

1 Concrete Semantics of Scheme CESKt*

Syntax Domains:

$e \in \text{Exp} ::= \mathfrak{x}$

| (if e e e)
 | (let ($[x$ $e]$...) e)
 | (call/cc e)
 | (set! x e)
 | (prim op e ...)
 | (apply-prim op e)
 | (apply e e)
 | (e e ...)

$\mathfrak{x} \in \text{AExp} ::= x \mid lam \mid \mathbb{Z} \mid \#t \mid \#f$
 | (quote e)

$lam \in \text{Lam} ::= (\lambda (x \dots) e) \mid (\lambda x e)$

$x \in \text{Var}$ A set of identifiers

$op \in \text{Prim}$ A set of primitives

Atomic Evaluation:

$\mathcal{A} : \Sigma_E \times \sigma \rightarrow \text{Val}$

$\mathcal{A}(\langle lam, \rho, -, - \rangle, -) \triangleq (lam, \rho)$

$\mathcal{A}(\langle (\text{quote } e), -, -, - \rangle, -) \triangleq \text{quote}(e)$

$\mathcal{A}(\langle x, \rho, -, - \rangle, \sigma) \triangleq \sigma(\rho(x))$

$\mathcal{A}(\langle \mathfrak{x}, -, -, - \rangle, -) \triangleq \mathfrak{x}$

Tick/Alloc:

$tick : \text{Eval} \times \mathbb{N} \rightarrow \text{Time}$

$tick(E \langle -, -, -, t \rangle, n) \triangleq (t + n)$

$alloc : \Sigma \times \mathbb{N} \triangleq \text{Addr}$

$alloc(E \langle -, -, -, t \rangle, n) \triangleq (t + n)$

Injection:

$\mathcal{I} : \text{Exp} \rightarrow \Sigma$

$\mathcal{I}(e) \triangleq (e, \emptyset, \emptyset, \{0 : \mathbf{mt}\}, 0, 1)$

Transition:

Collecting Semantics:

Semantic Domains:

$\varsigma \in \Sigma \triangleq E \langle \text{Eval} \rangle + A \langle \text{Apply} \rangle$

$\text{Eval} \triangleq \text{Exp} \times \text{Env} \times \text{Store}$
 $\times K\text{Store} \times \text{Addr} \times \text{Time}$

$\text{Apply} \triangleq \text{Val} \times \text{Store}$
 $\times K\text{Store} \times \text{Addr} \times \text{Time}$

$\rho \in \text{Env} \triangleq \text{Var} \rightarrow \text{Addr}$

$\sigma \in \text{Store} \triangleq \text{Addr} \rightarrow \text{Val}$

$\sigma \in \text{Store} \triangleq \text{Addr} \rightarrow \text{Kont}$

$v \in \text{Val} \triangleq \text{Clo} + \text{Kont} + \mathbb{Z}$
 $+ \{\#t, \#f, \text{Null}, \text{Void}\}$
 $+ \{\text{quote}(e), \text{cons}(v, v)\}$

$clo \in \text{Clo} \triangleq \text{Lam} \times \text{Env}$

$a, a_\kappa \in \text{Addr} \triangleq \mathbb{N}$

$t \in \text{Time} \triangleq \mathbb{N}$

$\kappa \in \text{Kont} ::= \mathbf{mt}$

| ifk(e, e, ρ, a)

| callcck(a)

| setk(x, ρ, a)

| appappk($val?, e, \rho, a$)

| appk($done, todo, \rho, a$)

| appprimk(op, a)

| primk($op, done, todo,$
 ρ, a)

| letk($vars, done, todo,$
 e, ρ, a)

