

IX. SUPPLEMENTARY INFORMATION: QISKIT CODES FOR OPTIMIZATION.

For Energy estimation Z_0I , IZ_1 and Z_0Z_1 terms of Hamiltonian respectively are coded then optimized through `scipy.optimize` which contain 'powell', 'nelder-mead' or 'cobyta'. The QASM code for the same is as follows:

Code for $Z_1(Z_0I)$:

```

1  # -*- coding: utf-8 -*-
2  """
3  Created on Wed Feb 13 20:04:16 2019
4
5  @author: Rahul
6  """
7
8  from scipy.optimize import minimize
9  from qiskit import QuantumCircuit,
10     ClassicalRegister, QuantumRegister
11  import numpy as np
12  from qiskit import execute
13  from qiskit import BasicAer
14  backend = BasicAer.get_backend('qasm-simulator')
15  T=8192
16  def Z1(theta):
17      # Create a Quantum Register called "q" with
18      # 3 qubits
19      q = QuantumRegister(2)
20
21      # Create a Classical Register called "c"
22      # with 3 bits
23      c = ClassicalRegister(2)
24      qc = QuantumCircuit(q,c)
25
26      qc.u1(theta[0],q[0])
27      qc.u3(theta[1],-np.pi/2,np.pi/2,q[0])
28      qc.u1(theta[2],q[0])
29      qc.cx(q[0], q[1])
30      qc.u1(theta[3],q[1])
31      qc.u3(theta[4],-np.pi/2,np.pi/2,q[1])
32      qc.u1(theta[5],q[1])
33      qc.cx(q[1], q[0])
34      qc.u1(theta[6],q[0])
35      qc.u1(theta[7],q[1])
36      qc.u3(theta[8],-np.pi/2,np.pi/2,q[0])
37      qc.u3(theta[9],-np.pi/2,np.pi/2,q[1])
38      qc.u1(theta[10],q[0])
39      qc.u1(theta[11],q[1])
40      qc.z(q[0])
41
42      qc.measure(q[0], c[0])
43      qc.measure(q[1], c[1])
44
45      #print(qc)
46      #print(i)
47      shots = T # Number of shots to run the
48      program (experiment); maximum is 8192 shots.
49      max_credits = 3 # Maximum number of
50      credits to spend on executions.
51
52      job_hpc = execute(qc, backend=backend, shots
53      =shots, max_credits=max_credits)
54      result_hpc = job_hpc.result()
55      counts11 = result_hpc.get_counts(qc)
56      #print(counts11)
57      Z=0
58      if '00' in list(counts11):
59          Z=Z+counts11['00']/T

```

```

54      #print(Z)
55      if '01' in list(counts11):
56          Z=Z+counts11['01']/T
57      #print(Z)
58      if '10' in list(counts11):
59          Z=Z-counts11['10']/T
60      #print(Z)
61      if '11' in list(counts11):
62          Z=Z-counts11['11']/T
63      #print(Z)
64
65      return Z
66  theta0=[0,np.pi/2,0,0,np.pi/2,0,0,0,np.pi/2,np.
67  pi/2,0,0]
68  res = minimize(Z1, theta0, method='powell',
69  options={'xtol': 1e-8, 'disp': True})
70  #print(res, 'Z1')
71  #x=Z1([ 1.53637770e-03, 1.55027283e+00,
72  8.09473120e-04, 1.66340534e-04,-8.32420733e
73  -03, -2.28166637e-05, -8.94140209e-04,
74  5.61023344e-04,1.77675920e+00, 1.58425510e
75  +00, 1.09455026e-03, 8.51956374e-04])
76  #print(x)

