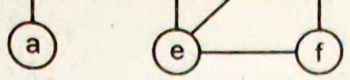


(b) Apply backtracking algorithm. Solve the instance of the sum of subset problem. **6M**

$s = \{1, 3, 4, 5\}$ and $m = 11$.

Find all possible subset of s that generates sum = m and draw solution space tree for same.

Q.11.(a) Explain P, NP, NP - complete and NP - hard with suitable example. **8M**

(b) Differentiate between decision and optimization problems with suitable example. **5M**

OR

Q.12.(a) Explain the following NP problem with respect to graph : **9M**

(1) Clique.

(2) Graph partitioned into triangle.

(3) Independent set problem.

(b) Prove that : $P \leq NP$. **4M**

SUMMER EXAM. - 2018

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 26)

Q.1.(a) Define algorithm in detail. Explain its characteristics. **6M**

Ans. See P.1-32, Q.56, P.PS-58, Q.1(c).

Q.1.(b) Solve the following recurrence by using substitution method : **7M**

(1) $T(n) = 2T\left(\frac{n}{2}\right) + n$

(2) $T(n) = T\left(\frac{n}{2}\right) + 1$

Ans. See P.1-22, Q.40.

Q.2.(a) Solve the following recurrence relation using master theorem. **8M**

(1) $T(n) = 9T\left(\frac{n}{3}\right) + n$

(2) $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$

(3) $T(n) = 2T\left(\frac{n}{4}\right) + n^2$

(4) $T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$

Ans. **(1), (2), (3) See P.PS-141, Q.2(a).**

(4) $T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$

$$= 4T\left(\frac{n}{2} + n^2 \times \frac{1}{n}\right)$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

See P.1-19, Q.32(3)

Q.2.(b) Find the time complexity for following algorithm : **5M**

```
Algorithm sum (a [ ], n)
{
    S = 0.0;
    for j = 1 to n do
    s = s + a [i];
    return s;
}
```

Ans. **See P.PS-66, Q.1(b).**

UNIT - II

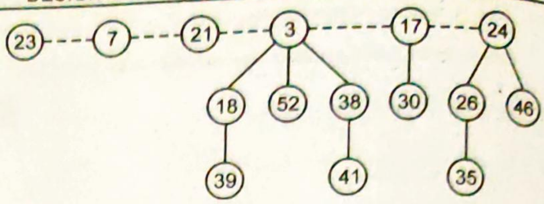
(Total Marks : 28)

Q.3.(a) What are the different asymptotic notation? Explain them briefly for the following equation find the values of constants using various approach : **7M**

(i) $3n + 2$ (ii) $10n^2 + 4n + 2$

Ans. **See P.2-4, 6, Q.1, 3.**

Q.3.(b) Explain the process of deleting a node from Fibonacci heap structure. Draw all the modification if minimum value is deleted from tree. **7M**



Ans. **See P.2-14, Q.17.**

Q.4.(a) Explain the worst case complexity of quick sort by using its recurrence equation. **5M**

Ans. **See P.3-18, Q.11.**

Q.4.(b) Binary search follows divide and conquer algorithm design strategies whose complexity is $\log_2^n$ if it is applied, an input size of n. **9M**

Write a modified binary search algorithm, which can reduce complexity up to $\log_3^n$.

Ans. **Ternary search :**

Modified binary search algorithms :

```
int binarysearch (int arr[ ], int 1, int r, int X)
{
    if (r > =1)
    {
        int mid = 1 + (r – 1) /2;
        if (arr [mid] = = X)
        return mid;
        if arr [mid] > x)
          return
        binary search(arr, 1, mid–1, X) .
        return
        binary serach (arr, mid+1, r, x)
    };
    return – 1;
}
```

In ternary search there are $4\log_3^{n+1}$ comparison in worst case

Time complexity = $4(\log_3^n + O(1))$

UNIT - III

(Total Marks : 26)

Q.5.(a) Find the optimal solution to the fractional knapsack. If the knapsack capacity W = 60 kg. **7M**

Item	I_1	I_2	I_3	I_4	I_5
Weight	5	10	15	22	25
Cost	30	40	45	77	90

Ans.

Item	Weight	Profit	Capacity Left
I_5	25	90	$60 - 25 = 35$
I_4	22	77	$35 - 22 = 13$
* I_3	15	45	$13 - 13 = 0$
		212	

Total profit = 212

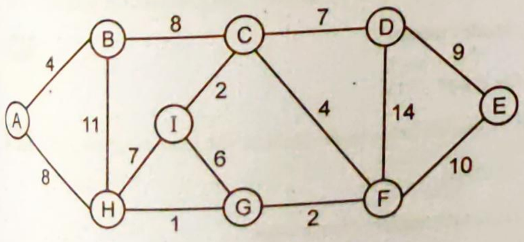
Object I_3 is partially filled.

Q.5.(b) Explain job scheduling approach find the optimal schedule for following job with n = 6. **6M**

Job (n)	Profit (P_i)	Deadline (D_i)
1	20	3
2	15	1
3	10	1
4	7	3
5	5	1
6	3	3

Ans. See P.3-44, Q.52.

Q.6.(a) Write algorithm to find out minimum cost spanning tree for the following graph implement PRIMS algorithm. **7M**



Ans. See P.3-50, 53, Q.65, 69.

Q.6.(b) What is optimal merge pattern? Implement optimal binary merge pattern for ten files whose length are 38, 42, 22, 15, 94, 63, 101, 43, 13, 21. **6M**

Ans. See P.3-59, Q.80

Given : [38, 42, 22, 15, 94, 63, 101, 43, 13, 21]

Sort = [13, 15, 21, 22, 38, 42, 43, 63, 99, 101]

[28, 21, 22, 38, 42, 43, 63, 94, 101]

sort [21 22 28 38 42 43 63 94 101]

[43 28 38 42 63 94 101]

sort [28 38 42 43 43 63 94 101]

[66 42 43 43 63 94 101]

sort [42 43 43 63 66 94 101]

[85 43 63 66 94 101]

sort [43 63 66 85 94 101]

[106 66 85 94 101]

sort [66 85 94 101 106]

[151 94 101 106]

sort [94 101 106 151]

[195 106 151]

