\frac{\pi}{2}\right) - cos\left(\frac{\pi}{2}\right)\right\} - \{sin0 - cos0\}\right]$$

$$= \frac{1}{3\pi}\,[\{1 - 0\} - \{0 - 1\}]$$

$$= \frac{1}{3\pi}\,[\{1\} - \{-1\}]$$

$$= \frac{1}{3\pi}\,[1 + 1]$$

$$= \frac{2}{3\pi}$$

Since the function is symmetric across the four quadrants, the total expected Manhattan distance will be 4 times this value:

So,

$$E[d_M] = 4 \times \frac{2}{3\pi}$$

$$= \frac{8}{3\pi}$$

Hence, the expected Manhattan distance $E[d_M]$ denoted as the average Manhattan distance $\overline{d_M}$ in the circle with radius R, is given by,

$$E[d_M] = \frac{8}{3\pi}R = 0.85R \;=\; \overline{d_M}$$

- # Concept of nodes within the radius R:

It seems like you're describing a concept related to network or spatial analysis, where you're assessing the density of nodes within a certain radius R and then assigning costs based on that density. Let's break down the concept:

High Density Nodes:

If the number of nodes within the radius R is greater than 2, it's considered high density.

The cost for high density nodes is calculated as $(\mathbf{n_1} \times \mathbf{0.67} \times \mathbf{R})$, where $\mathbf{n_1}$ is the number of nodes within the radius R,

$$\boldsymbol{Cost\ of\ high\ density\ nodes} = (\boldsymbol{n_1} \times \mathbf{0.67} \times \boldsymbol{R})$$

Low Density Nodes:

If the number of nodes within the radius R is less than or equal to 2, it's considered low density.

The cost for low density nodes is calculated as $(\mathbf{n_1} \times \mathbf{0.85} \times \mathbf{R})$, where $\mathbf{n_2}$ is the number of nodes within the radius R,

$$\boldsymbol{Cost\ of\ low\ density\ nodes} = (\boldsymbol{n_2} \times \mathbf{0.85} \times \boldsymbol{R})$$

This concept suggests that high density nodes incur a lower cost per node (0.67 × R) compared to low density nodes (0.85 × R). This could be interpreted in various contexts, such as urban planning, telecommunications network deployment, or environmental monitoring. The cost calculation seems to take into account the density of nodes within a given area, with higher density nodes being more cost-effective.

- ## Calculating the cost of density nodes:

To calculate the total cost of density nodes, you would sum the cost of high density nodes and the cost of low density nodes.

Let's denote:

- $\mathbf{n_1}$ as the number of high density nodes within the radius R
- $\mathbf{n_2}$ as the number of low density nodes within the radius R
- R as the radius

$$\boldsymbol{Total\ cost = cost\ of\ high\ density\ nodes}$$

$$\boldsymbol{+ cost\ of\ low\ density\ nodes}$$

$$\boldsymbol{Total\ cost = (n_1 \times 0.67 \times R) + (n_2 \times 0.85 \times R)}$$

This equation gives you the total cost based on the number of high density nodes and low density nodes within the given radius R, each multiplied by their respective cost factors.

• The Total Cost Formula:

Now we have to find the total cost of the ring and density nodes:

$$\boldsymbol{Total\ cost = cost\ of\ ring\ nodes + \ cost\ of\ high\ density\ nodes + cost\ of\ low\ density\ nodes}$$

So the total cost is calculated as:

$$\boldsymbol{Total\ Cost = \sum_{For\ S\ no.of\ nodes\ on\ ring}^{.} . \sum_{k_2=1}^{.} c_{ij}X_{ij} + \sum_{i=1}^{m_1} a_i \times (k_1 \times\ 0.67 \times Radius) + \sum_{j=1}^{m_2} a_j \times (k_2 \times\ 0.85 \times Radius)}$$

Where,

$$k_1 = The\ number\ of\ nodes\ on\ ring\ supplying\ in\ a\ high\ density\ circular\ area$$

$$3 \leq k_1 < \infty$$

$$k_2 = The\ number\ of\ nodes\ in\ each\ circular\ low\ density\ area$$

$$1 \leq k_2 < 3$$

$$m_1 = The\ number\ nodes\ on\ ring\ supplying\ in\ a\ \ hig\ density\ circular\ area.$$

$$m_2 = The\ number\ of\ nodes\ in\ each\ circular\ low\ density\ area.$$

Also,

$$for\ 1 \leq i \leq m_1$$

And

$$for\ 1 \leq j \leq m_2$$

And

$$a_i = a_j = 1.$$

4. Results and Discussion

• Introduction:

In this project, I have developed a C source code to find the optimal cost, or minimum distances between cities, of a given standard 29×29 cost matrix. The approach utilizes Genetic Algorithms (GA) to iteratively improve solutions towards the optimal solution.

The population size is set to 30, ensuring a diverse pool of potential solutions. To initiate the process, a 30×7 matrix is chosen to generate the initial population. This matrix represents a collection of potential solutions, each containing a permutation of cities. The Genetic Algorithms then iteratively evolve and refine these solutions towards the optimal solution.

Combinatorial optimization problems, such as finding the optimal path between cities, do not always guarantee an exact solution due to their complexity. Therefore, the focus shifts towards finding the most optimal solution within a reasonable computational effort. The GA operates on the principle of survival of the fittest, where the solutions with better fitness (lower cost in this context) are more likely to survive and contribute to the next generation.

We know that in Combinatorial Optimization the solution depends on the no. of iterations that means higher no. of iterations gives the higher accuracy of optimal solution. But in some case the solution is obtained in between the iterations i.e. the optimal solution can be obtained before the iterations are completed.

The iterative nature of Genetic Algorithms implies that the accuracy of the optimal solution improves with a higher number of iterations. However, it's noteworthy that in some cases, the optimal solution may be reached before completing all iterations. This phenomenon highlights the dynamic and adaptive nature of Genetic Algorithms in exploring solution spaces efficiently.

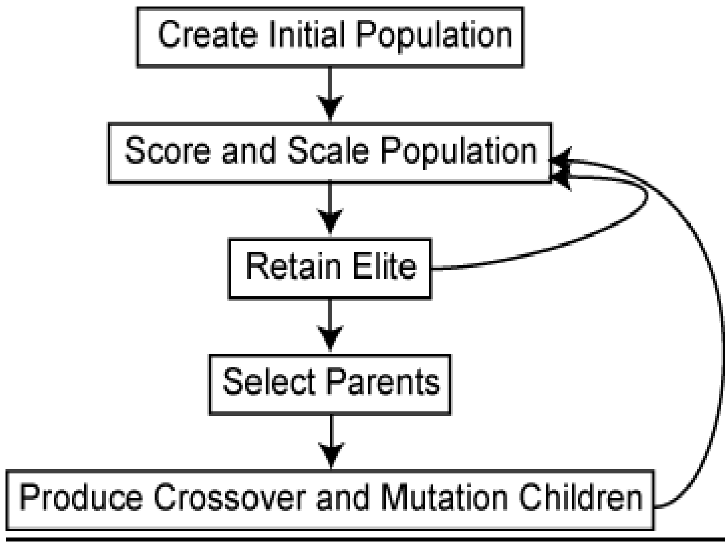
• Algorithm for whole concept:

The process involves several key steps:

1. Initialization: Generate an initial population of potential solutions. Each solution represents a permutation of cities.

2. Evaluation: Calculate the fitness of each solution in the population based on the cost function defined by the given cost matrix.

3. Selection: Choose individuals (solutions) from the current population to serve as parents for the next generation. Selection is typically biased towards individuals with higher fitness, but may also include mechanisms to maintain diversity.



4. Crossover: Create new solutions (offspring) by combining genetic material from selected parents. This process mimics natural reproduction and introduces diversity in the population.

5. Mutation: Introduce random changes in the offspring to further explore the solution space. Mutation prevents premature convergence and ensures exploration of new regions.

6. Replacement: Select individuals from the current population and the offspring to form the next generation. This step may include mechanisms such as elitism to preserve the best solutions.

By iteratively repeating these steps, Genetic Algorithms gradually converge towards optimal solutions. The optimal solution is determined by selecting the solution with the minimum cost among all generated solutions.

This project demonstrates the application of Genetic Algorithms in solving combinatorial optimization problems, specifically finding the optimal cost between cities. Despite the non-deterministic nature of Genetic Algorithms, they offer an effective and scalable approach to finding near-optimal solutions within reasonable computational resources.

• According to the source code the Algorithm works:

Step - 1. Initialization.

The process begins by creating an initial population. We start with a standard 29×29 matrix representing the cost between each pair of cities. The initial population is a matrix of size 30×7, representing 30 potential solutions (chromosomes), each consisting of a permutation of 7 cities.

