1 Concrete Semantics of Scheme CESKt*

Syntax Domains:

$$e \in \mathsf{Exp} ::= æ$$
$$| \; (\texttt{if } e \; e \; e)$$
$$| \; (\texttt{let } ([x \; e] \; ...) \; e)$$
$$| \; (\texttt{call/cc } e)$$
$$| \; (\texttt{set! } x \; e)$$
$$| \; (\texttt{prim } op \; e \; ...)$$
$$| \; (\texttt{apply-prim } op \; e)$$
$$| \; (\texttt{apply } e \; e)$$
$$| \; (e \; e \; ...)$$
$$æ \in \mathsf{AExp} ::= x \mid lam \mid \mathbb{Z} \mid \texttt{\#t} \mid \texttt{\#f}$$
$$| \; (\texttt{quote } e)$$
$$lam \in \mathsf{Lam} ::= (\lambda \; (x...) \; e) \mid (\lambda \; x \; e)$$
$$x \in \mathsf{Var} \qquad \text{A set of identifiers}$$
$$op \in \mathsf{Prim} \qquad \text{A set of primitives}$$

Atomic Evaluation:

$$\mathcal{A} : \Sigma_E \times \sigma \rightharpoonup Val$$
$$\mathcal{A}(\langle lam, \rho, \_, \_ \rangle, \_) \triangleq (lam, \rho)$$
$$\mathcal{A}(\langle (\texttt{quote } e), \_, \_, \_ \rangle, \_) \triangleq \mathbf{quote}(e)$$
$$\mathcal{A}(\langle x, \rho, \_, \_ \rangle, \sigma) \triangleq \sigma(\rho(x))$$
$$\mathcal{A}(\langle æ, \_, \_, \_ \rangle, \_) \triangleq æ$$

Tick/Alloc:

$$tick : Eval \times \mathbb{N} \to Time$$
$$tick(E\langle \_, \_, \_, t \rangle, n) \triangleq (t + n)$$
$$alloc : \Sigma \times \mathbb{N} \triangleq Addr$$
$$alloc(E\langle \_, \_, \_, t \rangle, n) \triangleq (t + n)$$

Injection:

$$\mathcal{I} : \mathsf{Exp} \to \Sigma$$
$$\mathcal{I}(e) \triangleq (e, \varnothing, \varnothing, \{0 : \mathbf{mt}\}, 0, 1)$$

Transition:
Collecting Semantics:

Semantic Domains:

$$\varsigma \in \Sigma \triangleq E\langle Eval \rangle + A\langle Apply \rangle$$
$$Eval \triangleq \mathsf{Exp} \times Env \times Store$$
$$\times KStore \times Addr \times Time$$
$$Apply \triangleq Val \times Store$$
$$\times KStore \times Addr \times Time$$
$$\rho \in Env \triangleq \mathsf{Var} \rightharpoonup Addr$$
$$\sigma \in Store \triangleq Addr \rightharpoonup Val$$
$$\sigma \in Store \triangleq Addr \rightharpoonup Kont$$
$$v \in Val \triangleq Clo + Kont + \mathbb{Z}$$
$$+ \{\texttt{\#t}, \texttt{\#f}, Null, Void\}$$
$$+ \{\mathbf{quote}(e), \mathbf{cons}(v, v)\}$$
$$clo \in Clo \triangleq \mathsf{Lam} \times Env$$
$$a, a_\kappa \in Addr \triangleq \mathbb{N}$$
$$t \in Time \triangleq \mathbb{N}$$
$$\kappa \in Kont ::= \mathbf{mt}$$
$$| \; \mathbf{ifk}(e, e, \rho, a)$$
$$| \; \mathbf{callcck}(a)$$
$$| \; \mathbf{setk}(x, \rho, a)$$
$$| \; \mathbf{appappk}(val?, e, \rho, a)$$
$$| \; \mathbf{appk}(done, todo, \rho, a)$$
$$| \; \mathbf{appprimk}(op, \rho, a)$$
$$| \; \mathbf{primk}(op, done, todo,$$
$$\rho, a)$$
$$| \; \mathbf{letk}(vars, done, todo,$$
$$e, \rho, a)$$
$$done \triangleq Val^*$$
$$todo \triangleq \mathsf{Exp}^*$$
$$vars \triangleq \mathsf{Var}^*$$

