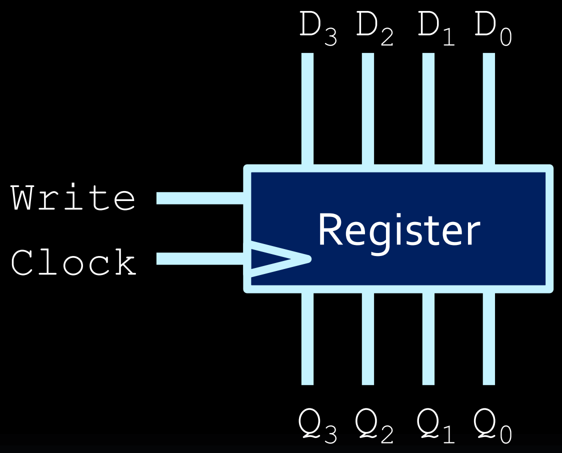
Week 6 Lectorial

Question #1

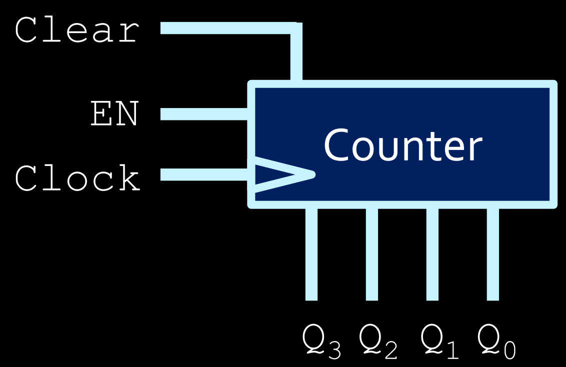
- Imagine you have access to a 4-bit register.



- What does the `Write` signal do?

Question #2

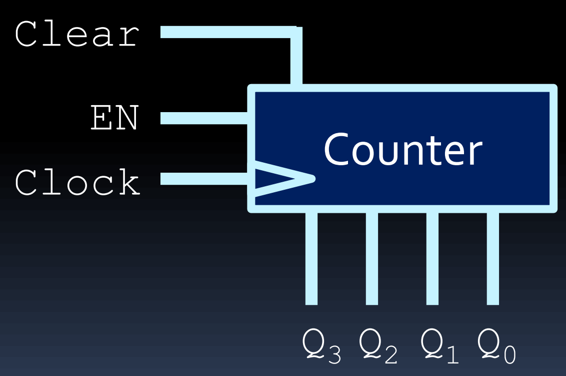
- Assume that you have access to a counter circuit:



- How do you make a signal that goes high after 10 clock cycles?
- How do you make a signal that goes high every 10 clock cycles?

Question #2 (cont'd)

- How do you make a signal that goes high every 100 clock cycles, only using 4-bit counters like the one below (and a few additional gates)?



