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Q.1 (b) $T(n) = 2$; if $n = 0$

$= 2t_{n-1} + 2^n + 3$, otherwise

$t_n(T(n)) = 2t_{n-1} + 2^n + 3$ — (i)

$t_n - 2t_{n-1} = 2^n + 3$

$2t_n - 4t_{n-1} = 2 \cdot 2^n + 6$

$2t_n - 4t_{n-1} = 2^{n+1} + 6$ — (1)

$n = n-1$

$2t_{n-1} - 4t_{n-2} = 2^n + 6$ — (2)

subtract (1) and (2)

$$\begin{array}{r} 2t_n - 4t_{n-1} = \cancel{2^n + 6} \\ - \quad 2t_{n-1} - 4t_{n-2} = \cancel{2^n + 6} \\ \hline 2t_n - 6t_{n-1} - 4t_{n-2} = 0 \end{array}$$

$2t_n$	$-$	$6t_{n-1}$	$-$	$4t_{n-2}$	$=$	0
$0=2$		$0=1$		$0=0$		
$2x^2$	$-$	$6x^1$	$-$	$4x^0$	$=$	0

$2x^2 - 6x - 4 = 0$

$x_1 = -0.56$, $x_2 = 3.56$

roots are real and distinct

$t_n = c_1(x_1)^n + c_2(x_2)^n$

$t_n = c_1(-0.56)^n + c_2(3.56)^n$ — (ii)

$0.56c_1 + 0.56c_2 = 1.12$

$\begin{array}{r} 0.56 \\ \times 2 \\ \hline 1.12 \end{array}$

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Put ~~n = 2~~ $n = 0$ in (ii)

$t_0 = c_1(-0.56)^0 + c_2(3.56)^0$

$2 = c_1 + c_2$ — (3)

Put $n = 1$ in (i)

$\cancel{t_n = 2t_1 - 6t_{1-1} - 4t_{1-2}}$

$\cancel{t(1) = 2t_1 - 6(t_0) - 4t_{-1}}$

$\cancel{t_1 - 2t_1}$

$t_n = 2t_{n-1} + 2^n + 3$

$t_1 = 2t_{1-1} + 2^1 + 3$

$t_1 = 9$

$\therefore$ Put $n = 1$ in (ii)

$t_1 = c_1(-0.56)^1 + c_2(3.56)^1$

$9 = -0.56c_1 + 3.56c_2$ — (4)

~~By~~ From (3) and (4)

$c_1 = -9$ $c_2 = 11$

$t_n = -9(-0.56)^n + 11(3.56)^n$

eliminate negitive

$t_n = \theta(3.56)^n$

Q.2 (a) Insertion Sort Best case & worst case

Algorithm:

For $j \to 2$ to length (A)
key = A[j]
$i = j-1$
while $i > 0$ and A[i] > key
A[i+1] = A[i]
$i = i-1$
A[i+1] = key.

cost	No. of times
c_1	n
c_2	$n-1$
c_3	$n-1$
c_4	$\sum_{j=2}^{n} t_j$
c_5	$\sum_{j=2}^{n} (t_j-1)$
c_6	$\sum_{j=2}^{n} (t_j-1)$
c_7	$n-1$

$\sum_{j=2}^{n}(1) = (n-1)$

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$$T(n) = c_1(n) + c_2(n-1) + c_3(n-1) + c_4\left(\sum_{j=2}^{n} t_j\right) + c_5\left(\sum_{j=2}^{n}(t_j-1)\right) + c_6\left(\sum_{j=2}^{n}(t_j-1)\right) + c_7(n-1) \quad \text{--- (1)}$$

For Best case:

If the array is already in ascending order then,

$\boxed{t_j = 1}$

$$T(n) = c_1(n) + c_2(n-1) + c_3(n-1) + c_4\left(\sum_{j=2}^{n}(1)\right) + c_5\left(\sum_{j=2}^{n}\underbrace{(1-1)}_{0}\right) + c_6\left(\sum_{j=2}^{n}\underbrace{(1-1)}_{0}\right) + c_7(n-1)$$

$$= c_1(n) + c_2(n-1) + c_3(n-1) + c_4(n-1) + c_7(n-1)$$

$$= nc_1 + nc_2 - c_2 + nc_3 - c_3 + nc_4 - c_4 + nc_7 - c_7$$

$$= nc_1 + nc_2 + nc_3 + nc_4 + nc_7 - c_2 - c_3 - c_4 - c_7$$

$$= (c_1 + c_2 + c_3 + c_4 + c_7)n + (-c_2 - c_3 - c_4 - c_7)$$

Now it is in the form of $an+b$.