$done \triangleq \text{Val}^*$

$todo \triangleq \text{Exp}^*$

$vars \triangleq \text{Var}^*$

Eval Rules

Rules for when the control is an expression

$$E\langle \mathfrak{x}, \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle \rightsquigarrow A\langle v, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

where $v \triangleq \mathcal{A}(\zeta)$

$$E\langle (\text{if } e_c \ e_t \ e_f), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e_c, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{ifk}(e_t, e_f, \rho, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\text{let } () \ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$E\langle (\text{let } bs \ e_b), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $bs = ([x_0 \ e_0] \ [x_s \ e_s] \ \dots)$

$$a'_\kappa \triangleq \text{alloc}(\zeta, 0)$$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{letk}(x_0 :: x_s, [], e_s, e_b, \rho, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\text{call/cc } e), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{callcck}(a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\text{set! } x \ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$a \triangleq \rho(x)$$

$$\kappa \triangleq \text{setk}(a, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\text{prim } op), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow A\langle v, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

where $v = op$ applied to 0 arguments

$$E\langle (\text{prim } op \ e_0 \ e_s \ \dots), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e_0, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{primk}(op, [], e_s, \rho, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\text{apply-prim } op \ e), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{appprimk}(op, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (\text{apply } e_f \ e_x), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e_f, \rho, \sigma, \sigma_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{appappk}(\emptyset, e_x, \rho, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

$$E\langle (e_f \ e_s \ \dots), \rho, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\rightsquigarrow E\langle e_f, \rho, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $a'_\kappa \triangleq \text{alloc}(\zeta, 0)$

$$t' \triangleq \text{tick}(\zeta, 1)$$

$$\kappa \triangleq \text{appk}([], e_s, \rho, a_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa]$$

Apply Rules

Rules for when the control is a value

$$\varsigma = A\langle v, \sigma, \sigma_\kappa, a_\kappa, t \rangle$$

$$\kappa \triangleq \sigma_\kappa(a_\kappa)$$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \varsigma$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_f, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t \rangle$$

where $v = \#f$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, a'_\kappa) \rightsquigarrow E\langle e_t, \rho_\kappa, \sigma, \sigma_\kappa, a'_\kappa, t \rangle$$

where $v \neq \#f$

$$\mathbf{letk}(\text{vars}, \text{done}, [], e_b, \rho_\kappa, a'_\kappa)$$

$$\rightsquigarrow E\langle e_b, \rho'_\kappa, \sigma', \sigma_\kappa, a'_\kappa, t' \rangle$$

where $a_i \triangleq \text{alloc}(\varsigma, i)$

$$t' \triangleq \text{tick}(\varsigma, n)$$

$$\text{done}' \triangleq \text{done} \# [v]$$

$$\rho'_\kappa \triangleq \rho_\kappa[\text{vars}_0 \mapsto a_0 \dots$$

$$\text{vars}_{n-1} \mapsto a_{n-1}]$$

$$\sigma' \triangleq \sigma[a_0 \mapsto \text{done}'_0 \dots$$

$$a_{n-1} \mapsto \text{done}'_{n-1}]$$

$$\mathbf{letk}(\text{vars}, \text{done}, e_h :: e_t, e_b, \rho_\kappa, a'_\kappa)$$

$$\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t' \rangle$$

where $a''_\kappa \triangleq \text{alloc}(\varsigma, 0)$

$$t' \triangleq \text{tick}(\varsigma, 1)$$

$$\kappa' \triangleq \mathbf{letk}(\text{vars}, \text{done} \# [v],$$

$$e_t, e_b, \rho_\kappa, a'_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$$

$$\mathbf{setk}(a, a'_\kappa) \rightsquigarrow A\langle \text{Void}, \sigma', \sigma_\kappa, a'_\kappa, t \rangle$$

where $\sigma' \triangleq \sigma[a \mapsto v]$

$$\mathbf{callcck}(a'_\kappa) \rightsquigarrow E\langle e, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t \rangle$$

where $v = ((\lambda (x) e), \rho_\lambda)$

$$a \triangleq \text{alloc}(\varsigma, 0)$$

$$t' \triangleq \text{tick}(\varsigma, 1)$$

$$\kappa' \triangleq \sigma_\kappa(a'_\kappa)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a]$$

