SUMMER EXAM. - 2015

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 26)

Q.1.(a) Explain the summation of arithmetic and geometric series. **7M**

Ans. **See P.1-7, Q.3, 6.**

Q.1.(b) Solve the following recurrence relations using master theorem :

(i) $T(n) = 9T(n/3) + n$

(ii) $T(n) = 2T(2n/3) + 1$

(iii) $T(n) = 3T(n/4) + n\lg n$ **6M**

Ans. **See P. PS-12, 1-29, Q.1(b), Q. 48(i).**

Q.2.(a) Solve the given recurrence relation using master method :

$T(n) = 3T(n/4) + \sqrt{n} + 5$ for $n \geq 4$

$T(1) = 2$ **7M**

Ans. **See P.1-23, Q.41.**

Q.2.(b) Draw recursion force for

$T(n) = T(n/3) + T(2n/3) + O(n)$ and find the upper bound for the recursion. Also comment on value generated using upper bound. **6M**

Ans. See P.1-30, Q.50.

UNIT - II

(Total Marks : 07)

Q.3.(a) Explain the asymptotic notations :

(i) Big O

(ii) Big Omega

(iii) Theta

(iv) Little O

(v) Little Omega **7M**

Ans. See P.2-4, Q.1.

(iv) Little O :

f(n) = O (g(n)), for any positive constant c > 0, ∃ a constant $n_0 > 0$ such that

$0 \le f(n) < cg(n) \; \forall n \ge n_0$

Example : Let $f(n) = 3n^2$

$g(n) = n^2$

$3n^2 = O(n^2)$

$3n^2 \ne O(n^2)$

So to represent the above situation we use little O notation

i.e. $3n = O(n^2)$

but $3n \ne O(n^2)$

(v) Little Omega (ω) :

- This is used to denote lower bound that is not asymptotically tight.

  ω (g(n)) = {f(n) : for any positive constant c >0, ∃ a constant $n_0 > 0$ such that $0 \le c\,g(n) < f(n) \;\; \forall n \ge n_0$

- Let g(n) be a set of function for the non negative integers into the positive real numbers. Then ω (g) is the set of function f(n) also from the non negative integers into the positive real numbers such that

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \text{ or } \lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$$

- We can define ω -notation as

  $f(n) \in \omega(g(n))$ if and only is $g(n) \in O(f(n))$

UNIT - III

(Total Marks : 47)

Q.3.(b) Explain quick sort algorithm with suitable example. Comment on time complexity of algorithm. **7M**

Ans. See P.3-16, Q.10.

Q.4.(a) Implement insertion sort on following array :

18 12 44 64 76 15 129 20

Write recursive algorithm for the same. Comment on stack size required to implement the algorithm. **7M**

Ans. See P. PS-23, Q.3(b).

Given array is 18, 12, 44, 64, 76, 15, 129, 20

Total elements are 8.

So, the no. of passes required are

$n - 1 = 8 - 1 = 7$

Pass 1 :

18	12
12	18
44	44
64	64
76	76
15	15
129	129
20	20

Pass 2:

12	12
18	18
44	44
64	64
76	76
15	15
129	129
20	20

Pass 3 :

12	12	12
18	18	18
44	44	44
64	64	64
76	76	76
15	15	15
129	129	129
20	20	20

Pass 4 :

12	12	12	12
18	18	18	18
44	44	44	44
64	64	64	64
76	76	76	76
15	15	15	15
129	129	129	129
20	20	20	20

Pass 5 :

12	12	12	12	12
18	18	15	15	15
44	44	18	18	18
64	64	44	44	44
76	76	64	64	64
15	15	76	76	76
129	129	129	129	129
20	20	20	20	20

Pass 6 :

12	12	12	12	12	12
15	15	15	15	15	15
18	18	18	18	18	18
44	44	44	44	44	44
64	64	64	64	64	64
76	76	76	76	76	76
129	129	129	129	129	129
20	20	20	20	20	20

Pass 7:

12	12	12	12	12	12	12
15	15	15	15	15	15	15
18	18	18	18	18	18	18
44	44	44	44	20	20	20
64	64	64	64	44	44	44
76	76	76	76	64	64	64
129	129	129	129	76	76	76
20	20	20	20	129	129	129

∴ The sorted array is,

12

15

18

20

44

64

76

129

The stack size required to implement the algorithm is $(n - 1)$.

Q.4.(b) Show the Strassen's matrix multiplication process on the matrix A and B given below :

$$A = \begin{bmatrix} 4 & 2 & 0 & 1 \\ 3 & 1 & 2 & 5 \\ 3 & 2 & 1 & 4 \\ 5 & 2 & 6 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 5 & 4 & 2 & 3 \\ 1 & 4 & 0 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

7M

Ans. **See P.3-34, Q.31.**

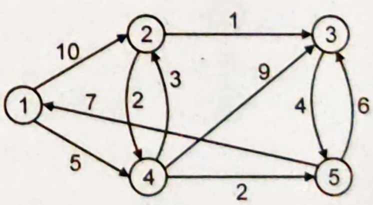
Q.5.(a) Illustrate the stepwise operation of heap sort on the input array :

A = < 5, 13, 2, 25, 7, 17, 20, 8, 3 >

Also find the recurrence relation for algorithm and discuss its complexity. **7M**

Ans. **See P.3-26, Q.24.**

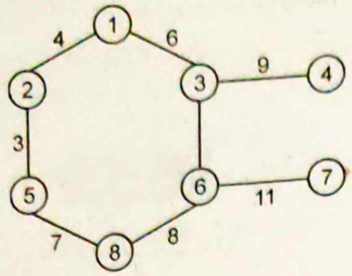
Q.5.(b) Find single source shortest path for the following diagram. Write the algorithm for the same. **6M**



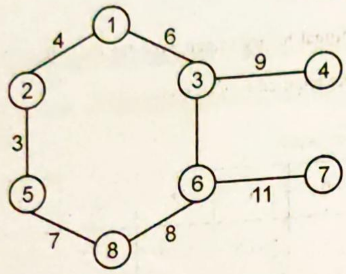
Ans. **See P.3-56, Q.74.**

Q.6.(a) What is minimum spanning tree? Show the snapshots of Prim's algorithm to find minimum cost spanning tree for the given graph :

7M



Ans. Minimum spanning tree : See P.3-45, Q.55



cost of 3 – 6 is not given

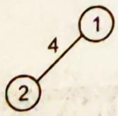
Let us consider cost of 3 – 6 = 5

Starting vertex (1)

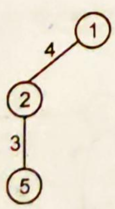
Adj of vertex (1) is (2) and (3)

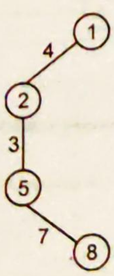
Vertex 2 = 4 vertex 3 = 6

4 is minimum

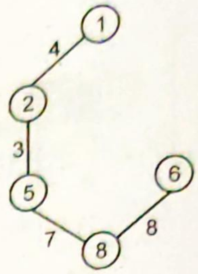


Next vertex → Adj of vertex 2 is (5)



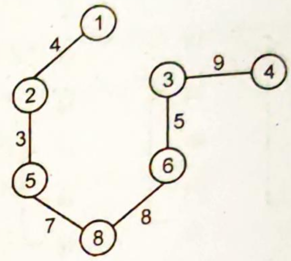


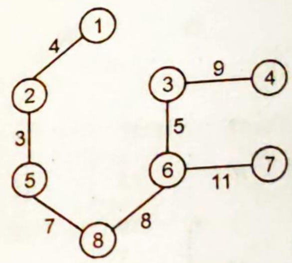
Adj of 8 is (6)



Adj of 6 is 3 and 11

3 is minimum





Total cost = 4+3+7+8+5+9+11= 47

Q.6.(b) Explain what is knapsack problem. Give algorithm for it and time complexity. Solve the following knapsack problem :

n = 3 m = 20

$(P_1, P_2, P_3) = (25, 24, 20)$

$(W_1, W_2, W_3) = (18, 15, 10)$ **6M**

Ans. **Knapsack problem :** **See P.3-39, Q.43.**

n = 3 m = 20

$(P_1, P_2, P_3) = (25, 24, 20)$

$(W_1, W_2, W_3) = (18, 15, 10)$

Object	O_1	O_2	O_3
Profit	25	24	20
Weight	18	15	10

Select the object with highest profit

Object	Profit	Weight	Capacity (20)
O_1	25	18	$20-18=2$
O_2*	2.083	2	$2-2=0$

$x = 24 \rightarrow 25$

$2 \rightarrow ?$

$24x = 25 \times 2$

$x = \frac{50}{24}$

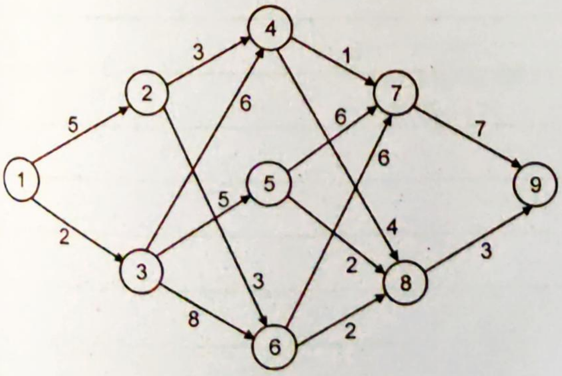
$x = \frac{25}{12} = 2.083$

Sr. No.	(x_1, x_2, x_3)	$\Sigma W_i \times i$	$\Sigma P_i \times i$
(1)	(1/2, 1/3, 1/4)	16.5	24.25
(2)	(1, 2/15, 0)	20	27.083
(3)	(0, 2/3, 1)	20	31
(4)	(0, 1, 1/2)	20	31 5

Hence resulting solution is 27.083

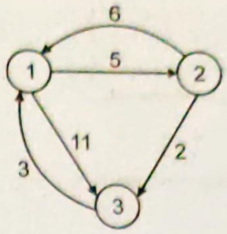
UNIT - IV
(Total Marks : 28)

Q.7.(a) Find the minimum cost path from vertex "s" to "t" in multistage graph shown. Write the algorithm for it. **7M**



Ans. See P.4-12, Q.5.

Q.7.(b) Find out shortest distance between all pairs of vertices and write the Flyod Washall all pair shortest path algorithm. **7M**



Ans. See P.4-16, Q.12.

