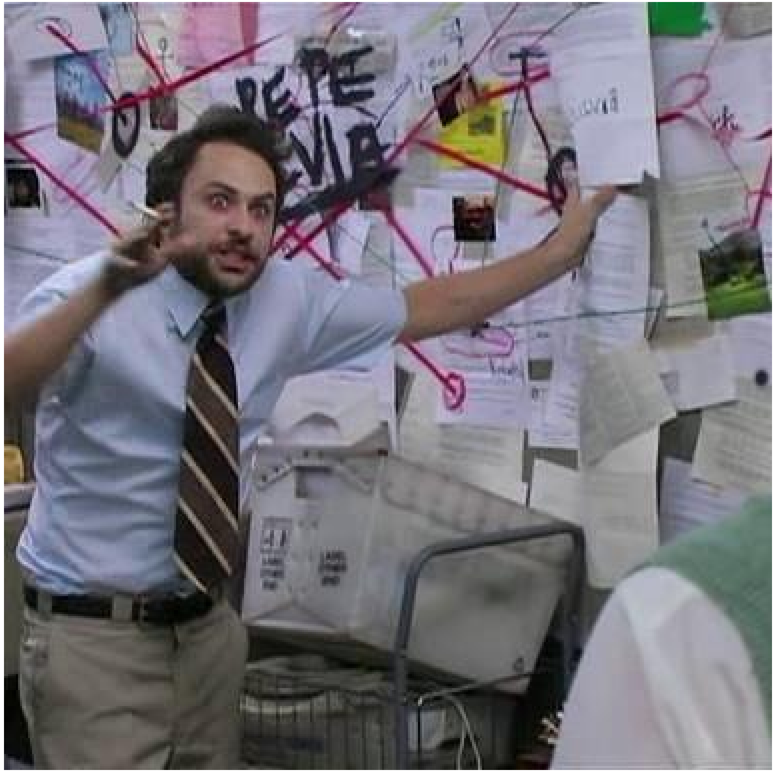
Traversing Trees the Hard Way

The Robson Traversal

By: Davis Silverman

How I felt learning how this algorithm works



Why not just do it the easy way?

- As Robson puts it, when using a stack, the size could be just as large as the tree!
    - AKA the space complexity for a basic Depth-first search is O(n).
- AKA, it may be memory inefficient
- The easy way is useful for understanding how traversals work, and for quick and dirty setups.
- When you need to traverse large trees, you need to break out the big guns.

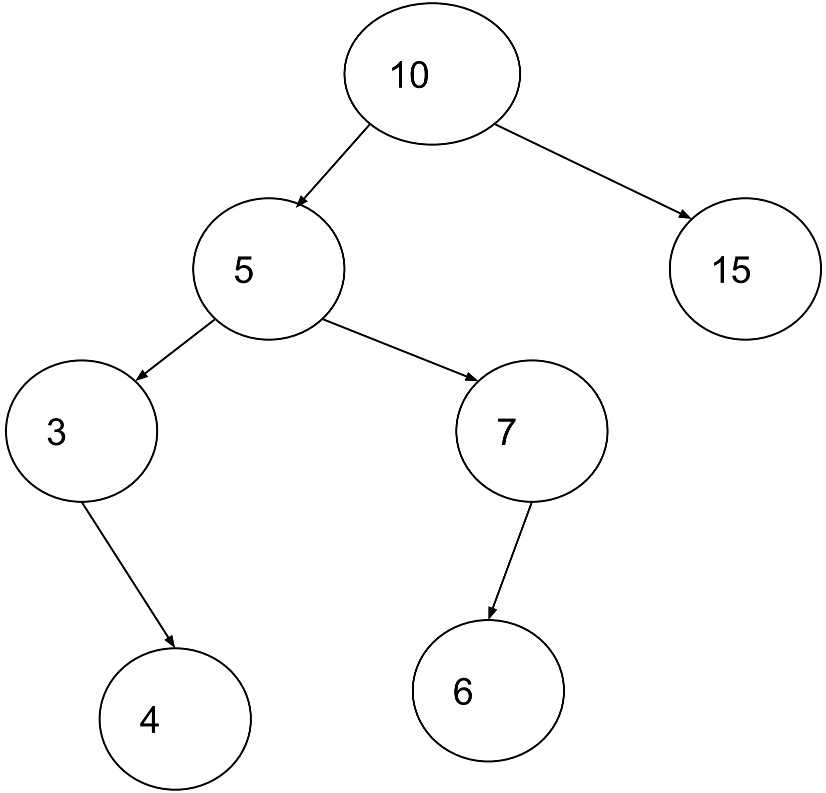
A Mixture of what we have learned

- Robson uses inverted links, similar to the link-inversion method
- It also uses the leaf-pointers, much like the threaded-tree model!
- However, instead of using an extra bit to detect if we should traverse right or up, we use the leaf pointers!
- If a leaf-pointer is pointing to the current node, and both subtrees are non-null, then we know the ascent was from the left, so go right.
- The leaf-pointers act as a pseudo stack!

