**Branch : CSE & IT** **Batch : Hinglish**

# Algorithms

## Analysis of Algorithms

DPP

**[MCQ]**

**1.** Sort the functions in ascending order of asymptotic(big-O) complexity.

$f_1(n) = n$, $f_2(n) = 80$, $f_3(n) = n^{\log n}$, $f_4(n) = \log\log^2 n$, $f_5(n) = (\log n)^{\log n}$

(a) $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$

(b) $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$

(c) $f_2(n), f_1(n), f_4(n), f_3(n), f_5(n)$

(d) $f_1(n), f_1(n), f_4(n), f_3(n), f_2(n)$

**[MCQ]**

**2.** Consider two function $f(n) = 10n+2\log n$ and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?

(a) $f(n) = \theta(g(n))$ (b) $f(n) = O(g(n))$

(c) $f(n) = \omega(g(n^2))$ (d) None of the above

**[MCQ]**

**3.** Consider two function $f(n) = \sqrt{n}$ and $g(n) = n \log n + n$ then $f(n)/g(n)$ is equivalent to how many of the following given below? _____.

(i) $o(n^{-1/2})$ (ii) $O(n^{-1/2})$

(iii) $\Omega(1/\log n)$ (iv) $\theta(n^{-1/2})$

**[MCQ]**

**4.** Consider the following C-code

```
void foo (int x)
    {
      int a = 1;
      if (n = = 1)
      return;
for (; a ≤ n; a++)
      {
         printf("GATEWALLAH");
         break;
       }
   }
```

What is the worst time complexity of above program?

(a) O(1) (b) O(n)

(c) O (log n) (d) $O\sqrt{n}$

**[MCQ]**

**5.** Find the time complexity of the following summation, assume that k is constant, $k > 0$

$$\sum_{x=1}^{n}\sum_{y=x+1}^{n}\frac{1}{k}$$

(a) $O(n^2)$ (b) O(n)

(c) $O(n^3)$ (d) None of the above

**[NAT]**

**6.** How many of the following expressions correctly describes $T(n) = n\log(n^2)$? _____

(a) $\theta(n^2)$ (b) O(n)

(c) $\Omega(n)$ (d) $O(n^2)$

**[MCQ]**

**7.** Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.

(a) $f_1(n) = (Of_2(n))$ (b) $f_1(n) = \theta(f_2(n))$

(c) $f_1(n) = \omega(f_2(n))$ (d) None of these

# Answer Key

1. **(a)**
2. **(a)**
3. **(2 to 2)**
4. **(a)**
5. **(a)**
6. **(2 to 2)**
7. **(c)**

# Hints & Solutions

**1. (a)**

$80 < n$

$\log \log^2 n < n$

put $n = 10^{100}$

$\log(\log n)^2 = 10^{100}$

$\log(100)^2 < 10^{100}$

$4 < 10^{100}$

$n < n^{\log n}$

taking log on both side

$\log n < \log n \log n$

we know that $(\log n)^2 > \log n$

now, $(\log n)^{\log n} < n^{\log n}$

as we can see that logn in LHS and n on RHS.

$n \quad (\log n)^{\log n}$

taking log on both sides

$\log n < \log n * \log \log n$

From above we conclude that growth of log*logn is higher than 1.

$\therefore$ option (a) is correct.

**2. (a)**

As we can see in above function, 'n' is the dominating factor in these 2 functions. Which means they also have similar growth rate.

$\therefore$ $f(n) = O(g(n))$

Hence option (a) is correct.

**3. (2 to 2)**

$$\frac{f(n)}{g(n)} = h(n)$$

Given

$$f(n) = \sqrt{n}, g(n) = n\log n + n$$

$$= \frac{\sqrt{n}}{n\log n + n}$$

$$= \frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{n}\log n + \sqrt{n}\right)}$$

$$= \frac{1}{\sqrt{n}\log n + \sqrt{n}}$$

and clearly $h(n) = O(n^{-0.5})$ and

$h(n) = o(n^{-0.5})$

**NOTE**: if small 'o' possible then Big 'O' is possible but if Big 'O' possible then small 'o' may or may not possible.

$\therefore$ (i) and (ii) are correct.

Hence 2 expressions are correct.