Eval Rules
Rules for when the control is an expression

$$E\langle \text{æ}, \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\text{where } v \triangleq \mathcal{A}(\varsigma)$$

$$E\langle (\texttt{if } e_c\ e_t\ e_f), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e_c, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{ifk}(e_t, e_f, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\texttt{let }()\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$

$$E\langle (\texttt{let } bs\ e_b), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } bs = ([x_0\ e_0]\ [x_s\ e_s]\ ...)$$
$$a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\texttt{call/cc } e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{callcck}(a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\texttt{set! } x\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$a \triangleq \rho(x)$$
$$\kappa \triangleq \textbf{setk}(a, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\texttt{prim } op), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\text{where } v = op \text{ applied to 0 arguments}$$

$$E\langle (\texttt{prim } op\ e_0\ e_s\ ...), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{primk}(op, [\,], e_s, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\texttt{apply-prim } op\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{appprimk}(op, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\texttt{apply } e_f\ e_x), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{appappk}(\varnothing, e_x, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (e_f\ e_s\ ...), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\rightsquigarrow E\langle e_f, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \textbf{appk}([\,], e_s, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

Apply Rules

Rules for when the control is a value

$$\varsigma = A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\kappa \triangleq \sigma_\kappa(a_\kappa)$$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \varsigma$$

$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_f, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$
$$\text{where } v = \texttt{\#f}$$

$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_t, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$
$$\text{where } v \neq \texttt{\#f}$$

$\mathbf{letk}(vars, done, [\,], e_b, \rho_\kappa, a'_\kappa)$
$\rightsquigarrow E\langle e_b, \rho'_\kappa, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$
$\text{where } a_i \triangleq alloc(\varsigma, i)$

$$t' \triangleq tick(\varsigma, n)$$
$$done' \triangleq done \mathbin{+\!\!+} [v]$$
$$\rho'_\kappa \triangleq \rho_\kappa[vars_0 \mapsto a_0 \ ...$$
$$vars_{n-1} \mapsto a_{n-1}]$$
$$\sigma' \triangleq \sigma[a_0 \mapsto done'_0 \ ...$$
$$a_{n-1} \mapsto done'_{n-1}]$$

$\mathbf{letk}(vars, done, e_h :: e_t, e_b, \rho_\kappa, a'_\kappa)$
$\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$
$\text{where } a''_\kappa \triangleq alloc(\varsigma, 0)$

$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa' \triangleq \mathbf{letk}(vars, done \mathbin{+\!\!+} [v],$$
$$e_t, e_b, \rho_\kappa, a'_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$$

$\mathbf{setk}(a, a'_\kappa) \rightsquigarrow A\langle \mathit{Void}, \sigma', \sigma_\kappa, a'_\kappa, t\rangle$
$$\text{where } \sigma' \triangleq \sigma[a \mapsto v]$$

$\mathbf{callcck}(a'_\kappa) \rightsquigarrow E\langle e, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t\rangle$
$$\text{where } v = ((\lambda\ (x)\ e), \rho_\lambda)$$

$$a \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa' \triangleq \sigma_\kappa(a'_\kappa)$$
$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]$$
$$\sigma' \triangleq \sigma[a \mapsto \kappa']$$

$\mathbf{callcck}(\_) \rightsquigarrow A\langle \kappa, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$
$$\text{where } v = \kappa'$$

$$a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa']$$

$\mathbf{appprimk}(op, \rho_\kappa, a'_\kappa) \rightsquigarrow A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t\rangle\rangle$
$$\text{where } v' \triangleq op \text{ applied to } v$$

