Social Networks
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Rich Get Richer Phenomenon
Lecture - 119
Rich Get Richer Phenomenon

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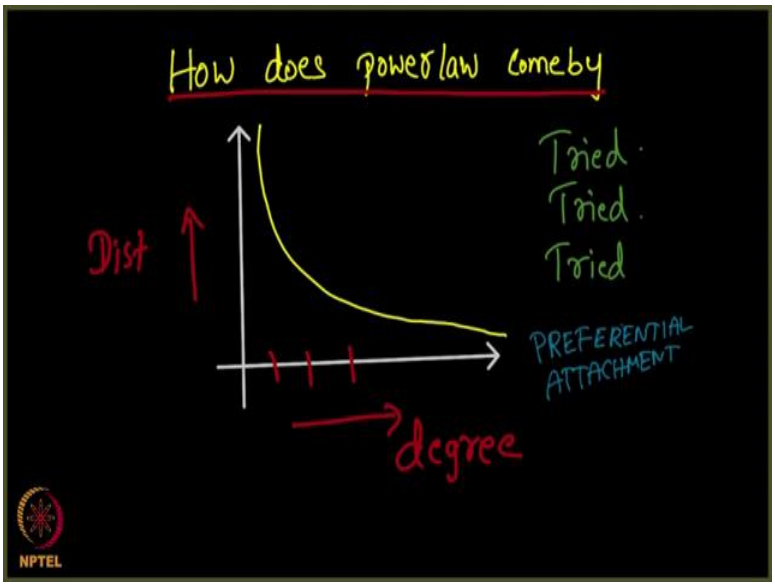

So, let us now make an attempt to answer the big question that we have been asking from quite some time. How does power law come by? What results in this power law ~~right.~~right? So, the interring question just to just for the sake of revision let me repose it. So, when you try to draw the degree let say on the x axis. So, how many vertices are there of degree 1, of degree 2 of, degree 3 if you plot that basically it is the distribution on the y axis you will observe drop like this. The big question is how ~~does this drop~~this drops combine, why does it come in place of something like this we observe something like this why ~~is this even~~this is even happening right.

So, many scientist have tried and tried to understand this, and they have come with their own set of hypothesis and one of the most convincing hypothesis is what is called the preferential attachment is called the preferential attachment hypothesis. It is a hypothesis people are said that there is something called preferential attachment that is happening because of which one observes power law. Now, let us see what exactly ~~is this~~

~~preferential attachment~~this preferential attachment is. So, assume there is a classroom alright.

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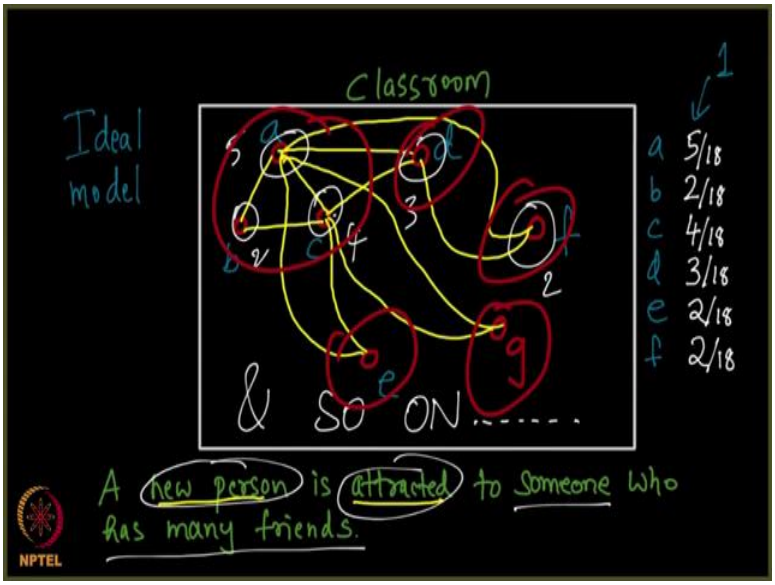

So, this is my classroom and there is a person who is sitting here another person another person. So, these 3 people are mutually friends with each other ok. This is a classroom let me write that down, this is a classroom and there only 3 people with only 3 people to begin with let say 1 2 and 3. So, a ~~fourth~~ 4th person comes; a ~~fourth~~ 4th person comes and he tries making friends with 2 of the existing people. So, which means he makes friends with let say 1 and 3. Now, wait a minute observe the degree of different vertices here his degree is 2 right and the degree of this is 3, this degree is 2 the degree of the node 1 is 3 correct. So, let me put this is a oval so, that you know that I am talking about degree here right fine.