$$\sum_{j=2}^{n} j = \frac{n(n+1)}{2} - 1 \qquad \sum_{j=2}^{n} (1) = (n-1)$$

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i.e. linear eqn of n

$T(n) = \Theta(n)$

$T(n) = \Omega(n)$

For worst case:

If the array is in decending order then,

$t_j = j$.

$$T(n) = c_1(n) + c_2(n-1) + c_3(n-1) + c_4\left(\sum_{j=2}^{n} j\right) + c_5\left(\sum_{j=2}^{n}(j-1)\right) + c_6\left(\sum_{j=2}^{n}(j-1)\right) + c_7(n-1)$$

$$= c_1(n) + c_2(n-1) + c_3(n-1) + c_4\sum_{j=2}^{n} j + c_5\sum_{j=2}^{n} j - c_5\sum_{j=2}^{n}(1) + c_6\sum_{j=2}^{n} j - c_6\sum_{j=2}^{n}(1) + c_7(n-1)$$

$$= nc_1 + nc_2 - c_2 + nc_3 - c_3 + c_4\left(\frac{n(n+1)}{2} - 1\right) + c_5\left(\frac{n(n+1)}{2} - 1\right) - c_5(n-1) + c_6\left(\frac{n(n+1)}{2} - 1\right) - c_6(n-1) + nc_7 - c_7$$

$$= nc_1 + nc_2 - c_2 + nc_3 - c_3 + c_4\left(\frac{n(n+1)}{2}\right) - c_4$$

$$+ c_5\left(\frac{n(n+1)}{2}\right) - c_5 - nc_5 + c_5 + c_6\left(\frac{n(n+1)}{2}\right)$$

$$- c_6 - nc_6 + c_6 + nc_7 - c_7.$$

$$= nc_1 + nc_2 - c_2 + nc_3 - c_3 + \frac{n^2c_4 + nc_4}{2} - c_4$$

$$+ \frac{n^2c_5 + nc_5}{2} - \cancel{c_5} - nc_5 + \cancel{c_5} + \frac{n^2c_6 + nc_6}{2}$$

$$- \cancel{c_6} - nc_6 + \cancel{c_6} + nc_7 - c_7$$

$$= \frac{n^2c_4}{2} + \frac{n^2c_5}{2} + \frac{n^2c_6}{2} + nc_1 + nc_2$$

$$+ nc_3 + \frac{nc_4}{2} + \frac{nc_5}{2} + \frac{nc_6}{2} + nc_7$$

$$- nc_5 - nc_6 - c_2 - c_3 - c_4 - c_7$$

$$= \frac{1}{2}(c_4 + c_5 + c_6)n^2 + n\left(c_1 + c_2 + c_3 + \frac{c_4}{2} + \frac{c_5}{2}\right.$$

$$\left. - c_5 + \frac{c_6}{2} - c_6 + c_7\right) + (-c_2 - c_3 - c_4 - c_7)$$

It is in the form of $an^2 + bn + c$
i.e. quadratic function $f(n)$

$T(n) = \Theta(n^2)$

$T(n) = O(n^2)$

Q.2 (b)

1. $T(n) = 4T(n/2) + n$

$a = 4$, $b = 2$, $f(n) = n$

LHS $= n^{\log_b a}$ RHS $= f(n)$

$n^{\log_2 4}$ n

n^2 $>$ n

$\therefore$ LHS $>$ RHS

$\therefore$ case i is applicable.

$\therefore T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a = 4$, $b = 2$, $f(n) = n^2$

LHS $= n^{\log_b a}$ RHS $= f(n)$

$n^{\log_2 4}$ n^2

n^2 $=$ n^2

LHS $=$ RHS

$\therefore$ case 2 is applicable.