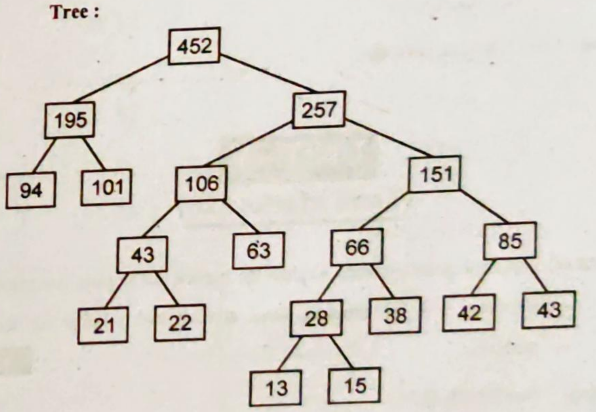
sort [106 151 195]

[257 195]

sort [195 257]

[452]

Tree :



UNIT - IV

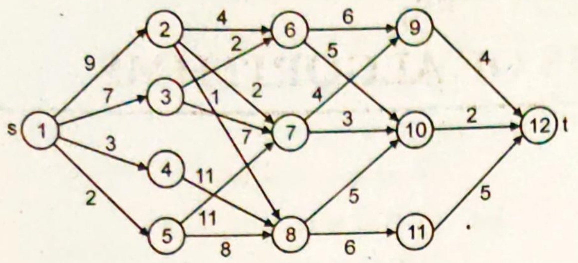
(Total Marks : 28)

Q.7.(a) What is traveling salesman problem? Implement traveling salesman problem for the following matrix representation of complete graph. **7M**

$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

Ans. See P.4-25, 27, Q.35, 38.

Q.7.(b) Find the minimum cost path from "s" to "t" in multistage graph shown, using forward approach. **7M**



Ans. See P.4-14, Q.9.

Q.8.(a) For the following probability value implement optimal binary search tree. Reconstruct the tree. **7M**

i	0	1	2	3	4
p_i	-	0.10	0.14	0.09	0.12
q_i	0.14	0.12	0.13	0.10	0.06

Ans. See P.4-25, Q.34.

Q.8.(b) Write an algorithm to generate Longest Common Subsequence (LCS). Apply the algorithm for the following string and generate LCS with the help of LCS matrix : **7M**

X = a, a, b, a, a, b, a, b, a, a.

Y = b, a, b, a, a, b, a, b.

Ans. See P.4-31, 33, Q.44, 46.

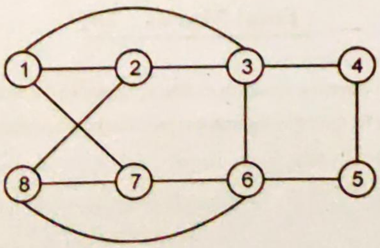
UNIT - V

(Total Marks : 26)

Q.9.(a) Explain 8 queen problem. Explain the explicit and implicit constraints associated with this problem. Give at least two solution for this problem. **8M**

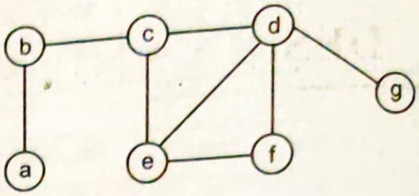
Ans. See P.5-21, Q.27.

Q.9.(b) What is the use of Hamiltonian cycle? Implement Hamiltonian cycle on the following graph. **5M**



Ans. See P.5-24, 26, Q.31, 32.

Q.10.(a) What is an approximation algorithm? Find approximate vertex cover of following graph. **7M**



Ans. See P.5-30, 32, Q.39, 40.

Q.10.(b) Apply backtracking algorithm. Solve the instance of the sum of subset problem. **6M**

s = {1, 3, 4, 5} and m = 11.

Find all possible subset of s that generates sum = m and draw solution space tree for same.

Soln. As value of m = 11

from the given subset

s = {1, 3, 4, 5}

We cannot satisfy the value of m = 11

So we cannot further solve the above problem.

UNIT - VI

(Total Marks : 26)

Q.11.(a) Explain P, NP, NP - complete and NP - hard with suitable example. **8M**

Ans. See P.6-4, Q.1.

Q.11.(b) Differentiate between decision and optimization problems with suitable example. **5M**

Ans. See P.6-10, Q.13.

Q.12.(a) Explain the following NP problem with respect to graph : **9M**

(1) Clique

(2) Graph partitioned into triangle

(3) Independent set problem

Ans. See P.6-12, Q.17.

Q.12.(b) Prove that : $P \le NP$. **4M**

Ans. See P.6-7, Q.7.

WINTER EXAMINATION - 2018
B.E. V SEM. (CT) (CBS)
DESIGN AND ANALYSIS OF ALGORITHMS

Statistical Analysis

Unit Number	Marks
Unit - I	26 M
Unit - II	28 M
Unit - III	26 M
Unit - IV	28 M
Unit - V	26 M
Unit - VI	26 M
Questions covered in VBD	160 M (100%)

Q.12.(a) Illustrate following class of problems with suitable example : 8M

(1) P class of problems

(2) NP class of problems

(3) NP-Hard class of problems.

(4) NP-Complete class of problems.

(b) Write short notes on : **5M**

(1) Decision problem.

(2) Optimization problem.

WINTER EXAM. - 2018

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 26)

Q.1.(a) Explain summation of arithmetic, geometric and telescopic series. **6M**

Ans. **See P.1-7, Q.3, 4, 6, 7.**

Telescopic series :

In mathematics, a telescoping series is a series whose partial sums eventually only have a fixed number of terms after cancellation. The cancellation technique, with part of each term cancelling with part of the next term, is known as the method of differences.

For example, the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

(The series of reciprocals of pronic numbers) simplifies as

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \lim_{N\to\infty} \sum_{n=1}^{N} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \lim_{N\to\infty} \left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\dots\dots+\left(\frac{1}{N}-\frac{1}{N+1}\right)\right]$$

$$= \lim_{N\to\infty} \left[1+\left(-\frac{1}{2}+\frac{1}{2}\right)+\left(-\frac{1}{3}+\frac{1}{3}\right)+\dots\dots\right.$$

$$\left.+\left(-\frac{1}{N}+\frac{1}{N}\right)-\frac{1}{N+1}\right]$$

$$= \lim_{N\to\infty} \left[1-\frac{1}{N+1}\right] = 1$$

Q.1.(b) Solve the following recurrence : **7M**

$$T(n) = \begin{cases} 1 & \text{, if } n = 0 \\ 2T(n-1)+6 & \text{, otherwise} \end{cases}$$

Ans. **See P.PS-128, Q.2(b).**

Q.2.(a) Explain what do you mean by recurrence relation. Solve the following recurrence using master theorem : **8M**

(a) $T(n) = 2T\left(\frac{n}{2}\right)+n$.

