Social Networks
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Rich Get Richer Phenomenon - 2
Lecture - 141
Analyzing basic reproductive number – 4

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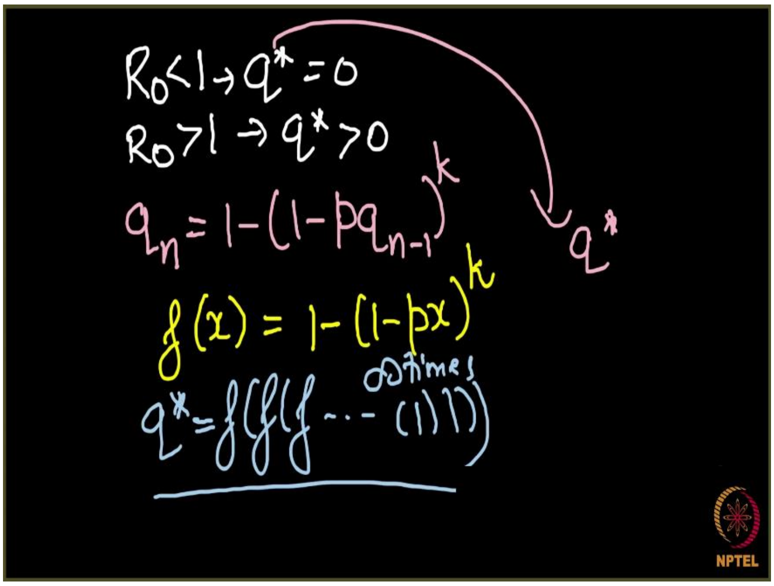

So, where were we? We will quickly recap from the beginning; our aim to show that if R0 < 1 q* = 0. And, to show that if R0 > 1 then q* > 0, this was our aim. And, then we found out a formula for $q_n = 1-(1-pq_{n-1})^k$, but our aim is to find q* right. And, we have defined the function, we have looked at a function and the function were $f(x) = 1-(1-px)^k$. And, we have looked at that this q* is nothing, but f(f(f(f(…(1)))) and this is the value which we are after. We need to find out what is this value, let us see how we find this value.

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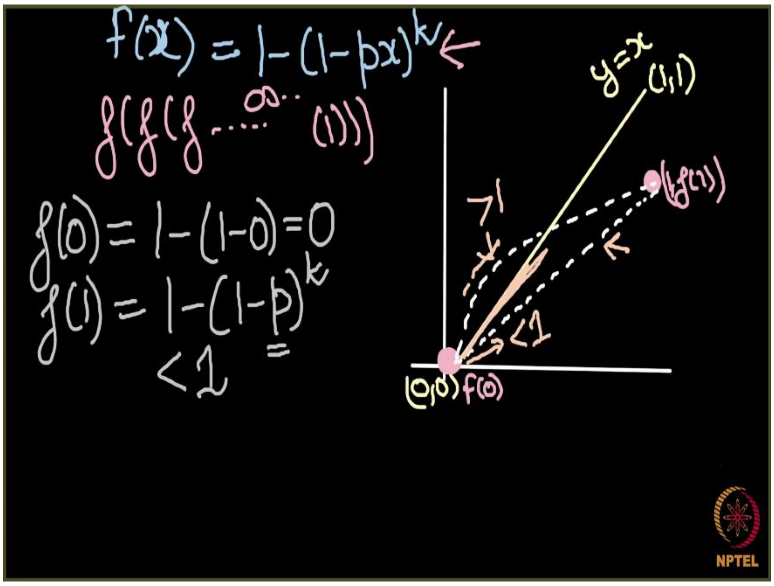

So, what all we are having $f(x) = 1 - (1 - px)^k$, this is our f(x) and our aim is to find f(f(f ….(1))). How do we find it? First, let us analyze what is this function, how does this function look. Let us see how this function look will. So, let us say, let us try to find f(0). What will f(0) be? It will be (1 – (1 – 0)) right which will be nothing, but 0. So, f(0) is 0 and let us look at what is f(1); $f(1) = 1 - (1 - p)^k$and we know that p is something positive.

So, this entire value is going to be less than 1. So, f(1) is something which is less than 1 f(0) is 0 and f(1) is something which is less than 1; let us try to see how this plot will look like. So, I draw a plot here let us say y axis and x axis and here it is 0, here it (0, 0) and I first of all I draw a line also here and this sorry and this is a line I say what is this line you all know. So, this line is y equals to x and this point is 1 comma 1 right. And, let us see how a function f(0) look will like, a function f(0) is first of all having a point here for sure because at 0 f(0) is 0.

And we know that f(1) is something less than 1. So, this point is f(0) and f(1) is going to lie somewhere here which is less than 1. So, this point is f(1), (1, f(1)). So, this point is (1, f(1)) ok. We have seen the starting point of this function and the ending point. What happens during middle? What happens here? How does how does this function look like? And, that is also easy to calculate, let us see how we can calculate that. Let us try to find out the slope of this function at this point, let us try to find out the slope of this function at this point.

So, what is this slope? This slope we know is 1 right, this slope is 1 which is your 45 degree; tan of 45 degree 1. So, if at this point so, now see this function f(x) can either go like this or go like this. When does this function go like this? When this slope of this function here is less than 1 right, if the slope is so, because this is slope equal to 1; so, if this is slope equal to 1, if slope less than 1 then this function will go from downwards of this function y = x. And, when will this function go like? This it will go like this when the slope of this function is greater than 1. So, let us first try to find out the slope of this function. So, what do we do to find out the slope of the function is to differentiate?

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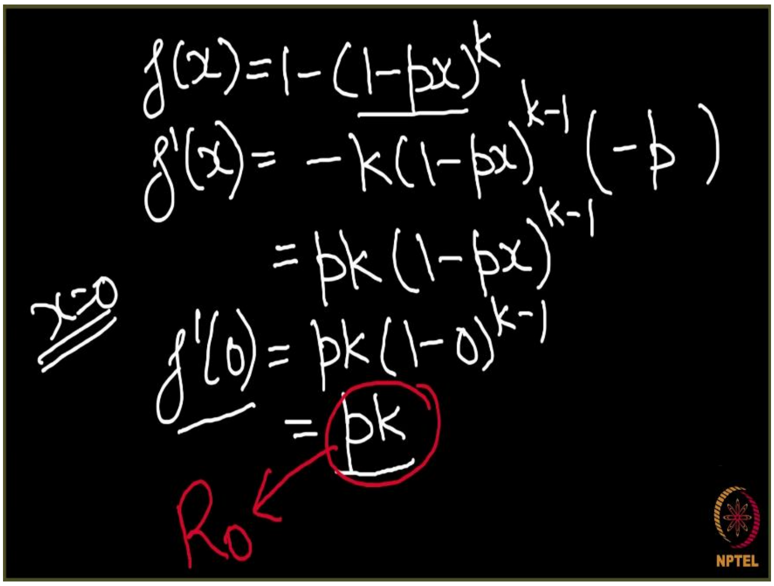

So, we have the function $f(x) = 1 - (1 - px)^k$, let us differentiate it. So, $f'(x) = -k(1 - px)^{k-1}$ right multiplied by differentiation of (1 – px) which is -p. So, this becomes equal to $pk(1 - px)^{k-1}$ right. So, this is the slope of this function and we are interested to find the slope of this function at x equals to 0 at origin.

What is $f'(0)$? $f'(0) = pk(1 - p * 0)^{k-1}$ which is nothing, but p * k; it is amazing. The slope of this function is p * k. What is p * k? If you remember p * k is nothing, but your basic reproductive number R0, from where we started and where we reached.

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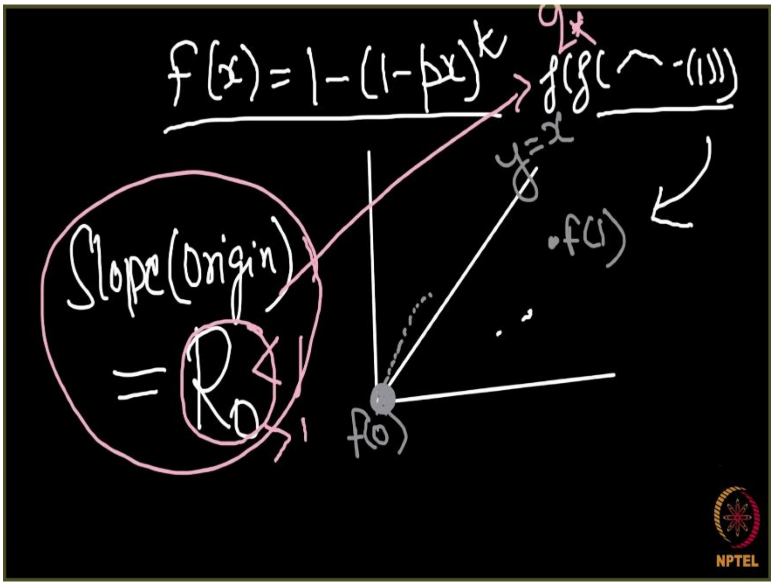

So, you see we started off with this function $f(x) = 1 - (1 - px)^k$and then we try to plot this function. And, this is the line y, and this is the line y equals to (Refer Time: 06:34) 1 second and this is the line y = x. This is the line y = x and then we saw that here will be our point f(0) and here will be our point f(1). And, then we were interested in finding the slope of this function at this point and it turned out that this slope of this function at origin is nothing, but the value of the basic reproductive number.

So, just remember all these we have this f(x) here and the function look something like this and we wanted to find out f(f(f(f(…...(1)))) and then we tried to look at this function. The function turned out something like this and then we saw something amazing; we saw that the slope of this function at origin is nothing, but R0. So, now we will see for different values of R0, that is R0 can be less than 1 or greater than 1 how does this value which is nothing, but q* turn out to be and that will be the end of this proof which will which will do in the coming up lecture.