Question #3

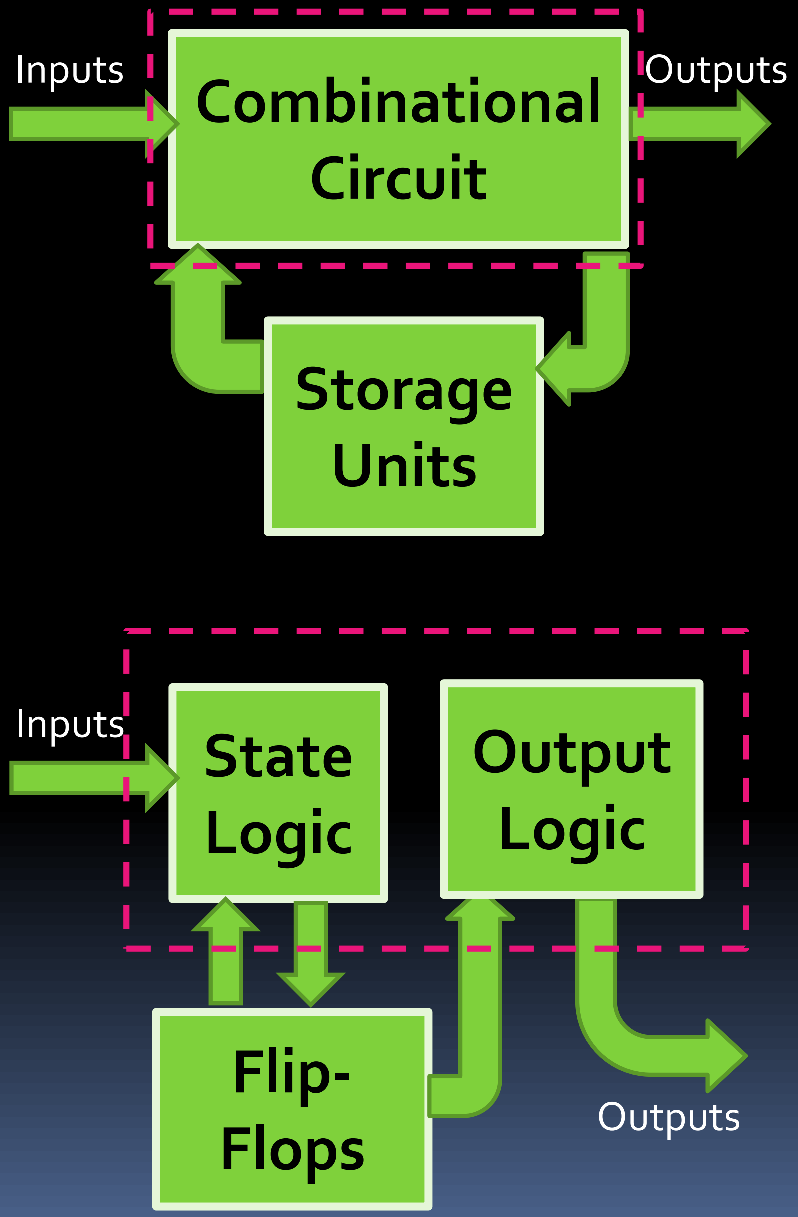
- How many flip-flops would you need to implement the following finite state machine (FSM)?
  - 11 states
  - # flip-flops = $\lceil \log_2 (\# \text{ of states}) \rceil$
  - # flip-flops = 4

Reminder: How to Design FSM

- As a brief reminder:
  1. Draw state diagram
  2. Derive state table from state diagram
  3. Assign flip-flop configuration to each state
     - Number of flip-flops needed is: $\lceil \log(\#\text{ of states}) \rceil$
  4. Redraw state table with flip-flop values
  5. Derive combinational circuit for output and for each flip-flop input.

