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Bayesian Learning

Q1 M : Probability of MANU win

A : Arsenal

P : pub packed

$P(M) = 0.7 \Rightarrow P(A) = 0.3$

$P(P|M) = 0.9$

$P(P|A) = 0.6$

$$P(M|P) = \frac{P(P|M).P(M)}{P(M).P(P|M) + P(A).P(P|A)}$$

$$= \frac{0.9 \times 0.7}{0.7 \times 0.9 + 0.3 \times 0.6}$$

$$= \frac{9 \times 7}{9 \times 7 + 3 \times 6} = \frac{3 \times 7}{3 \times 7 + 6} = \frac{21}{27}$$

$$= \frac{7}{9}$$

$P(M|P) = 0.78$

∴ Probability of MANU winning when packed pub is 0.78

There was 0.6 chance of people drowning themselves with sorrow so can't assume them to win.

Q2 F: Probability of forgetting to give pill.

D: dieing

$P(F) = 0.3 \Rightarrow P(F') = 0.7$

$P(D|F') = 0.1 \quad P(D|F) = 0.8$

$$P(F|D) = \frac{P(D|F) \cdot P(F)}{P(D|F) \cdot P(F) + P(D|F') \cdot P(F')}$$

$$= \frac{0.8 \times 0.3}{0.8 \times 0.3 + 0.1 \times 0.7}$$

$$= \frac{8 \times 3}{8 \times 3 + 7} = \frac{24}{24+7} = \frac{24}{31}$$

$$P(F|D) = 0.77$$

The chances of forgetting to give pill is 0.77 but, we know he would die if not receiving the pill is 0.8

$\therefore$ We can say she forgot to give pill.

Q3 G: Probability of Gold

C: Coal

P: test coming positive

$P(G) = 0.1$ $P(C) = 0.3$ $P(G' \cap C') = 0.6$

~~P(P|G) = P(G|P) =~~

$P(P|G) = 0.8$ $P(P|C) = 0.4$ $P(P|G' \cap C') = 0.2$

$$P(G|P) = \frac{P(P|G) \cdot P(G)}{P(P|G) \cdot P(G) + P(P|C) \cdot P(C) + P(P|G' \cap C') \cdot P(G' \cap C')}$$

$$= \frac{0.8 \times 0.1}{0.8 \times 0.1 + 0.4 \times 0.3 + 0.2 \times 0.6}$$

$$= \frac{8}{8+12+12} = \frac{8}{32} = \frac{1}{4} = 0.25$$

$\therefore$ Probability comes out to be 0.25

Q4 M: Patient of meningitis
P: Positive test for meningitis

$P(P|M) = 0.95$ $P(P'|M') = 0.70$ $P(M) = 0.05$

~~P(M)=0.05 P(M)=0.50~~ $\Rightarrow P(M') = 0.95$

$\Rightarrow P(P'|M) = 0.05$

1 $$P(M|P) = \frac{P(P|M).P(M)}{P(P|M).P(M) + P(P'|M').P(M')}$$

$$= \frac{0.95 \times 0.05}{0.95 \times 0.05 + 0.70 \times 0.95}$$

$$= \frac{5}{5+70} = \frac{5}{75} = \frac{1}{15} = 0.06$$

$\therefore$ 0.06 chance of having Meningitis given tested positive.

which is greater than national average, thus the patient has meningitis.

2 $$P(M|P') = \frac{P(P'|M).P(M)}{P(P'|M).P(M) + P(P'|M').P(M')}$$

$$= \frac{0.05 \times 0.05}{0.05 \times 0.05 + 0.70 \times 0.95}$$

$$= \frac{0.05}{0.05 + 0.70 \times 19} = \frac{5}{5 + 70 \times 19}$$

$$= \frac{5}{5 + 1330} = \frac{5}{1335}$$

$$= \frac{1}{267}$$

$P(M|P') = 0.0037 \simeq 0.004$

Significantly below national average.

$\therefore$ Not having Meningitis.