**4. (a)**

If we see carefully, loop will execute only one time because of break statement, therefore time complexity will be O(1)

**5. (a)**

$$\sum_{x=1}^{n}\sum_{y=x+1}^{n}\frac{1}{k} = \frac{1}{k}\sum_{x=1}^{n}\sum_{y=x+1}^{n}(1)$$

$$= \frac{1}{k}\sum_{x=1}^{n}\left[1+1+1+.....n-(x+1)+1 \text{ times}\right]$$

$$= \frac{1}{k}\sum_{x=1}^{n}\left[n-x\right]$$

$$= \frac{1}{k}\left[n\sum_{x=1}^{n}(1) - n\sum_{x=1}^{n}x\right] = \frac{1}{k}\left[n.n - \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{k}\left[n^2 - \frac{n^2+n}{2}\right] = \frac{1}{2k}\left[n^2 - n\right]$$

$= O(n^2)$

**6. (2 to 2)**

Given: $T(n) = n\log(n^2) = 2n\log n$

(i) $T(n) = \theta(n^2)$, which means the value of T(n) is exactly $\theta(n^2)$, but as we can see that T(n) is $n\log(n^{2)}$ so this is incorrect.

(ii) $T(n) = O(n)$: $T(n) \le k.n$, but value of T(n) is $n\log(n^2)$

So, this is also false.

(iii) $T(n) = \Omega(n)$

$T(n) >= k.n$ and the complexity given for T(n) is $n\log(n^2)$, so it is correct.

(iv) $O(n^2)$

$T(n) = O(n^2)$

$n\log(n^2) \le k.n^2$ which is correct.

Hence 2 expression out of 4 are correct.

**7. (c)**

$f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$

$n^{2^n} = n^{n^2}$

Taking log on both side

$2^n \log n$ $n^2 \log n$

as we can see that

$2^n$ has more growth rate than $n^2$ $\therefore$ we conclude

$f_2(n) < O(f_1(n))$ or

$f_1(n) = \omega(f_2(n))$

$\therefore$ (c) is correct.

# Branch : CSE & IT Batch : Hinglish

# Algorithms

## Design Strategies

DPP

**[MCQ]**

**1.** Consider an array containing the following elements in unsorted order (placed randomly) but 120 as first elements

120 160 30 190 14 24 70 180 110

Quick sort partitioning algorithm is applied by choosing first elements as pivot element. Then what is the total number of arrangements of array integers are possible preserving the effect of first pass of partitioning algorithm.

(a) 680 (b) 700

(c) 720 (d) 740

**[MCQ]**

**2.** Let $T(n) = [n(\log(n^3) - \log n) + \log n]n + \log n$.

complexity of T(n) is

(a) $O(n^2)$ (b) $O(n^3)$

(c) $O(n\log n)$ (d) $O(n^2\log n)$

**[MCQ]**

**3.** Assume that there are 4 sorted lists of $\frac{n}{4}$ elements each, if these lists are merged into a single sorted list of 'n' elements then how many key comparisons are required in the worst case using an efficient algorithm?

(a) $2n - 3$ (b) $\frac{7}{4}n - 3$

(c) $\frac{9}{4}n - 3$ (d) $\frac{6}{4}n - 3$

**[NAT]**

**4.** Consider the number in the sequence

2 5 11 17 19 21 26 33 39 40 51 65 79 88 99

Using binary search, the number of comparisons required to search elements '2' is____

**[MCQ]**

**5.** Merging 4 sorted files having 400, 100, 250, 50 records will take O (___) time?

(a) 800 (b) 400

(c) 200 (d) 100

**Ans. (a)**

**Sol.** Two sorted file of size m and n takes O(m + n) time for merging.

So, total time = 400 + 100 + 250 + 50 = 800

$\therefore$ (a) is correct.

**[NAT]**

**6.** Consider a machine which needs a minimum of 50 seconds to sort 500 names by quick sort, then what is the minimum time required to sort 50 names (approximately)is _____ (round off to 2 decimal)

**[NAT]**

**7.** What is the total number of comparisons that will be required in worst case to merge the following sorted files into a single sorted file into a single sorted file by merging together two files at a time____.

| Files | $F_1$ | $F_2$ | $F_3$ | $F_4$ |
|---|---|---|---|---|
| Number of records | 40 | 42 | 44 | 46 |

# Answer Key

1. **(c)**
2. **(d)**
3. **(a)**
4. **(4 to 4)**
5. **(a)**
6. **(3.14 to 3.14)**
7. **(341 to 341)**

# Hints & Solutions

**1. (c)**

We have to choose the I$^{st}$ elements as pivot. Here 120 is the I$^{st}$ element. After the I$^{st}$ pass I$^{st}$ elements goes to its exact location.