Step - 2. Selection:

Next, we score and scale the population. This involves evaluating the fitness of each chromosome in the population. To start the selection process for crossover, we randomly generate indices modulo 29, ensuring the selection of diverse chromosomes for further processing.

Step - 3. Fitness function(minimum cost chromosomes)

In this step we have 30×7 matrix, we use the concept of drayage distance and of fitness function choose the fittest chromosomes for crossover and hence start the crossover.
Using the concept of drayage distance, we calculate the fitness of each chromosome. The fitness function identifies the most suitable chromosomes for crossover, prioritizing those with the minimum cost. These chromosomes become the basis for initiating the crossover process.

Step - 4. Crossover

Focusing on the fittest chromosomes, we generate new populations through crossover. This process combines genetic material from selected parents, mimicking natural reproduction to create offspring with potentially improved fitness.

Step - 5. Mutation

After crossover, we filter the current populations to generate new parents for optimal solutions. Mutation introduces random changes, maintaining genetic diversity and preventing premature convergence. If the optimal solution is not achieved, the process repeats from steps 2 to 4 until convergence or a predefined termination condition is met.

Since combinatorial optimization does not guarantee an optimal solution, the process may require multiple iterations. By repeating the steps, we explore different regions of the solution space, increasing the likelihood of finding the optimal solution. The minimum optimal cost obtained represents the most appropriate solution for the given problem.

The Genetic Algorithm approach efficiently addresses the combinatorial optimization problem of finding the minimum cost path between cities. By iteratively refining solutions through selection, crossover, and mutation, the algorithm converges towards an optimal solution. The resulting minimum cost ensures efficient route planning for a salesman covering all cities within a fixed radius. Overall, Genetic Algorithms offer a robust and scalable approach to solving complex optimization problems with practical applications in various domains.

- **RESULT ACCORDING TO THE STANDERED COST MATRIX**

Result according to source code

Now we have to apply the whole algorithm for numerical calculation according to source code.

1. **Initialization:**

The cost matrix that we are using as the slandered 29×29 cost matrix. Now we have to select the initial population that is a matrix of size 30×7, representing 30 potential solutions (chromosomes), each consisting of a permutation of 7 cities.

```
Select C:\Users\User\Desktop\New folder\csp.exe
enter radius100
The cost matrix is :
0   97   205   139   86   60   220   65   111   115   227   95   82   225   168   103   266   205   149   120   58   257   152   52   180   136   82   34   145
97   0   129   103   71   105   258   154   112   65   204   150   87   176   137   142   204   148   148   49   41   211   226   116   197   89   153   124   74
205   129   0   219   125   175   386   269   134   184   313   201   215   267   248   271   274   236   272   160   151   300   350   239   322   78   276   220   60
139   103   219   0   167   182   180   162   208   39   102   227   60   86   34   96   129   69   58   60   120   119   192   114   110   192   136   176   173
86   71   125   167   0   51   296   150   42   131   268   88   131   245   201   175   275   218   202   119   50   281   238   131   244   51   166   95   69
60   105   175   182   51   0   279   114   56   150   278   46   133   266   214   162   302   242   203   146   67   300   205   111   238   98   139   52   120
220   258   386   180   296   279   0   178   328   206   147   308   172   203   165   121   251   216   122   231   249   209   111   169   72   328   144   237   331

65   154   269   162   150   114   178   0   169   151   227   133   104   242   182   84   290   230   146   165   121   270   91   48   158   200   39   64   210
111   112   134   208   42   56   328   169   0   172   309   68   169   286   242   208   315   259   240   160   90   322   260   160   281   57   192   107   90
115   65   184   39   131   150   206   151   172   0   140   195   51   112   72   104   153   93   88   25   85   152   200   104   139   154   134   149   135
227   204   313   102   268   278   147   227   309   140   0   320   146   64   68   143   106   88   81   159   219   63   216   187   88   293   191   258   272
95   150   201   227   88   46   308   133   68   195   320   0   174   311   258   196   347   288   243   192   113   345   222   144   274   124   165   71   153
82   87   215   60   131   133   172   104   169   51   146   174   0   144   86   57   189   128   71   71   82   176   150   56   114   168   83   115   160
225   176   267   86   245   266   203   242   286   117   64   311   144   0   61   165   51   32   105   127   201   36   254   196   136   260   212   258   234
168   137   248   34   201   214   165   182   242   72   68   258   86   61   0   106   110   56   49   91   153   91   197   136   94   225   151   201   205
103   142   271   96   175   162   121   84   208   104   143   196   57   165   106   0   215   159   64   126   128   190   98   53   78   218   48   127   214
266   204   274   129   275   302   251   290   315   153   106   347   189   51   110   215   0   61   155   157   235   47   305   243   186   282   261   300   252
205   148   236   69   218   242   216   230   259   93   88   288   128   32   56   159   61   0   105   100   176   66   253   183   146   231   203   239   204
149   148   272   58   202   203   122   146   240   88   81   243   71   105   49   64   155   105   0   113   152   127   150   106   52   235   112   179   211
120   49   160   60   119   146   231   165   160   25   159   192   71   127   91   126   157   100   113   0   79   163   220   119   164   135   152   153   114
58   41   151   120   50   67   249   129   90   85   219   113   82   201   153   128   235   176   152   79   0   236   201   90   195   90   127   84   91
257   211   300   119   281   209   300   270   322   152   63   345   176   36   91   190   47   66   127   163   236   0   273   226   148   296   238   291   269
152   226   350   192   238   205   111   91   260   200   216   222   150   254   197   98   305   253   150   220   201   273   0   112   130   286   74   155   291
52   116   239   114   131   111   169   48   160   104   187   144   56   196   136   53   243   183   106   119   90   226   112   0   130   178   38   75   180
180   197   322   110   244   238   72   158   281   139   88   274   114   136   94   78   186   146   52   164   195   148   130   130   0   281   120   205   270
136   89   78   192   51   98   328   200   57   154   293   124   168   260   225   218   282   231   235   135   90   296   286   178   281   0   213   145   36
82   153   276   136   166   139   144   39   192   134   191   165   83   212   151   48   261   203   112   152   127   238   74   38   120   213   0   94   217
34   124   220   176   95   52   237   64   107   149   258   71   115   258   201   127   300   239   179   153   84   291   155   75   205   145   94   0   162
145   74   60   173   69   120   331   210   90   135   272   153   160   234   205   214   252   204   211   114   91   269   291   180   270   36   217   162   0
```

2. Selection:

In this step to start the selection process for crossover, we randomly generate indices modulo 29, ensuring the selection of diverse chromosomes for further processing.

```
 The initial populations:
 13  25  15  27  5  12  1
11  23  20  25  28  13  21
28  10  26  17  18  4  24
17  10  5  3  26  4  8
25  7  13  0  18  24  9
26  14  8  6  7  5  19
25  5  0  11  3  13  15
17  0  15  27  2  14  26
6  17  1  8  26  16  15
0  14  28  2  6  16  20
14  6  13  0  19  8  15
8  12  14  9  13  23  20
28  22  26  27  24  17  5
0  22  15  3  20  11  5
5  8  15  23  11  6  17
1  24  14  7  13  3  25
26  3  23  5  22  9  12
4  19  11  23  7  6  27
1  3  13  10  24  20  6
14  8  17  13  4  0  2
0  4  13  12  21  11  8
17  18  13  10  24  2  16
8  2  17  13  6  5  15
17  9  27  2  4  24  10
17  18  1  12  6  20  26
3  14  21  2  27  16  7
19  6  7  2  23  21  20
26  21  8  15  24  23  22
22  26  23  28  4  8  13
20  10  1  18  0  19  5
```

3. Fitness function(minimum cost chromosomes)

In this step we have 30×7 matrix, we use the concept of drayage distance and of fitness function choose the fittest chromosomes for crossover and hence start the crossover.

Using the concept of drayage distance, we calculate the fitness of each chromosome. The fitness function identifies the most suitable chromosomes for crossover, prioritizing those with the minimum cost. These chromosomes become the basis for initiating the crossover process.

4. Crossover

Focusing on the fittest chromosomes, we generate new populations through crossover. This process combines genetic material from selected parents, mimicking natural reproduction to create offspring with potentially improved fitness.