Now, when a person comes when a person comes look at the story that I am telling you might question why ~~is this story~~this story is true, for the time being you please assume with the story happens to be true alright. So, this there is this person who come, and he observes that there are 1 2 3 4 there are 4 people here of different degrees correct and he prefers to make friends with. So, let me do one thing I think will remove 1 2 3 4 here for clarity say let me instead call it a b c ok, let me call this a and let me remove 2 here and 3 here and 4 here. Because, it is confusing when I call when I name the nodes as a numbers and then also talk about their degree ok.

Let me call it a b c d and now there is a new fellow e, he looks at these people a b c d and e and looks at their degrees; a has degree 3, b has degree 2, c has degree 3, d has degree 2. And, e must make friends with let say 2 people every person who comes inside makes friends with 2 people, just the way d made friends. He also makes friends with 2 people and who will he choose the story goes like this he will be attracted to people.

So, let me write that down this is a sort of important he will be attracted a new person, a new person is attracted is attracted to someone is attracted to someone who has many who has many friends who has many friends. So, what do I mean by this? It does not means that he is not attracted to people who are less friends. The point I am trying to make is he will be attracted to you know what it is self explanatory has I has I do this small exercise 3 plus 2 is 5 plus 3 is 8 plus 2 is 10.

So, now what I will do is I will write 3 by 10 probability 2 by 10 probability 3 by 10 probability 2 by 10 probability, you will see the sum of all this is actually 1. And, this will be the probability with which e will be attracted to b c d or let say a ok; this is a probability with which you will be attracted. So, e will be attracted to a with probability 3 by 10, b with probability 2 by 10, c with probability 3 by 10, d with probability 2 by 10.

So, what does it even mean? It means that I when I enter classroom, I will be attracted to people proportionately to they with respecting their degree. I will be attracted to c with probability 3 by 10 because the degree is c and then the total number of degree is 10. So, I will be attracted with degree 3 by 10.

So, you probably are confused you are you are not understanding what ~~has this~~this has to do with degree, what has friendship to do with degree. So, the point is simple when you talk to people a person who has a lot of friends will quickly come to the limelight, he is visible to you, he or she is visible to you and you would want to be friends with her more than anyone else, the person who is very visible.

So, the probability of making a new person having a friend is directly proportional to the number of friends that they already have alright. So, let me assume that e actually makes friends ends up making friends with c and in fact, a which means what does it means. I should now update the degree of c it is no more 3 it will be 1 2 3 4 ~~right,~~right; the degree of c is 4 and the degree of a also happens to b 4 degree of a is also 4 ok.

So, this requires me to erase the degree of a here and the degree of c also is to be removed right. And, then the degree of a happens to be 4 and the degree of c happens to be 4 and as you would have guessed I may want to remove all the denominators here. So, it is no more 10 you see what else is it done, it is going to be some of the degrees which is it 12 correct it is going to be 12 here. So, I am going to write 12 12 12 and 12 e also has 2 degree. So, e contributes again 2/ ~~divided by~~ 12 ok. So, let me count the total degree right now e is 2 2 ~~plus~~ + 2 ~~plus~~ + 4 ~~plus~~ + 2 ~~plus~~ + 2 which is oh no, it is not 12 it is 14 you see it is 14. So, let me remove 12 and make it 14 it is not 12 it is 14 good.

So, 14 so, now you are getting what I am doing right. So, 14 for c denominator and d will be again 14, the entire process is actually quite self explanatory, e is also 14. So, entire thing is very self explanatory, why that is because you are just following a simple rule that whenever a new person comes, he becomes attracted to someone who has many friends. It will become very clear one side I writing a new node let say f.

Now, the story becomes slightly complex, but it is easy for you to understand. So, what happens think about it, let me just remove these whatever the degrees here, any way we have a degrees here. So, that is should be enough I am just removing them. So, now tell me where exactly wi~~ll~~l ~~f~~ become friends with, f has now several options a b c d e.

But let us assume f become friends with only 2 people. Now, now you might me why is this restriction to see, it does not look like a ~~real life~~real-life scenario. What do you mean every person has only 2 friends? This is not have it happens in real life, but all I am trying to say is let us for a moment consider an ideal model right. Not all of us will make only 2 friends , but we all know that our friendship budget is sort of the same. We cannot make a lot of ~~friends,~~friends; we cannot make no friends also.

There is some average number and let us have a ideal world here let say each person makes friends with only 2 people ok; f comes here and he needs to makes make friends with 2 people. So, he makes friends with a with probability 4/ ~~by~~ 14, b with probability 2/~~2 by~~ 14, c with probability 4/ ~~by~~ 14, d with 2/ ~~by~~ 14 and e with 2/ ~~by~~ 14 anything can happen. And when he so, this is sort of tossing a coin and then deciding, but you see there is higher probability for f to become friends with a and c right.

So, he problem probably becomes friends with a and with let say d, see I just said these 2 are high probability. Probability does not mean that you always will hit on nodes with

the same with a with a high probability, you might become friends with the anyone here right. You might become friends with anybody here good . So, now let us recompute this probability since see what happens ok. So, here goes my next question why ~~should I~~I should recomputed because, there will be new nodes that is coming namely g ok.

