Lecture - 1

An atom consists of electrons and a compact nucleus of protons and neutrons.

Electrons → The mass of the electron is 1/2000 mass of proton or neutron. So electrons contribute very minimally to the total mass of the atom. They have electric charge of -1. All atoms have same number of electrons and protons. So, the positive and negative charges cancel out, making atoms electrically neutral.

Electrons are found outside the nucleus. Since opposite electric charges attract each other, negative electrons are attracted to the positive nucleus. The force of attraction keeps electrons constantly moving through the empty space around the nucleus.

Protons → Protons are found in the nucleus of the atom. This (Nucleus) is a tiny dense region at the centre of the atom. Protons have positive electric charge of (+1) and a mass of 1 amu. $m_p = 1.007825$ amu

Neutrons → Neutrons have no charge and are found in the nucleus of the atom. The mass of the neutron is slightly greater than the mass of a proton. Like protons, neutrons are bound into atom's nucleus as a result of a strong attractive force called Nuclear Force.

$$m_n = 1.008665 \text{ amu}$$

* Note 1 amu is a unit of mass equal to 1/12th mass of C-12 atom.

Nuclear force → is the force that acts between the protons and neutrons of the atom. Neutrons and protons, both nucleons (Neutrons and protons together are called as Nucleons) are affected by the nuclear force identically. Since protons have charge +1, they experience an electric force that tends to push them apart, but at short range the attractive force is strong enough to overcome the electro magnetic force. The Nuclear force binds the nucleons into atomic nuclei.

The nuclear force is powerfully attractive between nucleons at distances of about 1 fm or 10^{-15} ms but, it rapidly decreases to insignificance at distance beyond 2.5 fm. At distances less than 0.7 fm, the nuclear forces become repulsive. This repulsive component is responsible for the physical size of the nuclei, since the nucleons can come no closer than the force allows.

Size of the Nucleus

$$\rightarrow R = R_0 A^{1/3}$$

R_0 → Radius Constant $= 1.25$ fm

1 fm $= 10^{-15}$ metres

A → Mass No. of the nuclei

Nuclear size varies from 1.2 fm to about 6.7 fm in the heaviest nuclei.

Size of an Atom → $\sim 1 \,\text{Å} = 10^{-10}$ ms (5 times larger than Nucleus)

The fact that Nuclei do not clump together under normal conditions suggest that the nuclear force must be weaker than the electric repulsion at larger distances, but stronger at close range. Therefore, it has short range characteristics. An analogy to Nuclear force is the force between two small magnets, are very difficult to seperate when stuck together, but once pulled a short distance apart, the force between them drops almost to zero.

Isotopes → Isotoposes are two or more types of atoms that have same atomic number but different mass number due to different number of neutrons in their nuclei.

eg. U_{92}^{238} U_{92}^{235}

H_1^1 H_1^2 H_1^3

Protium Deuterium Tritium

Uranium fuel contains 0.72% of U_{92}^{235} atoms and 99.28% of U_{92}^{238} atoms.

Binding Enegy →

The enegy released when a nucleus is formed from its constituents particles or the minimum enegy required to divide the nucleus into its constituents particles is called the 'Binding Enegy' of the nucleus.

Neutrons and protons which constitute the nucleus have their respective masses. The mass of the nucleus formed should be equal to the sum of the masses of the neutrons and protons. But, it was observed that ~~when~~ ~~a~~ the mass of the nucleus ~~is~~ formed is less than the sum of the masses of neutrons and protons. So, there is a loss in mass. This is called as Mass Defect.

The reason for mass defect was explained by the famous scientist Albert Einstien.

During 20th century, Albert Einstien came up with the revolutionary theory that the mass and enegy are inter convertible ie. the mass can be converted into enegy and enegy can be converted into mass. He gave a famous equation,

$$E = mc^2$$

which stats that mass is related to enegy by a constant called c^2, where c is the speed of light. $c = 3 \times 10^8$ m/sec.

