LIGO INDIA

GRAVITATIONAL WAVE DATA ANALYSIS - I

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GW Data Analysis workshop
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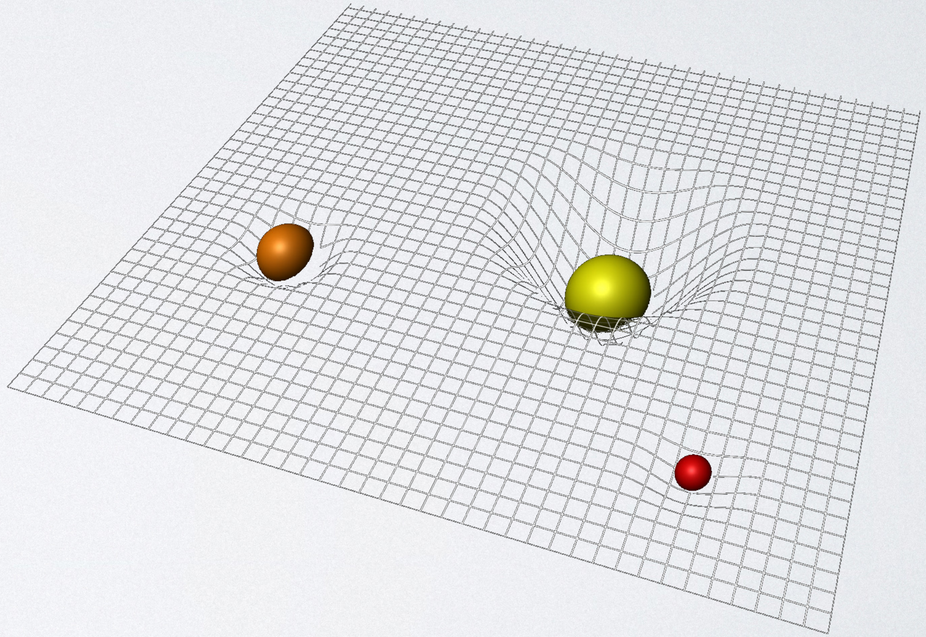
ICTS
International Centre for Theoretical Sciences
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

EINSTEIN'S GRAVITY

- Gravity is a manifestation of curvature of the space-time geometry (*not a Force*).
- *Geodesic equation:* $\frac{d^2x^\mu}{du^2} + \Gamma^\mu_{\alpha\beta}(\overleftrightarrow{\mathbf{g}})\frac{dx^\alpha}{du}\frac{dx^\beta}{du} = 0$
  - Set of 4 ODEs
- *Einstein field equations*: $\overleftrightarrow{\mathbf{G}} = \frac{8\pi G_N}{c^4}\overleftrightarrow{\mathbf{T}}$
  - Set of 10 coupled PDEs for the metric $\overleftrightarrow{\mathbf{g}}$
- Spacetime is dynamic.