$\mathbf{primk}(op, done, [\,], \rho_\kappa, a'_\kappa)$
$\rightsquigarrow A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t\rangle$
$\text{where } v' \triangleq op \text{ applied to } (done \mathbin{+\!\!+} [v])$

$\mathbf{primk}(op, done, e_h :: e_t, \rho_\kappa, a'_\kappa)$
$\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$
$\text{where } a''_\kappa \triangleq alloc(\varsigma, 0)$

$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa' \triangleq \mathbf{primk}(op, done \mathbin{+\!\!+} [v],$$
$$e_t, \rho_\kappa, a'_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$$

More Apply Rules
Rules for when the control is a value

$$\mathbf{appappk}(v_f, \text{-}, \text{-}, a'_\kappa)$$
$$\leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$
where $v_f = ((\lambda \ (x_s...) \ e_b), \rho_\lambda)$
$$a_i \triangleq alloc(\varsigma, i)$$
$$t' \triangleq tick(\varsigma, n)$$
$$\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto a_0 \ ...$$
$$x_{n-1} \mapsto a_{n-1}]$$
$$\sigma' \triangleq \sigma[a_0 \mapsto v_0 \ ...$$
$$a_{n-1} \mapsto v_{n-1}]$$

$$\mathbf{appappk}(v_f, \text{-}, \text{-}, a'_\kappa)$$
$$\leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$
where $v_f = ((\lambda \ x \ e_b), \rho_\lambda)$
$$a \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]$$
$$\sigma' \triangleq \sigma[a \mapsto v]$$

$$\mathbf{appappk}(\varnothing, e, \rho_\kappa, a'_\kappa)$$
$$\leadsto E\langle e, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where $a''_\kappa \triangleq alloc(\varsigma, 0)$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa' \triangleq \mathbf{appappk}(v, e, \rho_\kappa, a'_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$$

$$\mathbf{appk}(done, [\,], \text{-}, a'_\kappa)$$
$$\leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$
where $done = ((\lambda \ (x_s...) \ e_b), \rho_\lambda) :: v_s$
$$a_i \triangleq alloc(\varsigma, i)$$
$$t' \triangleq tick(\varsigma, n)$$
$$v'_s \triangleq v_s \mathbin{+\!\!+} [v]$$
$$\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto a_0 \ ...$$
$$x_{n-1} \mapsto a_{n-1}]$$
$$\sigma' \triangleq \sigma[a_0 \mapsto v'_0 \ ...$$
$$a_{n-1} \mapsto v'_{n-1}]$$

$$\mathbf{appk}(done, [\,], \text{-}, a'_\kappa)$$
$$\leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$
where $done = ((\lambda \ x \ e_b), \rho_\lambda) :: v_s$
$$a \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$v'_s \triangleq (v_s \mathbin{+\!\!+} [v])$$
$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]$$
$$\sigma' \triangleq \sigma[a \mapsto v'_s]$$

$$\mathbf{appk}([\kappa_\lambda], [\,], \rho_\kappa, a'_\kappa) \leadsto A\langle v, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where $a''_\kappa \triangleq alloc(\varsigma, 0)$
$$t' \triangleq tick(\varsigma, 1)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa_\lambda]$$

$$\mathbf{appk}(done, e_h :: e_t, \rho_\kappa, a'_\kappa)$$
$$\leadsto E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where $a''_\kappa \triangleq alloc(\varsigma, 0)$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa' \triangleq \mathbf{appk}(done \mathbin{+\!\!+} [v], e_t, \rho_\kappa, a'_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$$

