

# Unit 4

Semantics

# Requirements for Representation.

**Expressiveness:** Expressiveness refers to the ability of the representation system to capture a wide range of meanings, nuances, and contexts.

- **Rich Vocabulary:** The system should have a rich set of symbols to describe various entities, actions, properties, and relationships.
- **Complex Structures:** It should support the representation of complex structures such as nested and hierarchical relationships.
- **Context Sensitivity:** The representation should capture context to accurately reflect meaning (e.g., the difference between "bank" in "river bank" and "financial bank").
- **Ambiguity Resolution:** It should handle and resolve ambiguities in natural language.

**Formalism:** Formalism ensures that the representation has a well-defined syntax and semantics, allowing for unambiguous interpretation and processing.

- Clear Syntax: The rules for forming valid expressions should be precise and unambiguous.
- Defined Semantics: Each expression should have a clear meaning, defined in a way that machines can interpret consistently.
- Standardization: Use of standard representation languages (e.g., First-Order Logic, Description Logic) can help in maintaining consistency and interoperability.

**Inference:** Inference capability allows the system to derive new information from existing knowledge, which is essential for reasoning and decision-making.

- **Logical Deduction:** The system should support logical inference mechanisms such as modus ponens, modus tollens, and syllogism.
- **Probabilistic Inference:** For handling uncertainty, the system might need to support probabilistic reasoning (e.g., Bayesian networks).
- **Temporal Reasoning:** For applications involving time, the system should support reasoning about temporal relationships and events.

# Propositional Logic

1. "If it is raining, then the ground is wet." -  $P \rightarrow Q$
2. "Either the system is down, or the network is slow." -  $P \vee Q$
3. "If the user is logged in and the session is active, then the user can access the dashboard." -  $(P \wedge Q) \rightarrow R$
4. "The application will fail unless it is updated." -  $\neg Q \rightarrow P$  or  $P \vee Q$
5. "If the file is not found, an error message is displayed." -  $\neg P \rightarrow Q$
6. "The server is running if and only if the power is on." -  $P \leftrightarrow Q$
7. "If the temperature is above 30°C, then the air conditioner turns on." -  $P \rightarrow Q$
8. "If the database is backed up, the system will restart, and the logs will be cleared." -  $P \rightarrow (Q \wedge R)$
9. "If the user presses the submit button, then the form is validated and the data is sent." -  $P \rightarrow (Q \wedge R)$
10. "The document is either saved or discarded." -  $P \vee Q$

# First-Order Logic (FOL),

- **First-Order Logic (FOL)**, also known as Predicate Logic or First-Order Predicate Calculus, is a formal system used to express statements about objects, their properties, and their relationships. FOL extends propositional logic by incorporating quantifiers and predicates, making it much more expressive.

# Key Components of First-Order Logic

## **Terms:**

- Constants: Represent specific objects in the domain (e.g., a, b, Raju).
- Variables: Represent any object in the domain (e.g., x, y, z).
- Functions: Map a set of objects to another object
  - e.g., 'fatherOf(Raju)'

**Predicates:** Represent properties or relationships between objects. For example,

- Student(Raju) means "Raju is a student," and Likes(Raju, Pizza) means "John likes pizza."

## Logical Connectives:

- Conjunction ( $\wedge$ ):  $P \wedge Q$  means "P and Q."
- Disjunction ( $\vee$ ):  $P \vee Q$  means "P or Q."
- Negation ( $\neg$ ):  $\neg P$  means "not P."
- Implication ( $\rightarrow$ ):  $P \rightarrow Q$  means "if P then Q."
- Biconditional ( $\leftrightarrow$ ):  $P \leftrightarrow Q$  means "P if and only if Q."



## Quantifiers:

- Universal Quantifier ( $\forall$ ):  $\forall x P(x)$  means "for all  $x$ ,  $P(x)$  is true."
- Existential Quantifier ( $\exists$ ):  $\exists x P(x)$  means "there exists an  $x$  such that  $P(x)$  is true."

- "All humans are mortal."
- "Some students are brilliant." –
- "No dogs can fly."
- "If a person is a parent, then they have a child." -
- "There is a cat that is black."
- "Every student loves some book." –
- "Someone likes everyone." -
- "If an animal is a bird, then it can fly." -
- "There exists a unique number that is even."
- "For every number, there is a greater number." -

1. "All humans are mortal." -  $\forall x(\text{Human}(x) \rightarrow \text{Mortal}(x))$   $x$ : human
2. "Some students are brilliant." -  $\exists x(\text{Student}(x) \wedge \text{Brilliant}(x))$   $x$ : student
3. "No dogs can fly." -  $\forall x(\text{Dog}(x) \rightarrow \neg \text{CanFly}(x))$   $x$ : dog
4. "If a person is a parent, then they have a child." -  $\forall x(\text{Parent}(x) \rightarrow \exists y(\text{Child}(y, x)))$   $x$ : parent,  $y$ : child
5. "There is a cat that is black." -  $\exists x(\text{Cat}(x) \wedge \text{Black}(x))$   $x$ : cat
6. "Every student loves some book." -  $\forall x(\text{Student}(x) \rightarrow \exists y(\text{Book}(y) \wedge \text{Loves}(x, y)))$   $x$ : student,  $y$ : book
7. "Someone likes everyone." -  $\exists x(\forall y(\text{Person}(y) \rightarrow \text{Likes}(x, y)))$   $x$ : someone,  $y$ : person
8. "If an animal is a bird, then it can fly." -  $\forall x(\text{Bird}(x) \rightarrow \text{CanFly}(x))$   $x$ : bird