He explained through this equation that the deficit in mass of the nucleus is released in the form of energy. This energy is called as Binding Energy (B.E.)

Every nucleus has a B.E. associated with it. Binding Energy is the minimum energy required to dismantle the nucleus into its individual constituent neutrons and protons. It was found that the Nucleus of the iron atom has the highest Binding Energy and ~~its~~ is the most stable nucleus. That's why the Iron is the most abundant element found in the Universe.

In general, if a nucleus of atomic mass M is composed of A nucleons of which Z are protons, the average B.E. per nucleon (B.E/A) is given by,

$$\boxed{\underset{(\text{in MeV})}{B/A} = \frac{931}{A}\left[1.00785\,Z + 1.008665\,(A-Z) - M\right]}$$

eg. B/A of U^{235}_{92}

$$B/A = \frac{931}{235}\left[(1.00785 \times 92) + (1.008665 \times 143) - 235.0439\right]$$

$$= 7.59 \text{ MeV/nucleon}$$

b) Sn_{50}^{120} for which $M = 119.9022$ amu

$m_p = 1.007825$ amu , $m_n = 1.008665$ amu

$$B/A = \frac{931}{120}\left[(1.007825 \times 50) + (1.008665 \times 70) - 119.9022\right]$$

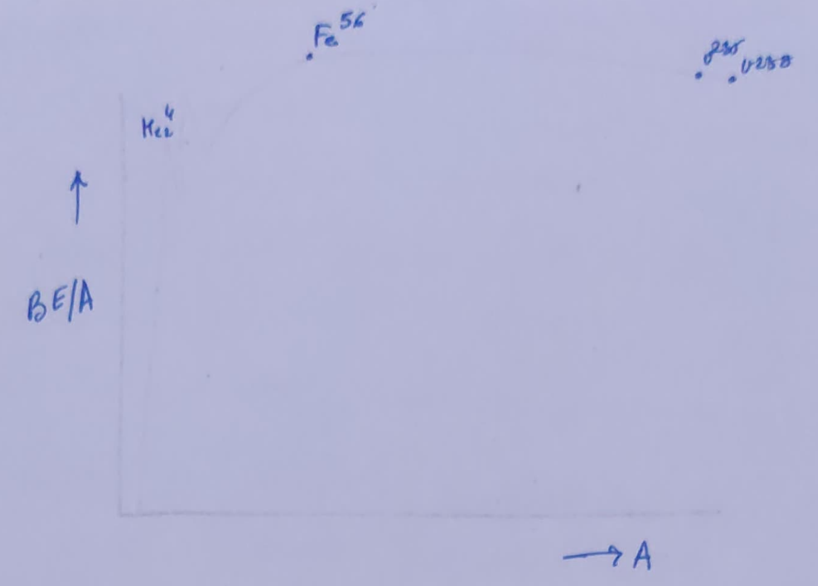
$= 8.50$ MeV/nucleon

c) Fe_{26}^{56} , $M = 55.9349375$ amu

$B/A = 8.8$ MeV/nucleon

d) Li_3^7 , $M = 7.016005$ amu

$B/A = 7.736$ MeV/nucleon



The curve has a flat maximum in the mass no. region 50 to 75. The average binding energy being about 8.6 Mev. These nuclei are relatively stable. Fe_{26}^{56} has the highest B.E/A of 8.8 Mev/A.

→ When two or more lighter nuclei combine together through fusion to form a medium atomic weight nucleus the average binding energy per nucleon increases. Therefore, energy will be released in the fusion process.

→ When a heavy nucleus splits or fissions into two or more medium weight nuclei, the average binding energy per nucleon again increases and so energy is released.

Liquid Drop Model →

Liquid Drop Model of the nucleus describes forces in atomic nuclei as if a tiny liquid drop formed the atomic nucleus. The liquid drop model considers that the forces on the nucleons on the surface are different from those on nucleons on the interior, where other attracting nucleons completly surround them. This is similar to taking into account surface tension as a contributor to the enegy of a tiny liquid drop.

