Digital Image Processing

Chapter 1: Introduction

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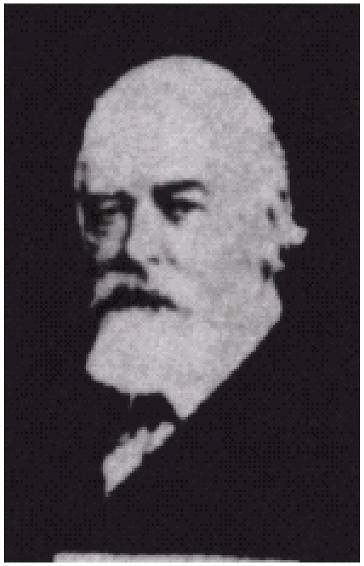
Digital Images in Early Era

1921 Telegraphing image

Printing industrial

Textile industrial



1922: image from Photographic reproduction Using punched tape

These images are not computerized processed.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Digital Images in Early Era

FIGURE 1.3 Unretouched cable picture of Generals Pershing and Foch, transmitted in 1929 from London to New York by 15-tone equipment. (McFarlane.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Digital Image Processing in Early Space Projects

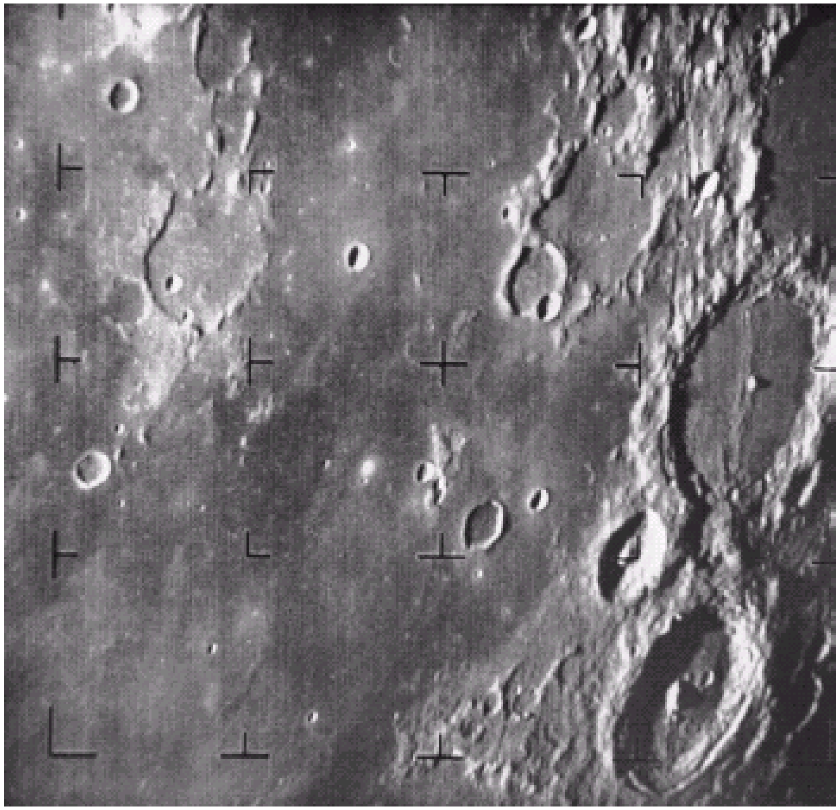


FIGURE 1.4 The first picture of the moon by a U.S. spacecraft. *Ranger 7* took this image on July 31, 1964 at 9:09 A.M. EDT, about 17 minutes before impacting the lunar surface. (Courtesy of NASA.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Energy Sources for Images

Acoustic, Ultrasonic and Electronic (in the form of electron beams)

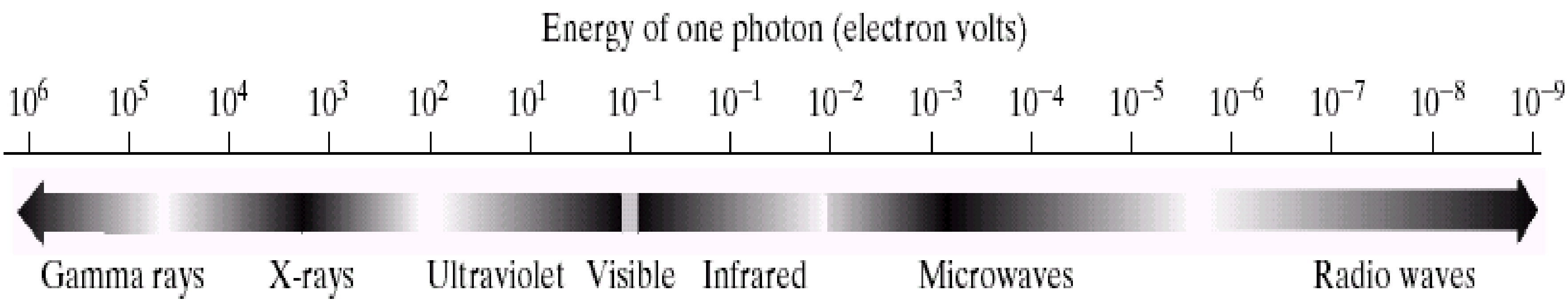


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

Images based on radiation from the Electromagnetic Spectrum are the most familiar

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gamma Ray Imaging

Applications

Nuclear Medicine and astronomical observations

Principle – Inject a patient with a radioactive isotope that emits gamma ray as it decays. Images are produced from the emission collected by gamma ray detector

Positron Emission Tomography

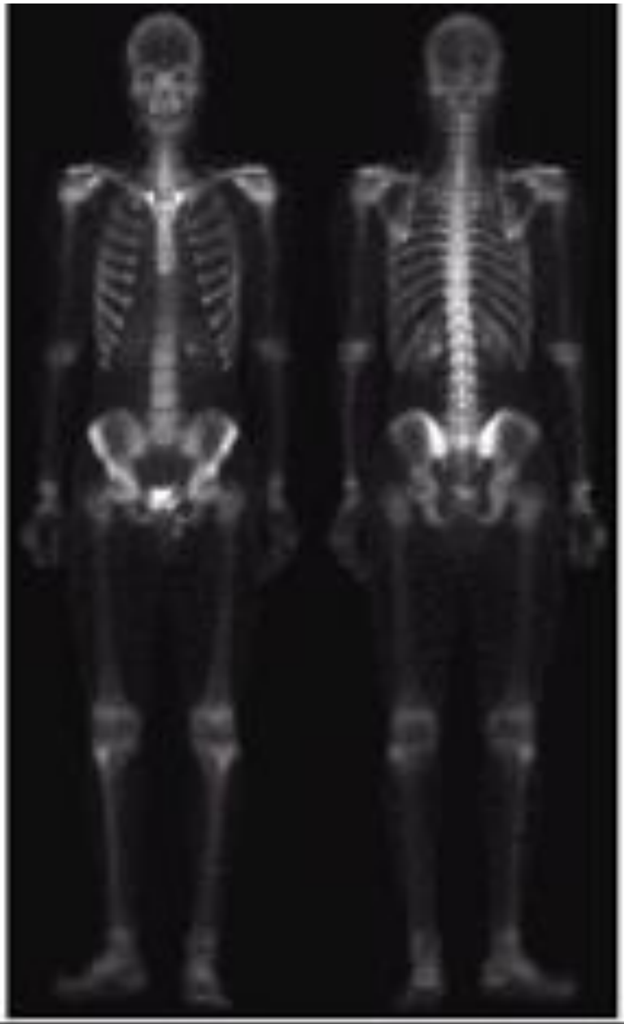
Principle is same as X-ray tomography. Patient is given a radioactive isotope that emits positrons as it decays. When a positron meets an electron, both are annihilated and two gamma rays are given off. These are detected and a tomographic image is created.

Cygnus loop- A star in the constellation of Cygnus exploded about 15000 years ago, generating a superheated stationary gas cloud known as Cygnus loop.

Bone Scan

Gamma-ray imaging

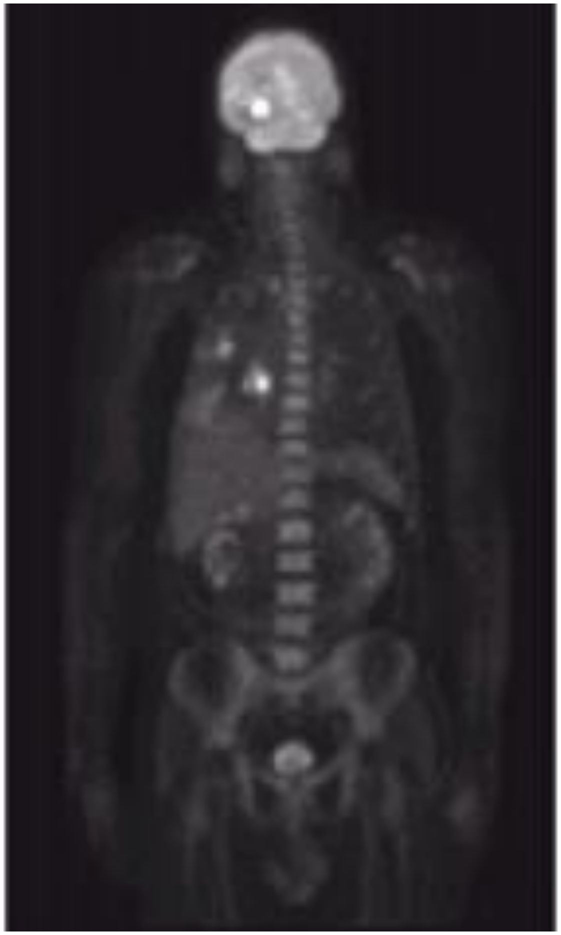
A radioactive isotope is injected, which emits positrons as it decays; when a positron meets an electron, they annihilate and two gamma rays are generated.



PET Image

Positron-Emission Tomography

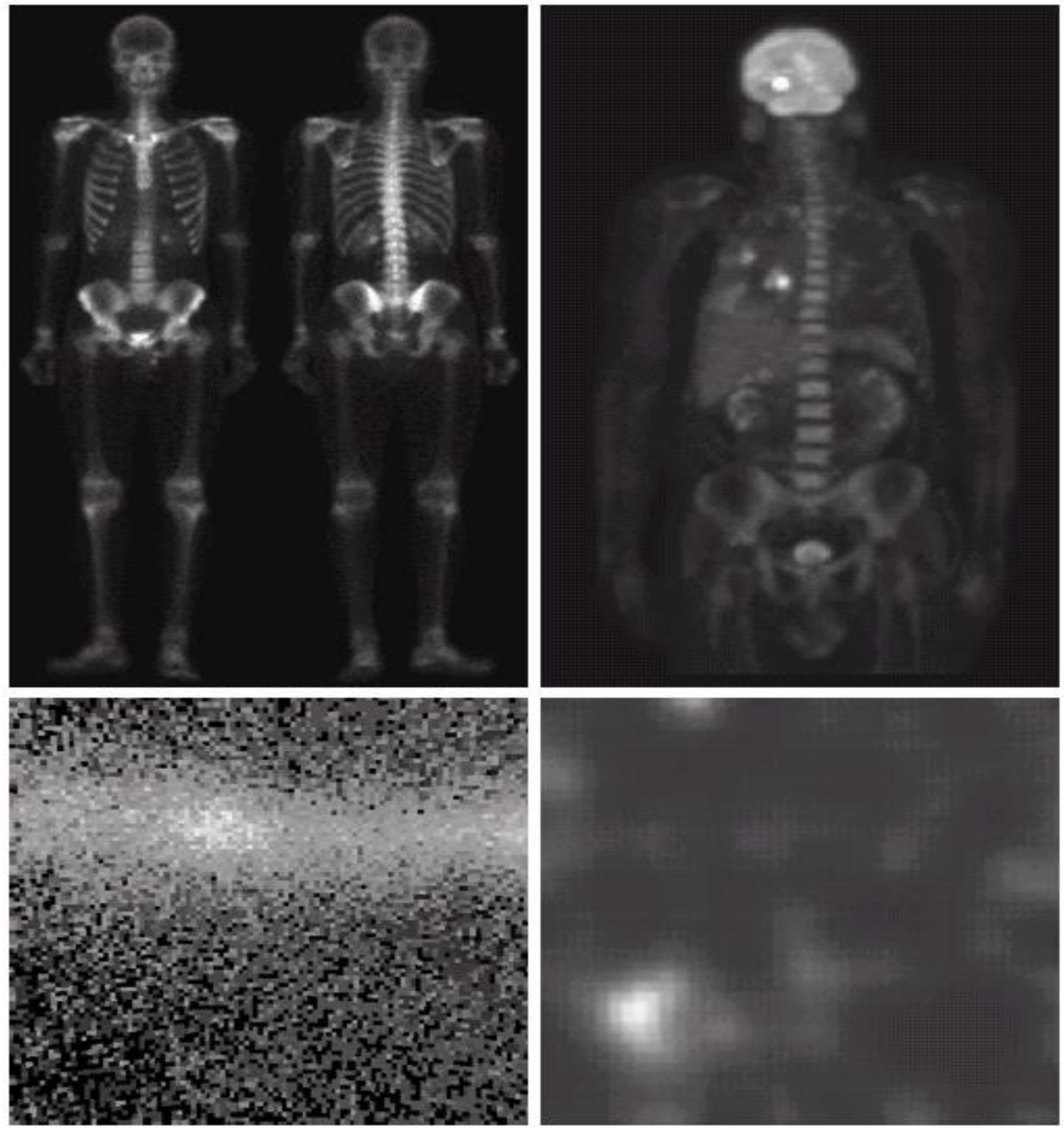
The collected gamma rays are used to construct a CT



Cygnus loop

Natural gamma ray source

Superheated stationary gas cloud in the constellation of Cygnus ("Cygnus Loop", 15,000 light-years from earth)



a	b
c	d

FIGURE 1.6 Examples of gamma-ray imaging. (a) Bone scan. (b) PET image. (c) Cygnus Loop. (d) Gamma radiation (bright spot) from a reactor valve. (Images courtesy of (a) G.E. Medical Systems, (b) Dr. Michael E. Casey, CTI PET Systems, (c) NASA, (d) Professors Zhong He and David K. Wehe, University of Michigan.)

X-Ray Imaging

X-ray are among the oldest sources of EM radiation used for imaging

Applications- medical diagnostics, astronomy, industry

Principle-

-X-rays are generated using an X-ray tube, which is vacuum tube with a cathode and anode. Cathode is heated causing release of free electrons.

-High speed electrons when strike a nucleus, energy is released in the form of X-ray radiation.

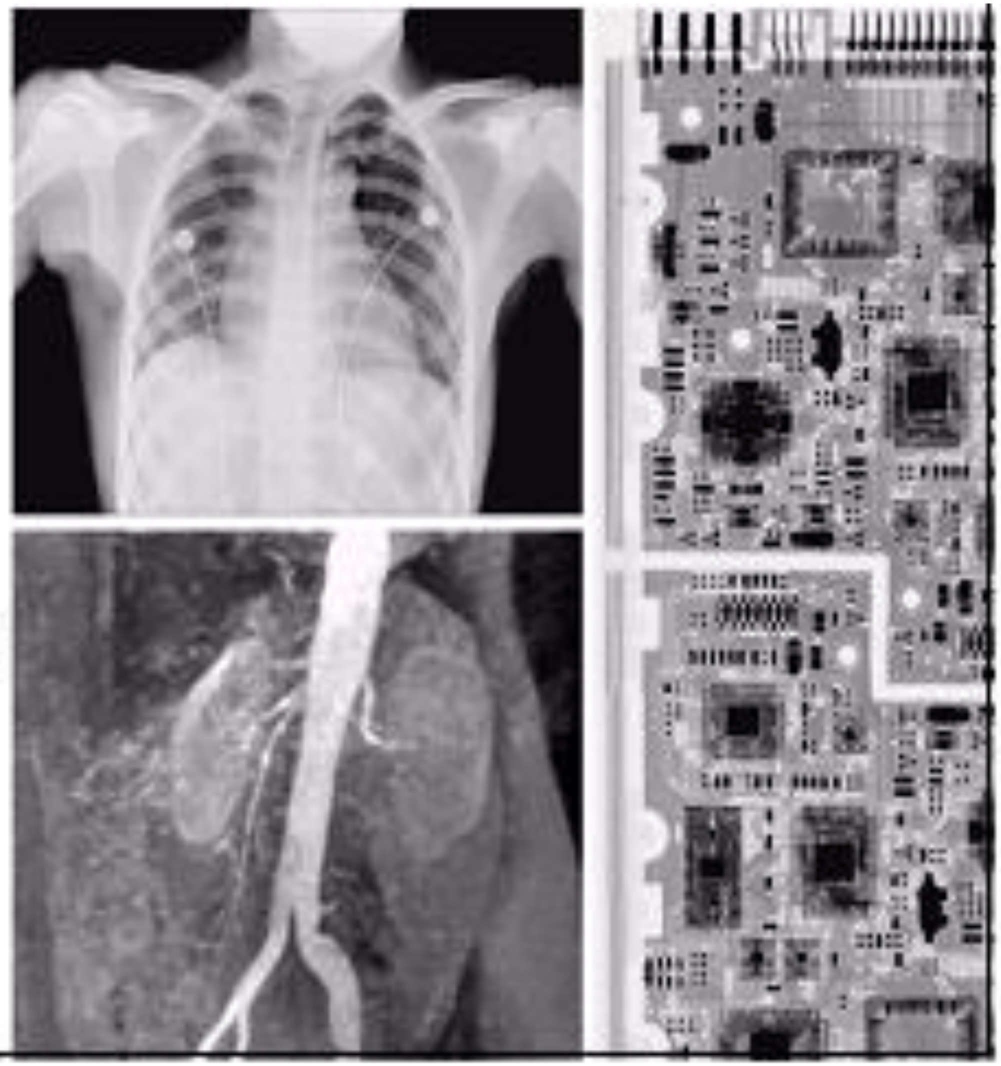
-Energy of X-ray is controlled by a current applied to the filament in the cathode

X-ray imaging

Chest; aortic angiogram; circuit boards.

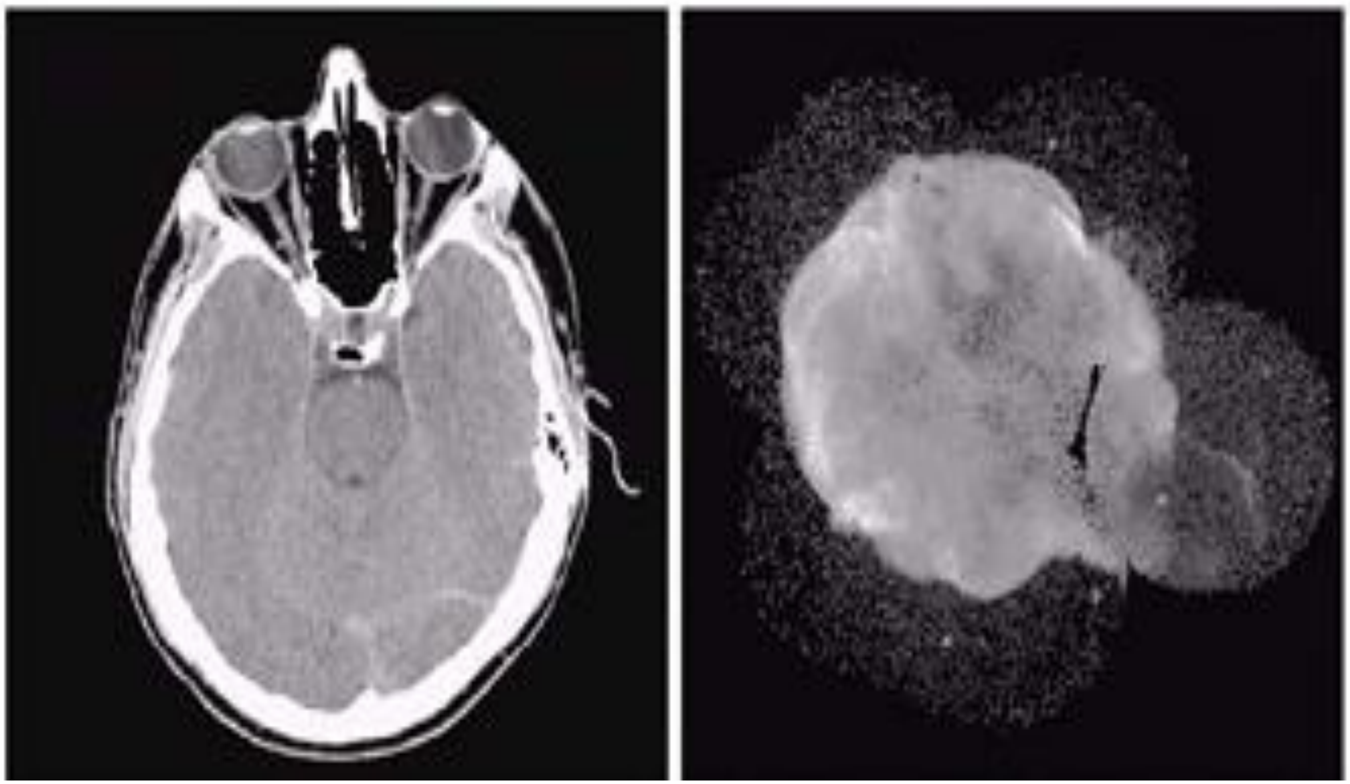
Electrons are emitted from a heated cathode with an energy such that their impact on a nucleus generates X-rays. A film or a digital sensor collects the transmitted rays.

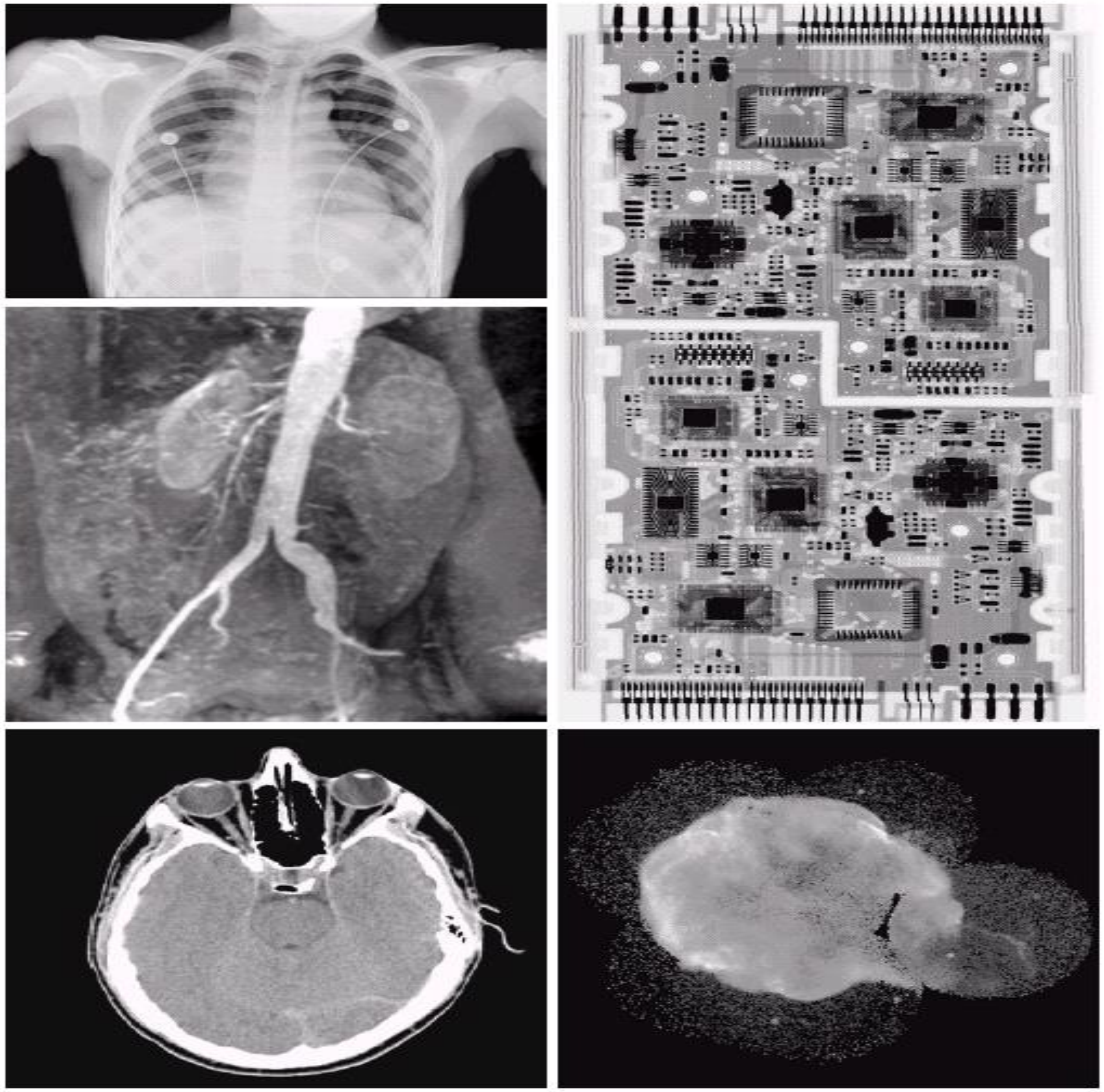
In angiography, a contrast medium is injected.



X-ray imaging

Head CT; Cygnus Loop.





a b c d e

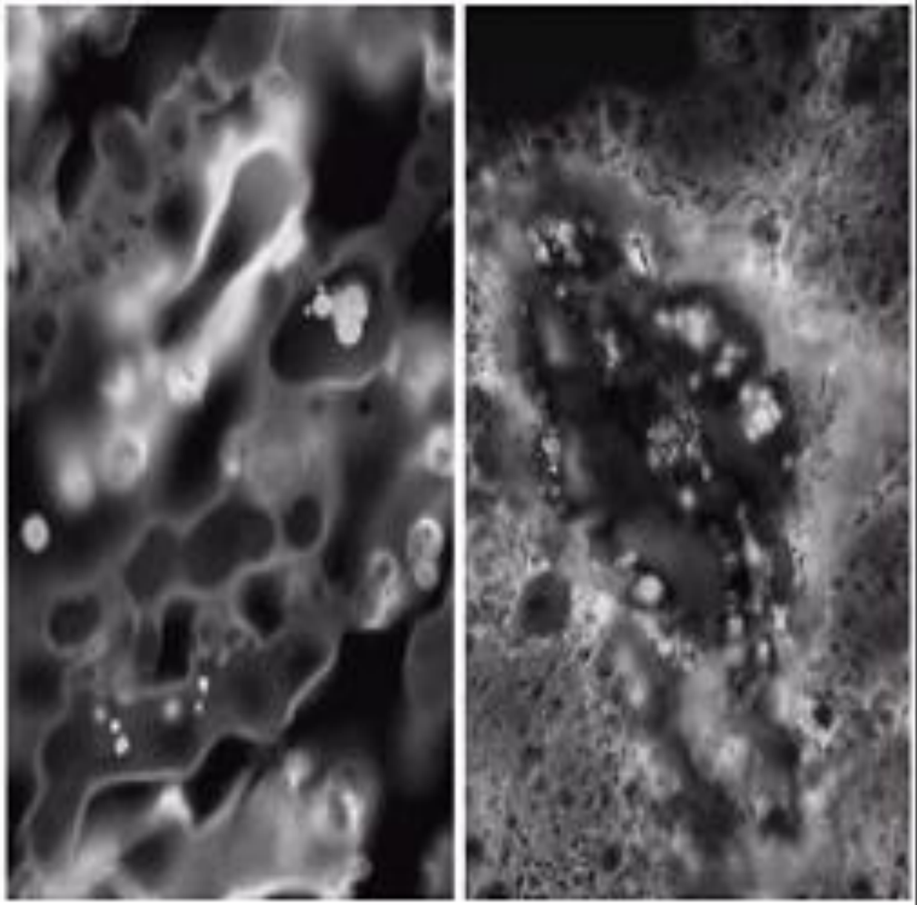
FIGURE 1.7 Examples of X-ray imaging. (a) Chest X-ray. (b) Aortic angiogram. (c) Head CT. (d) Circuit boards. (e) Cygnus Loop. (Images courtesy of (a) and (c) Dr. David R. Pickens, Dept. of Radiology & Radiological Sciences, Vanderbilt University Medical Center; (b) Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School; (d) Mr. Joseph E. Pascente, Lixi, Inc.; and (e) NASA.)

Ultraviolet Imaging

Ultraviolet imaging

Microphotography; normal corn and corn infected by parasites

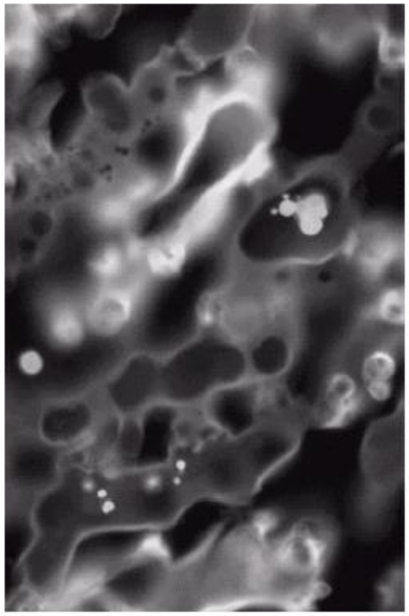
Visible radiation is excited by UV light (fluorescence)



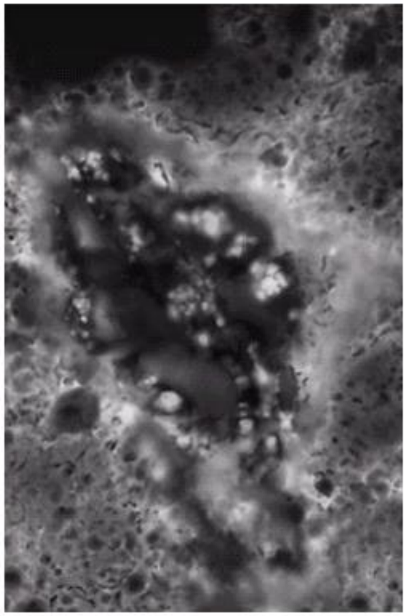
Ultraviolet

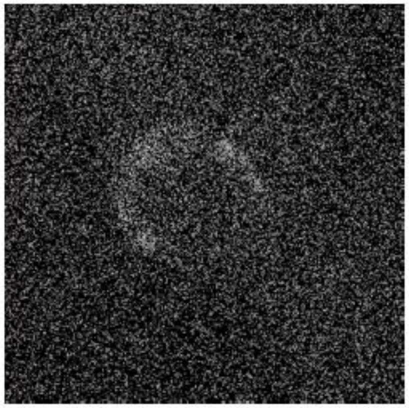
Fluorescence phenomenon

Normal corn



Smut corm



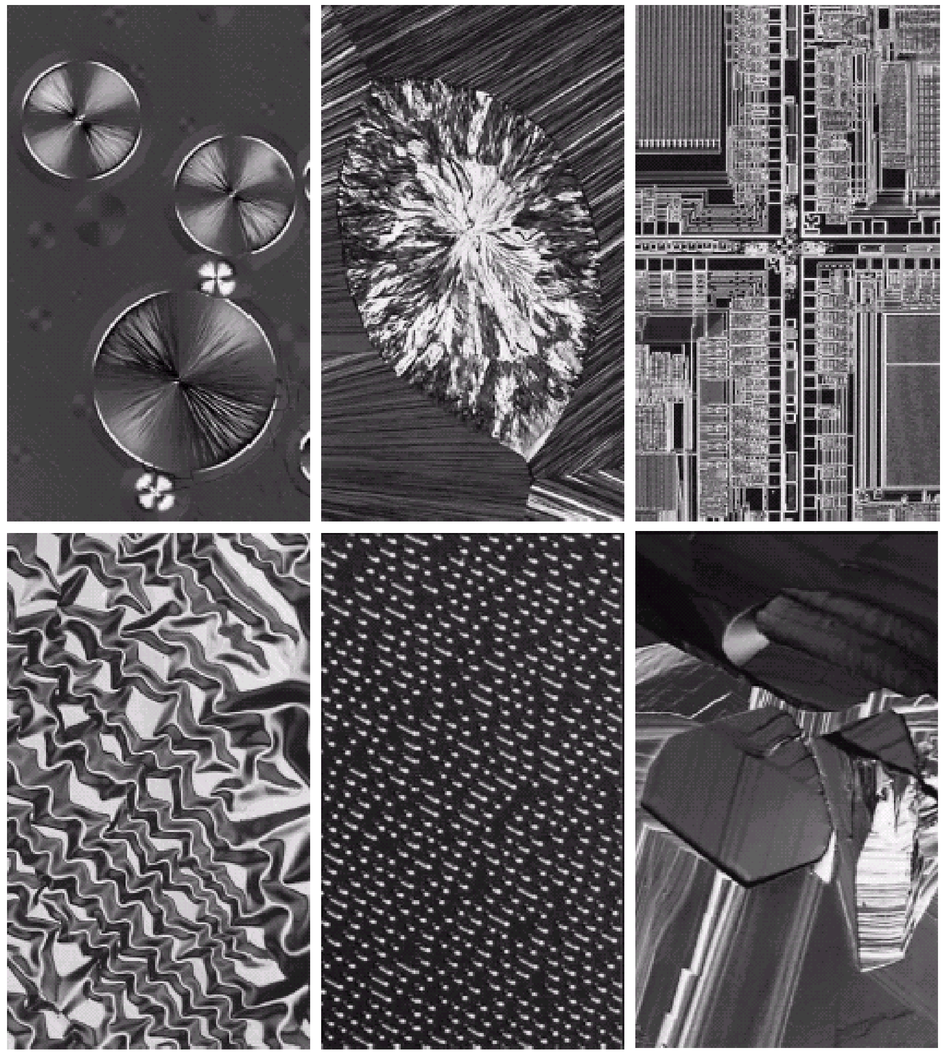


Cygnus Loop

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Visible Light and Infrared

Cholesterol



Taxol

Microprocessor

Nickel oxide
Thin film

Organic
superconductor

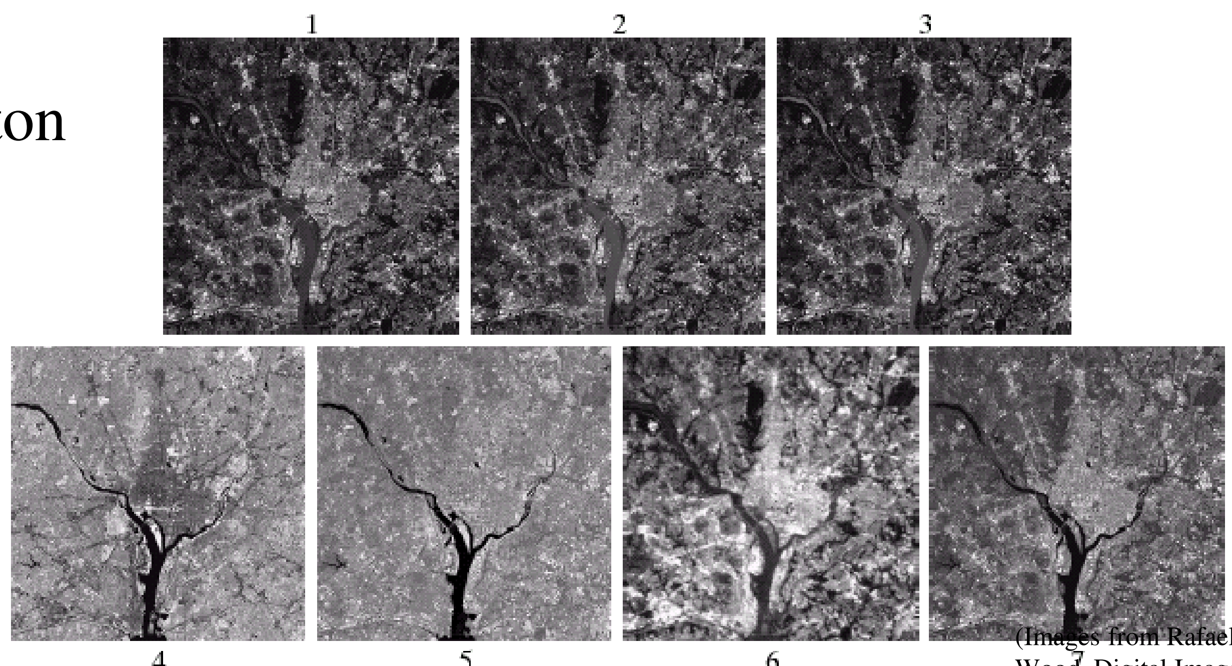
?

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Visible Light and Infrared

Band No.	Name	Wavelength (μm)	Characteristics and Uses
1	Visible blue	0.45–0.52	Maximum water penetration
2	Visible green	0.52–0.60	Good for measuring plant vigor
3	Visible red	0.63–0.69	Vegetation discrimination
4	Near infrared	0.76–0.90	Biomass and shoreline mapping
5	Middle infrared	1.55–1.75	Moisture content of soil and vegetation
6	Thermal infrared	10.4–12.5	Soil moisture; thermal mapping
7	Middle infrared	2.08–2.35	Mineral mapping

Washington D.C.



