1 Concrete Semantics of Scheme

Syntax Domains:

$$
\begin{aligned}
e \in \mathsf{Exp} ::=\ & æ^\ell \\
& \mid (\texttt{if}\ e^\ell\ e^\ell\ e^\ell)^\ell \\
& \mid (\texttt{let}\ ([x\ e^\ell]\ ...)\ e^\ell)^\ell \\
& \mid (\texttt{call/cc}\ e^\ell)^\ell \\
& \mid (\texttt{set!}\ x\ e^\ell)^\ell \\
& \mid (\texttt{prim}\ op\ e^\ell\ ...)^\ell \\
& \mid (\texttt{apply-prim}\ op\ e^\ell)^\ell \\
& \mid (\texttt{apply}\ e^\ell\ e^\ell)^\ell \\
& \mid (e^\ell\ e^\ell\ ...)^\ell \\
æ \in \mathsf{AExp} ::=\ & x \mid lam \mid \mathbb{Z} \mid \texttt{\#t} \mid \texttt{\#f} \\
& \mid (\texttt{quote}\ e) \\
lam \in \mathsf{Lam} ::=\ & (\lambda\ (x...)\ e^\ell) \mid (\lambda\ x\ e^\ell) \\
x \in \mathsf{Var} \quad & \text{A set of identifiers} \\
op \in \mathsf{Prim} \quad & \text{A set of primitives} \\
\ell \in \mathsf{Label} \quad & \text{A set of labels}
\end{aligned}
$$

Atomic Evaluation:

$$
\begin{aligned}
&\mathcal{A} : \Sigma_E \rightharpoonup Val \\
&\mathcal{A}(\langle lam, \rho, ...\rangle) \triangleq (lam, \rho) \\
&\mathcal{A}(\langle (\texttt{quote}\ e), ...\rangle) \triangleq \mathbf{quote}(e) \\
&\mathcal{A}(\langle x, \rho, \sigma, ...\rangle) \triangleq \sigma(\rho(x)) \\
&\mathcal{A}(\langle æ, ...\rangle) \triangleq æ
\end{aligned}
$$

Injection:

$$
\begin{aligned}
&\mathcal{I} : \mathsf{Exp} \to \Sigma_E \\
&\mathcal{I}(e) \triangleq E\langle e, \varnothing, \varnothing, \{(0,0) : \mathbf{mt}\}, (0,0)\rangle
\end{aligned}
$$

Store Joining:

$$
\begin{aligned}
\sigma \sqcup [a \mapsto v] &\triangleq \sigma[a \mapsto v] \\
\sigma_\kappa \sqcup [a_\kappa \mapsto \kappa] &\triangleq \sigma_\kappa[a_\kappa \mapsto \kappa]
\end{aligned}
$$

Collecting Semantics:

$$eval(e) = \{\varsigma \mid \mathcal{I}(e) \mapsto^* \varsigma\}$$

Semantic Domains:

$$
\begin{aligned}
\varsigma \in \Sigma &\triangleq E\langle Eval\rangle + A\langle Apply\rangle \\
Eval &\triangleq \mathsf{Exp} \times Env \times Store \\
&\quad \times KStore \times Addr \\
Apply &\triangleq Val \times Store \\
&\quad \times KStore \times Addr \\
\rho \in Env &\triangleq \mathsf{Var} \rightharpoonup Addr \\
\sigma \in Store &\triangleq Addr \rightharpoonup Val \\
\sigma_\kappa \in KStore &\triangleq Addr \rightharpoonup Kont \\
v \in Val &\triangleq Clo + Kont + \mathbb{Z} \\
&\quad + \{\texttt{\#t}, \texttt{\#f}, Null, Void\} \\
&\quad + \{\mathbf{quote}(e), \mathbf{cons}(v, v)\} \\
clo \in Clo &\triangleq \mathsf{Lam} \times Env \\
a, a_\kappa \in Addr &\triangleq Kont \times \mathsf{Label} \times \mathbb{N} \\
\kappa \in Kont &::= \mathbf{mt} \\
&\mid \mathbf{ifk}(e, e, \rho, a) \\
&\mid \mathbf{setk}(x, \rho, a) \\
&\mid \mathbf{callcck}(a, \ell) \\
&\mid \mathbf{appappk}(val?, e, \rho, a, \ell) \\
&\mid \mathbf{appk}(done, todo, \rho, a, \ell) \\
&\mid \mathbf{appprimk}(op, a) \\
&\mid \mathbf{primk}(op, done, todo, \\
&\qquad \rho, a, \ell) \\
&\mid \mathbf{letk}(vars, done, todo, \\
&\qquad e, \rho, a, \ell) \\
done &\triangleq Val^* \\
todo &\triangleq \mathsf{Exp}^* \\
vars &\triangleq \mathsf{Var}^*
\end{aligned}
$$

