Recursion

→ Recursion is not just a computer science topic
It corelates with maths as well.

Mathematically, recursion means a function calling itself.

$$f(x) = f(f(x'))$$

$f(a,b) = a \times f(a,b-1)$

↳ func^n definition

This function calculates a^b

$$a^b = a \times a^{b-1}$$

Ex $f(2,3) \to 2^3 \to 8$

$f(2,4) \to 2^4 \to 16$

$2^4 = 2 \times 2^3$

In computer science, recursion means function calling itself.

```
function fun(x) {
    ...
    ...
    ...
    fun(x-1);
    ...
    ...
}
```

```
function gun(x) {
    console.log(x);
}

function fun(x) {
    ...
    gun(x-1)
    fun(x-1);
    ...
}
```

gun(x-1) → This is not recursion

fun(x-1); → This is recursion

Example-1

Factorial

factorial of any value n ⇒ $n \times (n-1) \times (n-2) \cdots \times 2 \times 1$

↳ n!

$5! = 5 \times 4 \times 3 \times 2 \times 1 \rightarrow 120$

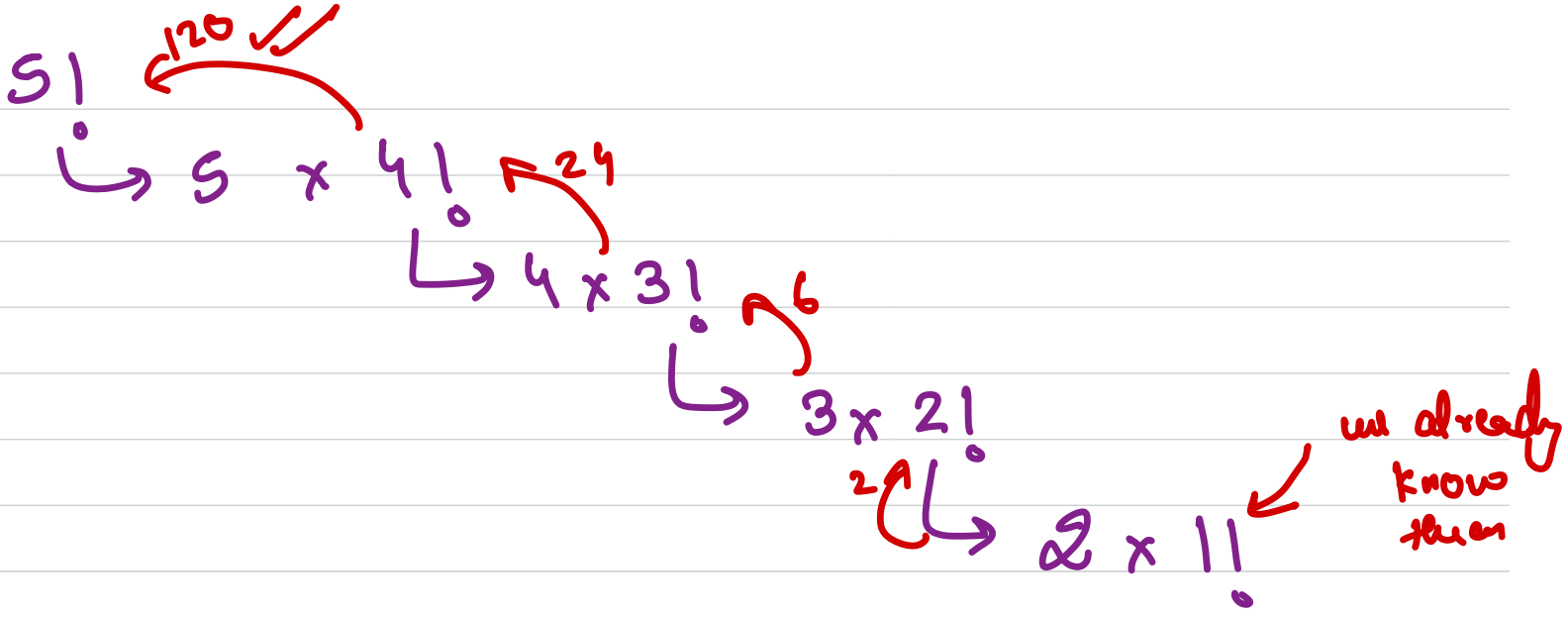
$4! = 4 \times 3 \times 2 \times 1 \rightarrow 24$

$3! = 3 \times 2 \times 1 \rightarrow 6$

$f(n) = n \times f(n-1)$

this function returns $n!$

The mathematical representation of a recursive function is called Recurrence Relation



What is $1!$? $\Rightarrow 1$

Recursion → In recursion, a function calls itself, where it tries to solve a smaller problem and then calc value of a bigger problem

PMI (principal of Mathematical Induction)

Q → What is the Sum of first N natural numbers ??

Ans → $\frac{n \times (n+1)}{2}$

3 step process

1) Base Case → the smallest value for which we can directly verify the ans.

2) Assumption

3) Verification / self task

for $n=1$ $\longrightarrow$ $\frac{1\times(1+1)}{2}$ $\longrightarrow$ is correct

↳ Base case

for some $n=k$ $\longrightarrow$ the formula is correct

i.e. $\frac{k\times(k+1)}{2}$

Let's prove ourselves that formula is correct for $n = k+1$

$$1 + 2 + 3 \text{ ------- } k + k+1$$

$$\rightarrow \frac{k \times (k+1)}{2} + (k+1)$$

$$\frac{k(k+1) + 2(k+1)}{2} \rightarrow \frac{(k+1)(k+2)}{2}$$

By formula → $n=(k+1)$

$$\frac{n\times(n+1)}{2} \rightarrow \frac{(k+1)(k+1+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

Components Of Recursion

1) Base Case
2) Assumption
3) Self work

Write a recursive sol^n for factorial of n

We will write a func^n

$f(n)$

↓

returns n!

Base Case → $(n==1)$ $f(1) \rightarrow 1$

```
if (n==1) return 1
```

Assumption → assume the function works fine for n-1

f(n-1) correctly calcs (n-1)!

Selfwork → f(n) = n x f(n-1)

Power function a^b

$f(a, b)$

↓

returns a^b

Base Case → if $(b == 0)$ then ans is 1

Assumption → assum func^n works fine for $b-1$

i.e. $f(a, b-1)$ is correct.

