Backtesting

September 21, 2023 10:53 AM

Machine Learning Applied to Financial Data

PROJECT 1

Backtesting

BACKTESTING_PROCEDURE

1) Assume we have 4 years of weekly data (W)

$\Rightarrow (4y) \times (52W/y) = \boxed{208W} = N$

2) fix a run window size of $\boxed{N_w = 52W}$

3) Calculate the number of windows/runs, with a slide of $s=4$

$$N^{runs} = \left\lfloor \frac{N - N_w}{s} \right\rfloor$$

(**Note**: Should divide exactly when N and N_w are multiples of 4)

$\Rightarrow$ in our case: $\boxed{N^{runs} = \left\lfloor \frac{208-52}{4} \right\rfloor = 37}$

4) **for** RUN $\tau = 0, 1, \dots, (N^{runs}-1)$ **do**:

5) └ CLOSE any current positions + P/L

6) └ **for** SECTOR $G = 1, \dots, 5$ **do**:

7) └ SECTOR_PROCEDURE(G, τ)

8) Function above should return:

└ Top 3 best stocks to long for next month (forecast)

9) └ $\boxed{\text{ADD tickers to portfolio}}$

10) └ OPTIMIZE_PORTFOLIO(portfolio $= \{\hat{S}_1, \hat{S}_2, \dots, \hat{S}_{15}\}$)

tickers + xts features

11) └ LONG_PORTFOLIO(portfolio)

SECTOR_PROCEDURE(G, τ)

1) Sector G contains tickers $\{S_1, S_2, \dots, S_{15}\}$

2) **for** each ticker $S_i \in \{S_1, S_2, \dots, S_{15}\}$ stocks/ETFs $\in G$

└ FIX current window

$[\, t_0 = \text{week } W_{4\tau},\; t_{52} = \text{week } W_{4\tau+52} \,]$

start point — end-point

t_0] date index — date idx [t_{52}

3) └ EXTRACT_STATIC_FEATURES(S_i)

- → Ticker Prices $(S_t^{(c)})$
- → Basic features
- → Technical Indicators
- → Lag Features

Should be precomputed

4) └ SPLIT the data into (train, val)

- N_{48}^{train} weeks for train set
- N_{4}^{val} weeks for validation set

train — val

t_0 — t_{48} — t_{52}

$|train| = 48$ — $|val| = 4$

5) └ PACK into a list of xts dataframes

$$\text{sector\_features} = \begin{cases} S_1 = (\text{train} = \text{train\_set},\ \text{val} = \text{val\_set}) & \text{xts} \\ \vdots \\ S_{15} = (\text{train} = \text{train\_set},\ \text{val} = \text{val\_set}) \end{cases}$$

6) MODELLING_PROCEDURE(sector_stocks)

└ RETURN $\boxed{\text{Top 3 best tickers}}$

Feature Engineering

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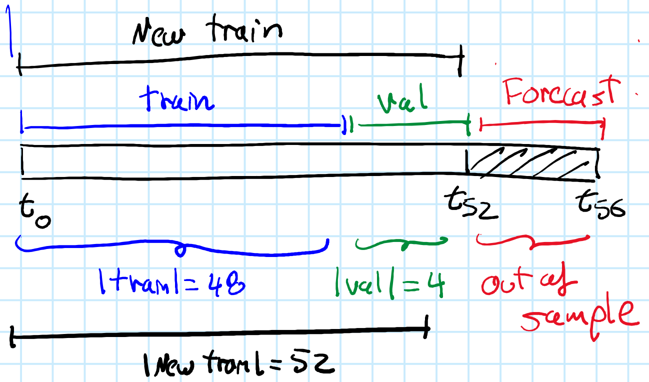
Note:
- train set: Used to Train all models
- validation set: Used for { feature reduction / Model selection
  - ↳ E.g. To select λ_1, λ_2 hyperparams.
- (train_set + validation_set) = New train
  - ↳ Used to re-train best models, forecast returns for next month, and choose best stocks



Feature Engineering

xts static / Only one stock

EXTRACT_DYNAMIC_FEATURES (static_features):

1) static_features: xts train (or new_train) feature matrix for some stock i

Note: Check SECTOR_PROCEDURE section.

