Social Networks
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Link Analysis (Continued)
Lecture – 109
Matrix Multiplication (Pre-requisite 1)

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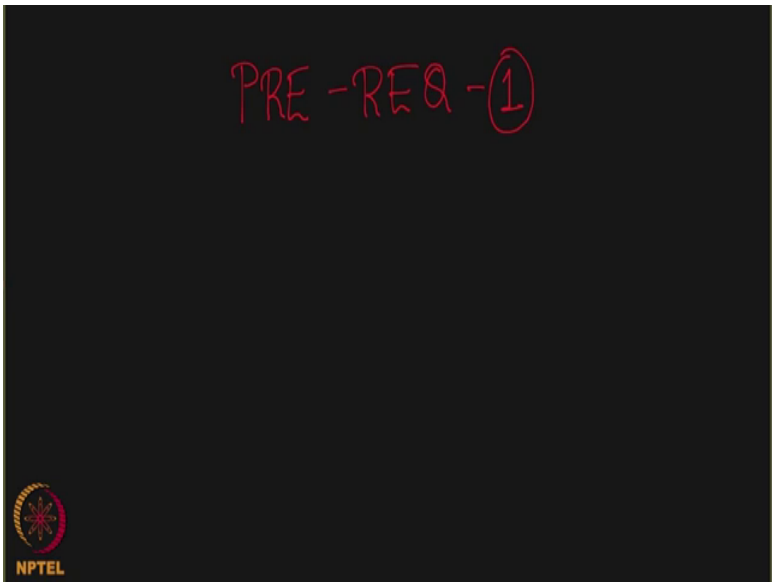

So, let us look at a few Pre-requisites before going further. These are nothing, but elementary let us say high school level mathematics precisely speaking it is matrix multiplication and we will understand what happens when you multiply a matrix. Although you would have understood what matrix multiplication is. You would not have seen this part that I am explaining next. Observe carefully, it is an interesting observation you can make about matrices ok.

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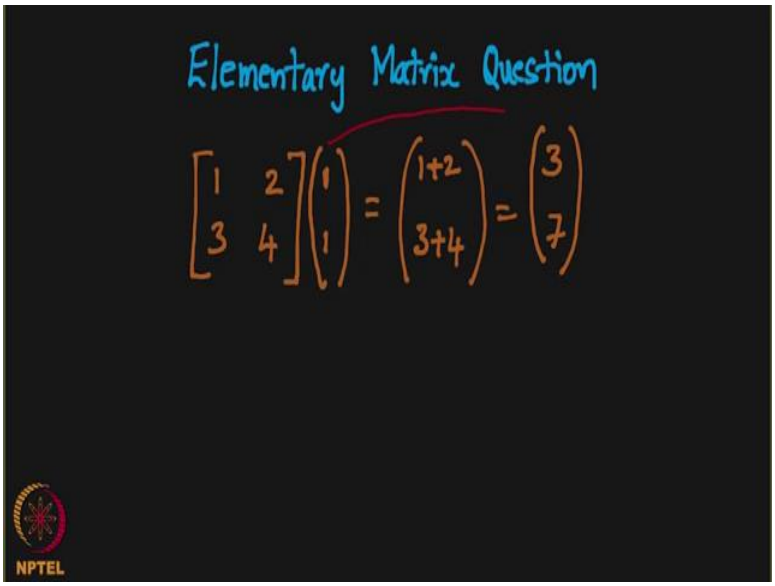

So, let us look at this very elementary matrix question you take a matrix $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ all right. We are going to play around this matrix only a simple 2×2 matrix right. Multiply it with this vector $\binom{1}{1}$, what do you get? You simply get 1 + 2 and then 3 + 4 right fine. We all understand this much of matrices at least right next. This is equal to $\binom{3}{7}$, so, $\binom{1}{1}$, is giving rise to $\binom{3}{7}$, right good and then.

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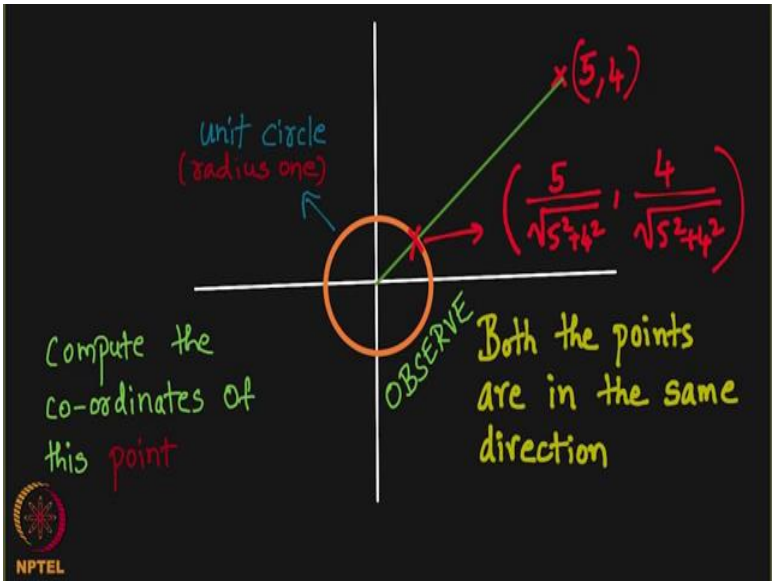

