Social Network
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Rich Get Richer Phenomenon - 2
Lecture - 139
Analyzing basic reproductive number 2

So, we have formulated the problem statement. I will again revise it.

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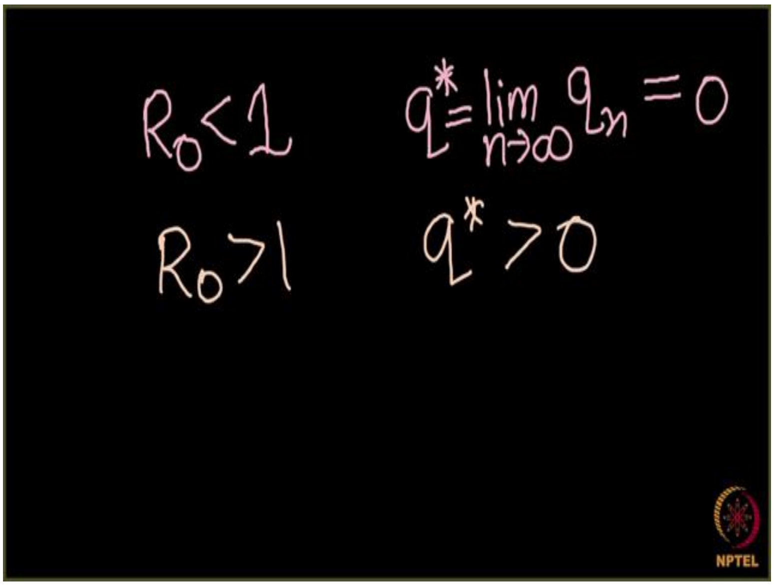

I keep revising things again and again so that we should not get confused whether we are in proof. At problem statement was simply we have to prove that if the value of R0 basic reproductive number is less than 1, then our q*, q* which is nothing but $\lim_{n \to \infty} q_n = 0$ and if our R0 it is greater than 1 then the same q* > 0. And you remember what is q_n; let us quickly see what is q_n.

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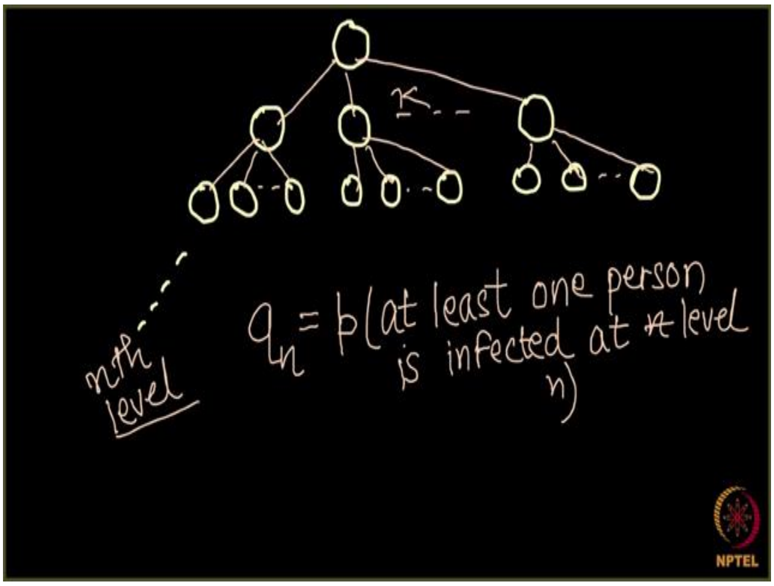

So, if I have a network over here let us say I have a network here, this node has k neighbors and each of these again have some k neighbors and so on. And it we make which is here and these are the edges k neighbors here. And this has k neighbors here, k neighbors here k neighbors here and so on.

And we got up to the nth level nth level. So, what was q_n ? q_n was the probability that at least at least one person is infected at the nth level. One person is infected at level n. Let us now see how we can find out this value of q_n. So, I make a clearer network over here; so, I need some time.