Q.8.(a) Draw optimal binary search Tree for the following. Also generate the matrics required for tree construction.

n = 5

i	0	1	2	3	4	5
p_i	-	0.08	0.05	0.12	0.20	0.10
q_i	0.05	0.10	0.05	0.11	0.10	0.04

7M

Ans. See P.4-24, Q.32.

Q.8.(b) Determine LCS of X = < p, o, l, y, n, o, m, i, a, l > and Y = < e, x, p, o, n, e, n, t, i, a, l> **7M**

Ans. See P.4-32, Q.45.

UNIT - V
(Total Marks : 26)

Q.9.(a) Give generalized schema for recursive backtracking algorithm and explain in brief. **6M**

Ans. See P.5-16, Q.18.

Q.9.(b) Explain how backtracking can be applied to solve 4-Queen's problem. **7M**

Ans. See P.5-19, Q.26.

Q.10.(a) Explain graph coloring method with suitable example. Give algorithm for it. **7M**

Ans. See P.5-22, Q.29.

Q.10.(b) Explain Knight's tour problem and give algorithm for it. 6M

Ans. Knight's tour problem :

A popular solution start at some point P_0 and then walks to its nearest neighbor P_1 first then repeat P_1 and so on until completion.

Algorithm :

(1) Pick and visit an initial point P_0.

(2) $P = P_0$

(3) $i = 0$

(4) While there are still unvisited points.

(5) Set $i = i + 1$

(6) Let P_i be the closed unvisited point to P_{i-1} then.

(7) Visit P_i

(8) Return to P_0 from P_i

UNIT - VI

(Total Marks : 26)

Q.11.(a) Explain the following NP problems and relation between them :

(i) Clique

(ii) Graph portioned into triangle

(iii) Independent Set Problem (ISP). **8M**

Ans. See P.6-12, Q.17.

Q.11.(b) Write an algorithm for non-deterministic sorting. **5M**

Ans. See P.6-9, Q.11.

Q.12.(a) Comment on $P \subseteq NP$. **4M**

Ans. See P.6-7, Q.7.

Q.12.(b) Give definition of NP hard and NP complete class of problems. **4M**

Ans. See P.6-6, Q.3.

Q.12.(c) How polynomial reduction can be used for showing NP-completeness of a problem? **5M**

Ans. See P.6-11, Q.16.

WINTER EXAM. - 2015

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 26)

Q.1.(a) State and explain the Master's theorem for complexity analysis. Also give its limitations. **6M**

Ans. See P.1-10, Q.15.

Limitations of master theorem are :

(1) If f (n) is big enough function, the top call can be bigger than the sum of all the little calls.

(2) Logs popup i.e. divide and conquer.

Q.1.(b) Derive a closed form solution for the summation :

$$S_k = \sum_{i=1}^{k-1}(i.a^i)$$ **7M**

Ans. See P.1-7, Q.5.

Q.2.(a) Solve the following recurrence relation with the help of characteristic equation method.

$$t_n = \begin{cases} 1 \text{ if } n = 0 \\ 4t_{n-1} - 2^n \text{ otherwise} \end{cases}$$ **6M**

Ans. **Step I :** Generate recurrence

$t_n - 4t_n - 1 = -2^n$

L. H. S. = $t_n - 4t_n - 1$

$(x - 4) = 0$

$r_1 = 4$

Step II : R. H. S. $= -2^n$

$= b^n . P(n)$

$b^n = 2^n$

$\therefore b = 2$

$P(n) = -1$

$d = 0$

Root repeated once

Step III : $t_n = C_1 r_1^n + C_2 r_2^n$

Roots are real and distinct

$t_n = C_1 4^n + C_2 . 2^n$ ..... (1)

Using initial condition

At $n = 0$ $t_o = 1$

$C_1 + C_2 = 1$ .... (2)

Using given recurrence

$t_n = 4t_n - 1 - 2^n$

At n = 1

$t_1 = 4t_o - 2^1$

$= 4 - 2 = 2$

At n = 1 in eqn (1)

$4C_1 + 2C_2 = 2$ .....(3)

Solving equation (2) and (3) we get

$C_1 = 0$
$C_2 = 1$

$\therefore$ $t_n = 2^n$

Q.2.(b) Solve following recurrence relation using change of variable method.

$T(n) = 2T(\sqrt{n}) + \log n$ **7M**

Ans. **See P.1-17, Q.30(i).**

UNIT - II

(Total Marks : 28)

Q.3.(a) State and explain in detail about asymptotic notations which are used for analysis of algorithms. **7M**

Ans. **See P.2-4, Q.1.**

Q.3.(b) Illustrate the stepwise execution of quicksort on the following array A. Also give its complexity by analyzing the recurrence relation.

A = [1, 3, 5, 8, 7, 6, 4] **7M**

Ans. **See P.3-16, Q.10.**

Given array is,

A = [1, 3, 5, 8, 7, 6, 4]

Step 1 : First element = 1

Middle element = 8

Last element = 4

Therefore the median on [1, 8, 4] is 4.

Step 2 : Partitioning

(a) First get the pivot out of the way and swapping it with the last number

1 3 5 8 7 6 4

(b) Now we want the elements greater than pivot to be on right side of it and similarly the elements less than the pivot to be on left side of it.

(c) Consider 2 pointers i and j

i → first index

j → last index of an array

(i) While i is less than j we keep incrementing i until we find an element greater than pivot.

(ii) Similarly while j > i keeps decrementing j until we find an element less than pivot.

(iii) After both i and j stop we swap the elements at the indexes of i and j respectively.

(iv) When i and j have crossed no swap is performed, scanning stops and the element pointed to by i is swapped with the pivot.

1	3	5	8	7	6	4	
i					j		
1	3	5	8	7	6	4	Interchange i and pivot
	j	i					
1	3	4	8	7	6	5	

Step 3 : Restore the pivot

After restoring the pivot we obtain the following partitioning into 3 groups

[1 3] [4] [8 7 6 5]

$\underbrace{[1\ 3]}_{\text{left subgroup}}$ [4] $\underbrace{[8\ 7\ 6\ 5]}_{\text{right subgroup}}$ Recursively quick sort the right subpart.

Step 4 : Consider,

8 7 6 5

First element = 8

Middle element = 7

Last element = 5

Median on [8 7 5] is 7

8	6	5	7
i		j	

8	6	5	7	Interchange i and pivot
j	i			

[8] [7] [5 6]

Now we want the elements greater than pivot on the R.H.S. and elements less than pivot to be on L.H.S. of i – 1

[5 6] [7] [8]

Hence the sorted array is,

1 3 4 5 6 7 8

Q.4.(a) What do you mean by amortized analysis of algorithm? Explain any one method with suitable example. **7M**

Ans. See P.2-9, Q.7.

Q.4.(b) Write an algorithm for binary search using divide and conquer strategy. Also give its stepwise execution for searching element X = 4 in following input array. **7M**

A = [19, 3, 15, 8, 1, 6, 4]

Ans. **See P.3-12, Q.3.**

Given array of A is,

1	2	3	4	5	6	7
19	3	15	8	1	6	4

Apply binary search for x = 4

Step 1 : Find the value of i and j

i = 1

j = 7

Now, find value of K as

$$K = \frac{i+j}{2} = \frac{1+7}{2} = \frac{8}{2} = 4$$

$\therefore$ K = 4

Step 2 : T [K] = T [4] = 8

8 > 4 then

i = K + 1

i = 4 + 1 = 5

Step 3 : Again

5	6	7
1	6	4

$$K = \frac{i+j}{2} = \frac{5+7}{2} = \frac{12}{2} = 6$$

$\therefore$ K = 6

Step 4 : T [K] = T [6] = 6

6 > 4 then

i = K + 1

i = 6 + 1

i = 7

Step 5 : Again

$$K = \frac{i+j}{2} = \frac{7+7}{2} = \frac{14}{2} = 7$$

$\therefore$ K = 7

6	7
6	4

T [K] = T [7] = 4

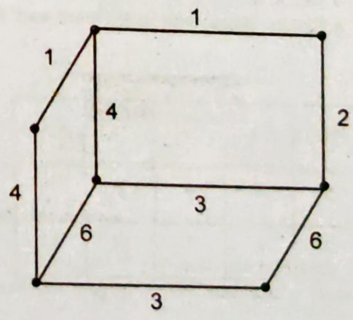
$\therefore$ 4 = 4

Hence search is successful at location 7.

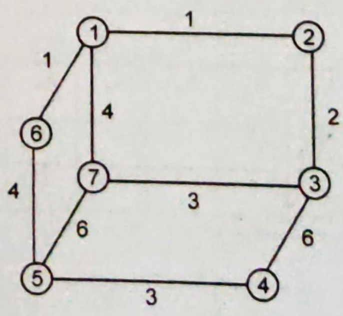
UNIT - III

(Total Marks : 26)

Q.5.(a) Find out Minimum Spanning Tree with its cost for given undirected graph as follows. Use kruskal's algorithm. **6M**



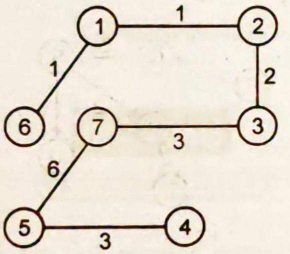
Ans.



Arrange the edges in ascending order of their weight.

{1, 2} {1, 6} {2, 3} {3, 7} {4, 5} {1, 7} {5, 6} {5, 7} {3, 4}

Edge	Cost	Accepted/ Rejected	Tree
-	-	-	{1} {2} {3} {4} {5} {6} {7}
{1, 2}	1	Accepted	{1, 2} {3}, {4}, {5}, {6}, {7}
{1, 6}	1	Accepted	{1, 2, 6} {3} {4} {5} {7}
{2, 3}	2	Accepted	{1, 2, 3, 6} {4} {5} {7}
{3, 7}	3	Rejected	{1, 2, 3, 6} {4} {5} {7}
{4, 5}	3	Accepted	{1, 2, 3, 6} {4, 5} {7}
{1, 7}	4	Rejected	{1, 2, 3, 6} {4, 5} {7}
{5, 6}	4	Rejected	{1, 2, 3, 6} {4, 5} {7}
{5, 7}	6	Accepted	{1, 2, 3, 4, 5, 6, 7}
{3, 4}	6	Rejected	{1, 2, 3, 4, 5, 6, 7}



Total cost = 1 + 2 + 6 + 3 + 3 + 1 = 16

Q.5.(b) Given 8 activities along with their start and finish time as follows :

Ai	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
Si	1	2	3	4	8	9	11	12
Fi	5	2	4	7	11	12	13	16

Then compute a schedule where largest number of activities takes place using Greedy approach. **7M**

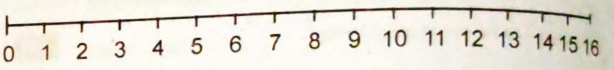
Ans.