Self work → $a^b = a \times a^{b-1}$

$f(a,b) \longleftarrow a \times f(a,b-1)$

(correct)

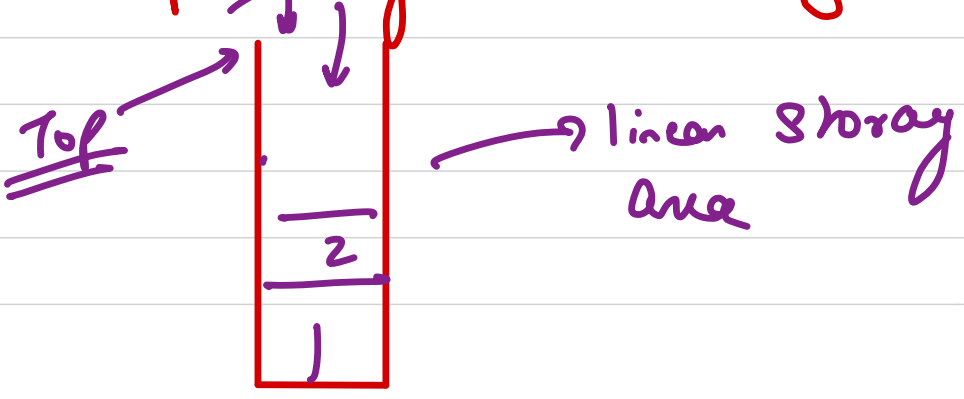
27^{51} $\quad 27 \times (27^{50}) =$

$27^{50} = y$

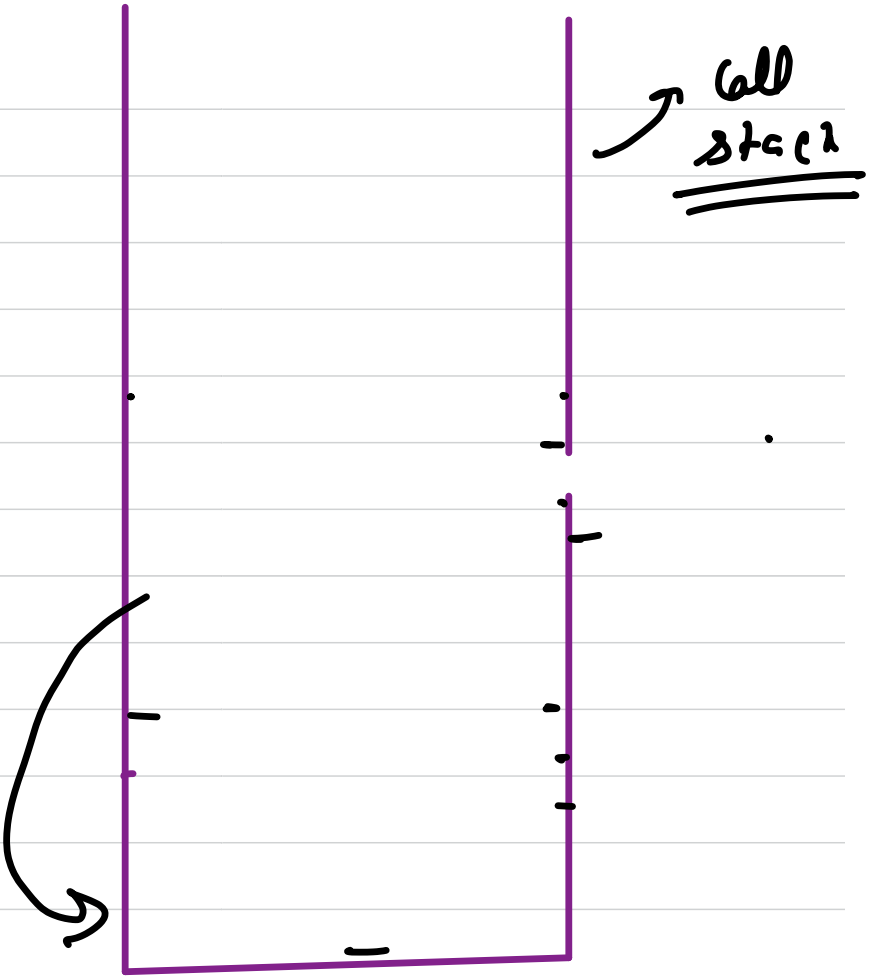
Q: What happens when we call a function??

In our memory, we have a lot of things going on.

One part of the memory is **call stack**



```
1 function fun(x)
2     let a = x + 10;
3     return a;
4 }
5 function gen(y)
6     let b = y + 20
7     let z = fun(b);
8     return z;
9 }
10 console.log(gen(10))
```



Whenever we call a new function it adds a new entry in the stack.

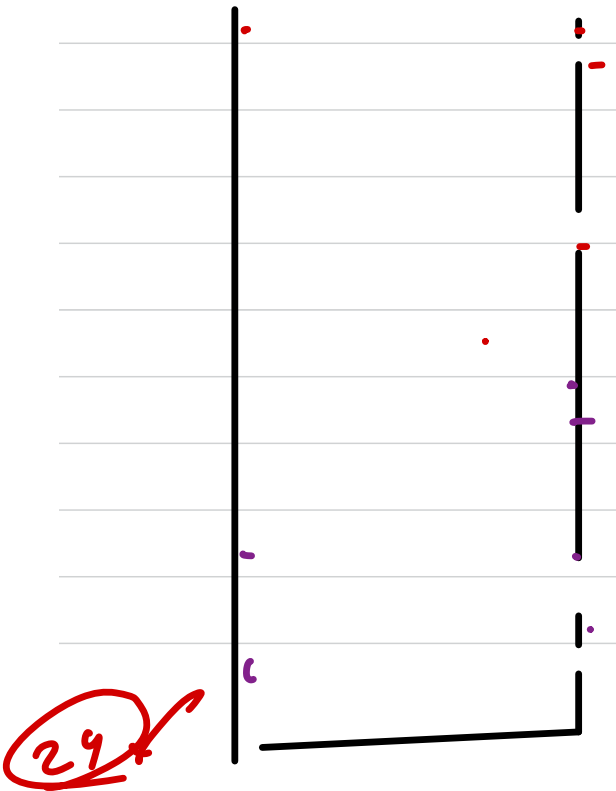
This new entry is called stack trace

Inside a trace we store all the variables and more data like current line of execution of function.

When we hit a return, the stack trace is removed & value is given back to the caller.

```
function fact(n) { // this function should calculate n!
    // Base case
    if(n == 1) return 1;
    let assume = fact(n-1); // that fact of n-1 is correct
    return n*assume;; // self work
}
```

fact (4)



Q. Given a value n, calc n^{th} fibonacci.

→ 0, 1, 1, 2, 3, 5, 8, 13, 21 ....