By using crossover we get the matrix:

```
17  18  13  10  24  8   13
22  26  23  28  4   2   16
0   22  15  5   22  9   12
26  3   23  3   20  11  5
4   19  11  23  7   6   21
11  23  20  25  28  13  27
8   12  14  9   13  13  15
25  5   0   11  3   23  20
14  6   13  0   19  23  22
26  21  8   15  24  8   15
1   3   15  27  5   12  1
13  25  13  10  24  20  6
0   4   1   12  6   20  26
17  18  13  12  21  11  8
17  10  5   13  6   5   15
8   2   17  3   26  4   8
1   24  14  2   27  16  7
3   14  21  7   13  3   25
17  9   27  2   4   24  17
5   8   15  23  11  6   10
```

5. Mutation

Mutation introduces random changes, maintaining genetic diversity and preventing premature convergence. If the optimal solution is not achieved, the process repeats from steps 2 to 4 until convergence or a predefined termination condition is met.

By using crossover we get

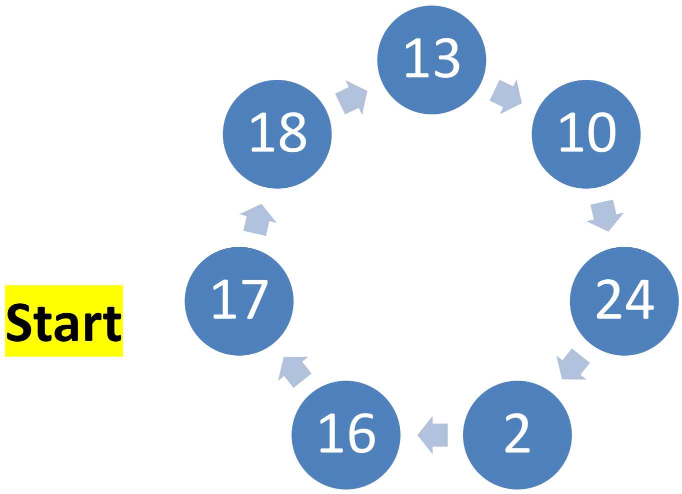
```
Mutation
7
    17

The row number is7

The max element in that row is25
 The position of mutation  in that row is
0

The row number is17

The max element in that row is27
 The position of mutation  in that row is
2
```

6. <u>The optimal tour:</u>

After completed all the steps we get the cost of each tour and the final optimal tour is the minimum of the each tour cost that in known as the optimal tour cost.

```
cost is of :  2833
cost is of :  2558
cost is of :  3370
cost is of :  2845
cost is of :  2945
cost is of :  3028
cost is of :  2747
cost is of :  3079
cost is of :  2948
cost is of :  3045
cost is of :  2770
cost is of :  2592
cost is of :  2945
cost is of :  2376
cost is of :  2925
cost is of :  2882
cost is of :  2384
cost is of :  2462
cost is of :  2799
cost is of :  2992
cost is of :  2834
cost is of :  2087
cost is of :  2847
cost is of :  2897
cost is of :  2836
cost is of :  2897
cost is of :  3120
cost is of :  2791
cost is of :  2263

Minimum Optimal Cost : 2087.
```

Hence, the optimal tour is given by according to the path of minimum cost:



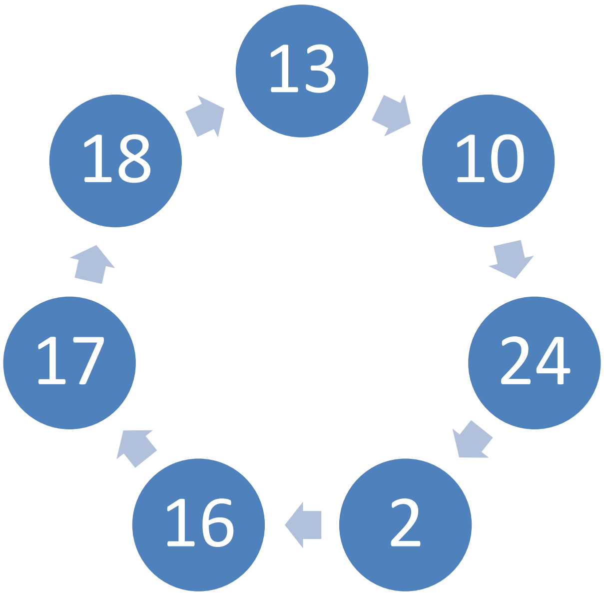
Hence we get according to source code the optimal tour cost = 2087

Result according to Model formulations:

The total optimal cost according to model formulation is given by,

$$\boldsymbol{Total\ cost = cost\ of\ ring\ nodes + \ cost\ of\ high\ density\ nodes}$$
$$\boldsymbol{+ cost\ of\ low\ density\ nodes}$$

Since the optimal tour according to the source code is given by



Now first we have to find the ring cost.
According to the given standard 29×29 matrix we have to find the cost between the nodes,

Node address	Paths of the nodes	cost
1	17-18	105
2	18-13	105
3	13-10	64
4	10-24	88
5	24-2	322
6	2-16	274
7	16-17	110

So we get the total ring cost,

$$\textbf{The total ring cost} = \mathbf{105 + 105 + 64 + 88 + 322 + 274 + 110 = 1068}$$

Now

We have to find the cost of ring nodes within the radius,

For these nodes we divides in to two types

1. High density nodes
2. Low density nodes.

The nodes of high density follow the Euclidean distance.

The nodes of low density follow the Manhattan distance.

Now we have to find these high density and low density nodes.

Since here

Here the fixed radius according to source code $\boldsymbol{R = 100}$

Using the concept of nodes within the radius $\boldsymbol{R}$ **,**

The number of nodes within the radius $\boldsymbol{r\ is\ n = 2}$,

$$\boldsymbol{n_1 = 3\,, n_2 = 4}$$

Since $\boldsymbol{n \leq 3}$

So these two nodes having low density.

$$\boldsymbol{The\ cost\ of\ low\ density\ nodes}$$
$$\boldsymbol{= \sum_{i=1}^{2} a_i \times (n_i \times 0.85 \times R)}$$
$$\boldsymbol{= 1 \times (n_1 \times 0.85 \times 100) + 1 \times (n_2 \times 0.85 \times 100) = 7 \times 85}$$
$$\boldsymbol{= 595}$$

If it is greater than 3 and the number of nodes within the radius **r** is $\boldsymbol{k = 0}$ that are low density nodes then,

$$\boldsymbol{k_1 = k_2 = k_3 = k_4 = k_5 = 0}$$

$$The\ cost\ of\ high\ density\ nodes$$

$$= \sum_{j=1}^{5} a_i \times (k_i \times 0.67 \times R) = 1.(k_1 \times 0.67 \times 100) + 1.(k_2 \times 0.67 \times 100) + 1.(k_3 \times 0.67 \times 100) + 1.(k_4 \times 0.67 \times 100) + 1.(k_5 \times 0.67 \times 100) = 67 \times 0 = 0$$

So the total cost is given by

$$Total\ cost = distance\ of\ ring\ nodes + distance\ of\ high\ dense\ nodes + distance\ of\ low\ nodes = 1068 + 0 + 595 = 1663$$

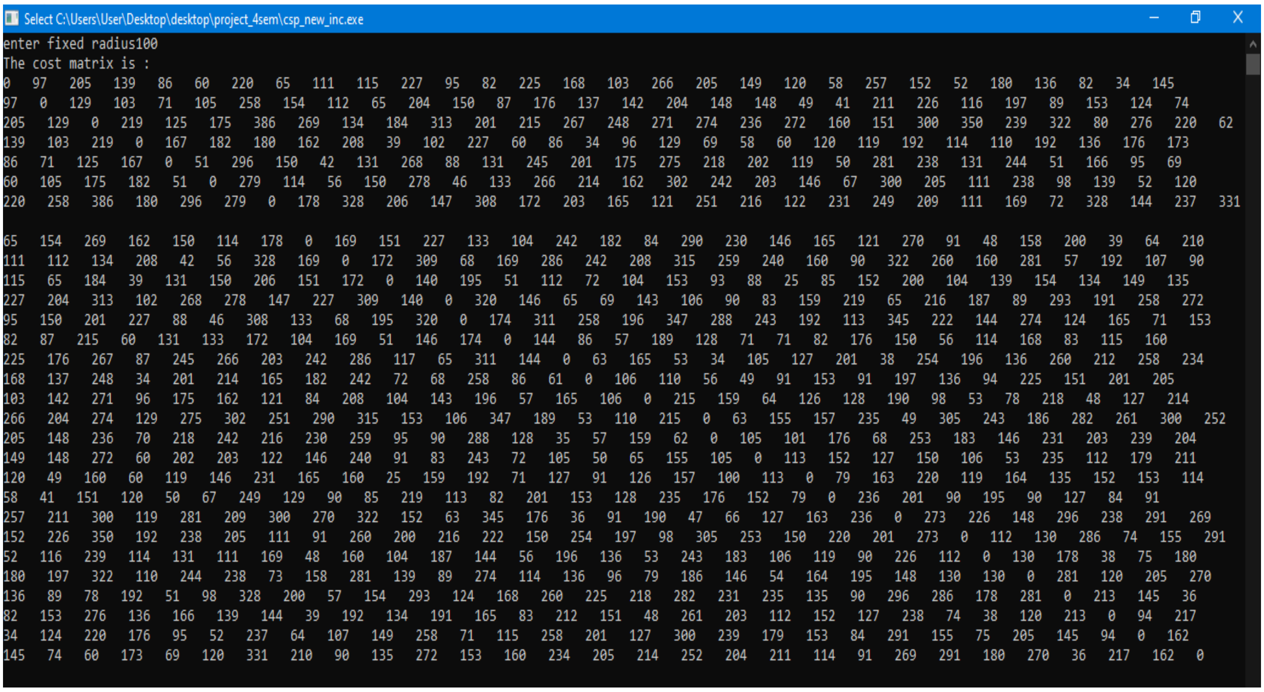
Now we have to change some entries of the cost matrix and then analyse the cost of each section.