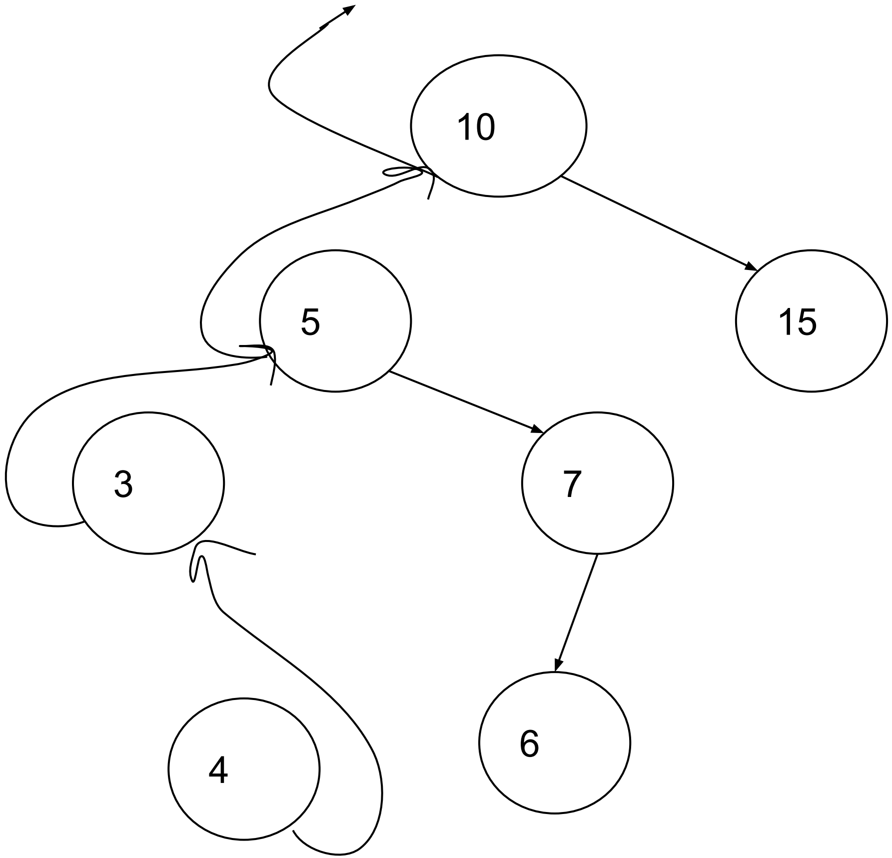
The key to Robson

- Careful understanding of what each stacks job is.
- The leaf stack tells you to traverse right or up
- The inner stack (of inverted links) are for traversal and discovery
- The leaf stack is used rarely

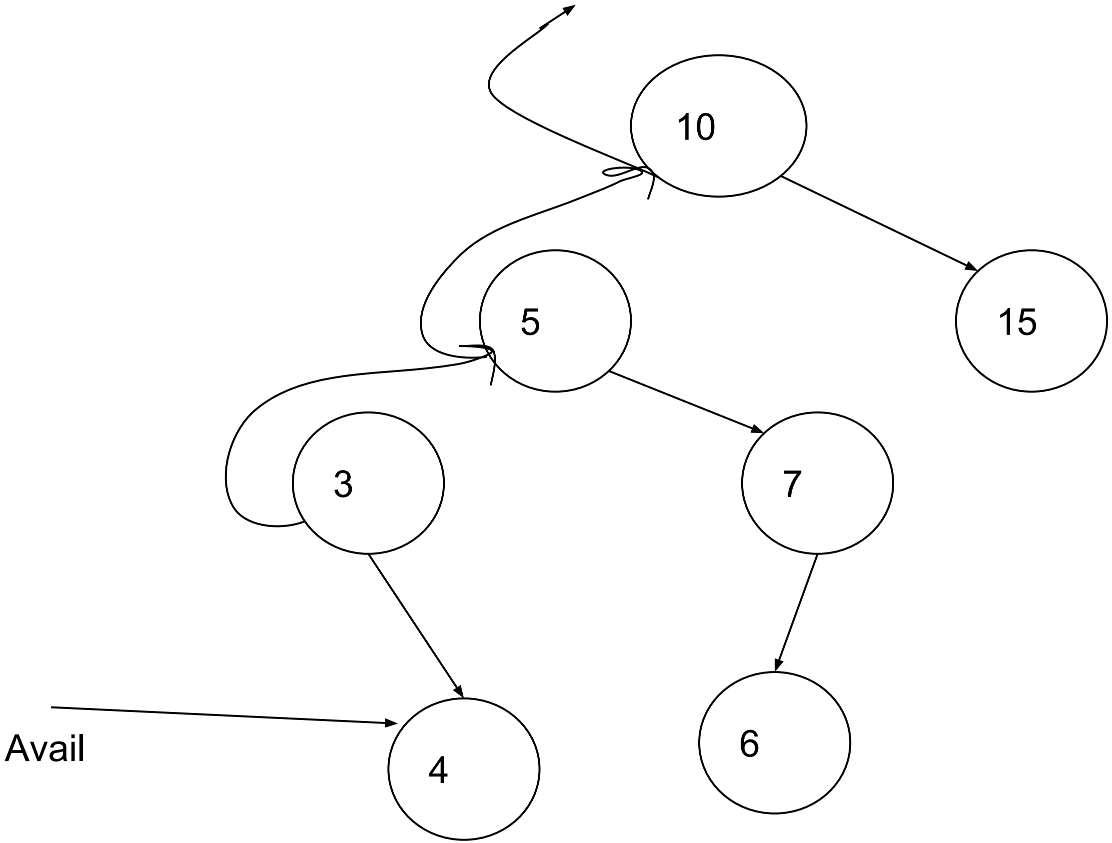
Lets run through this algorithm!



