

1) According to the Taylor Series

$$\begin{aligned} f(x) &= \lim_{\Delta \rightarrow 0} f(x+\Delta) \\ &= \lim_{\Delta \rightarrow 0} \left(f(x) + f'(x)\Delta + \frac{f''(x)}{2}\Delta^2 + \frac{f'''(x)}{3}\Delta^3 + \dots + \frac{f^{(k)}(x)}{k}\Delta^k + \dots \right) \\ &= \lim_{\Delta \rightarrow 0} (f(x) + f'(x)\Delta) \end{aligned}$$

When $\Delta \rightarrow 0$, it will easily get $f(x+\Delta) = f(x) + f'(x)\Delta$

In the example 2-11, $f(x) = \frac{1}{2}x^2$, $f'(x) = x$, $\Delta = 0.2, 0.1, 0.01, 0.001$

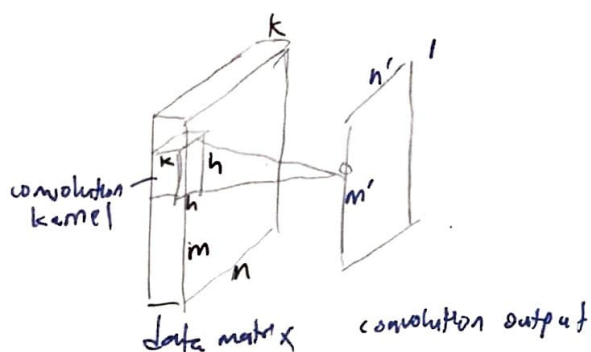
Δ	$f(x+\Delta)$	$f(x) + f'(x)\Delta$	$ f(x+\Delta) - f(x) + f'(x)\Delta $
$\Delta = 0.2$	0.72	0.7	0.02
$\Delta = 0.1$	0.605	0.6	0.005
$\Delta = 0.01$	0.5101	0.51	0.0001
$\Delta = 0.001$	0.501	0.501	0

\therefore From here, we deduce that the lesser the Δ , the lesser the error.

Q2) For Figure 4.15(a), 4.15(b), both are data with three-dimensional structure.

For Figure 4.15(a), it is 2D convolution on multiple frames

the data matrix is $Z \in \mathbb{R}^{k \times m \times n}$, convolution kernel is $U \in \mathbb{R}^{k \times h \times h}$, the convolution output is $S \in \mathbb{R}^{k \times m' \times n'}$, which resulted from the sliding of the convolution kernel U on the data matrix Z and calculate the element $s(j,i)$ of S



data matrix
in-height \times in-width \times n-channels.
($m \times n \times k$)

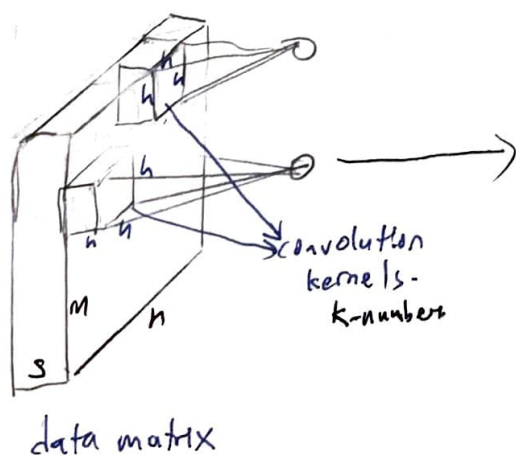
convolution kernel
filter-height \times filter-width \times in-channel \times out-channel.
($h \times h \times k \times 1$)

$$s(j,i) = Z \otimes U$$

$$= \sum_{w=1}^k \sum_{y=-1(h-1)/2}^{(h-1)/2} \sum_{x=-1(h-1)/2}^{(h-1)/2} z(j+y, i+x, w) u(y, x, w)$$

convolution output
out-height \times out-width \times out-channel.
($m' \times n' \times 1$)

For Figure 4.15(b) the data matrix is $Z \in \mathbb{R}^{k \times s \times m \times n}$, convolution kernel is $U \in \mathbb{R}^{k \times h \times h \times h}$, the convolution output is $S \in \mathbb{R}^{s' \times m' \times n'}$, which resulted from the sliding of the convolution kernel U on the data matrix Z and calculate the element $s(j,i,l)$ of S .



data matrix
in-channel, in-depth \times in-height \times in-width
($k \times s \times m \times n$)

convolution kernel
filter-depth \times filter-height \times filter-width \times in-channel \times out-channel
($h \times h \times h \times k \times s'$)

convolution output
out-depth \times out-height \times out-width \times out-channel
($s' \times m' \times n' \times k$)

— | element $s(j, i, l)$ of S

$$s(j, i, l) = z \otimes u$$

$$= \sum_{w=1}^k \sum_{y=-(h-1)/2}^{(h-1)/2} \sum_{x=-(h-1)/2}^{(h-1)/2} \sum_{v=-(h-1)/2}^{(h-1)/2} z(j+y, i+x, l+v, w) u(y, x, v, w)$$