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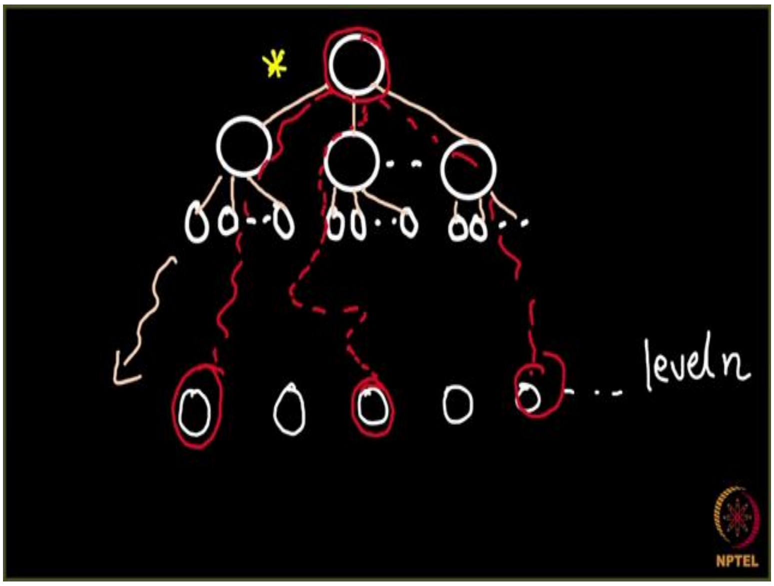

And here I can be a little big and here are the edges fine. This is my network right and somewhere down is our nth level. So, let us say this is our level n ok. Now what do we do? I define an event here. Now carefully watch, I define an event here and the event I say that it is a star event, star event.

What do I mean by a star event? We know that one of the node here should be infected right and for this node here to be infected there should be a path right, there should be a path from this node which was initially infected. So, this node here is obviously, infected. For some of the nodded level end to be infected there should be a path from this node to this node to somewhere here to somewhere here some path should be here right.

So, now what does even star means that that path can come from here, that path can come from here, and in fact some node here or that path can come from here, and in fact some node here. But there is at least some path and this path should runs through should go through at least one of these edges either through this either through this or either through this right. So, my star event says that it is the event that your infection reaches this level n through a path which involves this edge.

So, infection goes from this edge after this edge I do not know what happens, but it reaches level n, but this edge is for sure is infected. So, star event is the event that your infection reaches level n involving this edge and something happens after this edge any path. So, after this edge infection can go through this edge and have some path, can go through this edge and have some path can go through this edge and have some path, but for sure it comes from here.

That is my event star ok. What do you think is the probability of this event star? What is the probability that your infection reaches level n through this path, through some of the path here? It is easy how will infection reach here. So, we know that first for infection to reach here may take back all my paths ok.

So, for my infection to come through this path first of all this thing should be infected right. This edge should be infected and then something happens afterwards.

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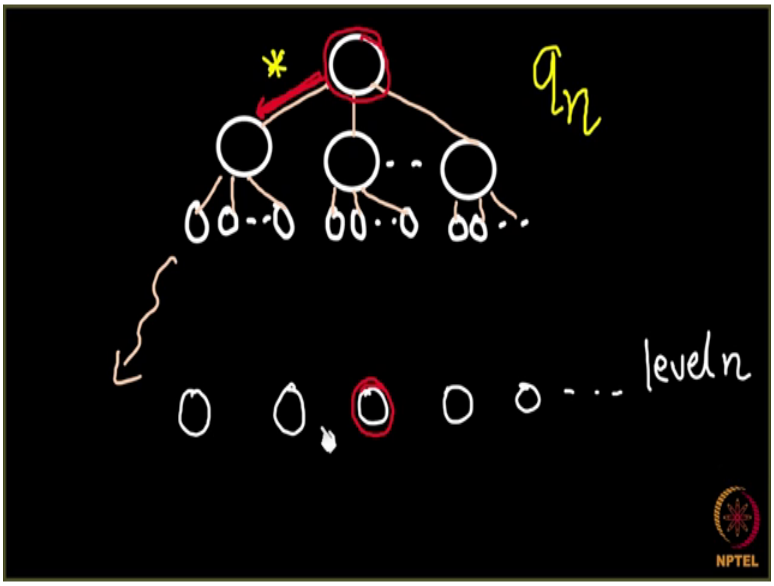

Now, see what was q_n? q_n was the probability that starting from here your infection reaches this level right that is q_n. You start from this level from this node and your infection reaches this right ok.

