CS 106B, Lecture 13
Recursive Backtracking

Plan for Today

- More backtracking!
  - Make sure to practice, in section, on CodeStepByStep, with the book
- Some notes on the midterm

"Arm's length" recursion

- Arm's length recursion: a poor style where unnecessary tests are performed before performing recursive calls
- Typically, the tests try to avoid making a call into what would otherwise be a base case
- Can lead to **functionality bugs** as well as **less readable code**
- Applies to all recursive code but **especially backtracking**

Backtracking Model

Choosing

1. We generally iterate over **decisions**. What are we iterating over here? What are the **choices** for each decision? Do we need a for loop?

Exploring

2. How can we *represent* that choice? How should we **modify the parameters** and **store our previous choices** (avoiding *arms-length* recursion)?
   a) Do we need to use a **wrapper** due to extra parameters?
3. How should we **restrict** our choices to be valid?
4. How should we use the **return value** of the recursive calls? Are we looking for all solutions or just one?

Un-choosing

5. How do we **un-modify** the parameters from step 3? Do we need to explicitly un-modify, or are they copied? Are they un-modified at the same level as they were modified?

Base Case

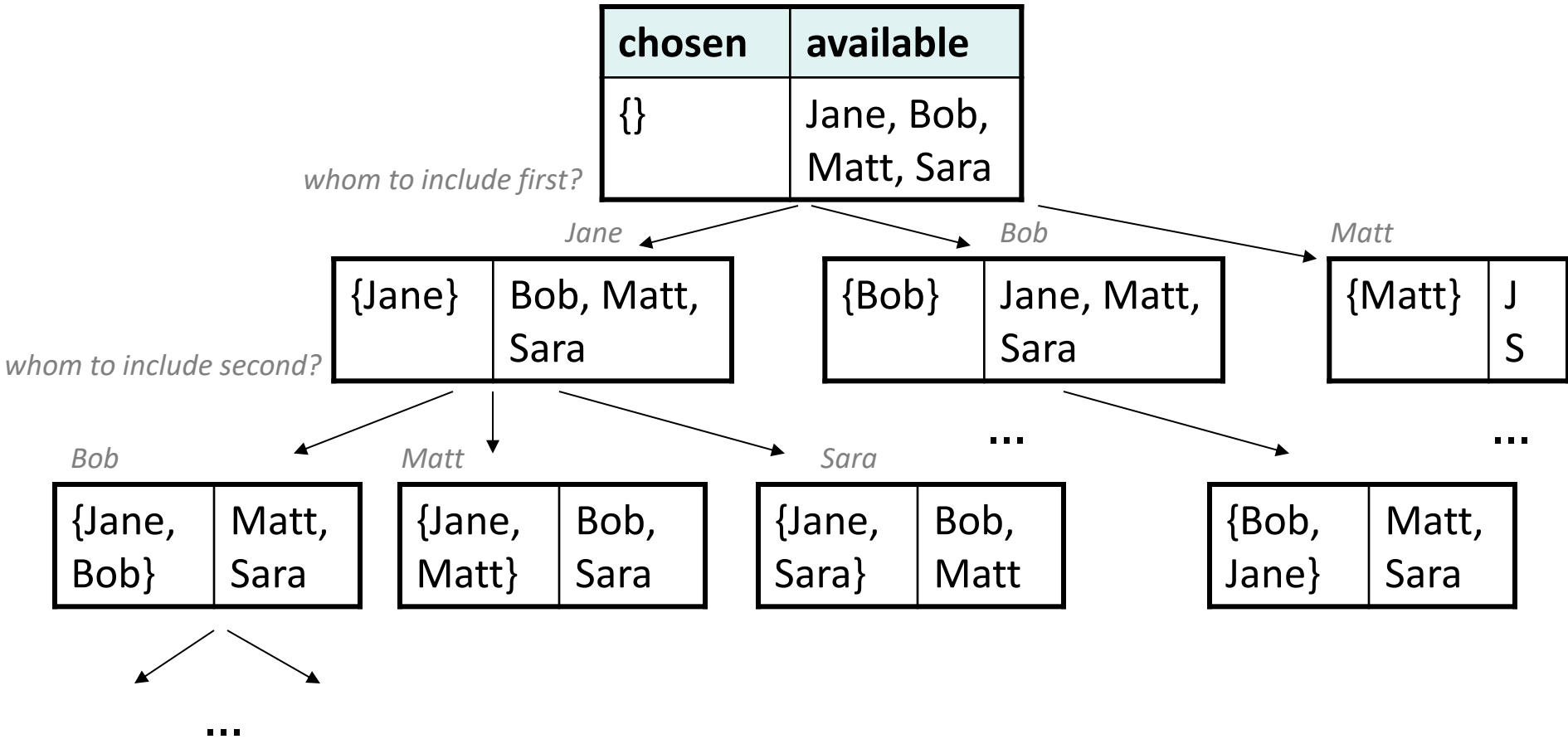
6. What should we do in the base case when we're **out of decisions** (usually return true)?
7. Is there a case for when there **aren't any valid choices left** or a "bad" state is reached (usually return false)?
8. Are the base cases ordered properly? Are we avoiding **arms-length** recursion?

