CS 106B

Lecture 24: Depth First and Breadth First Searching

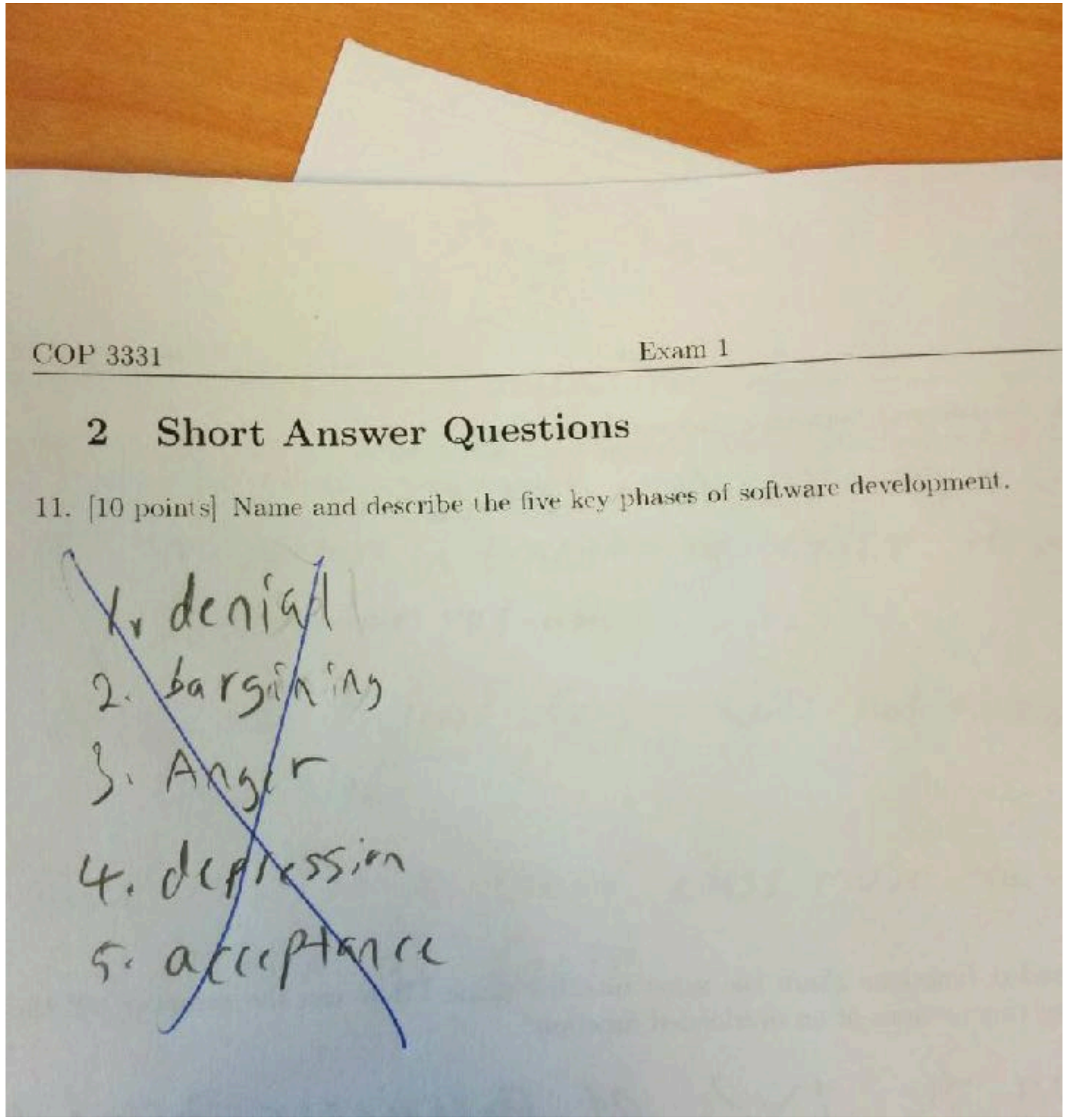
Friday, May 26, 2017

Programming Abstractions
Spring 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 18.6

At this point in the quarter…



https://i.redd.it/e5uylwsqzizx.jpg

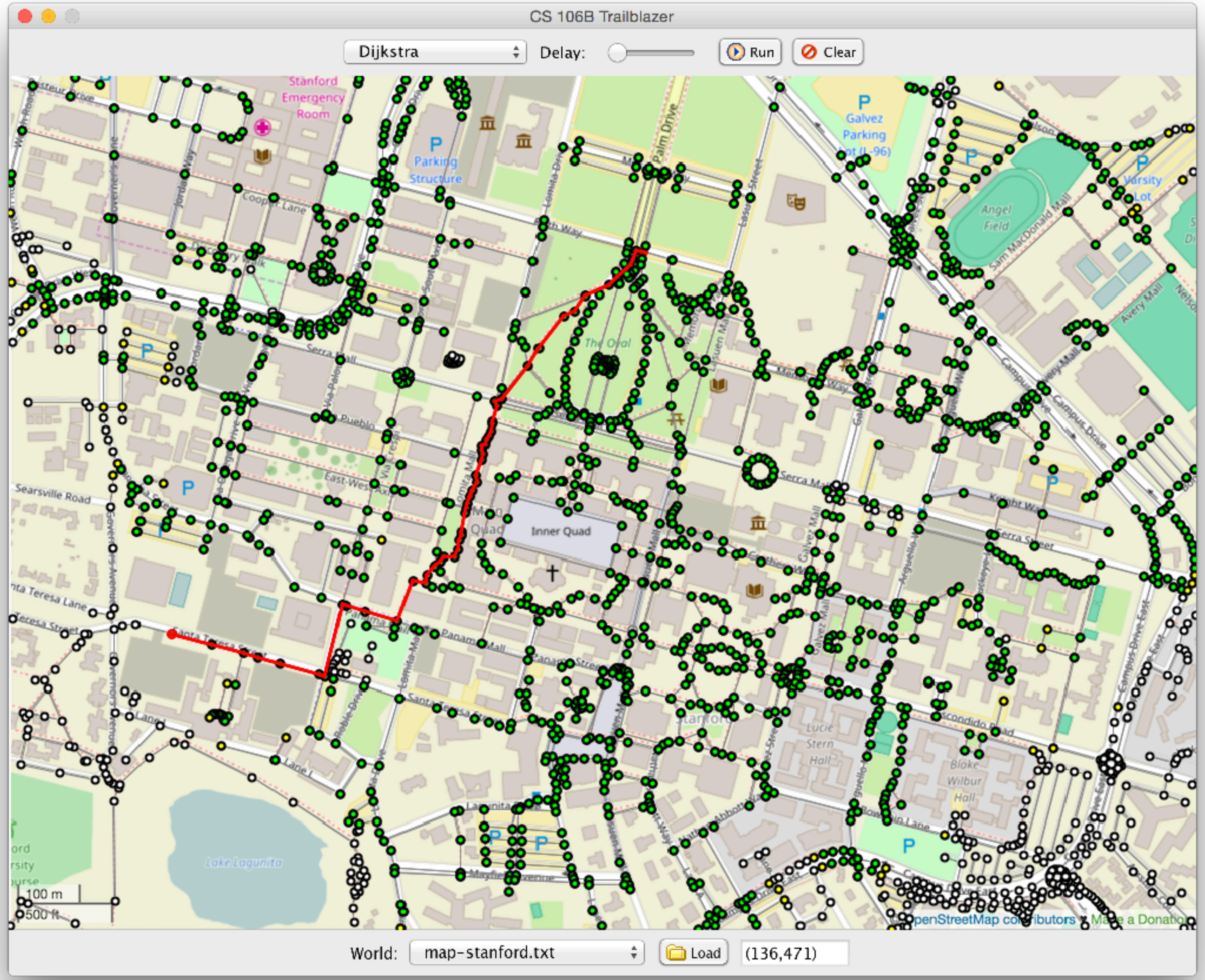
Today's Topics

- Logistics
  - Trailblazer: Will be due on the last day of classes, no late days allowed.

  - There was some Tiny Feedback about an SL who "*typically delays the grading and feedback process a few weeks after the homework is due*…" and "*told us that in the beginning of the quarter that we can use late credit for the Assignment 7*" — this is **bad**. We need to know as early as possible if SLs aren't holding IGs or doing grading on time, or is passing on mis-information. Please email me directly, or use sayat.me/chrisgregg to let me know these things!

- More on Graphs (and a bit on Trees)
  - Depth First Search
  - Breadth First Search

Trailblazer

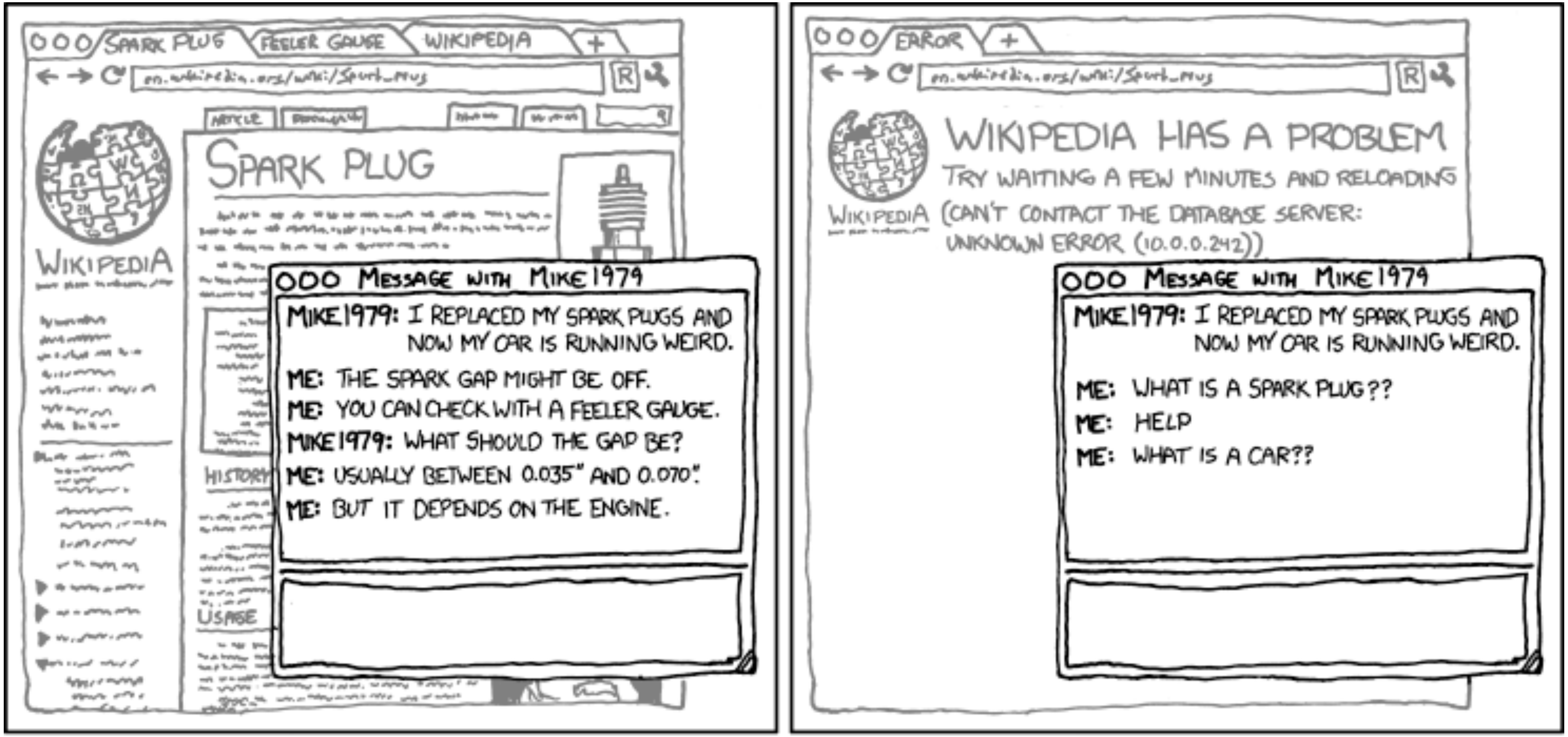


You create Google Maps!

You need to implement four different (but related) types of searches:

- Breadth First Search (today)
- Dijkstra (Wednesday, but will have an additional video by Saturday)
- A* (Wednesday, will also be covered in additional video)
- Alternate (you must determine algorithm)

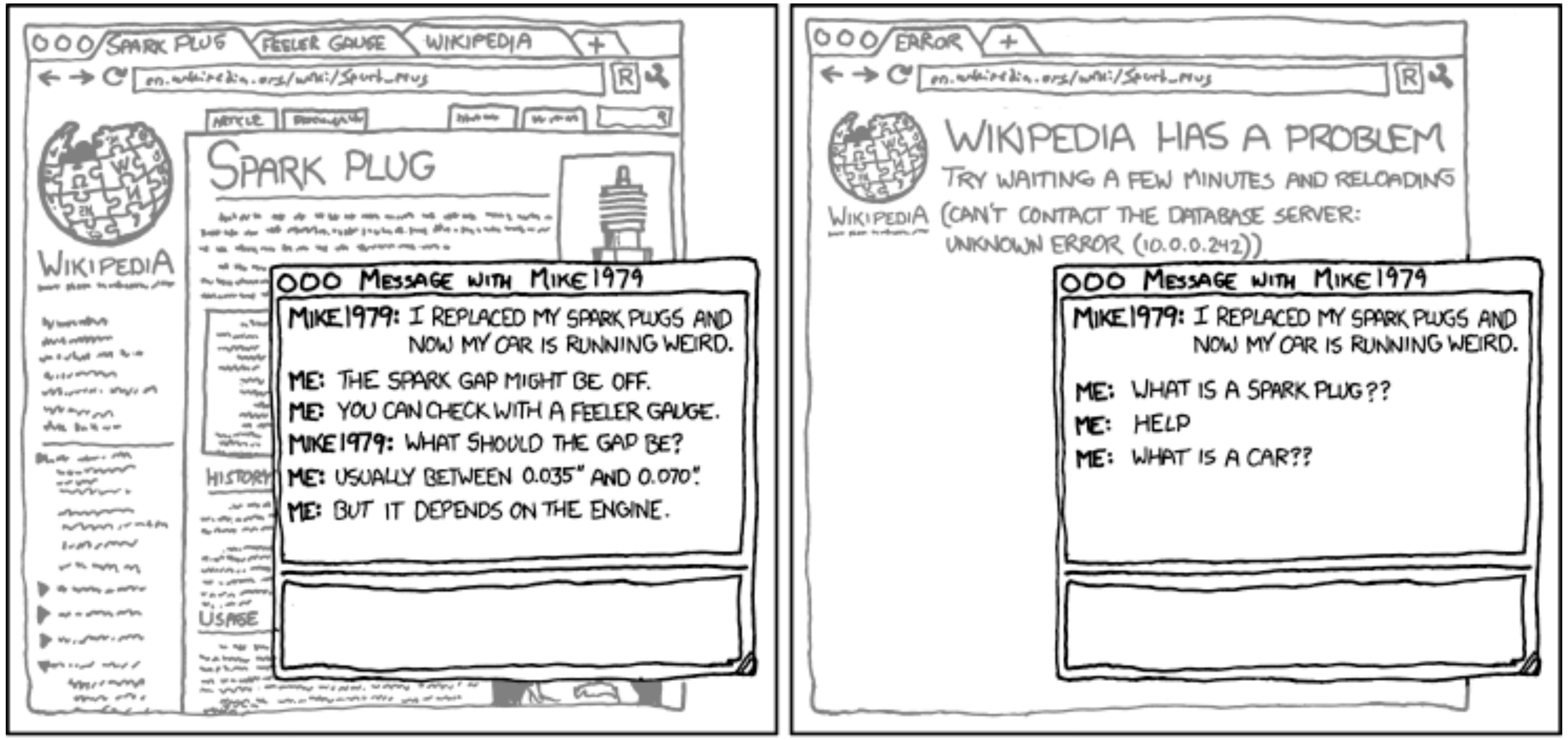
Wikipedia



WHEN WIKIPEDIA HAS A SERVER OUTAGE, MY APPARENT IQ DROPS BY ABOUT 30 POINTS.

XKCD 903, Extended Mind, http://xkcd.com/903/

Wikipedia



WHEN WIKIPEDIA HAS A SERVER OUTAGE, MY APPARENT IQ DROPS BY ABOUT 30 POINTS.

When you hover over an XKCD comic, you get an extra joke:

Wikipedia trivia: if you take any article, click on the first link in the article text not in parentheses or italics, and then repeat, you will eventually end up at "Philosophy".

XKCD 903, Extended Mind, http://xkcd.com/903/

Wikipedia

Wikipedia trivia: if you take any article, click on the first link in the article text not in parentheses or italics, and then repeat, you will eventually end up at "Philosophy".

Is this true??

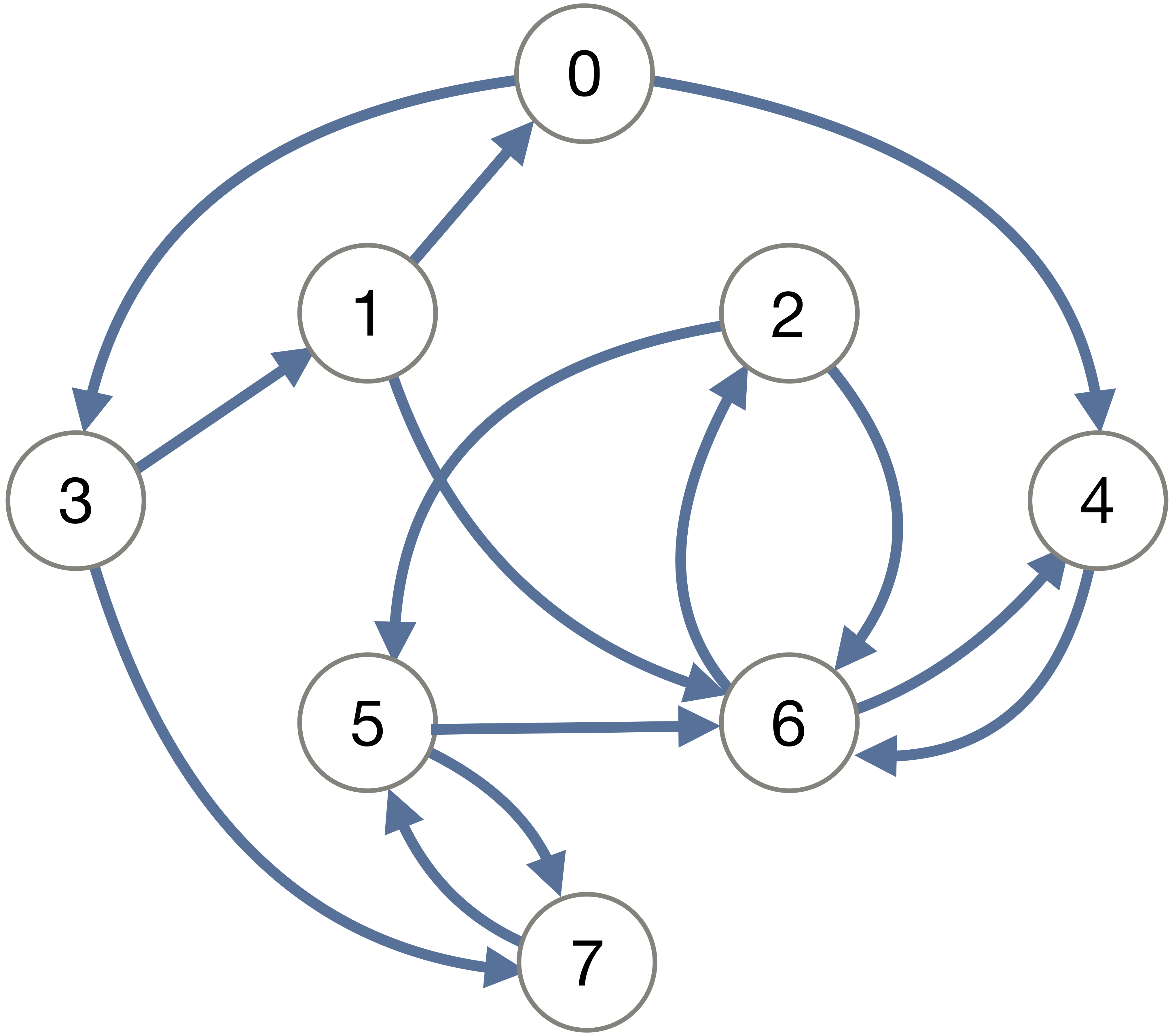
According to the Wikipedia article "Wikipedia:Getting to Philosophy" (so meta), (https://en.wikipedia.org/wiki/Wikipedia:Getting_to_Philosophy):

As of February 2016, 97% of all articles in Wikipedia eventually lead to the article Philosophy.

How can we find out? We shall see!

Graph Searching

Recall from the last couple of lectures that a *graph* is the "wild west of trees" — graphs relate *vertices* (nodes) to each other by way of *edges*, and they can be directed or undirected. Take the following directed graph:



A search on this graph starts at one vertex and attempts to find another vertex. If it is successful, we say there is a path from the start to the finish vertices.

What paths are there from 0 to 6?

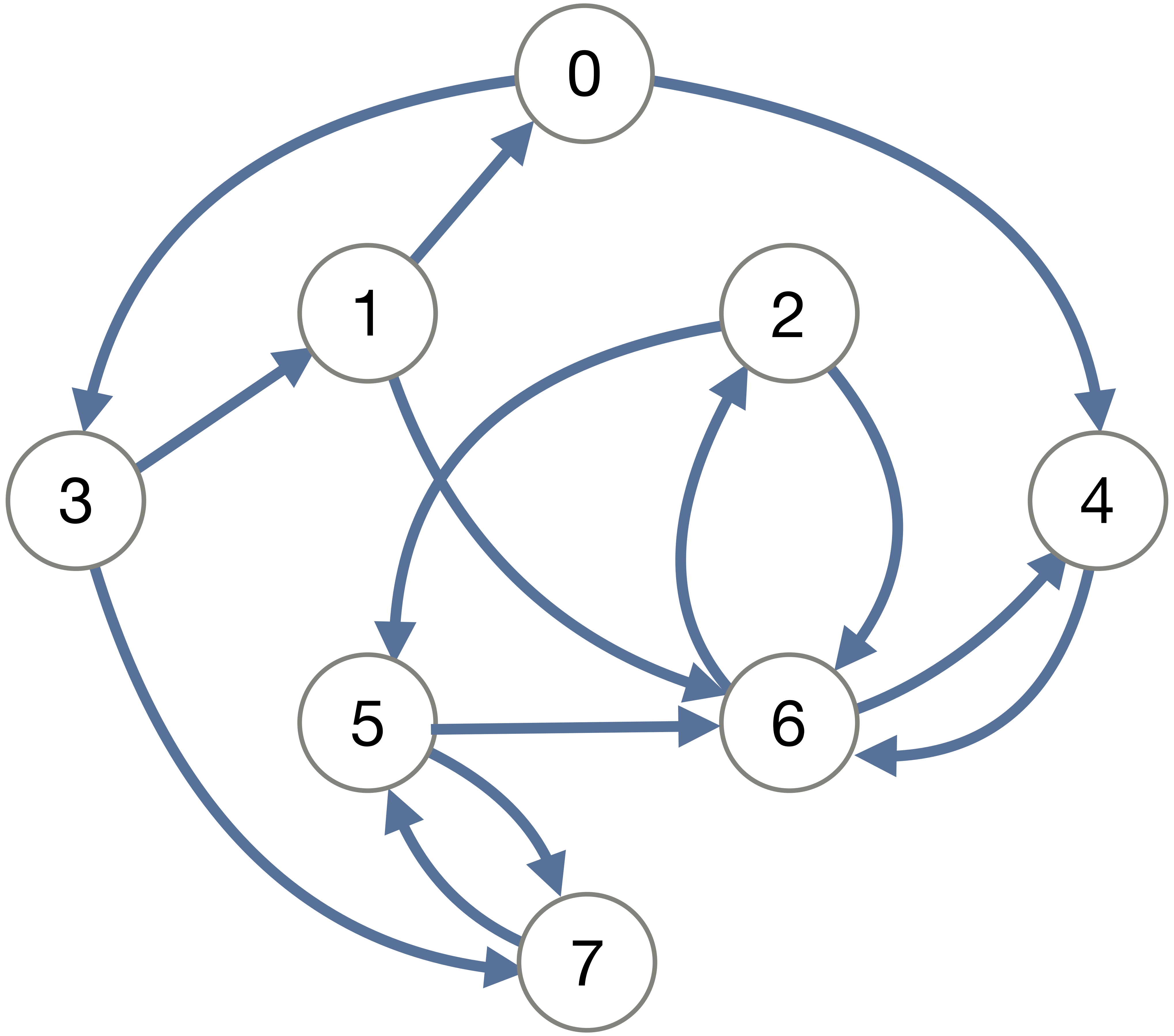
0☞4☞6

0☞3☞1☞6

0☞3☞7☞5☞6

Graph Searching

What paths are there from 3 to 2?



