

1 Concrete Semantics of Scheme CESKt*

Syntax Domains:

$e \in \text{Exp} ::= \text{\texttt{\ae}}$
 $\quad | (\text{if } e \ e \ e)$
 $\quad | (\text{let } ([x \ e] \ \dots) \ e)$
 $\quad | (\text{call/cc } e)$
 $\quad | (\text{set! } x \ e)$
 $\quad | (\text{prim } op \ e \ \dots)$
 $\quad | (\text{apply-prim } op \ e)$
 $\quad | (\text{apply } e \ e)$
 $\quad | (e \ e \ \dots)$
 $\text{\texttt{\ae}} \in \text{AExp} ::= x \mid lam \mid \mathbb{Z} \mid \text{\texttt{\#t}} \mid \text{\texttt{\#f}}$
 $\quad | (\text{quote } e)$
 $lam \in \text{Lam} ::= (\lambda (x \dots) \ e) \mid (\lambda x \ e)$
 $x \in \text{Var} \quad \text{A set of identifiers}$
 $op \in \text{Prim} \quad \text{A set of primitives}$

Atomic Evaluation:

$\mathcal{A} : \Sigma_E \times \sigma \rightarrow Val$
 $\mathcal{A}(\langle lam, \rho, -, - \rangle, -) \triangleq (lam, \rho)$
 $\mathcal{A}(\langle (\text{quote } e), -, -, - \rangle, -) \triangleq \text{\texttt{quote}}(e)$
 $\mathcal{A}(\langle x, \rho, -, - \rangle, \sigma) \triangleq \sigma(\rho(x))$
 $\mathcal{A}(\langle \text{\texttt{\ae}}, -, -, - \rangle, -) \triangleq \text{\texttt{\ae}}$

Tick/Alloc:

$tick : \Sigma \times \mathbb{N} \rightarrow Time$
 $tick(\langle -, -, -, t \rangle, n) \triangleq (t + n)$
 $alloc : \Sigma \times \mathbb{N} \triangleq Addr$
 $alloc(\langle -, -, -, t \rangle, n) \triangleq (t + n)$

Injection:

$\mathcal{I} : \text{Exp} \rightarrow \Sigma$
 $\mathcal{I}(e) \triangleq (e, \emptyset, 0, 1)$

Initial σ state $\triangleq \{0 : \text{\texttt{mt}}\}$

Transition: Collecting Semantics:

Semantic Domains:

$\varsigma \in \Sigma \triangleq E\langle Eval \rangle + A\langle Apply \rangle$
 $Eval \triangleq \text{Exp} \times Env$
 $\quad \times Addr \times Time$
 $Apply \triangleq Val \times Env$
 $\quad \times Addr \times Time$
 $\rho \in Env \triangleq \text{Var} \rightarrow Addr$
 $\sigma \in Store \triangleq Addr \rightarrow Val$
 $v \in Val \triangleq Clo + Kont + \mathbb{Z}$
 $\quad + \{\text{\texttt{\#t}}, \text{\texttt{\#f}}, Null, Void\}$
 $\quad + \{\text{\texttt{quote}}(e), \text{\texttt{cons}}(v, v)\}$
 $clo \in Clo \triangleq Lam \times Env$
 $a, b, c \in Addr \triangleq \mathbb{N}$
 $t, u \in Time \triangleq \mathbb{N}$
 $\kappa \in Kont ::= \text{\texttt{mt}}$
 $\quad | \text{\texttt{ifk}}(e, e, \rho, a)$
 $\quad | \text{\texttt{callcck}}(\rho, a)$
 $\quad | \text{\texttt{setk}}(x, \rho, a)$
 $\quad | \text{\texttt{appappk}}(val?, e, \rho, a)$
 $\quad | \text{\texttt{appk}}(done, todo, \rho, a)$
 $\quad | \text{\texttt{appprimk}}(op, \rho, a)$
 $\quad | \text{\texttt{primk}}(op, done, todo,$
 $\quad \quad \rho, a)$
 $\quad | \text{\texttt{letk}}(vars, done, todo,$
 $\quad \quad e, \rho, a)$
 $done \triangleq Val^*$
 $todo \triangleq \text{Exp}^*$
 $vars \triangleq \text{Var}^*$

