<u>Revision</u>

$$\Phi[cm^{-2}s^{-1}]=n(\vec{r},\hat{\Omega},E,t)v=\Phi(\vec{r},\hat{\Omega},E,t)$$

$$\sigma[cm^2\, or\, barn];1\, barn=10^{-24}\, cm^2$$

$$\Sigma=N\times\sigma[cm^{-1}]$$

$$N[atoms\ per\ cc\ or\ atoms\ per(parn-cm)]$$

Reaction rate per unit volume: $\underset{per\,cm|per\,cm^2\,per\,s}{\overset{\Sigma\Phi}{\downarrow\downarrow}}$

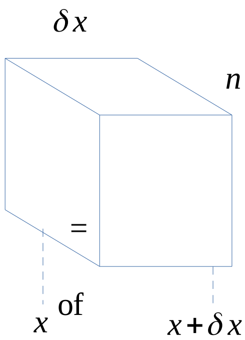
In a V volume, reaction rate will be: $\Sigma\Phi V[s^{-1}]$

In a V volume, number of reaction in time dt will be : $\Sigma\Phi V\,dt$

To denote cross-section of type x: $\sigma_x|\Sigma_x$

Definition of macroscopic cross-section

Consider a material with no neutron source inside. Some neutron is introduced on the left side of a simple cube of the material going in direction +ve x.



number of neutrons crossing left surface (area A) x in 1 second = $n(x)Av$

number of neutrons crossing right surface (area A) $x+\delta x$ in 1 second = $n(x+\delta x)Av$

Number of neutrons that are absorbed inside the volume $A\,\delta x$ in 1 second = $n(x+\delta x)Av-n(x)Av$

For conservation of neutron number, this amount should be equal to number of reactions that are happening inside the volume.

Thus,

$$n(x+\delta x)Av-n(x)Av=-\Sigma_{tot}\Phi(x)A\,\delta x$$

$$\Rightarrow\frac{dn}{dx}Av\,\delta x=-\Sigma_{tot}\Phi(x)A\,\delta x$$

$$\Rightarrow\frac{dn}{dx}v=-\Sigma_{tot}n(x)v$$

$$\Rightarrow\frac{1}{dx}\left(\frac{dn}{n}\right)=-\Sigma_{tot}$$

<u>**The macroscopic cross-section is the probability that a neutron will undergo a reaction per unit path length travelled in the material.**</u>

In actual case, the length traversed by the neutron should be extremely small so that the above derivation remains valid. Otherwise there would be scattering and other reactions which would act as neutron source in the material.

<u>**Exponential distribution:**</u> The solution of the above equation is,

$$n(x)=n(0)\exp[-\Sigma_{tot}x]$$

Rate of collision per unit area between $x\,and\,d\,x+\delta x=\Phi(x)\Sigma_{tot}dx=\Phi(0)\Sigma_{tot}\exp[-\Sigma_{tot}x]dx$

Probability of collision per unit area between $x\,and\,d\,x+\delta x=C\times\Sigma_{tot}\exp[-\Sigma_{tot}x]dx$

Where C is some normalization constant.

$$\int_0^\infty C\times\Sigma_{tot}\exp[-\Sigma_{tot}x]dx=1$$

$$\Rightarrow C\,\Sigma_{tot}\int_0^\infty\exp[-\Sigma_{tot}x]dx=1$$

$$\Rightarrow C=1$$

The probability of collision per unit area between $x \, and \, d \, x+\delta x=\Sigma_{tot} \exp[-\Sigma_{tot} x] dx$: Exponential distribution

Average distance a neutron can travel before getting absorbed:

$$\int_0^{\infty} x \Sigma_{tot} \exp[-\Sigma_{tot} x] dx$$

$$=-\Sigma_{tot} \frac{d}{d\Sigma_{tot}} \int_0^{\infty} \exp[-\Sigma_{tot} x] dx$$

$$=-\Sigma_{tot} \frac{d}{d\Sigma_{tot}} \left[\frac{1}{\Sigma_{tot}}\right] \text{.........................} \, Since \int_0^{\infty} \Sigma_{tot} \exp[-\Sigma_{tot} x] dx=1$$

$$=\frac{1}{\Sigma_{tot}}$$

This distance is mean free path of the neutron.

<u>Mean Free Path:</u> $\lambda_{tot}=\frac{1}{\Sigma_{tot}}$

Similar mean free path is defined for other type of reactions as well.

Boltzmann Transport Equation:

How particles probability density function should change under collision and external field (For electrons this would be some electric potential etc.):

$n(\vec{r}, \vec{v}, t) d^3 r \, d^3 v \, dt$: num of neutrons at ...

What happens to these neutrons after a very small time δt :

$\vec{r}$ to $\vec{r}+\vec{v}\delta t$

$\vec{v}$ to $\vec{v}+\frac{\vec{f}}{m}\delta t$

$$n(\vec{r}+\vec{v}\delta t, \vec{v}+\frac{\vec{f}}{m}\delta t, t+\delta t)-n(\vec{r}, \vec{v}, t)=dn|_{collision+ext}$$

Here we have used Liouville’s theorem regarding conservation of phase space. If the first term is expanded about $(\vec{r}, \vec{v}, t)$:

$$\vec{\nabla} n(\vec{r}, \vec{v}, t) \cdot \vec{v} \delta t+\frac{\partial n(\vec{r}, \vec{v}, t)}{\partial \vec{v}} \cdot \frac{\vec{f}}{m} \delta t+\frac{\partial n(\vec{r}, \vec{v}, t)}{\partial t} \delta t=dn|_{collision+ext}$$

$$\Rightarrow \vec{\nabla} n(\vec{r}, \vec{v}, t) \cdot \vec{v}+\frac{\partial n(\vec{r}, \vec{v}, t)}{\partial \vec{v}} \cdot \frac{\vec{f}}{m}+\frac{\partial n(\vec{r}, \vec{v}, t)}{\partial t}=\left.\frac{dn}{dt}\right|_{collision+ext}$$

This is **integro-differential** equation.