Exercise: sublists

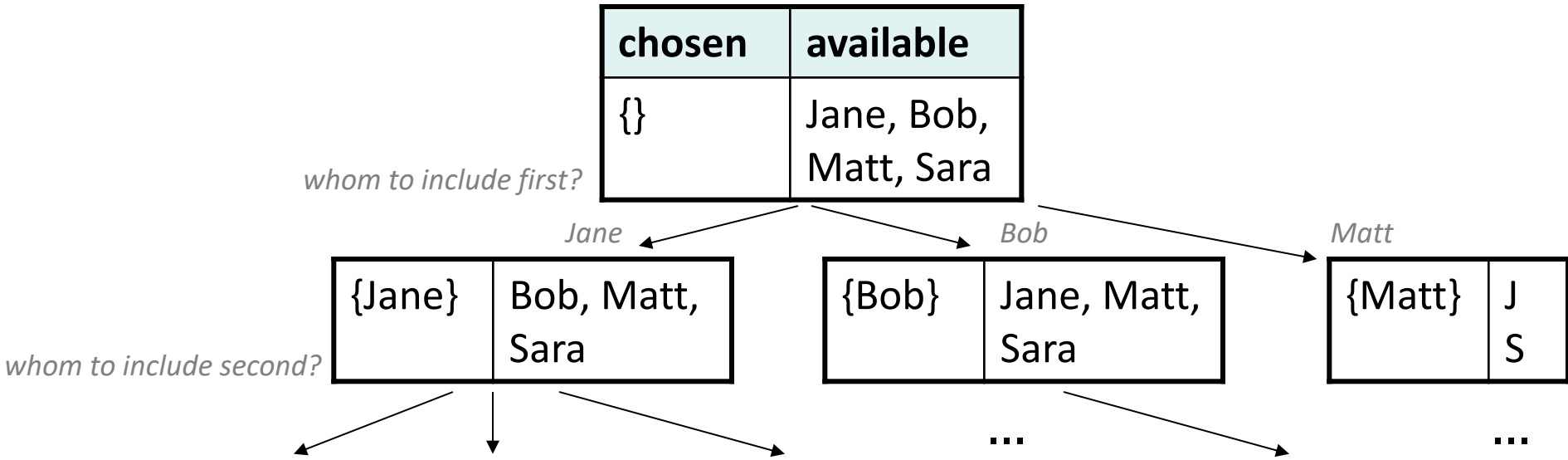
- Write a function **`sublists`** that finds every possible sub-list of a given vector. A sub-list of a vector *V* contains ≥ 0 of *V*'s elements.
  - Example: if *V* is `{Jane, Bob, Matt, Sara}`, then the call of **`sublists`**`(V);` prints:

```
{Jane, Bob, Matt, Sara}
{Jane, Bob, Matt}
{Jane, Bob, Sara}
{Jane, Bob}
{Jane, Matt, Sara}
{Jane, Matt}
{Jane, Sara}
{Jane}
{Bob, Matt, Sara}
{Bob, Matt}
{Bob, Sara}
{Bob}
{Matt, Sara}
{Matt}
{Sara}
{}
```

Decision tree?