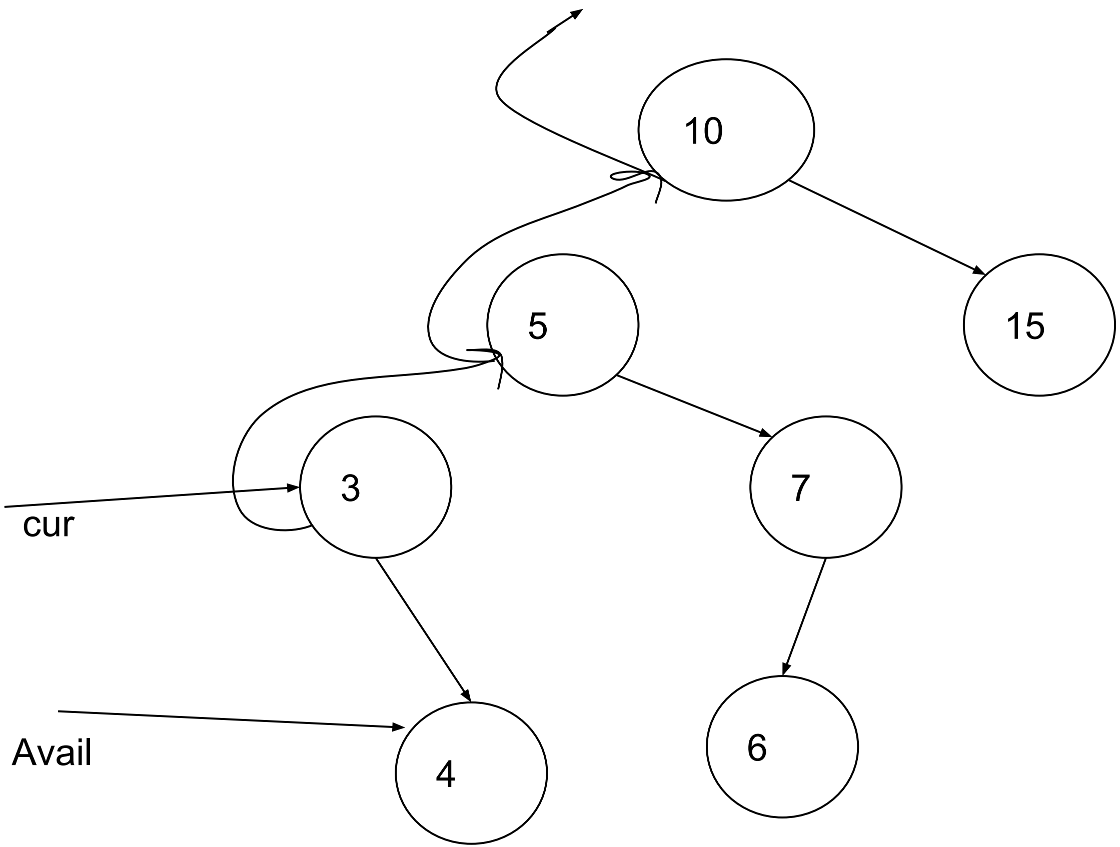
Step 1: Link Inversion!

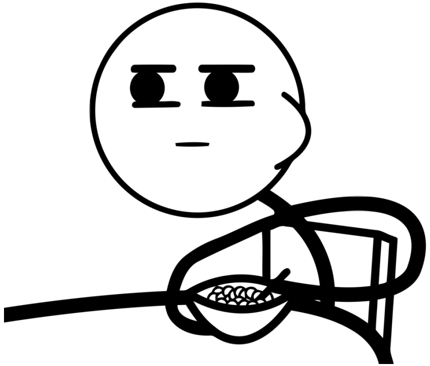


Step 2: Found Leaf, mark it available

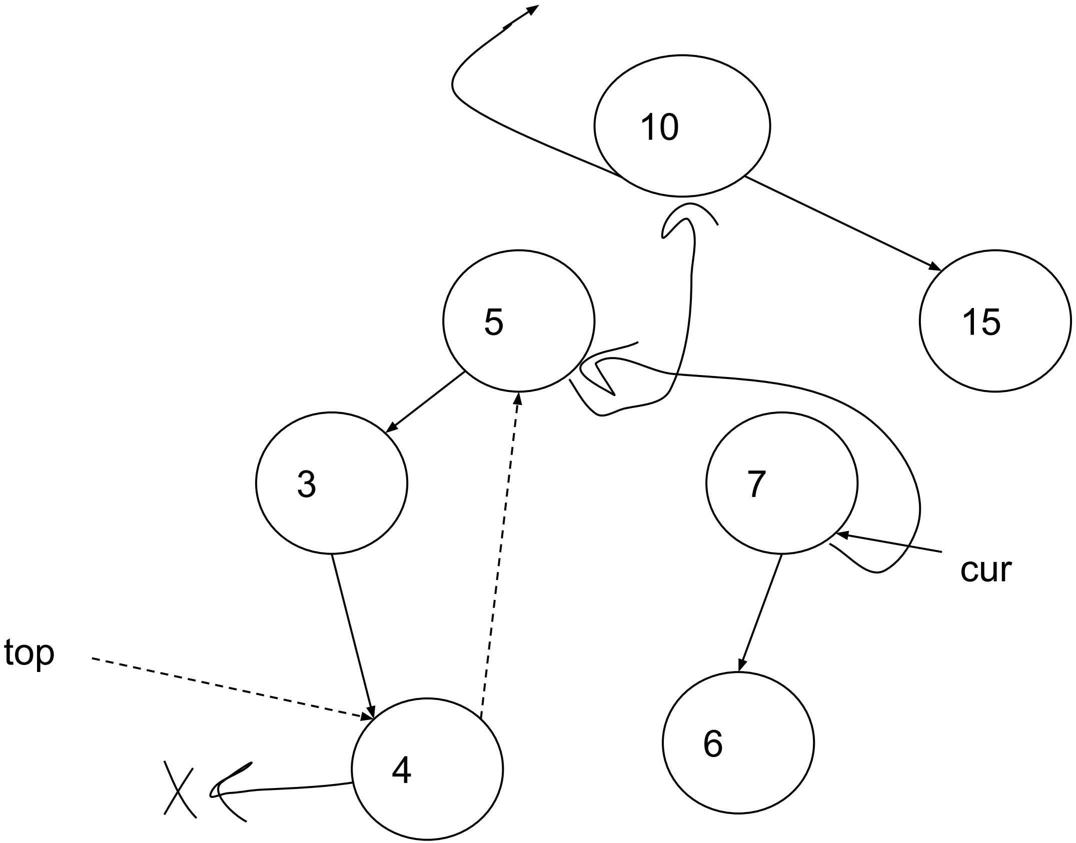


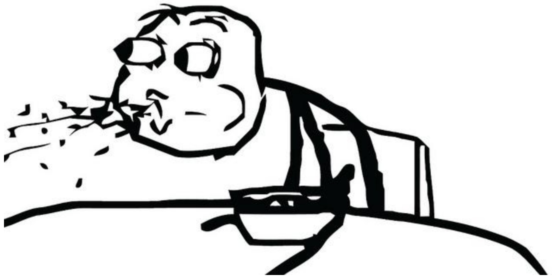
Step 3: Exchange Time!