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Multispectral Imaging

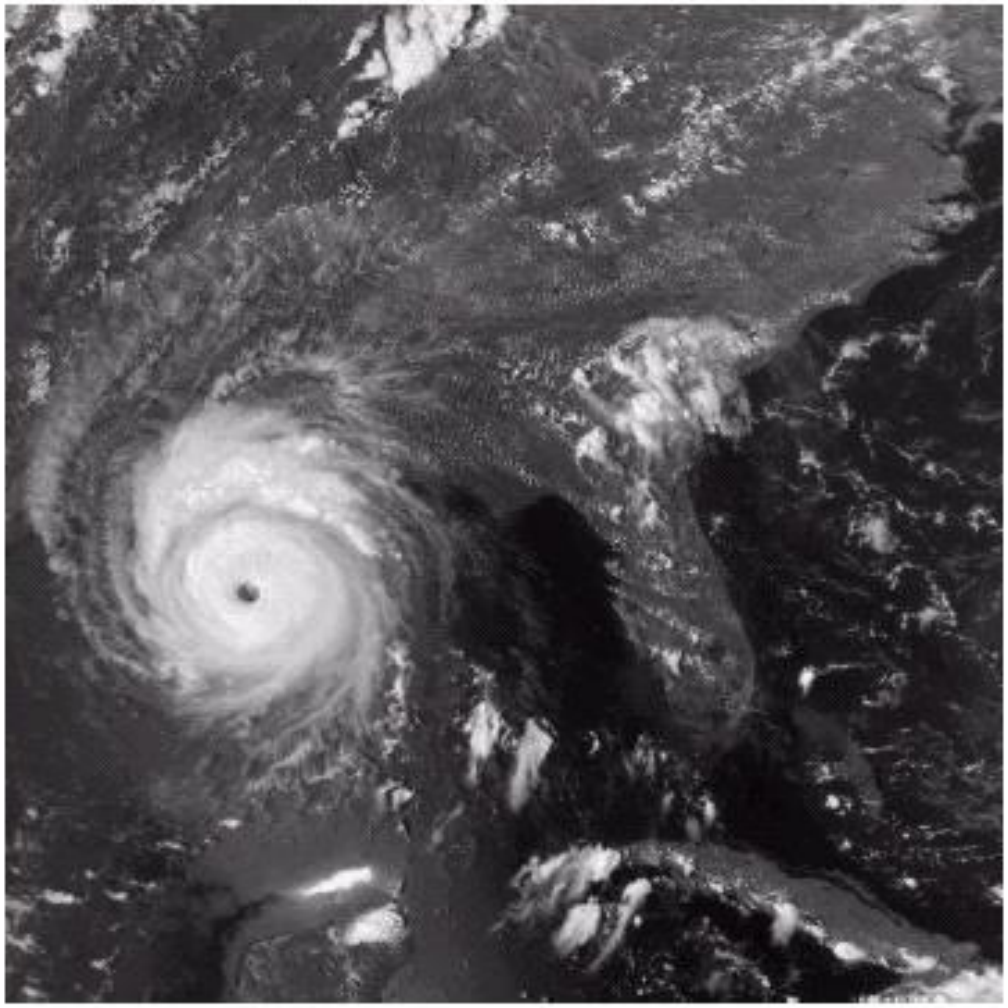


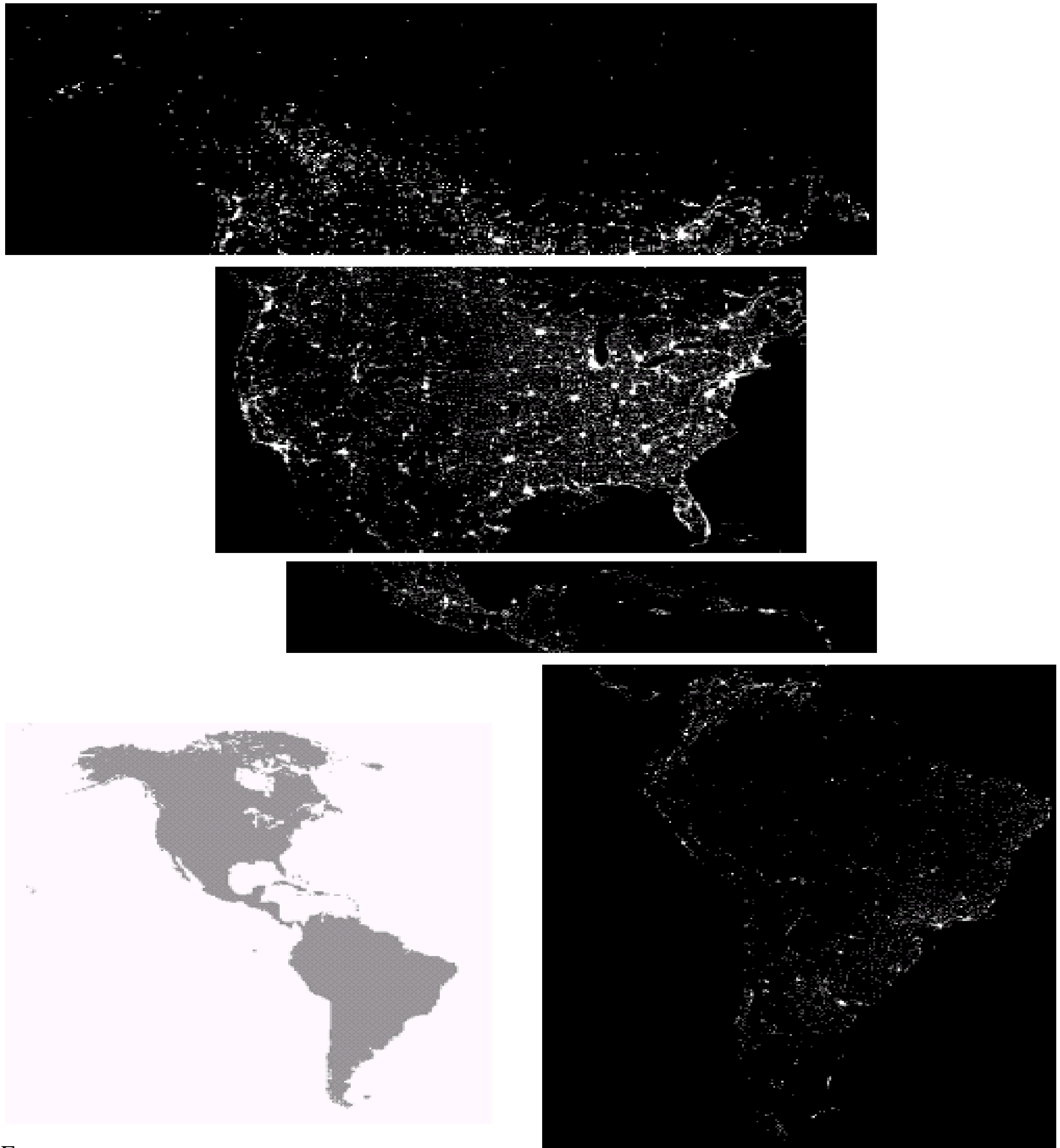
Image is produced by sensors that measure reflected energy from different sections of the EM spectrum

Hurricane Andrew

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Night time light of the world

FIGURE 1.12 Infrared satellite images of the Americas. The small gray map is provided for reference. (Courtesy of NOAA.)



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Nighttime light of the world (cont.)

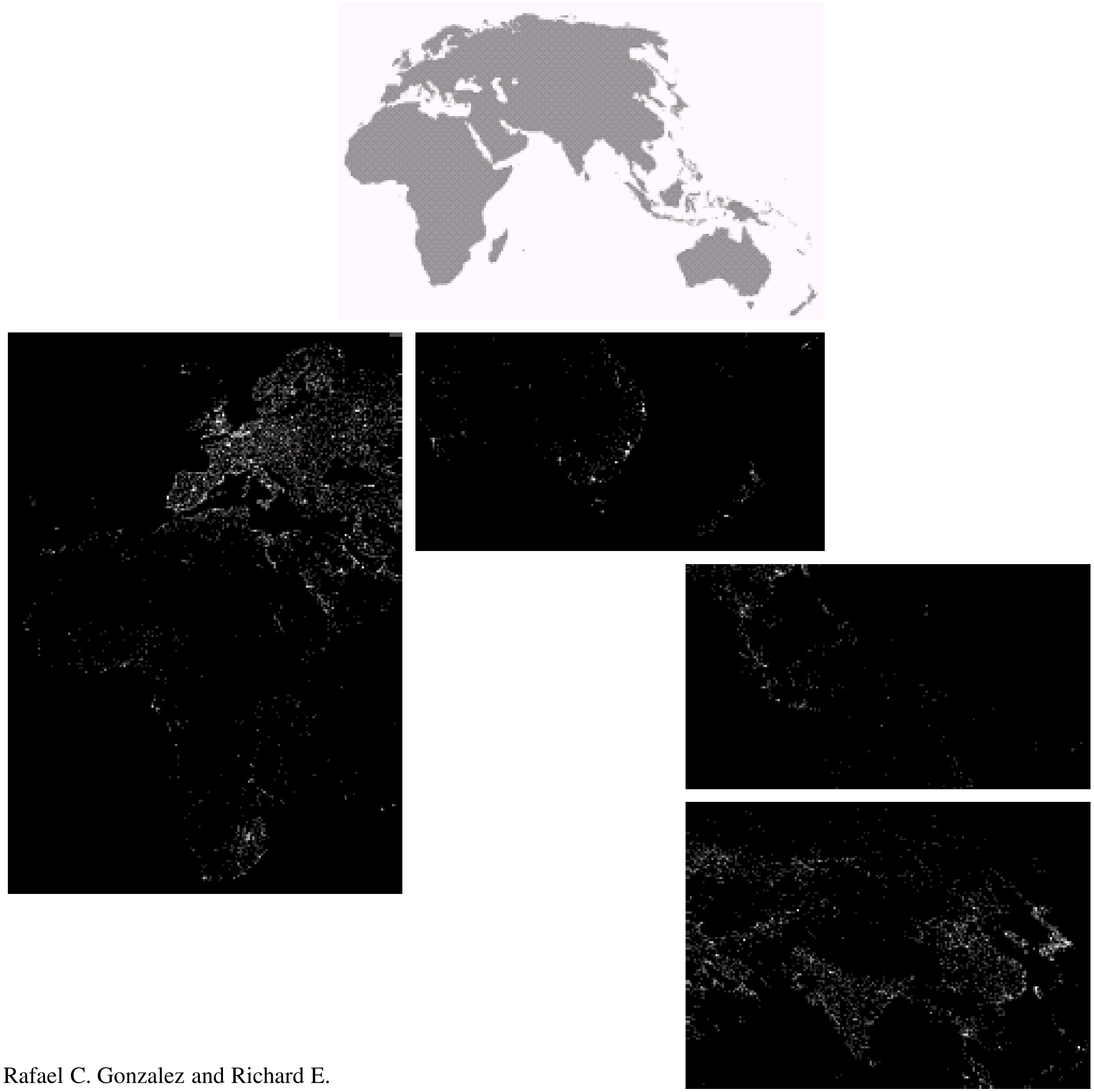


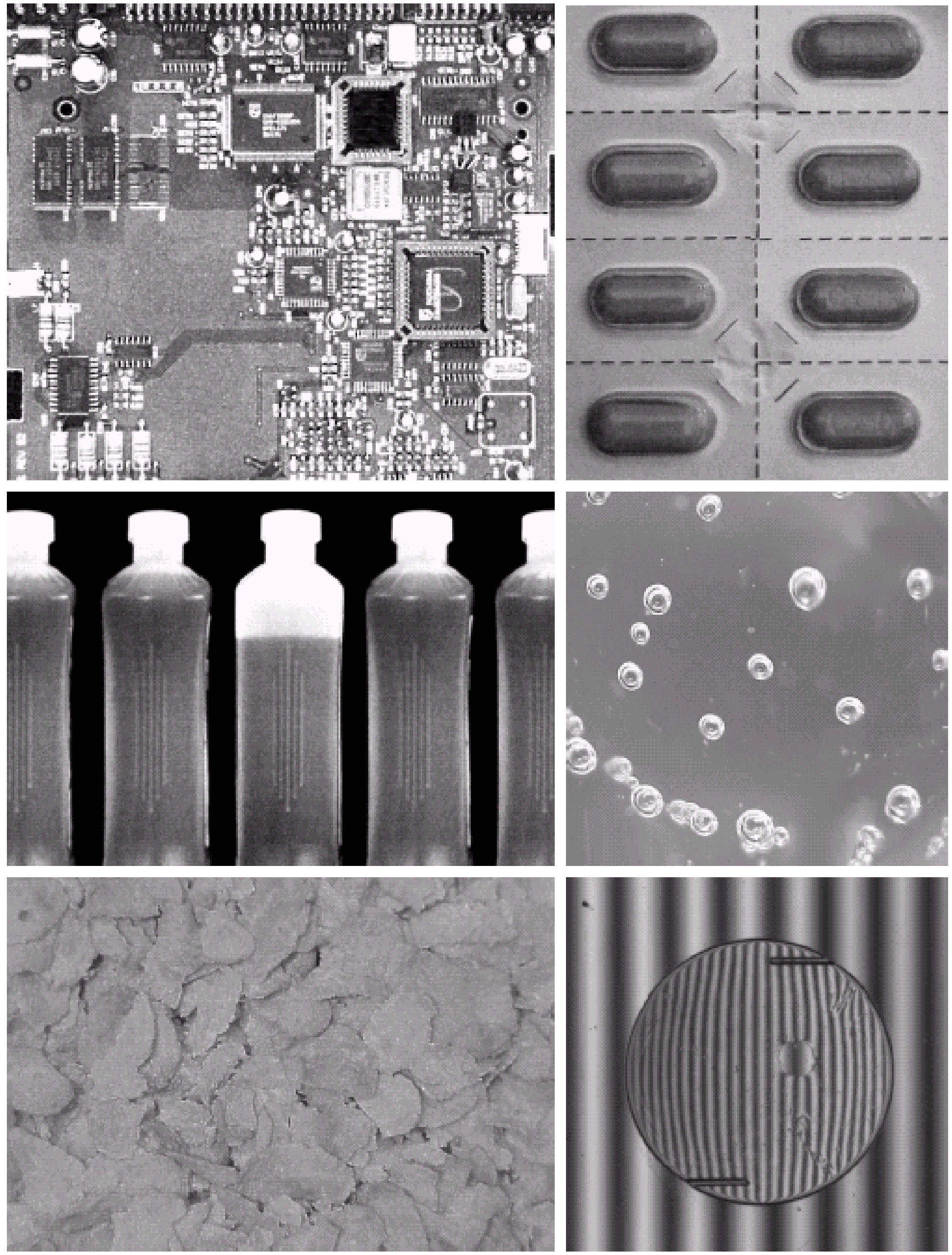
FIGURE 1.13 Infrared satellite images of the remaining populated part of the world. The small gray map is provided for reference. (Courtesy of NOAA.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Automated Visual Inspection

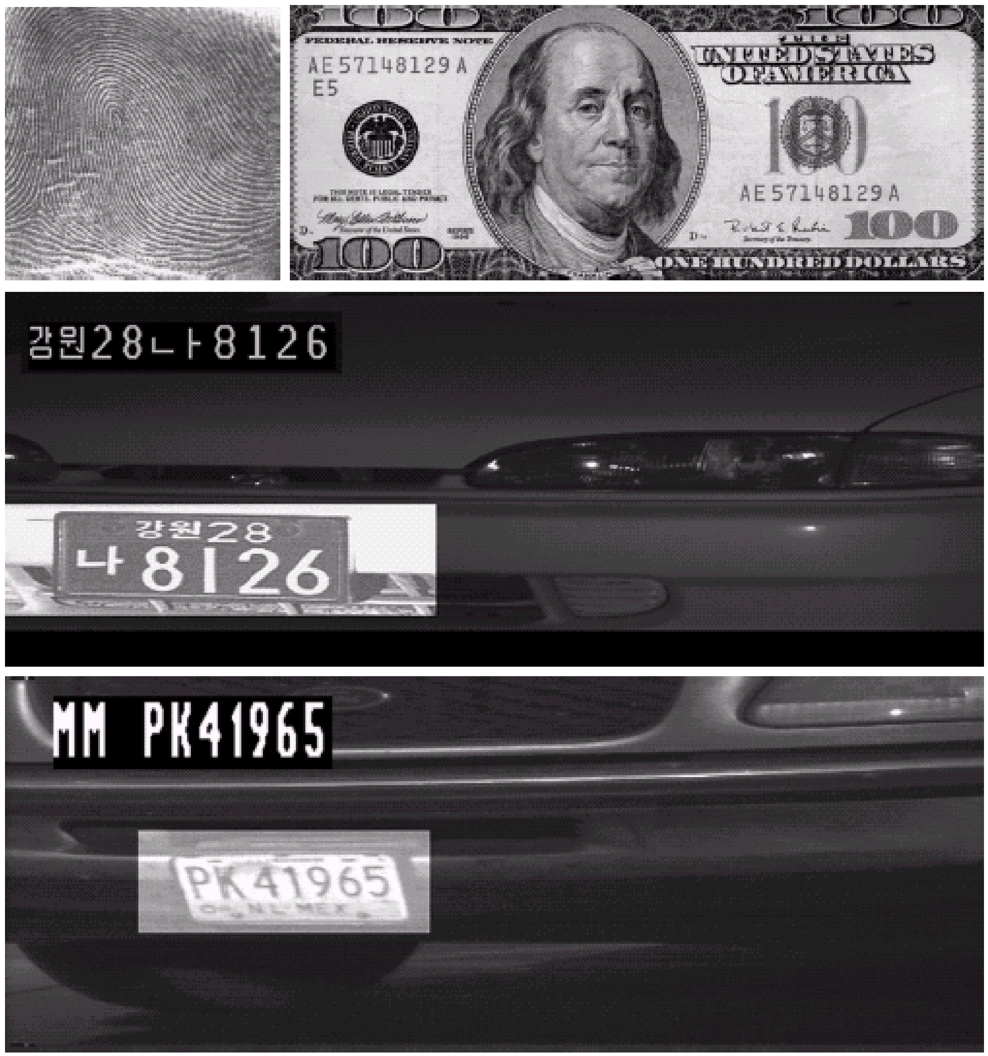
a b
c d
e f

FIGURE 1.14 Some examples of manufactured goods often checked using digital image processing. (a) A circuit board controller. (b) Packaged pills. (c) Bottles. (d) Bubbles in clear-plastic product. (e) Cereal. (f) Image of intraocular implant. (Fig. (f) courtesy of Mr. Pete Sites, Perceptics Corporation.)



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Automated Visual Inspection (cont.)



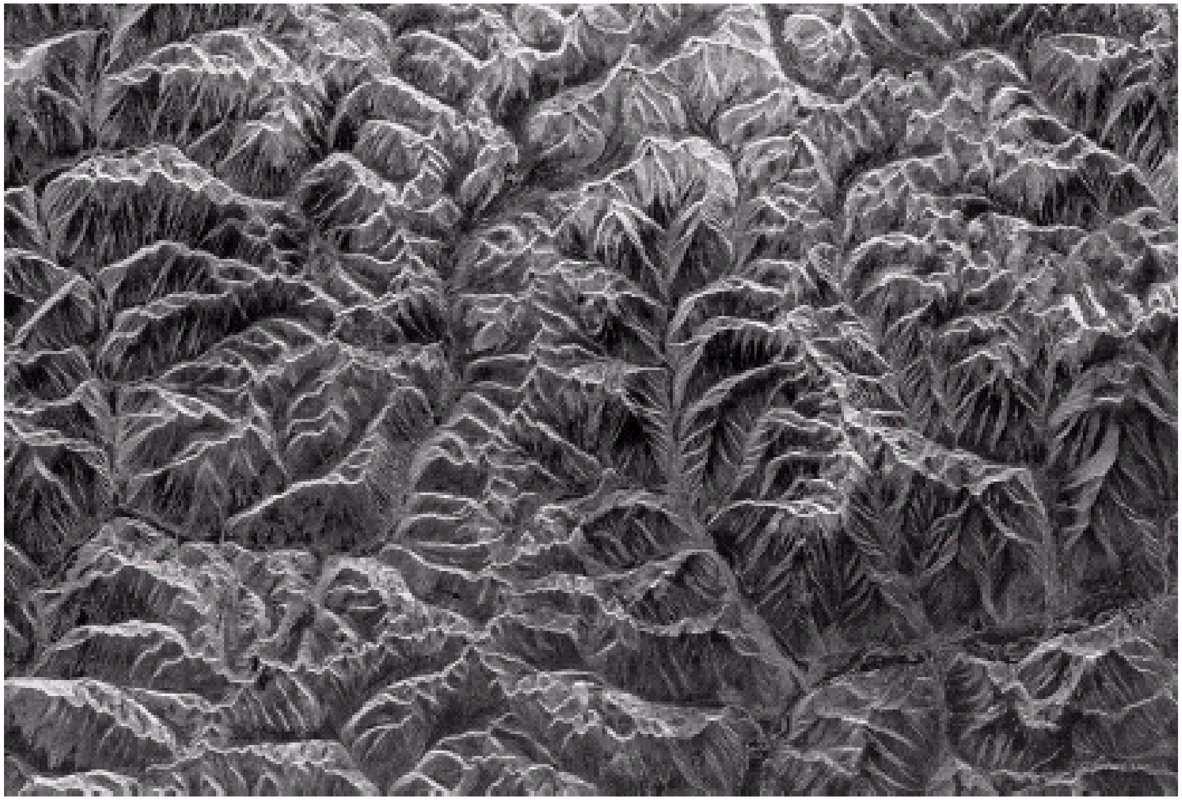
a b
c
d

FIGURE 1.15 Some additional examples of imaging in the visual spectrum. (a) Thumb print. (b) Paper currency. (c) and (d). Automated license plate reading. (Figure (a) courtesy of the National Institute of Standards and Technology. Figures (c) and (d) courtesy of Dr. Juan Herrera, Perceptics Corporation.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Microwave

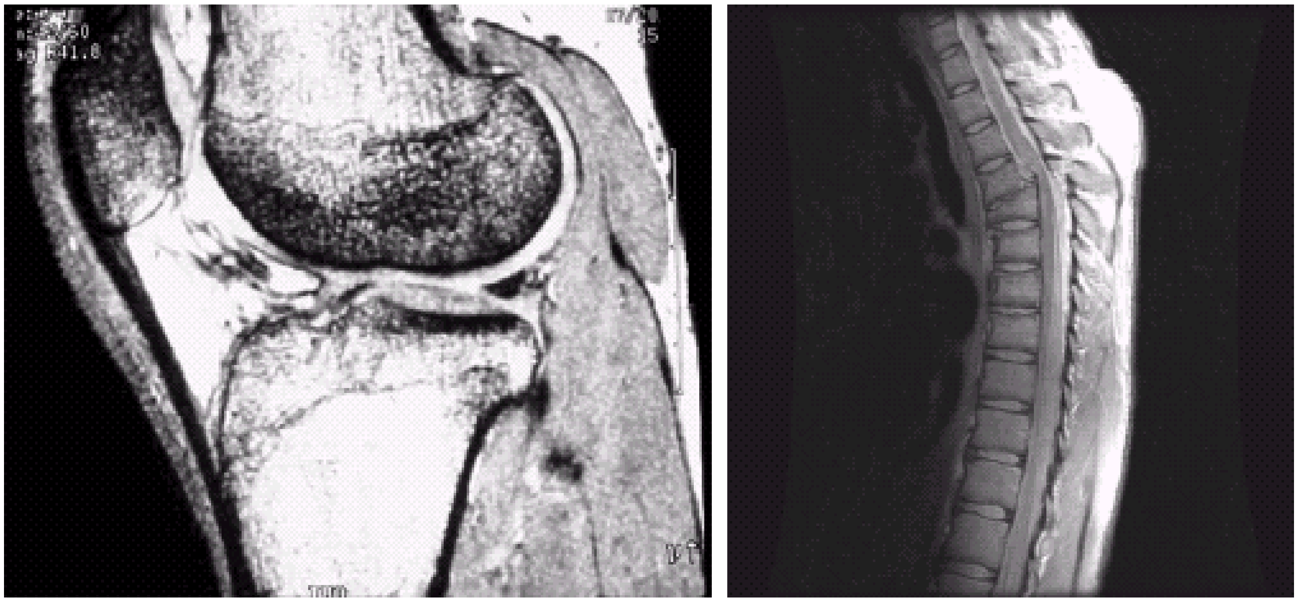
FIGURE 1.16 Spaceborne radar image of mountains in southeast Tibet. (Courtesy of NASA.)



Spaceborne Radar image

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Magnetic

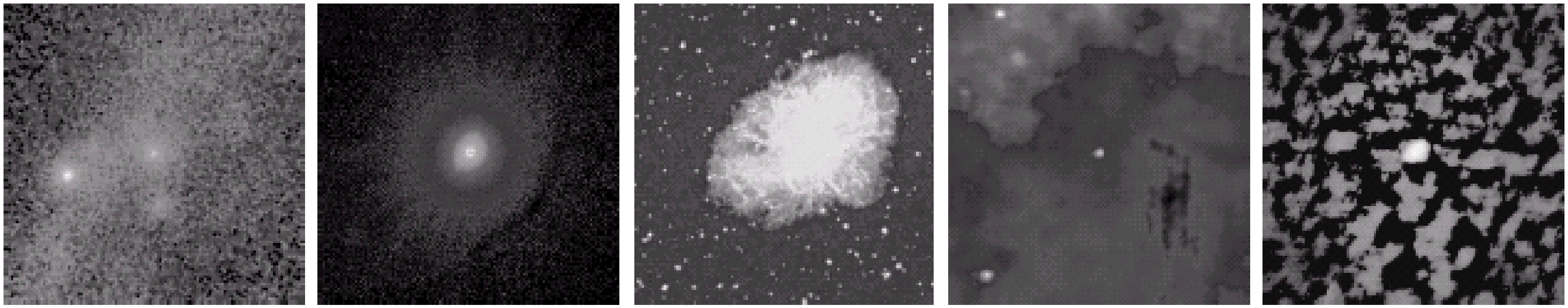


a b

FIGURE 1.17 MRI images of a human (a) knee, and (b) spine. (Image (a) courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School, and (b) Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Multispectral images



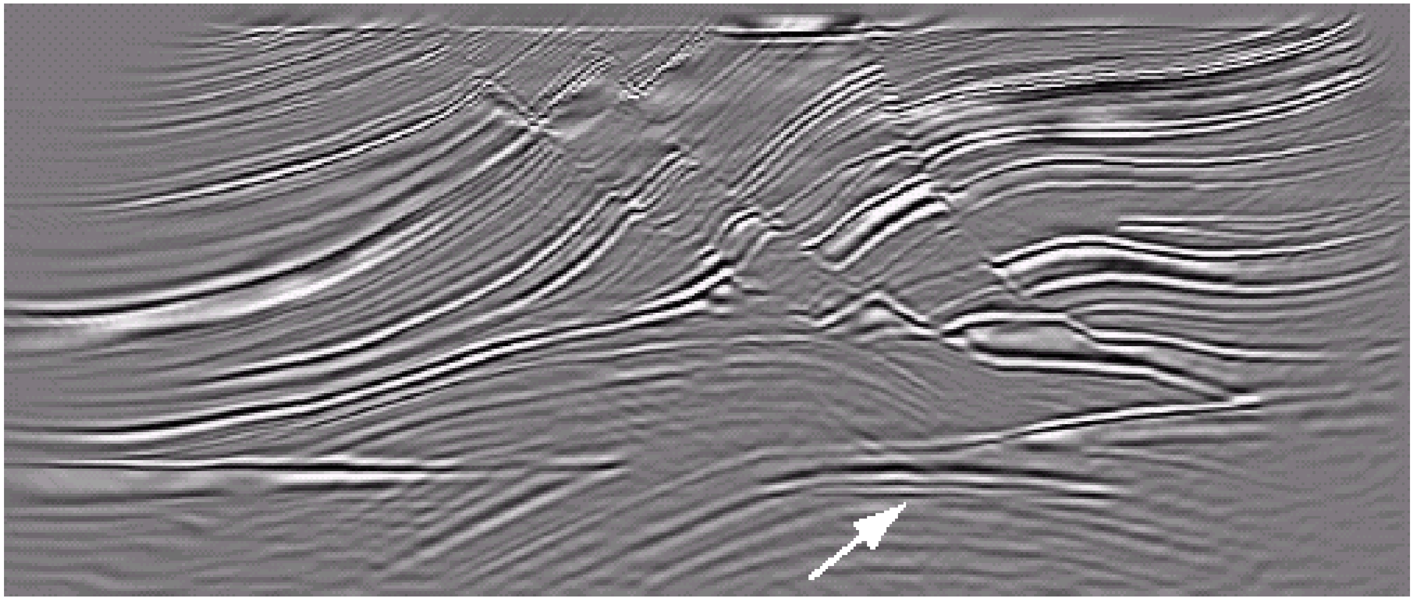
Gamma X-ray Optical Infrared Radiox

FIGURE 1.18 Images of the Crab Pulsar (in the center of images) covering the electromagnetic spectrum. (Courtesy of NASA.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

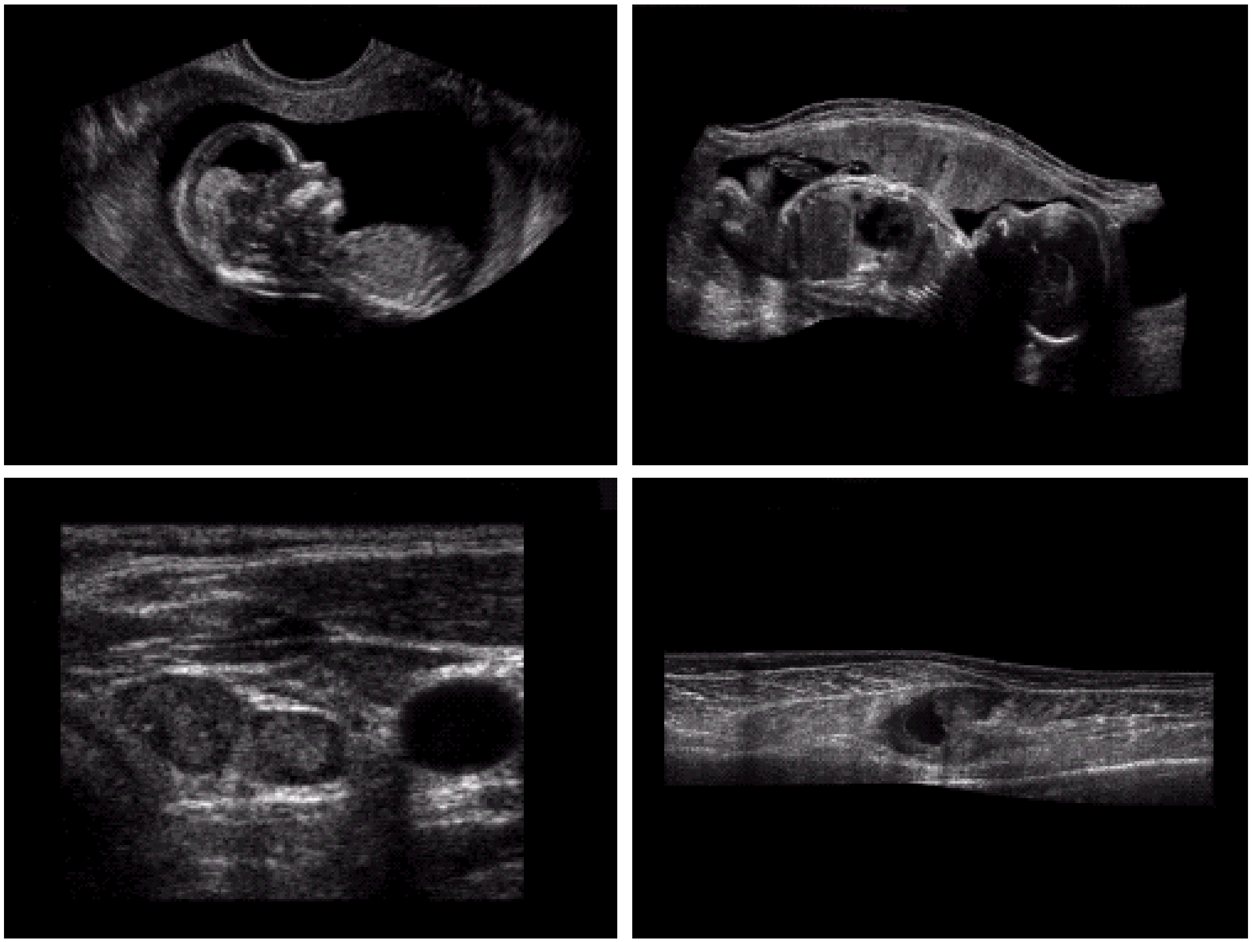
Seismic imaging

FIGURE 1.19 Cross-sectional image of a seismic model. The arrow points to a hydrocarbon (oil and/or gas) trap. (Courtesy of Dr. Curtis Ober, Sandia National Laboratories.)



