Branch : CSE & IT **Batch : Hinglish**

WEEKLY TEST – 02

Discrete Mathematics

Graph theory

Maximum Marks 15

Q.1 to 5 Carry ONE Mark Each

[MCQ]

1. Which of the following simple graphs is necessarily connected

(a) A graph with 7 vertices and 15 edges
(b) A graph with 8 vertices and 20 edges
(c) A graph with 9 vertices and 29 edges
(d) A graph with 10 vertices and 34 edges

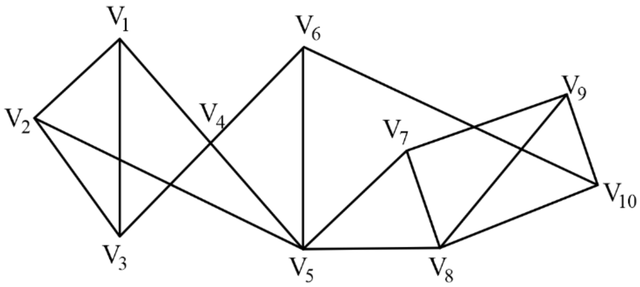
[MCQ]

2. Assume a graph G, which is simple disconnected graph with 16 vertices and the maximum number of edges are 66. Which of the following is minimum number of edges of the above graph G?

(a) 30
(b) 15
(c) 11
(d) None of these

[MCQ]

3. What is the vertex connectivity (VC) and edge connectivity (EC) of the graph shown below.



(a) VC = 1 and EC = 3
(b) VC = 2 and EC = 3
(c) VC = 3 and EC = 4
(d) VC = 4 and EC = 3

[NAT]

4. If G is a Euler graph with 11 vertices and degree of each vertex is at most 5.
The maximum number of edges possible in G is ___.

[MCQ]

5. The complete Bipartite Graph Km, n has a Hamiltonian cycle iff

(a) $m \geq 2$ and $n \geq 2$
(b) $m \geq 2$, $n \geq 2$ and $m = n$
(c) $m = n$
(d) $m \leq 2$ or $n \leq 2$

Q.6 to 10 Carry TWO Marks Each

[MCQ]

6. Consider a tree T with (n – 1) edges and n vertices. We define a term called cyclic cardinality of tree T as the number of cycles created when any two vertices of T are joined by an edge. Given a tree with 30 vertices, what is the cyclic cardinality of this tree?

(a) 30

(b) 29

(c) 435

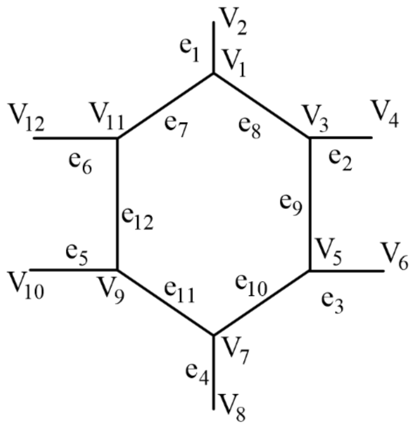
(d) 360

[NAT]

7. Let G be a simple graph with 30 vertices and 12 components. If we add an edge in G, then x is the minimum number of components and y is the maximum number of components. Find the value of x + y?

[NAT]

8. Consider the simple undirected graph G.



Find the number of cut set for the above graph G?

[MCQ]

9. Which of the following is not true?

(a) The vertex chromatic number of star graph with n vertices ($n \geq 2$) = 2.

(b) The vertex chromatic number of bipartite graph (with at least one edge) = 2.

(c) The vertex chromatic number of tree with n vertices ($n \geq 2$) = 2.

(d) If G is a simple graph in which all the cycles are of even length then the vertex chromatic number of G is 3.

[NAT]

10. If G is bipartite graph with 9 vertices and have maximum number of edges then find the chromatic number of $\overline{G}$ = ________.

Answer Key

1. (c)
2. (c)
3. (b)
4. (22)
5. (b)
6. (c)
7. (23)
8. (21)
9. (d)
10. (5)

Hints and solutions

1. (c)
A simple graph with n vertices is necessarily connected, if $|E| > \frac{(n-1)(n-2)}{2}$
$\therefore$ Only option (c) is correct.

