Revision

(1)

$$\Phi[cm^{-2}s^{-1}]=n(\vec{r},\hat{\Omega},E,t)v=\Phi(\vec{r},\hat{\Omega},E,t)$$
$$\sigma[cm^2\, or\, barn];1\,barn=10^{-24}cm^2$$
$$\Sigma=N\times\sigma[cm^{-1}]$$
$$N[atoms\ per\ cc\ or\ atoms\ per(parn-cm)]$$

Reaction rate per unit volume: $\underset{per\ cm}{\underset{\downarrow}{\Sigma}}\ \underset{per\ cm^2\ per\ s}{\underset{\downarrow}{\Phi}}$

In a V volume, reaction rate will be: $\Sigma\,\Phi\,V[s^{-1}]$

In a V volume, number of reaction in time dt will be : $\Sigma\,\Phi\,V\,dt$

To denote cross-section of type x: $\sigma_x|\Sigma_x$

Σ is defined as probability a neutron will undergo a reaction per unit path length. It is also inverse of mean free path.

(2)

Boltzmann transport equation:

$$\frac{1}{v}\frac{\partial\Phi(\vec{r},\hat{\Omega},E,t)}{\partial t}+\hat{\Omega}\cdot\vec{\nabla}\Phi(\vec{r},\hat{\Omega},E,t)+\Sigma_{tot}(\vec{r})\Phi(\vec{r},\hat{\Omega},E,t)=\frac{\chi(E)}{4\pi}\int\nu\Sigma_{fis}(\vec{r},E')\Phi(\vec{r},\hat{\Omega}',E',t)dE'd\hat{\Omega}'$$
$$+\int\Sigma_{scat}(\vec{r},\hat{\Omega}'\rightarrow\hat{\Omega},E'\rightarrow E)\Phi(\vec{r},\hat{\Omega}',E',t)dE'd\hat{\Omega}'$$
$$+Q_{ext}(\vec{r},\hat{\Omega},t)$$

LEFT SIDE:
TERM 1: temporal part
TERM 2: Streaming term, leak of neutron because of spatial variation in reaction rates
TERM 3: Absorption of neutrons

RIGHT SIDE: all terms are source terms
TERM 1: Fission term
TERM 2: Scattering term
TERM 3: External source term

If delayed neutron is to be taken, then fission term is divided into two parts.

1. This is linear integro-differential equation. Linearity because no n-n interaction.
2. This is mean behaviour of an underlying statistical phenomena. The mean behaviour comes from cross-section of n-nucleus reaction.
3. It cant be written in differential form because the underlying interactions inhibit continuity over energy and angle.

Integral form of transport equation

NTE:

$$\frac{1}{v}\frac{\partial\phi}{\partial t}+\hat{\Omega}\cdot\vec{\nabla}\phi+\Sigma_{tot}(\vec{r},E)\phi=q;$$
$$\phi\equiv\phi(\vec{r},\hat{\Omega},E,t);$$
$$q\equiv q(\vec{r},\hat{\Omega},E,t)$$

And q source terms has (1) fission term, (2) scattering term and (3) external source term.

Method of charactersitic to solve a PDE by reducing it to a ODE.

$$s = s(\vec{r}, t)$$

$$\frac{d\phi}{ds} = \frac{\partial \phi}{\partial t}\frac{dt}{ds} + \vec{\nabla}\phi \cdot \frac{d\vec{r}}{ds}$$

Comparing to the differential terms:

$$\frac{dt}{ds} = \frac{1}{v} \Rightarrow t = t_0 + \frac{s}{v}$$

$$\frac{d\vec{r}}{ds} = \hat{\Omega} \Rightarrow \vec{r} = \vec{r}_0 + s\hat{\Omega}$$

Put this to the NTE:

$$\frac{d\phi}{ds} + \Sigma_t \phi = q$$

$$\phi \equiv \phi(\vec{r}_0 + s\hat{\Omega}, \hat{\Omega}, E, t_0 + \frac{s}{v});$$

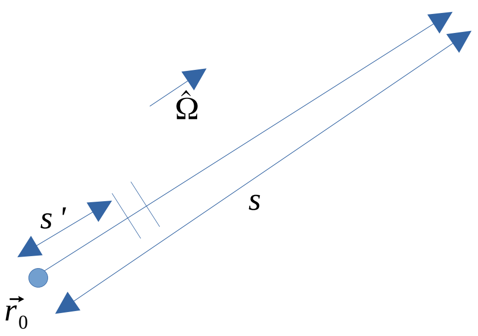
$$\Sigma_t \equiv \Sigma_t(\vec{r}_0 + s\hat{\Omega}, E)$$

