<u>Revision</u>

(1)

$$\Phi[cm^{-2}s^{-1}]=n(\vec{r},\hat{\Omega},E,t)v=\Phi(\vec{r},\hat{\Omega},E,t)$$
$$\sigma[cm^{2}\,or\,barn];1\,barn=10^{-24}\,cm^{2}$$
$$\Sigma=N\times\sigma[cm^{-1}]$$
$$N[atoms\ per\ cc\ or\ atoms\ per\,(parn-cm)]$$

Reaction rate per unit volume:
$$\begin{array}{c|c}\Sigma & \Phi\\ \downarrow & \downarrow\\ per\ cm & per\ cm^{2}\ per\ s\end{array}$$

In a V volume, reaction rate will be: $\Sigma\,\Phi\,V[s^{-1}]$
In a V volume, number of reaction in time dt will be : $\Sigma\,\Phi\,V\,dt$

To denote cross-section of type x: $\sigma_x|\Sigma_x$
Σ is defined as probability a neutron will undergo a reaction per unit path length. It is also inverse of mean free path.

(2)

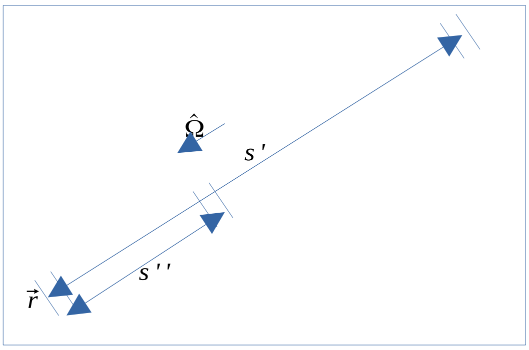
Boltzmann transport equation:

$$\frac{1}{v}\frac{\partial\Phi(\vec{r},\hat{\Omega},E,t)}{\partial t}+\hat{\Omega}\cdot\vec{\nabla}\Phi(\vec{r},\hat{\Omega},E,t)+\Sigma_{tot}(\vec{r})\Phi(\vec{r},\hat{\Omega},E,t)=\frac{\chi(E)}{4\pi}\int\nu\Sigma_{fis}(\vec{r},E')\Phi(\vec{r},\hat{\Omega}',E',t)\,dE'\,d\hat{\Omega}'$$
$$+\int\Sigma_{scat}(\vec{r},\hat{\Omega}'\rightarrow\hat{\Omega},E'\rightarrow E)\Phi(\vec{r},\hat{\Omega}',E',t)\,dE'\,d\hat{\Omega}'$$
$$+Q_{ext}(\vec{r},\hat{\Omega},t)$$

LEFT SIDE:
TERM 1: temporal part
TERM 2: Streaming term, leak of neutron because of spatial variation in reaction rates
TERM 3: Absorption of neutrons

RIGHT SIDE: all terms are source terms
TERM 1: Fission term
TERM 2: Scattering term
TERM 3: External source term

(3)



$$\phi(\vec{r},\hat{\Omega},E,t)=\int_0^{\infty}d\bar{s}'\,q\left(\vec{r}-\bar{s}'\hat{\Omega},\hat{\Omega},E,t-\frac{\bar{s}'}{v}\right)e^{\int_0^{\bar{s}'}-\Sigma_t(\vec{r}-\bar{s}''\hat{\Omega},E)\,d\bar{s}''}$$

This cant be integrated over angle mainly due to the scattering term, which in general is not isotropic in nature.

Isotropic assumption and angle integrated flux:

Under the isotropic assumption:

$$q(\vec{r},\hat{\Omega},E,t)\,d\hat{\Omega}=\frac{d\hat{\Omega}}{4\pi}q(\vec{r},E,t)$$

By putting this and integrating over the angle, the integral transport equation becomes:

$$\begin{aligned}\Phi(\vec{r},E,t)&=\int_0^{\infty} ds'\int_{4\pi} d\hat{\Omega}\,q(\vec{r}-s'\hat{\Omega},\hat{\Omega},E,t-\frac{s'}{v})e^{\int_0^{s'}-\Sigma_t(\vec{r}-s''\hat{\Omega},E)ds''}\\ &=\int_0^{\infty} ds'\int_{4\pi}\frac{d\hat{\Omega}}{4\pi}q(|\vec{r}-s'\hat{\Omega}|,E,t-\frac{s'}{v})e^{\int_0^{s'}-\Sigma_t(|\vec{r}-s''\hat{\Omega}|,E)ds''}\end{aligned}$$

The source of flux at $\vec{r}$ has been converted from a point at distance s’ from direction $\hat{\Omega}$ to all points around $\vec{r}$ which are at distance s', i.e. they lie on the surface of a sphere of radius s', the center of the sphere at the point $\vec{r}$.