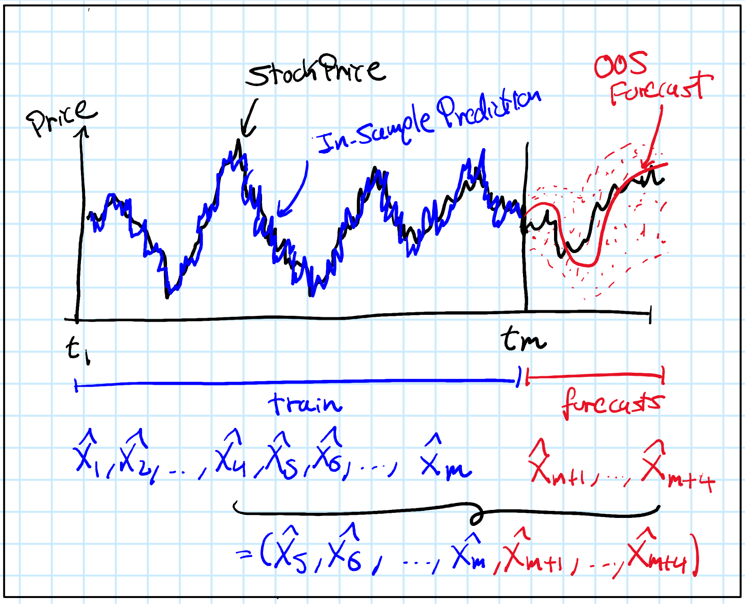
Note: static_features design:

	Date	Basic feats	P/E	⋯	Lag Features	⋯
train OR new_train	t_1					
	⋮					
	t_m					

Note: train set passed to function could be: new_train = (train, val) combined.

⇒ $\boxed{m = 48 \text{ or } 52.}$

2) Compute ARIMA FEATURES
- └ EXTRACT stock/ETF prices, say $X = (X_1, X_2, \dots, X_m)$
- └ for every combination of (p, d, q), where $p, d, q \in \{0, 1\}$:
  - └ FIT an ARIMA(p,d,q) model on $(X_1, X_2, \dots, X_m)$
  - └ PREDICT in_sample $(\hat{X}_1, \hat{X}_2, \dots \hat{X}_m)$
  - └ FORECAST out-of-sample (OOS) $(\hat{X}_{m+1}, \hat{X}_{m+2}, \hat{X}_{m+3}, \hat{X}_{m+4})$



- └ ATTACH features $(\hat{X}_5, \hat{X}_6, \dots, \hat{X}_{m+1}, \hat{X}_{m+4})$ to feature matrix of static_features

Date	Price X	Static Features	X ARIMA(1,0,0)	X ARIMA(0,1,0)	⋯
t_1	X_1		$\hat{X}_5$	$\hat{X}_5$	
t_2	X_2		$\hat{X}_6$	$\hat{X}_6$	
t_3	X_3		$\hat{X}_7$	$\hat{X}_7$	
⋮	⋮	⋮	⋮	⋮	
t_{m-1}	X_{m-1}		$\hat{X}_{m+4-1}$	$\hat{X}_{m+4-1}$	
t_m	X_m		$\hat{X}_{m+4}$	$\hat{X}_{m+4}$	

Modelling

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3) └── Compute GARCH(1,1) Features

  └ FIT an GARCH(1,1) model

    on $(\sigma_1^2, \sigma_2^2, \sigma_3^2, \ldots, \sigma_m^2)$

  └ PREDICT in-sample $(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \ldots, \hat{\sigma}_m^2)$

  └ FORECAST out-of-sample (OOS)

    $(\hat{\sigma}_{m+1}^2, \hat{\sigma}_{m+2}^2, \hat{\sigma}_{m+3}^2, \hat{\sigma}_{m+4}^2)$

