1 Concrete Semantics of Scheme CESKt*

Syntax Domains:

$$
\begin{aligned}
e \in \mathsf{Exp} ::= {} & æ \\
& \mid (\texttt{if}\ e\ e\ e) \\
& \mid (\texttt{let}\ ([x\ e]\ ...)\ e) \\
& \mid (\texttt{call/cc}\ e) \\
& \mid (\texttt{set!}\ x\ e) \\
& \mid (\texttt{prim}\ op\ e\ ...) \\
& \mid (\texttt{apply-prim}\ op\ e) \\
& \mid (\texttt{apply}\ e\ e) \\
& \mid (e\ e\ ...) \\
æ \in \mathsf{AExp} ::= {} & x \mid lam \mid \mathbb{Z} \mid \texttt{\#t} \mid \texttt{\#f} \\
& \mid (\texttt{quote}\ e) \\
lam \in \mathsf{Lam} ::= {} & (\lambda\ (x...)\ e) \mid (\lambda\ x\ e) \\
x \in \mathsf{Var} \quad & \text{A set of identifiers} \\
op \in \mathsf{Prim} \quad & \text{A set of primitives}
\end{aligned}
$$

Atomic Evaluation:

$$
\begin{aligned}
\mathcal{A} : \Sigma_E \times \sigma \rightharpoonup {} & Val \\
\mathcal{A}(\langle lam, \rho, \_, \_\rangle, \_) \triangleq {} & (lam, \rho) \\
\mathcal{A}(\langle (\texttt{quote}\ e), \_, \_, \_\rangle, \_) \triangleq {} & \mathbf{quote}(e) \\
\mathcal{A}(\langle x, \rho, \_, \_\rangle, \sigma) \triangleq {} & \sigma(\rho(x)) \\
\mathcal{A}(\langle æ, \_, \_, \_\rangle, \_) \triangleq {} & æ
\end{aligned}
$$

Tick/Alloc:

$$
\begin{aligned}
tick : Eval \times \mathbb{N} \to {} & Time \\
tick(E\langle \_, \_, \_, t\rangle, n) \triangleq {} & (t + n) \\
alloc : \Sigma \times \mathbb{N} \triangleq {} & Addr \\
alloc(E\langle \_, \_, \_, t\rangle, n) \triangleq {} & (t + n)
\end{aligned}
$$

Injection:

$$
\mathcal{I} : \mathsf{Exp} \to \Sigma
$$

$$
\mathcal{I}(e) \triangleq (e, \varnothing, \varnothing, \{0 : \mathbf{mt}\}, 0, 1)
$$

Transition:
Collecting Semantics:

Semantic Domains:

$$
\begin{aligned}
\varsigma \in \Sigma \triangleq {} & E\langle Eval\rangle + A\langle Apply\rangle \\
Eval \triangleq {} & \mathsf{Exp} \times Env \times Store \\
& \times KStore \times Addr \times Time \\
Apply \triangleq {} & Val \times Store \\
& \times KStore \times Addr \times Time \\
\rho \in Env \triangleq {} & \mathsf{Var} \rightharpoonup Addr \\
\sigma \in Store \triangleq {} & Addr \rightharpoonup Val \\
\sigma \in Store \triangleq {} & Addr \rightharpoonup Kont \\
v \in Val \triangleq {} & Clo + Kont + \mathbb{Z} \\
& + \{\texttt{\#t}, \texttt{\#f}, Null, Void\} \\
& + \{\mathbf{quote}(e), \mathbf{cons}(v, v)\} \\
clo \in Clo \triangleq {} & \mathsf{Lam} \times Env \\
a, b, c \in Addr \triangleq {} & \mathbb{N} \\
t, u \in Time \triangleq {} & \mathbb{N} \\
\kappa \in Kont ::= {} & \mathbf{mt} \\
& \mid \mathbf{ifk}(e, e, \rho, a) \\
& \mid \mathbf{callcck}(a) \\
& \mid \mathbf{setk}(x, \rho, a) \\
& \mid \mathbf{appappk}(val?, e, \rho, a) \\
& \mid \mathbf{appk}(done, todo, \rho, a) \\
& \mid \mathbf{appprimk}(op, \rho, a) \\
& \mid \mathbf{primk}(op, done, todo, \\
& \qquad \rho, a) \\
& \mid \mathbf{letk}(vars, done, todo, \\
& \qquad e, \rho, a) \\
done \triangleq {} & Val^* \\
todo \triangleq {} & \mathsf{Exp}^* \\
vars \triangleq {} & \mathsf{Var}^*
\end{aligned}
$$

