

CS 106B

Lecture 23: Graphs II

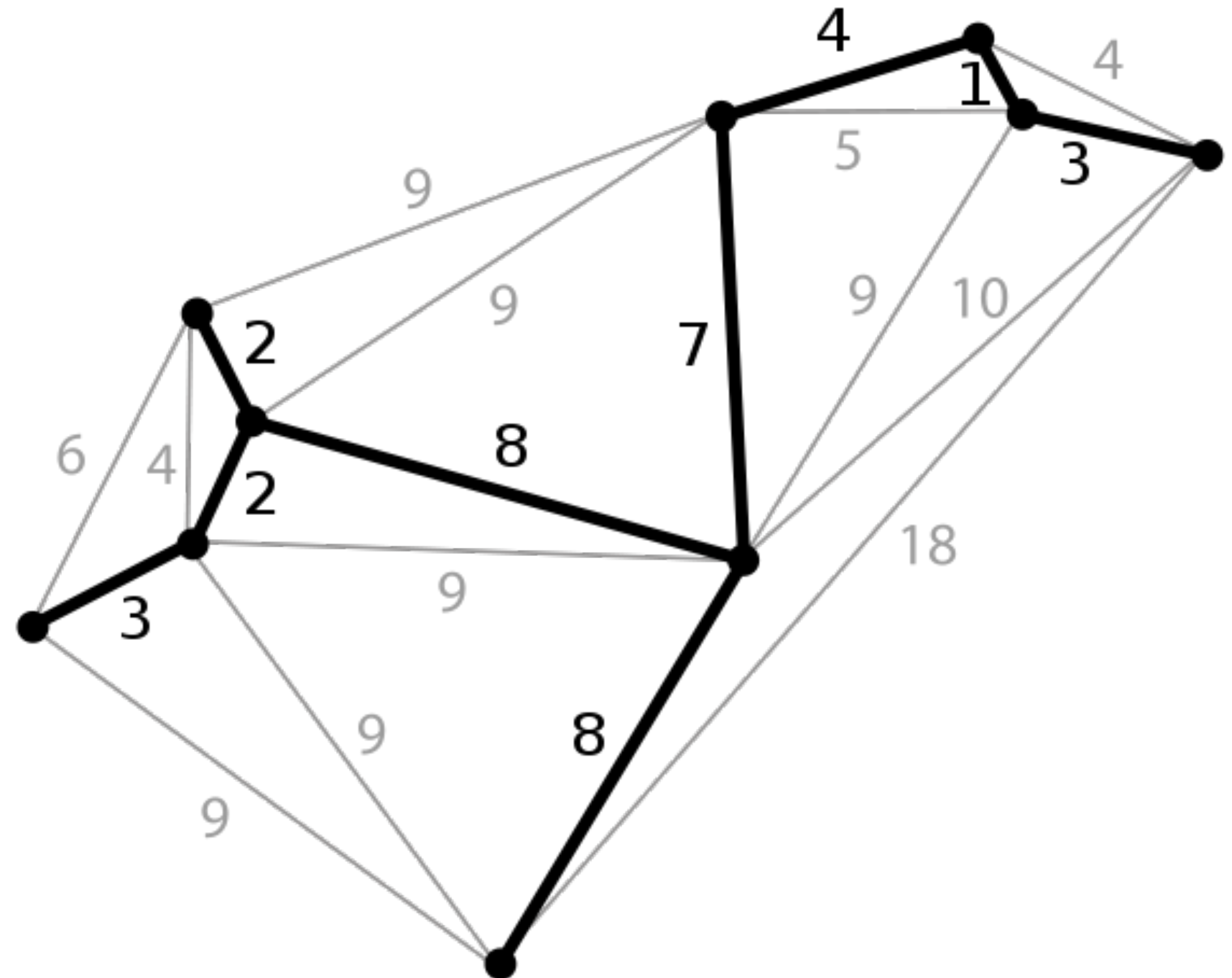
Wednesday, May 24, 2017

Programming Abstractions
Spring 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:

Programming Abstractions in C++, Chapter 18



Today's Topics

- Logistics
 - Huffman Encoding: due Thursday, 12:00pm -- last change to use late credits
 - Final Assignment YEAH hours tomorrow, 3:30pm, B03
 - CS+Social Good Interest form
 - How to Declare EE Lunch, Thursday, May 25th
- Real Graphs:
 - Internet routers and traceroute
- Topological Sort
- Minimum Spanning Trees
 - Kruskal's algorithm



CS+Social Good Interest Form

<https://goo.gl/mOcBEu>

CS53SI: Discussions in Social Tech Interest Form

CS+Social Good is planning on organizing a course aiming on discussing the intersection of technology and social impact through speakers and student discussions. If you are interested in taking such a class, we would love to hear what kind of class you would like to take!

DESCRIPTION: This course introduces students to various intersections of social good and technology through a weekly discussion and speaker series. Students will be given a space to exchange ideas and experiences regarding a certain social issue. Invited speakers come from industry, academia, and non-profit organizations. They will share their career paths, what drove them to these fields, and advice for students. The topics examined will span a broad variety of social issues -- from race and class to mental health to sustainability -- and help students better understand how to kick off their journey in using computer science for social good.



How to Declare EE Lunch

Fuse, the Electrical Engineering undergraduate student group, presents

How to Declare EE

Come walk through the process for declaring a major or minor in Electrical Engineering

We will have professors and student advisors who know all about the process!

Lunch from Asian Box will be provided!

Thursday, May 25th at noon

Packard 318



Real Graphs!

There was a Tiny Feedback from the first graphs lecture that said,

“I would give more examples of how graphs are used, or, in general, give a wider variety of examples of applications to the methods being used. I think it would be interesting to give examples of how certain topics can be used in cross-sections of CS and other majors/departments.”

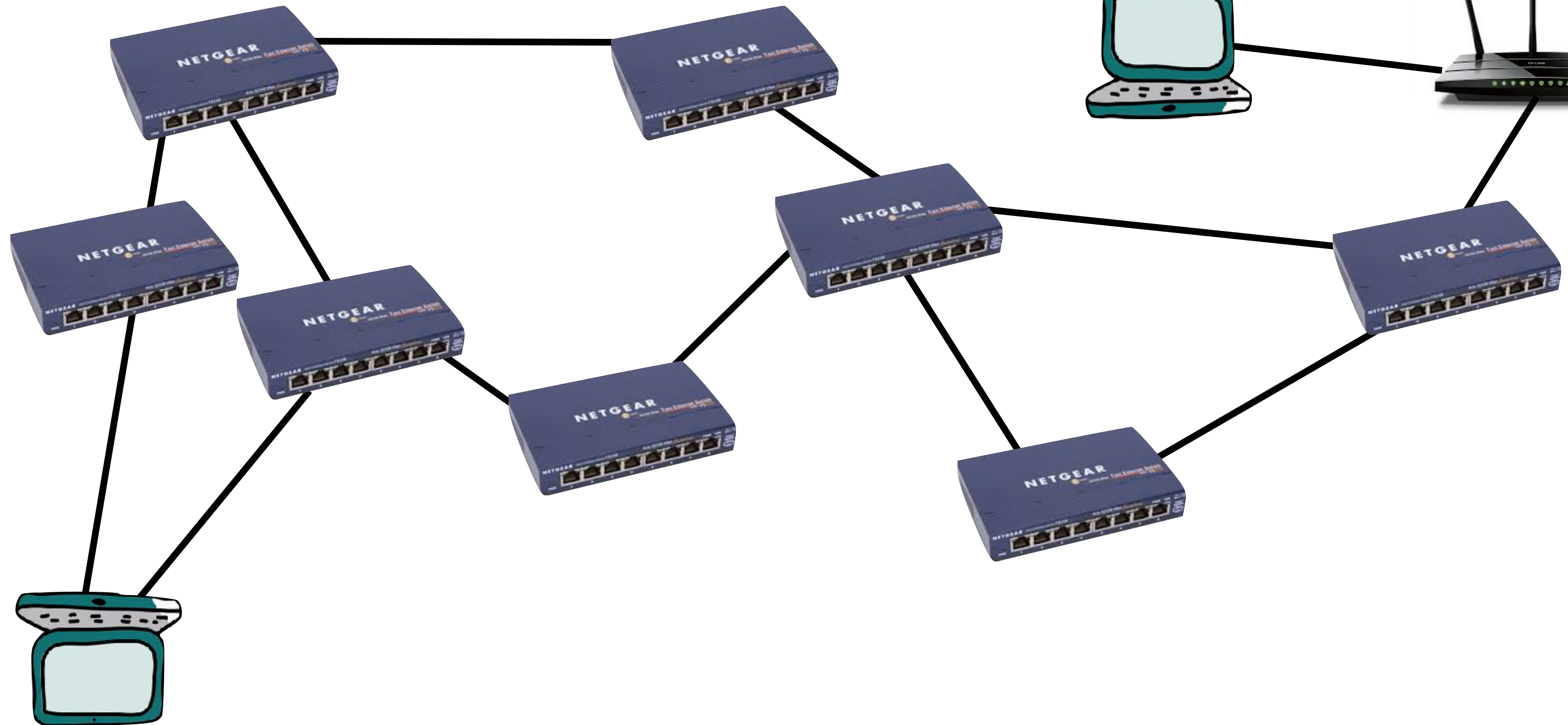
Let's dig a bit deeper into how the Internet is a real graph by analyzing internet routers, or:

How does a message get sent from your computer to another computer on the Internet, say in Australia?



The Internet: Computers connected through routers

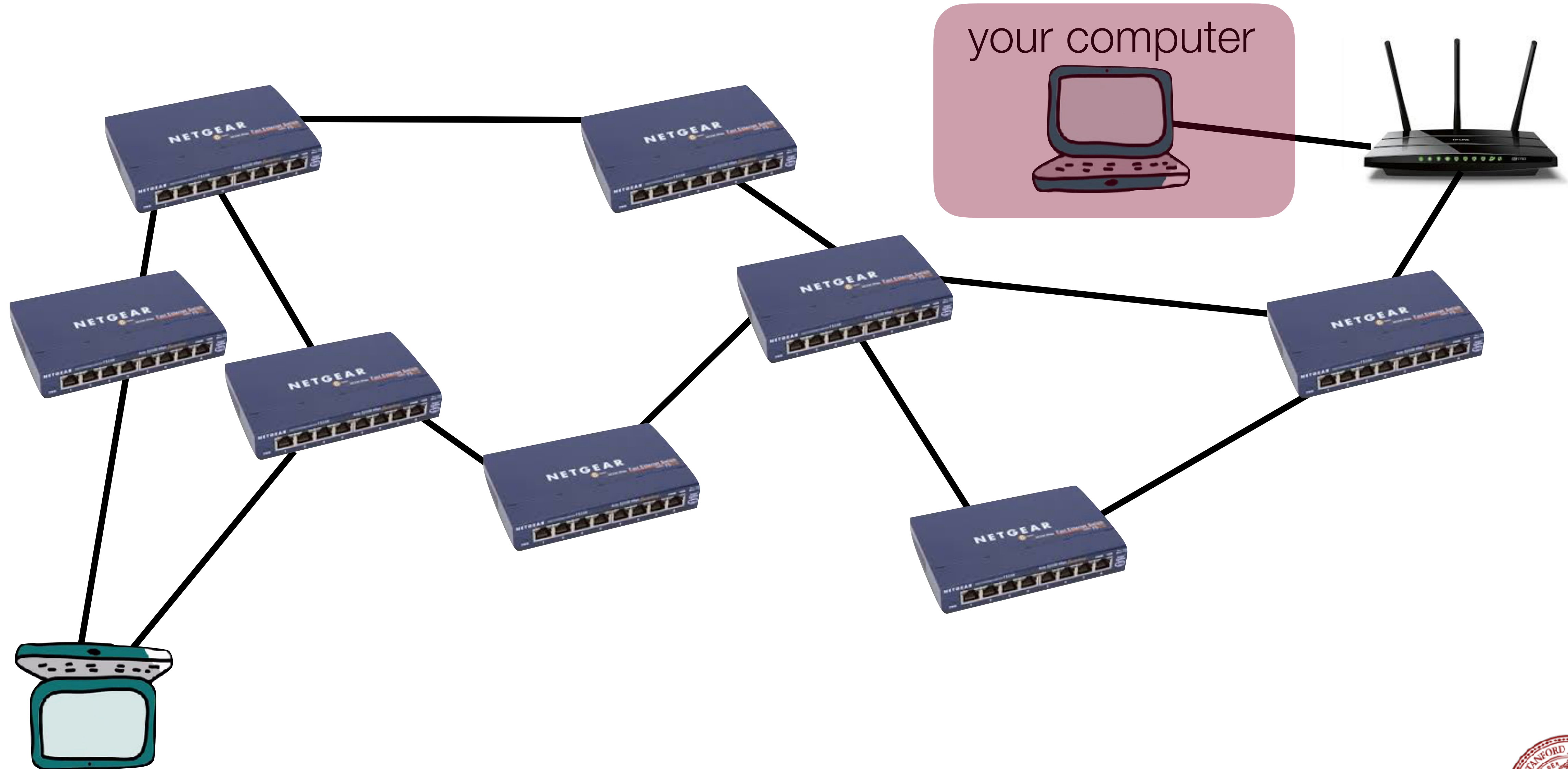
your computer



computer in Australia



The Internet: Computers connected through routers



The Internet: Let's simplify a bit

The destination computer has a name and an IP address, like this:

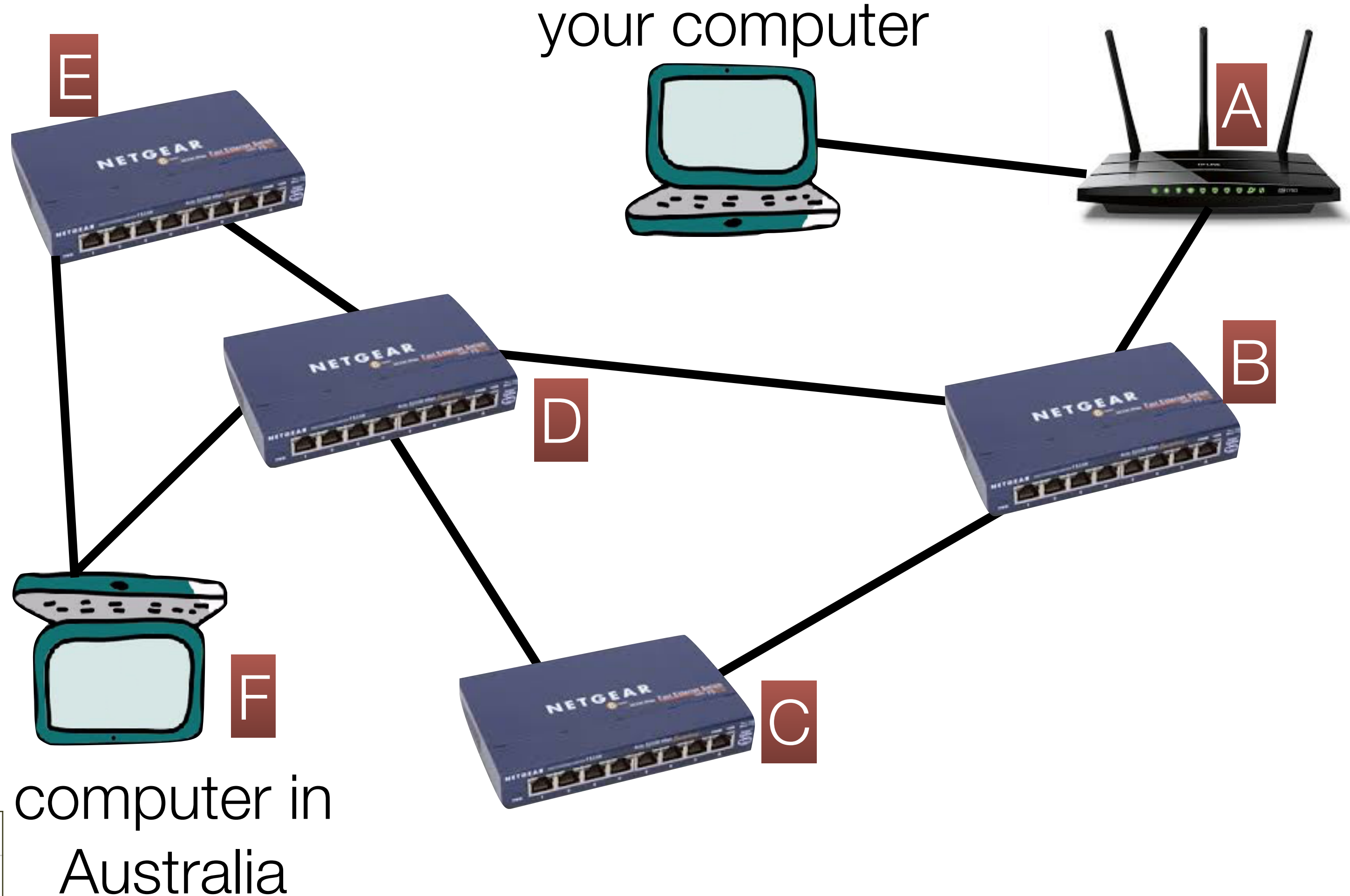
www.engineering.unsw.edu.au

IP address: 149.171.158.109

The first number denotes the "network address" and routers continually pass around information about how many "hops" they think it will take for them to get to all the networks.

