Branch : CSE & IT **Batch:Hinglish**

WEEKLY TEST – 03

Subject : Discrete Mathematics

Topic : Domination Number, Matching & Planarity

Maximum Marks 15

Q.1 to 5 Carry ONE Mark Each

[MCQ]

1. Which of the following is not True?

(a) Number of perfect matching in $k_{2n} = \frac{(2n)!}{2^n n!}$

(b) Number of perfect matching in $K_{n,n} = n!$

(c) Number of perfect matching in C_n (n is even) = 2

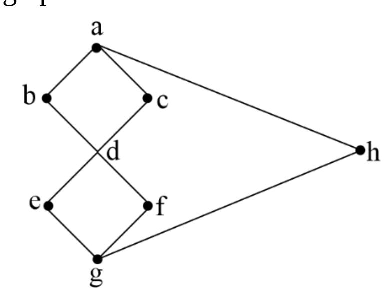
(d) Number of perfect matching in $w_{2n} = 2n$

[NAT]

2. If G is a bipartite graph with 6 vertices and maximum number of edges then find the total members of perfect matching ___?

[MSQ]

3. For the graph shown below:



Which of the following is / are True?

(a) Dominating set = {d, a, g}

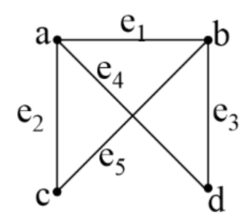
(b) Domination number is 4

(c) Dominating set = {d, h}

(d) Domination number is 2.

[NAT]

4. Number of maximal matchings in the graph shown below is ____?



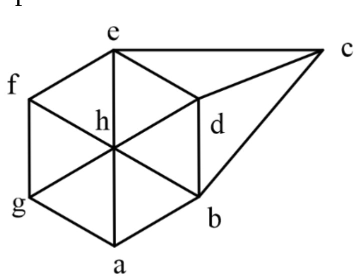
[NAT]

5. Find the total number of distinct Hamiltonian cycles for the complete graph K_5?

Q.6 to 10 Carry TWO Mark Each

[MCQ]

6. For the graph is shown below



Chromatic number of G + matching number of G = ____.

(a) 6 (b) 7

(c) 5 (d) 8

[MCQ]

7. Consider a 5-regular graph with number of vertices 10. How many closed faces are in planar embedding for connected planar?

(a) 15 (b) 16

(c) 17 (d) 18

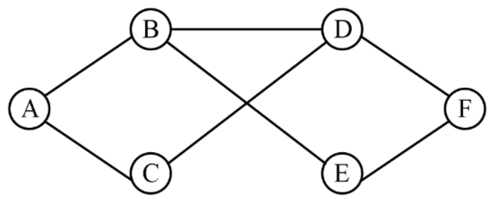
[NAT]

8. If G is a disconnected graph with 11 vertices and maximum number of edges, then matching number of G + chromatic number of G = ________ .

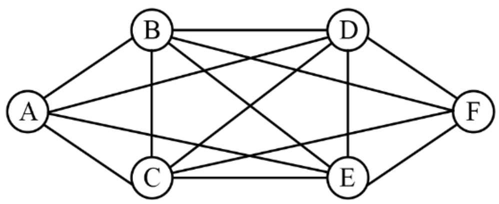
[MCQ]

9. Consider the following graphs G_1 and G_2:

G_1:



G_2:



Which of the following options are true regarding the above graphs:

(a) G_1 is planar

(b) G_2 is bipartite

(c) G_1 is planar and bipartite

(d) None of these

[MCQ]

10. Let G be a simple undirected planar graph on 20 vertices with 25 edges. If G is a connected graph, then the number of bounded forces is any embedding of G on the plane is equal to____?

(a) 3 (b) 7

(c) 5 (d) 6

Answer Key

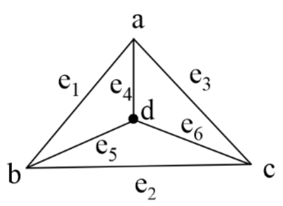
1. **(d)**
2. **(6)**
3. **(a, c, d)**
4. **(3)**
5. **(12)**
6. **(b)**
7. **(b)**
8. **(15)**
9. **(c)**
10. **(d)**

Hints and Solutions

1. (d)

The total number of perfect matchings in $w_{2n} = (2n - 1)$.