Let me see how one can represent points on this two-dimensional plane, two-dimensional coordinate system ok. So, we got $\binom{3}{7}$, right. Now let me take a point (5, 4) somewhere in this coordinate axis. I will tell you what 1 means by pulling this to a circle ok. You will understand it very soon it is a quite easy concept. So, look at the vector of this typically by this point (5, 4). It is simply the line joining the origin and (5, 4) right and now what I do? I draw a unit circle. What do you mean by unit circle? By this you mean what is the unit circle? By a unit circle we mean, a circle with radius 1 with center as the origin all right.

So, what happens next? Here is a quiz question for you all. Can you compute what is this point? This very point here. What is this point? Pause the video and then compute this. Please note I have given you a point (5, 4) and I am giving you a unit circle and I am asking you where does this line joining the origin and (5, 4) intersect the unit circle right. This is some basic thing. You compute the coordinates of this point extremely basic right. What you do is you first write 5 and then divide this by the square root of the sum of the squares right.

We all have seen this. It takes a minutes pause and then observe this if you unable to see this. It is an extremely high school level coordinate system mathematics nothing beyond that. And then you have similarly $4/\sqrt{5^2 + 4^2}$ and this is your point on the unit circle.

Given any point (a, b), you can write the corresponding point intersecting the unit circle in the same direction as $a/\sqrt{a^2 + b^2}$. Here a is 5 and b is 4. So, $4/\sqrt{5^2 + 4^2}$ all right? Particularly I showed it for the example (5, 4) so far so good you are understand what has happened so far.

Let us observe something an obvious observation is that both the points are in the same direction right. They are both in the same direction correct. From origin if you can see, these two vectors (5, 4) and $5/\sqrt{5^2 + 4^2}$ both are in the same direction. Now let me look at what just happened before.

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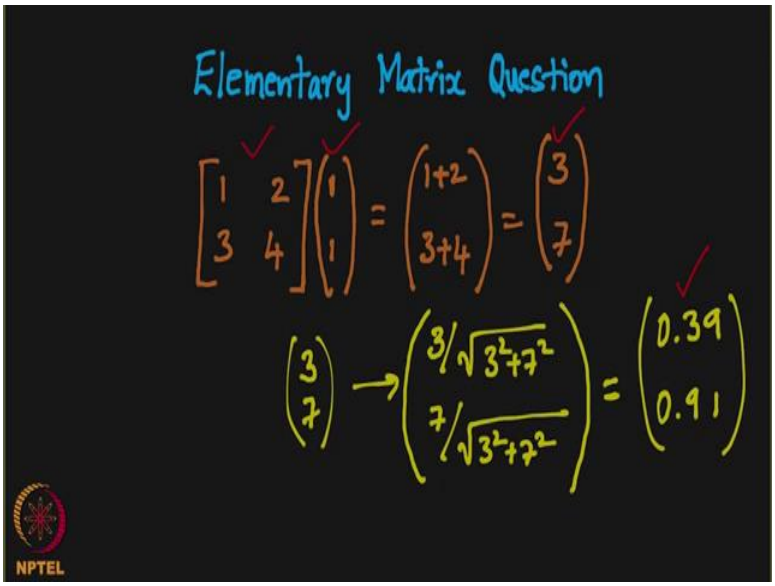

We took let me repeat this, it was an elementary matrix question and we took matrix $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and multiplied with $\begin{pmatrix}1\\1\end{pmatrix}$. We got $\begin{pmatrix}3\\7\end{pmatrix}$, so, right.

Now, what do I do? I take this $\begin{pmatrix}3\\7\end{pmatrix}$, and then try to see what the point on the unit circle in the same direction is. What do I mean by this? Just now we did that before right. Before the previous video that I showed you whatever we did here, I am going to ask you to do the same thing for the next slide. 3 by 7 when it is pulled to the unit circle will give you so much right. Again, $a/\sqrt{a^2+b^2}$ and $b/\sqrt{a^2+b^2}$ is the corresponding point on the unit circle.