9. "There exists a unique number that is even." -  $\exists x(\text{Even}(x) \wedge \forall y(\text{Even}(y) \rightarrow x = y))$   $x$ : number,  $y$ : number
10. "For every number, there is a greater number." -  $\forall x(\text{Number}(x) \rightarrow \exists y(\text{Number}(y) \wedge \text{Greater}(y, x)))$   $x$ : number,  $y$ : number

# CMSC 471

## First-Order Logic

### Chapter 8.1-8.3

Adapted from slides by  
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Marie desJardins.

Some material adopted from notes  
by Andreas Geyer-Schulz

# Outline

- First-order logic
  - Properties, relations, functions, quantifiers, ...
  - Terms, sentences, axioms, theories, proofs, ...
- Extensions to first-order logic
- Logical agents
  - Reflex agents
  - Representing change: situation calculus, frame problem
  - Preferences on actions
  - Goal-based agents

# First-order logic

- First-order logic (FOL) models the world in terms of
  - **Objects**, which are things with individual identities
  - **Properties** of objects that distinguish them from other objects
  - **Relations** that hold among sets of objects
  - **Functions**, which are a subset of relations where there is only one “value” for any given “input”
- Examples:
  - Objects: Students, lectures, companies, cars ...
  - Relations: Brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
  - Properties: blue, oval, even, large, ...
  - Functions: father-of, best-friend, second-half, one-more-than ...

# User provides

- **Constant symbols**, which represent individuals in the world
  - Mary
  - 3
  - Green
- **Function symbols**, which map individuals to individuals
  - father-of(Mary) = John
  - color-of(Sky) = Blue
- **Predicate symbols**, which map individuals to truth values
  - greater(5,3)
  - green(Grass)
  - color(Grass, Green)



# FOL Provides

- **Variable symbols**

- E.g.,  $x$ ,  $y$ ,  $\text{foo}$

- **Connectives**

- Same as in PL: not ( $\neg$ ), and ( $\wedge$ ), or ( $\vee$ ), implies ( $\rightarrow$ ), if and only if (biconditional  $\leftrightarrow$ )

- **Quantifiers**

- Universal  $\forall x$  or **(Ax)**
- Existential  $\exists x$  or **(Ex)**

# Sentences are built from terms and atoms

- A **term** (denoting a real-world individual) is a constant symbol, a variable symbol, or an  $n$ -place function of  $n$  terms.  
 $x$  and  $f(x_1, \dots, x_n)$  are terms, where each  $x_i$  is a term.  
A term with no variables is a **ground term**
- An **atomic sentence** (which has value true or false) is an  $n$ -place predicate of  $n$  terms
- A **complex sentence** is formed from atomic sentences connected by the logical connectives:  
 $\neg P, P \vee Q, P \wedge Q, P \rightarrow Q, P \leftrightarrow Q$  where  $P$  and  $Q$  are sentences
- A **quantified sentence** adds quantifiers  $\forall$  and  $\exists$
- A **well-formed formula (wff)** is a sentence containing no “free” variables. That is, all variables are “bound” by universal or existential quantifiers.  
 $(\forall x)P(x,y)$  has  $x$  bound as a universally quantified variable, but  $y$  is free.

# Quantifiers

- **Universal quantification**

- $(\forall x)P(x)$  means that  $P$  holds for **all** values of  $x$  in the domain associated with that variable
- E.g.,  $(\forall x) \text{dolphin}(x) \rightarrow \text{mammal}(x)$

- **Existential quantification**

- $(\exists x)P(x)$  means that  $P$  holds for **some** value of  $x$  in the domain associated with that variable
- E.g.,  $(\exists x) \text{mammal}(x) \wedge \text{lays-eggs}(x)$
- Permits one to make a statement about some object without naming it

# Quantifiers

- Universal quantifiers are often used with “implies” to form “rules”:  
 $(\forall x) \text{ student}(x) \rightarrow \text{smart}(x)$  means “All students are smart”
- Universal quantification is *rarely* used to make blanket statements about every individual in the world:  
 $(\forall x) \text{ student}(x) \wedge \text{smart}(x)$  means “Everyone in the world is a student and is smart”
- Existential quantifiers are usually used with “and” to specify a list of properties about an individual:  
 $(\exists x) \text{ student}(x) \wedge \text{smart}(x)$  means “There is a student who is smart”
- A common mistake is to represent this English sentence as the FOL sentence:  
 $(\exists x) \text{ student}(x) \rightarrow \text{smart}(x)$ 
  - But what happens when there is a person who is *not* a student?