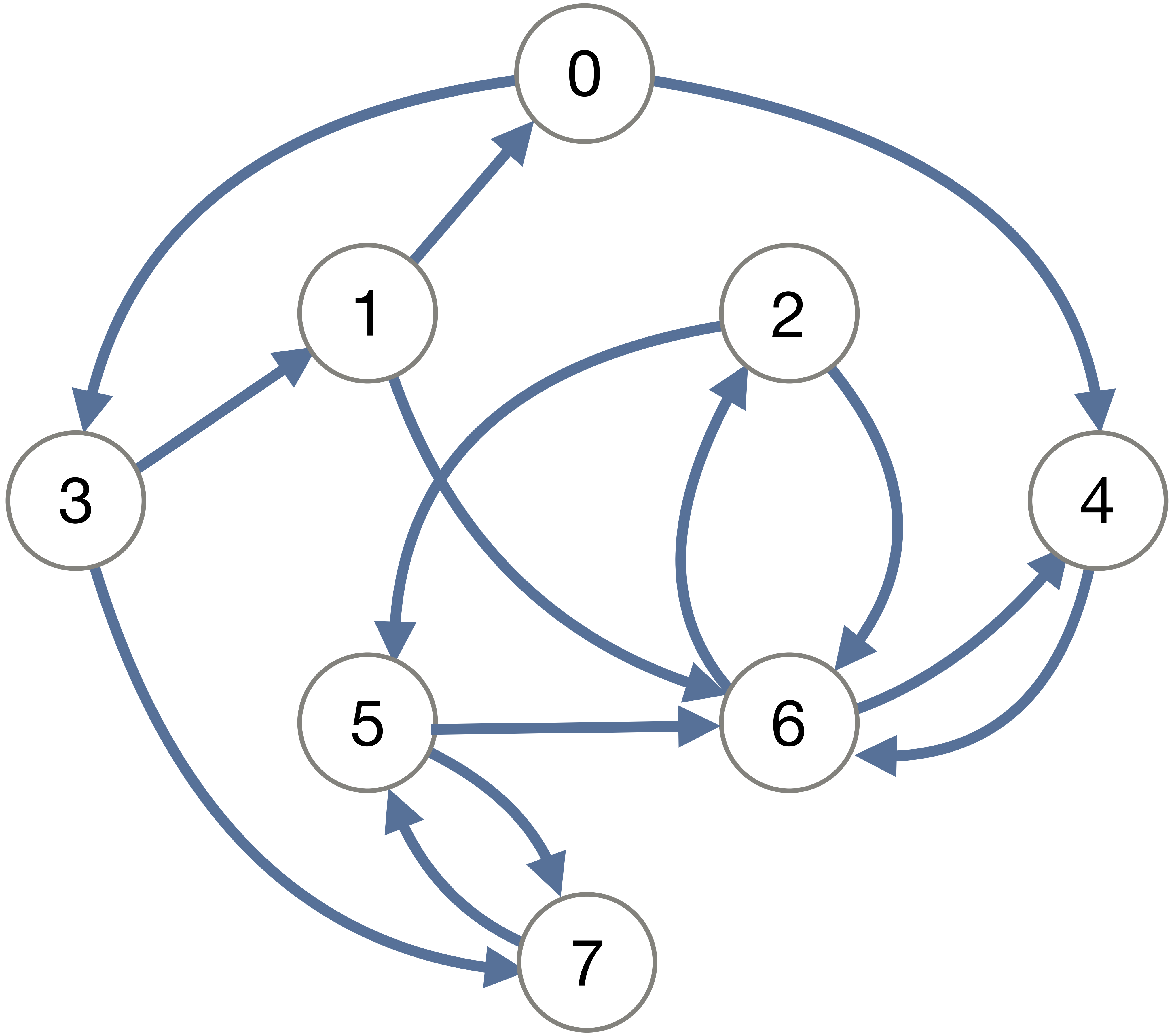
3 ☞ 1 ☞ 6 ☞ 2

3 ☞ 7 ☞ 5 ☞ 6 ☞ 2

3 ☞ 1 ☞ 0 ☞ 4 ☞ 6 ☞ 2

Graph Searching

What paths are there from 4 to 1?



`None!  :(`

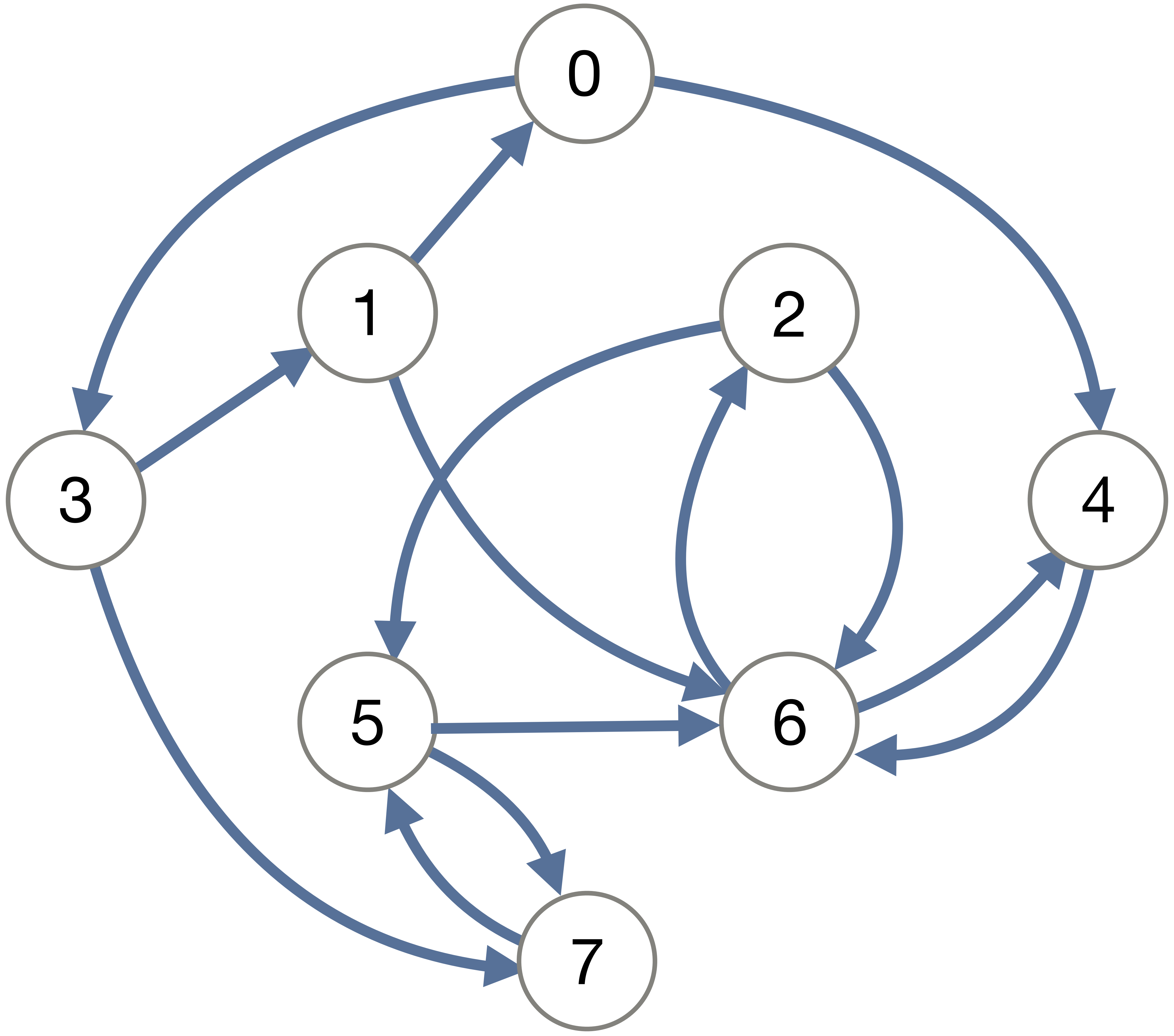
Graph Searching

We have different ways to search graphs:

- **Depth First Search**: From the start vertex, explore as far as possible along each branch before backtracking.

- **Breadth First Search**: From the start vertex, explore the neighbor nodes first, before moving to the next level neighbors.

Both methods have pros and cons — let's explore the algorithms.



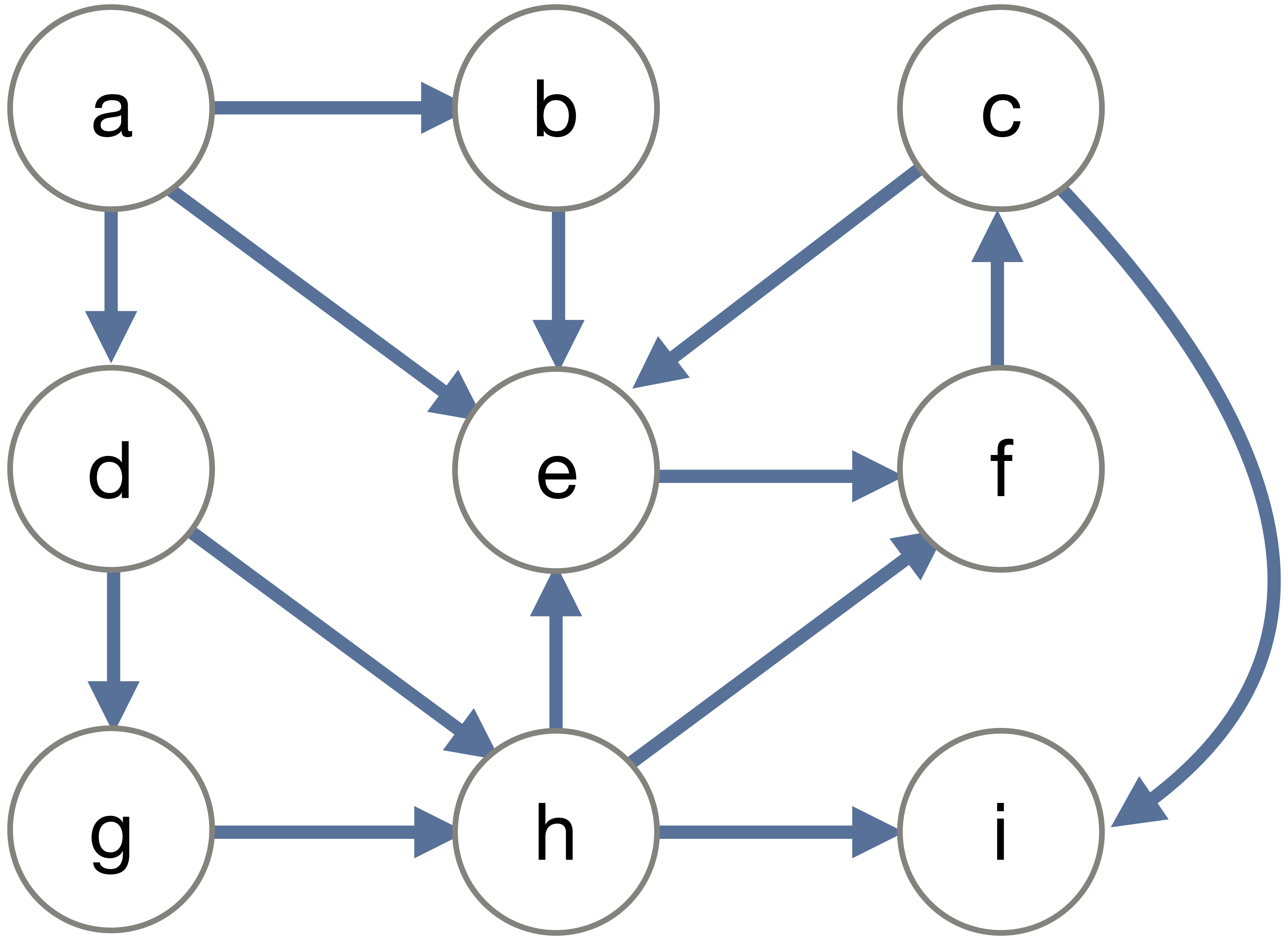
Depth First Search (DFS)

From the start vertex, explore as far as possible along each branch before backtracking.

This is often implemented recursively. For a graph, you *must mark visited vertices*, or you might traverse forever (e.g., c☞e☞f☞c☞e…)

DFS from a to h (assuming a-z order) visits:

a☞
b☞
e☞
f☞
c☞
i (dead end — back to c,f,e,b,a)
d☞
g☞
h



Notice: not the shortest!

path found: a☞d☞g☞h

Depth First Search (DFS): Recursive pseudocode

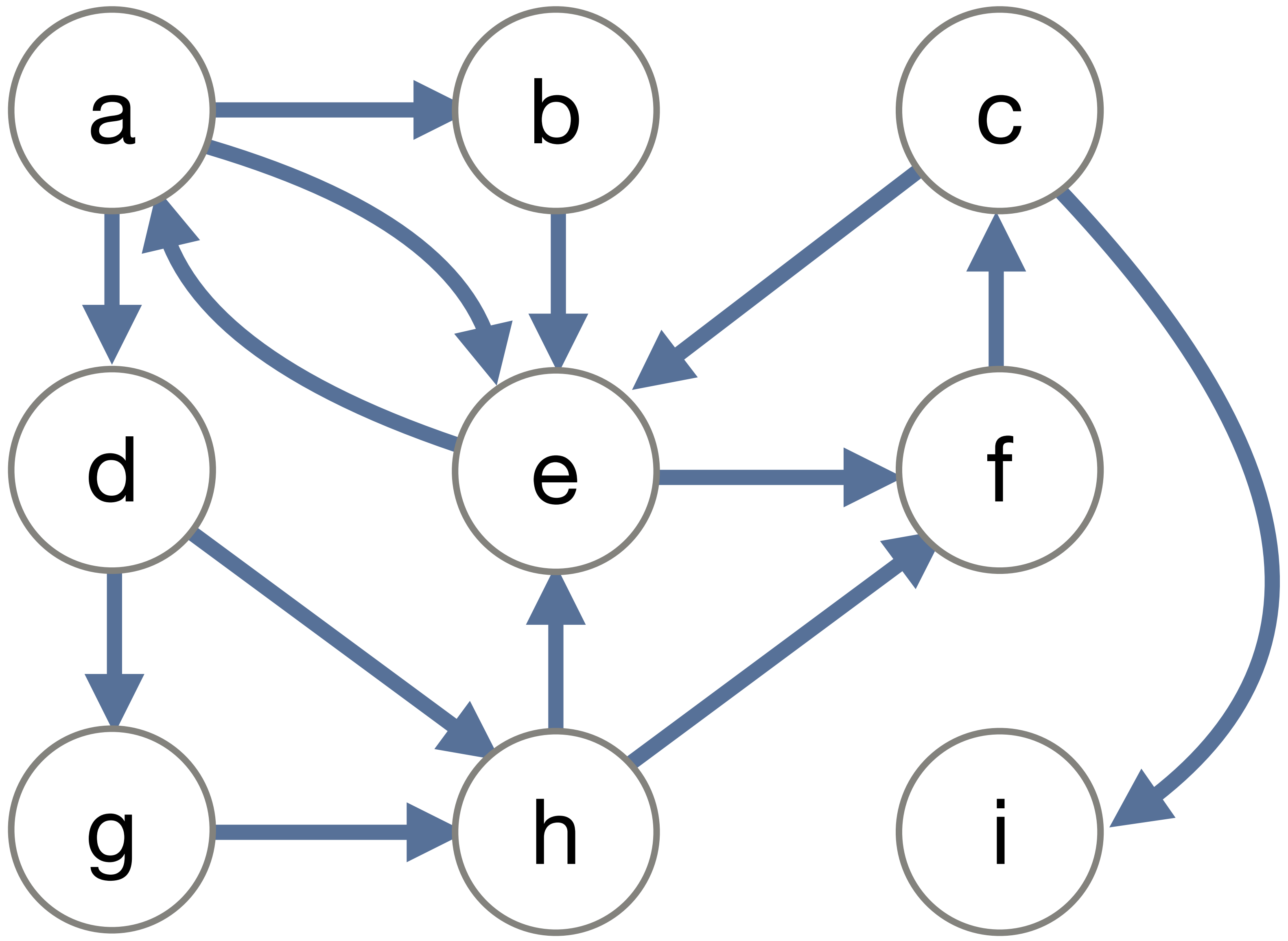
dfs from v_1 to v_2:

base case: if at v_2, found!

mark v_1 as visited.

for all edges from v_1 to its neighbors:

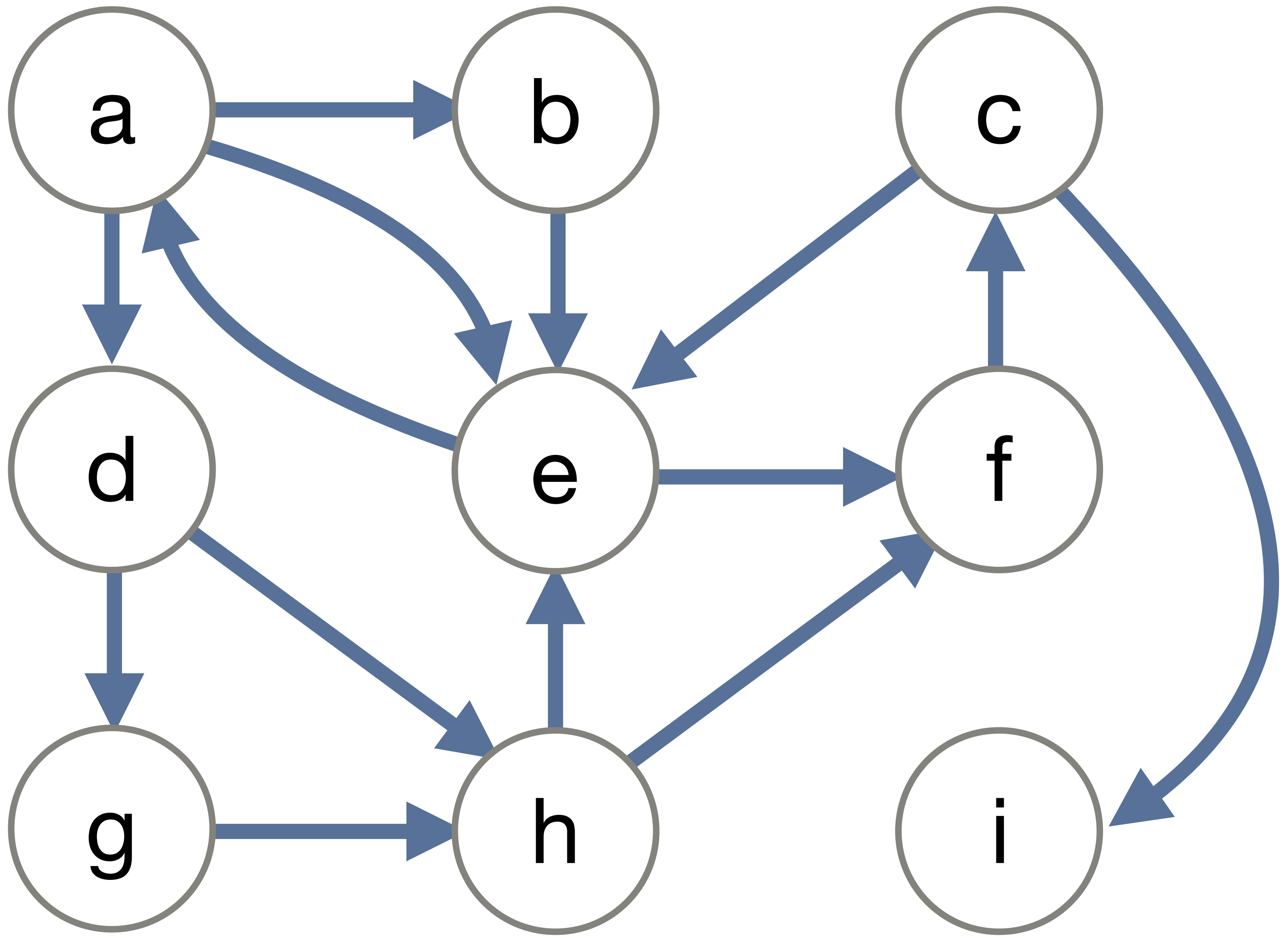
if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:
  mark v_1 as visited.
  for all edges from v_1 to its neighbors:
    if neighbor n is unvisited, recursively call **dfs**(n, v_2).

