Social Networks
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Rich Get Richer Phenomenon
Lecture - 118
Detecting the Presence of Powerlaw

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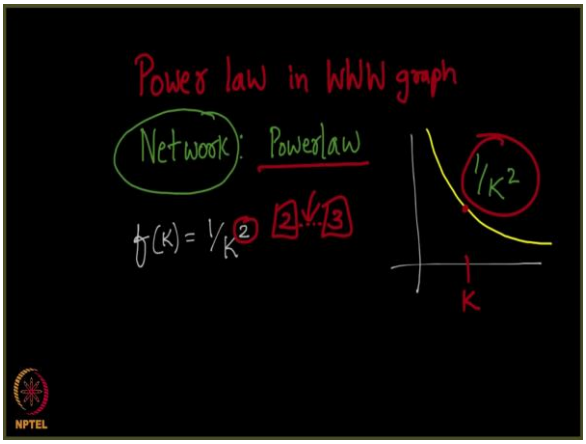

So, we saw in the previous lecture that Powerlaw emerges right. We observe powerlaw in the network of weblets right called the WWW graph the web graph, we observe this we saw this ok. Now, assume you have a network alright. So, before going any further ~~ah~~ I am going to address this question on how to detect power law in given network. Assume you had a network you are given a network and you have this question in mind. The question is in is this network exhibiting powerlaw right. If so, then what exactly is the exponent~~.~~ ~~So~~so, x and y axis and if you can see something like this what exactly is the exponent of this.

I told you that the exponent in case of the web graph happens to be ~~1 over K square~~$1/K^2$ right. So, how will you find out whether your network that you are looking at exhibits powerlaw or not. So, here is a very easy litmus test that you all can quickly observe and sort of node your head saying it is nothing, but obvious. This is basically as I told you this is your f~~ (of~~ (x) rather ~~f of K~~f(K), if this is K this becomes ~~f of K~~f(K) which is ~~1 over~~

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~~K square~~1/K^2. So, let me note that down and say ~~f of K~~f(K) is equal to ~~1 over K square~~1/K^2.

By the way this exponent here needed necessarily be 2, it is observed to be between 3 and 3, in case of some networks it is 3, in case of some networks it is 2 and so on right. In some networks it is greater than 2 and less than 3, it is somewhere in the middle right in many case, I would like to find out if a network is exhibiting powerlaw or not.

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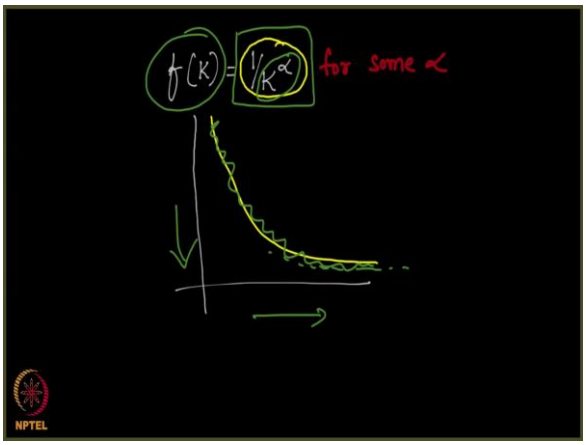

So, how do I do that? It is it is ~~fairly straight~~straight forward a process. If ~~f of K~~f(K) ~~is equal to~~= 1 over let us say K^{α} ~~to the power of alpha~~ for some α~~alpha~~, I do not know what that α ~~alpha~~ is for some α. ~~alpha l~~Let us say some alpha how do I find this. So, I have a plot given remember you can plot the distribution and you observe that it resembles something like this. And, you know it is something like 1~~ over~~ K^{α}~~K to the alpha~~, but you do not know for sure whether it is a reciprocal of a polynomial like this K^{α} ~~K to the alpha~~ or something.

So, what you do is you, what ~~is~~does this curve observe this curve this curve is nothing else, but your ~~f of K~~f(K). And, you ask the question is at of the form 1/K^{α} ~~over K to the alpha a~~As you see as the denominator increases the value sort of decreases that is why it becomes small right ok. How do I see this? I write so, let me make some space from myself here how do I see this?~~.~~

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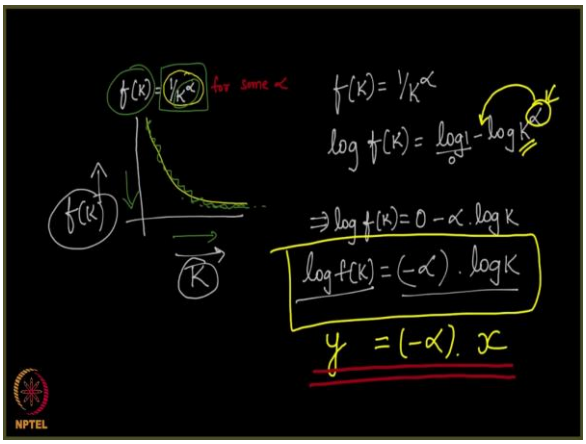

Let me write on this side. So, how do I see this? Given that ~~f of K~~f(K) ~~is equal to~~= 1/-K^α. ~~over K to the alpha w~~What I do is I apply log on both sides very straight forward math, I am sure I am not using anything beyond let us say high school level math which is equal to log 1 ~~minus -~~ log K^α ~~K to the alpha~~. So, basics of logarithms I am sure all of you are familiar this simply becomes log ~~of f of K~~f(K) is equal to we all know log of 1 is 0 0 right.

This becomes 0 ~~minus -~~ log K^α ~~K to the alpha~~ can be brought here that is the property of logarithms, I am sure you all remember minus alpha times log of K so, look at this; look at this. So, what does this mean? Log ~~of f of K~~f(K) ~~is equal to~~= ~~minus -~~ α ~~alpha times~~* log ~~of~~ K, as you know the x axis is your K and y axis was your ~~f of K~~f(K). Now, you see if you were to plot the log of it ok. So, what you will do is you will plot instead of K you will plot log K, instead of ~~f of K~~f(K) you will plot log ~~of f of K~~f(K) ok.

If you observe it, what do you see here? You see that look this is a variable that we call it some y ~~is equal to~~= ~~minus -~~ α~~alpha~~, please note this is a constant minus alpha is a constant because it is the power of K right. K is the variable here right times log ~~of~~ K which I am going to replace by let us say variable x. So, as you can see you are getting equation of a ~~straight line~~straight-line y equals minus alpha times x. So, what does it mean this simply means that when you are trying to plot the log of the graph; the log log of the ~~graph.~~graph?

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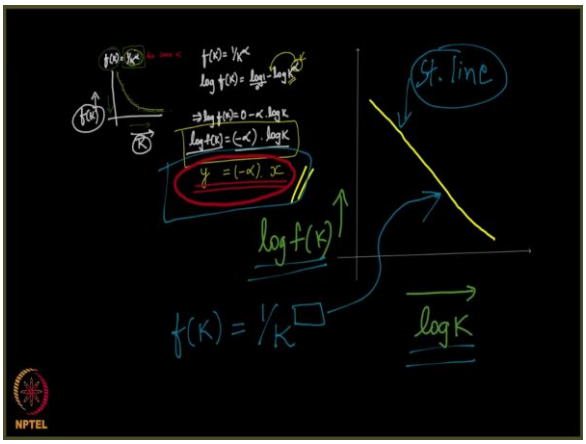

Let us say you are not plotting the typical K and ~~f of K~~f(K). You are plotting be clear here I am writing a plot here y axis this is my x axis and if I am trying to plot instead of K, I plot here log K and here I plot log ~~of f of K~~f(K). So, what I will trying to say look at this look at this, I am trying to say that this will ~~actually be~~be a straight line a straight line right. So, actually it should pass through a origin here, but do not worry I am just giving you an example. So, this will be a straight line with slope let us say minus alpha ok, if get a straight line with slope minus alpha.

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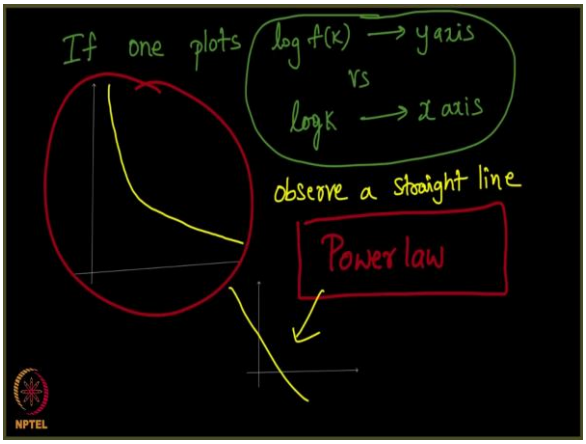

So, let me write down a straight line. If you getting a straight line which means that log K versus log f(-K) rather log f(-K) versus log K that you wrote was of this form, which means ~~f of K~~f(K) was indeed of the form 1/~~ over~~ K to the power of something and that something happens to be the slope of this line. So, all in all the litmus test is the following it might sound very technical, but all I am saying is the following let me write that down neatly.

If one plots log of ~~f of K~~f(K) in the y axis; in the y axis versus log K in my x axis, just in case after plotting this if you observe a straight line then you can possibly conclude that there is powerlaw happening here. Now, you might ask me this question when you are given a plot like this let us say like this why ~~cannot I~~I cannot observe this plot and then simply say there is powerlaw happening here. ~~But,~~But then this particular plot can be anything it need not necessarily be 1/-K^{α} ~~over K to the alpha~~ it can be anything right.

So, you never know if it is really of the form 1/~~ divided by~~ a variable to the power of a constant let us say α~~alpha~~. You would not know if it is like this only right. So, what you do is you instead do not take chances when you have when you get a curve like this you try to draw the log log plot; you try to draw the log log plot. This is called a log log plot y axis make it log x axis, make it log values and then try to see you will get a straight line. When you get a straight ~~line~~line, it means that this is powerlaw happening here perfect.