Q5 R: Probability of raining

P: positive test

$P(R) = 0.8 \Rightarrow P(R') = 0.2$

$P(P|R) = 0.75 \quad P(P|R') = 0.15$

$\Rightarrow P(P'|R) = 0.25 \quad P(P'|R') = 0.85$

$$P(R'|P') = \frac{P(P'|R').P(R')}{P(P')}$$

$$= \frac{P(P'|R').P(R')}{P(P'|R').P(R') + P(P'|R).P(R)}$$

$$= \frac{0.85 \times 0.2}{0.85 \times 0.2 + 0.25 \times 0.8}$$

$$= \frac{0.85}{0.85 + 0.25 \times 4} = \frac{0.85}{0.85 + 1}$$

$$= \frac{85}{185} = \frac{17}{37}$$

$$= 0.45$$

∴ By this test 0.45 chance of not raining which is significantly higher than actual probability.

∴ He should predict no-rain.

Q6 C: Probability of contracting chickungunya
J: 'Joint Pain'
P: test being positive.

$P(C) = 0.0001 \Rightarrow P(C') = 0.9999$
$P(J|C) = 0.64 \Rightarrow P(J'|C) = 0.36$
$P(J|C') = 0.6 \Rightarrow P(J'|C') = 0.4$
$P(P|C) = 0.99 \Rightarrow P(P'|C) = 0.01$
$P(P|C') = 0.04 \Rightarrow P(P'|C') = 0.96$

~~P(C|P,J)~~

~~$P(P, J) = P(P, J|C).P(C) + P(P, J|C').P(C')$~~

$$P(C|(P, J)) = \frac{P((P, J)|C).P(C)}{P(P, J)}$$
$$= \frac{P(P|C).P(J|C).P(C)}{P(P, J)}$$
$$= \frac{P(P|C).P(J|C).P(C)}{P(C).P(P|C).P(J|C) + P(C').P(P|C').P(J|C')}$$
$$= \frac{0.99 \times 0.64 \times 0.0001}{0.99 \times 0.64 \times 0.0001 + 0.9999 \times 0.04 \times 0.6}$$
$$= \frac{0.99 \times 0.64}{0.99 \times 0.64 + 9999 \times 0.04 \times 0.6}$$
$$= \frac{\cancel{99} \times 0.64}{\cancel{99} \times 0.64 + \cancel{9999}^{101} \times 4 \times 0.6}$$
$$= \frac{64}{64 + 101 \times 4 \times 60} = \frac{16}{16 + 6060} = \frac{4\cancel{16}}{\cancel{6076}_{1519}} = \frac{4}{1519}$$

$P(C|(P, J)) = 0.0026$

which is higher than average chance.
$\therefore$ He has Chickungunya.

ii $P(C|(P^n, J)) = \dfrac{P((P^n, J)|C) \cdot P(C)}{P((P^n, J))}$

$= \dfrac{[P(P|C)]^n \cdot P(J|C) \cdot P(C)}{P(C) \cdot P(P|C)^n \cdot P(J|C) + P(C') \cdot P(P|C')^n \cdot P(J|C')}$

$= \dfrac{(0.99)^n \times 0.64 \times \cancel{0.0001}}{\cancel{0.0001} \times (0.99)^n \times 0.64 + (0.9999) \times (0.4)^n \times 0.6}$

$= \dfrac{(0.99)^n \times 0.64}{(0.99)^n \times 0.64 + 9999 \times 0.6 \times (0.4)^n}$

$= \dfrac{0.64}{0.64 + 9999 \times 0.6 \times \left(\frac{0.40}{0.99}\right)^n}$

$P(C|(P^n, J)) = \dfrac{64}{64 + 60 \times 9999 \times \left(\frac{40}{99}\right)^n}$ — (1)

iii Find n such that
$\boxed{P(C|(P^n, J)) > P(C'|(P^n, J))}$

$P(C'|(P^n, J)) = \dfrac{P((P^n, J)|C') \cdot P(C')}{P((P^n, J))}$

$= \dfrac{[P(P|C')]^n \cdot P(J|C') \cdot P(C')}{[P(P|C')]^n \cdot P(J|C') \cdot P(C') + P(C) \cdot P(P|C)^n \cdot P(J|C)}$