Example: $w_4 \equiv w_{2*2}$



$PM(M_1) = \{e_1, e_6\}$
$PM(M_2) = \{e_2, e_4\}$
$PM(M_3) = \{e_3, e_5\}$

2. (6)

A bipartite graph G is given with maximum number of edges and 6 vertices.

So, maximum number of edges possible when both partition have equal number of vertices

$\therefore$ $K_{3,3}$(Number of edges) = 3 × 3 = 9 edges

Now,

The number of perfect matchings for

$k_{3,3}$ = 3!

= 3 × 2

= 6

3. (a, c, d)

The set of vertices from which the whole graph can be covered in the single move.

So, the dominating set {d, a, g} and {d, h} are correct dominating set as it covers all the vertices of graph.

domination number of the given graph is 2, as the smallest dominating set is {d, h}.

4. (3)

Matching is a set of edges in which none of them are adjacent to each other.

$\therefore$ Maximal matching $M_1 = \{e_4, e_5\}$

$M_2 = \{e_3, e_2\}$

Hence, we have 3 maximal matching set for the given graph.

$M_3 = \{e_1\}$.

5. (12)

As we know that a graph G is Hamiltonian if and only if it contains a Hamiltonian Cycle (Closed Path) that cover every vertex exactly once.

$\therefore$ The Hamiltonian Cycle for complete graph

$$K_n = \frac{(n-1)!}{2}$$

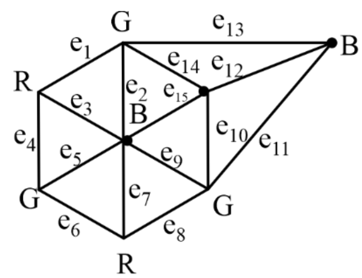
So, $$K_5 = \frac{(5-1)!}{2}$$

$$= \frac{4 \times 3 \times 2}{2}$$

$$= 12$$

6. (b)

I. The given graph has complete graph of 3 vertices (K_3) so, the chromatic number will be at least 3.



Hence, the chromatic number is 3.

II. Matching number is the cardinality of the maximum matching set.

$\therefore$ maximal matching set = $\{e_1, e_5, e_8, e_{12}\}$

Hence, the matching number of the graph is 4.

So, find value: 3 + 4 = 7

7. (b)

I: Find the total number of edges for the given 5 - regular graph with 10 vertices.

$\therefore$ Number of edges $= \dfrac{nk}{2} = \dfrac{5*10}{2}$

$= 25$ edges

II. Now, we know that for a connected planar graph, number of faces are:

$r = e - n + 2$

$\therefore\ r = 25 - 10 + 2 = 17$

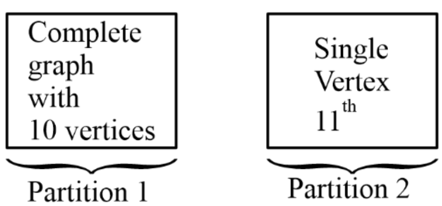
So, the total number of closed faces are = 17- 1

= 16 closed faces.

8. (15)

The given graph *G* is disconnected graph with 11 vertices and maximum edges.

So,



Now,

The chromatic number of $K_{10} = 10$

And

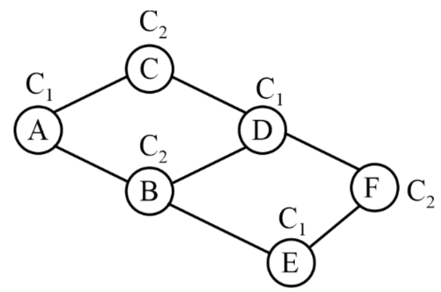
The matching number of complete graph

$$K_{10} = \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{10}{2} \right\rfloor \quad = 5$$

$\therefore$ Final value = 5 + 10 = 15

9. (c)

G_1: The given graph is planar graph as we can flip " A — C — D" part to obtain the planar graph.



The above graph is also bipartite since we can color all the vertices with two colors.

10. (d)

For any planar graph using Euler's formula:

$V - E + R = 2$

Here V = Vertices

E = Number of edges

R = Region

$\therefore\ 20 - 25 + R = 2$

$R = 2 + 5$

$R = 7$

Hence, we have total 7 regions in the given graph, out of this, one region is unbounded and the other 6 are bounded or closed region.