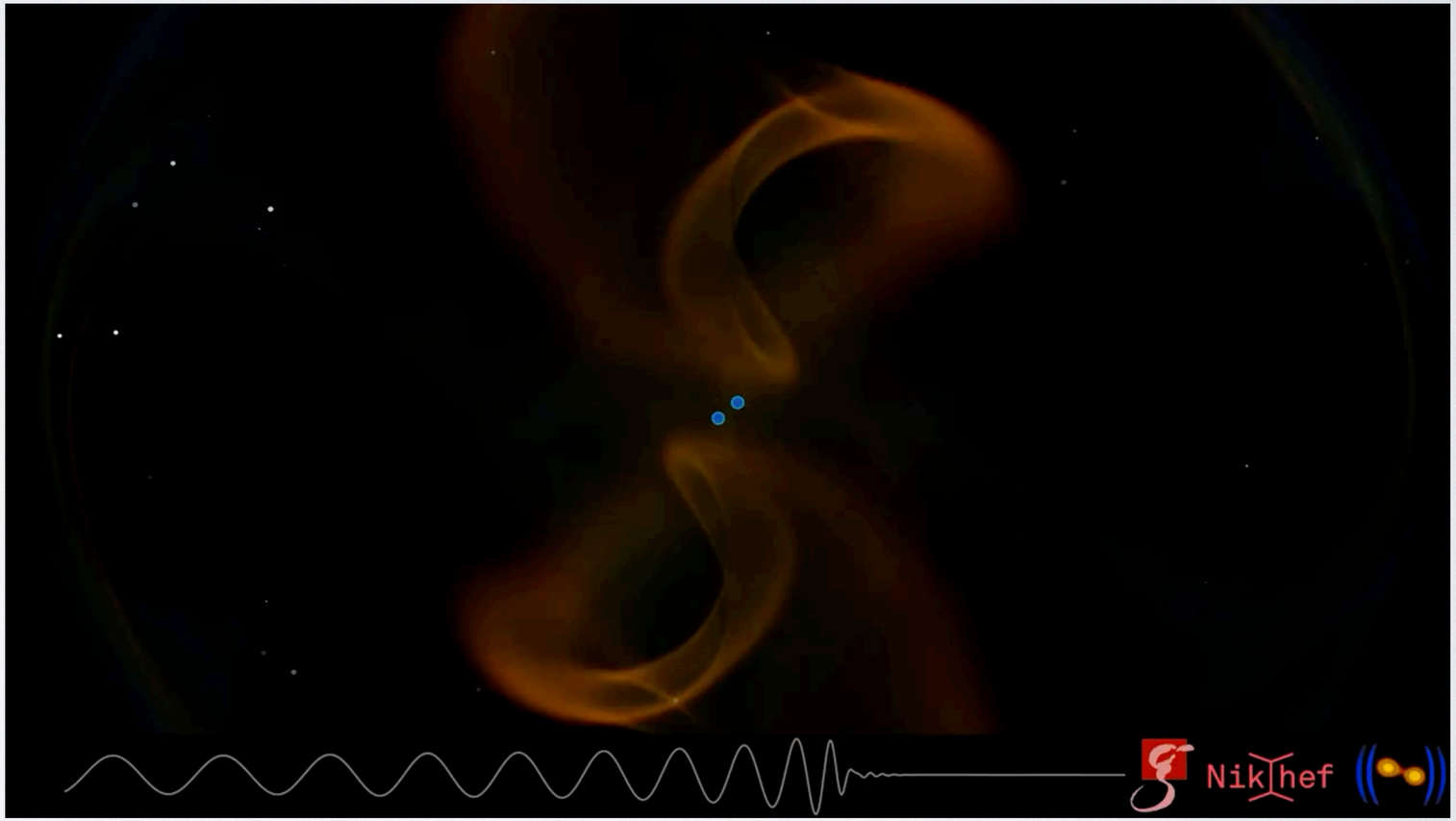


Spacetime geometry due to presence of masses. Courtesy.

GRAVITATIONAL WAVES

- Existence of Gravitational Waves (GWs) : one of the fundamental predictions of General Relativity.
- GWs are freely propagating *ripples* in the fabric of spacetime.
- Solutions to linearized Einstein's field equations

$$\Box \bar{\mathbf{h}} = -\frac{16\pi G_N}{c^4} \overleftrightarrow{\mathbf{T}}$$



Simulation of the binary neutron star (BNS) coalescence GW190425. Courtesy.

AMPLITUDE AND FREQUENCY OF GW

The GW amplitude to the leading order multipole moment in TT-gauge:

GW amplitude

$$h_{ij}(t) \sim \frac{2}{d}\ddot{Q}_{ij}(t - d/c)$$

Distance to the source

Second time derivative
Of the mass quadrupole moment

$$Q_{ij} = \int \rho x_i x_j d^3x$$

Accelerating charges
(time-varying dipole moment) → Electromagnetic waves

Accelerating masses
(time-varying quadrupole moment) → Gravitational waves

AMPLITUDE AND FREQUENCY OF GW

Schwarzschild radius of the reduced mass

$$R_s' = 2G_N\mu/c^2$$

Compactness of the Source (< 0.5)

$$\frac{R_s}{R} = \frac{2G_N M}{c^2 R} \sim \left(\frac{v}{c}\right)^2$$

$$h \sim \left(\frac{R_s'}{d}\right)\left(\frac{R_s}{R}\right)\cos(2\omega t)$$

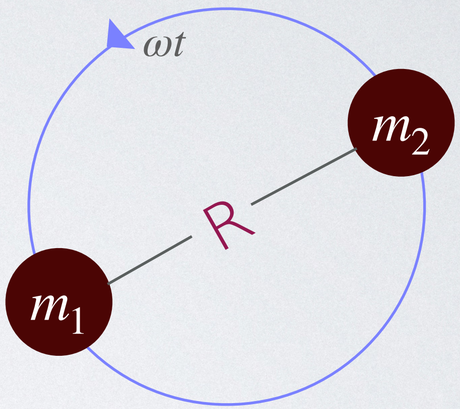
Distance to the source
(100 **Mpc** $\sim 10^{21}$**km**)