Eval Rules

Rules for when the control is an expression

$$E\langle æ, \rho, \sigma, \sigma_\kappa, a, t\rangle \leadsto A\langle v, \sigma, \sigma_\kappa, a, t\rangle$$
$$\text{where } v \triangleq \mathcal{A}(\varsigma)$$

$$E\langle(\texttt{if}\ e_c\ e_t\ e_f), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e_c, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{ifk}(e_t, e_f, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(\texttt{let}\ ()\ e), \rho, \sigma, \sigma_\kappa, a, t\rangle \leadsto E\langle e, \rho, \sigma, \sigma_\kappa, a, t\rangle$$

$$E\langle(\texttt{let}\ ([x_0\ e_0]\ [x_s\ e_s]\ ...)\ e_b), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(\texttt{call/cc}\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{callcck}(a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(\texttt{set!}\ x\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{setk}(x, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(\texttt{prim}\ op), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$$
$$\text{where } v = op \text{ applied to 0 arguments}$$

$$E\langle(\texttt{prim}\ op\ e_0\ e_s\ ...), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{primk}(op, [\,], e_s, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(\texttt{apply-prim}\ op\ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{appprimk}(op, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(\texttt{apply}\ e_f\ e_x), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{appappk}(\varnothing, e_x, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle(e_f\ e_s\ ...), \rho, \sigma, \sigma_\kappa, a_\kappa, t\rangle \leadsto E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa, t'\rangle$$
$$\text{where } a'_\kappa \triangleq alloc(\varsigma, 0)$$
$$t' \triangleq tick(\varsigma, 1)$$
$$\kappa \triangleq \mathbf{appk}([\,], e_s, \rho, a_\kappa)$$
$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

Apply Rules

Rules for when the control is a value

$\varsigma = A\langle v, \sigma, \sigma_\kappa, a_\kappa, t\rangle$

$\kappa \triangleq \sigma_\kappa(a_\kappa)$

Proceed by matching on κ

$$\mathbf{mt} \leadsto \varsigma$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \leadsto E\langle e_f, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v = \texttt{\#f}$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \leadsto E\langle e_t, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v \neq \texttt{\#f}$

$$\mathbf{letk}(vars, done, [\,], e_b, \rho_\kappa, a'_\kappa) \leadsto E\langle e_b, \rho'_\kappa, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$
where
$$\begin{aligned} a_i &\triangleq alloc(\varsigma, i) \\ t' &\triangleq tick(\varsigma, n) \\ \rho'_\kappa &\triangleq \rho_\kappa[vars_0 \mapsto a_0 \ldots \\ &\qquad vars_{n-1} \mapsto a_{n-1}] \\ done' &\triangleq done +\!\!+ [v] \\ \sigma' &\triangleq \sigma[a_0 \mapsto done'_0 \ldots \\ &\qquad a_{n-1} \mapsto done'_{n-1}] \end{aligned}$$

$$\mathbf{letk}(vars, done, e_h :: e_t, e_b, \rho_\kappa, a'_\kappa) \leadsto E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where
$$\begin{aligned} a''_\kappa &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \kappa' &\triangleq \mathbf{letk}(vars, done +\!\!+ [v], \\ &\qquad e_t, e_b, \rho_\kappa, a'_\kappa) \\ \sigma'_\kappa &\triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa'] \end{aligned}$$

$$\mathbf{callcck}(a'_\kappa) \leadsto E\langle e, \rho'_\lambda, \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v = ((\lambda\ (x)\ e), \rho_\lambda)$
$$\begin{aligned} a &\triangleq tick(\varsigma, 0) \\ t' &\triangleq alloc(\varsigma, 1) \\ \kappa' &\triangleq \sigma_\kappa(a'_\kappa) \\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto a] \\ \sigma' &\triangleq \sigma[a \mapsto \kappa'] \end{aligned}$$