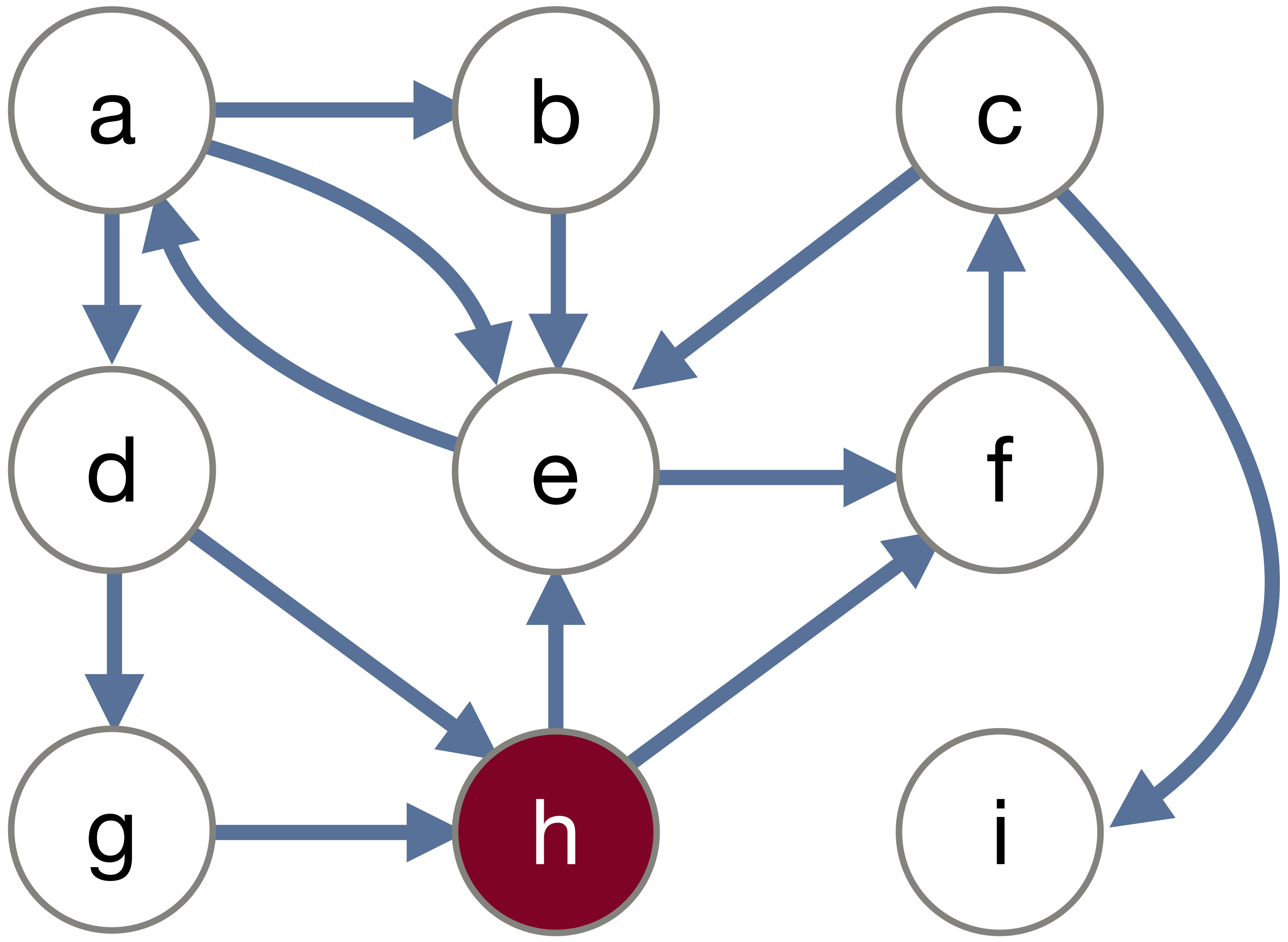
Let's look at **dfs** from h to c:



Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	false
i	false

Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:
    mark v_1 as visited.
    for all edges from v_1 to its neighbors:
        if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(h,c)

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	true
i	false

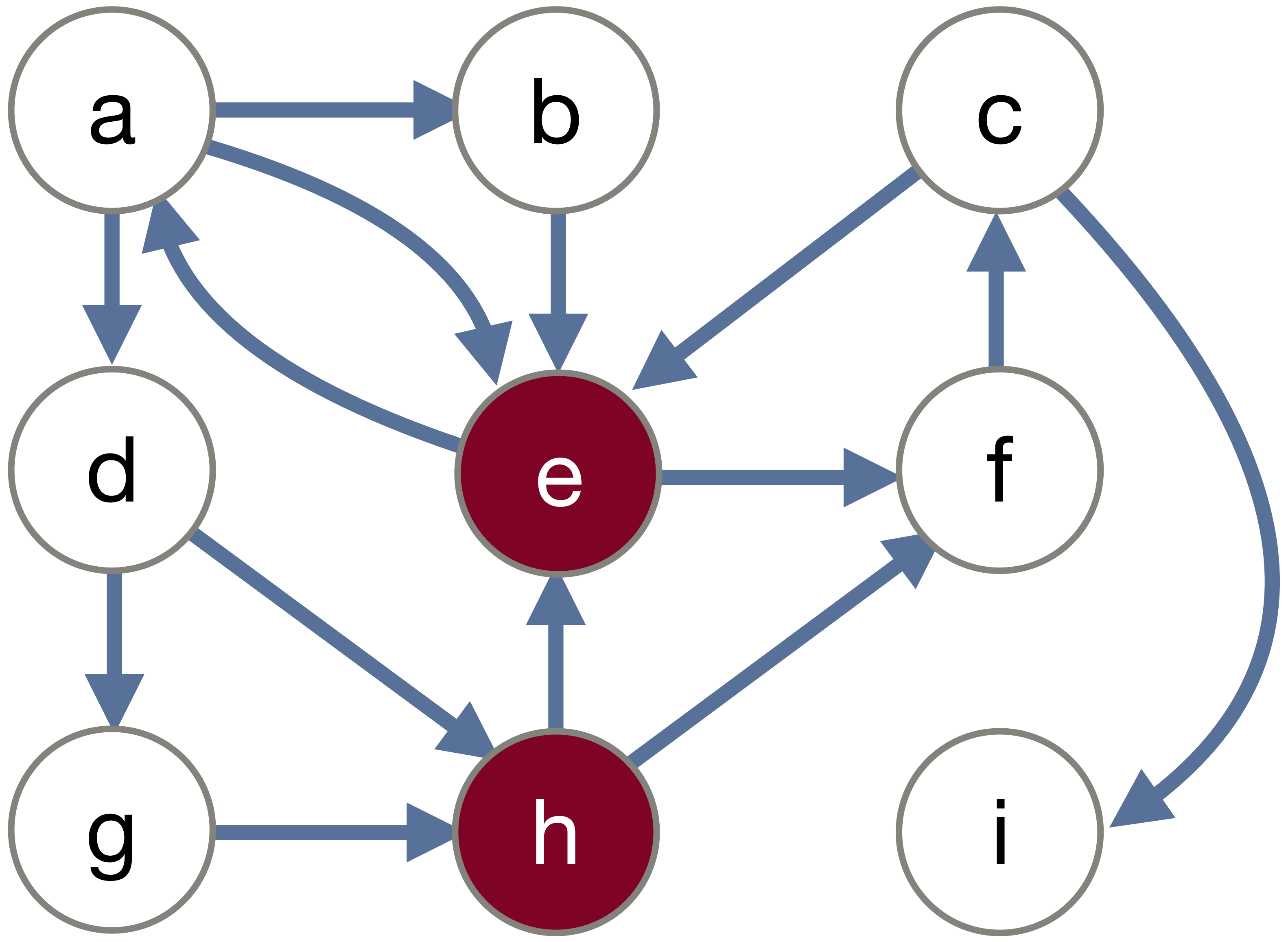
Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(e,c)
dfs(h,c)

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	true
f	false
g	false
h	true
i	false

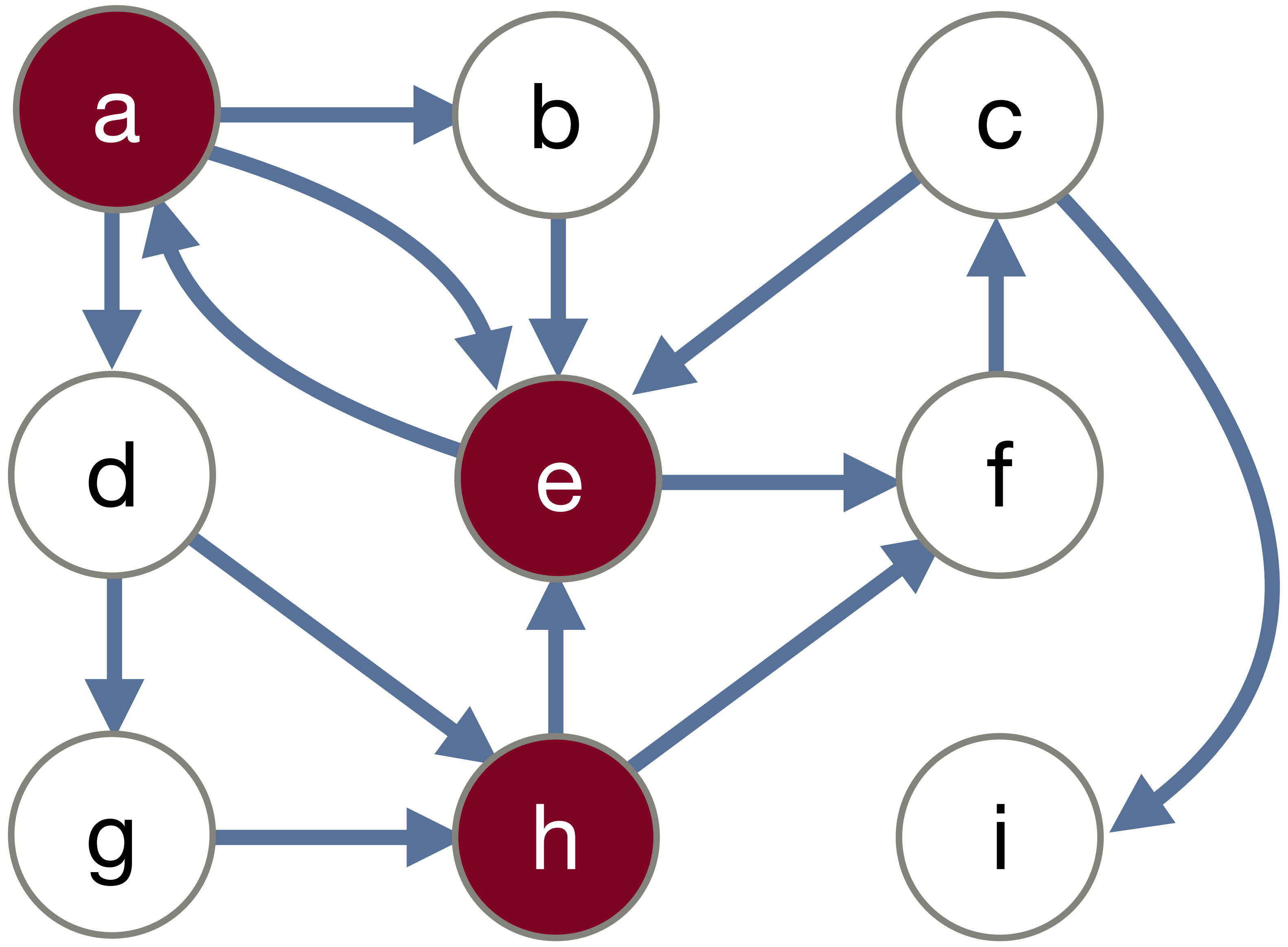
Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex Map

Vertex	Visited?
a	true
b	false
c	false
d	false
e	true
f	false
g	false
h	true
i	false

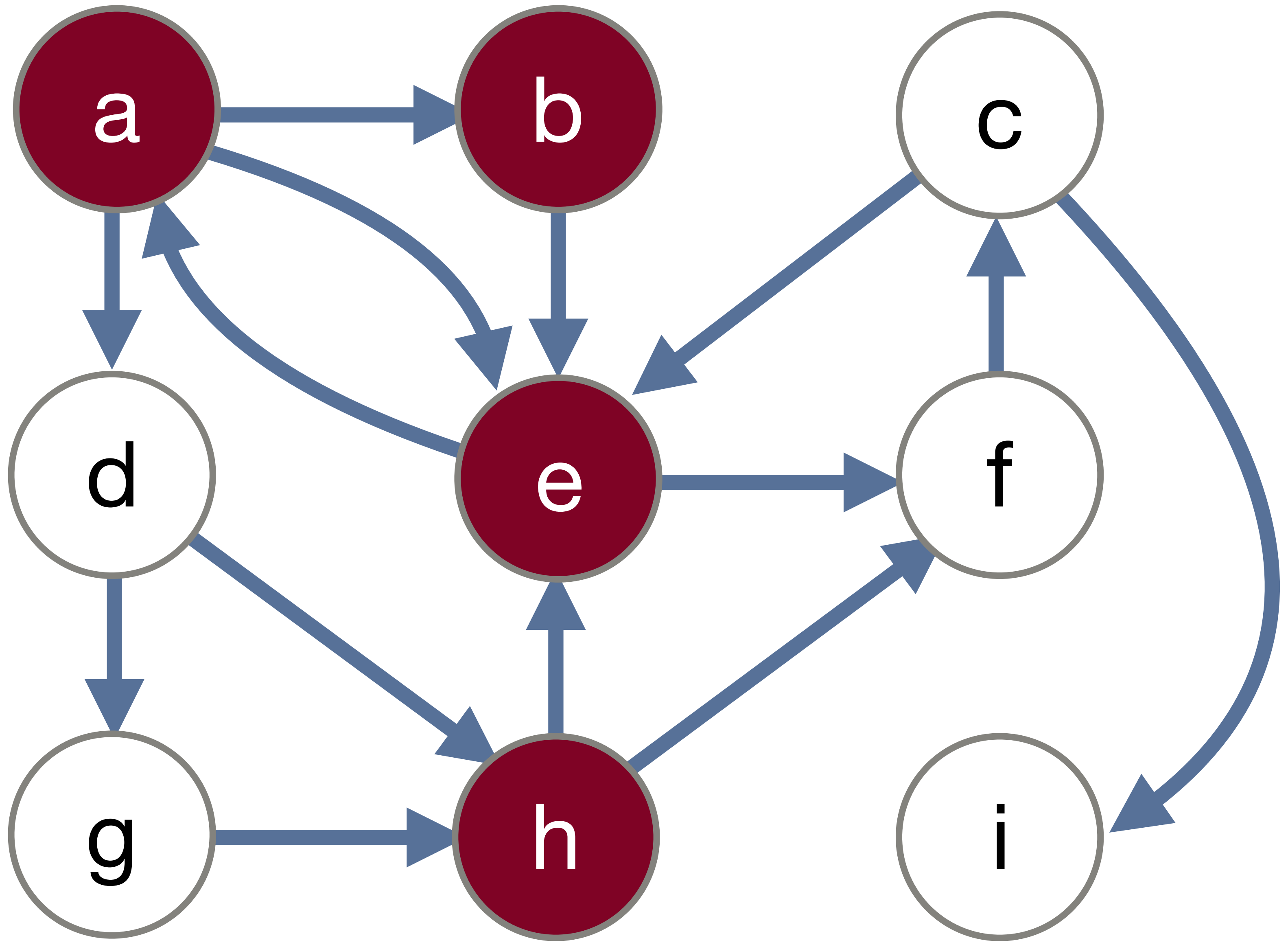
Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(b,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	false
e	true
f	false
g	false
h	true
i	false

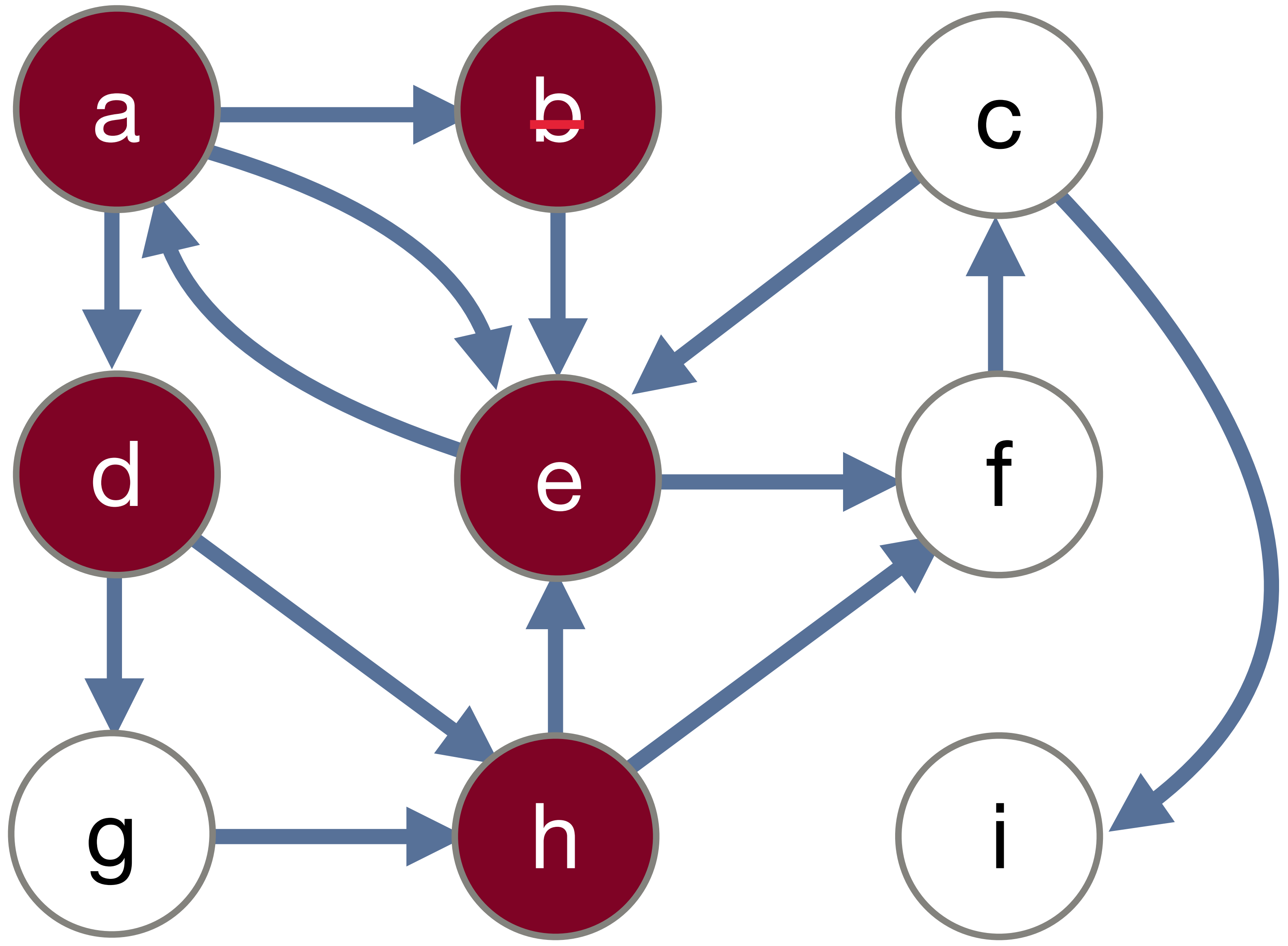
Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	false
g	false
h	true
i	false

Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

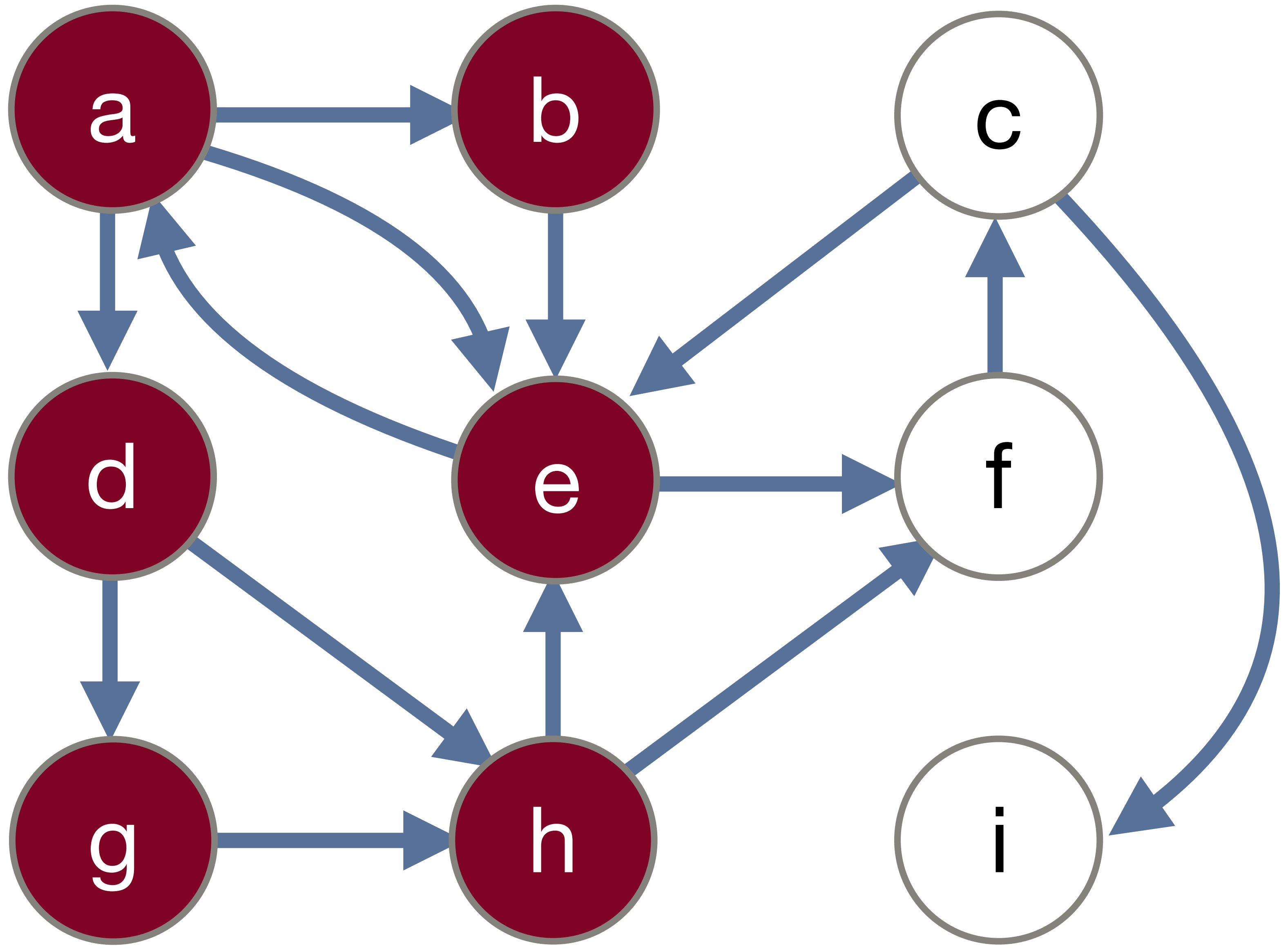
Let's look at **dfs** from h to c:

call stack:

dfs(g,c)
dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	false
g	true
h	true
i	false



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

  mark v_1 as visited.

  for all edges from v_1 to its neighbors:

    if neighbor n is unvisited, recursively call **dfs**(n, v_2).