Address Allocation:

$$
\begin{aligned}
&alloc : Kont \times \mathsf{Label} \times \mathbb{N} \to Addr \\
&alloc(\kappa, \ell, n) \triangleq (\kappa, \ell, n)
\end{aligned}
$$

*Labels may be omitted for brevity if unneeded

Eval Rules

Rules for when the control is an expression

$$E\langle æ, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto A\langle v, \sigma, \sigma_\kappa, a_\kappa\rangle$$
where $v \triangleq \mathcal{A}(\varsigma)$

$$E\langle(\texttt{if}\ e_c\ e_t\ e_f)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e_c, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{ifk}(e_t, e_f, \rho, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{let}\ ()\ e), \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle$$

$$E\langle(\texttt{let}\ bs\ e_b)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $bs = ([x_0\ e_0]\ [x_s\ e_s]\ ...)$

$a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \rho, a_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{call/cc}\ e)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{callcck}(a_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{set!}\ x\ e)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$a \triangleq \rho(x)$

$\kappa \triangleq \mathbf{setk}(a, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{prim}\ op), \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto A\langle v, \sigma, \sigma_\kappa, a_\kappa\rangle$$
where $v = op$ applied to 0 arguments

$$E\langle(\texttt{prim}\ op\ e_0\ e_s\ ...)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{primk}(op, [\,], e_s, \rho, a_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{apply-prim}\ op\ e)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{appprimk}(op, a_\kappa)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(\texttt{apply}\ e_f\ e_x)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{appappk}(\varnothing, e_x, \rho, a_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

$$E\langle(e_f\ e_s\ ...)^\ell, \rho, \sigma, \sigma_\kappa, a_\kappa\rangle \leadsto E\langle e_f, \rho, \sigma, \sigma'_\kappa, a'_\kappa\rangle$$
where $a'_\kappa \triangleq alloc(\sigma_\kappa(a_\kappa), \ell, 0)$

$\kappa \triangleq \mathbf{appk}([\,], e_s, \rho, a_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa]$

Apply Rules

Rules for when the control is a value

$\varsigma = A\langle v, \sigma, \sigma_\kappa, a_\kappa \rangle$

$\kappa \triangleq \sigma_\kappa(a_\kappa)$

Proceed by matching on κ

$$\textbf{mt} \leadsto \varsigma$$

$$\textbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \leadsto E\langle e_f, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa \rangle$$
where $v = \texttt{\#f}$

$$\textbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \leadsto E\langle e_t, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa \rangle$$
where $v \neq \texttt{\#f}$

$$\textbf{letk}(vars, done, [\,], e_b, \rho_\kappa, a'_\kappa, \ell) \leadsto E\langle e_b, \rho'_\kappa, \sigma', \sigma_\kappa, a'_\kappa \rangle$$
where $a_i \triangleq alloc(\kappa, \ell, i)$

$done' \triangleq done +\!\!+ [v]$

$\rho'_\kappa \triangleq \rho_\kappa[vars_0 \mapsto a_0 \; ... \; vars_{n-1} \mapsto a_{n-1}]$

$\sigma' \triangleq \sigma \sqcup [a_0 \mapsto done'_0 \; ... \; a_{n-1} \mapsto done'_{n-1}]$

$$\textbf{letk}(vars, done, e_h :: e_t, e_b, \rho_\kappa, a'_\kappa, \ell) \leadsto E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa \rangle$$
where $a''_\kappa \triangleq alloc(\kappa, \ell, 0)$

$\kappa' \triangleq \textbf{letk}(vars, done +\!\!+ [v], e_t, e_b, \rho_\kappa, a'_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a''_\kappa \mapsto \kappa']$

$$\textbf{setk}(a, a'_\kappa) \leadsto A\langle Void, \sigma', \sigma_\kappa, a'_\kappa \rangle$$
where $\sigma' \triangleq \sigma \sqcup [a \mapsto v]$

$$\textbf{callcck}(a'_\kappa, \ell) \leadsto E\langle e, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa \rangle$$
where $v = ((\lambda\ (x)\ e), \rho_\lambda)$

$a \triangleq alloc(\kappa, \ell, 0)$

$\kappa' \triangleq \sigma_\kappa(a'_\kappa)$

$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]$

$\sigma' \triangleq \sigma \sqcup [a \mapsto \kappa']$

$$\textbf{callcck}(\_, \ell) \leadsto A\langle \kappa, \sigma, \sigma'_\kappa, a'_\kappa \rangle$$
where $v = \kappa'$

