Branch : CSE & IT Batch : Hinglish

WEEKLY TEST – 05

Subject : Discrete Mathematics

Maximum Marks 15

Q.1 to 5 Carry ONE Mark Each

1. Consider the following statements:
S_1: $[(p \wedge \sim q) \rightarrow r] \rightarrow [p \rightarrow (q \vee r)]$
S_2: $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$
Which of the following is true?
(a) S_1 is valid and S_2 is not valid
(b) S_2 is valid and S_1 is not valid
(c) Both S_1 and S_2 are valid
(d) Neither S_1 nor S_2 is valid

2. $\{(p \rightarrow \neg q) \wedge (r \rightarrow q) \wedge r\} \rightarrow p$ is
(a) Tautology (b) Contingency
(c) Contradiction (d) None

3. $\neg (S \leftrightarrow (P \rightarrow S))$ is
(a) Tautology (b) Contingency
(c) Contradiction (d) Can't be determined

4. The game of logic has 2 assumptions
1. Logic is difficult or not many students like logic.
2. If mathematics is easy, then logic is not difficult.

Which of the following is conclusion for the given assumption?
(a) Mathematics is not easy, if many students like logic
(b) Not many students like logic, if mathematics is not easy
(c) Mathematics is not easy or logic is difficult
(d) None of the above

5. P: $\neg(A \wedge B) \vee (\neg A \rightarrow B)$ is
(a) Tautology (b) Contingency
(c) Contradiction (d) Can't be determined.

Q.6 to 10 Carry TWO Marks Each

6. $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$ is
(a) Tautology (b) Contingency
(c) Contradiction (d) Can's be determined.

7. Which of the following statements are True?
(a) $(P \rightarrow Q) \vee (Q \rightarrow P)$ is always True
(b) $(P \rightarrow Q) \vee (Q \rightarrow R)$ is always True
(c) $(P \rightarrow Q) \vee (\neg P \rightarrow R)$ is always True
(d) $(P \rightarrow Q) \vee (R \rightarrow Q)$ is always True

8. Suppose $P \rightarrow \neg q$ is false.
What is number of all possible combinations of truth value of r and s for which $(\neg q \rightarrow r) \wedge (\neg p \vee s)$ is true?

9. Which of the following is not true?
(a) $\exists x \exists y [(2x + y = 5) \wedge (x - 3y = -8)]$
(b) $\exists x \exists y [xy = 1]$
(c) $\exists x \forall y [xy = 1]$
(d) None

10. $p(x)$: $x^2 - 8x + 15 = 0$
$q(x)$: x is odd
$r(x)$: $x > 0$
Which of the following is true?
(a) $\exists x[p(x) \rightarrow q(x)]$
(b) $\forall x[q(x) \rightarrow p(x)]$
(c) $\exists x[r(x) \rightarrow p(x)]$
(d) $\forall x[p(x) \rightarrow q(x)]$

Answer Key

1. **(c)**
2. **(b)**
3. **(b)**
4. **(a)**
5. **(a)**
6. **(c)**
7. **(a, b, c)**
8. **(2)**
9. **(c)**
10. **(a, c)**

Hints and solutions

1. (c)

both are true

S_1:

pqr	$[p \to (q \vee r)]$	$p \wedge \sim q$	$[(p \wedge \sim q) \to r]$	$[(p \wedge \sim q) \to r] \to [p \to (q \vee r)]$
TFF	F	T	F	T

S_2: 2 times disjunctive syllogism

2. (b)

$\{(p \to \neg q) \wedge (r \to q) \wedge r\} \to p$

p = F
q = } Output is dependent on q
r = T

q = T ⇒ o/p = T

q = F ⇒ o/p = F

∴ Contingency

3. (b)

P	S	P → S	x: S↔P→S	¬x
T	T	T	T	F
T	F	F	T	F
F	T	T	T	F
F	F	T	F	T

4. (a)

(a) "Mathematics is not easy, if many students like logic" can be represented mathematically as $q \to \neg r$. We note that the proposition $q \to \neg$ r is logical equivalence, thus the conclusion is valid.

5. (a)

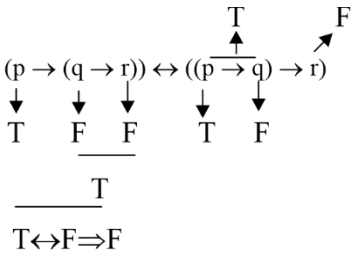
$(\neg A \vee \neg B) \vee (\neg A \vee B)$

$\neg A \vee \neg B \vee \neg A \vee B$

$\neg A \vee \quad 1 \quad = 1$

6. (c)

Assume RHS = F, LHS becomes T



7. (a, b, c)

(d) $(P \to Q) \vee (R \to Q)$ is always True

T F ∨ T F

F ∨ F = F

(a) $(P \to Q) \vee (Q \to P)$ P = T, Q = F

F F T = T

T

(c) $(P \to Q) \vee (Q \to R)$

If Q is true, $P \to Q$ can never be false

If Q is false, $Q \to R$ can never be false

(d) $(P \to Q) \vee (\neg P \to R)$

$\neg P \vee Q \vee P \vee R$

$1 \vee Q \vee R = 1$

8. (2

)

$p \to \neg q$ is false

p = T

$\neg$ q = F ⇒ q = T

9. (c)

(a) x = 1, y = 3 } 2x + y = 2 * 1 + 3 = 5; x – 3y = 1 * 1 – 3 * 3 = -8 } True

(b) x = 1, y = 1 } There exists atleast One value which makes xy = 1 } True

(c) False, Not valid for all

10. (a, c)

(a) $\exists x[p(x) \rightarrow q(x)] \Rightarrow x = 5, 3$

(b) $\forall x[q(x) \rightarrow p(x)] \Rightarrow x = 7 \Rightarrow q(x) = \text{true}, p(x) = \text{False} \therefore$ (b) is false

(c) $\exists x[r(x) \rightarrow p(x)] \Rightarrow x = 5, 3$

(d) $\forall x[p(x) \rightarrow q(x)$