```

Code for $Z_2(IZ_1)$:

```

1  # -*- coding: utf-8 -*-
2  """
3  Created on Wed Feb 13 20:04:17 2019
4
5  @author: Rahul
6  """
7
8  from scipy.optimize import minimize
9  from qiskit import QuantumCircuit,
10     ClassicalRegister, QuantumRegister
11  import numpy as np
12  from qiskit import execute
13  from qiskit import BasicAer
14  backend = BasicAer.get_backend('qasm-simulator')
15  T=8192
16  def Z2(theta):
17      # Create a Quantum Register called "q" with
18      # 3 qubits
19      q = QuantumRegister(2)
20
21      # Create a Classical Register called "c"
22      # with 3 bits
23      c = ClassicalRegister(2)
24      qc = QuantumCircuit(q,c)
25
26      qc.u1(theta[0],q[0])
27      qc.u3(theta[1],-np.pi/2,np.pi/2,q[0])
28      qc.u1(theta[2],q[0])
29      qc.cx(q[0], q[1])
30      qc.u1(theta[3],q[1])
31      qc.u3(theta[4],-np.pi/2,np.pi/2,q[1])
32      qc.u1(theta[5],q[1])
33      qc.cx(q[1], q[0])
34      qc.u1(theta[6],q[0])
35      qc.u1(theta[7],q[1])
36      qc.u3(theta[8],-np.pi/2,np.pi/2,q[0])
37      qc.u3(theta[9],-np.pi/2,np.pi/2,q[1])
38      qc.u1(theta[10],q[0])
39      qc.u1(theta[11],q[1])
40      qc.z(q[1])
41
42      qc.measure(q[0], c[0])
43      qc.measure(q[1], c[1])
44
45      #print(qc)

```

```

43 #print(i)
44 shots = T # Number of shots to run
the program (experiment); maximum is 8192
shots.
45 max_credits = 3 # Maximum number of
credits to spend on executions.
46
47 job_hpc = execute(qc, backend=backend, shots
=shots, max_credits=max_credits)
48 result_hpc = job_hpc.result()
49 counts22 = result_hpc.get_counts(qc)
50
51 Z=0
52 if '00' in list(counts22):
53     Z=Z+counts22['00']/T
54     print(Z)
55 if '01' in list(counts22):
56     Z=Z-counts22['01']/T
57     print(Z)
58 if '10' in list(counts22):
59     Z=Z+counts22['10']/T
60     print(Z)
61 if '11' in list(counts22):
62     Z=Z-counts22['11']/T
63     print(Z)
64
65 return Z
66 theta0=[0,np.pi/2,0,0,np.pi/2,0,0,np.pi/2,np.
pi/2,0,0]
67 res = minimize(Z2, theta0, method='nelder-mead',
options={'xtol': 1e-8, 'disp': True})
68 print(res, 'Z2')
69 #z=Z2([-5.64200351e-01, 1.61554986e+00,
1.56821823e+00, 1.61219634e-03, 1.81902336e
+00, 5.96777406e+00, -6.66574506e-03,
4.63725496e+00, 1.32156756e+00, 3.78827977e
-01, 9.59049221e+00, 3.90466495e+00])
70 #print(z)

```

Code for $Z_3(Z_0 Z_I)$:

```

1 # -*- coding: utf-8 -*-
2 """
3 Created on Wed Feb 13 20:04:15 2019
4
5 @author: Rahul
6 """
7
8 from scipy.optimize import minimize
9 from qiskit import QuantumCircuit,
ClassicalRegister, QuantumRegister
10 import numpy as np
11 from qiskit import execute
12 from qiskit import IBMQ
13 backend =IBMQ.get_backend('ibmqx4')
14 T=8192
15 def Z3(theta):
16     # Create a Quantum Register called "q" with
3 qubits
17     q = QuantumRegister(2)
18
19     # Create a Classical Register called "c"
with 3 bits
20     c = ClassicalRegister(2)
21     qc = QuantumCircuit(q,c)
22
23     qc.u1(theta[0],q[0])
24     qc.u3(theta[1],-np.pi/2,np.pi/2,q[0])
25     qc.u1(theta[2],q[0])
26     qc.cx(q[0], q[1])
27     qc.u1(theta[3],q[1])