(b) $T(n) = 7T\left(\frac{n}{2}\right)+n^2$.

(c) $T(n) = 8T\left(\frac{n}{2}\right)+n^3$.

(d) $T(n) = 4T\left(\frac{6n}{18}\right)+\log n$.

Ans.

(a) $T(n) = 2T\left(\frac{n}{2}\right)+n$: **See P.1-28, Q.47.**

(b) $T(n) = 7T\left(\frac{n}{2}\right)+n^2$

Here $a = 7,\ b = 2\ f(n) = n^2$

$\log_b^a = \log_2^7 = 2.8$

$n^{\log_b^a} = n^{2.8}$

$f(n) = n^2$

$\therefore\ \theta(T(n)) = \theta\left(n^{\log_b^a}\right) \Rightarrow n^{\log_2^7} = n^{2.81}$

$\theta(n\log_2^7 - \epsilon) = \theta(n^{2.81} - 0.81) f(n) = n^2$

$T(n) = \theta(n^{\log_b^a})$

$\therefore\ T(n) = \theta(n^{2.81})$

(c) $T(n) = 8T\left(\frac{n}{2}\right)+n^3$

Here $a = 8,\ b = 2\ f(n) = n^3$

$\log_b^a = \log_2^8 = 3$

$n^{\log_b^a} = n^3$

$f(n) = n^3$

$\therefore\ n^{\log_b^a} = f(n)$

$n^{\log_2^8} = n^2$

$n^3 > n^2$

$\therefore$ Checking whether $af\left(\frac{n}{b}\right) < cf(n)$ for any constant c.

$\therefore\ \theta(T(n)) = \theta(f(n)).\theta(n^3)$

(d) $T(n) = 4T\left(\frac{6n}{18}\right)+\log n$:

See P.PS-128, Q.2(a)(1).

Q.2.(b) Solve the following recurrence relation : **5M**

$$T(n) = 3T\left(\frac{n}{4}\right)+\theta(n^2)$$

Using recursion tree method.

Ans. **See P.1-21, Q.36.**

UNIT - II

(Total Marks : 28)

Q.3.(a) Explain different asymptotic notations : **5M**

(1) Big oh.

(2) Theta.

(3) Big omega

Ans. **See P.2-4, Q.1.**

(4) Little omega

Ans. **See P.PS-32, Q.3(a)(v).**

(5) Little oh :

Little oh notation is used to describe an upper bound that cannot be tight.

Let f(n) and g(n) be functions that map positive integer to positive real numbers such that f(n) = 0 (g(n)) if there exists an integer constant $\eta_o > 1$ such that $0 \le f(n) < c * g(n)$.

Q.3.(b) Explain divide and conquer strategy for binary-search algorithm. Write its recurrence relation and comment on its complexity. Also comment on change in recurrence relation and complexity if array is divided in three equal parts, for searching **9M**

Ans. See P.3-10, 15, 19, 20, Q.1, 9, 15, 16.

Q.4.(a) Define amortized analysis of algorithm. Explain any one method with suitable example. **7M**

Ans. See P.2-9, Q.7.

Q.4.(b) Use Strassen's algorithm to compute matrix product. Show the steps for following matrices : **7M**

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$$

Ans. $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$

$P (A_{11} + A_{22}) (B_{11} + B_{22})$

$= (1 + 7) (6 + 2) = 8 \times 8 = 64$

$Q = (A_{21} + A_{22}) B_{11}$

$= (5 + 7) \times 6 = 72$

$R = A_{11} (B_{12} - B_{22})$

$= 1 (4 - 2) = 2$

$S = A_{22} (B_{21} - B_{11})$

$= 7 (3 - 6) = - 21$

$T = (A_{11} + A_{12}) B_{22}$

$= (1 + 3) (2) = 8$

$U = (A_{21} - A_{11}) (B_{11} + B_{12})$

$= (5 - 1) (6 + 4) = 4 \times 10 = 40$

$V = (A_{12} - A_{22}) (B_{21} + B_{22})$

$= (3 - 7) (3 + 2) = - 4 \times 5 = - 20$

$C_{11} = P + S - T + V = 64 - 21 - 8 + (- 20) = 15$

$C_{12} = R + T = 2 + 8 = 10$

$C_{21} = Q + S = 72 - 21 = 51$

$C_{22} = P + R - Q + U = 64 + 2 - 72 + 40 = 34$

$$\therefore \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 10 \\ 51 & 34 \end{bmatrix}$$ **Ans.**

UNIT - III

(Total Marks : 26)

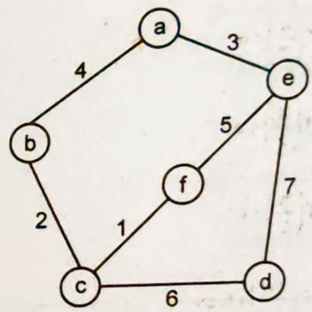
Q.5.(a) Find out solution for fractional knapsack problem using Greedy Strategy for following instance n = 7 m = 15. **6M**

$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10, 5, 15, 7, 6, 18, 3)$

$(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (2, 3, 5, 7, 1, 4, 1)$

Ans. **See P.3-41, Q.47.**

Q.5.(b) Obtain minimum spanning tree for given undirected graph using Prim's algorithm. Assume vertex 'a' as source : **7M**



Ans. **See P.PS-88, Q.5(b).**

Q.6.(a) Given eight activities along with their start and finish time as follows : **8M**

A_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
S_i	1	2	3	4	8	9	11	12
F_i	5	2	4	7	11	12	13	16

Compute a schedule where largest number of activities takes place using Greedy approach.