All the elements greater than 120 goes to right of 120 and lesser elements goes to left side of 120 after I$^{st}$ pass.

| 110 | 30 | 14 | 24 | 70 | 120 | 160 | 180 | 190 |
|---|---|---|---|---|---|---|---|---|
| 5! | | | | | | 3! | | |

5! × 3!

⇒ 720 possible arrangement

∴ C is correct option.

**2. (d)**

Given

$T(n) = [n(\log(n^3) - \log n) + \log n]\, n + \log n$

$= \left[ n\left( \log \frac{n^3}{n} \right) + \log n \right] n + \log n$

$= \left[ n \log n^2 + \log n \right] n + \log n$

$= n^2 \cdot 2\log n + n\log n + \log n$

$= 2n^2 \log n + n\log n + \log n$

$= O(n^2 \log n)$

∴ option (d) is correct.

**3. (a)**

$\underbrace{\frac{n}{4}\ \frac{n}{4}}_{\left\{\frac{n}{2}-1\right\}}$ $\underbrace{\frac{n}{4}\ \frac{n}{4}}_{\left\{\frac{n}{2}-1\right\}}$

Comparisons Comparisons

n –1 Comparisons

Total number of comparisons = $\left(\frac{n}{2}-1\right) + \left(\frac{n}{2}-1\right) + (n-1)$

= 2n – 3

∴ option (a) is correct.

**4. (4 to 4)**

Low = 0, high = 15

Mid = $\frac{0+15}{2} = 7$

A[7] = 33

33 > 2

Low =0, high = 6;

Mid = $\frac{0+6}{2} = 3$

A[3] = 17

17 > 2

Low = 0, high = 2, mid = $\frac{0+2}{2} = 1$

A[1] = 5

5 > 2

Low = 0, high = 0, mid = $\frac{0+0}{2} = 0$

A[0] = 2

2 = = 2

**5. (a)**

Two sorted file of size m and n takes O(m + n) time for merging.

So, total time = 400 + 100 + 250 + 50 = 800

∴ (a) is correct.

**6. (3.14 to 3.14)**

In best case quick sort algorithm takes nlogn comparisons,

So, $500 \times \log_2 500 \cong 4482.89$

Which takes 50 seconds.

To sort 50 names a minimum of $50(\log_2 50) = 282.19$ comparisons

∴ 282.19 comparisons are needed.

This takes = $50 \times \frac{282.19}{4482.89}$ = 3.14 second

**7. (341 to 341)**

Given files

40, 42, 44, 46} 82 – 1 = 81 comparisons

82, 44, 46} 90 – 1 = 89 comparisons

82, 90} 172 – 1 = 171 comparisons

So, 81 + 89 + 171 = 341 comparisons

**Any issue with DPP, please report by clicking here:-** https://forms.gle/t2SzQVvQcs638c4r5

**For more questions, kindly visit the library section: Link for web:** https://smart.link/sdfez8ejd80if


**PW Mobile APP:** https://smart.link/7wwosivoicgd4

# Branch : CSE & IT  Batch : Hinglish

# Algorithms

## 'Dynamic Programming' & 'Graph Algorithm'  DPP

**[MCQ]**

**1.** What is the time complexity of dynamic programming for matrix chain multiplication problem?

(a) $O(n^2)$
(b) $O(n^3)$
(c) O(nlogn)
(d) None of these

**[NAT]**

**2.** Consider the matrices x, y and z with dimension 10 × 20, 20 × 30 respectively. Then what is the minimum number of multiplications required to multiply the matrices? _____

**[MCQ]**

**3.** What is the length of the LCS for the pair of subsequences given below.

P = ATGACTATAA
Q = GACTAATA

(a) 5  (b) 6
(c) 7  (d) 8

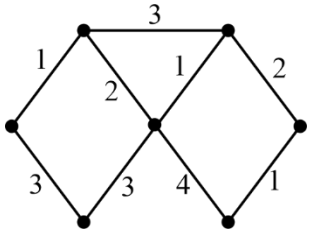
**[MCQ]**

**4.** Consider a connected weighted graph G = (V, E), where |V| = n, |E| = m, if all the edges have distinct positive integer weights, then the maximum number of minimum weight spanning trees in the graph is ?

