Branch : CSE & IT **Batch : Hinglish**

WEEKLY TEST – 01

Subject : Discrete Mathematics

Topic : Basics of Graphs, Degree Sequence in Graphs & Types of Graphs

Maximum Marks 15

Q.1 to 5 Carry ONE Mark Each

1. Let G be a simple graph with 11 vertices. If degree of each vertex is at least 5 and at most 7, then the number of edges in G should lie between ____and ___.

(a) 29 and 39 (b) 28 and 39
(c) 28 and 38 (d) 28 and 35

2. If G be simple graph with 40 edges and degree of each vertex is 4, then number of vertices in G is ____.

3. Suppose a graph G has the degree sequence 1, 1, 2, 2, 3, 3, 3, 3, 4, 5, 5. Then find the degree sequence of the complement of G?

(a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
(b) 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 9
(c) 5, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9
(d) None of these

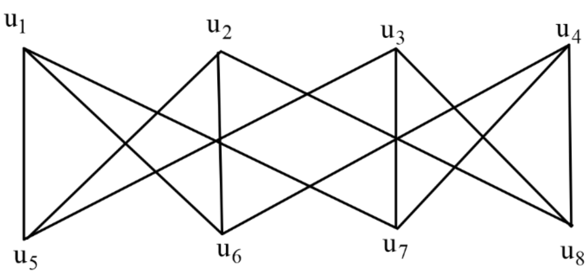
4. Suppose all vertices in a graph G have degree K, where K is an odd number, then number of vertices in G is _____.

(a) odd
(b) even
(c) a multiple of K
(d) None of these

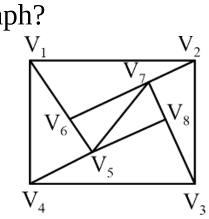
5. If G is a simple graph with degree sequence {5, 4, 4, 4, 4, 1} then number of edges in the complement $\overline{G}$ = ____.

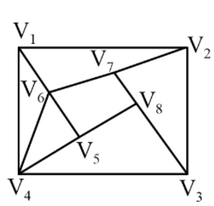
Q.6 to 10 Carry TWO Mark Each

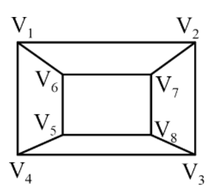
6. Consider the graph given bellows:



Which one of the following is isomorphic to the above graph?

(a)


(b)


(c)


(d) None of these

7. If W_n is a wheel graph with 'n' vertices then, the number of edge in the complement of W_n is:

(a) $\frac{n(n-3)}{2}$ (b) $\frac{n(n-2)}{2}$
(c) $\frac{(n-1)(n-3)}{2}$ (d) $\frac{(n-1)(n-4)}{2}$

8. If G is a simple graph with 6 vertices of degree 2, 3 vertices of degree 3 and 3 vertices of degree 5, then the number of edges possible in G is ____.

9. Consider a star graph with 12 edges, then the minimum and maximum degree of the complemented graph is ____.

(a) minimum = 0 and maximum = 11

(b) minimum = 0 and maximum = 10

(c) minimum = 1 and maximum = 9

(d) minimum = 1 and maximum = 8

10. Consider a graph with order 7. The degree of each vertex is defined as $\deg(V_i) = \lceil$mean of the factors of 'i'$\rceil$. Assume X is the number of edges and Y is the degree sequence of the complement of the given graph. Find X and Y?

(a) X = 10 and Y = 5, 3, 3, 3, 2, 2, 2

(b) X = 12 and Y = 5, 4, 4, 3, 3, 3, 2

(c) X = 14and Y = 5, 5, 4, 4, 4, 4, 2

(d) X = 16 and Y = 6, 5, 5, 5, 5, 3, 3

Answer Key

1. (c)
2. (20)
3. (c)
4. (b)
5. (4)
6. (c)
7. (d)
8. (18)
9. (a)
10. (b)

Hints and solutions

1. (c)

The simple graph have 11 vertices, minimum degree (δ) = 5 and maximum degree (Δ) is 7.

The relation between vertices, δ, and Δ is:

$n.\delta(G) \le 2 * |E| \le n.\Delta(G)$

$\therefore\ 11 * 5 \le 2 * |E| \le 11 * 7$

$\therefore\ 55 \le 2 * |E| \le 77$

$$\left\lceil \frac{55}{2} \right\rceil \le |E| \le \left\lfloor \frac{77}{2} \right\rfloor$$

Hence the number of edges should lie between 28 and 38.

2. (20)

Using handshaking lemma:

Sum of degrees = $2 * |E|$

Now, assume we have x number of vertices in the graph with degree of each vertex is 4.

So, $x * 4 = 2 * |E|$

$4x = 2 * 40$

$$\therefore\ x = \frac{2 * 40}{4} = 20$$

Hence, the total number of vertices is 20.

3. (c)

The given graph have 11 vertices and in K_n complete graph, the degree of each vertex is $(n - 1)$ So, for K_{11} complete graph the degree of each vertex would be 10.

$\therefore$ The degree sequence of complement of G.

