CSE 473/573 Home work 2

Problem 1: Determine the padding needed so that we can
a) keep the image size unchange before and after convolution

Solution: Given: input image $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

kernal: $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\therefore$ N be the image size $= 3$
F be the kernal size $= 3$
P is the padding $= ?$
stride movement in image $= 1$

$\therefore$ Output Image size $= 3$

$$= (N - F) + \frac{2P}{\text{stride}} + 1$$

$$3 = (3-3) + \frac{2P}{1} + 1$$

$$2 = 2P$$

Ans where $P = 1$ $\therefore$ Padding of 1 will keep the image size unchanged.

b) Determine the output of convolution

Assuming: stride = 1
padding = 0

$\therefore = (1 \times 3) + 1 \times 2$

$= 3 + 2 = \underline{\underline{5}}$

k_1 k_2 k_3

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Input Image

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

kernal

out put image

Ans $= \begin{bmatrix} 4 & 2 & 0 \\ 2 & 5 & 3 \\ 0 & 3 & 6 \end{bmatrix}$

Problem 2 Design a 5x5 filter which can shift image up by 2 pixels and to the left by 1 pixel

Solution:

5x5 filter which can shift image up by 2 pixel and to the left by 1 pixel

Assume:

~~Given~~:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad 5\times5$$

To move 2 pixel ~~down~~ up

To move 1 pixel left

As we know pixel movement works opposite

$\therefore$

Ans

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Problem 3

Solution

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-x^2 y^2}{2\sigma^2}}$$

$$G(x) = \frac{1}{2\pi\sigma^2} e^{-x^2/2\sigma^2}$$

$$G(y) = \frac{1}{2\pi\sigma^2} e^{-y^2/2\sigma^2}$$

$$2\pi\sigma^2 = 289$$
$$\sigma^2 = 6.78$$

Filter in x direction	Filter in Y direction
$= \frac{1}{6.78\sqrt{2\pi}} e^{-x^2/2.46}$	$= \frac{1}{6.78\sqrt{2\pi}} e^{-y^2/92.46}$
$= \frac{1}{6.78\sqrt{2\pi}} e^{-x^2/92}$	$= \frac{1}{6.78\sqrt{2\pi}} e^{-y/92}$

Now, 1D filter in x direction will be

$$x = \frac{1}{17}\begin{bmatrix} 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

1D filter in y direction will b

$$y = \frac{1}{17}\begin{bmatrix} 1 \\ 4 \\ 7 \\ 4 \\ 1 \end{bmatrix}$$

Example: if we multiply x and y we will get same matrix

$$x * y = \frac{1}{289}\begin{bmatrix} 1 \\ 4 \\ 7 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Ans

$$= \frac{1}{289}\begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 28 & 16 & 4 \\ 7 & 28 & 49 & 28 & 7 \\ 4 & 16 & 28 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Problem 4:

a) What is the value range for square difference and cross-corelation? Explain your Answer

Solution:

From eqn we know

$$\text{Range of SSD} = \sum_{i=1}^{n} \left(y^{(i)} - x^{(i)} \right)^2 \geq 0$$

$$= 0 \leq SSD < \infty$$

Here Y and x has similar pixel because

$$SSD \approx 0$$

$\therefore$ the difference b/w them $= 0$

Cross-correlation CC:

$$G(i,j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v)$$

$$= \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$\therefore$ it range from $\Rightarrow \underline{-1 \leq CC \leq +1}$

as max value is 1 and min value is -1 after normalized

where -1 shows dissimilar pixel b/w H and F

$+1$ show similar pixel b/w H and F