02-425/725 HW5

March 27, 2024

1 Markov Chain

Consider the 100x100 symmetric probability transition matrix A from `transition_matrix_a.txt`.

1. Argue what is the equilibrium distribution for this probability transition matrix.

2. Compute the first left eigenvector of matrix A (corresponding to the largest eigenvalue). Compare this eigenvector to the equilibrium distribution (you can use publicly available packages for computation of eigenvalues and eigenvectors).

3. Simulate a markov chain using the above matrix A by

   (a) Sampling the first state X_0 (e.g. based on uniform distribution)

   (b) Jumping from state X_i to state X_{i+1}, according to the transition probability defined by 1.1.

4. For $M = 10,000$, what is the portion of time the Markov chain spends in each of the 100 states? Compare this distribution to the expected equilibrium distribution using Euclidian distance between the two distributions as a metric. How fast the distribution approaches the expected distribution as M increases?

2 MCMC

1. Use the Metropolis-Hasting approach to MCMC, for introducing rejection probabilities in the probability transition matrix such that the equilibrium probability distribution is equal to $P_{desired}$ from `desired_equilibrium_distribution.txt`. In Metropolis-Hasting approach, the rejection probabilities are defined as:

$$R_{ij} = 1 - min(1, \frac{p_j.A_{ij}}{p_i.A_{ji}})$$

2. Compute the probability transition matrix B after applying the rejection probabilities.

3. Compute the first left eigenvector of matrix B (corresponding to the largest eigenvalue). Compare this eigenvector to the desired equilibrium distribution.

4. Simulate a Markov chain using matrix B and approximate equilibrium distribution after M=10,000 samples. Compare this equilibrium distribution to the desired equilibrium distribution (using Euclidean distance metric).