National Forensic Sciences University

School of Cyber Security and Digital Forensics

Course Name: M.Tech Artificial Intelligence and Data Science (Batch: 2024-26)
Semester - I Exam: TA - I (September- 2024)

Subject Code: CTMTAIDS SI P1 **Time:** 10.30 am to 11.15 am
Subject Name: Mathematical and Computational Foundation for Artificial Intelligence
Date: 09-09-2024

Q1. Which of the following is a valid definition of a vector space?
- A) A set of elements with a defined magnitude and direction
- B) A set with two operations: addition and scalar multiplication, satisfying specific axioms
- C) A collection of linearly dependent vectors
- D) A set of vectors all lying on a single plane

Q2. What does it mean for a set of vectors to be linearly independent?
- A) All vectors have the same magnitude
- B) No vector in the set can be written as a linear combination of the others
- C) The vectors all lie in the same direction
- D) The vectors all lie in the same plane

Q3. Which of the following is true about the matrix representation of data?
- A) A matrix can only represent a set of vectors in a two-dimensional space
- B) The rows of a matrix always represent the features of a dataset
- C) The columns of a matrix can represent vectors in a vector space
- D) Matrix representations are only useful for square matrices

Q4. In a vector space, the norm of a vector refers to:
- A) The direction of the vector
- B) The length or magnitude of the vector
- C) The angle between two vectors
- D) The inner product of the vector with itself

Q5. In a vector space, two vectors are orthogonal if:
- A) Their inner product is zero
- B) Their magnitudes are equal
- C) They lie on the same line
- D) They have the same direction

Q6. Draw each of the following vectors in standard position in R^2
(a) $v = (3,2)$ (b) $x = (1,-3)$ (c) $w = (-0.5,3)$ (d) $y = (-2,-1)$

Q7. Compute the dot product $v \cdot w$ of each of the following pairs of vectors.
- (a) $v = (-2,4)$, $w = (2,1)$
- (b) $v = (1,2,3)$, $w = (-3,2,-1)$
- (c) $v = (3,-1,0,1)$, $w = (0,2,1,3)$
- (d) $v = (\sqrt{2}, \sqrt{3}, \sqrt{5})$, $w = (\sqrt{2}, \sqrt{3}, \sqrt{5})$

Q8. Prove that the following two vectors form the vector space
(1) v1= (3,4)
(2) v2=(−1,2)
(3) Scale a = 2

Q9. Let V be a vector space of all 2-dimensional real vectors. Consider the following two vectors:
v1 = (1,2)
v2 = (3,4)
Determine if v1 and v2 form a basis for R2. If they form a basis, express the vector v3=(5,6) as a linear combination of v1 and v2.

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Course Name: M.Tech Artificial Intelligence and Data Science (Batch: 2024-26)
Semester – III
Subject Code: CTMTAIDS SI P1 **Time:** 11:00-12.30 pm
Subject Name: Mathematical and Computational Foundation for Artificial Intelligence
Exam: Mid Semester Examination (October - 2024) **Date:** 7-10-2024

Q1. Find k so that u and v are orthogonal **5 marks**

(a) u = (1, k, 3) and v = (2, -5, 4)
(b) u = (2, 3k, -4, 1,5) and v = (6, -1, 3, 7, 2k)

Q2. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & -4 & 13 \\ 2 & 1 & -5 \end{bmatrix}$. Perform LU decomposition on the matrix **7 marks**

Q3. Solve the following system of linear equations using Gaussian Elimination **8 marks**

$$-3x_1 + 2x_2 - x_3 = -1$$
$$6x_1 - 6x_2 + 7x_3 = -7$$
$$3x_1 - 4x_2 + 4x_3 = -6$$

Q4. Which of the following matrices are diagonalizable with reasons? Show the decomposition as well

(a) $B = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (b) $C = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **10 marks**

Q5. Calculate the singular value decomposition of

$$D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$ **10 marks**

Q6. Perform Cholesky decomposition of the following system of equations

$$4x_1 + 2x_2 + 14x_3 = 14$$
$$2x_1 + 17x_2 - 5x_3 = -101$$
$$14x_1 - 5x_2 + 83x_3 = 155$$ **10 marks**

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School of Cyber Security and Digital Forensics

Course Name: M.Tech Artificial Intelligence and Data Science (Batch: 2024-26)

Semester – III

Subject Code: CTMTAIDS SI P1 **Time:** 11:00-12.30 pm

Subject Name: Mathematical and Computational Foundation for Artificial Intelligence

Exam: Block-Mid Semester Examination (October - 2024) **Date:** 22-10-2024

Q1. Compute the dot product $v \cdot w$ of each of the following pairs of vectors. **10 marks**

(a) $v = (-2,4)$, $w = (2,1)$

(b) $v = (1,2,3)$, $w = (-3,2,-1)$

(c) $v = (3,-1,0,1)$, $w = (0,2,1,3)$

(d) $v = (\sqrt{2}, \sqrt{3}, \sqrt{5})$, $w = (\sqrt{2}, \sqrt{3}, \sqrt{5})$

Q2. Let $A = \begin{bmatrix} 2 & -1 & -2 \\ -4 & 6 & 3 \\ -4 & -2 & 8 \end{bmatrix}$. Perform LU decomposition on the matrix **10 marks**

Q3. Solve the following system of linear equations using Gaussian Elimination **10 marks**

$$2x_1 + 3x_2 - x_3 = 5$$
$$4x_1 + 7x_2 + 2x_3 = 11$$
$$-2x_1 + 4x_2 + 5x_3 = -1$$

Q4. Draw each of the following vectors in standard position in R^2

(a) $v = (3,2)$ (b) $x = (1,-3)$ (c) $w = (-0.5,3)$ (d) $y = (-2,-1)$

10 marks

Q5. Calculate the singular value decomposition of

$$D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

10 marks