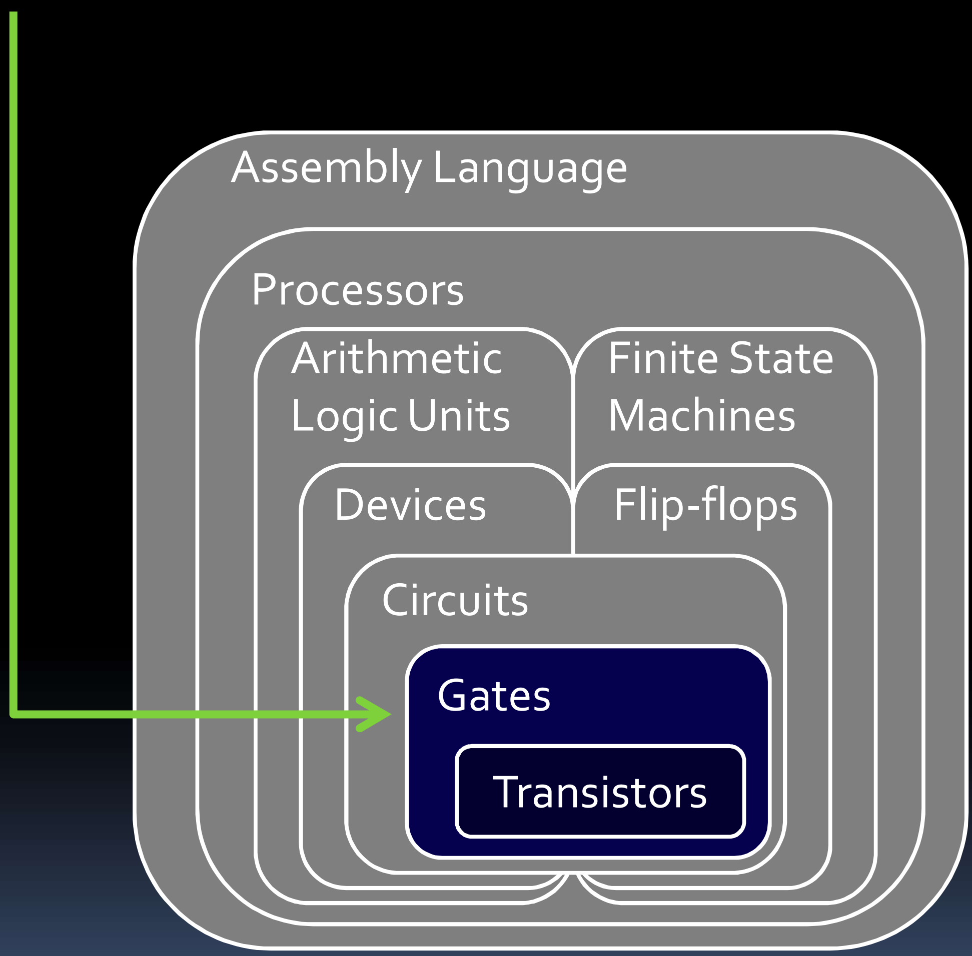
# Circuit Creation

# You are here



Assembly Language
Processors
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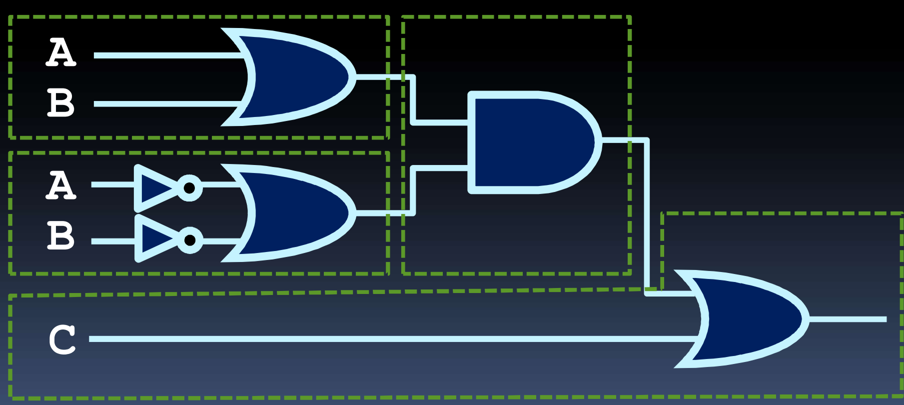
Devices
Flip-flops
Circuits
Gates
Transistors

# Making boolean expressions

- So how would you represent boolean expressions using logic gates?

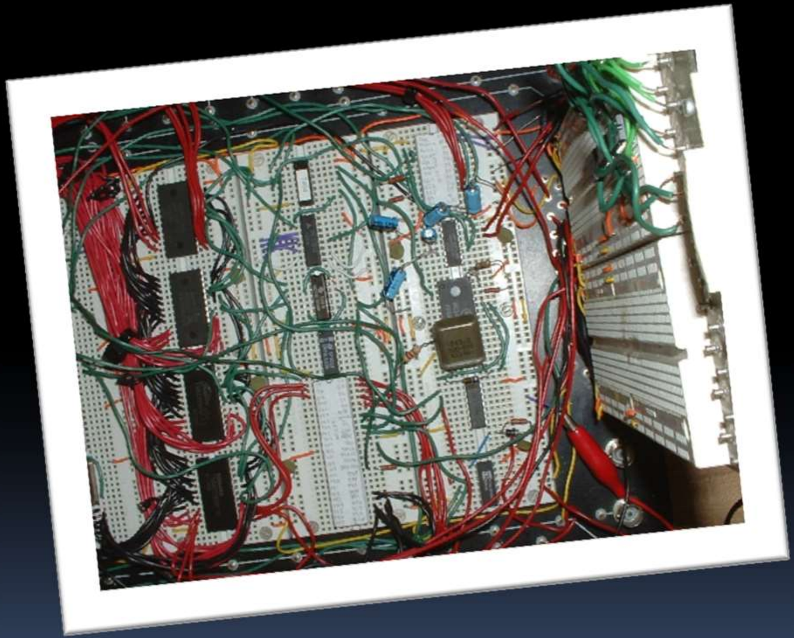
```
Y = (A or B) and (not A or not B) or C
```

