SC - 205 Discrete Mathematics Project

Application of Graph Theory in Traffic Control

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July 3, 2021

1 Introduction

In this project we worked on one of the ways of reducing waiting time of vehicles at traffic signals. While working on this we are using concept of **Graph Theory** in **Discrete Mathematics** and solving some equations with multiple variables.

Here we have used some terms which might not have been encountered by you before, for that we have given a quick idea of the pre - requisites in the start. This project excludes the volume of traffic or the size of the road into consideration. This is done using a mere assumption of ideal roads and ideal traffic in all of the directions at the signal point.

2 Problem Statement

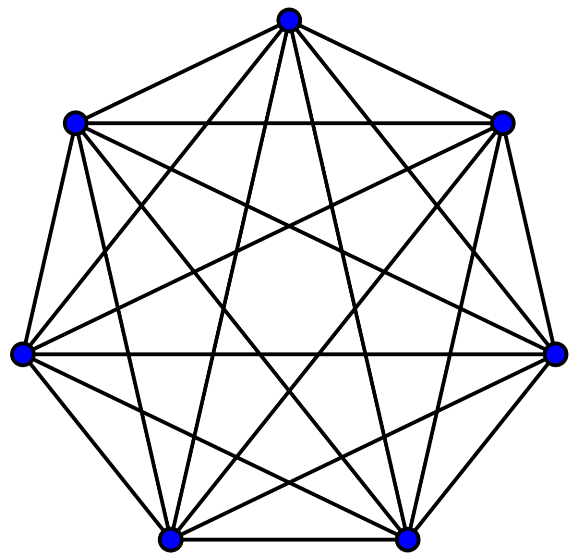
Evolve a method to minimize the red light time of all vehicular streams coming to a junction at a traffic signal by implicitly reducing the red light time using suitable mathematical theories. The inclusion of volume of roads and size of the road in the problem may be neglected. It may be solved by keeping ideal conditions in mind.

3 Pre - Requisite Knowledge

3.1 Complete Graph:

An undirected graph in which every pair of distinct vertices is connected by a unique edge.

- Its property is that it's a symmetric graph.



3.2 Graph Notation:

- G(V,E) notation implies 'V' number of vertices and 'E' number of edges.
- In the further discussion, we are gonna discuss about 'H' which is a subgraph of 'G'.

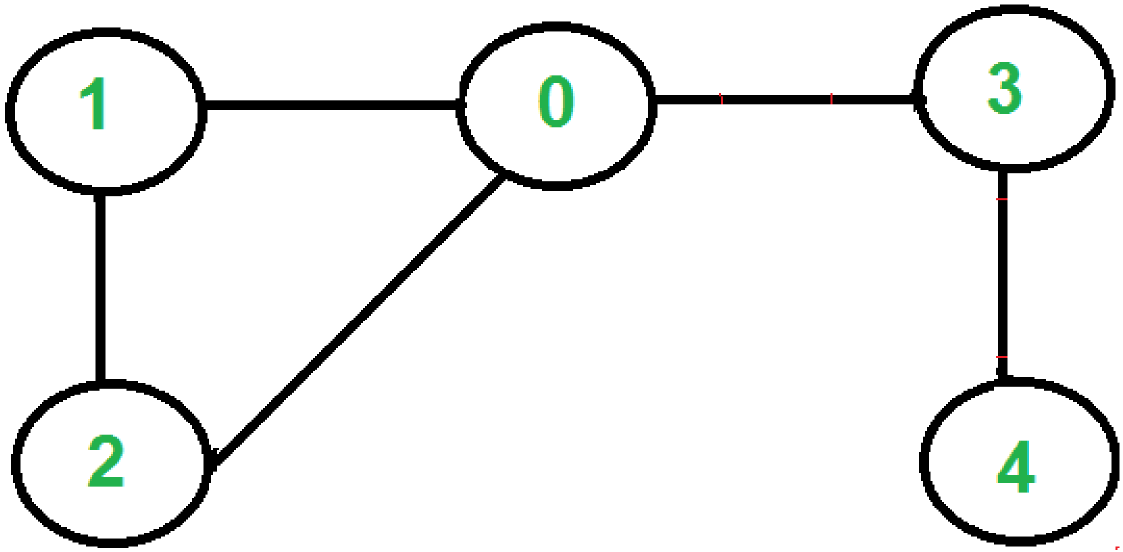
3.3 Subgraph:

It is a combination of certain edges and vertices picked from the main graph.