$$\mathbf{callcck}(\_) \leadsto A\langle \kappa, \sigma, \sigma'_\kappa, a'_\kappa, t'\rangle$$
where $v = \kappa'$
$$\begin{aligned} a'_\kappa &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \sigma'_\kappa &\triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa'] \end{aligned}$$

$$\mathbf{setk}(x, \rho_\kappa, a'_\kappa) \leadsto A\langle Void, \sigma', \sigma_\kappa, a'_\kappa, t\rangle$$
where $\sigma' \triangleq \sigma[\rho_\kappa(x) \mapsto v]$

$$\mathbf{appprimk}(op, \rho_\kappa, a'_\kappa) \leadsto A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t)\rangle$$
where $v' \triangleq op$ applied to v

$$\mathbf{primk}(op, done, [\,], \rho_\kappa, a'_\kappa) \leadsto A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t\rangle$$
where $v' \triangleq op$ applied to $(done +\!\!+ [v])$

$$\mathbf{primk}(op, done, e_h :: e_t, \rho_\kappa, a'_\kappa) \leadsto E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$
where
$$\begin{aligned} a''_\kappa &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \kappa' &\triangleq \mathbf{primk}(op, done +\!\!+ [v], \\ &\qquad e_t, \rho_\kappa, a'_\kappa) \\ \sigma'_\kappa &\triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa'] \end{aligned}$$

More Apply Rules

Rules for when the control is a value

$$\mathbf{appappk}(v_f, \_, \_, a'_\kappa) \leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } v_f &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) \\ a_i &\triangleq alloc(\varsigma, i) \\ t' &\triangleq tick(\varsigma, n) \\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto a_0 \ ... \\ &\qquad x_{n-1} \mapsto a_{n-1}] \\ \sigma' &\triangleq \sigma[a_0 \mapsto v_0 \ ... \\ &\qquad a_{n-1} \mapsto v_{n-1}] \end{aligned}$$

$$\mathbf{appappk}(v_f, \_, \_, a'_\kappa) \leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } v_f &= ((\lambda\ x\ e_b), \rho_\lambda) \\ a &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto a] \\ \sigma' &\triangleq \sigma[a \mapsto v] \end{aligned}$$

$$\mathbf{appappk}(\varnothing, e, \rho_\kappa, a'_\kappa) \leadsto E\langle e, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } a''_\kappa &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \kappa' &\triangleq \mathbf{appappk}(v, e, \rho_\kappa, a'_\kappa) \\ \sigma'_\kappa &\triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa'] \end{aligned}$$

$$\mathbf{appk}(done, [\,], \_, a'_\kappa) \leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } done &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) :: v_s \\ a_i &\triangleq alloc(\varsigma, i) \\ t' &\triangleq tick(\varsigma, n) \\ v'_s &\triangleq v_s \mathbin{+\!\!+} [v] \\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto a_0 \ ... \\ &\qquad x_{n-1} \mapsto a_{n-1}] \\ \sigma' &\triangleq \sigma[a_0 \mapsto v'_0 \ ... \\ &\qquad a_{n-1} \mapsto v'_{n-1}] \end{aligned}$$

$$\mathbf{appk}(done, [\,], \_, a'_\kappa) \leadsto E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } done &= ((\lambda\ x\ e_b), \rho_\lambda) :: v_s \\ a &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ v'_s &\triangleq (v_s \mathbin{+\!\!+} [v]) \\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto a] \\ \sigma' &\triangleq \sigma[a \mapsto v'_s] \end{aligned}$$

$$\mathbf{appk}([\kappa_\lambda], [\,], \rho_\kappa, a'_\kappa) \leadsto A\langle v, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } a''_\kappa &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \sigma'_\kappa &\triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa_\lambda] \end{aligned}$$

$$\mathbf{appk}(done, e_h :: e_t, \rho_\kappa, a'_\kappa) \leadsto E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t'\rangle$$

$$\begin{aligned} \text{where } a''_\kappa &\triangleq alloc(\varsigma, 0) \\ t' &\triangleq tick(\varsigma, 1) \\ \kappa' &\triangleq \mathbf{appk}(done \mathbin{+\!\!+} [v], e_t, \rho_\kappa, a'_\kappa) \\ \sigma'_\kappa &\triangleq \sigma[a''_\kappa \mapsto \kappa'] \end{aligned}$$