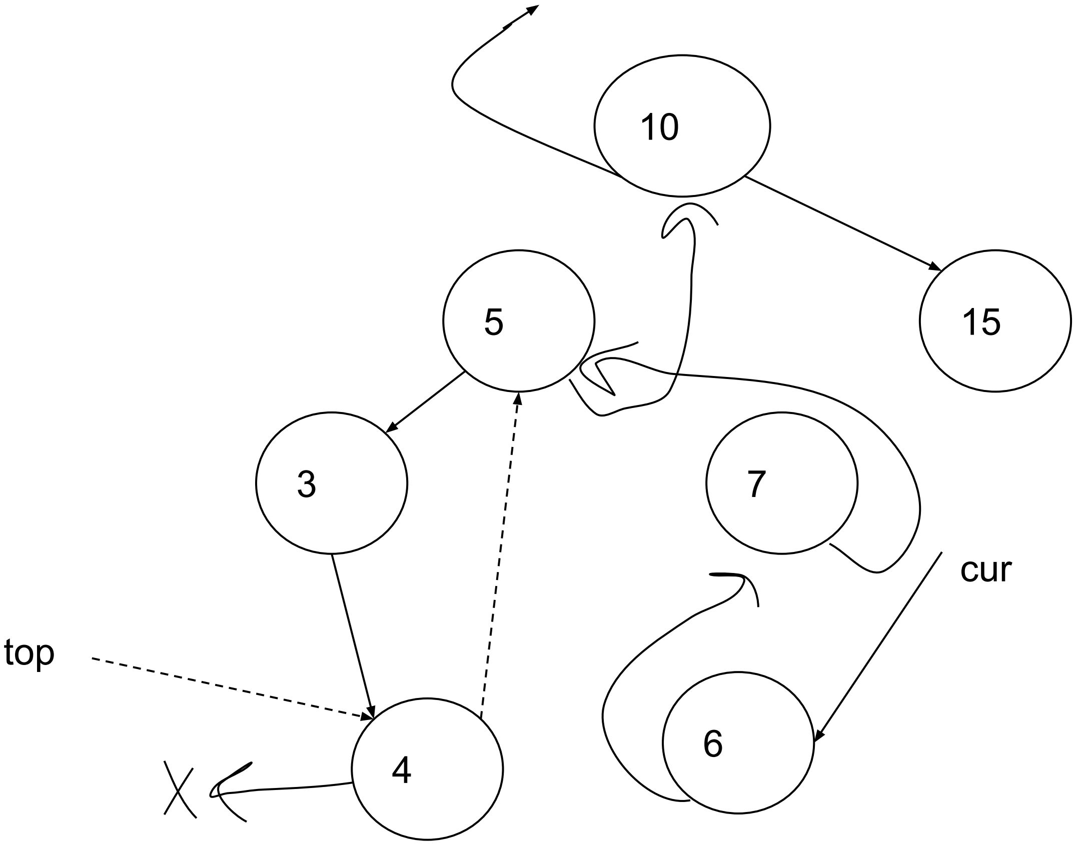


Step 3: Exchange Time!