$$\sigma' \triangleq \sigma[a \mapsto \kappa']$$

$$\mathbf{callcck}(-) \rightsquigarrow A\langle \kappa, \sigma, \sigma'_\kappa, a'_\kappa, t' \rangle$$

where $v = \kappa'$

$$a'_\kappa \triangleq \text{alloc}(\varsigma, 0)$$

$$t' \triangleq \text{tick}(\varsigma, 1)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a'_\kappa \mapsto \kappa']$$

$$\mathbf{appprimk}(op, a'_\kappa) \rightsquigarrow A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t \rangle$$

where $v' \triangleq op$ applied to v

$$\mathbf{primk}(op, \text{done}, [], \rho_\kappa, a'_\kappa)$$

$$\rightsquigarrow A\langle v', \sigma, \sigma_\kappa, a'_\kappa, t \rangle$$

where $v' \triangleq op$ applied to $(\text{done} \# [v])$

$$\mathbf{primk}(op, \text{done}, e_h :: e_t, \rho_\kappa, a'_\kappa)$$

$$\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t' \rangle$$

where $a''_\kappa \triangleq \text{alloc}(\varsigma, 0)$

$$t' \triangleq \text{tick}(\varsigma, 1)$$

$$\kappa' \triangleq \mathbf{primk}(op, \text{done} \# [v],$$

$$e_t, \rho_\kappa, a'_\kappa)$$

$$\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa']$$

More Apply Rules

Rules for when the control is a value

$$\begin{array}{ll}
\mathbf{appappk}(v_f, -, -, a'_\kappa) & \mathbf{appk}(done, [], -, a'_\kappa) \\
\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t' \rangle & \rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t' \rangle \\
\text{where } v_f = ((\lambda (x_s \dots) e_b), \rho_\lambda) & \text{where } done = ((\lambda (x_s \dots) e_b), \rho_\lambda) :: v_s \\
a_i \triangleq alloc(\varsigma, i) & a_i \triangleq alloc(\varsigma, i) \\
t' \triangleq tick(\varsigma, n) & t' \triangleq tick(\varsigma, n) \\
\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto a_0 \dots & v'_s \triangleq v_s \# [v] \\
& \quad x_{n-1} \mapsto a_{n-1}] \\
\sigma' \triangleq \sigma[a_0 \mapsto v_0 \dots & \rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto a_0 \dots \\
& \quad a_{n-1} \mapsto v_{n-1}] & \quad x_{n-1} \mapsto a_{n-1}] \\
& \sigma' \triangleq \sigma[a_0 \mapsto v'_0 \dots \\
& \quad a_{n-1} \mapsto v'_{n-1}] \\
\\
\mathbf{appappk}(v_f, -, -, a'_\kappa) & \mathbf{appk}(done, [], -, a'_\kappa) \\
\rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t' \rangle & \rightsquigarrow E\langle e_b, \rho'_\lambda, \sigma', \sigma_\kappa, a'_\kappa, t' \rangle \\
\text{where } v_f = ((\lambda x e_b), \rho_\lambda) & \text{where } done = ((\lambda x e_b), \rho_\lambda) :: v_s \\
a \triangleq alloc(\varsigma, 0) & a \triangleq alloc(\varsigma, 0) \\
t' \triangleq tick(\varsigma, 1) & t' \triangleq tick(\varsigma, 1) \\
\rho'_\lambda \triangleq \rho_\lambda[x \mapsto a] & v'_s \triangleq (v_s \# [v]) \\
\sigma' \triangleq \sigma[a \mapsto v] & \rho'_\lambda \triangleq \rho_\lambda[x \mapsto a] \\
& \sigma' \triangleq \sigma[a \mapsto v'_s] \\
\\
\mathbf{appappk}(\emptyset, e, \rho_\kappa, a'_\kappa) & \mathbf{appk}([\kappa_\lambda], [], \rho_\kappa, a'_\kappa) \rightsquigarrow A\langle v, \sigma, \sigma'_\kappa, a''_\kappa, t' \rangle \\
\rightsquigarrow E\langle e, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t' \rangle & \text{where } a''_\kappa \triangleq alloc(\varsigma, 0) \\
\text{where } a''_\kappa \triangleq alloc(\varsigma, 0) & t' \triangleq tick(\varsigma, 1) \\
t' \triangleq tick(\varsigma, 1) & \sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa_\lambda] \\
\kappa' \triangleq \mathbf{appappk}(v, e, \rho_\kappa, a'_\kappa) & \\
\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa'] & \\
\\
\mathbf{appk}(done, e_h :: e_t, \rho_\kappa, a'_\kappa) & \\
\rightsquigarrow E\langle e_h, \rho_\kappa, \sigma, \sigma'_\kappa, a''_\kappa, t' \rangle & \\
\text{where } a''_\kappa \triangleq alloc(\varsigma, 0) & \\
t' \triangleq tick(\varsigma, 1) & \\
\kappa' \triangleq \mathbf{appk}(done \# [v], e_t, \rho_\kappa, a'_\kappa) & \\
\sigma'_\kappa \triangleq \sigma_\kappa[a''_\kappa \mapsto \kappa'] &
\end{array}$$