# Quantifier Scope

- Switching the order of universal quantifiers *does not* change the meaning:
  - $(\forall x)(\forall y)P(x,y) \leftrightarrow (\forall y)(\forall x) P(x,y)$
- Similarly, you can switch the order of existential quantifiers:
  - $(\exists x)(\exists y)P(x,y) \leftrightarrow (\exists y)(\exists x) P(x,y)$
- Switching the order of universals and existentials *does* change meaning:
  - Everyone likes someone:  $(\forall x)(\exists y) \text{ likes}(x,y)$
  - Someone is liked by everyone:  $(\exists y)(\forall x) \text{ likes}(x,y)$

# Connections between All and Exists

We can relate sentences involving  $\forall$  and  $\exists$  using De Morgan's laws:

$$(\forall x) \neg P(x) \leftrightarrow \neg(\exists x) P(x)$$

$$\neg(\forall x) P \leftrightarrow (\exists x) \neg P(x)$$

$$(\forall x) P(x) \leftrightarrow \neg (\exists x) \neg P(x)$$

$$(\exists x) P(x) \leftrightarrow \neg(\forall x) \neg P(x)$$

# Quantified inference rules

1. Universal instantiation: If a property is true for all elements in a domain, it is true for any specific element in that domain.
  1.  $\forall x P(x) \therefore P(A)$
2. Universal generalization: If a property is true for an arbitrary element in a domain, then it is true for all elements in the domain.
  1.  $P(A) \wedge P(B) \dots \therefore \forall x P(x)$
3. Existential instantiation: If there exists an element in a domain for which a property is true, we can infer that the property is true for some specific element.
  1.  $\exists x P(x) \therefore P(F)$
4. Existential generalization: If a property is true for some specific element in a domain, then there exists an element in the domain for which the property is true.
  1.  $P(A) \therefore \exists x P(x)$

**1. Statement:** "All dogs bark."

**Inference:** "Rover is a dog, so Rover barks."

**2. Observation:** "If any student studies hard, they pass the exam."

**Generalization:** "Therefore, all students who study hard pass the exam."

**3. Statement:** "There is someone in the office who can help you."

**Inference:** "Let's call this person Alex. Alex can help you."

**4. Observation:** "Maria can solve the puzzle."

**Generalization:** "Therefore, there exists someone who can solve the puzzle."



# Universal instantiation (a.k.a. universal elimination)

- If  $(\forall x) P(x)$  is true, then  $P(C)$  is true, where  $C$  is *any* constant in the domain of  $x$
- Example:  
 $(\forall x) \text{ eats}(\text{Ziggy}, x) \Rightarrow \text{eats}(\text{Ziggy}, \text{IceCream})$
- The variable symbol can be replaced by any ground term, i.e., any constant symbol or function symbol applied to ground terms only

# Existential instantiation (a.k.a. existential elimination)

- From  $(\exists x) P(x)$  infer  $P(c)$
- Example:
  - $(\exists x) \text{ eats}(\text{Ziggy}, x) \rightarrow \text{eats}(\text{Ziggy}, \text{Stuff})$
- Note that the variable is replaced by a **brand-new constant** not occurring in this or any other sentence in the KB
- Also known as skolemization; constant is a **skolem constant**
- In other words, we don't want to accidentally draw other inferences about it by introducing the constant
- Convenient to use this to reason about the unknown object, rather than constantly manipulating the existential quantifier

# Existential generalization (a.k.a. existential introduction)

- If  $P(c)$  is true, then  $(\exists x) P(x)$  is inferred.
- Example  
 $\text{eats}(\text{Ziggy}, \text{IceCream}) \Rightarrow (\exists x) \text{eats}(\text{Ziggy}, x)$
- All instances of the given constant symbol are replaced by the new variable symbol
- Note that the variable symbol cannot already exist anywhere in the expression

# Translating English to FOL

**Every gardener likes the sun.**

$$\forall x \text{ gardener}(x) \rightarrow \text{likes}(x, \text{Sun})$$

**You can fool some of the people all of the time.**

$$\exists x \forall t \text{ person}(x) \wedge \text{time}(t) \rightarrow \text{can-fool}(x, t)$$

**You can fool all of the people some of the time.**

$$\forall x \exists t (\text{person}(x) \rightarrow \text{time}(t) \wedge \text{can-fool}(x, t))$$

$$\forall x (\text{person}(x) \rightarrow \exists t (\text{time}(t) \wedge \text{can-fool}(x, t)))$$

← Equivalent

**All purple mushrooms are poisonous.**

$$\forall x (\text{mushroom}(x) \wedge \text{purple}(x)) \rightarrow \text{poisonous}(x)$$

**No purple mushroom is poisonous.**

$$\neg \exists x \text{ purple}(x) \wedge \text{mushroom}(x) \wedge \text{poisonous}(x)$$

$$\forall x (\text{mushroom}(x) \wedge \text{purple}(x)) \rightarrow \neg \text{poisonous}(x)$$

← Equivalent

**There are exactly two purple mushrooms.**

$$\begin{aligned} \exists x \exists y \text{ mushroom}(x) \wedge \text{purple}(x) \wedge \text{mushroom}(y) \wedge \text{purple}(y) \wedge \neg(x=y) \wedge \forall z \\ (\text{mushroom}(z) \wedge \text{purple}(z)) \rightarrow ((x=z) \vee (y=z)) \end{aligned}$$