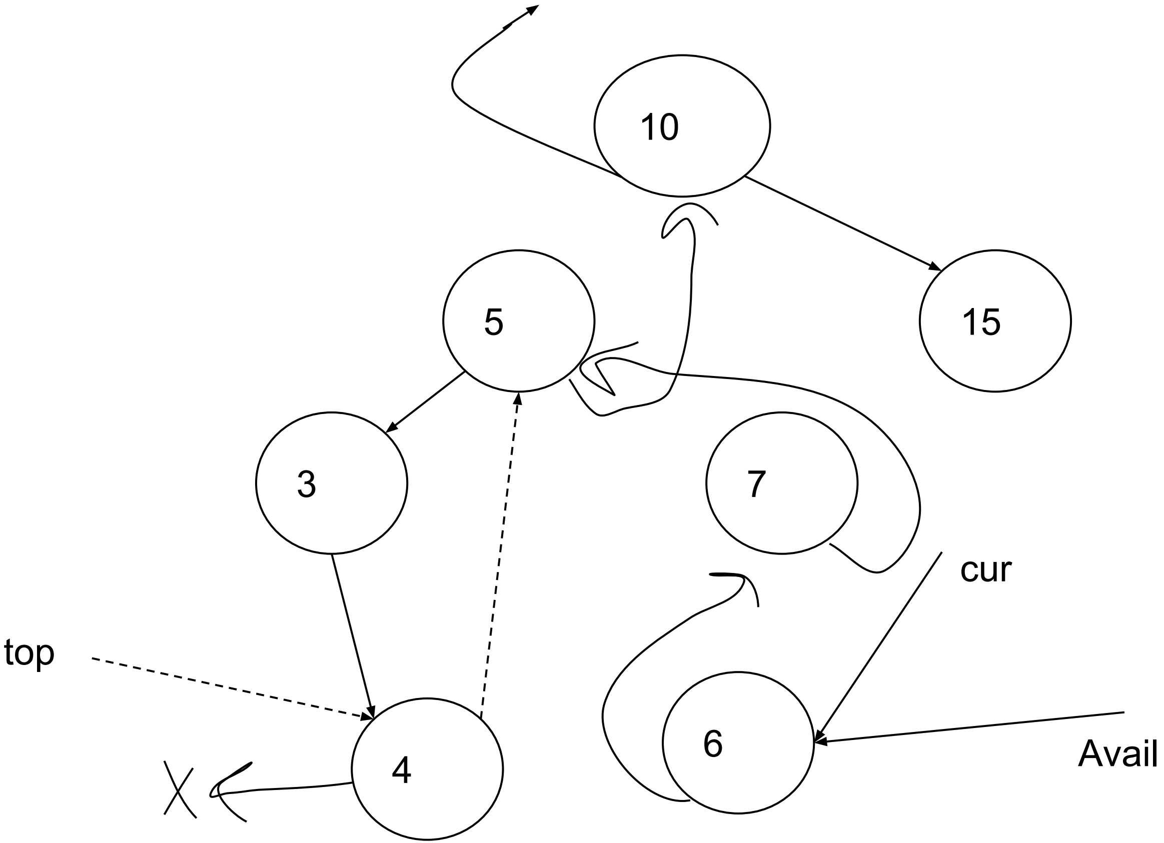




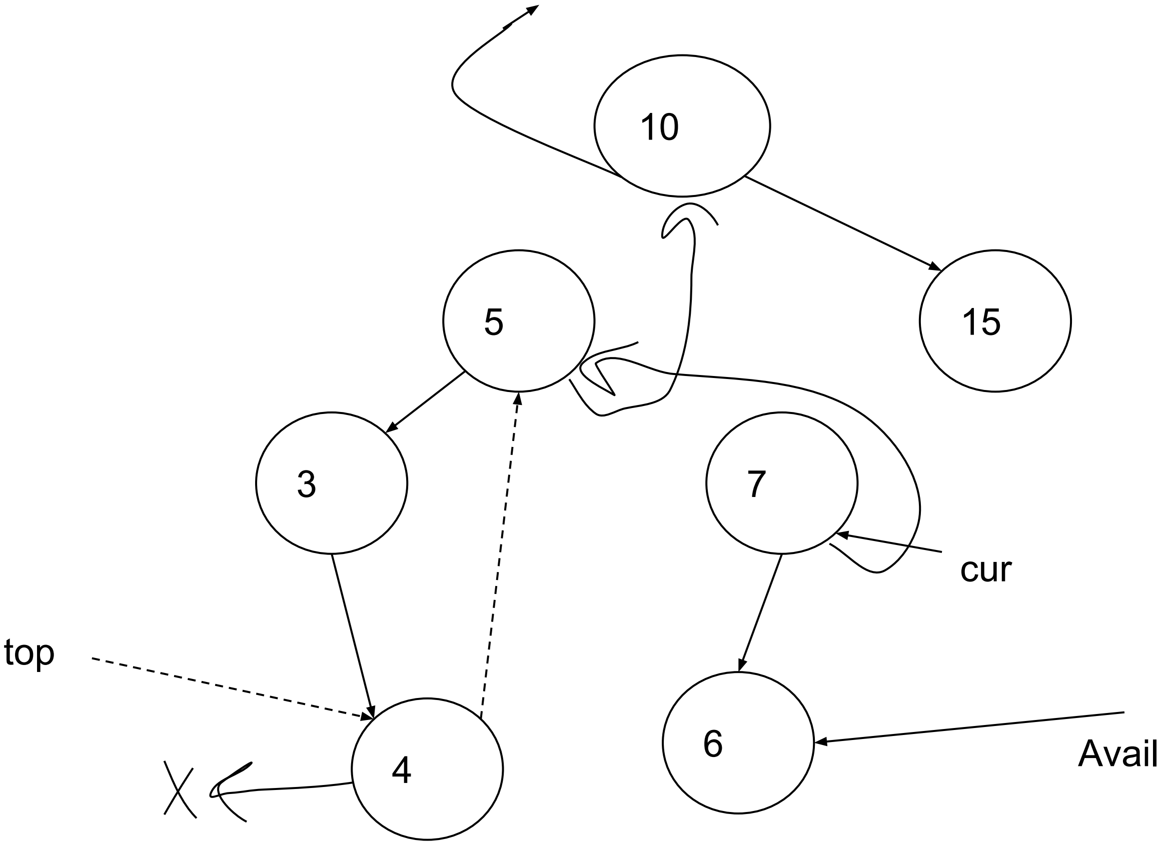
Step 4: Back to Link Inversion!



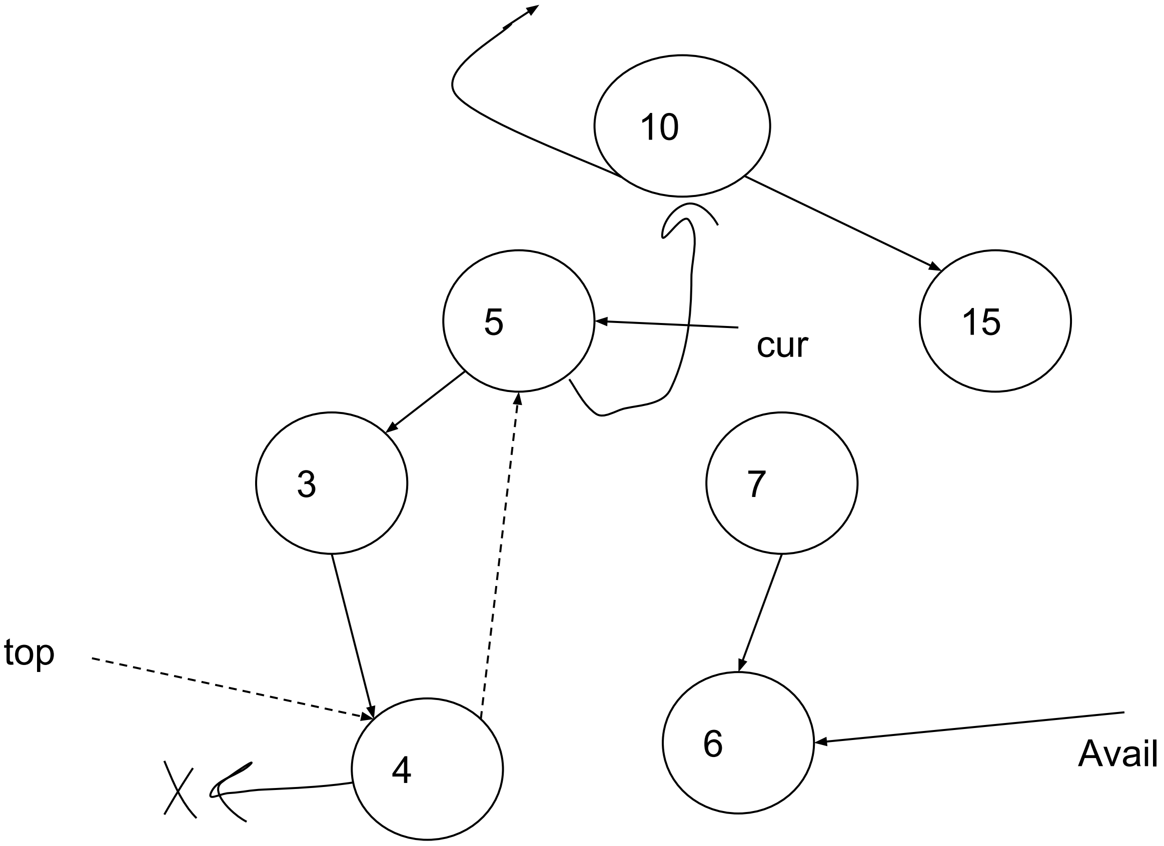
Step 5: Another leaf!