0^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	.....
fib	fib	fib	fib	fib	fib	fib	

n = 7

Ans = 13

Solve it recursively

n^{th} fib

$f(n)$

↓

returns n^{th} fib

Base Cases

$f(0) \rightarrow 0$

$f(1) \rightarrow 1$

} → stacks

Assumption → we assume $f(n-1)$ works

for & $f(n-2)$ works for

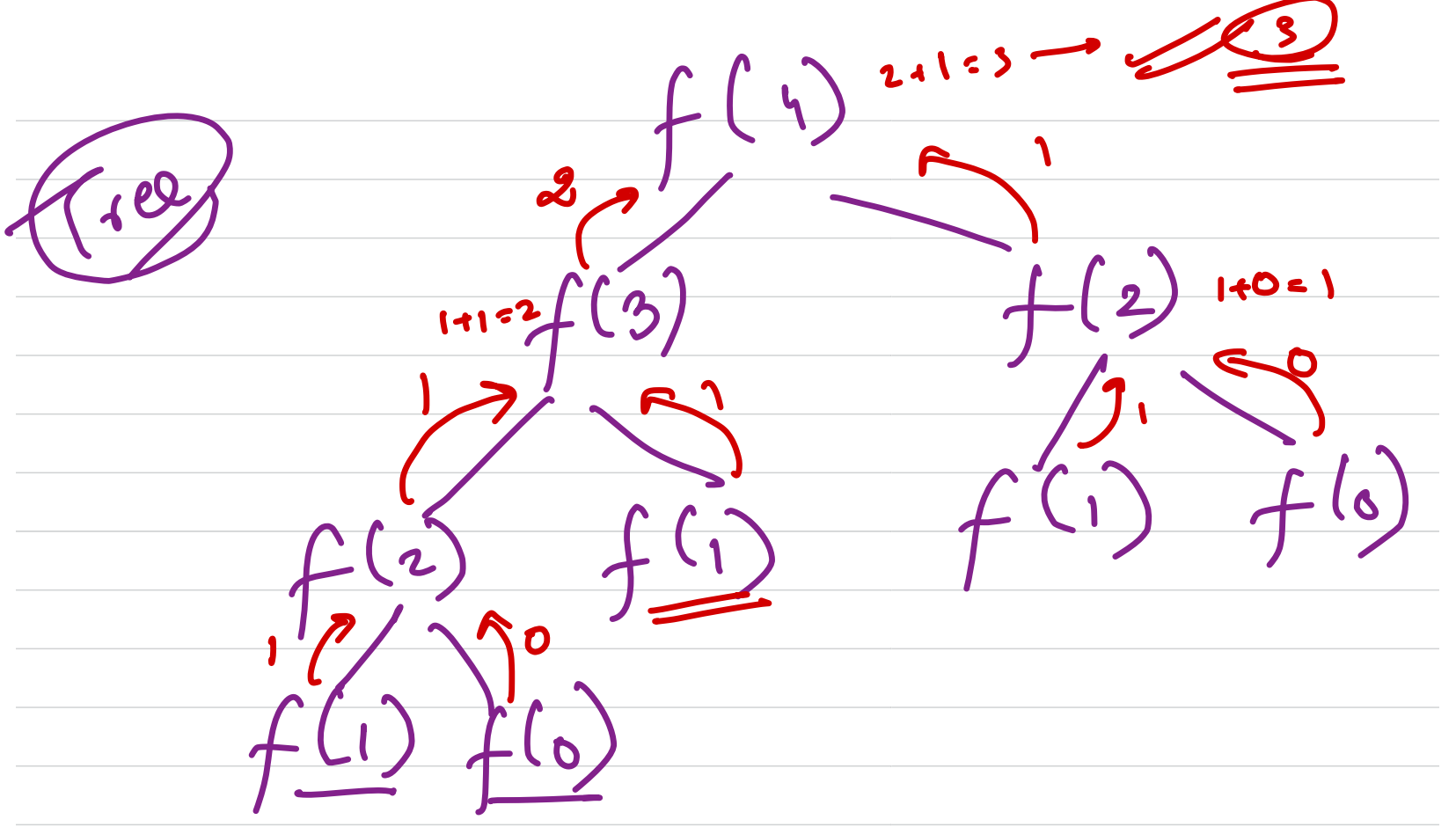
Self work → $f(n-1) + f(n-2)$

$f(n) = f(n-1) + f(n-2)$

```
function fib(n) {
    // base case
    if(n == 0) return 0;
    if(n == 1) return 1;
    // assumption
    let a = fib(n-1);
    let b = fib(n-2);
    // self work
    return a+b;
}

console.log(fib(3))
```

Recursion takes Extra space in memory


f(4)
2+1=3
3
2
1
f(3)
1+1=2
f(2)
1+0=1
1
1
1
0
f(2)
f(1)
f(1)
f(1)
f(0)
1
0
f(1)
f(0)

$$f(n) + f(n+1) = f(n+2)$$

$$f(n-1) + f(n) = f(n+1)$$

$$f(n-2) + f(n-1) = f(n)$$

Say $n+2 = x$

$$f(x-2) + f(x-1) = f(x)$$

Q→ Given a number n, print the first N natural no. in increasing order, recursively

n = 5

Base → n == 1 → console.log(1)

ans → 1
2
3
4
5

assum → if someone can give me the first 4 natural no.'s then we can do something

self → print 5

f(n) → f(n-1)

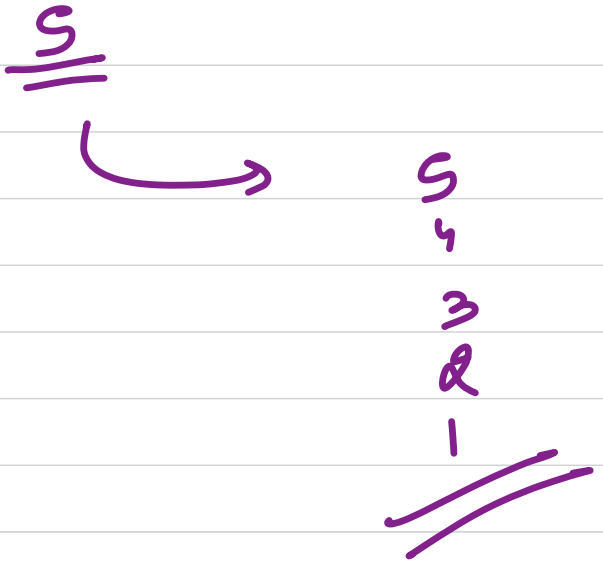
console.log(n)

which print

first n natural

no.s

assum f(n-1) works correctly



$f(n)$

n = 1

```
console.log(n)
f(n-1)
```

which prints first n natural no in decreasing order

f(n)