Frequency of the source

$$\omega = \sqrt{\frac{G_N M}{R^3}}$$

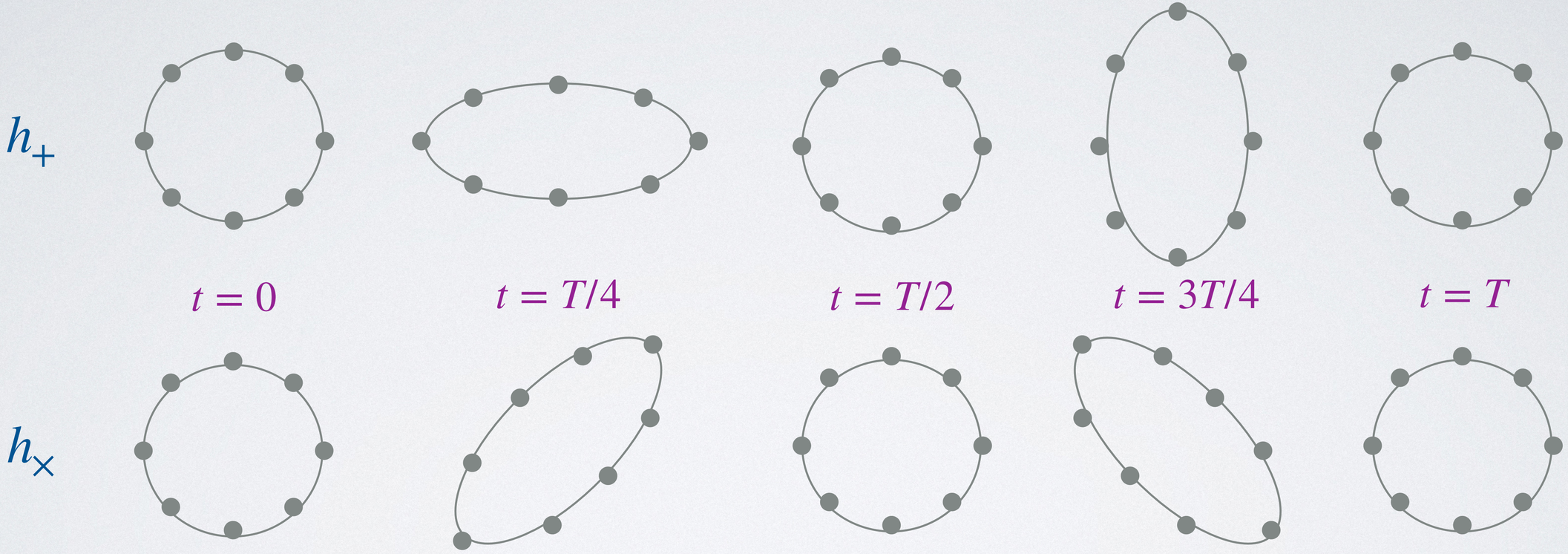
$$M = m_1 + m_2$$

$$\mu = m_1 m_2 / M$$

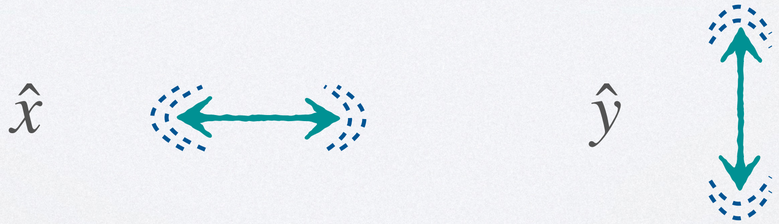


Most promising sources are **relativistic** phenomena Involving **massive** and **compact** objects : *Compact Binary Coalescences*

