1 Concrete Semantics of Scheme CESKt*

Syntax Domains:

$$
\begin{aligned}
e \in \mathsf{Exp} ::=\ & æ \\
& \mid (\texttt{if}\ e\ e\ e) \\
& \mid (\texttt{let}\ ([x\ e]\ ...)\ e) \\
& \mid (\texttt{call/cc}\ e) \\
& \mid (\texttt{set!}\ x\ e) \\
& \mid (\texttt{prim}\ op\ e\ ...) \\
& \mid (\texttt{apply-prim}\ op\ e) \\
& \mid (\texttt{apply}\ e\ e) \\
& \mid (e\ e\ ...) \\
æ \in \mathsf{AExp} ::=\ & x \mid lam \mid \mathbb{Z} \mid \texttt{\#t} \mid \texttt{\#f} \\
& \mid (\texttt{quote}\ e) \\
lam \in \mathsf{Lam} ::=\ & (\lambda\ (x...)\ e) \mid (\lambda\ x\ e) \\
x \in \mathsf{Var}\ & \quad \text{A set of identifiers} \\
op \in \mathsf{Prim}\ & \quad \text{A set of primitives}
\end{aligned}
$$

Atomic Evaluation:

$$
\begin{aligned}
\mathcal{A} : \Sigma_E \times \sigma \rightharpoonup Val \\
\mathcal{A}(\langle lam, \rho, \_, \_\rangle, \_) &\triangleq (lam, \rho) \\
\mathcal{A}(\langle (\texttt{quote}\ e), \_, \_, \_\rangle, \_) &\triangleq \mathbf{quote}(e) \\
\mathcal{A}(\langle x, \rho, \_, \_\rangle, \sigma) &\triangleq \sigma(\rho(x)) \\
\mathcal{A}(\langle æ, \_, \_, \_\rangle, \_) &\triangleq æ
\end{aligned}
$$

Tick/Alloc:

$$
\begin{aligned}
tick : Eval \times \mathbb{N} \rightarrow Time \\
tick(E\langle \_, \_, \_, t\rangle, n) &\triangleq (t + n) \\
alloc : \Sigma \times \mathbb{N} &\triangleq Addr \\
alloc(E\langle \_, \_, \_, t\rangle, n) &\triangleq (t + n)
\end{aligned}
$$

Injection:

$$
\begin{aligned}
\mathcal{I} : \mathsf{Exp} \rightarrow \Sigma \\
\mathcal{I}(e) \triangleq (e, \varnothing, \varnothing, \{0 : \mathbf{mt}\}, 0, 1)
\end{aligned}
$$

Transition:
Collecting Semantics:

Semantic Domains:

$$
\begin{aligned}
\varsigma \in \Sigma &\triangleq E\langle Eval\rangle + A\langle Apply\rangle \\
Eval &\triangleq \mathsf{Exp} \times Env \times Store \\
& \quad \times KStore \times Addr \times Time \\
Apply &\triangleq Val \times Store \\
& \quad \times KStore \times Addr \times Time \\
\rho \in Env &\triangleq \mathsf{Var} \rightharpoonup Addr \\
\sigma \in Store &\triangleq Addr \rightharpoonup Val \\
\sigma \in Store &\triangleq Addr \rightharpoonup Kont \\
v \in Val &\triangleq Clo + Kont + \mathbb{Z} \\
& \quad + \{\texttt{\#t}, \texttt{\#f}, Null, Void\} \\
& \quad + \{\mathbf{quote}(e), \mathbf{cons}(v, v)\} \\
clo \in Clo &\triangleq \mathsf{Lam} \times Env \\
a, b, c \in Addr &\triangleq \mathbb{N} \\
t, u \in Time &\triangleq \mathbb{N} \\
\kappa \in Kont &::= \mathbf{mt} \\
& \quad \mid \mathbf{ifk}(e, e, \rho, a) \\
& \quad \mid \mathbf{callcck}(a) \\
& \quad \mid \mathbf{setk}(x, \rho, a) \\
& \quad \mid \mathbf{appappk}(val?, e, \rho, a) \\
& \quad \mid \mathbf{appk}(done, todo, \rho, a) \\
& \quad \mid \mathbf{appprimk}(op, \rho, a) \\
& \quad \mid \mathbf{primk}(op, done, todo, \\
& \quad \quad \rho, a) \\
& \quad \mid \mathbf{letk}(vars, done, todo, \\
& \quad \quad e, \rho, a) \\
done &\triangleq Val^* \\
todo &\triangleq \mathsf{Exp}^* \\
vars &\triangleq \mathsf{Var}^*
\end{aligned}
$$

