Homework #10

MSAE-E4201 Spring 2017

Due April 11 before 5:30 pm in course mailbox

1. (10 points) The specific heat of a gas of diatomic molecules has specific heat of 2.9R. Does this make sense? Why?

2. (15 points) We use a simplified form of the Debye model to show that the specific heat of solid due to phonons (vibrations, oscillators) varies at T^3 at low temperatures. Start with the Einstein result for the specific heat of one mode at frequency ω, multiply it by $\alpha\omega^2$ (to account for the number of modes at each frequency in a 3D solid, which is called the density of states), and then write the integral to account for all modes from frequency 0 to ω_{max}. Change variables to $u = \beta\hbar\omega$. Show that in the low temperature limit the integral extends from 0 to infinity and so is a definite integral and that the specific heat varies at T^3. (In the real Debye model there is more than one type of mode variation and the density of states factor is derived.)

3. (25 points) Derive the form of the thermodynamic function G in terms of the partition function Z, T and V, starting from the expression for F.

4. (25 points) There are N atoms per unit volume in volume V at temperature T. Those with magnetic moment $+\mu$ have energy $-\mu H$ in a magnetic field H. The others have magnetic moment $-\mu$ and energy $+\mu H$. Use the **canonical ensemble** method to find the average energy per unit volume, U, the specific heat, c, and the magnetization M, which is the magnetic moment per unit volume. ($dU = TdS - MdH$, and $dF = -SdT - MdH$, where the last term in each is the change in magnetic work by the sample (in this set of units), and is analogous to $-pdV$.)

5. (25 points) A polymer under tension $\mathcal{T}$ along the x direction has N identical units of length ℓ, which link to each other and are independent of each other. In this model, each can be in either the +x, -x, +y or – y direction and has respective energy called ε_{+x}, ε_{-x}, ε_{+y}, and ε_{-y}. For a given configuration there are N_{+x}, N_{-x}, N_{+y}, and N_{-y} of them, respectively, where $N = N_{+x} + N_{-x} + N_{+y} + N_{-y}$. The energy (due to interactions with the solvent) of each monomer in the +x or –x direction is 0, and for those in the +y or –y direction it is ε. The overall energy is that for these monomers in the y-direction plus $\mathcal{T}L_x$, where $\mathcal{T}$ is the tension and L_x is the length of the polymer (and from which you can obtain ε_{+x}, ε_{-x}, ε_{+y}, and ε_{-y}). ($dU = TdS + \mathcal{T}dL_x$, and $dF = -SdT + \mathcal{T}dL_x$, where the last term in each is the work done on the sample, and is analogous to $-pdV$.)

(a) Show the polymer length is $L_x = (N_{+x} - N_{-x})\,\ell$.

(b) Show the total energy $U = (N_{+y} + N_{-y})\varepsilon + (N_{+x} - N_{-x})\,\ell\,\mathcal{T}$

(c) Find the partition function for each polymer unit and for the whole polymer, and find F. (Here and below, do not use intermediate parameters, such as N_{+x}, N_{-x}, N_{+y}, and N_{-y}, in the final answers.)

(d) Find the mean length of the polymer

(e) Find the mean energy of the polymer.

(f) Find the mean entropy of the polymer.