**Clinton is not tall.**

$$\neg \text{tall}(\text{Clinton})$$

**X is above Y iff X is on directly on top of Y or there is a pile of one or more other objects directly on top of one another starting with X and ending with Y.**

$$\forall x \forall y \text{ above}(x, y) \leftrightarrow (\text{on}(x, y) \vee \exists z (\text{on}(x, z) \wedge \text{above}(z, y)))$$

# Example: A simple genealogy KB by FOL

- **Build a small genealogy knowledge base using FOL that**
  - contains facts of immediate family relations (spouses, parents, etc.)
  - contains definitions of more complex relations (ancestors, relatives)
  - is able to answer queries about relationships between people
- **Predicates:**
  - `parent(x, y)`, `child(x, y)`, `father(x, y)`, `daughter(x, y)`, etc.
  - `spouse(x, y)`, `husband(x, y)`, `wife(x, y)`
  - `ancestor(x, y)`, `descendant(x, y)`
  - `male(x)`, `female(y)`
  - `relative(x, y)`
- **Facts:**
  - `husband(Joe, Mary)`, `son(Fred, Joe)`
  - `spouse(John, Nancy)`, `male(John)`, `son(Mark, Nancy)`
  - `father(Jack, Nancy)`, `daughter(Linda, Jack)`
  - `daughter(Liz, Linda)`
  - etc.

- **Rules for genealogical relations**

- $(\forall x, y) \text{ parent}(x, y) \leftrightarrow \text{child}(y, x)$
- $(\forall x, y) \text{ father}(x, y) \leftrightarrow \text{parent}(x, y) \wedge \text{male}(x)$  (similarly for  $\text{mother}(x, y)$ )
- $(\forall x, y) \text{ daughter}(x, y) \leftrightarrow \text{child}(x, y) \wedge \text{female}(x)$  (similarly for  $\text{son}(x, y)$ )
- $(\forall x, y) \text{ husband}(x, y) \leftrightarrow \text{spouse}(x, y) \wedge \text{male}(x)$  (similarly for  $\text{wife}(x, y)$ )
- $(\forall x, y) \text{ spouse}(x, y) \leftrightarrow \text{spouse}(y, x)$  (**spouse relation is symmetric**)
- $(\forall x, y) \text{ parent}(x, y) \rightarrow \text{ancestor}(x, y)$
- $(\forall x, y)(\exists z) \text{ parent}(x, z) \wedge \text{ancestor}(z, y) \rightarrow \text{ancestor}(x, y)$
- $(\forall x, y) \text{ descendant}(x, y) \leftrightarrow \text{ancestor}(y, x)$
- $(\forall x, y)(\exists z) \text{ ancestor}(z, x) \wedge \text{ancestor}(z, y) \rightarrow \text{relative}(x, y)$   
(related by common ancestry)
- $(\forall x, y) \text{ spouse}(x, y) \rightarrow \text{relative}(x, y)$  (related by marriage)
- $(\forall x, y)(\exists z) \text{ relative}(z, x) \wedge \text{relative}(z, y) \rightarrow \text{relative}(x, y)$  (**transitive**)
- $(\forall x, y) \text{ relative}(x, y) \leftrightarrow \text{relative}(y, x)$  (**symmetric**)

- **Queries**

- $\text{ancestor}(\text{Jack}, \text{Fred})$  /\* the answer is yes \*/
- $\text{relative}(\text{Liz}, \text{Joe})$  /\* the answer is yes \*/
- $\text{relative}(\text{Nancy}, \text{Matthew})$   
\*/ /\* no answer in general, no if under closed world assumption
- $(\exists z) \text{ ancestor}(z, \text{Fred}) \wedge \text{ancestor}(z, \text{Liz})$

# Semantics of FOL

- **Domain M:** the set of all objects in the world (of interest)
- **Interpretation I:** includes
  - Assign each constant to an object in M
  - Define each function of n arguments as a mapping  $M^n \Rightarrow M$
  - Define each predicate of n arguments as a mapping  $M^n \Rightarrow \{T, F\}$
  - Therefore, every ground predicate with any instantiation will have a truth value
  - In general there is an infinite number of interpretations because  $|M|$  is infinite
- **Define logical connectives:**  $\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$  as in PL
- **Define semantics of  $(\forall x)$  and  $(\exists x)$** 
  - $(\forall x) P(x)$  is true iff  $P(x)$  is true under all interpretations
  - $(\exists x) P(x)$  is true iff  $P(x)$  is true under some interpretation

- **Model**: an interpretation of a set of sentences such that every sentence is *True*
- **A sentence is**
  - **satisfiable** if it is true under some interpretation
  - **valid** if it is true under all possible interpretations
  - **inconsistent** if there does not exist any interpretation under which the sentence is true
- **Logical consequence**:  $S \models X$  if all models of  $S$  are also models of  $X$