ds' is the increment in radius of sphere which is related to the volume element of the sphere as: $dV'=s'^2 ds'\,d\hat{\Omega}$. Thus,

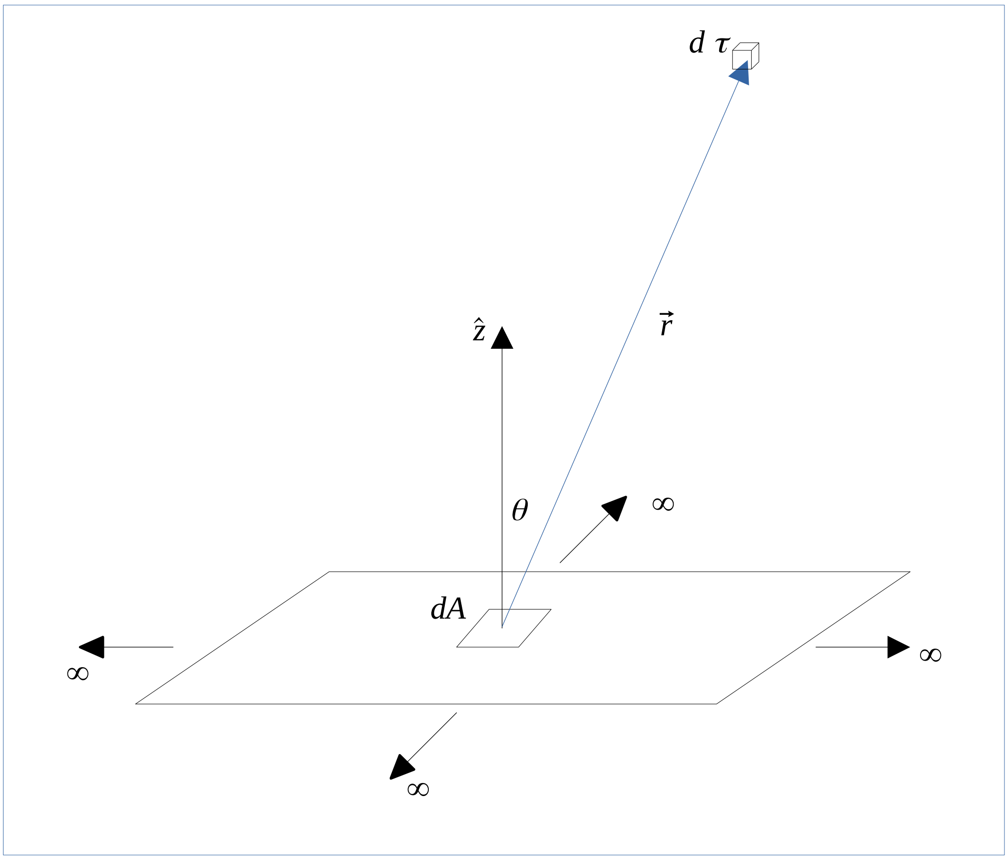
$$\Phi(\vec{r},E,t)=\int\frac{dV'}{4\pi s'^2}q(s',E,t-\frac{s'}{v})e^{\tau(E,dV'\rightarrow\vec{r})}$$

Where $\tau(E,dV'\rightarrow\vec{r})$ is the optical path length for energy E from the volume element to the point of consideration.

REMEMBER: $\Phi(\vec{r},E,t)=\int\frac{d^3r'}{4\pi|\vec{r}-\vec{r}'^2|}q(\vec{r}',E,t-\frac{|\vec{r}-\vec{r}'|}{v})e^{-\tau(E,\vec{r}'\rightarrow\vec{r})}$

Diffusion Approximation

Consider an isotropic scattering medium without fissile material and external sources.



Number of neutrons produced a second in $d\tau = \Sigma_s \Phi(\vec{r}) d\tau$

Number of neutrons coming from $d\tau$ going downwards

$$\perp dA = \frac{\Sigma_s \Phi(\vec{r}) d\tau}{4\pi r^2} dA \cos\theta \exp[-\Sigma_{tot} r] d\tau$$

i.e.

$$J_z^{downwards} = \int_{upper\ half\ space} \frac{\Sigma_s \phi(\vec{r})}{4\pi r^2} \cos\theta \exp[-\Sigma_{tot} r] r^2 \sin\theta d\theta d\phi$$

$$= \int_{upper\ half\ space} \frac{\Sigma_s}{4\pi r^2} \left[\phi(0) + \left.\frac{\partial\phi}{\partial x}\right|_0 x + \left.\frac{\partial\phi}{\partial y}\right|_0 y + \left.\frac{\partial\phi}{\partial z}\right|_0 z \right] \cos\theta \exp[-\Sigma_{tot} r] r^2 \sin\theta d\theta d\phi$$

