Pendulum Motion Prediction Using Ridge Regression

1 Problem Statement

This project aims to develop a predictive model for angular acceleration ($\ddot{\theta}$) in pendulum motion through the application of machine learning techniques. The theoretical model for a simple pendulum can be expressed as:

$$\ddot{\theta} = -\frac{g}{L}\sin(\theta)$$

where g is the acceleration due to gravity and L is the length of the pendulum. The model leverages features such as angle (θ), angular velocity ($\dot{\theta}$), and additional derived features to accurately predict angular acceleration.

2 Solution Procedure

2.1 Data Collection

The dataset utilized in this project was collected from pendulum experiments, incorporating features such as angle (θ), angular velocity ($\dot{\theta}$), and angular acceleration ($\ddot{\theta}$).

2.2 Data Preprocessing

The initial steps involved loading the dataset and separating the features from the target variable. Additionally, further features were created based on the initial dataset to expand the hypothesis space for the regression model.

2.3 Exploratory Data Analysis (EDA)

The correlation matrix was computed to elucidate the relationships between the features. Furthermore, the data was visualized through scatter plots, enabling the identification of underlying patterns.

2.4 Model Selection

The dataset was split into training and testing sets. Ridge regression was implemented to determine the optimal regularization parameter (α). A range of values between 0 and 2 was utilized, and the value that minimized the Mean Squared Error (MSE) was selected.

$$J(\mathbf{w}) = \frac{1}{2m}\left(\|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 + \alpha\|\mathbf{w}\|^2\right)$$

2.5 Model Evaluation

Mean Squared Error (MSE) served as the evaluation metric. The MSE values for different α values were visualized to identify the optimal parameter.

2.6 Results Visualization

Predictions were plotted against actual values to evaluate the model's effectiveness.

3 Results

The outcomes of the model evaluation steps are summarized as follows:

- Correlation Matrix: The correlation between angular acceleration and other features revealed a strong negative correlation with angle (θ).

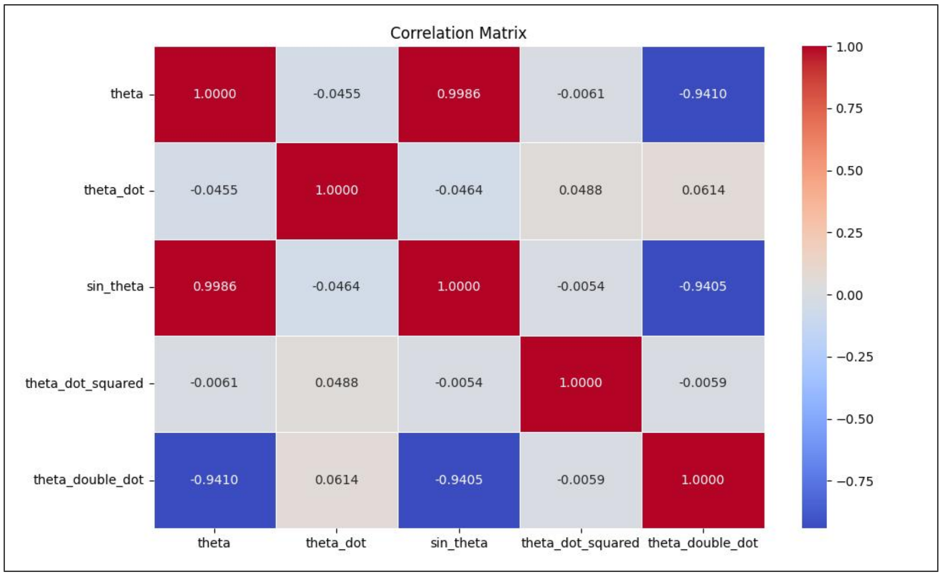

Figure 1: Correlation matrix of the features.

- Best alpha found through Ridge regression: $\alpha = 1.068$, determined by evaluating MSE across the range $[0, 2]$ and selecting the α value that minimized MSE.

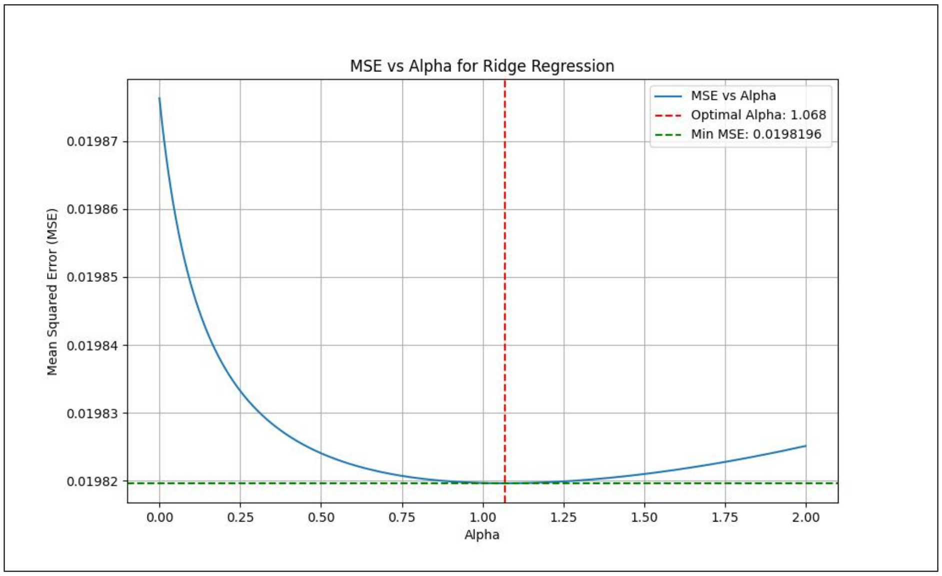

Figure 2: MSE vs Alpha for Ridge Regression. The optimal α is identified as the value where the MSE reaches its minimum.