POLARIZATION OF GW

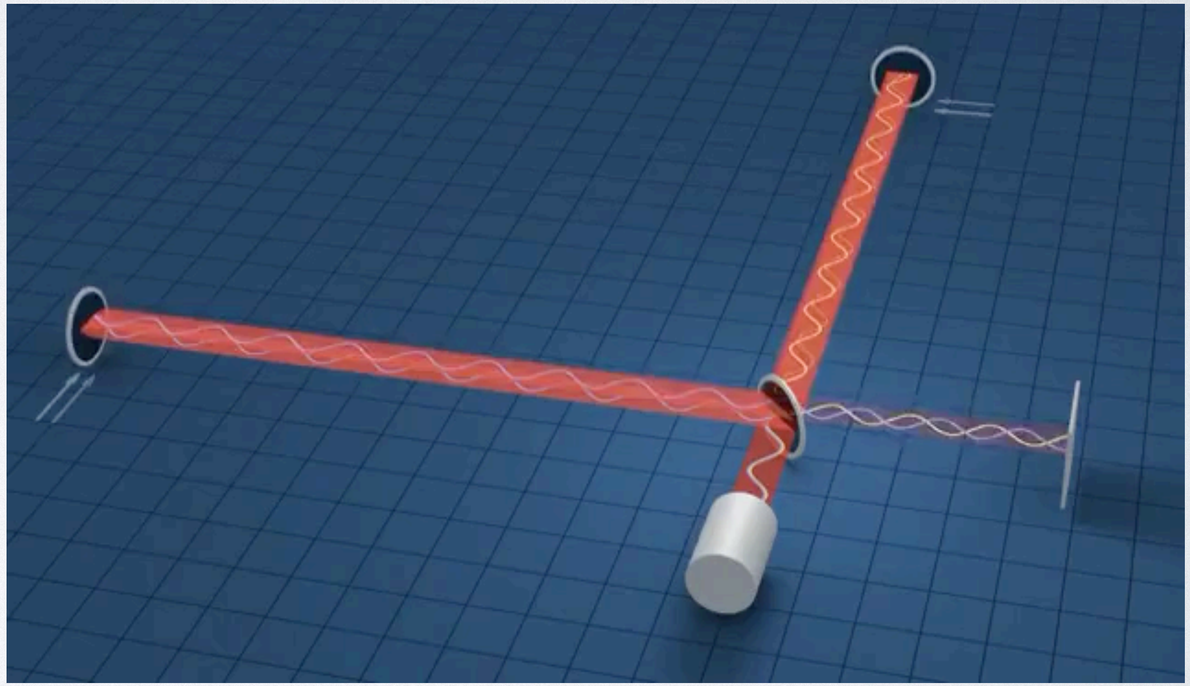


Contrast with EM dipole radiation



GROUND BASED GW DETECTORS

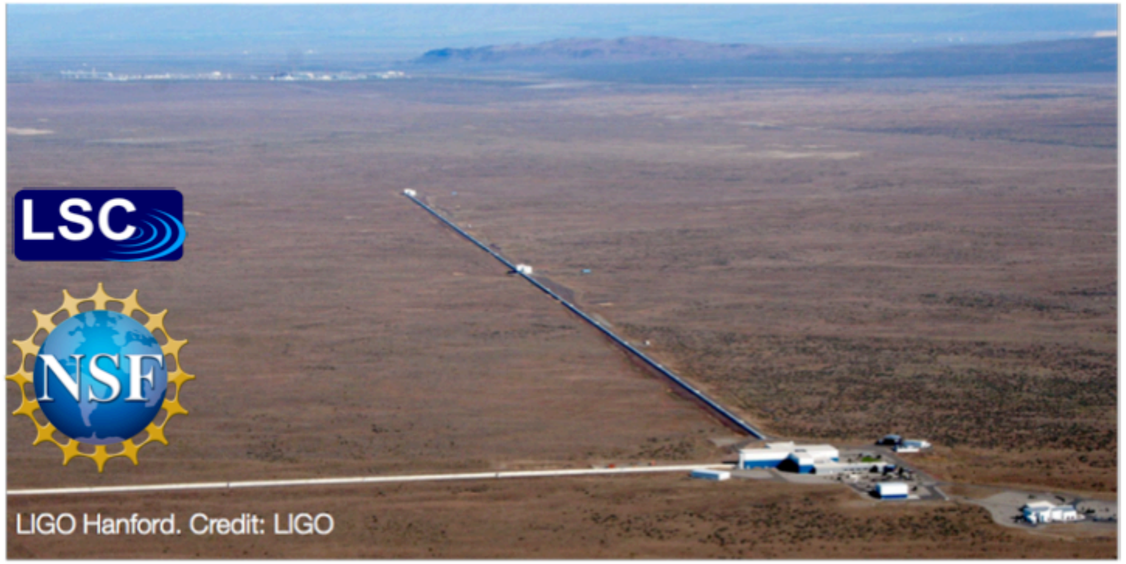
- When GWs pass through Earth, they produce a time-dependent change in the geometry of space.
- These changes can be detected with the help of laser interferometers.
- The amplitude of GW is awfully small! $h \sim \Delta L/L \sim 10^{-23}$



Interferometry to detect GWs. Courtesy.