→ Scattering Experiments suggest that the nuclei have approx. constant density.

→ Nuclei have their own volume and surface, where forces act differently.

→ In the ground state, nucleus is spherical.

→ If the sufficient kinetic or binding enegy is added, this spherical nucleus may be distorted to dumbbell shape and they may split into fragments.

The Weizasaecker formula is an empericalIy refined form of the liquid drop model for the binding enegy of nuclei. It has the following terms

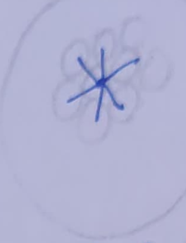
- Volume Term
- Surface Term
- Asymmetry Term
- Pairing Term

Using, the Weizaecker formula, the B.E. and also the enegy released per fission can be derived.

Volume Term

Each nucleon has a B.E. which binds it to the nucleus. Therefore, we get a term proportional to the volume ie. proportional to A.

$$\text{Volume Term} = a_v A$$



Volume of the nucleus is proportional R^3 and so is proportional to A (since $R = R_0 A^{1/3}$)
The basis of this term is the strong nuclear force
The strong force affects both protons and neutrons and so in independent of Z. Because the number of pair that can be taken from A particle ie. if a nucleon has interacted with all the other nucleons, volume term is proportional to A(A-1) but strong force has a very limited range and nucleon interacts only strongly with its nearest neighbors and is so proportional to A.

2. Surface Term

The nucleons at the surface of the liquid drop only interact with the other nucleons inside the nucleus so that their binding energy is reduced.
This leads to reduction of binding energy proportional to the surface area of the drop.

$$\text{Surface Term} = -a_s A^{2/3}$$

3. Columb Term → Although the B.E. is mainly due to the strong nuclear force, B.E. is reduced owing to the columb repulsion between the protons,

$$\text{Columb Term} = -\frac{a_c Z^2}{A^{1/3}} \quad \left(\text{Coloumb enegy} \propto 1/r\right)$$

4. Asymmetry term →

This is quantum effect arising from Pauli's Exclusion Principle which only allows two p or two n (with opposite spin direction) in each enegy state. If a nucleus contains the same number of proton and neutron then for each type of p and n fill to the same maximum enegy level (the Fermi level).

If on the other hand, we exchange neutron by a proton then that proton would be required by the exclusion principle to occupy higher enegy state. Since all of them below are already occupied.

Nuclides with $Z = N$ have a higher B.E. whereas for nuclei with different no. of p and n (for fixed A) the B.E. decreases as square of number difference.

$$\text{Asymmetry term} = -\frac{a_A (Z-N)^2}{A}$$

Pairing Term →

It is experimentally found that 2p or 2n bound more strongly than 1p or 1n. In order to account for this experimentally observation phenomenon we add a term to B.E. if number of protons and number of neutrons are both even, we substract if they are odd and do nothing if one is odd and other is even.

$$\delta(A, Z) = \frac{(-1)^Z + (-1)^N a_p A^{1/2}}{2}$$

Thus, the total B.E. →

$$= a_v A - a_s A^{2/3} - a_{sym} \frac{(A-2Z)^2}{A}$$

$$- a_c \frac{Z^2}{A^{1/3}} + \frac{a_p \delta}{A^{3/4}}$$

The emperically determined values of these Co-efficients are

$a_v = 14$, $a_s = 13$, $a_{symm} = 19.3$

$a_c = 0.585$, $a_p = 33$

Therefore total B.E. energy of U-235

$$A = 235 \;,\; Z = 92 \;,\; \delta = 0$$

$$\Rightarrow 1779 \text{ MeV}$$

$$B.E/A = \frac{1779}{235} = 7.57 \text{ MeV}$$