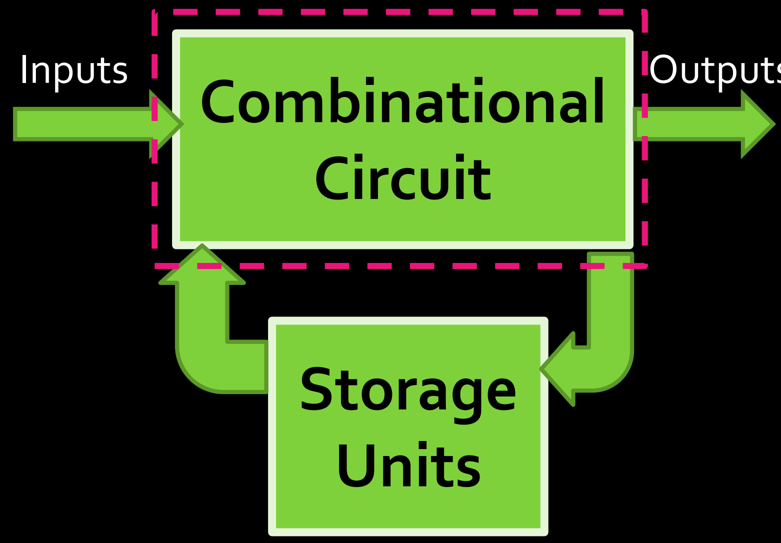
Review of FSMs

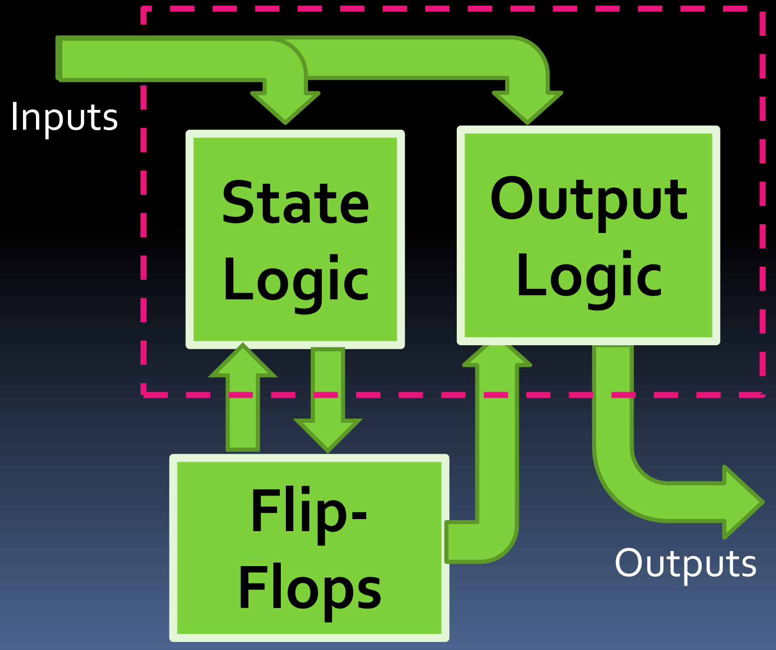
- Step 5 requires two combinational circuit design tasks.
  - For Moore machines (pictured bottom right), output is determined solely based on current state (i.e. flip-flop values).



Review of FSMs

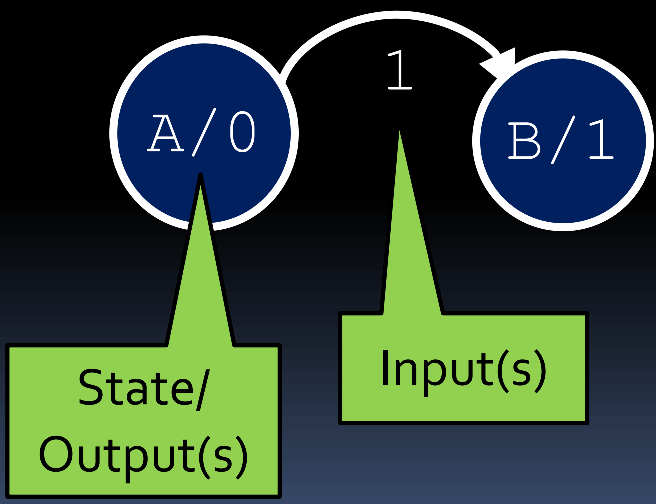
- For Mealy machines, output is determined by both the current state and the current input values.
  - For simplicity, most of our examples will focus on Moore machines.

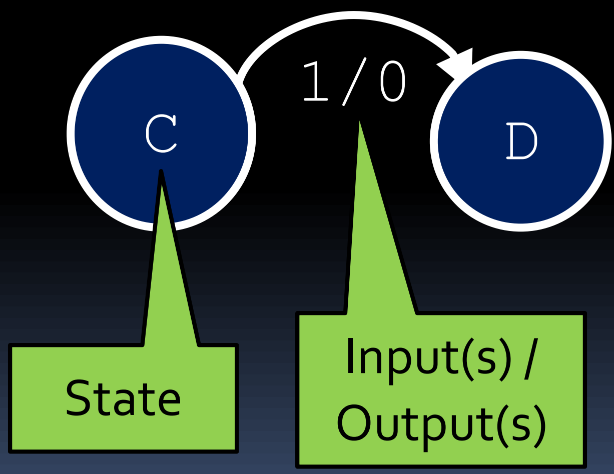




State diagrams with output

- Output values are incorporated into the state diagram, depending on the machine used.
  - Moore Machine
  - Mealy Machine