- Like so:



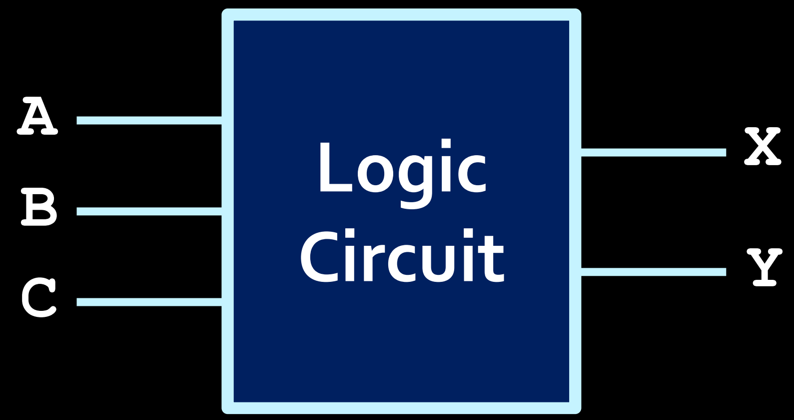
# Creating complex circuits

- What do we do in the case of more complex circuits, with several inputs and more than one output?
  - If you're lucky, a truth table is provided to express the circuit.
  - Usually the behaviour of the circuit is expressed in words, and the first step involves creating a truth table that represents the described behaviour.



# Circuit example

- The circuit on the right has three inputs (A, B and C) and two outputs (X and Y).




- *What logic is needed to set $X$ high when all three inputs are high?*
- *What logic is needed to set $Y$ high when the number of high inputs is odd?*

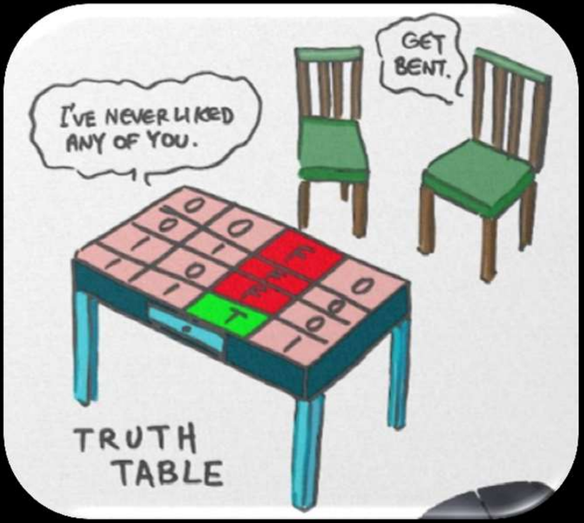
# Combinational circuits

- Small problems can be solved easily.

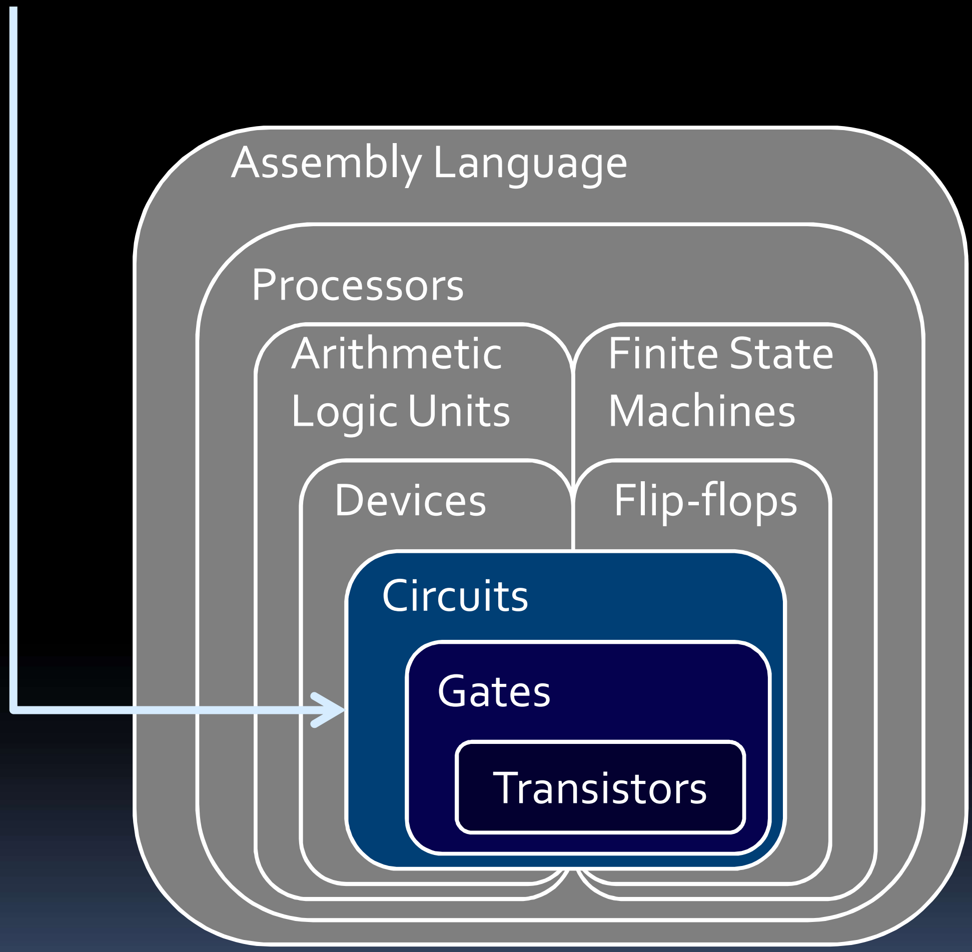

- Larger problems require a more systematic approach.
  - <u>Example:</u> Given three inputs A, B, and C, make output Y high in the case where all of the inputs are low, or when A and B are low and C is high, or when A and C are low but B is high, or when A is low and B and C are high.

