Branch : CSE/IT **Batch : Hinglish**

WEEKLY TEST – 06

Subject : Discrete Mathematics

Topic : Set Theory

Maximum Marks 17

Q.1 to 5 Carry ONE Mark Each

[MCQ]

1. If A is a proper sub set of B, then which of the following statements is not true

(a) $A \cap B = A$ (b) $B^C \subset A^C$

(c) $B \cup A^C = U$ (d) $B - A = \varnothing$

[MCQ]

2. The poset [{2, 3, 5, 30, 60, 120, 180, 360}] is

(a) a lattice

(b) a join semi lattice

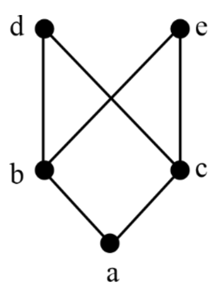
(c) a meet semi lattice

(d) neither a join semi lattice nor a meet semi lattice

[MCQ]

3. Consider the poset:

P = {a, b, c, d, e} shown below



Which of the following statements is false?

(a) P is not a lattice

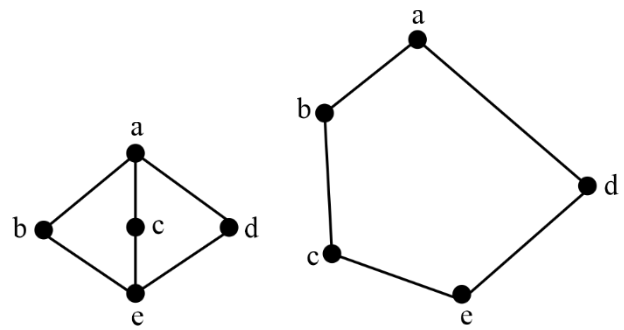
(b) The sub set {a, b, c, d} of P is a lattice

(c) The sub set {b, c, d, e} of P is a lattice

(d) The sub set {a, b, c, e} of P is a lattice

[MCQ]

4. Consider the following Lattices L_1* and L_2*



Which of the following is true?

(a) L_1* is distributive and L_2* is distributive

(b) L_1* is not distributive and L_2* is distributive

(c) Both Lattices are distributive

(d) Both Lattices are non-distributive

[MCQ]

5. The set of all strings under the operation concatenation of strings is

(a) a monoid but not a group

(b) an abelian group

(c) a group but not a abelian group

(d) not a semi group

Q.6 to 11 Carry TWO Mark Each

[MCQ]

6. If (G, *) is a group then which of the following is false

(a) $\{(a * b) = (a * c)\} \Rightarrow (b = c)$

(b) $\{(a * c) = (b * c)\} \Rightarrow (a = b)$

(c) $a * b = b * a$

(d) $(a * b)^{-1} = (b^{-1} * a^{-1})$

[MCQ]

7. Which of the following is false?

(a) A cyclic group with only one generator can have atmost 2 elements

(b) The order of a cyclic group is equal to the order of its generator

(c) The group $(\{1, 2, 3, 4\}, \otimes_5)$ is cyclic

(d) A group of order 4 is cyclic

[MCQ]

8. Let G be a reduced residue system modulo is say G = {1, 2, 4, 7, 8, 11, 13, 14} (i.e. the set of integers between 1 and 15 which are coprime to 15). Then G is a group under multiplication modulo 15. Which of the following is false

(a) inverse of 2 = 8

(b) inverse of 7 = 13

(c) inverse of 11 = 11

(d) inverse of 4 = 9

[MCQ]

9. If $X = \{x \mid x \text{ is a multiple of } 4\}$ and $Y = \{y \mid y \text{ is a multiple of } 6\}$. If $Z = X \cup Y$ and $z \in Z$ then z is a multiple of ______.

(a) 4 (b) 6

(c) 12 (d) 2

[MCQ]

10. Let 'A' is set of all non zero real numbers. For a, b $\in$ A, a relation R on A is defined as "$a \,^{R}\, b$ iff $\frac{a}{b} \in Q$" where Q is set of all rational numbers. Then 'R' is ______.

(a) An equivalence relation

(b) A partial ordering relation

(c) Symmetric but not transitive

(d) Transitive but not symmetric

[MCQ]

11. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (3, 4), (2, 1)} be a relation on A. The transitive closure of R is _____.

(a) $A \times A$

(b) {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 4)}

(c) $R \cup R^{-1}$

(d) $R \cup \Delta_A$

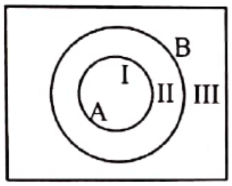
Answer Key

1. **(d)**
2. **(c)**
3. **(c)**
4. **(d)**
5. **(a)**
6. **(c)**
7. **(d)**
8. **(d)**
9. **(d)**
10. **(a)**
11. **(b)**

Hints and Solutions

1. (d)

The venn diagram to represent $A \subset B$ is



(a) $A \quad B = \{I\} \cap \{I, II\} = \{I\} = A$

(b) $B^c = \{III\}$ and $A^c = \{II, III\}$

$\therefore B^c \subset A^c$

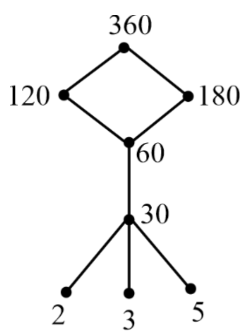
(c) $B \cup A^c = \{I, II\} \cup \{II, II\} = \{I, II, III\} = U$

(d) $B - A = \{II\} = \phi$

The statement (d) is false.