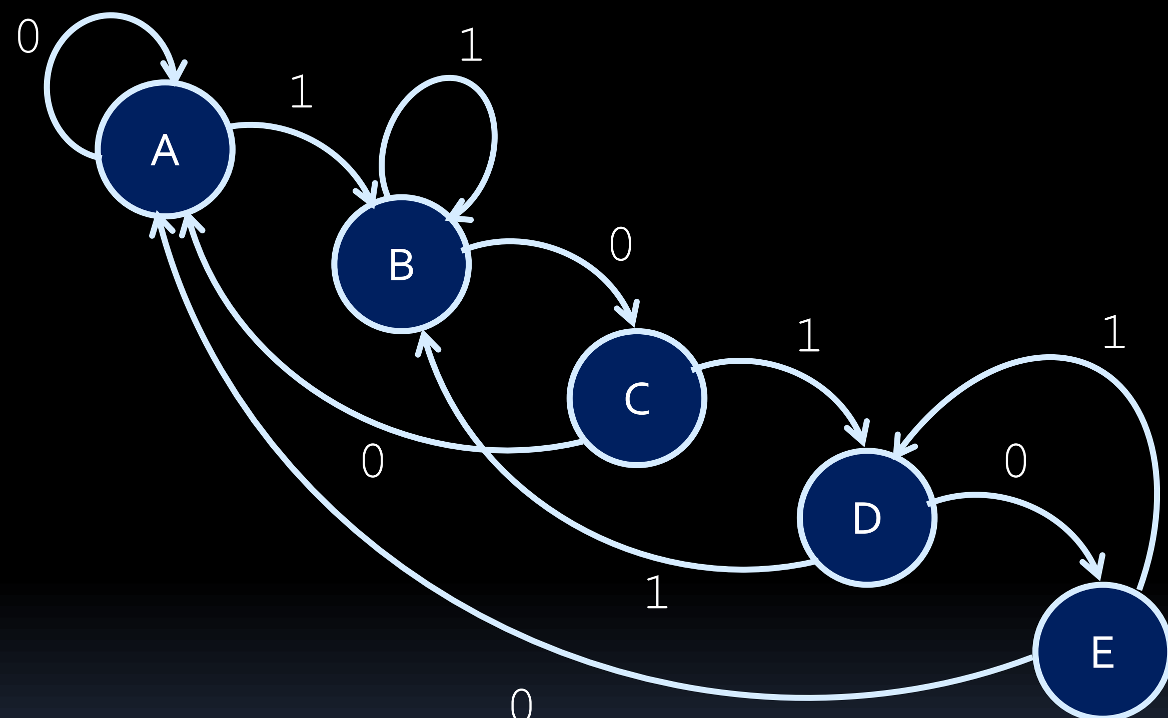




FSM Example: Barcode Reader

- When scanning UPC barcodes, the laser scanner looks for black and white bars that indicate the start of the code.
- If black is read as a `1` and white is read as a `0`, the start of the code (from either direction) has a `1010` pattern.
  - Can you create a state machine that detects this pattern?