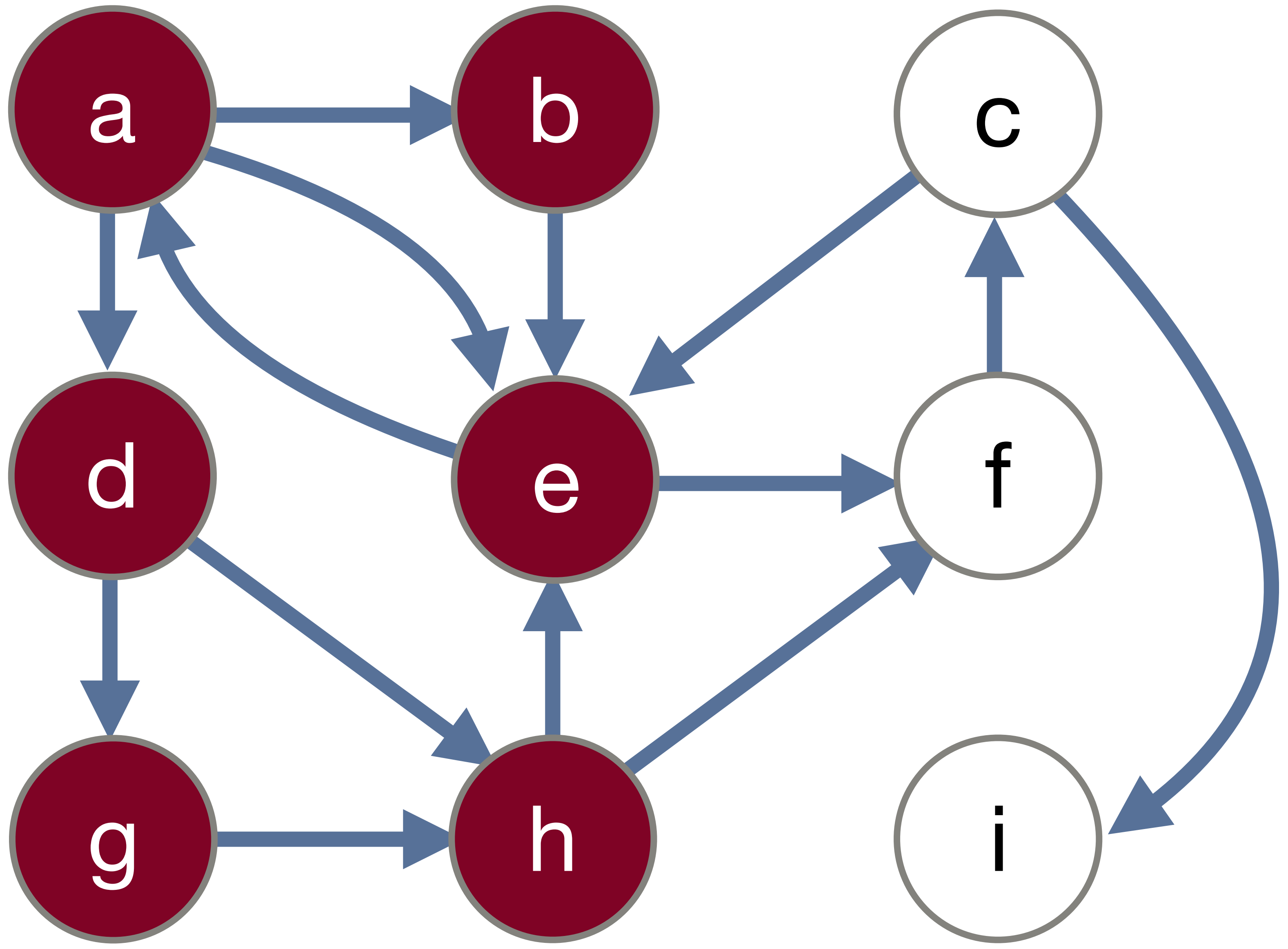
Let's look at **dfs** from h to c:

call stack:

~~dfs(g,c)~~
~~dfs(d,c)~~
~~dfs(a,c)~~
dfs(e,c)
dfs(h,c)

Vertex Map

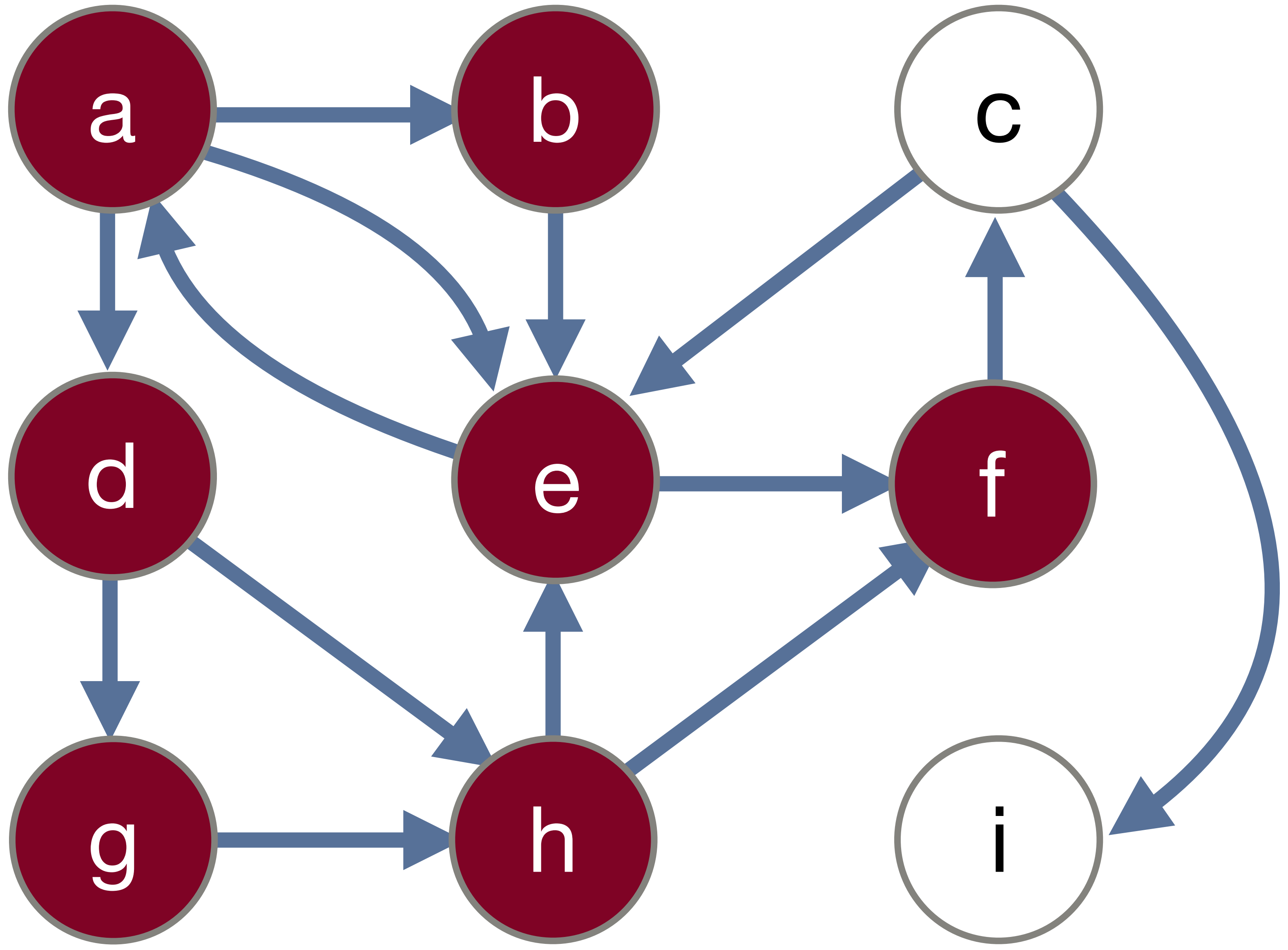
Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

    mark v_1 as visited.
    for all edges from v_1 to its neighbors:
        if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(f,c)
dfs(e,c)
dfs(h,c)

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false

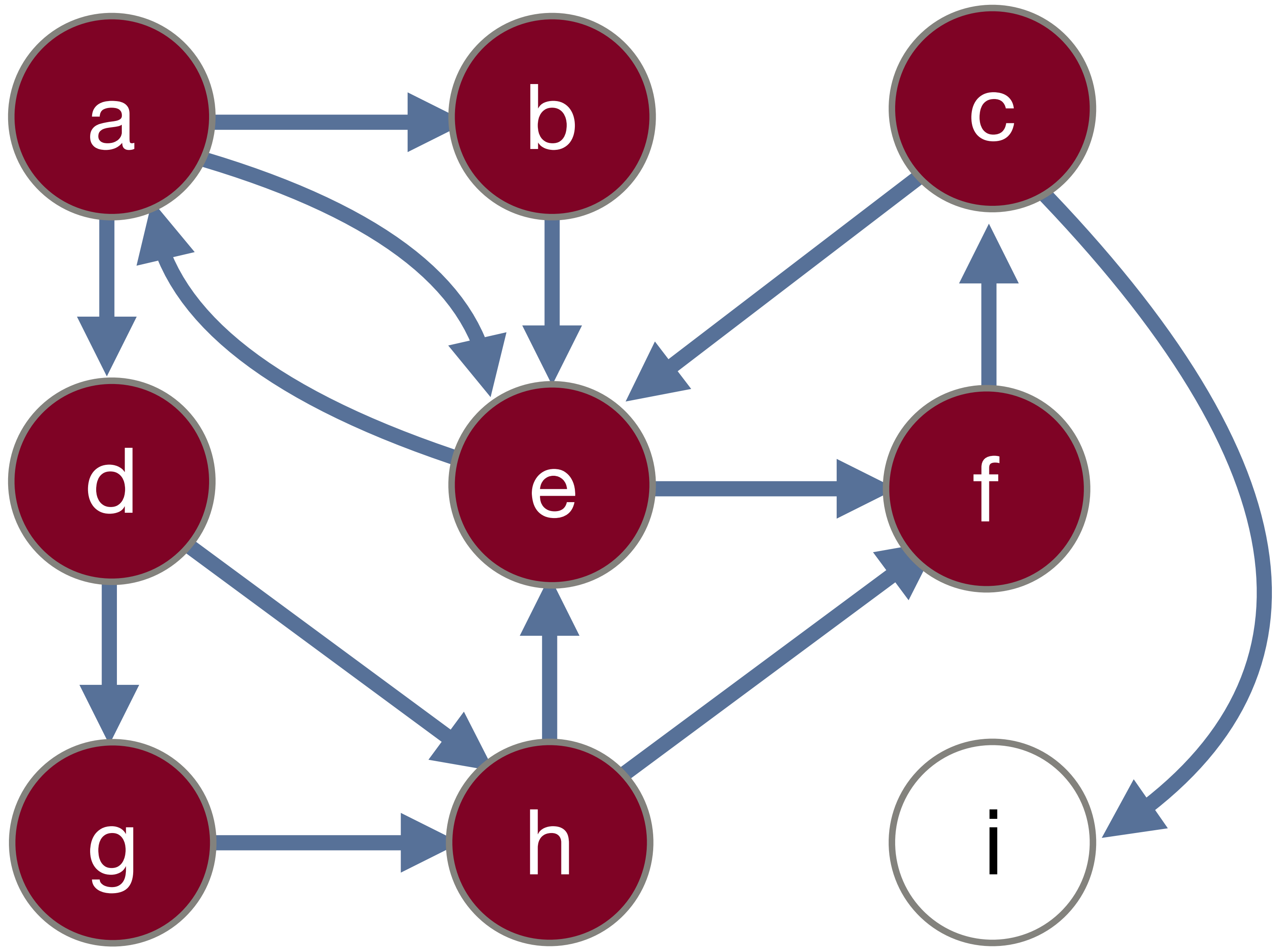
Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2:

  mark v_1 as visited.

  for all edges from v_1 to its neighbors:

    if neighbor n is unvisited, recursively call **dfs**(n, v_2).



Let's look at **dfs** from h to c:

call stack:

dfs(c,c)
dfs(f,c)
dfs(e,c)
dfs(h,c)

found!

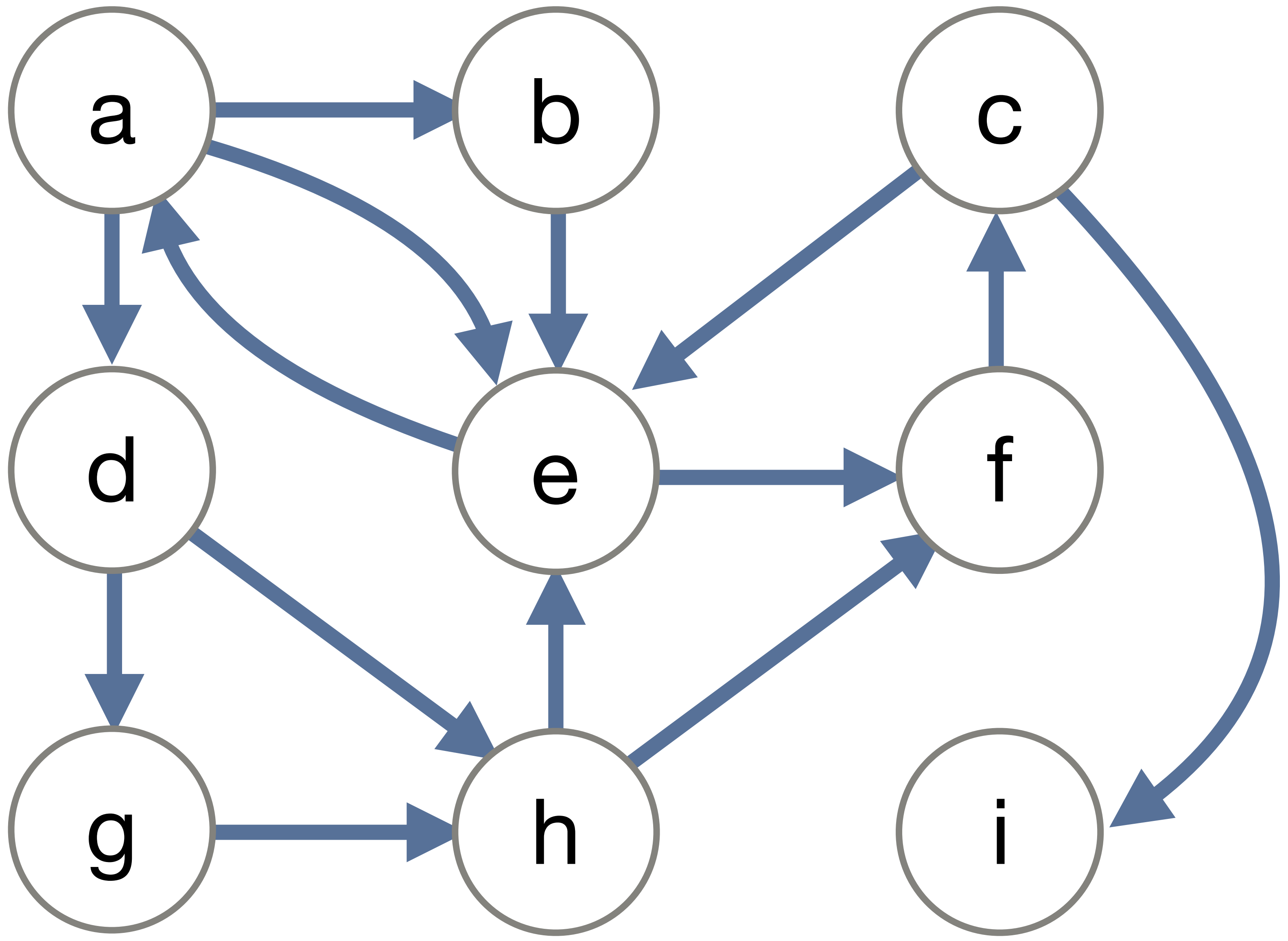
Vertex Map

Vertex	Visited?
a	true
b	true
c	true
d	true
e	true
f	true
g	true
h	true
i	false

Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:

- create a stack, s
- s.push(v_1)
- while s is not empty:
  - v = s.pop()
  - if v has not been visited:
    - mark v as visited
    - push all neighbors of v onto the stack



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:
- create a stack, s
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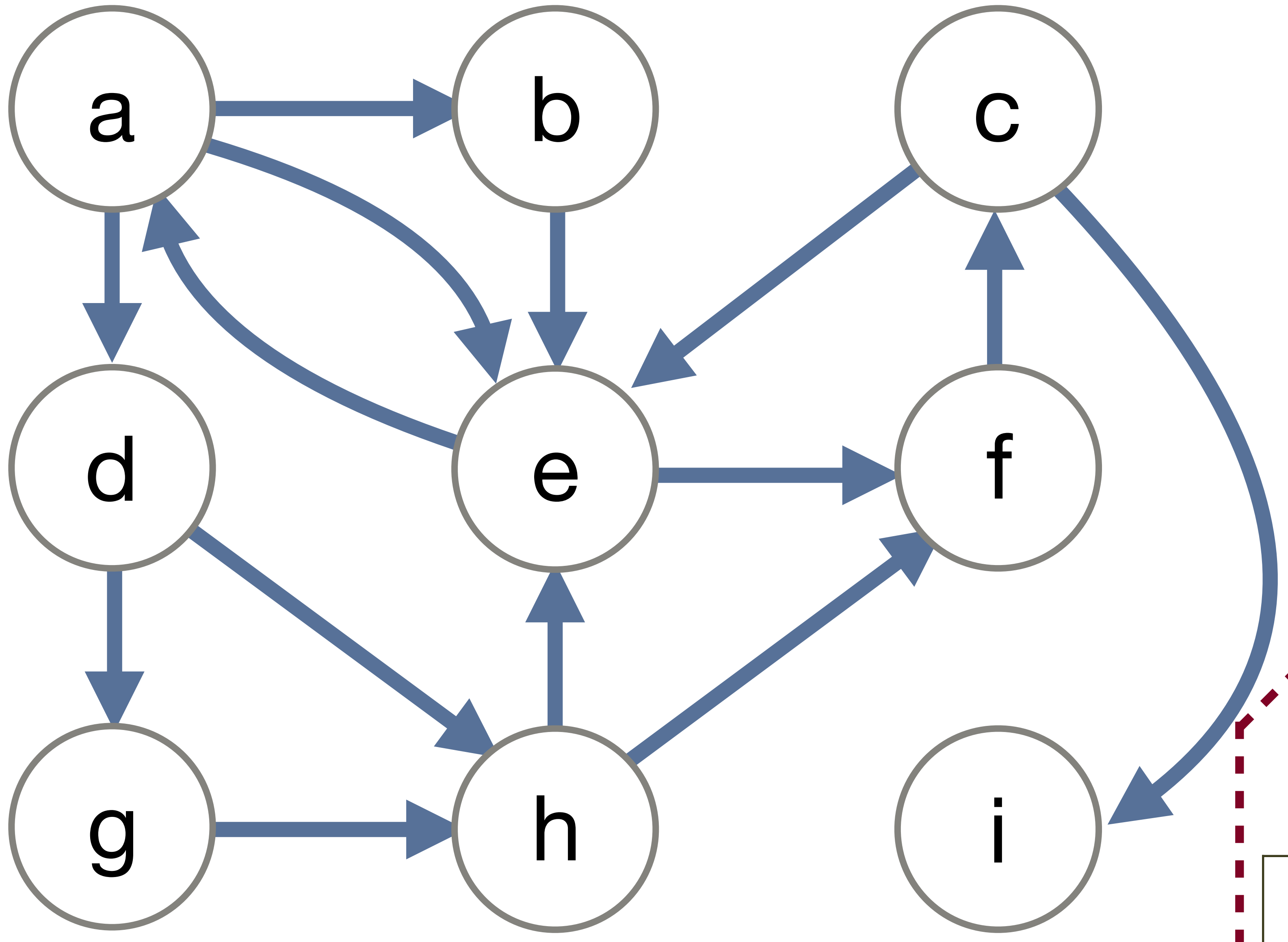


Let's look at **dfs** from h to c:

push h

stack s

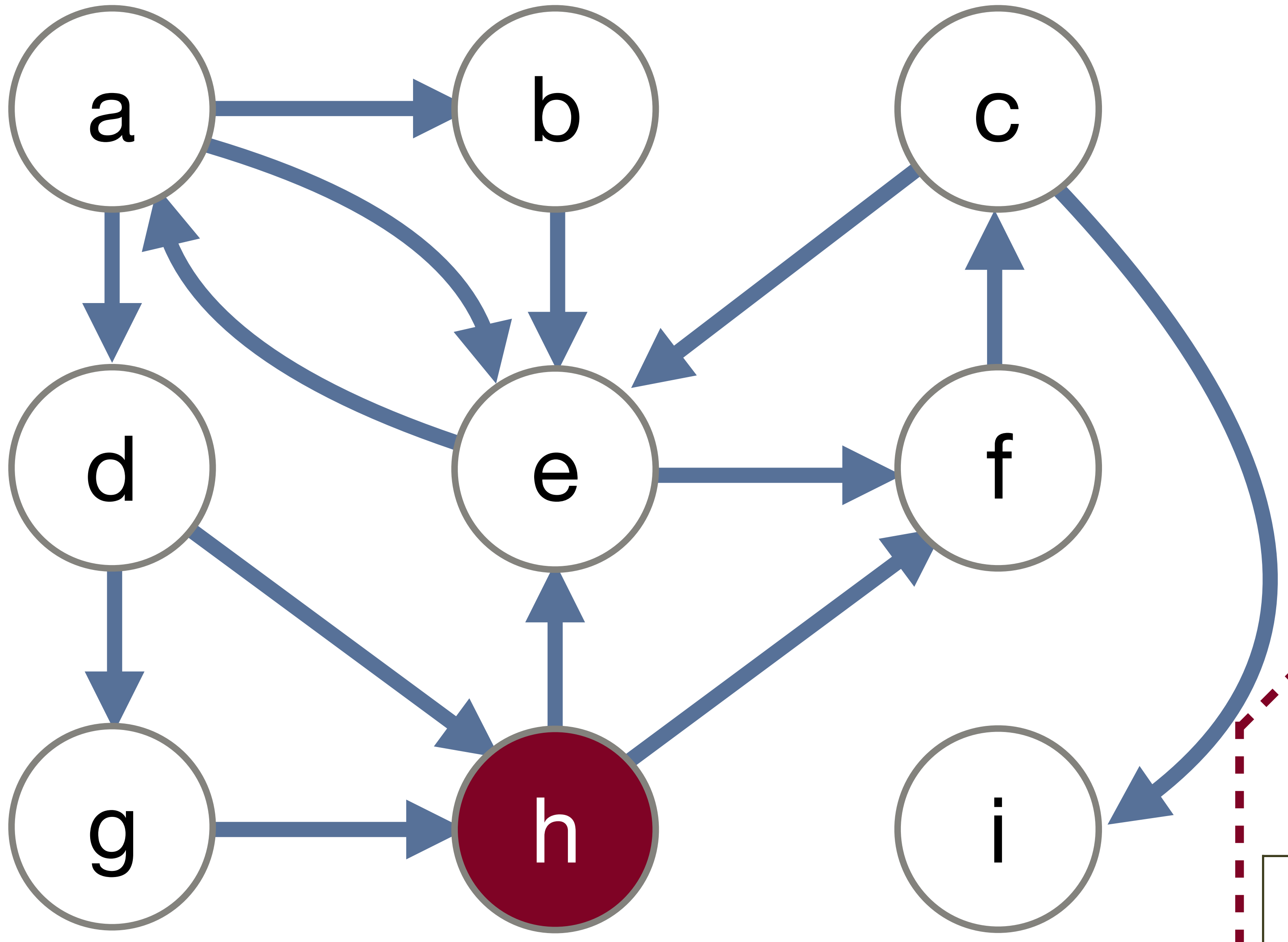
stack s
h

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	false
i	false

Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:
    create a stack, s
    s.push(v_1)
    while s is not empty:
        v = s.pop()
        if v has not been visited:
            mark v as visited
            push all neighbors of v onto the stack

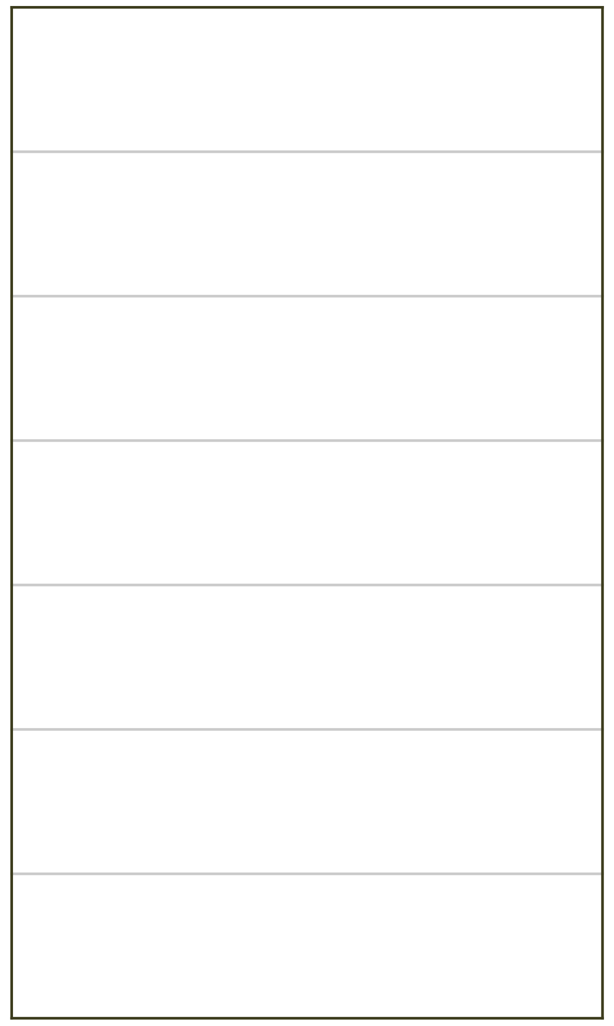


Let's look at **dfs** from h to c:

in while loop:
v = s.pop()

v: h

stack s

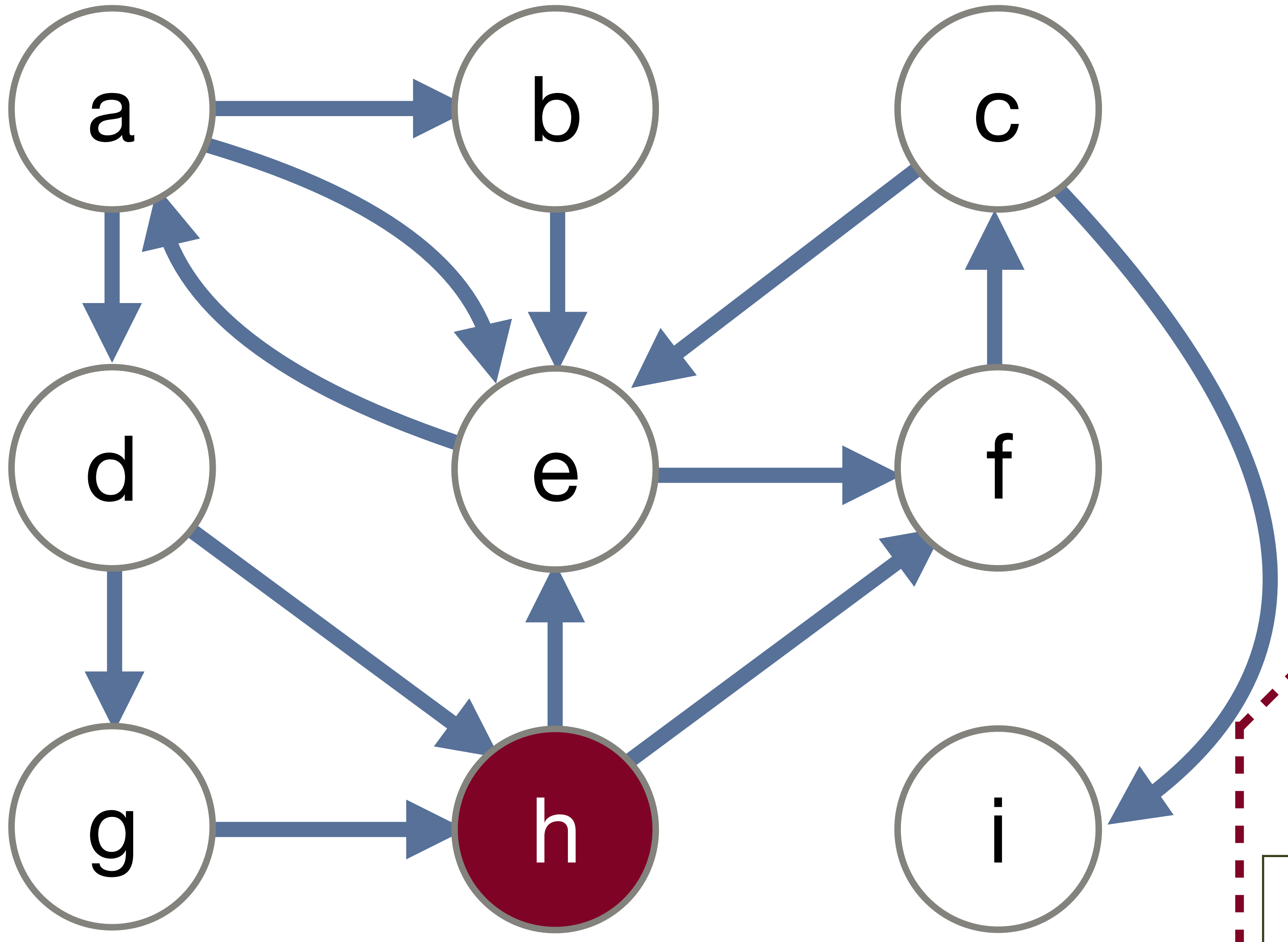


Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	true
i	false

Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:
    create a stack, s
    s.push(v_1)
    while s is not empty:
        v = s.pop()
        if v has not been visited:
            mark v as visited
            push all neighbors of v onto the stack