2 Abstract Semantics of Scheme CESKt*

Abstract Semantic Domains:

$$
\begin{aligned}
\hat{\varsigma} \in \hat{\Sigma} &\triangleq E\langle \mathit{Eval}\rangle + E\langle \mathit{Apply}\rangle \\
\mathit{Eval} &\triangleq \mathsf{Exp} \times \widehat{\mathit{Env}} \times \widehat{\mathit{Store}} \\
&\quad \times \widehat{\mathit{KStore}} \times \widehat{\mathit{Addr}} \times \widehat{\mathit{Time}} \\
\mathit{Apply} &\triangleq \widehat{\mathit{Val}} \times \widehat{\mathit{Store}} \\
&\quad \times \widehat{\mathit{KStore}} \times \widehat{\mathit{Addr}} \times \widehat{\mathit{Time}} \\
\hat{\rho} \in \widehat{\mathit{Env}} &\triangleq \mathsf{Var} \rightharpoonup \widehat{\mathit{Addr}} \\
\hat{\sigma} \in \widehat{\mathit{Store}} &\triangleq \widehat{\mathit{Addr}} \rightharpoonup \mathcal{P}(\mathit{Val}) \\
\hat{\sigma}_\kappa \in \widehat{\mathit{KStore}} &\triangleq \widehat{\mathit{Addr}} \rightharpoonup \mathcal{P}(\widehat{\mathit{Kont}}) \\
\hat{v} \in \widehat{\mathit{Val}} &\triangleq \widehat{\mathit{Clo}} + \widehat{\mathit{Kont}} + \mathbb{Z} \\
&\quad + \{\texttt{\#t}, \texttt{\#f}, \mathit{Null}, \mathit{Void}\} \\
&\quad + \{\mathbf{quote}(e), \mathbf{cons}(v, v)\} \\
\widehat{\mathit{clo}} \in \widehat{\mathit{Clo}} &\triangleq \mathsf{Lam} \times \widehat{\mathit{Env}} \\
\hat{a}, \hat{a}_{\hat{\kappa}} \in \mathit{Addr} &\triangleq \mathbb{N} \\
\hat{t} \in \mathit{Time} &\triangleq \mathbb{N} \\
\widehat{\mathit{done}} &\triangleq \widehat{\mathit{Val}}^*
\end{aligned}
$$

$$
\begin{aligned}
\hat{\kappa} \in \widehat{\mathit{Kont}} ::= &\ \mathbf{mt} \\
&\mid \mathbf{ifk}(e, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{callcck}(\hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{setk}(x, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{appappk}(\widehat{\mathit{val}}?, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{appk}(\widehat{\mathit{done}}, \mathit{todo}, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{appprimk}(\mathit{op}, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{primk}(\mathit{op}, \widehat{\mathit{done}}, \mathit{todo}, \\
&\qquad \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\mid \mathbf{letk}(\mathit{vars}, \widehat{\mathit{done}}, \mathit{todo}, \\
&\qquad e, \hat{\rho}, \hat{a}_{\hat{\kappa}})
\end{aligned}
$$

Tick/Alloc:

$$
\begin{aligned}
\widehat{\mathit{tick}} &: \hat{\Sigma} \times \widehat{\mathit{Kont}} \to \widehat{\mathit{Time}} \\
\widehat{\mathit{tick}}(\hat{\varsigma}, \hat{\kappa}) &\triangleq 0 \\
\widehat{\mathit{alloc}} &: \hat{\Sigma} \times \widehat{\mathit{Kont}} \to \widehat{\mathit{Addr}} \\
\widehat{\mathit{alloc}}(\hat{\varsigma}, \hat{\kappa}) &\triangleq 0
\end{aligned}
$$

Abstract Atomic Evaluation:

$$
\begin{aligned}
\mathcal{A} &: \mathit{Eval} \to \widehat{\mathit{Val}} \\
\mathcal{A}(E\langle \mathit{lam}, \hat{\rho}, \ldots\rangle) &\triangleq \{(\mathit{lam}, \hat{\rho})\} \\
\mathcal{A}(\text{æ}) &\triangleq \{\text{æ}\}
\end{aligned}
$$

