@= What is a random variable in probability theory?

A **random variable** is a variable that takes on numerical values based on the outcome of a random event. It maps outcomes of a random process to numbers.

;= What are the types of random variables?

There are **two main types**:

1. **Discrete Random Variable** – Takes countable values (e.g., 0, 1, 2,...).
2. **Continuous Random Variable** – Takes any value within a range (e.g., any real number between 0 and 1).

5= What is the difference between discrete and continuous distributions?

- **Discrete distribution**: Probabilities are assigned to specific values.
- **Continuous distribution**: Probabilities are represented over intervals; the probability of a specific value is 0.

= What are probability distribution functions (PDF)?

- For **discrete** variables: it's a **Probability Mass Function (PMF)**.
- For **continuous** variables: it's a **Probability Density Function (PDF)**, which shows the relative likelihood of values in a range.

= How do cumulative distribution functions (CDF) differ from PDFs?

- **PDF** gives the probability *density* at a point (for continuous).

- **CDF** gives the **probability that a variable is less than or equal to a value**.

-= What is a discrete uniform distribution?

A distribution where **all outcomes are equally likely**, such as rolling a fair die ($P(x) = 1/n$).

= What are the key properties of a Bernoulli distribution?

- Only **two outcomes**: Success (1) or Failure (0).
- One trial.
- Defined by one parameter **p** (probability of success).

%= What is the binomial distribution, and how is it used in probability?

Used when you have **n independent Bernoulli trials**.
Parameters: `n` (number of trials), `p` (probability of success).
Formula:
$P(X = k) = C(n, k) * p^k * (1-p)^{(n-k)}$

<= What is the Poisson distribution and where is it applied?

Describes the **number of events** occurring in a fixed interval (time/space) with a known constant mean rate λ.
Useful in: phone call arrivals, decay events, etc.

@#= What is a continuous uniform distribution?

Every value within a given interval `[a, b]` is **equally likely**.
PDF: $f(x) = 1 / (b - a)$, for x in $[a, b]$.

@@= What are the characteristics of a normal distribution?

- Bell-shaped curve.
- Symmetric about the mean.
- Defined by **mean (μ)** and **standard deviation (σ)**.

@;= What is the standard normal distribution, and why is it important?

A normal distribution with **μ = 0 and σ = 1**.
It's important because **any normal distribution can be converted to it** using Z-scores, simplifying calculations.

@5= What is the Central Limit Theorem (CLT), and why is it critical in statistics?

The CLT states:

> When independent random variables are added, their **sum tends toward a normal distribution**, regardless of the original distribution, as the sample size becomes large ($n \geq 30$).

This allows the use of normal approximation for hypothesis testing.

@= How does the Central Limit Theorem relate to the normal distribution?

The CLT **explains why normal distribution appears so frequently**. It enables use of **normal models** even when the data isn't normally distributed originally.

@= What is the application of Z statistics in hypothesis testing?

Z-statistics are used when population standard deviation is known and sample size is large to:

- Test means.

- Construct confidence intervals.
- Compare sample to population.

@-= How do you calculate a Z-score, and what does it represent?

$Z = (X - \mu) / \sigma$
It tells you **how many standard deviations a value X is from the mean μ**.

@= What are point estimates and interval estimates in statistics?

- **Point Estimate**: Single value (e.g., sample mean) as best guess of parameter.
- **Interval Estimate**: Range (e.g., confidence interval) that likely contains the parameter.

@%= What is the significance of confidence intervals in statistical analysis?

They give a **range of plausible values** for the population parameter and indicate the **degree of certainty** in estimation (e.g., 95% confidence).

@<= What is the relationship between a Z-score and a confidence interval?

Confidence intervals use **Z-scores** (for known σ) to set boundaries.
E.g., 95% CI:
`x̄ ± Z(0.025) * (σ / √n)`

;#= How are Z-scores used to compare different distributions?

Z-scores **standardize** different distributions to a common scale (mean = 0, σ = 1), allowing comparison across datasets.

;@= What are the assumptions for applying the Central Limit Theorem?

1. Random sampling.
2. Independent observations.
3. Finite mean and variance.
4. Sample size large enough ($n \geq 30$ generally).

;;= What is the concept of expected value in a probability distribution?

The **expected value** (mean) is the **long-run average** outcome.
$E[X] = \Sigma [x * P(x)]$ for discrete, or $\int x \cdot f(x)\, dx$ for continuous.

;5= How does a probability distribution relate to the expected outcome of a random variable?

The **probability distribution** determines how likely each outcome is, and the **expected value** summarizes the **average** of all possible outcomes weighted by their probabilities.