Social Networks Rich

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Rich Get Richer Phenomenon – 2
Lecture - 140
Analyzing basic reproductive number – 3

So, now in this lecture we are going to start from the same place where we left of in the last lecture. So, what did we see in the last lecture, first of all our problem statement.

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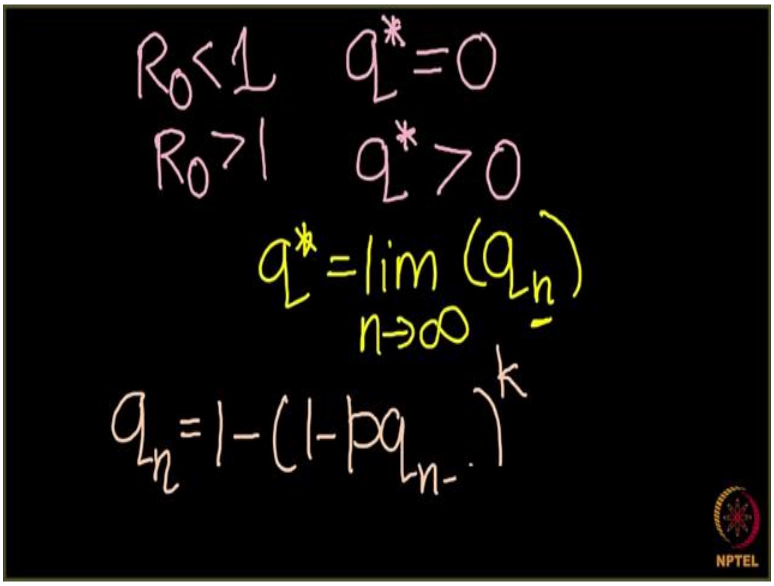

What is the problem statement? First, if R0 <1 then we have to prove that the value of q* equals to 0 and if R0 is greater than 1 then the value of q* is something which is greater than 0. This thing we have to prove and then we and we know what is q* right.

So, what is q*? q* is nothing but $\lim_{n \to \infty} q_n$ where q_n is the probability that your infection persist till the nth level; that is at least 1 person at the nth level is infected. And in the last lecture we have derived a formula for q_n and the formula was $1 - (1 - pq_{n-1})^k$. What we are going to do in this lecture is now we are going to analyze this formula further.

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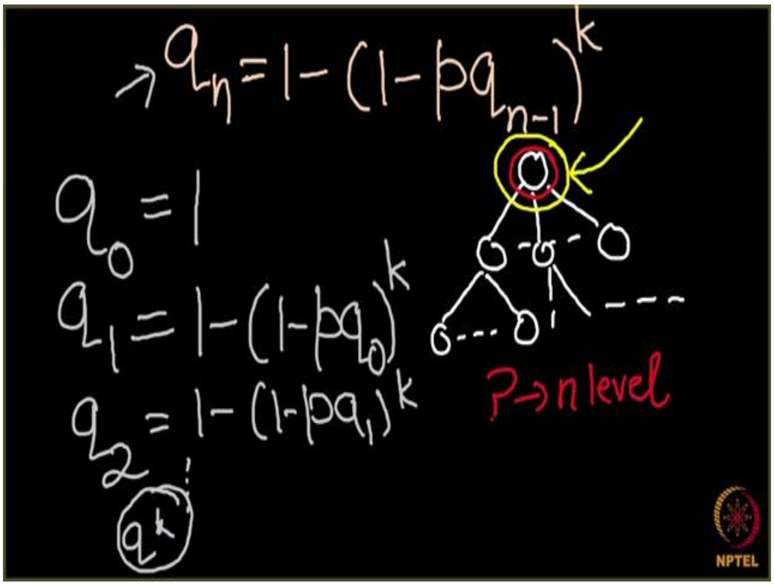

So, let me write this formula here and we will analyzing it $q_n = 1 - (1 - pq_{n-1})^k$ ok. Now what is q_0? Let us look at q_0, q_1, q_2, q_3 and so on. What you think is q_0? You will not get it from this formula. So, q_0 is what, the probability that your infection persists till the 0th level and what is a probability.

So, what was a problem statement? This was a guy who was here having k neighbors and this person was again having k neighbors. This person was again having k neighbors and so on. And your infection started from here and the question was what is the probability that infection reaches here to the nth level was our question.

What is level 0 here? This is level 0 and what is the probability that at least 1 person at this level is infected? It is obviously, 1 right for sure 1 this guy must be infected here. If this guy was not infected here, there was no problem we would be solving right. So, entire problem is because this person at the 0th level is infected it is infected with the probability 1. So, $q_0 = 1$. What is q_1? Finding q_1 is easy. Put the value in this formula what is q_1, $1 - (1 - pq_0)^k$, right.

What is q_2? q_2 is $(1 - pq_1)^k$ and what is our aim? Our aim is to find q* which will come after we will keep repeating this formula. So, we have q_0, from q_0 we can find q_1, from q_1 we can find q_2 and then we have to do this process infinite number of times and finally, we will find the value of q*. We can do it infinite number of times. So, let us see how do we find out this value.

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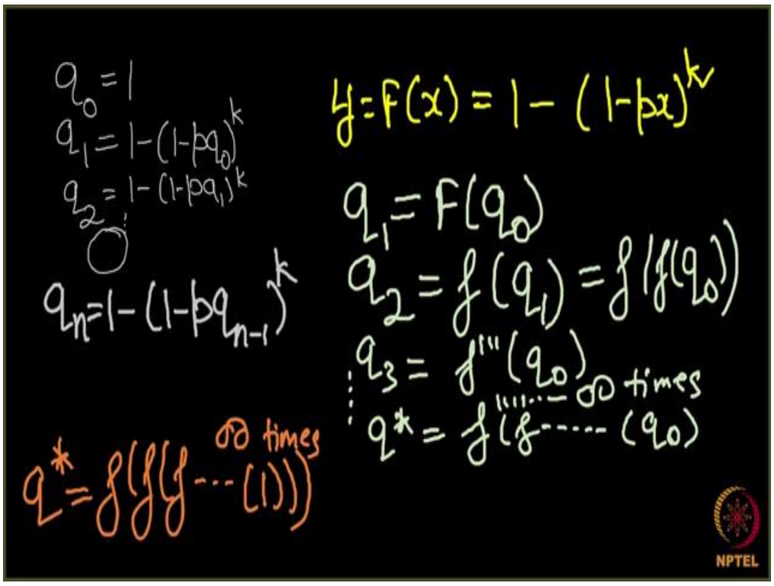

So, now we have all this we know q_0, q_1, q_2 and we know that what is q n, q n is nothing, but $(1 - pq_{n\,-\,1})^k$. I try to write it down in the form of a function. It is already in form of a function. If I just take a function y = f(x) we define this function as $1 - (1 - px)^k$.

Now in terms of this function can we see what is q_1, q_0 obviously is 1, what is q_1 according to this function? So, if we see what is q_1 here $(1 - pq_0)^k$. This and this are the same. I have just written this in the form of a function here. So, we can write q_1 as nothing, but 1 minus 1 minus oh sorry. So, we can simply write q_1 as; what $f(q_0)$ right. What is $f(q_0)$? $(1 - pq_0)^k$ which is same as this right. What is q_2 now? q_2 is $f(q_1)$, $(1 - pq_1)^k$ which is nothing, but I put the value of q_1 here. It becomes f''means f of f of so, to $f(f(f(q_0)))$.

Similarly, what is q_3? q_3 is nothing, but $f'''(q_0)$. And similarly, you see what is q* going to be? q* is going to be f dash dash dash dash infinite times means $f(f(f(f(\ldots(q_0)))))$. And we also know that the value of q_0 is 1. So, I can write it down as $q^* = f(f(f(f(\ldots(1)))))$ right.

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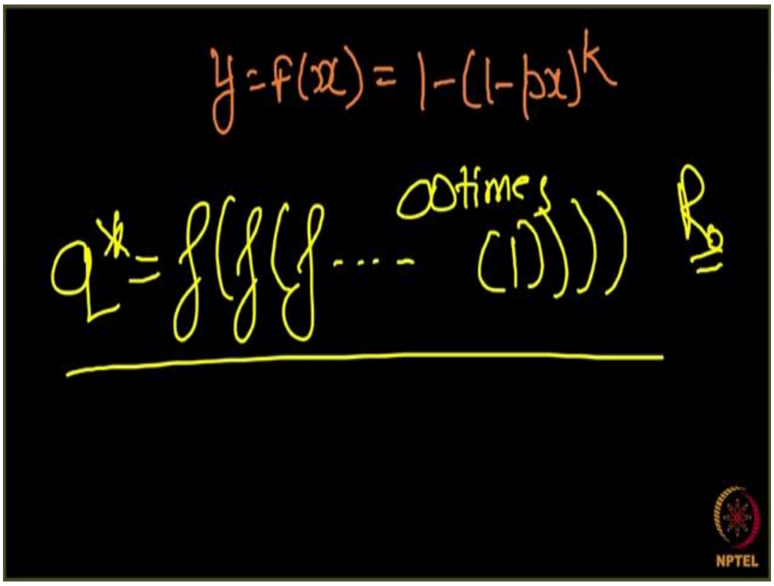

So, what is our overall aim now? Our aim now is we have a function y = f(x) which we define which is $1-(1-px)^k$and I know now the value q* which I have to find is nothing but f(f(f(f(….(1)))).

And how do I find out this value? Once I find out this value my task is done, our aim is to find out the value of q*, obviously given the value of R0 that is to come in the next lecture. Our overall aim is just to find this value of q* and we will be finding it in next lectures. Probably just 1 or 2 lecture more we will be finding what is q*.