Module 1 Graded Assignment and Quiz

Due Aug 29, 2021 at 11:59pm **Points** 4 **Questions** 4
Available Aug 16, 2021 at 12am - Feb 14 at 11:59pm 6 months
Time Limit None

Instructions

Read the problem below. Given an algorithm that is guaranteed to find an assignment of students to hospitals that avoids the two types of instability described below.

Once you have completed the problem, complete this assignment's companion quiz (next content item in the course) to submit your findings.

Problem

Gale and Shapley published their paper on the Stable Matching Problem in 1962; but a version of their algorithm had already been in use for ten years by the National Resident Matching Program, for the problem of assigning medical residents to hospitals.

Basically, the situation was the following. There were m hospitals, each with a certain number of available positions for hiring residents. There were n medical students graduating in a given year, each interested in joining one of the hospitals. Each hospital had a ranking of the students in order of preference, and each student had a ranking of the hospitals in order of preference. We will assume that there were more students graduating than there were slots available in the m hospitals.

The interest, naturally, was in finding a way of assigning each student to at most one hospital, in such a way that all available positions in all hospitals were filled. (Since we are assuming a surplus of students, there would be some students who do not get assigned to any hospital.)

We say that an assignment of students to hospitals is stable if neither of the following situations arises.

First type of instability: There are students s and s′, and a hospital h, so that

– s is assigned to h, and

– s′ is assigned to no hospital, and

– h prefers s′ to s.

Second type of instability: There are students s and s′, and hospitals h and h′, so that

– s is assigned to h, and

– s′ is assigned to h′, and

– h prefers s′ to s, and

– s′ prefers h to h′.

So, we basically have the Stable Matching Problem, except that (i) hospitals generally want more than one resident, and (ii) there is a surplus of medical students.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	1,075 minutes	4 out of 4

Score for this quiz: **4** out of 4
Submitted Aug 26, 2021 at 10:38am
This attempt took 1,075 minutes.

Question 1

1 / 1 pts

Explain how your algorithm is able to guarantee that no assignment will exhibit the second type of instability described in the prompt. Make sure your algorithm terminates in $O(mn)$ time.

Correct!

A hospital h generates offers in descending order of preference, thus it must have sent an offer to s', else it prefers all of its currently filled positions to s'. If s' preferred h, my algorithm would have allowed s' to cancel its current assignment to h' and become assigned to h.

A hospital h generates offers in descending order of preference, thus it must have sent an offer to s', else it prefers all of its currently filled positions to s'. If s' preferred h, my algorithm would have ensured h would have sent an offer to s' before any other hospital was able to, thus h and s' are guaranteed to be matched.

When deciding whether to assign resident s' to hospital h', my algorithm includes a check to see if some other hospital h prefers s' to any of its current assigned residents. If so, then the assignment of resident s' to hospital h' is not made.

Question 2

1 / 1 pts

Given the conditions of the residents and hospitals problem described in the assignment prompt, under what mechanisms will your algorithm terminate and still produce a stable matching? Assume hospitals are initiating proposals, and potential residents may accept or reject these proposals. (Also assume that hospitals issue proposals that residents may either accept or reject.)

Let p be the number of positions available at each hospital. Randomly remove medical students until n is equal to mp and terminate when all remaining medical students have been assigned.

Neither of the other responses is correct.

Correct!

Store the number of open positions for each hospital in an array A. Adjust the values of A as hospitals and residents become assigned and unassigned. Terminate when all values of A are zero.

Question 3

1 / 1 pts

The Stable Matching Problem, as discussed in lecture, assumes that all men and women have a fully ordered list of preferences. In this problem we will consider a version of the problem in which men and women can be indifferent between certain options. As before we have a set M of n men and a set W of n women. Assume each man and each woman ranks the members of the opposite gender, but now we allow ties in the ranking. For example (with $n = 4$), a woman could say that m_1 is ranked in first place; second place is a tie between m_2 and m_3 (she has no preference between them); and m_4 is in last place. We will say that w prefers m to m' if m is ranked higher than m' on her preference list (they are not tied).

With indifferences in the rankings, there could be two natural notions for stability. And for each, we can ask about the existence of stable matchings, as follows.

A strong instability in a perfect matching S consists of a man m and a woman w, such that each of m and w prefers the other to their respective partners in S. Does there always exist a perfect matching with no strong instability?

Correct!

- (selected) Yes
- No

Question 4

1 / 1 pts

Continued from question 3 above:

A weak instability in a perfect matching S consists of a man m and a woman w, such that their partners in S are w' and m', respectively, and one of the following holds:

– m prefers w to w', and w either prefers m to m' or is indifferent between these two choices; or

– w prefers m to m', and m either prefers w to w' or is indifferent between these two choices.

In other words, the pairing between m and w is either preferred by both or preferred by one while the other is indifferent. Does there always exist a perfect matching with no weak instability?

Correct!

- [x] No
- [ ] Yes

Quiz Score: **4** out of 4