Wrong decision tree



Q: Why isn't this the right decision tree for this problem?

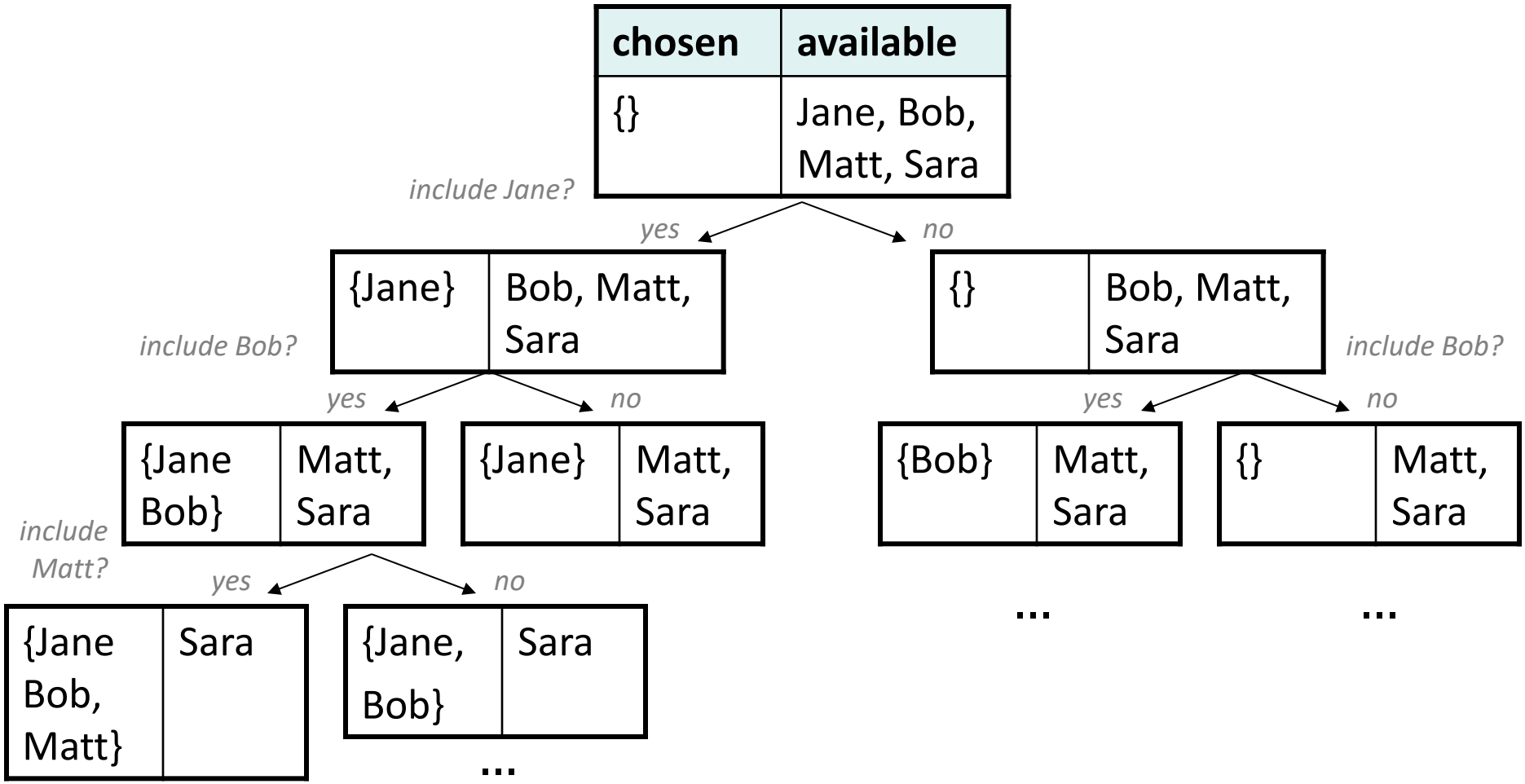
A. It does not actually end up finding every possible sublist.