E.g., for router C:

router	hops
A	2
B	1
C	-
D	1
E	2
F	2



The Internet: Let's simplify a bit

Each router knows its neighbors, and it has a copy of its neighbors' tables. So, B would have the following tables:

A

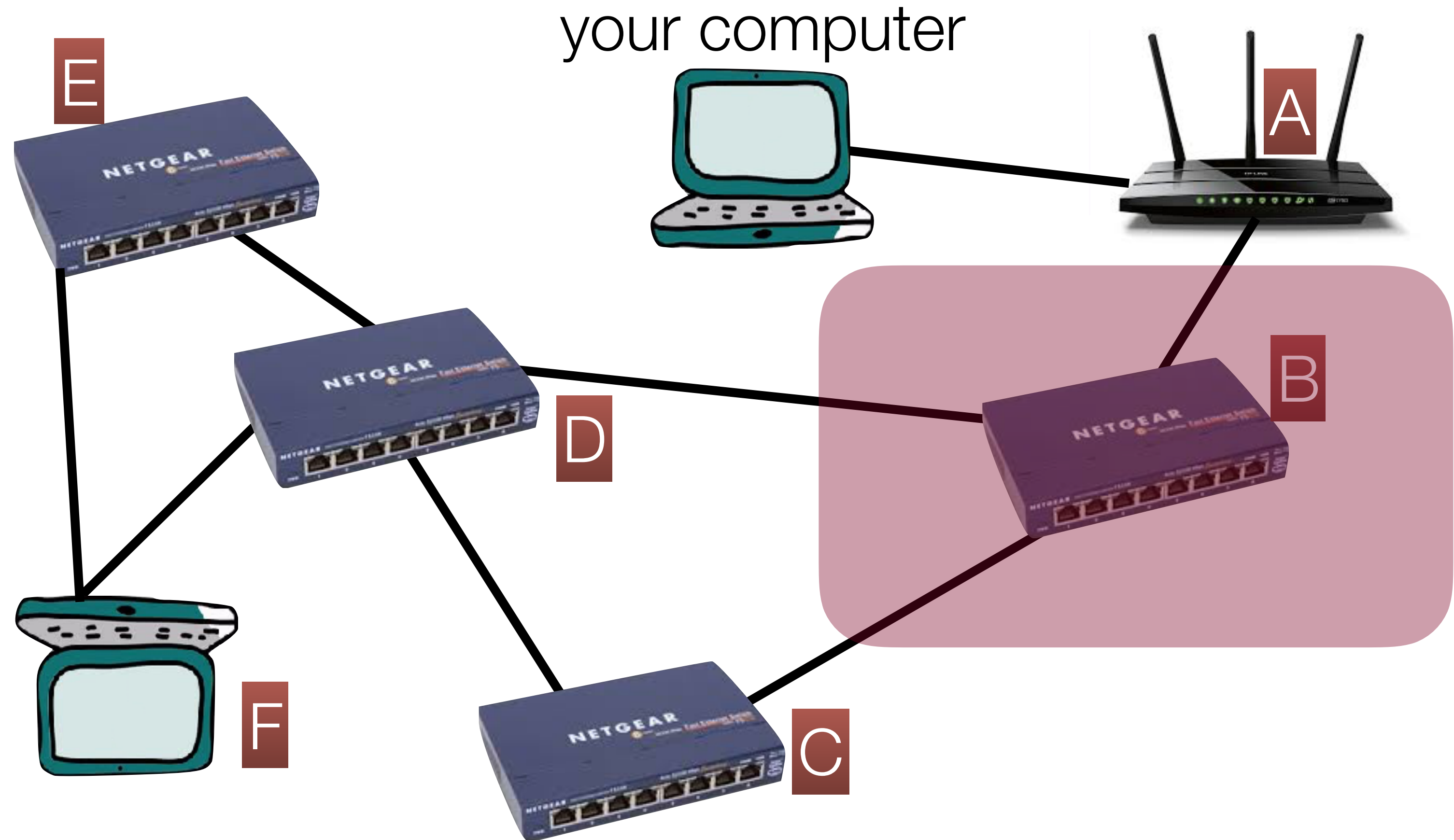
router	hops
A	-
B	1
C	3
D	2
E	3
F	3

C

router	hops
A	2
B	1
C	-
D	1
E	2
F	2

D

router	hops
A	2
B	1
C	1
D	-
E	1
F	1



The Internet: Let's simplify a bit

If B wants to connect to F, it connects through its neighbor that reports the shortest path to F. Which router would it choose?

A

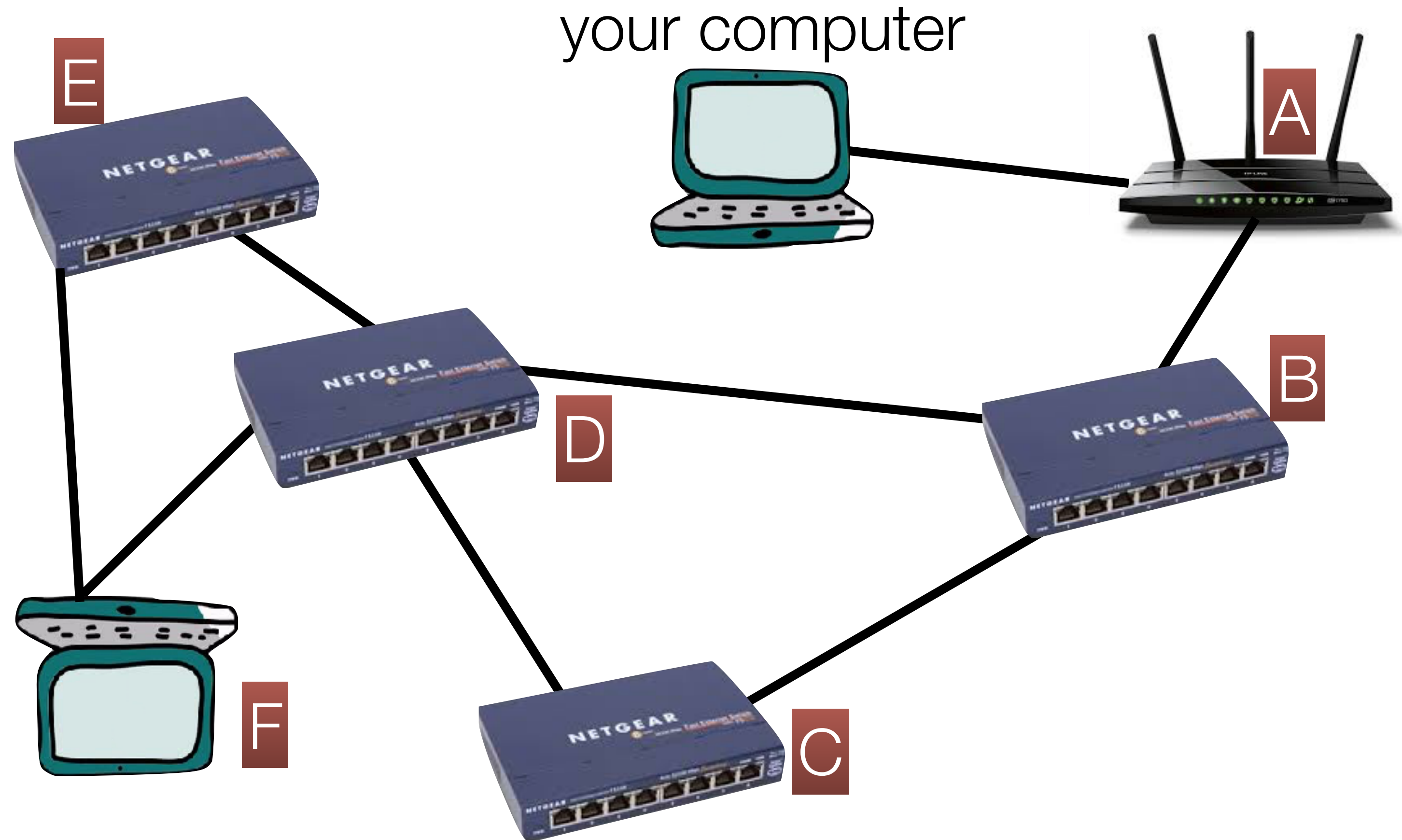
router	hops
A	-
B	1
C	3
D	2
E	3
F	3

C

router	hops
A	2
B	1
C	-
D	1
E	2
F	2

D

router	hops
A	2
B	1
C	1
D	-
E	1
F	1



The Internet: Let's simplify a bit

If B wants to connect to F, it connects through its neighbor that reports the shortest path to F. Which router would it choose? D.

A

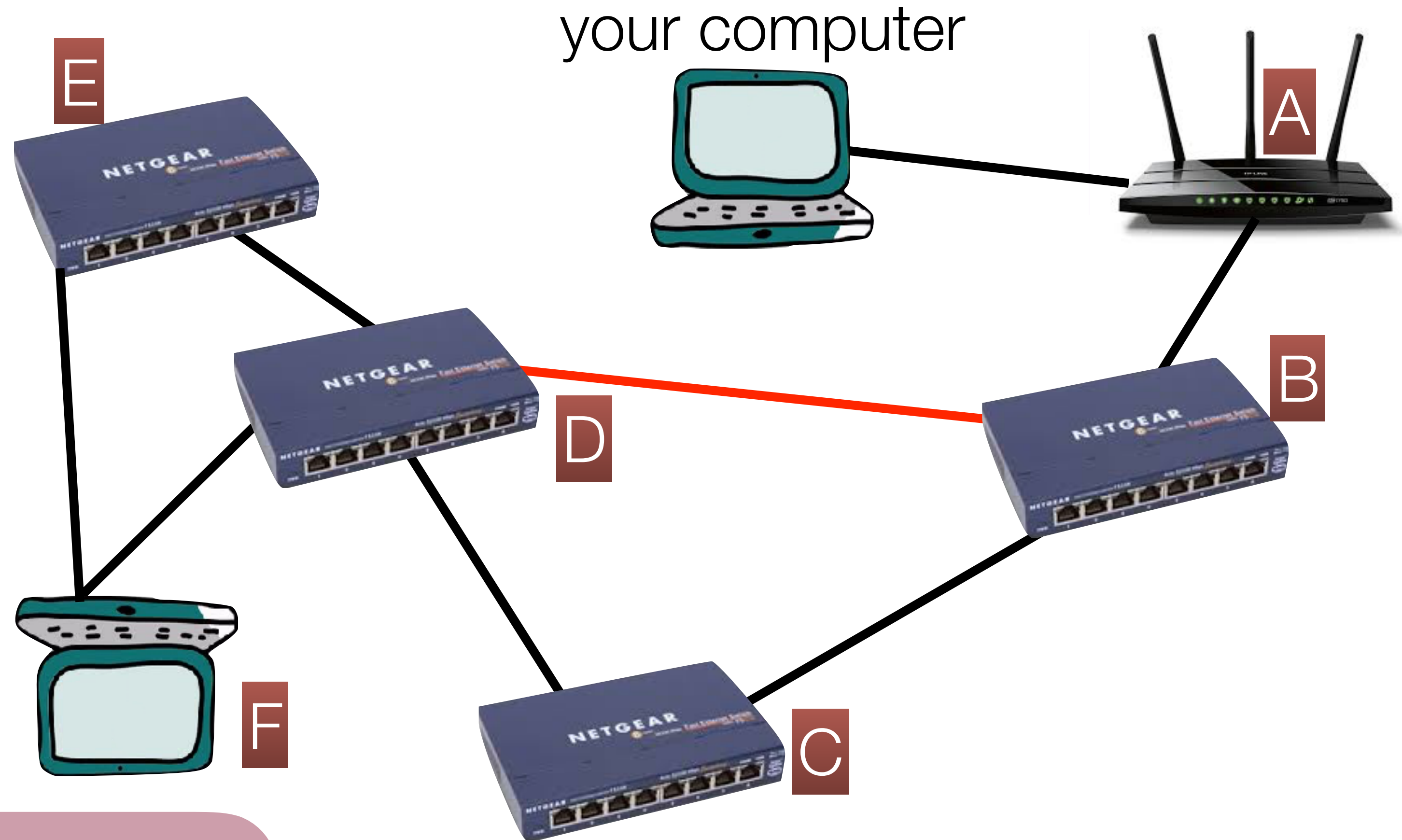
router	hops
A	-
B	1
C	3
D	2
E	3
F	3

C

router	hops
A	2
B	1
C	-
D	1
E	2
F	2

D

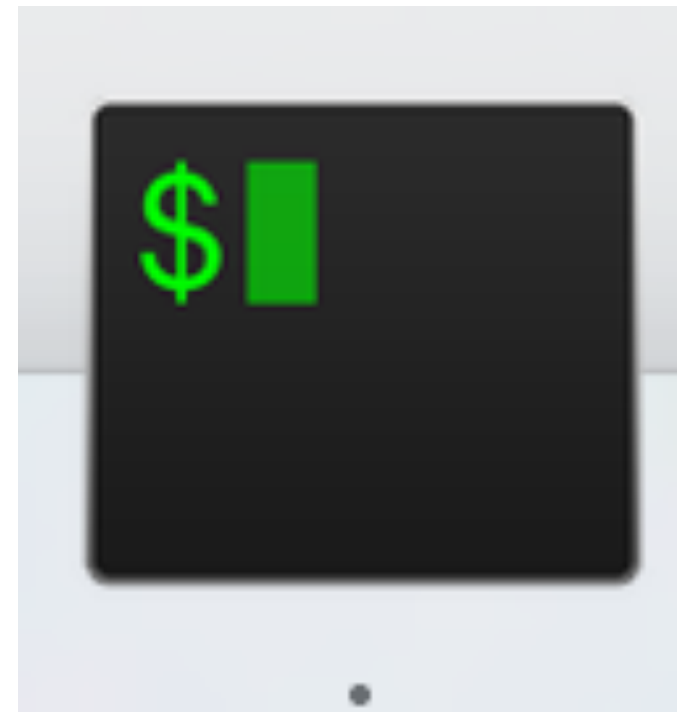
router	hops
A	2
B	1
C	1
D	-
E	1
F	1



Traceroute

We can use a program called "traceroute" to tell us the path between our computer and a different computer:

traceroute -I -e www.engineering.unsw.edu.au



Traceroute: Stanford Hops

```
traceroute -I -e www.engineering.unsw.edu.au
```

```
traceroute to www.engineering.unsw.edu.au (149.171.158.109), 64 hops max, 72 byte packets
```

```
1  csmx-west-rtr.sunet (171.67.64.2)  7.414 ms  9.155 ms  8.288 ms
2  gnat-2.sunet (172.24.70.12)  0.339 ms  1.532 ms  0.423 ms
3  csmx-west-rtr-vl3866.sunet (171.64.66.2)  38.916 ms  10.506 ms  8.402 ms
4  dca-rtr-vlan8.sunet (171.64.255.204)  0.530 ms  0.521 ms  0.713 ms
5  dc-svl-agg4--stanford-10ge.cenic.net (137.164.50.157)  1.554 ms  1.653 ms  2.828 ms
6  hpr-svl-hpr2--svl-agg4-10ge.cenic.net (137.164.26.249)  1.212 ms  1.161 ms  1.204 ms
7  aarnet-2-is-jmb-778.sttlwa.pacificwave.net (207.231.245.4)  17.994 ms  17.998 ms  18.319 ms
8  et-2-0-0.pe2.brwy.nsw.aarnet.net.au (113.197.15.98)  160.020 ms  160.234 ms  159.922 ms
9  et-3-3-0.pe1.brwy.nsw.aarnet.net.au (113.197.15.148)  160.285 ms  160.076 ms  160.118 ms
10 138.44.5.1 (138.44.5.1)  160.124 ms  160.138 ms  160.068 ms
11 ombcr1-te-1-5.gw.unsw.edu.au (149.171.255.106)  160.090 ms  160.381 ms  160.185 ms
12 r1dcdnex1-po-2.gw.unsw.edu.au (149.171.255.178)  160.909 ms  160.847 ms  160.921 ms
13 dcfw1-ae-1-3049.gw.unsw.edu.au (129.94.254.60)  160.592 ms  160.558 ms  160.949 ms
14 www.engineering.unsw.edu.au (149.171.158.109)  160.978 ms  161.184 ms  160.987 ms
```



Traceroute: CENIC

```
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The **Corporation for Education Network Initiatives in California (CENIC)** is a nonprofit corporation formed in 1996 to provide high-performance, high-bandwidth networking services to [California](#) universities and research institutions (source: Wikipedia)



Traceroute: Pacificwave (Seattle)

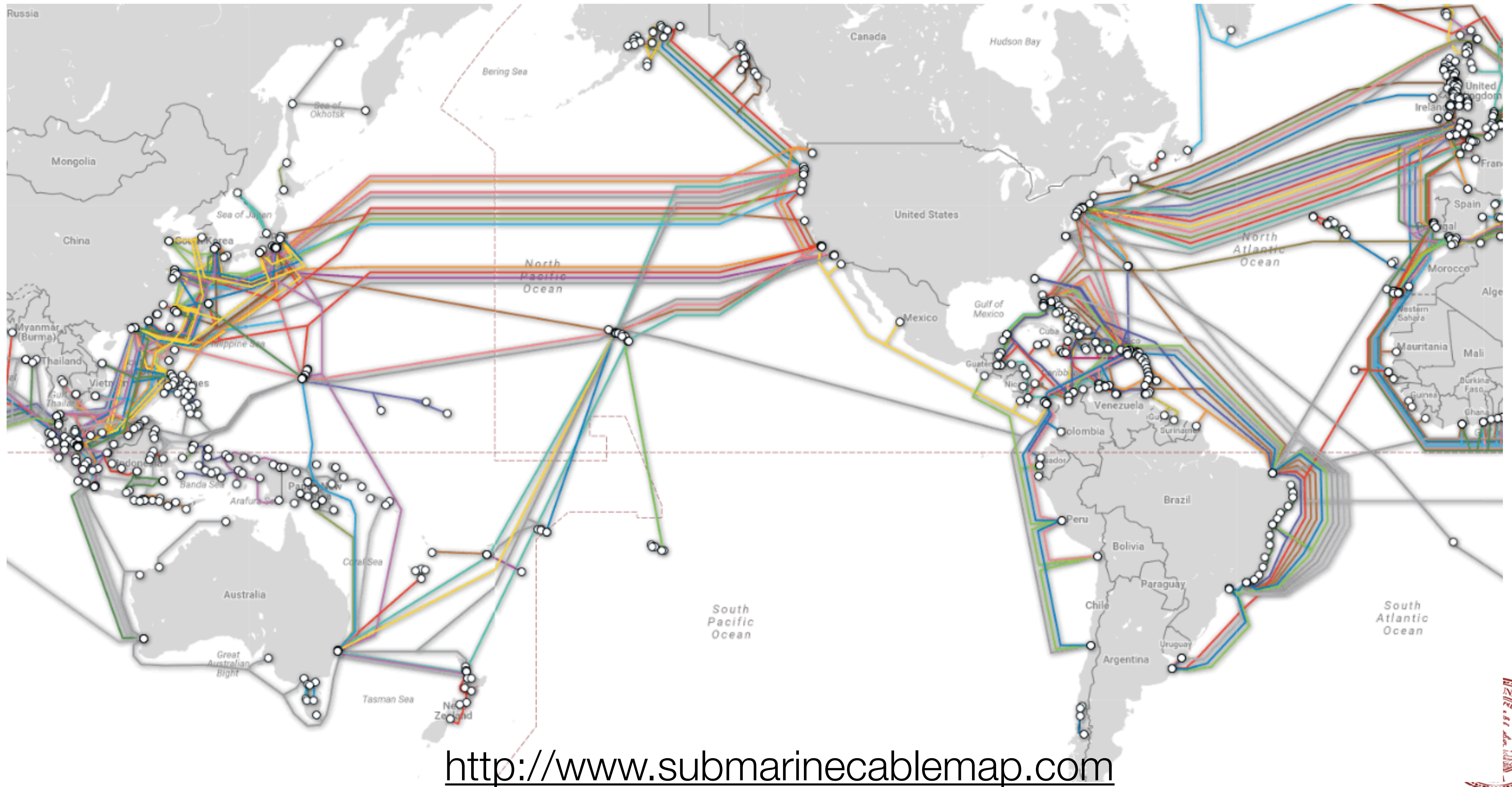
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```



Pass Internet traffic directly with other major national and international networks, including U.S. federal agencies and many Pacific Rim R&E networks (source: <http://www.pnwgp.net/services/pacific-wave-peering-exchange/>)



Traceroute: Oregon to Australia - underwater!



Traceroute: Australia

```
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```

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```



Traceroute: University of New South Wales

```
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14  www.engineering.unsw.edu.au (149.171.158.109)  160.978 ms  161.184 ms  160.987 ms
```

161 milliseconds to get to the final computer





Other Real Life Uses for Graphs

- Amazon.com -- Product relationships are graph-based


Sponsored products related to this item [\(What's this?\)](#)

Page 1 of 18







Go Pet Club IQ Busy Box Cat Tree, 26" x 21" x 45"
★★★★★ 62
\$58.75 ✓Prime



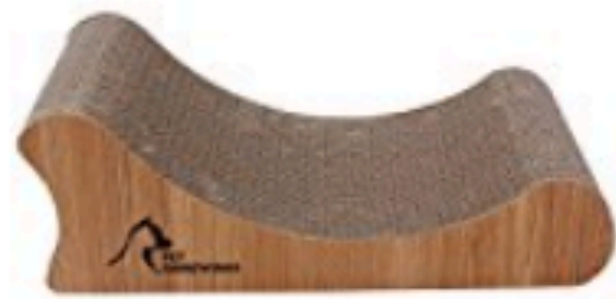
Go Pet Club F3019 Cat Scratcher Condo Furniture
★★★★★ 56
\$36.99 ✓Prime




Kurgo Heather Dog Booster Seat for Cars with Seat Belt Tether, Charcoal
★★★★★ 15
\$58.49 ✓Prime



Favorite Medium Cat Tunnel Foldable Lightweight Fun Dangling Ball Toy,...
★★★★★ 124
\$9.99 ✓Prime



PET SHINEWINGS- Basic Cat Scratcher Scratching Board Pad
★★★★★ 1
\$23.99 ✓Prime



- What product might this be related to?