```

```

28 qc.u3(theta[4],-np.pi/2,np.pi/2,q[1])
29 qc.u1(theta[5],q[1])
30 qc.cx(q[1], q[0])
31 qc.u1(theta[6],q[0])
32 qc.u1(theta[7],q[1])
33 qc.u3(theta[8],-np.pi/2,np.pi/2,q[0])
34 qc.u3(theta[9],-np.pi/2,np.pi/2,q[1])
35 qc.u1(theta[10],q[0])
36 qc.u1(theta[11],q[1])
37 qc.z(q[0])
38 qc.z(q[1])
39
40 qc.measure(q[0], c[0])
41 qc.measure(q[1], c[1])
42
43 #print(qc)
44 #print(i)
45 shots= T # Number of shots to run
the program (experiment); maximum is 8192
shots.
46 max_credits = 3 # Maximum number of
credits to spend on executions.
47
48 job_hpc = execute(qc, backend=backend, shots
=shots, max_credits=max_credits)
49 result_hpc = job_hpc.result()
50 counts12 = result_hpc.get_counts(qc)
51
52 Z=0
53 if '00' in list(counts12):
54     Z=counts12['00']/T
55     print(Z)
56 if '01' in list(counts12):
57     Z=Z-counts12['01']/T
58     print(Z)
59 if '10' in list(counts12):
60     Z=Z-counts12['10']/T
61     print(Z)
62 if '11' in list(counts12):
63     Z=Z+counts12['11']/T
64     print(Z)
65 return Z
66 theta0=[0,np.pi/2,0,0,np.pi/2,0,0,np.pi/2,np.
pi/2,0,0]
67 res = minimize(Z3, theta0, method='nelder-mead',
options={'xtol': 1e-8, 'disp': True})
68 print(res, 'Z3')
69 #y=Z3([-7.63609374, -0.31633082, 2.15909817,
4.31092763, 1.57470161, 1.6111563,
-0.13041641, 0.08880943, 1.58865914,
1.50475982, 0.59323763, 2.86816974])
70 #print(y)

```

The tabulated data of convergence is as shown below.

TABLE I. For Nelder Mead

Iterations	Theoretical	Experimental
1	-0.52952	-0.3658118514
2	-0.52952	-0.3687388385
3	-0.52952	-0.364456499
4	-0.52952	-0.3640980715
5	-0.52952	-0.3735459878
6	-0.52952	-0.3547868431
7	-0.52952	-0.3642996161
8	-0.52952	-0.3652836663
9	-0.52952	-0.3730938032
10	-0.52952	-0.3646086323
11	-0.52952	-0.3657974241
12	-0.52952	-0.3712501318
13	-0.52952	-0.3667064972
14	-0.52952	-0.3660282661
15	-0.52952	-0.3702449038
16	-0.52952	-0.3702913827
17	-0.52952	-0.3308180563
18	-0.52952	-0.3653252632
19	-0.52952	-0.3770401351
20	-0.52952	-0.3867578684
21	-0.52952	-0.3746037324
22	-0.52952	-0.3745729229
23	-0.52952	-0.375111539
24	-0.52952	-0.3759054584
25	-0.52952	-0.3819807294
26	-0.52952	-0.3751555337
27	-0.52952	-0.3774191599
28	-0.52952	-0.3790968918
29	-0.52952	-0.3830969824
30	-0.52952	-0.3842554535
31	-0.52952	-0.3821377446
32	-0.52952	-0.3852416368
33	-0.52952	-0.387955228
34	-0.52952	-0.3997053027
35	-0.52952	-0.3925481822
36	-0.52952	-0.3912610618
37	-0.52952	-0.3925515116
38	-0.52952	-0.39142132
39	-0.52952	-0.3948359592
40	-0.52952	-0.3993956196
41	-0.52952	-0.3929185069
42	-0.52952	-0.3984600897
43	-0.52952	-0.3981955975
44	-0.52952	-0.3995114402
45	-0.52952	-0.3982939261
46	-0.52952	-0.4039865324
47	-0.52952	-0.4040763391
48	-0.52952	-0.4061567579
49	-0.52952	-0.4068460868
50	-0.52952	-0.4075465596