B. It does find all sublists, but it finds them in the wrong order.

C. It does find all sublists, but it is inefficient.

D. None of the above

Better decision tree



- Each decision is: "Include Jane or not?" ... "Include Bob or not?" ...
  - The **order** of people chosen does not matter; only the **membership**.

Mental Model

- **Choose**: What decisions do we have to make? What are our choices?
- **Explore**: How should we modify our parameters after making a choice?
- **Un-Choose**: How do we revert our choice?
- **Base Case**: What should we do when we are out of decisions to make?

Mental Model

- **Choose**: What decisions do we have to make? What are our choices?
  - *Whether to include a person or not*
- **Explore**: How should we modify our parameters after making a choice?
  - *Build up a vector containing people chosen so far*
- **Un-Choose**: How do we revert our choice?
  - *Remove the person previously inserted into the vector*
- **Base Case**: What should we do when we are out of decisions to make?
  - *Print the result vector*

sublists solution

```
void sublists(Vector<string>& v) {
    Vector<string> chosen;
    sublistsHelper(v, 0, chosen);
}

void sublistsHelper(Vector<string>& v, int i,
                    Vector<string>& chosen) {
    if (i >= v.size()) {
        cout << chosen << endl;   // base case; nothing to choose
    } else {
        // there are two choices to explore:
        // the subset without i'th element, and the one with it

        sublistsHelper(v, i+1, chosen);  // choose/explore (without)

        chosen.add(v[i]);
        sublistsHelper(v, i+1, chosen);  // choose/explore (with)

        chosen.remove(chosen.size() - 1);   // "undo" our choice
    }
}
```

Announcements

- Assignment 4 goes out tonight. You should receive Assn2 feedback by end of the day
  - A small part of Assn. 4 uses structs which are covered tomorrow.
- Exam logistics
  - Midterm review session in class on 7/23
  - Midterm is on Wednesday, July 24, from 7:00-9:00PM
  - Midterm info (list of topics covered and study tips) online: https://web.stanford.edu/class/cs106b/exams/midterm.html
  - **Highly Recommended:** Complete assignment 4 (or parts of it) before the midterm – backtracking will be tested. Assignment 4 will not be due until July 25th though
  - Lectures 14 and 15 are NOT included on the midterm