$T(n) = \Theta(n^2 \log n)$

3. $T(n) = 4T(n/2) + n^3$

$a = 4$, $b = 2$, $f(n) = n^3$

LHS $= n^{\log_b a}$ RHS $= f(n)$

$n^{\log_2 4}$ n^3

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$n^2 < n^3$

LHS < RHS

$\therefore$ case 3 might be applicable

RC test

$a \cdot f(n/b)$ $\quad$ $c \cdot f(n)$

$4 \cdot f(n/2)^3$ $\quad$ $c \cdot n^3$

$4 \cdot f(n^3/8)$ $\quad$ $c \cdot n^3$

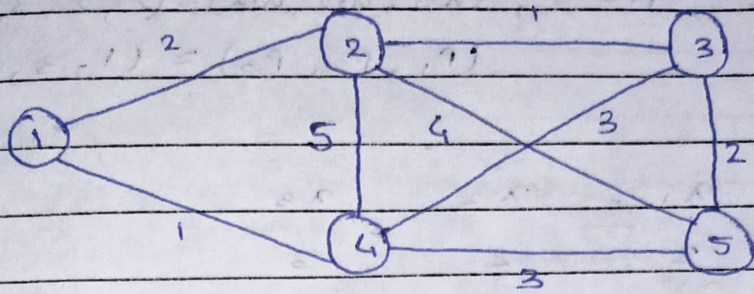
$\frac{1}{2} \cdot n^3$ $\quad$ $c \cdot n^3$

$\therefore$ $c = 1/2$ i.e. $0.5 < 1$

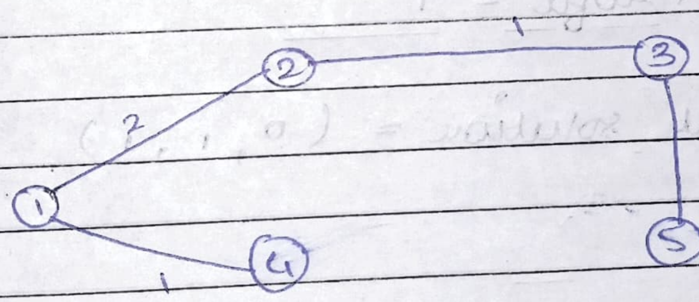
$\therefore$ case 3 is applicable.

$T(n) = \Theta(n^3)$.

4(a)



Q →	1	2	3	4	5
key →	0	∞	∞	∞	∞
	—	2	∞	1	∞
	—	2	3	—	3
	—	—	1	—	3
	—	—	—	—	2



$$MST = (w_1, w_4) + (w_1, w_2) + (w_2, w_3) + (w_3, w_5)$$

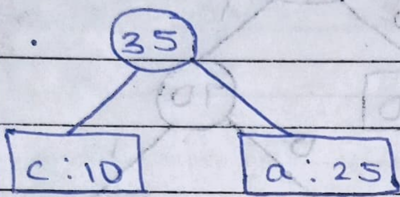
$$= 1 + 2 + 1 + 2$$

$$= 6$$

46 a : 25, b : 50, c : 10, d : 75, e : 35

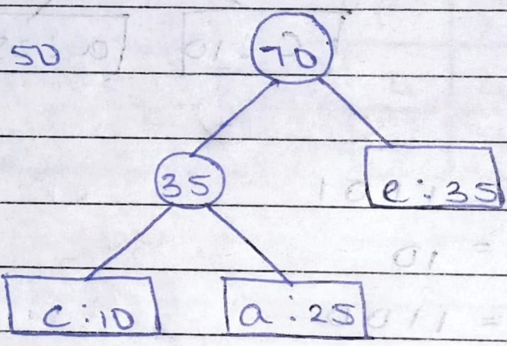
1. c : 10, a : 25, e : 35, b : 50, d : 75

2.



