

Arithmetic, Logical and Matrix operations in R

In this lecture

- Arithmetic
- Logical
- Matrix operations

Arithmetic operations in R

Arithmetic operations in R

Symbols	Operation
=, < [*] -	<i>Assignment</i>
+	<i>Addition</i>
-	<i>Subtraction</i>
*	<i>Multiplication</i>
/	<i>Division</i>
^, **	<i>Exponent</i>
%%	<i>Remainder</i>
%/%	<i>Integer division</i>

^{*} In R only ' $< -$ ' is valid for assignment operation where as in R Studio both $=$ and $< -$ will work

Hierarchy of operations

$$A = 7 - 2 \times \frac{27}{3^2} + 4$$

Order of Precedence	Operation
<i>Bracket</i>	()
<i>Exponent</i>	\wedge , **
<i>Division</i>	/
<i>Multiplication</i>	*
<i>Addition and subtraction</i>	+, -

Logical operations in R

Logical operations in R

Symbols	Operation	Examples
<	<i>Less than</i>	> 2>3 [1] FALSE
<=	<i>Less than equal to</i>	> 2<3 [1] TRUE
>	<i>Greater than</i>	> 2>=3 [1] FALSE
>=	<i>Greater than equal to</i>	> 2<=3 [1] TRUE
==	<i>Exactly equal to</i>	> 2==3 [1] FALSE
!=	<i>Not equal to</i>	> !2 [1] FALSE
!	<i>Not</i>	> 2!=3 [1] TRUE
	<i>Or</i>	>
&	<i>And</i>	
isTRUE	<i>Test if variable is TRUE</i>	

Matrix operations in R

Matrices

A matrix is a rectangular arrangement of numbers in rows and columns

Rows run horizontally and columns run vertically

$$\begin{pmatrix} 1 & 5 & 3 \\ 4 & 9 & 2 \\ 5 & 6 & 7 \end{pmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad [1 \ 4 \ 5]$$

Creating matrices

Follow these steps to create a matrix

1. Open a curve bracket,

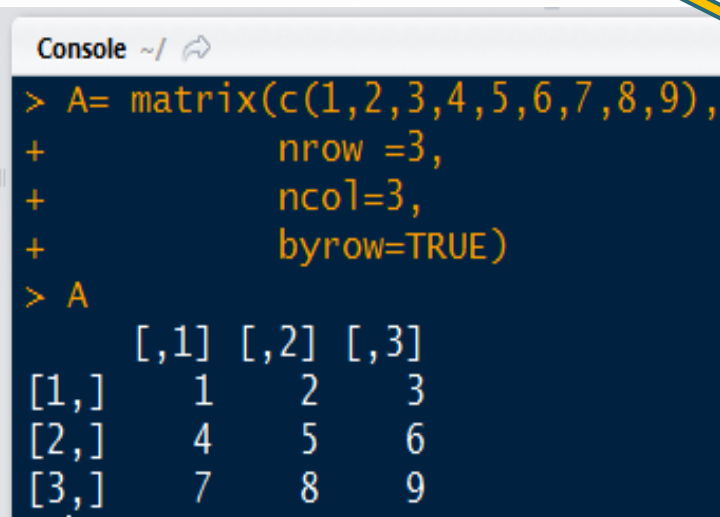
```
A = matrix()
```


2. Enter the sequence of elements,

```
A = matrix(c(1,2,3,4,5,6,7,8,9))
```

3. Specify the parameters nrow, ncol, byrow

```
A = matrix(c(1,2,3,4,5,6,7,8,9), nrow = 3,  
ncol = 3, byrow = TRUE)
```



```
Console ~/ 
> A = matrix(c(1,2,3,4,5,6,7,8,9),
+           nrow = 3,
+           ncol = 3,
+           byrow = TRUE)
> A
      [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

This parameter decides how values in the vector would be assigned i.e. “by row” or not

Creating special matrices

Different ways of creating matrices:

- a. **Matrix** where all rows and columns are filled by a single constant 'k'.

- For k=3, with 'm' rows & 'n' columns

Command: `matrix(3,m,n)`

- b. **Diagonal matrix:**

- Values in diagonal, similar to 'matrix()'.
- Mention 'k' as constant/array in first parameter.

- Command: `diag(k,m,n)`

- c. **Identity matrix:**

- Use 'diag()' command with k=1

