CS 106B, Lecture 11
Exhaustive Search and Backtracking

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Based on slides created by Marty Stepp, Chris Gregg, Keith Schwarz, Julie Zelenski, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, and others

Plan for Today

- New recursive problem-solving techniques
  - Exhaustive Search
  - Backtracking

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Exhaustive search

- **exhaustive search**: Exploring every possible combination from a set of choices.
  - often implemented recursively
  - Sometimes called *recursive enumeration*

  Applications:
  - producing all permutations of a set of values
  - enumerating all possible names, passwords, etc.

- Often the search space consists of many ***decisions***, each of which has several available ***choices***.
  - Example: When enumerating all 5-letter strings, each of the 5 letters is a *decision*, and each of those decisions has 26 possible *choices*.

Exhaustive search

A general pseudo-code algorithm for exhaustive search:

Explore(*decisions*):

– if there are no more decisions to make: Stop.

– else, let's handle one decision ourselves, and the rest by recursion.
for each available choice *C* for this decision:

- **Choose** *C* by modifying parameters.
- **Explore** the remaining decisions that could follow *C*.
- **Un-choose** *C* by returning parameters to original state (if necessary).

Exercise: printAllBinary

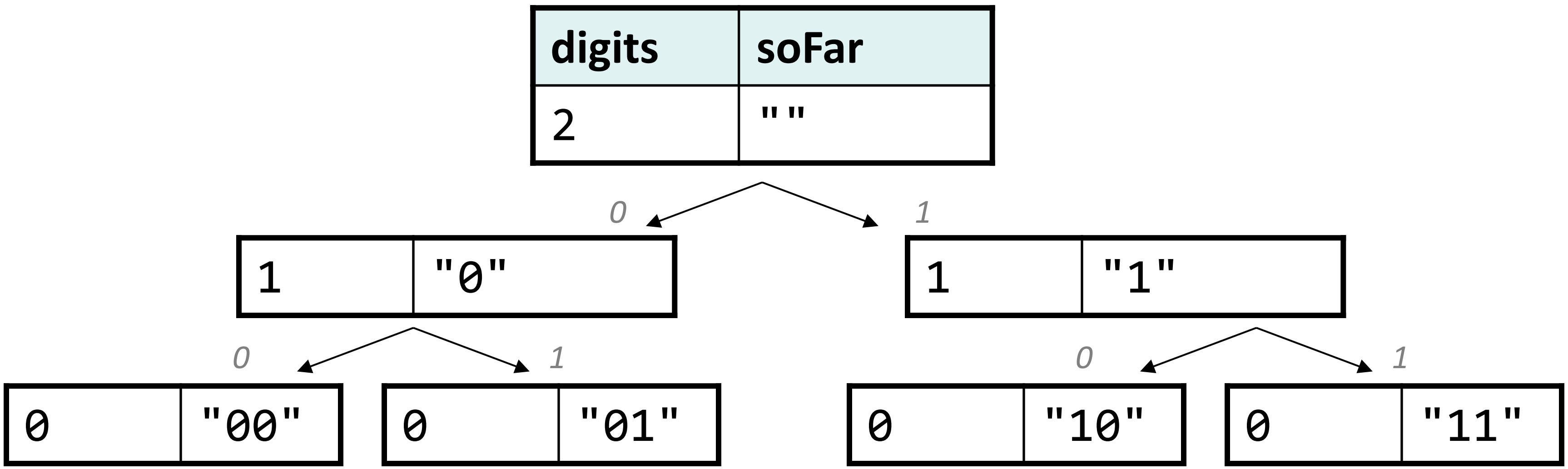
- Write a recursive function **`printAllBinary`** that accepts an integer number of digits and prints all binary numbers that have exactly that many digits, in ascending order, one per line.
  - `printAllBinary(2);`
    ```
    00
    01
    10
    11
    ```
    `printAllBinary(3);`
    ```
    000
    001
    010
    011
    100
    101
    110
    111
    ```

printAllBinary solution

```
void printAllBinary(int numDigits) {
    printAllBinaryHelper(numDigits, "");
}

void printAllBinaryHelper(int digits, string soFar) {
    if (digits == 0) {
        cout << soFar << endl;
    } else {
        printAllBinaryHelper(digits - 1, soFar + "0");
        printAllBinaryHelper(digits - 1, soFar + "1");
    }
}
```

A tree of calls

- `printAllBinary(2);`



  - This kind of diagram is called a *call tree* or *decision tree*.
  - Think of each call as a choice or decision made by the algorithm:
    - Should I choose 0 or 1 as the next digit?