• RESULT ACCORDING TO THE MINOR INCREASING IN THE COST MATRIX

Result according to source code

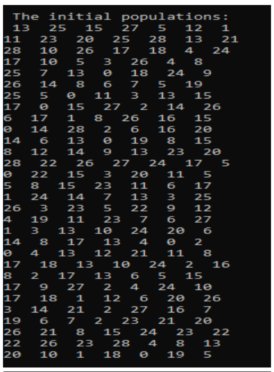
According to the previous result we put only results instead of all explanations:

1. **Initialization:**



```
Select C:\Users\User\Desktop\desktop\project_4sem\csp_new_inc.exe
enter fixed radius100
The cost matrix is :
0   97   205   139   86   60   220   65   111   115   227   95   82   225   168   103   266   205   149   120   58   257   152   52   180   136   82   34   145
97   0   129   103   71   105   258   154   112   65   204   150   87   176   137   142   204   148   148   49   41   211   226   116   197   89   153   124   74
205   129   0   219   125   175   386   269   134   184   313   201   215   267   248   271   274   236   272   160   151   300   350   239   322   80   276   220   62
139   103   219   0   167   182   180   162   208   39   102   227   60   86   34   96   129   69   58   60   120   119   192   114   110   192   136   176   173
86   71   125   167   0   51   296   150   42   131   268   88   131   245   201   175   275   218   202   119   50   281   238   131   244   51   166   95   69
60   105   175   182   51   0   279   114   56   150   278   46   133   266   214   162   302   242   203   146   67   300   205   111   238   98   139   52   120
220   258   386   180   296   279   0   178   328   206   147   308   172   203   165   121   251   216   122   231   249   209   111   169   72   328   144   237   331

65   154   269   162   150   114   178   0   169   151   227   133   104   242   182   84   290   230   146   165   121   270   91   48   158   200   39   64   210
111   112   134   208   42   56   328   169   0   172   309   68   169   286   242   208   315   259   240   160   90   322   260   160   281   57   192   107   90
115   65   184   39   131   150   206   151   172   0   140   195   51   112   72   104   153   93   88   25   85   152   200   104   139   154   134   149   135
227   204   313   102   268   278   147   227   309   140   0   320   146   65   69   143   106   90   83   159   219   65   216   187   89   293   191   258   272
95   150   201   227   88   46   308   133   68   195   320   0   174   311   258   196   347   288   243   192   113   345   222   144   274   124   165   71   153
82   87   215   60   131   133   172   104   169   51   146   174   0   144   86   57   189   128   71   71   82   176   150   56   114   168   83   115   160
225   176   267   87   245   266   203   242   286   117   65   311   144   0   63   165   53   34   105   127   201   38   254   196   136   260   212   258   234
168   137   248   34   201   214   165   182   242   72   68   258   86   61   0   106   110   56   49   91   153   91   197   136   94   225   151   201   205
103   142   271   96   175   162   121   84   208   104   143   196   57   165   106   0   215   159   64   126   128   190   98   53   78   218   48   127   214
266   204   274   129   275   302   251   290   315   153   106   347   189   53   110   215   0   63   155   157   235   49   305   243   186   282   261   300   252
205   148   236   70   218   242   216   230   259   95   90   288   128   35   57   159   62   0   105   101   176   68   253   183   146   231   203   239   204
149   148   272   60   202   203   122   146   240   91   83   243   72   105   50   65   155   105   0   113   152   127   150   106   53   235   112   179   211
120   49   160   60   119   146   231   165   160   25   159   192   71   127   91   126   157   100   113   0   79   163   220   119   164   135   152   153   114
58   41   151   120   50   67   249   129   90   85   219   113   82   201   153   128   235   176   152   79   0   236   201   90   195   90   127   84   91
257   211   300   119   281   209   300   270   322   152   63   345   176   36   91   190   47   66   127   163   236   0   273   226   148   296   238   291   269
152   226   350   192   238   205   111   91   260   200   216   222   150   254   197   98   305   253   150   220   201   273   0   112   130   286   74   155   291
52   116   239   114   131   111   169   48   160   104   187   144   56   196   136   53   243   183   106   119   90   226   112   0   130   178   38   75   180
180   197   322   110   244   238   73   158   281   139   89   274   114   136   96   79   186   146   54   164   195   148   130   130   0   281   120   205   270
136   89   78   192   51   98   328   200   57   154   293   124   168   260   225   218   282   231   235   135   90   296   286   178   281   0   213   145   36
82   153   276   136   166   139   144   39   192   134   191   165   83   212   151   48   261   203   112   152   127   238   74   38   120   213   0   94   217
34   124   220   176   95   52   237   64   107   149   258   71   115   258   201   127   300   239   179   153   84   291   155   75   205   145   94   0   162
145   74   60   173   69   120   331   210   90   135   272   153   160   234   205   214   252   204   211   114   91   269   291   180   270   36   217   162   0
```

2. **Selection:**



```
 The initial populations:
 13   25   15   27   5   12   1
11   23   20   25   28   13   21
28   10   26   17   18   4   24
17   10   5   3   26   4   8
25   7   13   0   18   24   9
26   14   8   6   7   5   19
25   5   0   11   3   13   15
17   0   15   27   2   14   26
6   17   1   8   26   16   15
0   14   28   2   6   16   20
14   6   13   0   19   8   15
8   12   14   9   13   23   20
28   22   26   27   24   17   5
0   22   15   3   20   11   5
5   8   15   23   11   6   17
1   24   14   7   13   3   25
26   3   23   5   22   9   12
4   19   11   23   7   6   27
1   3   13   10   24   20   6
14   8   17   13   4   0   2
0   4   13   12   21   11   8
17   18   13   10   24   2   16
8   2   17   13   6   5   15
17   9   27   2   4   24   10
17   18   1   12   6   20   26
3   14   21   2   27   16   7
19   6   7   2   23   21   20
26   21   8   15   24   23   22
22   26   23   28   4   8   13
20   10   1   18   0   19   5
```

3. Crossover and mutation:

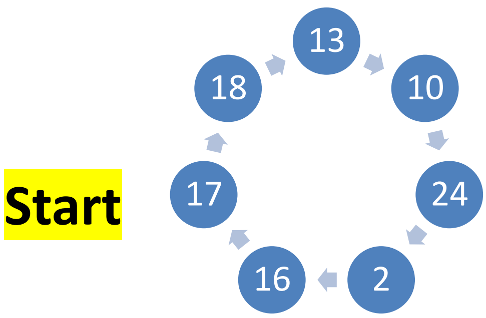
```
17   18   13   10   24   19   5
20   10   1    18   0    2    16
22   26   23   15   24   23   22
26   21   8    28   4    8    13
19   6    7    2    23   21   7
3    14   21   2    27   16   20
17   18   1    12   6    24   10
17   9    27   2    4    20   26
8    2    17   13   6    11   8
0    4    13   12   21   5    15
14   8    13   10   24   20   6
1    3    17   13   4    0    2
4    19   23   5    22   9    12
26   3    11   23   7    6    27
1    24   14   23   11   6    17
5    8    15   7    13   3    25
0    22   15   27   24   17   5
28   22   26   3    20   11   5
8    12   14   9    13   23   15
14   6    13   0    19   8    20
```

```
 Mutation
7
     17

 The row number is7

 The max element in that row is27
  The position of mutation  in that row is
 0

 The row number is17

 The max element in that row is28
  The position of mutation  in that row is
 2
```