Other Real Life Uses for Graphs



Roll over image to zoom in

Cat Scratching DJ Deck



248 customer reviews | 3 answered questions

Price: **\$24.00** ✓Prime | FREE Same-Day

Delivered today for FREE with qualifying orders over \$35. [Details](#)

In Stock.

Get it **TODAY, May 24.** Order within **2 hrs 53 mins** and choose **Same-Day Delivery** at checkout. [Details](#)

Ships from and sold by Amazon.com. Gift-wrap available.

Style: **DJ Deck**

Cabin

\$33.83 ✓Prime

Catillac

\$31.23 ✓Prime

DJ Deck

\$24.00 ✓Prime

Fire Engine

\$33.86 ✓Prime

Laptop

\$24.09 ✓Prime

Plane

\$44.95 ✓Prime

Tank

--



Teepee

--

- Cardboard scratching deck for cats
- Easy to assemble - comes with full instructions
- A fun distraction for your cats!
- Comes with spinable deck and posable arm
- Measures 35cm x 38.8cm x 14.4cm

Other Real Life Uses for Graphs

- Web page searching (we will discuss this on Friday!)
- Google Maps (your final assignment, Trailblazer!)



[All](#) [Maps](#) [News](#) [Images](#) [Videos](#) [More](#) [Settings](#) [Tools](#)

About 690,000,000 results (0.92 seconds)

San Francisco Travel | Visitor Information

www.sftravel.com/

San Francisco is home to a little bit of everything. Whether you're a first time visitor or a long-time local, San Francisco's Golden Gates welcome all. This is the ...

[Alcatraz](#) · [Fisherman's Wharf](#) · [Golden Gate Bridge](#) · [Explore](#)

San Francisco - Wikipedia

https://en.wikipedia.org/wiki/San_Francisco



San Francisco (initials SF) officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California. Located at the ...

[San Francisco Bay Area](#) · [Ed Lee](#) · [Northern California](#) · [History of San Francisco](#)

San Francisco 2017: Best of San Francisco, CA Tourism - TripAdvisor

<https://www.tripadvisor.com> › [United States](#) › [California \(CA\)](#)

San Francisco Tourism: TripAdvisor has 907213 reviews of San Francisco Hotels , Attractions, and Restaurants making it your best San Francisco resource.



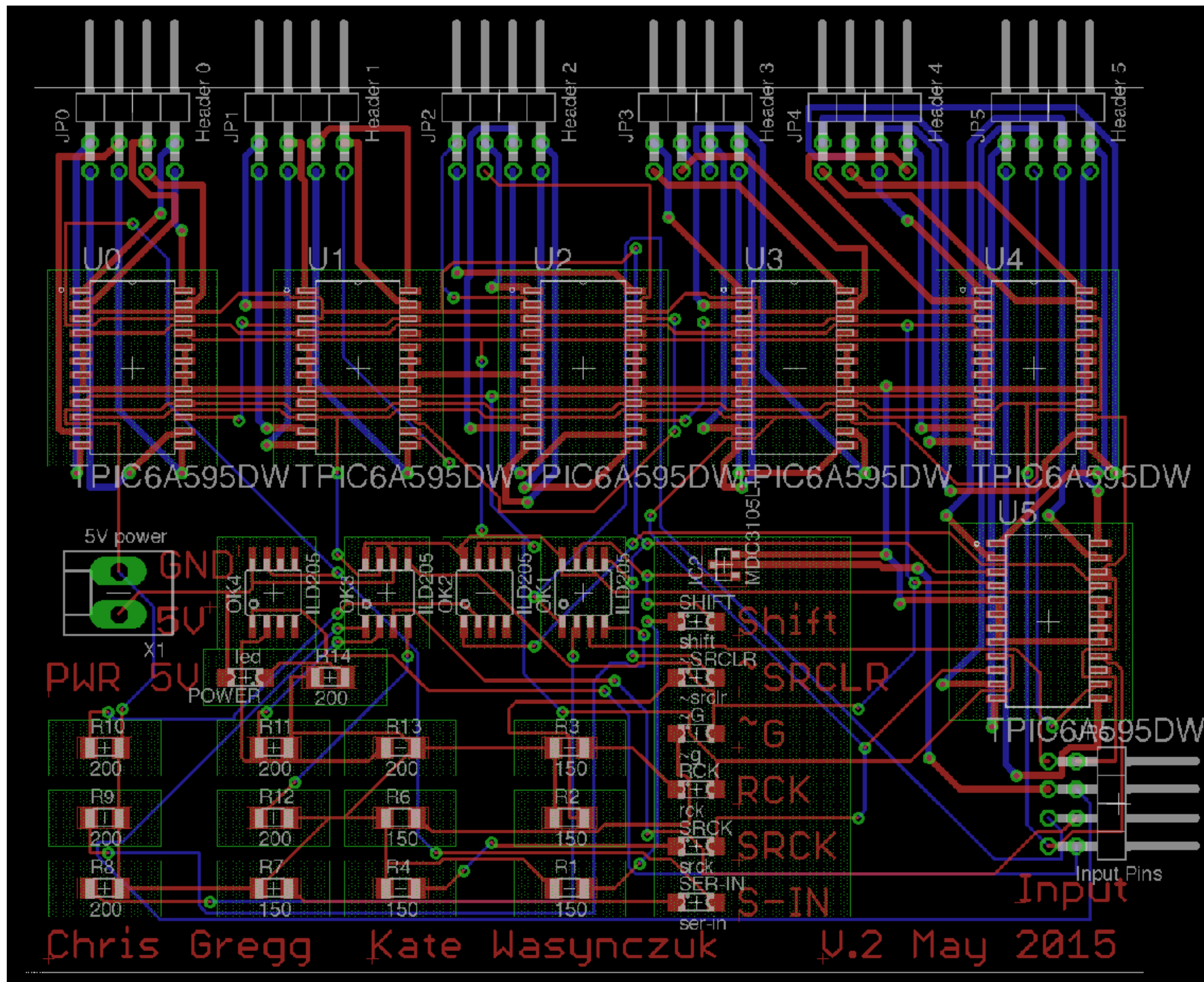
San Francisco

City in California

San Francisco, in northern California, is a hilly city on the tip of a peninsula surrounded by the Pacific Ocean and San Francisco Bay. It's known for its year-round fog, iconic Golden Gate Bridge, cable cars and colorful Victorian houses. The Financial District's Transamerica Pyramid is its most distinctive skyscraper. In the bay sits Alcatraz Island, site of the notorious former prison.

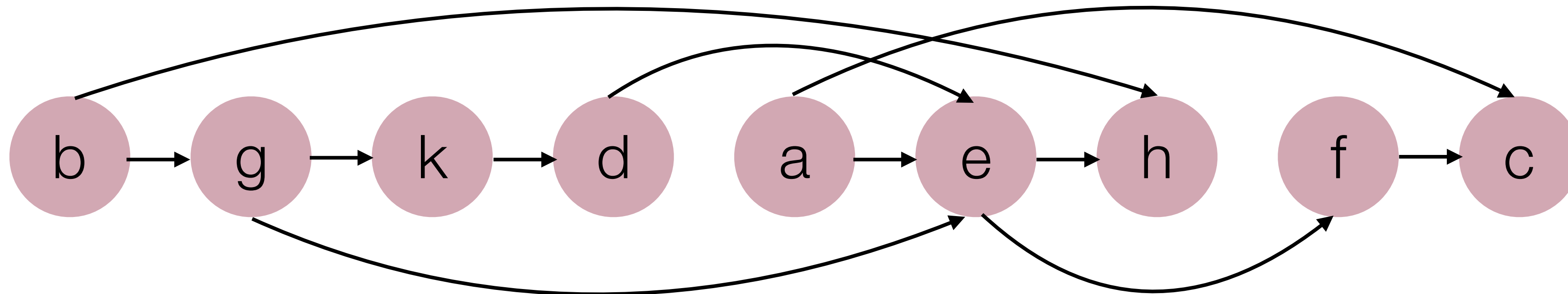
Other Real Life Uses for Graphs

- Routing circuits:



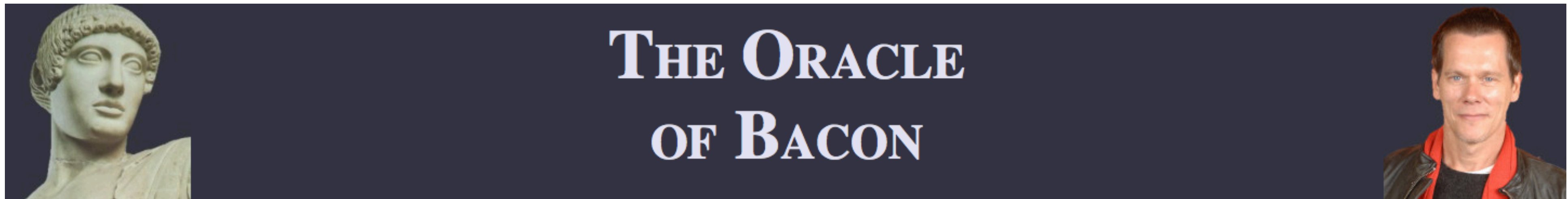
Other Real Life Uses for Graphs

- Scheduling work based on dependencies (e.g., when doing laundry, the washer must finish before the dryer, and before folding) -- this is called a "topological sort")



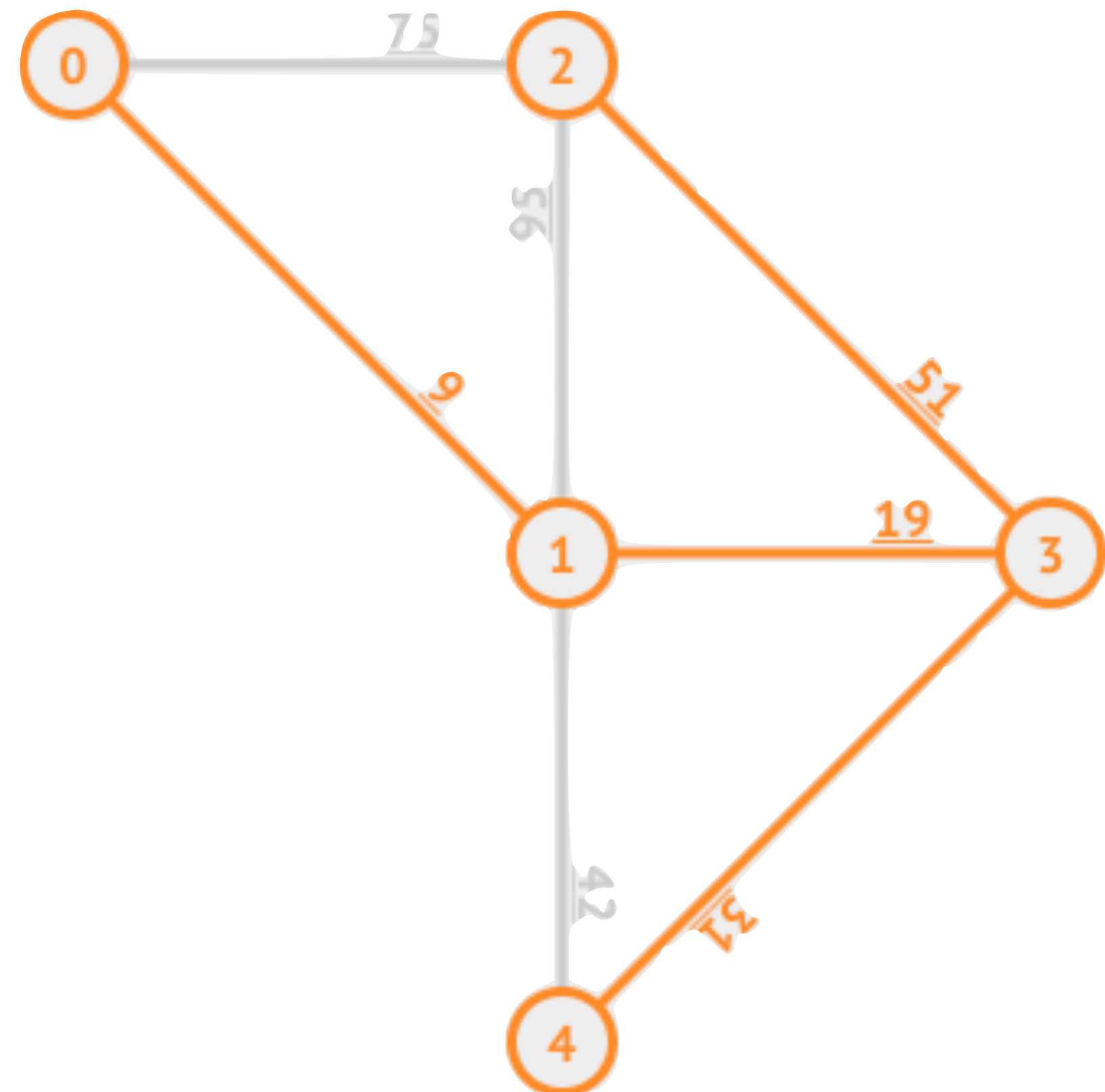
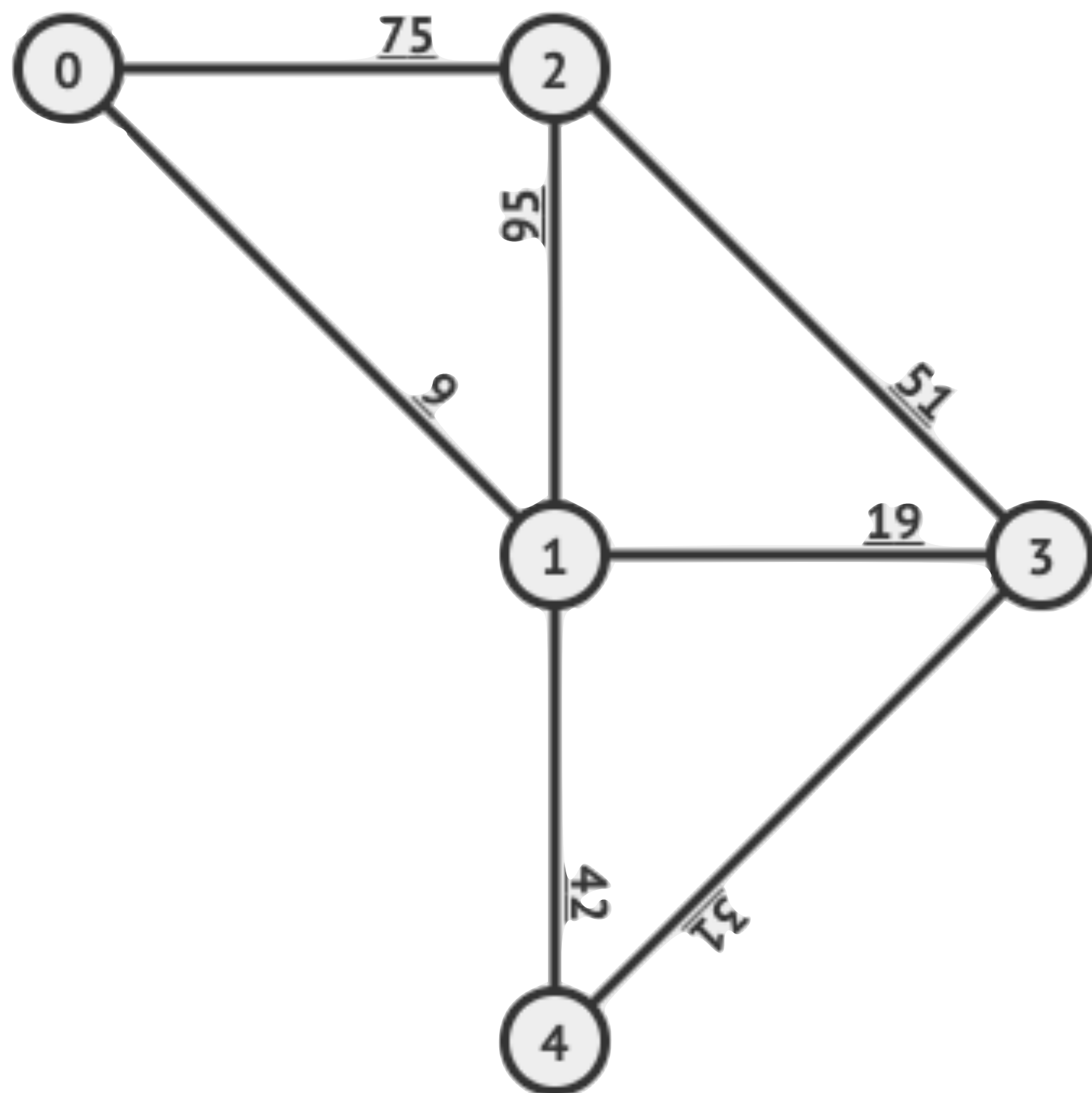
Other Real Life Uses for Graphs

- The "Oracle of Bacon": <https://oracleofbacon.org> (we will cover graph searching on Friday)