Ai	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
Si	1	2	3	4	8	9	11	12
Fi	5	2	4	7	11	12	13	16

Sr. No.	Activities (Fi)	Start time (Si)	Finish time (Fi)
1	A_1	1	5
2	A_2	2	2
3	A_3	3	4
4	A_4	4	7
5	A_5	8	11
6	A_6	9	12
7	A_7	11	13
8	A_8	12	16

Step 1 : Sort the input activities by increasing finish time.

$2 < 4 < 5 < 7 < 11 < 12 < 13 < 16$



Step 2 : Call a procedure greedy activity selector (S, f)

$n \leftarrow$ length [s]

$n \leftarrow 8$

$A \leftarrow [i]$

$A \leftarrow 1$

$j \leftarrow 1$

For $i \leftarrow 2$ to 8

Do if $S_i \geq f_i$

Then $A \leftarrow A \cup \{i\}$

$A = \{1\}$ $f_i = 5$

$A = \{1, 2\}$ $f_i = 2$

$A = \{1, 2, 3\}$ $f_i = 4$

$A = \{1, 2, 3, 4\}$ $f_i = 7$

$A = \{1, 2, 3, 4, 7\}$ $f_i = 13$

$A = \{1, 2, 3, 4, 7\}$

This algorithm is greedy because it always picks the activity with the earliest compatible finish.

Q.6.(a) Find out optimal solution for fractional knapsack problem using

Greedy strategy for following instances :

$n = 7$

$m = 15$

$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10, 5, 15, 7, 6, 18, 3)$

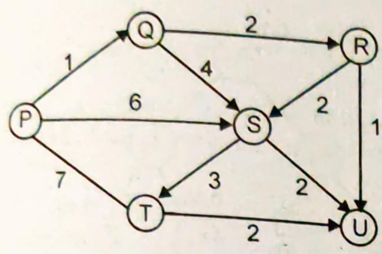
$(W_1, W_2, W_3, W_4, W_5, W_6, W_7)$

$= (2, 3, 5, 7, 1, 4, 1)$

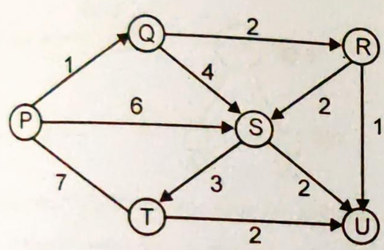
6M

Ans. See P.3-41, Q.47.

Q.6.(b) Suppose we run Dijkstra's algorithm on edge directed graph with vertex P as source on following graph. Then in what order do nodes get included into set of vertices for which shortest distance are finalized. **7M**

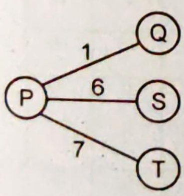


Ans.



Choose closest to node P. i.e. Q weight = 1

$\therefore$ adj P = {Q, T, S}= {1, 7, 6}

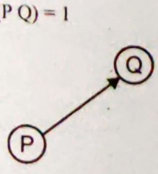


[Q] = 1

[T] = 7

d [S] = 6

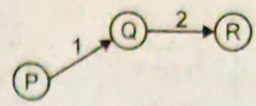
Consider Edge (P Q) = 1



adj Q = {R, S}

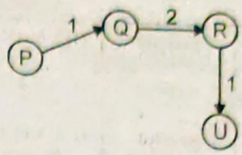
= {4, 2}

Min R = 2



adj R = {U, S}

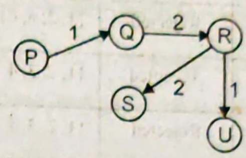
Min U = 1



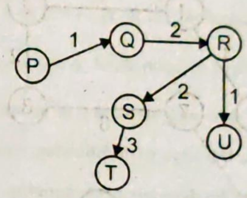
adj U = release U as there is no outgoing edge from U

Now, consider

adj R = S i.e. 2



adj S = T i.e. 3



Total cost = 1 + 2 + 1 + 2 + 3 = 9

UNIT - IV

(Total Marks : 28)

Q.7.(a) What is principle of optimality? Explain in brief. **4M**

Ans. See P.4-10, Q.2.

Principle of optimality :

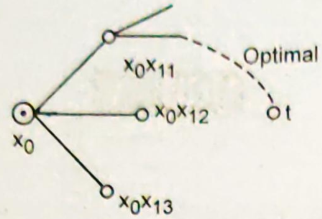
An optimal sequence of decisions has a property that whatever be the initial state and the decision, the remaining decisions must constitute an optimal decision sequence with regard to the state resulting from the first decision.

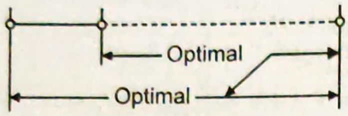
Example :

- Consider the problem of the shortest path from a node i to node j. Assume that $i, i_1, i_2, \ldots i_K, j$ is the shortest path.
- Starting from the initial node i, a decision has been made to go to the node i_1.
- Following this decision the problem state is defined by initial node i_1 and

final node j. Clearly, the path $i_1, i_2, \ldots i_K$, j must be the shortest path from i_1 to j.

- Though the principle of optimality is stated only for the initial state and the first decision by iteration it can be applied at any stage.





The principle can be applied in :

(1) The forward direction that is $x_i, i = 0$ to n be the problem state variables then take a decision about x_i from $x_{i+1}, x_{i+2}, \ldots x_n$

(2) The backward direction that is x_i, i = 0 to n be the problem state variables then take a decision about x_i from $x_0, x_1, \ldots x_{i-1}$.

Q.7.(b) Determine the cost and structure of an optimal binary search tree for set of n = 5 keys with following searching probabilities. Illustrate answer by dynamic programming. **10M**

i	0	1	2	3	4	5
pi	-	0.15	0.10	0.05	0.10	0.20
qi	0.05	0.10	0.05	0.05	0.05	0.10

Ans. **OBST :**

i	0	1	2	3	4	5
p_i	-	0.15	0.10	0.05	0.10	0.20
q_i	0.05	0.10	0.05	0.05	0.15	0.10
$\sum p_i q_i$	0.5	0.25	0.15	0.10	0.25	0.30

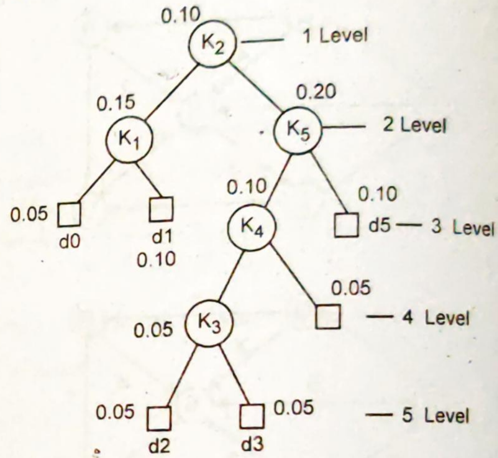
n = 5

$$W = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.05 & 0.30 & 0.45 & 0.55 & 1.80 & 2.10 \\ & 0.10 & 0.25 & 0.35 & 0.60 & 0.90 \\ & & 0.05 & 0.15 & 0.40 & 0.70 \\ & & & 0.05 & 0.30 & 0.60 \\ & & & & 0.15 & 0.45 \\ & & & & & 0.10 \end{bmatrix} \end{array}$$

$$c = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.05 & 0.30 & 0.70 & 1.05 & 3.10 & 4.75 \\ & 0.10 & 0.25 & 0.50 & 1.30 & 2.15 \\ & & 0.05 & 0.15 & 0.70 & 1.75 \\ & & & 0.05 & 0.30 & 1.05 \\ & & & & 0.15 & 0.45 \\ & & & & & 0.10 \end{bmatrix} \end{array}$$

$$r = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ & 2 & 2 & 2 & 2 \\ & & 3 & 3 & 3 \\ & & & 4 & 4 \\ & & & & 5 \end{bmatrix} \end{array}$$

Tree =



Q.8.(a) What is travelling salesman problem (TSP)? Compute optimal TSP tour for following distance matrix using dynamic programming approach. **7M**

	A	B	C	D
A	0	10	15	20
B	5	0	9	10
C	6	13	0	12
D	8	8	9	0

Ans. **See P.4-26, 27, Q.35, 38.**

Q.8.(b) Solve the following instance of 0/1 knapsack problem using dynamic programming :

Maximize : $x_1 + 2x_2 + 5x_3$

subject to constraints :

$2x_1 + 3x_2 + 4x_3 \le 6$

and restrictions,

$0 \le x_i \le 1, 1 \le i \le 3$ **7M**

Ans. $n = 3$

$(W_1, W_2, W_3) = \{2, 3, 4\}$

$(P_1, P_2, P_3) = \{1, 2, 5\}$

$m = 6$

For given problem we write

$f_0(y) = f_1(y) = f_2(y) = f_3(y) = -\infty \ \forall \ y < 0$

$f_0(y) = 0 \ \forall \ y \geq 0$

$f_1(0) = f_2 = (0) = f_3(0) = 0$

For $n = 1$

$f_1(1) = \max \{f_{1-1}(1), f_0(1 - w_1) + P_1$

$f_1(1) = \max \{0, -\infty + 1\} = 0$

$f_1(2) = \max [f_0(2), f_o(0) + 1]$

$= \max [0, 0 + 1]$

$= \max [0, 1] = 1$

$f_1(3) = 1$

$f_1(4) = 1$

$f_1(5) = 1$

$f_1(6) = 1$

For $n = 2$, $m = 1$

$f_2(1) = 0$

$f_2(2) = 1$

$f_2(3) = 2$

$f_2(4) = 2$

$f_2(5) = 3$

$f_2(6) = 3$

For $n = 3$ $m = 1$

$f_3(1) = \max \{f_2(1), f_2(-2) + 4\}$

$= \max \{0, -\infty + 4\} = 0$

$f_3(3) = 4$

$f_3(4) = 4$

$f_3(5) = \max \{f_2(5), f_2(2) + 4\}$

$= \max \{3, 1 + 4\}$

$= \max \{3, 5\} = 5$

$f_3(6) = \max \{f_2(6), f_2(3) + 4\}$

$= \max \{3, 2 + 4\}$

$= \max \{3, 6\} = 6$

Maximum profit = 6

Capacity = 6 – 3 = 4

Profit = 6 – 4 = 2

Optimal solution is (0, 1, 1)

UNIT - V

(Total Marks : 26)

Q.9.(a) Write and explain algorithms for iterative backtracking and recursive backtracking. **5M**

Ans. See P.5-18, Q.22.