(a) n  (b) m
(c) 1  (d) $n^{n-2}$

**[MCQ]**

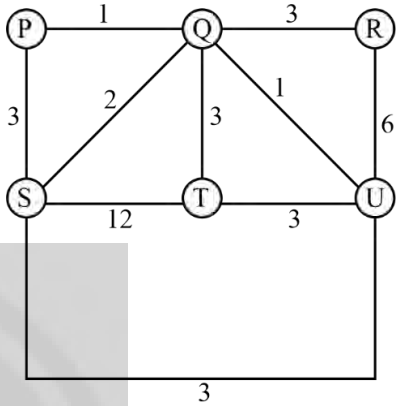
**5.** What is the weight of the minimum spanning tree for the graph shown below?




(a) 7  (b) 8
(c) 9  (d) 10

**[MCQ]**

**6.** How many minimum spanning tree does this graph have?




(a) 2  (b) 3
(c) 4  (d) 5

**[MCQ]**

**7.** Consider the following problem with knapsack capacity of 8

| Items | Profits | Weights |
|---|---|---|
| $I_1$ | 13 | 1 |
| $I_2$ | 8 | 5 |
| $I_3$ | 7 | 3 |
| $I_4$ | 3 | 4 |

Which of the following item is not selected in the optimal solution of 0/1, knapsack problem?

(a) $I_1$ only  (b) $I_2$ only
(c) $I_3$ only  (d) $I_4$ only

**[MCQ]**

**8.** Consider the following statements

S1: for every weighted graph and any two vertices p and q, Bellman ford algorithm starting at p will always return a shortest path to q.

S2: At the termination of Bellman ford algorithm even if graph has negative weight cycle, correct shortest path is found for vertex for which shortest path is well-defined.

Which of the statement is correct?

(a) only S1
(b) only S2
(c) Both S1 and S2 are true
(d) neither S1 nor S2 is true

# Answer Key

1. **(b)**
2. **(18000 to 18000)**
3. **(c)**
4. **(c)**
5. **(d)**
6. **(a)**
7. **(b)**
8. **(d)**

# Hints & Solutions

**1. (b)**

Time complexity of the dynamic programming approach is $O(n^3)$. Where n is the number of matrices. Because it contains nested loop iterating over the matrix dimension to fill in the optimal costs.

**2. (18000 to 18000)**

Given matrix dimension

x : 10 × 20

y : 20 × 30

z : 30 × 40

Optimal parentification is ((xy)z)

Minimum number of multiplications

= 10 × 20 × 30 + 10 × 30 × 40 = 18000

**3. (c)**

P=A T G A C T A T A A

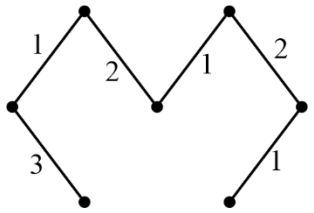
G A C T A A T A

G A C T A A A

**4. (c)**

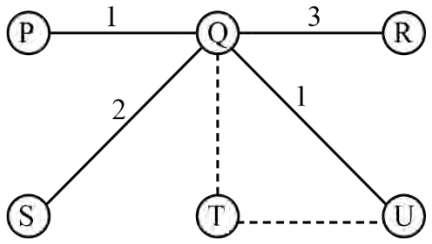
The maximum number of minimum weight spanning tree in a connected weighted graph G(V, E) with |V| = n and |E| = m, where all edges have distinct positive integer weight is 1.

**5. (d)**




1 + 2 + 1 + 2 + 3 + 1 = 10

**6. (a)**




There are 2 dotted lines are the only choices that it has,

∴ Hence are only 2 MST possible here.

**7. (b)**

w = 8(capacity)

Feasible solution

(i) $\{I_1, I_3, I_4\}$

Profit = 13 + 7 +3 = 23

(ii) $\{I_2, I_3\}$

Profit = 8 + 7 = 15

Optimal solution = $\{I_1, I_3, I_4\}$

With the capacity of 8 and maximum profit produced is 23.

$I_2$ is not selected in the solution.

∴ (b) is correct option.

**8. (d)**

Bellman ford algorithm may not return a shortest path from p to q

∴ $S_1$ is false

If graph has negative weight cycle, then Bellman ford given error, so 2nd statement is also false.

# Branch : CSE & IT  Batch : Hinglish

# Algorithms

## Greedy Method

DPP

**[MCQ]**

**1.** Consider the following statements.

$S_1$: Given a weighted declared graph with the distinct weights, the shortest path among any two vertices will be unique.

$S_2$: A minimum spanning tree can contain negative edges.

Choose the correct statements.