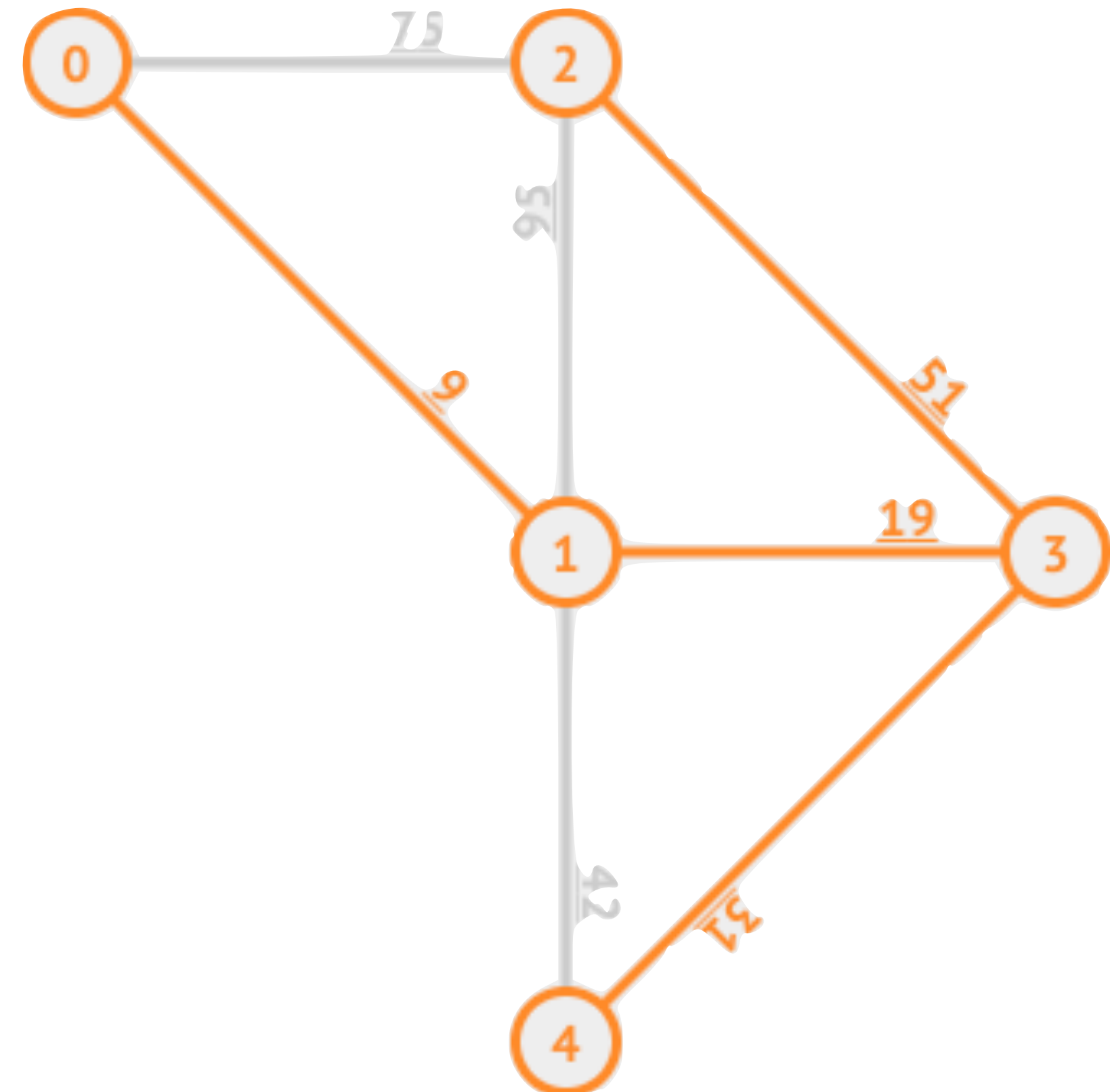
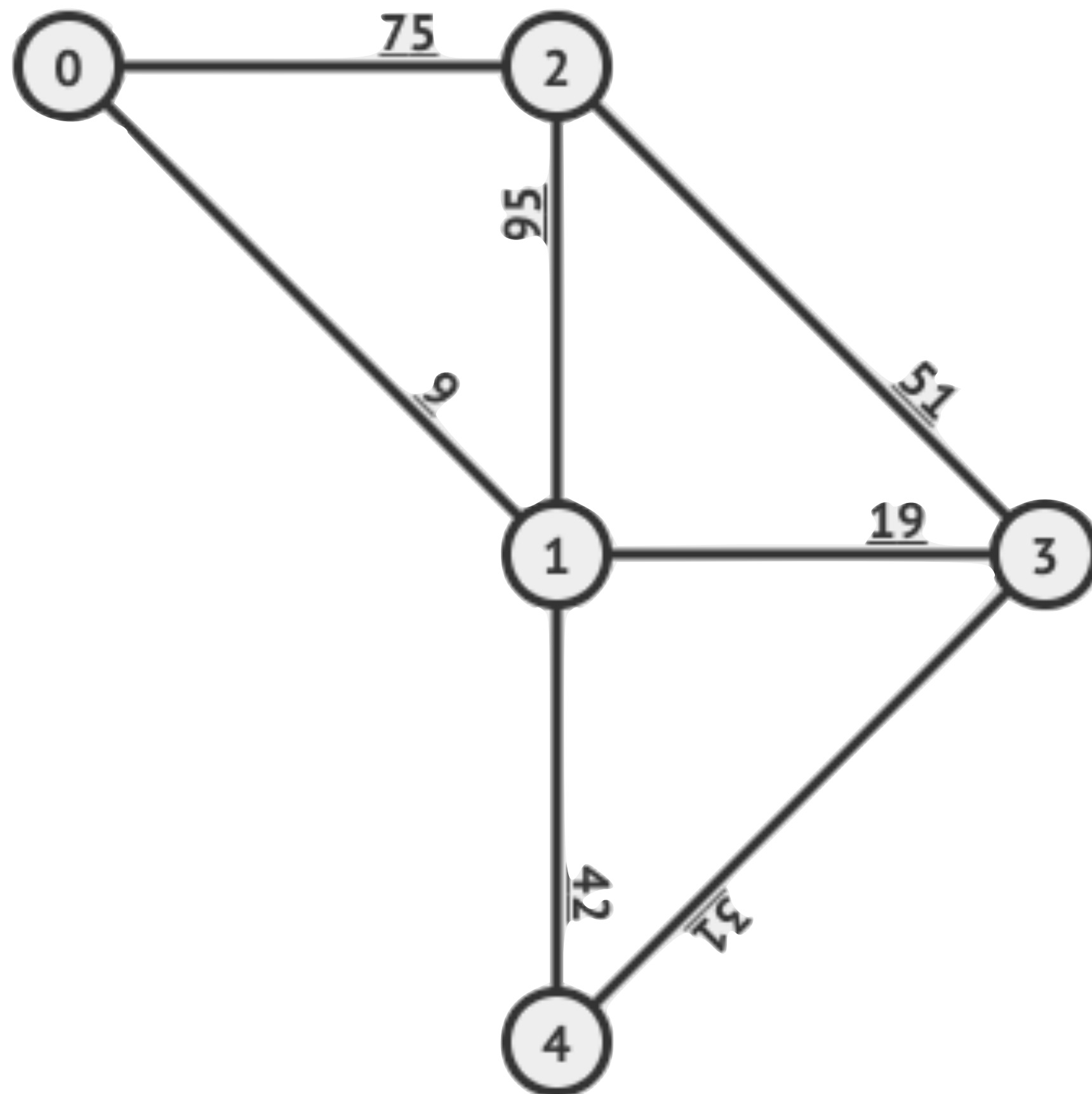
Other Real Life Uses for Graphs

- Telecommunications: find the least expensive way to lay out a set of cables for a telephone or cable TV system ("a Minimum Spanning Tree")



Spanning Trees and Minimum Spanning Trees

Definition: A **Spanning Tree (ST)** of a connected undirected weighted graph **G** is a subgraph of **G** that is a **tree** and **connects (spans) all vertices of G**. A graph **G** can have multiple STs. A **Minimum Spanning Tree (MST)** of **G** is a ST of **G** that has the **smallest total weight** among the various STs. A graph **G** can have multiple MSTs but the MST weight is unique.



Minimum Spanning Tree

Kruskal's Algorithm to find a Minimum Spanning Tree

- **Kruskal's algorithm:** Finds a MST in a given graph.

function **kruskal**(graph):

Remove all edges from the graph.

Place all edges into a **priority queue** based on their weight (cost).

While the priority queue is not empty:

 Dequeue an edge e from the priority queue.

 If e 's endpoints aren't already connected to one another,
 add that edge into the graph.

 Otherwise, skip the edge.

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

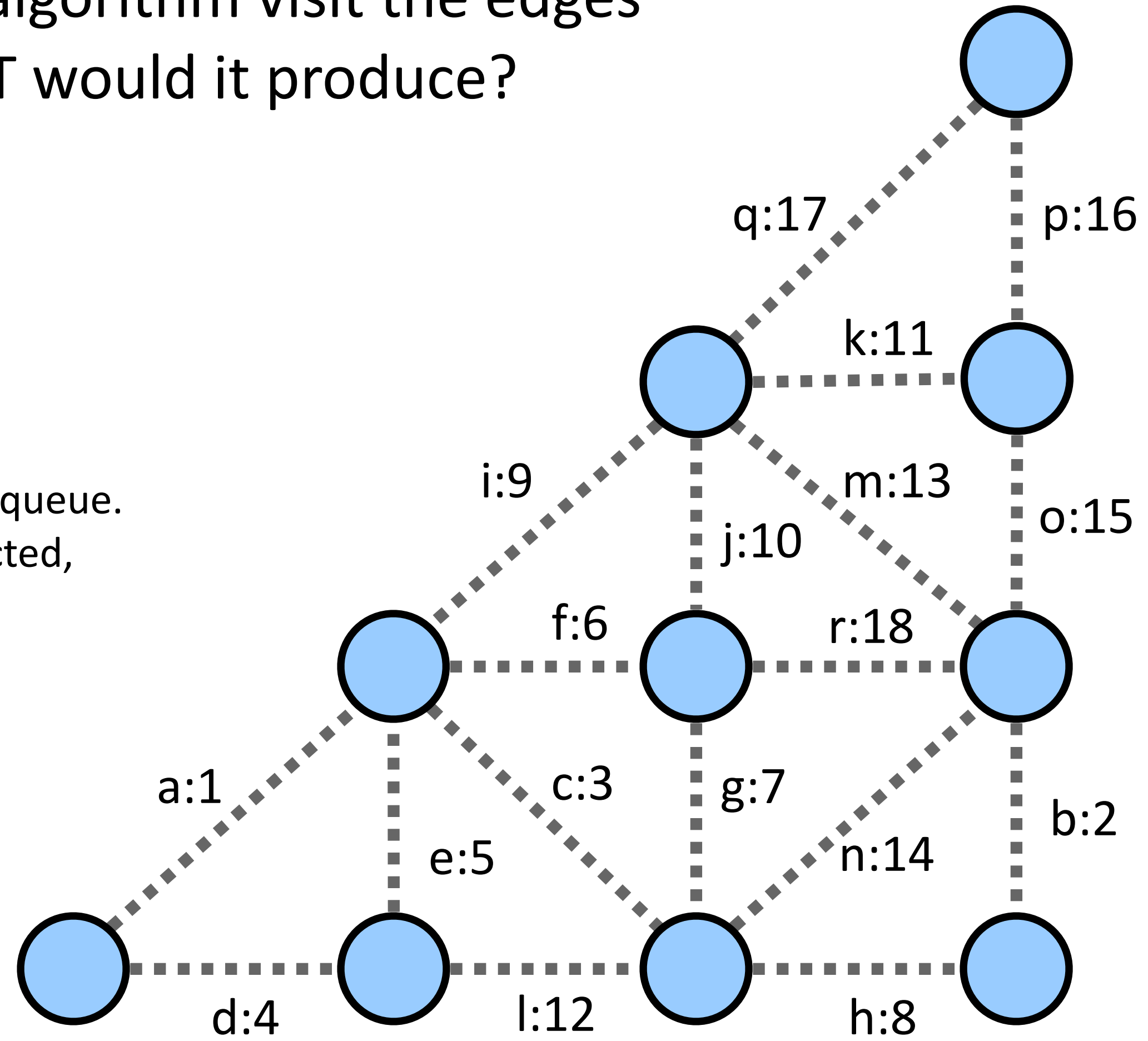
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based on their weight (cost).

While the priority queue is not empty:

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If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {a:1, b:2, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

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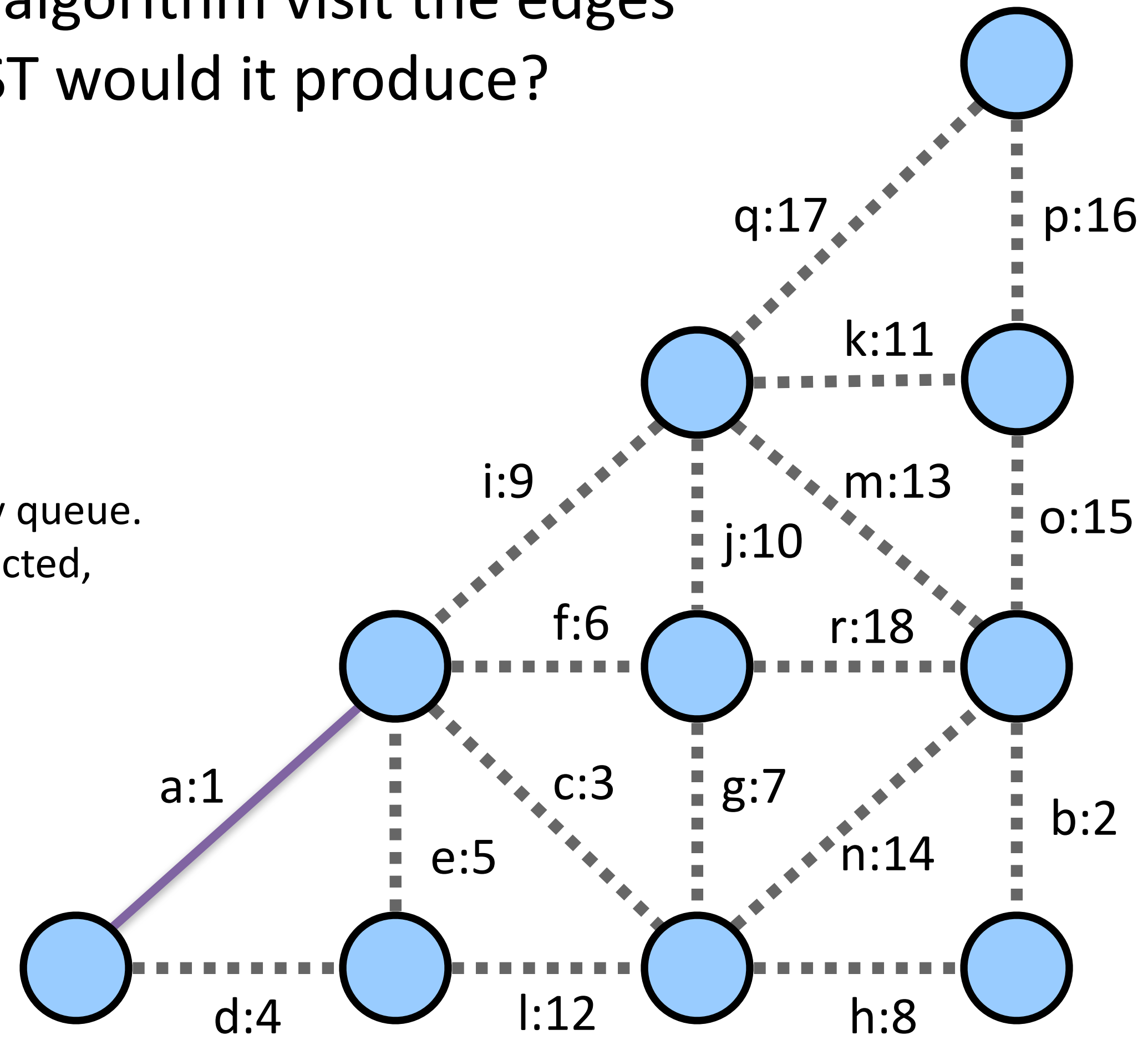
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add that edge into the graph.

Otherwise, skip the edge.



pq = {**a:1**, b:2, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

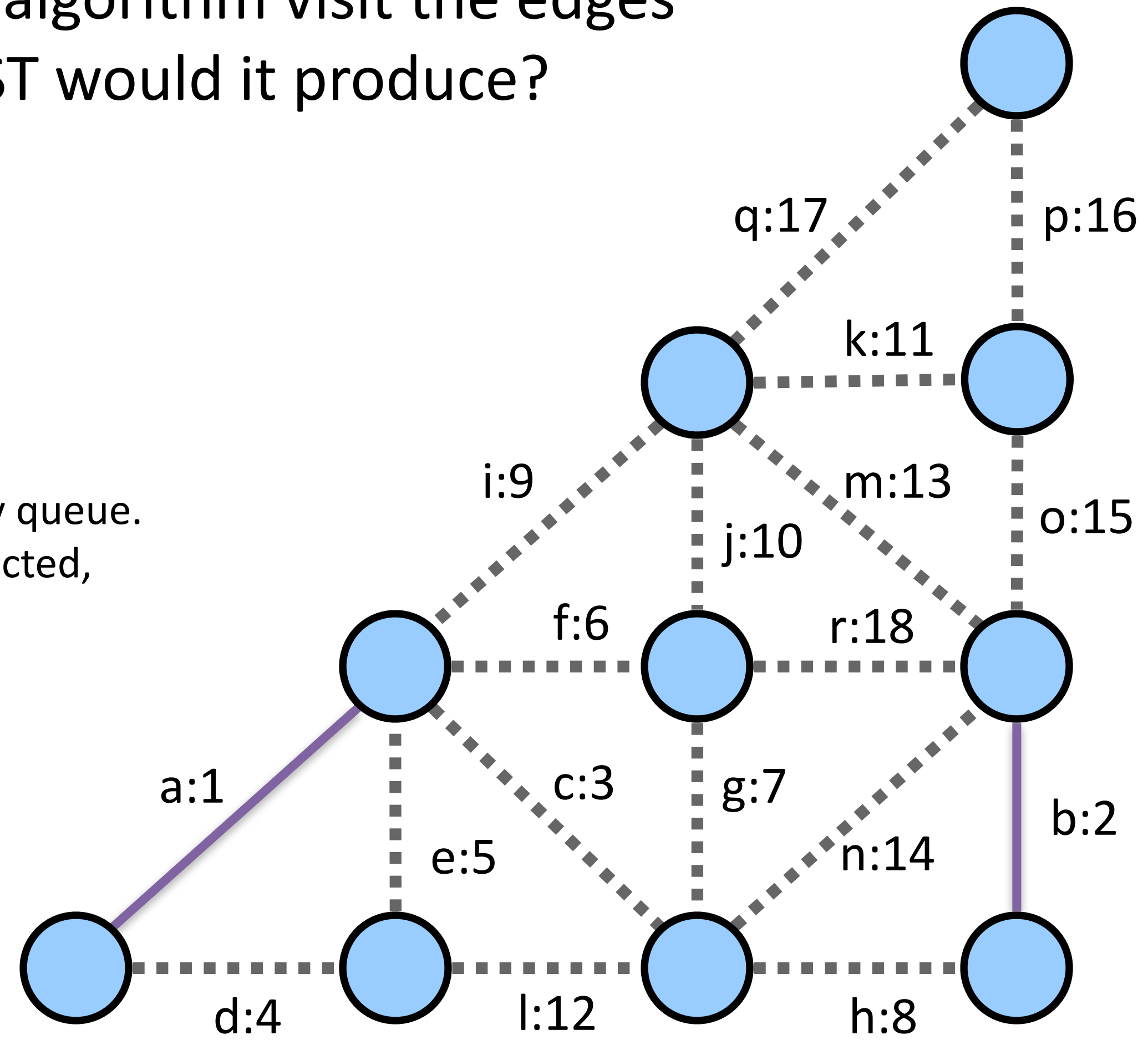
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**b:2**, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