4. The optimal tour:

```
cost is  :  2840
cost is  :  2563
cost is  :  3292
cost is  :  2854
cost is  :  2957
cost is  :  3028
cost is  :  2756
cost is  :  3085
cost is  :  2958
cost is  :  3051
cost is  :  2775
cost is  :  2602
cost is  :  2861
cost is  :  2376
cost is  :  2837
cost is  :  2892
cost is  :  2384
cost is  :  2462
cost is  :  2816
cost is  :  3005
cost is  :  2836
cost is  :  2040
cost is  :  2766
cost is  :  2917
cost is  :  2844
cost is  :  2907
cost is  :  3120
cost is  :  2797
cost is  :  2273

Minimum Optimal Cost  :  2040.
```

Hence, the optimal tour is given by according to the path of minimum cost



Hence we get according to source code the optimal tour cost = 2040

Result according to Model formulations:

The total optimal cost according to model formulation is given by,

$$\boldsymbol{Total\ cost = cost\ of\ ring\ nodes + \ cost\ of\ high\ density\ nodes}$$
$$\boldsymbol{+ cost\ of\ low\ density\ nodes}$$

Now first we have to find the ring cost.
According to the given slandered 20×29 matrix we have to find the cost between the nodes,

Node address	Paths of the nodes	cost
1	17-18	106
2	18-13	106
3	13-10	65
4	10-24	90
5	24-2	323
6	2-16	275
7	16-17	112

So we get the total ring cost,

$$\mathbf{The\ total\ ring\ cost = 106 + 106 + 65 + 90 + 323 + 275 + 112 = 1077}$$

Now

We have to find the cost of ring nodes within the radius,

For these nodes we divides in to two types

1. High density nodes
2. Low density nodes.

The nodes of high density follow the Euclidean distance.

The nodes of low density follow the Manhattan distance.

Now we have to find these high density and low density nodes.

Since here

Here the fixed radius according to source code $\boldsymbol{R = 100}$

Using the concept of nodes within the radius $\boldsymbol{R}$ **,**

The number of nodes within the radius $\boldsymbol{r\ is\ n = 2}$,

$$\boldsymbol{n_1 = 3\ , n_2 = 4}$$

Since $\boldsymbol{n \leq 3}$

So these two nodes having low density.

$$\boldsymbol{The\ cost\ of\ low\ density\ nodes}$$
$$\boldsymbol{= \sum_{i=1}^{2} a_i \times (n_i \times 0.85 \times R)}$$
$$\boldsymbol{= (n_1 \times 0.85 \times 100) + (n_2 \times 0.85 \times 100) = 7 \times 85 = 595}$$

If it is greater than 3 and the number of nodes within the radius **r** is $\boldsymbol{k = 5}$ that are low density nodes then,

$$\boldsymbol{k_1 = k_2 = k_3 = k_4 = k_5 = 0}$$

$$\boldsymbol{The\ cost\ of\ high\ density\ nodes}$$
$$\boldsymbol{= \sum_{i=1}^{5} a_i \times (k_i \times 0.67 \times R) = (k_1 \times 0.67 \times 100)}$$
$$\boldsymbol{+ (k_2 \times 0.67 \times 100) + (k_3 \times 0.67 \times 100)}$$
$$\boldsymbol{+ (k_4 \times 0.67 \times 100) + (k_5 \times 0.67 \times 100) = 67 \times 0 = 0}$$

So the total cost is given by

$$\boldsymbol{Total\ cost = distance\ of\ ring\ nodes}$$
$$\boldsymbol{+ distance\ of\ high\ dense\ nodes}$$
$$\boldsymbol{+ distance\ of\ low\ nodes = 1077 + 0 + 595 = 1672}$$

- # RESULT ACCORDING TO THE MINOR DECREASING IN THE COST MATRIX

Result according to source code

According to the previous result we put only results instead of all explainations:

1. Initialization:

```
Select C:\Users\User\Desktop\desktop\project_4sem\csp_new_dec.exe
enter fixed radius100
The cost matrix is :
0   97   205   139   86   60   220   65   111   115   227   95   82   225   168   103   266   205   149   120   58   257   152   52   180   136   82   34   145
97   0   129   103   71   105   258   154   112   65   204   150   87   176   137   142   204   148   148   49   41   211   226   116   197   89   153   124   74
205   129   0   219   125   175   386   269   134   184   313   201   215   267   248   271   274   236   272   160   151   300   350   239   322   77   276   220   59
139   103   219   0   167   182   180   162   208   39   102   227   60   86   34   96   129   69   58   60   120   119   192   114   110   192   136   176   173
86   71   125   167   0   51   296   150   42   131   268   88   131   245   201   175   275   218   202   119   50   281   238   131   244   51   166   95   69
60   105   175   182   51   0   279   114   56   150   278   46   133   266   214   162   302   242   203   146   67   300   205   111   238   98   139   52   120
220   258   386   180   296   279   0   178   328   206   147   308   172   203   165   121   251   216   122   231   249   209   111   169   72   328   144   237   331

65   154   269   162   150   114   178   0   169   151   227   133   104   242   182   84   290   230   146   165   121   270   91   48   158   200   39   64   210
111   112   134   208   42   56   328   169   0   172   309   68   169   286   242   208   315   259   240   160   90   322   260   160   281   57   192   107   90
115   65   184   39   131   150   206   151   172   0   140   195   51   112   72   104   153   93   88   25   85   152   200   104   139   154   134   149   135
227   204   313   102   268   278   147   227   309   140   0   320   146   62   66   143   106   85   80   159   219   61   216   187   86   293   191   258   272
95   150   201   227   88   46   308   133   68   195   320   0   174   311   258   196   347   288   243   192   113   345   222   144   274   124   165   71   153
82   87   215   60   131   133   172   104   169   51   146   174   0   144   86   57   189   128   71   71   82   176   150   56   114   168   83   115   160
225   176   267   85   245   266   203   242   286   117   62   311   144   0   60   165   50   30   105   127   201   35   254   196   136   260   212   258   234
168   137   248   34   201   214   165   182   242   72   68   258   86   61   0   106   110   56   49   91   153   91   197   136   94   225   151   201   205
103   142   271   96   175   162   121   84   208   104   143   196   57   165   106   0   215   159   64   126   128   190   98   53   78   218   48   127   214
266   204   274   129   275   302   251   290   315   153   106   347   189   50   110   215   0   60   155   157   235   45   305   243   186   282   261   300   252
205   148   236   67   218   242   216   230   259   91   87   288   128   30   55   159   59   0   105   98   176   65   253   183   146   231   203   239   204
149   148   272   57   202   203   122   146   240   87   80   243   70   105   48   63   155   105   0   113   152   127   150   106   50   235   112   179   211
120   49   160   60   119   146   231   165   160   25   159   192   71   127   91   126   157   100   113   0   79   163   220   119   164   135   152   153   114
58   41   151   120   50   67   249   129   90   85   219   113   82   201   153   128   235   176   152   79   0   236   201   90   195   90   127   84   91
257   211   300   119   281   209   300   270   322   152   63   345   176   36   91   190   47   66   127   163   236   0   273   226   148   296   238   291   269
152   226   350   192   238   205   111   91   260   200   216   222   150   254   197   98   305   253   150   220   201   273   0   112   130   286   74   155   291
52   116   239   114   131   111   169   48   160   104   187   144   56   196   136   53   243   183   106   119   90   226   112   0   130   178   38   75   180
180   197   322   110   244   238   71   158   281   139   87   274   114   136   93   76   186   146   50   164   195   148   130   130   0   281   120   205   270
136   89   78   192   51   98   328   200   57   154   293   124   168   260   225   218   282   231   235   135   90   296   286   178   281   0   213   145   36
82   153   276   136   166   139   144   39   192   134   191   165   83   212   151   48   261   203   112   152   127   238   74   38   120   213   0   94   217
34   124   220   176   95   52   237   64   107   149   258   71   115   258   201   127   300   239   179   153   84   291   155   75   205   145   94   0   162
145   74   60   173   69   120   331   210   90   135   272   153   160   234   205   214   252   204   211   114   91   269   291   180   270   36   217   162   0
```

2. Selection:

```
 The initial populations:
 13  25  15  27  5  12  1
11  23  20  25  28  13  21
28  10  26  17  18  4  24
17  10  5  3  26  4  8
25  7  13  0  18  24  9
26  14  8  6  7  5  19
25  5  0  11  3  13  15
17  0  15  27  2  14  26
6  17  1  8  26  16  15
0  14  28  2  6  16  20
14  6  13  0  19  8  15
8  12  14  9  13  23  20
28  22  26  27  24  17  5
0  22  15  3  20  11  5
5  8  15  23  11  6  17
1  24  14  7  13  3  25
26  3  23  5  22  9  12
4  19  11  23  7  6  27
1  3  13  10  24  20  6
14  8  17  13  4  0  2
0  4  13  12  21  11  8
17  18  13  10  24  2  16
8  2  17  13  6  5  15
17  9  27  2  4  24  10
17  18  1  12  6  20  26
3  14  21  2  27  16  7
19  6  7  2  23  21  20
26  21  8  15  24  23  22
22  26  23  28  4  8  13
20  10  1  18  0  19  5
```