Q.9.(b) Consider $S = \{S_1, S_2, S_3, S_4\}$ and weight vector $W = \{10, 25, 5, 10\}$ and $M = 25$. Then compute all possible subsets of w that sum to m. Draw the portion of state space tree that generates a fixed length tuple using backtracking algorithm. **8M**

Ans. $S = \{S_1, S_2, S_3, S_4\}$

$W = \{10, 25, 5, 10\}$

$M = 25$

$[S_1 - S_3 - S_4] = 25$

$[S_2] = 25$

i.e. [1 0 1 1]

[0 1 0 0]

1 = used

0 = unused

Positive sequence

(1) 1 - 2 - 3 - 4

1-2-4

1-3-4

1- 4

2-3-4

2-4

3-4

4

} 8 Sequences

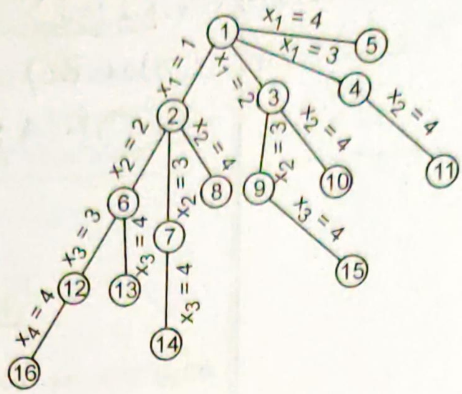
The solution is defined by all the path from root node terminating at leaf node

Solution node :

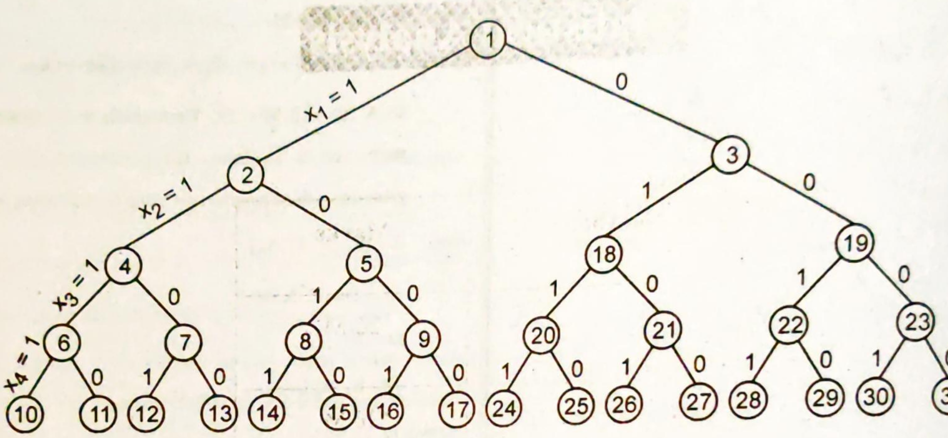
1-3-4

2

Variable size tuple :



Fixed size tuple :



Q.10.(a) What is M-colorability optimization problem in context of graph coloring? Explain the backtracking algorithm for graph coloring problem. **6M**

Ans. **M-colorability optimization problem :**

A graph G and a set of m colours are given. We want to determine if the nodes of the graph can be painted with distinct colours, so that no two adjacent nodes have the same color. This is called the M-colourability decision problem.

See P.5-22, Q.29.

Q.10.(b) What is Hamiltonian cycle? Explain back-tracking algorithm for it. **7M**

Ans. **See P.5-24, Q.31.**

UNIT - VI

(Total Marks : 26)

Q.11.(a) How polynomial reduction can be used for showing NP-completeness of a problem? **7M**

Ans. **See P.6-11, Q.16.**

Q.11.(b) Explain in detail about Cook's theorem. **6M**

Ans. **See P.6-10, Q.15.**

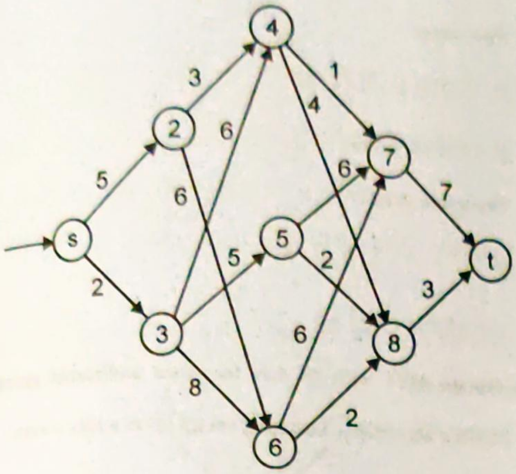
Q.12.(a) What is NP-hard and NP-complete problem? Explain in detail. **6M**

Ans. **See P.6-6, Q.3.**

Q.12.(b) What is clique? Comment about its NP-completeness. **7M**

Ans. **See P.6-12, Q.17.**

Q.11.(a) For a given graph, find out the shortest distance from vertex 's' to vertex 't'. **7M**



(b) Differentiate between DFS and BFS. **6M**

OR

Q.12.(a) Comment on P = NP. **3M**

(b) Explain polynomial reduction. **3M**

(c) Give the definition of NP hard and NP-complete class of problems. **3M**

(d) How polynomial reduction can be used for showing NP-completeness of a problem? **4M**

SUMMER EXAM. - 2016
SOLUTION
(Unit-Wise)

UNIT - I
(Total Marks : 22)

Q.1.(b) Solve the given recurrences using master theorem. **8M**

(i) $T(n) = 2T(\sqrt{n}) + \lg n$

(ii) $T(n) = T(\sqrt{n}) + 1$

Ans. **See P.1-17, 23, Q.30(i), 40(iii).**

Q.2.(a) What is an algorithm? Explain the properties of algorithm. **6M**

Ans. **See P.1-32, Q.56, PS-58, Q.1(c).**

Q.2.(b) Solve the following recurrence relation, **8M**

$T(n) = 3T(n/4) + \theta(n^2)$

using recursion tree method.

Ans. **See P.1-21, Q.36.**

UNIT - II
(Total Marks : 32)

Q.1.(a) State and explain asymptotic notations used for analyzing the algorithms. **6M**

Ans. **See P.2-4, Q.1.**

Q.3.(a) Write an algorithm of insertion sort. Derive its best case and worst case time complexity. **6M**

Ans. **See P.PS-23, Q.3(b).**

Q.3.(b) Use Strassen's algorithm to compute the matrix product and find the recurrence relation and its time complexity. **7M**

$$\begin{pmatrix} 3 & 2 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} 4 & 5 \\ 7 & 6 \end{pmatrix}$$

Ans. **See P.3-33, Q.29.**

$$A = \begin{bmatrix} 3 & 2 \\ 6 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 \\ 7 & 9 \end{bmatrix}$$

$$P = (A_{11} + A_{22})(B_{11} + B_{22})$$
$$= (4)(13) = 52$$

$$Q = (A_{21} + A_{22})B_{11}$$
$$= (7)4 = 28$$

$$R = A_{11}(B_{12} - B_{22})$$
$$= 3(5 - 9)$$
$$R = -12$$

$$S = A_{22}(B_{21} - B_{22})$$
$$= (1)(7 - 9) = -2$$

$$T = (A_{11} + A_{12})B_{22}$$
$$= (3 + 2)9$$
$$= 5 \times 9 = 45$$

$$U = (A_{21} - A_{11})(B_{11} + B_{12})$$
$$= (6 - 3)(4 + 5)$$
$$= 3(9) = 27$$

$$V = (A_{12} - A_{22})(B_{21} + B_{22})$$
$$= (2 - 1)(7 + 9) = 16$$

$C_{11} = P + S - T + V$

$= 52 + (-2) - 45 + 16 = 21$

$C_{12} = R + T$

$= (-12) + 45 = 33$

$C_{21} = Q + S$

$= 28 - 2 = 26$

$C_{22} = P + R - Q + U$

$= 52 - 12 - 28 + 27 = 39$

$$\begin{bmatrix} 21 & 33 \\ 26 & 39 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 7 & 9 \end{bmatrix}$$

Q.5.(a) Define amortized analysis of algorithm. Explain any one method with suitable example. **4M**

Ans. **See P.2-9, Q.7.**

Q.9.(a) For the following four matrices, find the order of parenthesization for the optimal chain multiplication. **9M**

$A_1 = 15 \times 5$

$A_2 = 5 \times 10$

$A_3 = 10 \times 20$

$A_4 = 20 \times 25$

Ans. **See P.PS-7, Q.9(b).**

UNIT - III

(Total Marks : 35)

Q.4.(a) Prove that the n-element heap has height h = lg n. **4M**

Ans. **See P.3-33, Q.27.**

Q.4.(b) Show the snapshots of merge sort when supplied with the input (1, 3, 5, 8, 9, 6, 4, 2). Also find the recurrence relation and time complexity of merge sort in best and worst cases. **9M**

Ans. **See P.3-20, Q.16.**

[1 3 5 8 9 6 4 2]

Array is divided into 2 parts

[1 3 5 8] [9 6 4 2]

Again divide until single element

[1 3] [5 8] [9 6] [4 2]

[1 3] [5 8] [6 9] [2 4]

Now merge

[1 3] [5 8] [6 9] [2 4]

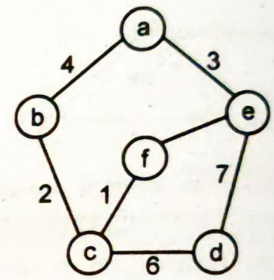
[1 3 5 8] [6 9 2 4]

The sorted array is

[1 2 3 4 5 6 7 8]

Complexity = $\log_2^8 = 3$

Q.5.(b) Obtain MST with its cost for given undirected graph using PRIM's algorithm. Assuming vertex 'a' as a root vertex. **9M**

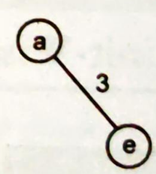


Ans. Starting vertex a

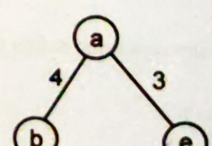
adj of a = b, e

min (e) i.e. 3

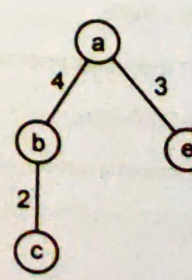
∴ next vertex is e and edge ae.