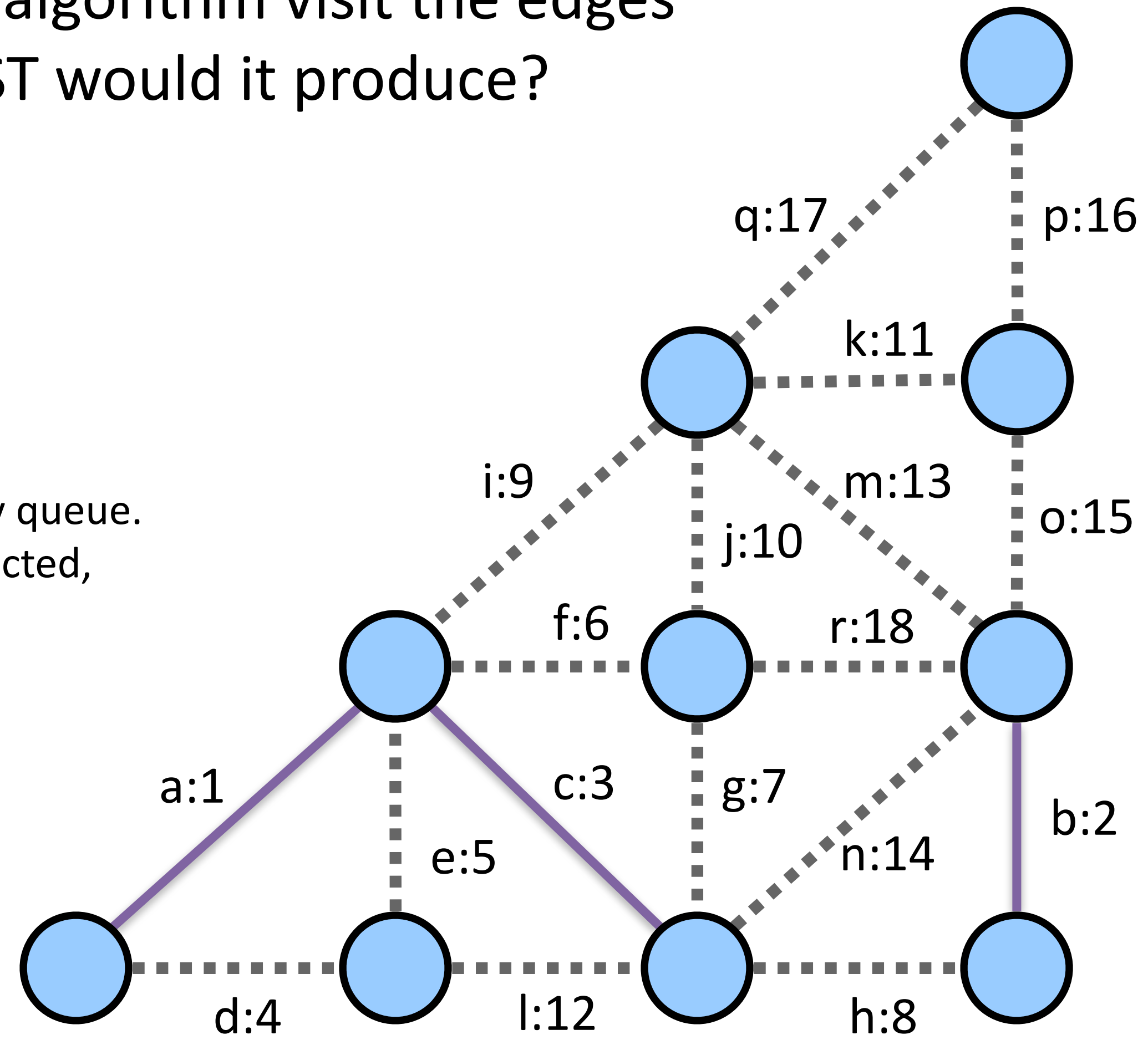
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**c:3**, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

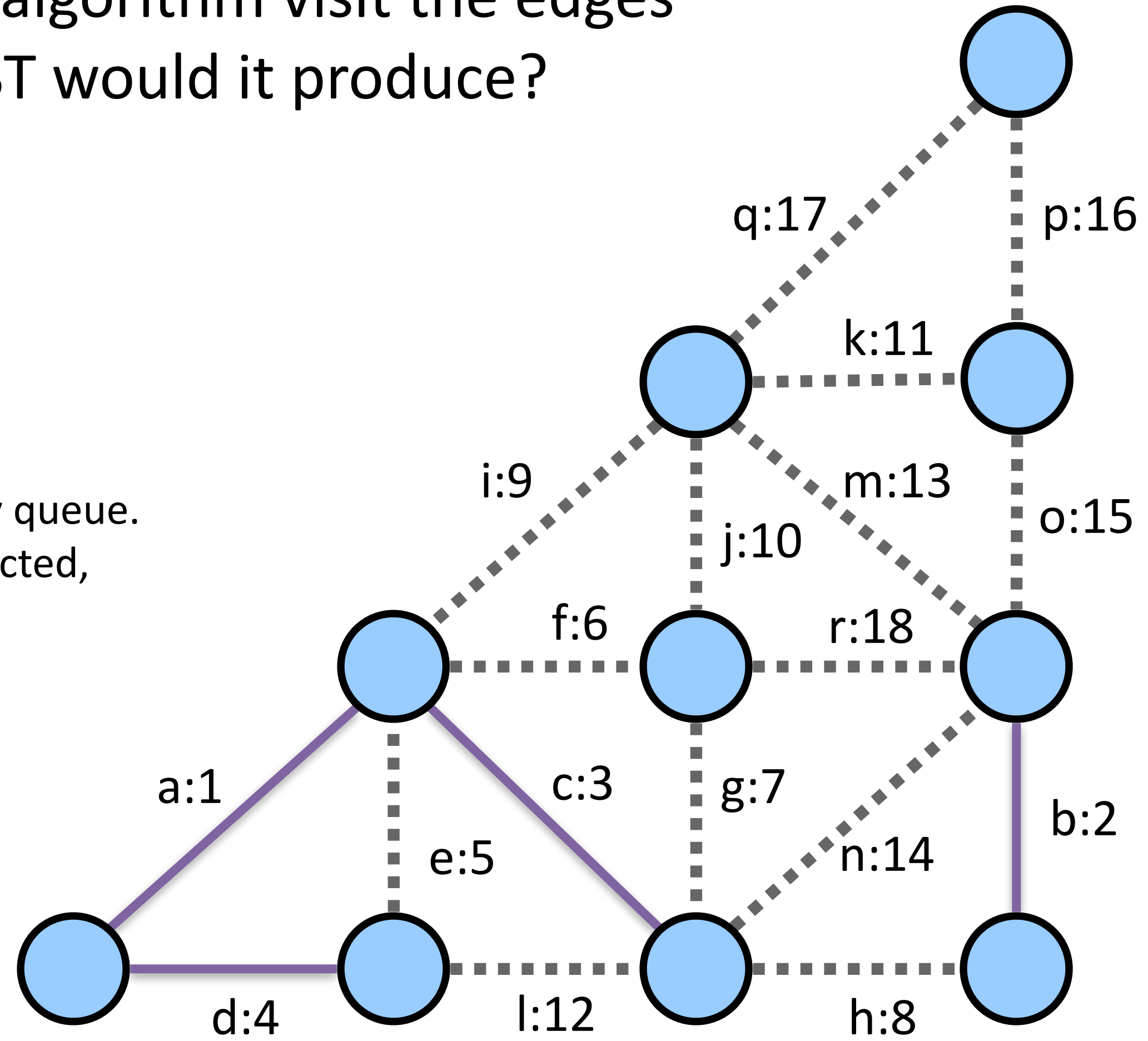
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**d:4**, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

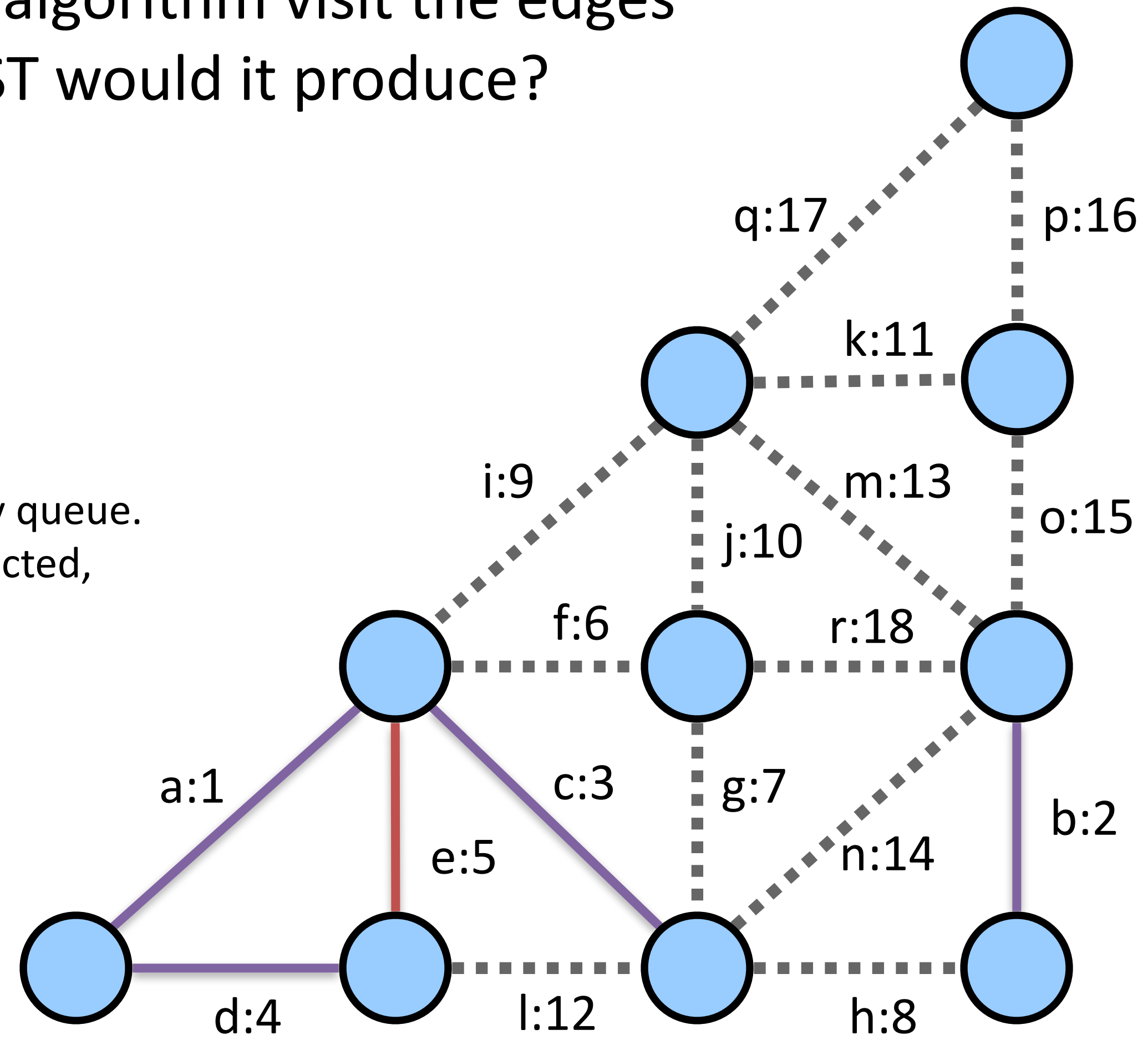
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**e:5**, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

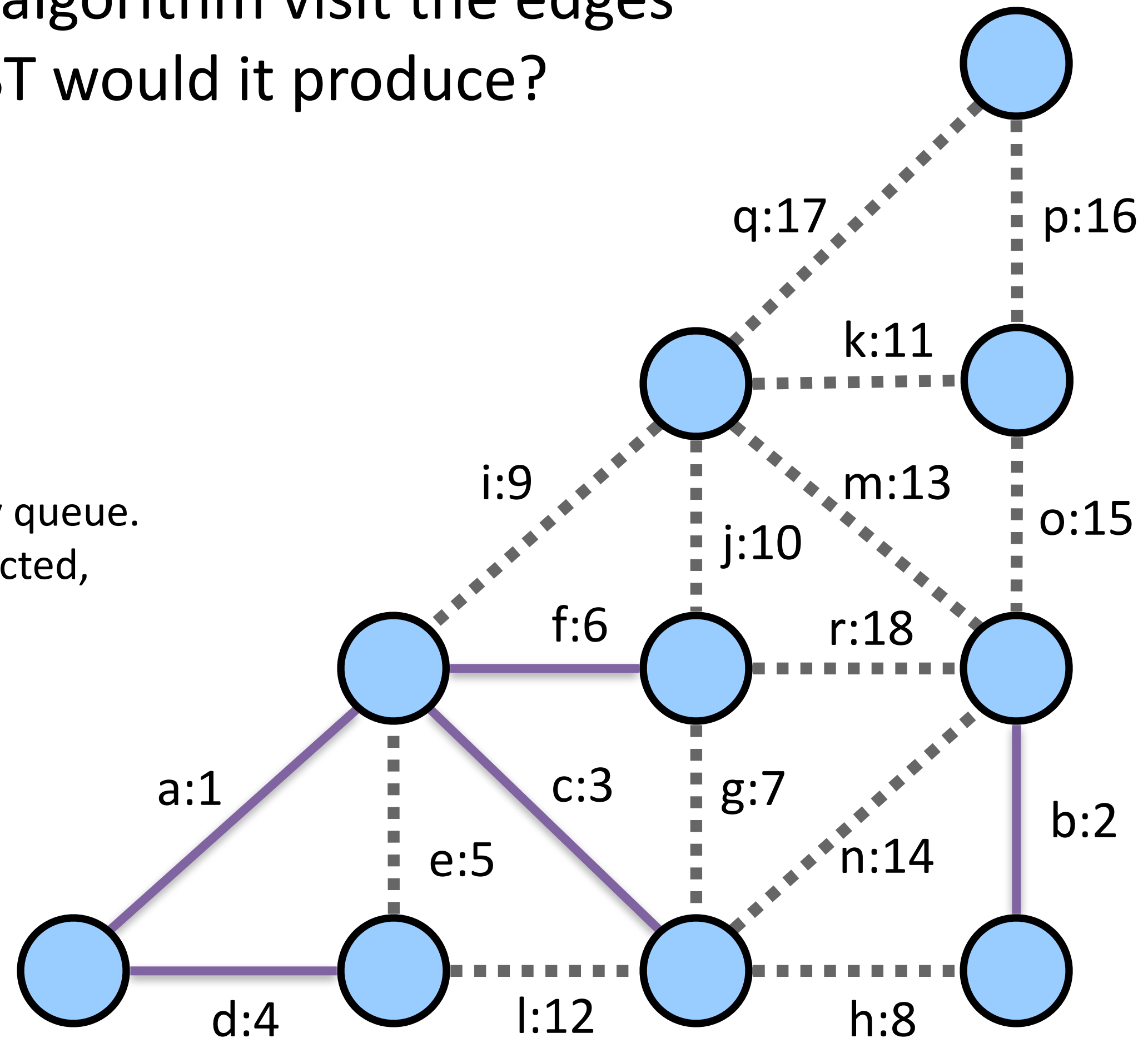
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**f:6**, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

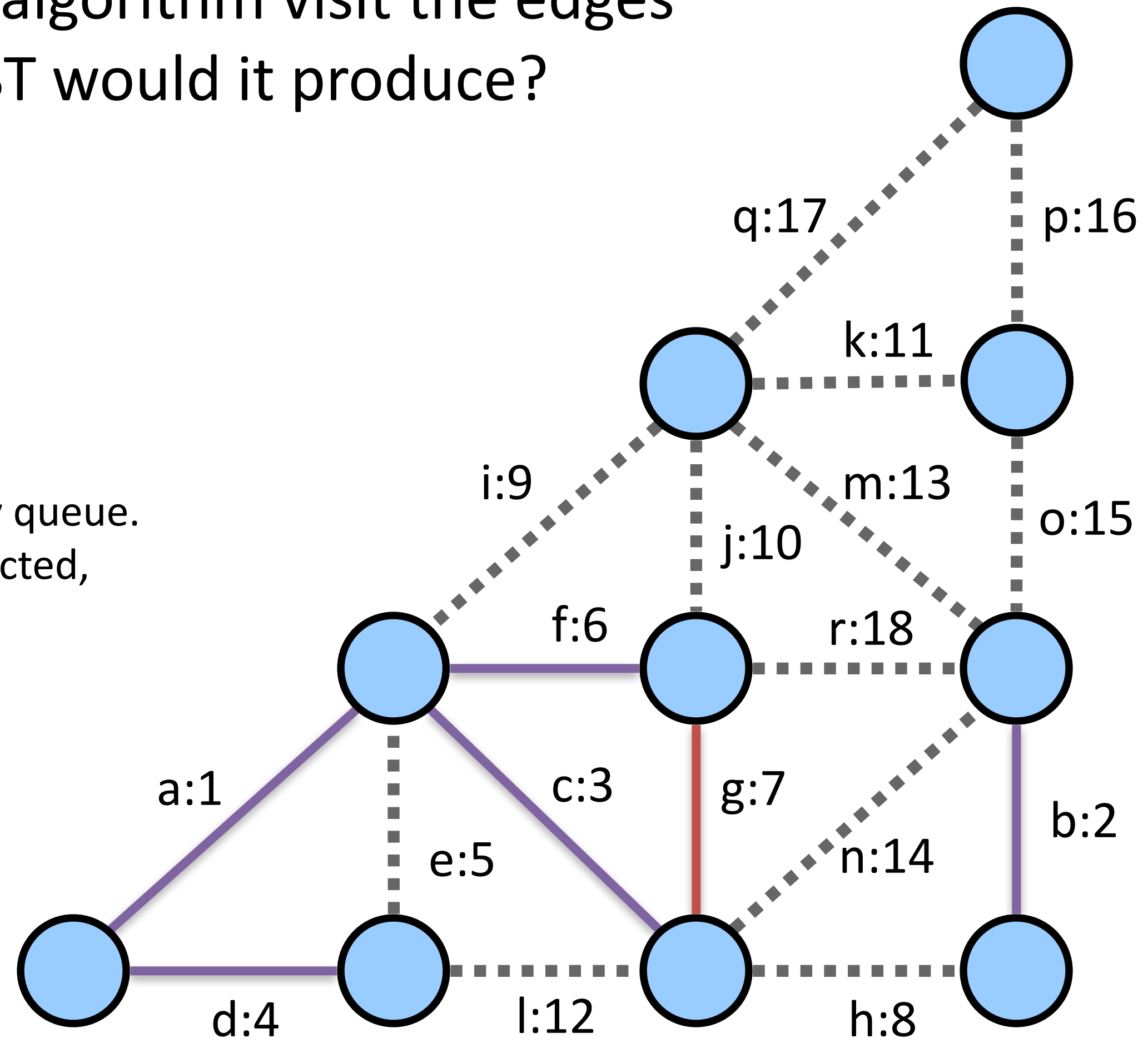
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**g:7**, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

