Assumptions in NRP

1. Neutrons are taken as

- **classical,**
- **identical,**
- **non relativistic,**
- **point particles**.

Atom number density is $\sim 10^{22}$ per cc and neutron density is $\sim 10^{10}$: chance of neutron-neutron interaction is negligible. Only neutron-nucleus interaction is taken.

Criteria for classical:

De-Broglie wavelength should be less compared to inter-atomic distance.

Consider U235 atoms. Its density ~ 19 g/cc. Hence atom number density $\sim \frac{6.023\times 10^{23}\times 19}{235}$ atoms/cc. $\sim 6\times 10^{22}$ atoms/cc.

If interatomic distance is d m then,

$$6\times 10^{22}\times (d\,m)^3 \sim (10^{-2}m)^3 \Rightarrow d^3 \sim \frac{1}{6\times 10^{28}} \Rightarrow d \sim \frac{10^{-9}}{\sqrt[3]{60}} \sim \frac{10^{-9}}{4} \sim 2.5\times 10^{-10}$$

i.e., wavelength of the neutron should be less than 2.5 $\mathring{A}$

If v is the velocity of the neutron then,

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{m_n c^2 \frac{v}{c}}$$

$$\Rightarrow \frac{v}{c} = \frac{hc}{m_n c^2 \times \lambda} > \frac{1240[MeV - fm]}{940[MeV]\times 2.5\mathring{A}} = \frac{1240\,fm}{940\times 2.5\times 10^5 fm} \sim \frac{4}{3\times 2.5\times 10^5} = \frac{16}{3\times 10^6} \sim 5\times 10^{-6}$$

$$\Rightarrow v > 5\times 10^{-6}\times 3\times 10^8 [m/s] \sim 1000\, m/s$$

i.e. $\boldsymbol{v_{min} > 1[km/s]}$

Criteria for non relativistic:

The speed of the neutron should be far less than speed of light.

Range of speed in nuclear reactors:

Min speed:

The number of neutrons in the nuclear reactor is huge. If for a moment if we assume that neutrons are classical identical particles then like theory of thermal gasses, the neutrons should follow Maxwell-Boltzmann distribution. The neutrons will collide with atoms continuously and their kinetic energy will be exchanged with the vibrational energy of the atoms (temperature).

In the Maxwell Boltzmann distirbution the peak probability has the energy $k_B T$. In equilibrium the lowest a neutron energy can go with the collision phenomena is corresponding to the temperature of the material in which the neutrons are moving. In reactors this temperature can be atleast ~ 300 K. The corresponding energy will be $k_B \times 300\,K = 8.617\times 10^{-5}[eV/K]\times 300[K] \sim 0.0253\,eV$. Max neutrons will have energy very close to his value. Very few will be having energy less than this energy. Corresponding neutron velocity:

$$\frac{1}{2} m v^2 = 0.0253[eV]$$

$$\Rightarrow v^2 = \frac{2\times 0.0253[eV]}{m} = \frac{2\times 0.0253[eV]}{mc^2}\times c^2$$
$$\Rightarrow \left(\frac{v}{c}\right)^2 = \frac{2\times 0.0253[eV]}{940\,MeV} \sim \frac{5\times 10^{-2}[eV]}{9.40\times 10^8[eV]} \sim \frac{5}{9}\times 10^{-10}$$
$$\Rightarrow \frac{v}{c} = \frac{2.2}{3}\times 10^{-5}$$
$$\Rightarrow v = \frac{2.2}{3}\times 10^{-5}\times 3\times 10^8[m/s] = 2200\,m/s > 1\,km/s$$ **Hence neutron can be assumed classical**

Max speed:

The neutrons that are produced in fission will have maximum energy. (**Exceptions?ADS)**

This is a probabilistic event following Watt spectrum of energy distribution. The peak is at around 700keV and average at 1.2MeV. Even the maximum if we consider is 10MeV (though it has very very low probability), the velocity of that neutron:

$$\frac{1}{2} m v_{max}^2 = 10[MeV]$$

Clearly, $\frac{v^2}{E}$ is a constant. Hence,

$$\frac{v_{max}^2}{10[MeV]} = \frac{(2.2[km/s])^2}{0.0253\,eV}$$
$$\Rightarrow v_{max} = \sqrt{\frac{10^7}{0.0253}}\times 2.2[km/s] = \sqrt{\frac{10^{10}}{25.3}}\times 2.2[km/s] \sim \frac{10^5}{5}\times 2.2[km/s] \sim 4.4\times 10^4[km/s] < 3\times 10^5[km/s]$$

Hence neutron can be assumed non-relativistic.

