

Swap_Pricing

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1 Get discount factor for JPY

- input: MoneyMarket (short term interest rate), Swap rate.
- output: discount factors for each tenor listed by MoneyMarket and Swap rate.

1.1 Pricing

1.1.1 Swap pricing formula

The value of the exchange between a float and a fixed side is given by

$$V = \sum_{i=1}^N L(t_{i-1}, t_i) \times DF(t_i) \times \delta_i - \sum_{i=1}^N SwapRate \times DF(t_i) \times \delta_i,$$

where $L(t_{i-1}, t_i)$ is the float interest rate between t_{i-1} and t_i , $DF(t_i)$ is a discount factor, δ_i is a day-count-fraction and $SwapRate$ is a Swap rate which means a par rate for a swap trade.

1.1.2 Bootstrap method for getting discount factors

Discount factors as of today can be estimated from a par swap trade which corresponds to $V = 0$ under swap pricing formula. For example, let us consider a swap trade with maturity of 1.5 year. The discount factor for 1.5 year $DF(t_{1.5Y})$ is calculated by solving the following equation:

$$\sum_{i=1}^3 L(t_{i-1}, t_i) \times DF(t_i) \times \delta = \sum_{i=1}^3 SwapRate(1.5Y) \times DF(t_i) \times \delta$$

where a quoted swap rate is used for $SwapRate(1.5Y)$, the day-count-fraction δ is assumed 6 month and the float side interest rate is assumed that a following model expressed as

$$L(t_{i-1}, t_i) = \frac{1}{\delta} \left(\frac{DF(t_{i-1})}{DF(t_i)} - 1 \right).$$

The above equation can be solved by using $DF(t_{0.5Y})$, $DF(t_{1.0Y})$ and the float interest rate which is defined as above equation. As a result, the discount factor $DF(t_{1.5Y})$ is given by

$$DF(t_{1.5Y}) = \frac{1}{(1 + \delta \times SwapRate(1.5Y))} \left(DF(t_0) - SwapRate(1.5Y) \times \delta \times (DF(t_{0.5Y}) + DF(t_{1.0Y})) \right),$$

where $DF(t_{0.5Y})$ and $DF(t_{1.0Y})$ is calculated by using a quoted LIBOR (the rate of Money Market). The short rate of Money Market means spot rate, where the cashflows is expressed as only two terms. For example, $DF(t_{0.5Y})$ is given by

$$DF(t_{0.5Y}) = \frac{1}{(1 + \delta \times L(0.0Y, 0.5Y))},$$

where $L(0.0Y, 0.5Y)$ is the LIBOR rate between today and 6 month later. Discount factors after $t_{1.5Y}$ can be calculated by the same way as the derivation of $DF(t_{1.5Y})$. This method of getting discount factors gradually is called Bootstrap method.