LIGO Livingston. Credit: LIGO

Virgo observatory. Credit: Virgo

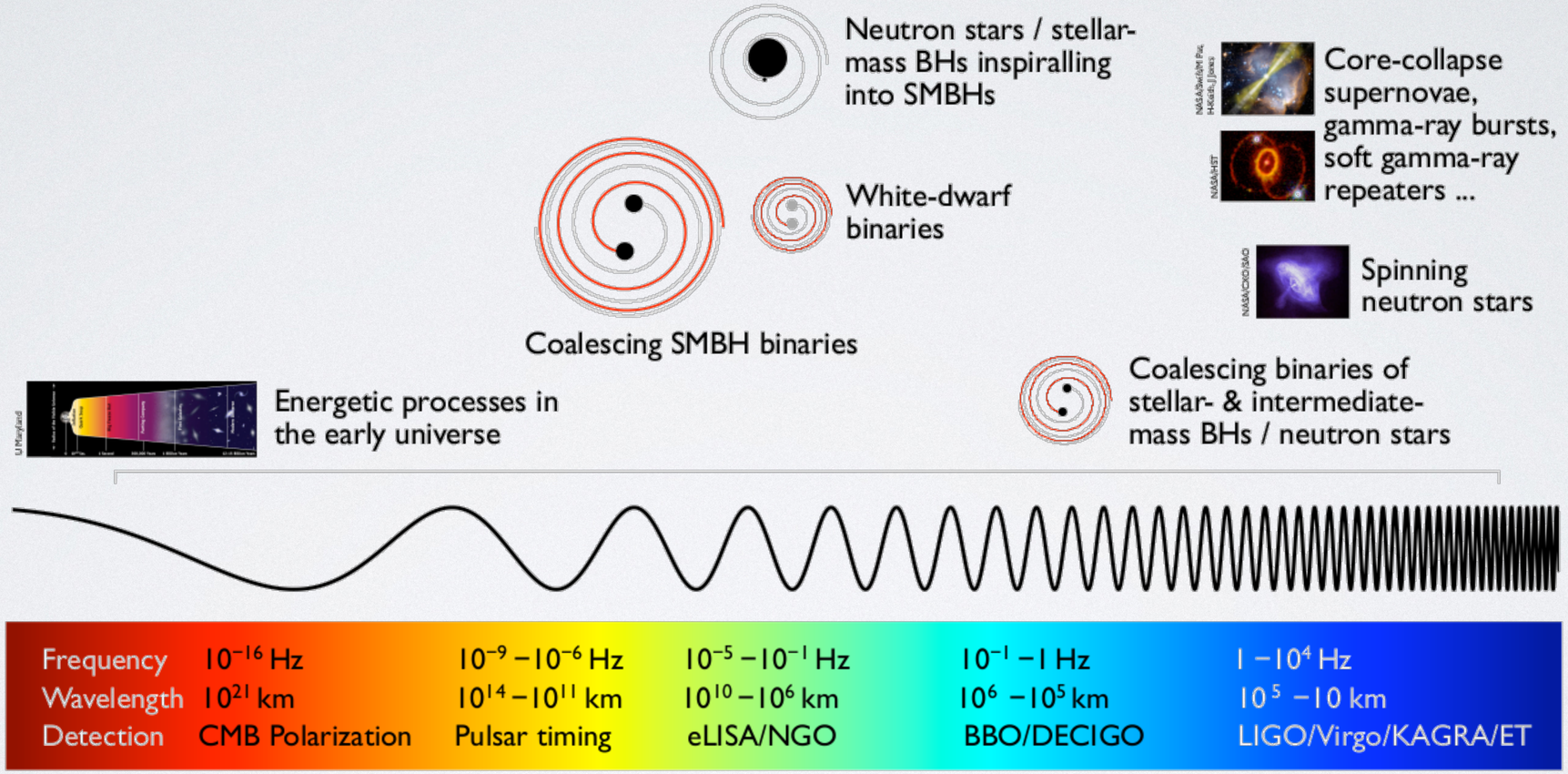


LIGO Hanford. Credit: LIGO

GEO observatory. Credit: GEO

Gravitational wave observatories (Not in picture: KAGRA, Japan). Upcoming: LIGO-India.

SOURCES OF GRAVITATIONAL WAVES

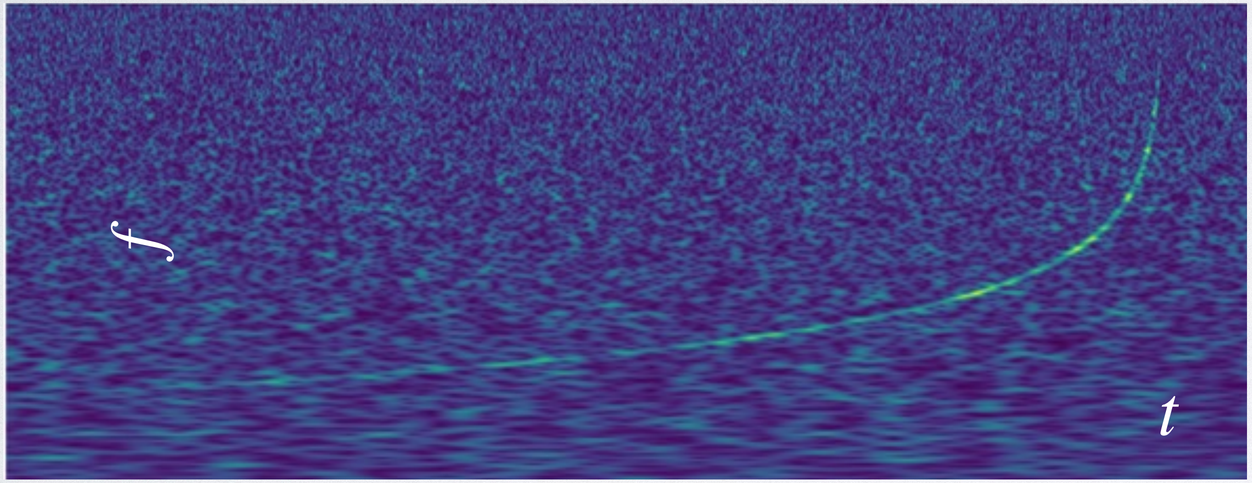


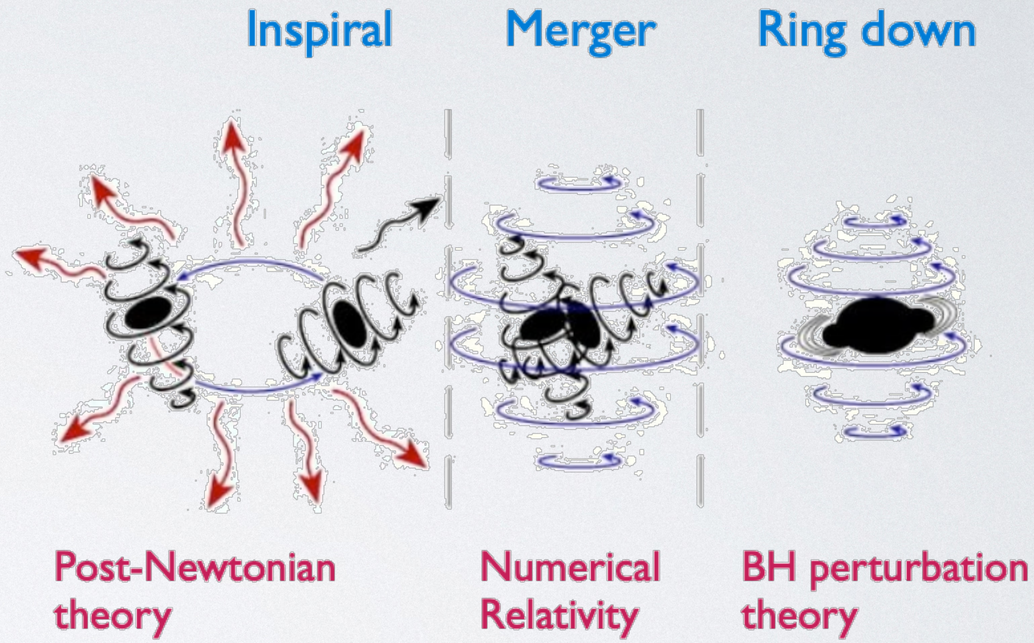
Gravitational Wave spectrum. Courtesy.