Ans. **See P.PS-42, Q.5(b).**

Q.6.(b) Write Huffman code algorithm. Also find optimal Huffman code for following set of frequencies :

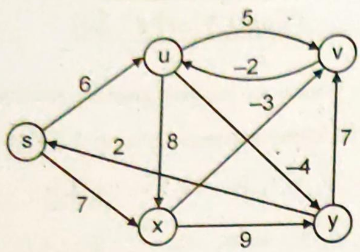
a : 20, b : 15, c : 5, d : 25, e : 35, f : 11 **5M**

Ans. **See P.PS-94, Q.6(a).**

UNIT - IV

(Total Marks : 28)

Q.7.(a) Explain Bellman - Ford algorithm and find its complexity. Find the shortest distance using Bellman Ford algorithm for given graph : **7M**



Ans. See P.4-20, Q.20.

Q.7.(b) Write an algorithm to find longest common subsequence. Also find its complexity. Find optimal solution for given sequence :

$X = \langle A, B, C, D, B, A, B \rangle$

$Y = \langle B, C, D, A, B, A \rangle$ **7M**

Ans. See P.4-31, Q.44.

$X = \langle A, B, C, D, B, A, B \rangle$

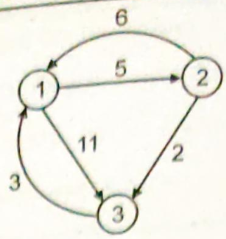
$Y = \langle B, C, D, A, B, A \rangle$

	y_i	B	C	D	A	B	A
x_i	0	0	0	0	0	0	0
A	0	0	0	0	1	1	1
B	0	1	1	1	1	2	2
C	0	1	2	2	2	2	2
D	0	1	2	3	3	3	3
B	0	1	2	3	3	4	4
A	0	1	2	3	4	4	5
B	0	2	2	3	4	5	5

Length of string = 5

Common sequence = B C D B A

Q.8.(a) Find out shortest distance between all pairs of vertices and write the Floyd-Warshall all pair shortest path algorithm. **6M**



Ans. See P.4-16, Q.12.

Q.8.(b) Draw optimal binary search tree for the following. Also generate matrices required for tree construction : **8M**

i	0	1	2	3	4	5
p_i	--	0.08	0.05	0.12	0.20	0.10
q_i	0.05	0.11	0.05	0.11	0.10	0.04

Ans. See P.PS-112, Q.8(a).

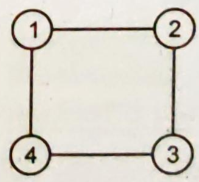
UNIT - V

(Total Marks : 26)

Q.9.(a) Give generalized schema for recursive back-tracking algorithm and explain in brief. **6M**

Ans. See P.5-16, Q.18.

Q.9.(b) Implement graph coloring on following graph and generate solution space tree for number of permitted colors = 3. Also write algorithm for graph coloring. **7M**



Ans. See P.5-22, Q.29.

Q.10.(a) Explain Knight's tour problem and give algorithm for it. **6M**

Ans. See P.PS-35, Q.10(b).

Q.10.(b) Consider $S = \{S_1, S_2, S_3, S_4\}$ and weight vector $W = \{10, 25, 5, 10\}$ and $M = 25$. Then compute all possible subsets of W that sum to M. Draw the portion of state tree that generates a fixed length tuple using backtracking algorithm. **7M**

Ans. See P.PS-45, Q.9(b).

UNIT - VI

(Total Marks : 26)

Q.11.(a) Explain non-deterministic algorithm. Give non-deterministic algorithm for searching and sorting problem. **6M**

Ans. **See P.6-9, Q.11, P.PS-71, Q.11(a)(ii).**

Q.11.(b) Explain the concept of polynomial reduction and how it can be used for showing NP completeness of problem. **7M**

Ans. **See P.6-11, Q.16.**

Q.12.(a) Illustrate following class of problems with suitable example : **8M**

(1) P class of problems

(2) NP class of problems

(3) NP-Hard class of problems.

(4) NP-Complete class of problems.

Ans. **See P.6-5, 6, Q.1, 3.**

Q.12.(b) Write short notes on : **5M**

(1) Decision problem.

(2) Optimization problem.

Ans. **See P.6-9, Q.12.**

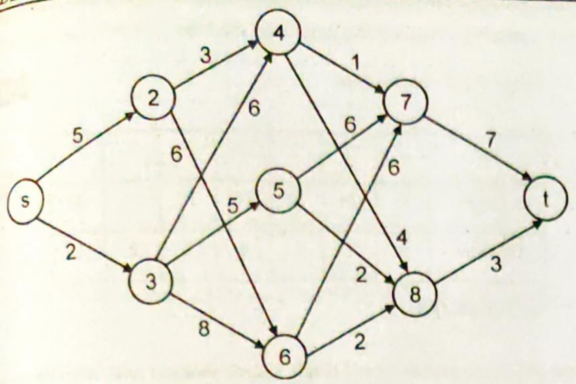
SUMMER EXAMINATION - 2019

B.E. V SEM. (CT) (CBS)

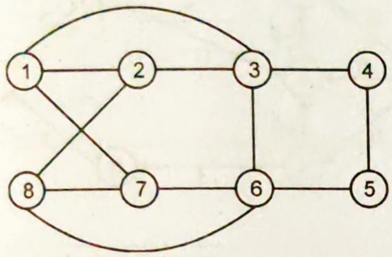
DESIGN AND ANALYSIS OF ALGORITHMS

Statistical Analysis

Unit Number	Marks
Unit - I	28 M
Unit - II	26 M
Unit - III	33 M
Unit - IV	28 M
Unit - V	19 M
Unit - VI	26 M
Questions covered in VBD	160M (100%)



(b) What is the use of Hamiltonian cycle? Implement Hamiltonian cycle on following graph? **6M**



OR

Q.10.(a) Discuss 4-queen problem and give its algorithm using backtracking method. **7M**

(b) Let W = {11, 13, 24, 7} and m = 31 find all possible subset of w such that its sum to M. Draw the portion of the state space tree. **6M**

Q.11.(a) What is clique? Comment about its NP-Completeness. **6M**

(b) Comment on $P \subseteq NP$. **6M**

OR

Q.12.(a) What is non - deterministic algorithm? Explain primality testing. **7M**

(b) How polynomial reduction can be used for showing NP - Completeness of a problem? **6M**

SUMMER EXAM. - 2019

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 28)

Q.1.(a) What is an algorithm? Explain the properties of algorithm. **6M**

Ans. See P.1-32, Q.56.

Q.1.(b) Explain geometric and harmonic series with one example of each. **8M**

Ans. See P.1-7, PS-135, Q.6, 1(a).