What is this equal to? Let me compute I use my calculator and I saw this comes out to be $\begin{pmatrix}0.39\\0.91\end{pmatrix}$, good. What next? What now? I multiplied $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$with $\begin{pmatrix}1\\2\end{pmatrix}$. I am sorry $\begin{pmatrix}1\\1\end{pmatrix}$ and I got $\begin{pmatrix}3\\7\end{pmatrix}$ and I sort of this is called normalization. I normalized it; I got this value on the unit circle.

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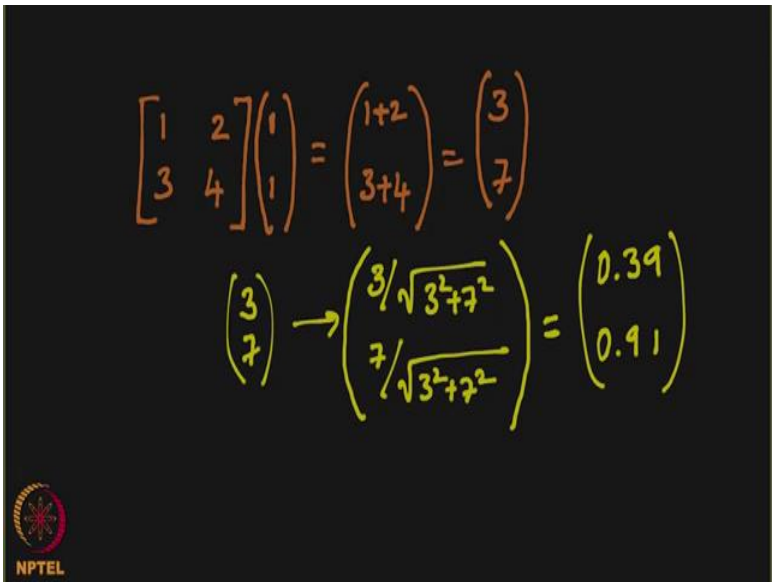

What if I continue doing this process right? I got this $\begin{pmatrix}3\\7\end{pmatrix}$. I continue this I pull it to the unit circle I got so much. What if I take these two things look at this from the previous slide, I simply take this $\begin{pmatrix}0.39\\0.91\end{pmatrix}$.

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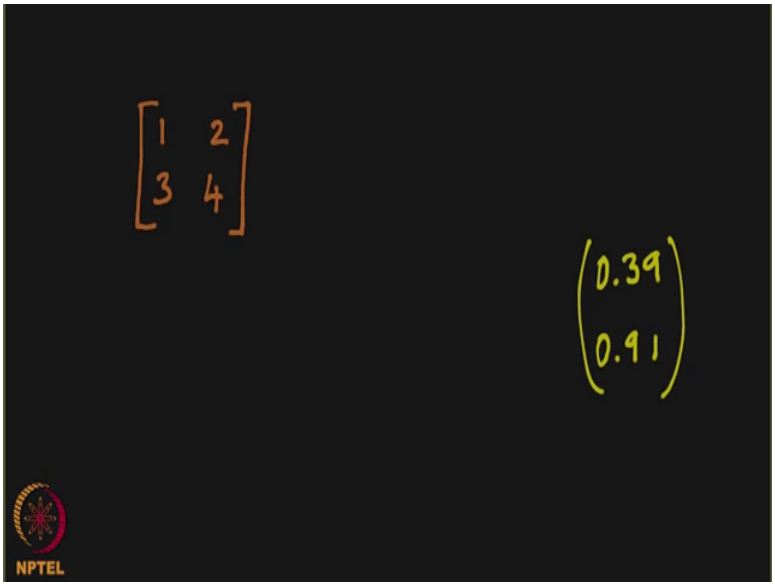

And I apply $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ matrix on it ok.

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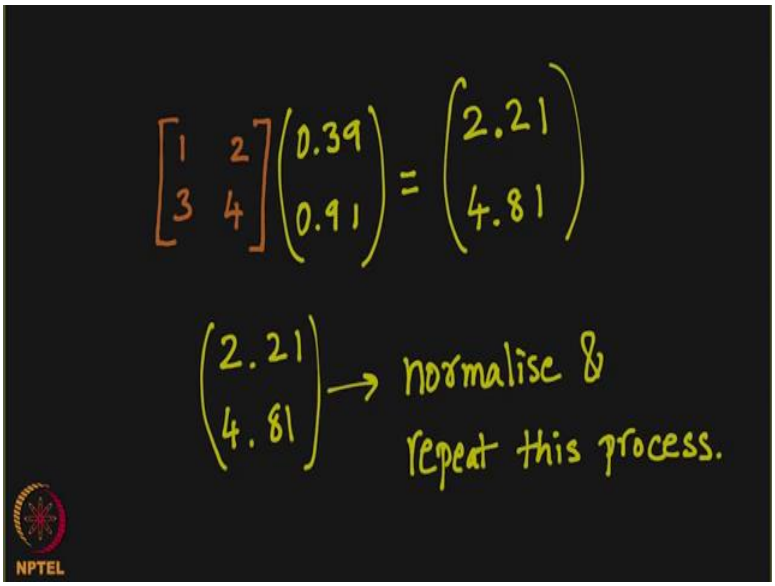

What do I get? I get my calculator says I get $\begin{pmatrix}2.21\\4.81\end{pmatrix}$,. Again, I repeat this process, I normalize this, and I see as I keep repeating this process, what exactly do I get.