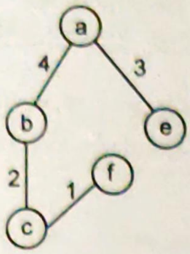
Step 2 : Next minimum connected to ae is ab i.e. 4



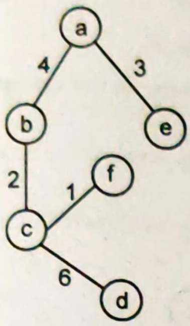
Step 3 :



Step 4 :



Step 5 :



Total cost = 16

Q.6.(a) Given 10 activities along with their start and finish time as :

$S = (A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10})$

$S_i = (1, 2, 3, 4, 7, 8, 8, 9, 11, 12)$

$F_i = (3, 5, 4, 6, 10, 11, 13, 12, 14, 9)$

Compute a schedule where largest number of activities take place. 6M

Ans. $S = (A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10})$

$S_i = (1, 2, 3, 4, 7, 8, 8, 9, 11, 12$

$F_i = (3, 5, 4, 6, 10, 11, 13, 12, 14, 9)$

Sr. No.	Activities	Start time (S_i)	Finish time (F_i)
(1)	A_1	1	3
(2)	A_2	2	5
(3)	A_3	3	4
(4)	A_4	4	6
(5)	A_5	7	10
(6)	A_6	8	11
(7)	A_7	8	13
(8)	A_8	9	12
(9)	A_9	11	14
(10)	A_{10}	12	9

Sort input activities according to finish time.

$3 < 4 < 5 < 6 < 9 < 10 < 11 < 12 < 13 < 14$

η = length [S]

n = 10

A = [i]

A = 1

j = 1

for i ← 2 to 10

Do if $S_i \geq F_i$

Then A ← A U {i}

A {1} $f_i = 3$

A {1, 2} $f_i = 5$

A {1, 2, 3} $f_i = 4$

A {1, 2, 3, 4} $f_i = 6$

A {1, 2, 3, 4, 12} $f_i = 9$

A = {1, 2, 3, 4, 12}

Q.6.(b) Write the algorithm of Optimal Huffman code. Find Optimal Huffman codes for following set of frequencies : 7M

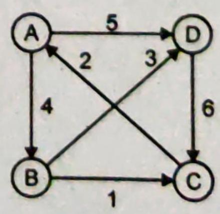
a : 25, b : 50, c : 15, d : 75, e : 40

Ans. See P.3-58, Q.77, 78.

UNIT - IV

(Total Marks : 33)

Q.7.(a) Find all pairs shortest paths using Floyd Warshall algorithm for given graph : 8M



Ans. See P.4-18, Q.15.

Q.8.(a) Find optimal solution using 0/1 Knapsack problem for given data **8M**

$M = 6, n = 3, (w_1, w_2, w_3) = (3, 2, 3)$

$(p_1, p_2, p_3) = (2, 1, 4)$

Ans. **See P.PS-6, Q.8(a).**

Q.8.(b) Determine LCS of **6M**

$X = (a, b, a, b, a, a, b)$

$Y = (a, b, a, b, b, a, a)$

Ans.

	Yi	a	b	a	b	b	a	a
Xi	0	0	0	0	0	0	0	0
a	0	↖1	←1	↖1	←1	←1	↖1	↖1
b	0	1↑	↖2	←2	↖2	↖2	←2	←2
a	0	↖1	2↑	↖3	←3	←3	↖3	↖3
b	0	1↑	↖2	3↑	↖4	↖4	←4	←4
a	0	↖1	2↑	↖3	4↑	4↑	↖5	↖5
a	0	↖1	2↑	↖3	4↑	4↑	↖5	↖6
b	0	1↑	↖2	3↑	↖4	↖5	5↑	6↑

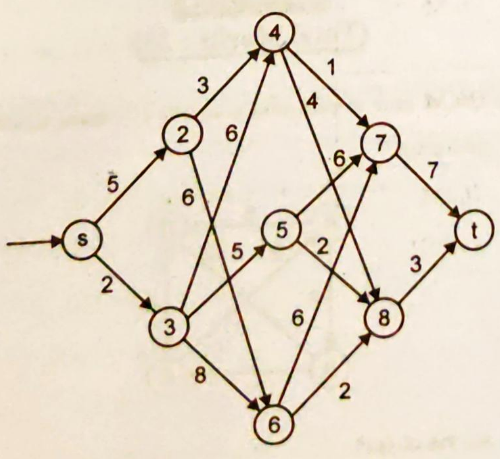
Length of LCS = 6

LCS = a b a b a a

Q.9.(b) Explain the principle of optimality and show how it can be applied on optimal binary search tree problem. **4M**

Ans. **See P.4-10, 23, Q.2, 29.**

Q.11.(a) For a given graph, find out the shortest distance from vertex 's' to vertex 't'. **7M**



Ans. **See P.4-12, Q.5.**

UNIT - V

(Total Marks : 25)

Q.7.(b) What is articulation point of an graph? Write a dynamic programming algorithm to find articulation point in a graph. **6M**

Ans. **See P.5-13, Q.10.**

Q.10.(a) Discuss 4-Queen problem and give its algorithm using backtracking method. **7M**

Ans. **See P.5-19, 21, Q.26, 27.**

Q.10.(b) Explain graph coloring method with example. Give algorithm for it. **6M**

Ans. **See P.5-22, Q.29.**

Q.11.(b) Differentiate between DFS and BFS. **6M**

Ans. **See P.5-13, Q.9.**

UNIT - VI

(Total Marks : 13)

Q.12.(a) Comment on P = NP. **3M**

Ans. **See P.6-7, Q.6.**

Q.12.(b) Explain polynomial reduction. **3M**

Ans. **See P.6-11, Q.16.**

Q.12.(c) Give the definition of NP hard and NP-complete class of problems. **3M**

Ans. **See P.6-6, Q.3.**

Q.12.(d) How polynomial reduction can be used for showing NP-completeness of a problem? **4M**

Ans. **See P.6-11, Q.16.**

(b) Discuss Hamiltonian cycle. Also write an algorithm for finding Hamiltonian cycle for a graph. **6M**

Q.11.(a) Explain Knight's tour problem and give algorithm for it. **7M**

(b) Explain following terms : **6M**

(1) P class of problems.

(2) NP complete class of problem.

(3) NP hard class of problems.

OR

Q.12.(a) Explain non deterministic algorithm. Give non deterministic algorithm for searching and sorting problem. **7M**

(b) Explain the concept of polynomial reduction and how it can be used for showing NP completeness of problem. **6M**

WINTER EXAM. - 2016
SOLUTION
(Unit-Wise)

UNIT - I
(Total Marks : 26)

Q.1.(a) Explain what do you mean by recurrence relation. Solve the following recurrences using master theorem. **7M**

(a) $T(n) = 4T(n/2) + n$

(b) $T(n) = 4T(n/2) + n^2$

(c) $T(n) = 4T(n/2) + n^3$

Ans. **See P.1-9, 19, Q.13, 32(3),(1),(2).**

Q.1.(b) Draw the recursion tree for the following recurrence relation.

$T(n) = 2T(n/2) + n^2$ **6M**

Ans. **See P.1-20, Q.34.**

Q.2.(a) Explain summation of arithmetic and geometric series. **6M**

Ans. **See P.1-7, Q.3, 6.**

Q.2.(b) Solve the following recurrence using changing variable method.

$T(n) = 2T\left(\left\lfloor\sqrt{n}\right\rfloor\right) + l_g\, n$ **7M**

Ans. **See P.1-17, Q.30.**

UNIT - II
(Total Marks : 35)

Q.3.(a) Explain different asymptotic notations :

(1) Big Oh (2) Theta

(3) Big omega (4) Little Oh

(5) Little omega **7M**

Ans. **See P.2-4, PS-32, Q.1, 3(a)(iv),(v).**

Q.3.(b) What do you mean by complexity of an algorithm? State different approaches of algorithm analysis. Also comment on worst case complexity of Quicksort is n^2. **7M**

Ans. **See P.1-30, 32, Q.51, 57,**

and P.3-16, Q.10.

Q.4.(a) Explain amortized analysis with example. Give its application for analyzing stack operations. **7M**

Ans. **See P.2-9, 10, Q.7, 8.**

Q.4.(b) Explain divide and conquer strategy for binary search algorithm. Write its recurrence relation and comment on its complexity. **7M**

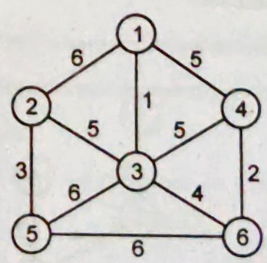
Ans. **See P.3-12, Q.3.**

Q.5.(a) Explain Strassen's matrix multiplication algorithm. Also perform its complexity analysis. **7M**

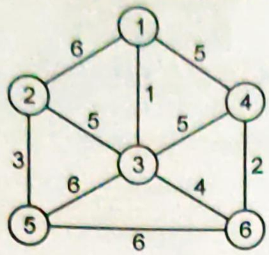
Ans. **See P.3-33, Q.29.**

UNIT - III
(Total Marks : 19)

Q.5.(b) What is minimum cost spanning tree? Show the snapshots of Prim's algorithm to find minimum cost spanning tree for the given graph. **6M**

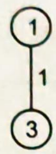


Ans. Minimum cost spanning tree : **See P.3-45, Q.55.**



Minimum distance = 1

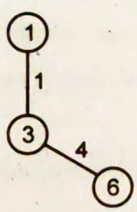
i.e. Edge 1 – 3



Adjacent of vertex 3 is 2, 5, 4, 6

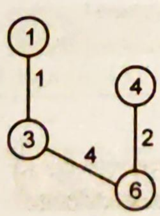
2 – 3 = 5, 3 – 4 = 5, 3 – 5 = 6, 3 – 6 = 4

Minimum is 3 – 6 i.e. 4 is next edge.