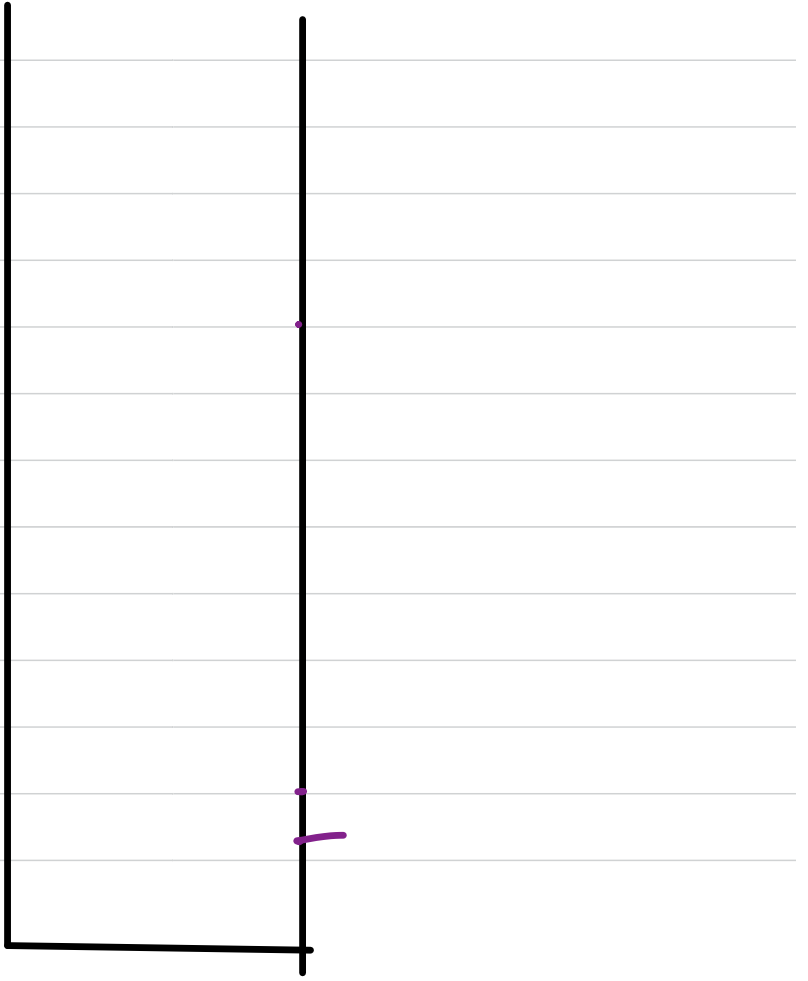
5
4
3
2
1

Console.log(n)

f(n-1) → assume it works fine

```
1   function print_inc(n) {
2       // base case
3       if(n == 1) {
4           console.log(1);
5           return;
6       }
7       print_inc(n-1);
8       console.log(n);
9   }
10
11  print_inc(3)
```

1
2
3



```
function print_dec(n) {
    // base case
    if(n == 1) {
        console.log(1);
        return;
    }
    console.log(n); // self task
    print_dec(n-1); // recursive assumption
}

print_dec(3)
```

3
2
1

Q. There are n friends, who want to go to a party. They can either go alone or go in a pair. Calc the no. of ways in which n friends can go to party. (Try Recursively)

Ex n = 3

ans → 4

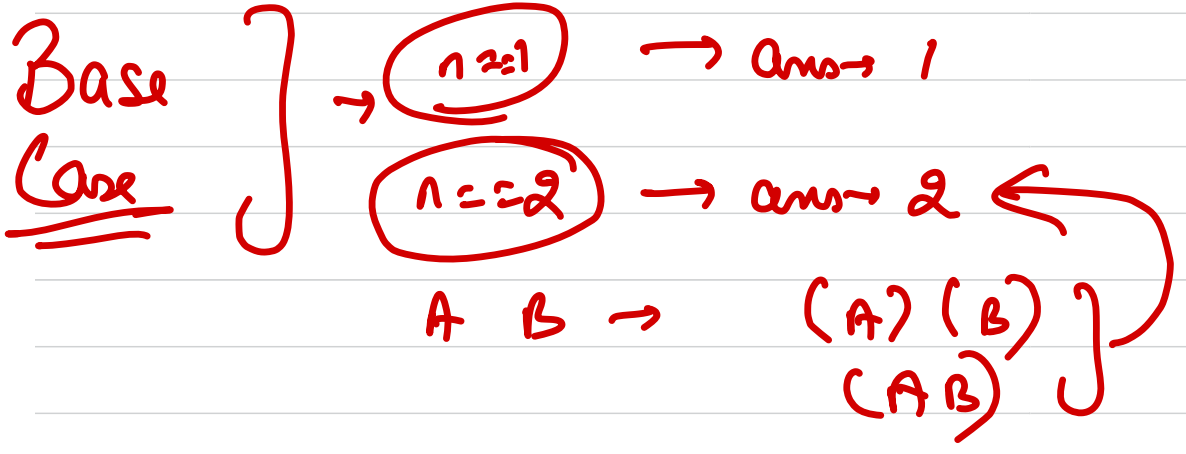
(A) (B) (C) →

(A B) (C) →

(A C) (B) →

(A) (B C) →

n=4 A B C D



n=4

A B C D

$f(n) = f(n-1) + (n-1) \times f(n-2)$

$f(n)$ ↓

returns the no. of ways in which n friends can go

$f(n-1)$: n^{th} friend said to go alone

$(n-1) \times f(n-2)$: n^{th} friend said to make a pair

n=4 A B C D → nth friend

D: alone / pair

what if D goes alone ??

(A) (B) (C) (D)

(AB) (C) (D)

(AC) (B) (D)

(BC) (A) (D)

What if n^{th} friend asked to make a pair ??

n^{th} friend

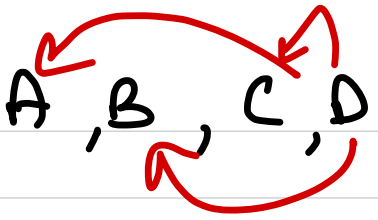
n=4 A B C D

In how many ways n^{th} friend can make a pair ?? → (n-1)

After making a pair how many elements left ?? → (n-2)

(n-1) x f(n-2)

3x2
=6



(DC) (A)(B)]
(DC) (AB)

(DB) (A)(C)]
(DB) (AC)

(AD) (B)(C)]
(AD) (BC)