Eval Rules

Rules for when the control is an expression

$$E\langle æ, \rho, \sigma, \sigma_\kappa, a, t\rangle \rightsquigarrow A\langle v, \sigma, \sigma_\kappa, a, t\rangle$$
where $v \triangleq \mathcal{A}(\varsigma)$

$$E\langle(\texttt{if}\ e_c\ e_t\ e_f), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e_c, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{ifk}(e_t, e_f, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{let}\ ()\ e), \rho, \sigma, \sigma_\kappa, a, t\rangle \rightsquigarrow E\langle e, \rho, \sigma, \sigma_\kappa, a, t\rangle$$

$$E\langle(\texttt{let}\ ([x_0\ e_0]\ [x_s\ e_s]\ ...)\ e_b), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{call/cc}\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{callcck}(a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{set!}\ x\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{setk}(x, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{prim}\ op), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
where $v = op$ applied to 0 arguments

$$E\langle(\texttt{prim}\ op\ e_0\ e_s\ ...), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{primk}(op, [\,], e_s, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{apply-prim}\ op\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{appprimk}(op, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{apply}\ e_f\ e_x), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{appappk}(\varnothing, e_x, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

$$E\langle(e_f\ e_s\ ...), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \rightsquigarrow E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa, t'\rangle$$
where $a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa \triangleq \mathbf{appk}([\,], e_s, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$

Apply Rules

Rules for when the control is a value

$\varsigma = A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$

$\kappa \triangleq \sigma_\kappa(a_\kappa)$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \varsigma$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_f, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v = \texttt{\#f}$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_t, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v \neq \texttt{\#f}$

$$\mathbf{letk}(vars, done, [\,], e_b, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_b, \rho'_\kappa, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$
where $a_i \triangleq alloc(\varsigma, i)$

$t' \triangleq tick(\varsigma, n)$

$\rho'_\kappa \triangleq \rho_\kappa[vars_0 \mapsto a_0 \; ... \; vars_{n-1} \mapsto a_{n-1}]$

$done' \triangleq done +\!\!+ [v]$

$\sigma' \triangleq \sigma[a_0 \mapsto done'_0 \; ... \; a_{n-1} \mapsto done'_{n-1}]$

$$\mathbf{letk}(vars, done, e_h :: e_t, e_b, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where $a''_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa' \triangleq \mathbf{letk}(vars, done +\!\!+ [v], e_t, e_b, \rho_\kappa, a'_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$

$$\mathbf{setk}(x, \rho_\kappa, a'_\kappa) \rightsquigarrow A\langle Void, \sigma', \sigma_\kappa, a'_\kappa, t\rangle$$
where $\sigma' \triangleq \sigma[\rho_\kappa(x) \mapsto v]$

$$\mathbf{callcck}(a'_\kappa) \rightsquigarrow E\langle e, \rho'_\lambda, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v = ((\lambda\ (x)\ e), \rho_\lambda)$

$a \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa' \triangleq \sigma_\kappa(a'_\kappa)$

$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]$

$\sigma' \triangleq \sigma[a \mapsto \kappa']$

$$\mathbf{callcck}(\_) \rightsquigarrow A\langle \kappa, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $v = \kappa'$

$a'_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa']$

$$\mathbf{appprimk}(op, \rho_\kappa, a'_\kappa) \rightsquigarrow A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t)\rangle$$
where $v' \triangleq op$ applied to v

$$\mathbf{primk}(op, done, [\,], \rho_\kappa, a'_\kappa) \rightsquigarrow A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v' \triangleq op$ applied to $(done +\!\!+ [v])$

$$\mathbf{primk}(op, done, e_h :: e_t, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where $a''_\kappa \triangleq alloc(\varsigma, 0)$

$t' \triangleq tick(\varsigma, 1)$

$\kappa' \triangleq \mathbf{primk}(op, done +\!\!+ [v], e_t, \rho_\kappa, a'_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$

More Apply Rules

Rules for when the control is a value

$$\begin{aligned}&\mathbf{appappk}(v_f, \_, \_, a'_\kappa)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } v_f &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda)\\ a_i &\triangleq alloc(\varsigma, i)\\ t' &\triangleq tick(\varsigma, n)\\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto a_0\ ...\\ &\qquad x_{n-1} \mapsto a_{n-1}]\\ \sigma' &\triangleq \sigma[a_0 \mapsto v_0\ ...\\ &\qquad a_{n-1} \mapsto v_{n-1}]\end{aligned}$$

$$\begin{aligned}&\mathbf{appappk}(v_f, \_, \_, a'_\kappa)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } v_f &= ((\lambda\ x\ e_b), \rho_\lambda)\\ a &\triangleq alloc(\varsigma, 0)\\ t' &\triangleq tick(\varsigma, 1)\\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto a]\\ \sigma' &\triangleq \sigma[a \mapsto v]\end{aligned}$$

$$\begin{aligned}&\mathbf{appappk}(\varnothing, e, \rho_\kappa, a'_\kappa)\\ &\rightsquigarrow E\langle e, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } a''_\kappa &\triangleq alloc(\varsigma, 0)\\ t' &\triangleq tick(\varsigma, 1)\\ \kappa' &\triangleq \mathbf{appappk}(v, e, \rho_\kappa, a'_\kappa)\\ \sigma'_\kappa &\triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']\end{aligned}$$

$$\begin{aligned}&\mathbf{appk}(done, [\,], \_, a'_\kappa)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } done &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) :: v_s\\ a_i &\triangleq alloc(\varsigma, i)\\ t' &\triangleq tick(\varsigma, n)\\ v'_s &\triangleq v_s +\!\!+ [v]\\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto a_0\ ...\\ &\qquad x_{n-1} \mapsto a_{n-1}]\\ \sigma' &\triangleq \sigma[a_0 \mapsto v'_0\ ...\\ &\qquad a_{n-1} \mapsto v'_{n-1}]\end{aligned}$$

$$\begin{aligned}&\mathbf{appk}(done, [\,], \_, a'_\kappa)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } done &= ((\lambda\ x\ e_b), \rho_\lambda) :: v_s\\ a &\triangleq alloc(\varsigma, 0)\\ t' &\triangleq tick(\varsigma, 1)\\ v'_s &\triangleq (v_s +\!\!+ [v])\\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto a]\\ \sigma' &\triangleq \sigma[a \mapsto v'_s]\end{aligned}$$

$$\mathbf{appk}([\kappa_\lambda], [\,], \rho_\kappa, a'_\kappa) \rightsquigarrow A\langle v, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$

$$\begin{aligned}\text{where } a''_\kappa &\triangleq alloc(\varsigma, 0)\\ t' &\triangleq tick(\varsigma, 1)\\ \sigma'_\kappa &\triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa_\lambda]\end{aligned}$$

$$\begin{aligned}&\mathbf{appk}(done, e_h :: e_t, \rho_\kappa, a'_\kappa)\\ &\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } a''_\kappa &\triangleq alloc(\varsigma, 0)\\ t' &\triangleq tick(\varsigma, 1)\\ \kappa' &\triangleq \mathbf{appk}(done +\!\!+ [v], e_t, \rho_\kappa, a'_\kappa)\\ \sigma'_\kappa &\triangleq \sigma[a''_\kappa \mapsto \kappa']\end{aligned}$$

2 Abstract Semantics of Scheme CESKt*

Abstract Semantic Domains:

$$
\begin{aligned}
\hat{\varsigma} \in \hat{\Sigma} &\triangleq E\langle Eval\rangle + E\langle Apply\rangle \\
Eval &\triangleq \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \\
&\quad \times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time} \\
Apply &\triangleq \widehat{Val} \times \widehat{Store} \\
&\quad \times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time} \\
\hat{\rho} \in \widehat{Env} &\triangleq \mathsf{Var} \rightharpoonup \widehat{Addr} \\
\hat{\sigma} \in \widehat{Store} &\triangleq \widehat{Addr} \rightharpoonup \mathcal{P}(Val) \\
\hat{\sigma}_\kappa \in \widehat{KStore} &\triangleq \widehat{Addr} \rightharpoonup \mathcal{P}(\widehat{Kont}) \\
\hat{v} \in \widehat{Val} &\triangleq \widehat{Clo} + \widehat{Kont} + \mathbb{Z} \\
&\quad + \{\texttt{\#t}, \texttt{\#f}, Null, Void\} \\
&\quad + \{\mathbf{quote}(e), \mathbf{cons}(v, v)\} \\
\widehat{clo} \in \widehat{Clo} &\triangleq \mathsf{Lam} \times \widehat{Env} \\
\hat{a}, \hat{a}_{\hat{\kappa}} \in Addr &\triangleq \mathbb{N} \\
\hat{t} \in Time &\triangleq \mathbb{N} \\
\widehat{done} &\triangleq \widehat{Val}^*
\end{aligned}
$$

$$
\begin{aligned}
\hat{\kappa} \in \widehat{Kont} ::= &\ \mathbf{mt} \\
&\mid \mathbf{ifk}(e, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{callcck}(\hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{appappk}(\widehat{val}?, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{appk}(\widehat{done}, todo, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{appprimk}(op, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{primk}(op, \widehat{done}, todo, \\
&\qquad \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{letk}(vars, \widehat{done}, todo, \\
&\qquad e, \hat{\rho}, \hat{a}_{\hat{\kappa}})
\end{aligned}
$$

Tick/Alloc:

$$
\begin{aligned}
\widehat{tick} &: \hat{\Sigma} \times \widehat{Kont} \to \widehat{Time} \\
\widehat{tick}(\hat{\varsigma}, \hat{\kappa}) &\triangleq 0 \\
\widehat{alloc} &: \hat{\Sigma} \times \widehat{Kont} \to \widehat{Addr} \\
\widehat{alloc}(\hat{\varsigma}, \hat{\kappa}) &\triangleq 0
\end{aligned}
$$

Abstract Atomic Evaluation:

$$
\begin{aligned}
\mathcal{A} &: Eval \to \widehat{Val} \\
\mathcal{A}(E\langle lam, \hat{\rho}, ...\rangle) &\triangleq \{(lam, \hat{\rho})\} \\
\mathcal{A}(\text{æ}) &\triangleq \{\text{æ}\}
\end{aligned}
$$

Abstract Eval Rules

Rules for when the control is an expression

$$E\langle æ, \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow A\langle \hat v, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle$$
$$\text{where } \hat v \triangleq \hat{\mathcal{A}}(\hat\varsigma, \hat\sigma)$$

$$E\langle(\texttt{if}\ e_c\ e_t\ e_f), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e_c, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{ifk}(e_t, e_f, \hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(\texttt{let}\ ()\ e), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e, \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle$$

$$E\langle(\texttt{let}\ bnds\ e_b), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e_0, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } bnds = ([x_0\ e_0]\ [x_s\ e_s]\ ...)$$
$$\hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(\texttt{call/cc}\ e), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{callcck}(\hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(\texttt{set!}\ x\ e), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat a \triangleq \hat\rho(x)$$
$$\hat\kappa \triangleq \mathbf{setk}(\hat a, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(\texttt{prim}\ op), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow A\langle \hat v, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle$$
$$\text{where } \hat v = op \text{ applied to 0 arguments}$$

$$E\langle(\texttt{prim}\ op\ e_0\ e_s\ ...), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e_0, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{primk}(op, [\,], e_s, \hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(\texttt{apply-prim}\ op\ e), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{appprimk}(op, \hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(\texttt{apply}\ e_f\ e_x), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e_f, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{appappk}(\varnothing, e_x, \hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

$$E\langle(e_f\ e_s\ ...), \hat\rho, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\,\rangle \rightsquigarrow E\langle e_f, \hat\rho, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
$$\text{where } \hat a'_{\hat\kappa} \triangleq \widehat{alloc}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{tick}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa \triangleq \mathbf{appk}([\,], e_s, \hat\rho, \hat a_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \hat\kappa]$$

Apply Rules

Rules for when the control is a value

$\hat{\varsigma} = A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\rangle$

$\hat{\kappa} \in \hat{\sigma}_{\hat{\kappa}}(\hat{a}_{\hat{\kappa}})$

Proceed by matching on $\hat{\kappa}$

$\mathbf{mt} \rightsquigarrow \hat{\varsigma}$

$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_f, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\,\rangle$
where $\hat{v} = \texttt{\#f}$

$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_t, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\rangle$
where $\hat{v} \neq \texttt{\#f}$

$\mathbf{letk}(vars, \widehat{done}, [\,], e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$
$\rightsquigarrow E\langle e_b, \hat{\rho}'_{\hat{\kappa}}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$
where $\hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$

$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$

$\hat{\rho}'_{\hat{\kappa}} \triangleq \hat{\rho}_{\hat{\kappa}}[vars_0 \mapsto \hat{a}_0 \; ... \; vars_{n-1} \mapsto \hat{a}_{n-1}]$

$\widehat{done}' \triangleq \widehat{done} +\!\!+ [\hat{v}]$

$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \widehat{done}'_0 \; ... \; \hat{a}_{n-1} \mapsto \widehat{done}'_{n-1}]$

$\mathbf{letk}(vars, \widehat{done}, e_h :: e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$
$\rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$
where $\hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$

$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$

$\hat{\kappa}' \triangleq \mathbf{letk}(vars, \widehat{done}, +\!\!+[\hat{v}], e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$

$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$

$\mathbf{setk}(x, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle\, Void, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\,\rangle$
where $\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{\rho}_{\hat{\kappa}}(x) \mapsto \hat{v}]$

$\mathbf{callcck}(\hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\,\rangle$
where $\hat{v} = ((\lambda\ (x)\ e), \hat{\rho}_{\lambda})$

$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$

$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$

$\hat{\kappa}' \triangleq \hat{\sigma}_{\hat{\kappa}}(\hat{a}'_{\hat{\kappa}})$

$\hat{\rho}'_{\lambda} \triangleq \hat{\rho}_{\lambda}[x \mapsto \hat{a}]$

$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{\kappa}']$

$\mathbf{callcck}(\_) \rightsquigarrow A\langle \hat{\kappa}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$
where $v = \kappa'$

$\hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$

$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$

$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \kappa']$

$\mathbf{appprimk}(op, \_, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\,\rangle$
where $\hat{v}' \triangleq op$ applied to $\hat{v}$

$\mathbf{primk}(op, \widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}})$
$\rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}\,\rangle$
where $\hat{v}' \triangleq op$ applied to $(\widehat{done} +\!\!+ [\hat{v}])$

$\mathbf{primk}(op, \widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$
$\rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$
where $\hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$

$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$

$\hat{\kappa}' \triangleq \mathbf{primk}(op, \widehat{done} +\!\!+ [\hat{v}])\; e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}$

$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$

More Apply Rules

Rules for when the control is a value

$$\begin{aligned}&\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}})\\ &\rightsquigarrow E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } \hat{v}_f &= ((\lambda\ (x_s...)\ e_b), \hat{\rho}_\lambda)\\ \hat{a}_i &\triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})\\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0\ ...\\ &\qquad x_{n-1} \mapsto \hat{a}_{n-1}]\\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}_0\ ...\\ &\qquad \hat{a}_{n-1} \mapsto \hat{v}_{n-1}]\end{aligned}$$

$$\begin{aligned}&\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}})\\ &\rightsquigarrow E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } \hat{v}_f &= ((\lambda\ x\ e_b), \hat{\rho}_\lambda)\\ \hat{a} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})\\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]\\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{v}]\end{aligned}$$

$$\begin{aligned}&\textbf{appappk}(\varnothing, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})\\ &\rightsquigarrow E\langle e, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } \hat{a}''_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})\\ \hat{\kappa}' &\triangleq \textbf{appappk}(\hat{v}, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})\\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']\end{aligned}$$

$$\begin{aligned}&\textbf{appk}(\widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}})\\ &\rightsquigarrow E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } \widehat{done} &= ((\lambda\ (x_s...)\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s\\ \hat{a}_i &\triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})\\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0\ ...\\ &\qquad x_{n-1} \mapsto \hat{a}_{n-1}]\\ \hat{v}'_s &\triangleq \hat{v}_s +\!\!+ [\hat{v}]\\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}'_0\ ...\\ &\qquad \hat{a}_{n-1} \mapsto \hat{v}'_{n-1}]\end{aligned}$$

$$\begin{aligned}&\textbf{appk}(\widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}})\\ &\rightsquigarrow E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } \widehat{done} &= ((\lambda\ x\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s\\ \hat{a} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})\\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]\\ \hat{v}'_s &\triangleq \hat{v}_s +\!\!+ [\hat{v}]\\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{v}'_s]\end{aligned}$$

$$\textbf{appk}([\hat{\kappa}_\lambda], [\,], \hat{\rho}_{\hat{\kappa}}, \_) \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$

$$\begin{aligned}\text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})\\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}_\lambda]\end{aligned}$$

$$\begin{aligned}&\textbf{appk}(\widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})\\ &\rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle\end{aligned}$$

$$\begin{aligned}\text{where } \hat{a}''_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})\\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})\\ \hat{\kappa}' &\triangleq \textbf{appk}(\widehat{done} +\!\!+ [\hat{v}], e_t, \hat{\rho}_\kappa, \hat{a}'_{\hat{\kappa}})\\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']\end{aligned}$$