Solving Polynomial Equations Using Complex Roots

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December 19, 2025

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Catalogue

Fundamental Theorem of Algebra

Theorem 1

Let $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ be a polynomial of degree n with coefficient in $\mathbb{C}$. Then the polynomial equation $P(z) = 0$ has a root in $\mathbb{C}$.

Corollary

Theorem 2

Let $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ be a polynomial of degree n with coefficient in $\mathbb{C}$. Then the polynomial equation $P(z) = 0$ has n roots in $\mathbb{C}$, counting multiplicities.

Theorem

Theorem 3

Let $P(z)$ be a polynomial of degree n with coefficients in $\mathbb{R}$. If $\alpha = a + bi$ is a root of $P(z) = 0$ with $b \neq 0$, then $\overline{\alpha} = a - bi$ is also a root of $P(z) = 0$.

Viète Theorem

Theorem

Let $z_1, z2, \cdots, z_n$ be the n roots of the equation $a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = 0$, where $a_n \neq 0$. Then

$$z_1 + z_2 + \cdots + z_n = -\frac{a_{n-1}}{a_n}$$

$$\sum_{1 \leqslant j < k \leqslant n} = \frac{a_{n-2}}{a_n}$$

$$\vdots$$

$$\prod_{j=1}^{n} z_j = (-1)^n \frac{a_0}{a_n}$$

Examples

Example 1

Given that $2 - i$ is a root of the equation $z^4 - 3z^2 + az + 40 = 0$, where $a \in \mathbb{R}$. Find the value of a and the other roots of the equation.

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Answer

Since $2 - i$ is a root, $2 + i$ is a root.

$$\begin{aligned}
P(z) &= [z - (2 - i)][z - (2 + i)]Q(z) \\
&= (z^2 - 4z + 5)Q(z) \\
&= (z^2 - 4z + 5)(z^2 + 4z + 8) + (a + 12)z \\
&= [z - (2 - i)][z - (2 + i)][z - (-2 + i)][z - (-2 - i)]
\end{aligned}$$

So $a = -12$ and $z = 2 \pm i, -2 \pm i$

Examples

Example 2

Given that the three roots of the equation $z^3 + 6z^2 + 4z - 3 = 0$ are α, β, and γ. Find $\alpha + \beta + \gamma$.

Answer

Apply the Viete Theorem to get

$$\begin{aligned}\alpha + \beta + \gamma &= -\frac{a_{n-1}}{a_n} \\ &= -\frac{6}{-1} = -6\end{aligned}$$

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