1 Concrete Semantics of Scheme CESK*

Syntax Domains:

$$
\begin{aligned}
e \in \mathsf{Exp} ::=\ & æ \\
\mid\ & (\texttt{if}\ e\ e\ e) \\
\mid\ & (\texttt{let}\ ([x\ e]\ ...)\ e) \\
\mid\ & (\texttt{call/cc}\ e) \\
\mid\ & (\texttt{set!}\ x\ e) \\
\mid\ & (\texttt{prim}\ op\ e\ ...) \\
\mid\ & (\texttt{apply-prim}\ op\ e) \\
\mid\ & (\texttt{apply}\ e\ e) \\
\mid\ & (e\ e\ ...)
\end{aligned}
$$

$$æ \in \mathsf{AExp} ::= x \mid lam \mid \mathbb{Z} \mid \texttt{\#t} \mid \texttt{\#f} \mid (\texttt{quote}\ e)$$

$$lam \in \mathsf{Lam} ::= (\lambda\ (x...)\ e) \mid (\lambda\ x\ e)$$

$x \in \mathsf{Var}$ A set of identifiers

$op \in \mathsf{Prim}$ A set of primitives

Atomic Evaluation:

$$\mathcal{A} : \Sigma_E \times \sigma \rightharpoonup Val$$
$$\mathcal{A}(\langle lam, \rho, \_, \_\rangle, \_) \triangleq (lam, \rho)$$
$$\mathcal{A}(\langle (\texttt{quote}\ e), \_, \_, \_\rangle, \_) \triangleq \mathbf{quote}(e)$$
$$\mathcal{A}(\langle x, \rho, \_, \_\rangle, \sigma) \triangleq \sigma(\rho(x))$$
$$\mathcal{A}(\langle æ, \_, \_, \_\rangle, \_) \triangleq æ$$

Tick/Alloc:

$$tick : \Sigma \times \mathbb{N} \to Time$$
$$tick(\langle \_, \_, \_, t\rangle, n) \triangleq (t + n)$$
$$alloc : \Sigma \times \mathbb{N} \triangleq Addr$$
$$alloc(\langle \_, \_, \_, t\rangle, n) \triangleq (t + n)$$

Injection:

$$\mathcal{I} : \mathsf{Exp} \to \Sigma$$
$$\mathcal{I}(e) \triangleq (e, \varnothing, 0, 1)$$

Initial σ state $\triangleq \{0 : \mathbf{mt}\}$

Transition:

Collecting Semantics:

Semantic Domains:

$$
\begin{aligned}
\varsigma \in \Sigma &\triangleq E\langle Eval\rangle + A\langle Apply\rangle \\
Eval &\triangleq \mathsf{Exp} \times Env \\
&\quad \times Addr \times Time \\
Apply &\triangleq Val \times Env \\
&\quad \times Addr \times Time \\
\rho \in Env &\triangleq \mathsf{Var} \rightharpoonup Addr \\
\sigma \in Store &\triangleq Addr \rightharpoonup Val \\
v \in Val &\triangleq Clo + Kont + \mathbb{Z} \\
&\quad + \{\texttt{\#t}, \texttt{\#f}, Null, Void\} \\
&\quad + \{\mathbf{quote}(e)\} \\
&\quad + \{\mathbf{cons}(v, v)\} \\
clo \in Clo &\triangleq \mathsf{Lam} \times Env \\
a, b, c \in Addr &\triangleq \mathbb{N} \\
t, u \in Time &\triangleq \mathbb{N} \\
\kappa \in Kont &::= \mathbf{mt} \\
&\mid \mathbf{ifk}(e, e, \rho, a) \\
&\mid \mathbf{callcck}(\rho, a) \\
&\mid \mathbf{setk}(x, \rho, a) \\
&\mid \mathbf{appappk}(val?, e, \rho, a) \\
&\mid \mathbf{appk}(done, todo, \rho, a) \\
&\mid \mathbf{appprimk}(op, \rho, a) \\
&\mid \mathbf{primk}(op, done, todo, \\
&\qquad \rho, a) \\
&\mid \mathbf{letk}(vars, done, todo \\
&\qquad e, \rho, a) \\
done &\triangleq Val^* \\
todo &\triangleq \mathsf{Exp}^* \\
vars &\triangleq \mathsf{Var}^*
\end{aligned}
$$