- Hypothesis Space: The scatter plot depicting the hypothesis space allows for a better understanding of the underlying relationships among the features.

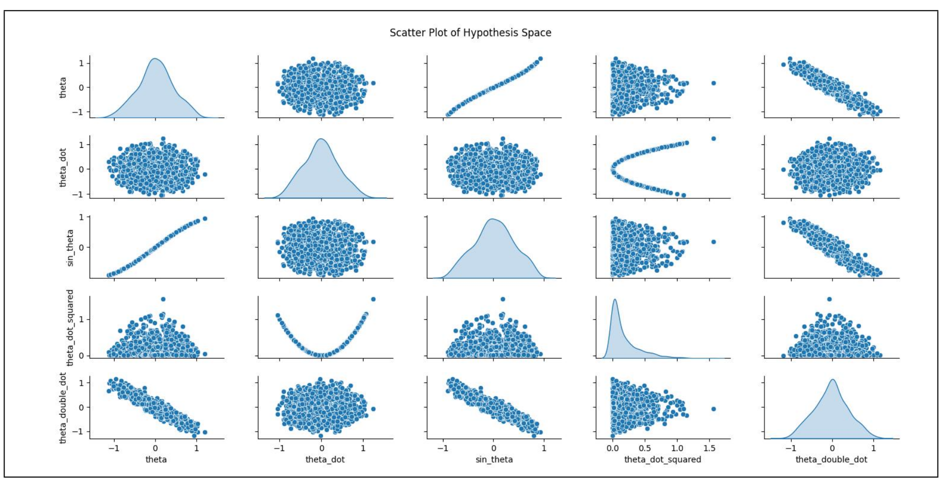

Figure 3: Scatter plot of the hypothesis space.

- Coefficients of the final model:

$$\ddot{\theta} = -0.4874\,\theta + 0.0304\,\dot{\theta} - 0.4611\,\sin\theta - 0.0374\,\dot{\theta}^2$$

- Cross-validated Mean Squared Error (MSE): 0.0206.
- The final predictions were plotted against the actual values to evaluate the model's effectiveness.

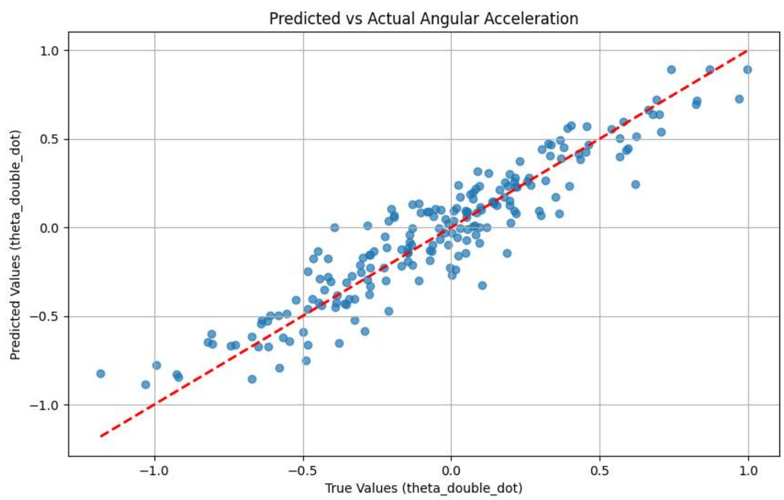

Figure 4: Predicted vs Actual Angular Acceleration.

4 Discussion

Several factors contributed to the deviations from the theoretical model, including:

- Measurement noise during data collection affected the accuracy of the angles and velocities.
- Nonlinear dynamics present in pendulum motion may not have been fully captured by the linear regression approach.
- The model's assumptions might not perfectly align with the physical behavior of the system. It was assumed that the cost function of Ridge regression has only one global minimum, which may not hold true in more complex scenarios where multiple local minima exist. This assumption potentially limits the model's ability to accurately represent the underlying dynamics.
- The effects of air friction and viscosity introduce additional damping forces that influence the pendulum's motion and are not accounted for in the current model.

5 Conclusion

Developed a predictive model for angular acceleration in pendulum motion using Ridge regression. The final mechanical model is expressed as follows:

$$\ddot{\theta} = -0.4874\,\theta + 0.0304\,\dot{\theta} - 0.4611\,\sin\theta - 0.0374\,\dot{\theta}^2$$

The model demonstrates a strong relationship with the provided features, and the predictions were validated against actual values. A significant cause for the deviations from the theoretical model was identified as the impact of air friction and viscosity, which introduces additional damping forces affecting the pendulum's motion and remain unaccounted for in the regression analysis.