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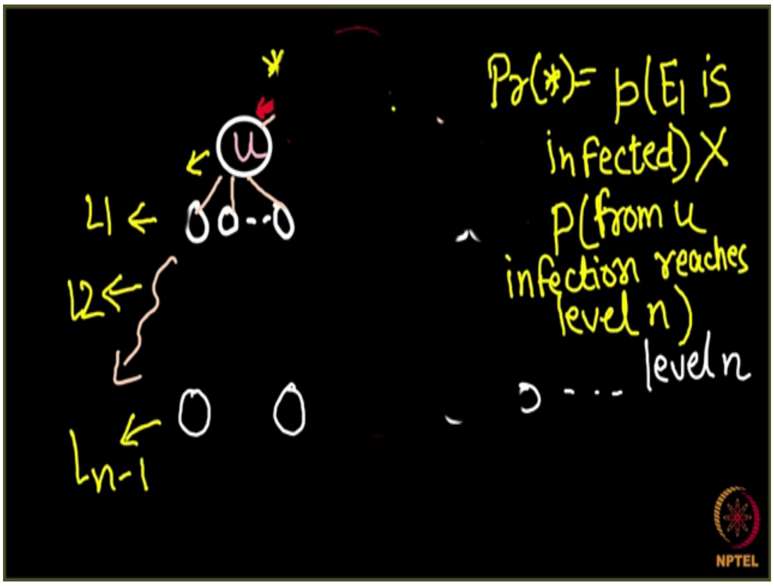

Now, tell me can I write the probability of event star as probability that let us call this edges E1 and probability that E1 is infected. The infection passes through E1 multiplied by, what should come after multiplication? After multiplication is the probability that and let us say that this node over here is u. So, into probability that from u infection reaches level n.

Probability that E1 is infected into probability that from u infection reaches level n. What is this probability that from u infection reaches level n, do you see? If you start from u it is the same as if you as if you are starting from here, just one level has reduced. What is what was q_n? You start from here and your infection should reach nth level. What is q_n - 1? That you start from here and your infection reaches level n - 1.

So, if you look at this node u over here, so assume that this node is not here, all these nodes are not here, I will even remove this, we will take them back wait; we will just rub all these portion. Let us say all this is not here all this is not here; I forget about all these and what I want to know is that you start from u and infection reaches level n.

So, this was here level n, what is this level for u? So, if I start from u, so for u this is level 1, next will be level 2 and this will be level n - 1. So, this thing, probability that your infection starts from u and reaches level n is nothing, but q_n - 1 ok.

So, I will just take everything back right. So, how can I write my probability of event star, what is the probability that E1 is infected? The probability that E1 is infected let us take a new slide and write it there, I will quickly draw all these things here.

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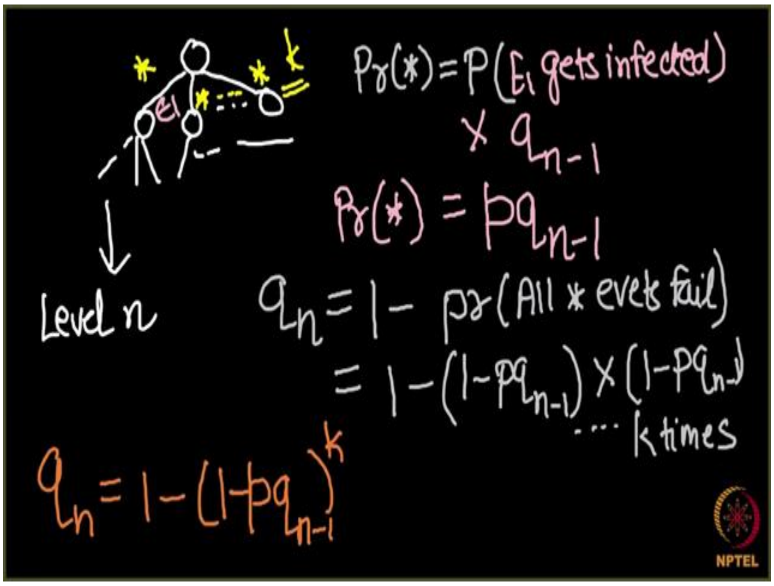