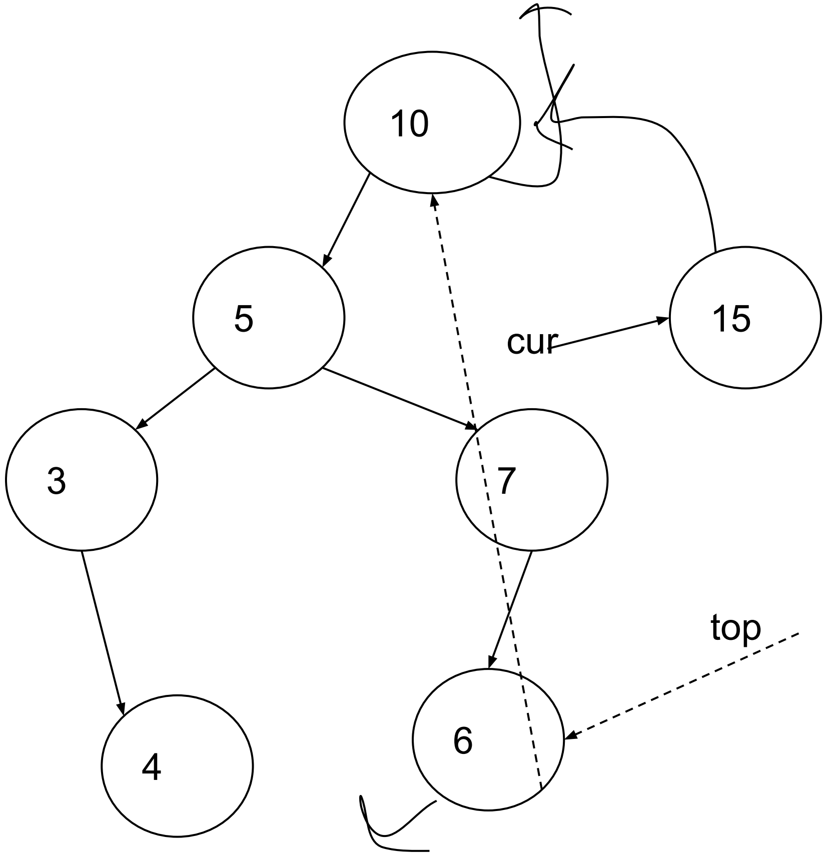
Step 6: Unwinding the inner stack again.



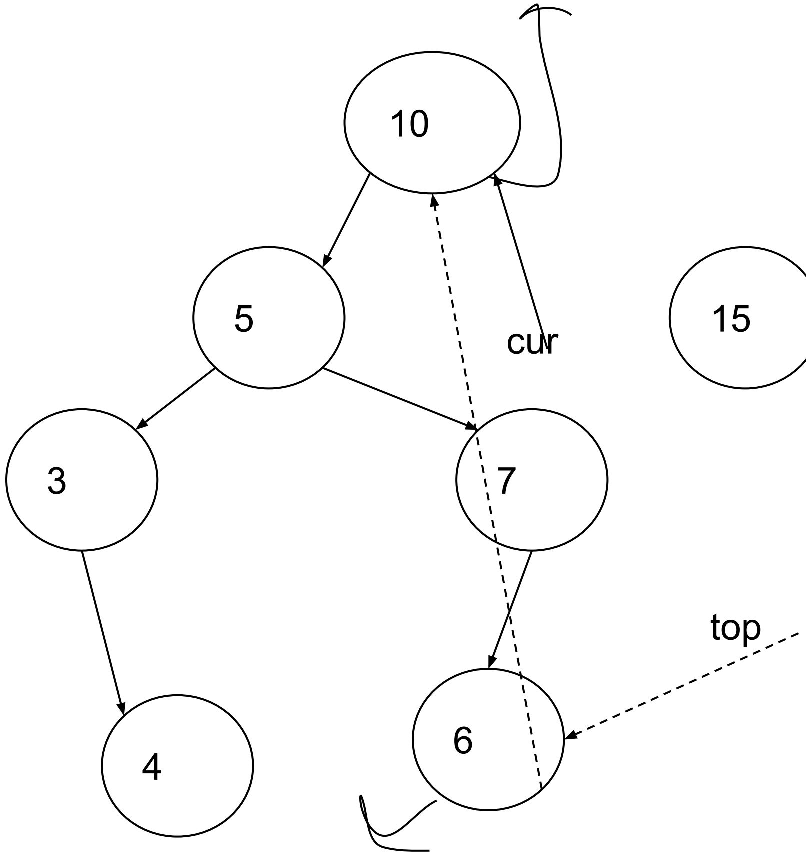
Step 7: Arrived at Leaf Stack Top!