  └ ATTACH features $(\hat{\sigma}_5^2, \ldots, \hat{\sigma}_{m+1}^2, \ldots, \hat{\sigma}_{m+4}^2)$ to feature matrix of static_features

Date	Price X	Vol σ^2	Static Features	ARIMA Features	GARCH Features
t_1	X_1	σ_1^2			$\hat{\sigma}_5^2$
t_2	X_2	σ_2^2			$\hat{\sigma}_6^2$
t_3	X_3	σ_3^2			$\hat{\sigma}_7^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t_{m-1}	X_{m-1}	σ_4^2			$\hat{\sigma}_{m-1+4}^2$
t_m	X_m	σ_5^2			$\hat{\sigma}_{m+4}^2$

└ RETURN Fully prepared feature matrix.

Note: This function will be used by the MODELLING_PROCEDURE function.

Modelling and Stock Picking

MODELLING_PROCEDURE(sector_stocks)

1) sector_stocks is a named list which contains train_set and val_set for all stocks in sector G.

$$\text{sector\_features} = \begin{pmatrix} S_1 = \begin{pmatrix} \text{train} = \text{train\_set}, \\ \text{val} = \text{val\_set}, \end{pmatrix} \\ \vdots \\ S_{15} = \begin{pmatrix} \text{train} = \text{train\_set}, \\ \text{val} = \text{val\_set}, \end{pmatrix} \end{pmatrix}$$

xts_0   xts

2) Objective Metric: Modified Sharpe Ratio

$$\boxed{MSR = \frac{E[R] - R_f}{ES_\alpha[R]}}$$

where,

- $E[\hat{R}]$ = Predicted Expected return
- $R_f = 5\%$ (Ignore for now)
- $ES_\alpha[\hat{R}]$ = Expected Shortfall (α) of returns.
- $\alpha = 95\%$

Note:

$$\begin{cases} ES[\hat{R}] = \text{mean}(\text{Returns} < VaR) \\ VaR_\alpha = (\alpha)\text{-th quantile of returns} \end{cases}$$

3) for all stocks $S_1, \ldots, S_{15}$ do:

4) └ Call

EXTRACT_DYNAMIC_FEATURES(static_features)

to update the current features for train_set and val_set

Modelling

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5) └ PERFORM_FEATURE_SELECTION:

I) Use MSR as the criterion

II) Use backwards selection

↳ the Training set is train_set

↳ $\boxed{\text{select features}}$ that do best on val_set (best MSR)

III) Pack features back into a named list

$$\text{best\_stocks\_feats} = \begin{cases} S_1 = \text{best\_feats}(xts) \\ S_2 = \text{best\_feats}(xts) \\ \vdots \\ S_{15} = \underbrace{\text{best\_feats}}_{\text{subset of best feats from static\_feats}}(xts) \end{cases}$$

6) └ TRAIN_ML_MODELS:

I) For each stock, train a Elasticnet model

$$\hat{\beta} = \min_{\beta} \underbrace{\frac{1}{2} \sum_{i=1}^{|train|} (y_i - x_i^T \beta)^2}_{RSS} + \underbrace{\lambda_1 \|\beta\|}_{\text{Lasso Penalty}} + \underbrace{\lambda_2 \|\beta\|_2^2}_{\text{Ridge Penalty}}$$

- y_i = historical returns for fixed data
- x_i = feature vector

II) Hypertune λ_1, λ_2 using val_set

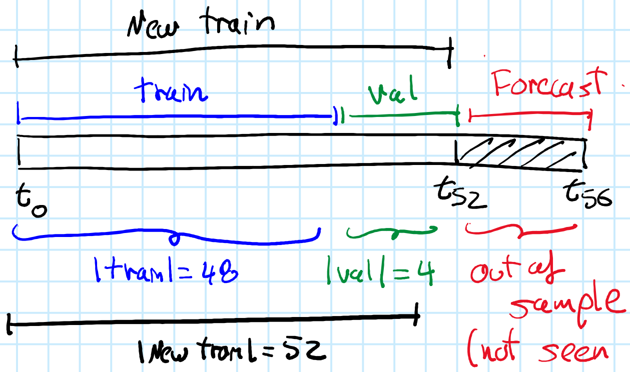
III) RETRAIN model on (train, val) = new_train