(a) Only $S_1$ is true
(b) Only $S_2$ is true
(c) Both $S_1$ and $S_2$ are true
(d) neither $S_1$ nor $S_2$ is true

**[MCQ]**

**2.** Which of the statement is/are correct?

(a) First edge added by Kruskal's algorithm can be the last edge added by prim's algorithm
(b) In a graph, if one raises the length of all edge to the power of 3, the minimum spanning tree will stay the same.
(c) The heaviest edge in a graph cannot belong to the minimum spanning tree.
(d) The maximum spanning tree (spanning tree of maximum cost) can be computed by negating the cost of all the edges in the graph and then computing minimum spanning tree.

**[NAT]**

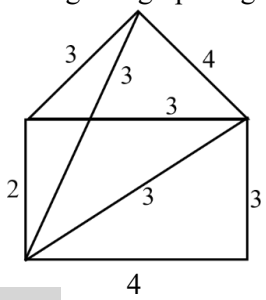
**3.** Consider the following instantons of the job for-scheduling problem with deadlines (Note: every Job takes one unit time)

| Job | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ | $J_7$ |
|---|---|---|---|---|---|---|---|
| Deadline | 1 | 3 | 4 | 3 | 2 | 1 | 2 |
| Profit | 3 | 5 | 20 | 18 | 1 | 6 | 30 |

What is the maximum profit generated by greedy algorithm _____?

**[NAT]**

**4.** Consider is the weighted graph G given by




How many MST does G Have?

**[MCQ]**

**5.** Let's suppose, we want to merge some sorted files where the number of records in each file is given below. (15, 18, 20, 21, 24, 28, 30, 32, 35, 40, 45, 50) then what is the minimum number of comparisons required to merge the following files?

(a) 1200  (b) 1225
(c) 1251  (d) 1255

**[MCQ]**

**6.** Greedy algorithm fails to give an optimal solution to which of the following problems?

(p) Travelling salesman problem
(q) Job scheduling with deadlines and penalty
(r) Shortest path algorithm
(s) optimal merge pattern
(t) Huffman encoding

(a) p, q, r  (b) r, s, t
(c) p, q, r, s, t  (d) All of the above

# Answer Key

1. **(b)**
2. **(a, b, d)**
3. **(74 to 74)**
4. **(4 to 4)**
5. **(c)**
6. **(a)**

# Hints & Solutions

**1. (b)**

(i) multiple spanning trees may exist, we have kirchoff method to count them.

(ii) Spanning tree can't have negative edge, it can have any real value.

**2. (a, b, d)**

True(a): The graph d(A, B) = 1, d(B, C) = 2 and we start running prim's at 'C'

True (b): The MST algorithm care about relative edge lengths and raising all edge lengths, the 3rd power pressure this relationship.

False(c): This edge may be connecting two otherwise dis-connected sub-graphs.

Ture(d): This work, and none of the proofs on MST algorithm depends on edge weight being non-negative.

**3. (74 to 74)**

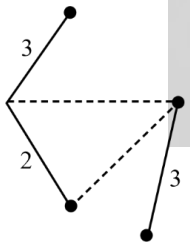
Put earliest deadline schedule with maximum profit first.

| J6 | J7 | J4 | J3 |
|---|---|---|---|
| 1 | 2 | 3 | 4 |

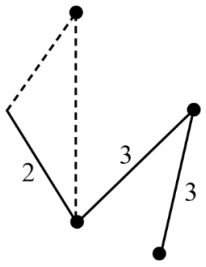
total profit = 6 + 30 + 18 + 20 = 74

∴ 74

**4. (4 to 4)**



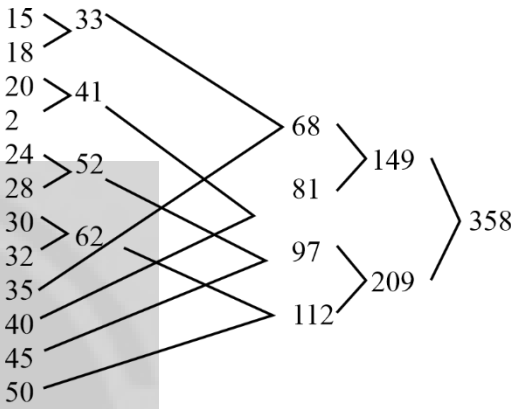

Choose any 1 dotted line here




Choose any one dotted line here

∴ Totals 4 different MST's

**5. (c)**




Total comparisons = m = n – 1

= 32 + 40 + 51 + 61 + 67 + 80 + 96 + 111 + 148 + 208 + 357

**6. (a)**

Greedy algorithm fails to give an optimal solution for

(p) travelling salesman problem

(q) Job scheduling with deadline and penalty (Greedy fails when there is infinite number of jobs and jobs are arriving continuously)

(r) Shortest path algorithm in this if we follow the greedy approach, the algorithm fails when there are negative edge weight in the graph.

