

# Module 7 Graded Quiz

**Due** Nov 19, 2021 at 11:59pm

**Points** 15

**Questions** 15

**Available** Nov 6, 2021 at 12am - Jan 21 at 11:59pm 3 months

**Time Limit** 120 Minutes

## Attempt History

	Attempt	Time	Score
<b>LATEST</b>	<u><b>Attempt 1</b></u>	29 minutes	15 out of 15

Score for this quiz: **15** out of 15

Submitted Nov 19, 2021 at 6:35pm

This attempt took 29 minutes.

### Question 1

1 / 1 pts

Which of the following problems does not likely have a polynomial-time algorithm? Select all that apply.

☐ Shortest Path

☐ Minimum Cut

☒ Knapsack

☒ Vertex Cover

**Correct!**

**Correct!**

### Question 2

1 / 1 pts

What is a decision problem?

**Correct!**

- ☐ A problem that is in P or NP.
- ☒ A problem in which the answer is always “yes” or “no”.
- ☐ A problem that requires any algorithm for it to return a result that is not a boolean value.
- ☐ A problem that requires any algorithm for it to make a decision.

**Question 3****1 / 1 pts**

Why, informally, is P a subset of NP?

**Correct!**

- ☒ Because the certificate could be the algorithm itself, and the certifier could be the execution of the algorithm.
- ☐ Because any polynomial-time algorithm obviously runs in polynomial-time.
- ☐ Because it follows directly from the definition.

**Question 4****1 / 1 pts**

Which of the following is a valid statement about problem Y for when  $X \leq_P Y$  and X is guaranteed to be polynomial-time solvable?

**Correct!**

- ☒ Nothing can be guaranteed about Y.
- ☐ Y is not in P.

- ☐ Y is in P.
- ☐ Y is equivalent to X.

**Question 5****1 / 1 pts**

If problems X and Y are equivalent (i.e. X reduces in poly-time to Y and Y reduces in poly-time to X), what can we conclude about their run times?

- ☐ They are within a constant factor.
- ☒ They are at most a polynomial factor different.
- ☐ They are at most an exponential factor different.
- ☐ They are identical.

**Correct!****Question 6****1 / 1 pts**

Which of the following is a valid set of constraints on the inputs to the VERTEX-COVER problem?

- ☒ A graph, and a nonnegative integer k.
- ☐ A directed graph, and an arbitrary integer k.
- ☐ A directed graph, and a nonnegative integer k.
- ☐ A graph, and an arbitrary integer k.

**Correct!**

**Question 7****1 / 1 pts**

In the SAT problem, what does “conjunctive normal form” mean?

**Correct!**

The formula is a conjunction of clauses, which are disjunctions of literals.



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**Question 8****1 / 1 pts**

Consider the following algorithm for VERTEX-COVER: given the graph  $G$  and integer  $k$ , test all  $O(n^k)$  sets of  $k$  vertices, and for each determine if it is a vertex cover. Is this a polynomial-time algorithm? Why/why not?



No, because no polynomial time algorithm exists for VERTEX-COVER.



Yes, because  $n^k$  is a polynomial.



No, since  $k$  is part of the input and it is not constrained to be a constant



Yes, because there are polynomially many subsets of the vertices.

**Correct!**

**Question 9****1 / 1 pts**

What was the “clause gadget” in the 3-SAT to INDEPENDENT-SET reduction?

**Correct!**

- ☒ A triangle (cycle of length 3).
- ☐ A path of length 3.
- ☐ A complete graph of size  $O(n)$  ( $n$  is the number of variables).
- ☐ A single vertex with no edges.

**Question 10****1 / 1 pts**

Why is the  $\leq_P$  relation transitive? (i.e.,  $X \leq_P Y$  and  $Y \leq_P Z$  implies that  $X \leq_P Z$ )

**Correct!**

- ☐ Because a polynomial plus another is also a polynomial.
- ☐ None of the other answers is correct.
- ☐ Because a polynomial times another is also a polynomial.
- ☒ Because the reductions can be composed together into a polynomial time reduction from problem  $X$  to problem  $Z$ .

**Question 11****1 / 1 pts**

What if in the 3-SAT problem you were allowed to have at most 3 literals per clause (the original problem specified exactly 3)? Please mark which of the following are correct.

☐

The modified version of the 3-SAT problem is the same as the 2-SAT problem.

☐

The modified version of the 3-SAT problem is in P, and 3-SAT is NP complete.

☐

The modified version of the 3-SAT problem is in P, and 3-SAT is not in P.

☒

The modified version of the 3-SAT problem is polynomially equivalent to the original version of 3-SAT.

**Correct!**

## Question 12

1 / 1 pts

Which of the following is not an example of a known NP-complete problem?

☐

KNAPSACK

☒

PRIMES

☐

TRAVELING SALESMAN PROBLEM

☐

SUBSET-SUM

**Correct!**

**Question 13****1 / 1 pts**

My friend is trying to show  $X \leq_P Y$  for problems  $X, Y$ . He has given a polynomial time function that transforms an instance  $I$  of  $X$  into an instance  $\text{poly}(I)$  of  $Y$ . Lastly, he has shown that if the decision is yes for  $\text{poly}(I)$ , then the decision is always yes for  $I$ . Is my friend's proof complete?

☐

Yes, this completes all the steps to show a polynomial time reduction from  $X$  to  $Y$  since yes decisions of  $Y$  are mapped to no decisions for  $X$ .

☐

No, your friend must show a decision of no for  $\text{poly}(I)$  corresponds to a decision of yes for  $I$ .

☐

Yes, this completes all the steps to show a polynomial time reduction from  $X$  to  $Y$  since yes decisions of  $Y$  are mapped to yes decisions for  $X$ .

☒

No, your friend must show a decision of no for  $\text{poly}(I)$  corresponds to a decision of no for  $I$ .

**Correct!****Question 14****1 / 1 pts**

The statements below concern the classes  $P$  and  $NP$ . Mark all correct statements.

☒

$P$  and  $NP$  consist only of decision problems.

☐

The version of the Knapsack problem wherein one must find a subset of maximum profit subject to weight constraints is in  $NP$ .

**Correct!**

**Correct!**☒  $|P| \leq |NP|$ **Correct!**☒

The version of the Traveling Salesman problem wherein one must determine whether a tour of weight at least  $W$  exists is in NP.

**Question 15****1 / 1 pts**

If an instance  $I$  of a problem  $X$  undergoes a transformation to an instance  $I'$  of a problem  $Y$ , and this transformation terminates in time polynomial in the size of  $I$ , then must the size of  $I'$  be polynomial in the size of  $I$ ?

☐

No, problem  $X$  might be NP-complete, and thus any transformation of  $I$  will produce a result exponential in the size of  $I$ .

**Correct!**☒

Yes, a Turing machine that runs for  $|I|^c$  transitions for some constant  $c$  can only access at most  $|I|^c$  tape cells.

☐

No, even if the transformation terminates in polynomial time, it could have used an exponential amount of space in its computation.

☐

Yes, since the size of  $I'$  must be equal to the size of  $I$ .

**Quiz Score: 15 out of 15**