CHEM 361B - Lecture 4 Activity

The First Modern Model of the Atom

1. One of the most powerful modern techniques for studying structure is neutron diffraction. This technique involves generating a collimated beam of neutrons at a particular temperature from a high-energy neutron source. The speed of a neutron is given by

$$v_n = \sqrt{\frac{3k_B T}{m_n}}$$

   where m_n is the mass of the neutron. If we want the beam of neutrons to have a a de Broglie wavelength of 50 pm, show that its temperature 2531 K.

2. There is an uncertainty principle for energy and time:

$$\Delta E \Delta t \geq h$$

   (a) Show that both sides of this expression have the same units

   (b) This relationship has been interpreted to mean that a particle of mass m ($E = mc^2$) can materialise from nothing provided that it returns to nothing within a time $\Delta t \leq \frac{h}{mc^2}$. Particles that last for time Δt or more are called *real particles*; particles that last less than time Δt are called *virtual particles*. The mass of a charged pion, a subatomic particle, is 2.5×10^{-28} kg. Show that the minimum lifetime of a real pion is $2.95 \times 10^{-23} s$.

3. When deriving the Bohr radius for a hydrogen atom, it was assumed that the nucleus (a proton) was fixed and the electron revolved around it. A better representation of reality is that both the electron and proton are rotating about each other as shown in Figure 1.

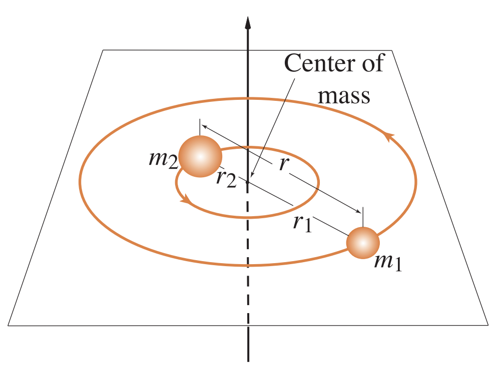

Figure 1: Two masses rotating about their centre of mass.

(a) The effective mass of the system can be expressed as

$$\mu = \frac{m_p m_e}{m_p + m_e}$$

Show that the reduced mass of the hydrogen atom is $9.104431272 \times 10^{-31}$ kg.

(b) (Challenge) Given that the Rydberg constant, R_H can now be expressed as

$$R_H = \frac{\mu e^4}{8\epsilon_0^2 c h^3}$$

show that using the reduced mass of the Hydrogen atom (μ) gives a Rydberg constant value of 109677.66 cm^{-1}. This is a much better estimation of the accepted experimental value of 109677.581 cm^{-1} over using the mass of the electron alone. To get an acceptable answer, use a precision to at least **7 decimal points** for all values.

4. A hydrogen-like ion is any atom with all of its electrons stripped away except for one (eg. Li^{2+}).

   (a) (Challenge) Starting with Newton's Second Law, and using Bohr's first postulate show that the radius of the orbit of an electron in the n^{th} Bohr orbit of a hydrogen-like ion with a nuclear charge Z is

   $$r = \frac{\epsilon_0 n^2 h^2}{\pi \mu Z e^2}$$

   (b) Show that the circumference of the ground state orbit of He^+ is 1.66×10^{-10} m.

   (c) Using the uncertainty principle, show the uncertainty on the speed of the electron is 2.817×10^8 ms^{-1} if the precision of measuring its position is 2% of the ground state orbit's circumference.

   (d) If the velocity of an electron in a hydrogen-like ion is given by

   $$v = \frac{Ze^2}{2\epsilon_0 n h}$$

   show that the velocity of the electron in the ground state orbit is 4.37×10^6 ms^{-1}. Compare this value to its uncertainty and comment on what the result means.

   (e) (Challenge) Show that the Bohr formula for $1/\lambda$ for a hydrogen-like atom of atomic number Z is

   $$\frac{1}{\lambda} = \frac{Z^2 e^4 \mu}{8\epsilon_0^2 c h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

   (f) Show that the wavelength of the photon that is required to ionize the electron from He^+ in the ground state is 2.279×10^{-8} m?