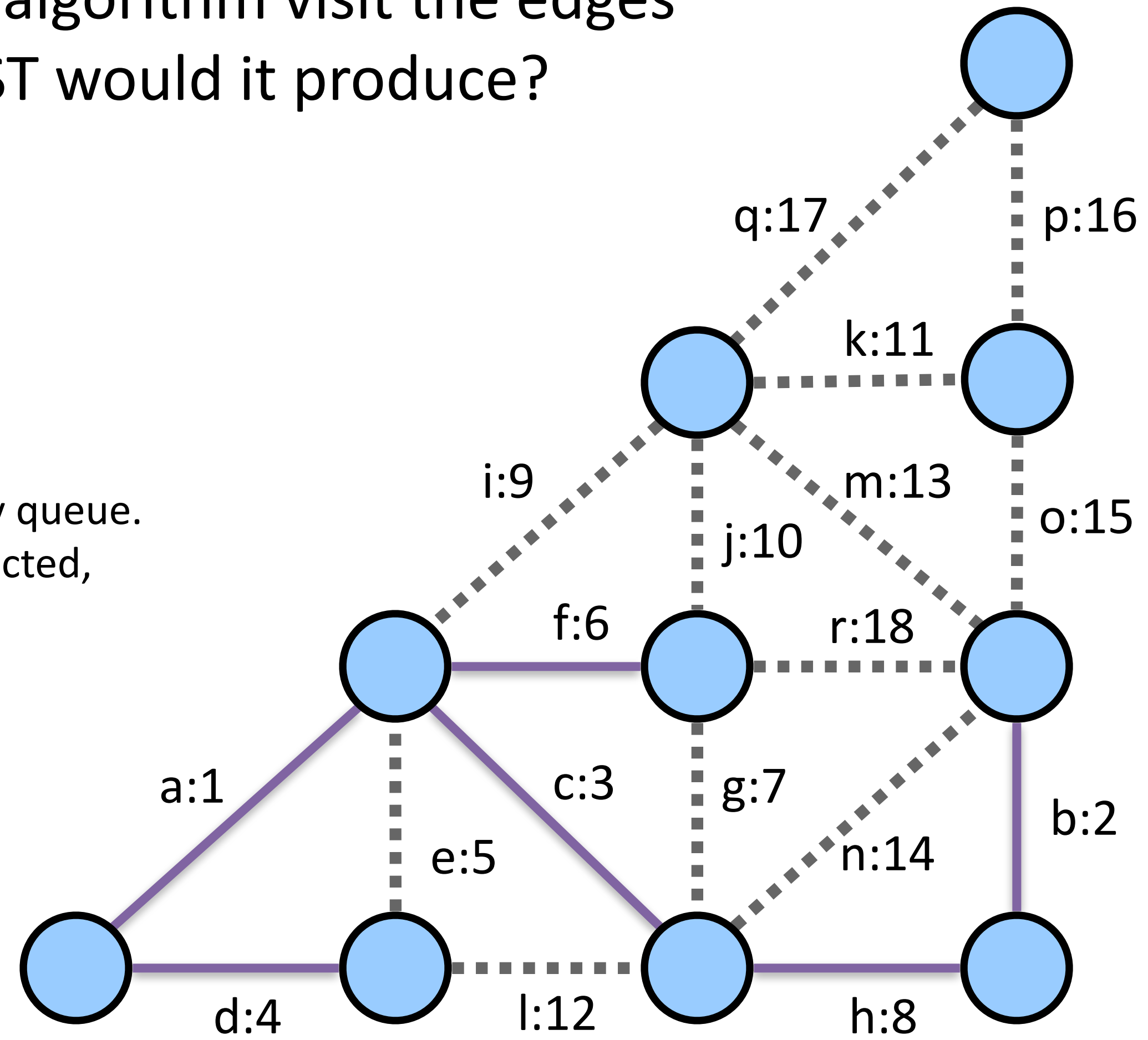
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**h:8**, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

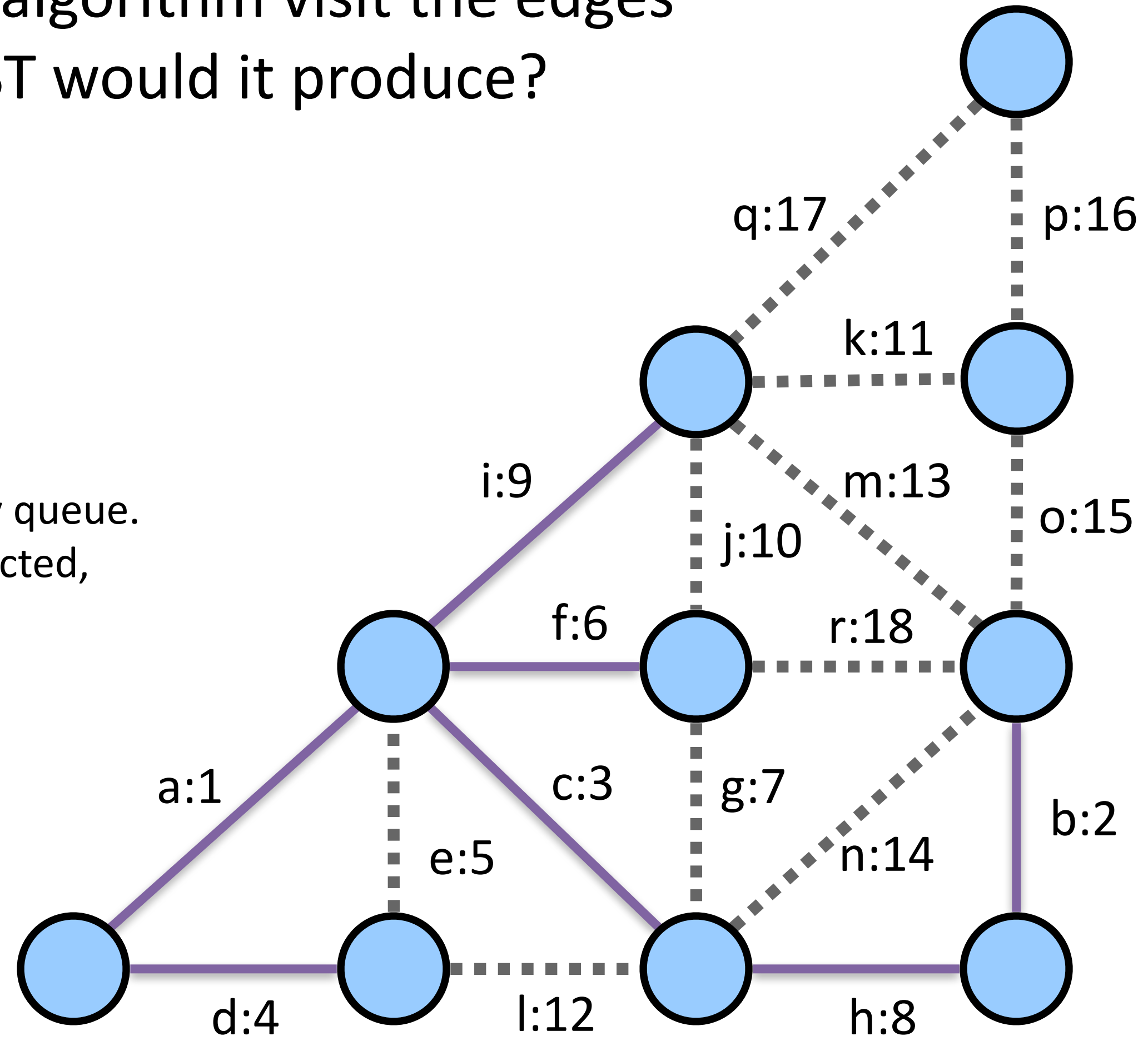
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**i:9**, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

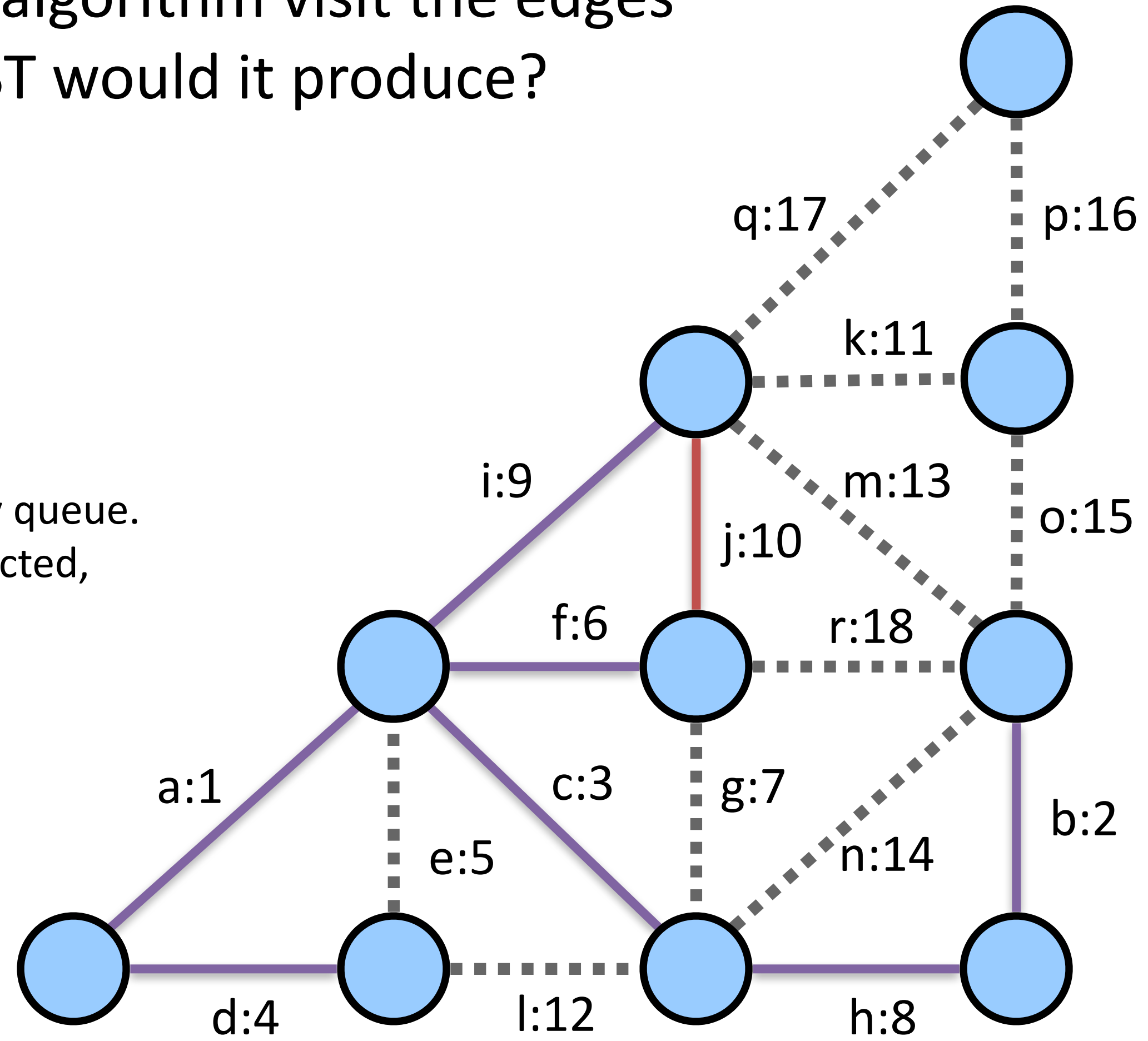
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**j:10**, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

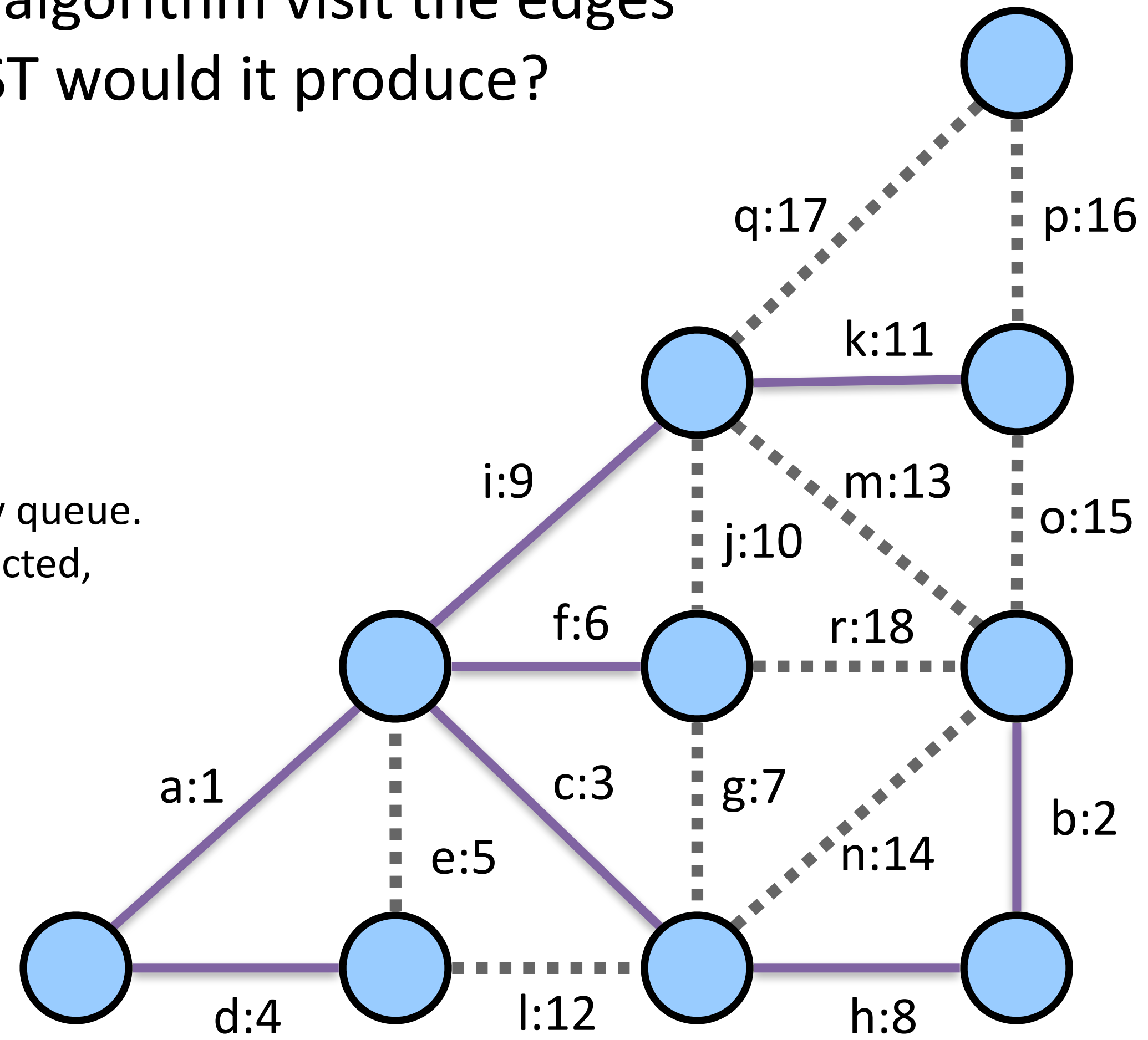
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**k:11**, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

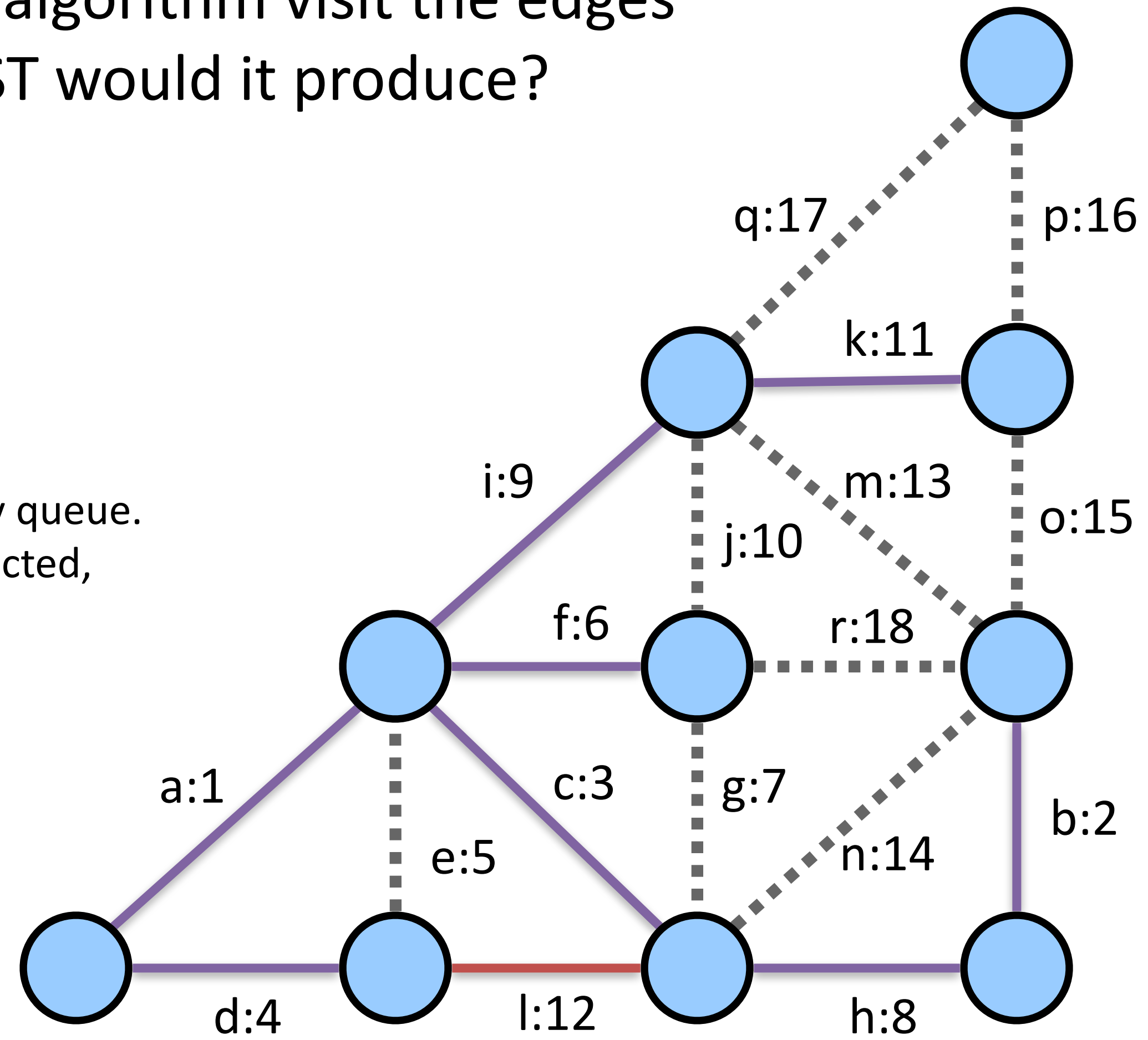
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**l:12**, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