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Ultrasound imaging

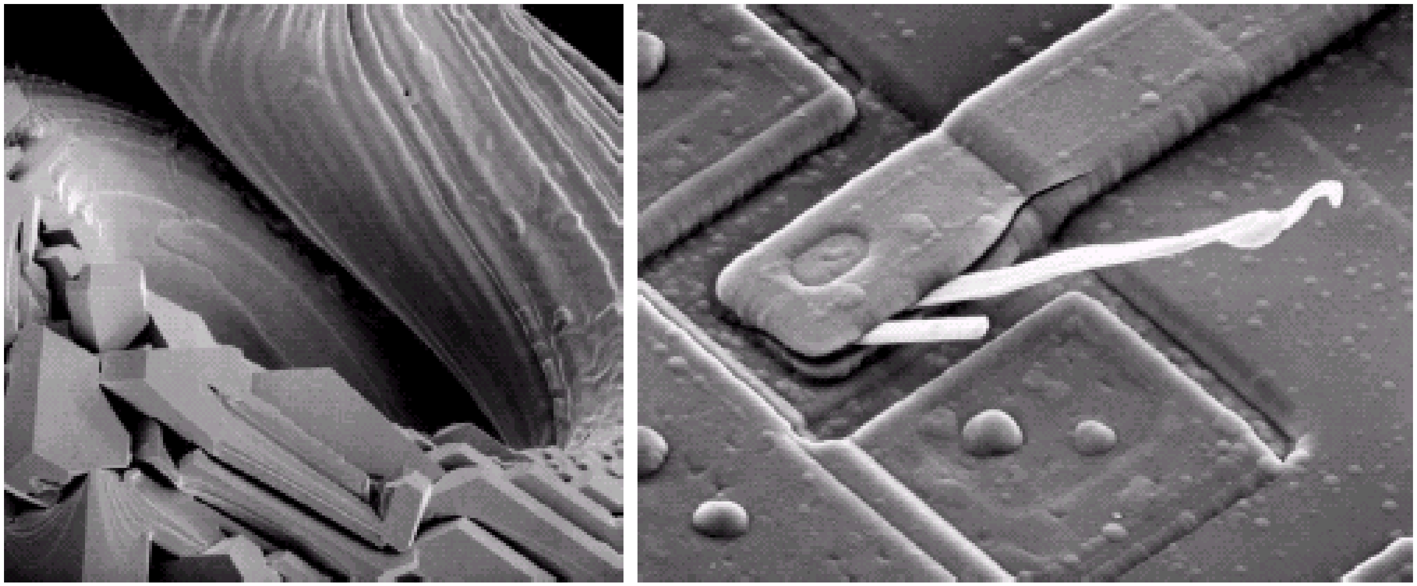


a b
c d

FIGURE 1.20 Examples of ultrasound imaging. (a) Baby. (2) Another view of baby. (c) Thyroids. (d) Muscle layers showing lesion. (Courtesy of Siemens Medical Systems, Inc., Ultrasound Group.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Electron Microscope Images

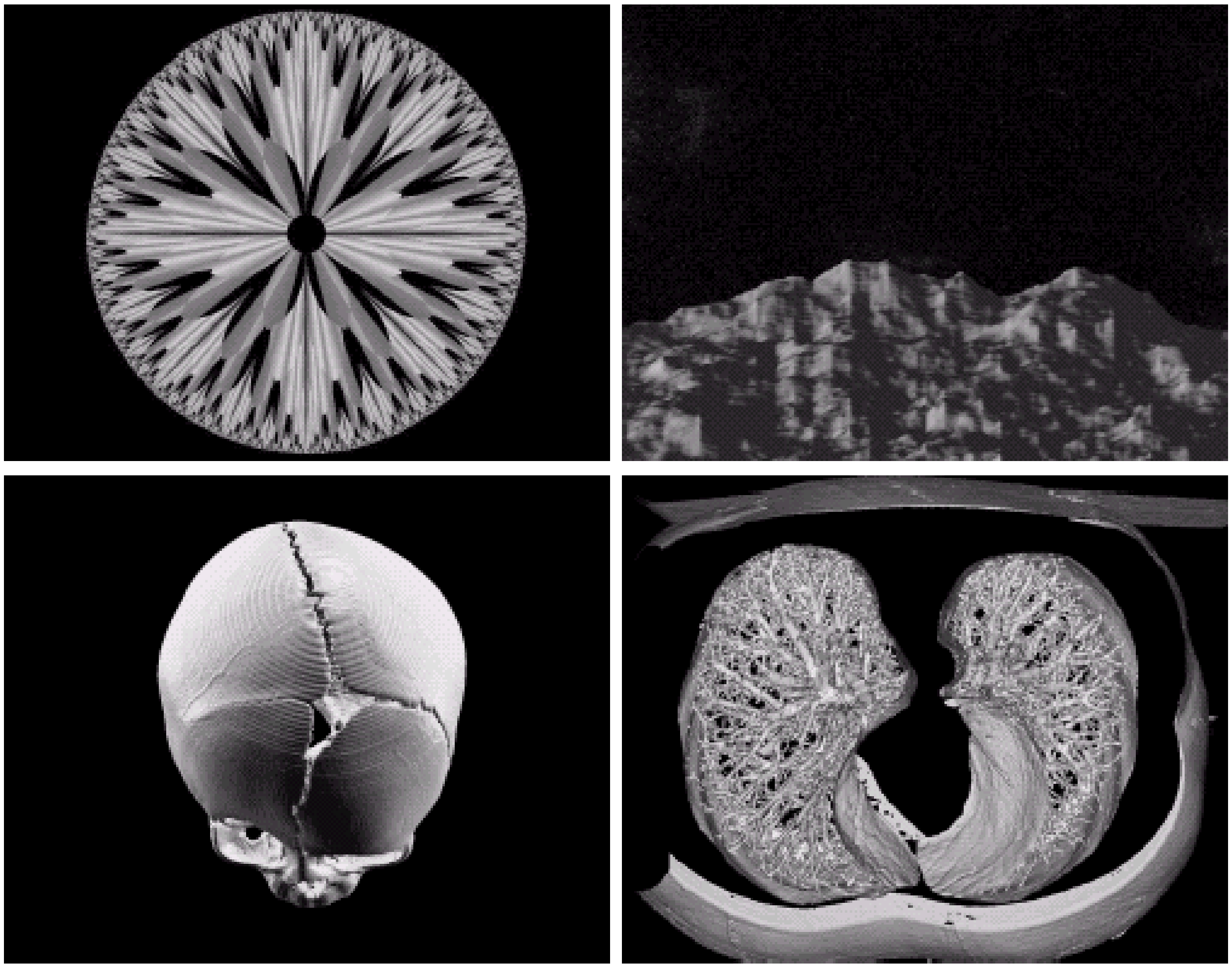


a b

FIGURE 1.21 (a) 250× SEM image of a tungsten filament following thermal failure. (b) 2500× SEM image of damaged integrated circuit. The white fibers are oxides resulting from thermal destruction. (Figure (a) courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene; (b) courtesy of Dr. J. M. Hudak, McMaster University, Hamilton, Ontario, Canada.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Synthesis Images

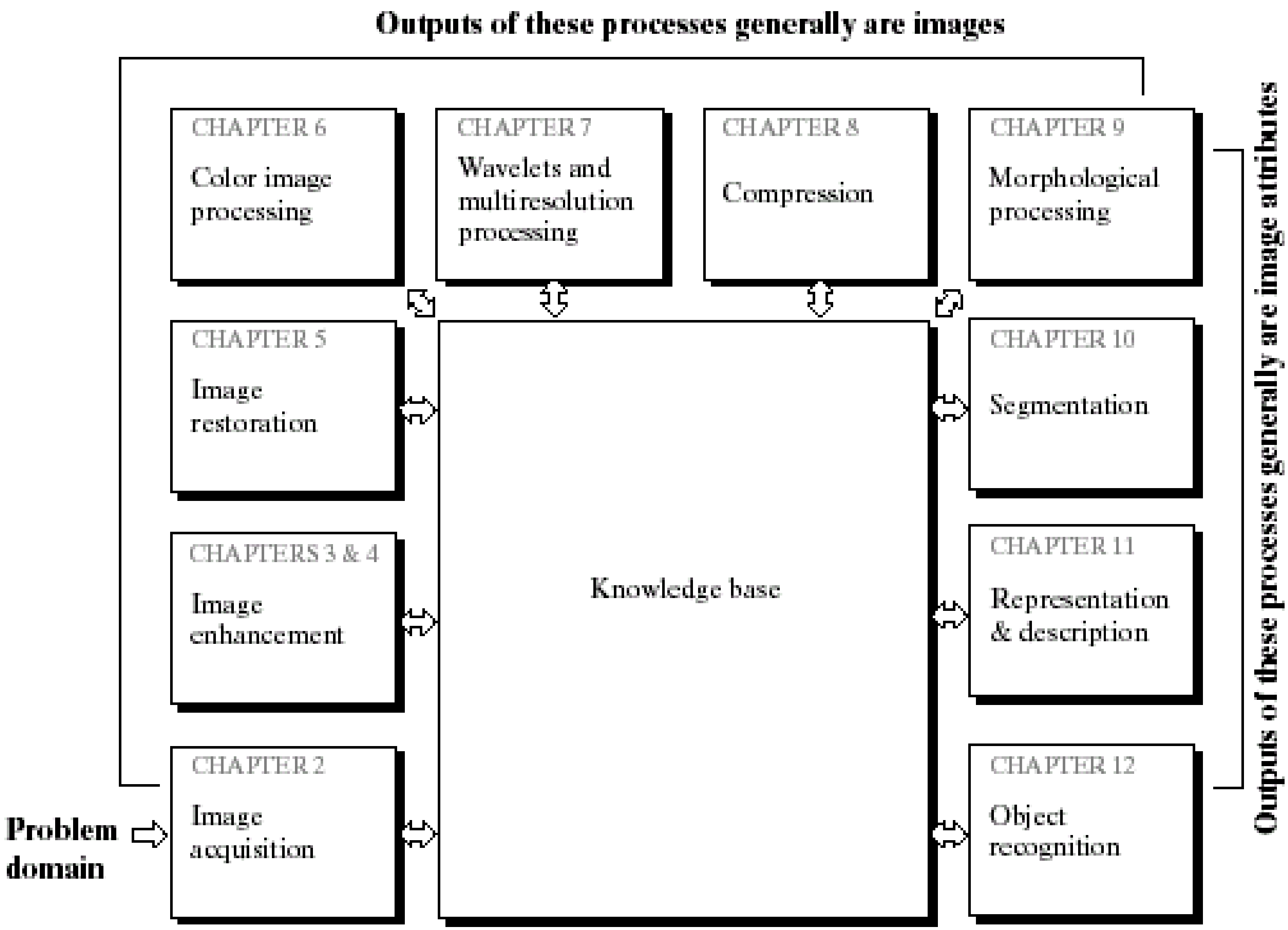


a	b
c	d

FIGURE 1.22 (a) and (b) Fractal images. (c) and (d) Images generated from 3-D computer models of the objects shown. (Figures (a) and (b) courtesy of Ms. Melissa D. Binde, Swarthmore College, (c) and (d) courtesy of NASA.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Contents in the book



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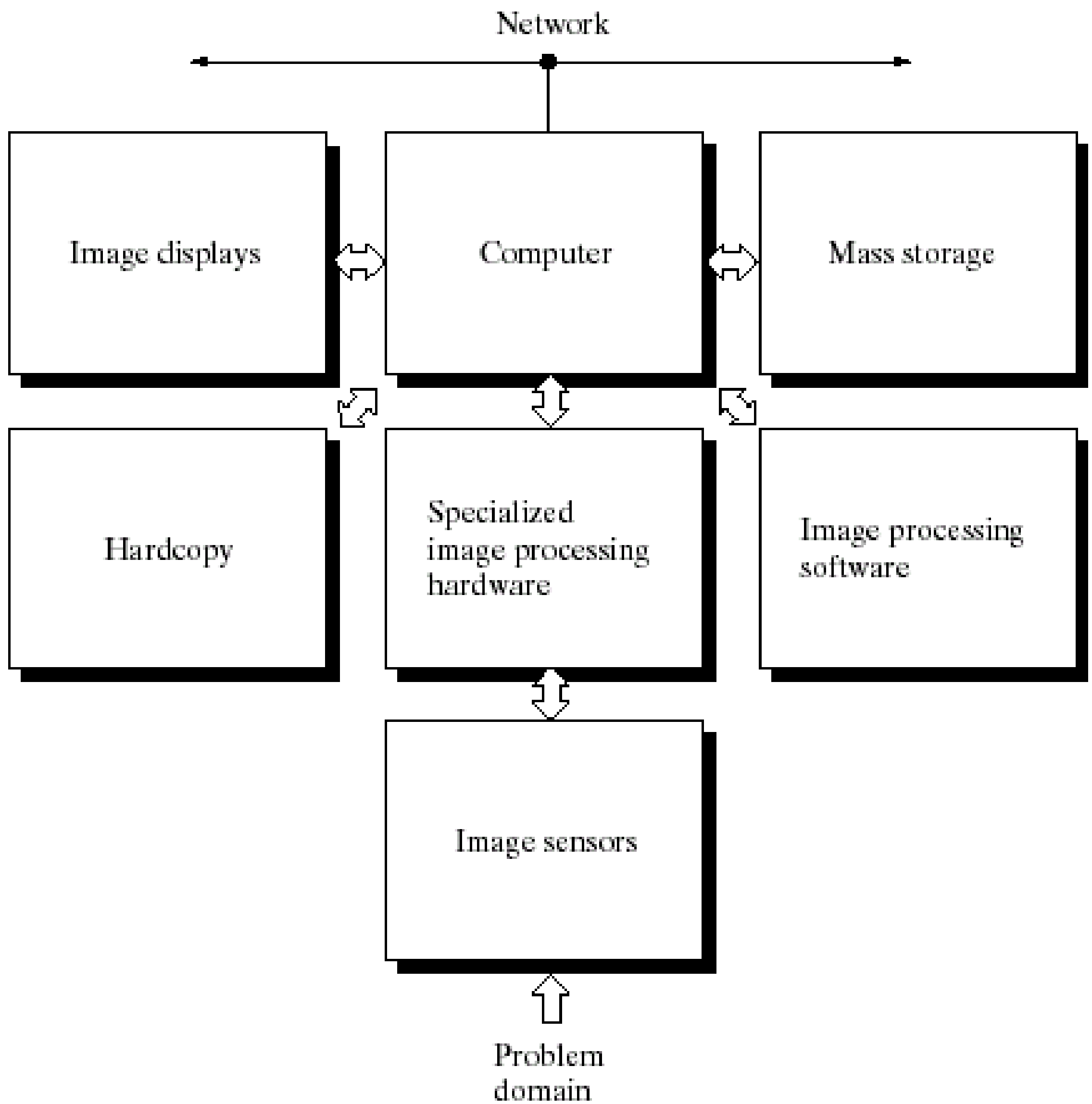
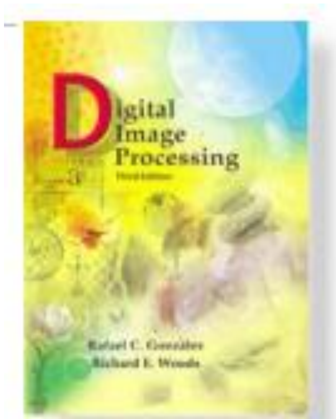
General Purpose Image Processing System



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.



Chapter- 2

Digital Image Fundamentals

Digital Image Processing, 3rd ed.
Gonzalez & Woods

Motilal Nehru National Institute of Technology
Allahabad

Chapter 2

Digital Image Fundamentals

2.1 Elements of Visual Perception

2.1.1 Structure of the Human Eye

Contracts & expands to control the amount of light entering the eye

Central opening of the Iris: Pupil (diameter: ~ 2 to 8 mm)

Concentric layers of fibrous cells
Absorb ~8% of the visible light spectrum

Innermost membrane of the eye
Light from object imaged on retina

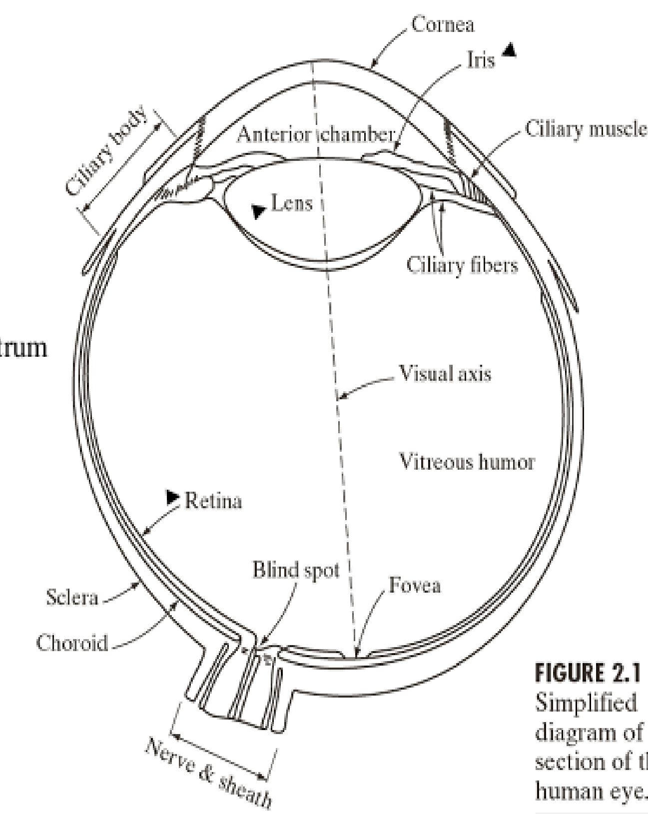


FIGURE 2.1 Simplified diagram of a cross section of the human eye.

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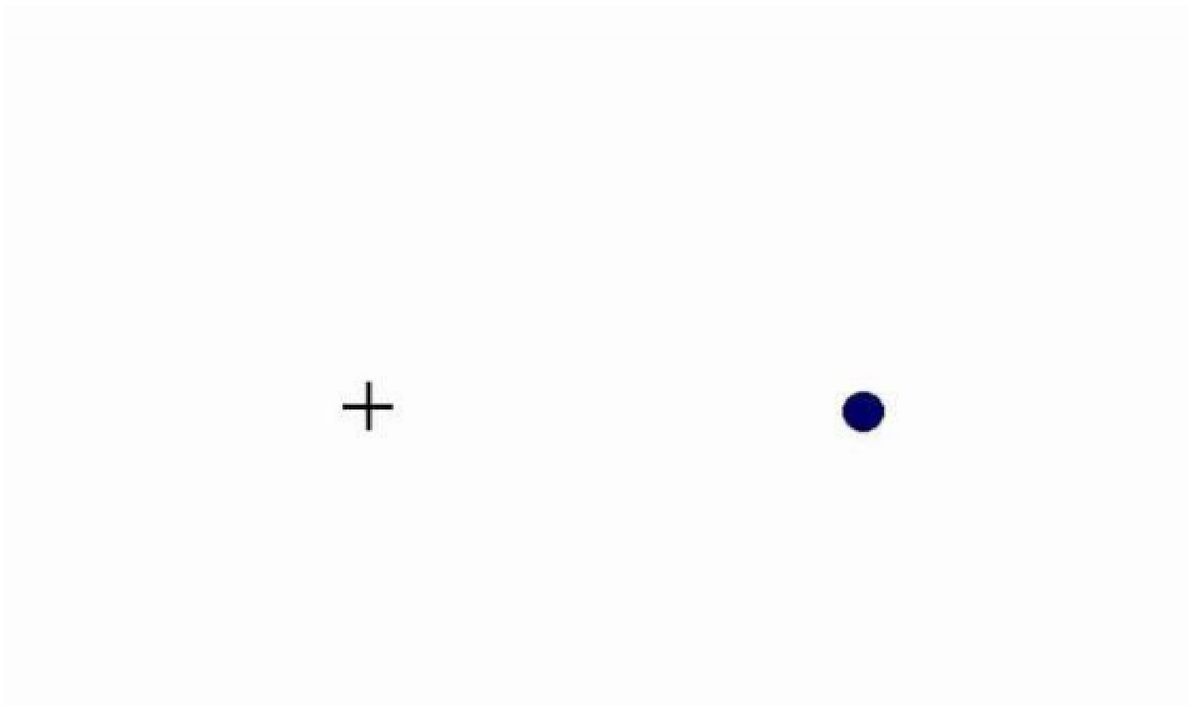
Eye’s Blind Spot

2.1.1 Structure of the Human Eye



Experimentation to illustrate the eye’s *Blind Spot*:

Close your left eye and stare at the cross. Get your head closer (or further) to the image. In a particular position, the dot should disappear. If you get even closer (or further) the dot appears again.

Distribution of Light Receptors

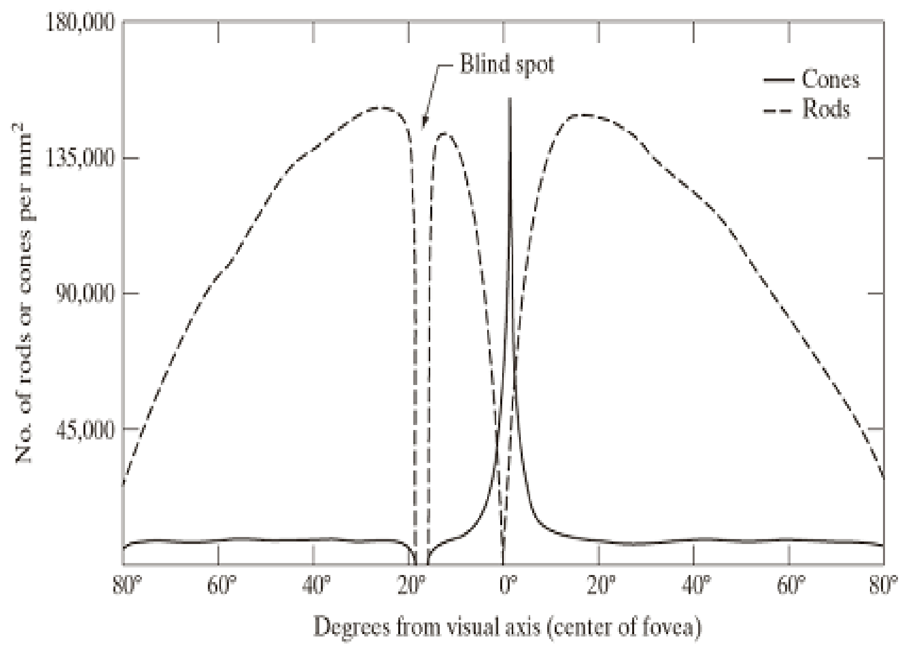
Distribution of discrete light receptors over the surface of the retina

2 classes of receptors: *cones* and *rods* :

• **Cones**: 6-7 million in each eye, mainly located in the fovea. Highly sensitive to colour, fine details.
"Photopic" or bright-light vision

• **Rods**: 75-150 million, distributed. Sensitive to low level of illumination, not involved in colour vision.
"Scotopic" or dim-light vision

FIGURE 2.2
Distribution of rods and cones in the retina.



Distribution of receptors is radially symmetric about the fovea, except the so-called "blind spot"

Distribution of Light Receptors (Contd.)

Approximation: fovea ≈ square sensor array of size 1.5 mm x 1.5 mm.

Density of cones in this area: 150,000 elements/mm^2

=> Number of cones in the region of highest acuity in the eye: ~337,000 elements.

Just in term of raw resolving power, a CCD can have this number of elements in a receptor array no larger than 5mm x 5mm.

=> basic ability of the eye to resolve detail is comparable to current electronic imaging sensors

Image Formation in the Eye

2.1.2 Image Formation in the Eye

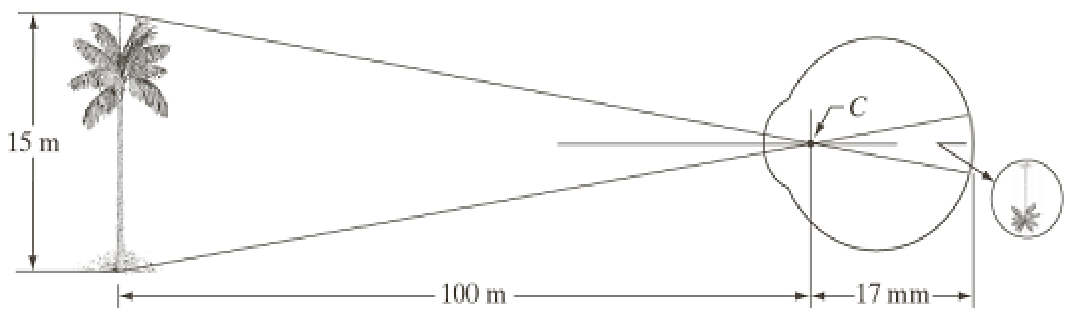


FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point *C* is the optical center of the lens.

<u>Photo camera</u>: lens has *fixed focal length*. Focusing at various distances by *varying distance* between lens and imaging plane (location of film or chip)

<u>Human eye</u>: converse. *Distance* lens-imaging region (retina) is *fixed*. *Focal length* for proper focus obtained by *varying* the shape of the lens.

Perception takes place by the relative excitation of light receptors, which transform radiant energy into electrical impulses that ultimately are decoded by the brain

HVS Characteristics

2.1.3 Brightness Adaptation and Discrimination

Eye's ability to discriminate between different intensity levels

Range of light intensity levels to which the human visual system can adapt: on the order of 10^{10}

-Subjective brightness is a logarithmic function of the light incident on the eye

-Total range of distinct intensity levels the eye can discriminate simultaneously is small compared to the total adaptation range

-Current sensitivity of a visual system is called the brightness adaptation levels

- At Ba adaptation level , intersecting curve represents the range of subjective brightness that eye can perceive

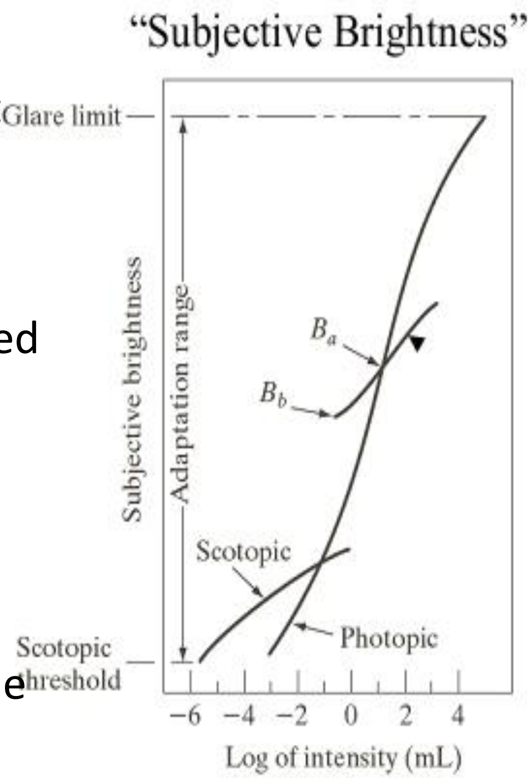


FIGURE 2.4 Range of subjective brightness sensations showing a particular adaptation level.

Range of *subjective brightness* the eye can perceive when adapted to this level B_a

mL- mili Lambert

Brightness Discrimination

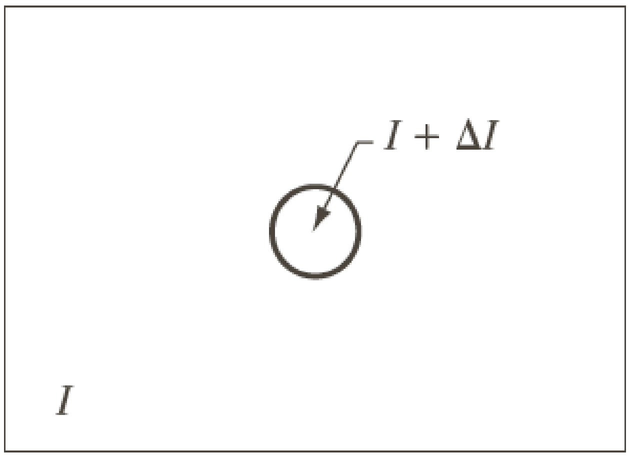


FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

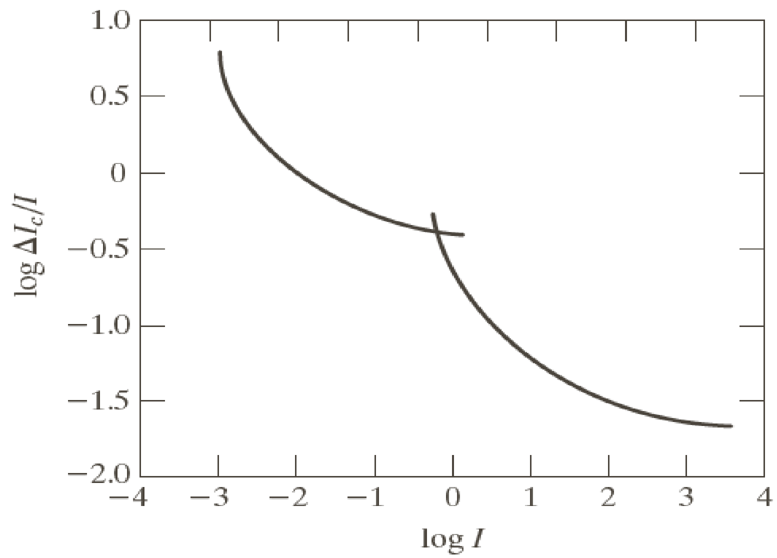


FIGURE 2.6 Typical Weber ratio as a function of intensity.

-The ability of the eye to discriminate between changes in light intensity at any specific adaptation level

- The quantity ΔIc/I , ΔIc is the increment of illumination discriminable 50 % of the time with background illumination I, is called the **Weber ratio**

Brightness Discrimination

Small weber ratio --- small percentage change in intensity is discriminable --- good brightness discrimination
Large weber ratio----- a large percentage change in intensity is required ---- poor brightness discrimination

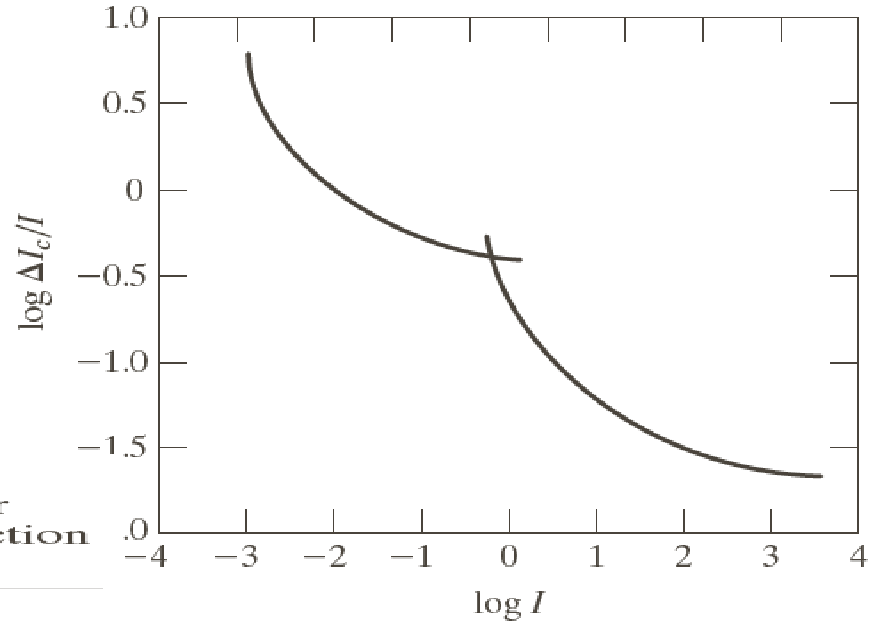


FIGURE 2.6 Typical Weber ratio as a function of intensity.

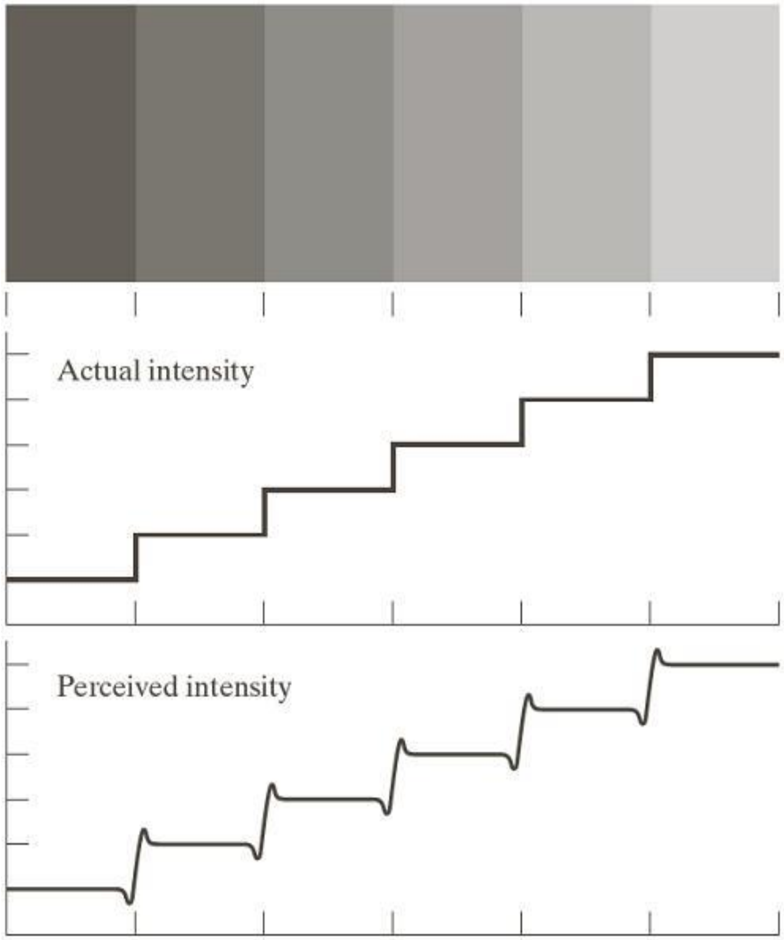
- The curve shows that the brightness discrimination is poor at low level of illumination and it improves significantly as background illumination increases
- At low level illumination – vision is carried out by rods
- At high level illumination – vision is carried out by cones
- The number of different intensities a person can see at any point of time in a monochrome image is 1 to 2 dozens

Visual Perception

Perceived brightness is not a simple function of intensity

- Mach bands: visual system tends to undershoot or overshoot around the boundary of regions of different intensities
- Simultaneous contrast: region's perceived brightness does not depend on its intensity
- Optical illusions: eye fills in non-existing information or wrongly perceives geometrical properties of objects

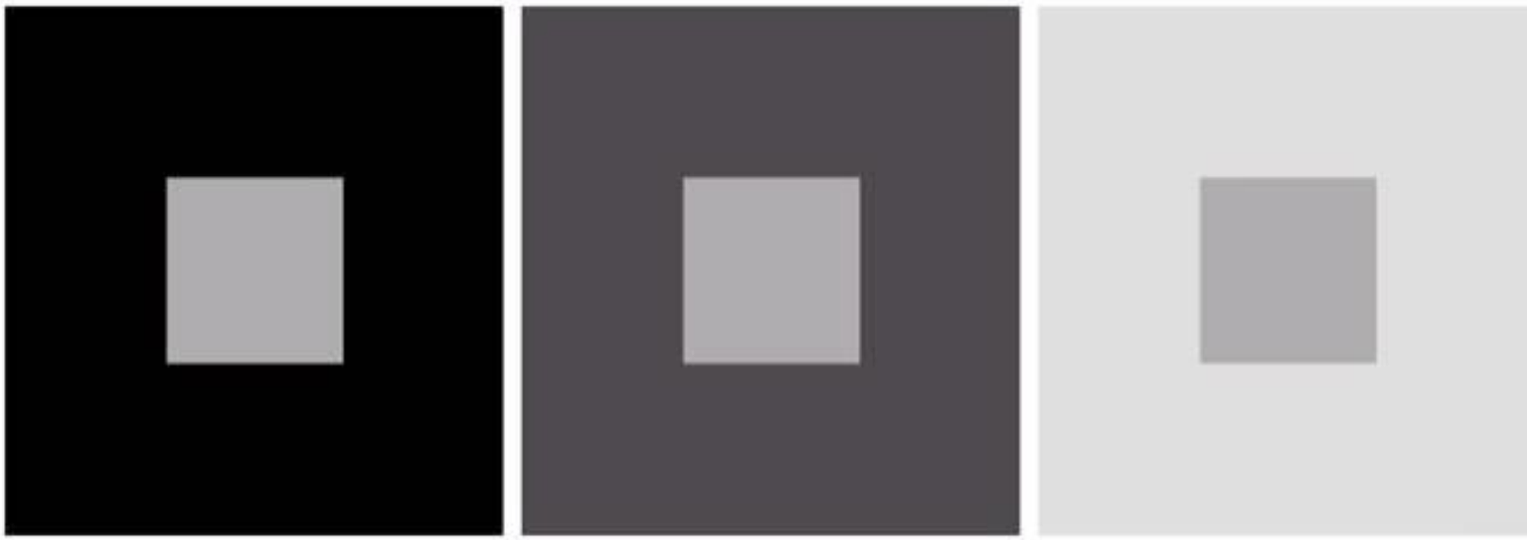
Mach Band Effect



a
b
c

FIGURE 2.7 Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.