"You must be the change you want to see in the world." —Mahatma Gandhi.

$$= \frac{(0.04)^n \times 0.6 \times 0.9999}{(0.04)^n \times 0.6 \times 0.9999 + \cancel{0.0001} \times (0.99)^n \times 0.64}$$

$$= \frac{(0.04)^n \times 0.6 \times 9999}{(0.04)^n \times 0.6 \times 9999 + (0.99)^n \times 0.64}$$

$$= \frac{0.6 \times 9999}{0.6 \times 9999 + \left(\frac{99}{4}\right)^n \times 0.64}$$

$$= \frac{60 \times 9999}{60 \times 9999 + \left(\frac{99}{4}\right)^n \times 64} \quad - \; 2$$

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Now, ① > ②

$$\frac{64}{64 + 60\times 9999 \times \left(\frac{4}{99}\right)^n} > \frac{60\times 9999}{60\times 9999 + \left(\frac{99}{4}\right)^n \times 64}$$

$$\Rightarrow \cancel{64\times 60\times 9999} + \left(\frac{99}{4}\right)^n \times (64)^2 > \cancel{64\times 60\times 9999} + (60\times 9999)^2 \times \left(\frac{4}{99}\right)^n$$

$$\Rightarrow (64)^2 \times \left(\frac{99}{4}\right)^n > (60\times 9999)^2 \times \left(\frac{4}{99}\right)^n$$

$$\left(\frac{\frac{99}{4}}{\frac{4}{99}}\right)^n > \left(\frac{60\times 9999}{64}\right)^2$$

$$\left(\frac{99}{4}\right)^{\cancel{2}n} > \left(\frac{60\times 99\times 101}{64}\right)^{\cancel{2}}$$

$$\left(\frac{99}{4}\right)^n > \left(\frac{\overset{15}{\cancel{60}}\times 99\times 101}{\underset{16}{\cancel{64}}}\right)$$

$$> (15\times 101)\times \frac{99}{16}$$

$$\left(\frac{99}{4}\right)^n \times \left(\frac{16}{99}\right) > 15\times 101$$

$$\frac{99^{n-1}}{4^{n-\cancel{2}}} \times \cancel{4^{2}}\frac{1}{4} > 15\times 101$$

$$\left(\frac{99}{4}\right)^{n-1} > 60\times 101$$

$$\left(\frac{99}{4}\right)^{n-1} > 6060$$

$$(24.75)^{n-1} > 6060$$

$$24.75^n > 149985$$

$$(24.75)^2 < 149985,\ (24.75)^3 < 149985$$

$$(24.75)^4 > 149985$$

$$\therefore \boxed{n = 4 \text{ min}}$$

Q1, Frequency table for Handset, Churning
(1)

Handset	Churning Yes	Churning No	
New	3	5	~~8/10~~
Old	2	0	~~2/10~~
	~~5/10~~	~~5/10~~	

$P(New) = 8/10$ $P(old) = 2/10$ $P(Yes) = 5/10$

$P(New|Yes) = 3/5$ $P(old|Yes) = 2/5$ $P(No) = 5/10$

$P(old|No) = 0$ $P(New|No) = 5/5$

II Frequency table for Time, Churning

Time	Churning Yes	Churning No
<1	2	0
[1, 2.5]	1	2
>2.5	2	3

$P(<1) = 2/10$ $P(1, 2.5) = 3/10$ $P(>2.5) = 5/10$

$P(<1|Yes) = 2/5$ $P(1, 2.5|Yes) = 1/5$ $P(>2.5|Yes) = 2/5$

$P(Yes) = 5/10$ $P(No) = 5/10$

$P(<1|No) = 0$ $P(1, 2.5|No) = 2/5$ $P(>2.5|No) = 3/5$

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III Frequency table for Age & Churning