6

$f(n-2) + f(n-2) + f(n-2)$

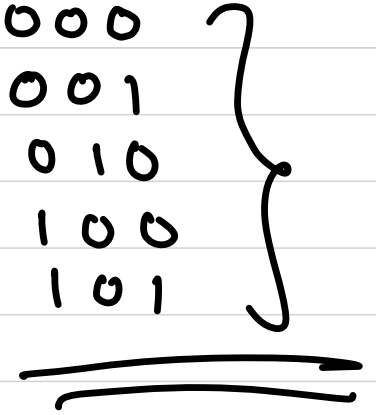
$(n-1) \times f(n-2)$

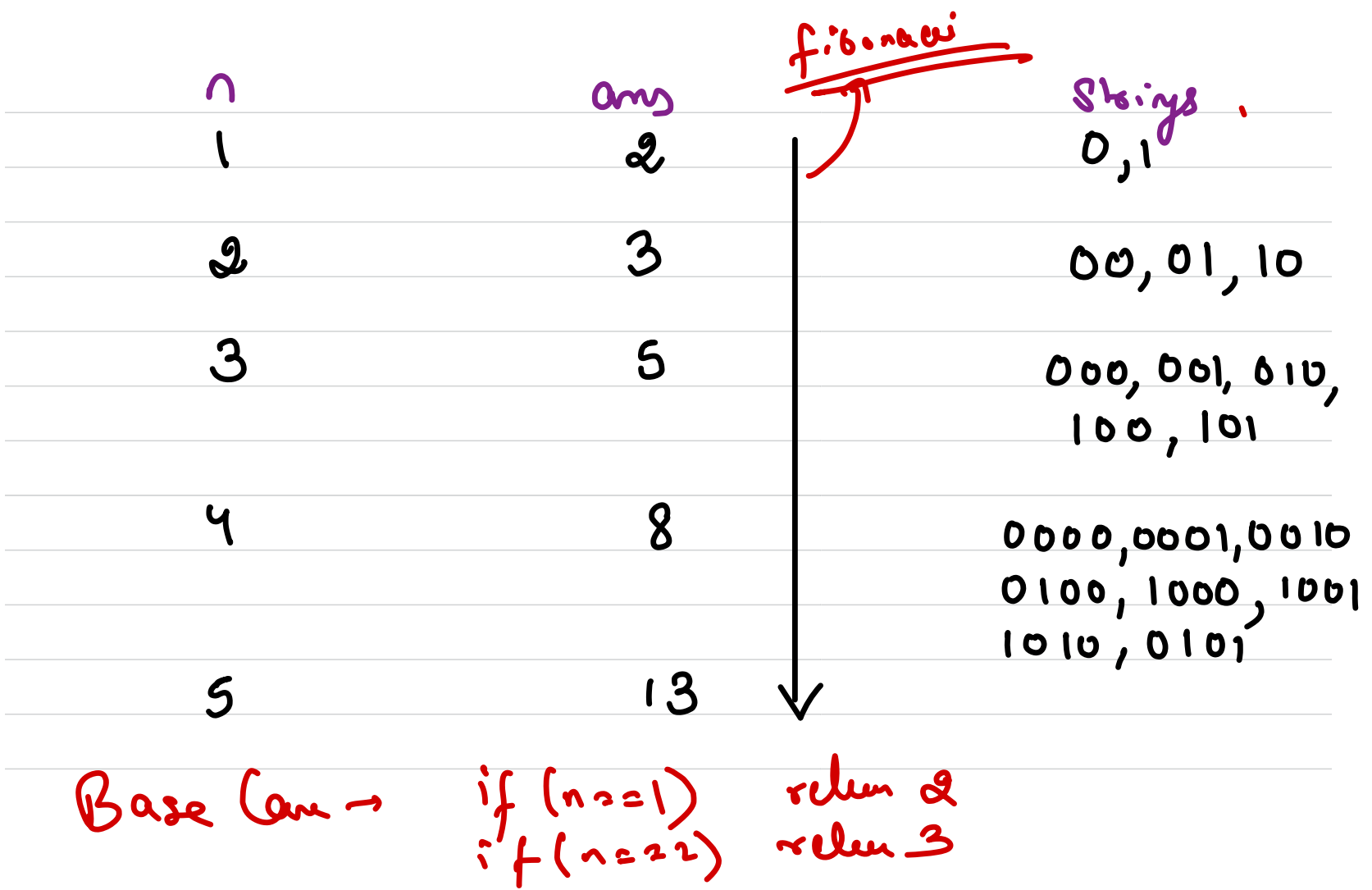
Q. Given a number n, calculate the no. of binary strings of length n with no consecutive one.

n = 3

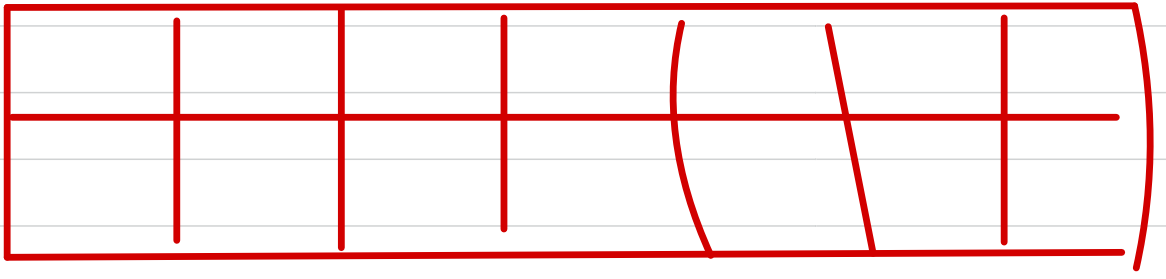
Ans → 5

000
001
010
100
101

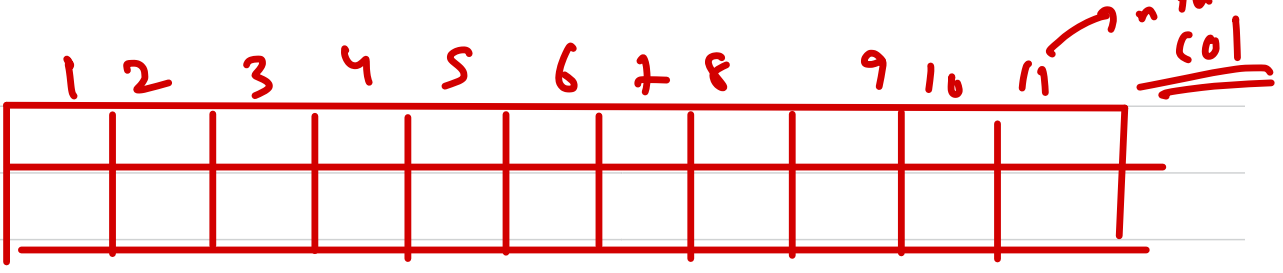



fibonacci
n
ans
Strings
1
2
0,1
2
3
00,01,10
3
5
000,001,010,
100,101
4
8
0000,0001,0010
0100,1000,1001
1010,0101
5
13
Base Case →
if (n==1)
if (n==2)
return 2
return 3

$$f(n) = f(n-1) + f(n-2)$$


2x1


2x1



2x1

On the n^{th} column you can put a tile either vertically or horizontally

$f(n) = f(n-1) + f(n-2)$

$f(n)$: return the no. of ways to put tiles on $2 \times n$ board

$f(n-1)$: n^{th} board we put a vertical tile

$f(n-2)$: n^{th} board with $(n-1)^{th}$ board filled with horizontal tile

Base Case

$n == 1 \rightarrow 1$

$n == 2 \rightarrow 2$


2
3
1

Q) Given an array, print all subsets of the array.

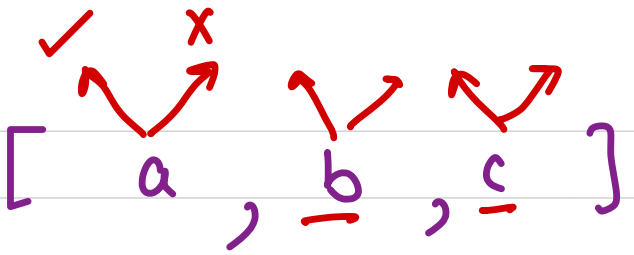
→ [1, 2, 3]

1
2
3
1 2
1 3
2 3
1 2 3
empty

If there is a set of length n,
how many subsets are possible ??