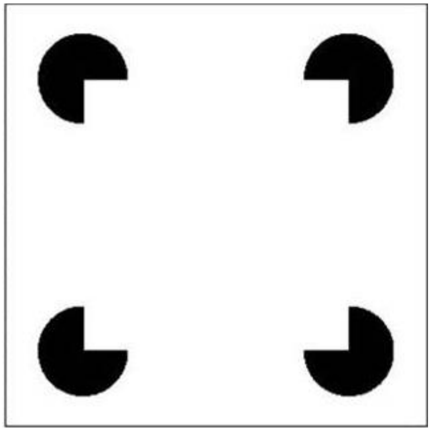
Simultaneous Contrast



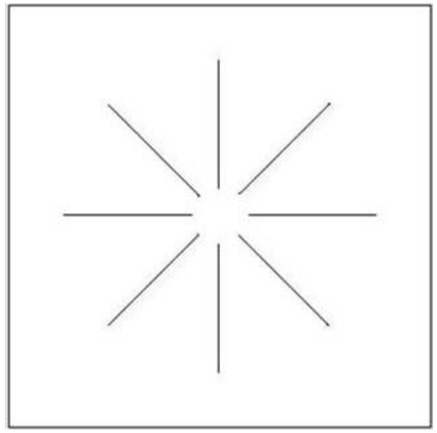
a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

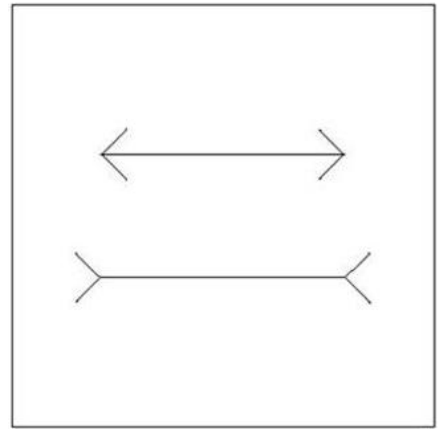
Some Optical Illusions



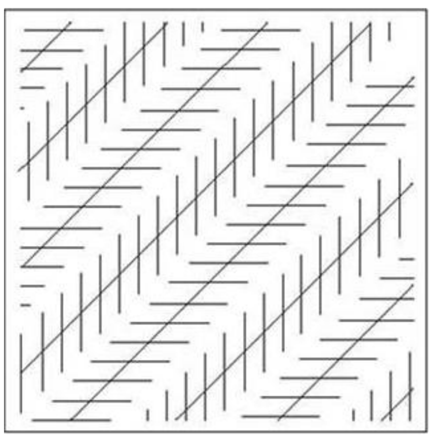
(a) Outline of a square is seen clearly



(b) Outline of a circle is seen



(c) Two horizontal line segments are of the same length, but one appears shorter than the other



(d) All lines that are oriented at 45 degree are equidistant and parallel

Optical illusions: Eye fills in non-existing information or wrongly perceives geometrical properties of objects

Electromagnetic Spectrum

2.2 Light and the Electromagnetic Spectrum

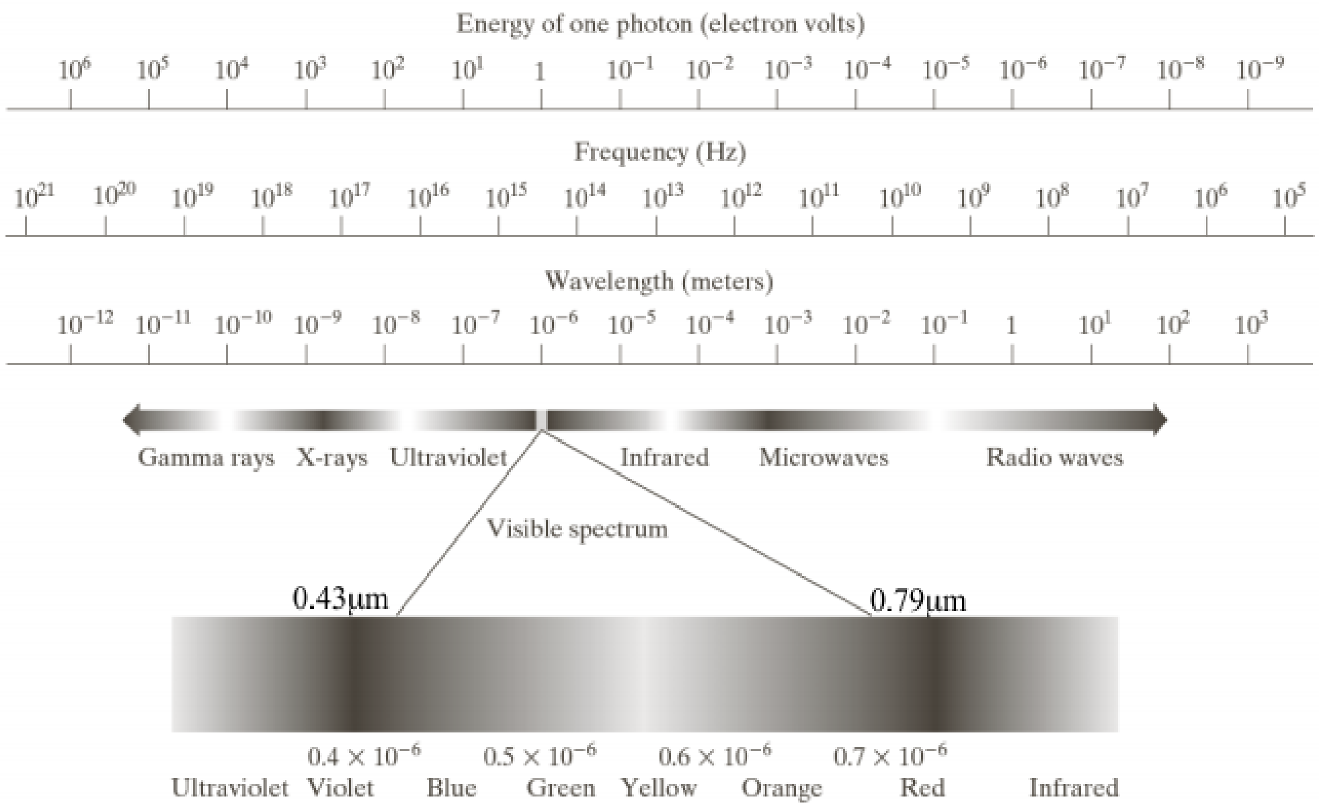
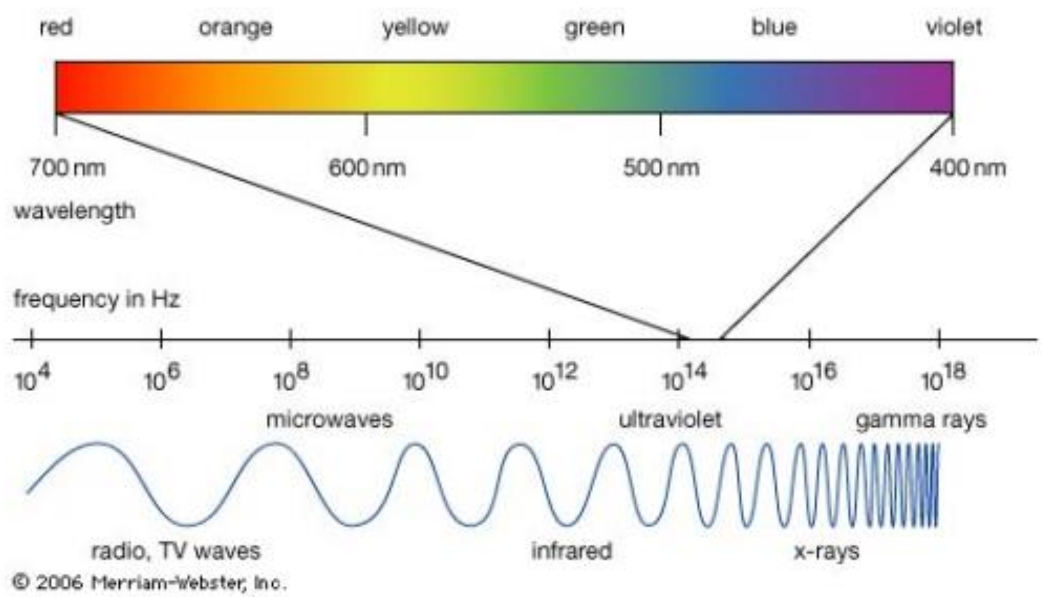


FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

Color Lights



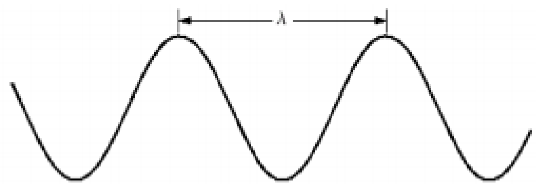
Wavelength (λ) and frequency (ν) related by: $$\lambda = \frac{c}{\nu}$$

$c \approx 2.998 \times 10^8$ m/s
λ in *microns* (μm=10^{-6} m) or *nanometers* (nm=10^{-9} m)

Energy (eV): $$E = h\nu$$ (h: Planck's constant)

Properties of Light

FIGURE 2.11 Graphical representation of one wavelength.



- Light void of colour = ***monochromatic*** (or *achromatic*) light

  => only attribute : ***intensity*** or ***gray level***

- Range of measured values = *gray scale*
- Monochromatic images = ***gray-scale images***

Chromatic light source: frequency + radiance, luminance, brightness

- *Radiance* = total amount of energy that flows from the light source (W)
- *Luminance* (in lumens, lm) = measure of the amount of energy an observer *perceives* from a light source
- *Brightness* = subjective descriptor of light perception practically impossible to measure

Image Sensing

Images are generated by the combination of the illumination source and the reflection or absorption of energy from that source by elements of the scene being imaged

2.3 Image Sensing and Acquisition

Transform of illumination energy into digital images:

The incoming energy is transformed into a voltage by the combination of input electrical power and sensor material.

Output voltage waveform = response of the sensor(s)

A digital quantity is obtained from each sensor by *digitizing* its response.

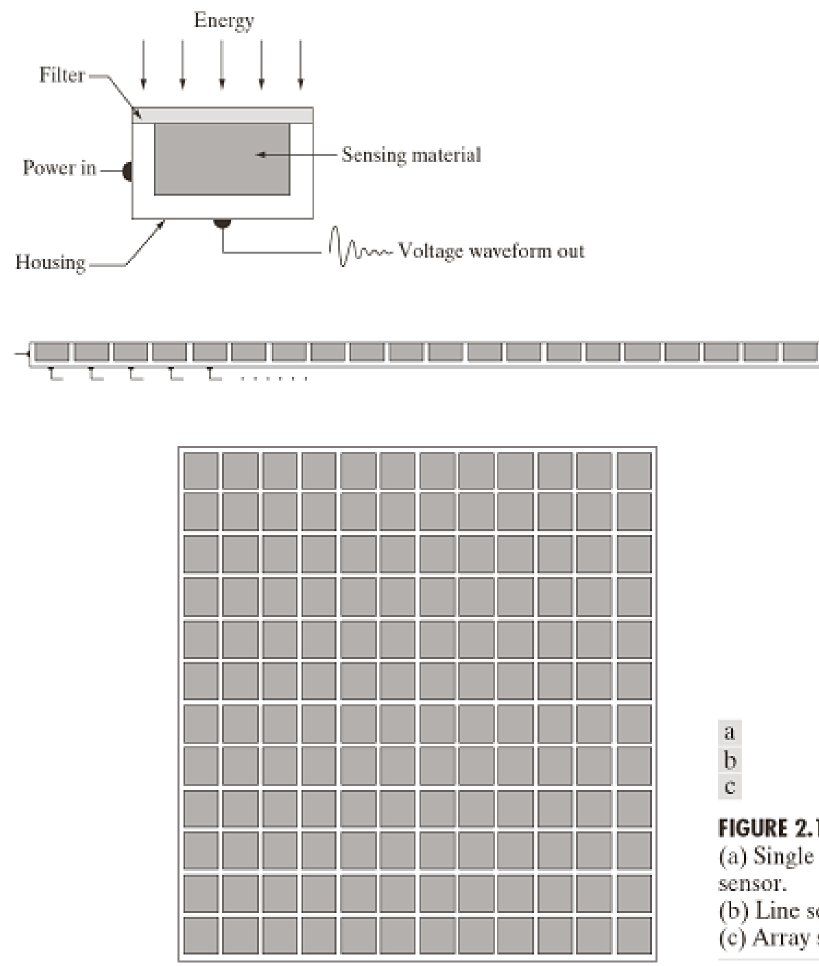
Imaging Sensors

Ex: Photodiode

Made of silicon

Output voltage waveform proportional to light

Filter in front: increase selectivity



a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image Acquisition

2.3.1 Image acquisition using a single sensor

Arrangement for high precision scanning

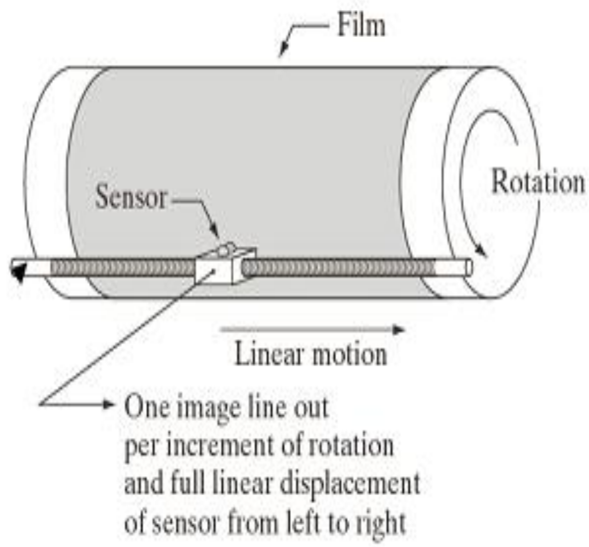


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

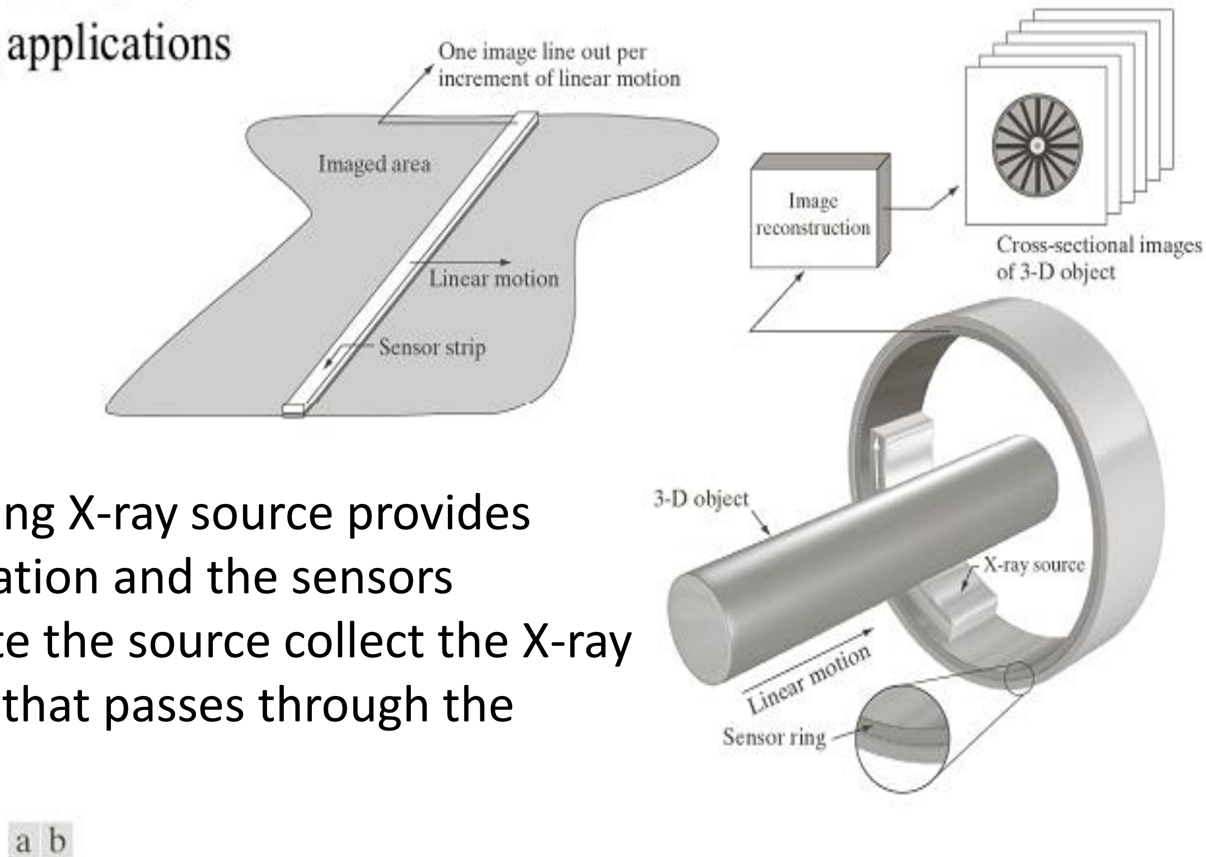
Lead screw

In-expensive (but slow) way to obtain high-resolution images

Image Acquisition

2.3.2 Image acquisition using sensor strips

Ex of use: airborne imaging applications



Ring configuration

Medical (CAT) and industrial imaging

(cross-sectional (slice) images of 3D objects

A rotating X-ray source provides illumination and the sensors opposite the source collect the X-ray energy that passes through the object

Image Acquisition using Sensor Arrays

2.3.3 Image acquisition using sensor arrays

- Illumination source reflected from a scene element
- Imaging system collects the incoming energy and focus it onto an image plane (sensor array)
- Response of each sensor proportional to the integral of the light energy projected
- Sensor output: analog signal → digitized

NB1: Motion not necessary
NB2: Predominant arrangement for digital cameras (e.g. CCD array)

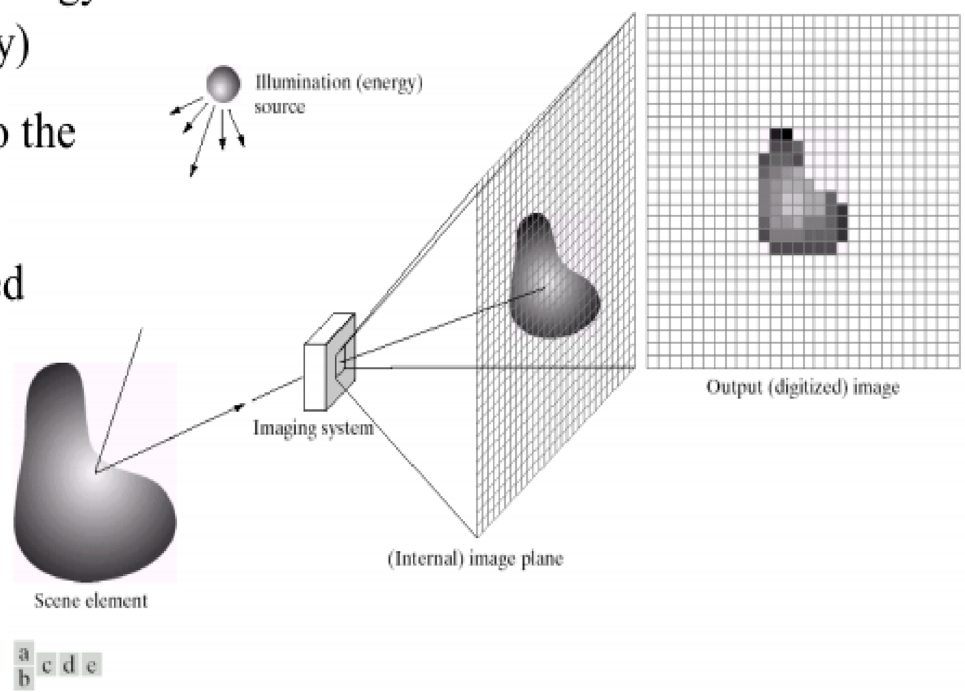


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

CCD Camera

2.3.3 Image acquisition using sensor arrays

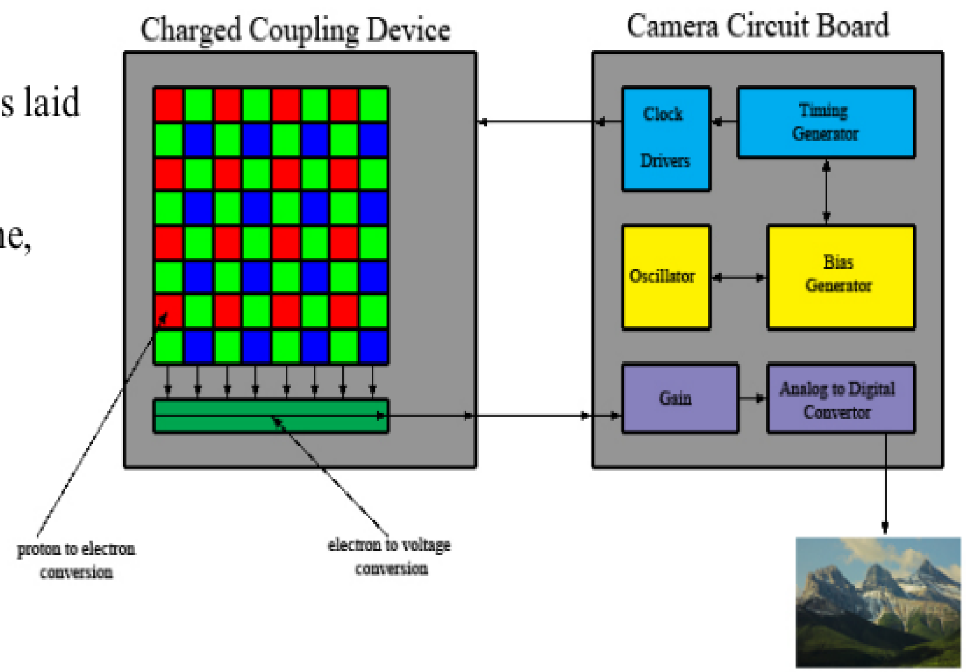
CCD cameras: widely used in modern applications: private consumers, industry, astronomy…

CCD: Charge Couple Device

© sensorcleaning.com

Rectangular grid of electron-collection sites laid over a thin silicon wafer

Image readout of the CCD one row at a time, each row transferred in parallel to a serial output register



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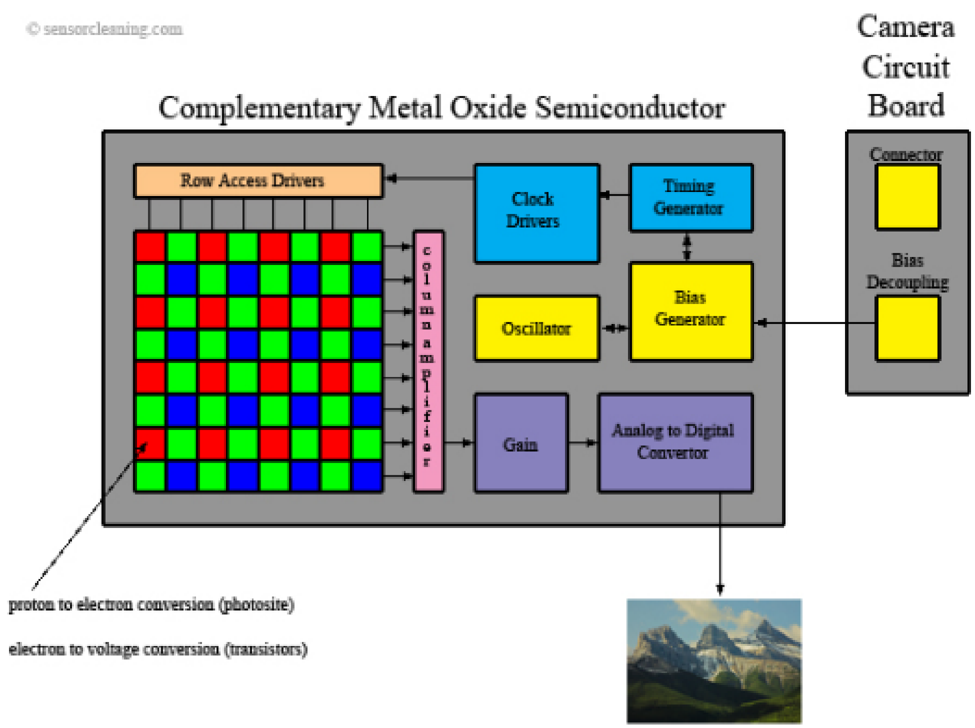
CCD Vs CMOS

2.3.3 Image acquisition using sensor arrays

Alternative to CCD cameras: **CMOS** technology

CMOS: Complementary Metal-Oxyde-Semiconductor



CMOS chip : active pixel sensor made using CMOS semiconductor

CMOS can potentially be implemented with fewer components, use less power and provide data faster than CCDs

CCD: more mature technology

NB: a CMOS-based camera can be significantly smaller than a comparable CCD camera

Image Formation Model

2.3.4 A Simple Image Formation Model

Images denoted by two-dimensional functions $f(x,y)$

Value of amplitude of f at (x,y): positive scalar quantity

Image generated by physical process: intensity values are proportional to the energy radiated by a physical source => $0 < f(x,y) < \infty$

$f(x,y)$ may be characterized by 2 components:

(1) The amount of source illumination *incident* on the scene: *illumination* $i(x,y)$

(2) The amount of illumination *reflected* by the objects of the scene: *reflectance* $r(x,y)$

$$f(x,y) = i(x,y)\, r(x,y), \quad \text{where } 0 < i(x,y) < \infty \text{ and } 0 < r(x,y) < 1$$

total absorption

total reflectance

Image Formation Model

2.3.4 A Simple Image Formation Model

Example of typical ranges of illumination *i(x,y)* for visible light (average values):

- Sun on a clear day: ~ 90,000 lm/m^2, down to 10,000 lm/m^2 on a cloudy day
- Full moon on a clear evening: ~0.1 lm/m^2
- Typical illumination level in a commercial office: ~1000 lm/m^2

Typical values of reflectance *r(x,y):*

- 0.01 for black velvet
- 0.65 for stainless steel
- 0.8 for flat white wall paint
- 0.9 for silver-plated metal
- 0.93 for snow

Monochromatic Image

2.3.4 A Simple Image Formation Model

Monochrome image

Intensity l: $L_{min} \leq l \leq L_{max}$. In practice: $L_{min}=i_{min}\, r_{min}$ and $L_{max} = i_{max}\, r_{max}$

Typical limits for indoor values in the absence of additional illumination: $L_{min} \approx 10$ and $L_{max} \approx 1000$

$[L_{min}, L_{max}]$ is called the *gray* (or *intensity*) *scale*

Common practice: shift to $[0, L-1]$, where $l=0$ is considered black and $l=L-1$ is considered white

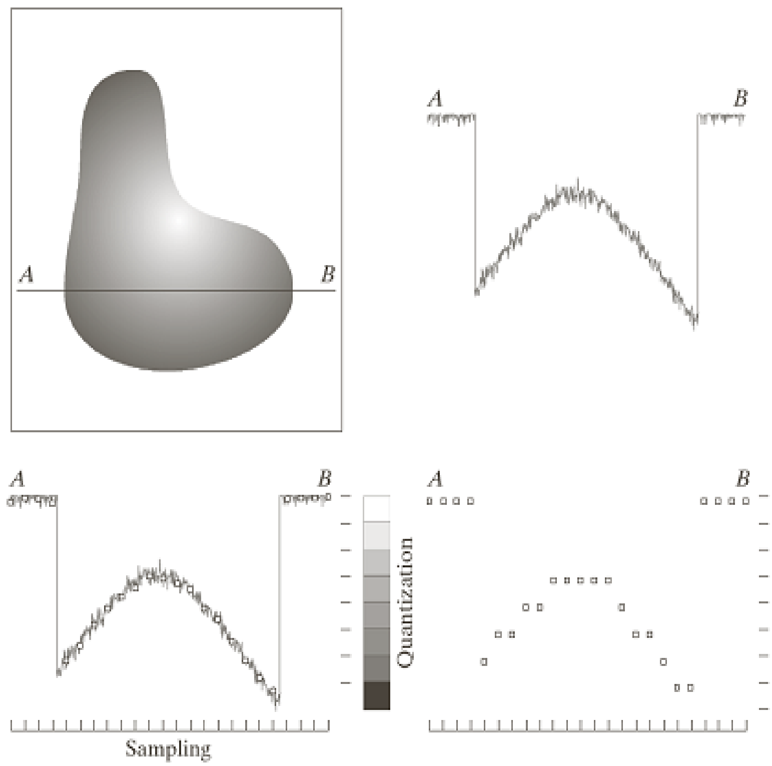
Image Sampling and Quantization

2.4.1 Basic Concepts in Sampling and Quantization

Discretizing coordinate values is called Sampling
Discretizing the amplitude values is called Quantization

Method of **sampling** determined by the sensor arrangement:

- **Single sensing element combined with motion:** spatial sampling based on number of individual mechanical increments
- **Sensing strip:** the number of sensors in the strip establishes the sampling limitations in one image direction; in the other: same value taken in practice
- **Sensing array:** the number of sensors in the array establishes the limits of sampling in both directions



(a) Continuous image (b) A scan line from A to B (c) Sampling (d) Quantization

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Image Sampling and Quantization

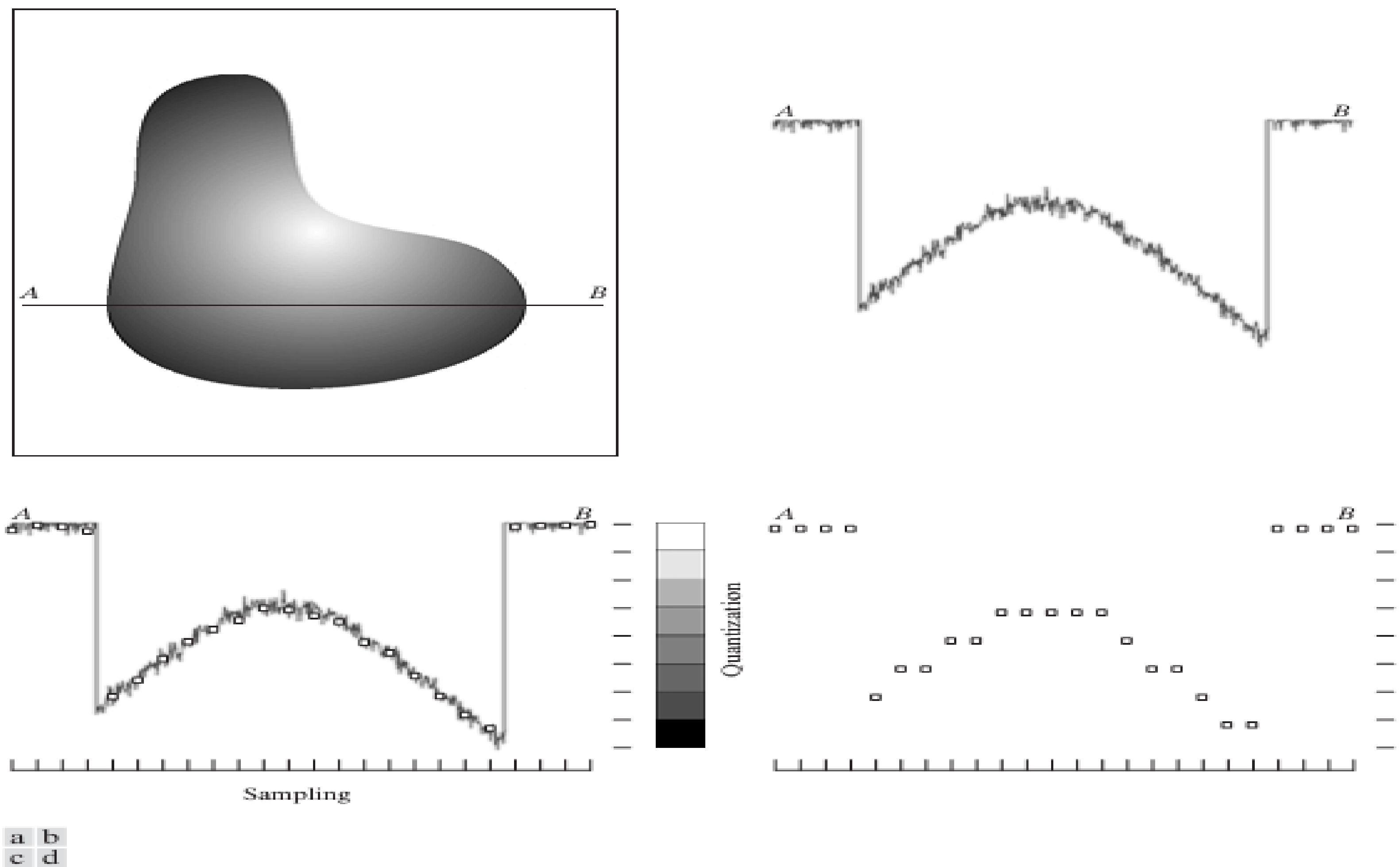


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digital Image

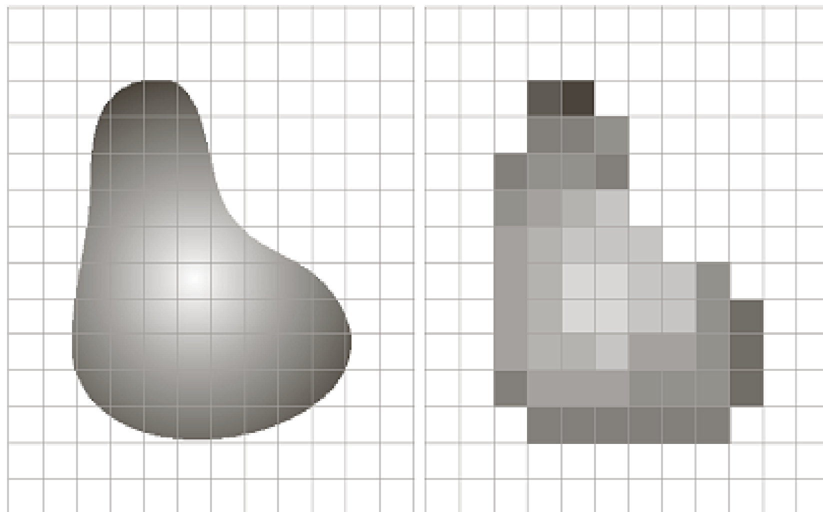

a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

The *quality of a digital image* is determined to a large degree by the number of samples and discrete intensity levels used in sampling and quantization.

However image content is also an important consideration in choosing these parameters

Digital Image Representation

2.4.2 Representing Digital Images

Continuous image: function of 2 continuous variables $f(s,t)$

→ *digital image* by sampling and quantization

→ 2D array $f(x,y)$, M rows and N columns, (x,y) = discrete coordinates

$x = 0, 1, 2, \ldots, M\text{-}1$ and $y = 0, 1, 2 \ldots, N\text{-}1$

Section of the real plane spanned by the coordinates of an image = *spatial domain*

x and y are called *spatial variables* or *spatial coordinates*

Digital Image Representation

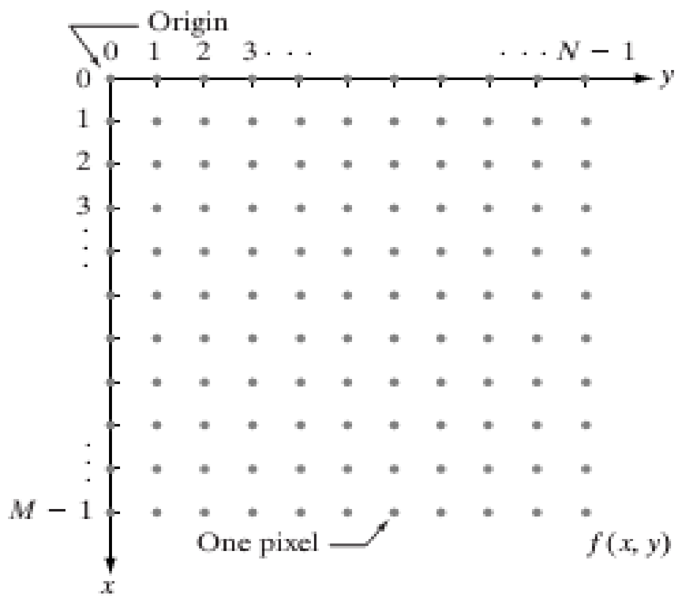


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

The notation introduced in the preceding paragraph allows us to write the complete $M \times N$ digital image in the following compact matrix form:

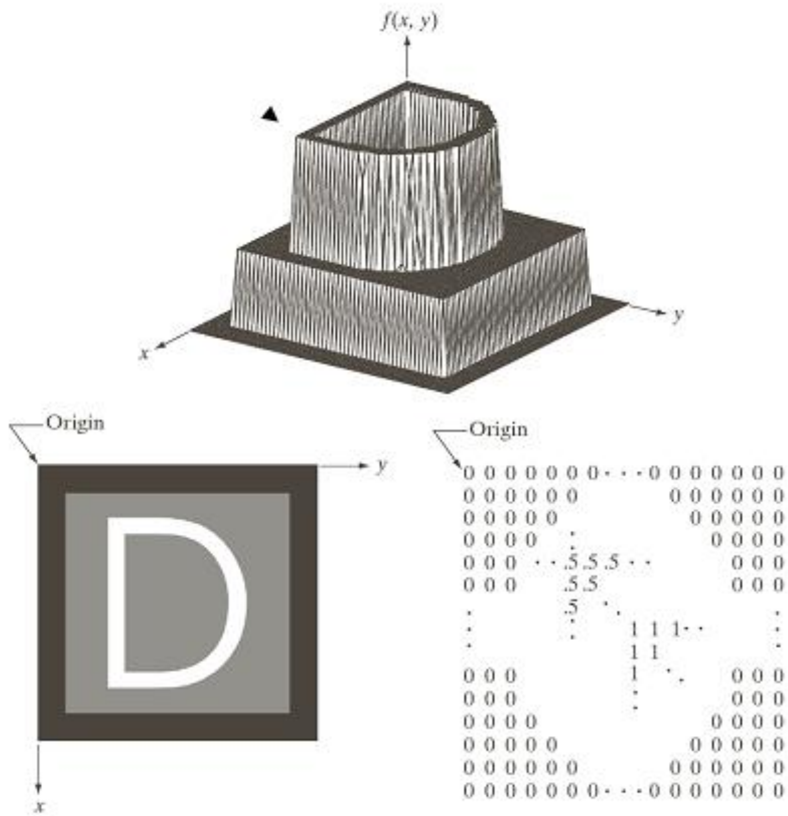
$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1, N-1) \end{bmatrix}. \quad (2.4\text{-}1)$$

Digital Image Representation

2.4.2 Representing Digital Images

Representation useful for gray-scale images

NB: Origin and axes
→ TV + matrix



a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

F of size 600x600 here
= 360,000 numbers…
Useful for algorithms

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Representing Digital Images

- Digital image
  - $M \times N$ array
  - L discrete intensities – power of 2
    - $L = 2^k$
    - Integers in the interval [$0, L - 1$]
    - Dynamic range: ratio of maximum / minimum intensity
      - Low: image has a dull, washed-out gray look
    - Contrast: difference between highest and lowest intensity
      - High: image have high contrast

Saturation and Noise

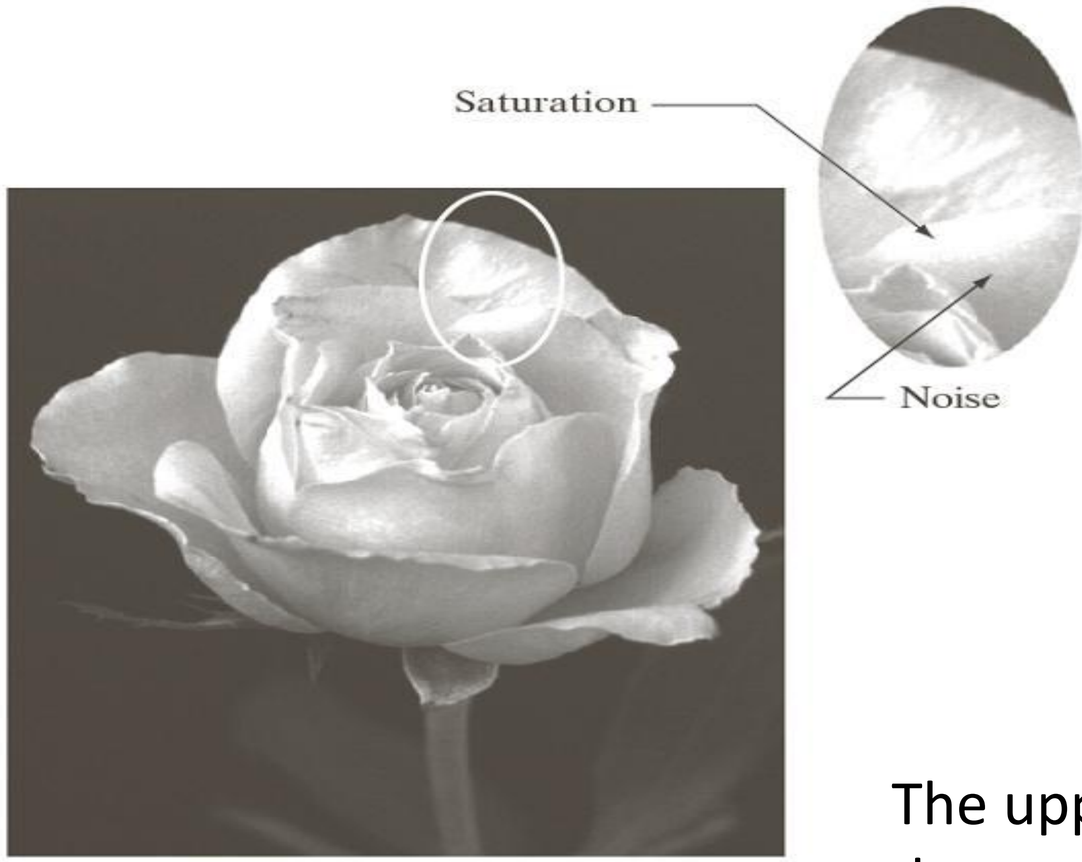


FIGURE 2.19 An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.