Let's look at **dfs** from h to c:

in while loop:
    push all
        neighbors of h

stack s

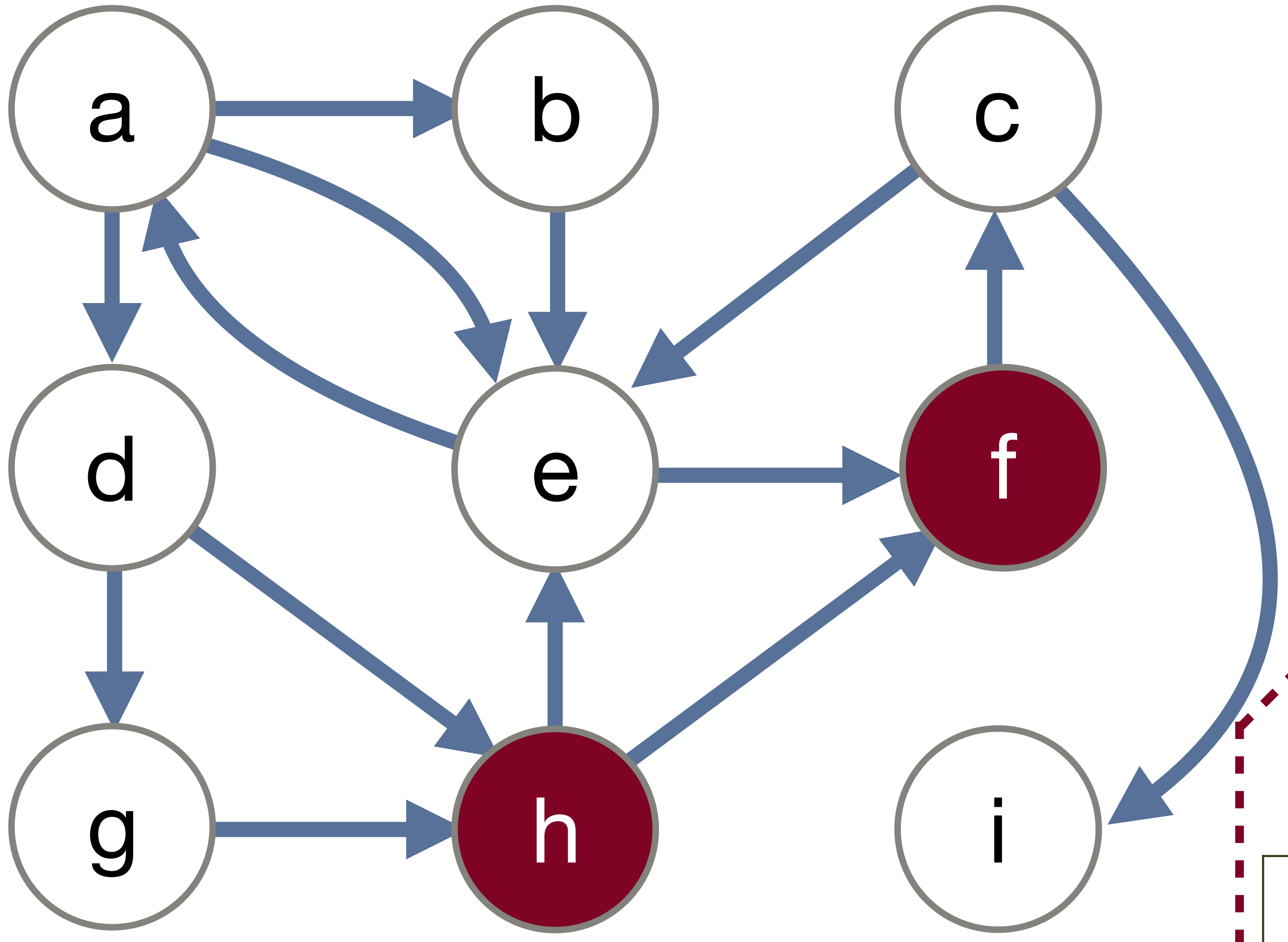
f
e

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	true
i	false

Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:
    create a stack, s
    s.push(v_1)
    while s is not empty:
        v = s.pop()
        if v has not been visited:
            mark v as visited
            push all neighbors of v onto the stack



Let's look at **dfs** from h to c:

in while loop:
v = s.pop()

v: f

stack s

e

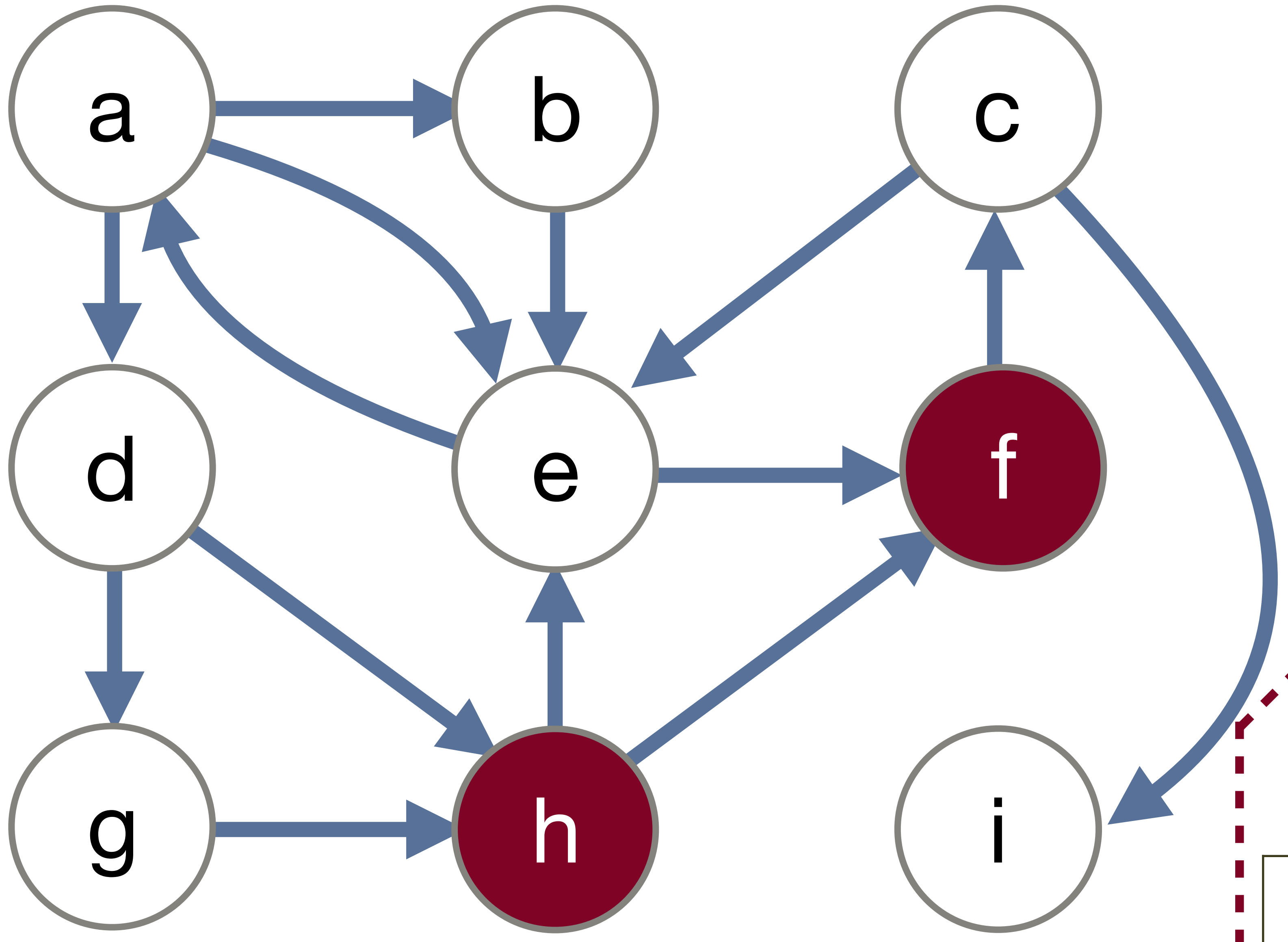
Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	true
g	false
h	true
i	false

Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:

```
create a stack, s
s.push(v1)
while s is not empty:
    v = s.pop()
    if v has not been visited:
        mark v as visited
        push all neighbors of v onto the stack
```



Let's look at **dfs** from h to c:

in while loop:
push all
neighbors of f

stack s

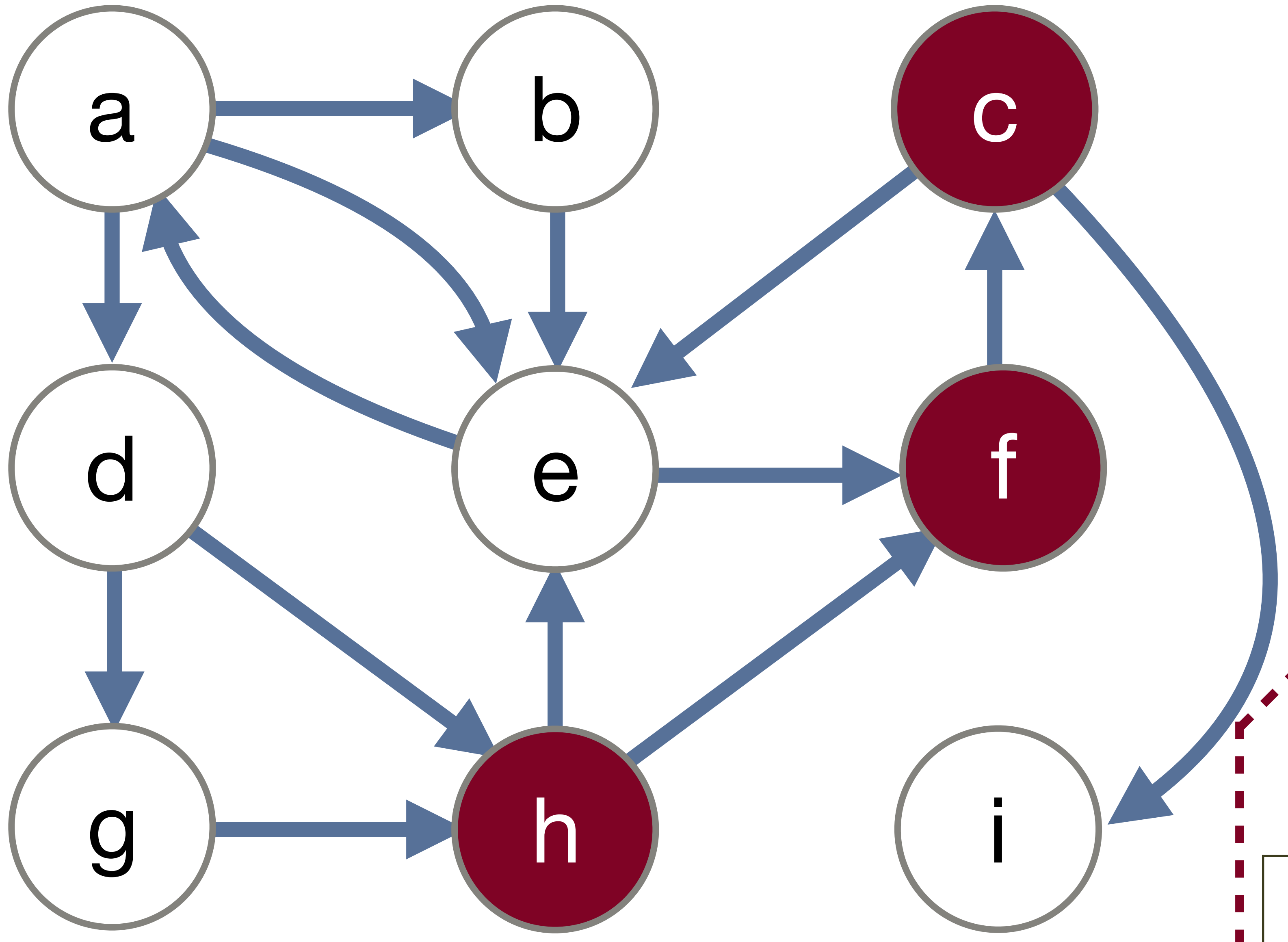
c
e

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	true
g	false
h	true
i	false

Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2:
    create a stack, s
    s.push(v_1)
    while s is not empty:
        v = s.pop()
        if v has not been visited:
            mark v as visited
            push all neighbors of v onto the stack



Let's look at **dfs** from h to c:

in while loop:
v = s.pop()

v: c
found — stop!

stack s

c
e

Vertex Map

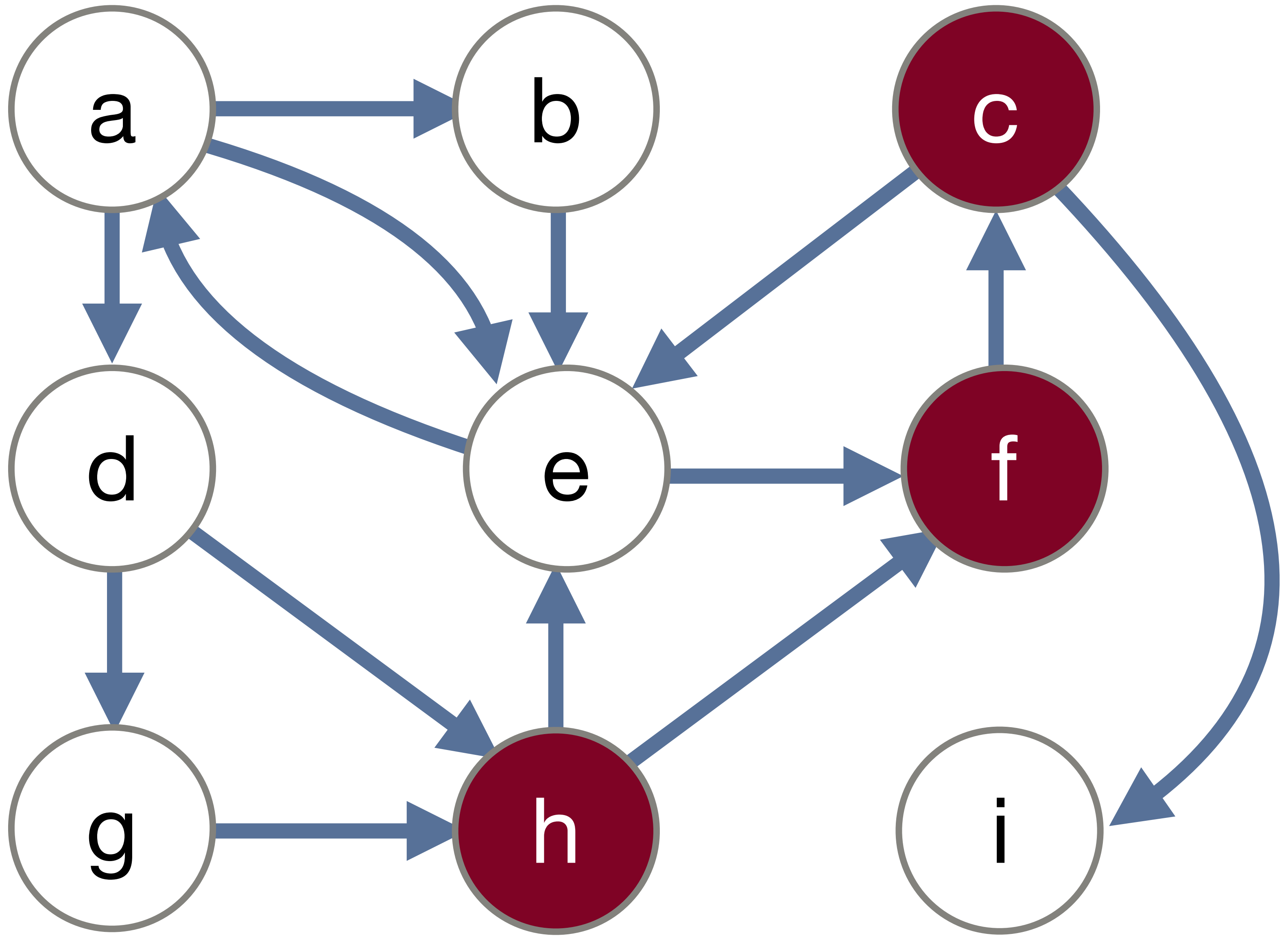
Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	true
g	false
h	true
i	false

Depth First Search (DFS)

Both the recursive and iterative solutions to DFS were correct, but because of the subtle differences in recursion versus using a stack, they traverse the nodes in a different order.

For the h to c example, the iterative solution happened to be faster, but for different graphs the recursive solution may have been faster.

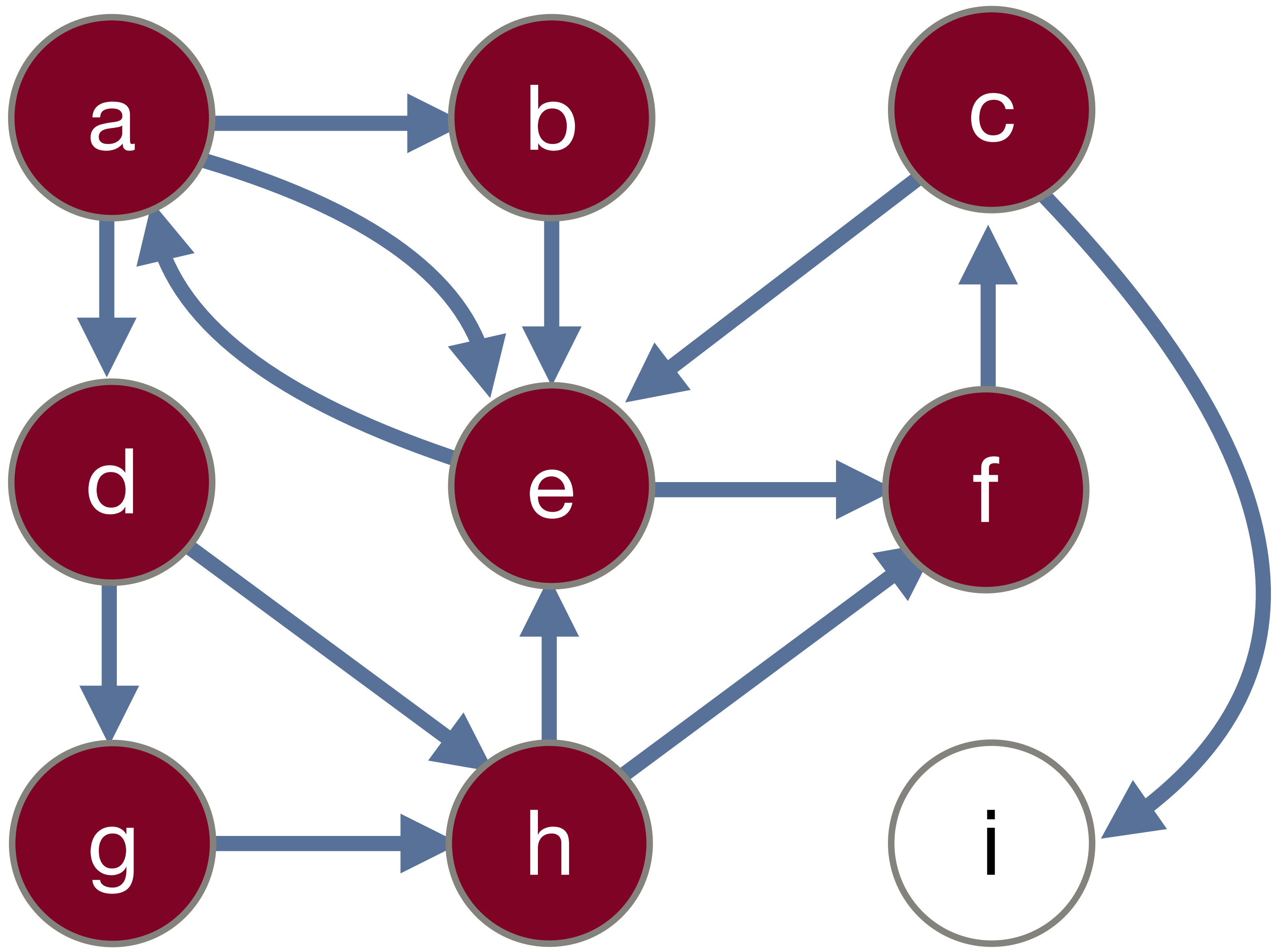
To retrieve the DFS path found, pass a collection parameter to each cell (if recursive) and choose-explore-unchoose (our old friend, recursive backtracking!)