Iterations	Theoretical	Experimental
51	-0.52952	-0.4042369078
52	-0.52952	-0.4167492066
53	-0.52952	-0.4100107529
54	-0.52952	-0.4121749407
55	-0.52952	-0.4123393275
56	-0.52952	-0.4153778284
57	-0.52952	-0.4201981629
58	-0.52952	-0.416993412
59	-0.52952	-0.4075684502
60	-0.52952	-0.4216554432
61	-0.52952	-0.4229979668
62	-0.52952	-0.4196147123
63	-0.52952	-0.4253503764
64	-0.52952	-0.4249917301
65	-0.52952	-0.4299162527
66	-0.52952	-0.4330937475
67	-0.52952	-0.4257880472
68	-0.52952	-0.4038551852
69	-0.52952	-0.424561212
70	-0.52952	-0.4276609296
71	-0.52952	-0.4244919108
72	-0.52952	-0.4281787243
73	-0.52952	-0.4255140557
74	-0.52952	-0.4291707672
75	-0.52952	-0.4413052833
76	-0.52952	-0.4357576628
77	-0.52952	-0.4398903991
78	-0.52952	-0.4376459041
79	-0.52952	-0.4434627664
80	-0.52952	-0.4494898276
81	-0.52952	-0.445895972
82	-0.52952	-0.4480489295
83	-0.52952	-0.448329137
84	-0.52952	-0.4582959119
85	-0.52952	-0.4535441657
86	-0.52952	-0.4562919823
87	-0.52952	-0.458321483
88	-0.52952	-0.4572042524
89	-0.52952	-0.4542379797
90	-0.52952	-0.4615284535
91	-0.52952	-0.4637654447
92	-0.52952	-0.4654706567
93	-0.52952	-0.4647630364
94	-0.52952	-0.4646672379
95	-0.52952	-0.4587078335
96	-0.52952	-0.4644073631
97	-0.52952	-0.458071464
98	-0.52952	-0.4648104928
99	-0.52952	-0.455790524
100	-0.52952	-0.4643548465

Iterations	Theoretical	Experimental
101	-0.52952	-0.4597123945
102	-0.52952	-0.4614880122
103	-0.52952	-0.4640232781
104	-0.52952	-0.4631510063
105	-0.52952	-0.4633038065
106	-0.52952	-0.4607956644
107	-0.52952	-0.4640445423
108	-0.52952	-0.4648928415
109	-0.52952	-0.464328522
109	-0.52952	-0.4654582708
110	-0.52952	-0.4653556353
111	-0.52952	-0.4637653123
112	-0.52952	-0.4627551564
113	-0.52952	-0.4616840943
114	-0.52952	-0.4639483874
115	-0.52952	-0.4658633096
116	-0.52952	-0.4652171301
117	-0.52952	-0.4662359862
118	-0.52952	-0.4657969867
119	-0.52952	-0.4654209807
120	-0.52952	-0.4644860719
121	-0.52952	-0.4655182454
122	-0.52952	-0.4639068364
123	-0.52952	-0.4650023139
124	-0.52952	-0.4634304145
125	-0.52952	-0.4670682595