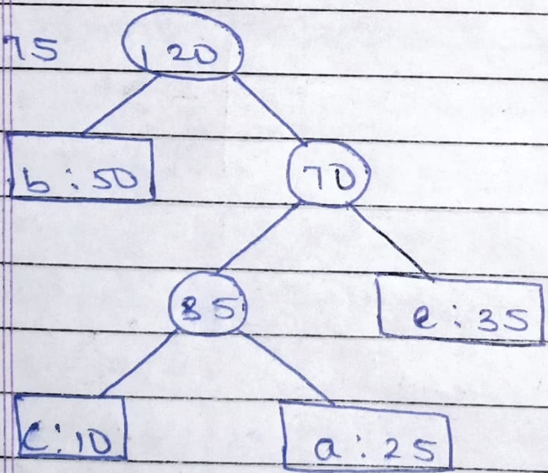
e : 35, b : 50, d : 75

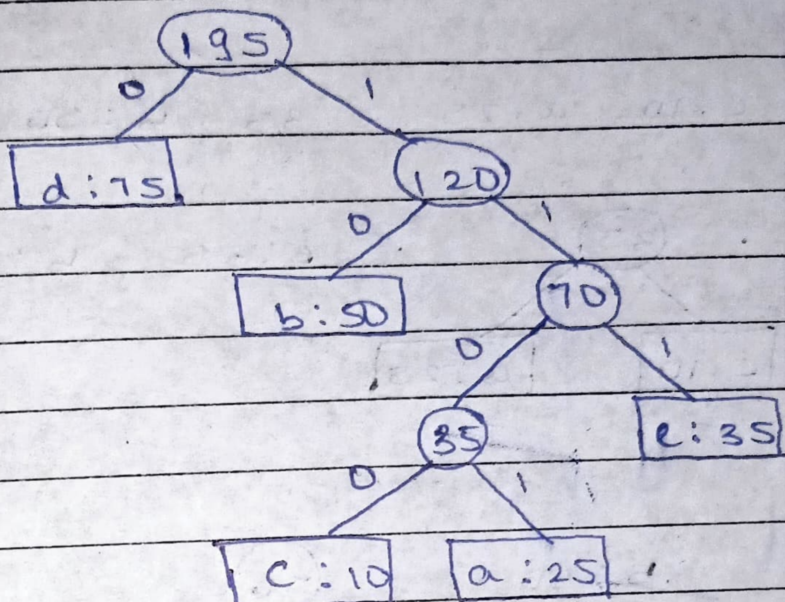
3. b : 50



d : 75

4. d : 75





a : 25 = 1101

b : 50 = 10

c : 10 = 1100

d : 75 = 0

e : 35 = 111

5a. $x = (A, B, C, B, D, A, B)$, $y = (B, D, C, A, B, A)$

	x	A	B	C	B	D	A	B
y	0	0	0	0	0	0	0	0
B	0	$\uparrow 0$	$\nwarrow 1$	$\leftarrow 1$	$\nwarrow 1$	$\leftarrow 1$	$\leftarrow 1$	$\nwarrow 1$
D	0	$\uparrow 0$	$\uparrow 1$	$\uparrow 1$	$\uparrow 1$	$\nwarrow 2$	$\leftarrow 2$	$\leftarrow 2$
C	0	$\uparrow 0$	$\uparrow 1$	$\nwarrow 2$	$\leftarrow 2$	$\uparrow 2$	$\uparrow 2$	$\uparrow 2$
A	0	$\nwarrow 1$	$\uparrow 1$	$\uparrow 2$	$\uparrow 2$	$\uparrow 2$	$\nwarrow 3$	$\leftarrow 3$
B	0	$\uparrow 1$	$\nwarrow 2$	$\uparrow 2$	$\nwarrow 3$	$\leftarrow 3$	$\uparrow 3$	$\nwarrow 4$

$LCS(x, y) = (B, D, A, B)$

5(b) $m = 6$, $n = 3$, $(w_1, w_2, w_3) = (2, 2, 3)$

$(P_1, P_2, P_3) = (1, 3, 4)$

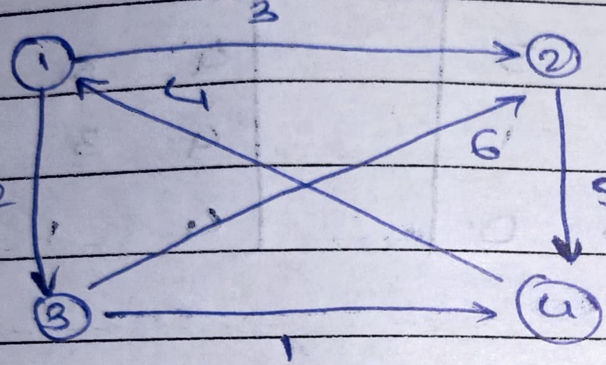
$i \rightarrow$	x_1	x_2	x_3
$w_i \rightarrow$	2	2	3
$p_i \rightarrow$	1	3	4

n \ W	0	1	2	3	4	5	6	
0	0	0	0	0	0	0	0	(w_i, p_i)
1	0	0	1	1	1	1	1	(2, 1)
2	0	0	3	3	4	4	4	(2, 3)
3	0	0	3	4	4	7	$\boxed{7}$	(3, 4)

Max Profit = 7

Optimal solution = (0, 1, 1)

6a



$$D^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 2 & \infty \\ 2 & \infty & 0 & \infty & 5 \\ 3 & \infty & 6 & 0 & 1 \\ 4 & 4 & \infty & \infty & 0 \end{array}$$

$$\pi^0 = \begin{bmatrix} N & 1 & 1 & N \\ N & N & N & 2 \\ N & 3 & N & 3 \\ 4 & N & N & N \end{bmatrix}$$