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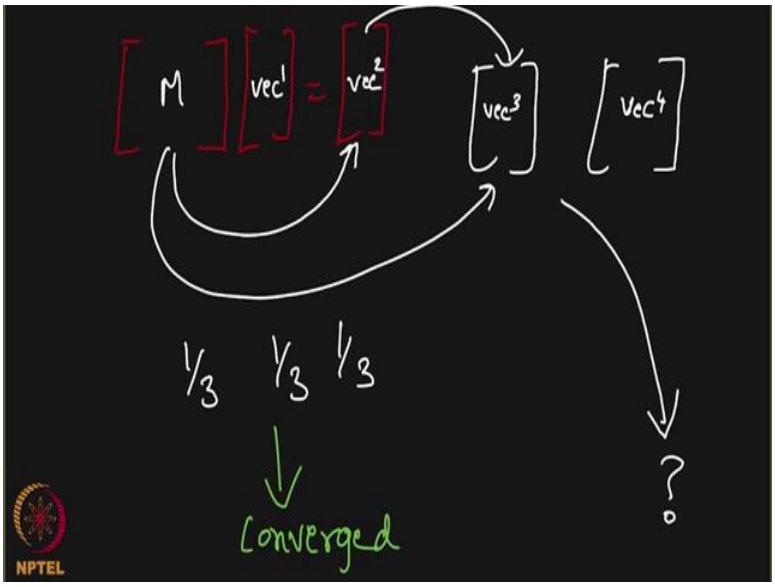

So, what I see I took a matrix, I took a matrix applied it to a vector; I got another vector. So, a vector a matrix I got another vector. What I do is I take this matrix and apply it to the same vector. Now before that I sort of, I normalized it normalizing is just to ensure that you do not get bigger values. All you care about is the direction I will tell you what I am trying to do, and I get a new vector another vector. Let us say this was vector 1, this is

vector 2. Now you get a vector 3 all right and then again apply this matrix on this vector, you get a vector 4. At every instance you are normalizing by normalizing I mean; you pull it to the circle, and you see where exactly this is going where exactly this is going.

Somehow, we seem to be interested in repeating the same process again and again and we see what happens. We saw what happened with the iterated value of this matrix. Remember 1/3, 1/3, 1/3 not of the matrix that of the graph. We took a graph with 3 nodes, we are saying the values 1/3, 1/3, 1/3 and we saw that it converged. And now we are asking a matrix question; it converged right now we are asking a matrix question right.

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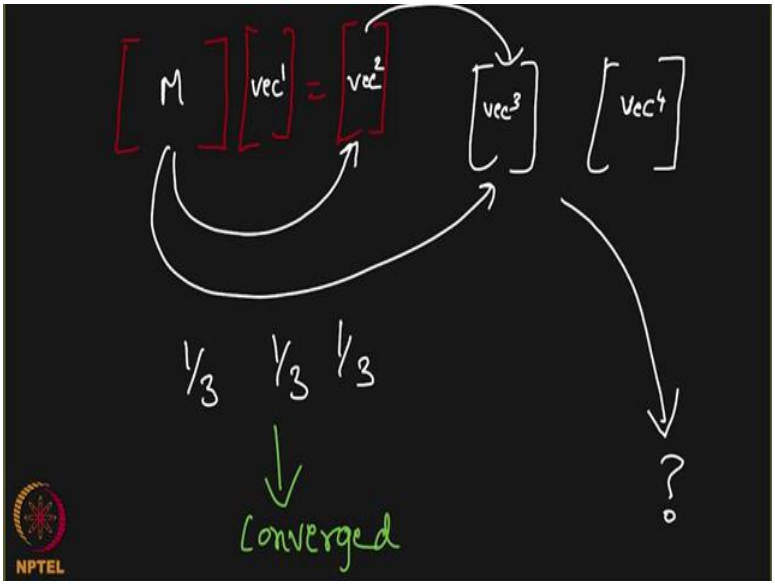

When you apply a matrix repeatedly on a vector, what happens to it? Will it also converge or not? Obviously when you apply a matrix like $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ on a vector the values become bigger and bigger and that is why I am trying to pull it back to the unit circle right. I am normalizing it, what I will do is, I will switch to screen cast mode and I will try to understand this matrix multiplication process nicely.