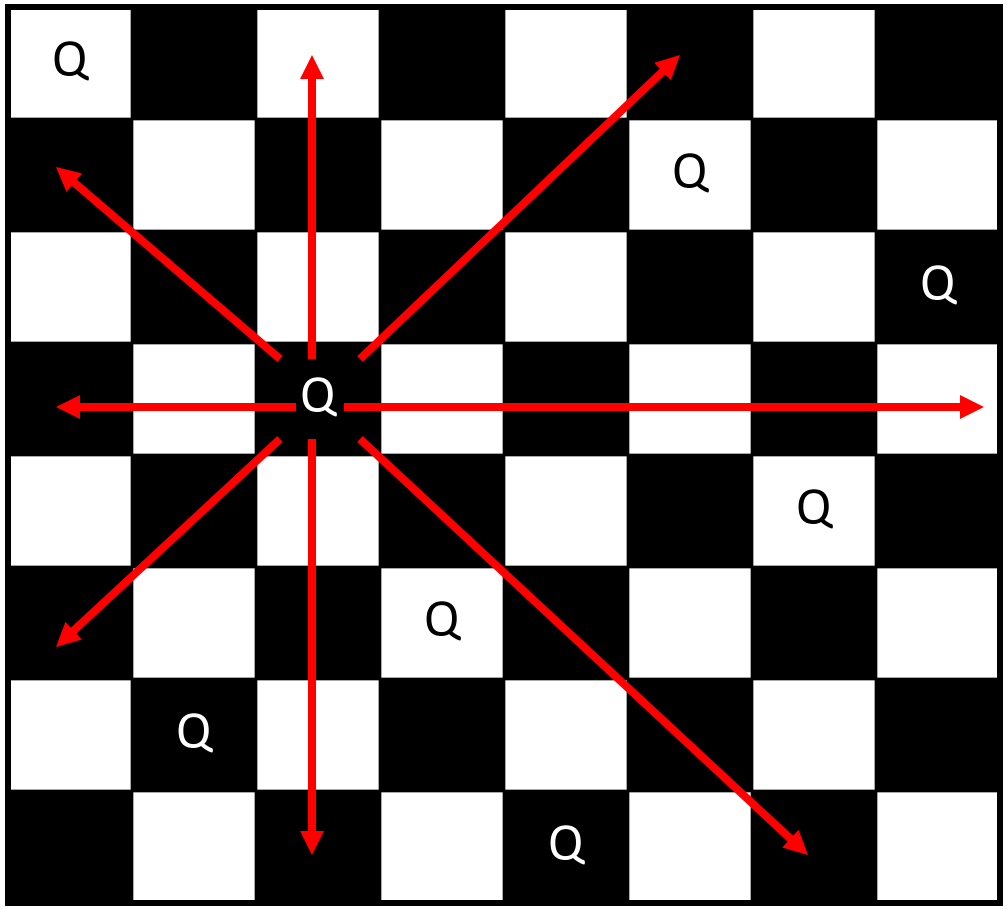
Announcements

- Practice midterm is released. You need BlueBook to use it.
  - Download the file as a .json file
  - Open BlueBook, follow instructions

- Save your answers in a separate document to compare to the practice midterm answers.
  - Practice midterm answers will be released in a few days

The "8 Queens" problem

- Consider the problem of trying to place 8 queens on a chess board such that no queen can attack another queen.



Exercise

- Suppose we have a Board class with the following methods:

Member	Description
`Board b(size);`	construct empty board
`b.isSafe(row, column)`	`true` if a queen could be safely placed here (0-based)
`b.isValid()`	`true` if all current queens are safe
`b.place(row, column);`	place queen here
`b.remove(row, column);`	remove queen from here
`cout << b << endl;` or `b.toString()`	print/return a text display of the board state

- Write a function **`solveQueens`** that accepts a Board as a parameter and tries to place 8 queens on it safely.
  - Your method should return a board with the queens placed if it's possible.

Mental Model

- **Choose**: What decisions do we have to make? What are our choices?
- **Explore**: How should we modify our parameters after making a choice?
- **Un-Choose**: How do we revert our choice?
- **Base Case**: What should we do when we are out of decisions to make?

Naive algorithm

- for (each board square):
  - Place a queen there.
  - Try to place the rest of the queens.
  - Un-place the queen.

Q: How large is the solution space for this algorithm?