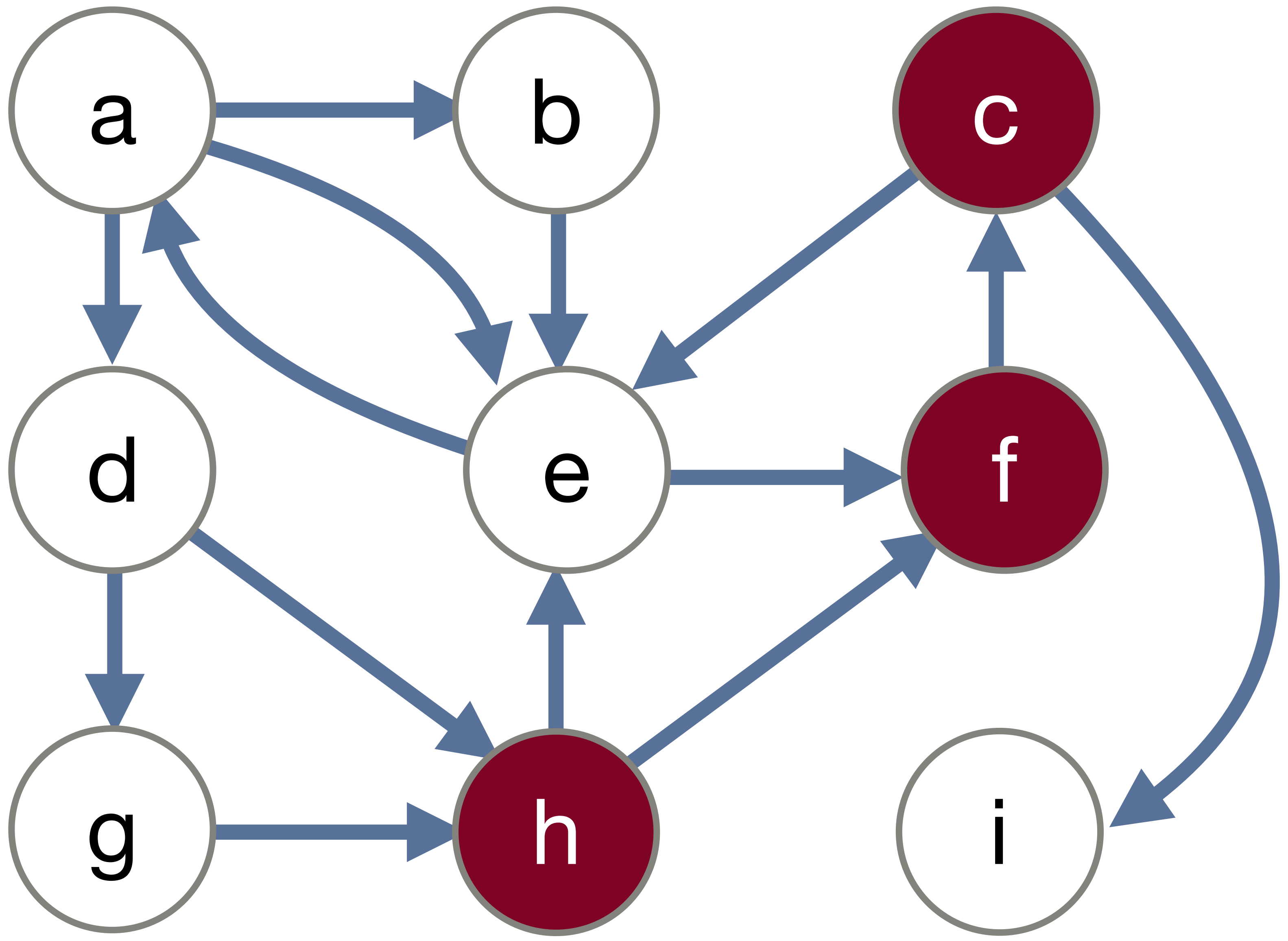
Depth First Search (DFS)

DFS is guaranteed to find a path if one exists.

It is *not* guaranteed to find the best or shortest path! (i.e., it is not optimal)



vs.



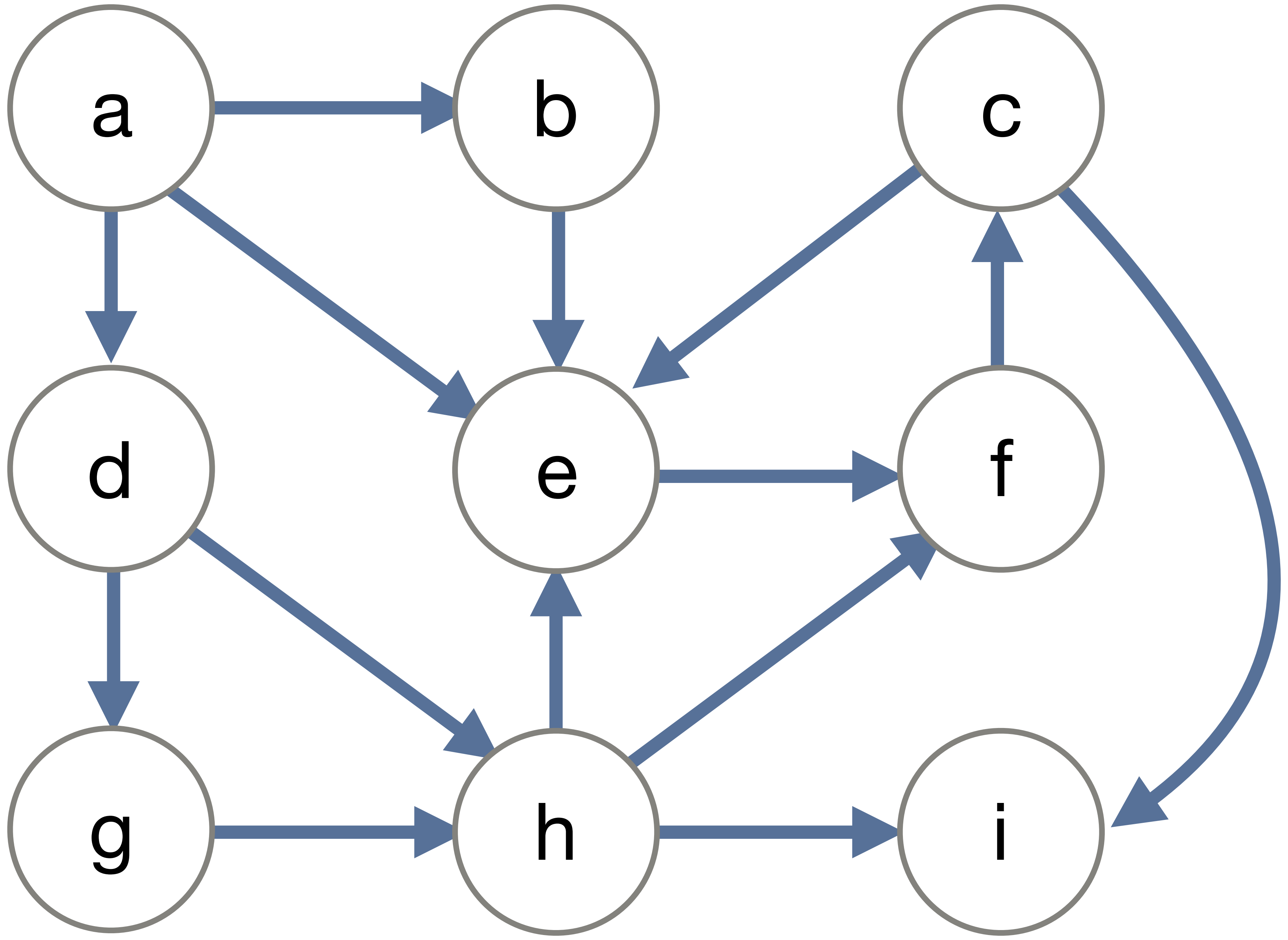
Breadth First Search (BFS)

- From the start vertex, explore the neighbor nodes first, before moving to the next level neighbors.

This *isn't easy to implement* recursively. The iterative algorithm is very similar to the DFS iterative, except that we use a queue.

BFS from a to i (assuming a-z order) visits:

a
a☞b
a☞d
a☞e
neighbors of a
a☞d☞g
a☞d☞h
neighbors of d
a☞e☞f
a☞d☞h☞i



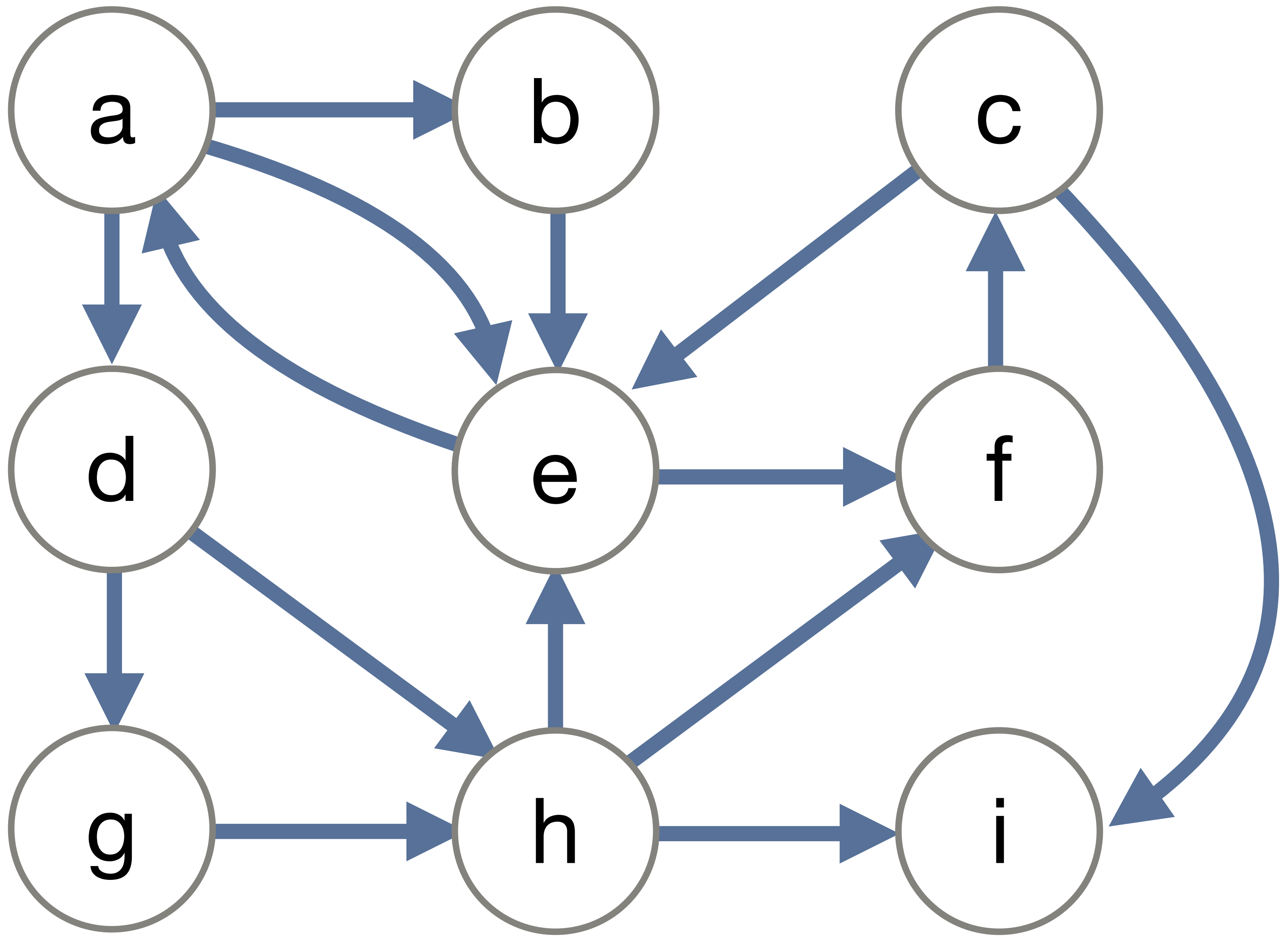
Notice: the shortest!

path: a☞d☞h☞i

Breadth First Search (BFS): Iterative pseudocode

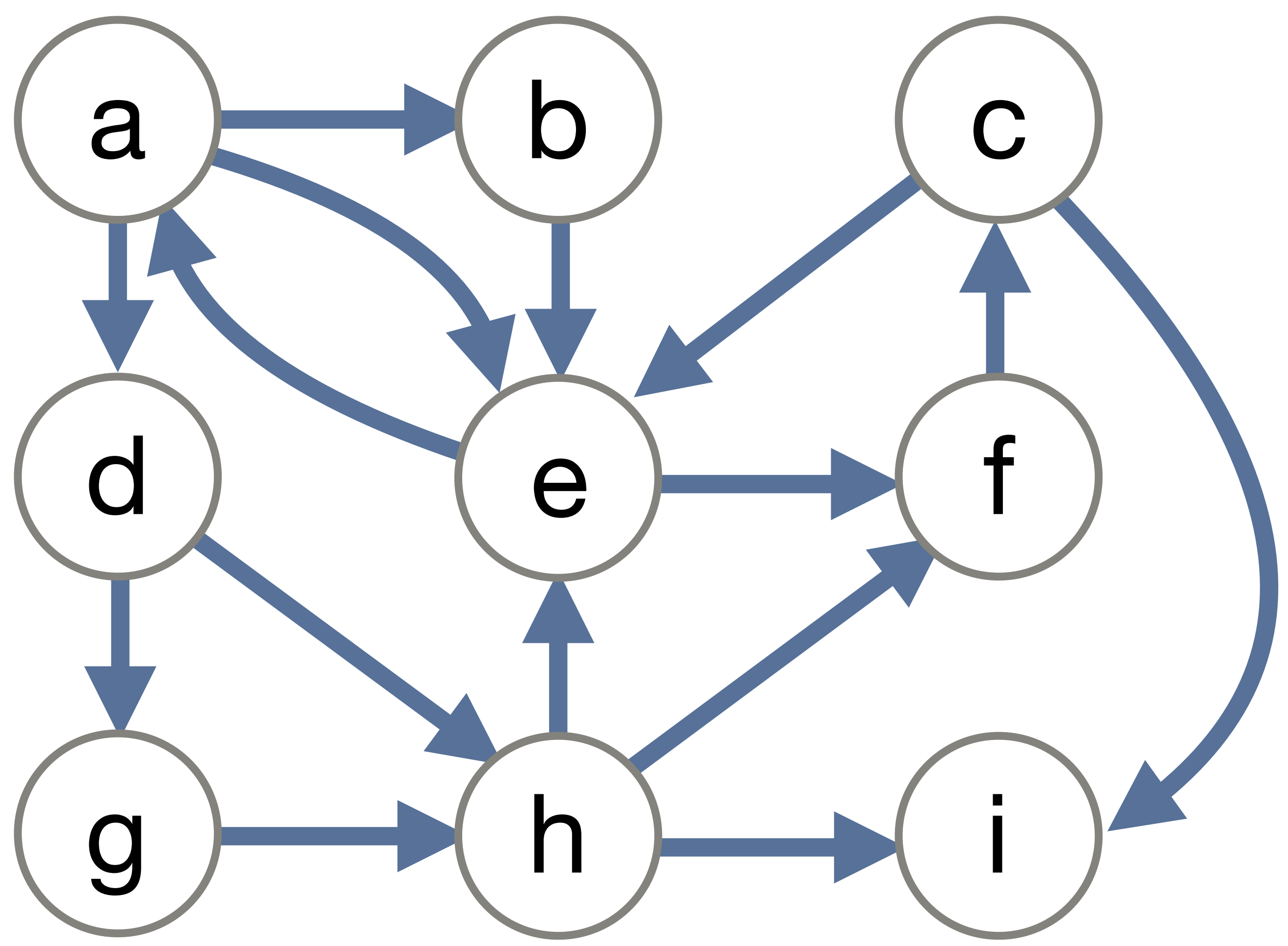
bfs from v_1 to v_2:

```
create a queue of paths (a vector), q
q.enqueue(v1 path)
while q is not empty and v2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    if v is the end vertex, we can stop after adding to the
          current path.
    for each unvisited neighbor of v:
        make new path with v's neighbor as last element
        enqueue new path onto q
```



Breadth First Search (BFS): Iterative pseudocode

```
bfs from v1 to v2:
    create a queue of paths (a vector), q
    q.enqueue(v1 path)
    while q is not empty and v2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v as last element
            enqueue new path onto q
```



Visited Set:
(empty)

Let's look at **bfs** from a to i:

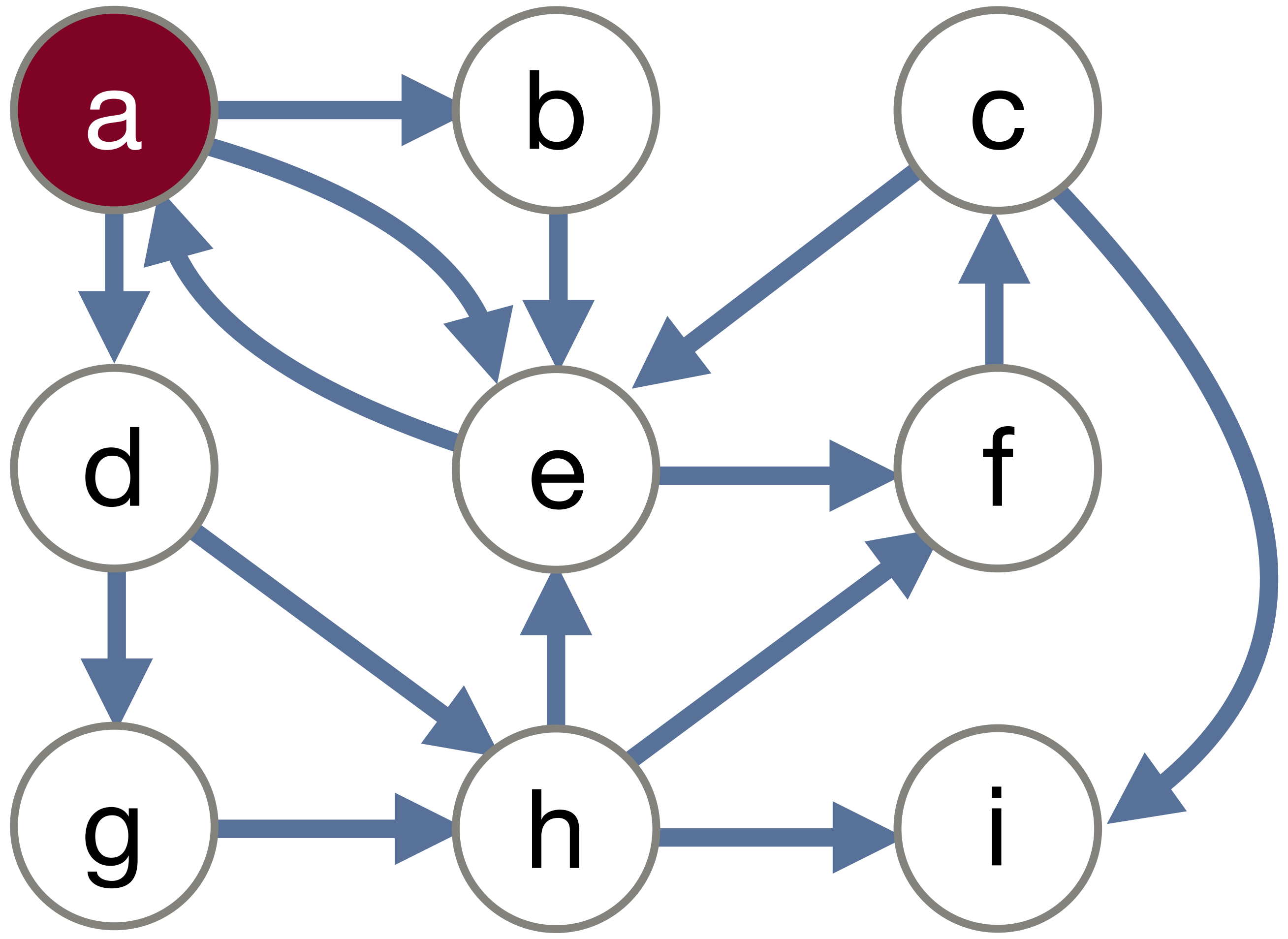
queue:

								front
								a

```
Vector<Vertex *> startPath
startPath.add(a)
q.enqueue(startPath)
```

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
  create a queue of paths (a vector), q
  q.enqueue(v_1 path)
  while q is not empty and v_2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    for each unvisited neighbor of v:
      make new path with v's neighbor as last element
      enqueue new path onto q



Visited Set:
a

Let's look at **bfs** from a to i:

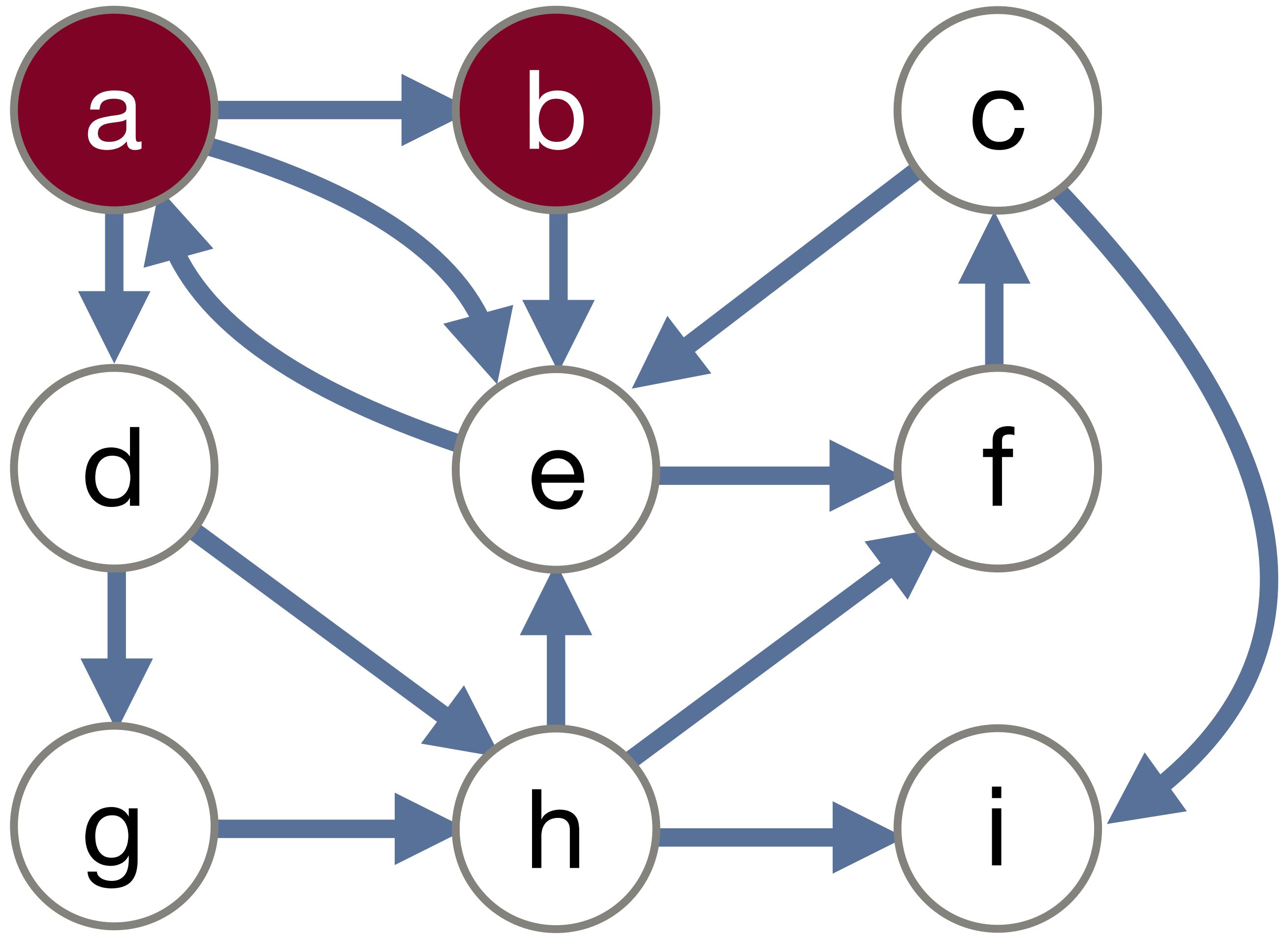
queue:

								front
						ae	ad	ab

in while loop:
  curPath = q.dequeue() (path is a)
  v = last element in curPath (v is a)
  mark v as visited
  enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
    create a queue of paths (a vector), q
    q.enqueue(v_1 path)
    while q is not empty and v_2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v's neighbor as last element
            enqueue new path onto q



Visited Set:
a
b

Let's look at **bfs** from a to i:

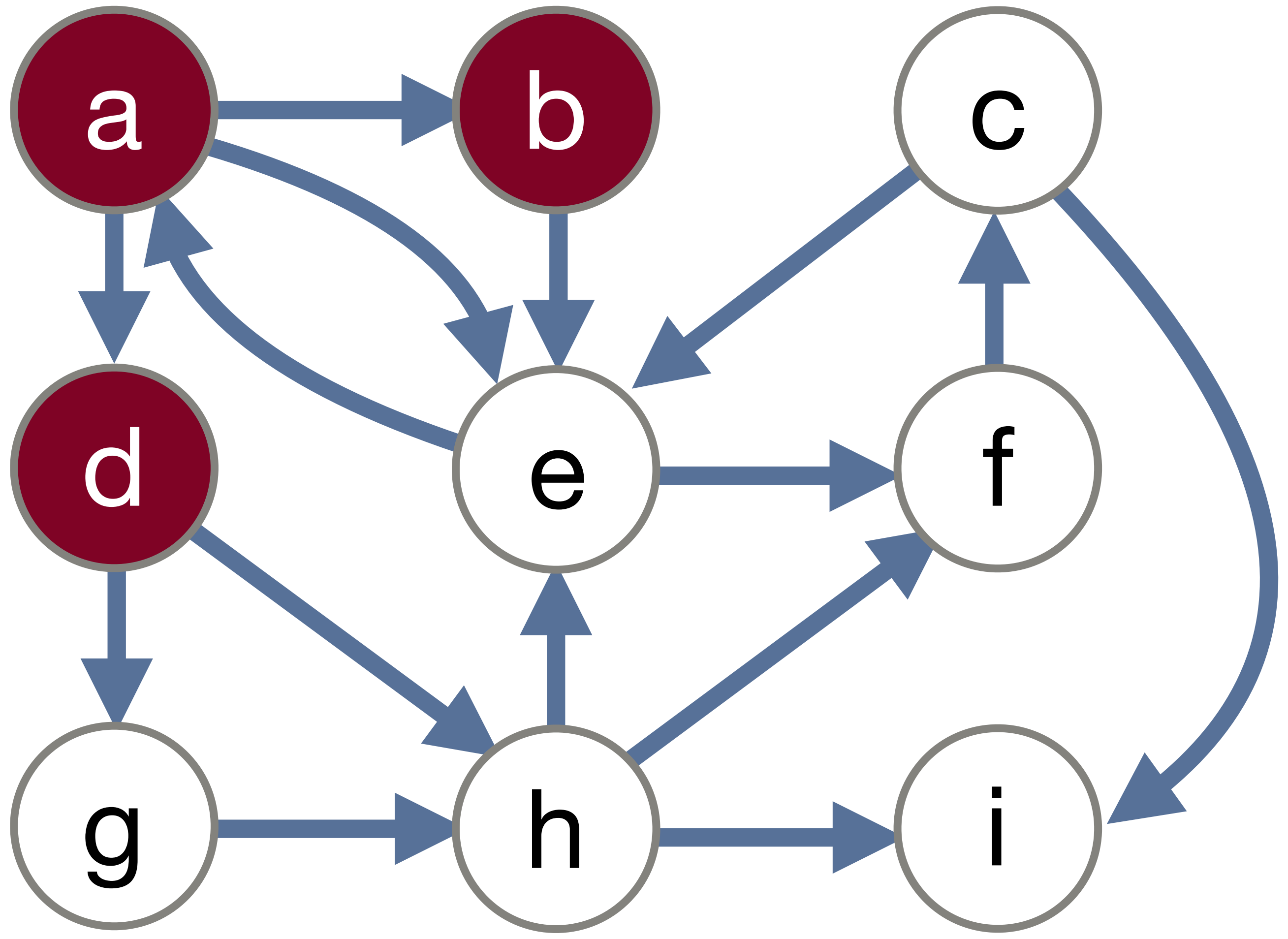
queue:

								front
						abe	ae	ad

in while loop:
  curPath = q.dequeue() (path is ab)
  v = last element in curPath (v is b)
  mark v as visited
  enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
    create a queue of paths (a vector), q
    q.enqueue(v_1 path)
    while q is not empty and v_2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v's neighbor as last element
            enqueue new path onto q



Visited Set:
a
b
d

Let's look at **bfs** from a to i:

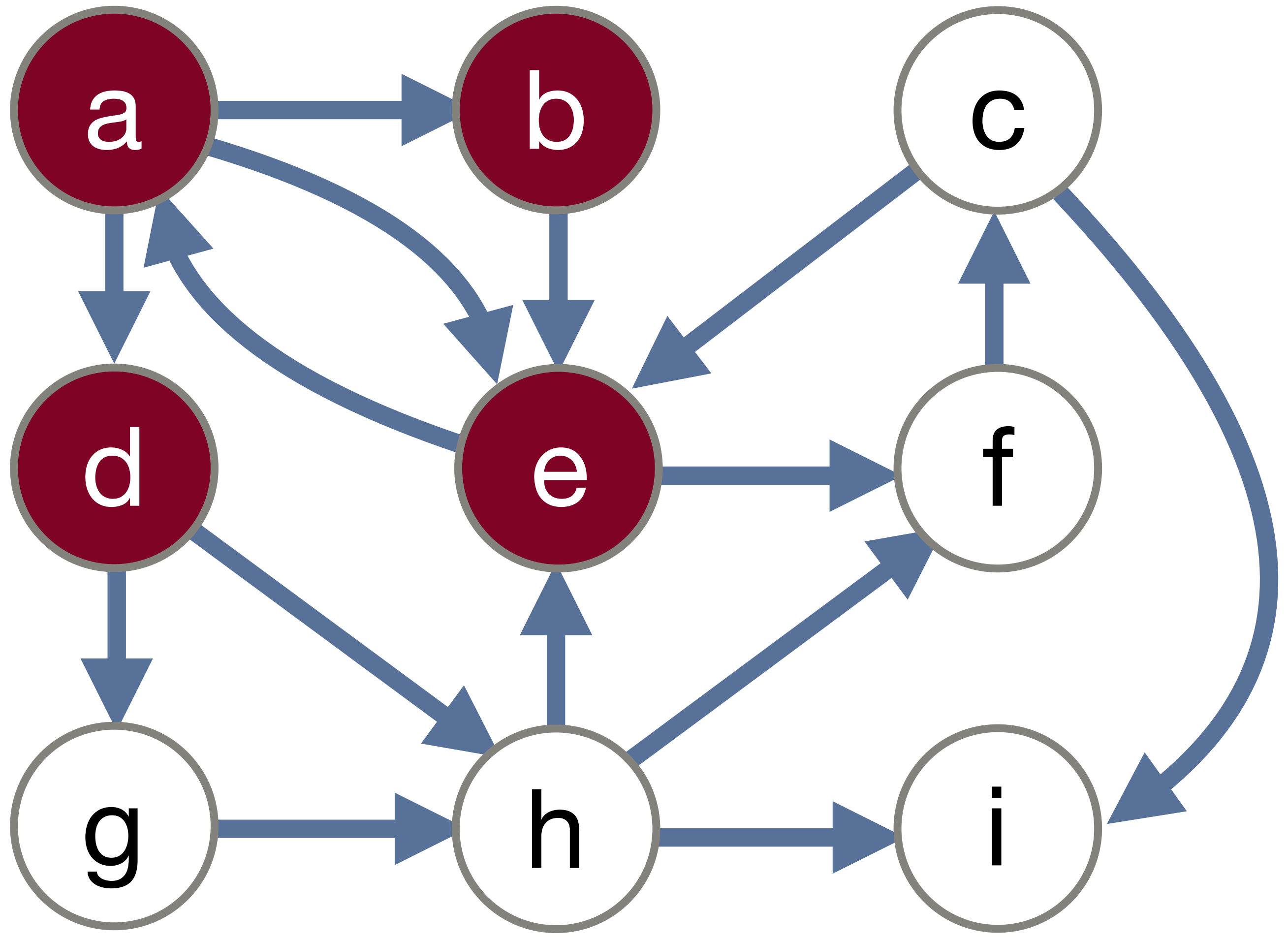
queue:

								front
					adh	adg	abe	ae

in while loop:
    curPath = q.dequeue() (path is ad)
    v = last element in curPath (v is d)
    mark v as visited
    enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
    create a queue of paths (a vector), q
    q.enqueue(v_1 path)
    while q is not empty and v_2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v's neighbor as last element
            enqueue new path onto q



Visited Set:
a
b
d
e

Let's look at **bfs** from a to i:

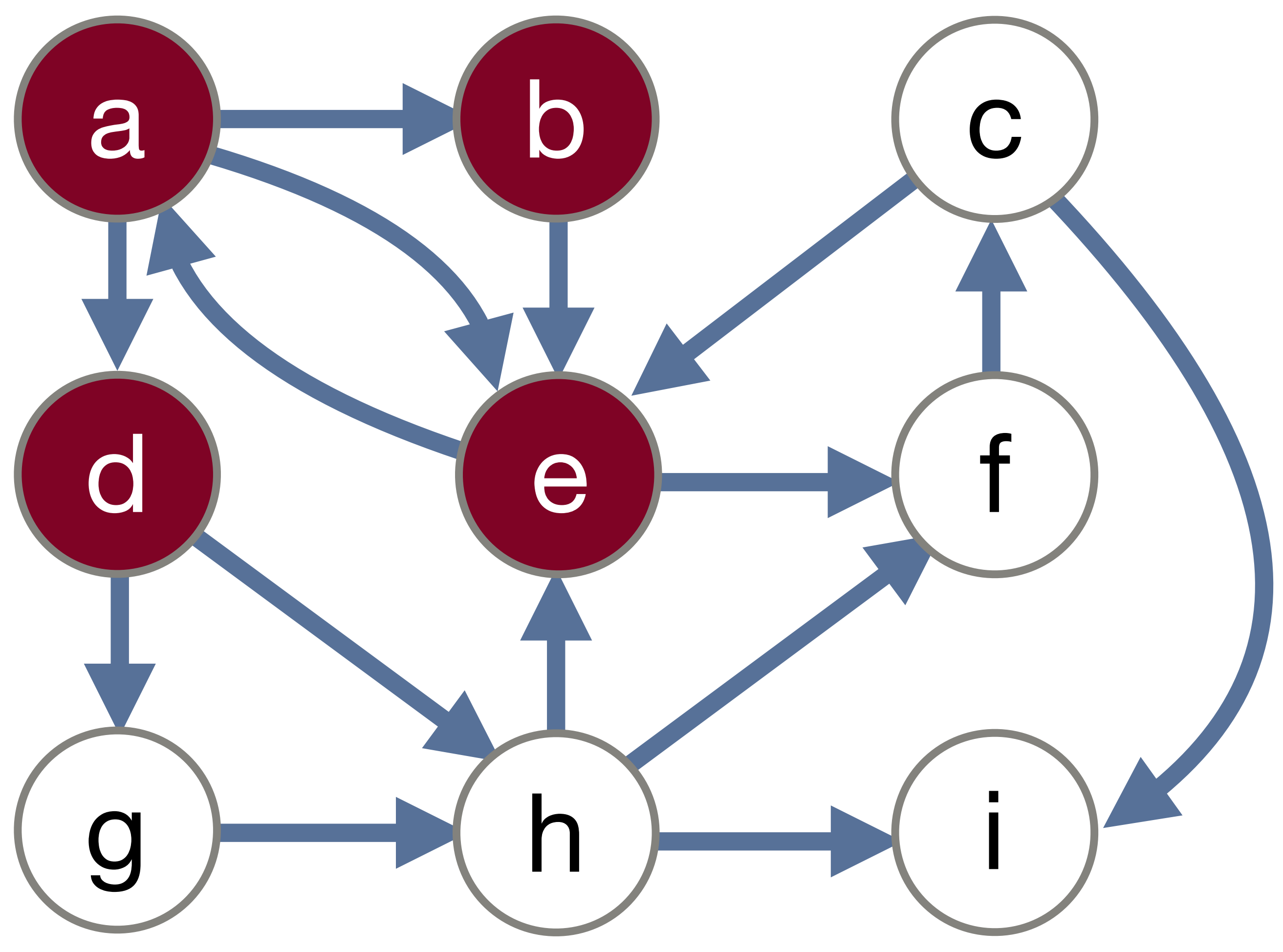
queue:

								front
					aef	adh	adg	abe

in while loop:
    curPath = q.dequeue() (path is ae)
    v = last element in curPath (v is e)
    mark v as visited
    enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
    create a queue of paths (a vector), q
    q.enqueue(v_1 path)
    while q is not empty and v_2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v's neighbor as last element
            enqueue new path onto q



Visited Set:
a
b
d
e

Let's look at **bfs** from a to i:

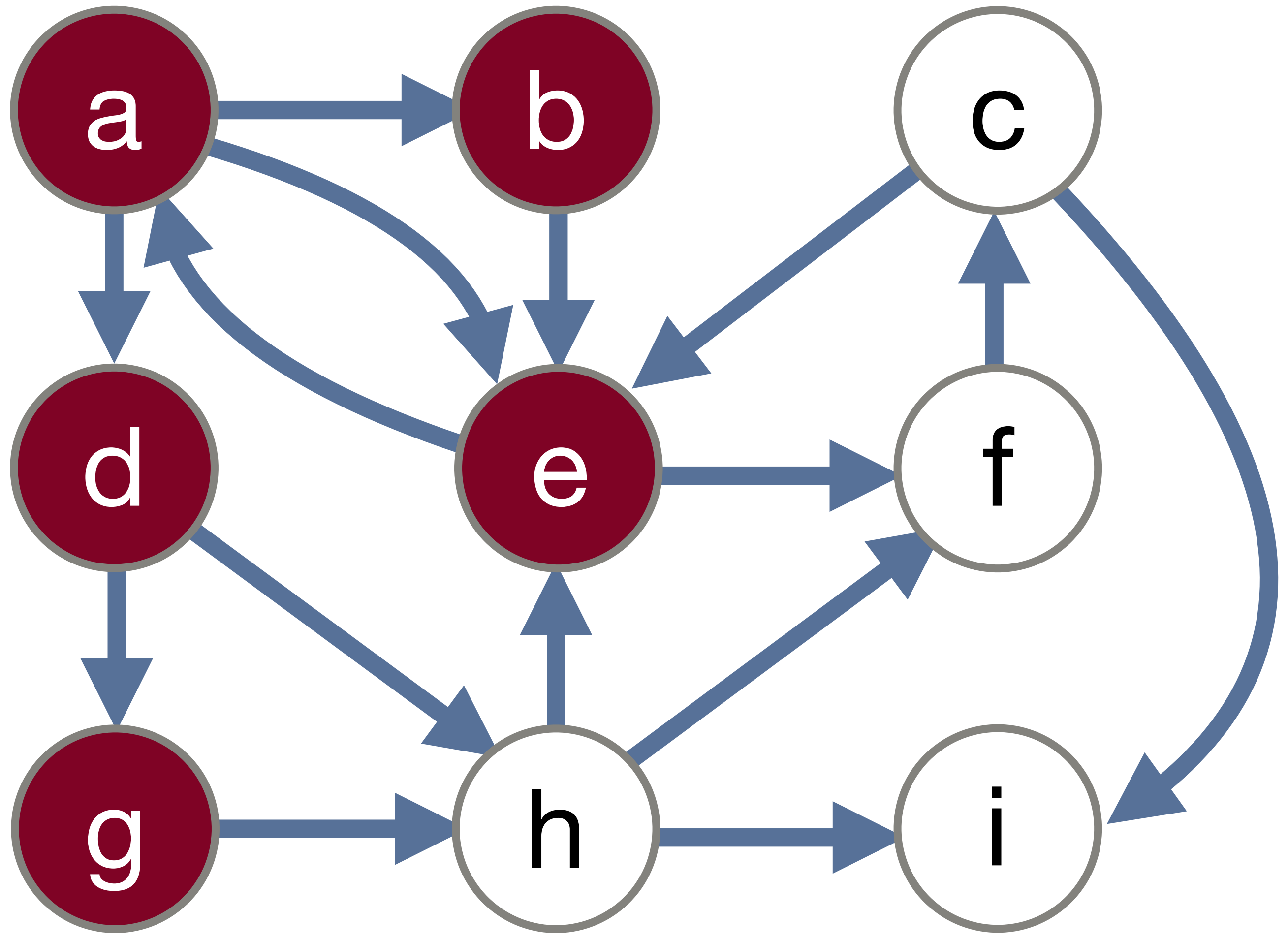
queue:

								front
					abef	aef	adh	adg

in while loop:
  curPath = q.dequeue() (path is abe)
  v = last element in curPath (v is e)
  mark v as visited (already been marked)
  enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
  create a queue of paths (a vector), q
  q.enqueue(v_1 path)
  while q is not empty and v_2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    for each unvisited neighbor of v:
      make new path with v's neighbor as last element
      enqueue new path onto q



Visited Set:
a
b
d
e
g

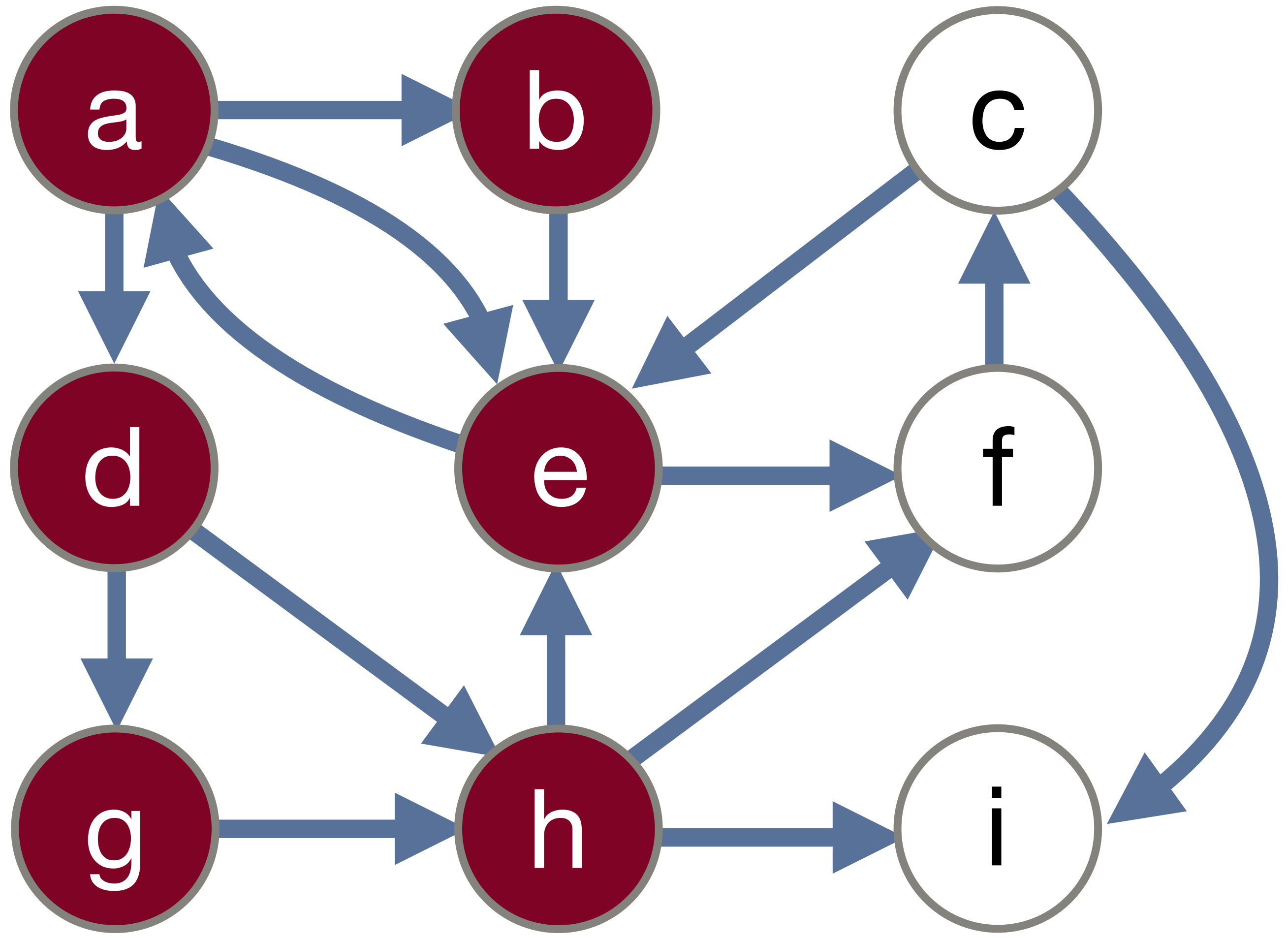
Let's look at **bfs** from a to i:

queue:									front
						adgh	abef	aef	adh

in while loop:
  curPath = q.dequeue() (path is adg)
  v = last element in curPath (v is g)
  mark v as visited
  enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
  create a queue of paths (a vector), q
  q.enqueue(v_1 path)
  while q is not empty and v_2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    for each unvisited neighbor of v:
      make new path with v's neighbor as last element
      enqueue new path onto q



Visited Set:
a
b
d
e
h

Let's look at **bfs** from a to i:

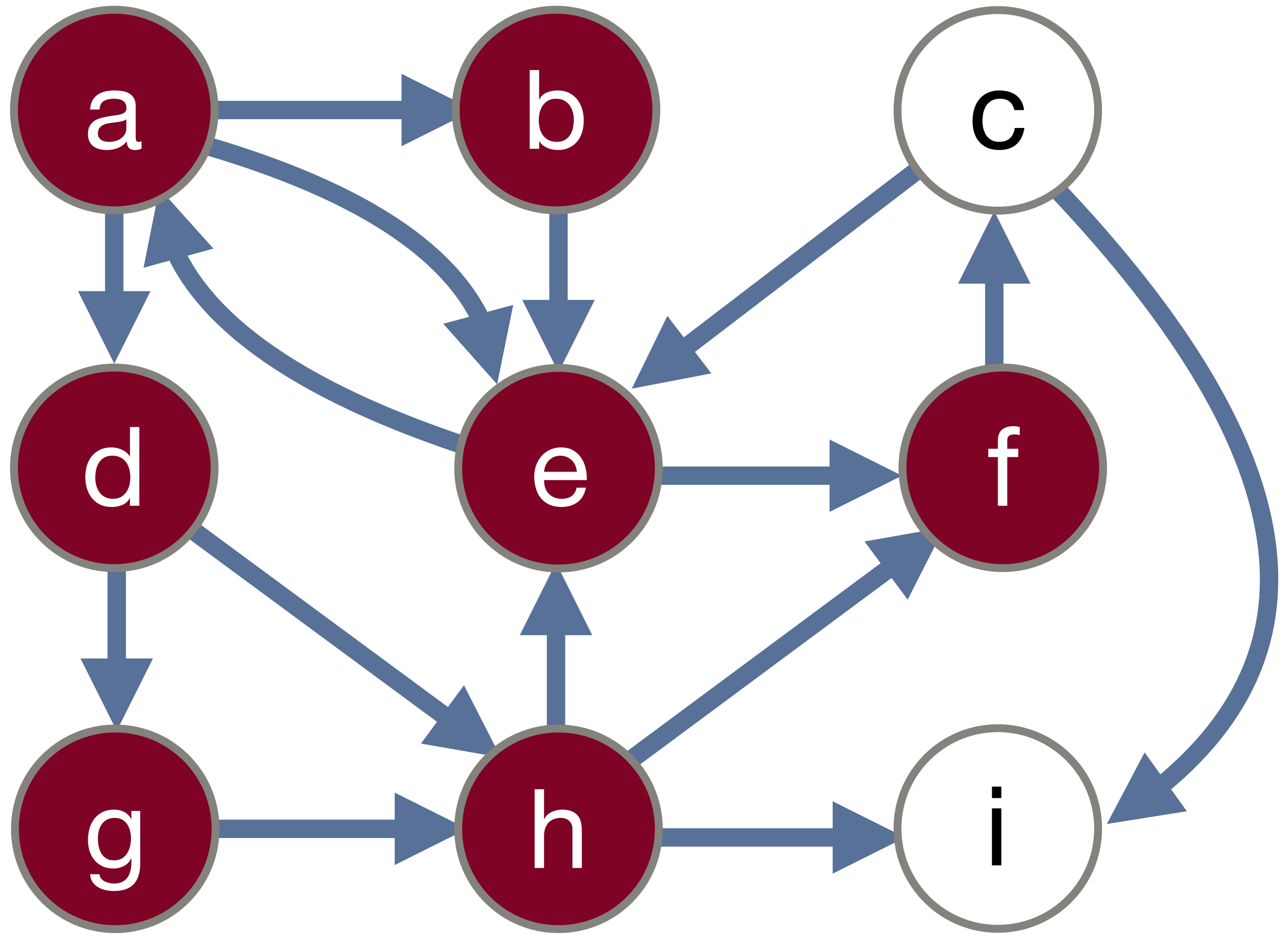
queue:

								front
				adhi	adhf	adgh	abef	aef

in while loop:
  curPath = q.dequeue() (path is adh)
  v = last element in curPath (v is h)
  mark v as visited
  enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
  create a queue of paths (a vector), q
  q.enqueue(v_1 path)
  while q is not empty and v_2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    for each unvisited neighbor of v:
      make new path with v's neighbor as last element
      enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

