

Q ⇒ Given an array of characters, print all possible subsequences of the given array.

example → ['a', 'b', 'c']

ans →

'a'

'b'

'c'

'ab'

'bc'

'ac'

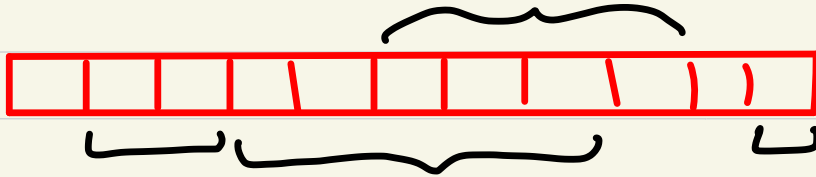
'abc'

''

→ empty

# Subsets vs Subsequence vs Subarray vs Substring

# Subarray → A subarray is a contiguous cross-section of an array and it maintains the relative order of the elements.



Ex  $\rightarrow [1, 2, 3, 4]$

1  
2  
3  
4

1, 2

2, 3

3, 4

1, 2, 3

2, 3, 4

1, 2, 3, 4

for an array of size  $n$ ,  
there are approx  $\frac{n \times (n+1)}{2}$

non empty subarrays.

$x, \dots, y \rightarrow [x, \dots, y]$

# Substrings → A substring is a contiguous cross-section of a string and it maintains the relative order of the elements.

# Subsets  $\rightarrow$  A subset contains elements of the given array, where they are not bound to be in contiguous order and the relative ordering of the elements doesn't matter.

ex  $\rightarrow$   $[1, 2, 3, 4]$  <sup>2 2 2 2</sup>

	$\{1\}$	$\{2\}$	$\{3\}$	$\{4\}$
	$\{1, 2\}$	$\{3, 1\}$	$\{4, 1\}$	$\{2, 3\}$
	$\{2, 4\}$	$\{3, 4\}$	$\{1, 2, 3\}$	$\{1, 2, 4\}$
	$\{3, 1, 4\}$	$\{2, 3, 4\}$	$\{4, 2, 3\}$	$\{1\}$
	$\{\}$ <u>empty</u>			

element  
pick  $\swarrow$  avoid  $\searrow$

for a given array  $\rightarrow$  we have  $2^n$  subsets  
 where  $n$  is the length of the  
 given array.

$[1, 2, 3]$

✓	✓	✓
✓	x	✓
✓	x	x
x	✓	x
x	x	✓
✓	✓	x
x	✓	✓
x	x	x

$\rightarrow \{1, 2, 3\}$

$\rightarrow \{1, 3\}$

$\{1\}$

$\{2\}$

$\{3\}$

$\{1, 2\}$

$\{2, 3\}$

$\{1, 2, 3\}$

empty

$\{\phi\}$   
 $\rightarrow \phi$

# Subsequences  $\rightarrow$  A subset contains elements of the given array, where they are not bound to be in contiguous order and the relative ordering of the elements does matter.

$[1, 2, 3] \rightarrow \{3, 1\}$  is not a subsequence.

$\{1\}$   $\{2\}$   $\{3\}$

$\{1, 2\}$   $\{2, 3\}$   $\{1, 3\}$

$\{1, 2, 3\}$   $\{\}$  empty

→ subsets & subsequences can be defined for both arrays & strings.

str = "abc"

→  
"a"  
"b"  
"c"  
"ab"  
"bc"  
"ac"  
"abc"  
"" → empty.



Q<sub>2</sub> Given an array of characters, print all possible subsequences of the given array.

example → ['a', 'b', 'c']

ans →

'a'  
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'c'  
'ab'  
'bc'  
'ac'  
'abc'  
''

→ empty

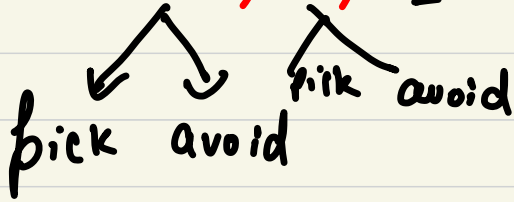
↙  
Recursively

⇒ We have an array of length  $n$ , then

total no. of subsequences will be  $2^n$ .

↪ picking elements in non contiguous order

$[a_1, a_2, a_3, \dots, a_n]$



pick  $\nwarrow$  avoid  $\nearrow$  pick  $\nwarrow$  avoid  $\nearrow$   
 $[1, 2, 3]$   
pick  $\nwarrow$  avoid  $\nearrow$

$2 \quad 2 \quad 2 \quad \dots \quad 2$   
 $[a_1 \quad a_2 \quad a_3 \quad \dots \quad a_n]$   $\rightarrow \underline{\underline{2^n}}$

We want to recursively write a func<sup>n</sup>  $f$ , to  
print all subsequences.

Subsequences of [a, b, c]

[a b c]

[a c]

[a b]

[a]

[b]

[c]

[b c]

[ ]

a ✓

a x

[b]

[b c]

[c]

[ ]

b ✓

b x

[c]

[ ]

c ✓

c x

Subsequences of empty array

↑

Subsequences of c

all subsequences of bc

if we want to get all the subsequences of  
a given array, by considering the property that  
every element got 2 choices  $\rightarrow$  pick  
 $\rightarrow$  avoid

$$f(arr, i, result) = f(arr, i+1, result + arr[i])$$

↑ pick

function prints all  
subsequences of

a given array. from index  $[i, n-1]$

result  $\rightarrow$  string  $\rightarrow$  storing all  
the subsequences

$i \rightarrow$  int  $\rightarrow$  to iterate on the  
array

arr  $\rightarrow$  array under consideration

( $n \rightarrow$  length of  
array)

avoid

Base Case

when array is  
exhausted  
if  $li == arr.length$   
print(result)  
return  
}

2'

