Module 8 Graded Homework

Due Nov 29, 2021 at 11:59pm **Points** 5 **Questions** 5
Available Nov 6, 2021 at 12am - Jan 13 at 11:59pm 2 months
Time Limit None

Instructions

Suppose you're acting as a consultant for the Port Authority of a small Pacific Rim nation. They're currently doing a multi-billion-dollar business per year, and their revenue is constrained almost entirely by the rate at which they can unload ships that arrive in the port.

Here's a basic sort of problem they face. A ship arrives, with n con- tainers of weight w1, w2, . . . , wn. Standing on the dock is a set of trucks, each of which can hold K units of weight. (You can assume that K and each wi is an integer.) You can stack multiple containers in each truck, subject to the weight restriction of K; the goal is to minimize the number of trucks that are needed in order to carry all the containers. This problem is NP-complete (you don't have to prove this).

A greedy algorithm you might use for this is the following. Start with an empty truck, and begin piling containers 1, 2, 3, . . . into it until you get to a container that would overflow the weight limit. Now declare this truck "loaded" and send it off; then continue the process with a fresh truck. This algorithm, by considering trucks one at a time, may not achieve the most efficient way to pack the full set of containers into an available collection of trucks.

a.) Give an example of a set of weights, and a value of K, where this algorithm does not use the minimum possible number of trucks.

b.) Show, however, that the number of trucks used by this algorithm is within a factor of 2 of the minimum possible number, for any set of weights and any value of K.

This quiz was locked Jan 13 at 11:59pm.

Attempt History

	Attempt	Time	Score

	Attempt	Time	Score
LATEST	Attempt 1	23 minutes	5 out of 5

Score for this quiz: **5** out of 5
Submitted Nov 29, 2021 at 9:10pm
This attempt took 23 minutes.

Question 1

1 / 1 pts

Regarding part a. of the prompt, which of the following is a valid counterexample to the notion that the greedy algorithm described is optimal? Responses are given in the form of a set of item weights and a truck capacity K.

- {w_1,w_2,w_3}={1,2,2}, K=2
- Correct! {w_1,w_2,w_3}={1,2,1}, K=2
- None of these is a valid counterexample
- {w_1,w_2,w_3}={2,2,2}. K=2

Question 2

1 / 1 pts

If W is the sum of all weights in the set of weights {w_1,...,w_n}, identify which of the following expressions represents a lower bound of the number of trucks necessary to fit all n items in terms of W and K.

- $\frac{K}{W}$
- Correct! $\frac{W}{K}$
- $\sqrt[K]{W}$

- WK

Question 3

1 / 1 pts

For part b. of the prompt, the remaining questions of this quiz will guide you through the proof strategy. The first part of the proof is as follows:

Suppose the number of trucks is the odd number m=2q+1 for some q in {0,1,2,...}, and we denote the set of trucks as {t_1,t_2,...,t_m}. We then divide the trucks into consecutive groups of 2 such that group 1 is {t_1,t_2}, group 2 is {t_3,t_4} and so on until the last group consists only of {t_m}, which is necessary since the total number of trucks is odd.

In this construction, which of the following expressions represents the total number of groups created in terms of q?

- q
- q/2
- Correct! q+1 (selected)
- q/2+1

Question 4

1 / 1 pts

Continuing from question 3, consider all groups except the last group that contains only one truck. We note that for any fixed truck group of these remaining groups, the combined weight of both trucks must be at least K, else the greedy algorithm would not have required splitting the items between these two trucks. Which of the following expressions does this observation imply?

- W>(q+1)k
- W<qk
- Correct! W>qK
- W<(q/2)k

Question 5

1 / 1 pts

Use your results from questions 2 through 4 to finish the proof. Which of the following statements completes the proof? (Exercise: use a similar argument to show this is true for an even number of trucks)

- Thus, the greedy algorithm always uses the minimum number of trucks necessary.
- Thus, the greedy algorithm uses exactly q+1 trucks.
- Thus, the optimum solution uses at least q+1 trucks, which is at most half the number of trucks greedy uses.
- Correct! Thus, the optimum solution uses at least q+1 trucks, which is at least half the number of trucks greedy uses.

Quiz Score: **5** out of 5