Q.2.(a) Solve the following recurrence relation using master method. **6M**

(i) $T(n) = 3T(n/4) + n \log n$

(ii) $T(n) = 3T(\sqrt{n}) + \log n$

Ans. See P.1-29, 17, Q.48, 30(i).

(i) $T(n) = 3T(n/4) + n\log n$

$$aT\left(\frac{n}{b}\right) + \theta(n \log^p n)$$

Here $f(n) = n \log n$

$a = 3$

$b = 4$

$k = 1$

$p = 1$

$p^k = 4' = 4$

$a = 3 < 4$

$T(n) = \theta(n^k \log^p n)$

$\boxed{T(n) = \theta(n \log n)}$

(ii) $T(n) = 3T(\sqrt{n}) + \log n$ $\left[\because \sqrt{n} = \frac{n}{2}\right]$

Here $a = 3$, $b = 2$ $f(n) = \log n$

$k = 0$, $p = 1$

Here $b^k = 2^0 = 1$

$q > b^k$ $[\because 3 > 1]$

$T(n) = \theta(n \log_b^a)$

$= \theta(n \log_2^3)$

Q.2.(b) Solve the following recurrence: **8M**

$T(n) = 0$ if $n = 0$

$T(n) = 5$ if $n = 1$

$T(n) = 3t(n-1) + 4t(n-2)$ otherwise

Ans. See P.1-21, Q.37.

UNIT - II

(Total Marks : 26)

Q.3.(a) What are the different asymptotic notations? Explain them briefly for the following equations, find the values of constants using various approaches. **8M**

(i) $20n^2 + 8n + 10$

(ii) $2n + 5$

Ans. See P.2-4, 5, Q.1, 2(ii), 2(i).

Q.3.(b) Write an algorithm for merge sort. Derive its best case and worst case time complexity. **5M**

Ans. See P.3-20, Q.16.

Q.4.(a) Explain min max search procedure using divide and conquer strategy. **7M**

Ans. See P.3-36, Q.33.

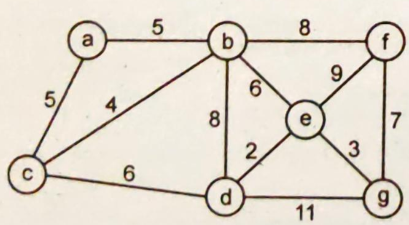
Q.4.(b) Define amortized analysis of algorithm. Explain any two method with suitable example. **6M**

Ans. See P.2-9, Q.7.

UNIT - III

(Total Marks : 33)

Q.5.(a) Obtain MST with its given undirected graph using PRIM's algorithm. Assume vertex 'a' as a root vertex. **7M**



Ans. See P.PS-5, Q.5(b)

Q.5.(b) Find out optimal solution for fractional knapsack problem using greedy strategy for following instances : **6M**

$n = 7$

$m = 15$

$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10, 5, 15, 7, 6, 18, 3)$

$(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (2, 3, 5, 7, 1, 4, 1)$

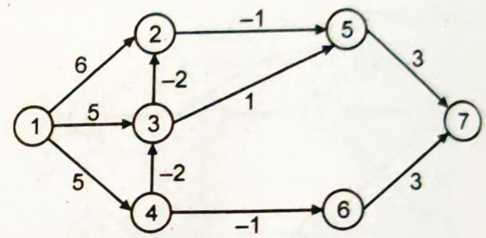
Ans. See P.3-41, Q.47.

Q.6.(a) Explain job sequencing approach. Find the best possible sequence for the following deadline: **6M**

Job	1	2	3	4	5	6	7
Gain	35	20	18	16	12	10	08
Deadline	3	1	3	4	2	2	1

Ans. See P.3-44, Q.53.

Q.6.(b) Illustrate greedy based single source shortest path algorithm on following graph. **7M**



Ans.

Step 1	1	2	3	4	5	6	7
d = 0	0	∞	∞	∞	∞	∞	∞
Step 2	1	2	3	4	5	6	7
d = 1	0	6	5	5	∞	∞	∞
Step 3	1	2	3	4	5	6	7
d = 2	0	3	3	5	5	4	∞
d = 3	0	1	3	5	2	4	∞
d = 4	0	1	3	5	2	4	7
d = 5	0	1	3	5	2	4	7
d = 6	0	1	3	5	2	4	7
d = 7	0	1	3	5	2	4	7

Q.8.(b) Explain Knapsack problem with one simple example. **7M**

Ans. See P.3-40, Q.44.

UNIT - IV

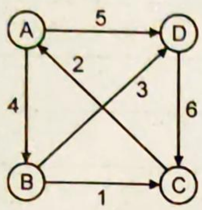
(Total Marks : 28)

Q.7.(a) Compute optimal TSP tour for following distance matrix using dynamic programming approach. **8M**

	A	B	C	D
A	0	10	15	20
B	5	0	9	10
C	6	13	0	12
D	8	8	9	0

Ans. **See P.4-26, 27, Q.35, 38.**

Q.7.(b) Find all pair shortest paths using Floyd Warshall algorithm for given graph. **6M**



Ans. **See P.4-18, Q.15.**

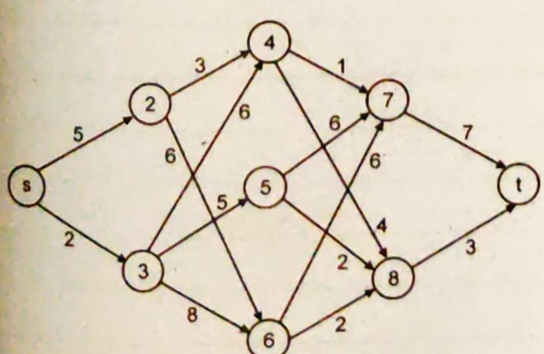
Q.8.(a) Determine LCS of x = {a, b, a, b, a, a, b, a} and y = {a, b, a, a, b, a, b}. **7M**

Ans.

		a	b	a	a	b	a	b
	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
a 1	0	1	1	1	1	1	1	1
b 2	0	1↑	2	2	2	2	2	2
a 3	0	1	2↑	3	3	3	3	3
b 4	0	1↑	2	3↑	3↑	4	4	4
a 5	0	1	2↑	3	4	4↑	5	5
a 6	0	1	2↑	3	4	4↑	5	6
b 7	0	1↑	2	3↑	4↑	5	5↑	6
a 8	0	1	2↑	3	4	5↑	6	6↑