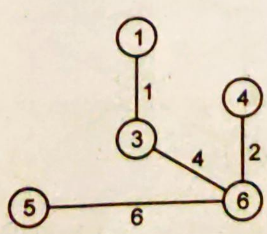
Adjacent to vertex 6 is 4, 5,

Next edge is 6 – 4, i.e. 2

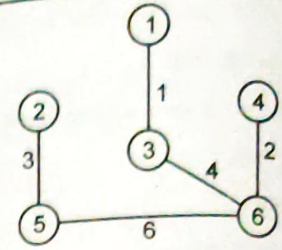


Adjacent to vertex 6 is 5

Next edge is 6 – 5 i.e. 6



Next edge is 5 – 2 i.e. 3



Minimum distance = 16

Total cost = 16

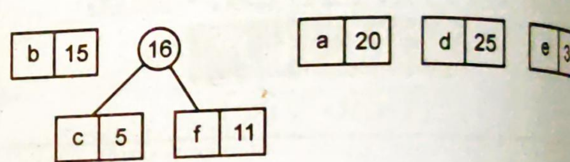
1 + 4 + 2 + 6 + 3 = 16.

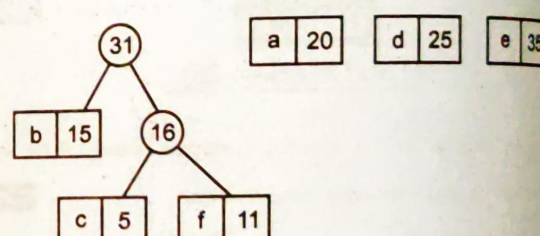
Q.6.(a) Write Huffman code algorithm. Also find optimal Huffman code for following set of frequencies.

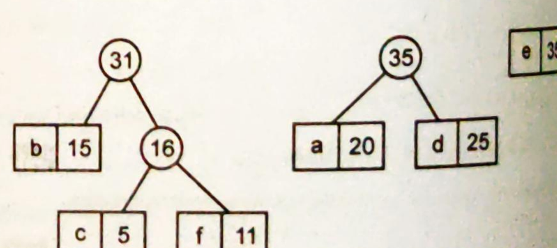
a : 20, b : 15, c : 5, d : 25, e : 35, f : 11.

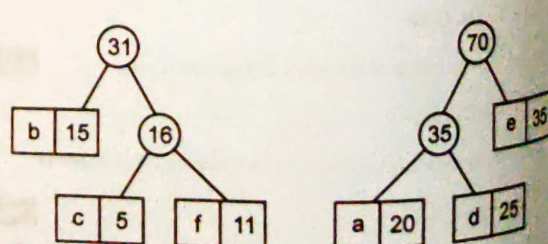
Ans. **See P.3-58, Q.77**

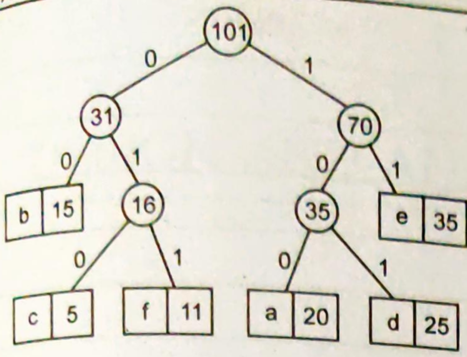
a : 20 b : 15 c : 5 d : 25 e : 35 f : 11











Prefix code : a = 100, b = 00, c = 010, d = 101,

e = 11, f = 011

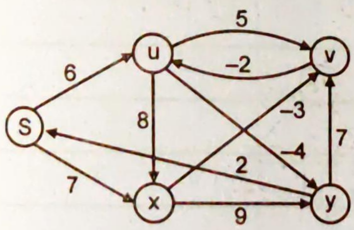
Q.6.(b) Find optimal solution to knapsack instance n = 3 m = 20 (p_1, p_2, p_3) = (25, 24, 15) and $w_1, w_2, w_3 = (18, 15, 10)$. Also find all feasible solution. **6M**

Ans. See P.3-40, Q.45.

UNIT - IV

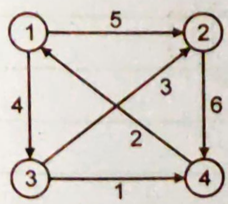
(Total Marks : 28)

Q.7.(a) Explain Bellman-Ford algorithm and find its complexity. Find the shortest distance using Bellman-Ford algorithm for given graph. **7M**



Ans. **See P.4-20, Q.20.**

Q.7.(b) Find shortest distance between all pairs of vertices and write Floyd-Warshall all pairs shortest path algorithm. **7M**



Ans. **See P.4-18, 15, Q.15, 10.**

Q.8.(a) Determine LCS of x = (e, x, p, o, n, e, n, t, i, a, l) and y = (p, o, l, y, n, o, m, i, a, l) by using dynamic programming algorithm. **7M**

Ans. **See P.4-32, Q.45.**

Q.9.(a) Find OBST with its cost and show all necessary matrices for following data. **7M**

i	0	1	2	3	4
p_i	-	0.1	0.05	0.15	0.2
q_i	0.1	0.2	0.05	0.05	0.1

Ans.

i	0	1	2	3	4
p_i	-	0.1	0.05	0.15	0.2
q_i	0.1	0.2	0.05	0.05	0.1
$\Sigma p_i pq_i$	0.1	0.3	0.10	0.20	0.3

$$W = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.1 & 0.4 & 0.50 & 0.70 & 1.0 \\ & 0.3 & 0.40 & 0.60 & 0.90 \\ & & 0.10 & 0.30 & 0.60 \\ & & & 0.20 & 0.50 \\ & & & & 0.3 \end{bmatrix} \end{array}$$

$$e = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.1 & 0.8 & 1.40 & 2.10 & 3.30 \\ & 0.3 & 0.80 & 1.50 & 2.70 \\ & & 0.10 & 0.60 & 1.50 \\ & & & 0.20 & 1.00 \\ & & & & 0.3 \end{bmatrix} \end{array}$$

Cost of tree = 3.30

$$r = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 \text{ or } 2 & 2 & 2 \\ & 2 & 1 & 1, 2 \text{ or } 3 \\ & & 3 & 2 \\ & & & 4 \end{bmatrix} \end{array}$$

UNIT - V

(Total Marks : 26)

Q.8.(b) Give generalized schema for recursive backtracking algorithm. Explain explicit and implicit constraints with example. **7M**

Ans. See P.5-16, Q.18.

Q.9.(b) Given an algorithm to obtain DFS and analyse its complexity. **6M**

Ans. See P.5-9, Q.4.

Q.10.(a) Explain backtracking algorithm for sum of subsets problem. State its implicit and explicit constraints. **7M**

Ans. See P.5-27, Q.35.

Q.10.(b) Discuss Hamiltonian cycle. Also write an algorithm for finding Hamiltonian cycle for a graph. **6M**

Ans. See P.5-24, Q.31.

UNIT - VI

(Total Marks : 26)

Q.11.(a) Explain Knight's tour problem and give algorithm for it. **7M**

Ans. See P.PS-35, Q.10(b).

Q.11.(b) Explain following terms : **6M**

(1) P class of problems.

(2) NP complete class of problem.

(3) NP hard class of problems.

Ans. See P.6-6, Q.2, 3.

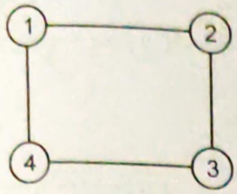
Q.12.(a) Explain non deterministic algorithm. Give non deterministic algorithm for searching and sorting problem. **7M**

Ans. See P.6-8, 9, PS-71, Q.9, 11, 11(a)(ii).

Q.12.(b) Explain the concept of polynomial reduction and how it can be used for showing NP completeness of problem. **6M**

Ans. See P.6-11, Q.16.

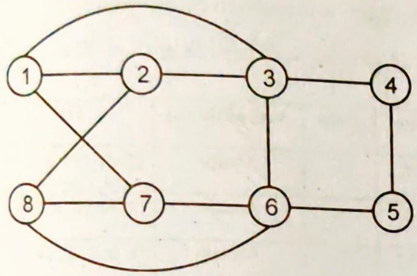
(b) What is planner graph? Implement graph coloring on following graph and generate solution space tree of no. of permitted colors = 3. Also write algorithm for the graph coloring. **7M**



OR

Q.10.(a) What is backtracking? Explain the application in which backtracking principle can be used to design a solution. **6M**

(b) What is the use of Hamiltonian cycle? Implement Hamiltonian cycle on following graph. **7M**



Q.11.(a) Write a note on :

(i) Clique **3M**

(ii) NP-Hard **3M**

(iii) NP-Complete **3M**

(b) Explain polynomial reduction. **4M**

OR

Q.12.(a) What is non-deterministic algorithm? Explain primality testing. **7M**

(b) Write a brief note on NP-Complete problem. **6M**

SUMMER EXAM. - 2017
SOLUTION
(Unit-Wise)

UNIT - I
(Total Marks : 26)

Q.1.(a) Explain geometric and harmonic series with one example of each. **7M**

Ans. **See. P.1-7, Q.6.**

Harmonic series : For positive integers n, the n^{th} harmonic number is

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{n}$$

$$= \sum_{k=1}^{n} \frac{1}{k}$$

$$= \ln n + 0(11)$$

Q.1.(b) Solve the following recurrence.