Boltzmann Neutron Transport Equation:

1. No external field that affects neutron motion
2. No n-n interaction, only n-nucleus interaction: linearity in Transport equation

$$\left.\frac{dn}{dt}\right|_{collision+ext}=\left.\frac{dn}{dt}\right|_{fission}+\left.\frac{dn}{dt}\right|_{scattering}+\left.\frac{dn}{dt}\right|_{absorption}+\left.\frac{dn}{dt}\right|_{ext}$$

$$\left.\frac{dn}{dt}\right|_{fission} = \frac{\chi(E)}{4\pi} \int \nu \Sigma_{fis}(\vec{r}, E') \Phi(\vec{r}, \hat{\Omega}', E', t) dE' d\hat{\Omega}'$$

$$\left.\frac{dn}{dt}\right|_{scattering} = \int \Sigma_{scat}(\vec{r}, \hat{\Omega}' \rightarrow \hat{\Omega}, E' \rightarrow E) \Phi(\vec{r}, \hat{\Omega}', E', t) dE' d\hat{\Omega}'$$

$$\left.\frac{dn}{dt}\right|_{absorption} = -\Sigma_{tot}(\vec{r}) \Phi(\vec{r}, \hat{\Omega}, E, t)$$

$$\left.\frac{dn}{dt}\right|_{ext} = Q_{ext}(\vec{r}, \hat{\Omega}, t)$$

By putting back all these terms in the Boltzmann transport equation, the neutron transport equation is obtained:

$$\hat{\Omega} \cdot \vec{\nabla} n(\vec{r}, \vec{v}, t) v + \frac{v}{v} \frac{\partial n(\vec{r}, \vec{v}, t)}{\partial t} = \left.\frac{dn}{dt}\right|_{collision+ext}$$

$$\begin{aligned} \Rightarrow \hat{\Omega} \cdot \vec{\nabla} \Phi(\vec{r}, \vec{v}, t) + \frac{1}{v} \frac{\partial \Phi(\vec{r}, \vec{v}, t)}{\partial t} = \frac{\chi(E)}{4\pi} \int \nu \Sigma_{fis}(\vec{r}, E') \Phi(\vec{r}, \hat{\Omega}', E', t) dE' d\hat{\Omega}' \\ + \int \Sigma_{scat}(\vec{r}, \hat{\Omega}' \rightarrow \hat{\Omega}, E' \rightarrow E) \Phi(\vec{r}, \hat{\Omega}', E', t) dE' d\hat{\Omega}' \\ - \Sigma_{tot}(\vec{r}) \Phi(\vec{r}, \hat{\Omega}, E, t) + Q_{ext}(\vec{r}, \hat{\Omega}, t) \end{aligned}$$

Rearranging:

$$\begin{aligned} \frac{1}{v} \frac{\partial \Phi(\vec{r}, \hat{\Omega}, E, t)}{\partial t} + \hat{\Omega} \cdot \vec{\nabla} \Phi(\vec{r}, \hat{\Omega}, E, t) + \Sigma_{tot}(\vec{r}) \Phi(\vec{r}, \hat{\Omega}, E, t) = \frac{\chi(E)}{4\pi} \int \nu \Sigma_{fis}(\vec{r}, E') \Phi(\vec{r}, \hat{\Omega}', E', t) dE' d\hat{\Omega}' \\ + \int \Sigma_{scat}(\vec{r}, \hat{\Omega}' \rightarrow \hat{\Omega}, E' \rightarrow E) \Phi(\vec{r}, \hat{\Omega}', E', t) dE' d\hat{\Omega}' \\ + Q_{ext}(\vec{r}, \hat{\Omega}, t) \end{aligned}$$

Discussion:

- Limitation of neutron transport equation:
  - Neutrons are classical, non relativistic, point particles
  - Mean behaviour of the statistical phenomena (cross-section)
  - Delayed neutron contribution is not considered here. A form of NTE exists with delayed neutrons.
- Can it be written completely in form of differentials?
  - No, whole range of energy and directions were necessary to consider in scattering and fission terms. In this events, energy and angle change in a discontinuous manner, hence it is not possible to define a derivative over these variables.
- Can it be written completely in form of integrals? Yes. Will be seen later.

<u>Continuity at the boundary</u>: $n(\vec{r}, \hat{\Omega}, E, t)$ is neutron density and this will not change if this packet crosses boundary and enters another media. Neither their velocity is going to change. Hence $\Phi(\vec{r}, \hat{\Omega}, E, t)$ is also continuous at the boundary. However, in some cases the neutron source is assumed to be concentrated. In such a case, neutron angular density will not be constant. A similar situation will happen if a thin strongly absorbing region is considered as a surface.

Some Important Conceptual Questions for science students:

1. What are cold neutrons/very cold/ ultra cold neutrons? How are they produced? Places where quantum mechanics needs to be taken into account in NRP.
2. How successful is neutron transport equation which do not take fluctuations into account? Where is fluctuation important?
3. A brief idea of MC methods.