# Creating complex logic

- How do we approach problems like these (and circuit problems in general)?
- Basic steps:
  1. Create truth tables.
  2. Express as boolean expression.
  3. Convert to gates.
- The key to an efficient design?
  - Spending extra time on Step #2.



# Now you are here



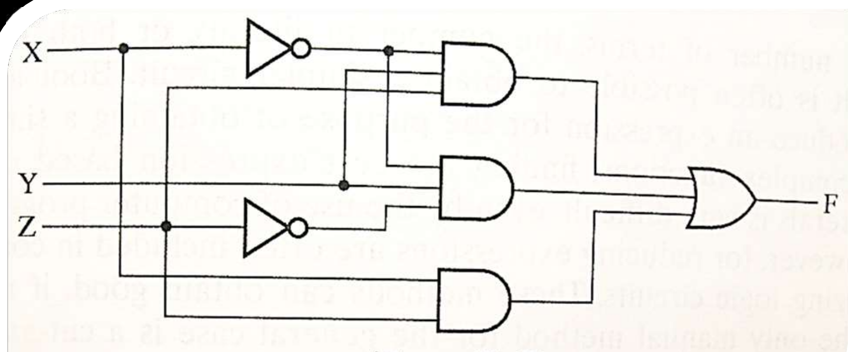
# Lecture Goals

- After this lecture, you should be able to:
  - Create a truth table that represents the behaviour of a circuit you want to create.
  - Translate the minterms from a truth table into gates that implement that circuit.
  - Use Karnaugh maps to reduce the circuit to the minimal number of gates.

# Lecture Goals

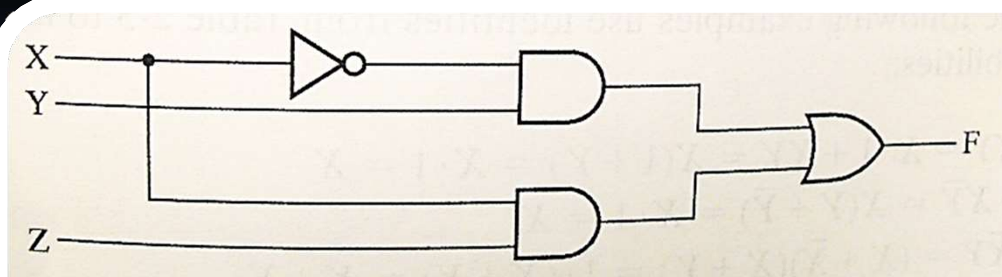
- Which implementation do you prefer? Why?

A.




(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$

B.



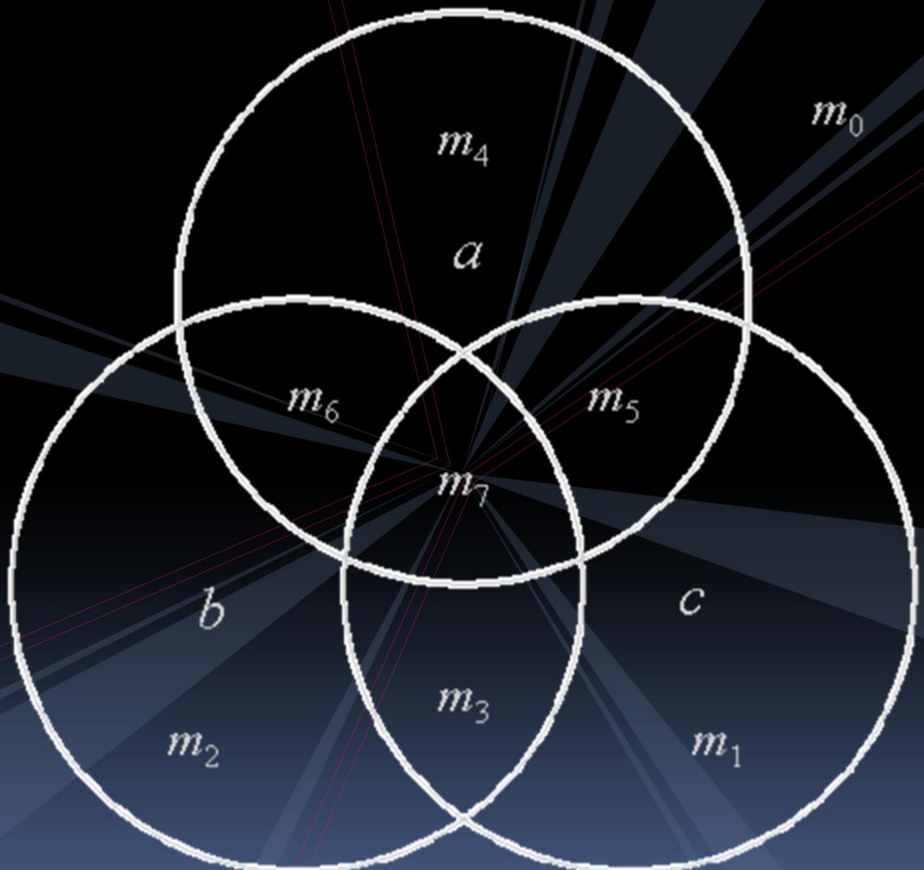

(b) $F = \overline{X}Y + XZ$

# Example truth table

- Consider the following example:
  - *"Given three inputs $A$, $B$, and $C$, make output $Y$ high wherever any of the inputs are low, except when all three are low or when $A$ and $C$ are high."*
- This leads to the truth table on the right.
  - Is there a better way to describe the cases when the circuit's output is high?

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Minterms and Maxterms



# Minterms

- An easier way to express circuit behaviour is to assume the standard truth table format, and then list which input rows cause high output.
  - These rows are referred to as minterms.