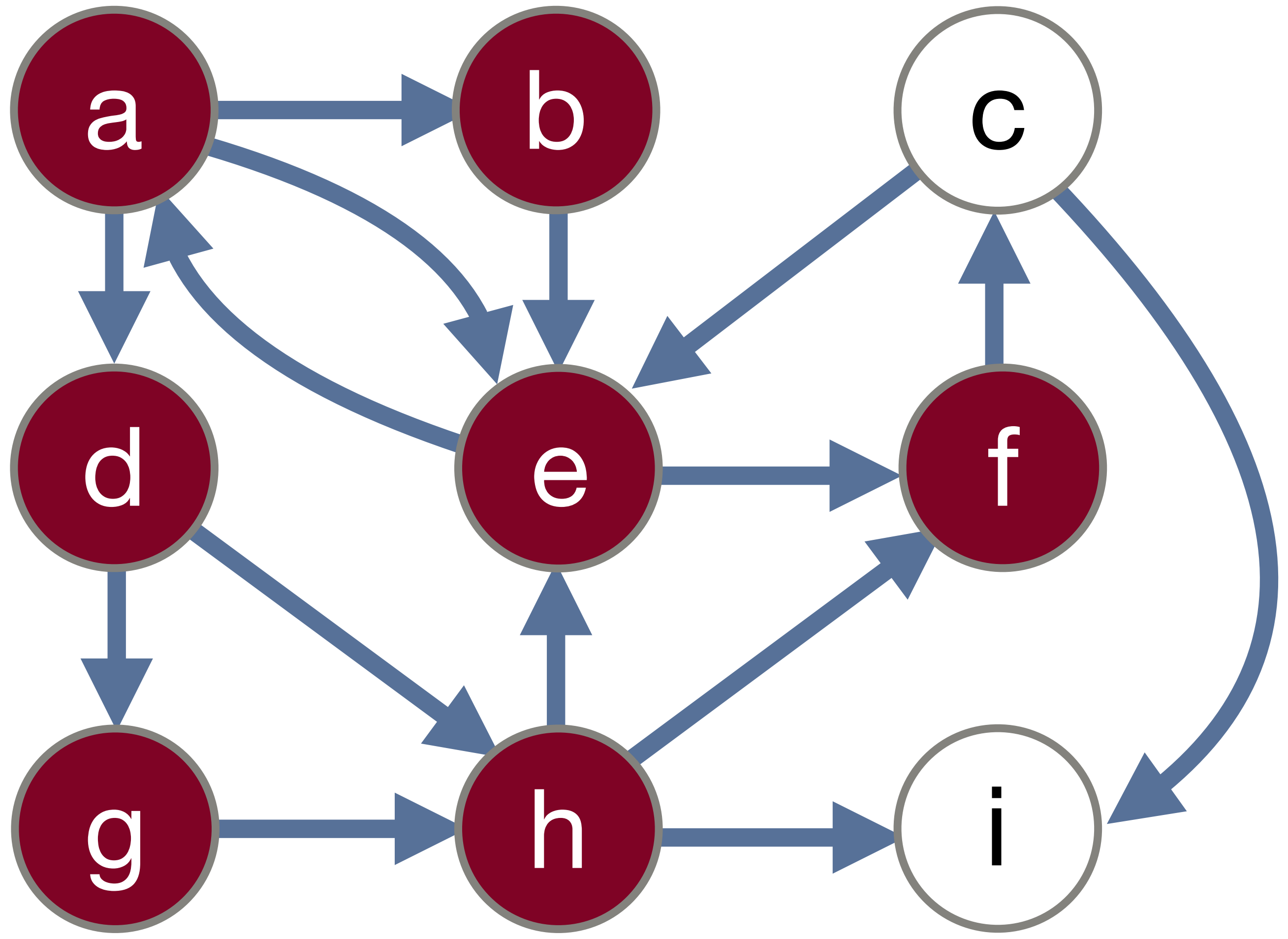
								front
				aefc	adhi	adhf	adgh	abef

in while loop:
  curPath = q.dequeue() (path is aef)
  v = last element in curPath (v is f)
  mark v as visited
  enqueue all unvisited neighbor paths onto q

Visited Set:
a
b
d
e
f

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
    create a queue of paths (a vector), q
    q.enqueue(v_1 path)
    while q is not empty and v_2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v's neighbor as last element
            enqueue new path onto q



Visited Set:
a
b
d
e
f

Let's look at **bfs** from a to i:

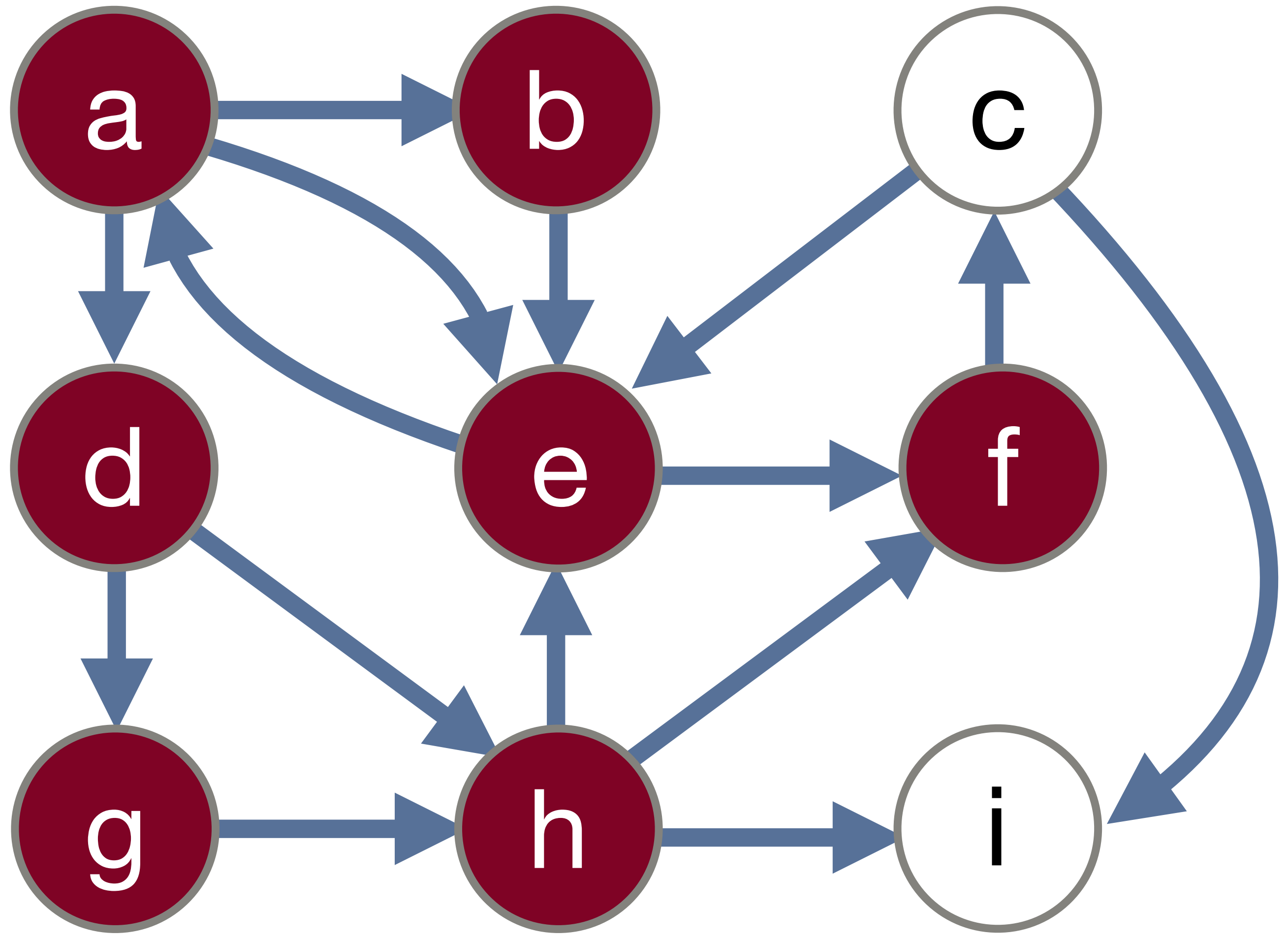
queue:

								front
				abefc	aefc	adhi	adhf	adgh

in while loop:
    curPath = q.dequeue() (path is abef)
    v = last element in curPath (v is f)
    mark v as visited (already been marked)
    enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
  create a queue of paths (a vector), q
  q.enqueue(v_1 path)
  while q is not empty and v_2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    for each unvisited neighbor of v:
      make new path with v's neighbor as last element
      enqueue new path onto q



Visited Set:
a
b
d
e
f
h

Let's look at **bfs** from a to i:

queue:

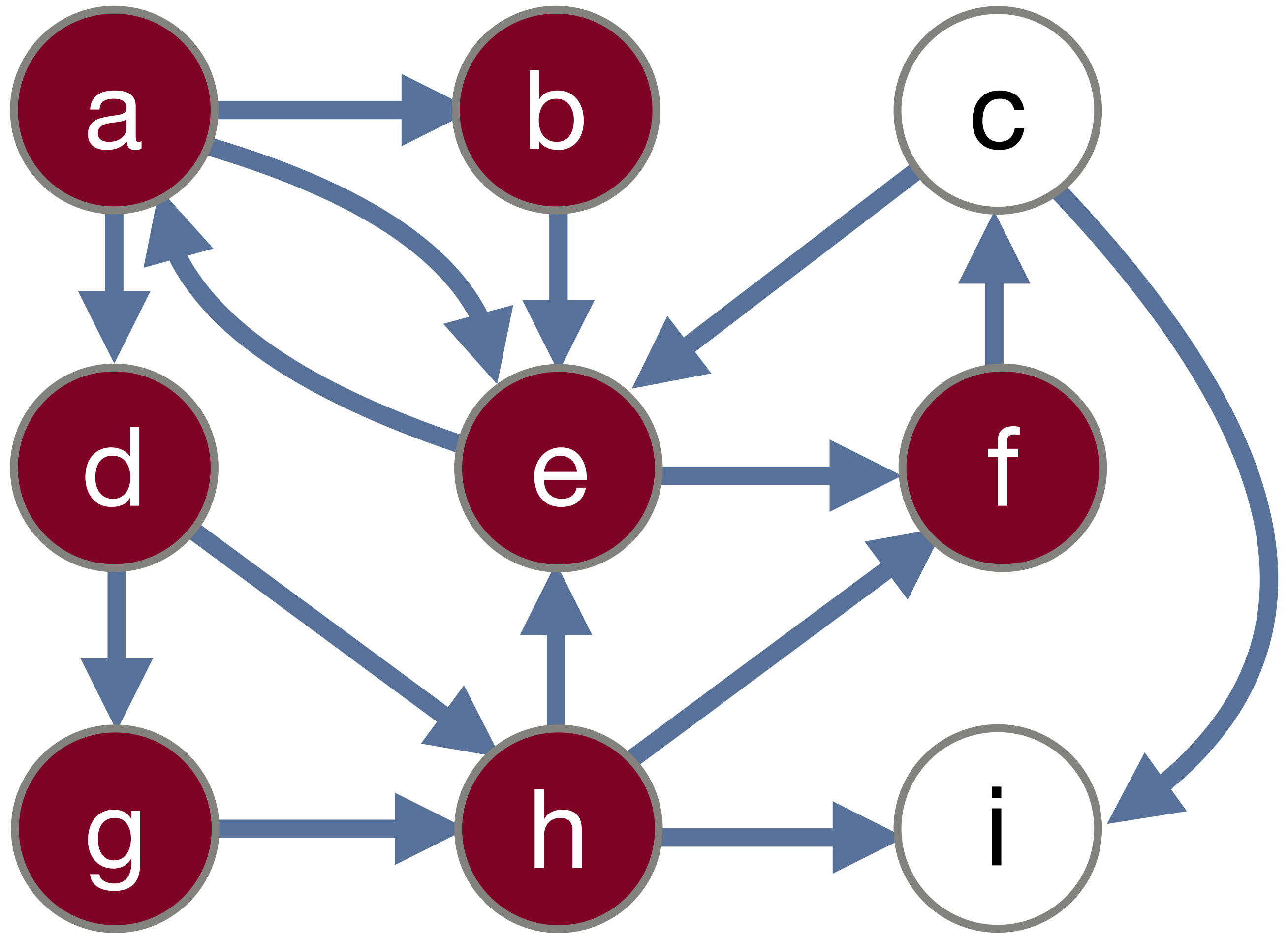
								front
				adghi	abefc	aefc	adhi	adhf

in while loop:
  curPath = q.dequeue() (path is adgh)
  v = last element in curPath (v is h)
  mark v as visited (already been marked)
  enqueue all unvisited neighbor paths onto q

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:

```
create a queue of paths (a vector), q
q.enqueue(v1 path)
while q is not empty and v2 is not yet visited:
    path = q.dequeue()
    v = last element in path
    mark v as visited
    for each unvisited neighbor of v:
        make new path with v's neighbor as last element
        enqueue new path onto q
```



Let's look at **bfs** from a to i:

queue:

								front
				adhfc	adghi	abefc	aefc	adhi

in while loop:

curPath = q.dequeue() (path is adhf)

v = last element in curPath (v is f)

mark v as visited (already been marked)

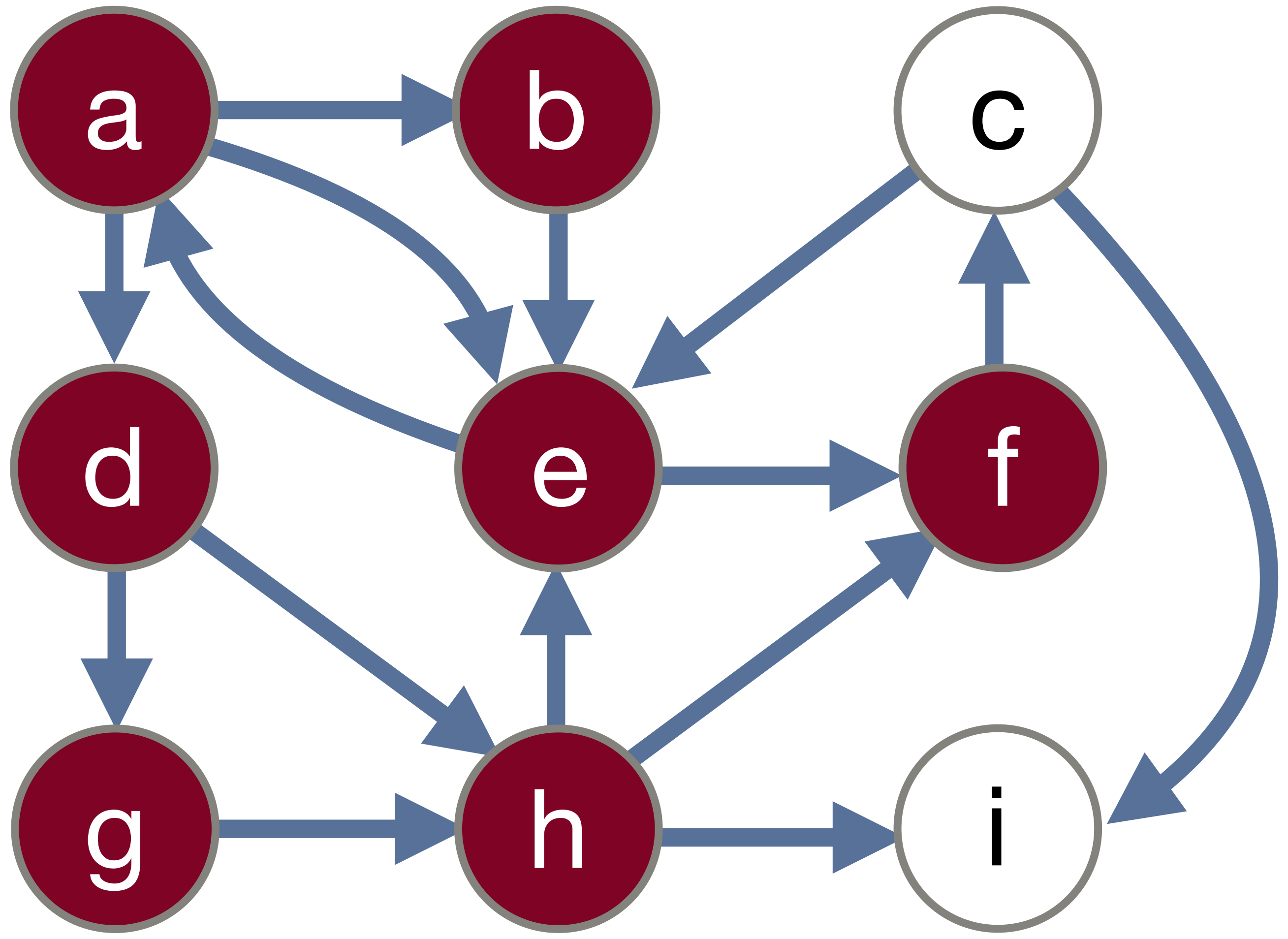
enqueue all unvisited neighbor paths onto q

Visited Set:

a
b
d
e
f
h

Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2:
    create a queue of paths (a vector), q
    q.enqueue(v_1 path)
    while q is not empty and v_2 is not yet visited:
        path = q.dequeue()
        v = last element in path
        mark v as visited
        for each unvisited neighbor of v:
            make new path with v's neighbor as last element
            enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
				adhfc	adghi	abefc	aefc	adhi

in while loop:
  curPath = q.dequeue() (path is adhi)
  v = last element in curPath (v is i)
  found!

Visited Set:
a
b
d
e
f
h
i

Wikipedia: Getting to Philosophy

WIKIPEDIA
The Free Encyclopedia

So I downloaded Wikipedia…

It turns out that you *can* download Wikipedia, but it is > 10 Terabytes (!) uncompressed. The reason Wikipedia asks you for money every so often is because they have lots of fast computers with lots of memory, and this is expensive (so donate!)

But, the Internet is just a graph...so, Wikipedia pages are just a graph...let's just do the searching by taking advantage of this: download pages as we need them.

Wikipedia: Getting to Philosophy

What kind of search is the "getting to philosophy" algorithm?

"Clicking on the first lowercase link in the main text of a Wikipedia article, and then repeating the process for subsequent articles, usually eventually gets one to the Philosophy article."

This is a depth-first search! To determine if a Wikipedia article will get to Philosophy, we just select the first link each time. If we ever have to select a second link (or if a first-link refers to a visited vertex), then that article doesn't get to Philosophy.

Wikipedia: Getting to Philosophy

We can also perform a Breadth First Search, as well. How would this change our search?

A BFS would look at all links on a page, then all links for each link on the page, etc. This has the potential of taking a long time, but it will find a shortest path.

References and Advanced Reading

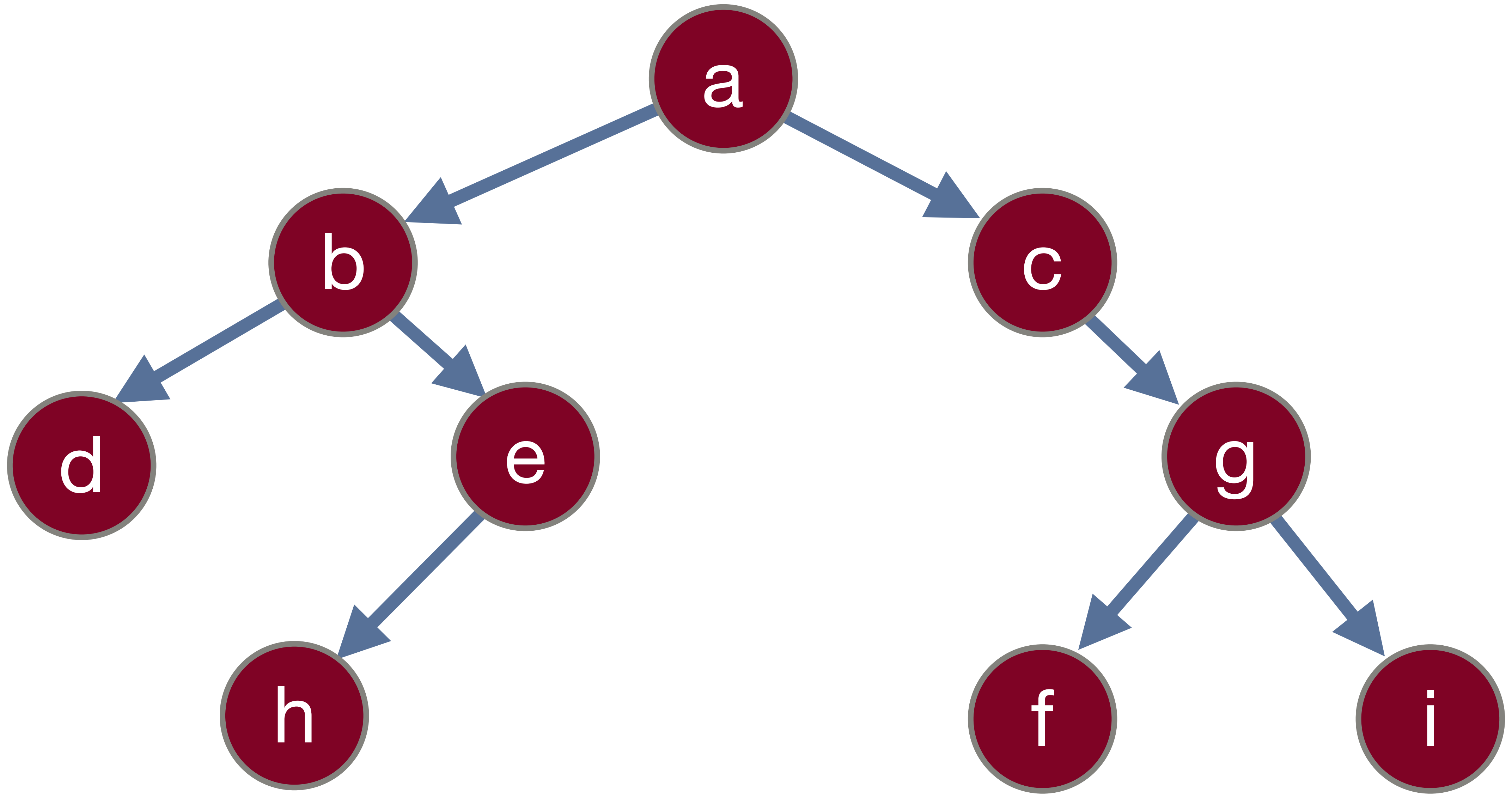
- **References:**
  - Depth First Search, Wikipedia: https://en.wikipedia.org/wiki/Depth-first_search
  - Breadth First Search, Wikipedia: https://en.wikipedia.org/wiki/Breadth-first_search

- **Advanced Reading:**
  - Visualizations:
  - https://www.cs.usfca.edu/~galles/visualization/DFS.html
  - https://www.cs.usfca.edu/~galles/visualization/BFS.html

Extra Slides

Breadth First Search (BFS): Tree searching

A Breadth First Search on a tree will produce a "level order traversal":



Breadth First Search: a☞b☞c☞d☞e☞g☞h☞f☞i

This is necessary if we want to print the tree to the screen in a pretty way, such that it retains its tree-like structure.