The base case

```
void printAllBinaryHelper(int digits, string soFar) {
    if (digits == 0) {
        cout << soFar << endl;
    } else {
        printAllBinaryHelper(digits - 1, soFar + "0");
        printAllBinaryHelper(digits - 1, soFar + "1");
    }
}
```

- The **base case** is where the code stops after doing its work.
  - `pAB(3) -> pAB(2) -> pAB(1) -> pAB(0)`
- Each call should keep track of the work it has done.

- Base case should print the result of the work done by prior calls.
  - Work is kept track of in some variable(s) - in this case, `string soFar`.

Exercise: printDecimal

- Write a recursive function **`printDecimal`** that accepts an integer number of digits and prints all base-10 numbers that have exactly that many digits, in ascending order, one per line.
  - `printDecimal(2);`

    ```
    00
    01
    02
    ..
    98
    99
    ```

    `printDecimal(3);`

    ```
    000
    001
    002
    ...
    997
    998
    999
    ```

printDecimal solution

```
void printDecimal(int digits) {
    printDecimalHelper(digits, "");
}

void printDecimalHelper(int digits, string soFar) {
    if (digits == 0) {
        cout << soFar << endl;
    } else {
        for (int i = 0; i < 10; i++) {
            printDecimalHelper(digits - 1, soFar +
                               integerToString(i));
        }
    }
}
```

Announcements

- Homework 3 due on Wednesday at **5PM**

- **Midterm next Wednesday,** 7/24 7-9PM

Plan for Today

- New recursive problem-solving techniques
  - Exhaustive Search
  - Backtracking

Backtracking

- **Backtracking**: Finding solution(s) by trying all possible paths and then abandoning them if they are not suitable.
  - a "brute force" algorithmic technique
  - often implemented recursively
  - Could involve looking for **one** solution
    - If any of the paths found a solution, a solution exists! If none find a solution, no solution exists
  - Could involve finding **all solutions**
  - Idea: it's exhaustive search **with conditions**

Applications:

- games: anagrams, crosswords, word jumbles, 8 queens, sudoku
- combinatorics and logic programming
- escaping from a maze

Backtracking: One Solution

A general pseudo-code algorithm for backtracking problems searching for one solution

Backtrack(*decisions*):

- if there are no more decisions to make:
  - if our current solution is valid, return **true**
  - else, return **false**
- else, let's handle one decision ourselves, and the rest by recursion. for each available **valid** choice *C* for this decision:
  - **Choose** *C* by modifying parameters.
  - **Explore** the remaining decisions that could follow *C*. If any of them find a solution, return **true**
  - **Un-choose** *C* by returning parameters to original state (if necessary).
- If no solutions were found, **return false**

Backtracking: All Solutions

A general pseudo-code algorithm for backtracking problems searching for all solutions

Backtrack(*decisions*):

- if there are no more decisions to make:
  - if our current solution is valid, add it to our list of found solutions
  - else, do nothing or return
- else, let's handle one decision ourselves, and the rest by recursion. for each available **valid** choice *C* for this decision:
  - **Choose** *C* by modifying parameters.
  - **Explore** the remaining decisions that could follow *C*. Keep track of which solutions the recursive calls find.
  - **Un-choose** *C* by returning parameters to original state (if necessary).
- Return the list of solutions found by all the helper recursive calls.

Exercise: Dice roll sum

- Write a function **`diceSum`** that accepts two integer parameters: a number of dice to roll, and a desired sum of all die values. Output all combinations of die values that add up to exactly that sum.

```
diceSum(2, 7);

{1, 6}
{2, 5}
{3, 4}
{4, 3}
{5, 2}
{6, 1}
```

```
diceSum(3, 7);

{1, 1, 5}
{1, 2, 4}
{1, 3, 3}
{1, 4, 2}
{1, 5, 1}
{2, 1, 4}
{2, 2, 3}
{2, 3, 2}
{2, 4, 1}
{3, 1, 3}
{3, 2, 2}
{3, 3, 1}
{4, 1, 2}
{4, 2, 1}
{5, 1, 1}
```

