1 Concrete Semantics of Scheme CESK*

Syntax Domains:

$$
\begin{aligned}
e \in \mathsf{Exp} &::= æ \\
&\mid (\texttt{if}\ e\ e\ e) \\
&\mid (\texttt{let}\ ([x\ e]\ ...)\ e) \\
&\mid (\texttt{call/cc}\ e) \\
&\mid (\texttt{set!}\ x\ e) \\
&\mid (\texttt{prim}\ op\ e\ ...) \\
&\mid (\texttt{apply-prim}\ op\ e) \\
&\mid (\texttt{apply}\ e\ e) \\
&\mid (e\ e\ ...) \\
æ \in \mathsf{AExp} &::= x \mid lam \mid \mathbb{Z} \mid \texttt{\#t} \mid \texttt{\#f} \\
&\qquad \mid (\texttt{quote}\ e) \\
lam \in \mathsf{Lam} &::= (\lambda\ (x...)\ e) \mid (\lambda\ x\ e) \\
x \in \mathsf{Var} &\quad \text{A set of identifiers} \\
op \in \mathsf{Prim} &\quad \text{A set of primitives}
\end{aligned}
$$

Atomic Evaluation:

$$
\begin{gathered}
\mathcal{A} : \Sigma_E \times \sigma \rightharpoonup \mathit{Val} \\
\mathcal{A}(\langle lam, \rho, \_, \_\rangle, \_) \triangleq (lam, \rho) \\
\mathcal{A}(\langle(\texttt{quote}\ e), \_, \_, \_\rangle, \_) \triangleq \mathbf{quote}(e) \\
\mathcal{A}(\langle x, \rho, \_, \_\rangle, \sigma) \triangleq \sigma(\rho(x)) \\
\mathcal{A}(\langle æ, \_, \_, \_\rangle, \_) \triangleq æ
\end{gathered}
$$

Tick/Alloc:

$$
\begin{gathered}
tick : \Sigma \times \mathbb{N} \to \mathit{Time} \\
tick(\langle \_, \_, \_, t\rangle, n) \triangleq (t+n) \\
alloc : \Sigma \times \mathbb{N} \triangleq \mathit{Addr} \\
alloc(\langle \_, \_, \_, t\rangle, n) \triangleq (t+n)
\end{gathered}
$$

Injection:

$$
\begin{gathered}
\mathcal{I} : \mathsf{Exp} \to \Sigma \\
\mathcal{I}(e) \triangleq (e, \varnothing, 0, 1) \\
\text{Initial } \sigma \text{ state} \triangleq \{0 : \mathbf{mt}\}
\end{gathered}
$$

Transition:

Collecting Semantics:

Semantic Domains:

$$
\begin{aligned}
\varsigma \in \Sigma &\triangleq E\langle \mathit{Eval}\rangle + A\langle \mathit{Apply}\rangle \\
\mathit{Eval} &\triangleq \mathsf{Exp} \times \mathit{Env} \\
&\quad \times \mathit{Addr} \times \mathit{Time} \\
\mathit{Apply} &\triangleq \mathit{Val} \times \mathit{Env} \\
&\quad \times \mathit{Addr} \times \mathit{Time} \\
\rho \in \mathit{Env} &\triangleq \mathsf{Var} \rightharpoonup \mathit{Addr} \\
\sigma \in \mathit{Store} &\triangleq \mathit{Addr} \rightharpoonup \mathit{Val} \\
v \in \mathit{Val} &\triangleq \mathit{Clo} + \mathit{Kont} + \mathbb{Z} \\
&\quad + \{\texttt{\#t}, \texttt{\#f}, \mathit{Null}, \mathit{Void}\} \\
&\quad + \{\mathbf{quote}(e)\ ,\ \mathbf{cons}(v, v)\} \\
clo \in \mathit{Clo} &\triangleq \mathsf{Lam} \times \mathit{Env} \\
a, b, c \in \mathit{Addr} &\triangleq \mathbb{N} \\
t, u \in \mathit{Time} &\triangleq \mathbb{N} \\
\kappa \in \mathit{Kont} &::= \mathbf{mt} \\
&\mid \mathbf{ifk}(e, e, \rho, a) \\
&\mid \mathbf{callcck}(\rho, a) \\
&\mid \mathbf{setk}(x, \rho, a) \\
&\mid \mathbf{appappk}(val?, e, \rho, a) \\
&\mid \mathbf{appk}(done, todo, \rho, a) \\
&\mid \mathbf{appprimk}(op, \rho, a) \\
&\mid \mathbf{primk}(op, done, todo, \\
&\qquad \rho, a) \\
&\mid \mathbf{letk}(vars, done, todo \\
&\qquad e, \rho, a) \\
done &\triangleq \mathit{Val}^* \\
todo &\triangleq \mathsf{Exp}^* \\
vars &\triangleq \mathsf{Var}^*
\end{aligned}
$$