3.4 Clique:

A clique is a subset of vertices of an undirected graph such that every two distinct vertices in the cliques are adjacent.

- A clique is a subgraph of graph which is complete.

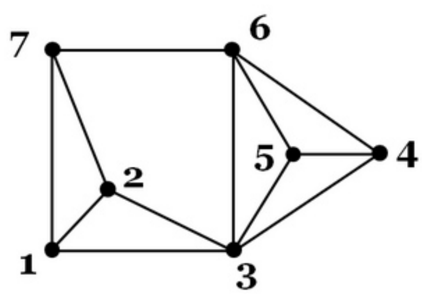


This Graph can be divided in two Cliques : One Clique contains {0, 1, 2} and other Clique contains {3, 4}

3.5 Maximal Clique:

A Clique that cannot be extended by including one more adjacent vertex.

- Also, it is a maximal clique if it is not contained in another clique of graph.

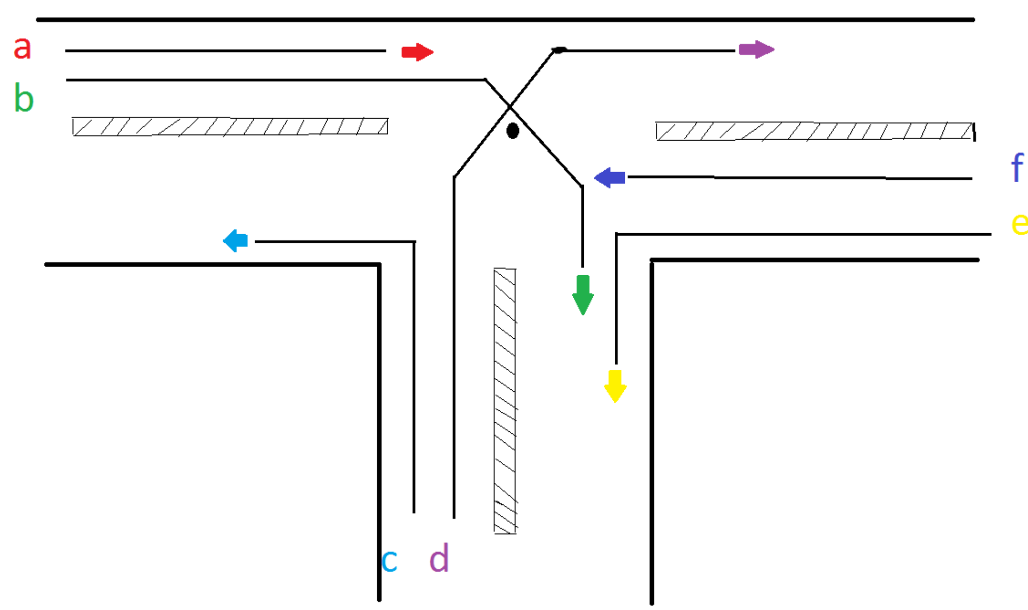


In this graph the maximal cliques are $\{6, 3, 4, 5\}, \{7, 2, 1\}, \{2, 1, 3\}$, These flower brackets indicate each maximal clique, Here, $\{7, 2, 1\}$ is a maximal clique because if you try to include an adjacent vertex like 6, then it will not become a clique.

4 Formulation of Mathematics

1. The problem can be solved by a particular method which is applicable to all such sort of problems.
2. We look at the vehicles streams first, then make a compatibility graph out of it.
3. Then we have to divide the graph into cliques which would be called as phases.
4. Each phase will be having a particular amount of time for green light.
5. The phase consists of certain streams.
6. We set some constraints like total time of 1 cycle and minimum amount of time of green light for each phase according to our needs.
7. Then giving variable names for phases, we follow the mathematics which gives us the least red light time of all streams.