Easier: Dice rolls

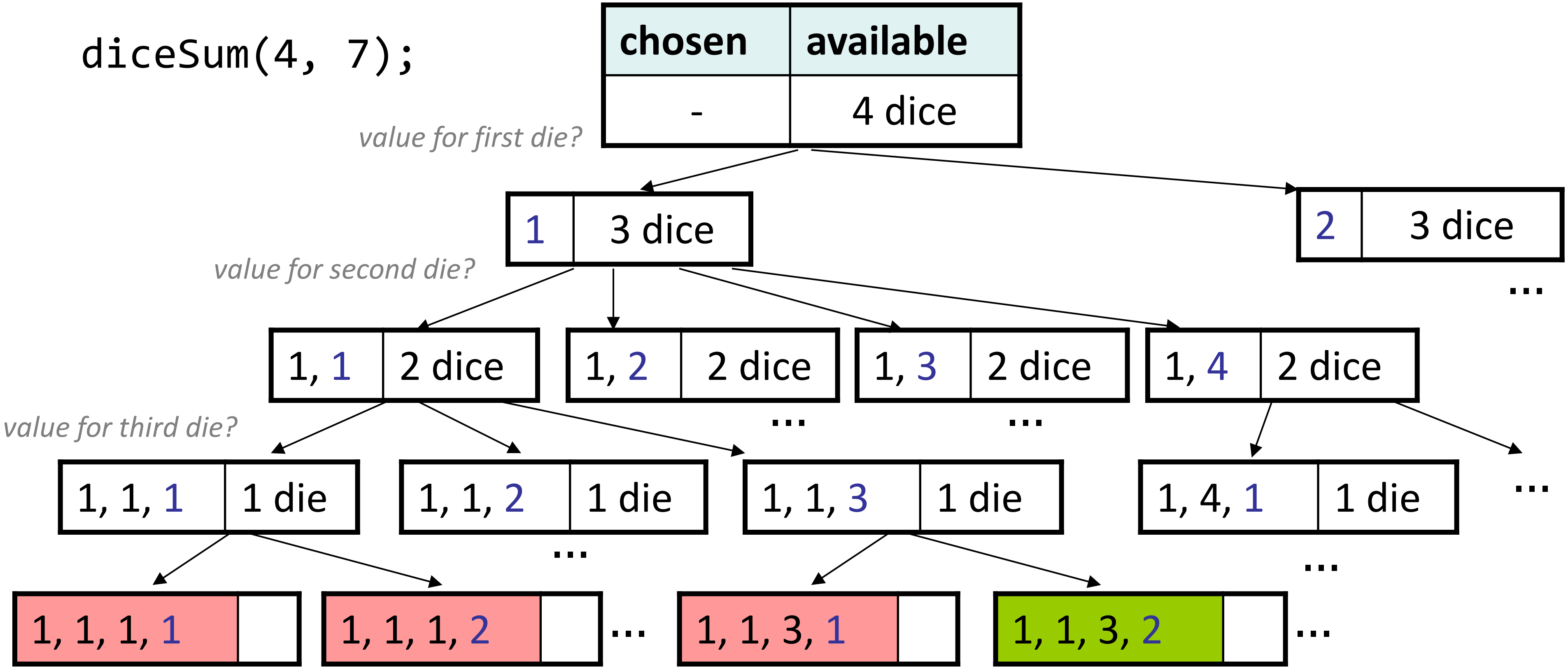
- **Suggestion:** First just output all possible combinations of values that could appear on the dice.
- This is just exhaustive search!
- In general, starting with exhaustive search and then adding conditions is not a bad idea

```
diceSum(2, 7);

{1, 1}    {3, 1}    {5, 1}
{1, 2}    {3, 2}    {5, 2}
{1, 3}    {3, 3}    {5, 3}
{1, 4}    {3, 4}    {5, 4}
{1, 5}    {3, 5}    {5, 5}
{1, 6}    {3, 6}    {5, 6}
{2, 1}    {4, 1}    {6, 1}
{2, 2}    {4, 2}    {6, 2}
{2, 3}    {4, 3}    {6, 3}
{2, 4}    {4, 4}    {6, 4}
{2, 5}    {4, 5}    {6, 5}
{2, 6}    {4, 6}    {6, 6}
```

```
diceSum(3, 7);

{1, 1, 1}
{1, 1, 2}
{1, 1, 3}
{1, 1, 4}
{1, 1, 5}
{1, 1, 6}
{1, 2, 1}
{1, 2, 2}
   ...
{6, 6, 4}
{6, 6, 5}
{6, 6, 6}
```

