Module 8 Graded Quiz

Due Nov 29, 2021 at 11:59pm **Points** 11
Questions 11
Available Nov 6, 2021 at 12am - Feb 2 at 11:59pm 3 months
Time Limit 120 Minutes

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	102 minutes	11 out of 11

Score for this quiz: **11** out of 11
Submitted Nov 29, 2021 at 10:28pm
This attempt took 102 minutes.

Question 1

1 / 1 pts

Consider the MAX-SAT problem, where we are given a boolean formula Φ in conjunctive normal form (i.e. a conjunction of disjoint literals), and we seek a satisfying assignment of variables that maximizes the number of clauses satisfied in Φ.

Suppose we have invented an approximation algorithm that finds a suboptimal solution to the MAX-SAT problem and I have somehow proven that this algorithm is a ρ-approximation of the optimal solution. What can one then conclude about the value of ρ?

- Nothing meaningful can be concluded about the value of ρ from the information provided.

Correct!

- $0 < \rho < 1$

- $\rho > 1$

- $\rho = 1$

Question 2

1 / 1 pts

Consider an arbitrary, connected, weighted graph. The professor presents this graph to her students and tasks them with finding whether there exists a hamiltonian cycle of a certain total weight. To which complexity class does this task belong?

- It is strictly NP-Hard since it is a combinatorial optimization problem.
- It is in NP but not NP-complete since there exists a polynomial time certifier for any instance of the problem.

Correct!

- It is strictly NP-complete since it is a decision problem to which all problems in NP can be reduced and the problem is in NP.
- It is in P since it is known that the problem can be solved in polynomial time.

Question 3

1 / 1 pts

Recall the proof of correctness for the 2-approximation algorithm that solves the traveling salesman problem seen in lecture. Which of the following statements justifies the claim $w(TSP) \leq 2w(MST)$? Assume all positive edge weights.

Given n nodes, the optimal hamiltonian tour has n edges, and the tour our algorithm gives will have 2(n-1) edges. In addition, $n \leq 2(n-1)$ for all $n \geq 2$ and all edge weights are positive by assumption.

Correct!

The tour found by the approximation algorithm is a valid hamiltonian tour. Since $w(TSP)$ is the weight of the optimal hamiltonian tour, it cannot be any greater than the weight of the tour found by our algorithm. Finally, the cost of the tour found by our algorithm has twice the weight of a minimum spanning tree.

None of the other responses is correct.

Clearly, doubling the weight of any spanning tree will always be at least the weight of any spanning cycle.

Question 4

1 / 1 pts

Regarding the new dynamic programming framework given for the knapsack problem, which of the following statements justifies $V^* \leq nv_{\max}$?

This represents the total value of any subset containing the item with maximum value.

We enforce this condition to ensure the algorithm runs in pseudo-polynomial time.

Correct!

The largest possible total value a subset can have is found when the algorithm considers a subset containing all possible items, and all items are associated with the same value. Any other subset considered would have a total value less than this.

We enforce this condition to ensure the algorithm runs in polynomial time.

Question 5

1 / 1 pts

Consider the following instance of the knapsack problem in the table below:

Item	**Value**	**Weight**
1	155,053	5
2	851,200	8
3	21,939,416	10
4	27,326,369	12

Use the technique shown in lecture to derive the vector:

$$\begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \\ \hat{v}_4 \end{bmatrix}$$

Assume $\varepsilon = 0.015$.

- $\begin{bmatrix} 112 \\ 178 \\ 223 \\ 267 \end{bmatrix}$

Correct!

- (selected) $\begin{bmatrix} 2 \\ 9 \\ 215 \\ 267 \end{bmatrix}$

- $\begin{bmatrix} 2.5131 \\ 9.3065 \\ 215.0976 \\ 266.6667 \end{bmatrix}$

- $\begin{bmatrix} 3 \\ 10 \\ 216 \\ 267 \end{bmatrix}$

Question 6

1 / 1 pts

Regarding the proof of correctness for the knapsack PTAS seen in lecture, which of the following statements justifies the claim

$$\sum_{i \in S} v_i + n\theta \leq (1+\epsilon) \sum_{i \in S} v_i$$?