Eval Rules

Rules for when the control is an expression

$$E\langle æ, \rho, a, t\rangle \rightsquigarrow A\langle v, \rho, a, t\rangle$$
$$\text{where } v \triangleq \mathcal{A}(\varsigma, \sigma)$$

$$E\langle(\texttt{if}\ e_c\ e_t\ e_f), \rho, a, t\rangle \rightsquigarrow E\langle e_c, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{ifk}(e_t, e_f, \rho, a)]$$

$$E\langle(\texttt{let}\ ()\ e), \rho, a, t\rangle \rightsquigarrow E\langle e, \rho, a, u\rangle$$
$$\text{where } u \triangleq \mathit{tick}(\varsigma, 1)$$

$$E\langle(\texttt{let}\ ([x_0\ e_0]\ [x_s\ e_s]\ ...)\ e_b), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e_0, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \rho, a)]$$

$$E\langle(\texttt{call/cc}\ e), \rho, a, t\rangle \rightsquigarrow E\langle e, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{callcck}(\rho, a)]$$

$$E\langle(\texttt{set!}\ x\ e), \rho, a, t\rangle \rightsquigarrow E\langle e, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{setk}(x, a)]$$

$$E\langle(\texttt{prim}\ \mathit{op}), \rho, a, t\rangle \rightsquigarrow A\langle v, \rho, a, t\rangle$$
$$\text{where } v = \mathit{op} \text{ applied to 0 arguments}$$

$$E\langle(\texttt{prim}\ \mathit{op}\ e_0\ e_s\ ...), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e_0, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{primk}(\mathit{op}, [\,], e_s, \rho, a)]$$

$$E\langle(\texttt{apply-prim}\ \mathit{op}\ e), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{appprimk}(\mathit{op}, a)]$$

$$E\langle(\texttt{apply}\ e_f\ e_x), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e_f, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{appappk}(\varnothing, e_x, \rho, a)]$$

$$E\langle(e_f\ e_s\ ...), \rho, a, t\rangle \rightsquigarrow E\langle e_f, \rho, b, u\rangle$$
$$\text{where } b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{appk}([\,], e_s, \rho, a)]$$

Apply Rules

Rules for when the control is a value

$\varsigma = A\langle v, \rho, a, t\rangle$

$\kappa \triangleq \sigma(a)$

Proceed by matching on κ

$\mathbf{mt} \rightsquigarrow \varsigma$

$\mathbf{ifk}(e_t, e_f, \rho, c) \rightsquigarrow E\langle e_f, \rho, c, t\rangle$
where $v = \texttt{\#f}$

$\mathbf{ifk}(e_t, e_f, \rho, c) \rightsquigarrow E\langle e_t, \rho, c, t\rangle$
where $v \neq \texttt{\#f}$

$\mathbf{letk}(vars, done, [\,], e_b, \rho_\kappa, c)$
$\rightsquigarrow E\langle e_b, \rho'_\kappa, c, u\rangle$
where $b_i \triangleq alloc(\varsigma, i)$
$u \triangleq tick(\varsigma, n)$
$done' \triangleq done +\!\!+ [v]$
$\rho'_\kappa \triangleq \rho_\kappa[vars_0 \mapsto b_0 \ldots$
$vars_{n-1} \mapsto b_{n-1}]$
$\sigma[b_0 \mapsto done'_0 \ldots b_{n-1} \mapsto done'_{n-1}]$

$\mathbf{letk}(vars, done, e_h :: e_t, e_b, \rho_\kappa, c)$
$\rightsquigarrow E\langle e_h, \rho_\kappa, b, u\rangle$
where $b \triangleq alloc(\varsigma, 0)$
$u \triangleq tick(\varsigma, 1)$
$\sigma[b \mapsto \mathbf{letk}(vars, done +\!\!+ [v],$
$e_t, e_b, \rho_\kappa, c)]$

$\mathbf{callcck}(\_, c) \rightsquigarrow E\langle e, \rho'_\lambda, c, t\rangle$
where $v = ((\lambda\ (x)\ e), \rho_\lambda)$
$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto c]$

$\mathbf{callcck}(\rho', \_) \rightsquigarrow A\langle \kappa, \rho', b, u\rangle$
where $v = \kappa'$
$b \triangleq alloc(\varsigma, 0)$
$u \triangleq tick(\varsigma, 1)$
$\sigma[b \mapsto \kappa']$