2 Abstract Semantics of Scheme CESKt*

Abstract Semantic Domains:

$$\hat{\varsigma} \in \hat{\Sigma} \triangleq E\langle Eval\rangle + E\langle Apply\rangle$$

$$Eval \triangleq \mathsf{Exp} \times \widehat{Env} \times \widehat{Store}$$
$$\times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time}$$

$$Apply \triangleq \widehat{Val} \times \widehat{Store}$$
$$\times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time}$$

$$\hat{\rho} \in \widehat{Env} \triangleq \mathsf{Var} \rightharpoonup \widehat{Addr}$$

$$\hat{\sigma} \in \widehat{Store} \triangleq \widehat{Addr} \rightharpoonup \mathcal{P}(Val)$$

$$\hat{\sigma}_\kappa \in \widehat{KStore} \triangleq \widehat{Addr} \rightharpoonup \mathcal{P}(\widehat{Kont})$$

$$\hat{v} \in \widehat{Val} \triangleq \widehat{Clo} + \widehat{Kont} + \mathbb{Z}$$
$$+ \{\mathtt{\#t}, \mathtt{\#f}, Null, Void\}$$
$$+ \{\mathbf{quote}(e), \mathbf{cons}(v, v)\}$$

$$\widehat{clo} \in \widehat{Clo} \triangleq \mathsf{Lam} \times \widehat{Env}$$

$$\hat{a}, \hat{a}_{\hat{\kappa}} \in Addr \triangleq \mathbb{N}$$

$$\hat{t} \in Time \triangleq \mathbb{N}$$

$$\widehat{done} \triangleq \widehat{Val}^*$$

$$\hat{\kappa} \in \widehat{Kont} ::= \mathbf{mt}$$
$$| \;\mathbf{ifk}(e, e, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{callcck}(\hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{appappk}(\widehat{val?}, e, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{appk}(\widehat{done}, todo, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{appprimk}(op, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{primk}(op, \widehat{done}, todo,$$
$$\hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$| \;\mathbf{letk}(vars, \widehat{done}, todo,$$
$$e, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$

Tick/Alloc:

$$\widehat{tick} : \hat{\Sigma} \times \widehat{Kont} \to \widehat{Time}$$

$$\widehat{tick}(\hat{\varsigma}, \hat{\kappa}) \triangleq 0$$

$$\widehat{alloc} : \hat{\Sigma} \times \widehat{Kont} \to \widehat{Addr}$$

$$\widehat{alloc}(\hat{\varsigma}, \hat{\kappa}) \triangleq 0$$

Abstract Atomic Evaluation:

$$\mathcal{A} : Eval \to \widehat{Val}$$

$$\mathcal{A}(E\langle lam, \hat{\rho}, ...\rangle) \triangleq \{(lam, \hat{\rho})\}$$

$$\mathcal{A}(æ) \triangleq \{æ\}$$

Abstract Eval Rules
Rules for when the control is an expression

$$E\langle æ, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\text{where } \hat{v} \triangleq \hat{\mathcal{A}}(\hat{\varsigma}, \hat{\sigma})$$

$$E\langle (\texttt{if } e_c\ e_t\ e_f), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e_c, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{ifk}(e_t, e_f, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (\texttt{let } ()\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$

$$E\langle (\texttt{let } bnds\ e_b), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } bnds = ([x_0\ e_0]\ [x_s\ e_s]\ ...)$$
$$\hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (\texttt{call/cc } e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{callcck}(\hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (\texttt{set! } x\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{a} \triangleq \hat{\rho}(x)$$
$$\hat{\kappa} \triangleq \textbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (\texttt{prim } op), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\text{where } \hat{v} = op \text{ applied to 0 arguments}$$

$$E\langle (\texttt{prim } op\ e_0\ e_s\ ...), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{primk}(op, [\,], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (\texttt{apply-prim } op\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{appprimk}(op, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (\texttt{apply } e_f\ e_x), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{appappk}(\varnothing, e_x, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

$$E\langle (e_f\ e_s\ ...), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$
$$\rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa} \triangleq \textbf{appk}([\,], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$$