```
Console ~/
> matrix(3,3,4)
      [,1] [,2] [,3] [,4]
[1,]    3    3    3    3
[2,]    3    3    3    3
[3,]    3    3    3    3
> diag(c(4,5,6),3,3)
      [,1] [,2] [,3]
[1,]    4    0    0
[2,]    0    5    0
[3,]    0    0    6
```

```
> diag(1,3,3)
      [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

Exercise: Creating matrices

Create the following matrices in R

$$\begin{bmatrix} 3 & 5 \\ -2 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 10 \\ 3 & -1 \\ 7 & 5 \end{bmatrix}$$

and

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ -1 & -2 & -3 \\ 5 & 4 & 3 \end{bmatrix}$$

Matrix metrics

create a matrix A

```
A= matrix(c(1,2,3,4,5,6,7,8,9), nrow =3,  
ncol=3, byrow=TRUE )
```

Finding the size of the matrix, A :

`dim(A)` will return the size of the matrix

`nrow(A)` will return the number of rows

`ncol(A)` will return the number of columns

`prod(dim(A))` or `length(A)` will return the number of elements

Console ~/ ↩

```
> dim(A)
```

```
[1] 3 3
```

```
> nrow(A)
```

```
[1] 3
```

```
> ncol(A)
```

```
[1] 3
```

```
> length(A)
```

```
[1] 9
```

```
> |
```

Accessing, editing, deleting in elements in matrices

They follow the same convention as dataframes such as

- Array/value before “,” for accessing rows
- Array/value before “,” for accessing columns
- use of ‘-’ for removing rows/columns
- Strings can be assigned as names of rows and columns using:
 - rownames() and Colnames()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 1 \end{bmatrix}_{3 \times 3}$$

```
> A=matrix(c(1,2,3,4,5,6,8,9,1),
+          3,3,byrow = T)
>
> colnames(A) <- c("a","b","c")
> rownames(A) <- c("d","e","f")
> A
  a b c
d 1 2 3
e 4 5 6
f 8 9 1
> A[,1:2]
  a b
d 1 2
e 4 5
f 8 9
> A[,c("a","c")]
  a c
d 1 3
e 4 6
f 8 1
> A[c("d","f"),]
  a b c
d 1 2 3
f 8 9 1
> |
```

Accessing an entry of a matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

First row, second column

```
> A[1,2]
[1] 2
>
```

Second row, third column

```
> A[2,3]
[1] 6
>
```

The part before the comma should be the row number
 The part after the comma should be the column number

Accessing a column

- Specify the column index
- Leave the rows index unspecified
- This means accessing all row elements of the given column index

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

All rows in
first column

```
> A[,1]  
[1] 1 4 7  
>
```


Accessing a row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

```
> A[2,]  
[1] 4 5 6  
>
```

Leaving the column index empty
means choose all the columns

How do you access the last row ?

`A[nrow(A),]`

Accessing everything but one column

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- Access the column that has to be avoided and then put a '-' sign in front of it
 - For example: `A[,-2]`
 - This will fetch all the columns except the 2nd column

```
> A[,-2]
      [,1] [,2]
[1,]    1    3
[2,]    4    6
[3,]    7    9
```

Accessing everything but one row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

```
> A[-2,]
      [,1] [,2] [,3]
[1,]     1     2     3
[2,]     7     8     9
>
```

- Access the row that has to be avoided and then put a '-' sign in front of it
 - For example: `A[-2,]`
 - This will fetch all the row except the 2nd row

Exercise: Accessing elements of a matrix

Do the following in R

Assign the following matrix

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 4 & 4 & 6 \\ 4 & 7 & 12 \end{bmatrix}$$

- Change the element 12 to 13
- Access the second row and the third column
- List all the elements in the second column and third row

Colon operator

Colon operator can be used to create a row matrix

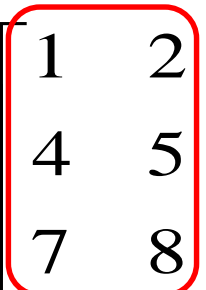
```
> 1:10  
[1] 1 2 3 4 5 6 7 8 9 10  
>  
> 10:1  
[1] 10 9 8 7 6 5 4 3 2 1  
>
```

Colon operator: sub matrices selection

The colon notation can also be used to pick sub-matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3 \times 3}$$

