Module 2 Graded Assignment and Quiz

Due Sep 10, 2021 at 11:59pm **Points** 4 **Questions** 4
Available Aug 28, 2021 at 12am - Jan 28 at 11:59pm 5 months
Time Limit None

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	1,595 minutes	4 out of 4

Score for this quiz: **4** out of 4
Submitted Sep 9, 2021 at 8:38pm
This attempt took 1,595 minutes.

Question 1

1 / 1 pts

For each of the following two statements, decide whether it is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

1. Suppose we are given an instance of the Minimum Spanning Tree Problem on a graph G, with edge costs that are all positive and distinct. Let *T* be a minimum spanning tree for this instance. Now suppose we replace each edge cost c_e by its square, c_e^2, thereby creating a new instance of the problem with the same graph but different costs. True or false? *T* must still be a minimum spanning tree for this new instance.
2. Suppose we are given an instance of the Shortest *s*-*t* Path Problem on a directed graph G. We assume that all edge costs are positive and distinct. Let P be a minimum-cost *s*-*t* path for this instance. Now suppose we replace each edge cost c_e by its square, c_e^2, thereby creating a new instance of the problem with the same graph but

different costs. True or false? P must still be a minimum-cost *s*-*t* path for this new instance.

Answer true or false for statements 1 and 2.

Correct!

- Statement 1: True, Statement 2: False
- Statement 1: False, Statement 2: True
- Statement 1: False, Statement 2: False
- Statement 1: True, Statement 2: True

Question 2

1 / 1 pts

Let's consider a long, quiet country road with houses scattered very sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road so that every house is within four miles of one of the base stations. Give an efficient algorithm that achieves this goal, using as few base stations as possible.

Suppose you are given this algorithm: start at the western end of the road and begin moving east until the first moment when there is a house h exactly four miles to the west. Place a base station at this point (if you go any further east without placing a base station, you would not cover h). Then delete all the houses covered by this base station and iterate this process on the remaining houses.

The algorithm is correct; which of the following demonstrates why?

Correct!

- An induction argument on the number of base stations placed, and an exchange argument with the optimal solution (similar to the proofs of other greedy methods).

It is equivalent to one of the other algorithms we have seen in lecture.

The algorithm always goes as far east as possible at each iteration.

Question 3

1 / 1 pts

One of the basic motivations behind the Minimum Spanning Tree Problem is the goal of designing a spanning network for a set of nodes with minimum *total* cost. Here we explore another type of objective: designing a spanning network for which the *most expensive* edge is as cheap as possible.

Specifically, let $G = (V, E)$ be a connected graph with n vertices, m edges, and positive edge costs. Let $T = (V, E')$ be a spanning tree of G; we define the *bottleneck edge* of T to be the edge of T with the greatest cost.

A spanning tree T of G is a *minimum-bottleneck spanning tree* if there is no spanning tree T' of G with a cheaper bottleneck edge.

Answer the following:

1. Is every minimum-bottleneck tree of G a minimum spanning tree of G? Prove or give a counterexample.
2. Is every minimum spanning tree of G a minimum-bottleneck tree of G? Prove or give a counterexample.

Is (1) true? If yes, why? If no, what is a counterexample?

Correct!

No; let G be the complete graph on 4 vertices, and each edge between a vertex with label $i \in \{1, 2, 3, 4\}$ and another vertex with label $j \in \{1, 2, 3, 4\}$ has weight $i + j$.

No, but any counter example must have more than 4 vertices.

Yes, since every minimum-bottleneck tree is already an MST.

Question 4

1 / 1 pts

In trying to understand the combinatorial structure of spanning trees, we can consider the space of *all* possible spanning trees of a given graph and study the properties of this space. This is a strategy that has been applied to many similar problems as well.

Here is one way to do this. Let G be a connected graph, and T and T' two different spanning trees of *G*. We say that T and T' are *neighbours* if T contains exactly one edge that is not in T', and T' contains exactly one edge that is not in T.

Now, from any graph G, we can build a (large) graph H as follows. The nodes of H each represent a unique spanning tree of G, and there is an edge between two nodes of H if the corresponding spanning trees are neighbours.

Is it true that, for any connected graph G, the resulting graph H is connected? Give a proof that H is always connected, or provide an example (with explanation) of a connected graph G for which H is not connected.

Is H connected and why/why not?

- ○ No, because there is an example of a tree T such that the constructed graph H has at least 2 connected components.
- ○ Yes, because the resulting graph H is the complete graph.
- Correct! ◉ Yes, let T and T' be two spanning trees of G each represented by distinct nodes in H. Argue by induction on the number of edges not common to both T and T' to infer the existence of a path between their corresponding vertices in H.

Quiz Score: **4** out of 4