Explanation of the Solution Algorithm for Logistic Regression and Visualization

1. Data Preparation

- **Input Data**: The dataset consists of three features: propellant age, storage temperature, and a binary outcome indicating pass or fail.
- **Feature Matrix**: Combine the age and temperature data into a matrix, including an additional column for the bias term (intercept). This matrix is used to perform linear operations in logistic regression.
- **Target Variable**: The binary outcome (pass or fail) is used as the target variable for classification.

2. Sigmoid Function

- **Purpose**: The sigmoid function is used to convert the linear combination of features into probabilities. This function maps any real-valued number into the range [0, 1].
- **Mathematical Formulation**: The sigmoid function h(z)=11+e−zh(z) = \frac{1}{1 + e^{-z}}h(z)=1+e−z1 is applied to the linear combination of features and weights.

3. Cost Function

- **Objective**: The cost function quantifies the error of the logistic regression model by comparing predicted probabilities to actual outcomes.
- **Logistic Regression Loss**: The cross-entropy loss function is used to measure how well the model's predictions match the actual labels. To avoid computational issues with zero probabilities, a small constant is added to probabilities.

4. Gradient Function

- **Purpose**: The gradient function computes the gradient of the cost function with respect to the model's weights.
- **Gradient Computation**: This involves calculating how changes in weights affect the cost function, which is used to adjust the weights during optimization.

5. Line Search

- **Objective**: To determine the optimal step size for updating weights during gradient descent.
- **Process**: An iterative approach is used to adjust the step size based on the Armijo condition, which checks if the step size leads to a sufficient decrease in the cost function.

6. Gradient Descent

- **Purpose**: To minimize the cost function by iteratively updating the model's weights.
- **Process**: Weights are initialized, and at each iteration, the gradient is computed and used to update the weights. The step size for updates is determined using the line search method. The process continues until convergence or a maximum number of iterations is reached.

7. Visualization

- **Probability Contour**: A contour plot is created to visualize the probability of passing (or failing) across different ages and temperatures. The contour levels are defined to show the probability regions in increments of 0.1.
- **Scatter Plot**: Data points are overlaid on the contour plot to show individual samples. Different colors (vibrant green for pass, vibrant red for fail) with black edges are used to highlight these points clearly against the contour plot.

8. Colormap and Plotting

- **Colormap**: A custom colormap is defined to represent probabilities from vibrant red to vibrant green, enhancing visual distinction between passing and failing probabilities.
- **Final Plot**: The contour plot, color bar, and scatter plot are combined to provide a comprehensive visual representation of the logistic regression results, highlighting the relationship between age, temperature, and the probability of passing.