Apply Rules

Rules for when the control is a value

$$\hat{\varsigma} = A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$$

$$\hat{\kappa} \in \hat{\sigma}_{\hat{\kappa}}(\hat{a}_{\hat{\kappa}})$$

Proceed by matching on $\hat{\kappa}$

$$\mathbf{mt} \rightsquigarrow \hat{\varsigma}$$

$$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_f, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$$

$$\text{where } \hat{v} = \texttt{\#f}$$

$$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_t, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$$

$$\text{where } \hat{v} \neq \texttt{\#f}$$

$$\mathbf{letk}(vars, \widehat{done}, [\,], e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\rightsquigarrow E\langle e_b, \hat{\rho}'_{\hat{\kappa}}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$
$$\hat{\rho}'_{\hat{\kappa}} \triangleq \hat{\rho}_{\hat{\kappa}}[vars_0 \mapsto \hat{a}_0 \ldots$$
$$vars_{n-1} \mapsto \hat{a}_{n-1}]$$
$$\widehat{done}' \triangleq \widehat{done} +\!\!+ [\hat{v}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \widehat{done}'_0 \ldots$$
$$\hat{a}_{n-1} \mapsto \widehat{done}'_{n-1}]$$

$$\mathbf{letk}(vars, \widehat{done}, e_h :: e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \mathbf{letk}(vars, \widehat{done}, +\!\!+[\hat{v}],$$
$$e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

$$\mathbf{setk}(x, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle Void, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$$
$$\text{where } \hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{\rho}_{\hat{\kappa}}(x) \mapsto \hat{v}]$$

$$\mathbf{callcck}(\hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$$
$$\text{where } \hat{v} = ((\lambda\ (x)\ e), \hat{\rho}_\lambda)$$
$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \hat{\sigma}_{\hat{\kappa}}(\hat{a}'_{\hat{\kappa}})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{\kappa}']$$

$$\mathbf{callcck}(\_) \rightsquigarrow A\langle \hat{\kappa}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } v = \kappa'$$
$$\hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \kappa']$$

$$\mathbf{appprimk}(op, \_, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$$
$$\text{where } \hat{v}' \triangleq op \text{ applied to } \hat{v}$$

$$\mathbf{primk}(op, \widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}})$$
$$\rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$$
$$\text{where } \hat{v}' \triangleq op \text{ applied to } (\widehat{done} +\!\!+ [\hat{v}])$$

$$\mathbf{primk}(op, \widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \mathbf{primk}(op, \widehat{done} +\!\!+ [\hat{v}])$$
$$e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

More Apply Rules
Rules for when the control is a value

$$\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}})$$
$$\leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{v}_f = ((\lambda\ (x_s...)\ e_b), \hat{\rho}_\lambda)$$
$$\hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0 ...$$
$$x_{n-1} \mapsto \hat{a}_{n-1}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}_0 ...$$
$$\hat{a}_{n-1} \mapsto \hat{v}_{n-1}]$$

$$\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}})$$
$$\leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{v}_f = ((\lambda\ x\ e_b), \hat{\rho}_\lambda)$$
$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{v}]$$

$$\textbf{appappk}(\varnothing, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\leadsto E\langle e, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \textbf{appappk}(\hat{v}, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

$$\textbf{appk}(\widehat{done}, [\ ], \_, \hat{a}'_{\hat{\kappa}})$$
$$\leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \widehat{done} = ((\lambda\ (x_s...)\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s$$
$$\hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0 ...$$
$$x_{n-1} \mapsto \hat{a}_{n-1}]$$
$$\hat{v}'_s \triangleq \hat{v}_s + [\hat{v}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}'_0 ...$$
$$\hat{a}_{n-1} \mapsto \hat{v}'_{n-1}]$$

$$\textbf{appk}(\widehat{done}, [\ ], \_, \hat{a}'_{\hat{\kappa}})$$
$$\leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \widehat{done} = ((\lambda\ x\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s$$
$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]$$
$$\hat{v}'_s \triangleq \hat{v}_s + [\hat{v}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{v}'_s]$$

$$\textbf{appk}([\hat{\kappa}_\lambda], [\ ], \hat{\rho}_{\hat{\kappa}}, \_) \leadsto A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}_\lambda]$$

$$\textbf{appk}(\widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\leadsto E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \textbf{appk}(\widehat{done} + [\hat{v}], e_t, \hat{\rho}_\kappa, \hat{a}'_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$