CLASSIFICATION OF GW SIGNALS

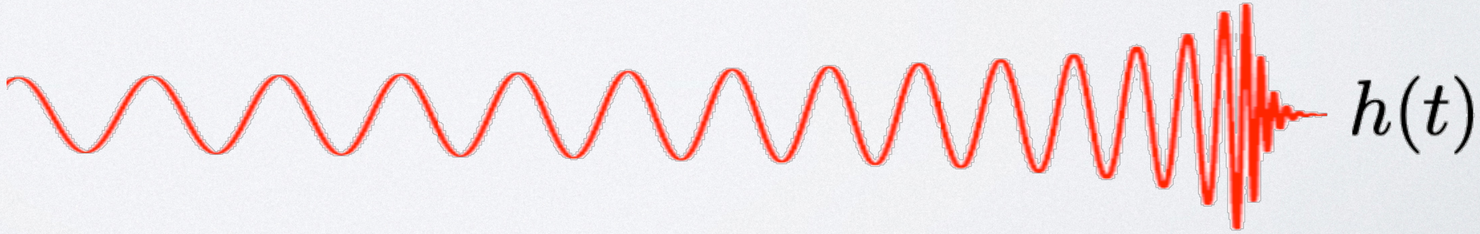
	Modelled	Unmodelled
Long	Continuous waves (e.g., Rotating neutron stars)	Stochastic Background (cosmological or astrophysical)
Short	Binary coalescence (Inspiral+Merger+Ringdown) (e.g., BH-BH, NS-BH, NS-NS)	Bursts (e.g., Supernova, BH merger, etc.)

GW SIGNAL FROM COMPACT BINARY COALESCENCE



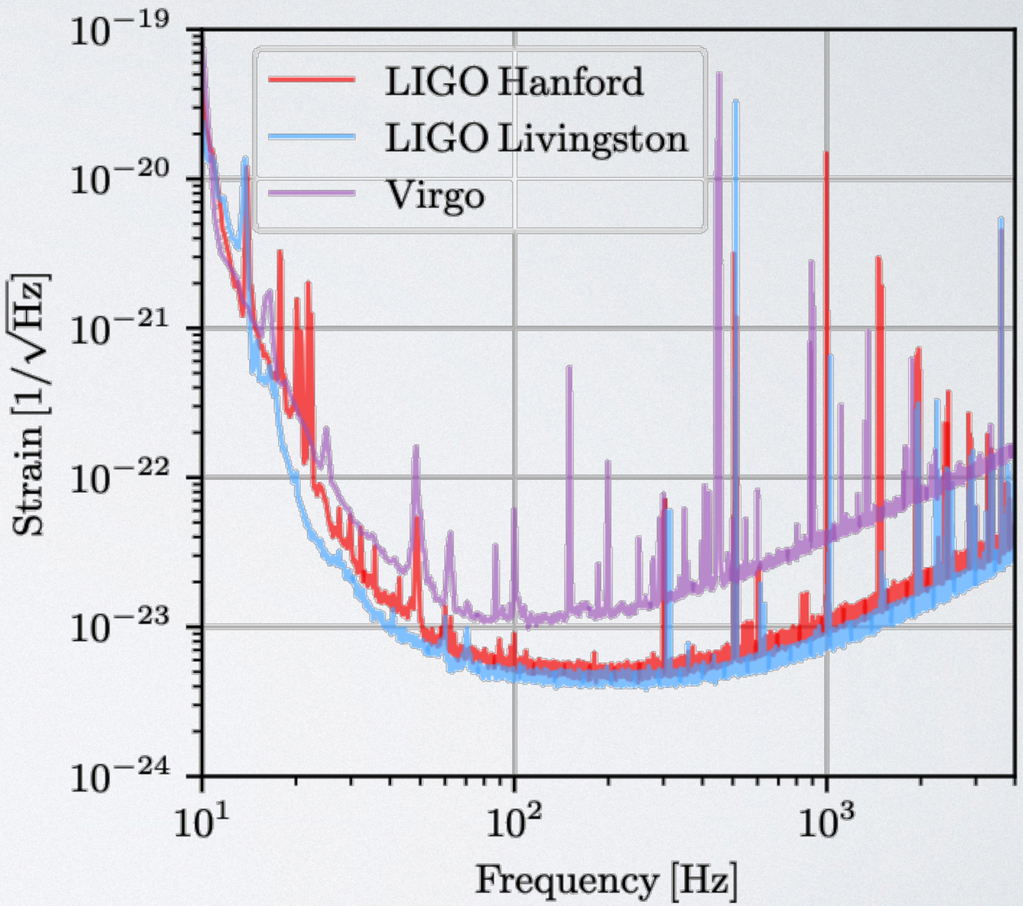


Top: Chirp signal spectrogram. Courtesy
Right: Signal at different phases. Pic: Kip Thorne



