NFSU

विद्यया अमृतं अश्नुते

National Forensic Sciences University, Goa

MTECH AIDS

Mathematical and Computational Foundation for AI

CTMSAIDS SI P1

Monday, 14-10-2024

2403477007003

Timing: 11:00 to 12:30 PM | I Semester | Max mark: 50

1. **Attempt any Four questions**

   (a) Find a basis for the eigenspace corresponding to each listed eigenvalue for the following matrices:
   $A = \begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix}$ Eigenvalues: $\lambda = 1, 5$ (5)

   (b) Find a basis for the eigenspace corresponding to each listed eigenvalue for the following matrices:
   $A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$ Eigenvalue: $\lambda = 4$ (5)

   (c)
   $$A = \begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$$
   (5)
   Determine whether $\lambda = 2$ is an eigenvalue of the matrix A. Explain your reasoning and show any necessary calculations to support your answer.

   (d) Define Linear Independence. Provide one suitable example. (5)

   (e) Write a brief over view of Simple Linear Regression. (5)

2. **Attempt all questions**

   (a) Write a short note on PCA and their applications. (5)

   (b) Given the vectors: (5)
   $$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$
   Compute the following quantities: $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{u}}\right)\mathbf{u}$

   (c) Solve the following system of linear equations using the row echelon method: (5)
   $$\begin{aligned} 2x + 3y - z &= 1 \\ 4x + y + z &= 5 \\ -2x + 2y + 3z &= 7 \end{aligned}$$

-56

$1-(-1)$

3. **Attempt any one questions**

(a) Determine which sets of vectors are orthogonal. (8)

$$\mathbf{u_1} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, \quad \mathbf{u_2} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{u_3} = \begin{pmatrix} 3 \\ -4 \\ -7 \end{pmatrix}$$

(b) Using Gram-Schmidt process how do you convert a set of linearly independent vectors into an orthonormal set. Write the Process of it and with one example. (8)

4. **Attempt any one questions**

(a) Define Orthogonal Matrices and key properties. Give one example. (7)

(b) Verify the parallelogram law for vectors $\mathbf{u}$ and $\mathbf{v}$ in R^n: (7)

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

Best wishes