Where we expanded flux at $\vec{r}$ in a Taylor series around $0 = (x=0, y=0, z=0)$ and assumed that the higher order variation of flux are negligible, i.e. $\frac{\partial^2\phi}{\partial \hat{n}^2} \sim 0, \frac{\partial^3\phi}{\partial \hat{n}^3} \sim 0, \ldots etc.$

First term:

$$= \frac{\Sigma_s \phi(0)}{4\pi} \int_0^\infty \exp\frac{[-\Sigma_{tot} r]}{r^2} r^2 dr \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta d\theta \int_0^{2\pi} d\phi$$

$$= \frac{\Sigma_s \phi(0)}{4\pi} \frac{1}{\Sigma_{tot}} 2\pi \int_0^1 \mu d\mu = \frac{\Sigma_s \phi(0)}{4\Sigma_{tot}}$$

Second term:

$$x=r\sin\theta\cos\phi,$$

$$\frac{\Sigma_s}{4\pi}\frac{\partial\phi}{\partial x}\bigg|_0 \int_0^\infty \exp\frac{[-\Sigma_{tot} r]}{r^2} r^3 dr \int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta d\theta \int_0^{2\pi}\cos\phi d\phi=0$$ Due to integration over ϕ

Similarly third term will also be zero as it will have a part : $\int_0^{2\pi}\sin\phi d\phi$
This is also expected because of rotational symmetry, choice of X and Y line are not unique and once term containing information of direction of X is zero, term containing the information of direction Y has to be zero.
Lastly, the X and Y components should also not contribute to the final current going downwards as the currents coming from opposite direction in XY plane would cancel out each other.

Last term:

$$z=r\cos\theta,$$

$$\frac{\Sigma_s}{4\pi}\frac{\partial\phi}{\partial z}\bigg|_0 \int_0^\infty \exp\frac{[-\Sigma_{tot} r]}{r^2} r^3 dr \int_0^{\frac{\pi}{2}} \sin\theta\cos^2\theta d\theta \int_0^{2\pi} d\phi$$

$$=\frac{\Sigma_s}{4\pi}\frac{\partial\phi}{\partial z}\bigg|_0 \frac{1}{\Sigma_{tot}}\int_0^1 \mu^2 d\mu\, 2\pi$$

$$=\frac{\Sigma_s}{6\Sigma_{tot}^2}\frac{\partial\phi}{\partial z}\bigg|_0$$

Thus, $J_z^{downwards}=\frac{\Sigma_s\phi(0)}{4\Sigma_{tot}}+\frac{\Sigma_s}{6\Sigma_{tot}^2}\frac{\partial\phi}{\partial z}\Big|_0$

A similar treatment with $J_z^{upwards}$ will have integration over θ from $\frac{-\pi}{2}$ to 0. The $\phi(0)$ term will be unaffected but $\frac{\partial\phi}{\partial z}\Big|_0$ term will have a negative sign.

$$J_z^{upwards}=\frac{\Sigma_s\phi(0)}{4\Sigma_{tot}}-\frac{\Sigma_s}{6\Sigma_{tot}^2}\frac{\partial\phi}{\partial z}\bigg|_0$$

Combining one can get:

$$J_z=J_z^{upwards}-J_z^{downwards}=\frac{-\Sigma_s}{3\Sigma_{tot}^2}\frac{\partial\phi}{\partial z}\bigg|_0$$

In vector form: $\vec{J}=-D\vec{\nabla}\Phi$ **where** $D=-\frac{\Sigma_s}{3\Sigma_t^2}$

This is Fick’s Law which appear in many branches of physics: $\vec{J}\propto-\vec{\nabla}\phi$
$\vec{J}=-D\vec{\nabla}\phi$

Physically : if there is a concentration gradient, there is net movement of particles. D is called diffusion coefficient.

Dimension of D: $[D]=\frac{[J(cm^{-2}s^{-1})]}{[\frac{\phi(cm^{-2}s^{-1})}{cm}]}=[cm]$

This dimension matches with mean free path. An associated **mean free path** and **macroscopic cross-section** is defined as:

$$\lambda_{transport} \stackrel{\text{def}}{=} D$$

$$\Sigma_{transport} \stackrel{\text{def}}{=} \frac{1}{\lambda_{transport}} = \frac{1}{D}$$

Diffusion Equation: $\dfrac{\partial \phi}{\partial t} + \vec{\nabla} \cdot \vec{J} = source - absorption$