Branch : CSE & IT **Batch:Hinglish**

WEEKLY TEST – 04

Subject : Discrete Mathematics

Topic : Planarity, Propositional Logic, Logic Equivalence & Inference Rule

Maximum Marks 15

Q.1 to 5 Carry ONE Mark Each

[MCQ]

1. What is the inverse of the conditional statement "The home team wins whenever it is raining"

(a) If it is raining, then the home does not win.
(b) If it is not raining, then the home team does not win.
(c) If the home team does not win, then it is not raining.
(d) None of these

[MCQ]

2. Consider the following given expression

$$[(p \vee q) \Rightarrow r \vee (s \Rightarrow (s \vee t))] \Rightarrow (p \vee \sim p)$$

Which of the following is correct?

(a) Tautology
(b) Contradiction
(c) Contingency
(d) None

[MCQ]

3. If $p \rightarrow q$ is false then what is truth value of $\sim p \Rightarrow q \vee r$?

(a) Tautology
(b) Contradiction
(c) Contingency
(d) None

[MCQ]

4. Which of the following is logically equivalent to given expression "$(p \rightarrow r) \wedge (q \rightarrow r)$"?

(a) $(p \vee q) \rightarrow r$
(b) $(p \wedge q) \rightarrow r$
(c) $(\sim p \wedge \sim q) \rightarrow r$
(d) None of these

[NAT]

5. Consider a connected simple planar graph with order 12. Find the number of edges such that the minimum degree of graph must be 4?

Q.6 to 10 Carry TWO Mark Each

[MSQ]

6. Consider the following logical expression

$(\sim q \wedge (p \Rightarrow q)) \rightarrow \sim p$

Which of the following is True?

(a) Tautology
(b) Contradiction
(c) Satisfiable
(d) Unsatisfiable

[NAT]

7. What is the probability for a propositional function to be contingency for n variable where $n = 4$? (round off upto 2 decimal)

[MCQ]

8. The formula or logical expression is equivalent to

$[(\sim p \wedge q) \vee (p \wedge \sim q) \vee (p \wedge q)]$

(a) $p \rightarrow q$
(b) $p \wedge q$
(c) $p \leftrightarrow q$
(d) $p \vee q$

[MCQ]

9. Consider the following statements:

S_1: $(a \leftrightarrow b) \rightarrow (a \wedge b)$

S_2: $(a \leftrightarrow b) \leftrightarrow ((a \wedge b) \vee (\sim a \wedge \sim b))$

Which of the following is true?

(a) S_1 is valid
(b) S_2 is valid
(c) Both S_1 and S_2 are valid
(d) Both S_1 and S_2 are not valid

[MCQ]

10. Consider the following logical statement

"I will stay only if you go"

Which of the following is converse of the above statement? **[Gate 1998]**

(a) I do not stay follows from you do not go.
(b) I stay is necessary for you to do not go.
(c) I stay is sufficient for you to go.
(d) I stay is follows from you go.

Answer Key

1. **(b)**
2. **(a)**
3. **(a)**
4. **(a)**
5. **(24)**
6. **(a, c)**
7. **(0.99 to 1)**
8. **(d)**
9. **(b)**
10. **(d)**

Hints and Solutions

1. (b)

I. As we know that one of the way to express the conditional statement $p \to q$ is "q whenever p".

II. So, the original statement can be re-written as:

"If it is raining, then the home team wins".

Thus, the inverse of this conditional statement will be:

"It is not raining, then the home team does not win".

Hence, option b is correct.

2. (a)

The given expression is:

$$[p \vee q \Rightarrow r \vee s \Rightarrow s \vee t] \Rightarrow \underbrace{(p \vee \sim p)}_{\text{It will always generate 1}}$$

$$\therefore \overline{[p \vee q \Rightarrow r \vee s \Rightarrow s \vee t]} + (p + \sim p)$$

$$= \overline{[p \vee q \Rightarrow r \vee s \Rightarrow s \vee t]} + 1 \equiv 1$$

Hence, it is tautology.

3. (a)

As we know that the conditional statement "$p \to q$" is false when "$p = 1, q = 0$"

So, if we substitute the value of p and q then:

$$\sim p \Rightarrow q \vee r$$
$$= \sim(\sim p) + q + r$$
$$= p + q + r$$
$$= 1 + 0 + r \equiv 1$$

Hence, the truth value for the given expression is 1 and it is tautology.