TABLE II. For Cobyly on real Chip

Iterations	Theoretical	Experimental
1	-0.52952	-0.3242458015
2	-0.52952	-0.312141158
3	-0.52952	-0.3000890929
4	-0.52952	-0.2012450742
5	-0.52952	-0.3431368385
6	-0.52952	-0.3354519405
7	-0.52952	-0.3605707275
8	-0.52952	-0.3092321975
9	-0.52952	-0.3421126117
10	-0.52952	-0.382317455
11	-0.52952	-0.365973851
12	-0.52952	-0.3070352964
13	-0.52952	-0.378163368
14	-0.52952	-0.372308508
15	-0.52952	-0.3760271897
16	-0.52952	-0.2783118205
17	-0.52952	-0.3703690787
18	-0.52952	-0.3461754311
19	-0.52952	-0.3659191554
20	-0.52952	-0.3757416573
21	-0.52952	-0.3401756666
22	-0.52952	-0.3805747747
23	-0.52952	-0.3638238258
24	-0.52952	-0.3789247876
25	-0.52952	-0.3807448887
26	-0.52952	-0.3913352043
27	-0.52952	-0.3779779816
28	-0.52952	-0.3963448298
29	-0.52952	-0.3759803137
30	-0.52952	-0.3827794955
31	-0.52952	-0.3805431712
32	-0.52952	-0.3928037149
33	-0.52952	-0.385364926
34	-0.52952	-0.3918327581
35	-0.52952	-0.3848496156
36	-0.52952	-0.3892537216
37	-0.52952	-0.3883246723
38	-0.52952	-0.3732819491
39	-0.52952	-0.3917883664
40	-0.52952	-0.3926158442
41	-0.52952	-0.3891585442
42	-0.52952	-0.3859512631
43	-0.52952	-0.3947456284
44	-0.52952	-0.3852036904
45	-0.52952	-0.3942924663
46	-0.52952	-0.3859246281
47	-0.52952	-0.3890963958
48	-0.52952	-0.4008480231
49	-0.52952	-0.4013480612
50	-0.52952	-0.3944512178

Iterations	Theoretical	Experimental
51	-0.52952	-0.3884351222
52	-0.52952	-0.3884351222
53	-0.52952	-0.3884351222
54	-0.52952	-0.3907311105
55	-0.52952	-0.3980456027
56	-0.52952	-0.3906448114
57	-0.52952	-0.4059641886
58	-0.52952	-0.388795591
59	-0.52952	-0.3965149438
60	-0.52952	-0.4046238846
61	-0.52952	-0.4025665527
62	-0.52952	-0.3938826373
63	-0.52952	-0.3762519993
64	-0.52952	-0.395794006
65	-0.52952	-0.3963359515
66	-0.52952	-0.4041835107
67	-0.52952	-0.4079657663
68	-0.52952	-0.3994302984
69	-0.52952	-0.3969173201
70	-0.52952	-0.4045947654
71	-0.52952	-0.3955795003
72	-0.52952	-0.396464158
73	-0.52952	-0.3954576879
74	-0.52952	-0.3922859202
75	-0.52952	-0.3903440066
76	-0.52952	-0.3997641322
77	-0.52952	-0.3953649947
78	-0.52952	-0.3912133919
79	-0.52952	-0.4025246453
80	-0.52952	-0.4028140875
81	-0.52952	-0.3936148616
82	-0.52952	-0.4070786244
83	-0.52952	-0.3972980299
84	-0.52952	-0.401852009
85	-0.52952	-0.4013366986
86	-0.52952	-0.4078261972
87	-0.52952	-0.3935246526
88	-0.52952	-0.4035197528
89	-0.52952	-0.4009673514
90	-0.52952	-0.4068310896
91	-0.52952	-0.4011399496
92	-0.52952	-0.407983523
93	-0.52952	-0.4008302665
94	-0.52952	-0.4089342389
95	-0.52952	-0.404329474
96	-0.52952	-0.4098480158
97	-0.52952	-0.4068971478
98	-0.52952	-0.3970568892
99	-0.52952	-0.4050681684
100	-0.52952	-0.4176714243

Iterations	Theoretical	Experimental
101	-0.52952	-0.4127125846
102	-0.52952	-0.4028407225
103	-0.52952	-0.397501173
104	-0.52952	-0.4120374641
105	-0.52952	-0.4085077118
106	-0.52952	-0.4095649677
107	-0.52952	-0.4097617167
108	-0.52952	-0.4122036682
109	-0.52952	-0.4182641555