National Forensic Sciences University

School of Cyber Security and Digital Forensics

Course Name: M.Tech Artificial Intelligence and Data Science (Batch: 2024-26)
Semester - I Exam: TA - I (September- 2024)

Subject Code: CTMTAIDS SI P1 **Time:** 10.30 am to 11.15 am
Subject Name: Mathematical and Computational Foundation for Artificial Intelligence
Date: 09-09-2024

Q1. Which of the following is a valid definition of a vector space?
- A) A set of elements with a defined magnitude and direction
- B) A set with two operations: addition and scalar multiplication, satisfying specific axioms
- C) A collection of linearly dependent vectors
- D) A set of vectors all lying on a single plane

Q2. What does it mean for a set of vectors to be linearly independent?
- A) All vectors have the same magnitude
- B) No vector in the set can be written as a linear combination of the others
- C) The vectors all lie in the same direction
- D) The vectors all lie in the same plane

Q3. Which of the following is true about the matrix representation of data?
- A) A matrix can only represent a set of vectors in a two-dimensional space
- B) The rows of a matrix always represent the features of a dataset
- C) The columns of a matrix can represent vectors in a vector space
- D) Matrix representations are only useful for square matrices

Q4. In a vector space, the norm of a vector refers to:
- A) The direction of the vector
- B) The length or magnitude of the vector
- C) The angle between two vectors
- D) The inner product of the vector with itself

Q5. In a vector space, two vectors are orthogonal if:
- A) Their inner product is zero
- B) Their magnitudes are equal
- C) They lie on the same line
- D) They have the same direction

Q6. Draw each of the following vectors in standard position in R^2
(a) $v = (3,2)$ (b) $x = (1,-3)$ (c) $w = (-0.5,3)$ (d) $y = (-2,-1)$

Q7. Compute the dot product $v \cdot w$ of each of the following pairs of vectors.
- (a) $v = (-2,4)$, $w = (2,1)$
- (b) $v = (1,2,3)$, $w = (-3,2,-1)$
- (c) $v = (3,-1,0,1)$, $w = (0,2,1,3)$
- (d) $v = (\sqrt{2}, \sqrt{3}, \sqrt{5})$, $w = (\sqrt{2}, \sqrt{3}, \sqrt{5})$

Q8. Prove that the following two vectors form the vector space
(1) $v1 = (3,4)$
(2) $v2 = (-1,2)$
(3) Scale $a = 2$

Q9. Let V be a vector space of all 2-dimensional real vectors. Consider the following two vectors:
$v1 = (1,2)$
$v2 = (3,4)$
Determine if v1 and v2 form a basis for R2. If they form a basis, express the vector $v3=(5,6)$ as a linear combination of v1 and v2.