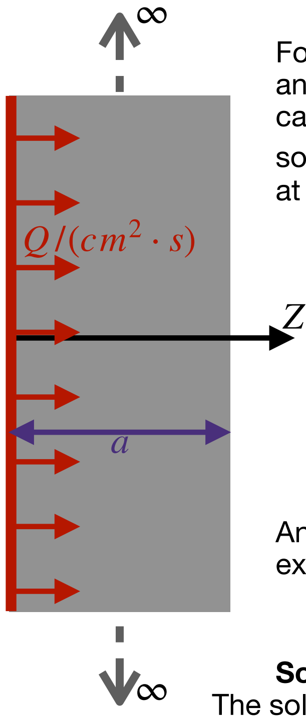
Solution of 1-g diffusion equation in 1D slab of non multiplying medium

By non multiplying medium, it is meant that the medium doesn't have fissile nuclides. Hence only scattering and external source can be present.



For simplicity, let us assume that the cross-section doesn't depend on space and there is not external source in the medium. The trivial solution in such a case is, total flux=0. However, let us consider the case where at z=0, a neutron source exists: Q neutrons per sq.cm per second are emitted in the z direction at that plane.

For such a case,

$$-D\frac{d^2\Phi(z)}{dz^2} + \Sigma_a \Phi(z) = 0 \qquad \text{at } z > 0$$

$$\lim_{z\to 0} J_z(z) = Q$$

And for a finite medium size, we can apply zero flux boundary condition at the extrapolated boundary of length a,

$$\Phi(a) = 0$$

Solution:

The solution can be written as,

$$\frac{d^2\Phi}{dz^2} - L^2\Phi = 0 \qquad \text{where, } L^2 = \frac{D}{\Sigma_a}$$

The quantity L is called diffusion length and in general a function of energy E. $L(E)$ is proportional to the average distance a neutron of energy E can travel through the medium.

The solution of the above equation is,

$$\Phi(z) = Ae^{z/L} + Be^{-z/L}$$

With the second boundary condition,

$$\Phi(a) = Ae^{a/L} + Be^{-a/L} = 0$$

$$\implies B = -Ae^{2a/L}$$

Hence,

$$\Phi(z) = A\left[e^{z/L} - e^{(2a-z)/L}\right]$$

The current becomes,

$$J_z(z) = -D\frac{d\Phi}{dz} = -\frac{DA}{L}\left[e^{z/L} + e^{(2a-z)/L}\right]$$

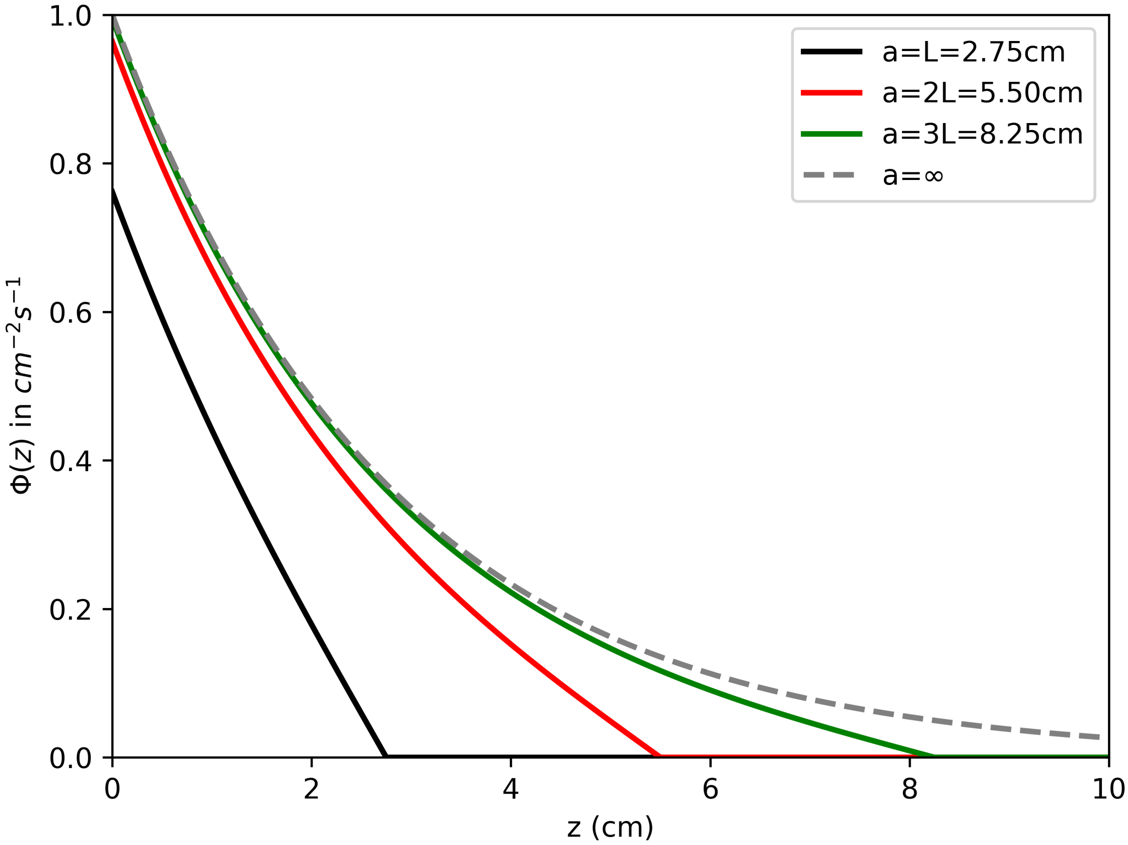
With the first boundary condition,

$$\lim_{z\to 0} J_z(z) = -\frac{DA}{L}\left[1 + e^{2a/L}\right] = Q$$

$$\implies A = -\frac{LQ}{D(1+e^{2a/L})}$$

Thus the complete solution gives,

$$\Phi(z) = -\frac{LQ}{D(1+e^{2a/L})}\left[e^{z/L} - e^{(2a-z)/L}\right] = -\frac{LQ}{De^{a/L}(e^{a/L}+e^{-a/L})}e^{a/L}\left[e^{(z-a)/L} - e^{-(z-a)/L}\right] = \frac{LQ}{D}\frac{\sinh\left[\frac{a-z}{L}\right]}{\cosh\left[\frac{a}{L}\right]}$$



Why this solution is important?

In reactor applications, neutron reflectors are placed outside the fuel locations to reduce neutron leakage from the core. In such a case, the interface between the fuel locations and reflector (which is mostly cylindrical) can be approximated as infinite (because the core dimensions are huge compared to neutron diffusion length), just like the case given above. Thus the above treatment gives the idea of how much reflector thickness is enough.

The result shown above clearly shows that with thickness~3 times diffusion length, the neutron flux at the interface (z=0) is same as the case where there is infinite reflector.