2 Abstract Semantics of Scheme CESKt*

Abstract Semantic Domains:

$$\begin{aligned}
\hat{\varsigma} \in \widehat{\Sigma} &\triangleq E\langle Eval \rangle + E\langle Apply \rangle & \hat{\kappa} \in \widehat{Kont} &::= \mathbf{mt} \\
Eval &\triangleq \mathbf{Exp} \times \widehat{Env} \times \widehat{Store} & &| \mathbf{ifk}(e, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&\times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time} & &| \mathbf{callcck}(\hat{a}_{\hat{\kappa}}) \\
Apply &\triangleq \widehat{Val} \times \widehat{Store} & &| \mathbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}}) \\
&\times \widehat{KStore} \times \widehat{Addr} \times \widehat{Time} & &| \mathbf{appappk}(\widehat{val?}, e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
\hat{\rho} \in \widehat{Env} &\triangleq \mathbf{Var} \rightarrow \widehat{Addr} & &| \mathbf{appk}(\widehat{done}, \widehat{todo}, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
\hat{\sigma} \in \widehat{Store} &\triangleq \widehat{Addr} \rightarrow \mathcal{P}(\widehat{Val}) & &| \mathbf{appprimk}(op, \hat{a}_{\hat{\kappa}}) \\
\hat{\sigma}_{\kappa} \in \widehat{KStore} &\triangleq \widehat{Addr} \rightarrow \mathcal{P}(\widehat{Kont}) & &| \mathbf{primk}(op, \widehat{done}, \widehat{todo}, \\
&&&&\hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
\hat{v} \in \widehat{Val} &\triangleq \widehat{Clo} + \widehat{Kont} + \mathbb{Z} & &| \mathbf{letk}(\widehat{vars}, \widehat{done}, \widehat{todo}, \\
&+ \{\#t, \#f, \mathbf{Null}, \mathbf{Void}\} & &e, \hat{\rho}, \hat{a}_{\hat{\kappa}}) \\
&+ \{\mathbf{quote}(e), \mathbf{cons}(v, v)\} \\
\widehat{clo} \in \widehat{Clo} &\triangleq \mathbf{Lam} \times \widehat{Env} \\
\hat{a}, \hat{a}_{\hat{\kappa}} \in \widehat{Addr} &\triangleq \mathbb{N} \\
\hat{t} \in \widehat{Time} &\triangleq \mathbb{N} \\
\widehat{done} &\triangleq \widehat{Val}^*
\end{aligned}$$