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

| Minterm | Y |
|---|---|
| $m_0$ | 0 |
| $m_1$ | 1 |
| $m_2$ | 1 |
| $m_3$ | 1 |
| $m_4$ | 1 |
| $m_5$ | 0 |
| $m_6$ | 1 |
| $m_7$ | 0 |

# Minterms and maxterms

- A more formal description:
  - Minterm = an AND expression with every input present in true or complemented form.
  - Maxterm = an OR expression with every input present in true or complemented form.
  - For example, given four inputs (A, B, C, D):
    - Valid minterms:
      - $A \cdot \overline{B} \cdot C \cdot D$, $\overline{A} \cdot B \cdot \overline{C} \cdot D$, $A \cdot B \cdot C \cdot D$
    - Valid maxterms:
      - $A+\overline{B}+C+D$, $\overline{A}+B+\overline{C}+D$, $A+B+C+D$
    - Neither minterm nor maxterm:
      - $A \cdot B+C \cdot D$, $A \cdot B \cdot D$, $A+B$

# Creating boolean expressions

- A quick aside about notation:
  - AND operations are denoted in these expressions by the multiplication symbol.
    - e.g. $A \cdot B \cdot C$ or $A*B*C \approx A \wedge B \wedge C$
  - OR operations are denoted by the addition symbol.
    - e.g. $A+B+C \approx A \vee B \vee C$
  - NOT is denoted by multiple symbols.
    - e.g. $\neg A$ or $A'$ or $\overline{A}$
  - XOR occurs rarely in circuit expressions.
    - e.g. $A \oplus B$

# The intuition behind minterms

- If you're confused about what a mintem means, consider how the expression behaves:
  - $m_{15} = A*B*C*D$
    - what is the behaviour?
  - A*B*C*D is low at all times, except when all four of the input values are high.

| A | B | C | D | $m_{15}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

# The intuition behind maxterms

- Similarly, consider how a maxterm expression works:
  - $M_0 = A+B+C+D$
    - what is the behaviour?
  - A+B+C+D is always high, except in the one case where all four input values are low.
- Try it with other input combinations!

| A | B | C | D | $M_0$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Specifying circuit behaviour

- Circuits are often described using minterms or maxterms, as a form of logic shorthand.
  - Given $n$ inputs, there are $2^n$ minterms and maxterms possible (same as rows in a truth table).
  - Naming scheme:
    - Minterms are labeled as $m_x$, maxterms are labeled as $M_x$
      - The $x$ subscript indicates the row in the truth table.
      - $x$ starts at $0$ (when all inputs are low), and ends with $2^n-1$.
  - Example: Given 3 inputs –
    - Minterms are $m_0$ ($\overline{A} \cdot \overline{B} \cdot \overline{C}$) to $m_7$ ($A \cdot B \cdot C$)
    - Maxterms are $M_0$ ($A+B+C$) to $M_7$ ($\overline{A}+\overline{B}+\overline{C}$)

# Quick Exercises

- Given 4 inputs A, B, C and D write:
  - $m_9$
  - $m_{15}$
  - $m_{16}$
  - $M_2$

- Which minterm is this?
  - $\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}$
- Which maxterm is this?
  - $A+B+C+\overline{D}$

# Using minterms and maxterms

- What are minterms used for?
  - A single minterm indicates a set of inputs that will make the output go high.
  - Example: $\mathtt{m}_2$
    - Output only goes high in third line of truth table.

| A | B | C | D | $\mathtt{m}_2$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

# Using minterms and maxterms

- What happens when you combine two minterms?
  - Using an OR operation, the result is an output that goes high in both minterm cases.
  - For $m_2+m_8$, both third and ninth lines of truth table result in high output.

| A | B | C | D | $m_2$ | $m_8$ | $m_2+m_8$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |

# Creating boolean expressions

- Two canonical forms of boolean expressions:
  - Sum-of-Minterms (SOM):
    - Since each minterm corresponds to a single high output in the truth table, the combined high outputs are a union of these minterm expressions.
    - Expressed in "Sum-of-Products" form.
  - Product-of-Maxterms (POM):
    - Since each maxterm only produces a single low output in the truth table, the combined low outputs are an intersection of these maxterm expressions.
    - Expressed in "Product-of-Sums" form.

$$Y = m_2 + m_6 + m_7 + m_{10} \quad (SOM)$$

| A | B | C | D | $m_2$ | $m_6$ | $m_7$ | $m_{10}$ | Y |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | | |
| 0 | 0 | 0 | 1 | | | | | |
| 0 | 0 | 1 | 0 | | | | | |
| 0 | 0 | 1 | 1 | | | | | |
| 0 | 1 | 0 | 0 | | | | | |
| 0 | 1 | 0 | 1 | | | | | |
| 0 | 1 | 1 | 0 | | | | | |
| 0 | 1 | 1 | 1 | | | | | |
| 1 | 0 | 0 | 0 | | | | | |
| 1 | 0 | 0 | 1 | | | | | |
| 1 | 0 | 1 | 0 | | | | | |
| 1 | 0 | 1 | 1 | | | | | |
| 1 | 1 | 0 | 0 | | | | | |
| 1 | 1 | 0 | 1 | | | | | |
| 1 | 1 | 1 | 0 | | | | | |
| 1 | 1 | 1 | 1 | | | | | |

# $Y = m_2 + m_6 + m_7 + m_{10}$ (SOM)

| A | B | C | D | $m_2$ | $m_6$ | $m_7$ | $m_{10}$ | Y |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

# Using Sum-of-minterms

- Sum-of-minterms is a way of expressing which inputs cause the output to go high.
  - Assumes that the truth table columns list the inputs according to some logical or natural order.
- Minterm and maxterm expressions are used for efficiency reasons:
  - More compact than displaying entire truth tables.
  - Sum-of-minterms are useful in cases with very few input combinations that produce high output.
    - Product-of-Maxterms useful when expressing truth tables that have very few low output cases…

$$Y = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14} \text{ (POM)}$$

| A | B | C | D | $M_3$ | $M_5$ | $M_7$ | $M_{10}$ | $M_{14}$ | Y |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | | | |
| 0 | 0 | 0 | 1 | | | | | | |
| 0 | 0 | 1 | 0 | | | | | | |
| 0 | 0 | 1 | 1 | | | | | | |
| 0 | 1 | 0 | 0 | | | | | | |
| 0 | 1 | 0 | 1 | | | | | | |
| 0 | 1 | 1 | 0 | | | | | | |
| 0 | 1 | 1 | 1 | | | | | | |
| 1 | 0 | 0 | 0 | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | |
| 1 | 0 | 1 | 0 | | | | | | |
| 1 | 0 | 1 | 1 | | | | | | |
| 1 | 1 | 0 | 0 | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | |
| 1 | 1 | 1 | 0 | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | |

$$Z = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14} \text{ (POM)}$$

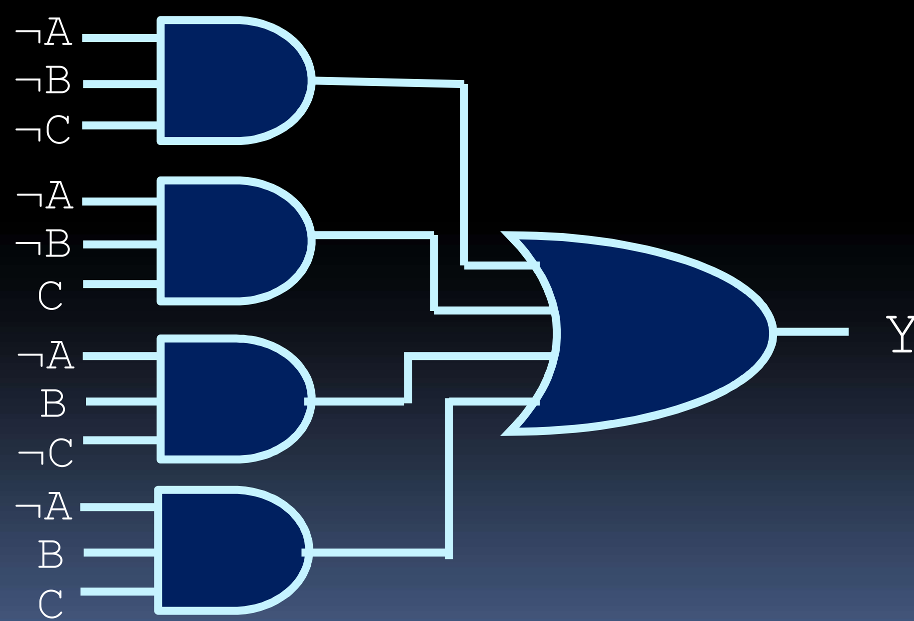
| A | B | C | D | $M_3$ | $M_5$ | $M_7$ | $M_{10}$ | $M_{14}$ | Z |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# Converting SOM to gates

- Once you have a Sum-of-Minterms expression, it is easy to convert this to the equivalent combination of gates:

$$m_0 + m_1 + m_2 + m_3 =$$

$$\overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot\overline{C} + \overline{A}\cdot B\cdot C =$$



# Example: 2-input XOR gate

- An interesting property: $m_x = \overline{M_x}$
  - Minterm x is the complement of maxterm x.
  - e.g., $m_0 = \overline{A} \cdot \overline{B}$ while $M_0 = A + B$

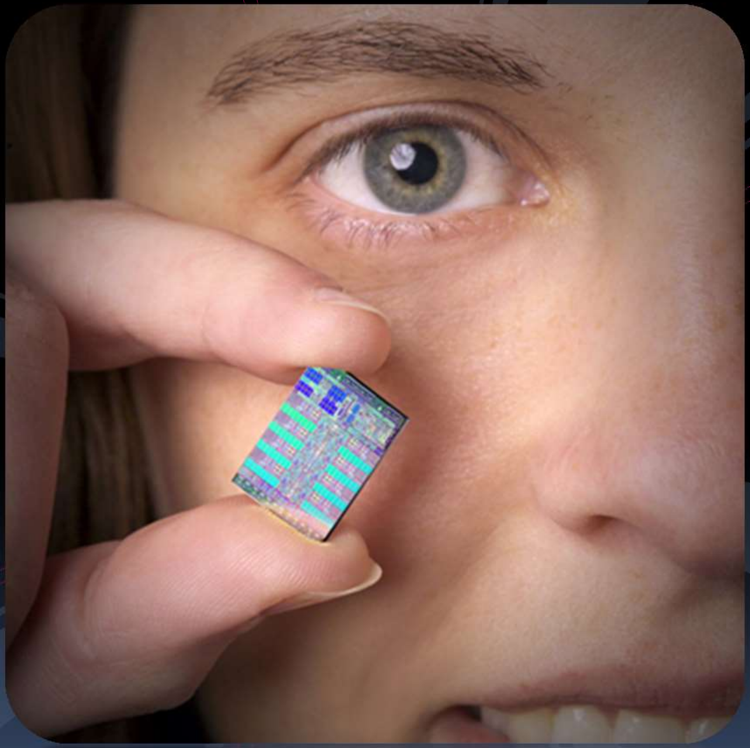
- 2-input XOR gate in SOM and POM form.
  - Sum-Of-Minterms: $F = m_1 + m_2$
  - Product-Of-Maxterms : $F = M_0 \cdot M_3$

- Write $\overline{F}$ in Sum-Of-Minterms form:
  - We need to include the minterms not present in F.
  - $\overline{F} = m_0 + m_3$