# Axioms, definitions and theorems

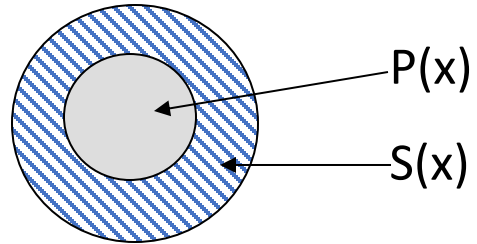
- **Axioms** are facts and rules that attempt to capture all of the (important) facts and concepts about a domain; axioms can be used to prove **theorems**
  - Mathematicians don't want any unnecessary (dependent) axioms –ones that can be derived from other axioms
  - Dependent axioms can make reasoning faster, however
  - Choosing a good set of axioms for a domain is a kind of design problem
- A **definition** of a predicate is of the form " $p(X) \leftrightarrow \dots$ " and can be decomposed into two parts
  - **Necessary** description: " $p(x) \rightarrow \dots$ "
  - **Sufficient** description " $p(x) \leftarrow \dots$ "
  - Some concepts don't have complete definitions (e.g.,  $\text{person}(x)$ )

## More on definitions

- A **necessary** condition must be satisfied for a statement to be true.
- A **sufficient** condition, if satisfied, assures the statement's truth.
- Duality: "P is sufficient for Q" is the same as "Q is necessary for P."
- Examples: define  $\text{father}(x, y)$  by  $\text{parent}(x, y)$  and  $\text{male}(x)$ 
  - $\text{parent}(x, y)$  is a necessary (**but not sufficient**) description of  $\text{father}(x, y)$ 
    - $\text{father}(x, y) \rightarrow \text{parent}(x, y)$
  - $\text{parent}(x, y) \wedge \text{male}(x) \wedge \text{age}(x, 35)$  is a **sufficient (but not necessary)** description of  $\text{father}(x, y)$ :
$$\text{father}(x, y) \leftarrow \text{parent}(x, y) \wedge \text{male}(x) \wedge \text{age}(x, 35)$$
  - $\text{parent}(x, y) \wedge \text{male}(x)$  is a **necessary and sufficient** description of  $\text{father}(x, y)$ 
$$\text{parent}(x, y) \wedge \text{male}(x) \leftrightarrow \text{father}(x, y)$$

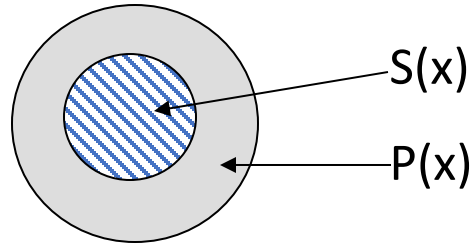
# More on definitions

$S(x)$  is a  
necessary  
condition of  $P(x)$



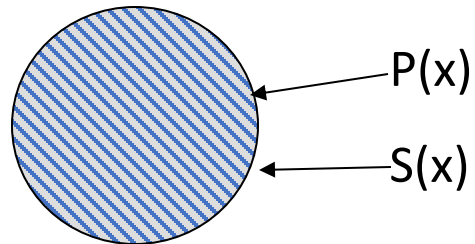
$$(\forall x) P(x) \Rightarrow S(x)$$

$S(x)$  is a  
sufficient  
condition of  $P(x)$



$$(\forall x) P(x) \Leftarrow S(x)$$

$S(x)$  is a  
necessary and  
sufficient  
condition of  $P(x)$



$$(\forall x) P(x) \Leftrightarrow S(x)$$

## Higher-order logic

- FOL only allows to quantify over variables, and variables can only range over objects.
- HOL allows us to quantify over relations
- Example: (quantify over functions)  
“two functions are equal iff they produce the same value for all arguments”  
$$\forall f \forall g (f = g) \leftrightarrow (\forall x f(x) = g(x))$$
- Example: (quantify over predicates)  
$$\forall r \text{ transitive}(r) \rightarrow (\forall xyz) r(x,y) \wedge r(y,z) \rightarrow r(x,z)$$
- More expressive, but **undecidable**. (there isn't an effective algorithm to decide whether all sentences are valid)
  - First-order logic is decidable only when it uses predicates with only one argument.

# Expressing uniqueness

- Sometimes we want to say that there is a single, unique object that satisfies a certain condition
- “There exists a unique  $x$  such that  $\text{king}(x)$  is true”
  - $\exists x \text{ king}(x) \wedge \forall y (\text{king}(y) \rightarrow x=y)$
  - $\exists x \text{ king}(x) \wedge \neg \exists y (\text{king}(y) \wedge x \neq y)$
  - $\exists! x \text{ king}(x)$
- “Every country has exactly one ruler”
  - $\forall c \text{ country}(c) \rightarrow \exists! r \text{ ruler}(c,r)$
- Iota operator: “ $\iota x P(x)$ ” means “the unique  $x$  such that  $p(x)$  is true”
  - “The unique ruler of Freedonia is dead”
  - $\text{dead}(\iota x \text{ ruler}(\text{freedonia},x))$

# Notational differences

- **Different symbols** for *and*, *or*, *not*, *implies*, ...