Abstract Atomic Evaluation:

$$\begin{aligned}
\mathcal{A} : Eval &\rightarrow \widehat{Val} \\
\mathcal{A}(E\langle lam, \hat{\rho}, \dots \rangle) &\triangleq \{(lam, \hat{\rho})\} \\
\mathcal{A}(\mathbf{\text{æ}}) &\triangleq \{\mathbf{\text{æ}}\}
\end{aligned}$$

Tick/Alloc:

$$\begin{aligned}
\widehat{tick} : \widehat{\Sigma} \times \widehat{Kont} &\rightarrow \widehat{Time} \\
\widehat{tick}(\hat{\varsigma}, \hat{\kappa}) &\triangleq 0 \\
\widehat{alloc} : \widehat{\Sigma} \times \widehat{Kont} &\rightarrow \widehat{Addr} \\
\widehat{alloc}(\hat{\varsigma}, \hat{\kappa}) &\triangleq 0
\end{aligned}$$

Abstract Eval Rules Rules for when the control is an expression

$$E\langle \text{æ}, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$$

where $\hat{v} \triangleq \hat{\mathcal{A}}(\hat{\zeta}, \hat{\sigma})$

| | |
|---|--|
| $E\langle (\text{if } e_c \ e_t \ e_f), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e_c, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{ifk}(e_t, e_f, \hat{\rho}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ $E\langle (\text{let } () \ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $E\langle (\text{let } bnds \ e_b), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $bnds = ([x_0 \ e_0] \ [x_s \ e_s] \ \dots)$</p> $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$ $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{letk}(x_0 :: x_s, [], e_s, e_b, \hat{\rho}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ $E\langle (\text{call/cc } e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{callcck}(\hat{\rho}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ $E\langle (\text{set! } x \ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{a} \triangleq \hat{\rho}(x)$ $\hat{\kappa} \triangleq \mathbf{setk}(\hat{a}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ | $E\langle (\text{prim } op), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ <p style="text-align: center;">where $\hat{v} = op$ applied to 0 arguments</p> $E\langle (\text{prim } op \ e_0 \ e_s \ \dots), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e_0, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{primk}(op, [], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ $E\langle (\text{apply-prim } op \ e), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{appprimk}(op, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ $E\langle (\text{apply } e_f \ e_x), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{appappk}(\emptyset, e_x, \hat{\rho}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ $E\langle (e_f \ e_s \ \dots), \hat{\rho}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$ $\rightsquigarrow E\langle e_f, \hat{\rho}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$ <p style="text-align: center;">where $\hat{a}'_{\hat{\kappa}} \triangleq \widehat{\text{alloc}}(\hat{\zeta}, 0, \hat{\kappa})$</p> $\hat{t}' \triangleq \widehat{\text{tick}}(\hat{\zeta}, 1, \hat{\kappa})$ $\hat{\kappa} \triangleq \mathbf{appk}([], e_s, \hat{\rho}, \hat{a}_{\hat{\kappa}})$ $\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}]$ |
|---|--|

Apply Rules

Rules for when the control is a value

$$\hat{\varsigma} = A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}_{\hat{\kappa}}, \hat{t} \rangle$$

$$\hat{\kappa} \in \hat{\sigma}_{\hat{\kappa}}(\hat{a}_{\hat{\kappa}})$$

Proceed by matching on $\hat{\kappa}$

$$\mathbf{mt} \rightsquigarrow \hat{\varsigma}$$

$$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_f, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$

$$\text{where } \hat{v} = \#f$$

$$\mathbf{ifk}(e_t, e_f, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_t, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$

$$\text{where } \hat{v} \neq \#f$$

$$\mathbf{letk}(\text{vars}, \widehat{done}, [], e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_b, \hat{\rho}'_{\hat{\kappa}}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\text{where } \hat{a}_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa})$$