5 Solution for the Mathematics

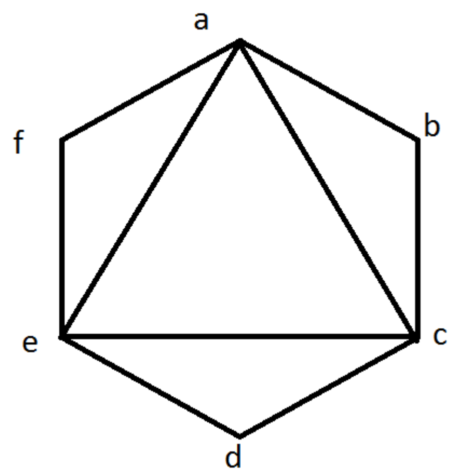


Signal point and Streams

Let us make a compatible graph for this.

Compatible Graph: Each stream is represented by a vertex in the graph. The streams which are compatible to each other are joined together. Compatibility here means, the stream which do not disturb each other when run at the same time.

H =



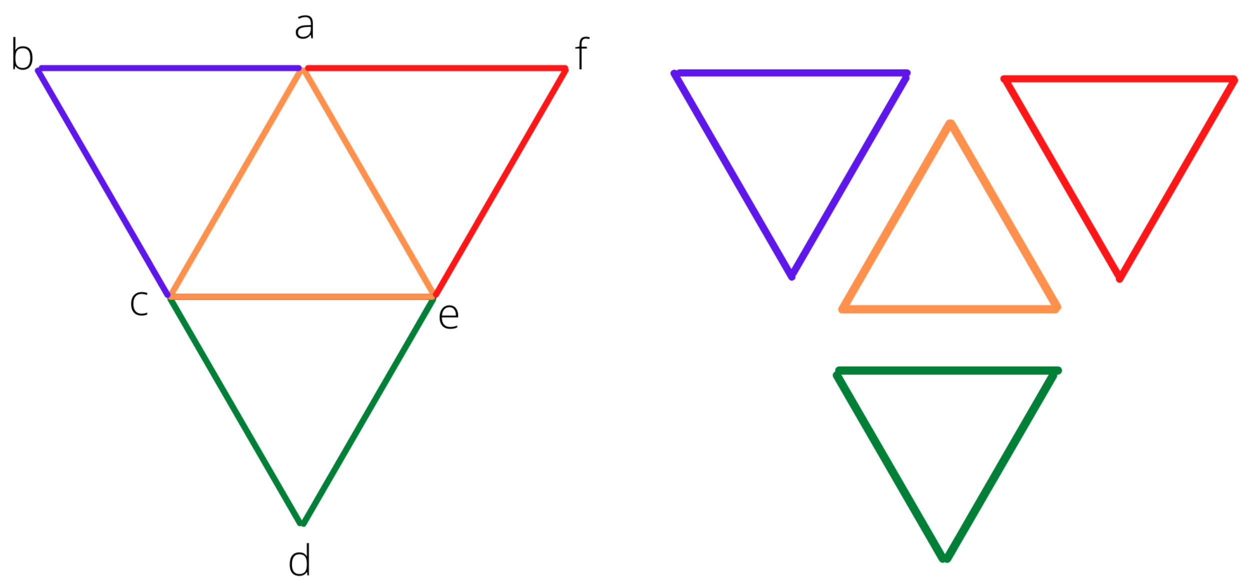
a is compatible with - b, c, f, e
b is compatible with - c, a
c is compatible with - d, f, a, b
d is compatible with - e, c
e is compatible with - f, a, d, c
f is compatible with - a, e

- We need to make a sub graph which has maximal cliques of the graph 'H'.
- This is because if we are able to do so, more streams may run at the same time, which will save time.
  - (More streams get green line at the same time)

So, it's subgraph G which has most maximal cliques, comes out to be H itself.

