Social Networks
Prof. S. R. S. Iyengar
Department of Computer Science
Indian Institute of Technology, Ropar

Rich Get Richer Phenomenon – 2
Lecture - 130
Simple Branching Process for Modelling Epidemics

So, now we have covered all the prerequisites, so the prerequisites were start. So till now we have covered all the prerequisites which we were required for understanding the modelling of epidemics and we have looked at the two most important factors there. The first one is the pathogen and second one is the network. So we are now all set to go ahead and model this spreading of a disease. So let us say how we can go about it.

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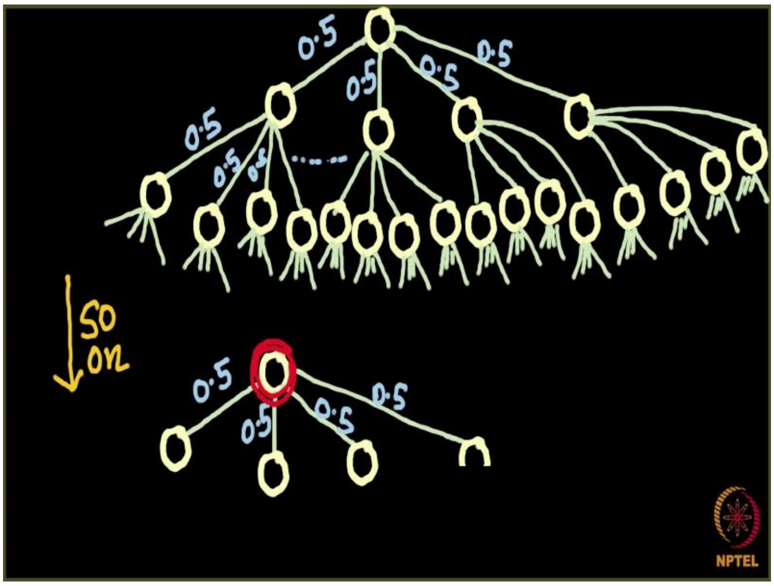

So, let us say here is a node, here is a person and now this person over here it has let us say four friends here as shown in this figure; so there is this person and this person has four friends here.

And then these four friends each of these has each of these have again four friends each. So you see we are taking a very simple example for modelling; so we will start from a very simple example and we will move towards a complex network. So we are taking the most simple network that is which is in the form of a tree; obviously, we know that social networks are not in the form of tree; because there will be triads on this network and these nodes will be connected to each other. But only for the sake of simplicity for the time being we are taking the network to be a tree network, so one node having four children each of these four children again having four children.

And then which of these again having four children each. So this is the kind of network on which we will be modelling a disease. So this is the network here now let us say this if you see this edge over there so I write a number on this 0.5. So as we have discussed previously there were two factors one was the network and another was the pathogen so the network we have seen here is in the form of a tree over here and second thing is the contagiousness of the pathogen. So we model the contagiousness of the pathogen in terms of probability.

For example, I have given this edge a probability of 0.5; what does it mean? It means that if this node here is infected then there is a 50 percent chance that this node will be able to infect this node. So with the 50 percent probability 1 by 2 probability the infection will transmit across this edge and if this node is infected it is this particular child will be infected. So I will say that the probability of infection across this edge is 0.5 and again for the sake of simplicity, I say that the probability of infection across all the edges is 0.5.

So, again if this node is infected it infects this node with the probability of 0.5; similarly, across this edge similarly across this edge and similarly across all of these edges. So every edge has a probability 0.5 of infection; what can we say from here? So let me just take the first level of this tree so I redraw this network here I have this node which is the same node here. So I take this node here and then this node has four children and it can infect each of these it infects each of this these children with the probability of 0.5.

Now if this node is infected so I show it with the red circle this node is infected. How many children do you think it will infect here? So each of its child it infects with the probability of 0.5 and when it infects these how many children do you think will become infected.