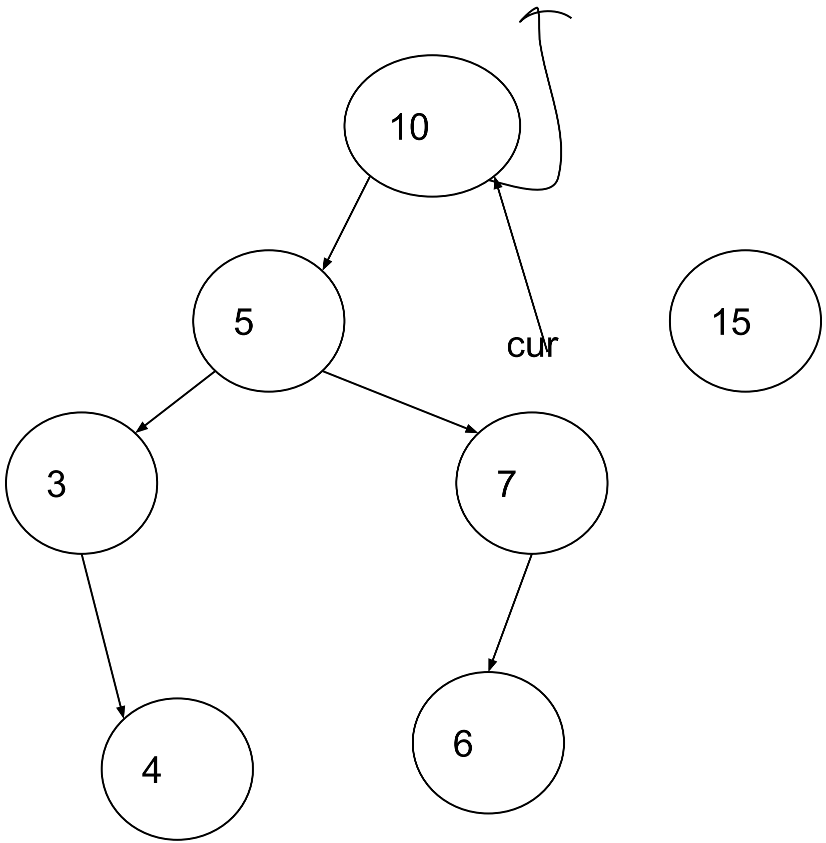
Step 8: Another Exchange



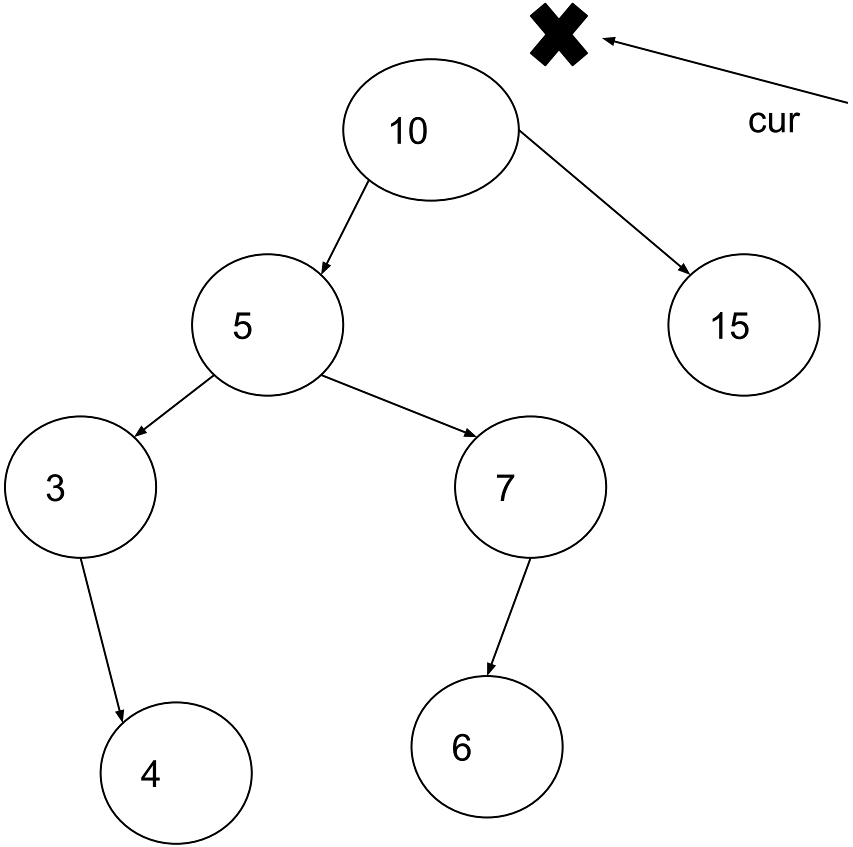
Step 9: Walking Up...