This is the subgraph after a little modulation.

The subgraph looks like:



Cliques:

$\{a, b, c\}$, $\{a, e, f\}$, $\{c, e, d\}$, $\{a, c, e\}$

Now let us give them 4 variables: K_1, K_2, K_3 and K_4 - these are four phases in which each clique will get a green light.

$$K_1 = \{a, b, c\} \qquad K_3 = \{c, b, c\}$$
$$K_2 = \{a, e, f\} \qquad K_4 = \{a, c, e\}$$

Let us now assign some times in which each phase will get green light:

$$K_1 - d_1 \qquad K_3 - d_3$$
$$K_2 - d_2 \qquad K_4 - d_4$$

d_1, d_2, d_3, d_4 - They are the respective time of green light for each phase.
Now, time of red light for each streams:

$a - d_1$
$b - d_2 + d_3 + d_4$
$c - d_3$
$d - d_1 + d_3 + d_4$

$e - d_1$
$f - d_1 + d_2 + d_4$

Let us denote the total red light time of all streams as Z.
So,

$$Z = (d_1) + (d_2 + d_3 + d_4) + (d_3) + (d_1 + d_3 + d_4) + (d_1) + (d_1 + d_2 + d_4)$$

$$Z = 3(d_1 + d_2 + d_3 + d_4)$$

Let us assume our traffic system has a minimum time of 20 seconds of green light for each stream. .... (a)

Now, we consider a cycle in which all four phases have shown green light once.

So, we assume our cycle is of 120 seconds which means

$$d_1 + d_2 + d_3 + d_4 = 120 \qquad ....(1)$$

and from equation (a),

$$d_1 + d_3 + d_4 \geq 20 \qquad ....(2)$$

$$d_1 \geq 20 \qquad ....(3)$$

$$d_1 + d_2 + d_4 \geq 20 \qquad ....(4)$$

$$d_2 \geq 20 \qquad ....(5)$$

$$d_2 + d_3 + d_4 \geq 20 \qquad ....(6)$$

$$d_3 \geq 20 \qquad ....(7)$$

finally we have,

$$d_1 + d_2 + d_3 + d_4 = 120 \qquad ....(1)$$

- Our main motive is to find d_1, d_2, d_3, d_4 such that Z is minimum. That is, the total red light time is minimum.
- So, from equation (1) we get Z = 360 seconds, which means, the Z is not varying for our chosen graph. The minimum and maximum both are 360 seconds.
- The values of d_1, d_2, d_3, d_4 for maintaining $Z_{min} = 360$ are,

$$100 \geq d_1 \geq 20$$

$$100 \geq d_2 \geq 20$$

$$100 \geq d_3 \geq 20$$

$$100 \geq d_4$$

- Here we, get our Z as a constant value but using this method, we can find Z_{min}.

- Solving all equations from equation (1) to equation (7) we get Z_{min}.

6 Practical Interpretation of Application

The final answer of this problem explains us that we can curb even the heaviest traffic on this planet but not all the situations will be handled in an exact manner. In some places we may reduce the red light time drastically whereas somewhere, there might be hardly a chance to reduce it. The graph theory implemented here was of great use in clubbing up the streams into one place and for having a close observation of their flows.

This may work practically in some situations, where the volume of traffic and size of the road are nearly same but not always. With this solution, the vehicles now will be lot of relieved because whenever a systematic approach or a disciplined manner is adopted, every job gets done with very few errors.

7 Conclusion

The value of Z which we derived is the least time of red light of all streams. This describes that when we sum up all the red light for each individual streams, it comes out to be 360 seconds.

The solution of our chosen problem mentioned previously , is in itself an explanation for the implementation in real life. This solution gives us an implicit way to come out of this problem and as said before this problem excludes the volume of traffic and the size of roads into consideration.

So this directly will not function properly for present real time situations. But it does provide an efficient way to showcase the beauty of mathematics and usage of it in real world problems.

Bibliography

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