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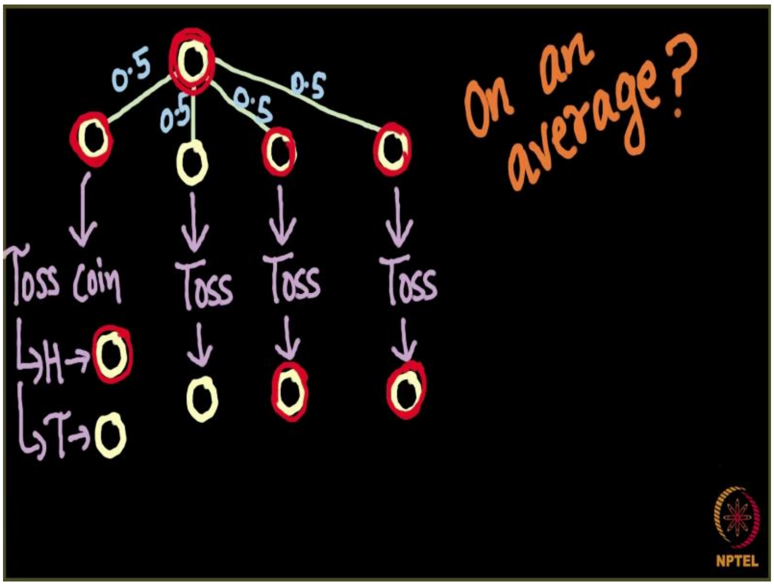

So, I redraw this network here just for the sake of simplicity and my question is how many of its children will it infect ok. So if we see here this node has a 0.5 probability of infecting this node what does that mean.

That means that if I toss a fair coin here and unbiased coin here I toss a coin I get head with the probability of 0.5 right. So if head turns up I will mark its the particular child to be infecting and if there is a tail I will say that this child is not infected. So how do I model this? If the probability of infection is 0.5, I will take a fair die toss it I get a head it means this node is infected else node.

And I do this for all of its children, so I do it for second child and let us say I get a tail and if I get a tail this node remains uninfected. And then I do it for the third child and let us say I get a head and I infect this and I do it for the fourth child let us say I again get a head.

So, I infect this fourth child also so three children are infected, so this happens for this particular case what do you so this three guys got infected over here. So what do you think happens on an average, on an expectation so what is the expected number of its children which will get infected here.

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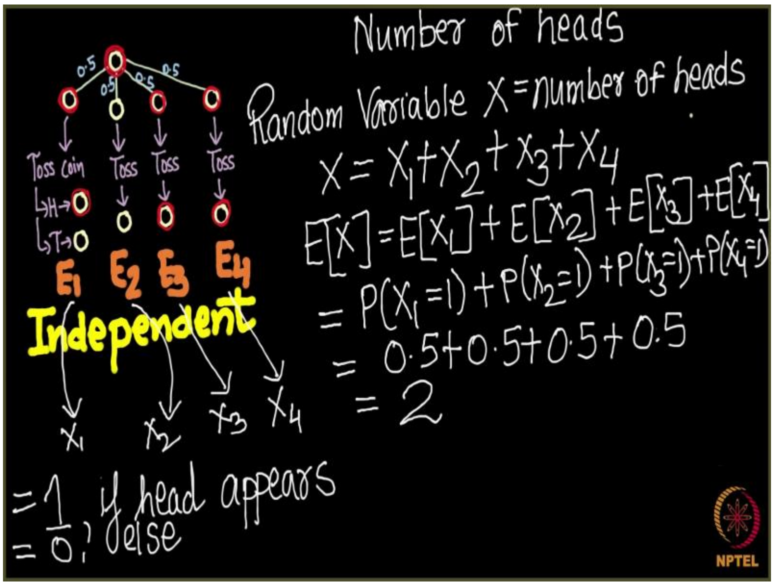

So, let us quickly see that; so I again redraw it so here I have four cases; in this case I am tossing the coin here, I am tossing the coin here, here, here. So four of these are different experiments, which are independent of each other so I have four experiments here.

I toss one coin for this, one coin for this, one coin for this and one coin for this and all these coin tosses are independent of each other. So I say that these all experiments are independent one has nothing to do with another; all of these are independent. And now our aim is to find the number of heads expected number of heads rather so if I the number of heads I get is equal to the number of nodes infected; because if I get a head I infect that particular node so our aim is to find the expected number of heads.