2^n

Permutation Combination

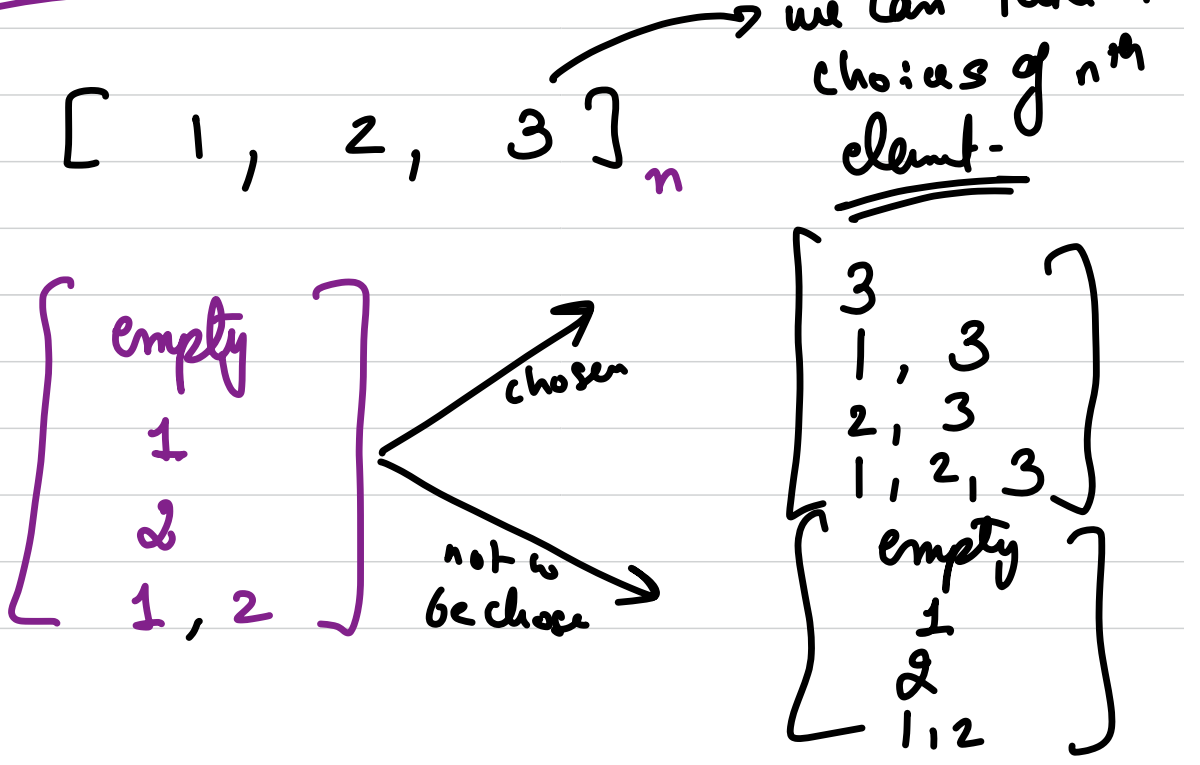


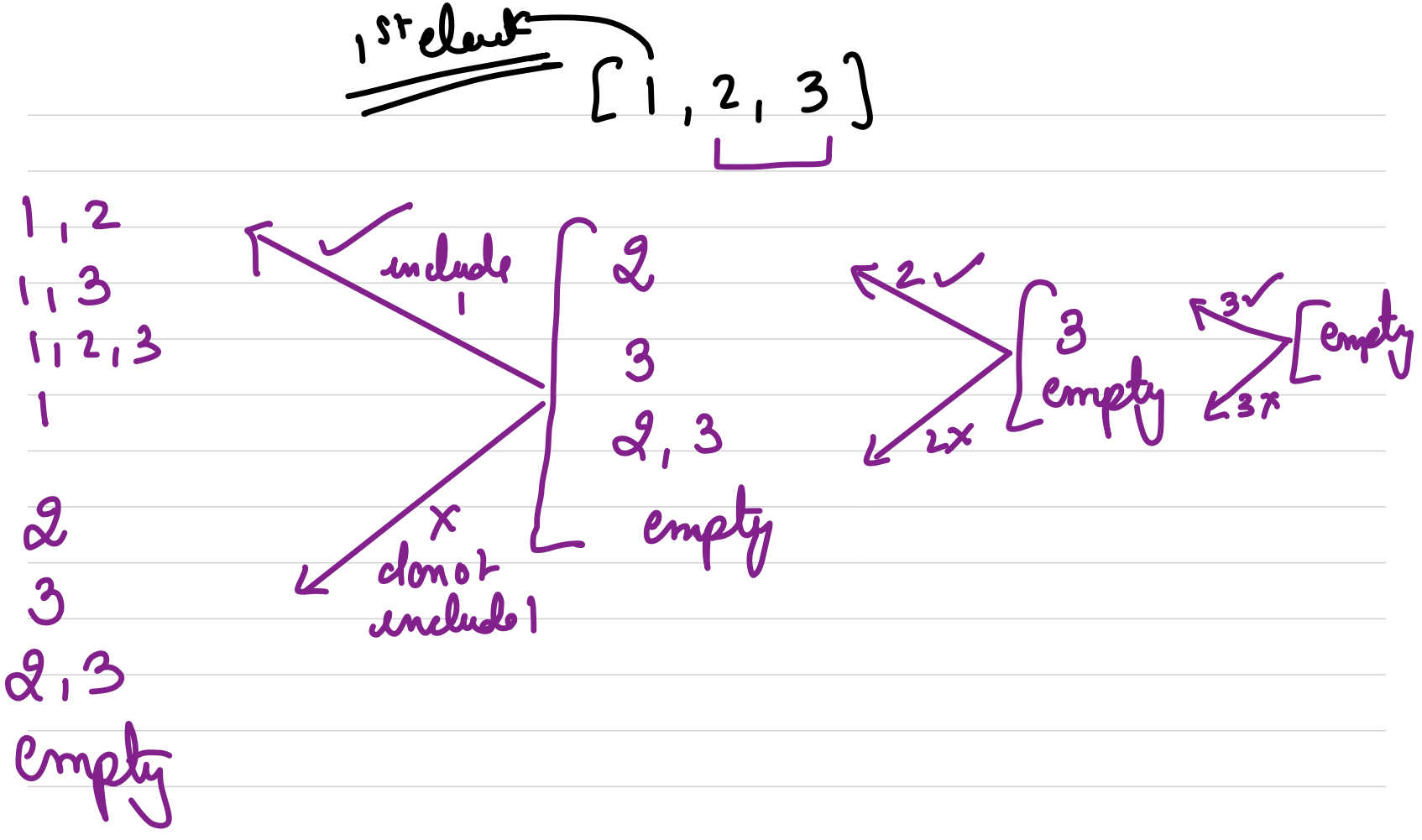
$2 \times 2 \times 2 \cdots$

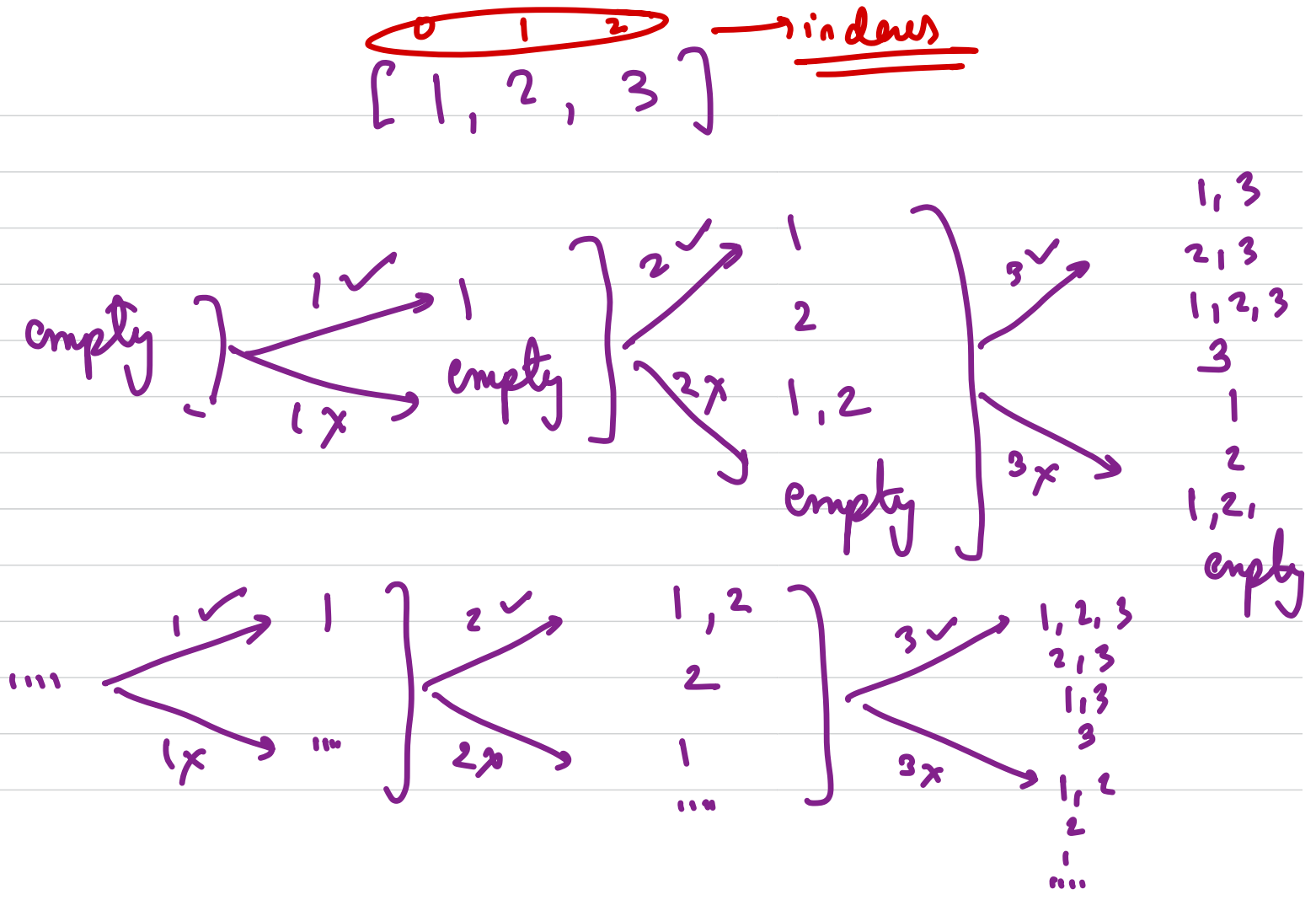
2^n

✗	✓	✓	→ b c
✓	✗	✓	→ a c
✗	✓	✗	→ b
✓	✓	✓	→ a b c
✗	✗	✗	→ empty
✓	✓	✗	→ a b
✗	✗	✓	→ c

We can use the same formula to build a recursive solution.




1st elem
[1, 2, 3]
include 1
x
donot include 1
2
3
2, 3
empty
2
2x
3
empty
3
3x
empty
1, 2
1, 3
1, 2, 3
1
2
3
2, 3
empty


0 1 2
indexes
[1, 2, 3]
empty
1 ✓
1 ✗
1
empty
2 ✓
2 ✗
1
2
1,2
empty
3 ✓
3 ✗
1, 3
2, 3
1, 2, 3
3
1
2
1, 2,
empty
.....
1 ✓
1 ✗
1
....
2 ✓
2 ✗
1, 2
2
1
....
3 ✓
3 ✗
1, 2, 3
2, 3
1, 3
3
1, 2
2
1
....

