Homework #9

MSAE-E4201

Spring 2017

Due April 4 before 5:30 pm in course mailbox

1. [30 points] In the microcanonical ensemble approach to the Einstein model of a crystal we assumed that the mode (or Einstein) frequency ω_0 did not depend on volume.

(a) In that model what is the isothermal compressibility?

Now assume that the mode frequency depends on molar volume v as $\omega(v) = \omega_0 - A \ln (v/v_0)$, where v is the molar volume and v_0 is the molar volume at 1 atm.

(b) Calculate the specific heat. If A > 0, is this larger than or smaller than that in the original Einstein model?

(c) Calculate the isothermal compressibility.

2. [35 points] There are N atoms per unit volume in volume V at temperature T. Those with magnetic moment $+\mu$ have energy $-\mu H$ in a magnetic field H. The others have magnetic moment $-\mu$ and energy $+\mu H$. Use the **microcanonical ensemble** method to find the average energy per unit volume, U, the specific heat, c, and the magnetization M, which is the magnetic moment per unit volume. ($dU = TdS - MdH$, and $dF = -SdT - MdH$, where the last term in each is the magnetic work done on the sample (in this set of units), and is analogous to -pdV.)

3. [35 points] Model a polymer (or rubber or rubber band) as N (>>1) molecules of length $\boldsymbol{\ell}$ that link one to the next, all in one dimension. Each can point in the +x (to the right) or –x (to the left) direction (see below diagram). The energy of each molecule does not depend on how it points. The overall length of the polymer is L.

(a) Find the entropy for N molecules, with N_R pointing to the right, and find the length of the polymer, using these same parameters.

(b) For a given tension force F the polymer will have length L. (This is analogous to pressure and volume in 3D.) Define F to be positive when the polymer is pulling inward. Find the thermodynamic expression that relates these parameters to the entropy.

(c) Using this thermodynamic expression, find the tension F in terms of L, T, N and $\boldsymbol{\ell}$.

(d) Show that when $L << N\boldsymbol{\ell}$, the tension force is proportional to L (as in Hooke's law).

(e) If you increase T, does this polymer (or rubber band) lengthen or contract?