- $\forall \exists \Rightarrow \Leftrightarrow \wedge \vee \neg \bullet \supset$
- $p \vee (q \wedge r)$
- $p + (q * r)$
- etc

- **Prolog**

`cat(X) :- furry(X), meows (X), has(X, claws)`

- **Lispy notations**

```
(forall ?x (implies (and (furry ?x)
                          (meows ?x)
                          (has ?x claws))
                  (cat ?x)))
```

# Logical agents for the Wumpus World

Three (non-exclusive) agent architectures:

- Reflex agents
  - Have rules that classify situations, specifying how to react to each possible situation
- Model-based agents
  - Construct an internal model of their world
- Goal-based agents
  - Form goals and try to achieve them

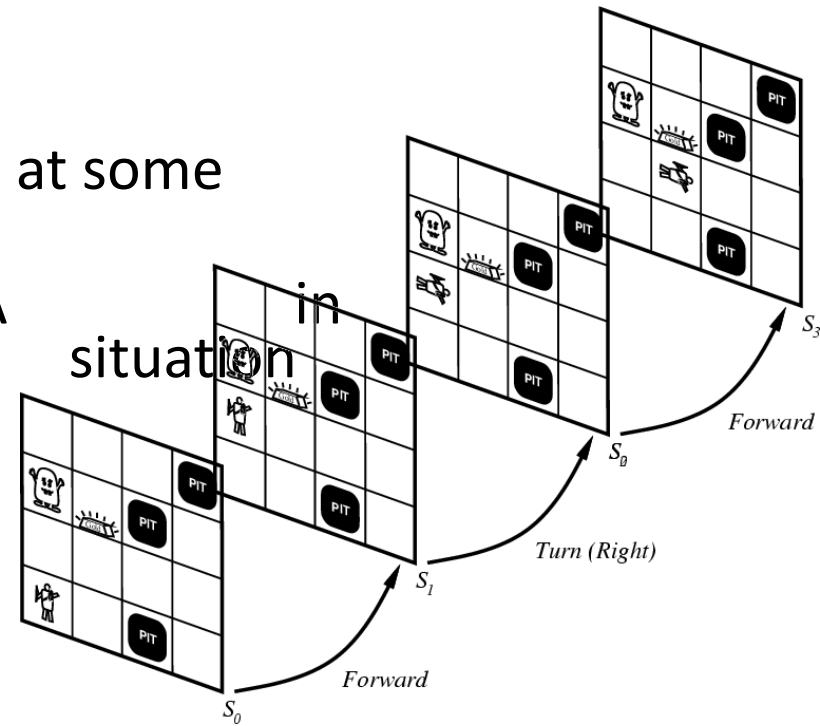
# A simple reflex agent

- Rules to **map percepts into observations**:
  - $\forall b,g,u,c,t \text{ Percept}([ \text{Stench}, b, g, u, c ], t) \rightarrow \text{Stench}(t)$
  - $\forall s,g,u,c,t \text{ Percept}([ s, \text{Breeze}, g, u, c ], t) \rightarrow \text{Breeze}(t)$
  - $\forall s,b,u,c,t \text{ Percept}([ s, b, \text{Glitter}, u, c ], t) \rightarrow \text{AtGold}(t)$
- Rules to **select an action given observations**:
  - $\forall t \text{ AtGold}(t) \rightarrow \text{Action}(\text{Grab}, t);$
- Some difficulties:
  - Consider Climb. There is no percept that indicates the agent should climb out – **position and holding gold are not part of the percept sequence**
  - Loops – the percept will be repeated when you return to a square, which should cause the same response (unless we maintain some **internal model of the world**)