Eval Rules

Rules for when the control is an expression

$$E\langle æ, \rho, a, t\rangle \rightsquigarrow A\langle v, \rho, a, t\rangle$$
$$\text{where } v \triangleq \mathcal{A}(\varsigma, \sigma)$$

$$E\langle(\texttt{if}\ e_c\ e_t\ e_f), \rho, a, t\rangle \rightsquigarrow E\langle e_c, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{ifk}(e_t, e_f, \rho, a)]$$

$$E\langle(\texttt{let}\ ()\ e), \rho, a, t\rangle \rightsquigarrow E\langle e, \rho, a, u\rangle$$
$$\text{where } u \triangleq tick(\varsigma, 1)$$

$$E\langle(\texttt{let}\ ([x_0\ e_0]\ [x_s\ e_s]\ ...)\ e_b), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e_0, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{letk}(x_0 :: x_s, [\,], e_s, e_b, \rho, a)]$$

$$E\langle(\texttt{call/cc}\ e), \rho, a, t\rangle \rightsquigarrow E\langle e, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{callcck}(\rho, a)]$$

$$E\langle(\texttt{set!}\ x\ e), \rho, a, t\rangle \rightsquigarrow E\langle e, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{setk}(x, a)]$$

$$E\langle(\texttt{prim}\ op), \rho, a, t\rangle \rightsquigarrow A\langle v, \rho, a, t\rangle$$
$$\text{where } v = op \text{ applied to 0 arguments}$$

$$E\langle(\texttt{prim}\ op\ e_0\ e_s\ ...), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e_0, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{primk}(op, [\,], e_s, \rho, a)]$$

$$E\langle(\texttt{apply-prim}\ op\ e), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{appprimk}(op, a)]$$

$$E\langle(\texttt{apply}\ e_f\ e_x), \rho, a, t\rangle$$
$$\rightsquigarrow E\langle e_f, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{appappk}(\varnothing, e_x, \rho, a)]$$

$$E\langle(e_f\ e_s\ ...), \rho, a, t\rangle \rightsquigarrow E\langle e_f, \rho, b, u\rangle$$
$$\text{where } b \triangleq alloc(\varsigma, 0)$$
$$u \triangleq tick(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{appk}([\,], e_s, \rho, a)]$$

Apply Rules

Rules for when the control is a value

$\varsigma = A\langle v, \rho, a, t\rangle$

$\kappa \triangleq \sigma(a)$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \varsigma$$

$$\mathbf{ifk}(e_t, e_f, \rho, c) \rightsquigarrow E\langle e_f, \rho, c, t\rangle$$
where $v = \texttt{\#f}$

$$\mathbf{ifk}(e_t, e_f, \rho, c) \rightsquigarrow E\langle e_t, \rho, c, t\rangle$$
where $v \neq \texttt{\#f}$

$$\mathbf{letk}(\mathit{vars}, \mathit{done}, [\,], e_b, \rho_\kappa, c) \rightsquigarrow E\langle e_b, \rho'_\kappa, c, u\rangle$$
where
$$b_i \triangleq \mathit{alloc}(\varsigma, i)$$
$$u \triangleq \mathit{tick}(\varsigma, n)$$
$$\mathit{done}' \triangleq \mathit{done} +\!\!+ [v]$$
$$\rho'_\kappa \triangleq \rho_\kappa[\mathit{vars}_0 \mapsto b_0 \ \ldots \ \mathit{vars}_{n-1} \mapsto b_{n-1}]$$
$$\sigma[b_0 \mapsto \mathit{done}'_0 \ \ldots \ b_{n-1} \mapsto \mathit{done}'_{n-1}]$$

$$\mathbf{letk}(\mathit{vars}, \mathit{done}, e_h :: e_t, e_b, \rho_\kappa, c) \rightsquigarrow E\langle e_h, \rho_\kappa, b, u\rangle$$
where
$$b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{letk}(\mathit{vars}, \mathit{done} +\!\!+ [v], e_t, e_b, \rho_\kappa, c)]$$

$$\mathbf{callcck}(\_, c) \rightsquigarrow E\langle e, \rho'_\lambda, c, t\rangle$$
where $v = ((\lambda\ (x)\ e), \rho_\lambda)$
$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto c]$$

$$\mathbf{callcck}(\rho', \_) \rightsquigarrow A\langle \kappa, \rho', b, u\rangle$$
where $v = \kappa'$
$$b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \kappa']$$