$T(n) = 3$ if $n = 0$

$= 2t_{n-1} + 2^n + 5$ otherwise **6M**

Ans. **See. P.1-27, Q.46.**

Q.2.(a) Prove that arithmetic series : **3M**

$$\sum_{k=1}^{n+1} k = \frac{1}{2} n(n+1)$$

Ans. $S = \sum_{k=1}^{n+1} k = \frac{1}{2} n(n+1)$

$$S_n = 1 + 2 + 3 + \ldots\ldots + (n-2) + (n-1) + n$$

$$S_n = n + (n-1) + (n-2) + \ldots\ldots + 3 + 2 + 1$$

If we add these two series we get

$$2S_n = (n+11)(n+1) + \ldots. + (n+1)$$

These are n of these $(n + 1)$'s so

$$2S_n = n(n+1)$$

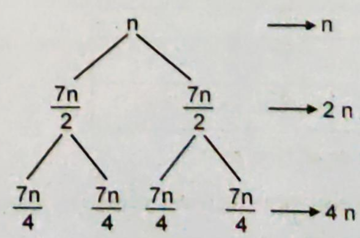
So

$$S_n = \frac{n(n+1)}{2}$$

$$\therefore \sum_{k=1}^{n+1} k = \frac{1}{2} n(n+1)$$

Q.2.(b) T(n) = 7T(n/2) + n solve this recurrence by using recursion tree method. **4M**

Ans.



$$\therefore T(n) = \theta(n^2)$$

Q.2.(c) Explain principles of designing an algorithm in brief. **6M**

Ans. **See. P.1-32, PS-110, Q.56, 2(b).**

UNIT - II

(Total Marks : 28)

Q.3.(a) What are the different asymptotic notations? Explain them briefly for the following equations, find the values of constants using various approaches. **8M**

(i) $3n + 2$ (ii) $10n^2 + 4n + 2$

Ans. **See. P.2-4, 6, Q.1, 3(i),(ii).**

Q.3.(b) Explain quick sort with example. **6M**

Ans. **See. P.3-16, Q.10.**

Q.4.(a) Write any two methods of amortized analysis in brief. **6M**

Ans. **See. P.2-9, Q.7.**

Q.4.(b) Use Strassen's algorithm to compute the matrix product. Show the process

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 4 \\ 6 & 2 \end{bmatrix}$$ **8M**

Ans. **See. P.PS-76, Q.5(a).**

UNIT - III

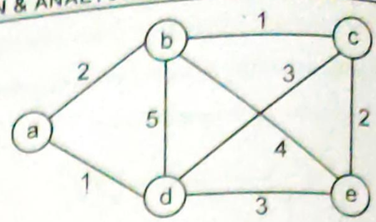
(Total Marks : 28)

Q.5.(a) Explain job scheduling approaches. Find the best possible sequence for the following deadlines. **7M**

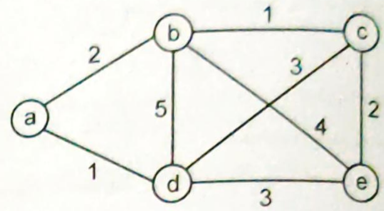
Jobs	Gain	Deadtime
1	35	3
2	20	1
3	18	3
4	16	4
5	12	2
6	10	2
7	8	1

Ans. **See. P.3-43, 44, Q.50, 53**

Q.5.(b) Write a KRUSKAL algorithm to generate spanning tree. Also implement algorithm on following graph. **7M**



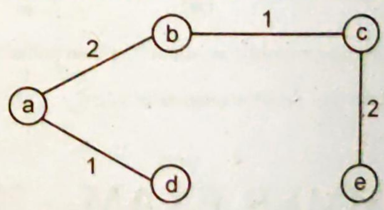
Ans. See. P.3-46, Q.56.



Arrange edges in ascending order of their weight

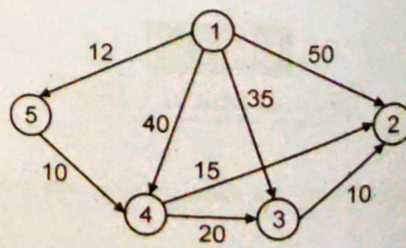
{a, d} {b, c} {a, b} {c, e} {c, d} {d, e} {b, e} {b, d}

Edge	Cost	Accept/Reject	Tree
(a, d)	1	Accept	{a, d} {b} {c} {e}
(b, c)	1	Accept	{a, d} {b, c} {e}
(a, b)	2	Accept	{a, b, c, d} {e}
(c, e)	2	Accept	{a, b, c, d, e}
(c, d)	3	Rejected	{a, b, c, d, e}
(d, e)	3	Rejected	{a, b, c, d, e}
(b, e)	4	Rejected	{a, b, c, d, e}
(b, d)	5	Rejected	{a, b, c, d, e}

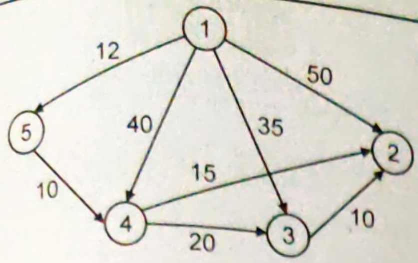


Total cost = 2 + 1 + 1 + 2 = 6

Q.6.(a) Write Greedy based single source shortest path algorithm. Implement the algorithm on following graph. **7M**

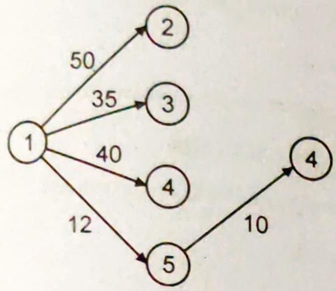


Ans. **See. P.3-56, 57, Q.73, 75.**

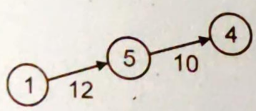


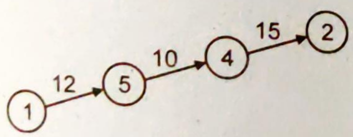
Minimum weight = 12

Edge 1 - 5 = 12



Minimum 12 i.e. 1 - 5





Steps	Cost	
1	1	-
2	1, 5	12
3	1, 5, 4	12 + 10 = 22
4	1, 5, 4, 2	12 + 10 + 15 = 37
5	1 - 3	37 + 15 = 52

Total cost = 52

Q.6.(b) Explain Knapsack problem with one simple example. **7M**

Ans. **See. P.3-39, 40, Q.43, 44.**

UNIT - IV

(Total Marks : 26)

Q.7.(a) Implement LCS for the following sequence.

x = a, a, b, a, a, b, a, b, a, a

y = b, a, b, a, a, b, a, b **6M**

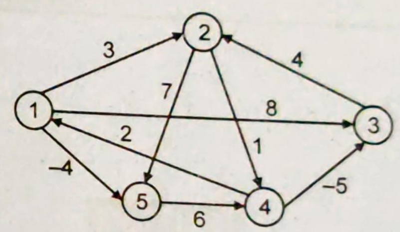
Ans. **See. P.4-33, Q.46.**

Q.7.(b) What is travelling salesman problem? Implement it for the following matrix. **7M**

$$\begin{bmatrix} 0 & 8 & 16 & 15 \\ 14 & 0 & 9 & 12 \\ 7 & 10 & 0 & 6 \\ 11 & 13 & 10 & 0 \end{bmatrix}$$

Ans. **See. P.4-26, 28, Q.35, 39.**

Q.8. Write Floyd Warshall algorithm for all pairs shortest path problem. Calculate distance matrix and path matrix for following graph. **13M**



Ans. **See. P.4-15, 18, Q.10, 17.**

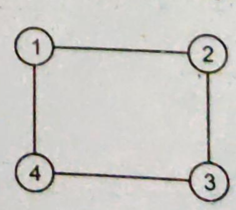
UNIT - V

(Total Marks : 26)

Q.9.(a) Write a algorithm for vertex cover problem using approximation approach. **6M**

Ans. **See. P.5-30, Q.39.**

Q.9.(b) What is planner graph? Implement graph coloring on following graph and generate solution space tree of no. of permitted colors = 3. Also write algorithm for the graph coloring. **7M**

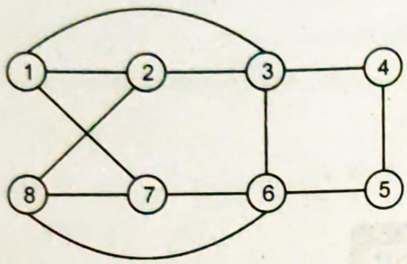


Ans. **See. P.5-22, 23, Q.29, 30.**

Q.10.(a) What is backtracking? Explain the application in which backtracking principle can be used to design a solution. **6M**

Ans. **See. P.5-16, Q.18.**

Q.10.(b) What is the use of Hamiltonian cycle? Implement Hamiltonian cycle on following graph. **7M**



Ans. **See. P.5-24, 26, Q.31, 32.**

UNIT - VI

(Total Marks : 26)

Q.11.(a) Write a note on :

(i) Clique **3M**

(ii) NP-Hard **3M**

(iii) NP-Complete **3M**

Ans. **See. P.6-12, 6, Q.17, 3.**

Q.11.(b) Explain polynomial reduction. **4M**

Ans. **See. P.6-11, Q.16.**

Q.12.(a) What is non-deterministic algorithm? Explain primality testing. **7M**

Ans. **See. P.6-9, PS-71, Q.11, 11(a).**

Q.12.(b) Write a brief note on NP-Complete problem. **6M**

Ans. **See. P.6-4, Q.1.**

WINTER EXAMINATION - 2017
B.E. V SEM. (CT) (CBS)
DESIGN AND ANALYSIS OF ALGORITHMS

Statistical Analysis

Unit Number	Marks
Unit - I	26 M
Unit - II	28 M
Unit - III	26 M
Unit - IV	28 M
Unit - V	26 M
Unit - VI	26 M
Questions covered in VBD	160M (100%)

WINTER EXAM. - 2017

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 26)

Q.1.(a) What is algorithm? Explain the characteristics of algorithm. **5M**

Ans. See P.1-32, Q.56 and P.PS-58, Q.1(c).

Q.1.(b) Solve the recurrence by characteristic equation.

$$f_n = \begin{cases} 0 \text{ if } n = 0 \\ 5 \text{ if } n = 1 \\ 3f_{n-1} + 4f_{n-2} \text{ otherwise} \end{cases}$$ **5M**

Ans. See P.1-21, Q.37.

Q.1.(c) Explain time complexity and space complexity with respect to any algorithm. **3M**

Ans. See P.1-30, Q.51.

Q.2.(a) Solve the following recurrence relation using master theorem : **7M**

(a) $T(n) = 2T\left(\frac{n}{2}\right) + n^3$.

(b) $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$.

(c) $T(n) = 9T\left(\frac{n}{3}\right) + n$.

Ans.

(a) $T(n) = 2T\left(\frac{n}{2}\right) + n^3$ ..... (1)

Comparing equation (1) with

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a = 2,\ b = 2,\ f(n) = n^3$

$f(n) = O\ (n^{\log_b^a})$

$n^3 = O\ (n^{\log_2^2})$

$n^3 = \Omega\ (n^{1+\epsilon})$

Here case 3 of master theorem applies

$\therefore \boxed{T(n) = \theta(f(n))}$

(b) $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$

$a = 3,\ b = 4,\ f(n) = n \log n$

Comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$f(n) = O(n^{\log_4^3})$

$n \log n = O(n^{\log_4^3})$

$= O(n^{\log_4^{3+\epsilon}})$

Here, case 3 of master theorem applies.

$\therefore \boxed{T(n) = \theta(n \log n)}$

(c) $T(n) = 9T\left(\frac{n}{3}\right) + n$

Comparing equation with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a = 9,\ b = 3,\ f(n) = n$

$f(n) = O(n^{\log_b^a}) = O(n^{\log_3^9})$

$n = O(n^{2-\epsilon})$

$T(n) = \theta(n^{\log_3^9})$

Here, case 1 of master theorem applies.