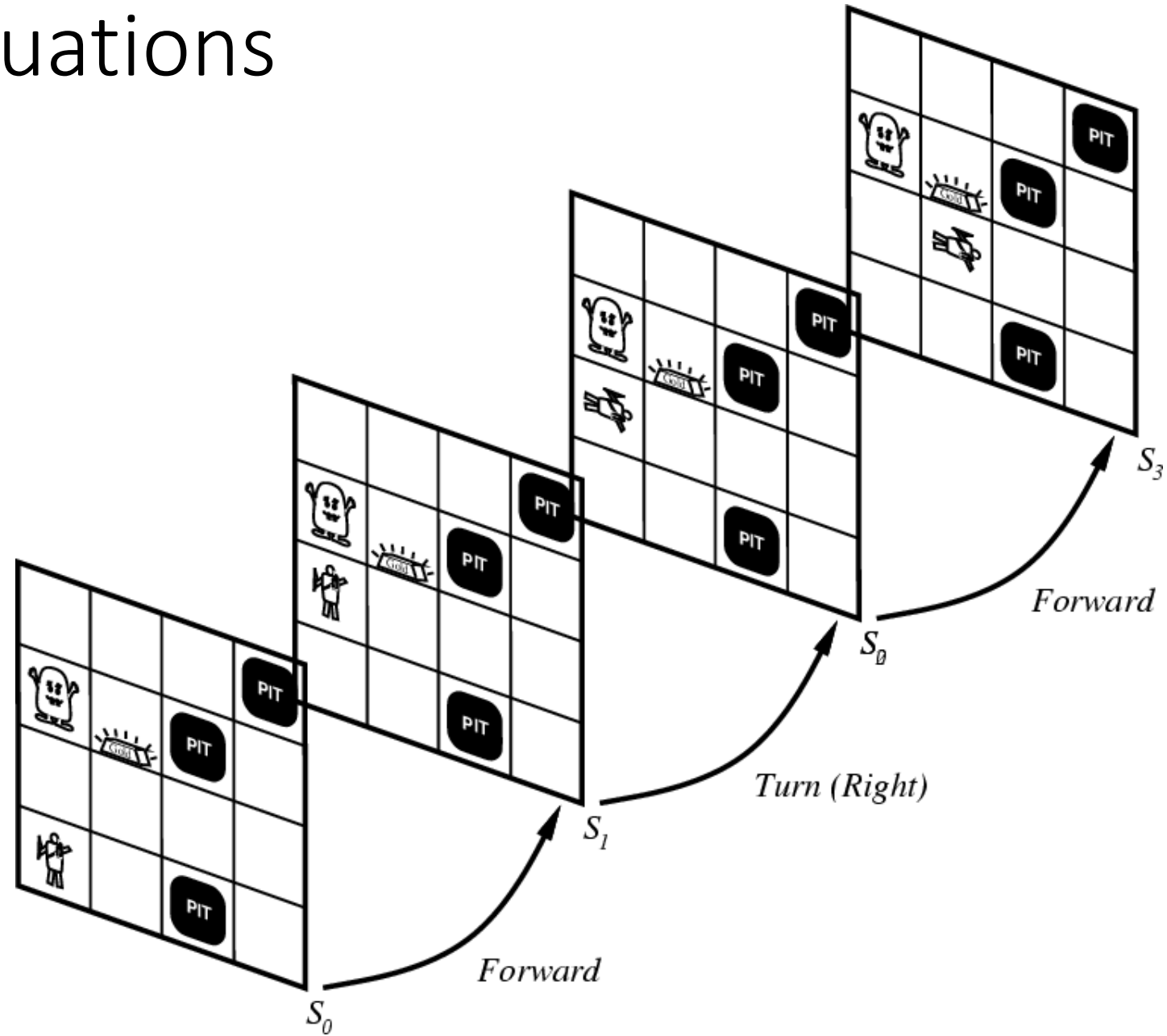


# Representing change

- Representing change in the world in logic can be tricky.
- One way is just to change the KB
  - Add and delete sentences from the KB to reflect changes
  - How do we remember the past, or reason about changes?
- **Situation calculus** is another way
- A **situation** is a snapshot of the world at some instant in time
- When the agent performs an action A in situation  $S_1$ , the result is a new  $S_2$ .



# Situations



# Situation calculus

- A **situation** is a snapshot of the world at an interval of time during which nothing changes
- Every true or false statement is made with respect to a particular situation.
  - Add **situation variables** to every predicate.
  - $\text{at}(\text{Agent}, 1, 1)$  becomes  $\text{at}(\text{Agent}, 1, 1, s_0)$ :  $\text{at}(\text{Agent}, 1, 1)$  is true in situation (i.e., state)  $s_0$ .
  - Alternatively, add a special 2<sup>nd</sup>-order predicate, **holds(f,s)**, that means “f is true in situation s.” E.g.,  $\text{holds}(\text{at}(\text{Agent}, 1, 1), s_0)$
- Add a new function, **result(a,s)**, that maps a situation s into a new situation as a result of performing action a. For example,  $\text{result}(\text{forward}, s)$  is a function that returns the successor state (situation) to s
- Example: The action agent-walks-to-location-y could be represented by
  - $(\forall x)(\forall y)(\forall s) (\text{at}(\text{Agent}, x, s) \wedge \neg \text{onbox}(s)) \rightarrow \text{at}(\text{Agent}, y, \text{result}(\text{walk}(y), s))$

# Deducing hidden properties

- From the perceptual information we obtain in situations, we can **infer properties of locations**

$\forall l, s \text{ at}(\text{Agent}, l, s) \wedge \text{Breeze}(s) \rightarrow \text{Breezy}(l)$

$\forall l, s \text{ at}(\text{Agent}, l, s) \wedge \text{Stench}(s) \rightarrow \text{Smelly}(l)$

- Neither Breezy nor Smelly need situation arguments because pits and Wumpuses do not move around

# Deducing hidden properties II

- We need to write some rules that relate various aspects of a single world state (as opposed to across states)
- There are two main kinds of such rules:
  - **Causal rules** reflect the assumed direction of causality in the world:
$$(\forall l1, l2, s) \text{ At(Wumpus, } l1, s) \wedge \text{ Adjacent}(l1, l2) \rightarrow \text{ Smelly}(l2)$$
$$(\forall l1, l2, s) \text{ At(Pit, } l1, s) \wedge \text{ Adjacent}(l1, l2) \rightarrow \text{ Breezy}(l2)$$
Systems that reason with causal rules are called **model-based reasoning systems**
  - **Diagnostic rules** infer the presence of **hidden properties** directly from the percept-derived information. We have already seen two diagnostic rules:
$$(\forall l, s) \text{ At(Agent, } l, s) \wedge \text{ Breeze}(s) \rightarrow \text{ Breezy}(l)$$
$$(\forall l, s) \text{ At(Agent, } l, s) \wedge \text{ Stench}(s) \rightarrow \text{ Smelly}(l)$$

