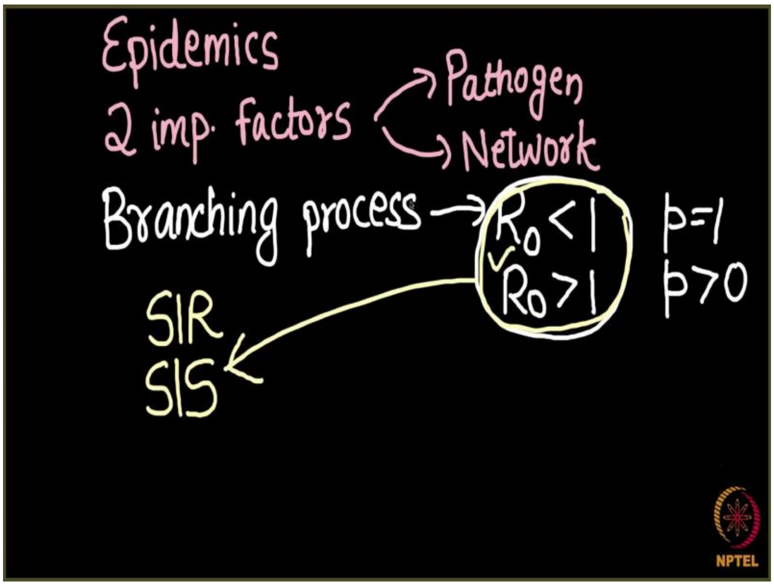
Social Networks
Prof. S. R. S. Iyengar
Department of Computer Science
Indian Institute of Technology, Ropar

Rich Get Richer Phenomenon - 2
Lecture - 136
Basic Reproductive Number Revisited for Complex Networks

Before going further let us quickly recap what all we have done in this chapter.

(Refer Slide Time: 00:12)

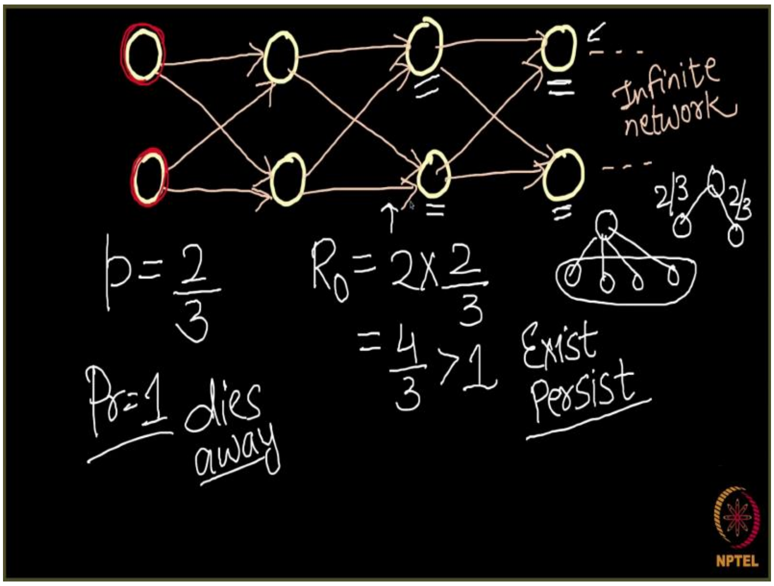


First of all we have looked at what are epidemics and how network scientists can study epidemics. Then we have looked at 2 important factors which are required for the modeling of a contagious disease. And what were those factors? The first one was a pathogen which was spreading and the second one was a network, network on which our contagion is spreading. After that we looked at a very simple process for modeling this spreading of a contagion, which is a branching model, branching process. And, one of the very interesting concept we looked in the branching process was the reproductive number R0.

We have seen that when R0 was less than 1 a disease it spreads sorry a disease it dies away with the probability equals to 1. And, when our R0 it is greater than 1 our disease persists in the network with some positive probability and R0 was responsible was very helpful in telling us whether a disease is going to be an epidemic or not. And, then we looked at 2 little bit complicated spreading models SIR model and SIS model.

Now, again I will pose a question to you and the question is like here we have seen that there is a reproductive number, which if less than 1 it means that disease for sure dies away. If it is greater than 1 then, then it is quite sure that the not completely sure, but quite sure that the disease will persist in the network. This theory which we did for the branching process, do you think that it will hold here, and the answer can be no, the answer is no. Why this theory does not hold here?

(Refer Slide Time: 02:14)