Length = 6 ababab

Q.9.(a) For a given graph, find out the shortest distance from vertex 's' to vertex 't'. **7M**

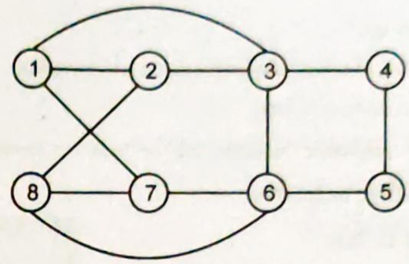


Ans. **See P.4-12, Q.5.**

UNIT - V

(Total Marks : 19)

Q.9.(b) What is the use of Hamiltonian cycle? Implement Hamiltonian cycle on following graph? **6M**



Ans. **See P.5-24, 26, Q.31, 32.**

Q.10.(a) Discuss 4-queen problem and give its algorithm using backtracking method. **7M**

Ans. **See P.5-19, Q.26.**

Q.10.(b) Let W = {11, 13, 24, 7} and m = 31 find all possible subset of w such that its sum to M. Draw the portion of the state space tree. **6M**

Ans. m = 31

$[x_1, x_2, x_4]$

$[x_3\ x_4]$

(1) [1 1 1 0]

(2) [0 0 1 1]

Positive sequences.

1 – 2 – 3 – 4

1 – 2 – 4

1 – 3 – 4

1 – 4

2 – 3 – 4

2 – 4

3 – 4

8 sequences

4

See P.5-28, Q.36.

Solution node = 1 – 2 – 4

3 – 4

UNIT - VI

(Total Marks : 26)

Q.11.(a) What is clique? Comment about its NP-Completeness. **7M**

Ans. **See P.6-12, Q.17.**

Q.11.(b) Comment on $P \subseteq NP$. **6M**

Ans. **See P.6-7, Q.7.**

Q.12.(a) What is non - deterministic algorithm? Explain primality testing. **7M**

Ans. **See P.6-9, PS-71, Q.11, 11(a).**

Q.12.(b) How polynomial reduction can be used for showing NP - Completeness of a problem? **6M**

Ans. **See P.6-11, Q.16.**

WINTER EXAMINATION - 2019

B.E. V SEM. (CT) (CBS)

DESIGN AND ANALYSIS OF ALGORITHMS

Statistical Analysis

Unit Number	Marks
Unit - I	26 M
Unit - II	26 M
Unit - III	28 M
Unit - IV	35 M
Unit - V	19 M
Unit - VI	26 M
Questions covered in VBD	160 M (100%)

WINTER EXAM. - 2019
QUESTION PAPER

Q.1.(a) Solve the following recurrence using master method:

$T(n) = T\left(\frac{n}{4}\right) + \sqrt{n} + 4$ for $n \geq 4$ and $T(1) = 4$. **7M**

(b) Explain the significance of solving recurrence in DAA? Also explain Homogenous, Non-Homogenous and Logarithmic recurrence. **6M**

OR

Q.2.(a) Solve the given recurrence: **3M**

$$t_n = \begin{cases} 0 & \text{if } n = 0 \\ 5 & \text{if } n = 1 \\ 3t_{n-1} + 4t_{n-2} & \text{otherwise} \end{cases}$$

(b) Solved the given recurrence using master method.

(a) $T(n) = T(\sqrt{n}) + 1$ **3M**

(b) $T(n) = 2T(\sqrt{n}) + \log n$ **3M**

(c) Explain various types of Algorithm design technique. **4M**

Q.3.(a) What is divide and conquer strategy? Explain binary search algorithm. **6M**

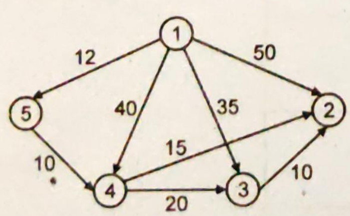
(b) Illustruse stepwise execution of quick sort on the following array A. Also give its complexity by analyzing the recurrence relation A = (1, 3, 5, 8, 7, 6, 4). **7M**

OR

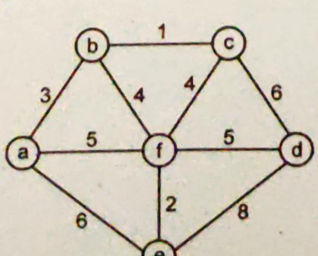
Q.4.(a) Implement insertion sort on following array A : (18, 12, 11, 22, 9, 7, 90, 77) write recursive algorithm. **6M**

(b) Define amortized analysis of algorithm. Explain any one method with suitable example. **7M**

Q.5.(a) Write Greedy based single source shortest path algorithm. Implement the algorithm on the following graph. **7M**



(b) Using Prim's algorithm, determine the minimum cost spanning tree for the graph. **7M**

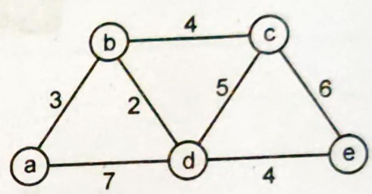


OR

Q.6.(a) Solve the following resistance of knapsack problem using greedy algorithm, Knapsack weight M = 20. **7M**

Item	1	2	3
Weight	18	15	10
Profit	25	24	15

(b) Design the Dijkstra's Algorithm? Apply the same to find the single source shortest paths problem for the graph taking vertex 'a' as source. **7M**



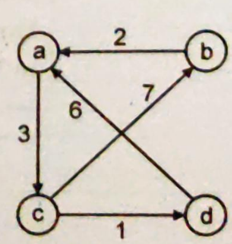
Q.7.(a) Differentiate between greedy approach and dynamic programming. **6M**

(b) What is TSP? Compute optimal TSP tour for the following matrix using dynamic programming approach. **8M**

	A	B	C	D
A	0	10	15	20
B	5	0	9	10
C	6	13	0	12
D	8	8	9	0

OR

Q.8.(a) State Floyd's Algorithm. Solve all pairs shortest path problem for the given graph using Floyd Algorithm. **7M**



(b) Find optimal solution using 0/1 Knapsack problem for given data m = 6, n = 3

$(w_1, w_2, w_3) = (3, 2, 3)$

$(p_1, p_2, p_3) = (2, 1, 4)$ **7M**

Q.9.(a) Explain the principle of optimality and show how it can be applied on optimal Binary Search tree problem. **7M**