3. Crossover:

```
17   18   13   10   24   19   5
20   10   1   18   0   2   16
22   26   23   15   24   23   22
26   21   8   28   4   8   13
19   6   7   2   23   21   7
3   14   21   2   27   16   20
17   18   1   12   6   24   10
17   9   27   2   4   20   26
8   2   17   13   6   11   8
0   4   13   12   21   5   15
14   8   13   10   24   20   6
1   3   17   13   4   0   2
4   19   23   5   22   9   12
26   3   11   23   7   6   27
1   24   14   23   11   6   17
5   8   15   7   13   3   25
0   22   15   27   24   17   5
28   22   26   3   20   11   5
8   12   14   9   13   23   15
14   6   13   0   19   8   20
```

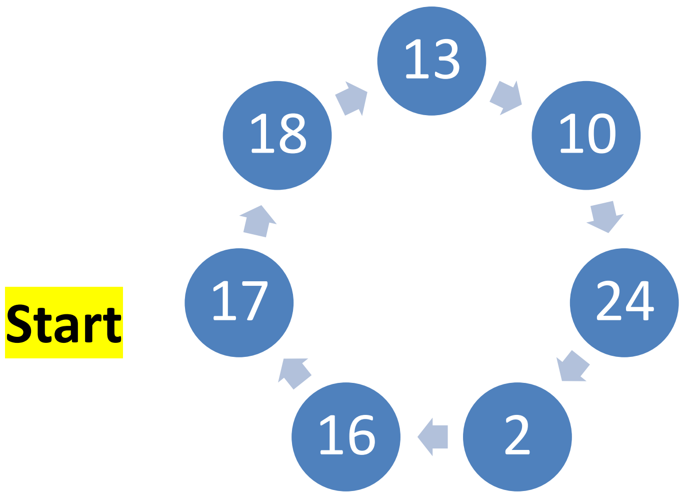
4. Mutation

```
Mutation
7
    17

The row number is7

The max element in that row is27
 The position of mutation  in that row is
0

The row number is17

The max element in that row is28
 The position of mutation  in that row is
2
```

5. The optimal tour:

```
cost is   :   2827
cost is   :   2555
cost is   :   3348
cost is   :   2835
cost is   :   2934
cost is   :   3028
cost is   :   2740
cost is   :   3074
cost is   :   2940
cost is   :   3041
cost is   :   2766
cost is   :   2584
cost is   :   2927
cost is   :   2376
cost is   :   2912
cost is   :   2874
cost is   :   2384
cost is   :   2462
cost is   :   2779
cost is   :   2974
cost is   :   2833
cost is   :   2060
cost is   :   2831
cost is   :   2871
cost is   :   2830
cost is   :   2891
cost is   :   3120
cost is   :   2786
cost is   :   2255

Minimum Optimal Cost : 2060
```

Hence, the optimal tour is given by according to the path of minimum cost



Hence we get according to source code the optimal tour cost = 2060

Result according to Model formulations:

The total optimal cost according to model formulation is given by,

$$\boldsymbol{Total\ cost = cost\ of\ ring\ nodes + \ cost\ of\ high\ density\ nodes}$$
$$\boldsymbol{+ cost\ of\ low\ density\ nodes}$$

Now first we have to find the ring cost.
According to the given slandered 20×29 matrix we have to find the cost between the nodes,

Node address	Paths of the nodes	cost
1	17-18	100
2	18-13	100
3	13-10	60
4	10-24	85
5	24-2	320
6	2-16	270
7	16-17	105

So we get the total ring cost,

$$\mathbf{The\ total\ ring\ cost} = \mathbf{100 + 100 + 60 + 85 + 320 + 270 + 105 = 1040}$$

Here the fixed radius according to source code $\boldsymbol{R = 100}$
Using the concept of nodes within the radius $\boldsymbol{R}$ **,**

The number of nodes within the radius $\boldsymbol{r\ is\ n = 4}$,

$$\boldsymbol{n_1 = 1\ , n_2 = 2\ , n_3 = 3\ , n_4 = 4}$$

Since $\boldsymbol{n \geq 3}$

So these two nodes having high density.

$$\boldsymbol{The\ cost\ of\ high\ density\ nodes} \\ = \sum_{i=1}^{4} a_i(n_i \times 0.67 \times R) \\ = (n_1 \times 0.67 \times 100) + (n_2 \times 0.67 \times 100) \\ + (n_3 \times 0.67 \times 100) + (n_4 \times 0.67 \times 100) = 10 \times 67 = 670$$

If it is less than 3 and the number of nodes within the radius **r** is $k = 5$ that are low density nodes then,

$$k_5 = k_6 = k_7 = 0$$

$$\boldsymbol{The\ cost\ of\ high\ density\ nodes} \\ = \sum_{i=1}^{3} a_i(k_i \times 0.85 \times R) = (k_1 \times 0.85 \times 100) \\ + (k_2 \times 0.85 \times 100) + (k_3 \times 0.85 \times 100) = 85 \times 0 = 0$$

So the total cost is given by

$$\boldsymbol{Total\ cost = distance\ of\ ring\ nodes} \\ \boldsymbol{+ distance\ of\ high\ dense\ nodes} \\ \boldsymbol{+ distance\ of\ low\ nodes} = 1040 + 670 + 0 = 1710$$

• RESULT ACCORDING TO RANDOMLY CHANGE IN THE COST MATRIX

Result according to source code

According to the previous result we put only results instead of all explainations:

1. **Initialization:**

```
Select C:\Users\User\Desktop\desktop\project_4sem\csp_new_random.exe
enter fixed radius100
The cost matrix is :
0   97   205   139   86   60   220   65   111   115   227   95   82   225   168   103   266   205   149   120   58   257   152   52   180   136   82   34   145
97   0   129   103   71   105   258   154   112   65   204   150   87   176   137   142   204   148   148   49   41   211   226   116   197   89   153   124   74
205   129   0   219   125   175   386   269   134   184   313   201   215   267   248   271   274   236   272   160   151   300   350   239   322   80   276   220   61
139   103   219   0   167   182   180   162   208   39   102   227   60   86   34   96   129   69   58   60   120   119   192   114   110   192   136   176   173
86   71   125   167   0   51   296   150   42   131   268   88   131   245   201   175   275   218   202   119   50   281   238   131   244   51   166   95   69
60   105   175   182   51   0   279   114   56   150   278   46   133   266   214   162   302   242   203   146   67   300   205   111   238   98   139   52   120
220   258   386   180   296   279   0   178   328   206   147   308   172   203   165   121   251   216   122   231   249   209   111   169   72   328   144   237   331

65   154   269   162   150   114   178   0   169   151   227   133   104   242   182   84   290   230   146   165   121   270   91   48   158   200   39   64   210
111   112   134   208   42   56   328   169   0   172   309   68   169   286   242   208   315   259   240   160   90   322   260   160   281   57   192   107   90
115   65   184   39   131   150   206   151   172   0   140   195   51   112   72   104   153   93   88   25   85   152   200   104   139   154   134   149   135
227   204   313   102   268   278   147   227   309   140   0   320   146   65   69   143   106   89   83   159   219   61   216   187   85   293   191   258   272
95   150   201   227   88   46   308   133   68   195   320   0   174   311   258   196   347   288   243   192   113   345   222   144   274   124   165   71   153
82   87   215   60   131   133   172   104   169   51   146   174   0   144   86   57   189   128   71   71   82   176   150   56   114   168   83   115   160
225   176   267   87   245   266   203   242   286   117   65   311   144   0   60   165   50   30   105   127   201   35   254   196   136   260   212   258   234
168   137   248   34   201   214   165   182   242   72   68   258   86   61   0   106   110   56   49   91   153   91   197   136   94   225   151   201   205
103   142   271   96   175   162   121   84   208   104   143   196   57   165   106   0   215   159   64   126   128   190   98   53   78   218   48   127   214
266   204   274   129   275   302   251   290   315   153   106   347   189   53   110   215   0   62   155   157   235   48   305   243   186   282   261   300   252
205   148   236   70   218   242   216   230   259   95   89   288   128   31   55   159   60   0   105   100   176   65   253   183   146   231   203   239   204
149   148   272   60   202   203   122   146   240   87   80   243   70   105   50   65   155   105   0   113   152   127   150   106   53   235   112   179   211
120   49   160   60   119   146   231   165   160   25   159   192   71   127   91   126   157   100   113   0   79   163   220   119   164   135   152   153   114
58   41   151   120   50   67   249   129   90   85   219   113   82   201   153   128   235   176   152   79   0   236   201   90   195   90   127   84   91
257   211   300   119   281   209   300   270   322   152   63   345   176   36   91   190   47   66   127   163   236   0   273   226   148   296   238   291   269
152   226   350   192   238   205   111   91   260   200   216   222   150   254   197   98   305   253   150   220   201   273   0   112   130   286   74   155   291
52   116   239   114   131   111   169   48   160   104   187   144   56   196   136   53   243   183   106   119   90   226   112   0   130   178   38   75   180
180   197   322   110   244   238   75   158   281   139   89   274   114   136   95   75   186   146   53   164   195   148   130   130   0   281   120   205   270
136   89   78   192   51   98   328   200   57   154   293   124   168   260   225   218   282   231   235   135   90   296   286   178   281   0   213   145   36
82   153   276   136   166   139   144   39   192   134   191   165   83   212   151   48   261   203   112   152   127   238   74   38   120   213   0   94   217
34   124   220   176   95   52   237   64   107   149   258   71   115   258   201   127   300   239   179   153   84   291   155   75   205   145   94   0   162
145   74   60   173   69   120   331   210   90   135   272   153   160   234   205   214   252   204   211   114   91   269   291   180   270   36   217   162   0
```