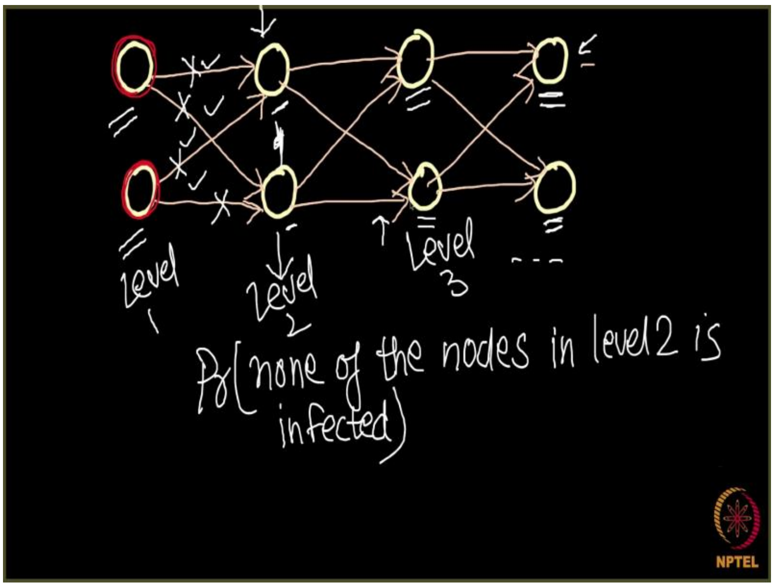
Let us take a small example and try to figure it out. Say we have a network here and let us say network is in this form I will give you a network and we adjust somewhat like this. So, this node is connected to 2 nodes on the next level and this node is connected to both the nodes on the next level. And similarly, this node here, this node here and this and this network goes on like this. So, it is an infinite network. And here I tell you that initially this node and these nodes. So, these 2 nodes are infected initially and then I tell you that the probability of your infection is spreading is 2/3. So, let us try to use the concept of basic reproductive number.

What was a basic reproductive number? What did R0 define? R0 told us the number of secondary infections. So, if a node is here and it has some neighbors here so, on an average how many of the people in this level will be infected, that was what R0 told us.

So, if we look at this network what is happening is every node. So, if we look at this node here this holds for any node in this network, if we look at this node here it is connected to this node and this node. So, 2 nodes in the next level, if I look at this node it is again connected to 2 nodes in the next level.

So, every node is having 2 children and each of these can be infected with the probability of 2 by 3. So, what is my basic reproductive number? It turns out to be 2 into 2 by 3 which is 4 by 3 which is greater than 1, which means that there should be a quite a high probability that my disease you know should exist on this network. There should be a high probability that my disease it should persist in the network, but it is not so, it does not happen like this. Rather, I show you that on this network this disease it for sure with a actually with a probability equals to 1 the disease dies away, which means that our old theory of basic reproductive number is not working here in the case of SIS and SIR model. So, let us see when will this disease die away. So, we will be taking this network.

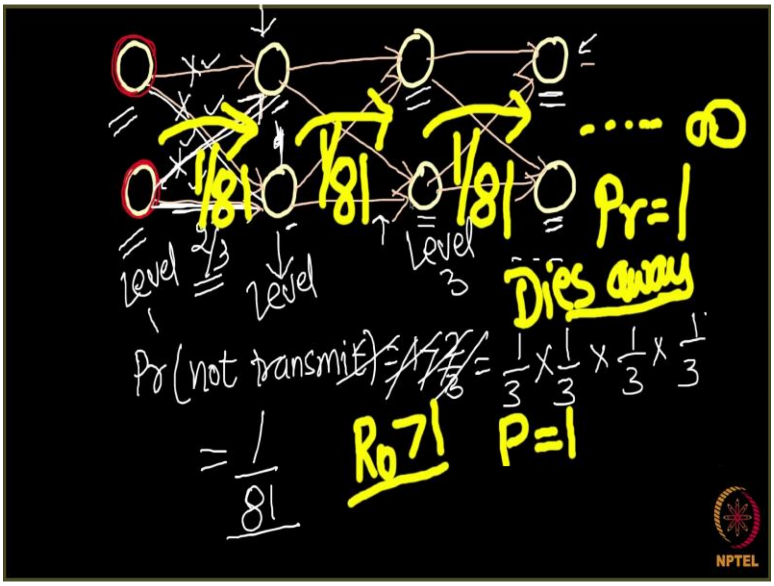
(Refer Slide Time: 05:05)



So, here is the here is our network, initially these 2 nodes are infected, let us look what is the probability that your infection does not come to this level. So, let us say that this is level 1 here, this is level 2, this is level 3 and so on. We are interested in looking at the probability that none of the nodes at level 2 is infected. What is the probability that none of the nodes in level 2 is infected and it is actually simple to calculate. So, when will be a node here infected?

If any of these four links workout then either this node will be infected, or this node will be infected right. So, when we none of these nodes infected when this link also fails, this link also fails, this link also fails and this link also fails, even if one of these links work my infection will come here to the second level. So, none of these links should work. What is the probability that none of these links will work? Let us see.

(Refer Slide Time: 06:27)



So, probability of infection here was 2/3 for every edge right, 2/3 is the probability that my infection will transmit across this edge. So, what is the probability that my infection will not transmit through this edge is nothing, but 1 – 2/3 which is equal to 1/3 right. And, then what is a probability that my disease does not get transmitted from here is again 1/3. So, I am going to multiply this, I am going to multiply this 1/3 and again 1/3 for the third edge and again 1/3 for the fourth edge.

So, it turns out to be 1/81, this 1/81 is the probability that my infection it does not get transmitted from my level 1 to level 2. 1/81 is the probability that my infection does not get transmitted from this level to this level, which is actually a high probability which is on a higher side. When you say there is a high probability that my infection will die away here, even if 1 of the node gets infected here again for going so, from here to here again the disease dies away with the probability 1/81. Again, from here to here the disease dies away with the probability 1/81 and since it is an infinite network with the probability equals to 1, your disease it dies away from the network.

So, even if you see your basic reproductive number was greater than 1, what is happening is counterintuitive, what is happening is with the probability equals to 1 your disease is dying away from this network.