The upper limit intensity is determined by **saturation** and the lower limit by **noise**

Storage bits for Various Values of N and K

Digital image

bits to store : $b = M \times N \times k$

When $M = N$: $b = N^2k$

k-bit image: e.g. an image with 256 possible discrete intensity values is called an 8-bit image

TABLE 2.1
Number of storage bits for various values of N and k.

(Square Image)

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial and Intensity Resolution

- Resolution: dots (pixels) per unit distance
- ***dpi***: dots per inch

2.4.3 Spatial and Intensity Resolution

Intuitively, *spatial resolution* = measure of the smallest discernible detail in an image

Quantitatively (most common measures): line pairs per unit distance or dots (pixels) per unit distance (printing and publishing industry). In the US: dots per inch (dpi)
e.g. newspapers: 75 dpi, magazines: 133 dpi, glossy brochures: 175 dpi, DIP book: 2400 dpi

Key point: to be meaningful, measures of spatial resolution must be stated *w.r.t. spatial units*

Intensity resolution = smallest discernible change in intensity level
Most common: 8bit. 16bit when needed. 32 bits rare. Exceptions: 10 or 12 bits

Spatial Resolution Example

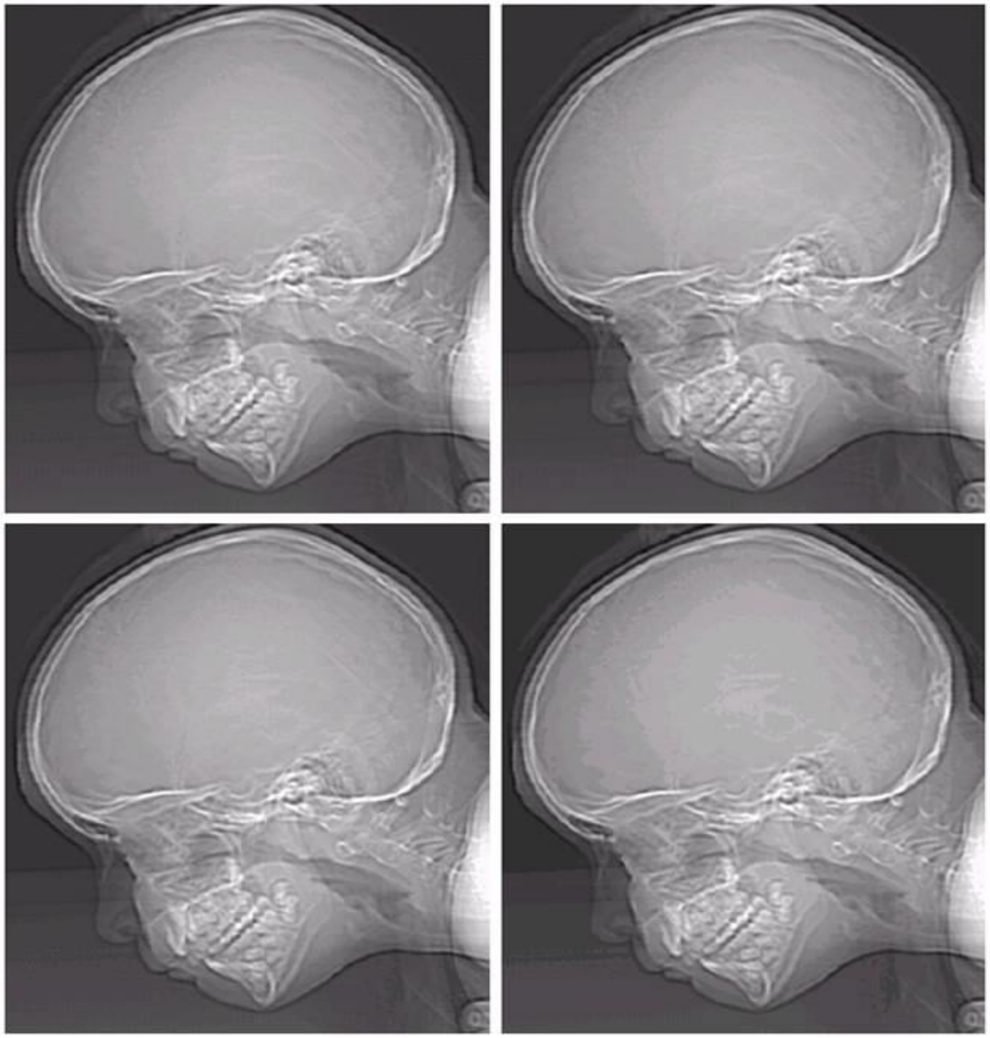
a	b
c	d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Variation of Number of Intensity Levels

- Reducing the number of bits from k=7 to k=1 while keeping the image size constant
- Insufficient number of intensity levels in smooth areas of digital image leads to false contouring

Effects of Varying the Number of Intensity Levels



a b
c d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Effects of Varying the Number of Intensity Levels

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

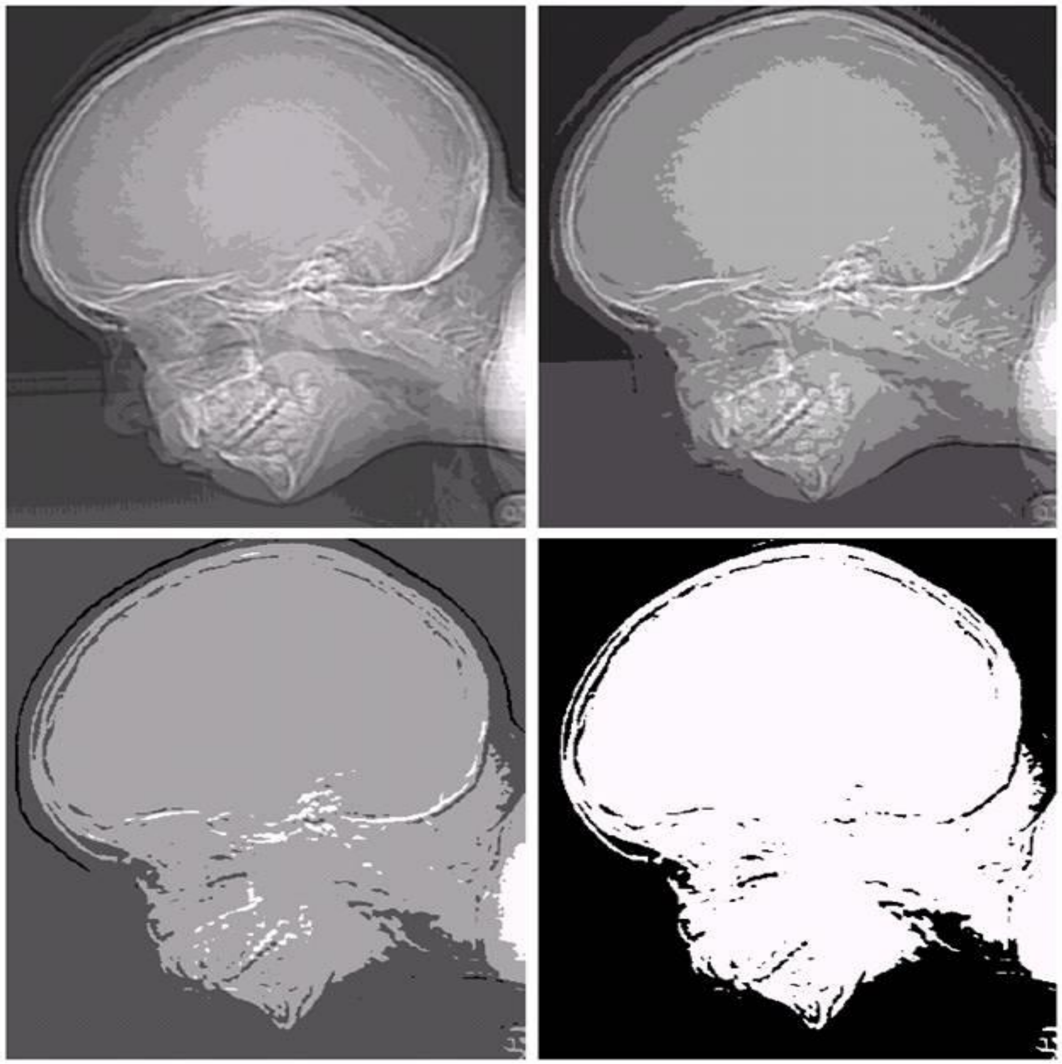


Image Interpolation

- Using known data to estimate values at unknown locations
- Used for zooming, shrinking, rotating, and geometric corrections
- Nearest Neighbor interpolation
  - Use closest pixel to estimate the intensity
  - simple but has tendency to produce artifacts
- Bilinear interpolation
  - use 4 nearest neighbor to estimate the intensity
  - Much better result
  - Equation used is

$$v(x, y) = ax + by + cxy + d$$

- Bicubic interpolation
  - Use 16 nearest neighbors of a point
  - Equation used is

$$v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

Zooming using Various Interpolation Techniques

a b c
d e f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).

Some Basic Relationships between Pixels

- Neighbors of a pixel
  - There are three kinds of neighbors of a pixel:
    - $N_4(p)$ 4-neighbors: the set of horizontal and vertical neighbors
    - $N_D(p)$ diagonal neighbors: the set of 4 diagonal neighbors
    - $N_8(p)$ 8-neighbors: union of 4-neighbors and diagonal neighbors

	O	
O	X	O
	O	

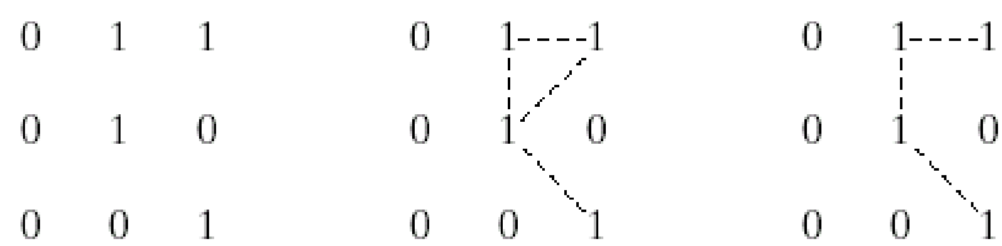
O		O
	X	
O		O

O	O	O
O	X	O
O	O	O

Some Basic Relationships between Pixels

- Adjacency:
  - Two pixels that are neighbors and have the same grey-level (or some other specified similarity criterion) are adjacent
  - Pixels can be 4-adjacent, diagonally adjacent, 8-adjacent, or m-adjacent.
- *m*-adjacency (mixed adjacency):
  - Two pixels *p* and *q* of the same value (or specified similarity) are *m*-adjacent if either
    - (i) *q* and *p* are 4-adjacent, or
    - (ii) *p* and *q* are diagonally adjacent and do not have any common 4-adjacent neighbors.
    - They cannot be both (i) and (ii).

- An example of adjacency:



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Some Basic Relationships Between Pixels

- Path:
  - The length of the path
  - Closed path
- Connectivity in a subset *S* of an image
  - Two pixels are connected if there is a path between them that lies completely within *S*.
- Connected component of *S*:
  - The set of all pixels in *S* that are connected to a given pixel in *S*.
- Region of an image
- Boundary, border or contour of a region
- Edge: a path of one or more pixels that separate two regions of significantly different gray levels.

Distance Measures

- Distance measures
  - Distance function: a function of two points, *p* and *q*, in space that satisfies three criteria

$$(a)\ D(p,q) \geq 0$$

$$(b)\ D(p,q) = D(q,p), \text{ and}$$

$$(c)\ D(p,z) \leq D(p,q) + D(q,z)$$

  - The Euclidean distance $D_e(p, q)$

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

  - The city-block (Manhattan) distance $D_4(p, q)$

$$D_4(p,q) = |x-s| + |y-t|$$

  - The chessboard distance $D_8(p, q)$

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

Distance Measures- Example

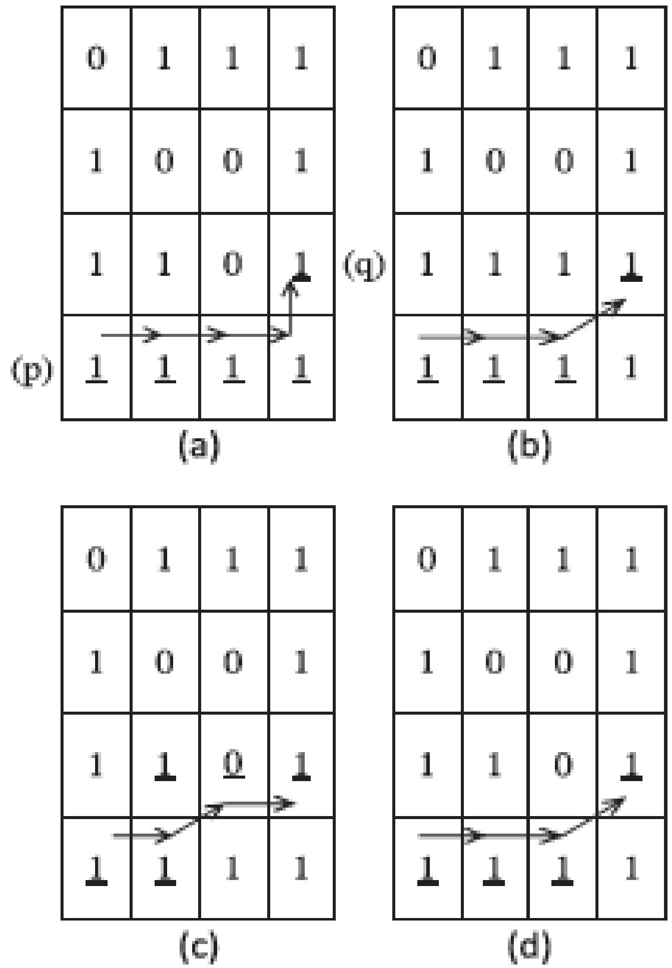


Fig. 3.11 Distance measures (a) Distance D_1 (b) Distance D_0 when $V = \{0, 1\}$ (c) Distance D_m when $V = \{0, 1\}$ (d) Distance D_m when $V = \{1\}$

Arithmetic Operations

- Array operations between images
- Carried out between corresponding pixel pairs
- Four arithmetic

$$s(x, y) = f(x, y) + g(x, y)$$
$$d(x, y) = f(x, y) - g(x, y)$$
$$p(x, y) = f(x, y) \times g(x, y)$$
$$v(x, y) = f(x, y) \div g(x, y)$$

- e.g. Averaging K different noisy images can decrease noise
  – Used in the field of astronomy

Averaging of Images

Averaging K different noisy images can decrease noise. Used in astronomy

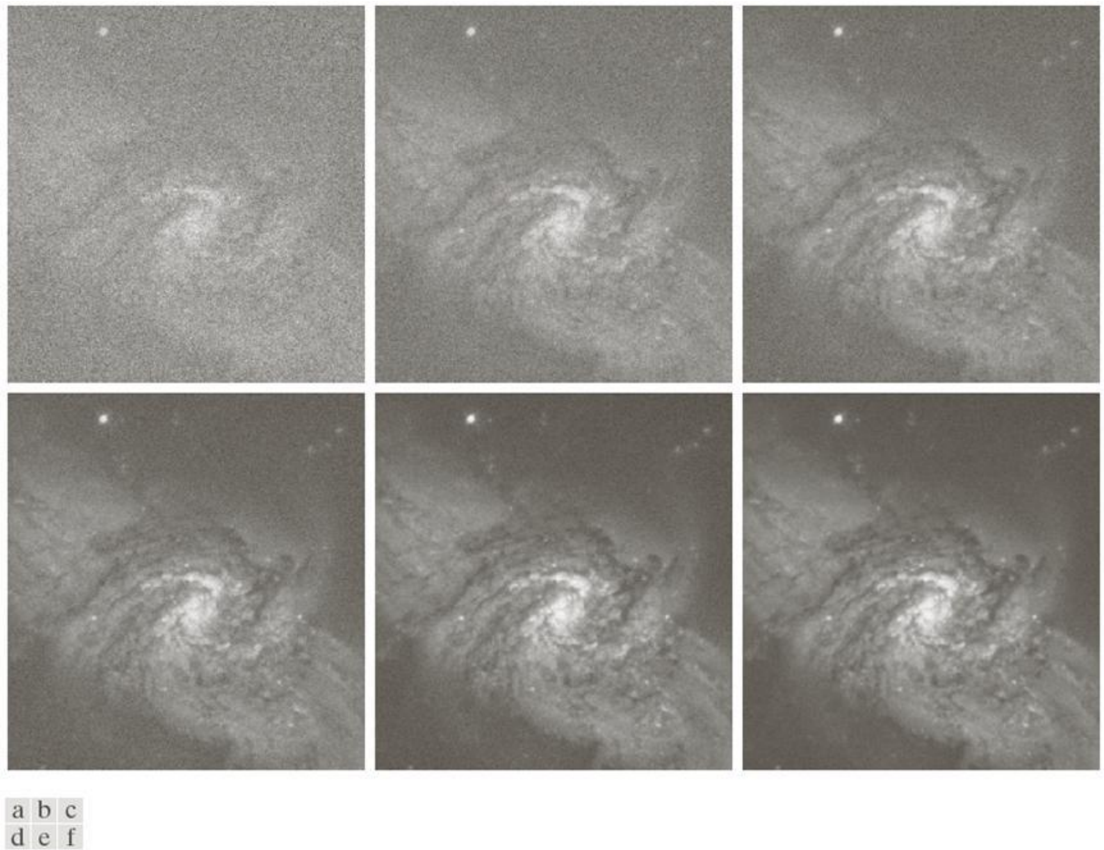
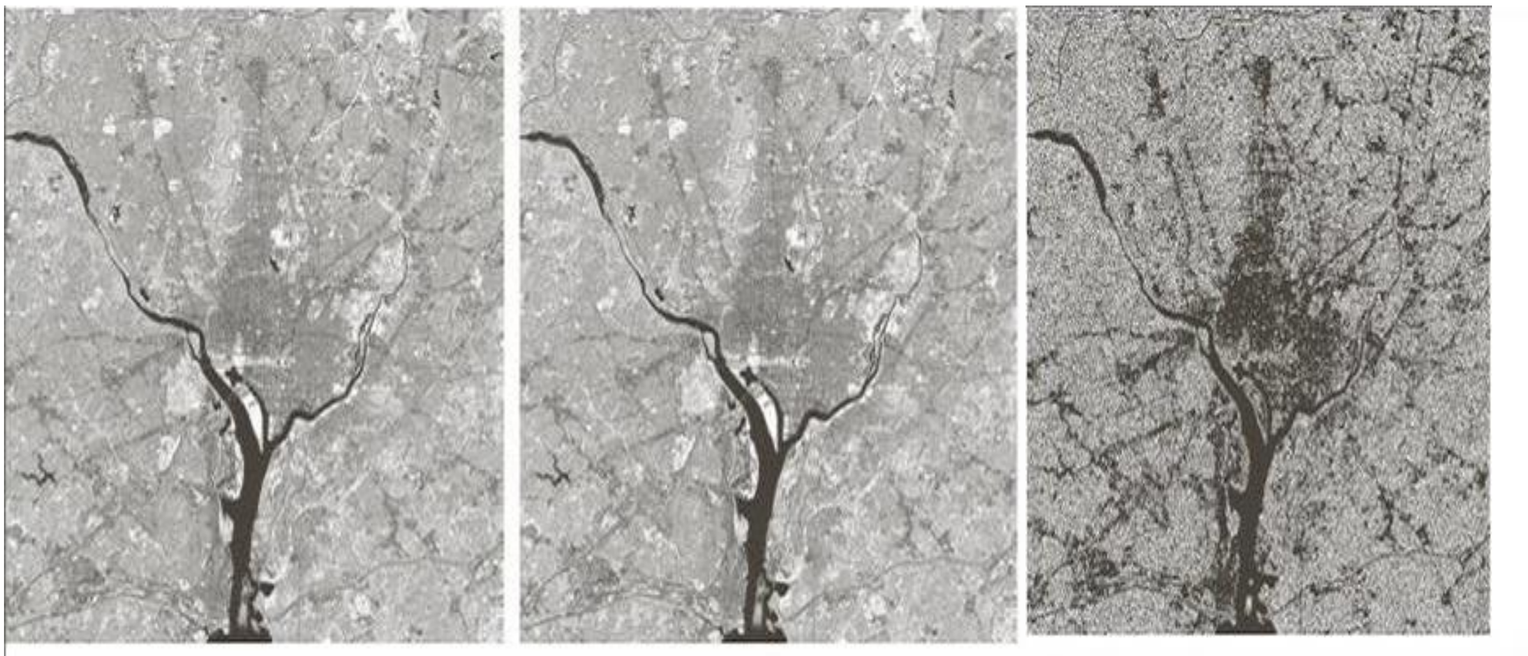


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Image Subtraction

Enhancement of difference between images using image subtraction

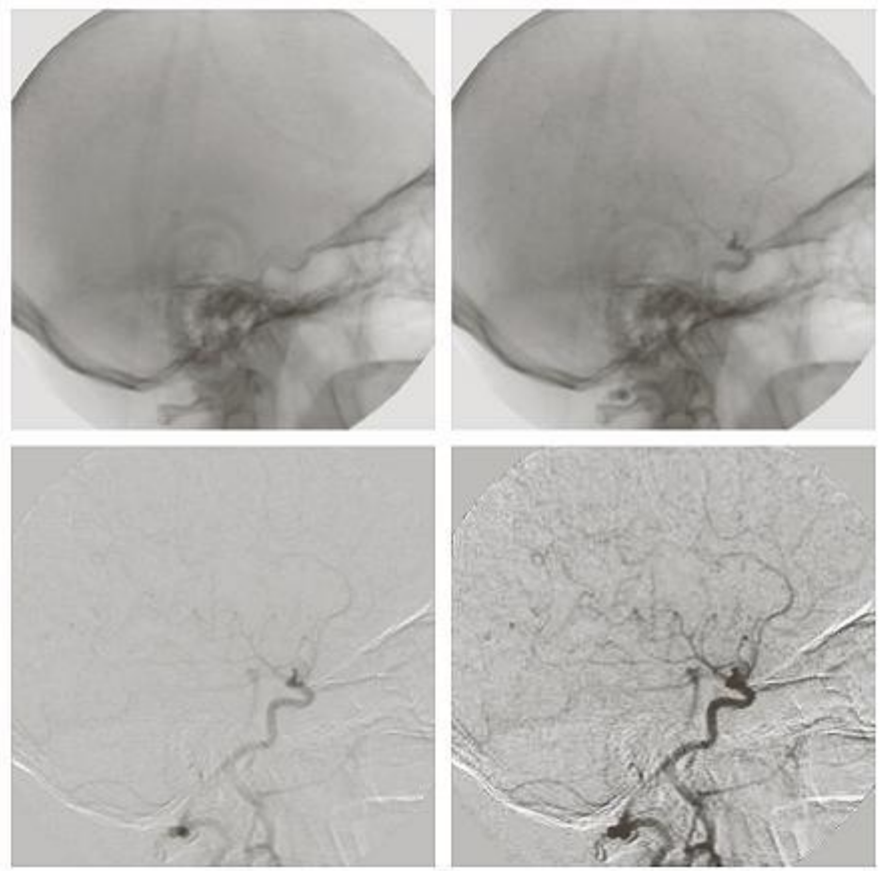


a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

Image Subtraction Application

Mask mode radiography



a	b
c	d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

Shading correction by image multiplication (and division)

$g(x, y) \quad = \quad h(x, y) \quad \times \quad f(x, y)$

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Masking (RIO) using image multiplication

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Arithmetic Operations

- To guarantee that the full range of an arithmetic operation between images is captured into a fixed number of bits, the following approach is performed on image f

  $$f_m = f - \min(f)$$

  which creates an image whose minimum value is 0. Then the scaled image is

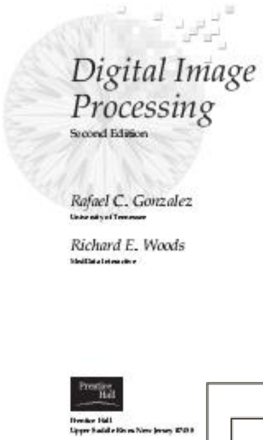
  $$f_s = K\,[\,f_m / max(f_m)]$$

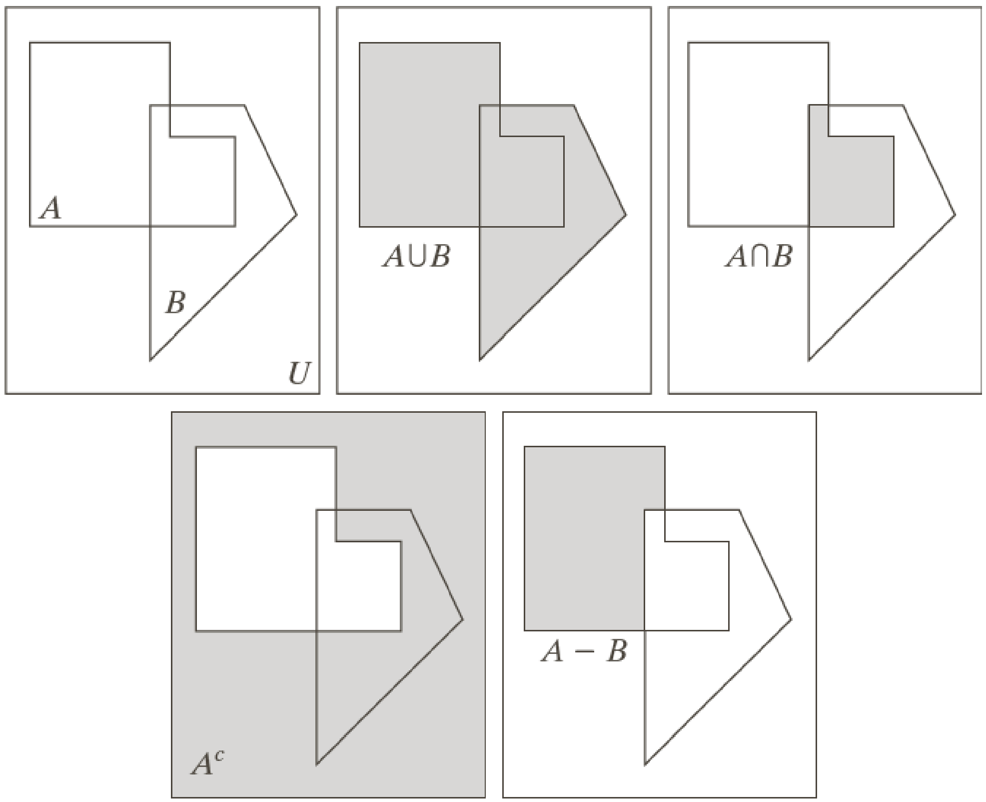
  whose value is in the range [0, *K*]

Example- for 8-bit image , setting K=255 gives scaled image whose intensities span from 0 to 255

Set and Logical Operations

- Elements of a sets are the coordinates of pixels (ordered pairs of integers) representing regions (objects) in an image
  - Union
  - Intersection
  - Complement
  - Difference
- Logical operations
  - OR
  - AND
  - NOT
  - XOR



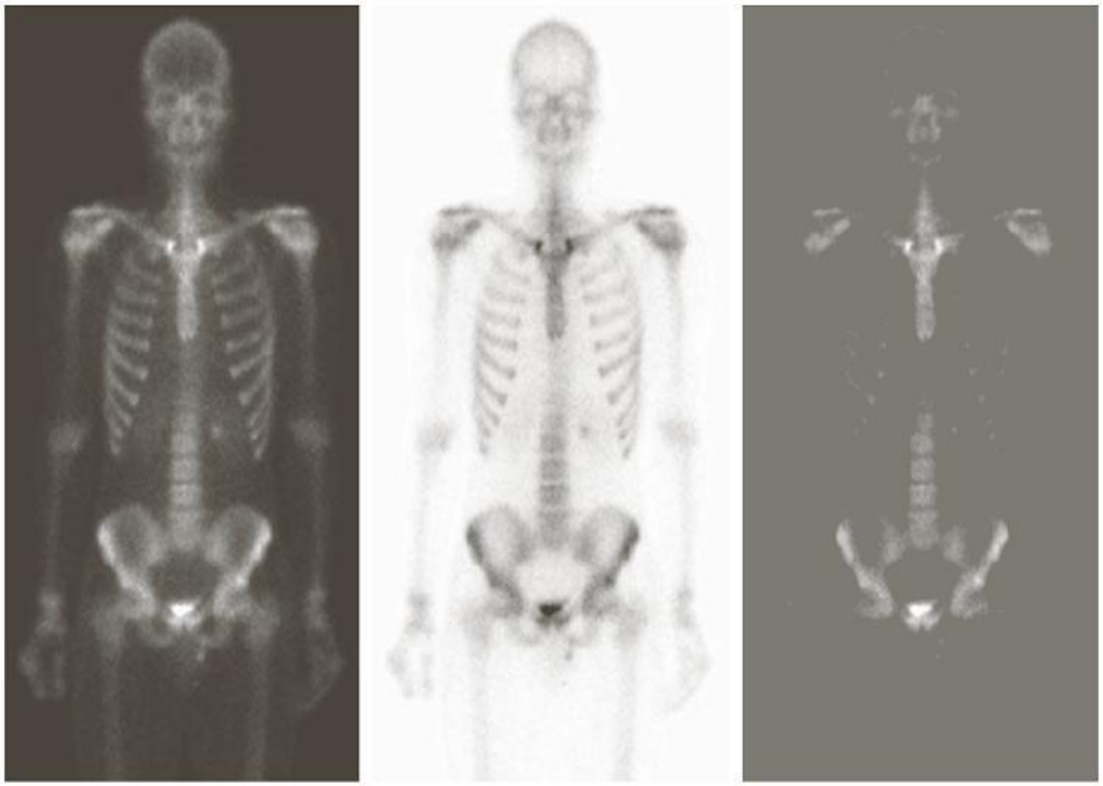


a b c
d e

FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

$A - B = A \cap B^c$

Set operations involving gray-scale images



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

$$A \cup B = \{\max(a,b)\; a \in A, b \in B$$

The union of two gray-scale sets is an array formed from the maximum intensity between pairs of spatially corresponding elements

Logical Operations

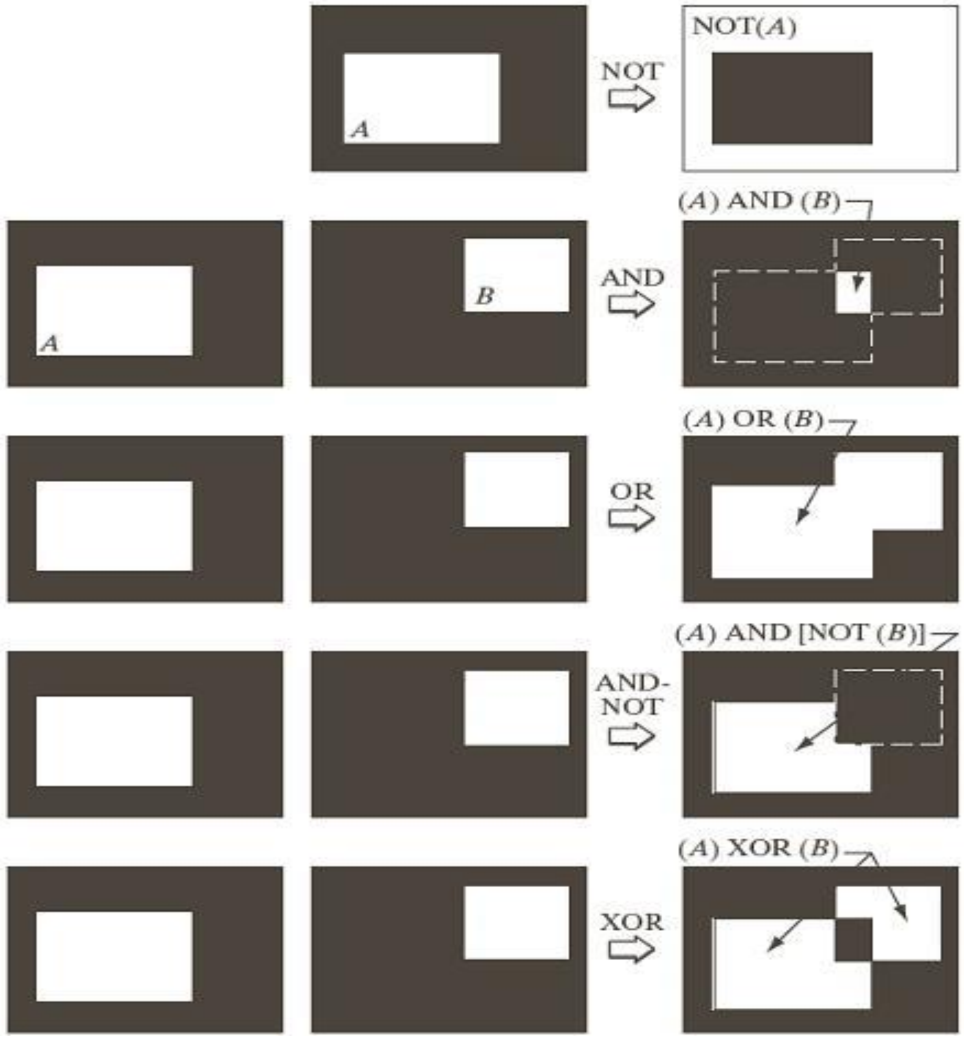


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Spatial Operations

- Single-pixel operations
  - For example, transformation to obtain the negative of an 8-bit image

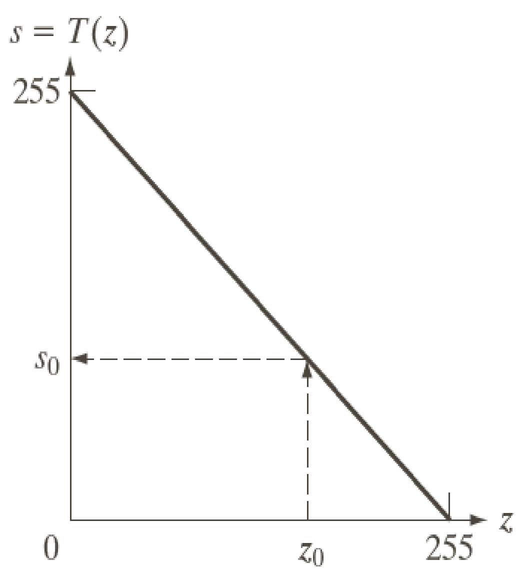
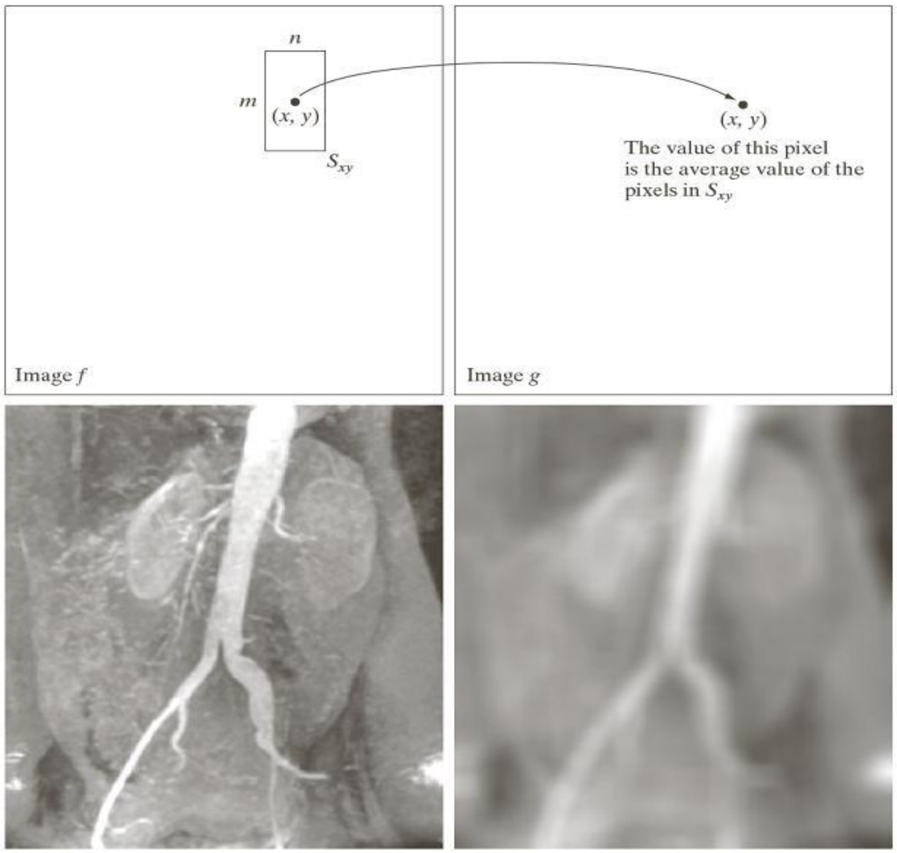