$([a,b,c], 0, "")$

$a\checkmark$

$ax$

$([a,b,c], 1, "a")$

$([a,b,c], 1, "")$

$b\checkmark$

$bx$

$b\checkmark$

$bx$

$([a,b,c], 2, ab)$

$([a,b,c], 2, a)$

$([a,b,c], 2, b)$

$([a,b,c], 2, "")$

$c\checkmark$

$cx$

$c\checkmark$

$cx$

$c\checkmark$

$cx$

$c\checkmark$

$cx$

$(arr, 3, abc)$

$(arr, 3, ab)$

$(arr, 3, ac)$

$(arr, 3, a)$

$(arr, 3, bc)$

$(arr, 3, b)$

$(arr, 3, c)$

$(arr, 3, "")$

baseca



2 → a b c

3 → d e f

⋮

9 → w x y z

i/b → 2 3

1 2 5 6

"23"

ad

ae

af

bd

be

bf

cd

ce

cf

~~a✓~~  
b✓

~~c✓~~

"3"

↓

"d"

"e"

"f"

"325"

daj, dak, dal  
dbj, ddk, ddl  
dcj, dck, dcl

eaj, eak, eal

- - - -

- - - -

- - - -

- - - -

- - - -

"25"

aj  
ak  
al  
bj  
bk  
bl  
cj  
ck  
cl

a

b

c

"5"

d

k

l

2 → abc  
3 → def  
4 → ghi  
5 → jkl

$$f(\text{str}, i, \text{result}) = f(\text{str}, i+1, \text{result} + \text{map}[\text{str}[i]][k])$$

point all the combinations  
for the given string str,  
from index i to n-1

$$\forall k \in [0, \text{map}[\text{str}[i]].\text{len}-1]$$

$$\text{for } (k=0; k < \text{len}; k++)$$

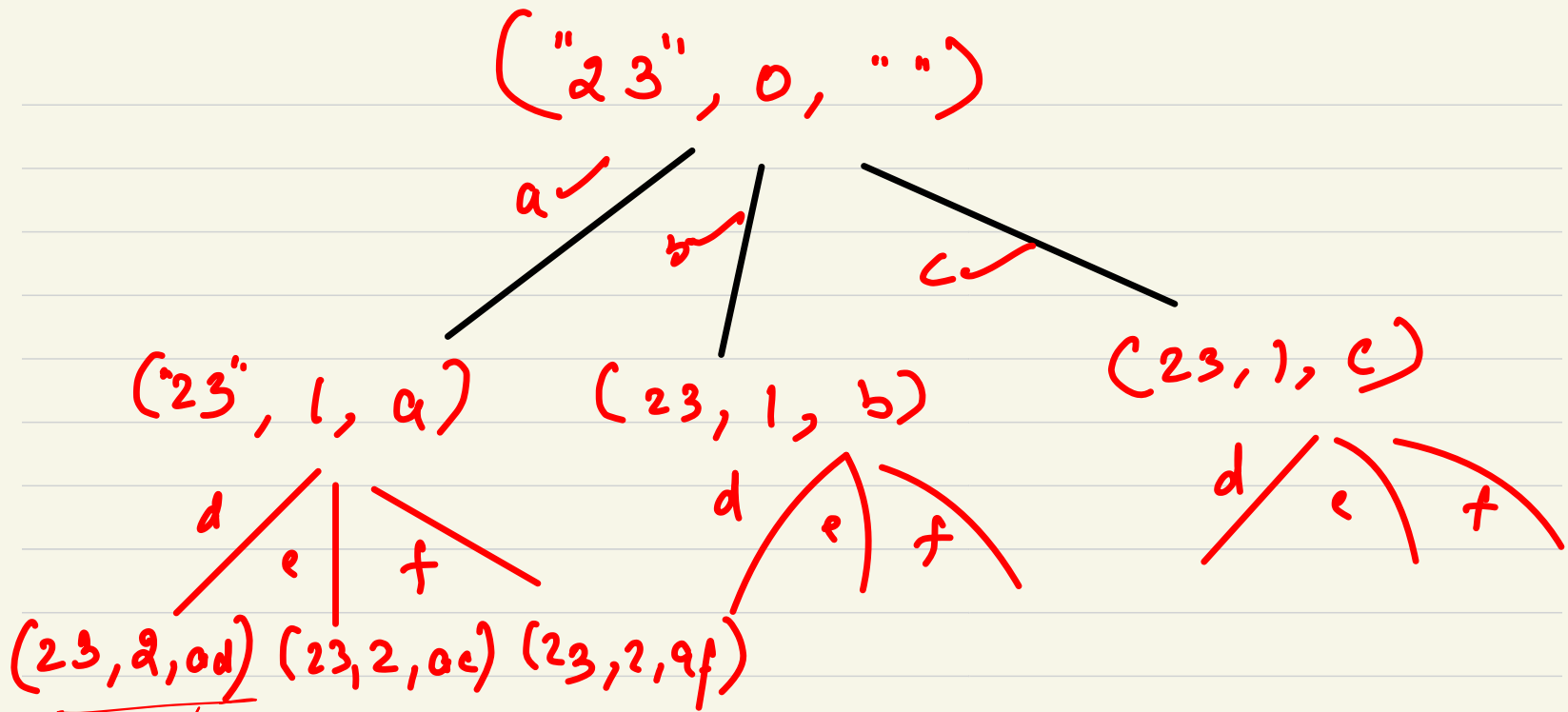
map = {  
2 : [a, b, c]  
3 : [d, e, f]  
:  
:  
9 : [w, x, y, z]

i=0



map[str[i]]

↓  
array of chars



Base  
Case

$\rightarrow$  if  $(i == \text{str.length})$