$a'_\kappa \triangleq alloc(\kappa, \ell, 0)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a'_\kappa \mapsto \kappa']$

$$\textbf{appprimk}(op, a'_\kappa) \leadsto A\langle v', \sigma, \sigma_\kappa, a'_\kappa)\rangle$$
where $v' \triangleq op$ applied to v

$$\textbf{primk}(op, done, [\,], \_, a'_\kappa, \_) \leadsto A\langle v', \sigma, \sigma_\kappa, a'_\kappa \rangle$$
where $v' \triangleq op$ applied to $(done +\!\!+ [v])$

$$\textbf{primk}(op, done, e_h :: e_t, \rho_\kappa, a'_\kappa, \ell) \leadsto E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa \rangle$$
where $a''_\kappa \triangleq alloc(\kappa, \ell, 0)$

$\kappa' \triangleq \textbf{primk}(op, done +\!\!+ [v], e_t, \rho_\kappa, a'_\kappa, \ell)$

$\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a''_\kappa \mapsto \kappa']$

More Apply Rules

Rules for when the control is a value

$$\begin{aligned}&\textbf{appappk}(v_f, \_, \_, a'_\kappa, \ell)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa\rangle\\ &\text{where } v_f = ((\lambda\ (x_s ...)\ e_b), \rho_\lambda)\\ &a_i \triangleq alloc(\kappa, \ell, i)\\ &\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto a_0\ ...\\ &\qquad x_{n-1} \mapsto a_{n-1}]\\ &\sigma' \triangleq \sigma \sqcup [a_0 \mapsto v_0\ ...\\ &\qquad a_{n-1} \mapsto v_{n-1}]\end{aligned}$$

$$\begin{aligned}&\textbf{appappk}(v_f, \_, \_, a'_\kappa, \ell)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa\rangle\\ &\text{where } v_f = ((\lambda\ x\ e_b), \rho_\lambda)\\ &a \triangleq alloc(\kappa, \ell, 0)\\ &\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]\\ &\sigma' \triangleq \sigma \sqcup [a \mapsto v]\end{aligned}$$

$$\begin{aligned}&\textbf{appappk}(\varnothing, e, \rho_\kappa, a'_\kappa, \ell)\\ &\rightsquigarrow E\langle e, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa\rangle\\ &\text{where } a''_\kappa \triangleq alloc(\kappa, \ell, 0)\\ &\kappa' \triangleq \textbf{appappk}(v, e, \rho_\kappa, a'_\kappa, \ell)\\ &\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a''_\kappa \mapsto \kappa']\end{aligned}$$

$$\begin{aligned}&\textbf{appk}(done, [\,], \_, a'_\kappa, \ell)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa\rangle\\ &\text{where } done = ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) :: v_s\\ &a_i \triangleq alloc(\kappa, \ell, i)\\ &v'_s \triangleq v_s +\!\!+ [v]\\ &\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto a_0\ ...\\ &\qquad x_{n-1} \mapsto a_{n-1}]\\ &\sigma' \triangleq \sigma \sqcup [a_0 \mapsto v'_0\ ...\\ &\qquad a_{n-1} \mapsto v'_{n-1}]\end{aligned}$$

$$\begin{aligned}&\textbf{appk}(done, [\,], \_, a'_\kappa, \ell)\\ &\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa\rangle\\ &\text{where } done = ((\lambda\ x\ e_b), \rho_\lambda) :: v_s\\ &a \triangleq alloc(\kappa, \ell, 0)\\ &v'_s \triangleq v_s +\!\!+ [v]\\ &\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]\\ &\sigma' \triangleq \sigma \sqcup [a \mapsto v'_s]\end{aligned}$$

$$\begin{aligned}&\textbf{appk}([\kappa_\lambda], [\,], \rho_\kappa, a'_\kappa, \ell) \rightsquigarrow A\langle v, \sigma, \sigma'_\kappa, a''_\kappa\rangle\\ &\text{where } a''_\kappa \triangleq alloc(\kappa, \ell, 0)\\ &\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a''_\kappa \mapsto \kappa_\lambda]\end{aligned}$$

$$\begin{aligned}&\textbf{appk}(done, e_h :: e_t, \rho_\kappa, a'_\kappa, \ell)\\ &\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa\rangle\\ &\text{where } a''_\kappa \triangleq alloc(\kappa, \ell, 0)\\ &\kappa' \triangleq \textbf{appk}(done +\!\!+ [v], e_t, \rho_\kappa, a'_\kappa, \ell)\\ &\sigma'_\kappa \triangleq \sigma_\kappa \sqcup [a''_\kappa \mapsto \kappa']\end{aligned}$$