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Spatial Operations

Neighborhood operations

For example, compute the average value of the pixels in a rectangular neighborhood of size $m \times n$ centered on (x, y)



a	b
c	d

FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

Spatial Operations

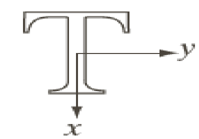
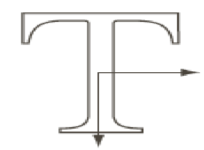
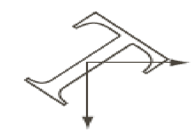
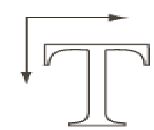
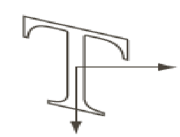
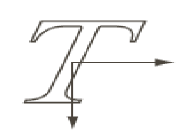
- Geometric spatial transformations
  - Called rubber-sheet transformations
  - Consists of two operations
    - Spatial transformation of coordinates

      e.g. $(x, y) = T\{(v, w)\} = (v/2, w/2)$
      - Affine transform: scale, rotate, transform, or sheer a set of points
    - Intensity interpolation
- Affine transform

  [x y 1]= [v w 1] [Affine Matrix, T]

AFFINE TRANSFORMATION

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

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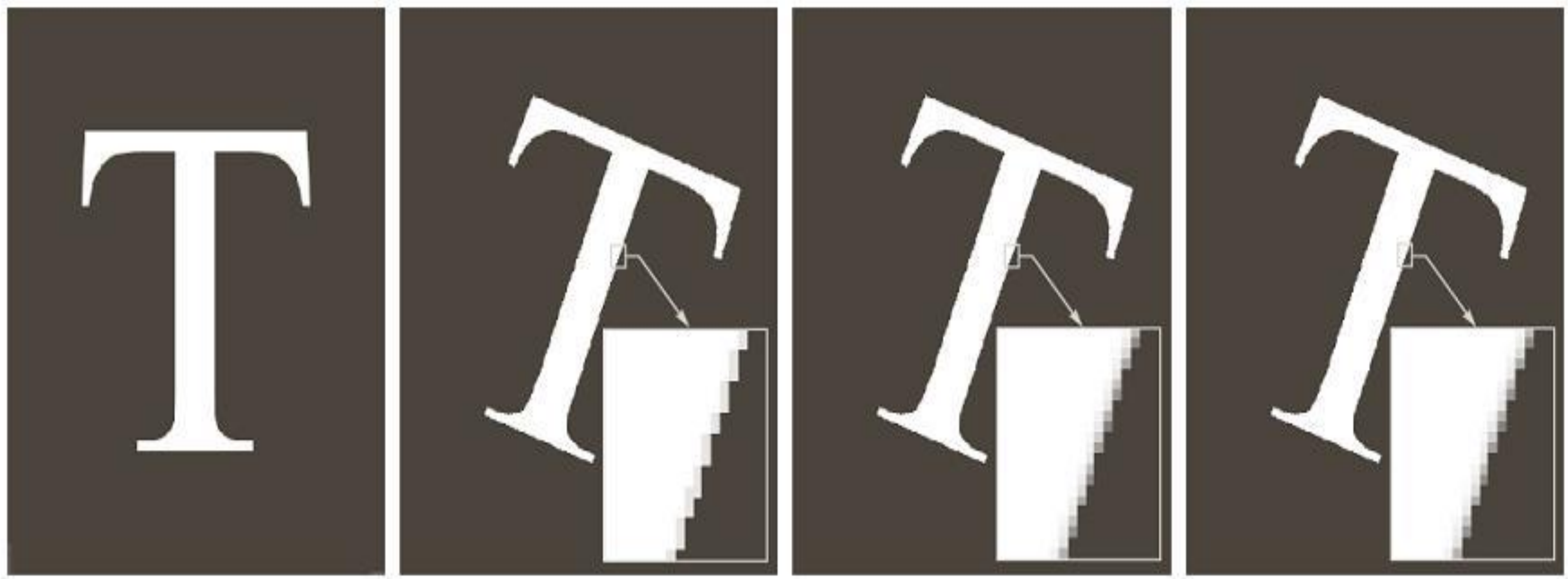
Interpolation



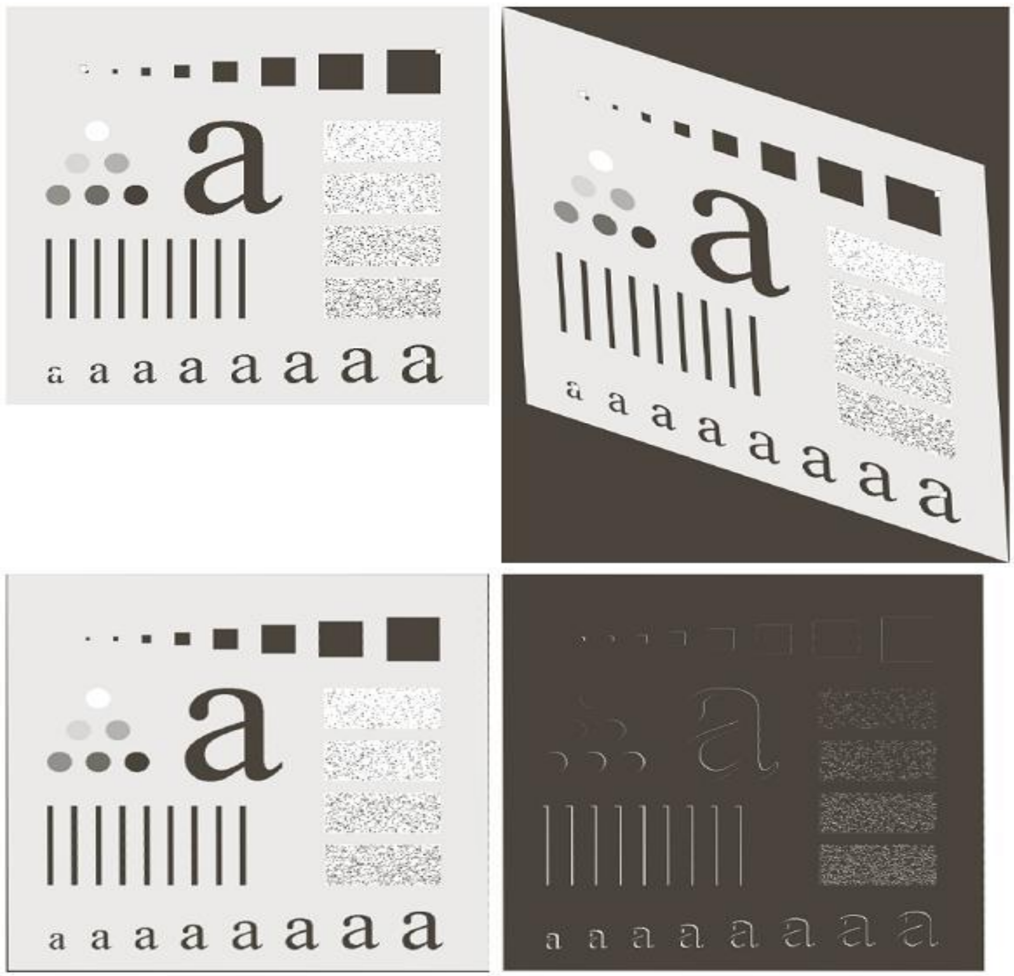
a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Image Registration

- Used for aligning two or more images of the same scene
- Input and output images available but the specific transformation that produced output is unknown
- Estimate the transformation function and use it to register the two images.
- **Input image-** image that we wish to transform
- **Reference image-** image against which we want to register the input
- **Principal approach-** use tie points (also called control points) ,which are corresponding points whose locations are known precisely in the input and reference images

Image Registration



a b
c d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

Vector and Matrix Operations

- RGB images
- Multispectral images

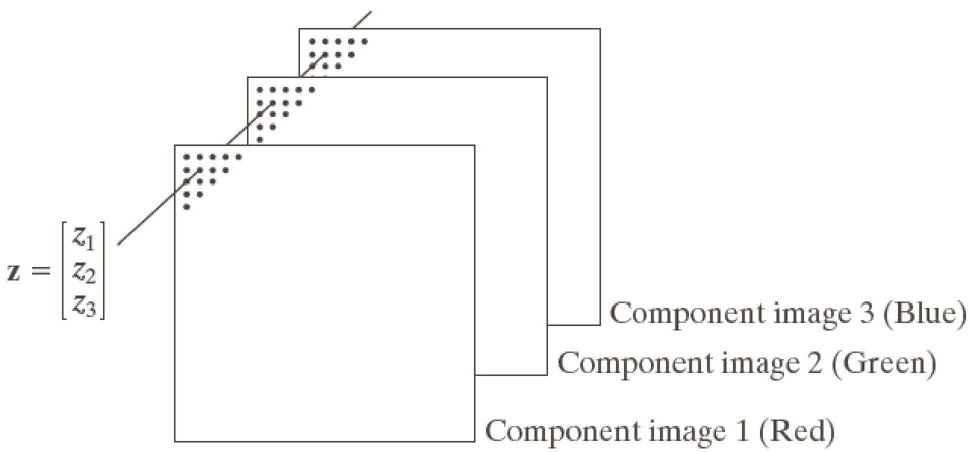


FIGURE 2.38 Formation of a vector from corresponding pixel values in three RGB component images.

Image Transforms

- Image processing tasks are best formulated by
  - Transforming the images
  - Carrying the specified task un a transform domain
  - Applying the inverse transform

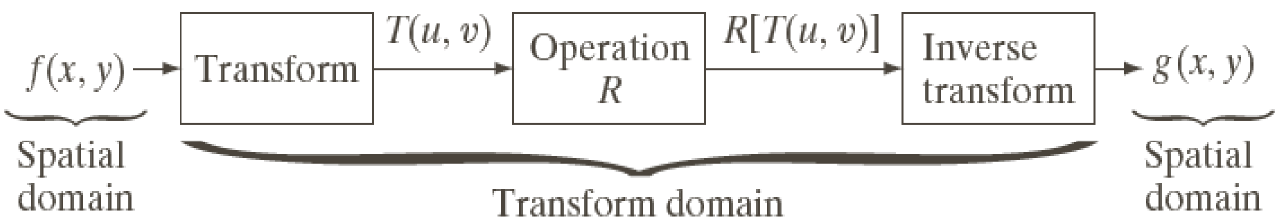
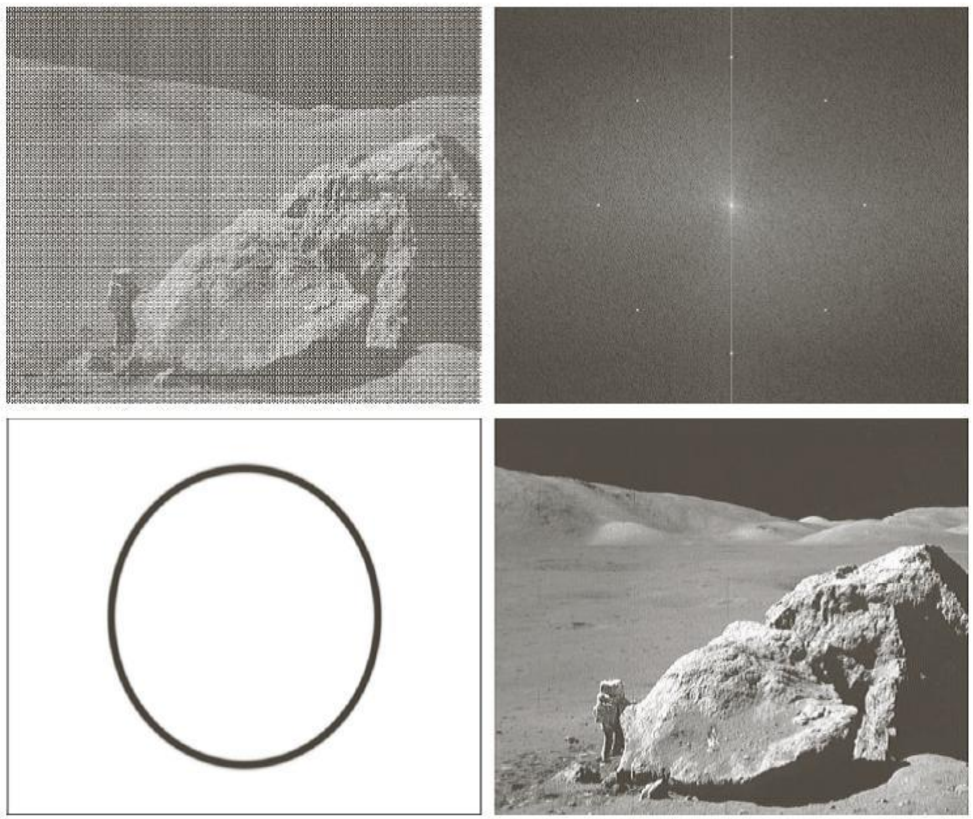


FIGURE 2.39 General approach for operating in the linear transform domain.

Image Transforms



a b
c d

FIGURE 2.40
(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

Intensity Transformations and Spatial Filtering

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Background

- Image domains

Spatial domain techniques -operate directly on the pixels of an image

Transform domain- operate on transform coefficients

Two main categories of spatial processing

Intensity transformation

- operation on single pixel of image

Ex- Contrast manipulation, image thresholding etc.

Spatial Filtering

- operation on a group of neighboring pixels
- Ex- image sharpening

Chapter 3

Intensity Transformations and Spatial Filtering

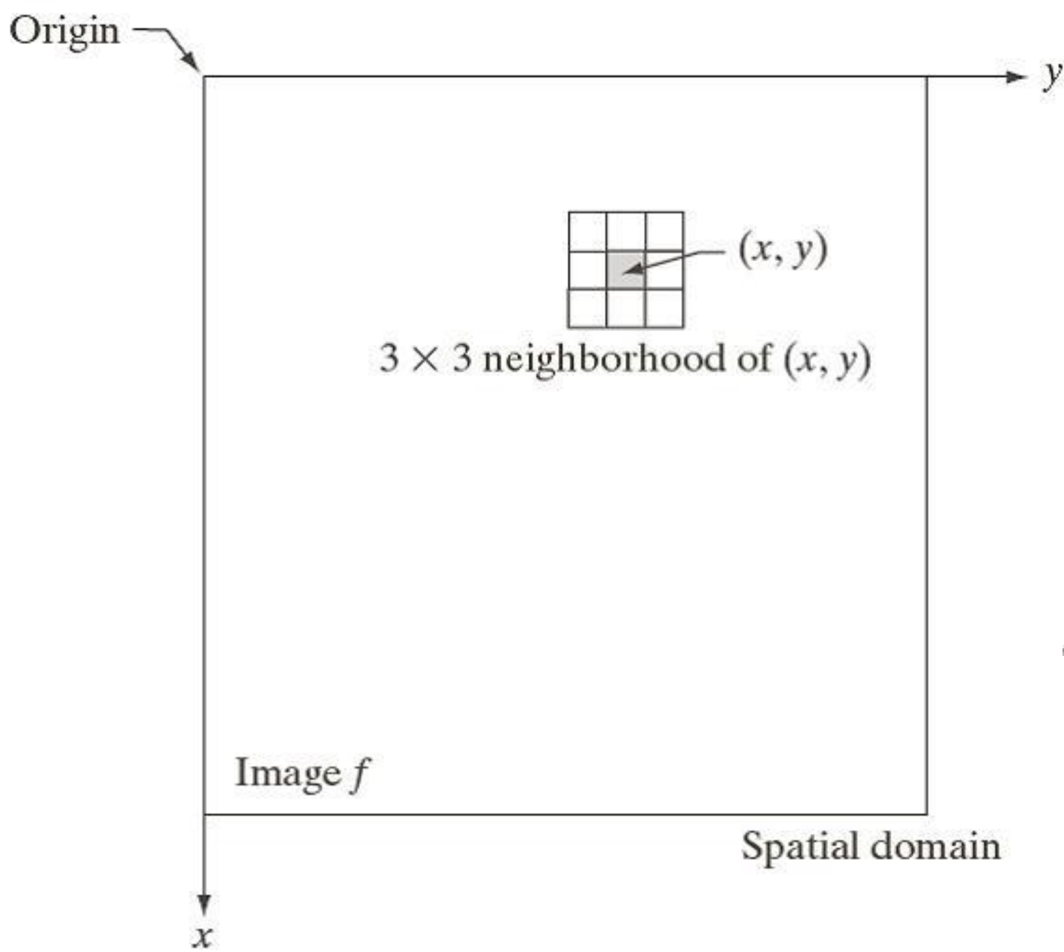


FIGURE 3.1 A 3 × 3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

$$g(x, y) = T\left[f(x, y)\right]$$

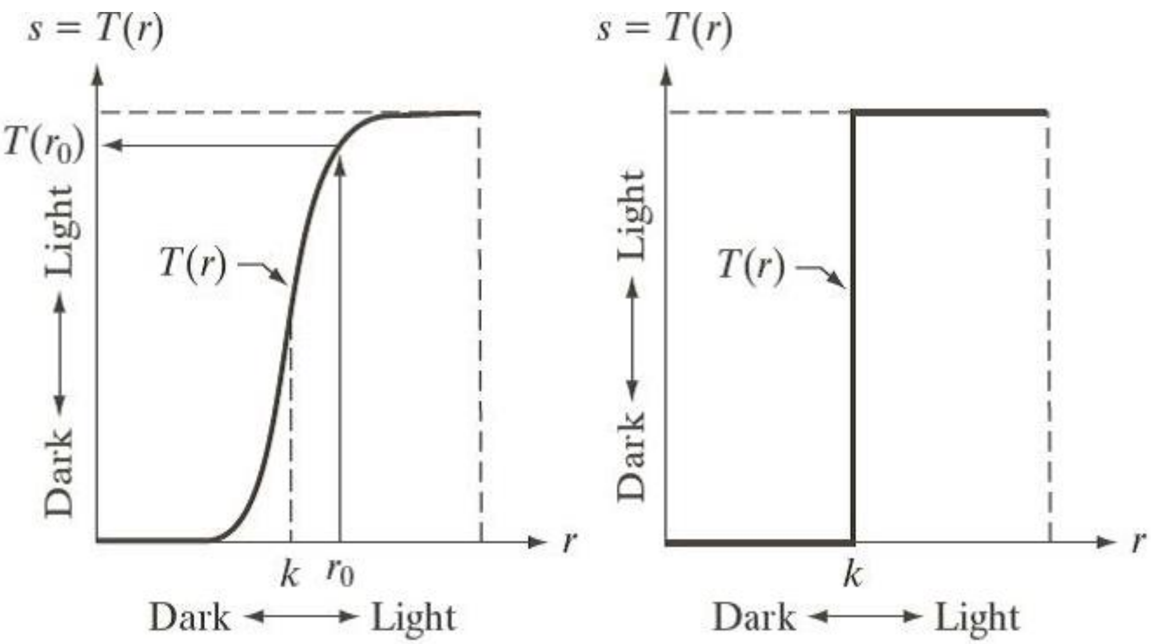
Basic Intensity Transformation Functions

- Values of pixels
  - before processing: ***r***
  - after processing: ***s***
- These value are related by an expression of the form

  $$s = T(r)$$

- Typically, process starts at the top left of the input image and proceeds pixel by pixel in horizontal scan, one row at a time
- Either ignore outside neighbors or pad the image with a border of 0s or some other specified intensity values

Intensity Transformations Functions



a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

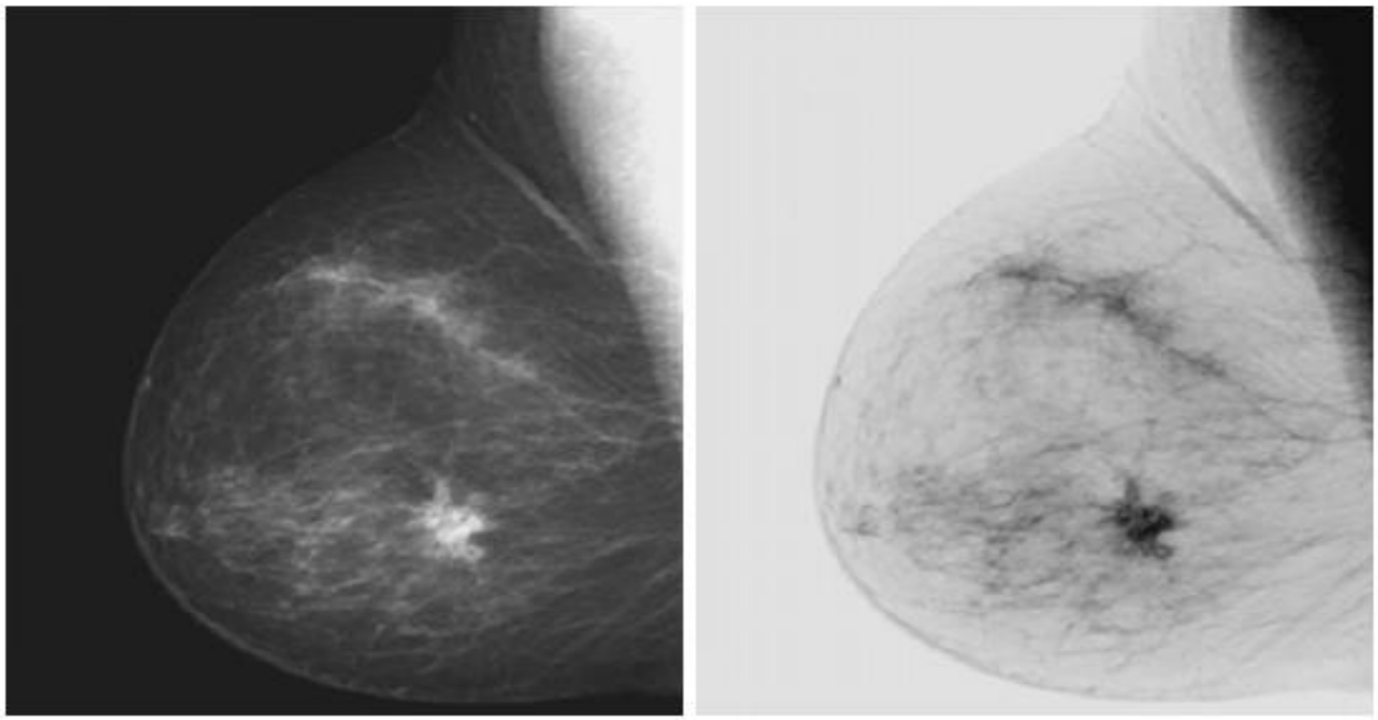
Higher contrast than original by darkening the intensity levels below k and brightening the levels above k

T(r) produces two-level , binary image

Image Negatives

- Negative of an image with intensity levels in the range [0, *L*-1] is obtained using the expression

$$s = L - 1 - r$$



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Suited for enhancing white or gray detail embedded in dark regions of an image specially when the black areas are dominant in size

Log Transformations

- General form of the log transformation is

  $$s = c \log(1 + r) \quad \text{(Eq. 3.2.2)}$$

  where ***c***: constant, ***r*** ≥ 0
- Transformation maps a narrow range of low intensity values in the input into a wider range of output levels. Opposite is true of higher values of input levels
- Expands the values of dark pixels in an image while compressing the higher level values
- Compresses the dynamic range of images with large variations in pixel values
  - Application: Fourier transform

Intensity Transformation Functions

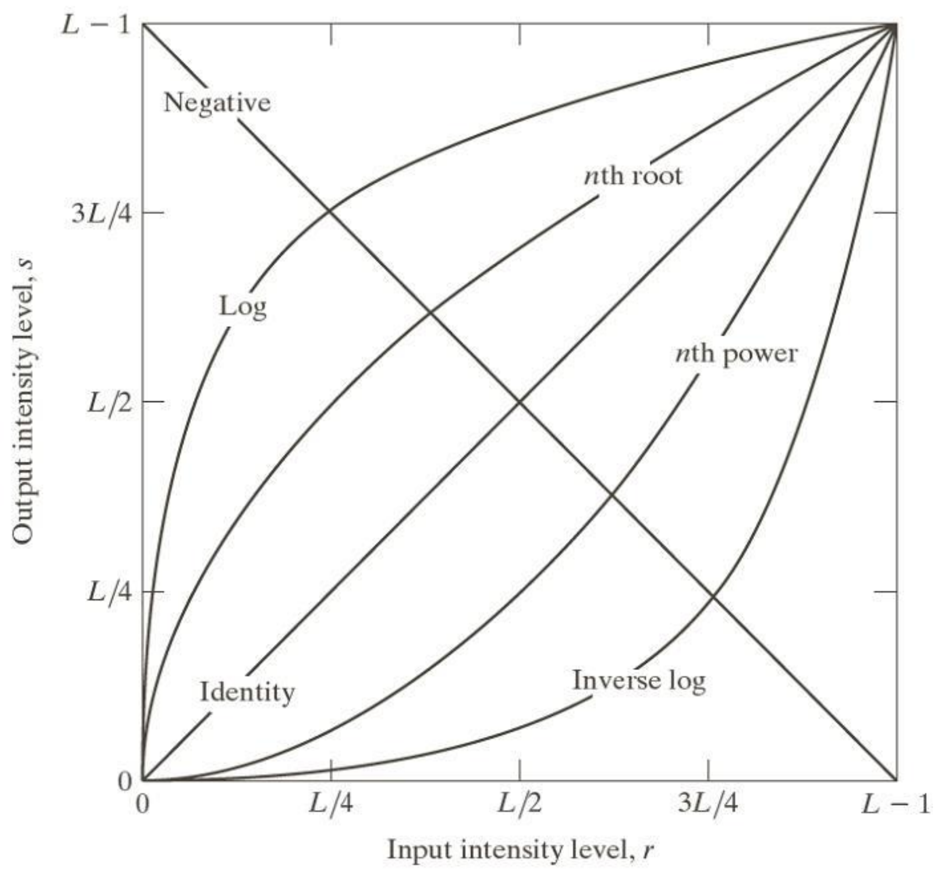
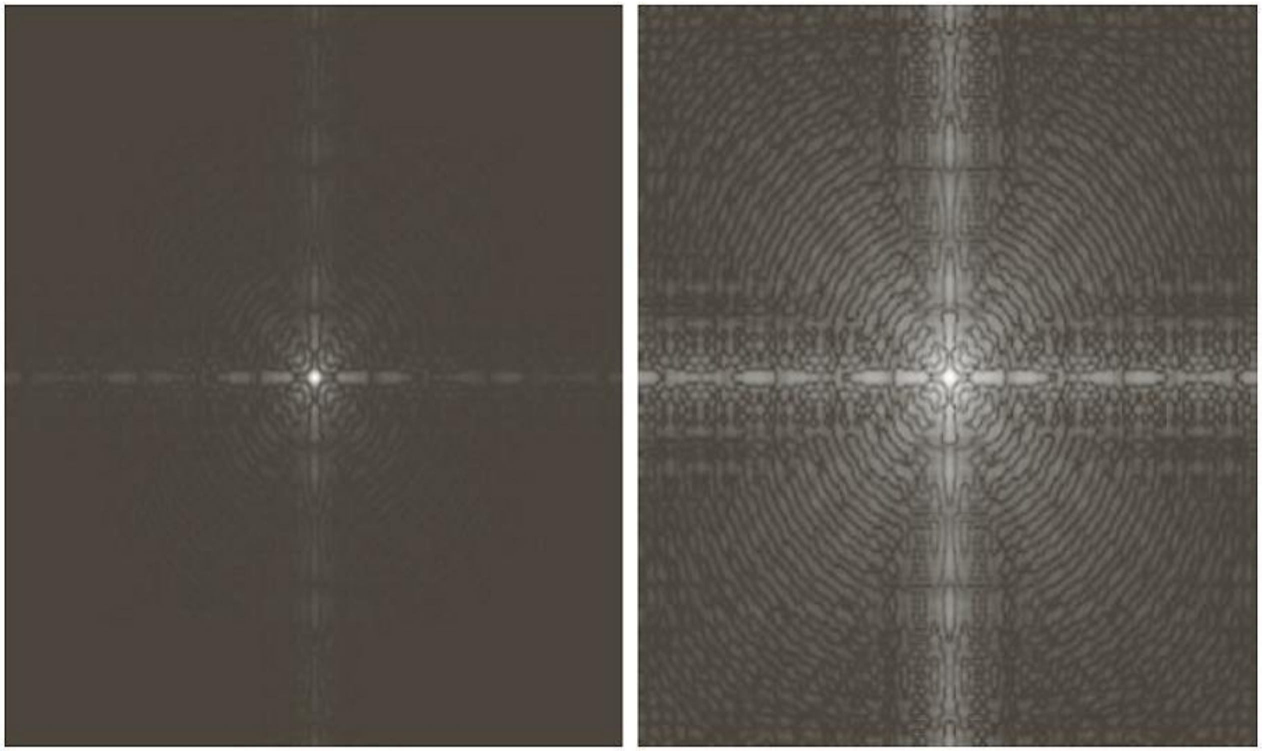


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Fourier Spectrum



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

3.5 (a) values in the range 0 to 1.5 x 10^{6}. When displayed by 8-bit system, brightest pixels will dominate at the expanse of lower values

3.5(b) Range is 0 to 6.2

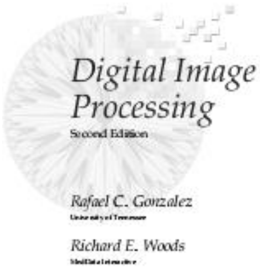
Power-Law (Gamma) Transformations

- Basic form is $s = c\, r^{\gamma}$ *(Eq. 3.2-3)*

  where *c* and γ: positive constants
- Transformation maps a narrow range of dark input values → into a wider ranger of output values, with the opposite being true for higher value of input levels (**When γ is fractional value**)
- Opposite is true for γ greater than 1
- Family of possible transformation curves obtained simply by varying γ
- Identity transformation when: $c = \gamma = 1$
- Gamma correction: for displaying an image accurately on a computer screen

  Too dark , Washed out

Power-Law (Gamma) Transformation Plots



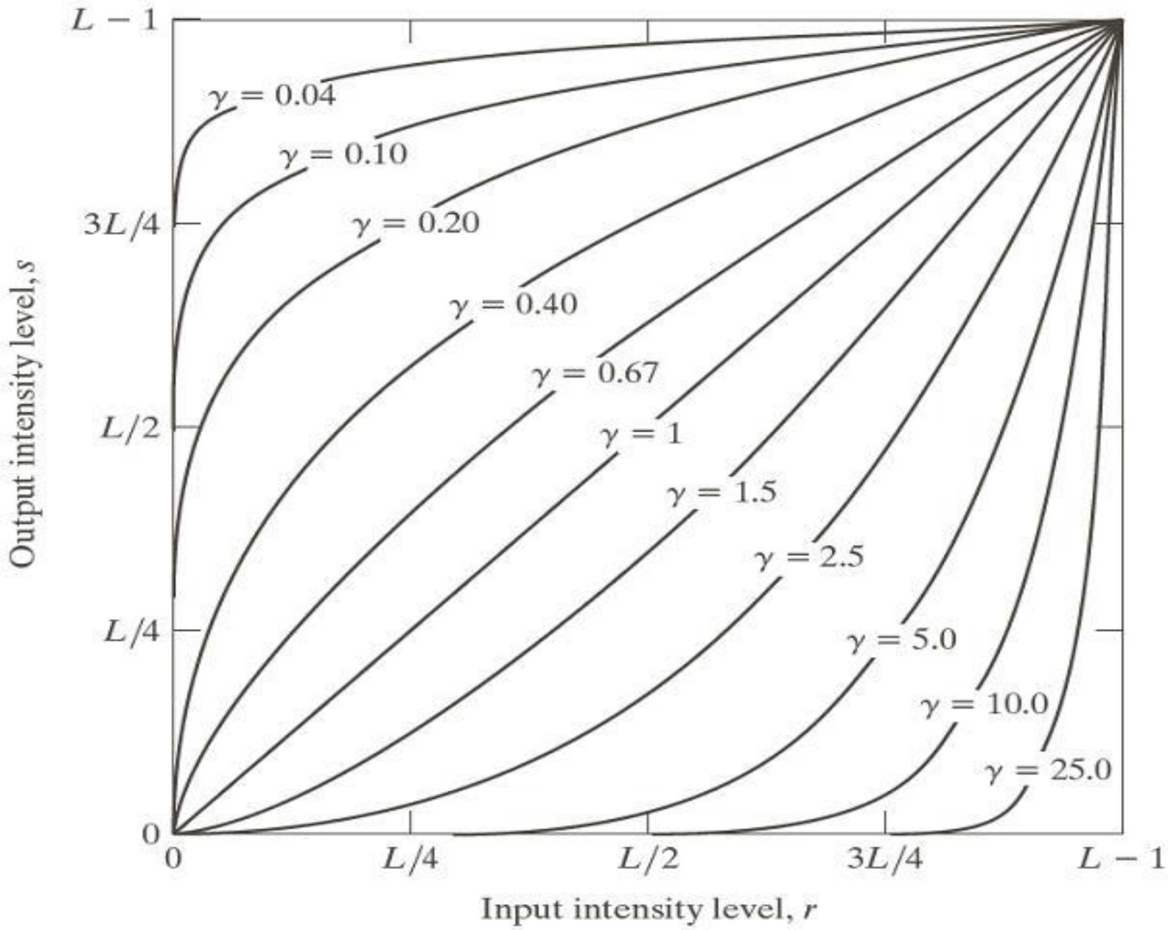
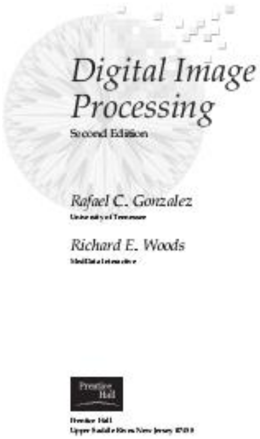


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

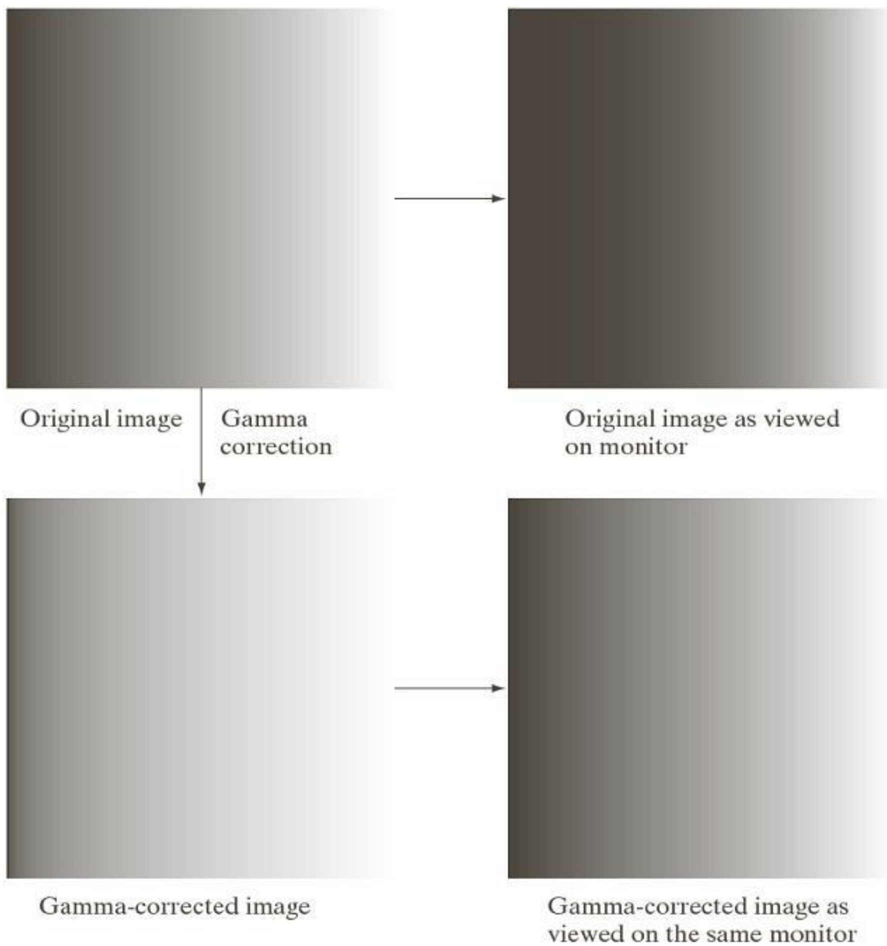
Power –law curve with fractional values of γ similar to log transformation

Gamma correction

- A variety of devices used for image capture, printing, and display respond according to a power law.
- The process used to correct these power-law response phenomena is called ***Gamma Correction***
- CRT device have an intensity-to-voltage response that is a power function with $\gamma = 1.8$ *to* 2.5.
- Different monitors / monitor setting require different gamma correction values
- Current image standards do not contain the value of gamma with which an image was created



