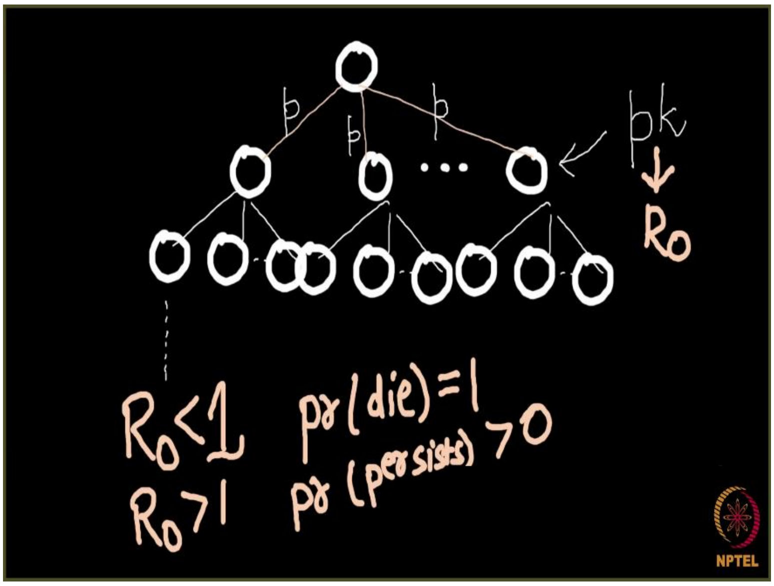
Social Networks
Prof. S. R. S. Iyengar
Department of Computer Science
Indian Institute of Technology, Ropar

Rich Get Richer Phenomenon – 2
Lecture - 138
Analysis of basic reproductive number in branching model (The problem statement)

In this lecture we are going to do the Analysis of basic reproductive number.

(Refer Slide Time: 00:18)



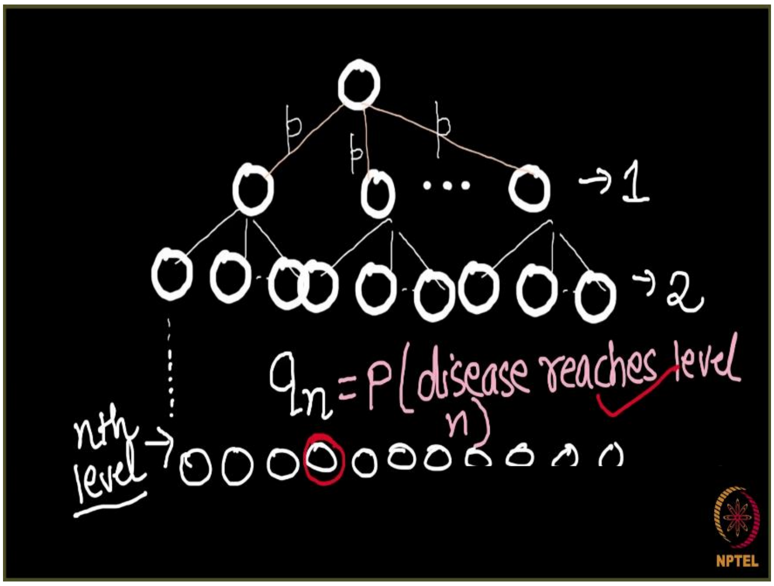
So, if you do remember when we talked about a branching model, so branching model where there is a node here and then this node is having k neighbors. So, this node here it is having k neighbors right and then each of these k neighbors, each of these k neighbors are again having some k neighbors. So, something like this, each of these k neighbors are again having k neighbors (Refer Time: 00:52) little bit of overlapping this and so on right.

So, this was our basic branching model and then we have looked at the number of expected people who get infected here was given as p into k, where p is the probability that this disease get transmitted across this edge. So, here we were having a number p into k and we call this number as basic reproductive number R0.

And then we made a claim and the claim was we said that if R0 < 1 then the probability that this disease die, the probability that this disease dies equals to 1. That is if R0 < 1 for sure this disease dies away and if R0 is greater than 1 then with the positive probability the disease persists in the network.

So, the probability that this disease persists in the network is greater than 0, we said this. And we will looked it how intuitively this is true, but we have not looked at a rigorous proof. So, in this lecture we will be doing a rigorous proof for this and the proof is actually very interesting. Let us see how we can do it. So, let us model it a little bit. So, what do we do is, we have this network over here ok.