THE NEED FOR GW DATA ANALYSIS

- GW amplitude is small $h \sim 10^{-23}$. Signal buried in the detector noise. Extraction of signal requires data analysis.
- If a signal is present, the next task is to estimate its parameters.



Representative ASD of the two LIGO detectors and the VIRGO detector during the first half of O3 run. Ref: O3a catalog paper

OPTIMAL DETECTION STATISTIC

- When the statistical properties of noise and exact form of signal are known, an optimal detection statistic can be constructed that distinguishes between the two hypotheses:

Null hypothesis $\mathcal{H}_0 : s(t) = n(t)$

Alternative hypothesis $\mathcal{H}_1 : s(t) = n(t) + h(t)$

data noise signal

Prior odds

Likelihood ratio, Λ (Bayes factor)

Odds ratio, $\mathcal{O} = \dfrac{P(\mathcal{H}_1 | s)}{P(\mathcal{H}_0 | s)} = \dfrac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)} \dfrac{P(s | \mathcal{H}_1)}{P(s | \mathcal{H}_0)}$

OPTIMAL DETECTION STATISTIC IN GAUSSIAN NOISE: THE MATCHED FILTER

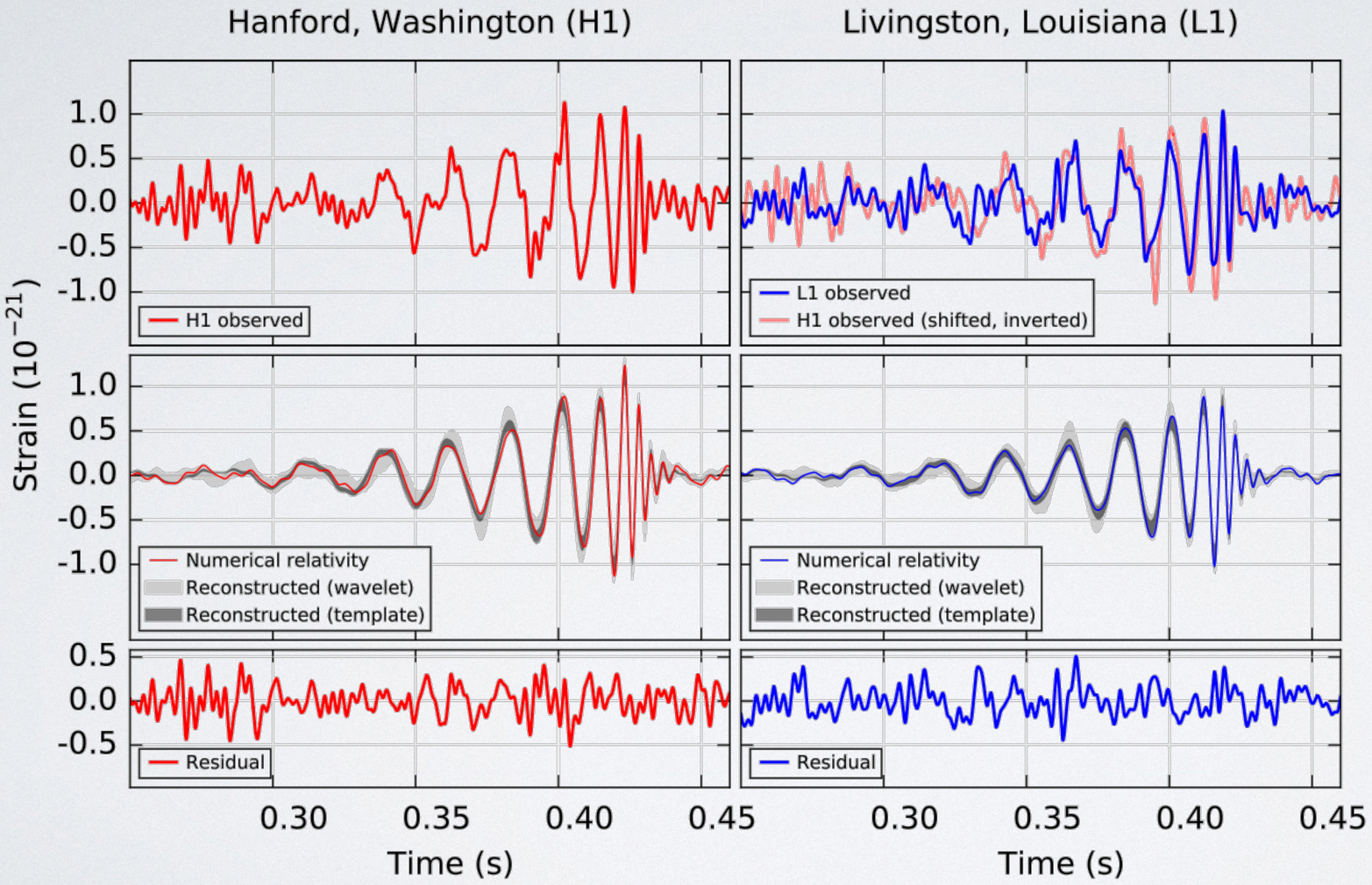
The likelihood ratio: $\Lambda = \dfrac{P(s \mid \mathscr{H}_1)}{P(s \mid \mathscr{H}_0)} = e^{(s,h)} e^{-(h,h)/2}$

