Social Networks
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Lecture – 67
Homophily (Continued) & Positive and Negative Relationships
Proof of Balance Theorem

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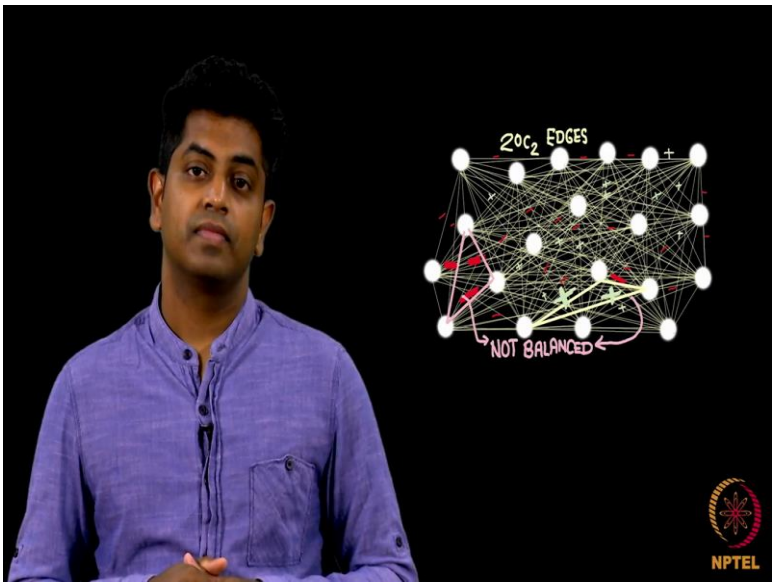

So, let us try to mathematically rigorously try to understand what is happening here. What I will do is firstly, I will develop the necessary machineries we will assume there is a complete graph, the graph on n nodes let us say some 20 nodes and you put edges between any 2 people, $^{20}C_2$ edges. Now there are some plus and there are some minus relationships, I put that 2 is this structurally balanced maybe not, you see there is a minus, minus, minus triangle there is also a plus, plus, plus minus triangle this is not structurally balanced. But what if it was structurally balanced let us see.

Let me now think of an example where is where a given graph is structurally balanced how we will look like this is an example of our graph that is structurally balanced see how it looks like.

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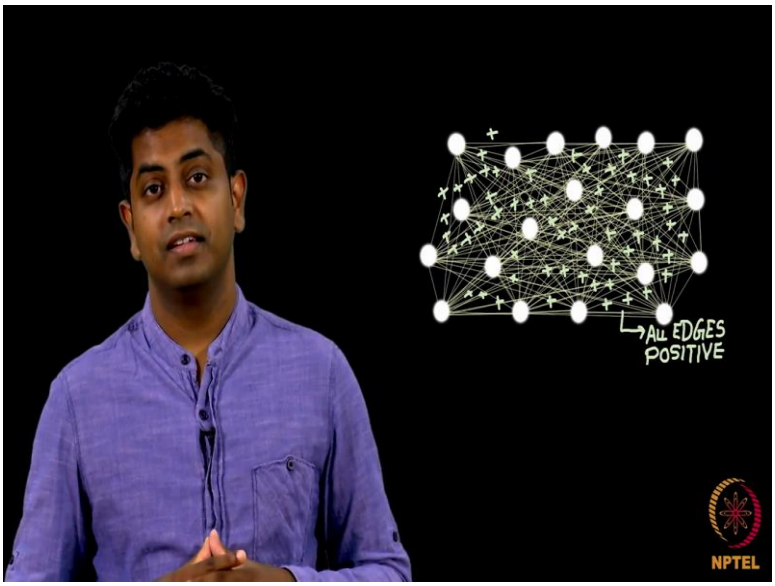

Obviously, all of them are plus I know this is structurally balanced what is so great about this. Is there any graph with negative edges as well that is structurally balanced?