Step #1: Draw state diagram



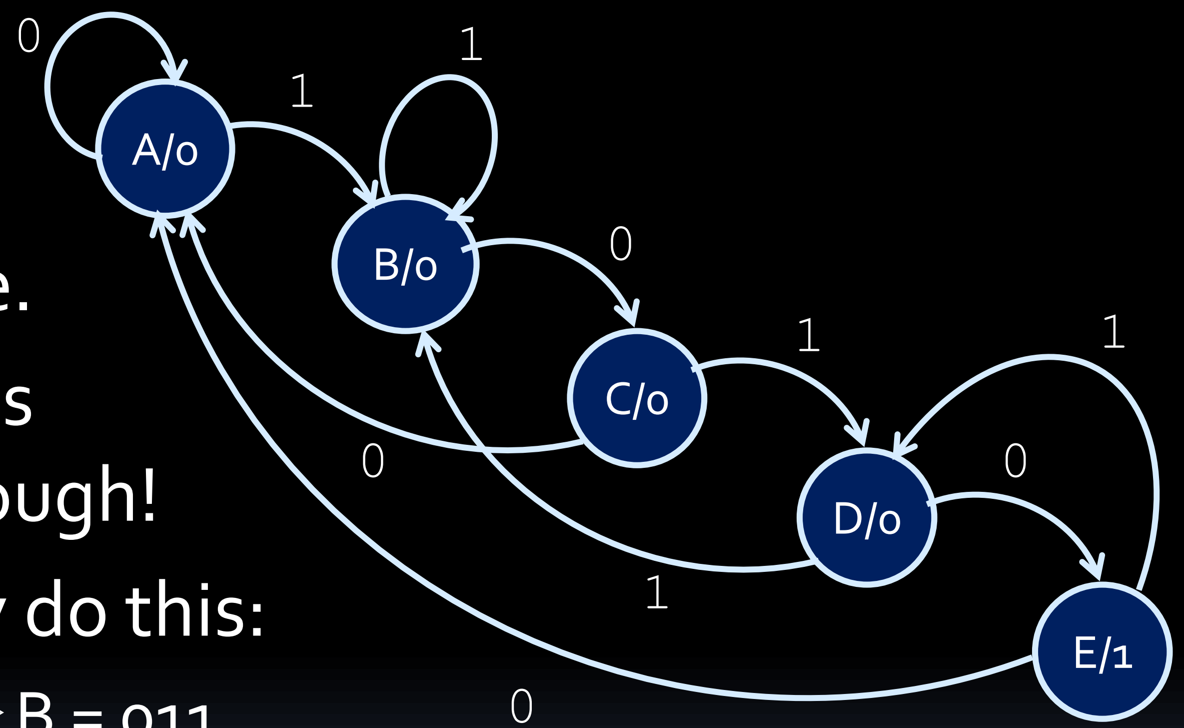
Step #2: State Table

- Output Z is determined by the current state.
  - Denotes Moore machine.
- Next step: allocate flip-flops values to each state.
  - How many flip-flops will we need for 5 states?
  - Recall:
    - # flip-flops = $\lceil \log(\text{\# of states}) \rceil$

Present State	Z	X	Next State
A	0	0	A
A	0	1	B
B	0	0	C
B	0	1	B
C	0	0	A
C	0	1	D
D	0	0	E
D	0	1	B
E	1	0	A
E	1	1	D