2. (c)

The hasse diagram of the poset is shown below



We have 3 minimal elements 2, 3 and 5.

For any two minimal elements glb does not exist.

$\therefore$ The given poset is not a meet semi lattice.

However, for every pair of elements in the poset, lub exists.

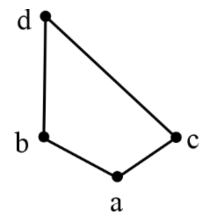
Hence, the poset is a join semi lattice.

3. (c)

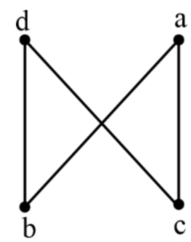
(a) The least upper bound of c and d does not exist.

$\therefore$ P is not a lattice.

(b) The subset {a, b, c, d} of P is a lattice whose Hasses diagram is shown below.



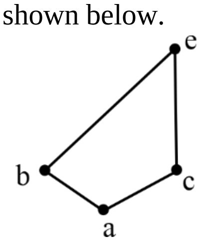
(c) The Hasses diagram of the subset {b, c, d, e} is shown below



We have, two minimal elements and two maximal elements.

$\therefore$ The poset is not a lattice.

(d) The subset {a, b, c, e} of P is a lattice whose Hasse diagram is shown below.



4. (d)

In the lattice L_1^*. The element b has two complements c and d.

$\therefore L_1^*$ is not a distributive lattice

In the lattice L_2^*, the element d has two complements b and c.

$\therefore L_2^*$ is not a distributive lattice

5. (a)

Let S = set of all bit strings and + denote string concatenation.

(i) concatenation of any two bit strings is also a bit string.

$\therefore$ + is a closed a closed operation on S.

(ii) string concatenatenation is associative

i.e., $(S_1 + S_2) + S_3 = S_1 + (S_2 + S_3)$

$\therefore$ + is associative on S.

(iii) The identity element of S with respect to + is the null string $\in$

(iv) The inverse of a non empty string does not exist with respect to +.

$\therefore$ (S, +) is a monoid but not a group

6. (c)

(a) Let $a * b = a * c$

$\Rightarrow a^{-1} * (a * b) = a^{-1} * (a * c)$

$\Rightarrow (a^{-1} *) * b = (a^{-1} * a) * c$

$\Rightarrow e * b = e * c$

$\Rightarrow b = c$

(b) Let $(a * c) = (b * c)$

$\Rightarrow (a * c) * c^{-1} = (b * c) * c^{-1}$

$\Rightarrow a*(c*c^{-1}) = b * (c * c^{-1})$

$\Rightarrow a * e = b * e$

$\Rightarrow a = b$

(c) (G, *) need not be an abelian group.

$\therefore$ option (c) is false.

(d) Consider $(a * b)*(b^{-1} * a^{-1})$

$= a*(b * b^{-1}) * a^{-1}$ (by associativity)

$= a*e*a^{-1} = a * a^{-1}$

$= e$

Similarly, $(b^{-1} * a^{-1})*(a*b) = e$

$\therefore (a*b)^{-1} = b^{-1} * a^{-1}$

7. (d)

G = {1, 3, 5, 7} is a group with respect to $\otimes_8$.

G is not cyclic, because the generating element does not exist.

8. (d)

(a) $2 \otimes_{15} 8 = 1$(identity element)
$\therefore$ Inverse of 2 is 8

(b) $7 \otimes_{15} 13 = 1$(identity element)
$\therefore$ Inverse of 7 is 13

(c) $11 \otimes_{15} 11 = 1$(identity element)
$\therefore$ Inverse of 11 is 11

(d) $4 \otimes_{15} 9 = 6$
$\therefore$ Inverse of 4 is 9.

Inverse of 4 = 4 ($\therefore 4 \otimes_{15} 4 = 1$)

9. (d)

Z = {4, 8, 12, 16,….6, 12, 18, 24, ….}

If $z \in Z$ then z is a multiple of 2, because z is a common divisor of 4 and 6.

10. (a)

A = R – {0}

$(a^R b) \Leftrightarrow \frac{a}{b}$ is a relational number

(a) $\frac{a}{a} = 1 \; (a \neq 0)$

$\Rightarrow a^R a \quad \forall\, a \in$

$\therefore$ r is reflexive

If $\frac{a}{b}$ = Rational number, then

$\frac{b}{a}$ = Rational number

$\therefore$ R is symmetric

Let $(a^R b)$ and $(b^R c)$

$\Rightarrow \left(\frac{a}{b}\right)$ and $\left(\frac{b}{c}\right)$ are rational number

$\frac{a}{c} = \left(\frac{a}{b}\right)\left(\frac{b}{c}\right) \in Q$

Q means set of all rational numbers

$\therefore$ R is transitive

(b) R is not anti-symmetric.

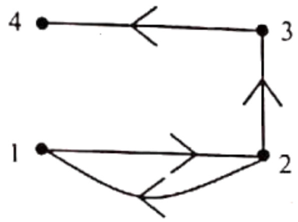
Ex: 2 R 3 and 3 R 2

$\therefore$ R is not a partial order

Options (c) and (b) are false.

11. (b)

The relation can be denoted buy the following diagram



From vertex 1, there is a path to all other vertices.

From vertex 2, there is a path to all other vertices.

From vertex 3, we can reach only vertex 4.

From vertex 4, there is not path to other vertices.

Transitive closing of R

= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 4)}