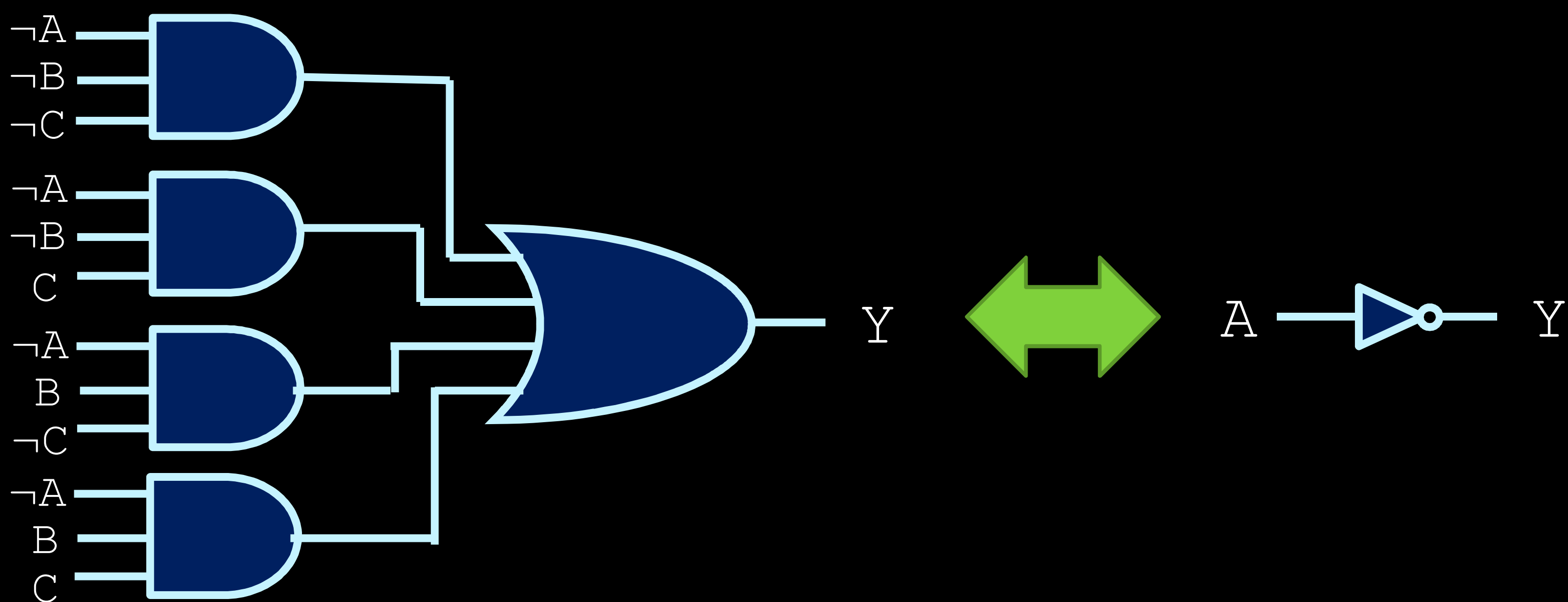
# Example: 2-input XOR gate (cont'd)

- Write $\overline{F}$ in Sum-Of-Minterms form:
  - We need to include the minterms not present in F.
  - $\overline{F} = m_0 + m_3$
- Now let's take the complement of $\overline{F}$.
  - $\overline{\overline{F}} = F = \overline{(m_0 + m_3)} = \overline{m}_0\overline{m}_3$
  - But $\overline{m}_0$ is $M_0$ and $\overline{m}_3$ is $M_3$
  - Therefore, $F = M_0 \cdot M_3$
- The canonical representations SOM and POM for a given function are equivalent! ☺

# Reducing circuits



# Reasons for reducing circuits




- Note example of Sum-of-Minterms circuit design.
- To minimize the number of gates, we want to reduce the boolean expression as much as possible from a collection of minterms to something smaller.
- This is where CSC165 skills come in handy ☺

# Boolean algebra review

- Axioms:

$$0 \cdot 0 = 0 \qquad 0 \cdot 1 = 1 \cdot 0 = 0$$
$$1 \cdot 1 = 1 \qquad \text{if } x = 1,\ \overline{x} = 0$$

- From this, we can extrapolate:

If one input of a 2-input AND gate is 1, then the output is whatever value the other input is.

$$x \cdot 0 = \qquad x+1 =$$
$$x \cdot 1 = \qquad x+0 =$$
$$x \cdot x = \qquad x+x =$$
$$x \cdot \overline{x} = \qquad x+\overline{x} =$$
$$\overline{\overline{x}} =$$

If one input of a 2-input OR gate is 0, then the output is whatever value the other input is.

# Boolean algebra review

- Axioms:

$$0 \cdot 0 = 0 \qquad 0 \cdot 1 = 1 \cdot 0 = 0$$
$$1 \cdot 1 = 1 \qquad \text{if } x = 1,\ \overline{x} = 0$$

- From this, we can extrapolate:

$$x \cdot 0 = 0 \qquad x + 1 = 1$$
$$x \cdot 1 = x \qquad x + 0 = x$$
$$x \cdot x = x \qquad x + x = x$$
$$x \cdot \overline{x} = 0 \qquad x + \overline{x} = 1$$
$$\overline{\overline{x}} = x$$

# Other Boolean identities

- Commutative Law:

$$x \cdot y = y \cdot x \qquad x+y = y+x$$

- Associative Law:

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$
$$x+(y+z) = (x+y)+z$$

- Distributive Law:

$$x \cdot (y+z) = x \cdot y + x \cdot z$$
$$x+(y \cdot z) = (x+y) \cdot (x+z)$$

Does this hold in conventional algebra?

# Consensus Law Proof -Venn diagram

- Consensus Law:

$$x \cdot y + \overline{x} \cdot z + y \cdot z = x \cdot y + \overline{x} \cdot z$$

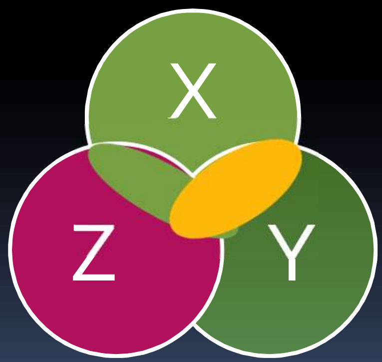
- Proof by Venn diagram:
  - $x \cdot y$
  - $\overline{x} \cdot z$
  - $y \cdot z$
    - Already covered!

# Consensus Law Proof -Venn diagram

- Consensus Law:

$$x \cdot y + \overline{x} \cdot z + y \cdot z = x \cdot y + \overline{x} \cdot z$$

- Proof by Venn diagram:
  - $x \cdot y$
  - $\overline{x} \cdot z$
  - $y \cdot z$
    - Already covered!