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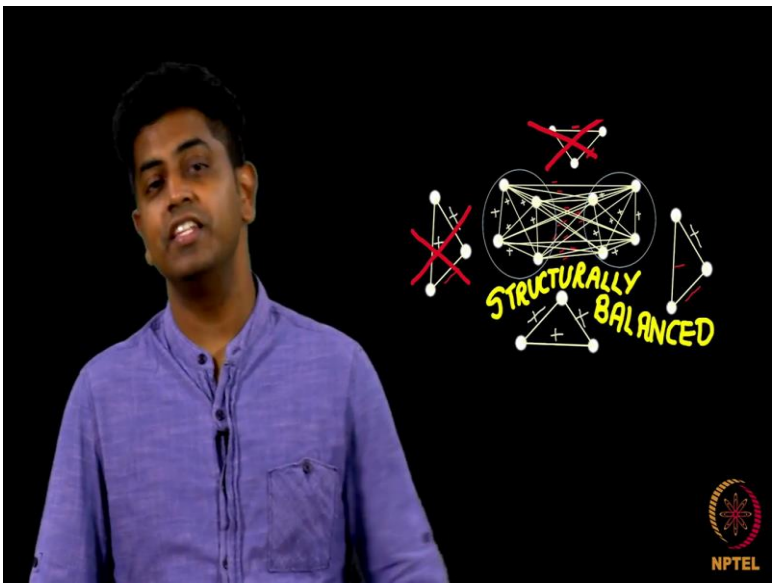

Let me think of finding one as I keep drawing. I converged to only structure like this do you see there are 2 clusters and edges inside these clusters are positive and edges across are negative. Basically there are 2 teams here 2 teams means what within the team the relationships is positive right you will see that in the figure relationships between 2 people inside the team is positive relationships between 2 people across the team is

negative. This is structurally balanced why, let me take a triangle here a triangle here is either positive, positive, positive as you observe or positive, negative, negative correct can you find a triangle here which has a positive, positive, negative plain impossible just observe can you give me a triangle here which is negative, negative, negative no impossible which means looks like this is structurally balanced.

So, when you have one team full of positivity structurally balanced when you have 2 teams where positivity is within negativity is across structurally balanced some basic observation that you did this structurally balanced. Now if I give you a graph g and college structurally balanced how will it look like we will it look like all positives or only 2 teams with positives inside negatives across is there are third type. So, we prove that there is no third type there is there only these 2 types the type one where there is only one team the type 2 where there are 2 teams.

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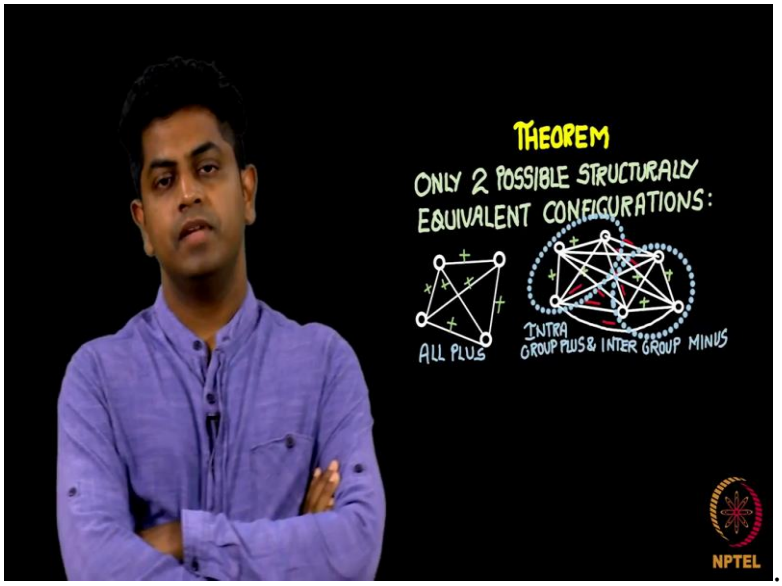

And period nothing else can happen. So, here is a theorem which states these are the only 2 possible structures when you say a given network is structurally balanced.

Let we let us try proving this now with good amount of rigger how do you go about it.