2. Selection:

```
 The initial populations:
 13  25  15  27  5  12  1
11  23  20  25  28  13  21
28  10  26  17  18  4  24
17  10  5  3  26  4  8
25  7  13  0  18  24  9
26  14  8  6  7  5  19
25  5  0  11  3  13  15
17  0  15  27  2  14  26
6  17  1  8  26  16  15
0  14  28  2  6  16  20
14  6  13  0  19  8  15
8  12  14  9  13  23  20
28  22  26  27  24  17  5
0  22  15  3  20  11  5
5  8  15  23  11  6  17
1  24  14  7  13  3  25
26  3  23  5  22  9  12
4  19  11  23  7  6  27
1  3  13  10  24  20  6
14  8  17  13  4  0  2
0  4  13  12  21  11  8
17  18  13  10  24  2  16
8  2  17  13  6  5  15
17  9  27  2  4  24  10
17  18  1  12  6  20  26
3  14  21  2  27  16  7
19  6  7  2  23  21  20
26  21  8  15  24  23  22
22  26  23  28  4  8  13
20  10  1  18  0  19  5
```

3. Crossover:

```
17   18   13   10   24   19   5
20   10   1    18   0    2    16
22   26   23   15   24   23   22
26   21   8    28   4    8    13
19   6    7    2    23   21   7
3    14   21   2    27   16   20
17   18   1    12   6    24   10
17   9    27   2    4    20   26
8    2    17   13   6    11   8
0    4    13   12   21   5    15
14   8    13   10   24   20   6
1    3    17   13   4    0    2
4    19   23   5    22   9    12
26   3    11   23   7    6    27
1    24   14   23   11   6    17
5    8    15   7    13   3    25
0    22   15   27   24   17   5
28   22   26   3    20   11   5
8    12   14   9    13   23   15
14   6    13   0    19   8    20
```

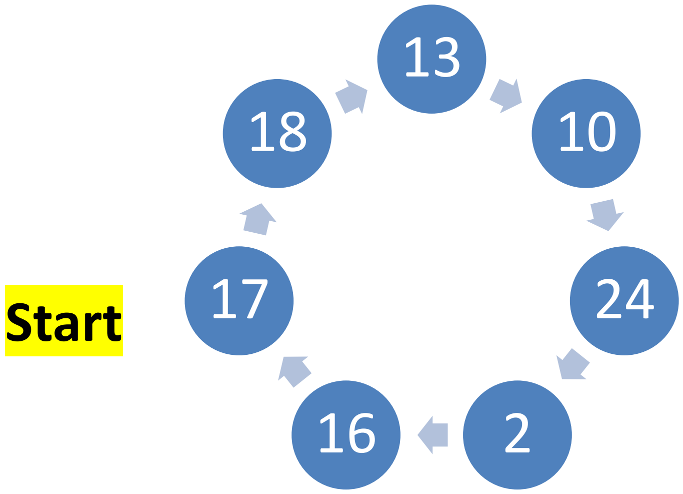
4. Mutation

```
 Mutation
7
    17

 The row number is7

 The max element in that row is27
  The position of mutation  in that row is
 0

 The row number is17

 The max element in that row is28
  The position of mutation  in that row is
 2
```

5. The optimal tour:

```
cost is  :  2830
cost is  :  2557
cost is  :  3372
cost is  :  2842
cost is  :  2945
cost is  :  3028
cost is  :  2743
cost is  :  3080
cost is  :  2951
cost is  :  3049
cost is  :  2769
cost is  :  2589
cost is  :  2946
cost is  :  2376
cost is  :  2925
cost is  :  2887
cost is  :  2384
cost is  :  2462
cost is  :  2800
cost is  :  2992
cost is  :  2833
cost is  :  2092
cost is  :  2844
cost is  :  2903
cost is  :  2832
cost is  :  2904
cost is  :  3120
cost is  :  2797
cost is  :  2260

 Minimum Optimal Cost : 2092.000000
--------------------------------
Process exited after 25.59 seconds with return value 0
Press any key to continue . . .
```

Hence, the optimal tour is given by according to the path of minimum cost



Hence we get according to source code the optimal tour cost = 2092

Result according to Model formulations:

The total optimal cost according to model formulation is given by,

$$\boldsymbol{Total\ cost = cost\ of\ ring\ nodes +\ cost\ of\ high\ density\ nodes + cost\ of\ low\ density\ nodes}$$

Now first we have to find the ring cost.
According to the given slandered 20×29 matrix we have to find the cost between the nodes,

Node address	Paths of the nodes	cost
1	17-18	104
2	18-13	104
3	13-10	60
4	10-24	85
5	24-2	320
6	2-16	275
7	16-17	115

So we get the total ring cost,

$$\mathbf{The\ total\ ring\ cost = 104 + 104 + 60 + 85 + 320 + 275 + 115 = 1063}$$

Since here
Here the fixed radius according to source code $\boldsymbol{R = 100}$
Using the concept of nodes within the radius $\boldsymbol{R}$ **,**

The number of nodes within the radius $\boldsymbol{r\ is\ n = 2}$,

$$\boldsymbol{n_1 = 3}\ ,\boldsymbol{n_2 = 4}$$

Since $\boldsymbol{n \leq 3}$

So these two nodes having low density.

$$\boldsymbol{The\ cost\ of\ low\ density\ nodes}$$
$$= \sum_{i=1}^{2} \boldsymbol{a_i(n_i \times 0.85 \times R)}$$
$$= \boldsymbol{(n_1 \times 0.85 \times 100) + (n_2 \times 0.85 \times 100) = 7 \times 85 = 595}$$

If it is greater than 3 and the number of nodes within the radius **r** is $\boldsymbol{k = 5}$ that are low density nodes then,

$$\boldsymbol{k_1 = \ k_2 = k_5 = k_6 = \ k_7 = 0}$$

$$\boldsymbol{The\ cost\ of\ high\ density\ nodes}$$
$$= \sum_{i=1}^{5} \boldsymbol{a_i(k_i \times 0.67 \times R) = \ (k_1 \times 0.67 \times 100)}$$
$$\boldsymbol{+ (k_2 \times 0.67 \times 100) + (k_3 \times 0.67 \times 100)}$$
$$\boldsymbol{+ (k_4 \times 0.67 \times 100) + (k_5 \times 0.67 \times 100) = 67 \times 0 = 0}$$

So the total cost is given by

$$\boldsymbol{Total\ cost = distance\ of\ ring\ nodes}$$
$$\boldsymbol{+ distance\ of\ high\ dense\ nodes}$$
$$\boldsymbol{+ distance\ of\ low\ nodes = 1063 + 595 + 0 = 1658}$$

Final results according to the whole work:

In our work after 1000 iterations are the different nodes on the ring and the number of nodes covered by the nodes on the ring represented by this table:

Nodes on ring	No. of nodes covered	High densed region	Low densed region
17	3	-	✓
0	9	✓	-
15	1	-	✓
27	3	-	✓
2	0	-	✓
14	5	✓	-
26	1	-	✓

And the total cost = 2484.00

5. CONCLUSIONS

1. Here, a set of conveyances is available at the above mentioned nodes and the salesman uses the appropriate conveyance for minimum cost.
2. Thus the problem reduces to finding the optimal covering set of nodes and the proper conveyance for travel so that total travel cost is minimum.
3. The proposed problem can be extended to include a restriction on some particular nodes for inclusion (due to some factors).
4. The proposed algorithm also can be tested with other types of uncertain parameters.
5. By using all process that we cover in the previous slides we found the shortest route.
6. As in real life different path options are available in high dense nodes, so Euclidean distance is calculated, but other types of representation of distances, costs considering external factors can be considered in future study.