$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa})$$

$$\hat{\rho}'_{\hat{\kappa}} \triangleq \hat{\rho}_{\hat{\kappa}}[\text{vars}_0 \mapsto \hat{a}_0 \dots \text{vars}_{n-1} \mapsto \hat{a}_{n-1}]$$

$$\widehat{done}' \triangleq \widehat{done} + [\hat{v}]$$

$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \widehat{done}'_0 \dots \hat{a}_{n-1} \mapsto \widehat{done}'_{n-1}]$$

$$\mathbf{letk}(\text{vars}, \widehat{done}, e_h :: e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$

$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$

$$\hat{\kappa}' \triangleq \mathbf{letk}(\text{vars}, \widehat{done}, +[\hat{v}], e_t, e_b, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$

$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

$$\mathbf{setk}(x, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle \text{Void}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$

$$\text{where } \hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{\rho}_{\hat{\kappa}}(x) \mapsto \hat{v}]$$

$$\mathbf{callcck}(\hat{a}'_{\hat{\kappa}}) \rightsquigarrow E\langle e, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$

$$\text{where } \hat{v} = ((\lambda (x) e), \hat{\rho}_{\lambda})$$

$$\hat{a} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$

$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$

$$\hat{\kappa}' \triangleq \hat{\sigma}_{\hat{\kappa}}(\hat{a}'_{\hat{\kappa}})$$

$$\hat{\rho}'_{\lambda} \triangleq \hat{\rho}_{\lambda}[x \mapsto \hat{a}]$$

$$\hat{\sigma}' \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{\kappa}']$$

$$\mathbf{callcck}(-) \rightsquigarrow A\langle \hat{\kappa}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\text{where } v = \kappa'$$

$$\hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$

$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$

$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \kappa']$$

$$\mathbf{appprimk}(op, \hat{a}'_{\hat{\kappa}}) \rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$

$$\text{where } \hat{v}' \triangleq op \text{ applied to } \hat{v}$$

$$\mathbf{primk}(op, \widehat{done}, [], -, \hat{a}'_{\hat{\kappa}})$$

$$\rightsquigarrow A\langle \hat{v}', \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t} \rangle$$

$$\text{where } \hat{v}' \triangleq op \text{ applied to } (\widehat{done} + [\hat{v}])$$

$$\mathbf{primk}(op, \widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}})$$

$$\rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}''_{\hat{\kappa}}, \hat{t}' \rangle$$

$$\text{where } \hat{a}''_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa})$$

$$\hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa})$$

$$\hat{\kappa}' \triangleq \mathbf{primk}(op, \widehat{done} + [\hat{v}])$$

$$e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}$$

$$\hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}''_{\hat{\kappa}} \mapsto \hat{\kappa}']$$

More Apply Rules

Rules for when the control is a value

$$\begin{aligned} & \mathbf{appappk}(\hat{v}_f, -, -, \hat{a}'_{\hat{\kappa}}) \\ & \rightsquigarrow E\langle e_b, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \\ & \text{where } \hat{v}_f = ((\lambda (x_s \dots) e_b), \hat{\rho}_{\lambda}) \end{aligned}$$

$$\begin{aligned} \hat{a}_i & \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa}) \\ \hat{t}' & \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa}) \\ \hat{\rho}'_{\lambda} & \triangleq \hat{\rho}_{\lambda}[x_0 \mapsto \hat{a}_0 \dots \\ & \quad x_{n-1} \mapsto \hat{a}_{n-1}] \\ \hat{\sigma}' & \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}_0 \dots \\ & \quad \hat{a}_{n-1} \mapsto \hat{v}_{n-1}] \end{aligned}$$

$$\begin{aligned} & \mathbf{appappk}(\hat{v}_f, -, -, \hat{a}'_{\hat{\kappa}}) \\ & \rightsquigarrow E\langle e_b, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \\ & \text{where } \hat{v}_f = ((\lambda x e_b), \hat{\rho}_{\lambda}) \end{aligned}$$