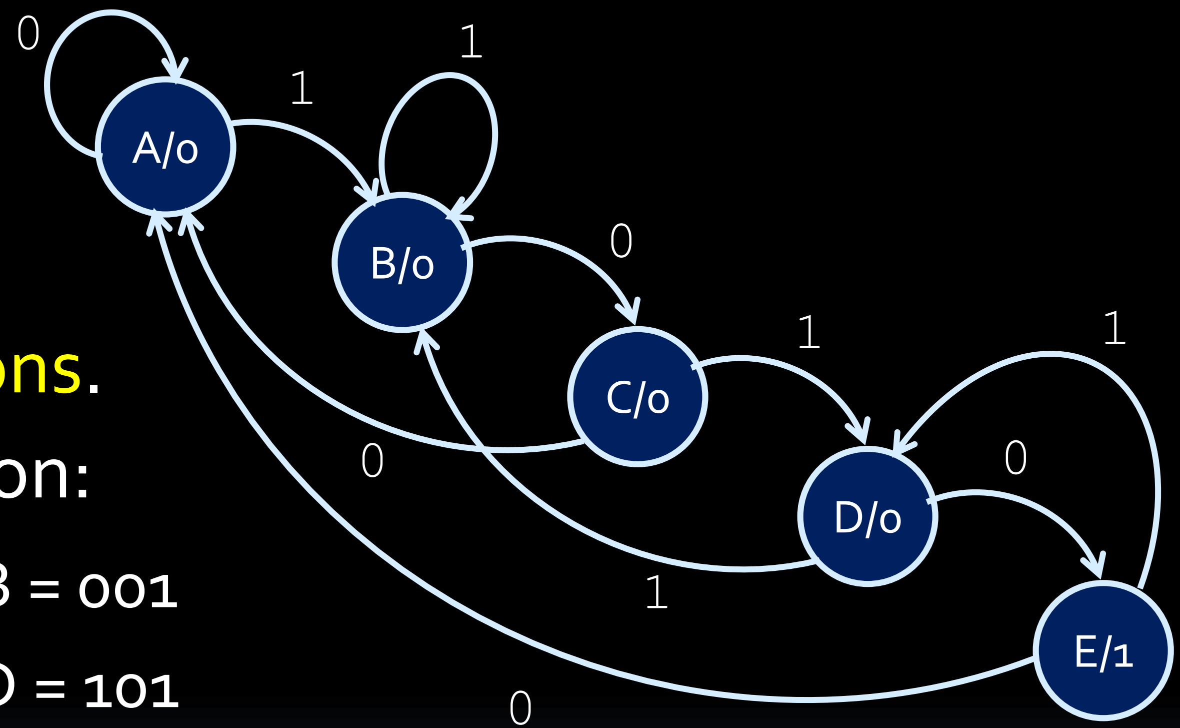
Step #3: Flip-Flop Assignment

- 3 flip-flops needed here.
- Assign states carefully though!
- Can't simply do this:
  - A = 100
  - B = 011
  - C = 010
  - D = 001
  - E = 000



Why not?

Step #3: Flip-Flop Assignment



- Be careful of race conditions.
- Better solution:
  - A = 000
  - B = 001
  - C = 011
  - D = 101
  - E = 100

- Does it still have race conditions?(e.g. C→D, or C→A, or D→B ?)
  - Sometimes, extra flip-flops may be necessary.

Step #4: Redraw State Table

- From here, we can construct the K-maps for the state logic combinational circuit.
  - Derive equations for each flip-flop value, given the previous values and the input X.
  - Three equations total, plus one more for Z (trivial for Moore machines).

Present State			Z	X	Next State		
0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1
0	0	1	0	0	0	1	1
0	0	1	0	1	0	0	1
0	1	1	0	0	0	0	0
0	1	1	0	1	1	0	1
1	0	1	0	0	1	0	0
1	0	1	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	0	1	1	1	0	1

Step 5: Circuit design

- Karnaugh map for F_2:

	$\overline{F_0}\cdot\overline{X}$	$\overline{F_0}\cdot X$	$F_0\cdot X$	$F_0\cdot\overline{X}$
$\overline{F_2}\cdot\overline{F_1}$	0	0	0	0
$\overline{F_2}\cdot F_1$	X	X	1	0
$F_2\cdot F_1$	X	X	X	X
$F_2\cdot\overline{F_1}$	0	1	0	1

$$F_2 = F_1X + F_2\overline{F_0}X + F_2F_0\overline{X}$$

Step 5: Circuit design

- Karnaugh map for F_1:

	$\overline{F_0} \cdot \overline{X}$	$\overline{F_0} \cdot X$	$F_0 \cdot X$	$F_0 \cdot \overline{X}$
$\overline{F_2} \cdot \overline{F_1}$	0	0	0	1
$\overline{F_2} \cdot F_1$	X	X	0	0
$F_2 \cdot F_1$	X	X	X	X
$F_2 \cdot \overline{F_1}$	0	0	0	0

$$F_1 = F_2 F_1 F_0 \overline{X}$$

Step 5: Circuit design

- Karnaugh map for F_0:

	$\overline{F_0} \cdot \overline{X}$	$\overline{F_0} \cdot X$	$F_0 \cdot X$	$F_0 \cdot \overline{X}$
$\overline{F_2} \cdot \overline{F_1}$	0	1	1	1
$\overline{F_2} \cdot F_1$	X	X	1	0
$F_2 \cdot F_1$	X	X	X	X
$F_2 \cdot \overline{F_1}$	0	1	1	0

$$\boxed{F_0 = X + \overline{F_2}\,\overline{F_1}\,F_0}$$

Step 5: Circuit design

- Output value Z goes high based on the following output equation:

$$Z = F_2\overline{F_1}\,\overline{F_0}$$

- Note: All of these equations would be different, given different flip-flop assignments!
  - Practice alternate assignment for the midterm ☺