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$$D^1 = \begin{bmatrix} 0 & 3 & 2 & \infty \\ \infty & 0 & \infty & 5 \\ \infty & 6 & 0 & 1 \\ 4 & 7 & 6 & 0 \end{bmatrix} \quad \pi^1 = \begin{bmatrix} N & 1 & 1 & N \\ N & N & N & 2 \\ N & 3 & N & 3 \\ 4 & 1 & 1 & N \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 0 & 3 & 2 & 8 \\ \infty & 0 & \infty & 5 \\ \infty & 6 & 0 & 1 \\ 4 & 7 & 6 & 0 \end{bmatrix} \quad \pi^2 = \begin{bmatrix} N & 1 & 1 & 2 \\ N & N & N & 2 \\ N & 3 & N & 3 \\ 4 & 1 & 1 & \text{[illegible]} \end{bmatrix}$$

$$D^3 = \begin{bmatrix} 0 & 3 & 2 & 3 \\ \infty & 0 & \infty & 5 \\ \infty & 6 & 0 & 1 \\ 4 & 7 & 6 & 0 \end{bmatrix} \quad \pi^3 = \begin{bmatrix} N & 1 & 1 & 3 \\ N & N & N & 2 \\ N & 3 & N & 3 \\ 4 & 1 & 1 & N \end{bmatrix}$$

$$D^4 = \begin{bmatrix} 0 & 3 & 2 & 3 \\ 9 & 0 & 11 & 5 \\ 5 & 6 & 0 & 1 \\ 4 & 7 & 6 & 0 \end{bmatrix} \quad \pi^4 = \begin{bmatrix} N & 1 & 1 & 3 \\ 4 & N & 1 & 2 \\ 4 & 3 & N & 3 \\ 4 & 1 & 1 & N \end{bmatrix}$$

4 (a)

$$\begin{bmatrix} 0 & 8 & 16 & 15 \\ 14 & 0 & 9 & 12 \\ 9 & 10 & 0 & 6 \\ 11 & 13 & 10 & 0 \end{bmatrix}$$

Formulae :

$$g(i, s) = \min_{k \in S} \{c_{ik} + g(k, S - \{k\})\}$$

Phase 1 :

$$g(1, \{2,3,4\}) = \min_{k \in \{2,3,4\}} \{c_{1k} + g(k, \{2,3,4\} - \{k\})\}$$

$$= \boxed{34}$$

Phase 2 :

$c_{12} + g(2, \{3,4\}) = 8 + g(2, \{3,4\})$
$= 8 + 26 = 34$

$c_{13} + g(3, \{2,4\}) = 16 + g(3, \{2,4\})$
$= 16 + 32 = 48$

$c_{14} + g(4, \{2,3\}) = 15 + g(4, \{2,3\})$
$= 15 + 31 = 44$

Phase 3 :

$c_{23} + g(3, \{4\}) = 9 + g(3, \{4\})$
$= 9 + 17 = 26$

$c_{24} + g(4, \{3\}) = 12 + g(4, \{3\})$
$= 12 + 19 = 31$

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$c_{32} + g(2, \{4\}) = 10 + g(2, \{4\})$
$= 10 + 23 = 33$

$c_{34} + g(4, \{2\}) = 6 + g(4, \{2\})$
$= 6 + 27 = 32$

$c_{42} + g(2, \{3\}) = 13 + g(2, \{3\})$
$= 13 + 18 = 31$

$c_{43} + g(3, \{2\}) = 10 + g(3, \{2\})$
$= 10 + 24 = 34$

Phase 4:

$c_{34} + g(4, \{\phi\}) = 6 + g(4, \{\phi\})$
$= 6 + 11 = 17$

$c_{43} + g(3, \{\phi\}) = 10 + g(3, \{\phi\})$
$= 10 + 9 = 19$

$c_{24} + g(4, \{\phi\}) = 12 + g(4, \{\phi\})$
$= 12 + 11 = 23$

$c_{42} + g(2, \{\phi\}) = 13 + g(2, \{\phi\})$
$= 13 + 14 = 27$

$c_{23} + g(3, \{\phi\}) = 9 + g(3, \{\phi\})$
$= 9 + 9 = 18$

$c_{32} + g(2, \{\phi\}) = 10 + g(2, \{\phi\})$
$= 10 + 14 = 24$

Phase 5:

$g(2, \phi) = 14$
$g(3, \phi) = 9$
$g(4, \phi) = 11$

Shortest Path :

$$1 \xrightarrow{8} 2 \xrightarrow{9} 3 \xrightarrow{6} 4 \xrightarrow{11} 1$$

~~Short~~ $= 8 + 9 + 6 + 11$

$= 34$