MCKV Institute of Engineering

Paper Code: ES-IT401

Discrete Mathematics

Time Allotted: 1 Hour

Full Marks: 30

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ***five*** of the following: 5×1

(i) The smallest set A such that $A \cup \{4,5\} = \{1,2,3,4,5\}$ is

(a) $\{1,2\}$
(b) $\{1,3\}$
(c) $\{1,2,3\}$
(d) $\{2,3\}$

(ii) $A - B$ contains elements in

(a) A but not in B
(b) B but not in A
(c) both A and B
(d) neither A nor B

(iii) If A' is the complement of the set A then $A \cup A'$ is

(a) the empty set $\emptyset$
(b) A'
(c) A
(d) the universal set U

(iv) If A' is the complement of the set A then $A \cup A'$ is

(a) the empty set $\emptyset$
(b) A'
(c) A
(d) the universal set U

(v) If A and B are sets then $A \cap (A \cup B) =$

(a) A
(b) B
(c) $\emptyset$
(d) None of these

(vi) The relation "$\leq$" on the set of Natural numbers is

(a) symmetric
(b) transitive
(c) not transitive
(d) None of these

Group – B

(Short Answer Type Questions)

Answer any ***two*** of the following 2×5

(2) A relation $\mathcal{R}$: $\mathbb{R} \to \mathbb{R}$ is defined as $a\mathcal{R}b$ if $b - a$ is divisible by 5. Show that $\mathcal{R}$ is an equivalence relation

(3) If $f: \mathbb{R}^+ \to \mathbb{R}^+$ and $g: \mathbb{R}^+ \to \mathbb{R}^+$ defined by $f(x) = \sqrt{x}$ and $g(x) = 3x + 1, \forall x \in \mathbb{R}^+$, find fog and gof. Is $fog = gof$?

(4) Show that the function $f(x) = x + 5$ from the set of real numbers $\mathbb{R}$ to $\mathbb{R}$ is injective.

Group – C

(Long Answer Type Questions)

Answer any ***one*** of the following 1×15

(5)

(a) If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ and $g(x) = \sin x, \forall x \in \mathbb{R}$, show that $fog \neq gof$

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = 3x - 4$. Find f^{-1}.

(c) Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = px + q$,for all x also $fof = I_R$, find the value of p & q.

(6)

(a) Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ $\forall x \in \mathbb{R}$ are inverses of one another.

(b) If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$ and $C = \{2, 5\}$ then find $(A - B) \times (B - C)$

(c) State and prove De'Morgan's law.

MCKV Institute of Engineering

Paper Code: ES-IT401

Paper Name: Discrete Mathematics

Time Allotted: *1* **Hour** ***Full Marks: 30***

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ***five*** of the following: 5×1=5

(i) The chromatic number of a bipartite graph is

(a) 3
(b) 2
(c) 4
(d) 1

(ii) The Boolean expression $(A + B + C)(D + E)' + (A + B + C)(D + E) =$

(a) $A + B + C$
(b) $D + E$
(c) $A'B'C'$
(d) $D'E'$

(iii) The chromatic number of an even cycle (circuit) with n number of vertices is

(a) 1
(b) 2
(c) 3
(d) n

(iv) In a planar graph with n no of vertices, r no of regions, and e no of edges $n - e + r$ equals

(a) 0
(b) 1
(c) 2
(d) 3

(v) In a ring, a zero divisor is always

(vi) How many unique colours will be required for proper vertex colouring of a line graph having vertices?

(a) 1
(b) 2
(c) $n-1$
(d) n

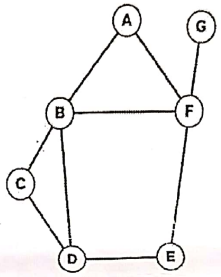
Group – B

(Short Answer Type Questions)

Answer any ***two*** of the following 2×5

(2.) Find the Hasse diagram of the positive divisor of 42 and also find the maximal & minimal elem of the Set. **(CO-3/APPLY/IOCQ/5)**

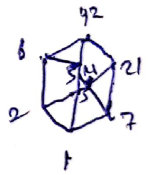
(3.) Find the chromatic number of the following graph



(CO-5/APPLY/IOCQ/5)

(4) Obtain the truth table for the following Boolean function:

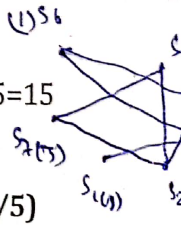
$f(x,y,z) = (x'+y')(x+z)(y+z')$ **(CO-4/APPLY/IOCQ/5)**



Group – C

(Long Answer Type Questions)

Answer any ***one*** of the following 1×15=15



(5)

(a) Show that a complete bipartite graph has perfect matching. **(CO-5/APPLY/IOCQ/5)**