2. (c)
In the problem we have a simple disconnected graph with 16 vertices and maximum 66 edges.

I. Now, first find the number of connected component of the graph G:

Maximum number of edges = $\frac{(n-K+1)(n-K)}{2}$

$\therefore$ $66 = \frac{(16-K+1)(16-K)}{2}$

So, by solving the above equation, we get 5 connected component.
So, K = 5.

II. Now, find the minimum number of edges possible for graph G with 16 vertices and 5 components.

$\therefore$ Minimum number of edges = n – K
= 16 – 5
= 11 edges.

Hence, we have minimum 11 edges.

3. (b)

I. The minimum degree for the given graph δ(G) is 3.

$\therefore$ $\delta(G) = 3$

II. Now, we know that the relation between the minimum degree δ(G), VC and EC is as follows:
$VC \le EC \le \delta(G)$

$\therefore$ $VC \le EC \le 3$

Form the above relation, we can conclude that the VC and EC would be less than or equal to 3.

III. Now, if we delete the vertices :
I. $\{V_2, V_4\}$ or
II. $\{V_6, V_5\}$ or
III. $\{V_5, V_{10}\}$
It will disconnect the given graph G.
Hence, the VC of the graph is 2.

IV. Now, to find the EC, try to delete the edges of minimum degree vertices.
EC = $\{(V_1, V_2), (V_2, V_3), (V_2, V_5)\}$
It will disconnect the given graph G.
Hence, the EC of the graph is 3.

4. (22)

I. A graph G is Euler graph iff it is connected and $\forall v \in G$ degree(V) = even .
Hence, the degree of each vertex will 4 that is even number.
So, the maximum number of edges possible with 11 vertices and degree of each vertex is 4.

$\therefore$ Sum of degree = 2 * |E|
$11 * 4 = 2\,|E|$

$\therefore$ $|E| = \frac{44}{2} = 22$ edges

Hence, the maximum number of edges is 22.

5. (b)
$K_{m,n}$ has Hamiltonian cycle iff m= n

($m \ge 2$ and $n \ge 2$)

6. (c)
The cyclic cardinality of a tree (T) is same as the fundamental cycle of tree T.
Now, if you select any two vertices and connect them with an edge, it will form cycle.
Hence, the cyclic cardinality = n_{c_2}

$= 30_{c_2}$

$= \frac{30 \times 29}{2}$

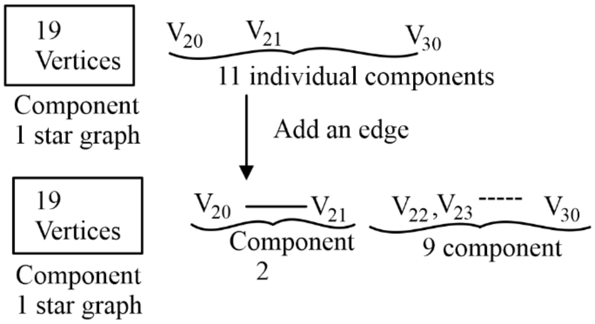
= 435

Thus, the cyclic cardinality of the tree (T) is 435.

7. **(23)**

I. We have 30 vertices and 12 component

Now,



So, the minimum number of components we get after adding 1 edge = 11

II. To get the maximum number of component, add the extra edge to the star graph.

So, the maximum number of components will be 12.

Thus x = 11 and y = 12

$\therefore$ x + y = 11 + 12 = 23

8. **(21)**

I. The vertices {V_2, V_4, V_6, V_8, V_{10}, V_{12}} are pendant vertices. Hence, all the edges which connect the pendant vertex will be cut edge. So, the edges {e_1, e_2, e_3, e_4, e_5, e_6} are the 6 cut set.

II. Now, in the above given graph, we have a cycle of length '6': {$V_1 - V_3 - V_5 - V_7 - V_9 - V_{11} - V_1$}

So, if we select any 2 edges from the cycle, it will disconnect the graph.