Sub-matrix



The sub-matrix occupies the **first three rows** and the **first two columns**

```
> A[1:3,1:2]
      [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
> A[1:3,-3]
      [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
> A[,1:2]
      [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

Accessing submatrices: Example 2

$$A = \begin{bmatrix} \boxed{1} & \boxed{2} & 3 \\ 4 & 5 & 6 \\ \boxed{7} & \boxed{8} & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix}$$

```
> A[c(1,3),1:2]
```

```
      [,1] [,2]
[1,]    1    2
[2,]    7    8
```

```
>
```

```
> A[c(1,3),c(1,2)]
```

```
      [,1] [,2]
[1,]    1    2
[2,]    7    8
```

```
>
```

```
.
```

Exercise: Accessing sub-matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

How do you access this sub-matrix $\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$

Matrix concatenation

- Matrix concatenation refers to merging of a row or column to a matrix
- Concatenation of a row to a matrix is done using **rbind()**
- Concatenation of a column to a matrix is done using **cbind()**
- Consistency of the dimensions between the matrix and the vector should be checked before concatenation

Matrix concatenation – rbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = [10 \quad 11 \quad 12]$$

Use rbind() to append B row vector to the rows of A

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$

```
> C = rbind(A,B)
>
> C
      [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
[4,]   10   11   12
>
```

Matrix concatenation – cbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

Use `cbind()` to append B column vector to the columns of A

$$C = [A \quad B]$$

```
> C = cbind(A,B)
>
> C
      [,1] [,2] [,3] [,4]
[1,]    1    2    3   10
[2,]    4    5    6   11
[3,]    7    8    9   12
> |
```

Dimension inconsistency –cbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}$$

Can these two matrices be merged to give

$$C = \begin{bmatrix} A & B \end{bmatrix}$$

```
> D = cbind(A,B)
Error in cbind(A, B) : number of rows of matrices must match (see arg 2)
```

Fixing the dimension inconsistency

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

$$C = [A \quad B]$$

```
> C = cbind(A,B)
>
> C
      [,1] [,2] [,3] [,4]
[1,]    1    2    3   10
[2,]    4    5    6   11
[3,]    7    8    9   12
> |
```

Deleting a column

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- Access the column that has to be deleted and then put a '-' sign in front of it
 - For example: `A=A[,-2]`
 - This will fetch all the columns except the 2nd column

```
> A[,-2]
      [,1] [,2]
[1,]    1    3
[2,]    4    6
[3,]    7    9
```

Deleting a row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- Access the row that has to be deleted and then put a '-' sign in front of it
 - For example: `A=A[-2,]`
 - This will fetch all the rows except the 2nd row

```
> A[-2, ]
      [,1] [,2] [,3]
[1,]    1    2    3
[2,]    7    8    9
>
```

Matrix algebra

- **Addition/subtraction**
- **Multiplication**
- **Matrix Operations in R**
- **Matrix Division**

Matrix addition/subtraction & multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 1 \end{bmatrix}_{3 \times 3}$$

$$B = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 1 \\ 5 & 1 & 2 \end{bmatrix}_{3 \times 3}$$

Element-wise multiplication is based on multiplication between corresponding elements of two matrices.

```

Console
> A+B
      [,1] [,2] [,3]
[1,]     4     3     6
[2,]     8     7     7
[3,]    13    10     3
> A-B
      [,1] [,2] [,3]
[1,]    -2     1     0
[2,]     0     3     5
[3,]     3     8    -1
> A*B # Regular matrix multiplication
      [,1] [,2] [,3]
[1,]    26     8    11
[2,]    62    20    29
[3,]    65    27    35
> A.*B # element-wise multiplication
      [,1] [,2] [,3]
[1,]     3     2     9
[2,]    16    10     6
[3,]    40     9     2
>
  
```

Matrix division



WARNING :

The following operation is not inverse of a matrix
but element wise division between matrices A & B.

$$A = \begin{bmatrix} 4 & 9 \\ 16 & 25 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$A / B = \frac{a_{ij}}{b_{ij}}$$

```

Console ~/
> A= matrix(c(4,9,16,25),2,2)
> B= matrix(c(2,3,4,5),2,2)
> A
      [,1] [,2]
[1,]    4   16
[2,]    9   25
> B
      [,1] [,2]
[1,]    2    4
[2,]    3    5
> A/B
      [,1] [,2]
[1,]    2    4
[2,]    3    5

```