$\mathbf{setk}(x, \rho_\kappa, c) \rightsquigarrow A\langle Void, \rho_\kappa, c, t\rangle$
where $\sigma[\rho_\kappa(x) \mapsto v]$

$\mathbf{appprimk}(op, \rho_\kappa, c) \rightsquigarrow A\langle v', \rho_\kappa, c, t\rangle$
where $v' \triangleq op$ applied to v

$\mathbf{primk}(op, done, [\,], \rho_\kappa, c)$
$\rightsquigarrow A\langle v', \rho_\kappa, c, t\rangle$
where $v' \triangleq op$ applied to $(done +\!\!+ [v])$

$\mathbf{primk}(op, done, e_h :: e_t, \rho_\kappa, c)$
$\rightsquigarrow E\langle e_h, \rho_\kappa, b, u\rangle$
where $b \triangleq alloc(\varsigma, 0)$
$u \triangleq tick(\varsigma, 1)$
$\sigma[b \mapsto \mathbf{primk}(op, done +\!\!+ [v],$
$e_t, \rho_\kappa, c)]$

More Apply Rules

Rules for when the control is a value

$$\textbf{appappk}(\varnothing, e, \rho_\kappa, c) \leadsto E\langle e, \rho_\kappa, b, u\rangle$$
$$\begin{aligned}\text{where } b &\triangleq alloc(\varsigma, 0)\\ u &\triangleq tick(\varsigma, 1)\\ &\sigma[b \mapsto \textbf{appappk}(v, e, \rho_\kappa, c)]\end{aligned}$$

$$\textbf{appappk}(v_f, \_, \rho_\kappa, c) \leadsto E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned}\text{where } v_f &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda)\\ b_i &\triangleq alloc(\varsigma, i)\\ u &\triangleq tick(\varsigma, n)\\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto b_0\ ...\\ &\qquad x_{n-1} \mapsto b_{n-1}]\\ &\sigma[b_0 \mapsto v_0\ ...\ b_{n-1} \mapsto v_{n-1}]\end{aligned}$$

$$\textbf{appappk}(v_f, \_, \rho_\kappa, c) \leadsto E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned}\text{where } v_f &= ((\lambda\ x\ e_b), \rho_\lambda)\\ b &\triangleq alloc(\varsigma, 0)\\ u &\triangleq tick(\varsigma, 1)\\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto b]\\ &\sigma[b \mapsto v]\end{aligned}$$

$$\textbf{appk}(done, [\,], \rho_\kappa, c) \leadsto E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned}\text{where } done &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) :: v_s\\ b_i &\triangleq alloc(\varsigma, i)\\ u &\triangleq tick(\varsigma, n)\\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto b_0\ ...\\ &\qquad x_{n-1} \mapsto b_{n-1}]\\ v'_s &\triangleq v_s +\!\!+ [v]\\ &\sigma[b_0 \mapsto v'_0\ ...\ b_{n-1} \mapsto v'_{n-1}]\end{aligned}$$

$$\textbf{appk}(done, [\,], \rho_\kappa, c) \leadsto E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned}\text{where } done &= ((\lambda\ x\ e_b), \rho_\lambda) :: v_s\\ b &\triangleq alloc(\varsigma, 0)\\ u &\triangleq tick(\varsigma, 1)\\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto b]\\ v'_s &\triangleq (v_s +\!\!+ [v])\\ &\sigma[b \mapsto v'_s]\end{aligned}$$

$$\textbf{appk}([\kappa_\lambda], [\,], \rho_\kappa, c) \leadsto A\langle v, \rho_\kappa, b, u\rangle$$
$$\begin{aligned}\text{where } b &\triangleq alloc(\varsigma, 0)\\ u &\triangleq tick(\varsigma, 1)\\ &\sigma[b \mapsto \kappa_\lambda]\end{aligned}$$

$$\textbf{appk}(done, e_h :: e_t, \rho_\kappa, c)$$
$$\leadsto E\langle e_h, \rho_\kappa, b, u\rangle$$
$$\begin{aligned}\text{where } b &\triangleq alloc(\varsigma, 0)\\ u &\triangleq tick(\varsigma, 1)\\ &\sigma[b \mapsto \textbf{appk}(done +\!\!+ [v],\\ &\qquad e_t, \rho_\kappa, c)]\end{aligned}$$

2 Abstract Semantics of Scheme CESK*