IV) FORECAST returns → $\mathbb{E}[\hat{R}_i]$

7) CALCULATE MSR for all stocks

8) RETURN $\boxed{\text{Top 3 tickers}}$ with best MSR

Note:



Note:

① Update Features [S_1 S_2 . . . S_{15}

↓

Feature Selection $\boxed{S_1' \quad S_2' \quad \dots \quad S_{15}'}$

↓

Trained Models (Elasticnet) [M_1 M_2 . . . M_{15}

↓

Forecasted Returns [$\hat{R}_1$ $\hat{R}_2$ . . . $\hat{R}_{15}$

↓

MSR [MSR_1 MSR_2 . . . MSR_{15}

↓

$\boxed{\text{Choose best 3 stocks/ETFs}}$

⇓

("TICKER1", "TICKER2", "TICKER3")

= features (step 0) = features (step 1) = (. . .)

Portfolio Optimization

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Portfolio Optimization

OPTIMIZE_PORTFOLIO(portfolio)

1) portfolio is a list containing the best 3 stocks for each sector, and their corresponding feature matrix (xts)

$$\text{portfolio} = \left(\begin{array}{l} \text{ticker1} = \text{feat\_mat1 (xts)} \\ \text{ticker2} = \text{feat\_mat2 (xts)} \\ \vdots \\ \text{ticker15} = \text{feat\_mat3 (xts)} \end{array} \right.$$

2) Extract the prices $\vec{X_j}$ for each ticker in the portfolio

3) Extract the GARCH(1,1) std. $\vec{\sigma_j}$ for each ticker

4) Compute the covariance matrix Σ

$$\Sigma := (\vec{\sigma_j})(\vec{\sigma_j})^T$$

5) Compute Min-Var Portfolio weights w,

$$\boxed{\begin{array}{l} \vec{w} = \min_{w} \; w \Sigma w^T \\ \text{s.t. } \sum_{k} w_k = 1 \\ \text{and } \forall k, \; w_k \geq 0.01 \\ \text{and no short sales} \end{array}}$$

Assumption of simulation

6) RETURN

$$\left(\begin{array}{l} \text{ticker\_1} = w_1 \\ \text{ticker\_2} = w_2 \\ \vdots \\ \text{ticker\_15} = w_{15} \end{array} \right.$$

Note: A weighted portfolio X_w, is s.t.

$$X_w = \sum_{k=1}^{K} w_k X_k = w_1 X_1 + \cdots + w_K X_K$$

(Note [Prateek])

sector, xts

$$\text{precomp\_features} = \left(\begin{array}{l} G_1 = \text{list}\left(\begin{array}{l} \text{ticker1} = \text{data}, \\ \text{ticker2} = \text{data}, \\ \vdots \end{array} \right), \\ \vdots \\ G_6 = \text{list}\left(\begin{array}{l} \text{ticker1} = \text{data} \\ \vdots \\ \text{ticker2} = \text{data} \end{array} \right) \end{array} \right)$$

2018-2022

precomp_features[[G1]]

xts

OUT list(ticker1 = data, ticker2 = data, ...)

precomp_features[[G1]][[ticker1]]

(idx)

