Quiz - 2

Instructions

- The following questions **may have more than one correct answers**.
- There is no negative marking for wrong answers.
- Correct answers are worth one point. Partially correct answers are worth half a point.
- There is no negative marking.
- 'i' represents the imaginary number, $i^2 = -1$.

Questions

1. The tensor product of $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ is:

   a. $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

   b. $\begin{pmatrix} ab & bc \\ cd & ad \end{pmatrix}$

   c. $\begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

   d. $\begin{pmatrix} ac \\ bd \\ bc \\ ad \end{pmatrix}$

2. The norm of a two qubit state vector is ________.

   a. $2n$
   b. 2^{2n}
   c. 2
   d. 1

3. Given two matrices $A = \begin{pmatrix} a_0 & a_1 \\ a_2 & a_3 \end{pmatrix}$ and $B = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$, which of the following matrices are valid tensor products of A and B?

   a. $\begin{pmatrix} b_0a_0 & b_0a_1 \\ b_0a_2 & b_0a_3 \\ b_1a_0 & b_1a_1 \\ b_1a_2 & b_1a_3 \end{pmatrix}$

   b. $\begin{pmatrix} b_0a_0 & b_0a_1 \\ b_1a_0 & b_1a_1 \\ b_0a_2 & b_0a_3 \\ b_1a_2 & b_1a_3 \end{pmatrix}$

   c. $\begin{pmatrix} b_0a_0 & b_0a_1 \\ b_1a_2 & b_1a_3 \\ b_1a_0 & b_1a_1 \\ b_0a_2 & b_0a_3 \end{pmatrix}$

   d. $\begin{pmatrix} b_0a_0 & b_0a_1 \\ b_1a_2 & b_1a_3 \\ b_0a_2 & b_0a_3 \\ b_1a_0 & b_1a_1 \end{pmatrix}$

4. If $C = A \otimes B$, where A is of size $m \times n$ and B is of size $p \times q$. What is the size of C?

   a. $mn \times pq$
   b. $mp \times nq$
   c. $mq \times np$
   d. none of the above

5. Which of the following state vectors represent an entangled state? [Assume the appropriate normalization constants]

   a. $\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

   b. $\begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

c. $\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

d. $\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$

6. A two-qubit system is initially in the state, $|+\rangle \otimes |1\rangle$ and the gate $Z \otimes H$ is applied to these qubits. What is the matrix of the resultant state vector? [Assume the appropriate normalization constants.]

   a. $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$

   b. $\begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$

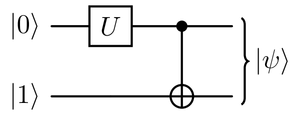
   c. $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$

   d. $\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$

7. A two-qubit system is in an unknown initial state. Applying the $H \otimes H$ gate, the state is seen to be entangled. What can be said of the initial state?

   a. The initial state is entangled.
   b. The initial state is NOT entangled.
   c. Nothing can be said of the initial state.
   d. Insufficient information has been provided.

8. Consider the following quantum circuit: [The two-qubit gate is the CNOT gate.]



The choice(s) of U for which the resultant $|\psi\rangle$ is entangled are

a. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

b. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

d. $\frac{1}{2}\begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$

9. Which of the following quantum state(s) is NOT entangled. [Ignore Normalization]

a. $|{+}{+}\rangle - \frac{1}{2}(|0\rangle\,|+\rangle)$

b. $|{+}{+}\rangle - \frac{1}{2}(|00\rangle + |11\rangle)$

c. $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

d. none of the above.

10. Consider the following questions:

Question - 1 Ignoring the global phase, how many states are orthonormal to a given single qubit state?

Question - 2 Ignoring the global phase, how many states are orthonormal to a given two-state?

Choose the correct set of answers:

a. 1 - One state only; 2 - One state only.

b. 1 - One state only; 2 - Infinitely many states.

c. 1 - Infinitely many states; 2 - Infinitely many states.

d. none of the above.