CHEM 361B - Lecture 8 Activity

Finite Potentials and Tunneling

1. A particle trapped in a finite potential. A plot of $\tan ka$, $-\cot ka$, and $\frac{\alpha}{k}$ gives:

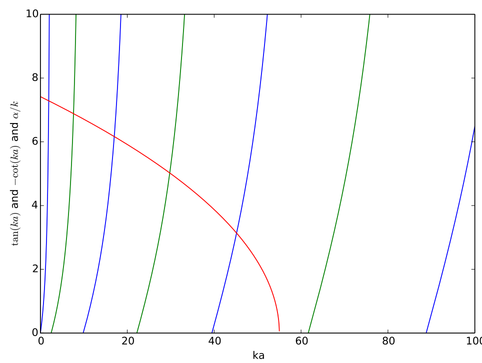

Figure 1: A plot which represents the energy of the bound states of a particle trapped in a finite potential well.

   (a) How many bound states are there?

   (b) If the depth of the well were to increase, would it increase or decrease the number of bound states? Does that shift the $\frac{\alpha}{k}$ line to the left or to the right?

   (c) What is the minimum number of bound states in a finite potential well? How do you know?

2. You are running an STM and set the surface/tip distance to be 2 Å. The tunneling current is given by

$$I \propto e^{-d}$$

   where d is the separation distance between the tip and the surface.

   (a) Show that the current will decrease by a factor of 149 to when the tip is moved away from the surface by an additional 5 Å.

   (b) You moved the tip too far by moving it to 7 Å away from the surface. Show that you should have moved it 5.91Åfrom the surface tip if you wanted to only decrease the tunneling current by a factor of 50 relative to the tunneling current at 2 Å.

3. A particle traveling in a region of space where there is no potential comes across a stepwise potential. The stepwise potential is defined by

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x > 0 \end{cases}$$

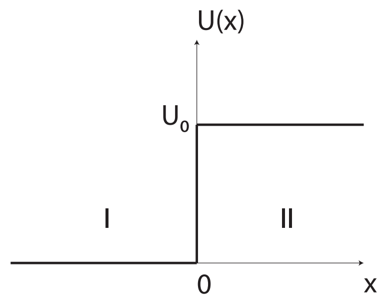

Figure 2: An image of the stepwise potential

(a) Assume that $E > U_0$ and solve the Schrödinger Equation in regions I and II to get

$$\begin{aligned} I: \quad & \psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x} \\ II: \quad & \psi_{II}(x) = Ce^{ik_2x} \end{aligned}$$

where

$$k_1 = \frac{\sqrt{2mE}}{\hbar} \quad \text{and} \quad k_2 = \frac{\sqrt{2m(E-U_0)}}{\hbar}$$

Why can we eliminate the e^{-ik_2x} term in region II?

(b) Apply the boundary conditions that $\psi_I(0) = \psi_{II}(0)$ and $\frac{d\psi_I}{dx}\Big|_{x=0} = \frac{d\psi_{II}}{dx}\Big|_{x=0}$ to get

$$A + B = C \quad \text{and} \quad k_1(A - B) = k_2C$$

(c) (Challenge) If the probability of the particle moving past the step is determined by $t = k_2|C|^2/k_1|A|^2$ and the probability of the particle being reflected by the barrier is $r = k_1|B|^2/k_1|A|^2$, show that

$$\begin{aligned} t =& \frac{4k_1k_2}{(k_1+k_2)^2} \\ r =& \frac{(k_1-k_2)^2}{(k_1+k_2)^2} \end{aligned}$$

(d) What happens when you add $t + r$. Does this result make sense? Explain.

(e) What happens to t and r as $U_0 \to 0$. Does this result make sense? Explain.