Swap_Pricing

December 18, 2017

1 Get discount factor for JPY

- input: MoneyMarket (short term interest rate), Swap rate.
- output: discount factors for each tenor listed by MoneMarket and Swap rate.

1.1 Pricing

1.1.1 Swap pricing formula

The value of the exchange between a floot and a fixed side is given by

$$V = \sum_{i=1}^{N} L(t_{i-1}, t_i) \times DF(t_i) \times \delta_i - \sum_{i=1}^{N} SwapRate \times DF(t_i) \times \delta_i,$$

where $L(t_{i-1}, t_i)$ is the floot interest rate between t_{i-1} and t_i, $DF(t_i)$ is a discount factor, δ_i is a day-count-fraction and $SwapRate$ is a Swap rate which means a par rate for a swap trade.

1.1.2 Bootstrap method for getting discount factors

Discount factors as of today can be estimated from a par swap trade which corresponds to $V = 0$ under swap pricing formula. For example, let us consider a swap trade with maturity of 1.5 year. The discount factor for 1.5 year $DF(t_{1.5Y})$ is calculated by solveing the following equation:

$$\sum_{i=1}^{3} L(t_{i-1}, t_i) \times DF(t_i) \times \delta = \sum_{i=1}^{3} SwapRate(1.5Y) \times DF(t_i) \times \delta$$

where a quoted swap rate is used for $SwapRate(1.5Y)$, the day-count-fraction δ is asuumed 6 month and the floot side interest rate is assumed that a following model expressed as

$$L(t_{i-1}, t_i) = \frac{1}{\delta}\Big(\frac{DF(t_{i-1})}{DF(t_i)} - 1\Big).$$

The above equation can be solved by using $DF(t_{0.5Y})$, $DF(t_{1.0Y})$ and the floot interest rate which is defined as above equation. As a result, the discount factor $DF(t_{1.5Y})$ is given by

$$DF(t_{1.5Y}) = \frac{1}{(1 + \delta \times SwapRate(1.5Y))}\Big(DF(t_0) - SwapRate(1.5Y) \times \delta \times \big(DF(t_{0.5Y}) + DF(t_{1.0Y})\big)\Big),$$

where $DF(t_{0.5Y})$ and $DF(t_{1.0Y})$ is calculated by using a quoted LIBOR (the rate of Money Market). The short rate of Money Market means spot rate, where the cashflows is expressed as only two terms. For example, $DF(t_{0.5Y})$ is given by

$$DF(t_{0.5Y}) = \frac{1}{(1 + \delta \times L(0.0Y, 0.5Y))},$$

where $L(0.0Y, 0.5Y)$ is the LIBOR rate between today and 6 month later. Discount factors after $t_{1.5Y}$ can be calculated by the same way as the derivation of $DF(t_{1.5Y})$. This method of getting discount factors gradually is called Bootstrap method.

In [11]:
```
import matplotlib.pyplot as plt
import numpy as np
import datetime

class getDF_moneymarket:
    def __init__(self, libor_rate, start_day, end_day):
        self.libor_rate = libor_rate
        self.start_day = start_day
        self.end_day = end_day
        self.datetime_obj_start = datetime.datetime.strptime(start_day, '%Y/%m/%d')
        self.datetime_obj_end = datetime.datetime.strptime(end_day, '%Y/%m/%d')
        self.daycount = (self.datetime_obj_end - self.datetime_obj_start).days / 360
        self.discount_factor = 0

    def getDF(self):
        self.discount_factor = [1 / (1 + self.daycount * self.libor_rate), self.start_d
        return self.discount_factor
```

In [12]:
```
DF = getDF_moneymarket(0.2, '2017/12/18', '2019/12/30')
print(DF.discount_factor)
print(DF.getDF())
print(DF.discount_factor)
```

```
0
[0.7081038552321007, '2017/12/18', '2019/12/30']
[0.7081038552321007, '2017/12/18', '2019/12/30']
```

In [3]:
```
DF1 = getDF_moneymarket(0.3, '2017/12/18', '2018/3/20')
DF1.getDF()
```

Out[3]:
```
0.9287925696594427
```