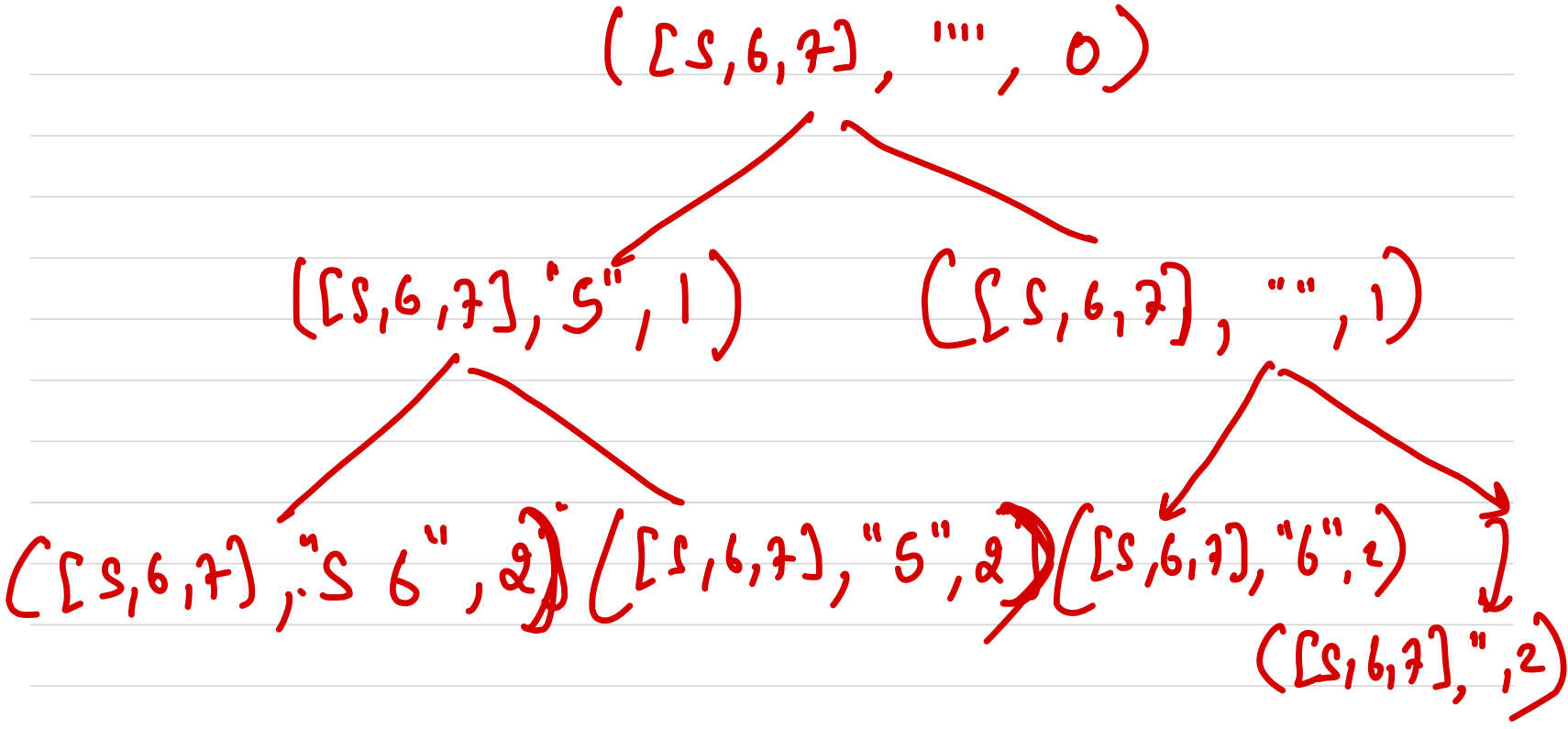
$f(arr, res, i) =$ $f(arr, res + arr[0], i+1)$ ← append

$f(arr, res, i+1)$

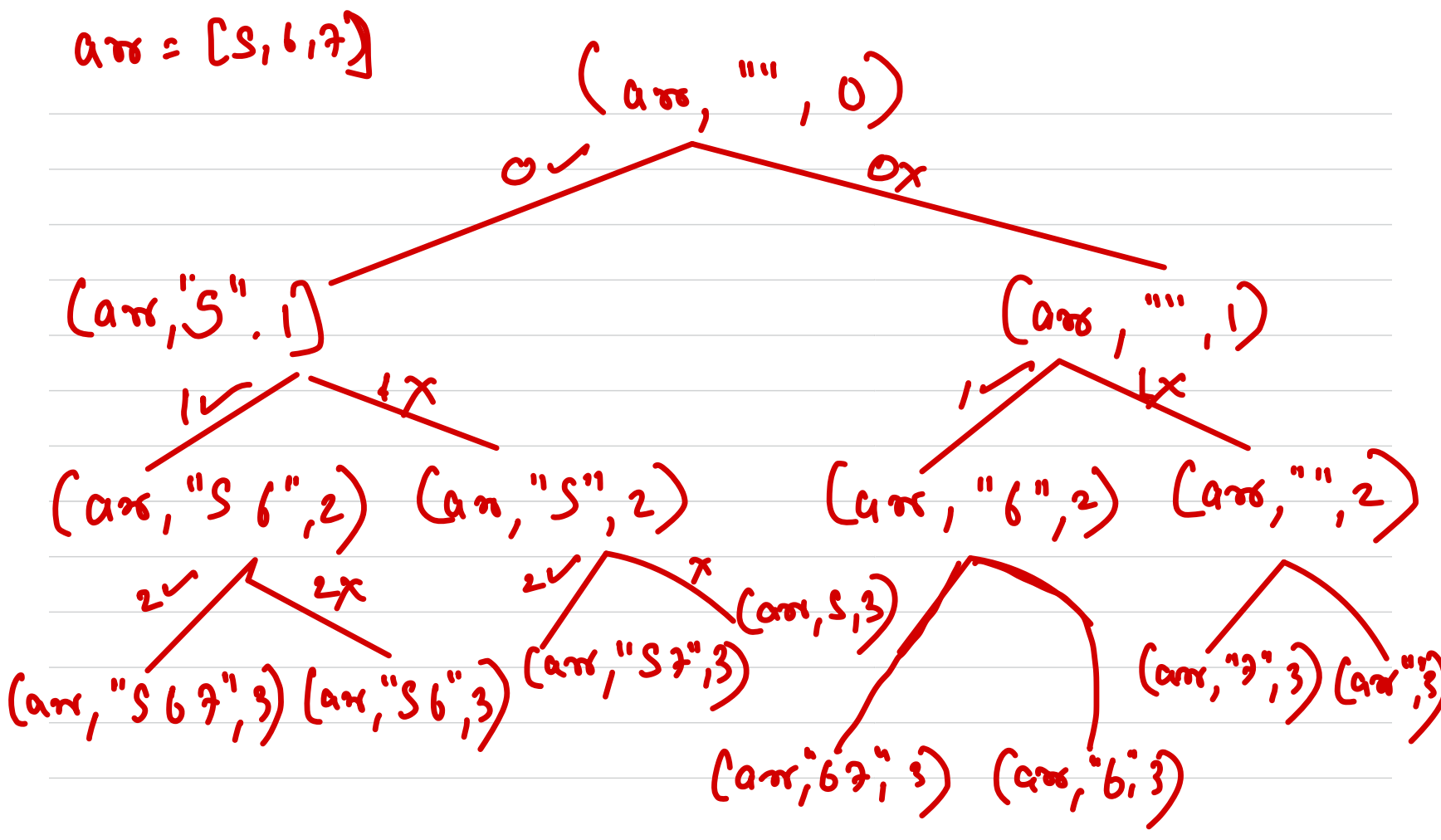
↳ this function prints all the subset of arr.

```
"3" + 7
"3 7"
```

Console.log(f([1,2,3], ""))


([5,6,7], "", 0)
([5,6,7], "5", 1)
([5,6,7], "", 1)
([5,6,7], "5 6", 2)
([5,6,7], "5", 2)
([5,6,7], "6", 2)
([5,6,7], "", 2)

arr = [5, 6, 7]


(arr, "", 0)
O✓
O✗
(arr, "5", 1)
(arr, "", 1)
1✓
1✗
1✓
1✗
(arr, "5 6", 2)
(arr, "5", 2)
(arr, "6", 2)
(arr, "", 2)
2✓
2✗
2✓
✗
(arr, "5 6 7", 3)
(arr, "5 6", 3)
(arr, "5 7", 3)
(arr, 5, 3)
(arr, "6 7", 3)
(arr, "6", 3)
(arr, "7", 3)