

# 1 Concrete Semantics of Scheme CESK\*

## Syntax:

$$\begin{aligned}
 e \in \text{Exp} &::= \mathfrak{x} \\
 &| (\text{if } e \ e \ e) \\
 &| (\text{let } (x \ e) \ e) \\
 &| (\text{prim } op \ e \ e \dots) \\
 &| (\text{call/cc } e) \\
 &| (e \ e \ \dots) \\
 \mathfrak{x} \in \text{AExp} &::= lam \mid \mathbb{Z} \mid \#t \mid \#f \\
 lam \in \text{Lam} &::= (\lambda (x \dots) \ e) \\
 x \in \text{Var} &\quad \text{A set of identifiers}
 \end{aligned}$$

## Semantics:

$$\begin{aligned}
 \varsigma \in \Sigma &\triangleq \text{Exp} \times \text{Env} \times \text{Kont} \\
 \rho \in \text{Env} &\triangleq \text{Var} \rightarrow \text{Addr} \\
 \sigma \in \text{Store} &\triangleq \text{Addr} \rightarrow \text{Val} \\
 v \in \text{Val} &\triangleq \text{Clo} + \text{Kont} + \mathbb{Z} + \{\#t, \#f\} \\
 clo \in \text{Clo} &\triangleq \text{Lam} \times \text{Env} \\
 \kappa \in \text{Kont} &\triangleq \mathbf{mt} \mid \mathbf{appk}(done, todo, \rho, a) \\
 &| \mathbf{ifk}(e, e, \rho, a) \\
 &| \mathbf{letk}(x, e, \rho, a) \\
 a, b, c \in \text{Addr} &\quad \text{A set of addresses} \\
 done &\triangleq \text{Val}^* \quad todo \triangleq \text{Exp}^*
 \end{aligned}$$

## Atomic Evaluation Function:

$$\begin{aligned}
 \mathcal{A}(x, \rho, \sigma) &\triangleq \sigma(\rho(x)) \\
 \mathcal{A}(lam, \rho, \sigma) &\triangleq (lam, \rho) \\
 \mathcal{A}(\mathfrak{x}, \rho, \sigma) &\triangleq \mathfrak{x}
 \end{aligned}$$

## Transition Function:

$$(\Sigma \times \text{Store}) \rightsquigarrow (\Sigma \times \text{Store})$$

$(\varsigma \times \sigma) \rightsquigarrow (\varsigma' \times \sigma)$ , where  $\kappa = \sigma(a)$ ,  $b = \text{alloc}(\varsigma)$   
 proceed by matching on  $\varsigma$

$\langle (\text{if } e_c \ e_t \ e_f), \rho, a \rangle$	$\langle e_c, \rho, b \rangle$ $\sigma[b \mapsto \text{ifk}(e_t, e_f, \rho, a)]$
$\langle (\text{let } (x \ e_x) \ e_b), \rho, a \rangle$	$\langle e_x, \rho, b \rangle$ $\sigma[b \mapsto \text{letk}(x, e_b, \rho, a)]$
$\langle (\text{prim } op \ e_0 \ es...), \rho, a \rangle$	$\langle e_0, \rho, b \rangle$ $\sigma[b \mapsto \text{appk}([op], es, \rho, a)]$
$\langle (\text{call/cc } e), \rho, a \rangle$	$\langle e, \rho, b \rangle$ $\sigma[b \mapsto \text{appk}([\text{call/cc}], [], \rho, a)]$
$\langle (e_f \ es...), \rho, a \rangle$	$\langle e_f, \rho, b \rangle$ $\sigma[b \mapsto \text{appk}([], es, \rho, a)]$
$\langle \mathfrak{x}, \rho, a \rangle$ let $v = \mathcal{A}(\mathfrak{x}, \rho, \sigma)$ match on $\kappa$ below	
<b>mt</b>	$\varsigma$
<b>ifk</b> $(e_t, e_f, \rho', c)$ when $v = \#f$	$\langle e_f, \rho', c \rangle$
<b>ifk</b> $(e_t, e_f, \rho', c)$ when $v \neq \#f$	$\langle e_t, \rho', c \rangle$
<b>letk</b> $(x, e_b, \rho', c)$	$\langle e_b, \rho'[x \mapsto b], c \rangle$ $\sigma[b \mapsto v]$
<b>appk</b> $(done, e_h :: e_t, \rho', c)$	$\langle e_h, \rho', b \rangle$ $\sigma[b \mapsto \text{appk}(done \# [v], e_t, \rho', c)]$
<b>appk</b> $(op :: v_s, [], \rho', c)$	$\langle v', \rho', c \rangle$ $v' = op \text{ applied to } (v_s \# [v])$
<b>appk</b> $(clo :: v_s, [], \rho', c)$ where $clo = ((\lambda (x \dots) e_b), \rho_\lambda)$	$\langle e_b, \rho_\lambda[x_{s0} \mapsto b_0 \dots x_{si} \mapsto b_i], c \rangle$ $v'_s = v_s \# [v]$ $\sigma[b_0 \mapsto v'_{s0} \dots b_i \mapsto v'_{si}]$
<b>appk</b> $([\text{call/cc}], [], \rho', c)$ where $v = ((\lambda (x) e), \rho_\lambda)$	$\langle e, \rho_\lambda[x \mapsto c], c \rangle$
<b>appk</b> $([\kappa_v], [], \rho', c)$	$\langle v, \rho', b \rangle$ $\sigma[b \mapsto \kappa_v]$

## 2 Abstract Semantics of Scheme CESK\*