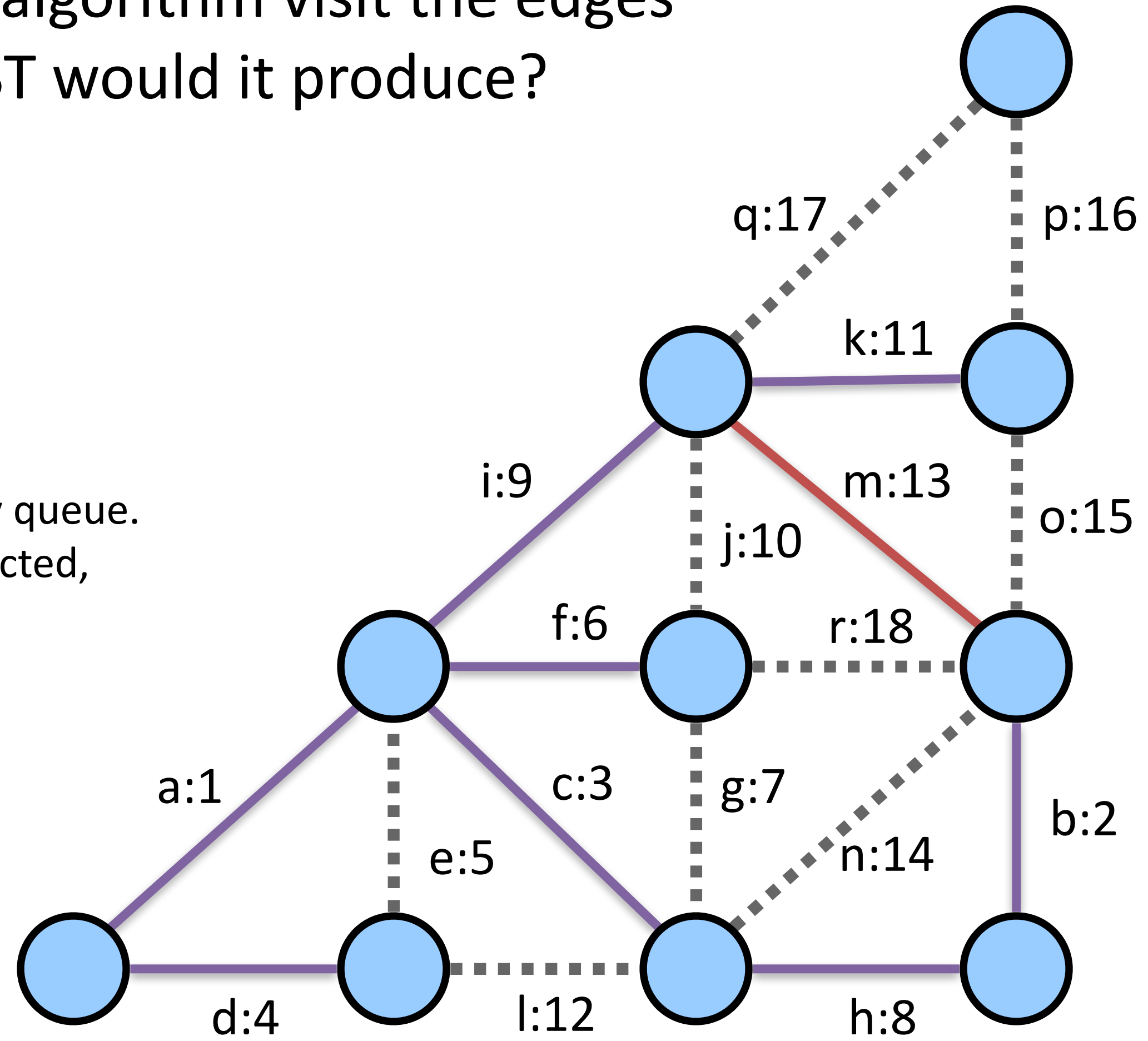
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**m:13**, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

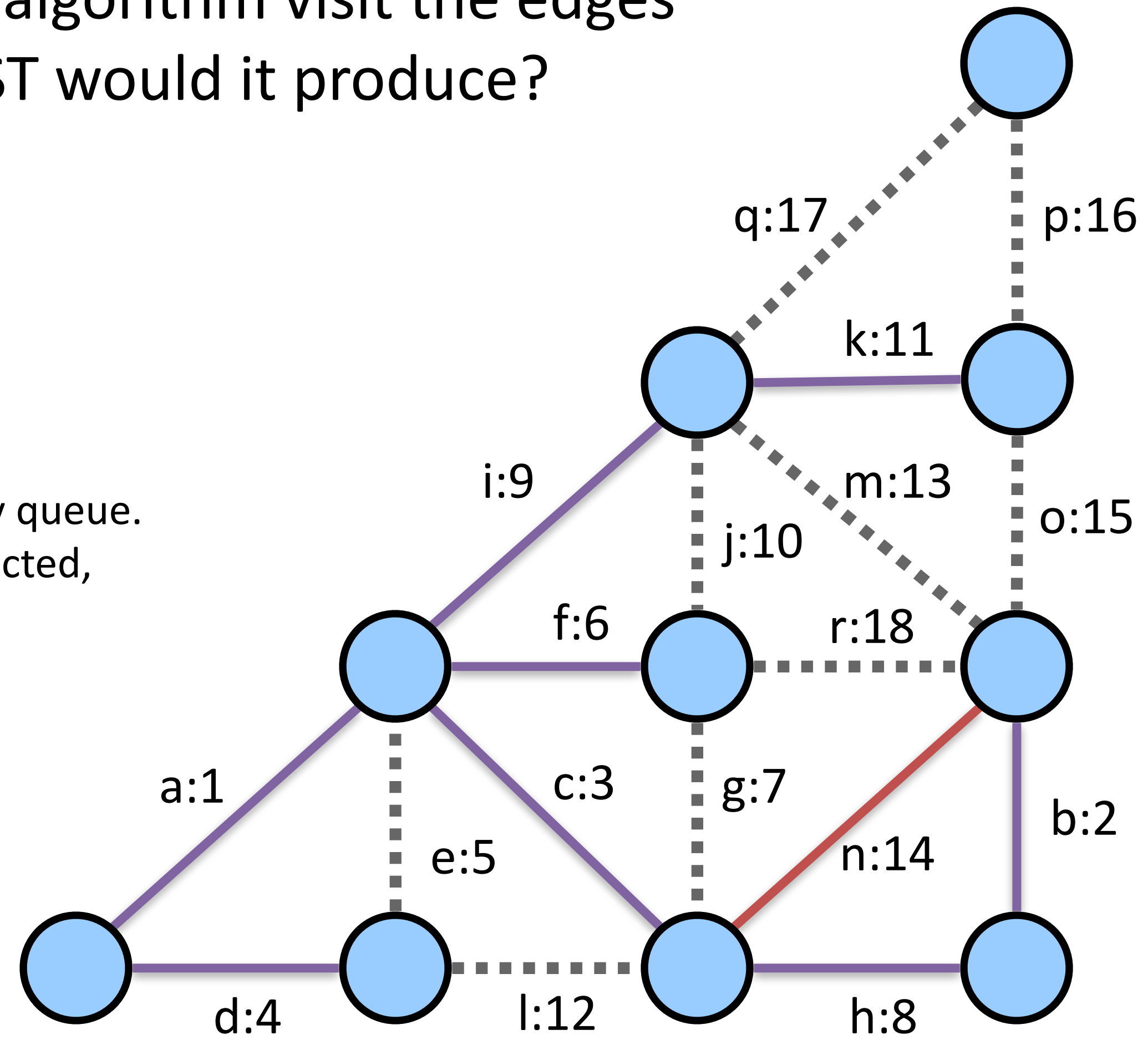
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**n:14**, o:15, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

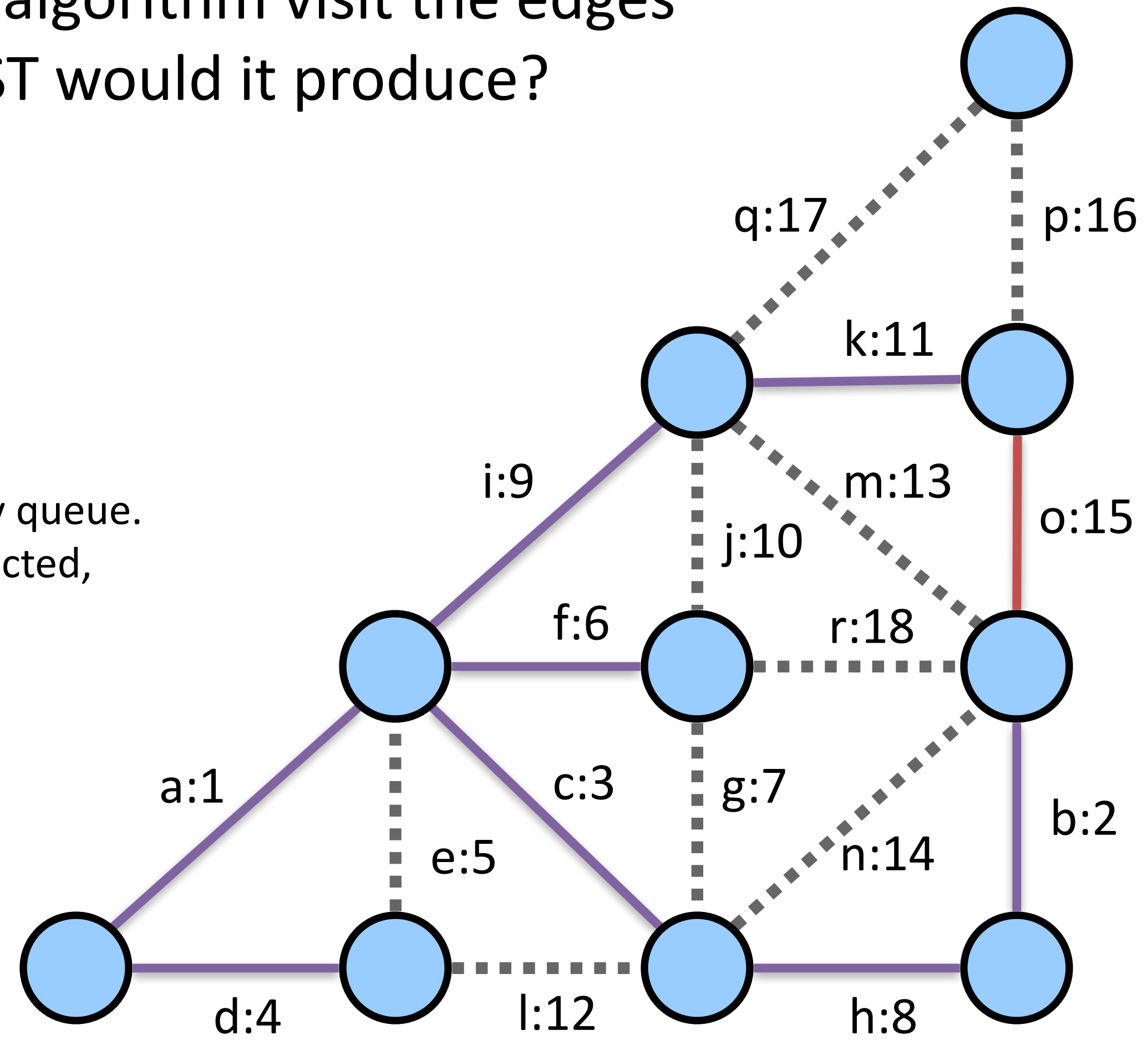
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**o:15**, p:16, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

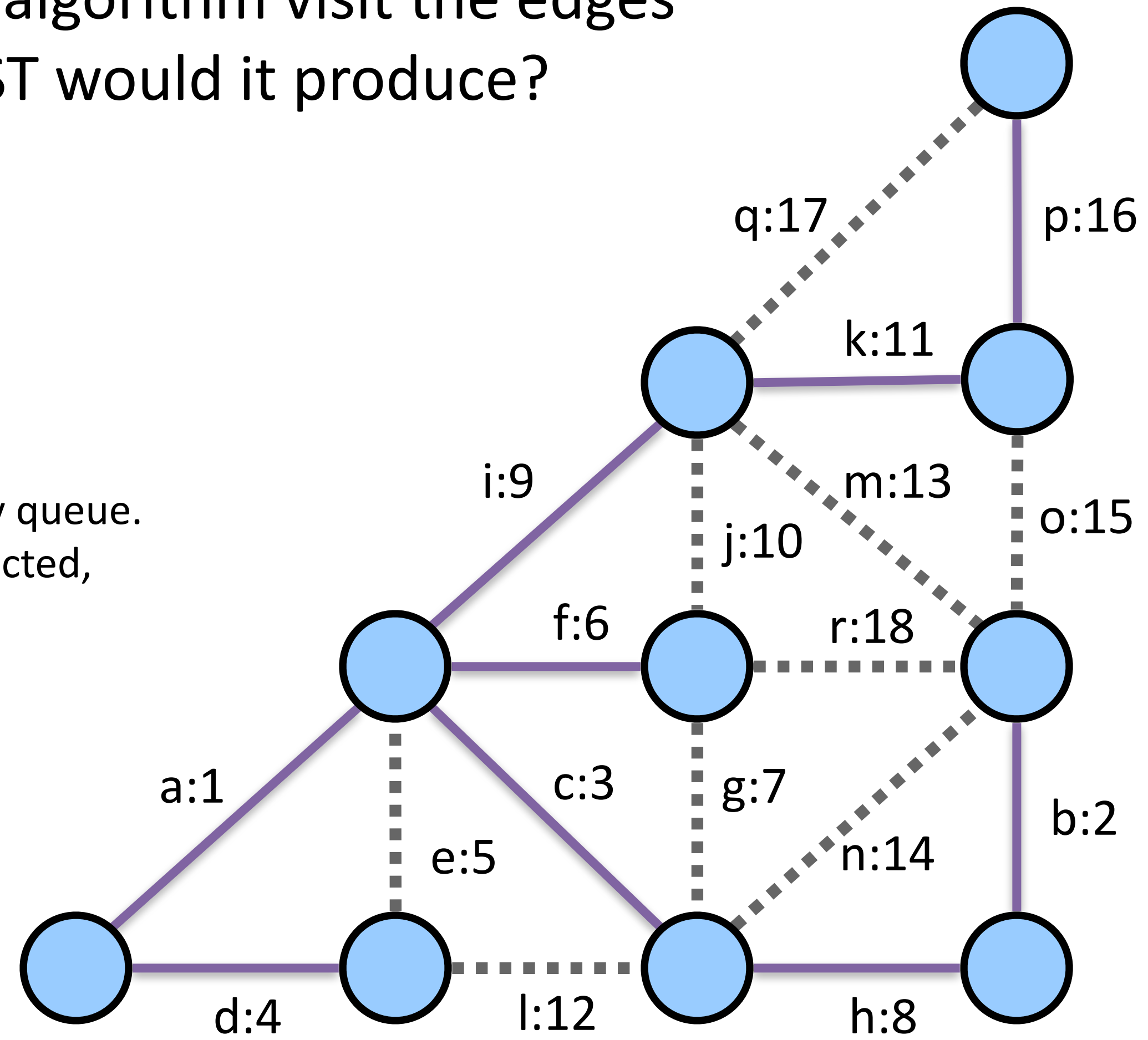
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**p:16**, q:17, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

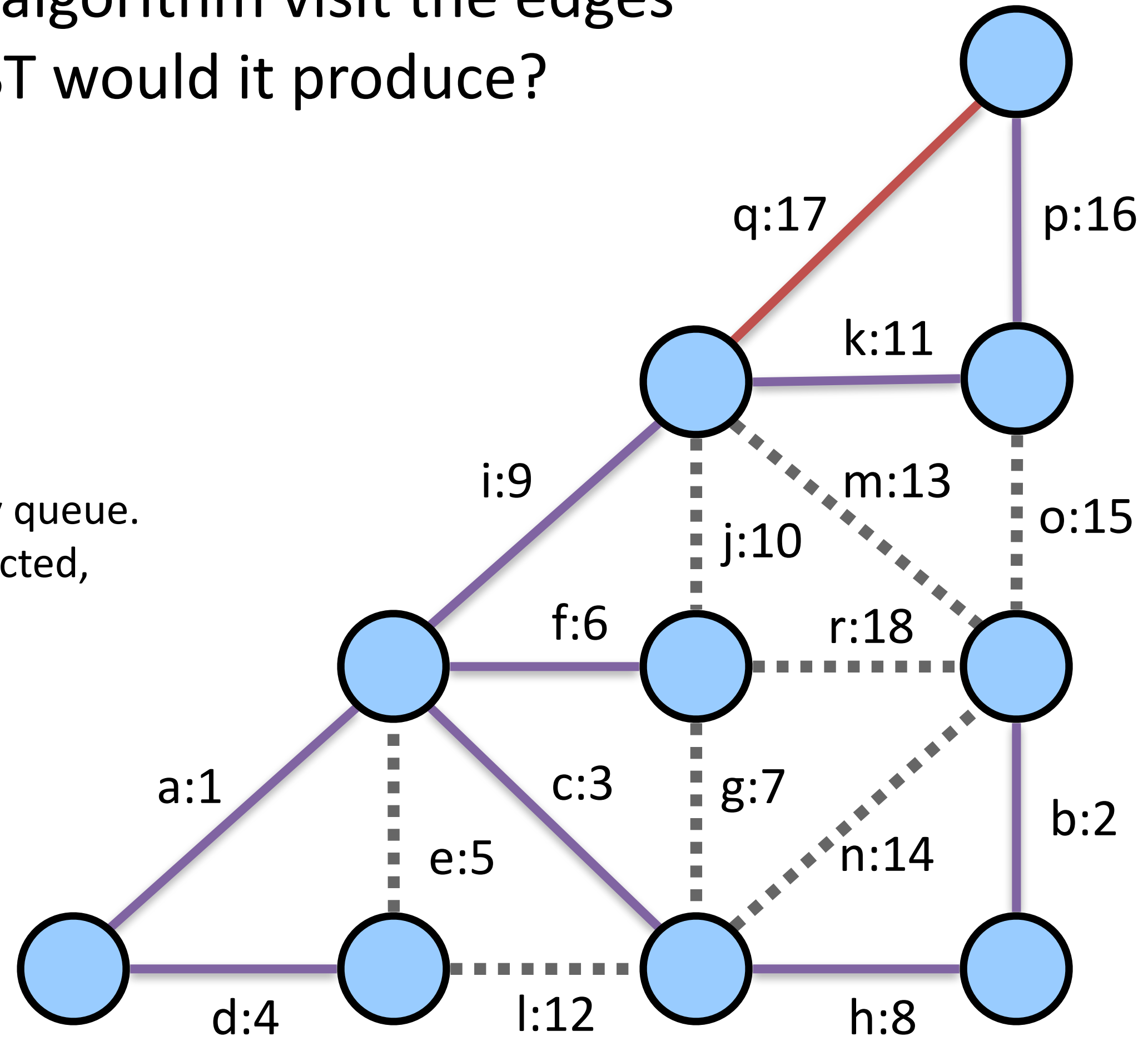
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**q:17**, r:18}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