4. (a)

Two statements forms are logical equivalent if and only if their resulting truth values are identical for each variation of statement variables..

So, $(p \to r) \wedge (q \to r)$

$$= (\bar{p} + r) \wedge (\bar{q} + r)$$
$$= \bar{p}\,\bar{q} + \bar{p}r + \bar{q}r + r$$
$$= \bar{p}\,\bar{q} + \bar{p}r + r$$
$$= \bar{p}\,\bar{q} + r$$
$$= \overline{(p \vee q)} + r \equiv (p \vee q) \to r$$

Hence, option a is logically equivalence to given statement.

5. (24)

I. As we know that the connected simple planar graph with 12 vertices can have at most "$3n - 6$" edges.

$\therefore$ No. of edges $\leq 3n - 6$

$$\leq 3 * 12 - 6$$
$$\leq 36 - 6$$
$$\leq 30$$

So, the number of edges must be less than or equal to 30.

II. Now, the relation between the minimum degree and number of edges is:

$$\delta(G) = \frac{2 * |E|}{n}$$

$$\delta(G) = \frac{2 * |E|}{12}$$

$$4 = \frac{2 * |E|}{12}$$

$$\therefore \quad |E| = \frac{12 * 4}{2} = 24 \text{ edges}$$

So, to get the minimum degree 4, the number of edges will be 24.

6. (a, c)

The given logical expression is:

$$(\sim q \wedge (p \to q) \to \sim p)$$
$$= (\bar{q} \cdot (\bar{p} + q)) \to \bar{p}$$
$$= (\bar{q}\,\bar{p} + \bar{q}q) \to \bar{p}$$
$$= (\bar{q}\,\bar{p} + 0) \to \bar{p}$$
$$= (\bar{q}\,\bar{p}) \to \bar{p}$$
$$= \overline{\bar{q}\,\bar{p}} + \bar{p}$$
$$= q + p + \bar{p}$$
$$= q + 1 \equiv 1$$

Hence, the given expression is tautolgy and every tautology is satisfiable.
So, option a and c are correct.

7. **(0.99 to 1)**
We know that the total number of contingency possible for n variable function is $2^{2^n} - 2$
So, the number of contigency for $n = 4$:

$$2^{2^n} - 2 = 2^{2^4} - 2 = 65536 - 2 = 65534$$

Now, probability $= \dfrac{\text{Total number of contigency}}{\text{Total functions}}$

$$= \frac{2^{2^n} - 2}{2^{2^n}} = \frac{65534}{65536}$$

$$= 0.99$$

8. **(d)**

$$[\overline{p}q + p\overline{q} + pq]$$
$$= [\overline{p}q + p(\overline{q} + q)]$$
$$= [\overline{p}q + p]$$
$$= [p \vee q]$$

Hence, option D is correct.

9. **(b)**
Statement S_1: Not valid

$$(a \leftrightarrow b) \to (a \wedge b)$$
$$= (\overline{a}\,\overline{b} + ab) \to ab$$
$$= \overline{(\overline{a}\,\overline{b} + ab)} + ab$$
$$= [(a + b)(\overline{a} + \overline{b})] + ab$$
$$= b\overline{a} + a\overline{b} + ab$$
$$= b\overline{a} + a(\overline{b} + b)$$
$$= b\overline{a} + a$$
$$= b + a \neq 1$$

Hence, it is not valid.
Statement S_2: valid

$$(a \leftrightarrow b) \leftrightarrow \big((a \wedge b) \vee (\sim a \wedge \sim b)\big)$$
$$(\overline{a}\,\overline{b} + ab) \leftrightarrow (ab + \overline{a}\,\overline{b})$$
$$T \Leftrightarrow T \equiv 1$$

Hence, S_2 is valid.

10. **(d)**
As we know that some of the way to express the conditional statement $p \to q$ are:
(i) q follows from p
(ii) q is necessary for p
(iii) p is sufficient for q
Hence, the logical equivalence for each option is as follows:
Option a: $\sim q \to \sim p$
Option b: $\sim q \to p$
Option c: $p \to q$
Option d: $q \to p$
So, option d is the converse of the given logical statement.