A decision tree

```
diceSum(4, 7);
```

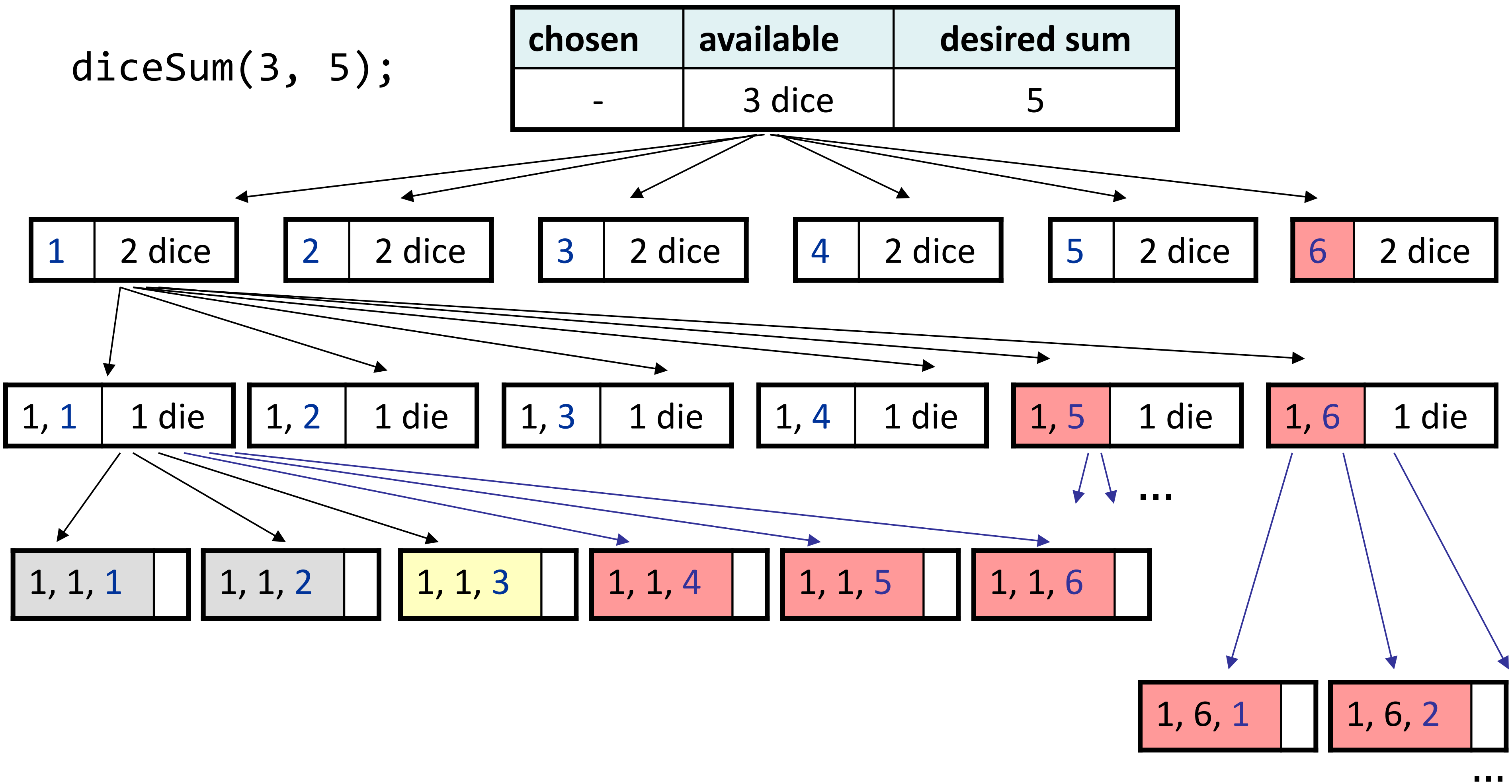


Initial solution

```
void diceSum(int dice, int desiredSum) {
    Vector<int> chosen;
    diceSumHelper(dice, desiredSum, chosen);
}

void diceSumHelper(int dice, int desiredSum, Vector<int>& chosen) {
    if (dice == 0) {
        if (sumAll(chosen) == desiredSum) {
            cout << chosen << endl;                         // base case
        }
    } else {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);                                  // choose
            diceSumHelper(dice - 1, desiredSum, chosen);    // explore
            chosen.remove(chosen.size() - 1);               // un-choose
        }
    }
}

int sumAll(const Vector<int>& v) {   // adds the values in given vector
    int sum = 0;
    for (int k : v) { sum += k; }
    return sum;
}
```

Wasteful decision tree

```
diceSum(3, 5);
```



Optimizations

- We need not visit every branch of the decision tree.
  - Some branches are clearly not going to lead to success.
  - We can preemptively stop, or **prune**, these branches.

- Inefficiencies in our dice sum algorithm:
  - Sometimes the current sum is already **too high**.
    - (Even rolling 1 for all remaining dice would exceed the desired sum.)
  - Sometimes the current sum is already **too low**.
    - (Even rolling 6 for all remaining dice would exceed the desired sum.)
  - The code must **re-compute** the sum many times.
    - (1+1+1 = ..., 1+1+2 = ..., 1+1+3 = ..., 1+1+4 = ..., ...)

diceSum solution

```
void diceSum(int dice, int desiredSum) {
    Vector<int> chosen;
    diceSumHelper(dice, desiredSum, chosen);
}

void diceSumHelper(int dice, int desiredSum, Vector<int>& chosen) {
    if (dice == 0 && desiredSum == 0) {
        cout << chosen << endl;
    } else if (dice > 0 && (dice <= desiredSum && desiredSum <= dice*6)) {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);
            diceSum(dice - 1, desiredSum - i, chosen);
            chosen.removeBack();
        }
    }
}
```