Eval Rules

Rules for when the control is an expression

$$E\langle \mathfrak{a}, \rho, a, t \rangle \rightsquigarrow A\langle v, \rho, a, t \rangle$$

where $v \triangleq \mathcal{A}(\zeta, \sigma)$

$E\langle (\text{if } e_c \text{ } e_t \text{ } e_f), \rho, a, t \rangle \rightsquigarrow E\langle e_c, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{ifk}(e_t, e_f, \rho, a)$</p> $E\langle (\text{let } () \text{ } e), \rho, a, t \rangle \rightsquigarrow E\langle e, \rho, a, t \rangle$ $E\langle (\text{let } ([x_0 \text{ } e_0] [x_s \text{ } e_s] \dots) e_b), \rho, a, t \rangle$ <p style="text-align: center;">$\rightsquigarrow E\langle e_0, \rho, b, u \rangle$</p> <p>where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{letk}(x_0 :: x_s, [], e_s, e_b, \rho, a)$</p> $E\langle (\text{call/cc } e), \rho, a, t \rangle \rightsquigarrow E\langle e, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{callcck}(\rho, a)$</p> $E\langle (\text{set! } x \text{ } e), \rho, a, t \rangle \rightsquigarrow E\langle e, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{setk}(x, a)$</p>	$E\langle (\text{prim } op), \rho, a, t \rangle \rightsquigarrow A\langle v, \rho, a, t \rangle$ <p style="text-align: center;">where $v = op$ applied to 0 arguments</p> $E\langle (\text{prim } op \text{ } e_0 \text{ } e_s \dots), \rho, a, t \rangle$ <p style="text-align: center;">$\rightsquigarrow E\langle e_0, \rho, b, u \rangle$</p> <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{primk}(op, [], e_s, \rho, a)$</p> $E\langle (\text{apply-prim } op \text{ } e), \rho, a, t \rangle$ <p style="text-align: center;">$\rightsquigarrow \langle e, \rho, b, u \rangle$</p> <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{appprimk}(op, a)$</p> $E\langle (\text{apply } e_f \text{ } e_x), \rho, a, t \rangle \rightsquigarrow E\langle e_f, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{appappk}(\emptyset, e_x, \rho, a)$</p> $E\langle (e_f \text{ } e_s \dots), \rho, a, t \rangle \rightsquigarrow E\langle e_f, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \text{alloc}(\zeta, 0)$</p> <p style="text-align: center;">$u \triangleq \text{tick}(\zeta, 1)$</p> <p style="text-align: center;">$\sigma[b] \triangleq \mathbf{appk}([], e_s, \rho, a)$</p>
--	--

Apply Rules

Rules for when the control is a value

$$\varsigma = A\langle v, \rho, a, t \rangle$$

$$\kappa \triangleq \sigma(a)$$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \varsigma$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, c) \rightsquigarrow E\langle e_f, \rho_\kappa, c, t \rangle$$

$$\text{where } v = \#f$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, c) \rightsquigarrow E\langle e_t, \rho_\kappa, c, t \rangle$$

$$\text{where } v \neq \#f$$

$$\mathbf{letk}(vars, done, [], e_b, \rho_\kappa, c)$$

$$\rightsquigarrow \langle e_b, \rho'_\kappa, c, u \rangle$$

$$\text{where } b_i \triangleq alloc(\varsigma, i)$$

$$u \triangleq tick(\varsigma, n)$$

$$\rho'_\kappa \triangleq \rho_\kappa[vars_0 \mapsto b_0 \dots$$

$$vars_{n-1} \mapsto b_{n-1}]$$

$$done' \triangleq done \# [v]$$

$$\sigma[b_0 \dots b_{n-1}] \triangleq done'_0 \dots done'_{n-1}$$

$$\mathbf{letk}(vars, done, e_h :: e_t e_b, \rho_\kappa, c)$$

$$\rightsquigarrow \langle e_h, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq alloc(\varsigma, 0)$$