❑❑❑

# Branch : CSE & IT Batch : Hinglish

# Algorithms
# 'Heap Algorithm' & 'Backtracking and Branch-Bound'

DPP

**[MCQ]**

**1.** Consider the statements.

S1: Merge-sort, quick-sort and bubble sort are comparison-based sorting algorithms

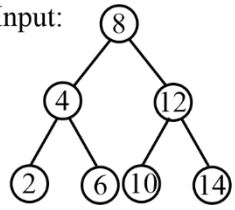
S2: A reverse-sorted array (ie. ……decreasing order) is always max-heap

(a) Only S1 is true
(b) Only S2 is true
(c) Both S1 and S2 are true
(d) Neither S1 nor S2 is true.

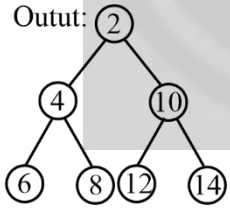
**[MCQ]**

**2.** Consider a binary search tree which is also a complete binary tree. The problem is to convert the BST which is given into a minheap with the condition that, all the values in the left subtree of a node should be less than all the values in the right subtree of the node. This condition is applied on all the nodes in the process of converting BST into minheap.

Input:




Outut:




What will be the worst-case time complexity (tightest) of given problem, if we can take auxiliary space of O(n)?

(a) O(n) (b) $O(n^2)$
(c) O(logn) (d) O(nlogn)

**[MCQ]**

**3.** How many different min-heap are possible with keys 1 2 3 4 5?

(a) 5 (b) 6
(c) 7 (d) 8

**[MCQ]**

**4.** What is the maximum number of exchanges required to order an array of 5 elements using the selection sort?

____

(a) 1 (b) 2
(c) 3 (d) 4

**[NAT]**

**5.** Number of undirected graph (not necessarily connected) can be constructed by given set V = [1, 2, 3, 4] of 4 vertices are ____.

**[NAT]**

**6.** The number of spanning trees of an undirected completed graph with 7 nodes is _____

**[MCQ]**

**7.** consider the following statements

**S1:** Backtracking is an algorithm technique for solving problems reclusively by trying to build a solution incrementally.

**S2:** Time complexity of N-Queens algorithm is O(n!)

Which statement is true?

(a) only S1
(b) only S
(c) Both S1 and S2 are true
(d) Neither S1 nor S2 is true

# Answer Key

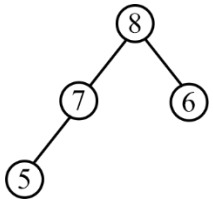
1. **(c)**
2. **(a)**
3. **(d)**
4. **(d)**
5. **(64 to 64)**
6. **(16807)**
7. **(c)**

# Hints & Solutions

**1. (c)**

S1(true): All are comparison-based sorting algorithms.

S2(true): 8, 7, 6, 5

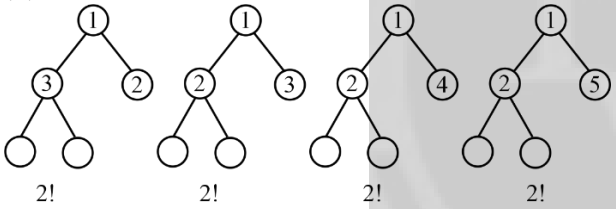



**2. (a)**

(i) Create an array x [] of size n. where n is the number of nodes in the given BST

2. Perform the inorder traversal of the BST and copy the node values in the x [] in sorted order.
3. Now perform the preorder traversed of tree.
4. While traversing the root during the preorder traversal, one by one copy the values from the x [] to the nodes.

**3. (d)**




**4. (d)**

In selection sort, with each pass, at most one swap is performed, so array with 5 elements will perform 4 passes and with each pass there will be 1 swap.

$\therefore$ d is correct.

**5. (64 to 64)**

As number of undirected graphs is

$2^{(n\,(n-1)/2)}$

**6. (16807)**

Number of spanning trees for an undirected complete graph with n nodes

$n = n^{n-2}$

**7. (c)**

True(S1): Backtracking is an algorithm technique for solving problems reclusively by trying to build a solution incrementally.

True (S2): Time complexity of N-Queens algorithm is O(n!)

❑❑❑