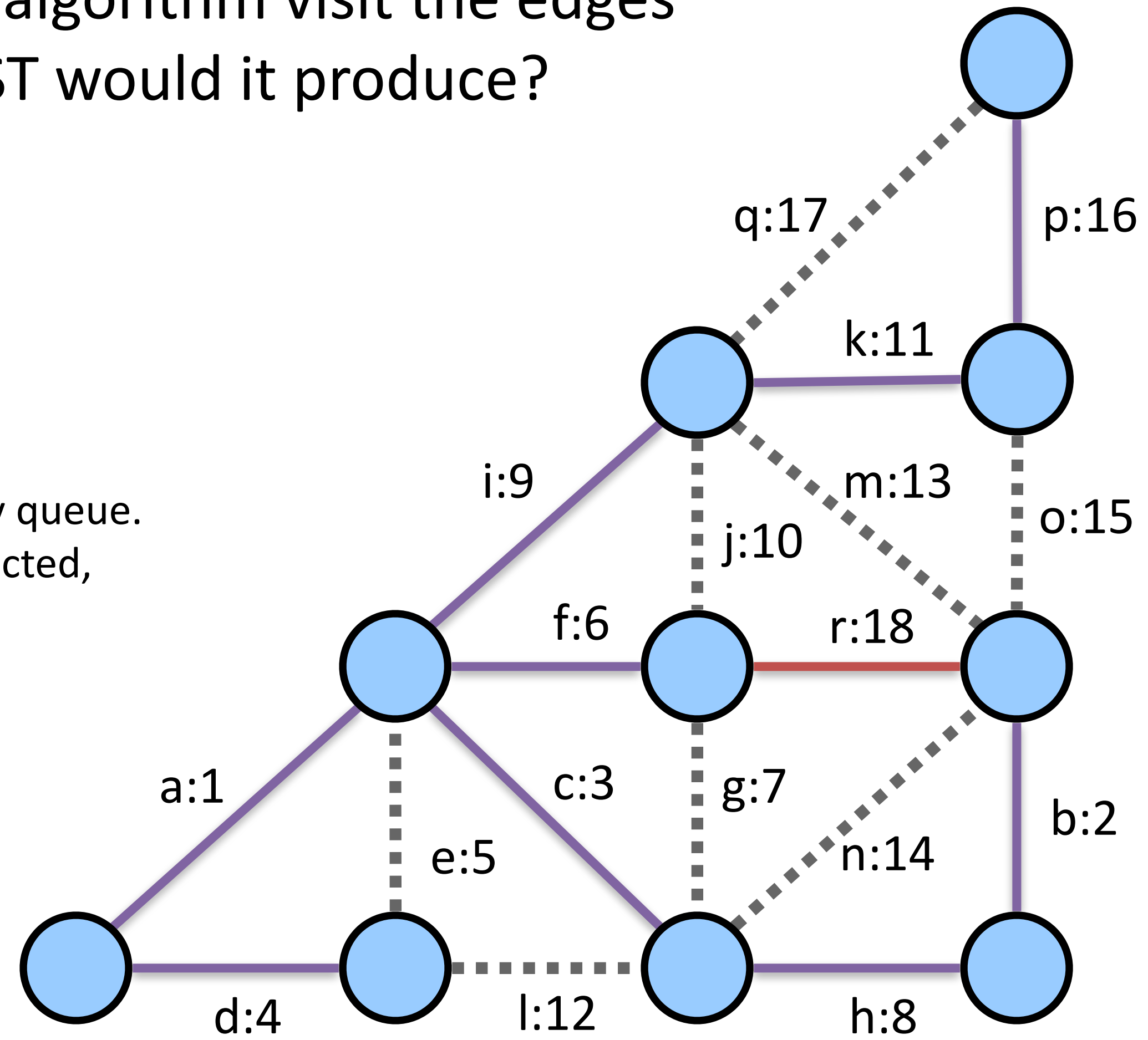
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {**r:18**}

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):

Remove all edges from the graph.

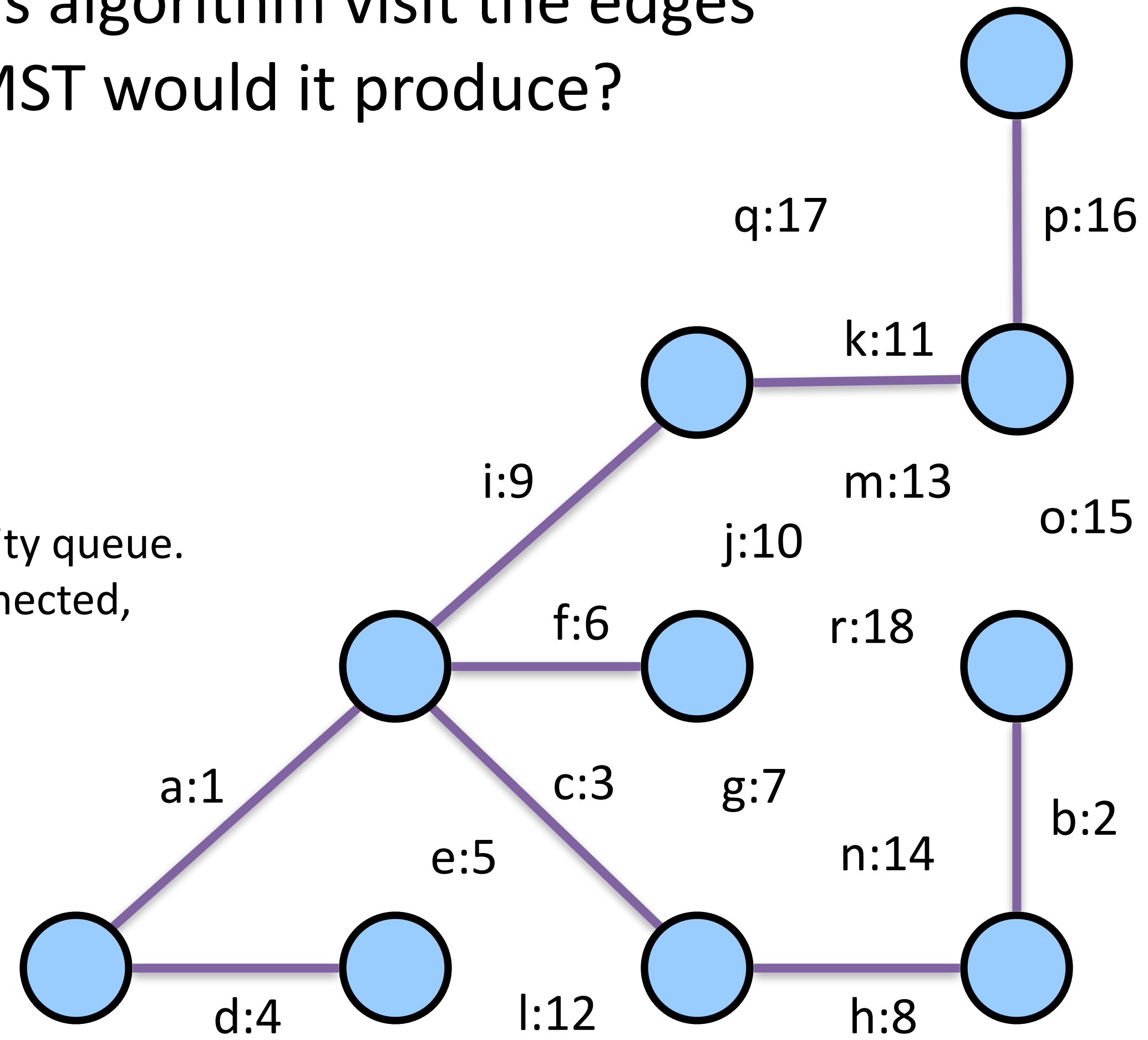
Place all edges into a priority queue
based on their weight (cost).

While the priority queue is not empty:

Dequeue an edge e from the priority queue.

If e 's endpoints aren't already connected,
add that edge into the graph.

Otherwise, skip the edge.



pq = {}

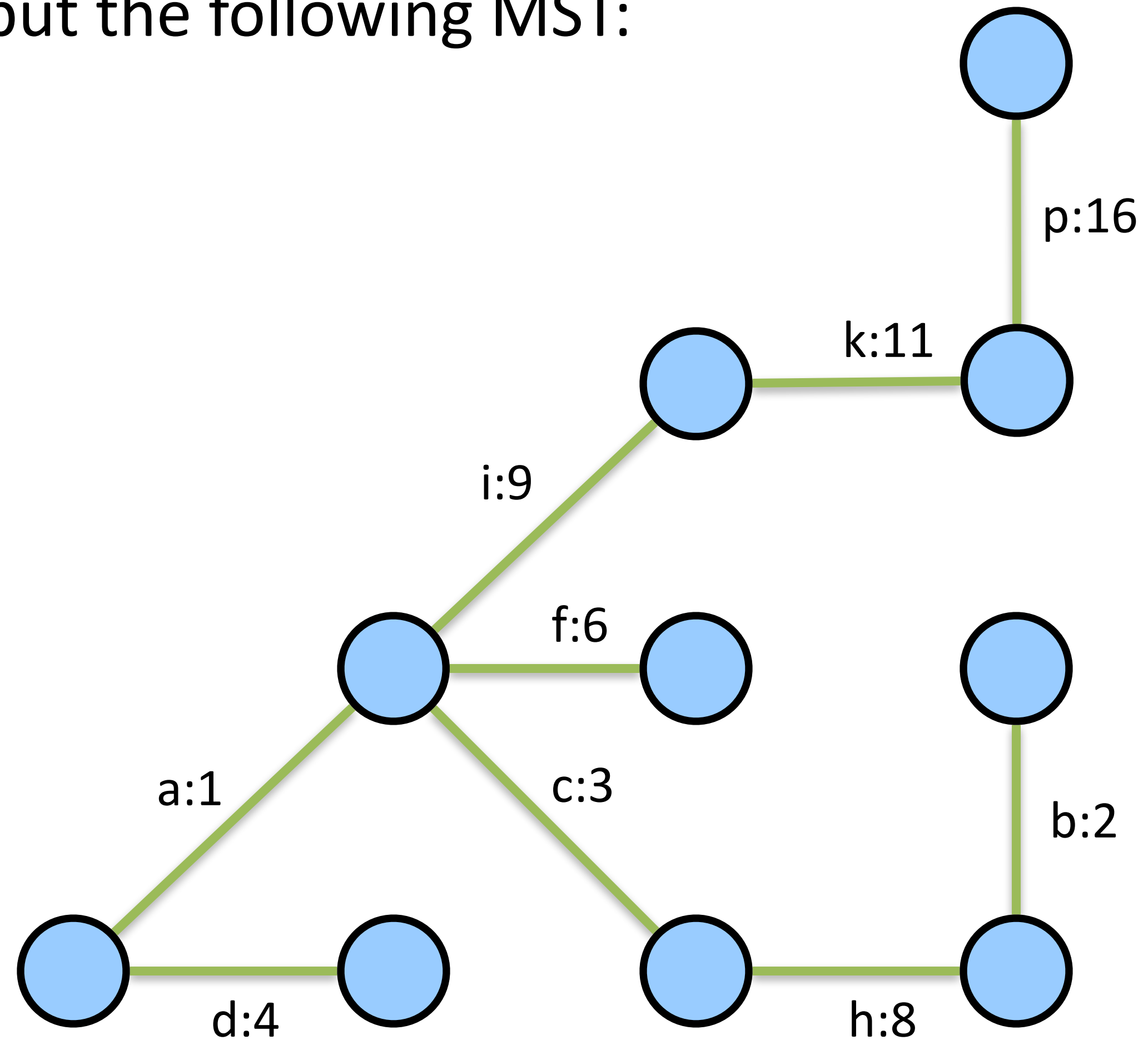
Kruskal Example

- Kruskal's algorithm would output the following MST:

– {a, b, c, d, f, h, i, k, p}

- The MST's total cost is:

$$1+2+3+4+6+8+9+11+16 = 60$$



- What data structures should we use to implement this algorithm?

function **kruskal**(graph):

Remove all edges from the graph.

Place all edges into a **priority queue**
based on their weight (cost).

While the priority queue is not empty:

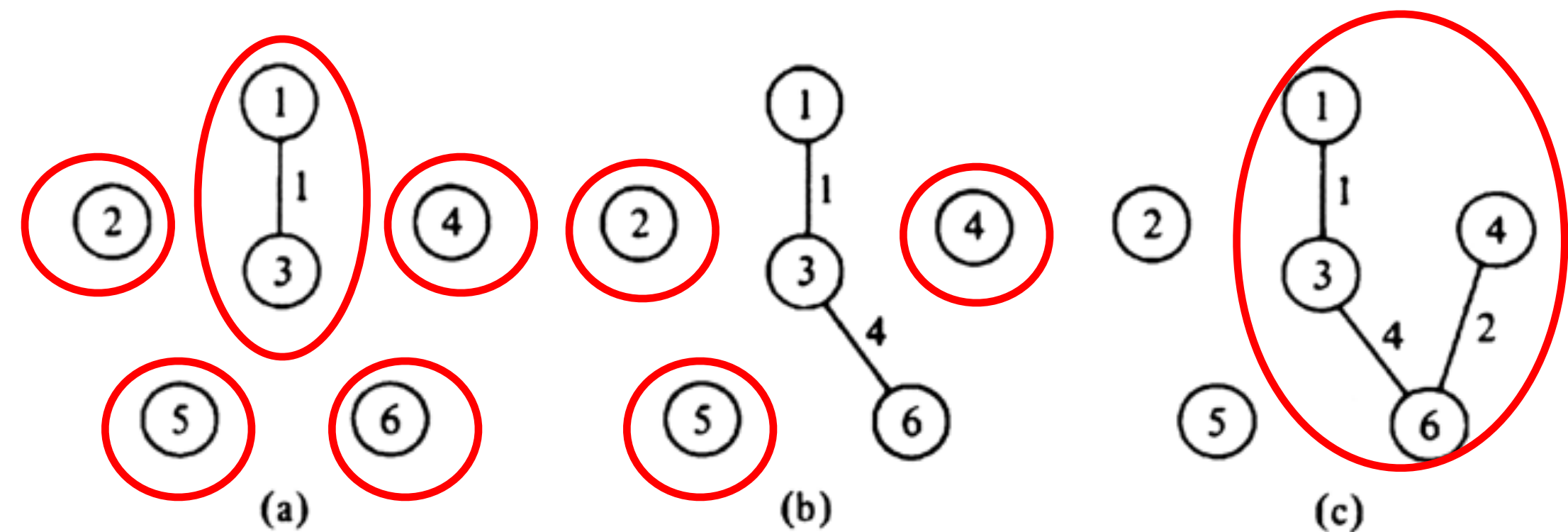
Dequeue an edge e from the priority queue.

**If e 's endpoints aren't already connected,
add that edge into the graph.**

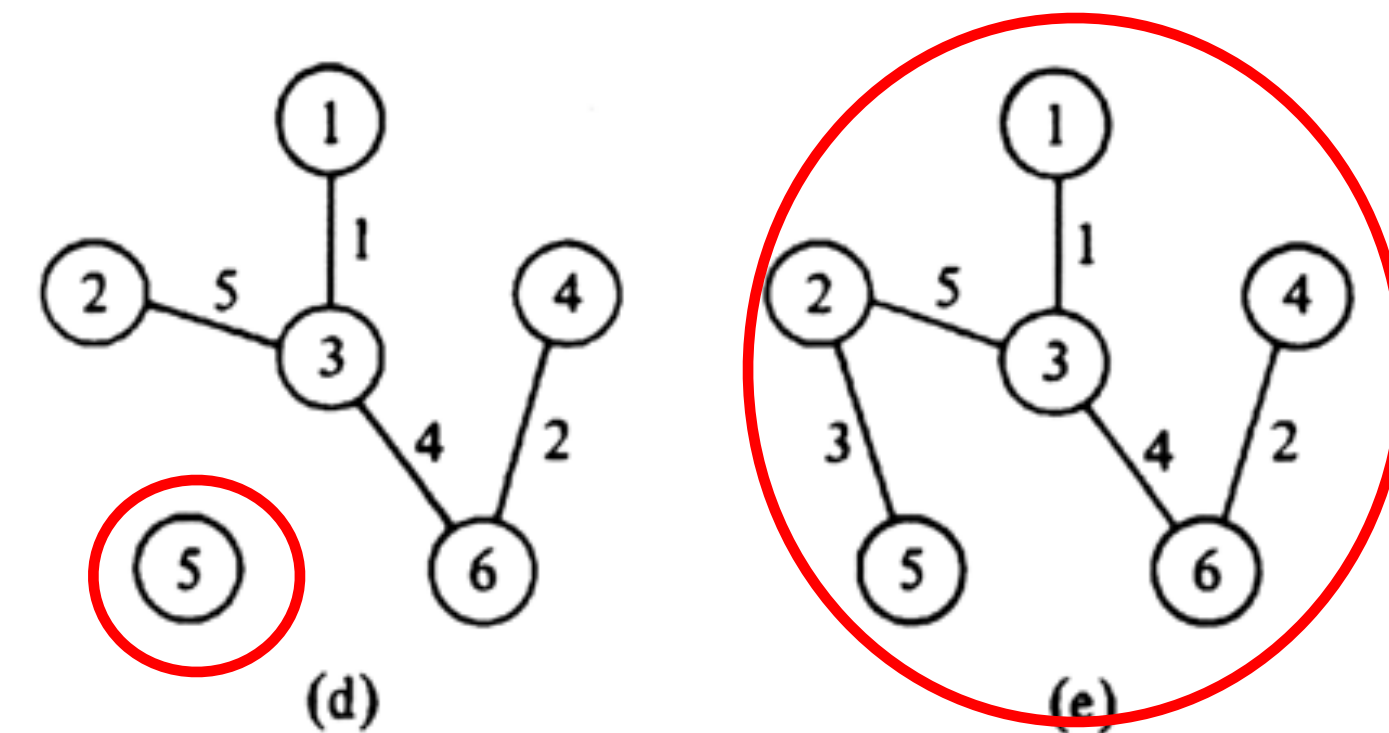
Otherwise, skip the edge.

- Need some way to identify which vertexes are "connected" to which other ones
 - we call these "**clusters**" of vertices

- Also need an efficient way to figure out which cluster a given vertex is in.



- Also need to **merge clusters** when adding an edge.



References and Advanced Reading

- **References:**

- Minimum Spanning Tree visualization: <https://visualgo.net/mst>
- Kruskal's Algorithm: https://en.wikipedia.org/wiki/Kruskal's_algorithm

- **Advanced Reading:**

- How Internet Routing works: <https://web.stanford.edu/class/msande91si/www-spr04/readings/week1/InternetWhitepaper.htm>
- <http://www.explainthatstuff.com/internet.html>



Extra Slides