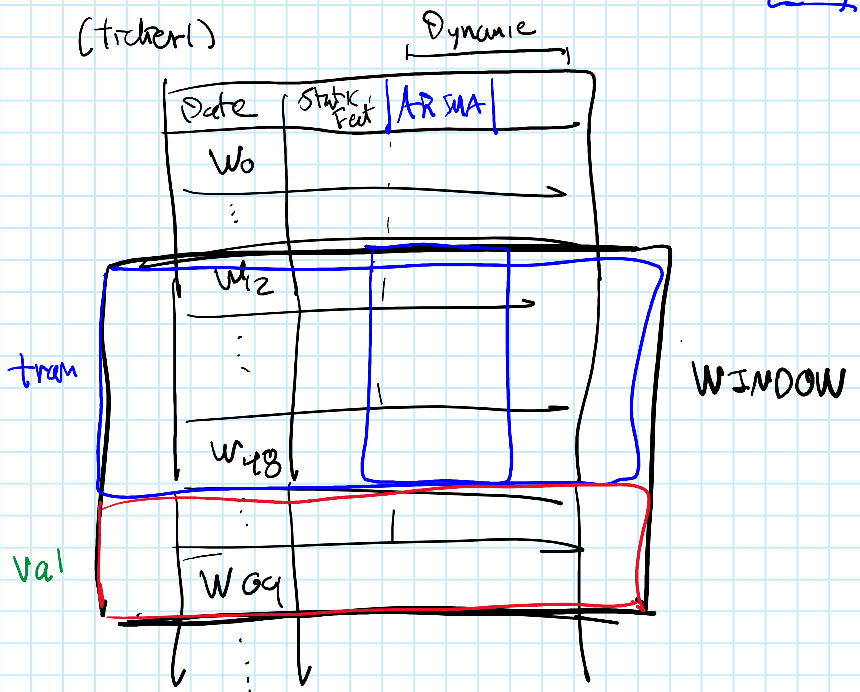
OUT

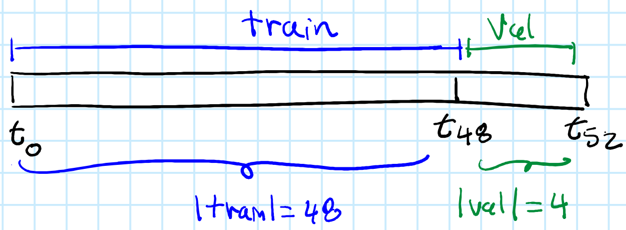
Date	H	O	C	...
W0				

2018-01-03

WINDOW, say $\tau = 3$, $\Rightarrow$ $t_0 = 3 \times 4 = \boxed{12}$

$t_{52} = 4 \times 3 + 52 = \boxed{64}$





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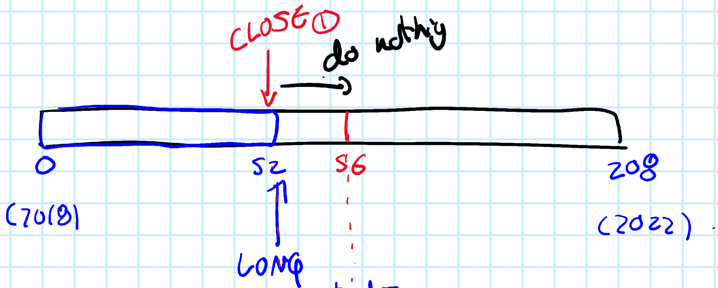
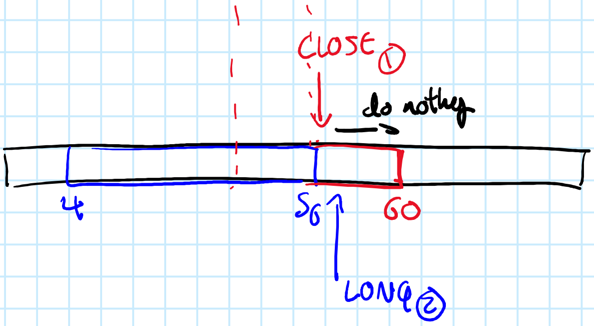


Chosen stocks with opt. weights

$$P^{T_0} = X_1 w + X_2 w + \dots + X_{15} w$$



$$P^{T_0} \neq P^{T_1} = X_1' w + X_2' w + \dots + X_{15}' w$$