CHEM 361B - Lecture 1 Activity

Calculus Review and Complex Numbers

1. Integrate

   (a) $\int_0^a \frac{2x}{a} \sin^2(\frac{3\pi x}{a})\ dx$ where a is a constant

   Hint:
   $$\int \sin^2 \alpha x\ dx = \frac{x}{2} - \frac{\sin(2\alpha x)}{4\alpha}$$

   (b) $\int_{-\infty}^{\infty} \sqrt{\frac{m\omega}{\pi\hbar}} x^2 e^{\frac{-m\omega}{\hbar}x^2}\ dx$ where m, ω and $\hbar$ are constants

   Hint:
   $$\int_{-\infty}^{\infty} x^2 e^{-ax^2} = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

   (c) $\int_0^{2\pi} \sin(\theta)\cos^2(\theta)\ d\theta$

2. Visualising complex numbers:

   (a) Draw an x and y axis and label the x-axis as 'Real', and the y-axis as 'Complex'.

   (b) On this plot, mark 1, 2 and 1+2.

   (c) On this plot, mark 1+i, 2+2i, and (1+i)+(2+2i)

3. If $z = 3 + 4i$, then show

   (a) Re(zz^*)=25

   (b) Im(zz^*)=0

4. Show that the following complex numbers in the form $re^{i\theta}$ are

   (a) $1 + 3i = \sqrt{10}e^{1.25i}$

   (b) $e + \pi i = \sqrt{e^2 + \pi^2}e^{0.86i}$

5. Show that $e^{-\pi i} + e^{6\pi i} = 0$

6. This problem offers a derivation of Euler's formula.

   (a) Start with
   $$f(\theta) = \ln(\cos\theta + i\sin\theta) \tag{1}$$

   Show that
   $$\frac{df}{d\theta} = i \tag{2}$$

   Hint: After you take the derivative of equation 1 you will find it useful to multiply it by $\frac{i}{i}$ to finish this step.

(b) Now integrate both sides of equation 2 to obtain equation 3:

$$f(\theta) = \ln(\cos\theta + i\sin\theta) = i\theta + C \tag{3}$$

where C is a constant of integration. Take the exponential of both sides of equation 3 to get something that looks like Euler's Formula. To complete the proof show that C is related to the radius of the vector that describes the complex number in polar coordinates. If the radius is equal to 1 (like it does for Euler's Formula) what must C be equal to?

7. Consider the set functions

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} \qquad \begin{cases} m = 0, \pm 1, \pm 2, \ldots \\ 0 \le \phi \le 2\pi \end{cases}$$

Show that

$$\int_0^{2\pi} \Phi_m^*(\phi)\Phi_n(\phi)\, d\phi = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$

8. Show that $i^i = e^{-\pi/2}$