$$\mathbf{setk}(x, \rho_\kappa, c) \rightsquigarrow A\langle \mathit{Void}, \rho_\kappa, c, t\rangle$$
where $\sigma[\rho_\kappa(x) \mapsto v]$

$$\mathbf{appprimk}(\mathit{op}, \rho_\kappa, c) \rightsquigarrow A\langle v', \rho_\kappa, c, t\rangle$$
where $v' \triangleq \mathit{op}$ applied to v

$$\mathbf{primk}(\mathit{op}, \mathit{done}, [\,], \rho_\kappa, c) \rightsquigarrow A\langle v', \rho_\kappa, c, t\rangle$$
where $v' \triangleq \mathit{op}$ applied to $(\mathit{done} +\!\!+ [v])$

$$\mathbf{primk}(\mathit{op}, \mathit{done}, e_h :: e_t, \rho_\kappa, c) \rightsquigarrow E\langle e_h, \rho_\kappa, b, u\rangle$$
where
$$b \triangleq \mathit{alloc}(\varsigma, 0)$$
$$u \triangleq \mathit{tick}(\varsigma, 1)$$
$$\sigma[b \mapsto \mathbf{primk}(\mathit{op}, \mathit{done} +\!\!+ [v], e_t, \rho_\kappa, c)]$$

More Apply Rules

Rules for when the control is a value

$$\textbf{appappk}(\varnothing, e, \rho_\kappa, c) \rightsquigarrow E\langle e, \rho_\kappa, b, u\rangle$$
$$\begin{aligned} \text{where } b &\triangleq alloc(\varsigma, 0) \\ u &\triangleq tick(\varsigma, 1) \\ &\sigma[b \mapsto \textbf{appappk}(v, e, \rho_\kappa, c)] \end{aligned}$$

$$\textbf{appappk}(v_f, \_, \rho_\kappa, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned} \text{where } v_f &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) \\ b_i &\triangleq alloc(\varsigma, i) \\ u &\triangleq tick(\varsigma, n) \\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto b_0\ ... \\ &\qquad x_{n-1} \mapsto b_{n-1}] \\ &\sigma[b_0 \mapsto v_0\ ...\ b_{n-1} \mapsto v_{n-1}] \end{aligned}$$

$$\textbf{appappk}(v_f, \_, \rho_\kappa, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned} \text{where } v_f &= ((\lambda\ x\ e_b), \rho_\lambda) \\ b &\triangleq alloc(\varsigma, 0) \\ u &\triangleq tick(\varsigma, 1) \\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto b] \\ &\sigma[b \mapsto v] \end{aligned}$$

$$\textbf{appk}(done, [\,], \rho_\kappa, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned} \text{where } done &= ((\lambda\ (x_s ...)\ e_b), \rho_\lambda) :: v_s \\ b_i &\triangleq alloc(\varsigma, i) \\ u &\triangleq tick(\varsigma, n) \\ \rho'_\lambda &\triangleq \rho_\lambda[x_0 \mapsto b_0\ ... \\ &\qquad x_{n-1} \mapsto b_{n-1}] \\ v'_s &\triangleq v_s +\!\!+ [v] \\ &\sigma[b_0 \mapsto v'_0\ ...\ b_{n-1} \mapsto v'_{n-1}] \end{aligned}$$

$$\textbf{appk}(done, [\,], \rho_\kappa, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u\rangle$$
$$\begin{aligned} \text{where } done &= ((\lambda\ x\ e_b), \rho_\lambda) :: v_s \\ b &\triangleq alloc(\varsigma, 0) \\ u &\triangleq tick(\varsigma, 1) \\ \rho'_\lambda &\triangleq \rho_\lambda[x \mapsto b] \\ v'_s &\triangleq (v_s +\!\!+ [v]) \\ &\sigma[b \mapsto v'_s] \end{aligned}$$

$$\textbf{appk}([\kappa_\lambda], [\,], \rho_\kappa, c) \rightsquigarrow A\langle v, \rho_\kappa, b, u\rangle$$
$$\begin{aligned} \text{where } b &\triangleq alloc(\varsigma, 0) \\ u &\triangleq tick(\varsigma, 1) \\ &\sigma[b \mapsto \kappa_\lambda] \end{aligned}$$

$$\textbf{appk}(done, e_h :: e_t, \rho_\kappa, c) \rightsquigarrow E\langle e_h, \rho_\kappa, b, u\rangle$$
$$\begin{aligned} \text{where } b &\triangleq alloc(\varsigma, 0) \\ u &\triangleq tick(\varsigma, 1) \\ &\sigma[b \mapsto \textbf{appk}(done +\!\!+ [v], \\ &\qquad e_t, \rho_\kappa, c)] \end{aligned}$$

2 Abstract Semantics of Scheme CESK*