2 Abstract Semantics of Scheme

Abstract Semantic Domains:

$$\hat{\varsigma} \in \hat{\Sigma} \triangleq E\langle Eval\rangle + E\langle Apply\rangle$$

$$Eval \triangleq \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time}$$

$$Apply \triangleq \widehat{Val} \times \widehat{Store} \times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time}$$

$$\hat{\rho} \in \widehat{Env} \triangleq \mathsf{Var} \rightharpoonup \widehat{Addr}$$

$$\hat{\sigma} \in \widehat{Store} \triangleq \widehat{Addr} \rightharpoonup \mathcal{P}(Val)$$

$$\hat{\sigma}_\kappa \in \widehat{KStore} \triangleq \widehat{Addr} \rightharpoonup \mathcal{P}(\widehat{Kont})$$

$$\hat{v} \in \widehat{Val} \triangleq \widehat{Clo} + \widehat{Kont} + \mathbb{Z} + \{\texttt{\#t}, \texttt{\#f}, Null, Void\} + \{\mathbf{quote}(e), \mathbf{cons}(\hat{v}, \hat{v})\}$$

$$\widehat{clo} \in \widehat{Clo} \triangleq \mathsf{Lam} \times \widehat{Env}$$

$$\hat{a}, \hat{a}_{\hat{\kappa}} \in Addr \triangleq \mathbb{N}$$

$$\hat{t} \in Time \triangleq \mathbb{N}$$

$$\widehat{done} \triangleq \widehat{Val}^*$$

$$\begin{aligned}\hat{\kappa} \in \widehat{Kont} ::= {} & \mathbf{mt} \\ & \mid \mathbf{ifk}(e, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{callcck}(\hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{appappk}(\widehat{val}?, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{appk}(\widehat{done}, todo, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{appprimk}(op, \hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{primk}(op, \widehat{done}, todo, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ & \mid \mathbf{letk}(vars, \widehat{done}, todo, e, \hat{\rho}, \hat{a}_{\hat{\kappa}})\end{aligned}$$

Tick/Alloc:

$$\widehat{tick} : \hat{\Sigma} \times \widehat{Kont} \to \widehat{Time}$$

$$\widehat{tick}(\hat{\varsigma}, \hat{\kappa}) \triangleq 0$$

$$\widehat{alloc} : \hat{\Sigma} \times \widehat{Kont} \to \widehat{Addr}$$

$$\widehat{alloc}(\hat{\varsigma}, \hat{\kappa}) \triangleq 0$$

Abstract Atomic Evaluation:

$$\mathcal{A} : Eval \to \widehat{Val}$$

$$\mathcal{A}(E\langle lam, \hat{\rho}, ...\rangle) \triangleq \{(lam, \hat{\rho})\}$$

$$\mathcal{A}(æ) \triangleq \{æ\}$$

Abstract Eval Rules

Rules for when the control is an expression

$$E\langle æ, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle$$
$$\text{where } \hat{v} \triangleq \hat{\mathcal{A}}(\hat{\varsigma}, \hat{\sigma})$$

$$E\langle (\texttt{if}\ e_c\ e_t\ e_f), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_c, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{ifk}(e_t, e_f, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{let}\ ()\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle$$

$$E\langle (\texttt{let}\ bnds\ e_b), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } bnds &= ([x_0\ e_0]\ [x_s\ e_s]\ ...) \\ \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{call/cc}\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{callcck}(\hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{set!}\ x\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{a} &\triangleq \hat{\rho}(x) \\ \hat{\kappa} &\triangleq \mathbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{prim}\ op), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle$$
$$\text{where } \hat{v} = op \text{ applied to 0 arguments}$$

$$E\langle (\texttt{prim}\ op\ e_0\ e_s\ ...), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{primk}(op, [\,], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{apply-prim}\ op\ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{appprimk}(op, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (\texttt{apply}\ e_f\ e_x), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{appappk}(\varnothing, e_x, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

$$E\langle (e_f\ e_s\ ...), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t}\,\rangle \rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\begin{aligned} \text{where } \hat{a}'_{\hat{\kappa}} &\triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' &\triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\kappa} &\triangleq \mathbf{appk}([\,], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\ \hat{\sigma}'_{\hat{\kappa}} &\triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}] \end{aligned}$$