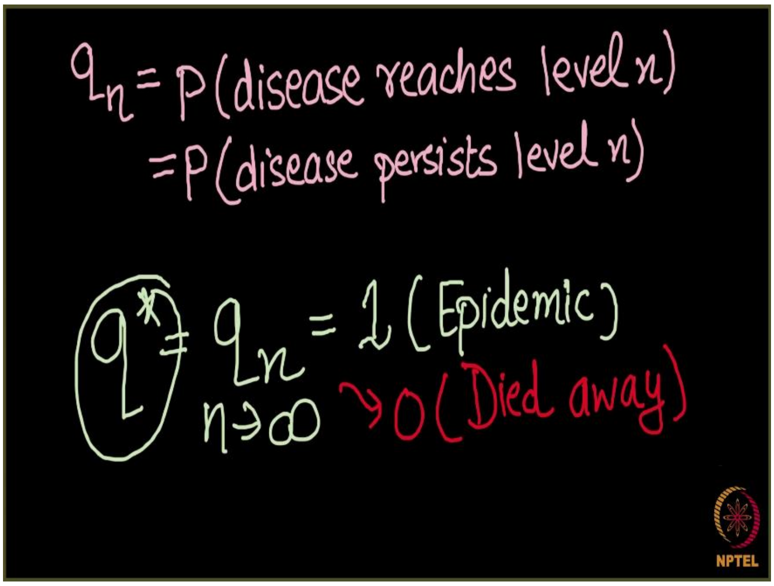
(Refer Slide Time: 02:32)



We have this network here right. First of all, let us see at when do I call this disease to become epidemic. So, let us see this is level 1, this is level 2 and somewhere down here is some level which is the nth level.

So, this is my nth level right and then I take a variable I take a notation let us say q n. And what is q_n? q_n is the probability that are disease it reaches level n. Reaches level n means at least 1 person at this level n should be infected. So, if these are the people at level n say, so at least 1 person in this level should be infected. And if 1 person at this level is infected, I say that my disease reaches level n. So, $q_n$becomes a probability that my disease reaches level n ok.

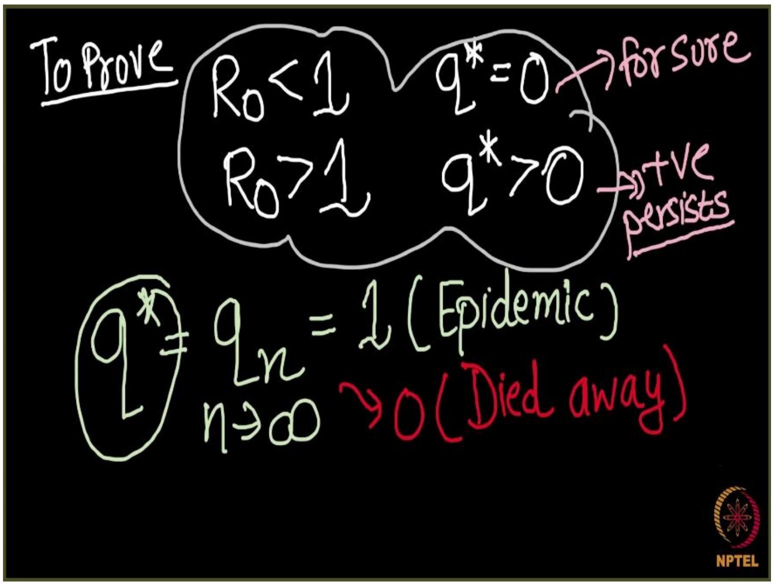
(Refer Slide Time: 03:45)



Let me write it down. So, q_n means the probability that disease reaches level n or I can also say that it is the probability that my disease persists still level n, so far so good. Now let me look at what does this value represent. I take here q* and I define q* as q_n as $n \to \infty$. What does it mean? It means that I keep going down, down, down, down, down and I go to the infinite level and I see there what is the probability that this disease is still persisting at that infinite level, that is my q*.

And you see what if, what does it mean if q* is 1? It means that is even if I go to the infinite level my disease is existing, it means that my disease is epidemic. If my q* is 1, I say that my disease is epidemic and if my q* is 0, I say that my disease has died away ok. So, now what do we have to prove? We have to prove that if our basic reproductive number R 0.

(Refer Slide Time: 05:23)



So, what do we have to prove? We have to prove that if our basic reproductive number R0, if it is less than 1, then we say that the disease for sure dies away, which means that then q* should be 0 right. What is q*? It is 0 when the disease dies away, q* is 0, when the disease dies away. So, when R0 < 1, the probability that your disease exists till your infinite level should be equal to 0; that is your q* should be equal to 0 and second thing when your R0 > 1, then q* should be greater than 0 right.

So, q* if it is greater than 0, it means that if the probability that your disease persists when you go to the infinite level in your network that probability is something positive. So, this is how we write our problem statement rigorously. It is essentially the same thing which we stated before that if R0 < 1, then our disease dies away for sure that is q* equals to 0. If R0 > 1, then with a positive probability our disease it persists in the network. So, we are going to do a proof which is a little bit lengthy.

So, at times you may want to look back and forth and see what is happening, although I try my best to keep it as clear as I can ok. This is what we have to prove. And we will continue in the next lecture. So, just look back and see how we have modeled this problem, what is q*, what is q_n and so on. And, we will continue in the next lecture.