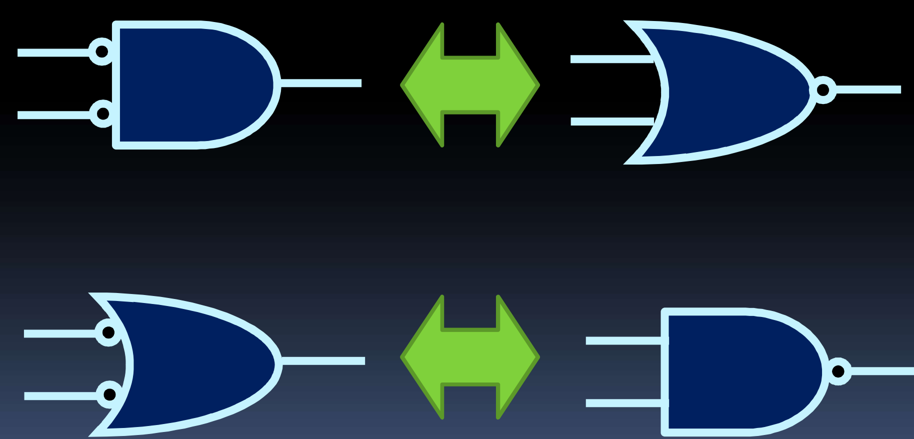
# Other boolean identities

- Absorption Law:

$$x \cdot (x+y) = x \qquad x + (x \cdot y) = x$$

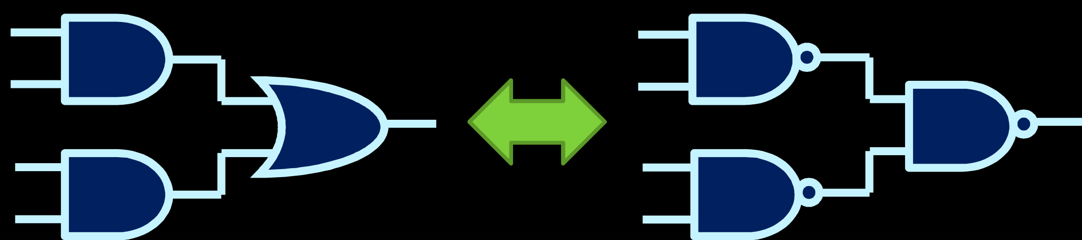
- De Morgan's Laws:

$$\overline{x} \cdot \overline{y} = \overline{x+y}$$
$$\overline{x} + \overline{y} = \overline{x \cdot y}$$

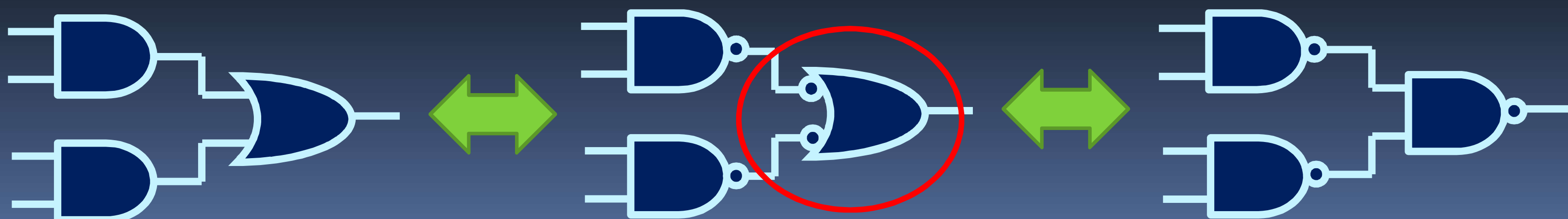


# Converting to NAND gates

- De Morgan’s Law is important because out of all the gates, NANDs are the cheapest to fabricate.
  - a Sum-of-Products circuit could be converted into an equivalent circuit of NAND gates:



- This is all based on de Morgan’s Law:



# Reducing boolean expressions

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

- Assuming logic specs at left, we get the following:

$$Y = \bar{A} \cdot B \cdot C + A \cdot \bar{B} \cdot \bar{C} + A \cdot B \cdot \bar{C} + A \cdot B \cdot C$$

- Now start combining terms, like the last two:

$$Y = \bar{A} \cdot B \cdot C + A \cdot \bar{B} \cdot \bar{C} \mathbf{+ A \cdot B}$$

# Reducing boolean expressions

- Different final expressions possible, depending on what terms you combine.
- For instance, given the previous example:

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

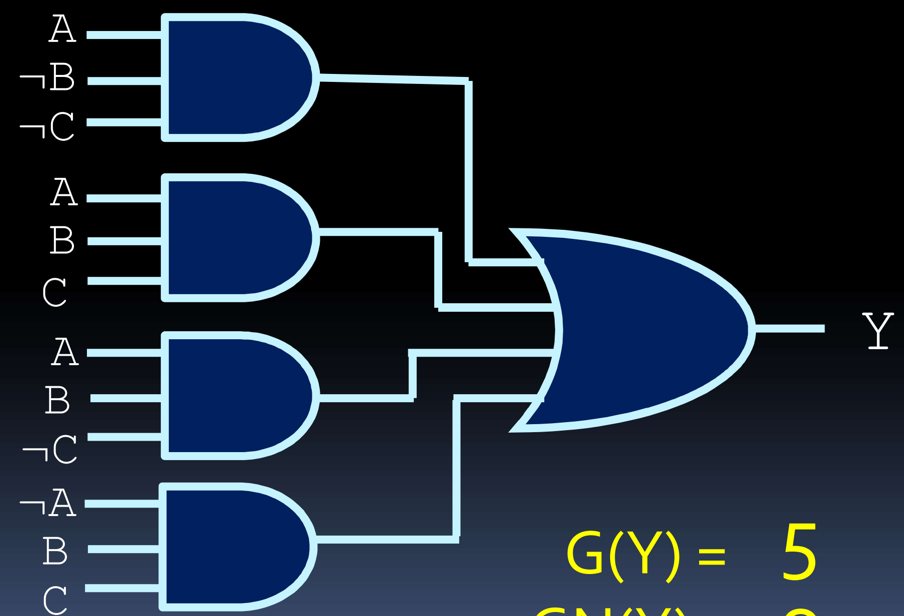
- If you combine the end and middle terms…

$$Y = B \cdot C + A \cdot \overline{C}$$

- Which reduces the number of gates and inputs!