Abstract Eval Rules

Rules for when the control is an expression

$$E\langle æ, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle$$
$$\text{where } \hat{v} \triangleq \hat{\mathcal{A}}(\hat{\varsigma}, \hat{\sigma})$$

$$E\langle (\texttt{if}\ e_c\ e_t\ e_f), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_c, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{ifk}(e_t, e_f, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{let}\ ()\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle$$

$$E\langle (\texttt{let}\ ([x_0\ e_0]\ [x_s\ e_s]\ ...)\ e_b), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{call/cc}\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{callcck}(\hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{set!}\ x\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{setk}(x, a) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{prim}\ op), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle$$
$$\text{where } \hat{v} = op \text{ applied to 0 arguments}$$

$$E\langle (\texttt{prim}\ op\ e_0\ e_s\ ...), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{primk}(op, [\,], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{apply-prim}\ op\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{appprimk}(op, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{apply}\ e_f\ e_x), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{appappk}(\varnothing, e_x, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (e_f\ e_s\ ...), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \textbf{appk}([\,], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

Apply Rules

Rules for when the control is a value

$\hat\varsigma = A\langle \hat v, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a_{\hat\kappa}, \hat t\rangle$

$\kappa \in \hat\sigma_{\hat\kappa}(\hat a_{\hat\kappa})$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \hat\varsigma$$

$$\mathbf{ifk}(e_t, e_f, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}) \rightsquigarrow E\langle e_f, \hat\rho_{\hat\kappa}, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t\,\rangle$$
where $\hat v =$ `#f`

$$\mathbf{ifk}(e_t, e_f, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}) \rightsquigarrow E\langle e_t, \hat\rho_{\hat\kappa}, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t\rangle$$
where $\hat v \neq$ `#f`

$$\mathbf{letk}(\mathit{vars}, \widehat{\mathit{done}}, [\,], e_b, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}) \rightsquigarrow E\langle e_b, \hat\rho'_{\hat\kappa}, \hat\sigma', \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
where
$$\hat a_i \triangleq \widehat{\mathit{alloc}}(\hat\varsigma, i, \hat\kappa)$$
$$\hat t' \triangleq \widehat{\mathit{tick}}(\hat\varsigma, n, \hat\kappa)$$
$$\hat\rho'_{\hat\kappa} \triangleq \hat\rho_{\hat\kappa}[\mathit{vars}_0 \mapsto \hat a_0 \ \dots \ \mathit{vars}_{n-1} \mapsto \hat a_{n-1}]$$
$$\widehat{\mathit{done}}' \triangleq \widehat{\mathit{done}} +\!\!+ [\hat v]$$
$$\hat\sigma' \triangleq \hat\sigma \sqcup [\hat a_0 \mapsto \widehat{\mathit{done}}'_0 \ \dots \ \hat a_{n-1} \mapsto \widehat{\mathit{done}}'_{n-1}]$$

$$\mathbf{letk}(\mathit{vars}, \widehat{\mathit{done}}, e_h :: e_t, e_b, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}) \rightsquigarrow E\langle e_h, \hat\rho_{\hat\kappa}, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a''_{\hat\kappa}, \hat t'\rangle$$
where
$$\hat a''_{\hat\kappa} \triangleq \widehat{\mathit{alloc}}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{\mathit{tick}}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa' \triangleq \mathbf{letk}(\mathit{vars}, \widehat{\mathit{done}}, +\!\!+[\hat v], e_t, e_b, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa})$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a''_{\hat\kappa} \mapsto \hat\kappa']$$

$$\mathbf{callcck}(\hat a'_{\hat\kappa}) \rightsquigarrow E\langle e, \hat\rho'_{\hat\lambda}, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t\,\rangle$$
where $\hat v = ((\lambda\ (x)\ e), \hat\rho_{\hat\lambda})$
$$\hat\rho'_{\hat\lambda} \triangleq \hat\rho_{\hat\lambda}[x \mapsto \hat a'_{\hat\kappa}]$$

$$\mathbf{callcck}(\_) \rightsquigarrow A\langle \hat\kappa, \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t'\rangle$$
where $v = \kappa'$
$$\hat a'_{\hat\kappa} \triangleq \widehat{\mathit{alloc}}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{\mathit{tick}}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a'_{\hat\kappa} \mapsto \kappa']$$