- We defined $\theta = \frac{v_{\max}\epsilon}{n}$ and there cannot be any instances where $v_{\max}$ is less than the sum of all values in the subset chosen by our algorithm.
- We defined $\theta = \frac{v_{\max}\epsilon}{n}$ and there cannot be any instances where $v_{\max}$ is more than the sum of all rounded values in the subset chosen by our algorithm.
- We defined $\theta = \frac{v_{\max}\epsilon}{n}$ and $v_{\max}$ may or may not be present in the subset chosen by our algorithm.

Correct!

- We defined $\theta = \frac{v_{\max}\epsilon}{n}$ and there cannot be any instances where $v_{\max}$ is more than the sum of all values in the subset chosen by our algorithm.

Question 7

1 / 1 pts

[Probability theory review] Suppose we have n containers with open lids. For each of these containers we decide to randomly close each lid with probability $\frac{3}{4}$. Let random variable X_i be equal to 1 if the lid of bin i becomes closed and zero otherwise. Then let $X = \sum_{i=1}^{n} X_i$. What is the value of $E[X]$? What does this value represent?

- None of the other responses is correct.
- $\frac{n}{4}$, the expected number of bins with open lids.

Correct!

- $\frac{3n}{4}$, the expected number of bins with closed lids.
- $\frac{3n}{4}$, the expected number of bins with open lids.
- $\frac{n}{4}$, the expected number of bins with closed lids.

Question 8

1 / 1 pts

In lecture, we saw as a preliminary a 2-tiered skip list and found that given the expected structure of this skip-list, the worst case number of nodes visited for a search operation is $\frac{n}{2}+2$, where n is the number of nodes in the original linked list. Suppose that while constructing the second tier, the probability that any node will be duplicated to the second tier is $\frac{1}{3}$ (i.e. any key appears in tier 2 with probability $\frac{1}{3}$). Give the worst case number of nodes visited for a search operation on the expected structure of such a list.

- $\frac{n}{3}+2$
- $\frac{2n}{3}+2$
- Correct! $\frac{n}{3}+3$
- $\frac{2n}{3}+3$

Question 9

1 / 1 pts

Give the worst case running time on a search operation on a skip-list with n nodes and a constant number of levels.

- Correct! $O(n)$
- ∞
- $O(\log n)$
- $O(n \log n)$

Question 10

1 / 1 pts

Suppose for some randomized algorithm, we have shown that the expected running time for this algorithm is $O(n)$, where n is the size of the input. We wish to show this expected running time will hold with "*high probability*". According to the definition given in the slides for this section, which of the following functions $f : \mathbb{N} \to [0, 1]$ are examples of a "high probability" result? Select all that apply.

Correct!

- [x] $f(n) = 1 - \frac{1}{2^n}$

Correct!

- [x] $f(n) = 1 - \frac{1}{n}$
- [ ] $f(n) = 1 - \frac{1}{\sqrt{n}}$
- [ ] $f(n) = 1 - \frac{1}{\log n}$

Question 11

1 / 1 pts

[Exploring the notion of high probability]

Inequality $1 + x \leq e^x$ holds for all real numbers x. Suppose you have an n-sided die with distinct faces. Let X be the number of times one must roll this die before we obtain a specific result $1 \leq r \leq n$. We know from the properties of geometric distributions, that, in expectation, we will have to roll this die n times before we obtain r. Use the inequality above (in the first sentence) to determine the number m of die rolls necessary such that the probability of rolling r in any of the m rolls is at least $1 - \frac{1}{n^c}$ for some constant c. **Select all that apply.**

Correct!

- [x] $m = n^2$

Correct!

- [x] $m = n \log n$

- [ ] $m = \log n$
- [ ] $m = n$

Quiz Score: **11** out of 11