# Representing change: The frame problem

- **Frame axioms:** If property  $x$  doesn't change as a result of applying action  $a$  in state  $s$ , then it stays the same.
  - $\text{On}(x, z, s) \wedge \text{Clear}(x, s) \rightarrow$   
 $\text{On}(x, \text{table}, \text{Result}(\text{Move}(x, \text{table}), s)) \wedge$   
 $\neg \text{On}(x, z, \text{Result}(\text{Move}(x, \text{table}), s))$
  - $\text{On}(y, z, s) \wedge y \neq x \rightarrow \text{On}(y, z, \text{Result}(\text{Move}(x, \text{table}), s))$
  - The proliferation of frame axioms becomes very cumbersome in complex domains

# The frame problem II

- **Successor-state axiom**: General statement that characterizes every way in which a particular predicate can become true:
  - Either it can be **made true**, or it can **already be true and not be changed**:
  - $\text{On}(x, \text{table}, \text{Result}(a, s)) \leftrightarrow$   
 $[\text{On}(x, z, s) \wedge \text{Clear}(x, s) \wedge a = \text{Move}(x, \text{table})] \wedge$   
 $[\text{On}(x, \text{table}, s) \wedge a \neq \text{Move}(x, z)]$
- In complex worlds, where you want to reason about longer chains of action, even these types of axioms are too cumbersome
  - Planning systems use special-purpose inference methods to reason about the expected state of the world at any point in time during a multi-step plan

# Qualification problem

- Qualification problem:
  - How can you possibly characterize every single effect of an action, or every single exception that might occur?
  - When I put my bread into the toaster, and push the button, it will become toasted after two minutes, unless...
    - The toaster is broken, or...
    - The power is out, or...
    - I blow a fuse, or...
    - A neutron bomb explodes nearby and fries all electrical components, or...
    - A meteor strikes the earth, and the world we know it ceases to exist, or...



# Ramification problem

- Similarly, it's just about impossible to characterize every side effect of every action, at every possible level of detail:
  - When I put my bread into the toaster, and push the button, the bread will become toasted after two minutes, and...
    - The crumbs that fall off the bread onto the bottom of the toaster over tray will also become toasted, and...
    - Some of the aforementioned crumbs will become burnt, and...
    - The outside molecules of the bread will become "toasted," and...
    - The inside molecules of the bread will remain more "breadlike," and...
    - The toasting process will release a small amount of humidity into the air because of evaporation, and...
    - The heating elements will become a tiny fraction more likely to burn out the next time I use the toaster, and...
    - The electricity meter in the house will move up slightly, and...

# Knowledge engineering!

- Modeling the “right” conditions and the “right” effects at the “right” level of abstraction is very difficult
- Knowledge engineering (creating and maintaining knowledge bases for intelligent reasoning) is an entire field of investigation
- Many researchers hope that automated knowledge acquisition and machine learning tools can fill the gap:
  - Our intelligent systems should be able to **learn** about the conditions and effects, just like we do!
  - Our intelligent systems should be able to learn when to pay attention to, or reason about, certain aspects of processes, depending on the context!

# Preferences among actions

- A problem with the Wumpus world knowledge base that we have built so far is that it is difficult to decide which action is best among a number of possibilities.
- For example, to decide between a forward and a grab, axioms describing when it is OK to move to a square would have to mention glitter.
- This is not modular!
- We can solve this problem by **separating facts about actions from facts about goals**. This way our **agent can be reprogrammed just by asking it to achieve different goals**.

# Preferences among actions

- The first step is to describe the desirability of actions independent of each other.
- In doing this we will use a simple scale: actions can be Great, Good, Medium, Risky, or Deadly.
- Obviously, the agent should always do the best action it can find:

$$(\forall a,s) \text{ Great}(a,s) \rightarrow \text{Action}(a,s)$$

$$(\forall a,s) \text{ Good}(a,s) \wedge \neg(\exists b) \text{ Great}(b,s) \rightarrow \text{Action}(a,s)$$

$$(\forall a,s) \text{ Medium}(a,s) \wedge (\neg(\exists b) \text{ Great}(b,s) \vee \text{ Good}(b,s)) \rightarrow \text{Action}(a,s)$$

...

# Preferences among actions

- We use this action quality scale in the following way.
- Until it finds the gold, the basic strategy for our agent is:
  - Great actions include picking up the gold when found and climbing out of the cave with the gold.
  - Good actions include moving to a square that's OK and hasn't been visited yet.
  - Medium actions include moving to a square that is OK and has already been visited.
  - Risky actions include moving to a square that is not known to be deadly or OK.
  - Deadly actions are moving into a square that is known to have a pit or a Wumpus.

# Goal-based agents

- Once the gold is found, it is necessary to change strategies. So now we need a new set of action values.
- We could encode this as a rule:
  - $(\forall s) \text{Holding}(\text{Gold}, s) \rightarrow \text{GoalLocation}([1,1], s)$
- We must now decide how the agent will work out a sequence of actions to accomplish the goal.
- Three possible approaches are:
  - **Inference**: good versus wasteful solutions
  - **Search**: make a problem with operators and set of states
  - **Planning**: to be discussed later