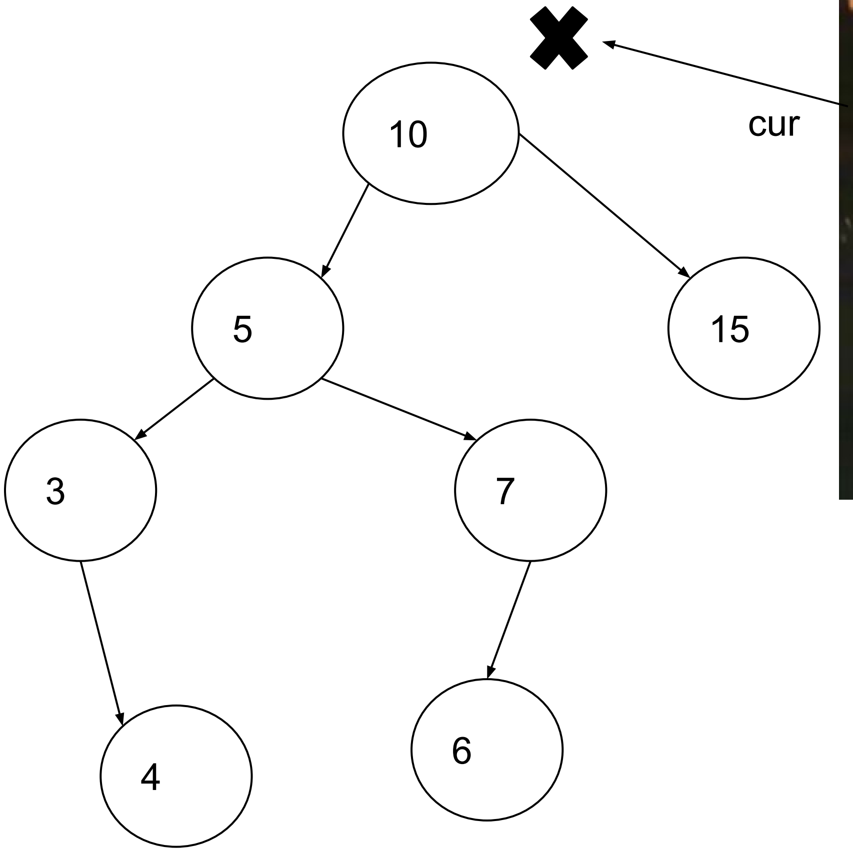
Step 10: Walking Up...



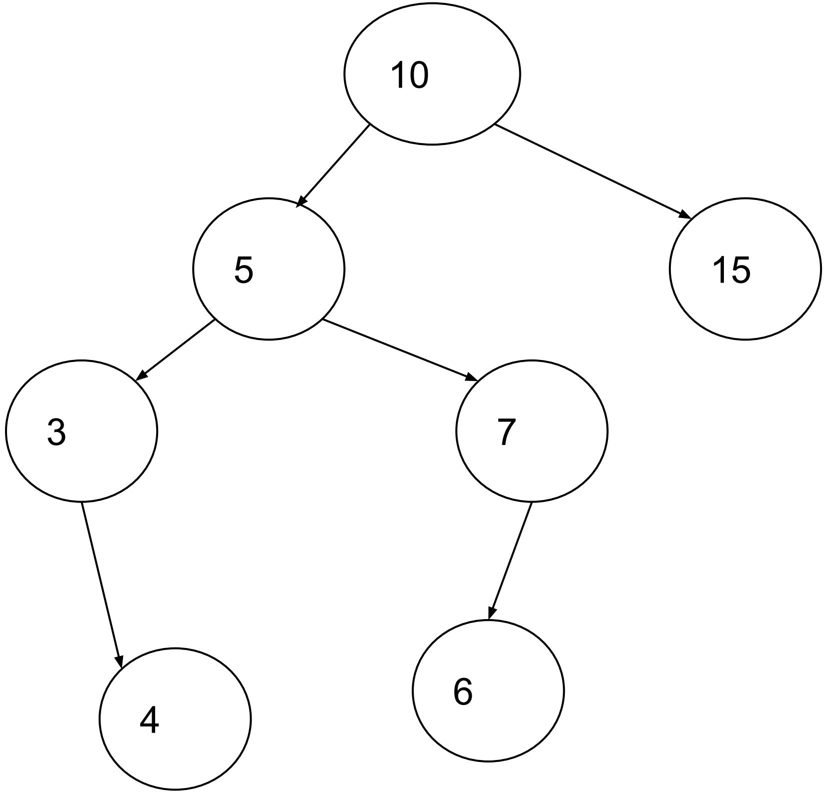
Step 10: Walking Up...



Step 11: cur == null → end



All clean again!



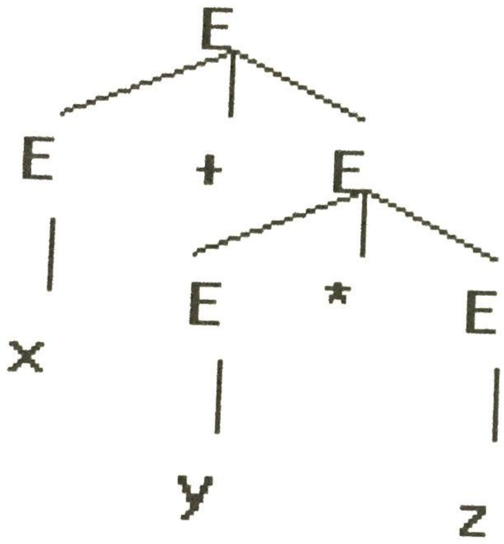
There you have it!

A traversed Binary Tree, in O(n) time, and O(1) space!

Great at parties!

When you can NOT use this traversal.

- When leaves point to things!
    - Expression trees! Where nodes are operators and leaves are variables or constants
    - Think 330 parse trees!



When you can NOT use this traversal.

- To find a single successor!
  - A threaded tree can find a successor in amortized O(1) time, but the Robson and link-inversion methods have no such ability.
  - Robson must complete a whole traversal, and can not be stopped midway through like a threaded tree.
  - Link-inversion and Robson traversals are ‘read-only’, and must be completed before being able to write to the tree again.

Sources

- J.M. Robson, An improved algorithm for traversing binary trees without auxiliary stack, Info. Process. Lett. 2 (1973) 12-14.
- D.E. Knuth, Fundamental Algorithms (Addison--Wesley, 1968). p. 562

Perlis and Thornton originally founded Threaded Trees, but I couldn't find a paper/source