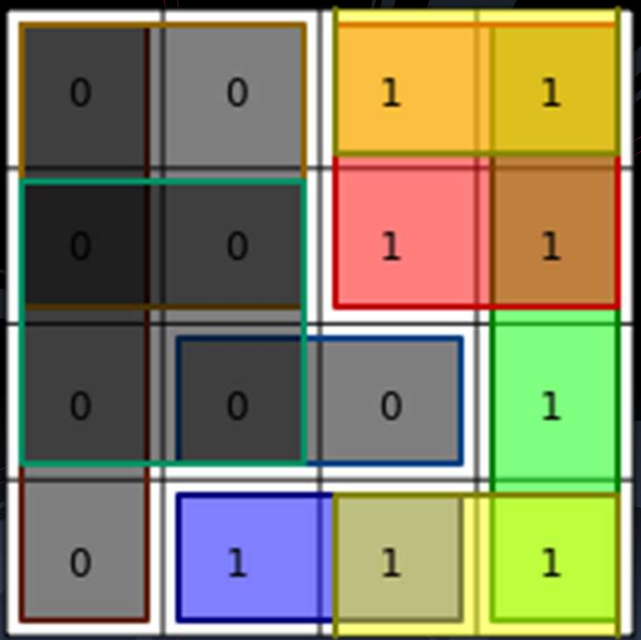
# Reducing boolean expressions

- What is considered the “simplest” expression?
  - In this case, “simple” denotes the lowest gate cost (G) or the lowest gate cost with NOTs (GN).
  - To calculate the gate cost, simply add all the gates together (as well as the cost of the NOT gates, in the case of the GN cost).




G(Y) = 5
GN(Y) = 8

# Karnaugh maps



# Reducing boolean expressions

- How do we find the “simplest” expression for a circuit?
  - Technique called Karnaugh maps (or K-maps).
  - Karnaugh maps are a 2D grid of minterms, where adjacent minterm locations in the grid differ by a single literal.
  - Values of the grid are the output for that minterm.

| | $\overline{B} \cdot \overline{C}$ | $\overline{B} \cdot C$ | $B \cdot C$ | $B \cdot \overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | 0 | 0 | 1 | 0 |
| $A$ | 1 | 0 | 1 | 1 |

# Karnaugh maps

- Karnaugh maps can be of any size, and have any number of inputs.
  - i.e. the 4-input example here.
- Since adjacent minterms only differ by a single value, they can be grouped into a single term that omits that value.

| | $\overline{C}\cdot\overline{D}$ | $\overline{C}\cdot D$ | $C\cdot D$ | $C\cdot\overline{D}$ |
|---|---|---|---|---|
| $\overline{A}\cdot\overline{B}$ | $m_0$ | $m_1$ | $m_3$ | $m_2$ |
| $\overline{A}\cdot B$ | $m_4$ | $m_5$ | $m_7$ | $m_6$ |
| $A\cdot B$ | $m_{12}$ | $m_{13}$ | $m_{15}$ | $m_{14}$ |
| $A\cdot\overline{B}$ | $m_8$ | $m_9$ | $m_{11}$ | $m_{10}$ |

# Using Karnaugh maps

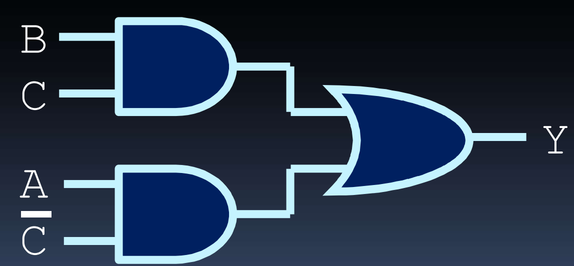
- Once Karnaugh maps are created, draw boxes over groups of high output values.
  - Boxes must be rectangular, and aligned with map.
  - Number of values contained within each box must be a power of 2.
  - Boxes may overlap with each other.
  - Boxes may wrap across edges of map.

| | $\overline{B} \cdot \overline{C}$ | $\overline{B} \cdot C$ | $B \cdot C$ | $B \cdot \overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | 0 | 0 | 1 | 0 |
| $A$ | 1 | 0 | 1 | 1 |

# Using Karnaugh maps

| | $\overline{B}\cdot\overline{C}$ | $\overline{B}\cdot C$ | $B\cdot C$ | $B\cdot\overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | 0 | 0 | 1 | 0 |
| $A$ | 1 | 0 | 1 | 1 |

- Once you find the minimal number of boxes that cover all the high outputs, create boolean expressions from the inputs that are common to all elements in the box.
- For this example:
  - Vertical box: $B\cdot C$
  - Horizontal box: $A\cdot\overline{C}$
  - Overall equation: $Y = B\cdot C + A\cdot\overline{C}$



# Karnaugh maps and maxterms

- Can also use this technique to group maxterms together as well.
- Karnaugh maps with maxterms involves grouping the zero entries together, instead of grouping the entries with one values.

| | C+D | C+$\overline{D}$ | $\overline{C}$+$\overline{D}$ | $\overline{C}$+D |
|---|---|---|---|---|
| **A+B** | $M_0$ | $M_1$ | $M_3$ | $M_2$ |
| **A+$\overline{B}$** | $M_4$ | $M_5$ | $M_7$ | $M_6$ |
| **$\overline{A}$+$\overline{B}$** | $M_{12}$ | $M_{13}$ | $M_{15}$ | $M_{14}$ |
| **$\overline{A}$+B** | $M_8$ | $M_9$ | $M_{11}$ | $M_{10}$ |

# Quick Exercise

| | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | 0 | 0 | 1 | 1 |
| $\bar{A}B$ | 1 | 1 | 0 | 0 |
| $AB$ | 1 | 1 | 0 | 0 |
| $A\bar{B}$ | 0 | 0 | 0 | 0 |

$$F = B \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot C$$