So, we have this node and it has some k neighbors which has some k neighbors again and so on. And here is your level n and then recall what was your event star? Event star was you reach level n through this path and we said that the probability of event star equals to probability that the edge E1 that the edge E1. So, if this is E1 probability that E1 gets infected multiplied by $q_n - 1$ right. And what is the probability that E1 gets infected is nothing but p.

So, it is pq_{n-1} is the probability of event star so far so good. Now what were we looking at? We start our infection from here and it should reach level n. For reaching level n how many star events are possible, it can go from here, it can go from here, it can go from here. So, there is a star event here, there is a star event here and there is a star event here.

So, there are k star events right and your infection can go through the happening of this star event or the happening of this star event or the happening this star event. If one of these at least one, so even if one of this star event succeed your infection will reach level n right because, what is this star event that your infection reaches level n through this path. So, for your infection node to reach level n all the star event should fail right. So, I can write q_n. $q_n$was my probability that my infection reaches level n.

And for reaching level n it must go through one of the star events. So, can I write q_n as 1 minus probability that all-star events fail? q_n is nothing, but the probability that all the star events fail ok. Now what is the probability that all the star events fail; 1 minus. What is the probability that the first star event fail? So, probability that first star event succeeds is this, probability that first star events will be E1 minus this, which is $1-(1-pq_{n-1})$.

What is the probability that second star event fail all of them should fail? So, I put a multiplication here and what is the probability that second star event fail it is a same. So, it is $1-(1-pq_{n-1})$ and I keep doing it how many times? How many star events are there that many times, so it is k times right?

So, I can write q_n as $1-(1-pq_{n-1})^k$ ok. So, I will quickly put back everything in place whatever we have done till now, we will do it at the end of every small lecture which is this proof is running So, let us quickly see what we have done.

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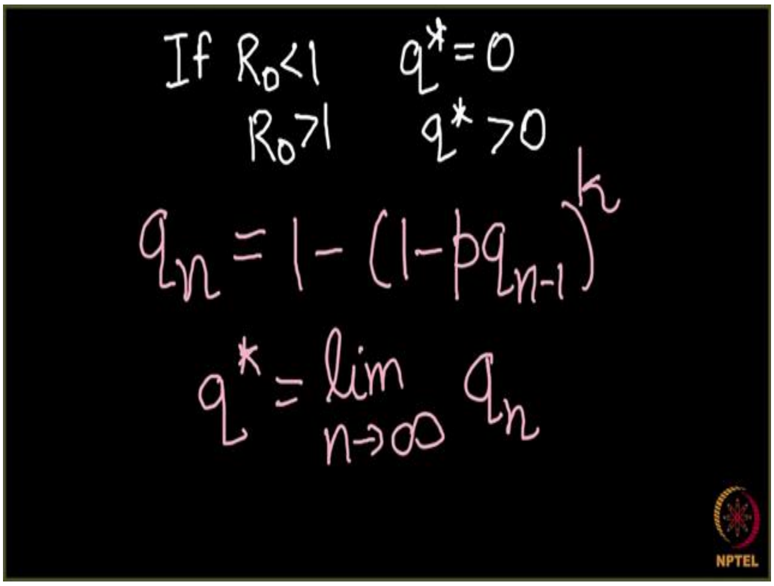

So, first what was our problem statement? Our problem statement was to prove that if R0 < 1 then q* = 0, if R0 > 1 then q* > 0, this was our problem statement.

Next in this lecture what we did? In this lecture we derived a formula for q_n and the formula is $1-(1-pq_{n-1})^k$. And we also know what is q^* from our first lecture: $q^* = \lim_{n\to\infty} q_n$. So, we will start from here in our next lecture.