CHEM 361B - Lecture 7 Activity

Operators

1. Suppose that the particle in an infinite square potential is represented by the following wavefunction

$$\psi(x) = \begin{cases} \sqrt{\frac{30}{a^5}}\, x(a-x) & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that this wavefunction is normalised

   (b) Show that

$$\langle E \rangle = \frac{5\hbar^2}{ma^2}$$

2. In this problem we will explore commutation of angular momentum. The angular momentum operators are

$$\begin{aligned} \hat{L}_x &= -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) \\ \hat{L}_y &= -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) \\ \hat{L}_z &= -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \end{aligned}$$

   (a) Show that $[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$
   Hint: You will apply the product rule when evaluating the differential of one of the terms.

   (b) Given that

$$\begin{aligned} [\hat{L}_x, \hat{L}_y] &= i\hbar\hat{L}_z \\ [\hat{L}_z, \hat{L}_x] &= i\hbar\hat{L}_y \end{aligned}$$

   what does these three commutation relationships say about the ability to measure the components of the angular momentum simultaneously?

   (c) If the operator for the total angular momentum can be determined by

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

   show that

$$[\hat{L}^2, \hat{L}_z] = 0$$

   What does this result mean? Hint: When solving this, keep the $\hat{L}$ notation and use the above commutation relations.