$$\begin{aligned} \hat{a} & \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' & \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\rho}'_{\lambda} & \triangleq \hat{\rho}_{\lambda}[x \mapsto \hat{a}] \\ \hat{\sigma}' & \triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{v}] \end{aligned}$$

$$\begin{aligned} & \mathbf{appappk}(\emptyset, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ & \rightsquigarrow E\langle e, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \\ & \text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ & \hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ & \hat{\kappa}' \triangleq \mathbf{appappk}(\hat{v}, e, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ & \hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}'] \end{aligned}$$

$$\begin{aligned} & \mathbf{appk}(\widehat{done}, [], -, \hat{a}'_{\hat{\kappa}}) \\ & \rightsquigarrow E\langle e_b, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \end{aligned}$$

$$\text{where } \widehat{done} = ((\lambda (x_s \dots) e_b), \hat{\rho}_{\lambda}) :: \hat{v}_s$$

$$\begin{aligned} \hat{a}_i & \triangleq \widehat{alloc}(\hat{\varsigma}, i, \hat{\kappa}) \\ \hat{t}' & \triangleq \widehat{tick}(\hat{\varsigma}, n, \hat{\kappa}) \\ \hat{\rho}'_{\lambda} & \triangleq \hat{\rho}_{\lambda}[x_0 \mapsto \hat{a}_0 \dots \\ & \quad x_{n-1} \mapsto \hat{a}_{n-1}] \\ \hat{v}'_s & \triangleq \hat{v}_s \# [\hat{v}] \\ \hat{\sigma}' & \triangleq \hat{\sigma} \sqcup [\hat{a}_0 \mapsto \hat{v}'_0 \dots \\ & \quad \hat{a}_{n-1} \mapsto \hat{v}'_{n-1}] \end{aligned}$$

$$\begin{aligned} & \mathbf{appk}(\widehat{done}, [], -, \hat{a}'_{\hat{\kappa}}) \\ & \rightsquigarrow E\langle e_b, \hat{\rho}'_{\lambda}, \hat{\sigma}', \hat{\sigma}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \\ & \text{where } \widehat{done} = ((\lambda x e_b), \hat{\rho}_{\lambda}) :: \hat{v}_s \end{aligned}$$

$$\begin{aligned} \hat{a} & \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ \hat{t}' & \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ \hat{\rho}'_{\lambda} & \triangleq \hat{\rho}_{\lambda}[x \mapsto \hat{a}] \\ \hat{v}'_s & \triangleq \hat{v}_s \# [\hat{v}] \\ \hat{\sigma}' & \triangleq \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{v}'_s] \end{aligned}$$

$$\begin{aligned} & \mathbf{appk}([\hat{\kappa}_{\lambda}], [], \hat{\rho}_{\hat{\kappa}}, -) \rightsquigarrow A\langle \hat{v}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \\ & \text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ & \hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ & \hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}_{\lambda}] \end{aligned}$$

$$\begin{aligned} & \mathbf{appk}(\widehat{done}, e_h :: e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ & \rightsquigarrow E\langle e_h, \hat{\rho}_{\hat{\kappa}}, \hat{\sigma}, \hat{\sigma}'_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}, \hat{t}' \rangle \\ & \text{where } \hat{a}'_{\hat{\kappa}} \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \hat{\kappa}) \\ & \hat{t}' \triangleq \widehat{tick}(\hat{\varsigma}, 1, \hat{\kappa}) \\ & \hat{\kappa}' \triangleq \mathbf{appk}(\widehat{done} \# [\hat{v}], e_t, \hat{\rho}_{\hat{\kappa}}, \hat{a}'_{\hat{\kappa}}) \\ & \hat{\sigma}'_{\hat{\kappa}} \triangleq \hat{\sigma}_{\hat{\kappa}} \sqcup [\hat{a}'_{\hat{\kappa}} \mapsto \hat{\kappa}'] \end{aligned}$$