Solution:

$$\frac{d}{ds}\left[\phi e^{\Sigma_t s}\right] = q e^{\Sigma_t s}$$

$$\Rightarrow \phi(\vec{r}_0 + s\hat{\Omega}, \hat{\Omega}, E, t_0 + \frac{s}{v}) = \int_{-\infty}^{s} ds' q(\vec{r}_0 + s'\hat{\Omega}, \hat{\Omega}, E, t_0 + \frac{s'}{v}) e^{-\Sigma_t(\vec{r}_0 + s'\hat{\Omega}, E)(s'-s)}$$

Explanation:



The equation is ok if the material doesn't change over the path s. If somewhere the material change, the cross-section will also change. Hence the general solution is formed by taking the term in power of x as integral from s' to s:

$$\Rightarrow \phi(\vec{r}_0 + s\hat{\Omega}, \hat{\Omega}, E, t_0 + \frac{s}{v}) = \int_{-\infty}^{s} ds' q(\vec{r}_0 + s'\hat{\Omega}, \hat{\Omega}, E, t_0 + \frac{s'}{v}) e^{\int_{s'}^{s} -\Sigma_t(\vec{r}_0 + s''\hat{\Omega}, E) ds''}$$

We are generally intersted in finding flux at a given position and time $(\vec{r}, t)$. For this a couple of change of variables are to be performed:

$$\begin{matrix} \vec{r}_0 + s\hat{\Omega} = \vec{r} \\ t_0 + \frac{s}{v} = t \end{matrix} \quad \Rightarrow \quad \begin{matrix} \vec{r}_0 + s'\hat{\Omega} = \vec{r} - (s - s')\hat{\Omega} \\ t_0 + \frac{s'}{v} = t - \frac{s - s'}{v} \end{matrix} \quad \text{Define} \quad \begin{matrix} \bar{s}' = s - s' \\ d\bar{s}' = -ds' \end{matrix} \quad \Rightarrow \quad \text{Limits:} \quad \begin{matrix} s' : [-\infty, s] \\ \bar{s}' : [\infty, 0] \end{matrix}$$

Similarly,

$$\begin{array}{c} \vec{r}_0 + s''\hat{\Omega} = \vec{r} - (s - s'')\hat{\Omega} \\ t_0 + \frac{s''}{v} = t - \frac{s - s''}{v} \end{array} \quad \text{Define} \quad \begin{array}{c} \bar{s}'' = s - s'' \\ d\bar{s}'' = -ds'' \end{array} \quad \Rightarrow \quad \text{Limits:} \quad \begin{array}{c} s'':[s', s] \\ \bar{s}':[\bar{s}', 0] \end{array}$$

By putting these, one gets:

$$\phi(\vec{r}, \hat{\Omega}, E, t) = \int_0^\infty d\bar{s}' q\left(\vec{r} - \bar{s}'\hat{\Omega}, \hat{\Omega}, E, t - \frac{\bar{s}'}{v}\right) e^{\int_0^{\bar{s}'} -\Sigma_t(\vec{r} - \bar{s}''\hat{\Omega}, E) d\bar{s}''}$$

The source term q contains fission and scattering terms, which in turn contains flux.
The equation can be thought of as:

$$\Phi = K\Phi + Q$$

K containing the fission, scattering and attenuation term, Q containg the external source and attenuation term.
A series of solution can be thought of as follows:

$$\begin{array}{l} \Phi_0 = Q \\ \Phi_1 = K\Phi_0 \\ \Phi_2 = K\Phi_1 \\ \Phi_3 = K\Phi_3 \\ \quad\vdots \end{array}$$

Physically

Φ_0 term is the flux which is a direct contribution from neutrons of external source without any collision inbetween

Φ_1 Neutrons of external source collides one time on its way, performs fission or scattering and the neutron produced in the process contributes to the flux; first collision neutrons

etc.

The total flux will thus be, $\Phi = \Phi_0 + \Phi_1 + \Phi_2 + \ldots$

Solution of the transport equation exists if the series converges, which is generally true because the interaction probabilities are small and the chances of neutrons produced due to high number of collision becomes negligible as the macroscopic cross-sections get multiplied.

If we try to integrate the flux over the directions, we need to see the direction dependence to each individual physical processes.
The fission cross-section is easy to deal with as these are mostly isotropic events: it doesn’t matter that from which direction the neutron came to nucleus and fission occured, the distribution of fission produced neutron over direction is uniform (unless it is hit by high energy neutron).
External sources are also easy to deal with for the same reason.
However, the scattering events are not always isotropic event. The distribution of scattered neutron largely depends on the direction of the projectile neutron, its energy and the target nucleus properties.
This inhibits the integral equation to be integrated over the angle variable.