So, the number of cut set from the cycle

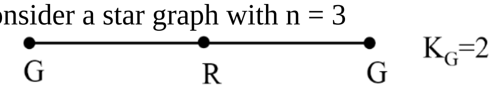
$$6_{c_2} = \frac{6 \times 5}{2} = 15$$

Hence, the total number of cut set will be 15 + 6 = 21

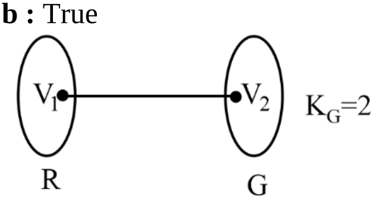
9. **(d)**

Option a : True

Consider a star graph with n = 3



Option b : True



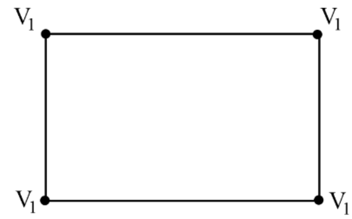
Note : A non – null graph is bipartite graph if and only if its bichromatic.

Option c : True

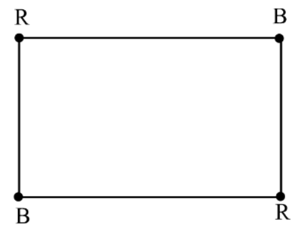
Every tree is a bipartite graph hence, it bichromatic.

Option d : False

Example



The above graph is cycle graph of even length but the vertex chromatic number is 2.



Hence, the given statement is false.

10. **(5)**

I. In the problem G is bipartite graph with 9 vertices.

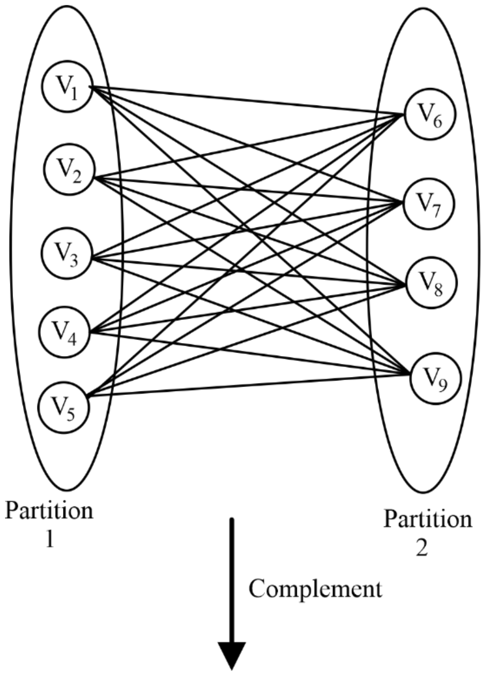
Number of vertices distribution for maximum number of edges = m + n = 5 + 4

Where m = 5 Number of vertices is partition 1

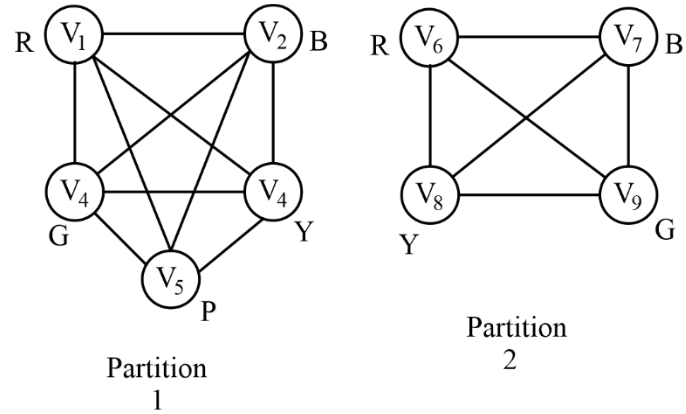
n = 4 Number of vertices is partition 2

II. The maximum number of edges of the given bipartite graph = m ∗ n

= 5 ∗ 4 = 20 edge.



The complement of the above bipartite graph will have complete graph with 5 vertices in partition 1 and complete graph with 4 vertices in partition 2.



The chromatic number of K_5 is 5.

Hence, the chromatic number of $\overline{G}$ is 5.