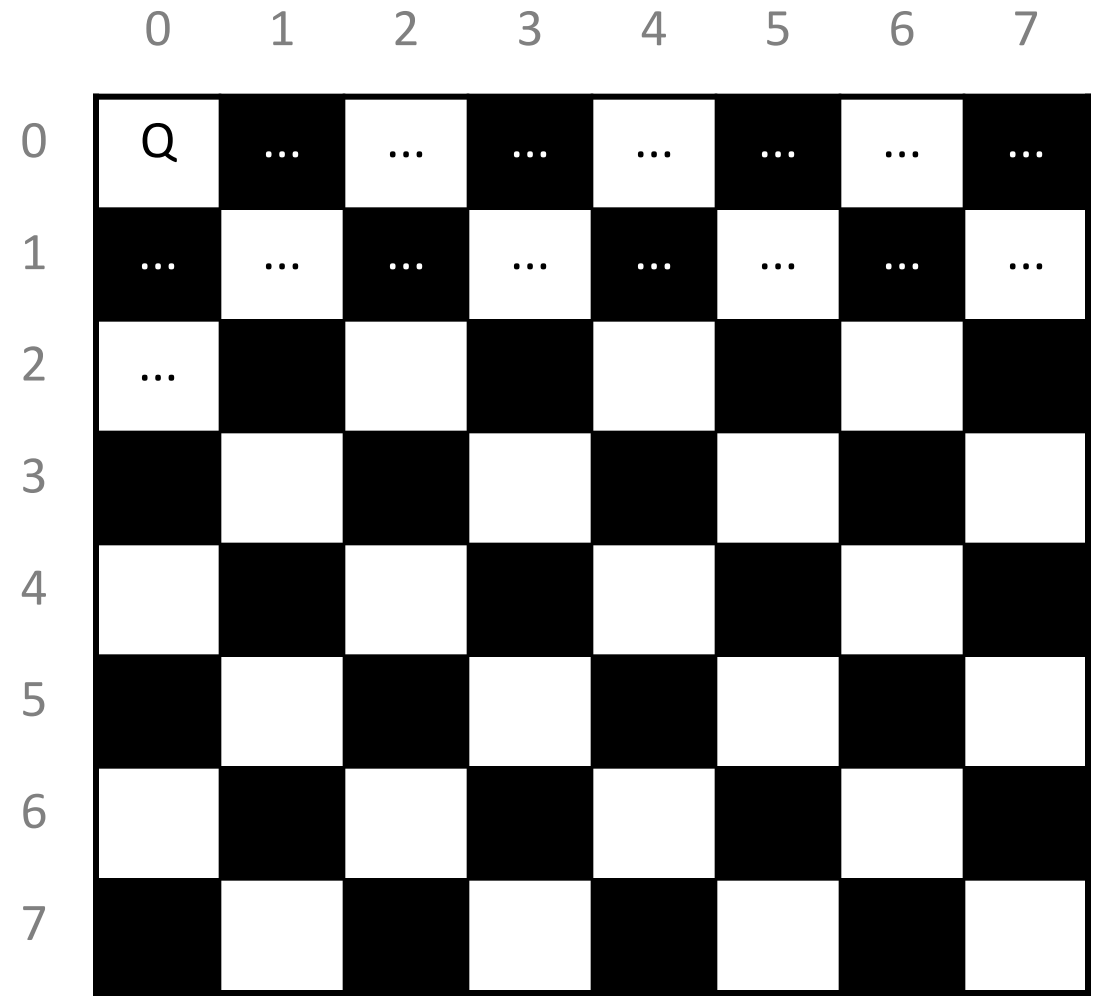
A. 64 choices

B. 64 * 8

C. 64^8

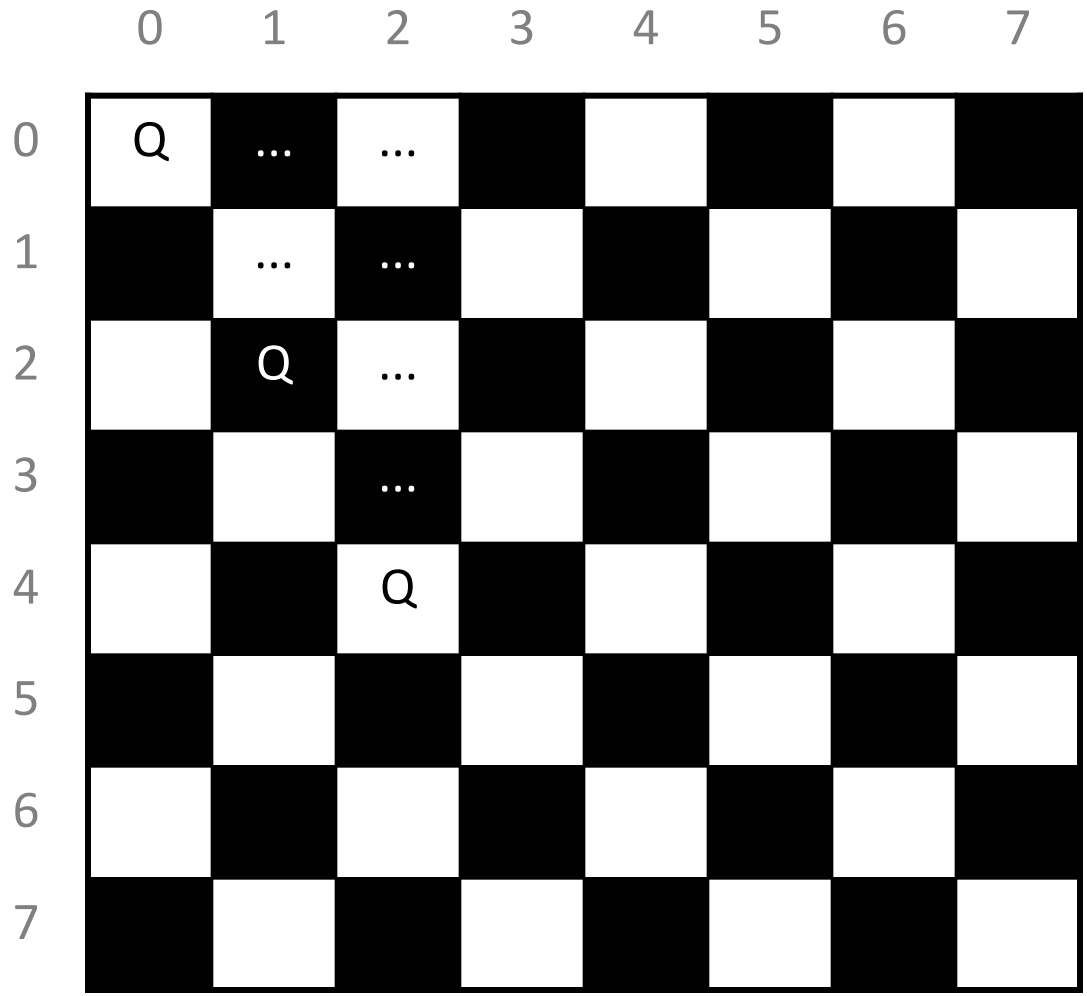
D. 64*63*62*61*60*59*58*57

E. none of the above



Better algorithm idea

- Observation: In a working solution, exactly 1 queen must appear in each row and in each column.
  - Redefine a "choice" to be valid placement of a queen in a particular column.
  - How large is the solution space now?
    - 8 * 8 * 8 * …



Mental Model

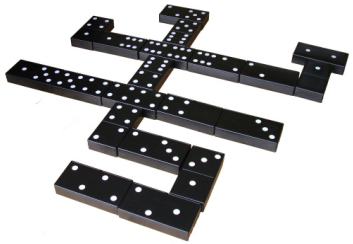
- **Choose**: What decisions do we have to make? What are our choices?
  - *Where in a column to place a queen*
- **Explore**: How should we modify our parameters after making a choice?
  - *Place the queen on the board, move on to the next column*
- **Un-Choose**: How do we revert our choice?
  - *Remove the queen that we placed previously*
- **Base Case**: What should we do when we are out of decisions to make?
  - *Return true*

8 Queens solution

```
// Recursively searches for a solutions to N queens
// on this board, starting with the given column.
// PRE: queens have been safely placed in columns 0 to (col-1)
bool solveHelper(Board& board, int col) {
    if (!board.isValid()) {
        return false;
    } else if (col >= board.size()) {
        return true; // base case: all columns placed
    } else {
        // recursive case: try to place a queen in this column
        for (int row = 0; row < board.size(); row++) {
            board.place(row, col);              // choose
            if (solveHelper(board, col + 1)) { // explore
                return true;
            }
            board.remove(row, col);             // un-choose
        }
    }
    return false;
}
bool solveQueens(Board& board) {
    solveHelper(board, 0);
}
```

Exercise: Dominoes

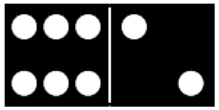
- Dominoes uses black tiles, each having 2 numbers of dots from 0-6. Players line up tiles to match dots.