Age	Churning Yes	Churning No
~~Yes~~ ≤ 55	2	3
~~No~~ >55	3	2

$P(\leq 55) = 5/10$ $\qquad P(>55) = 5/10$

$P(\leq 55 | Yes) = 2/5$ $\qquad P(>55 | Yes) = 3/5$

$P(Yes) = 5/10$ $\qquad P(No) = 5/10$

$P(Yes | Old, >2.5, \leq 55) =$

$= P(old | Yes) \times P(>2.5 | Yes) \times P(\leq 55 | Yes) \times P(Yes)$

$= \frac{2}{5} \times \frac{2}{5} \times \frac{\cancel{2}}{5} \times \frac{\cancel{5}}{\cancel{10}}$

$= \frac{4}{125}$

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(28) Frequency distribution of X, classification

X	Classification +	Classification −
1	0	1
2	0	1
3	1	1
4	1	0
5	2	0
7	0	1

$P(+) = 4/8$ $\quad$ $P(-) = 4/8$

$P(1|+) = 0$ $\quad$ $P(1|-) = 1$

$P(2|+) = 0$ $\quad$ $P(2|-) = 1$

$P(3|+) = 1/2$ $\quad$ $P(3|-) = 1/2$

$P(4|+) = 1$ $\quad$ $P(4|-) = 0$

$P(5|+) = 2$ $\quad$ $P(5|-) = 0$

$P(7|+) = 0$ $\quad$ $P(7|-) = 1$

Frequency distribution of Y, classification

Y	Classification +	Classification −
1	0	1
~~2~~	~~0~~	~~0~~
~~3~~	~~0~~	~~0~~
4	0	1
5	1	0
6	2	1
7	1	1

$P(+) = 4/8$

$P(-) = 4/8$

$P(1|-) = 1$

$P(4|-) = 1$

$P(5|+) = 1$

$P(6|+) = 2/3$ $\quad$ $P(6|-) = 1/3$

$P(7|+) = 1/2$ $\quad$ $P(7|-) = 1/2$

$P(+ \mid x=7, y=4) = P(+) \cdot P(x=7 \mid +) \cdot P(y=4 \mid +)$

$= \frac{4}{8} \times 0 \times 0 = 0$

Q9 Dataset : 2.8, 4.2, 5.3, 5.5, 2.1

	Gauss Distribution 1	Gauss Distribution 2
μ	2.3	6.8
σ^2	1.8	2.2
$P(x)$	$1/2$	$1/2$

$$P(x \mid \mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2} \Sigma (x_i - \mu)^2}$$

$$\ln[P(x \mid \mu, \sigma^2)] = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{j=1}^{n} [x_{(j)} - \mu]^2.$$

"For Gauss Distribution - 1"

$$\ln(P(X|\mu,\sigma^2)) = -0.57 - \frac{1}{3.6}\left[0.5^2 + 1.9^2 + 3^2 + 3.2^2 + 0.2^2\right]$$

$$= -0.57 - \frac{1}{3.6}\left[0.25 + 3.61 + 9 + 0.04 + 10.24\right]$$

$$= -0.57 - \frac{1}{3.6}\left[23.14\right]$$

$$= -0.57 - 6.43 = -7$$

"For Gauss Distribution - 2"

$$\ln(P(X|\mu,\sigma^2)) = -\frac{1}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{j=1}^{n}\left[x_j - \mu\right]^2$$

$$= -1.31 - \frac{1}{4.4}\left[4^2 + 2.6^2 + 1.5^2 + 1.3^2 + 4.7^2\right]$$

$$= -1.31 - \frac{1}{4.4}\left[16 + 6.76 + 2.25 + 1.69 + 22.09\right]$$

$$= -1.31 - \frac{1}{4.4}\left[48.79\right]$$

$$= -1.31 - 11.08 = -12.39$$

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