Probability

1. Using the data from the ages of the participants in the class, determine $\langle ages \rangle$, $\langle ages^2 \rangle$, σ^2_{ages} and σ_{ages}

2. Given the probability distribution

$$y(x) = \begin{cases} B^2 \sin^2\left(\frac{\pi x}{a}\right) & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

Show that

(a) $B = \sqrt{\frac{2}{a}}$

Hint:

$$\int \sin^2 \alpha x \; dx = \frac{x}{2} - \frac{\sin(2\alpha x)}{4\alpha}$$

(b) $\langle x \rangle = \frac{a}{2}$

(c) $\langle x^2 \rangle = a^2 \left(\frac{1}{3} - \frac{1}{2\pi^2}\right)$

Hint:

$$\int x^2 \sin^2(\alpha x) \; dx = \frac{x^3}{6} - \frac{x^2 \sin(2\alpha x)}{4\alpha} + \frac{\sin(2\alpha x)}{8\alpha^3} - \frac{x \cos(2\alpha x)}{4\alpha^2}$$

(d) $\sigma_x^2 = \frac{a^2}{2}\left(\frac{1}{6} - \frac{1}{\pi^2}\right)$

(e) $\sigma_x = a\sqrt{\frac{1}{2}\left(\frac{1}{6} - \frac{1}{\pi^2}\right)}$

(f) $\text{Prob}(\frac{a}{4} \leq x \leq \frac{3a}{4}) = \frac{1}{2} + \frac{1}{\pi}$

3. (Challenge) Show that

$$\sum_{j=1}^{n}(x_j - \langle x \rangle)^2 p(x_j) = \langle x^2 \rangle - \langle x \rangle^2$$