$$\mathbf{setk}(x, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}) \rightsquigarrow A\langle \mathit{Void}, \hat\sigma', \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t\,\rangle$$
where $\hat\sigma' \triangleq \hat\sigma \sqcup [\hat\rho_{\hat\kappa}(x) \mapsto \hat v]$

$$\mathbf{appprimk}(\mathit{op}, \_, \hat a'_{\hat\kappa}) \rightsquigarrow A\langle \hat v', \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t\,\rangle$$
where $\hat v' \triangleq \mathit{op}$ applied to $\hat v$

$$\mathbf{primk}(\mathit{op}, \widehat{\mathit{done}}, [\,], \_, \hat a'_{\hat\kappa}) \rightsquigarrow A\langle \hat v', \hat\sigma, \hat\sigma_{\hat\kappa}, \hat a'_{\hat\kappa}, \hat t\,\rangle$$
where $\hat v' \triangleq \mathit{op}$ applied to $(\widehat{\mathit{done}} +\!\!+ [\hat v])$

$$\mathbf{primk}(\mathit{op}, \widehat{\mathit{done}}, e_h :: e_t, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}) \rightsquigarrow E\langle e_h, \hat\rho_{\hat\kappa}, \hat\sigma, \hat\sigma'_{\hat\kappa}, \hat a''_{\hat\kappa}, \hat t'\rangle$$
where
$$\hat a''_{\hat\kappa} \triangleq \widehat{\mathit{alloc}}(\hat\varsigma, 0, \hat\kappa)$$
$$\hat t' \triangleq \widehat{\mathit{tick}}(\hat\varsigma, 1, \hat\kappa)$$
$$\hat\kappa' \triangleq \mathbf{primk}(\mathit{op}, \widehat{\mathit{done}} +\!\!+ [\hat v]) \quad e_t, \hat\rho_{\hat\kappa}, \hat a'_{\hat\kappa}$$
$$\hat\sigma'_{\hat\kappa} \triangleq \hat\sigma_{\hat\kappa} \sqcup [\hat a''_{\hat\kappa} \mapsto \hat\kappa']$$

More Apply Rules

Rules for when the control is a value

$$\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}}) \\ \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \hat{v}_f &= ((\lambda\ (x_s ...)\ e_b), \hat{\rho}_\lambda) \\ \hat{a}_i &\triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa}) \\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0\ ... \\ &\qquad x_{n-1} \mapsto \hat{a}_{n-1}] \\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}_0\ ... \\ &\qquad \hat{a}_{n-1} \mapsto \hat{v}_{n-1}] \end{aligned}$$

$$\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}}) \\ \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \hat{v}_f &= ((\lambda\ x\ e_b), \hat{\rho}_\lambda) \\ \hat{a} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}] \\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{v}] \end{aligned}$$

$$\textbf{appappk}(\varnothing, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ \leadsto E\langle e, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \hat{a}''_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa}' &\triangleq \textbf{appappk}(\hat{v}, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}'] \end{aligned}$$

$$\textbf{appk}(\widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}}) \\ \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \widehat{done} &= ((\lambda\ (x_s ...)\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s \\ \hat{a}_i &\triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa}) \\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0\ ... \\ &\qquad x_{n-1} \mapsto \hat{a}_{n-1}] \\ \hat{v}'_s &\triangleq \hat{v}_s +\!\!+ [\hat{v}] \\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}'_0\ ... \\ &\qquad \hat{a}_{n-1} \mapsto \hat{v}'_{n-1}] \end{aligned}$$

$$\textbf{appk}(\widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}}) \\ \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \widehat{done} &= ((\lambda\ x\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s \\ \hat{a} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\rho}'_\lambda &\triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}] \\ \hat{v}'_s &\triangleq \hat{v}_s +\!\!+ [\hat{v}] \\ \hat{\sigma}' &\triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{v}'_s] \end{aligned}$$

$$\textbf{appk}([\hat{\kappa}_\lambda], [\,], \hat{\rho}_{\hat{\kappa}}, \_) \leadsto A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}_\lambda] \end{aligned}$$

$$\textbf{appk}(\widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ \leadsto E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\begin{aligned} \text{where } \hat{a}''_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa}' &\triangleq \textbf{appk}(\widehat{done} +\!\!+ [\hat{v}], e_t, \hat{\rho}_\kappa, \hat{a}'_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}'] \end{aligned}$$