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### National Forensic Sciences University, Goa MTECH AIDS

## Mathematical and Computational Foundation for AI CTMSAIDS SI P1

Monday, 14-10-2024

240347007003

Timing: 11:00 to 12:30 PM

I Semester

Max mark: 50

#### 1. Attempt any Four questions

(a) Find a basis for the eigenspace corresponding to each listed eigenvalue for the following matrices:

$$A = \begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix}$$
 Eigenvalues:  $\lambda = 1, 5$  (5)

(b) Find a basis for the eigenspace corresponding to each listed eigenvalue for the following matrices:

$$A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix} \text{ Eigenvalue: } \lambda = 4 \tag{5}$$

(c)

$$A = \begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$$

Determine whether  $\lambda=2$  is an eigenvalue of the matrix A. Explain your reasoning and show any necessary calculations to support your answer.

- (d) Define Linear Independence. Provide one suitable example.
- (e) Write a brief over view of Simple Linear Regression.

(5)

(5)

(5)

#### 2. Attempt all questions

(b) Given the vectors:

(a) Write a short note on PCA and their applications.

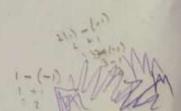
CA and their applications. (5)

$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

Compute the following quantities:  $(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{u}}) \mathbf{u}$ 

(c) Solve the following system of linear equations using the row echelon method:

$$2x + 3y - z = 1$$
$$4x + y + z = 5$$
$$-2x + 2y + 3z = 7$$



## 3. Attempt any one questions

(a) Determine which sets of vectors are orthogonal.

(8)

$$\mathbf{u_1} = \begin{pmatrix} -1\\4\\3 \end{pmatrix}, \quad \mathbf{u_2} = \begin{pmatrix} 5\\2\\1 \end{pmatrix}, \quad \mathbf{u_3} = \begin{pmatrix} 3\\-4\\-7 \end{pmatrix}$$

(b) Using Gram-Schmidt process how do you convert a set of linearly independent vectors into an orthonormal set. Write the Process of it and with one example.

(8)

#### 4. Attempt any one questions

(a) Define Orthogonal Matrices and key properties. Give one example.

(7)

(b) Verify the parallelogram law for vectors  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbb{R}^n$ :

(7)

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

Best wishes