So whenever we have to find the expected number of a variable we modulating the form of a random variable. So I take a random variable X here and the value of this random variable X is equal to the number of heads I am going to get. And how do I find the value X over here it is with the help of these four things so what I do is, I take four more random variables. One random variable for each experiment so for experiment 1 I have a random variable X1 rather we call it indicator random variable. So indicator random variable is a random variable, which takes the value only 1 or 0.

And then for E2 I have another indicator random variable, for E3 I have 1 for E4 I have 1. Indicator random variable is something very simple what is happening in experiment 1 and either getting a head or I am not getting a head. So I say that the value of X1 is 1 if I get a head and 0 if I do not get a head. So this is what do I mean by an indicator random variable it is 1 for success and 0 for failure. So I have X1 which is 1 if head appears, 0 if tail appears similarly X2 is 1 if head appears 0 if tail appears and same for X4 and X4.

So, now can you write this X in terms of X1, X2, X3 and X4 so what is my random variable X; it is the number of heads. So I know that X1 is what the number of heads I get from experiment 1 right. If it is a success, I get 1 head if it is a failure I get 0. Similarly, X2 is a number of heads I get from experiment 2 which again can be either 1 or 0 similarly I have X3 and then I have X4. So what do I can say is X equals to X1 plus X2 plus X3 plus X4. So wherever I get a head out of these experiments I can add that add that value and it becomes equal to X ok.

What is our aim? Our aim is to find the expected value of X so I will take an expectation on both the sides; left hand side and right hand side and I get expected value of X is expected value of whatever is there in the right hand side and I can actually distribute this expectation. So I can write expected value of this complete term X1 plus X2 plus X3 plus X4 as something like this. So this is called linearity of expectations I will not go in much detail so I get expected value of X is this and then again here is a you can take it as a black box. So just understand this if I am talking about the expected value of an indicator random variable.

So, here X1, X2, X3, X4 are indicator random variable either 0 or 1 so if I talk about the expected value of an indicator random variable it is nothing, but the probability that this variable is 1. So, I can write it like this expected value of X is probability that X1 equals to 1 plus probability that X2 equals to 1 plus probability that X is equals to 1 and so on. And then we know what is the probability that X1 is 1; when is X1 1? X1 is 1 when I get a head in the first coin toss and the probability of that happening is nothing, but 0.5 and same is for X2, X3 and X4.

So, overall I get the value 0.5 into 4 which is 2. So I have done it rigorously you even if you did not understand it is perfectly it comes to (Refer Time: 10:31) also that. If let us say I toss a coin four time so on an average how many heads will you get and intuitively your mind tells you that the answer is 2; 0.5 into 4 so even if you did not understand this calculation of expectations it is perfectly fine ok.

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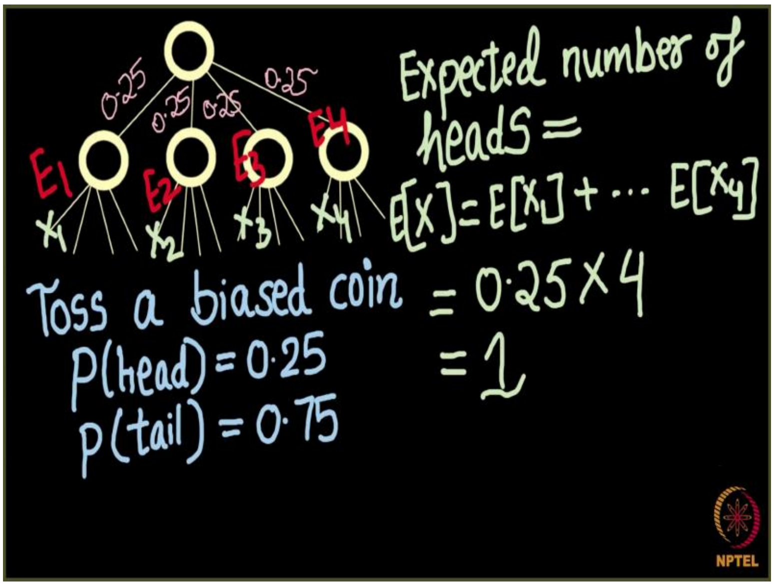

Now, let us take another example; again I have a node here and then this node has four neighbours and then each of them has four neighbours and then let us say the probability of infection across every edge this time is 0.25 instead of 5. So in the previous case we have tossed a coin so the probability was 0.5 we tossed a coin we looked at whether it is a head or it is a tail and accordingly we did what we have to do. Now what I do in this case can I toss a coin in this case; I do not know so coin gives me head with probability 0.5 tail with probability 0.5; what do I do in this case?