Apply Rules

Rules for when the control is a value

$$\hat{\varsigma} = A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$$

$$\hat{\kappa} \in \hat{\sigma}_{\hat{\kappa}}(\hat{a}_{\hat{\kappa}})$$

Proceed by matching on $\hat{\kappa}$

$$\mathbf{mt} \rightsquigarrow \hat{\varsigma}$$

$$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_f, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$
where $\hat{v} = \texttt{\#f}$

$$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_t, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$
where $\hat{v} \neq \texttt{\#f}$

$$\mathbf{letk}(vars, \widehat{done}, [\,], e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_b, \hat{\rho}'_{\hat{\kappa}}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$
where
$$\hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$
$$\hat{\rho}'_{\hat{\kappa}} \triangleq \hat{\rho}_{\hat{\kappa}}[vars_0 \mapsto \hat{a}_0 \ldots vars_{n-1} \mapsto \hat{a}_{n-1}]$$
$$\widehat{done}' \triangleq \widehat{done} + [\hat{v}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \widehat{done}'_0 \ldots \hat{a}_{n-1} \mapsto \widehat{done}'_{n-1}]$$

$$\mathbf{letk}(vars, \widehat{done}, e_h :: e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}' \rangle$$
where
$$\hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \mathbf{letk}(vars, \widehat{done}, +[\hat{v}], e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

$$\mathbf{setk}(x, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle Void, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$
where $\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{\rho}_{\hat{\kappa}}(x) \mapsto \hat{v}]$

$$\mathbf{callcck}(\hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$
where $\hat{v} = ((\lambda\ (x)\ e), \hat{\rho}_{\lambda})$
$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \hat{\sigma}_{\hat{\kappa}}(\hat{a}'_{\hat{\kappa}})$$
$$\hat{\rho}'_{\lambda} \triangleq \hat{\rho}_{\lambda}[x \mapsto \hat{a}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{\kappa}']$$

$$\mathbf{callcck}(\_) \rightsquigarrow A\langle \hat{\kappa}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$
where $v = \kappa'$
$$\hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \kappa']$$

$$\mathbf{appprimk}(op, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$
where $\hat{v}' \triangleq op$ applied to $\hat{v}$

$$\mathbf{primk}(op, \widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$
where $\hat{v}' \triangleq op$ applied to $(\widehat{done} + [\hat{v}])$

$$\mathbf{primk}(op, \widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}' \rangle$$
where
$$\hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \mathbf{primk}(op, \widehat{done} + [\hat{v}])\ e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

More Apply Rules

Rules for when the control is a value

$$\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}}) \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{v}_f = ((\lambda\ (x_s ...)\ e_b), \hat{\rho}_\lambda)$$
$$\hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0\ ...\ x_{n-1} \mapsto \hat{a}_{n-1}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}_0\ ...\ \hat{a}_{n-1} \mapsto \hat{v}_{n-1}]$$

$$\textbf{appappk}(\hat{v}_f, \_, \_, \hat{a}'_{\hat{\kappa}}) \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{v}_f = ((\lambda\ x\ e_b), \hat{\rho}_\lambda)$$
$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{v}]$$

$$\textbf{appappk}(\varnothing, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \leadsto E\langle e, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \textbf{appappk}(\hat{v}, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

$$\textbf{appk}(\widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}}) \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \widehat{done} = ((\lambda\ (x_s ...)\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s$$
$$\hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x_0 \mapsto \hat{a}_0\ ...\ x_{n-1} \mapsto \hat{a}_{n-1}]$$
$$\hat{v}'_s \triangleq \hat{v}_s +\!\!+ [\hat{v}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}'_0\ ...\ \hat{a}_{n-1} \mapsto \hat{v}'_{n-1}]$$

$$\textbf{appk}(\widehat{done}, [\,], \_, \hat{a}'_{\hat{\kappa}}) \leadsto E\langle e_b, \hat{\rho}'_\lambda, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \widehat{done} = ((\lambda\ x\ e_b), \hat{\rho}_\lambda) :: \hat{v}_s$$
$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\rho}'_\lambda \triangleq \hat{\rho}_\lambda[x \mapsto \hat{a}]$$
$$\hat{v}'_s \triangleq \hat{v}_s +\!\!+ [\hat{v}]$$
$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{v}'_s]$$

$$\textbf{appk}([\hat{\kappa}_\lambda], [\,], \hat{\rho}_{\hat{\kappa}}, \_) \leadsto A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}_\lambda]$$

$$\textbf{appk}(\widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \leadsto E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}'\rangle$$
$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$
$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$
$$\hat{\kappa}' \triangleq \textbf{appk}(\widehat{done} +\!\!+ [\hat{v}], e_t, \hat{\rho}_\kappa, \hat{a}'_{\hat{\kappa}})$$
$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$