- Given a class `Domino` with the following members:

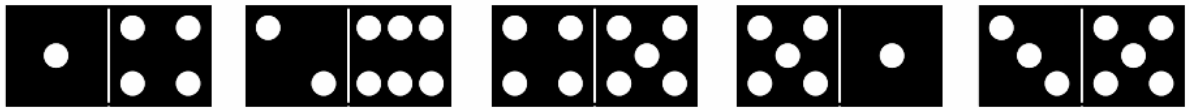
```
int first()                 // first dots value from 0-6
int second()                // second dots value from 0-6
void flip()                 // inverts 1st/2nd
bool contains(int n)        // true if 1st and/or 2nd == n
string toString()           // e.g. "(3|5)"
```

- Write a function **`chainExists`** that takes a `Vector` of dominoes and a starting/ending dot value, and returns whether the dominoes can be made into a chain that starts/ends with those values.

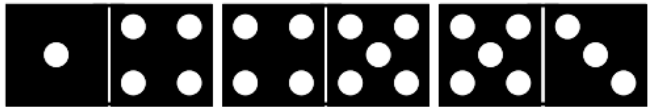


Domino chains

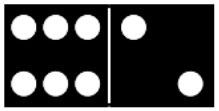
- Suppose we have the following dominoes:



- We can link them into a chain from 1 to 3 as follows:
  - Notice that the 3|5 domino had to be flipped.

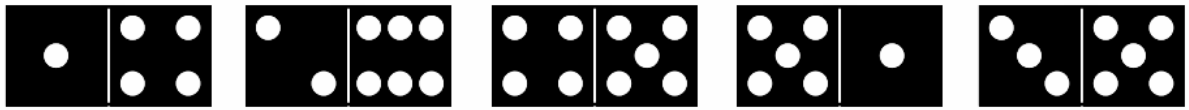


- We can "link" one domino into a "chain" from 6 to 2 as follows:



Enumerating choices

- If we have these dominoes, and we want a chain from 1 to 3:



Q: What are the "choices" your code should explore?

A. The numbers 0-6 that can appear on a domino.

B. The set of all of the dominoes above.

C. The set of dominoes above whose first number is 1.

D. The set of dominoes above whose second number is 3.

E. The set of dominoes above whose first or second number is 1.

hasChain pseudocode

```
function chainExists(dominoes, start, end):
    if dominoes is empty: nothing to do.
    if start == end:
        if any domino in dominoes contains start, return true.
    else:
        for each domino d in dominoes:
            if d contains start:
                choose d.
                if chainExists(dominoes):    // explore remaining dominoes.
                    return true.
                un-choose d.

        return false.   // no chain found
```

hasChain solution

```
bool chainExists(Vector<Domino>& dominoes, int start, int end) {
    if (start == end) {                          // base case
        for (Domino d : dominoes) {
            if (d.contains(start)) { return true; }
        }
        return false;
    } else {
        for (int i = 0; i < dominoes.size(); i++) {
            Domino d = dominoes[i];
            if (d.second() == start) {
                d.flip();
            }
            if (d.first() == start) {
                dominoes.remove(i);              // choose
                if (d.second() == end ||         // explore
                        chainExists(dominoes, d.second(), end)) {
                    dominoes.insert(i, d);
                    return true;
                }
                dominoes.insert(i, d);           // un-choose
            }
        }
        return false;
    }
}
```