For g to become friends with 2 people g needs to know the revised probability. So, let me remove it and let me again see what ~~is the total degree here 2 plus~~the total degree is here 2 + 3 2 ~~plus~~ + 3 because 1 2 3 4 4 ~~plus~~ + 2 ~~plus~~ + 1 2 3 4 5 for a ok. So, let me write it down 5 for a, 2 for b and 4 for c and 3 for d and 2 for e and 2 for my f.

So, what is the sum? The sum is 5 6 7 10 14 15 16 17 18 is the sum. So, the denominator will become 18 right now ok. I hope you are getting a hang of it right now, as I am doing this repeatedly. I am spending sometimes doing this manually. So, that you are understand what happening and later on we are going to write a piece of code and show you what exactly is happening here.

So, now g has come g has come, g must become friends with couple of people and he will respect this probabilities and become friends with 2 people. And, as you can see there are there is roughly one thirds probability that he will be friends with a right. And, good properly that he will be friends with c as well correct and also yeah 3 1 18 is 1 norm 6 right with d as well. So, g ends up becoming friends with let us say a and let say c and so on and so on dot ~~dot~~ dot ok. As I keep doing this the question that I would like to ask you all is the following, let me write that doubts that it seems into your mind.

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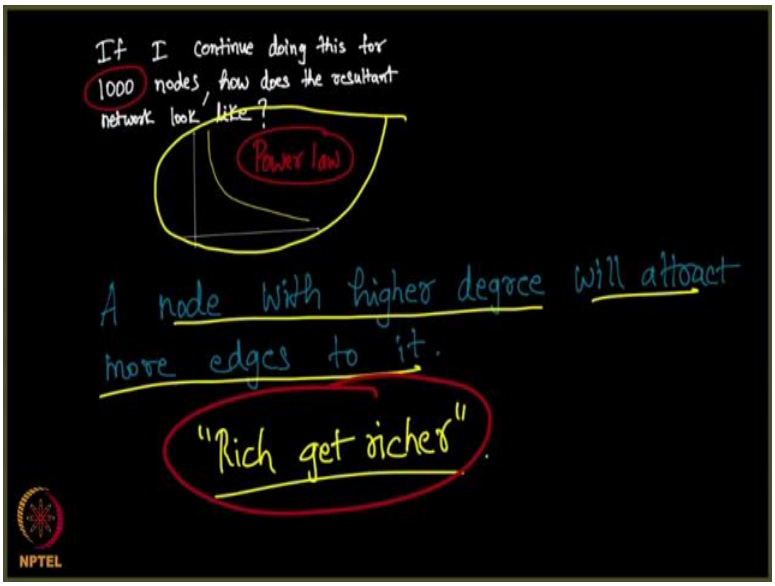

If I continue doing this process this for 1000 nodes let say, starting from the first 3 nodes as you know; let me go to the previous slide we start with this first 3 nodes. And, we went on adding more and more and more nodes correct right. As I keep doing this as I keep doing this if I keep doing this for 1000 nodes how does the graph look like, how does the resultant network look like.

So, what is happening let me go back and see what is happening. A new node always makes a 2 friends right, a new node makes friends with 2 people always and the next node that comes make friends with the existing people respecting the degree right. Any node that is coming becomes friends with the existing node by respecting the existing degree correct ok.

So, look like how does it look like? It looks like you will be started to observe that it looks like a network, where the degree distribution indeed follows powerlaw. Now that is the climax of a discussion, it indeed follows powerlaw. Such a network which grows by respecting the degrees of the existing people and then grow slowly node by node it follows powerlaw ok.

And, let us go back and then see what happened here, you see people nodes who already had higher probability had the propensity to attract new nodes right. It this continuous for 1000 nodes, you will observe that higher degree nodes will become more and more

rather it will become the degree will become higher and higher for nodes the whose degrees is already higher ok.

I repeat that I repeat that is that it is clear to you ok. So, let me is write that here a node with higher degree higher degree will attract more edges to it, why is that why is that isn't that obvious that is what we have been discussed. Because, a new node that comes that is comes will become friends with the existing a node that comes will become friends to the existing node that already has higher degree. Because, that is are that is the protocol that is the algorithm right. So, he will have many other options, but he will choose that node with higher degree already correct; that is what we discussed here right correct. So, a node with higher degree will attract more nodes to it.

Now, this process this process is called rich get richer process; I am sure it makes sense the very terminology make sense to you. What you mean by rich get richer? People who already have high degree are getting more and more degree because, that is what algorithm does you see. And surprisingly such an algorithms such of protocols such of mechanism of developing networks results in the emergence of powerlaw. It also a node with higher degree will attract more adjust to it. This phenomena is called the rich get richer phenomena and we observe powerlaw in such networks.