Meaning of point particle: No internal structure is assumed, hence no requirement of moment of inertia, rotational energy etc. Neutrons will have only mass, position and velocity. Thus the neutron density can be described by $n(\vec{r},\vec{v},t) = n(\vec{r},\hat{\Omega},v,t) = n(\vec{r},\hat{\Omega},E,t)$

2. Interactions:

Only neutron-nucleus interaction is considered.

i. Scattering of neutron :
   - elastic
   - inelastic

ii. Absorption of neutron by nucleus –
   - fission
   - capture

iii. Other reactions –
   - (n,2n), (n,3n) etc: taken into account by making capture negative
   - (n, α) require high energy, probability of occurance is negligible;
   - (n,p) or (n,β) reactions: no neutron is produced in exit channel, can be clubbed with capture

3. Quantification of interaction:

Reaction rate:

Reaction per second due to a homogenous shower of neutron on 1 nucleus of cross-section area σ :

$$RR/nucleus = (n\times v)\times \sigma = \phi\times \sigma$$

If a material has N atoms per cc then Reaction Rate per cc:

$$RR/cc = N \times \phi \times \sigma = \Sigma \phi$$

Discussion:
(1) Dimesions.
RR = [s^{-1}]
σ = [cm^2]
$\Rightarrow \phi$ = [cm^{-2}s^{-1}]
Σ = [cm^{-1}]

(2) Cross-section of mixtures of nuclides
$\Sigma_{mix} = N_1 \times \sigma_1 + N_2 \times \sigma_2 + \ldots$

Concept: Neutron flux against neutron density and neutron current? Difference of neutron flux from neutron source? Cross-section is a simplified theory of highly complex quantum phenomena which is probabilistic in nature. In actual case it is dependent upon incoming neutron energy, prob of forming compound nucleus, energy levels of compound nucleus etc.

Most of the time neutron flies freely without any interaction. Hence the energy of the neutron is about equal to its pure kinetic energy.

$$E[MeV] = \frac{1}{2} \times . \frac{940\, MeV}{9 \times 10^{20} [cm/s]^2} \times (v\, [cm/s])^2$$

Whenever neutron energy is used, it is assumed that its pure kinetic energy is being intended.

Probability density function

Since cross-section highly varies with material and energy of incoming neutron, the resulting neutron flux is a strong function of position and momentum (continuous variables) of neutron. Interaction is a probabilistic event =>

1. "probability density function" of neutron density:
$n(\vec{r}, \vec{p}, t) d^3 r\, d^3 p\, dt$ = expected number of neutrons which all have positions lying within volume element $d^3 r$ at $\vec{r}$ ... = $n(\vec{r}, \hat{\Omega}, E, t)$ = **neutron angular density**

2. probability density function of neutron flux:
$\phi(\vec{r}, \hat{\Omega}, E, t) = |\vec{v}| n(\vec{r}, \hat{\Omega}, E, t) d^3 r\, d^3 p\, dt$ = expected neutron flux which all have position lying within volume element ... and speed... = **neutron angluar flux**

$\vec{J}(\vec{r}, \hat{\Omega}, E, t) = \vec{v}\, n(\vec{r}, \hat{\Omega}, E, t) d^3 r\, d^3 p\, dt$ = expected neutron current which all have position lying within volume element ... and velocity... = **neutron current**

$\Phi(\vec{r}, E, t) = \int d\hat{\Omega}\, \phi(\vec{r}, \hat{\Omega}, E, t)$ = **neutron total flux**

Discussions:

Transfer probabilities:
In RP it is required to describe the probabiltiy that the neutrons emerging from a collison have various directions and enrgies. A form of differential cross section is defined for collisions, such that scattering, fission and (n,2n) reactions, from which neutrons emerge, as follows:

$$\sigma_x(\vec{r},E')f_x(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)$$

For elastic scatterings,

$$\int d\hat{\Omega}\int dE f_{ES}(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)=1$$

For fission, since the produced neutrons are emitted isotropically,

$$f_{FIS}(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)=\frac{1}{4\pi}\nu(\vec{r},E'\to E)d\hat{\Omega}dE=\frac{1}{4\pi}\bar{\nu}(\vec{r},E')\chi(E'\to E)d\hat{\Omega}dE$$

Which after integration over produced neutron direction and energy gives,

$$\int d\hat{\Omega}'\int dE f_{FIS}(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)=\bar{\nu}(\vec{r},E')$$

Definition: total probability of neutron transfer from $\hat{\Omega}',E'$ to $\hat{\Omega},E$

$$\sigma(\vec{r},E')f(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)=\sum_x \sigma_x(\vec{r},E')f_x(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)$$

Up on integrating over the produced neutron direction and energy one can get,

$$\int d\hat{\Omega}\int dE f(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)=\frac{\sigma_{ES}(\vec{r},E')+\sigma_{(n,n')}(\vec{r},E')+\sigma_{FIS}(\vec{r},E')+\ldots}{\sigma(\vec{r},E')}\stackrel{\text{def}}{=}c(\vec{r},E')$$

This is the number of neutrons produced from all reactions if a neutron interacts with energy E’ at $\vec{r}$

How to know the neutron source given neutron flux?
Flux is not neutron strength. A direct check is their dimensions are unequal.
Flux is used for counting the number of reactions that are occuring at a phasespace. This together with transfer probability gives the number of neutrons produced at that phasespace which have a definite momentum vector (direction and energy). If at the same phasespace point the external neutron source is known, the total neutron source at that point can be easily deduced.
Total neutron source:

$$Q(\vec{r},\hat{\Omega},E,t)=\int dE'\int d\hat{\Omega}'\Sigma(\vec{r},E')f(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)\phi(\vec{r},\hat{\Omega}',E',t)+Q_{ext}(\vec{r},\hat{\Omega},E,t)$$