(b) Explain graph colouring method with example, write algorithm for it. **6M**

OR

Q.10.(a) Give generalized schema for recursive backtracking algorithm and explain in brief. **6M**

(b) Write an algorithm to solve n-Queen problem. Explain implicit and explicit constraints associated with 8 queen problem. Give at least 2 solutions for 8 queen problem. **7M**

Q.11.(a) Define P, NP and NP - complete problems. How NP - complete is proved? **7M**

(b) Write an algorithm for non-deterministic sorting and searching. **6M**

OR

Q.12.(a) Explain the classes of NP hard and NP complete. **6M**

(b) Write non-deterministic algorithm to generate CLIQUE of size k form graph of n vertices. **7M**

WINTER EXAM. - 2019

SOLUTION

(Unit-Wise)

UNIT - I

(Total Marks : 26)

Q.1.(a) Solve the following recurrence using master method:

$T(n) = T\left(\frac{n}{4}\right) + \sqrt{n} + 4$ for $n \geq 4$ and $T(1) = 4$. **7M**

Ans. See P.1-24, Q.42.

Q.1.(b) Explain the significance of solving recurrence in DAA? Also explain Homogenous, Non-Homogenous and logarithmic recurrence. **6M**

Ans. A recurrence is an equation or inequality that describes a function in terms of its values on smaller inputs. To solve recurrence relation means to obtain a function defined on the natural numbers that satisfy the recurrence.

Homogenous and Non-homogenous recurrence: P.1-18, Q.31.

Logarithmic recurrence: If the updation of index (i) variable is by (*) multiplication or division operator then recurrence is logarighmic.

Q.2.(a) Solve the given recurrence: **3M**

$$t_n = \begin{cases} 0 & \text{if } n = 0 \\ 5 & \text{if } n = 1 \\ 3t_{n-1} + 4t_{n-2} & \text{otherwise} \end{cases}$$

Ans. See P.1-21, Q.37.

Q.2.(b) Solved the given recurrence using master method.

(a) $T(n) = T(\sqrt{n}) + 1$ **3M**

Ans. See P.1-23, Q.40(iii)

(b) $T(n) = 2T(\sqrt{n}) + \log n$ **3M**

Ans. See P.1-17, Q.30(i)

Q.2.(c) Explain various types of Algorithm design technique. **4M**

Ans. See P.1-32, Q.57.

UNIT - II

(Total Marks : 26)

Q.3.(a) What is divide and conquer strategy? Explain binary search algorithm. **6M**

Ans. See P.3-10, 12, Q.1, 3.

Q.3.(b) Illustruse stepwise execution of quick sort on the following array A. Also give its complexity by analyzing the recurrence relation A = (1, 3, 5, 8, 7, 6, 4). **7M**

Ans. A = (1, 3, 5, 8, 7, 6, 4)

1, 3, 5, 8, 7, 6, 4

1, 3, 5, 8, 7, 6, 4

(1) Compare all element form right to left pivot = 1

Find first element larger than pilot P

i.e. 3

next 5 compare with 4 swap.

1 3 4 8 9 6 5

↑ ↑

swap 5 8

1 3 4 5 9 6 8

↑ ↑

swap 8 9

1 3 4 5 8 6 9

swap 8, 6

[1 3 4 5 6 8 9]

sorted list.

Q.4.(a) Implement insertion sort on following array A : (18, 12, 11, 22, 9, 7, 90, 77) write recursive algorithm. **6M**

Ans. Insertion sort

A: (18, 12, 11, 22, 9, 7, 90, 77)

18 > 12

Swap 18 and 12

12 , 18 , 11 , 22 , 9 , 7 , 90 , 77

11 , 12 , 18 , 22 , 9 , 7 , 90 , 77

11 , 12 , 18 , 22 , 9 , 7 , 90 , 77

9 , 11 , 12 , 17 , 22 , 7 , 90 , 77

7 , 9 , 11 , 12 , 18 , 22 , 90 , 77

7 , 9 , 11 , 12 , 18 , 22 , 90 , 77

7 , 9 , 11 , 12 , 18 , 22 , 90 , 77

7 , 9 , 11 , 12 , 18 , 22 , 77 , 90

Algorithm:

Insertion sort (arr, n)

Loop from i = 1 to n – 1

(a) Pick element arr[i] and

insert it into sorted

sequence arr [0, ... i]

```
void insertion sort recursive (int arr, i j, int n)
{
    if (n <= 1)
        return i;
        insertionsort recursive (arr, n – 1)
            int last = arr [n – 1],
            int P = n – 2;
            while (i >= 0 && arr [j] > last)
            {
                arr[i + 1] = arr [j];
                j – –;
            }
            arr [j + 1] = last;
}
```

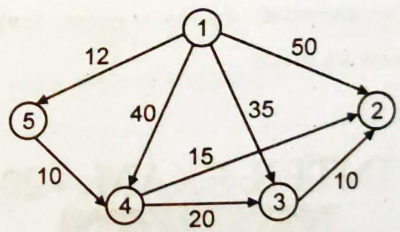
Q.4.(b) Define amortized analysis of algorithm. Explain any one method with suitable example. **7M**

Ans. See P.2-9, Q.7.

UNIT - III

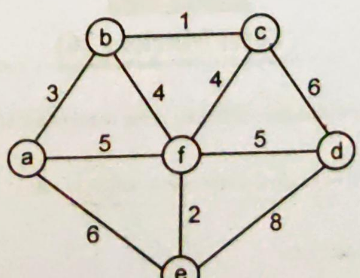
(Total Marks : 28)

Q.5.(a) Write Greedy based single source shortest path algorithm. Implement the algorithm on the following graph. **7M**

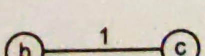


Ans. See P.PS-136, Q.6(a).