6. FUTURE SCOPE

In real life, suppose a production unit establishes its distribution centers, but in a span of 50 years of population density is changed, now road connectivity is established.

In this case Low density areas become high density area.

In own work we have incorporated the projected development and new nodes to supply commodity can be covered by previously taken distribution point, only the density of the region will be changed.

In the real-world scenario described, the evolution of population density over 50 years has led to significant changes in the landscape, particularly affecting road connectivity and distribution logistics. Initially low-density areas have transformed into high-density zones, necessitating a re-evaluation of distribution strategies for the production unit. However, the infrastructure previously established, including distribution centers, remains intact.

To adapt to these changes, the production unit must incorporate projected developments and new nodes into its distribution network. By leveraging advanced forecasting and planning techniques, it can optimize the utilization of existing distribution points to serve the evolving needs of the population. This means that while the physical locations of distribution centers remain unchanged, their operational scope and efficiency are adjusted to accommodate the shifting density patterns.

Essentially, the adaptation process involves recalibrating distribution routes and resource allocation based on the altered demographic landscape. Through strategic planning and flexibility, the production unit can continue to effectively supply commodities to the newly high-density areas, ensuring responsiveness to changing market dynamics while maximizing the utility of existing infrastructure.

7. APPLICATIONS OF CSP

The Covering Salesman Problem (CSP) finds applications in various domains where there's a need to efficiently cover a set of locations while minimizing the overall travel distance. Here are some notable applications:

Mobile Healthcare Services: In rural or underserved areas, mobile healthcare units aim to cover a set of villages or communities efficiently. The CSP helps in determining the optimal routes for these units to visit different locations while minimizing travel distance and ensuring adequate coverage.

Parcel Delivery Services: Companies providing parcel delivery services, such as courier companies or online retailers, aim to optimize their delivery routes to cover multiple delivery points while minimizing travel time and distance. The CSP can help in planning efficient delivery routes for their vehicles.

Emergency Response Planning: During emergency situations such as natural disasters or accidents, emergency response teams need to quickly reach affected areas while ensuring comprehensive coverage. The CSP assists in planning optimal routes for emergency vehicles to cover affected locations efficiently.

Garbage Collection: Municipalities and waste management companies aim to optimize garbage collection routes to cover all designated collection points while minimizing travel distance and time. The CSP helps in planning efficient routes for garbage collection trucks.

Surveillance and Security Patrols: Security agencies and law enforcement organizations need to patrol specific areas regularly to ensure security and surveillance. The CSP helps in planning optimal patrol routes to cover all designated areas while minimizing the overall distance travelled.

Field Service Management: Companies providing field services, such as maintenance, repair, and installation services, need to optimize the routes of their service technicians to cover multiple service calls efficiently. The CSP aids in planning optimal routes for technicians to visit different customer locations.

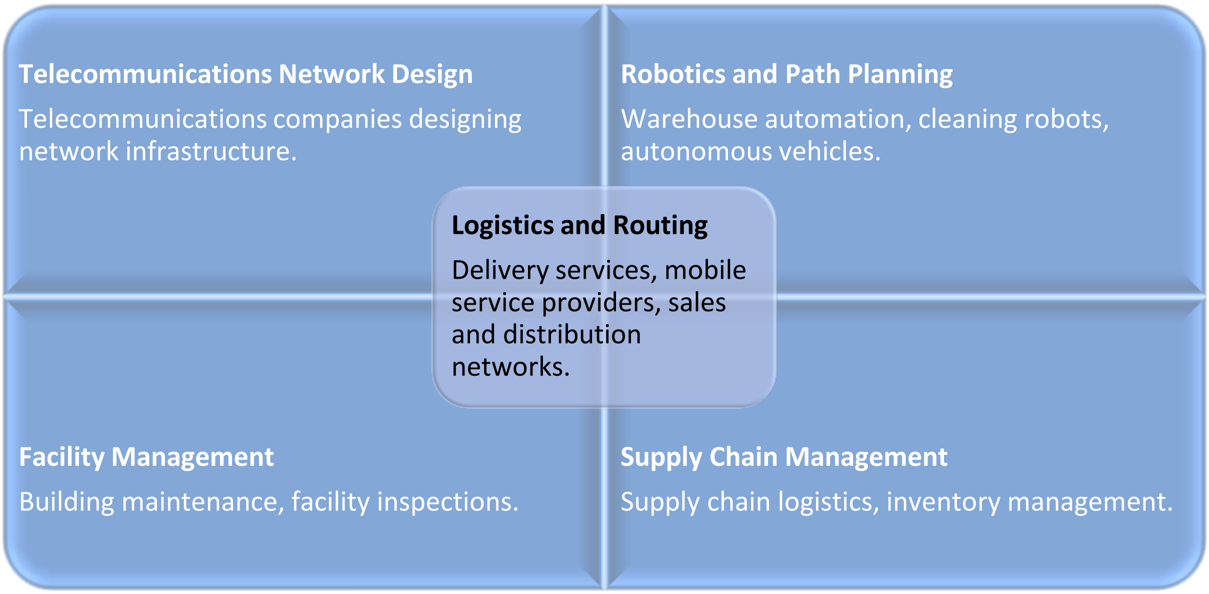
Agricultural Crop Spraying: In agriculture, crop spraying operations aim to cover entire fields with pesticides or fertilizers while minimizing the amount of product used and the time taken. The CSP helps in planning optimal routes for agricultural sprayers to cover fields efficiently.

Public Transportation: Public transportation agencies aim to optimize the routes of buses, trains, or other modes of transportation to cover various stops or stations while minimizing travel time and ensuring adequate coverage. The CSP assists in planning efficient routes for public transportation vehicles.

Search and Rescue Missions: During search and rescue missions in wilderness or disaster areas, search teams need to cover large areas to locate missing persons or survivors. The CSP helps in planning optimal search routes for search teams to cover the search area efficiently.

Tourism and Sightseeing: Tour operators and travel agencies aim to design efficient sightseeing routes for tourists to cover various attractions and points of interest while minimizing travel time and maximizing the tourist experience. The CSP assists in planning optimal sightseeing routes for tour buses or guided tours.

These applications demonstrate the versatility and practical significance of the Covering Salesman Problem in optimizing resource allocation and route planning across a wide range of domains and industries.



8. REFERENCES

- Current, J. R., & Schilling, D. A., The covering salesman problem. Transportation Science, 23(3), 208â-213 (1989).
- Constrained covering solid travelling salesman peoblem in uncertain environment by A Mukherjee, G. Panigrahi, S. Kar, M. Maiti - Journal of Ambient Intelligence and Humanized Computing 10(2) 2019 – Springer
- Golden, B. L., Naji-Azimi, Z., Raghavan, S., Salari, M., & Toth, P., The generalized covering salesman problem. INFORMS Journal on Computing, 24(4) (2012).
- Fischetti M, Salazar Gonzlez JJ, Toth P A branch and cut algorithm for the symmetric generalized traveling salesman problem. Operations Research 45(3) (1997)
- The generalized covering traveling salesman problem MH Shaelaie, M Salari, Z Naji-Azimi - Applied Soft Computing, 2014 – Elsevier
- A multi-objective covering salesman problem with 2-coverage SP` Tripathy, A Biswas, T Pal - Applied Soft Computing, 2021 – Elsevier
- Sheng, Y., & Mei, X. (2020). Uncertain random shortest path problem. *Soft Computing*, *24*, 2431-2440.
- Zgonc, B., Tekavčič, M., & Jakšič, M. (2019). The impact of distance on mode choice in freight transport. *European Transport Research Review*, *11*(1), 1-18.
- Ding SB (2014) Uncertain minimum cost flow problem. Soft Computing 18(11):2201–2207.
- Liu, B., & Liu, B. (2010). *Uncertainty theory* (pp. 1-79). Springer Berlin Heidelberg.
- Golden, B., Naji-Azimi, Z., Raghavan, S., Salari, M., & Toth, P. (2012). The generalized covering salesman problem. *INFORMS Journal on Computing*, *24*(4), 534-553.
- Bowerman, R., Hall, B., & Calamai, P. (1995). A multi-objective optimization approach to urban school bus routing: Formulation and solution method. *Transportation Research Part A: Policy and Practice*, *29*(2), 107-123.
- Gendreau, M., Laporte, G., & Semet, F. (1997). The covering tour problem. *Operations Research*, *45*(4), 568-576.