(10 – 1, 10 – 1, 10 – 2, 10 – 2, 10 – 3, 10 – 3, 10 – 3, 10 – 3, 10 – 4, 10 – 5, 10 – 5,)

= 9, 9, 8, 8, 7, 7, 7, 7, 6, 5, 5.

Thus, opton c is correct answer.

4. (b)

As we know that the handshaking lemma state that:

Sum of degree = $2 * |E|$

$\therefore\ \sum \text{even deg}(V) + \sum \text{odd deg}(V) = \text{Even}$

$\therefore\ \sum \text{odd deg}(V) = \text{Even} - \text{even} = \text{Even}$

Hence, the number of odd degree vertices must be even to get the sum of degree as even.

5. (4)

The degree sequence of the complement graph $\overline{G}$:

$$\begin{aligned} K_n &= 5,\ 5,\ 5,\ 5,\ 5,\ 5 \\ G &= 5,\ 4,\ 4,\ 4,\ 4,\ 1 \\ \hline \overline{G} &= 0,\ 1,\ 1,\ 1,\ 1,\ 4 \end{aligned}$$

Now, by using handshaking lemma:

Sum of degree = $2 * |E|$

$0 + 1 + 1 + 1 + 1 + 4 = 2 * |E|$

$8 = 2 * |E|$

$\therefore\ |E| = 4$

Hence, the number of edges will be 4.

6. (c)

A graph can exist in different forms having the same number of vertices, edges and also the same degree sequence.

The given graph is bipartite graph and the option c is also bipartite graph with same number of vertices, edges and degree sequence.

Hence, option c is the isomorphic graph of the given graph G.

7. (d)

The number of edges in the complement of W_n :

Number of edges in K_n – Number of edges in W_n.

$$\therefore\ \frac{n(n-1)}{2} - 2(n-1)$$

$$\Rightarrow\ \frac{n^2 - 5n + 4}{2}$$

$$\Rightarrow\ \frac{(n-1)(n-4)}{2}$$

Hence, option d is the correct answer.

8. (18)

By handshaking lemma:

Sum of degree = 2 ∗ |E|

$\therefore$ (6 ∗ 2) + (3 ∗ 3) + (3 ∗ 5) = 2 ∗ |E|

12 + 9 + 15 = 2 ∗ |E|

$\therefore \quad |E| = \frac{36}{2} = 18$

Hence, the number of edges is 18.

9. (a)

I. A star graph is given with 12 edges so, find the number of vertices using handshaking lemma:

Sum of degree = 2 * |E|

(n + 1) + (n –1) = 2 * |E|

$\therefore$ 2(n – 1) = 2 * 12

$\therefore$ n – 1 = 12 ⇒ n = 13

Hence, the number of vertices in star graph is 13.

II. Now, the number of edges in the completed graph will be:

$\frac{n(n-1)}{2} - 12$

$\frac{13 * 12}{2} - 12$

⇒ 78 – 12 = 66.

Hence, the number of edges in complement graph $(\overline{G})$ is 66.

Now, we know that

$n.\delta(\overline{G}) \le 2 * |E| \le n.\Delta(\overline{G})$

$13.\delta(\overline{G}) \le 2 * 66 \le 13.\Delta(\overline{G})$

$13.\delta(\overline{G}) \le 132 \le 13.\Delta(\overline{G})$

Hence, the minimum degree can be ‘0’ and the maximum degree will be at most 11.

10. (b)

I. The number of vertices of the graph G is 7 that is V_1, V_2, V_3, V_4, V_5, V_6, V_7.

Now, find the factors and mean for each vertex:

V_1 = Factor (1) = 1

$\therefore \quad \text{mean} = \left\lceil \frac{1}{1} \right\rceil = 1$

so, degree of V_1 = 1

V_2 = Factor (2) = 1, 2

$\therefore \quad \text{mean} = \left\lceil \frac{1+2}{2} \right\rceil = \left\lceil \frac{3}{2} \right\rceil = 2$

so, degree of V_2 = 2

V_3 = Factor (3) = 1, 3

$\therefore \quad \text{mean} = \left\lceil \frac{1+3}{2} \right\rceil = \left\lceil \frac{4}{2} \right\rceil = 2$

so, degree of V_3 = 2

Hence, degree of V_4 = 3

degree of V_5 = 3

degree of V_6 = 3

degree of V_7 = 4

II. The degree sequence for the given graph G is 4, 3, 3, 3, 2, 2, 1.

Now, the degree sequence of the complemented graph $\overline{G}$ will be as follows:

$K_7 = 6,\ 6,\ 6,\ 6,\ 6,\ 6, 6$

$\therefore \quad G \ = 4,\ 3,\ 3,\ 3,\ 2,\ 2, 1$

$\overline{G} \ = 2,\ 3,\ 3,\ 3,\ 4,\ 4,\ 5$

Hence, Y = 5, 4, 4, 3, 3, 3 2

III. To find the number of edges apply handshaking lemma:

Sum of degree = 2 ∗ |E|

$\therefore$ 5 + 4 + 4 + 3 + 3 +3 + 2 = 2 * |E|

$\therefore \quad |E| = \frac{24}{2} = 12$

Hence, X = 12.