Gamma correction

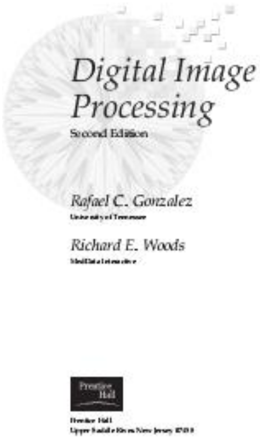


a b
c d

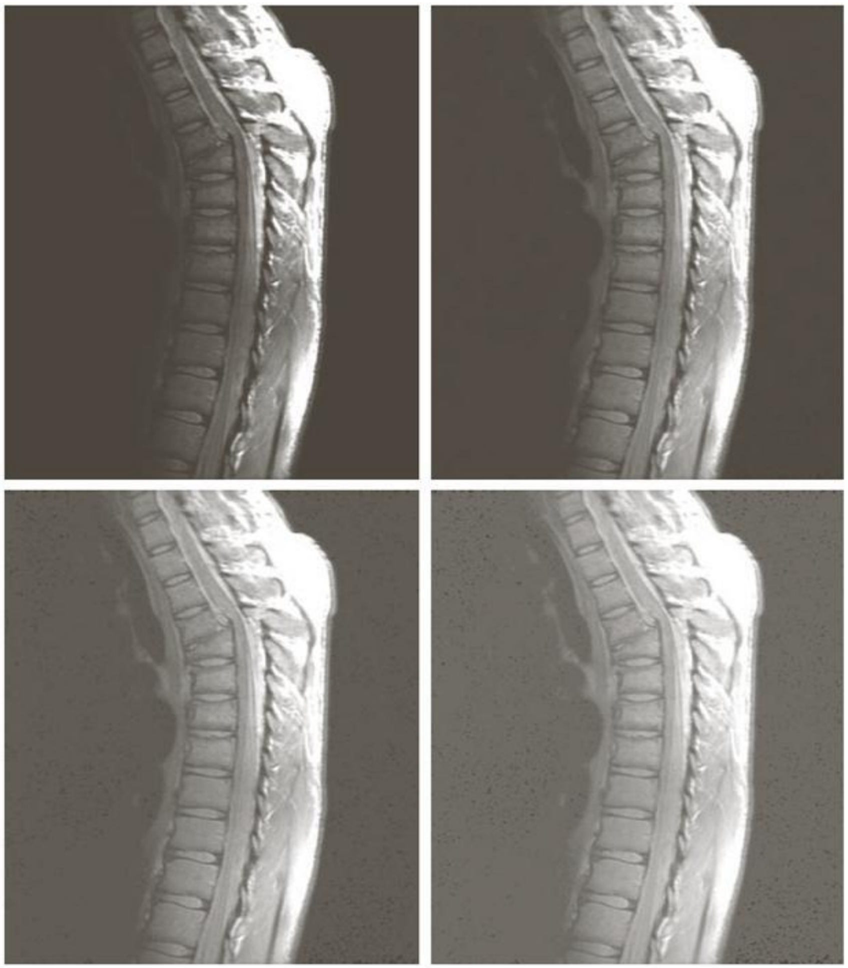
FIGURE 3.7 (a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

Gamma correction – by pre-processing the input image before inputting into monitor by performing transformation

$$s = r^{1/2.5} = r^{0.4}$$



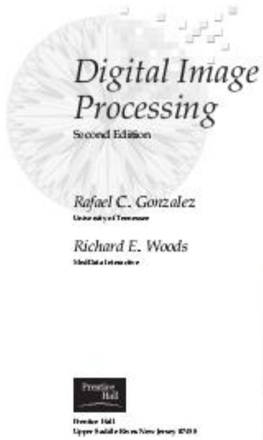
Contrast Enhancement using Power-law Transformation



a b
c d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

- ✓**For predominantly dark image, an expansion in intensity level is desirable**
- ✓**Fracture is more visible in subsequent image with decrease in fractional gamma value**



Contrast Enhancement using Power-law Transformation

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

- Aerial image to be processed has a washed-out appearance Suitable results in (b) and (c)
- In (d) areas are too dark, in which some detail is lost (dark region to the left of the main road in upper quadrant)

Piecewise-Linear Transformation Functions

- **Contrast Stretching:** expands the range of intensity levels in an image so that it spans the full intensity range
  - Points: (r_1, s_1) , (r_2, s_2)

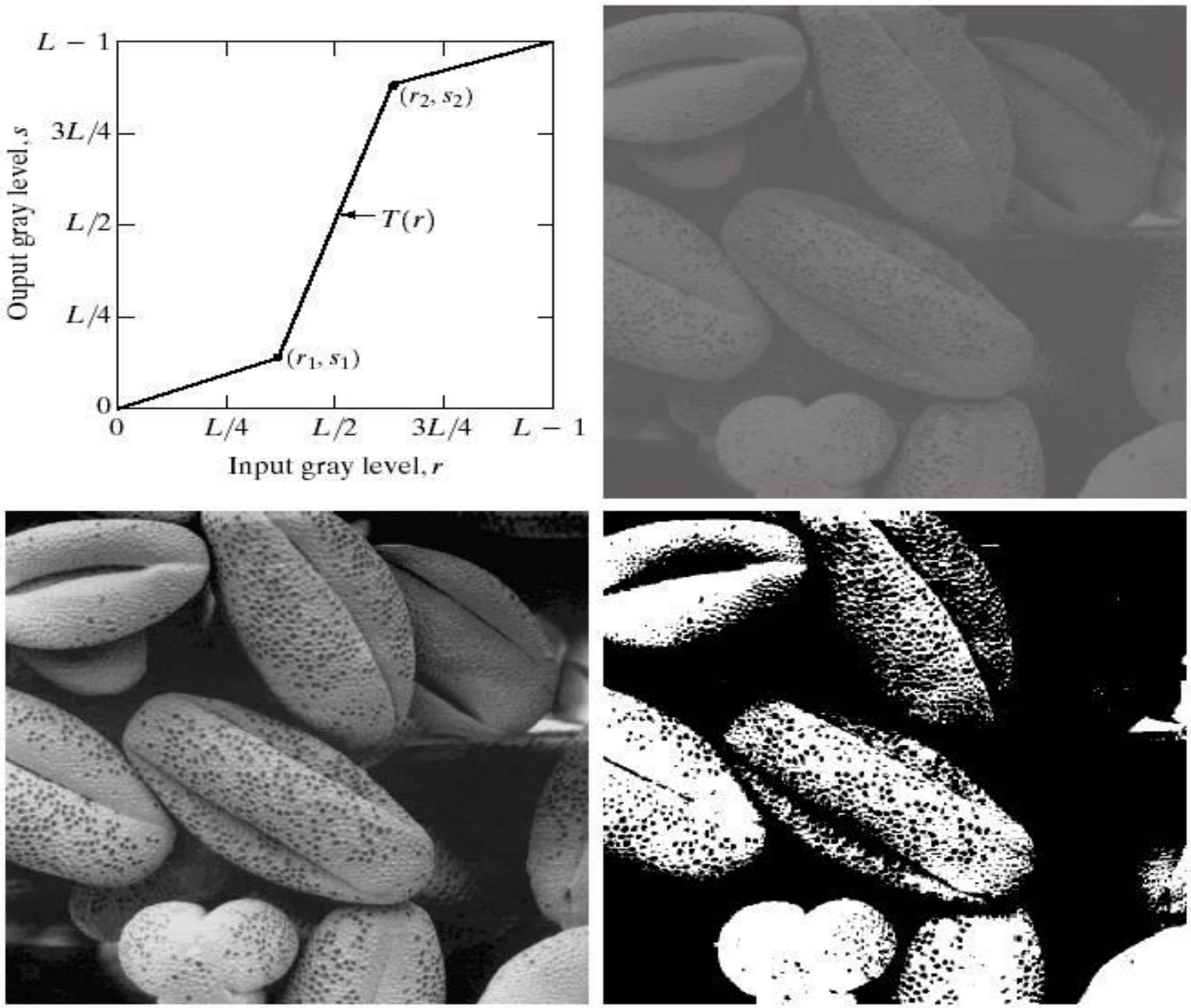
    if: $r_1 = s_1$ and $r_2 = s_2$ linear transformation that produces no changes in the intensity levels

    if: $r_1 = r_2$ and $s_1 = 0$ and $s_2 = L - 1$ the transformation becomes a **thresholding function** that creates a binary image **(Fig. 3.10 (d))**

    intermediate values of (r_1, s_1) , (r_2, s_2) produces various degrees of spread in the intensity levels of the output image, thus affecting its contrast

    Use: $(r_1, s_1) = (r_{min}, 0)$ and $(r_2, s_2) = (r_{max}, L - 1)$ **(Fig.3.10 (c))**

Contrast Stretching

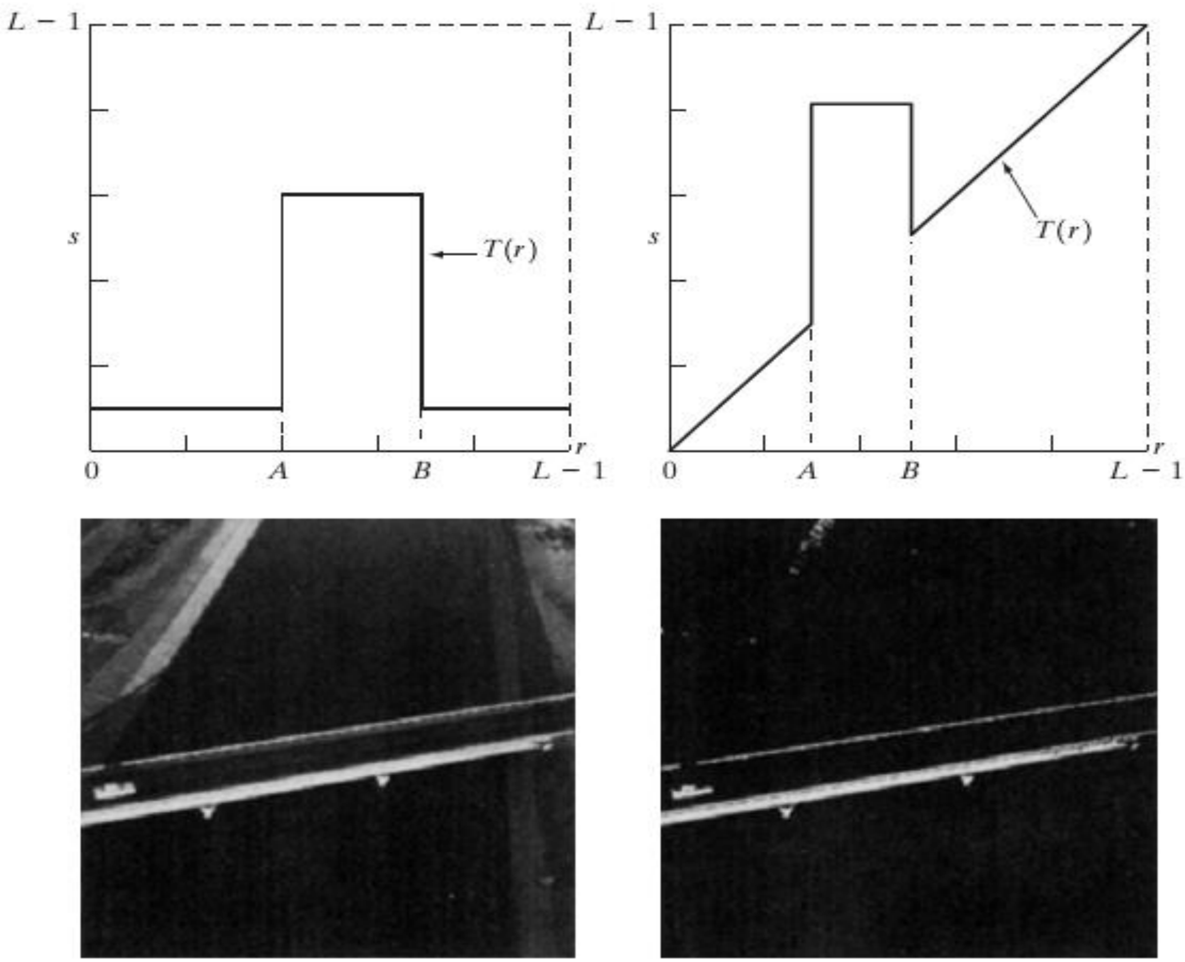


a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Intensity-level Slicing

Highlighting a specific range of intensities in an image



a b
c d

FIGURE 3.11 (a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Applications- enhancing features such as masses in satellite imagery and enhancing flaws in X-ray images

Bit-Plane Slicing

- Intensity of each pixel in a 256-level gray-scale image is composed of 8 bits
- Contribution made to total image appearance by specific bit is highlighted
- Useful for image compression , data hiding,, watermarking

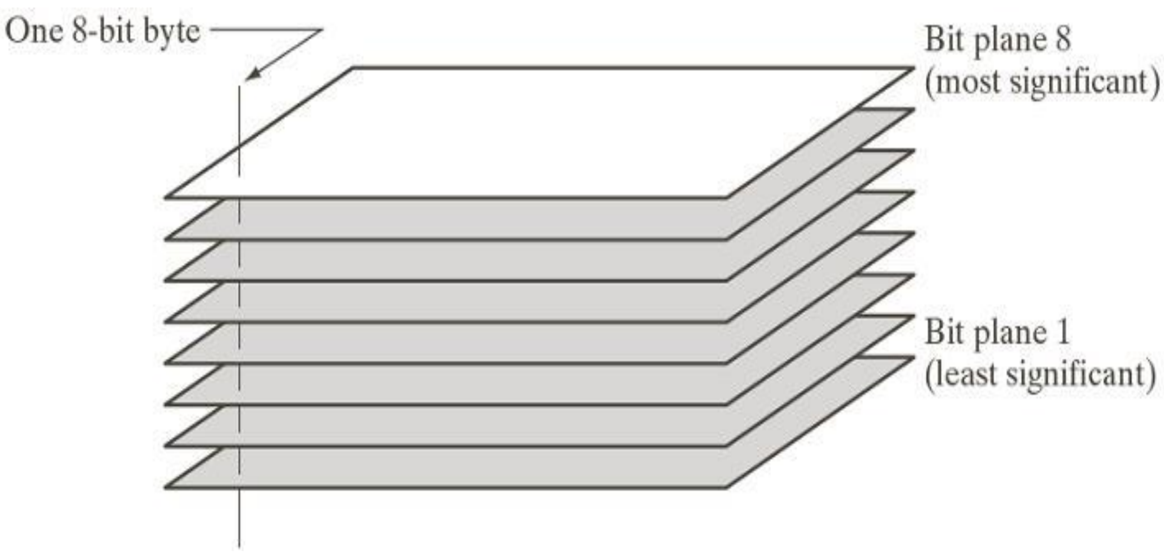


FIGURE 3.13 Bit-plane representation of an 8-bit image.

Bit-Plane Slicing- Example

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Gray border has intensity 194 (11000010)

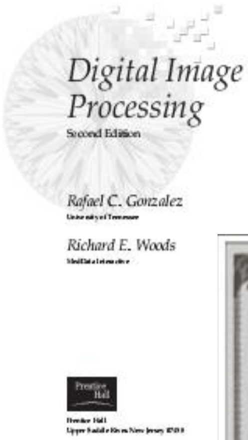


Image Reconstruction using Bit-Planes

a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

✓Reconstruction is done by multiplying the pixels of the nth plane by constant 2^{n-1}

✓Background of the image (b) has perceptible false contouring. This effect is reduced by adding 5^{th} plane

✓Using more bit planes in the reconstruction would not contribute significantly to the appearance of the image

Histogram Processing

- In probabilistic methods of image processing, intensity values are treated as random quantities
- Histogram of a digital image with intensity levels is a discrete function

$$h(r_k) = n_k$$

where r_k: the kth intensity value and n_k: number of pixels in the image with intensity r_k

- Normalized histogram

$$p(r_k) = n_k / MN \text{, for } k = 0, 1, 2, \dots , L-1$$

- Sum of all components of a normalized histogram is equal to 1
- High contrast image covers a wide range of the intensity scale and pixels distribution is almost uniform

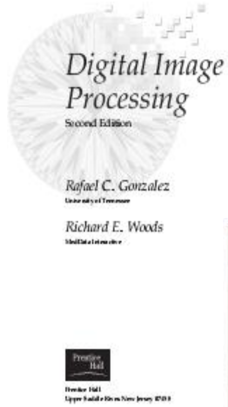


Image Histogram

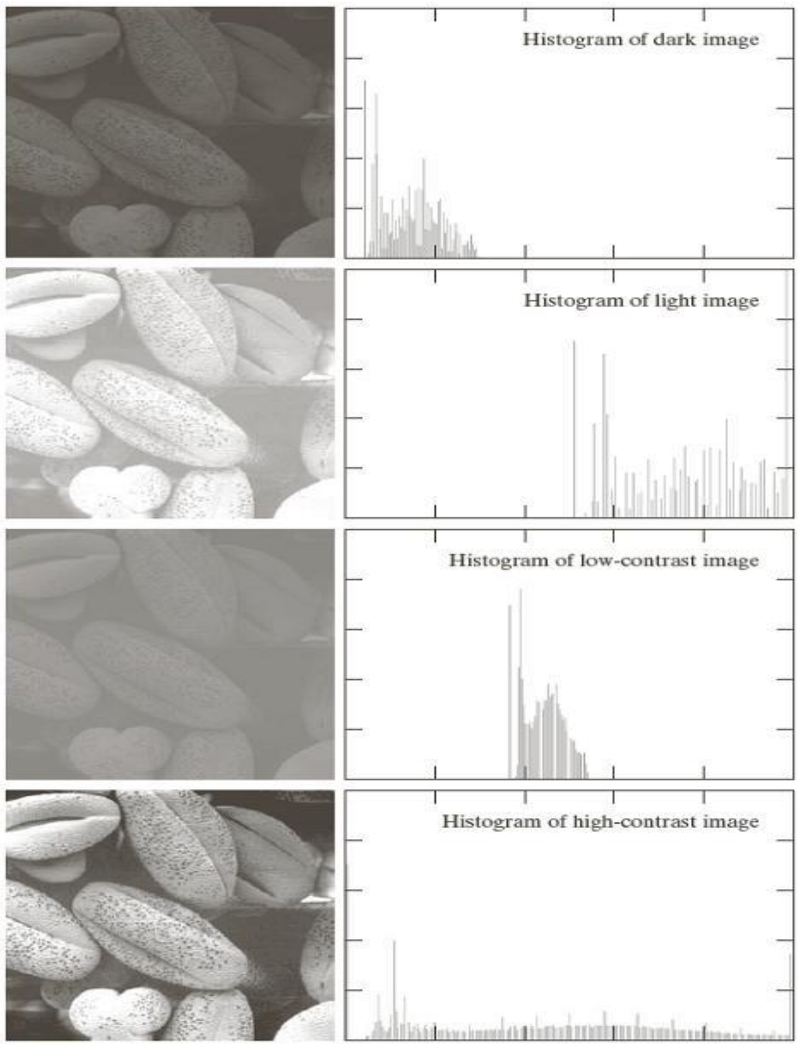


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

An image with pixel values occupying the entire range of possible intensity levels , tend to be distributed uniformly

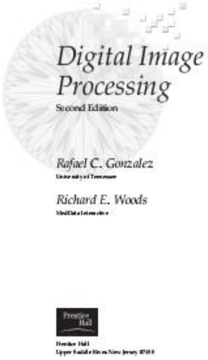
An appearance of high contrast exhibit a large variety of gray tones

Histogram Equalization

- ✓ Used to enhance the appearance of an image and to make certain features more visible
- ✓ An attempt to equalize the number of pixels with each particular value
- ✓ Allocate more gray levels where there are most pixels and allocate fewer levels where there are few pixels.
- ✓ Discrete form of the transformation is

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

$$k = 0, 1, 2, \cdots, L-1$$

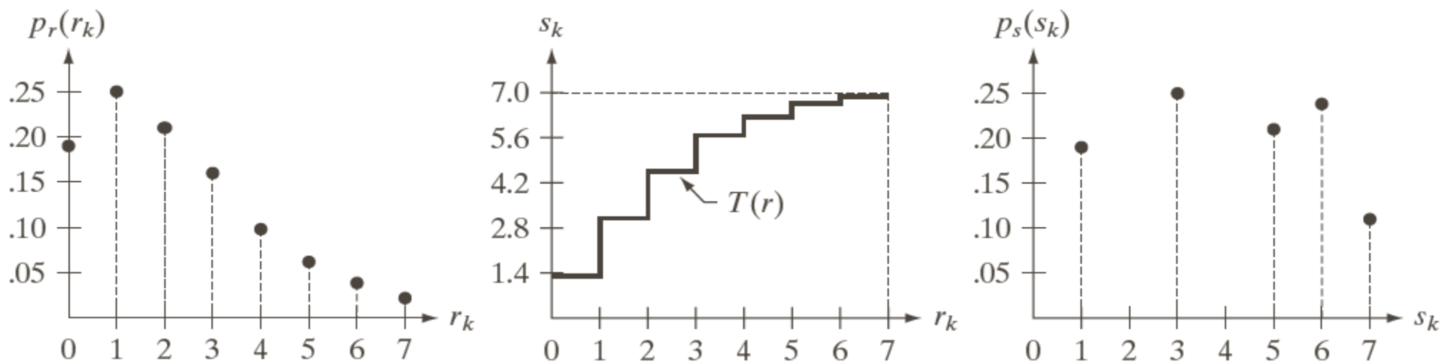


Histogram Equalization- Example

A 3-bit image (L=8) of size 64x64 pixels (MN=4096) has intensity distribution given in table 3.1, determine its equalized histogram

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization- Example

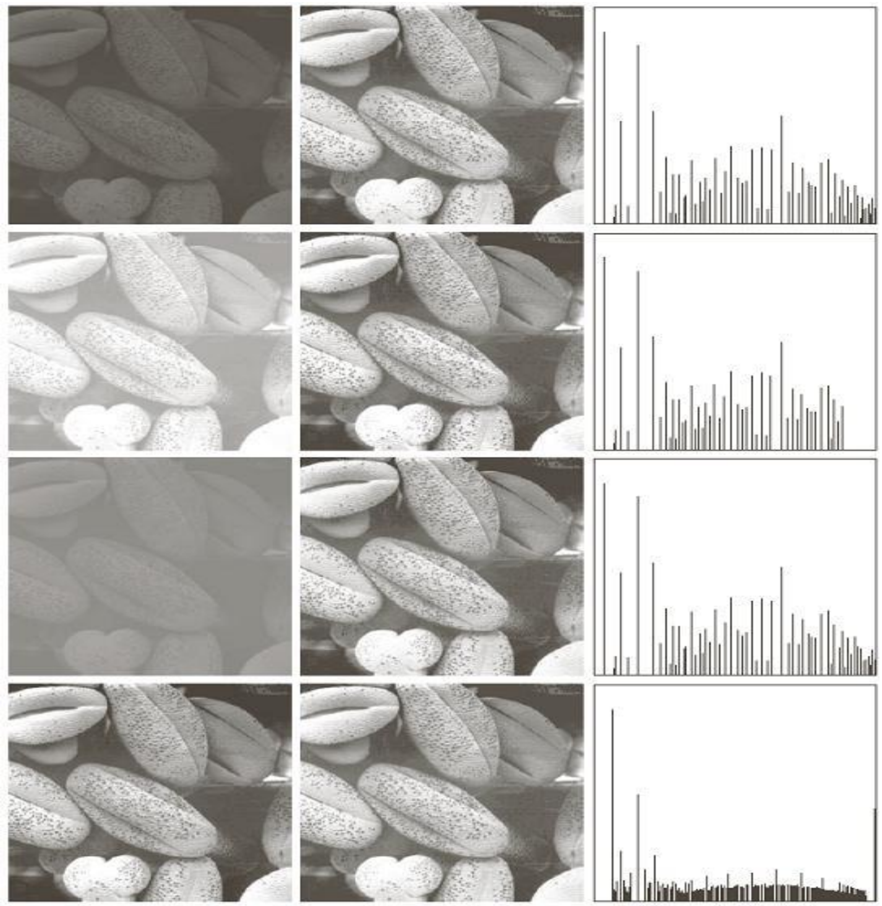


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Histogram Matching (Specification)

- Method to generate a processed image that has a specified histogram

Spatial Filtering

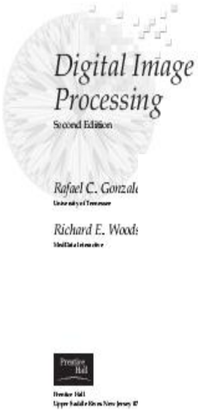
Filter consists of

- Neighborhood: typically a small rectangle
- Predefined operation
- Also called as **spatial masks, Kernels, templates,** and **Windows**)

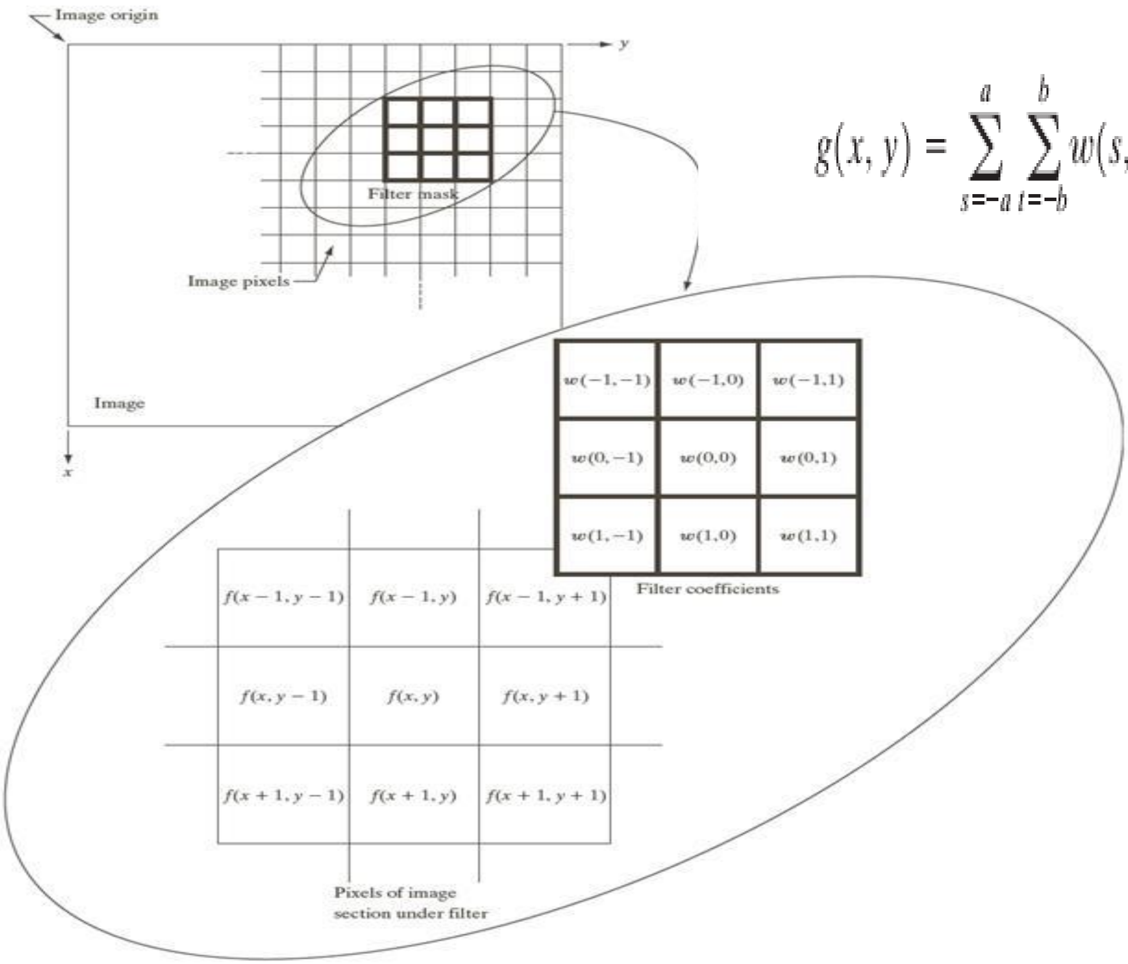
Filtering

- Accepting (passing) or rejecting certain frequency components

  e.g. Low-pass filter: blur (smooth) an image
- Creates a new pixel with coordinates equal to the coordinates of the center of the neighborhood, and whose value is the result of the filtering operation
- Use odd size filters (smallest being: $\mathbf{3} \times \mathbf{3}$)
- If the operation performed on the image pixels is linear, then the filter is called a ***linear spatial filter*** , otherwise filter is nonlinear



Linear Spatial Filtering



$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t) \qquad (3.5\text{-}1)$$

FIGURE 3.28 The mechanics of linear spatial filtering using a 3 × 3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

Spatial Filtering

- **Correlation**
  - Function of the displacement of the filter
  - Used for matching between images
- **Convolution**
  - Rotate filter by 180°
- Padding an with (filter size $m \times n$)
  - m-1 rows of **0**s at the top and bottom and
  - n-1 columns **0**s at the left and right
- **Cropping the result**

Correlation and Convolution of a 2-D filter

Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

(a)

Padded f

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(b)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c)

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

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Richard E. Woods

Correlation and Convolution of a 2-D filter

Origin $f(x, y)$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad w(x, y) \quad \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$$

(a)

Padded f

$$\begin{matrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{matrix}$$

(b)

Rotated w

$$\begin{matrix} \boxed{9} & \boxed{8} & \boxed{7} & 0&0&0&0&0&0 \\ \boxed{6} & \boxed{5} & \boxed{4} & 0&0&0&0&0&0 \\ \boxed{3} & \boxed{2} & \boxed{1} & 0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{matrix}$$

(f)

Full convolution result

$$\begin{matrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&2&3&0&0&0 \\ 0&0&0&4&5&6&0&0&0 \\ 0&0&0&7&8&9&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{matrix}$$

(g)

Cropped convolution result

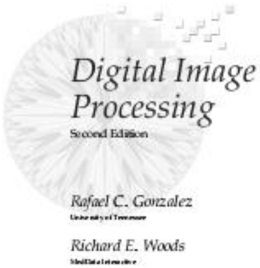
$$\begin{matrix} 0&0&0&0&0 \\ 0&1&2&3&0 \\ 0&4&5&6&0 \\ 0&7&8&9&0 \\ 0&0&0&0&0 \end{matrix}$$

(h)

FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

Spatial Filtering

- Generating Spatial Filter Masks
  - For example: 3×3 filter (mask)



Chapter 3
Intensity Transformations and Spatial Filtering

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

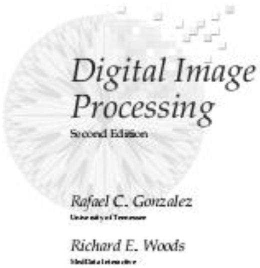
FIGURE 3.31 Another representation of a general 3×3 filter mask.

Smoothing Spatial Filters

- Used for blurring and for noise reduction
- Blurring is used in preprocessing tasks
  - Removal of small details prior to large object extraction
  - Bridging of small gaps in lines or curves
  - Smoothing false contouring
- Using averaging filters (lowpass filters)
- Replacing every pixel by the average intensity in the neighborhood which results in reduced sharp transitions in intensities
  - Undesirable side effect: blurring edges

Smoothing Spatial Filters

- Box filter
- Weighted average filter
  - Giving some pixels more importance (weight) at the expenses of others
  - Basic strategy: an attempt to reduce blurring in the smoothing process

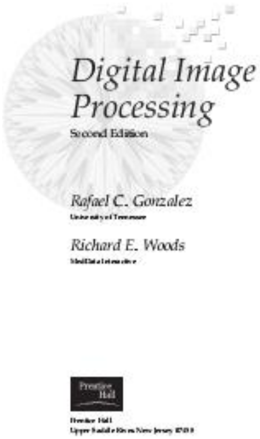


Chapter 3
Intensity Transformations and Spatial Filtering

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



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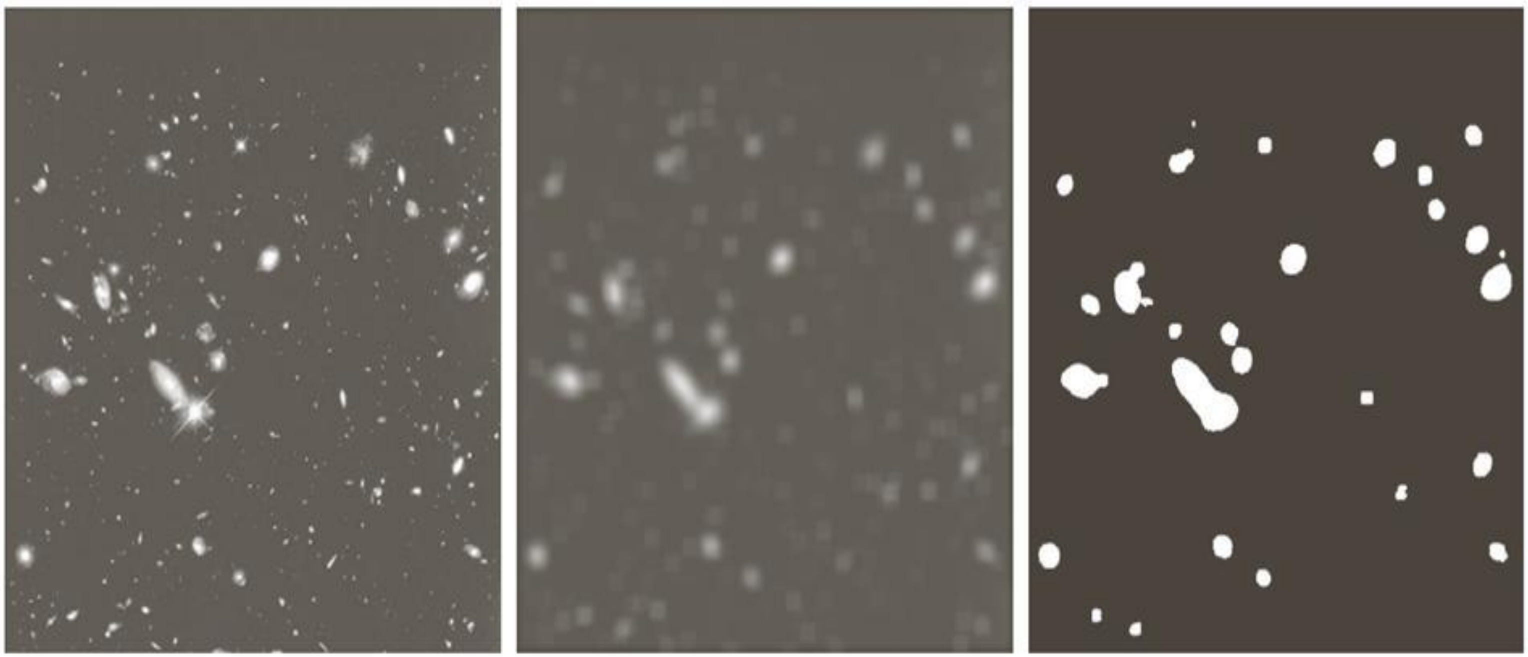
Chapter 3

Intensity Transformations and Spatial Filtering

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.



a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

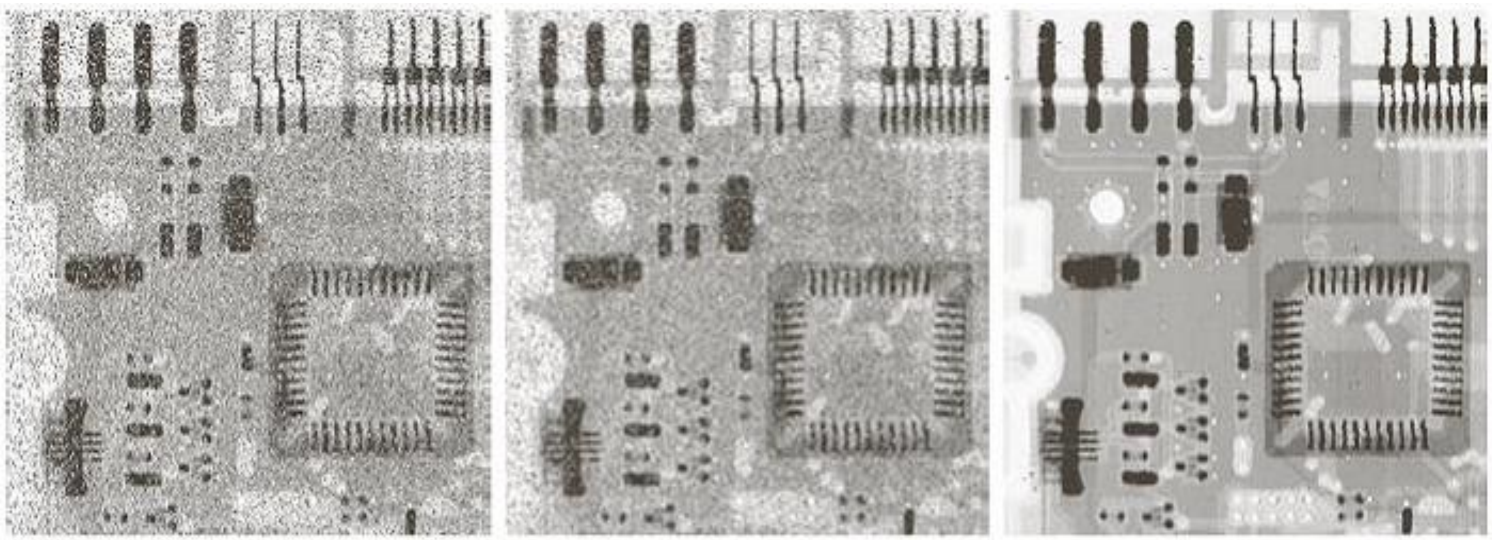
Order-Statistics Filters

- Based on ordering (ranking) pixels encompassed by the filter then replacing the center pixel with the ranking result
- Median filter
  - Provide excellent noise reduction with less blurring
- Median value half of the values less than or equal to median and half are greater than or equal to median
  - Sort pixels in the neighborhood
  - Determine the median
  - Assign median to the corresponding pixel