Q.5.(b) Using Prim's algorithm, determine the minimum cost spanning tree for the graph. **7M**



Ans. Consider minimum weight i.e. bc = 1

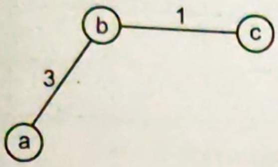


Adjacent of bc is {a, f, d}

ba = 3, bf = 5, cf = 4, cd = 6.

minimum = 3

∴ next ba = 3

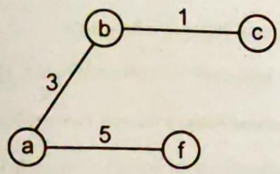


Adjacent of a = e. f.

ac = 6

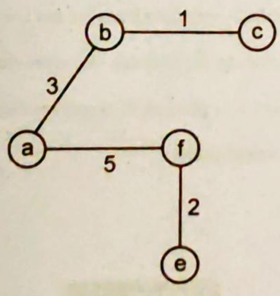
af = 5

minimum 5



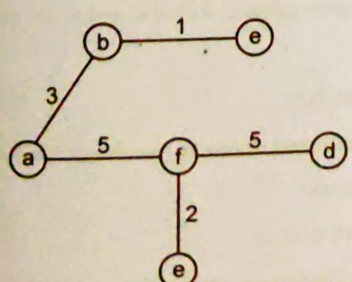
Adjacent of f is e, d, c minimum is 2

∵ ef



Next adjacent of e, f ed = 8 and fd = 5

∴ fd.



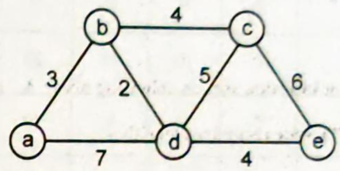
Total cost = 1 + 3 + 5 + 2 + 5 = 16

Q.6.(a) Solve the following resistance of knapsack problem using greedy algorithm. Knapsack weight M = 20. **7M**

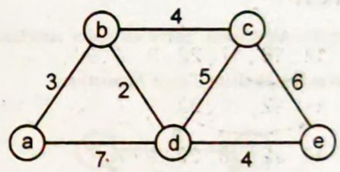
Item	1	2	3
Weight	18	15	10
Profit	25	24	15

Ans. **See P.3-40, Q.45.**

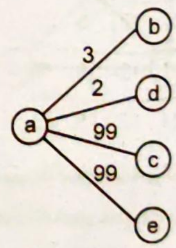
Q.6.(b) Design the Dijkstra's Algorithm? Apply the same to find the single source shortest paths problem for the graph taking vertex 'a' as source. **7M**

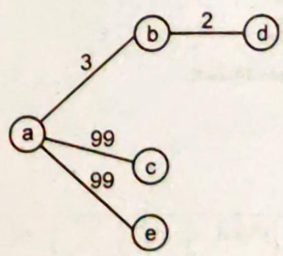


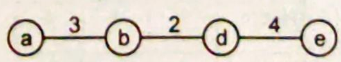
Ans. Dijkstra's algorithm is an algorithm for finding the shortest paths between nodes in a graph which may represent.



Source = a







Total cost = 9.

UNIT - IV

(Total Marks : 35)

Q.7.(a) Differentiate between greedy approach and dynamic programming. **6M**

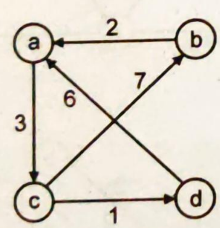
Ans. See P.PS-144, Q.7(b).

Q.7.(b) What is TSP? Compute optimal TSP tour for the following matrix using dynamic programming approach. **8M**

	A	B	C	D
A	0	10	15	20
B	5	0	9	10
C	6	13	0	12
D	8	8	9	0

Ans. See P.4-27, Q.38.

Q.8.(a) State Floyd's Algorithm. Solve all pairs shortest path problem for the given graph using Floyd Algorithm. **7M**



Ans. See P.4-15, Q.10.

As for all pair shortest path algorithm the graph should be complete all the nodes are connected. In the above graph the graph is not complete.

Q.8.(b) Find optimal solution using 0/1 Knapsack problem for given data **7M**

$m = 6,\ n = 3$

$(w_1, w_2, w_3) = (3, 2, 3)$

$(p_1, p_2, p_3) = (2, 1, 4)$

Ans. $m = 6$

Object	O_1	O_2	O_3
Profit	2	1	4
Weight	3	2	3

Select the object with highest profit.

Sr. No.	$x_1 x_2 x_3$	$\sum w_i x_i$	$\sum p_i x_i$
1	(1/2, 1/3, 1/4)	3.74	3.8
2	(1/3, 1, 0)	3	4.33
3	(0, 1, 1)	3	4.33
4	0, 1, 1/2	2.5	2.37

Object 1 has highest profit $P_1 = 4s$

Only 2 units of Knapsack capacity left.

object, has next congest profit $p_1 = 2$ However $w_1 = 3$. So it does not fit into Knapsack.

Using $x_1 = 1/3$ fill the Knapsack exactly with part of object and the value of resulting solution is 3.8.

Q.9.(a) Explain the principle of optimality and show how it can be applied on optimal Binary Search tree problem. **7M**

Ans. The dynamic programming works on principle of optimality.

The principle states that in an optimal sequence of decision on choices each subsequence must also be optimal.

Example - In OBST only the values we are interested is optional but all the other entries in the table are also represent optimal.

Optimal solution to a problem is a combination of optimal solution to some of its subproblems.

P.4-23, Q.29.

UNIT - V

(Total Marks : 19)

Q.9.(b) Explain graph colouring method with example, write algorithm for it. **6M**

Ans. See P.5-22, Q.29.

Q.10.(a) Give generalized schema for recursive backtracking algorithm and explain in brief. **6M**

Ans. See P.5-18, Q.22, 23.

Q.10.(b) Write an algorithm to solve n-Queen problem. Explain implicit and explicit constraints associated with 8 queen problem. Give at least 2 solutions for 8 queen problem. **7M**

Ans. See P.5-21, Q.27.

UNIT - VI

(Total Marks : 26)

Q.11.(a) Define P, NP and NP - complete problems. How NP - complete is proved. **7M**

Ans. **See P.6-4, 6, Q.1, 2, 3.**

Q.11.(b) Write an algorithm for non-deterministic sorting and searching. **6M**

Ans. **See P.6-9, PS-71, Q.11, 11(a)(ii)**

Q.12.(a) Explain the classes of NP hard and NP complete. **6M**

Ans. **See P.6-6, Q.3.**

Q.12.(b) Write non-deterministic algorithm to generate CLIQUE of size k form graph of n vertices, **7M**

Ans. **See P.6-12, Q.17.**