(b) Let D be the set of all diagonal matrices of order 2.Then show that D is a subring of the ring$M_2(R$ where $D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, a, b \in R$. **(CO-3/APPLY/IOCQ/5)**

(c) Show that the modulo 5 system is a field. **(CO-3/APPLY/IOCQ/5)**

(6)

(a) In an examination seven subjects are to be scheduled $S_1, S_2, S_3, S_4, S_5, S_6, S_7$. Following pairs of subjects have common students:

$$(S_1,S_2), (S_1,S_3), (S_1,S_4), (S_1,S_7), (S_2,S_3), (S_2,S_4), (S_2,S_5), (S_2,S_7), (S_3,S_4), (S_3,S_6), (S_3,S_7), (S_4,S_5),$$

$$(S_4,S_6), (S_5,S_6), (S_5,S_7), (S_6,S_7).$$

How can the examination be scheduled so that no student has two examination at the same day?

(CO-5/APPLY/HOCQ/5)

(b) Find the disjunctive normal form of the Boolean function $f(x,y,z)$ such that $(x,y,z) = 1$ if and only if two or more variables are 1 **(CO-4/APPLY/HOCQ/5)**

(c) Show that the set $\{S, +, .\}$ is a non-commutative ring with no unity element where the matrices of the form $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$, a, b, c are the even integers. **(CO-3/APPLY/HOCQ/5)**

MCKV Institute of Engineering

Paper Code: ES-IT401

Paper Name: Discrete Mathematics

Time Allotted: 1 Hour *Full Marks:* 30

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ***five*** of the following: 5×1=5

(i) The chromatic number of a bipartite graph is

(a) 3
(b) 2
(c) 4
(d) 1

(ii) The Boolean expression $(A + B + C)(D + E)' + (A + B + C)(D + E) =$

(a) $A + B + C$
(b) $D + E$
(c) $A'B'C'$
(d) $D'E'$

(iii) The chromatic number of an even cycle (circuit) with n number of vertices is

(a) 1
(b) 2
(c) 3
(d) n

(iv) In a planar graph with n no of vertices, r no of regions, and e no of edges $n - e + r$ equals

(a) 0
(b) 1
(c) 2
(d) 3

(v) In a ring, a zero divisor is always

(a) zero
(b) non-zero
(c) may be anything
(d) None of these

(vi) How many unique colours will be required for proper vertex colouring of a line graph havi vertices?

(a) 1
(b) 2
(c) $n-1$
(d) n

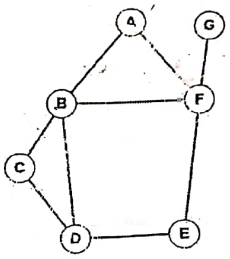
Group – B

(Short Answer Type Questions)

Answer any ***two*** of the following 2×5=10

(2.) Find the Hasse diagram of the positive divisor of 42 and also find the maximal & minimal elements of the Set. **(CO-3/APPLY/IOCQ/5)**

(3) Find the chromatic number of the following graph



(CO-5/APPLY/IOCQ/5)

(4) Obtain the truth table for the following Boolean function:

$f(x,y,z) = (x'+y')(x+z)(y+z')$ **(CO-4/APPLY/IOCQ/5)**

Group – C

(Long Answer Type Questions)

Answer any ***one*** of the following 1×15=15

(5)

(a) Show that a complete bipartite graph has perfect matching. **(CO-5/APPLY/IOCQ/5)**

(b) Let D be the set of all diagonal matrices of order 2.Then show that D is a subring of the ring$M_2(R)$ where $D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, a,b \in R$. **(CO-3/APPLY/IOCQ/5)**

(c) Show that the modulo 5 system is a field. **(CO-3/APPLY/IOCQ/5)**

(6)

(a) In an examination seven subjects are to be scheduled $S_1, S_2, S_3, S_4, S_5, S_6, S_7$. Following pairs of subjects have common students:

$$(S_1,S_2),(S_1,S_3),(S_1,S_4),(S_1,S_7),(S_2,S_3),(S_2,S_4),(S_2,S_5),(S_2,S_7),(S_3,S_4),(S_3,S_6),(S_3,S_7),(S_4,S_5),$$

$$(S_4,S_6),(S_5,S_6),(S_5,S_7),(S_6,S_7).$$

How can the examination be scheduled so that no student has two examination at the same day?

(CO-5/APPLY/HOCQ/5)

(b) Find the disjunctive normal form of the Boolean function $f(x,y,z)$ such that $(x,y,z) = 1$ if and only if two or more variables are 1 **(CO-4/APPLY/HOCQ/5)**

(c) Show that the set $\{S, +, .\}$ is a non-commutative ring with no unity element where the matrices of the form $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$, a, b, c are the even integers. **(CO-3/APPLY/HOCQ/5)**