Order-Statistics Filters

- Median
  - In 3×3 neighborhood is the 5^{th} largest value
  - In 5×5 neighborhood is the 13^{th} largest value
- Principal function of median filtering is to force pixels with distinct intensities to be more like their neighbors
- Other order-statistics filters
  - Max filter: find the brightest points in the image
  - Min filter

Chapter 3
Intensity Transformations and Spatial Filtering



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Spatial Filters

- Sharpening Filter: to highlight transitions in intensity
  - Applications
    - Medical imaging
    - Electronic printing
    - Industrial inspection
  - Enhances edges and other discontinuities and deemphasizes areas with slowly varying intensities

Sharpening Spatial Filters

- Accomplished by Spatial Differentiation
- Derivatives of a digital function are defined in terms of differences
- **First derivative (Conditions)**

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

  - must be zero in the area of constant intensity

  - must be nonzero at the onset of an intensity step or ramp

  - must be nonzero along ramp

- **Second derivative**

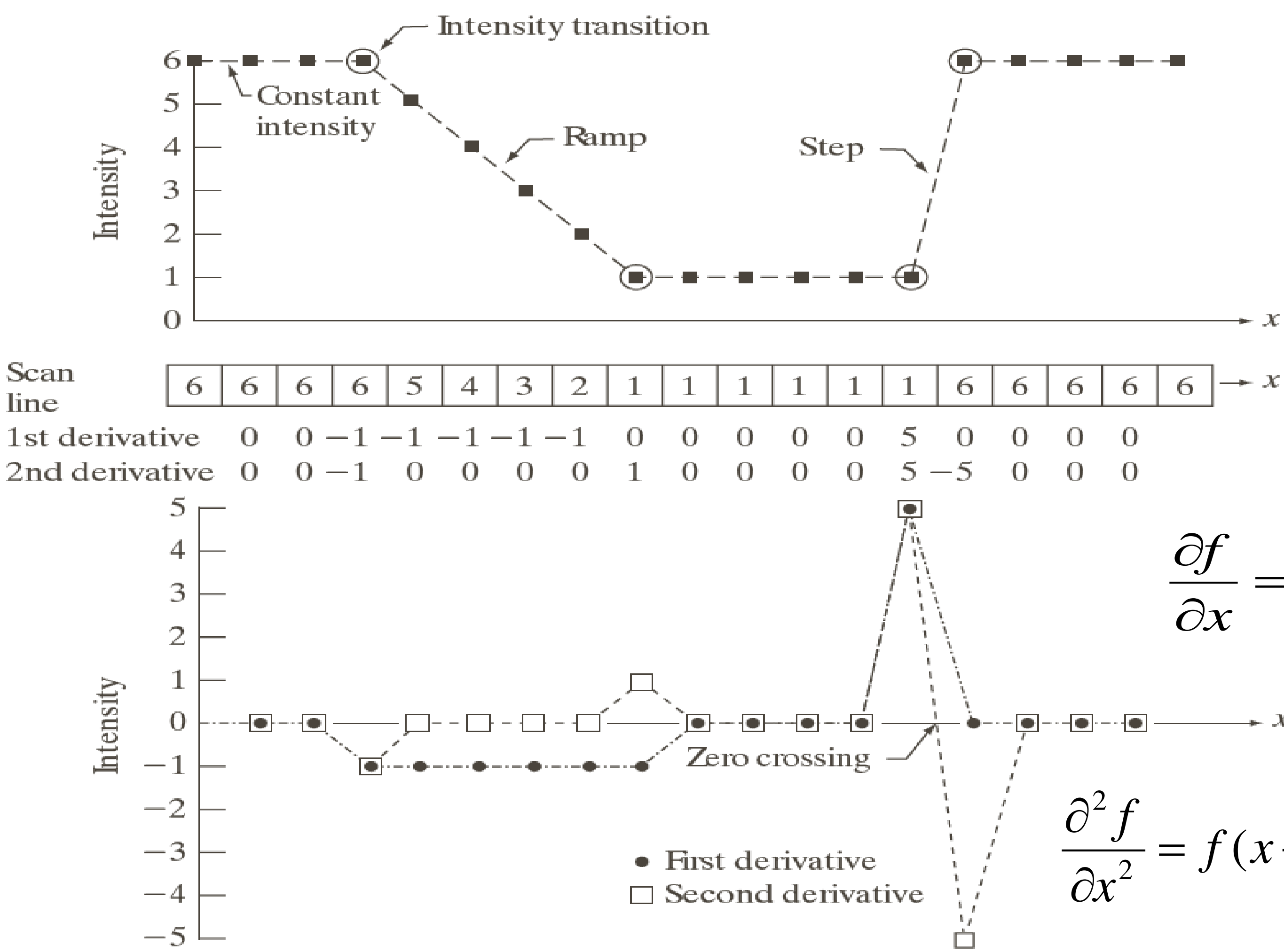
$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

- must be zero in constant intensity areas

- must be nonzero at the onset and end of an of an intensity step or ramp

-must be zero along ramps of constant slope

First and Second derivatives of a 1-D digital Function



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening Spatial Filters

- Edges in digital images often are ramp-like transitions in intensity

  - first derivative results in thick edges due to nonzero value along the ramp

  - Second derivative produces double edge one pixel thick separated by zero

Second derivative enhances fine details much better than the first derivative

Second derivative for image sharpening- The Laplacian

- Isotropic filter – response is independent of the direction of the discontinuities in the image (rotation invariant)
- Highlights intensity discontinuity in the image
- Laplacian for an image function f(x,y) is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

– I n the x-direction

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

– In the y-direction

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

-Final Laplacian of two variables is (eq. 3.6-6)

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Laplacian Filter Masks

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

-Basic way of image sharpening is done by adding the Laplacian image to the original image

-Resultant image g(x , y) is given by

$$g(x, y) = f(x, y) + c\left[\nabla^2 f(x, y)\right]$$

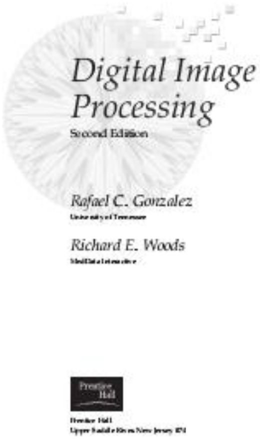
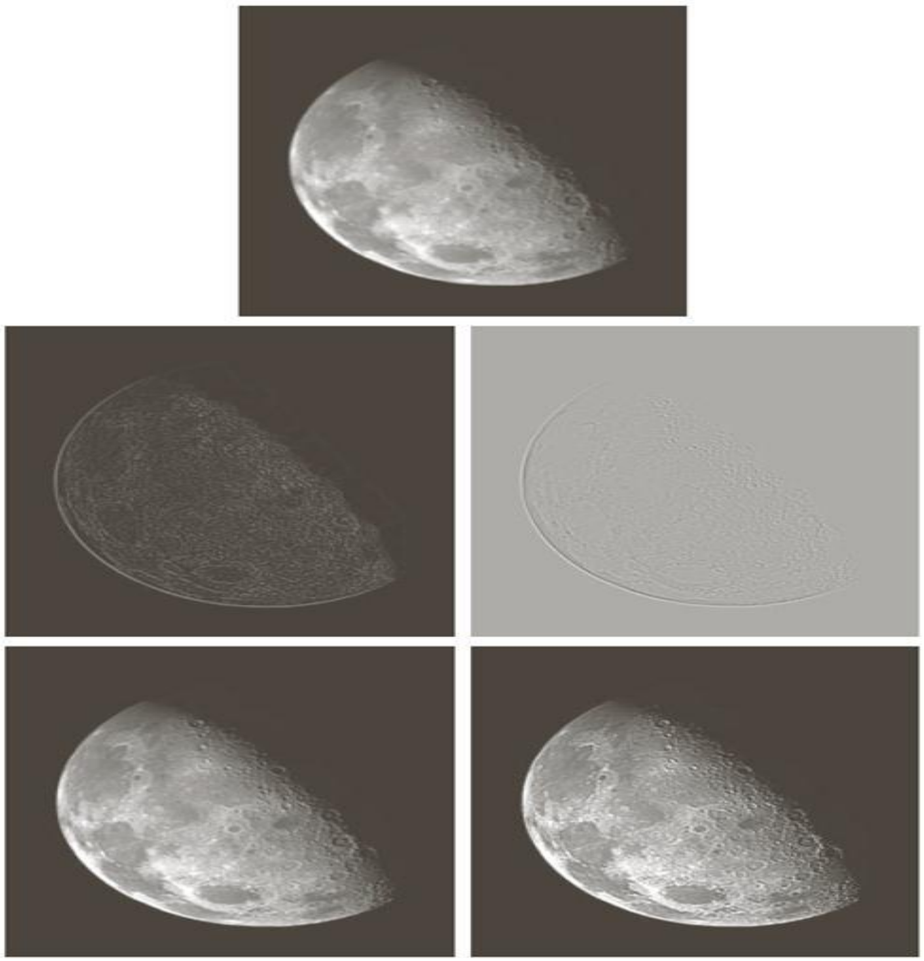


Image Sharpening using Laplacian



a
b c
d e

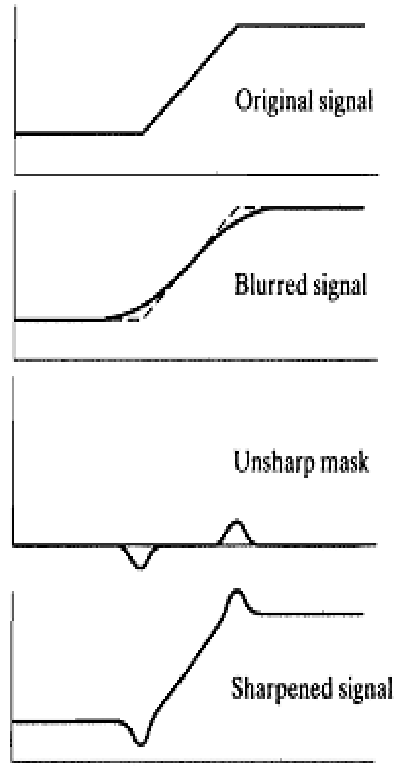
FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Unsharp Masking and Highboost Filtering

- **Unsharp Masking** - A process to sharpen images by subtracting an unsharp (smoothed) version of an image from the original image
- **Steps-**

  1. blur the original image
  2. Subtract the blur image from the original
  3. Add the mask to the original

1-D illustration of Unsharp Masking



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Mask image is given by

$G_{mask}(x, y) = f(x, y) – f^*(x, y)$

Sharpened image is given by

$G(x, y) = f(x, y) + G_{mask}(x, y)$

First Order derivative for image sharpening- The Gradient

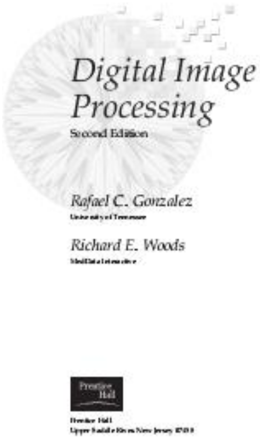
- First derivatives in image processing are magnitude of the gradient
- For a function **f(x, y),** the gradient of **f** at coordinates (x, y) is defined as 2-D column vector

$$\nabla f \equiv grad(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

  - Gradient vector points in the direction of the greatest rate of change of **f** at location (x, y)

The *magnitude* (*length*) of vector ∇f, denoted as $M(x, y)$, where

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$



Gradient Operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.41 A 3 × 3 region of an image (the *z*s are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

In some implementations, it is more suitable to approximate square root operation with absolute values

$$M(x, y) \approx |g_x| + |g_y|$$

example, the center point, z_5, denotes $f(x, y)$ at an arbitrary location, (x, y); z_1 denotes $f(x-1, y-1)$; and so on, using the notation introduced in Fig. 3.28. As indicated in Section 3.6.1, the simplest approximations to a first-order derivative that satisfy the conditions stated in that section are $g_x = (z_8 - z_5)$ and $g_y = (z_6 - z_5)$. Two other definitions proposed by Roberts [1965] in the early development of digital image processing use cross differences:

$$g_x = (z_9 - z_5) \quad \text{and} \quad g_y = (z_8 - z_6) \tag{3.6-13}$$

If we use Eqs. (3.6-11) and (3.6-13), we compute the gradient image as

$$M(x, y) = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2} \tag{3.6-14}$$

If we use Eqs. (3.6-12) and (3.6-13), then

$$M(x, y) \approx |z_9 - z_5| + |z_8 - z_6| \tag{3.6-15}$$

where it is understood that x and y vary over the dimensions of the image in the manner described earlier. The partial derivative terms needed in equation (3.6-13) can be implemented using the two linear filter masks in Figs. 3.41(b) and (c). These masks are referred to as the *Roberts cross-gradient operators*.

Masks of even sizes are awkward to implement because they do not have a center of symmetry. The smallest filter masks in which we are interested are of size 3×3. Approximations to g_x and g_y using a 3×3 neighborhood centered on z_5 are as follows:

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \tag{3.6-16}$$

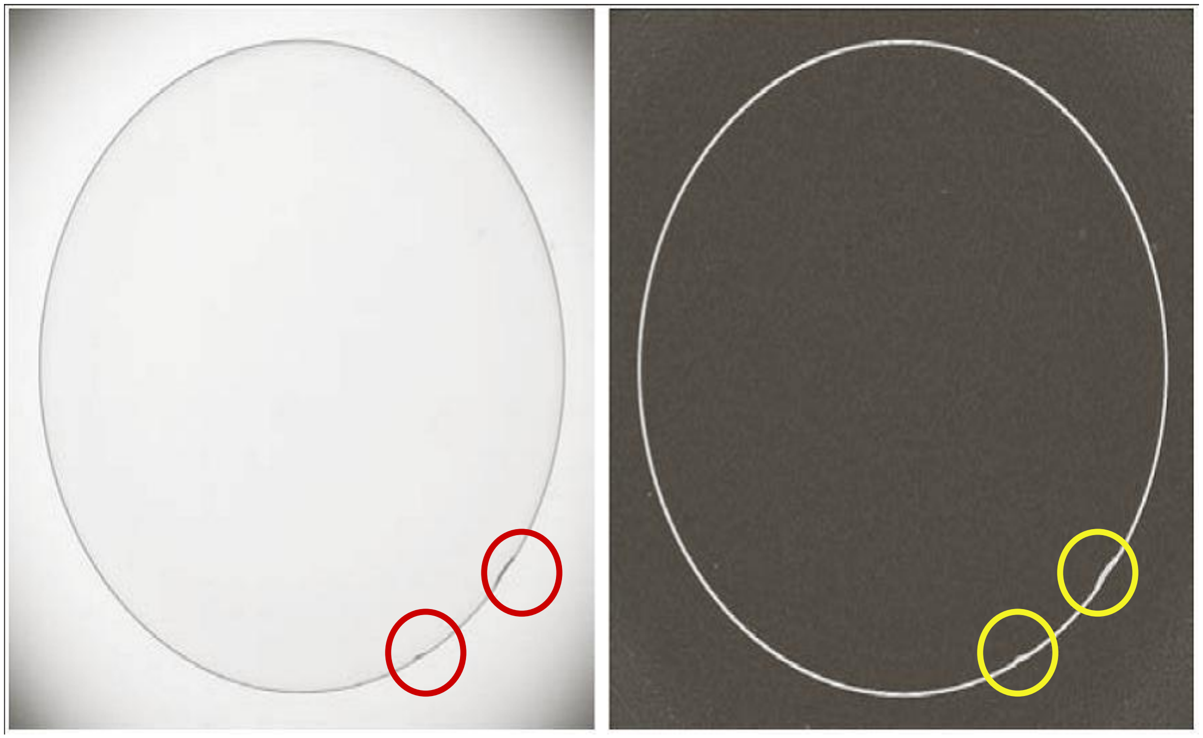
and

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \tag{3.6-17}$$

These equations can be implemented using the masks in Figs. 3.41(d) and (e). The difference between the third and first rows of the 3×3 image region implemented by the mask in Fig. 3.41(d) approximates the partial derivative in the x-direction, and the difference between the third and first columns in the other mask approximates the derivative in the y-direction. After computing the partial derivatives with these masks, we obtain the magnitude of the gradient as before. For example, substituting g_x and g_y into Eq. (3.6-12) yields

$$\begin{aligned} M(x, y) \approx\ & |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \\ & + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \end{aligned} \tag{3.6-18}$$

Gradient for Edge Enhancement



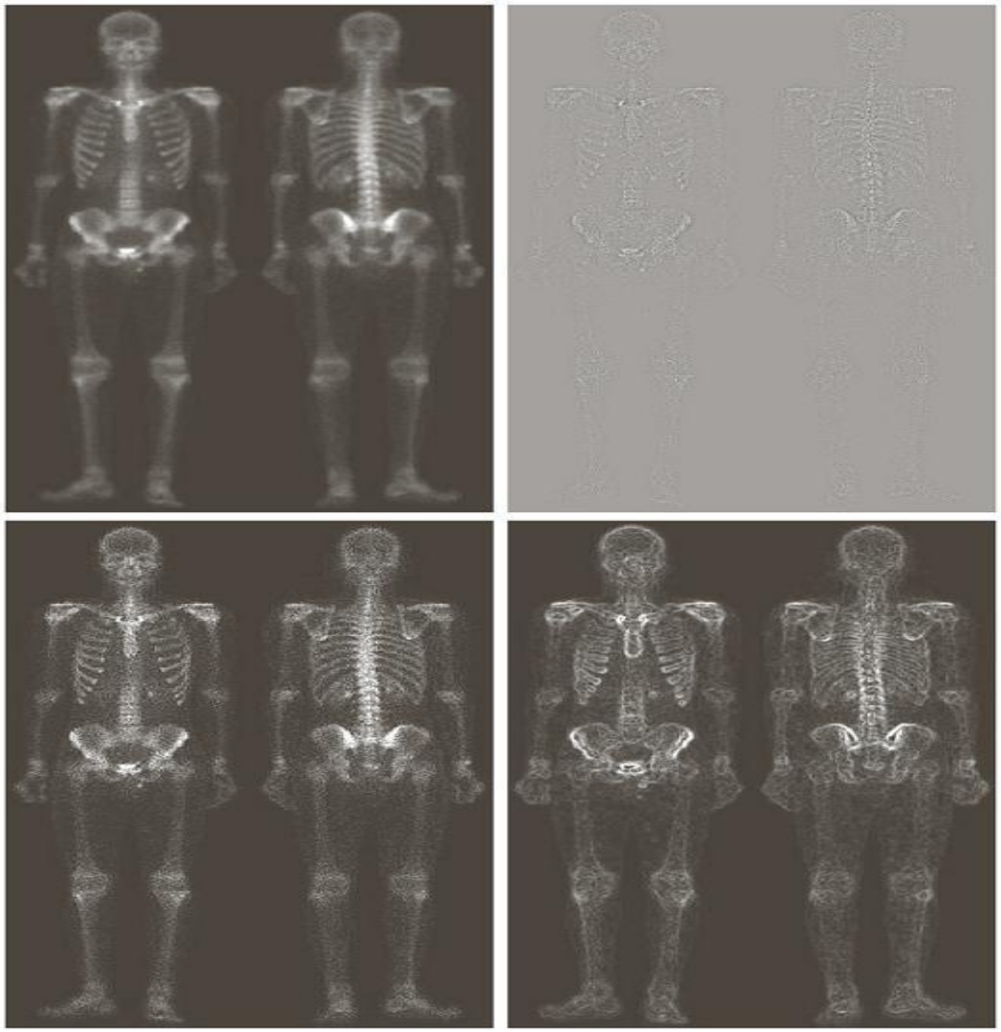
a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)

Combining Sharpening Enhancement Methods

- Frequently, a given task will require application of several techniques in order to achieve an acceptable result
- Image enhancement by
  - Smoothing
  - Sharpening
  - and so on

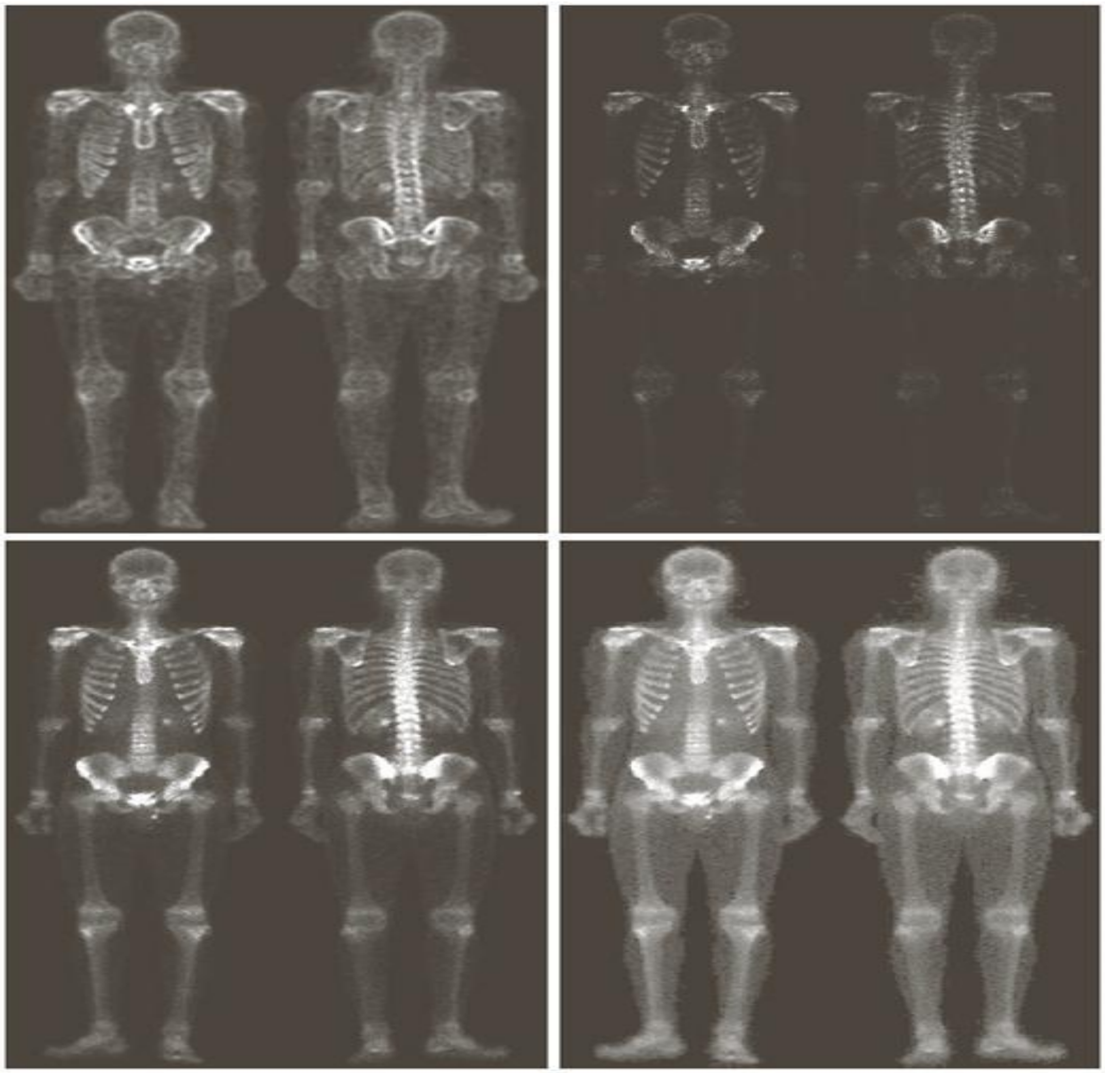
Combining Sharpening Enhancement Methods



a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).

Combining Sharpening Enhancement Methods



e	f
g	h

FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Filtering in the Frequency Domain

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Background

- Image domains
  - Spatial domain techniques operate directly on the pixels of an image
  - Transform domain
    - Fourier Transform (FT)
    - Wavelet Transform
    - Discrete Cosine Transform (DCT)

Background

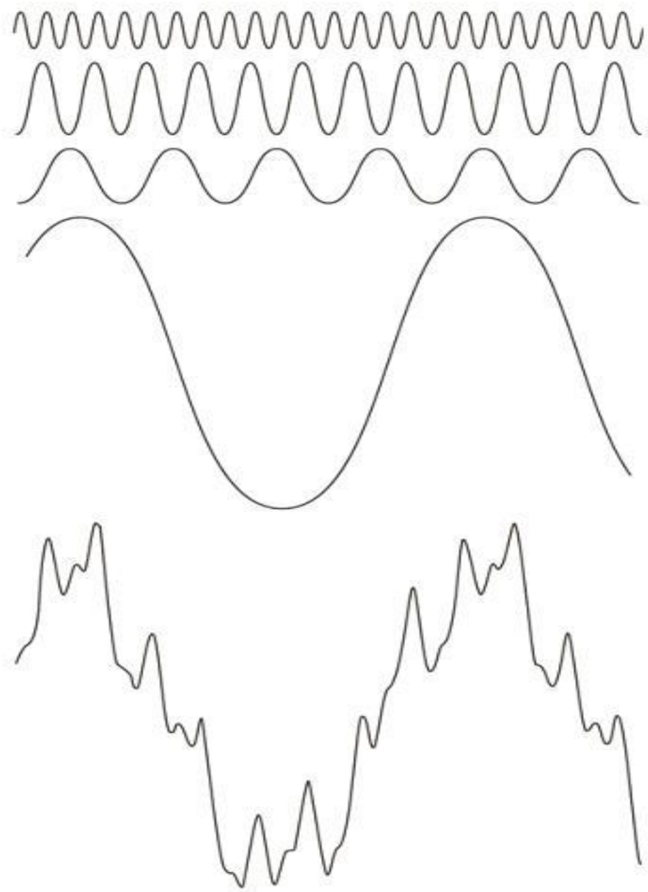


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Review

- Sampling and Fourier Transform of sampled functions

  - Sampling theorem, aliasing, reconstruction from sampled data
- Discrete Fourier Transform (DFT) of one variable
- 2-D DFT and its Inverse
- Properties of 2-D DFT

Sampling

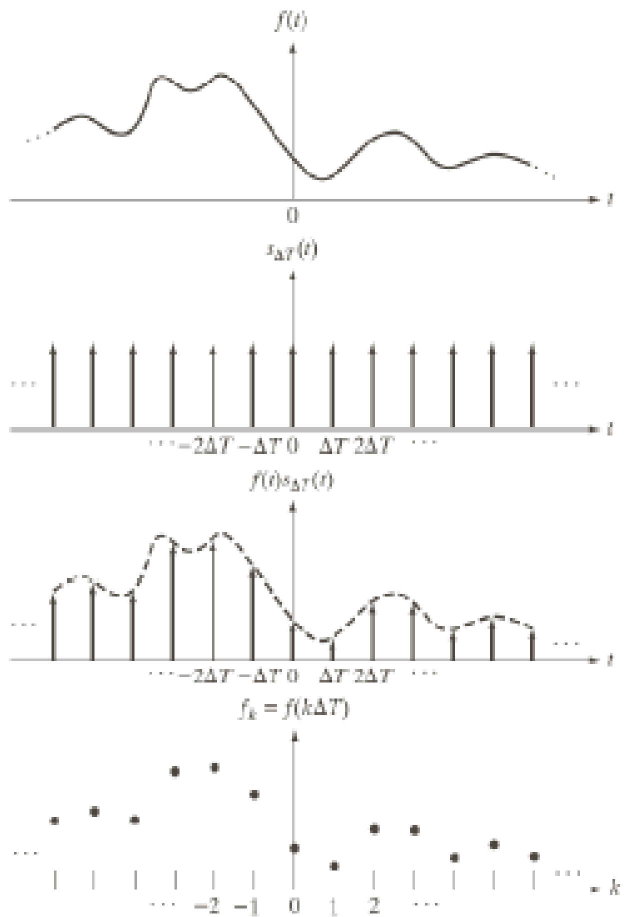
Sampling and Fourier Transform

1. Converting continuous function/signal into a discrete one.
2. The sampling is uniform at ΔT intervals

The sampled function and the value of each Sample are :

$$\tilde{f}(t) = f(t)s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

$$f_k = \int_{-\infty}^{\infty} f(t)\delta(t - k\Delta T)dt = f(k\Delta T)$$



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FT of Impulse Train

- Intermediate result
  - The Fourier transform of the impulse train.

$$\sum_{n=-\infty}^{+\infty} \delta(t - n\Delta T) \leftrightarrow \frac{1}{\Delta T} \sum_{n=-\infty}^{+\infty} \delta\left(\mu - \frac{n}{\Delta T} \right)$$

- It is also an impulse train in the frequency domain.
- Impulses are equally spaced every $1/\Delta T$.

Fourier Transform of Sampled Function

The Fourier of Sampled function

Let F(μ) and $\tilde{F}(\mu)$ be the Fourier transform of the continuous function f(t) and its equivalent Sampled function $\tilde{f}(t)$

$$\tilde{F}(\mu) = \Im\{\tilde{f}(t)\} = \Im\{f(t)s_{\Delta T}(t)\} = F(\mu) \circ S(\mu)$$

$$S(\mu) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta(\mu - \frac{n}{\Delta T})$$

$$\tilde{F}(\mu) = F(\mu) \circ S(\mu) = \int_{-\infty}^{\infty} F(\tau)S(\mu-\tau)d\tau$$

$$= \frac{1}{\Delta T}\int_{-\infty}^{\infty} F(\tau)\sum_{n=-\infty}^{\infty}\delta(\mu-\tau-\frac{n}{\Delta T})d\tau$$

$$= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} F(\tau)\delta(\mu-\tau-\frac{n}{\Delta T})d\tau$$

$$= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta T}\right)$$

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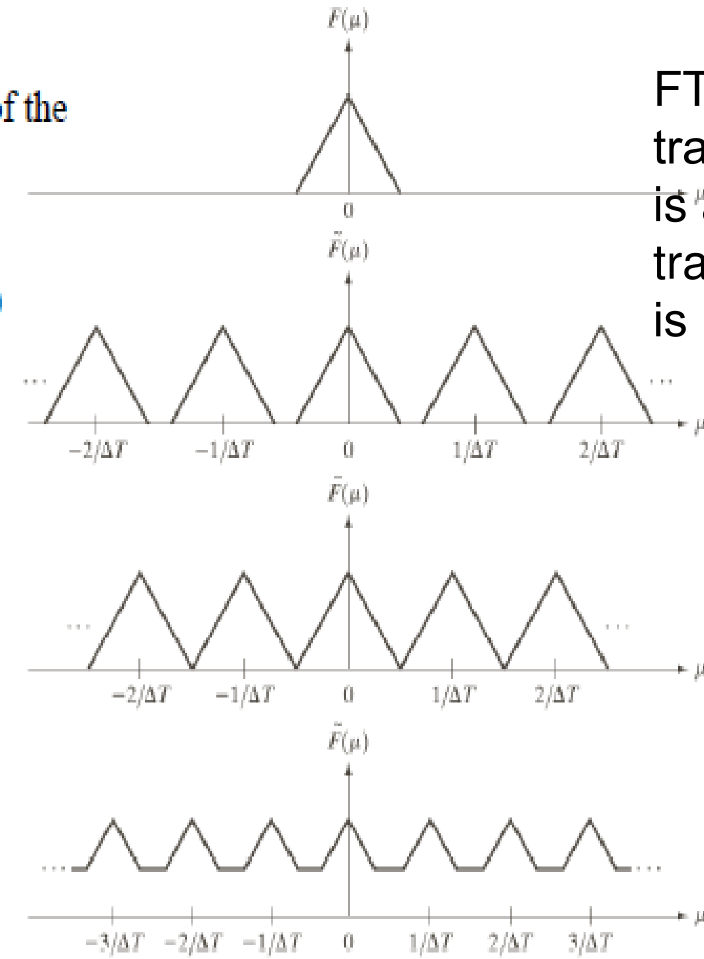


FT of an impulse train with period ΔT is also an impulse train, whose period is 1/ ΔT

Sampling Theorem

Sampling Theorem

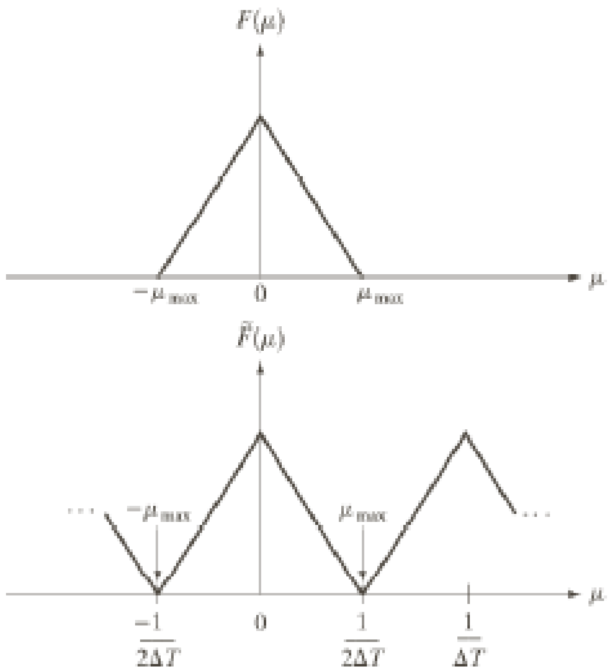
***Band limited*-**A function f(t) whose Fourier transform is zero out of the interval $[-\mu_{max}, \mu_{max}]$ is called band limited

We can recover a function f(t) from its sampled representation if we can isolate a copy of $F(\mu)$ from the periodic sequence of copies.

Extracting from $\tilde{F}(\mu)$ a single period that represents $F(\mu)$ is possible in the separation between copies is sufficient , which is guaranteed if $\frac{1}{2}\Delta T > \mu_{max}$

$$\frac{1}{\Delta T} > 2\mu_{max}$$

Which is call the Nyquist Rate



Extraction of F(μ)

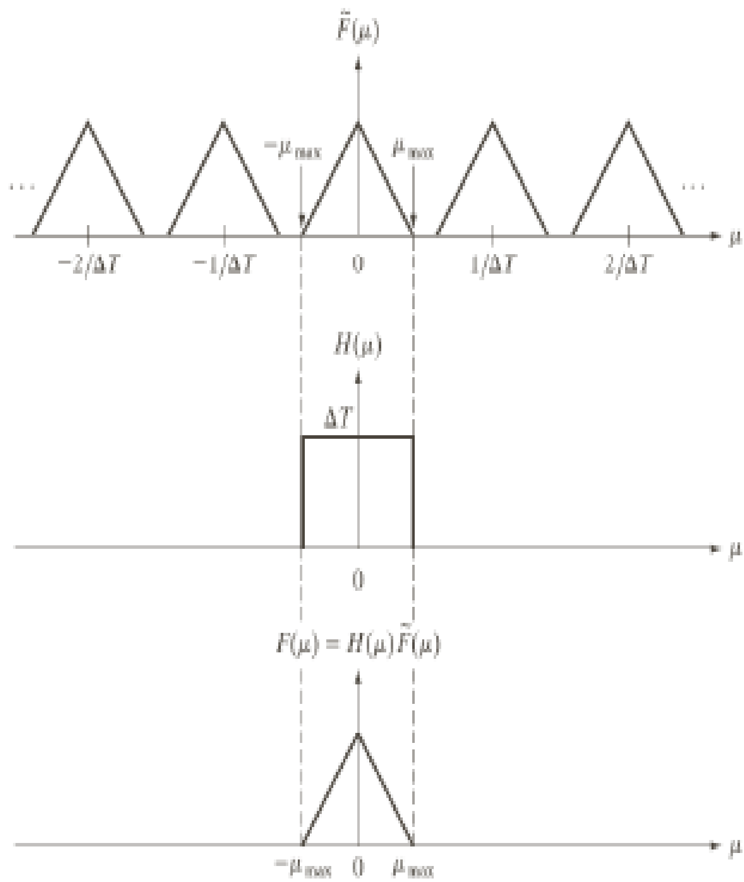
To extract a single copy we multiply $\tilde{F}(\mu)$ by H(μ).

$$H(\mu) = \begin{cases} \Delta T & -\mu_{\max} \le \mu \le \mu_{\max} \\ 0 & otherwise \end{cases}$$

$$F(\mu) = H(\mu)\tilde{F}(\mu)$$

Once we have F(μ) we can recover f(t)

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$



Aliasing

Aliasing

Appears when the sampling rate is less than the Nyquist rate – under-sampling. The inverse Fourier transform would then yield a corrupted function of t, which is known as frequency aliasing or just aliasing.