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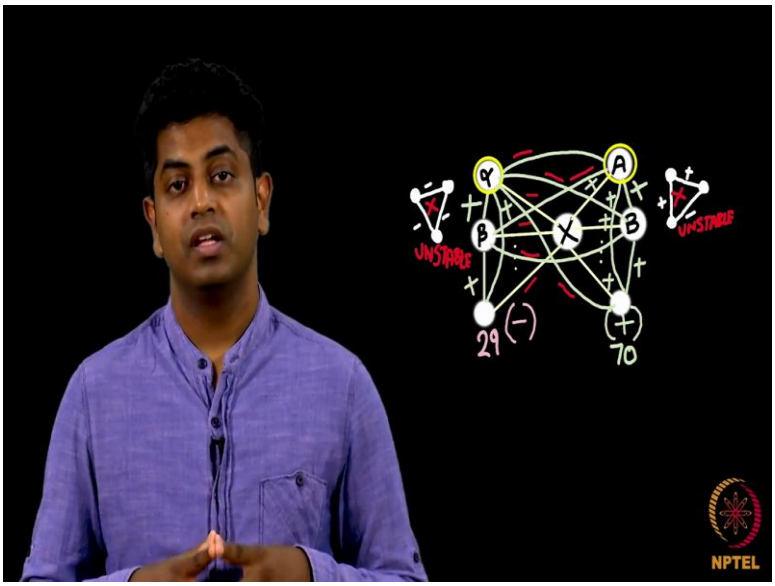

Let me take a given graph g with many nodes let us say 100 nodes and I pick one node from this network as you know in these graph with 100 nodes there are edges across any 2 nodes and there is a sign for every single edge plus or minus. As you can observe let me pick some node here call it let say x node x. x is a just sent to ninety-nine other nodes some of them positive some of them negative.

Let me say sorry roughly seventy of them is positive and twenty nine of them are negative seventy are positive twenty nine of them are negative now is you need obvious that this vertex x is friends with positive edges and the encompassing triangle on these edges as you can see x A B this triangle A B cannot be negative because I told you it is structurally balanced correct. Now look at this side x α, β x to α negative x to β negative now what can be the edge between alpha and beta it has to be positive because its structurally balanced correct think about it. This vertex x if it is a adjacent to 2 people positively the relationship between these 2 people as I told you in the example A and B should be positive with the vertex x is adjacent to α and β and x α is negative x beta is negative and what should be between alpha and beta enemy is enemy right it should be positive.

So, I observe that when you pick an element x and look at all its friends which are positive and all its friends which are which are negative you observe that amongst the positive friends of x relationships are all positive amongst the negative friends of x

relationships are all positive, but if you pick a friend A of x and a friend α of x. x A is positive, x α is negative then what is the relationship between A and α 1 positive one negative which means A and α should be negative that is straight forward. So, what do I gather all the friends that x is adjacent with they are friends with each other with positive friendship all the people that x is adjacent to with negative friendship negative relationship?

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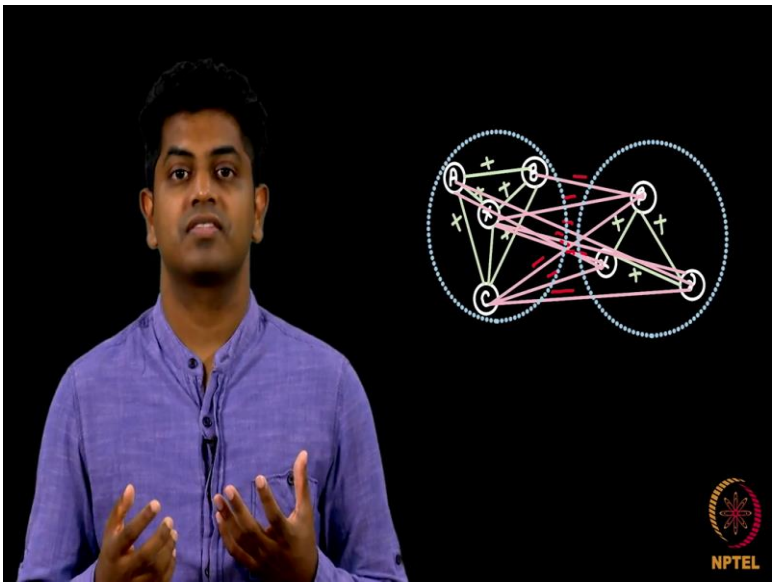

They are all friends with each other which means there are 2 clusters here what are those 2 clusters the clusters x and all his friends there all positive within and all the negative friends of x there all positive within, but negative across as simple as that that closes the theorem. So, the theorem just states that in case you have a structurally balanced network it better we type one all positive friendships within there is only one team type 2 there are precisely 2 teams where friendships within them is positive friendships occurs is negative.