Matched filter: $(s, h) = 4\mathrm{Re} \int_0^\infty \dfrac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df$

- Matched filter (s, h) is the optimal detection statistic in Gaussian noise in the sense that it maximises the SNR.

Optimal SNR: $\left(\dfrac{S}{N}\right) = (h \mid h)^{1/2}$

GW150914: THE FIRST DIRECT OBSERVATION



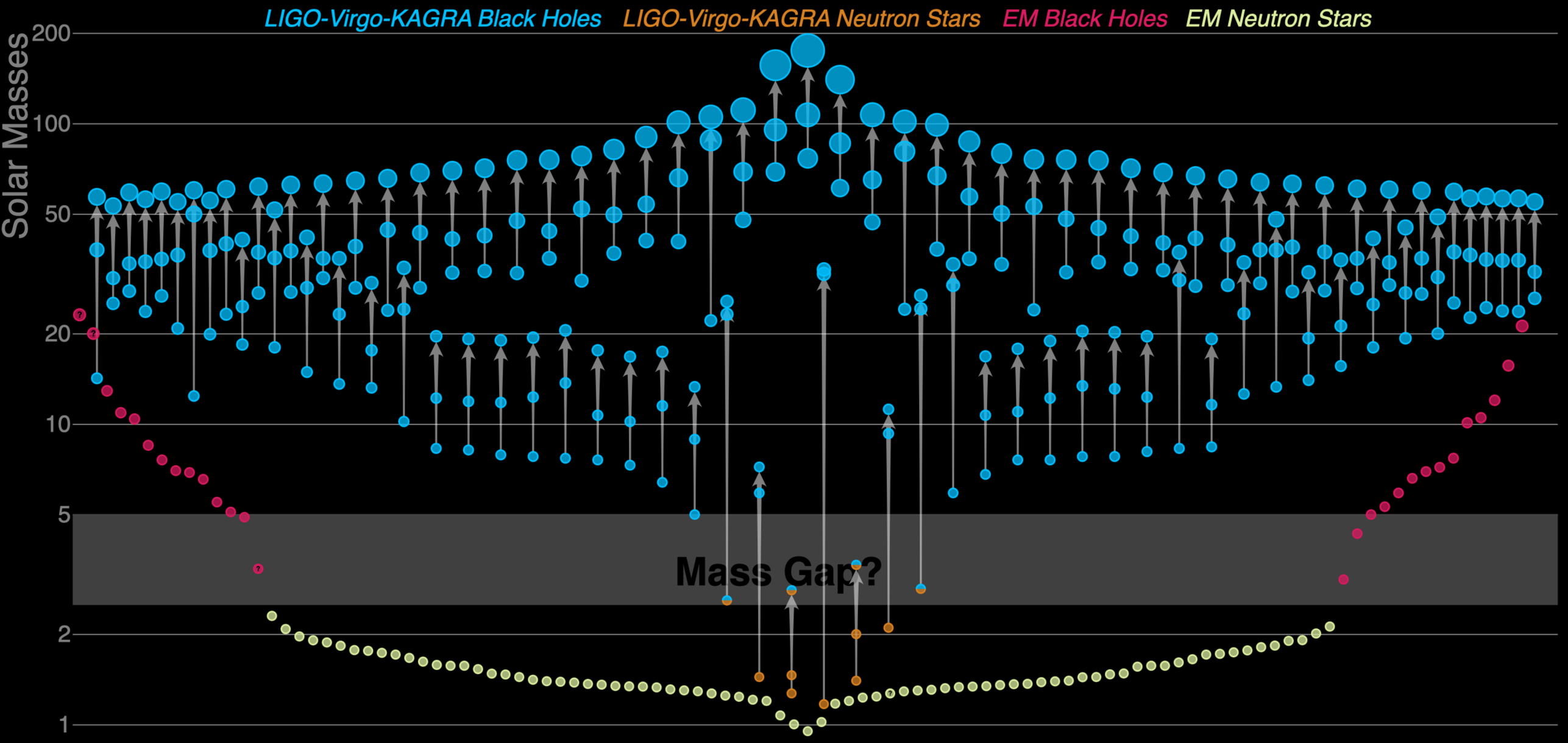
Observed data

Expected signal waveform

Residual
(consistent with noise)

Time series data from GW150914. Courtesy.

Masses in the Stellar Graveyard



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern