

1 Semantics of Basic Scheme CESK*

Syntax:

$$\begin{aligned}
 e \in \text{Exp} &::= \mathfrak{x} \\
 &\quad | (\text{if } e \ e \ e) \\
 &\quad | (\text{let } (x \ e) \ e) \\
 &\quad | (\text{prim } op \ e \ e \dots) \\
 &\quad | (e \ e \dots) \\
 \mathfrak{x} \in \text{AExp} &::= lam \mid \mathbb{Z} \mid \#\mathfrak{t} \mid \#\mathfrak{f} \\
 lam \in \text{Lam} &::= (\lambda (x \dots) \ e) \\
 x \in \text{Var} &\quad \text{A set of identifiers}
 \end{aligned}$$

Semantics:

$$\begin{aligned}
 \varsigma \in \Sigma &\triangleq \text{Exp} \times \text{Env} \times \text{Kont} \\
 \rho \in \text{Env} &\triangleq \text{Var} \rightarrow \text{Addr} \\
 \sigma \in \text{Store} &\triangleq \text{Addr} \rightarrow \text{Val} \\
 v \in \text{Val} &\triangleq \text{Clo} + \mathbb{Z} + \{\#\mathfrak{t}, \#\mathfrak{f}\} \\
 clo \in \text{Clo} &\triangleq \text{Lam} \times \text{Env} \\
 \kappa \in \text{Kont} &\triangleq \mathbf{mt} \mid \mathbf{appk}(done, todo, \rho, a) \\
 &\quad \mid \mathbf{ifk}(e, e, \rho, a) \\
 &\quad \mid \mathbf{letk}(x, e, \rho, a) \\
 a, b, c \in \text{Addr} &\quad \text{A set of addresses} \\
 done &\triangleq \text{Val}^* \quad todo \triangleq \text{Exp}^*
 \end{aligned}$$

Atomic Evaluation Function:

$$\begin{aligned}
 \mathcal{A}(x, \rho, \sigma) &\triangleq \sigma(\rho(x)) \\
 \mathcal{A}(lam, \rho, \sigma) &\triangleq (lam, \rho) \\
 \mathcal{A}(\mathfrak{x}, \rho, \sigma) &\triangleq \mathfrak{x}
 \end{aligned}$$

Transition Function:

$$(\Sigma \times \text{Store}) \rightsquigarrow (\Sigma \times \text{Store})$$

$(\varsigma \times \sigma) \rightarrow (\varsigma' \times \sigma)$, where $\kappa = \sigma(a)$, $b = \text{alloc}(\varsigma)$
 proceed by matching on ς

$\langle (\text{if } e_c \ e_t \ e_f), \rho, a \rangle$	$\langle e_c, \rho, b \rangle$ $\sigma[b \mapsto \text{ifk}(e_t, e_f, \rho, a)]$
$\langle (\text{let } (x \ e_x) \ e_b), \rho, a \rangle$	$\langle e_x, \rho, b \rangle$ $\sigma[b \mapsto \text{letk}(x, e_b, \rho, a)]$
$\langle (\text{prim } op \ e_0 \ es...), \rho, a \rangle$	$\langle e_0, \rho, b \rangle$ $\sigma[b \mapsto \text{appk}([op], es, \rho, a)]$
$\langle (e_f \ es...), \rho, a \rangle$	$\langle e_f, \rho, b \rangle$ $\sigma[b \mapsto \text{appk}([], es, \rho, a)]$
$\langle \mathfrak{x}, \rho, a \rangle$ let $v = \mathcal{A}(\mathfrak{x}, \rho, \sigma)$ match on κ below	
mt	ς
$\text{ifk}(e_t, e_f, \rho', c)$ when $v = \#f$	$\langle e_f, \rho', c \rangle$
$\text{ifk}(e_t, e_f, \rho', c)$ when $v \neq \#f$	$\langle e_t, \rho', c \rangle$
$\text{letk}(x, e_b, \rho', c)$	$\langle e_b, \rho'[x \mapsto b], c \rangle$ $\sigma[b \mapsto v]$
$\text{appk}(\text{done}, e_h :: e_t, \rho', c)$	$\langle e_h, \rho', b \rangle$ $\sigma[b \mapsto \text{appk}(\text{done} \# [v], e_t, \rho', c)]$
$\text{appk}(op :: v_s, [], \rho', c)$	$\langle v', \rho', c \rangle$ $v' = op$ applied to $(v_s \# [v])$
$\text{appk}(clo :: v_s, [], \rho', c)$ where $clo = ((\lambda (xs...) \ e_b), \rho_\lambda)$	$\langle e_b, \rho_\lambda[xs_0 \mapsto b_0 \dots xs_i \mapsto b_i], c \rangle$ $v'_s = v_s \# [v]$ $\sigma[b_0 \mapsto v'_{s_0} \dots b_i \mapsto v'_{s_i}]$

2 Formalization of an Abstract CESK* machine with basic Scheme features.