$\therefore \boxed{T(n) = \theta(n^2)}$

Q.2.(b) Explain summation of arithmetic and geometric series. **6M**

Ans. **See P.1-7, Q.3, 6.**

UNIT - II

(Total Marks : 28)

Q.3.(a) Explain the Asymptotic notation of analysis of algorithm. Also find θ notation for : **7M**

(i) $5n^3 + n^2 + 3n + 2$.

(ii) $27n^2 + 16n$.

Ans. **See P.2-4, Q.1.**

(i) $5n^3 + n^2 + 3n + 2$:

Sr. No.	Lower bound $\leq 5n^3$	Tight bound $\leq 5n^3 + n^2 + 3n + 2$	Upper bound $6n^3$
n = 1	5	11	6
n = 2	40	52	48
n = 3	135	155	162

Lower bound ≤ Tight bound ≤ Upper bound

or Big-oh ≤ omega ≤ Theta

i.e. for n = 3

θ -notation for $5n^3 + n^2 + 3n + 2$ is 162

(ii) $27n^2 + 16n$:

Sr. No.	Lower bound $\leq 27n^2$	Tight bound $\leq 27n^2 + 16n$	Upper bound $28n^2$
for n = 1	27	43	28
for n = 2	108	140	112
for n = 3	243	291	252
for n = 4	432	496	448
for n = 5	675	755	700
for n = 6	972	1068	1008
for n = 7	1323	1435	1372
for n = 8	1728	1856	1792
for n = 9	2187	2331	2268
for n = 10	2700	2860	2800
for n = 11	3267	3443	3388
n = 12	3888	4080	4032
n = 13	4563	4771	4732
n = 14	5292	5516	5488
n = 15	6075	6315	6300
n = 16	6912	≤ 7168	≤ 7168

for n = 16, θ -notation for $27n^2 + 16n$ is 7168

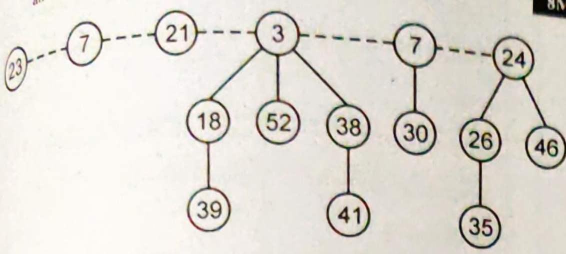
Q.3.(b) Explain Amortized analysis with example. Give its application for analyzing stack operation. **7M**

Ans. **See P.2-9, 10, Q.7, 8.**

Q.4.(a) What is divide and conquer strategy for binary search algorithm? Comment on its complexity. **6M**

Ans. See P.3-10, 11, Q.1, 2.

Q.4.(b) Explain the process of deleting a node from Fibonacci Heap structure. Draw all the modification if minimum value is deleted from the tree. **8M**



Ans. See P.2-14, Q.17.

UNIT - III

(Total Marks : 26)

Q.5.(a) Consider 5 items along their respective weight and value : **6M**

$I = (I_1, I_2, I_3, I_4, I_5)$

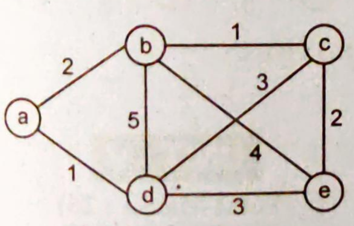
$W = (5, 10, 20, 30, 40)$

$V = (30, 20, 100, 90, 160)$

For capacity of knapsack W = 60. Find the solution of fractional knapsack problem.

Ans. See P.3-41, Q.46.

Q.5.(b) **Write a KRUSKAL algorithm to generate spanning tree. Also implement algorithm on following graph.** **7M**

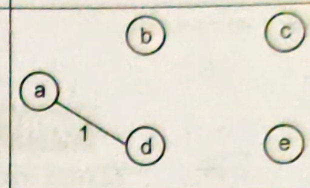
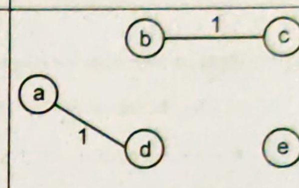
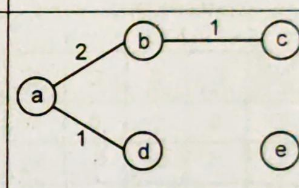
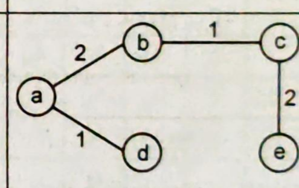


Ans. See P.3-46, Q.56.

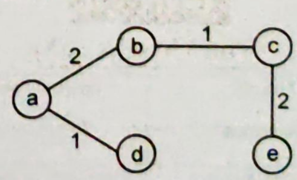
Step 1 : Arrange all the edges in ascending order of their weight.

Edges	Weight
(a, d)	1
(b, c)	1
(a, b)	2
(c, e)	2
(c, d)	3
(d, e)	3
(b, e)	4
(b, d)	5

Step 2 :

Edge	Cost	Action	Components	Connected graph
(a, d)	1	Accept	{a, d}, {b}, {c}, {e}	
(b, c)	1	Accept	{a, d} {b, c} {e}	
(a, b)	2	Accept	{a, b, c, d} {e}	
(c, e)	2	Accept	{a, b, c, d, e}	
(c, d)	3	Reject	{a, b, c, d, e}	

Step 3 : Minimum cost spanning tree :



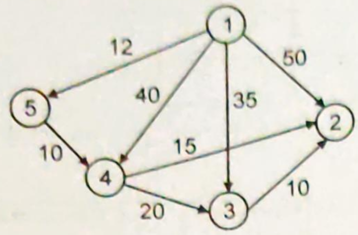
cost = 6

Q.6.(a) Write the algorithm of optimal Huffman code. Find optimal Huffman codes for following set of frequencies : **6M**

a : 25, b : 50, c : 15, d : 75, e : 40

Ans. **See P.3-58, Q.77, 78.**

Q.6.(b) Write greedy based single source shortest path algorithm. Implement the algorithm on following graph. **7M**



Ans. See P.3-56, 57, Q.73, 75.

UNIT - IV

(Total Marks : 28)

Q.7.(a) Write an algorithm for longest common subsequence. Determine LCS for the following X = "HOUSE" and Y = "COMPUTER." **8M**

Ans. See P.4-31, Q.44.

X \ Y	O	C	O	M	P	U	T	E	R
O	0	0	0	0	0	0	0	0	0
H	0	0_U	0_U	0_U	0_U	0_U	0_U	0_U	0_U
O	0	0_U	1_D	1_S	1_S	1_S	1_S	1_S	1_S
U	0	0_U	1_U	1_U	1_U	2_D	2_S	2_S	2_S
S	0	0_U	1_U	1_U	1_U	2_U	2_U	2_U	2_U
E	0	0_U	1_U	1_U	1_U	2_U	2_U	3_D	3_S
			O			U		E	

$\therefore$ LCS = OUE

Q.7.(b) Differentiate between greedy approach and dynamic approach. **6M**

Ans.

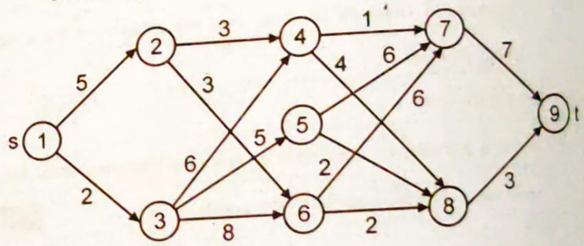
Sr. No.	Greedy algorithm	Dynamic programming
(1)	Greedy algorithms are short sighted algorithms.	Algorithms in dynamic programming are not short sighted i.e. every path in problem is evaluated first before making a decision.
(2)	Greedy algorithm makes local optimization at given point of time.	Dynamic programming breaks down the problem into smaller components.
(3)	Greedy algorithm have local choice of sub-problem that will lead to solution.	Dynamic programming solve all subproblems and then selects one that lead to optimal solution.
(4)	More efficient as compared to dynamic programming.	Less efficient as compared to greedy approach.

Q.8.(a) What is traveling salesman problem? Implement traveling salesman problem for the following matrix representation of complete graph. **7M**

$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

Ans. See P.4-26, 27, Q.35, 38.

Q.8.(b) Find a minimum cost path from s to t in the multistage graph using forward approach. **7M**



Ans. See P.4-11, 12, Q.4, 5.

UNIT - V

(Total Marks : 26)

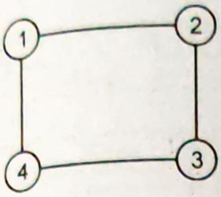
Q.9.(a) Let, W = {10, 25, 5, 10} and m = 25. Find all possible subset of W that generate sum = m and draw the portion of state space tree for the same. **7M**

Ans. See P.PS-45, Q.9(b).

Q.9.(b) Explain how backtracking can be applied to solve 4-queen problem and give two solution of 4-queen problem. **6M**

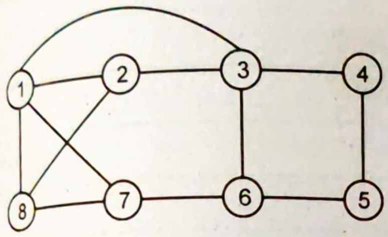
Ans. See P.5-19, Q.26.

(a) What is planner graph? Implement graph colouring on following graph and generate solution space tree if no. of permitted colors = 3 also write algorithm for graph coloring. **8M**



Ans. See P.5-23, 22, Q.30, 29.

(b) Discuss Hamiltonian cycle. Implement Hamiltonian cycle on following graph. **5M**



Ans. See P.5-24, 26, Q.31, 32.

UNIT - VI

(Total Marks : 26)

Q.11.(a) Explain the following terms : **8M**

(1) P class of problems.

(2) NP class of problems.

(3) NP complete problems.

(4) NP hard problems.

Ans. See P.6-6, Q.2, 3.

Q.11.(b) Explain the concept of polynomial reduction. **5M**

Ans. See P.6-11, Q.16.

Q.12.(a) Write short notes on decision and optimization problem. **7M**

Ans. See P.6-9, 10, Q.12, 13.

Q.12.(b) Explain in details about cook's theorem. **6M**

Ans. See P.6-10, Q.15.