So what we can do in this case is we toss a biased coin and my biased coin is such that it gives me head with the probability of 0.25 and gives me tail with the probability of 0.75. So biased coin is something like it is not fair coin, fair coin gives you head and tail with equal probability. So a biased coin can be something like if you remember the movie the movie Sholay; so there was a coin which has which had head on both the sides so that was a biased coin in that case probability of head was 1 probability of tail was 0.

No matter whatever you do you are going to get a head every time; that was a biased coin. Here I am using another form of biased coin which does not give you head always. It gives you head 25 percent of the times and gives you tail 75 percent of the times. And again what I am going to do is if I then again I will have these four experiments here for each experiment, I am going to toss a coin and then if I get an head I infect the particular node, if I get a tail I do not infect the particular node. And here what do you think is going to be the expected number of heads.

So, it is again very simple its almost the same what we have done previously so again we take four indicator random variables here X1 X2 X3 and X4 and we know that expected value of X is expected value of X1 + X2 + X3 + X4 which can be written like this. And we have looked at what is the expected value of a indicator random variable is nothing, but the probability of 1 which is probability of head which is 0.25 in this case. So I am going quiet I am going quiet quickly because it is similar to what we have done previously you can pause the video here and see what is actually happening.

So, we get here 0.25 into four which is nothing, but 1 so the expected number of heads here is 1 which means that if this guy here infects each of these nodes with the probability 0.5 on an expectation on an average it will infect one of its children ok.

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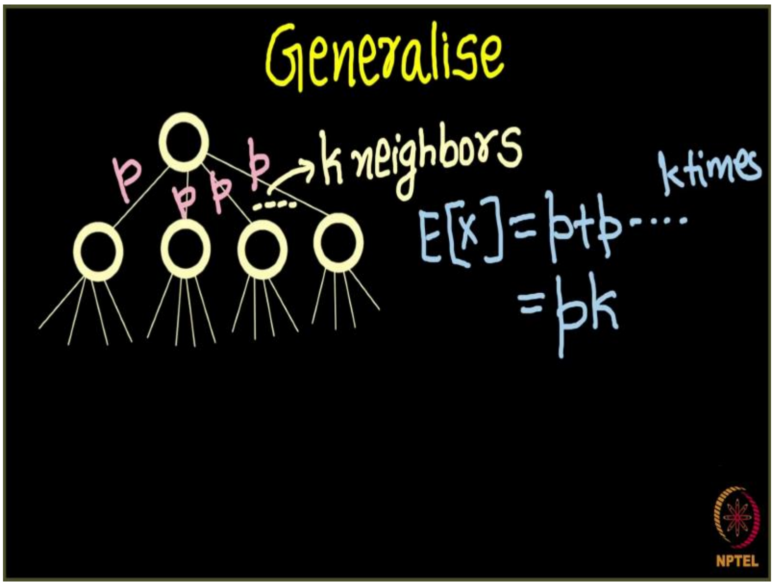

Now, what we are going to do is we are going to generalize what we have done till now. We have looked at two examples and now we are going to generalise whatever we have done.

And it is very easy how do you generalise it I take a network here and now I say instead of having let us say 4 children so instead of every node having four children, I say that every node is having k children or k naught neighbours. Every node is having k children so the node here it will have four children and then the node here it will have its own four children node here four children and so on I am very sorry not four it will be k.

So, this very sorry this node here will be having k children this node here will be having k children this node here will be having k children and so on. And let us say the probability of infection is p so we have taken 0.5 in the first example 0.25 in the second example here we generalise it and take this probability to be p. Now what do you think will be the expected number of infected people, so it will be simply be so you can write each of these.

So how many experiments are we doing here? We are doing k experiments here because there are k children and what is the probability of success in every experiment is p. So the expected value of X that is expected value of you know infected people here is going to turn out to be p + p + p k times which is going to be equal to p * k.