$$u \triangleq tick(\varsigma, 1)$$

$$\sigma[b] \triangleq \mathbf{letk}(vars, done \# [v],$$

$$e_t, e_b, \rho_\kappa, c)$$

$$\mathbf{callock}(-, c) \rightsquigarrow E\langle e, \rho'_\lambda, c, t \rangle$$

$$\text{where } v = ((\lambda (x) e), \rho_\lambda)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto c]$$

$$\mathbf{callock}(\rho_\kappa, -) \rightsquigarrow A\langle \kappa, \rho_\kappa, b, u \rangle$$

$$\text{where } v = \kappa'$$

$$b \triangleq alloc(\varsigma, 0)$$

$$u \triangleq tick(\varsigma, 1)$$

$$\sigma[b] \triangleq \kappa'$$

$$\mathbf{setk}(x, \rho_\kappa, c) \rightsquigarrow A\langle Void, \rho_\kappa, c, t \rangle$$

$$\text{where } \sigma[\rho_\kappa(x)] \triangleq v$$

$$\mathbf{appprimk}(op, \rho_\kappa, c) \rightsquigarrow A\langle v', \rho_\kappa, c, t \rangle$$

$$\text{where } v' \triangleq op \text{ applied to } v$$

$$\mathbf{primk}(op, done, [], \rho_\kappa, c)$$

$$\rightsquigarrow A\langle v', \rho_\kappa, c, t \rangle$$

$$\text{where } v' \triangleq op \text{ applied to } (done \# [v])$$

$$\mathbf{primk}(op, done, e_h :: e_t, \rho_\kappa, c)$$

$$\rightsquigarrow E\langle e_h, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq alloc(\varsigma, 0)$$

$$u \triangleq tick(\varsigma, 1)$$

$$\sigma[b] \triangleq \mathbf{primk}(op, done \# [v],$$

$$e_t, \rho_\kappa, c)$$

More Apply Rules

Rules for when the control is a value

$$\begin{array}{ll}
\mathbf{appappk}(\emptyset, e, \rho_\kappa, c) \rightsquigarrow E\langle e, \rho_\kappa, b, u \rangle & \mathbf{appk}(done, [], \rightarrow, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle \\
\text{where } b \triangleq alloc(\varsigma, 0) & \text{where } done = ((\lambda (x_s \dots) e_b), \rho_\lambda) :: v_s \\
u \triangleq tick(\varsigma, 1) & b_i \triangleq alloc(\varsigma, i) \\
\sigma[b] \triangleq \mathbf{appappk}(v, e, \rho_\kappa, c) & u \triangleq tick(\varsigma, n) \\
\mathbf{appappk}(v_f, \rightarrow, \rightarrow, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle & \rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto b_0 \dots \\
\text{where } v_f = ((\lambda (x_s \dots) e_b), \rho_\lambda) & \quad x_{n-1} \mapsto b_{n-1}] \\
b_i \triangleq alloc(\varsigma, i) & v'_s \triangleq v_s \# [v] \\
u \triangleq tick(\varsigma, n) & \sigma[b_0] \triangleq v'_0 \dots b_{n-1} \mapsto v'_{n-1} \\
\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto b_0 \dots & \mathbf{appk}(done, [], \rightarrow, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle \\
\quad x_{n-1} \mapsto b_{n-1}] & \text{where } done = ((\lambda x e_b), \rho_\lambda) :: v_s \\
\sigma[b_0 \dots b_{n-1}] \triangleq v_0 \dots v_{n-1} & b \triangleq alloc(\varsigma, 0) \\
\mathbf{appappk}(v_f, \rightarrow, \rightarrow, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle & u \triangleq tick(\varsigma, 1) \\
\text{where } v_f = ((\lambda x e_b), \rho_\lambda) & \rho'_\lambda \triangleq \rho_\lambda[x \mapsto b] \\
b \triangleq alloc(\varsigma, 0) & v'_s \triangleq (v_s \# [v]) \\
u \triangleq tick(\varsigma, 1) & \sigma[b] \triangleq v'_s \\
\rho'_\lambda \triangleq \rho_\lambda[x \mapsto b] & \mathbf{appk}([\kappa_\lambda], [], \rho_\kappa, c) \rightsquigarrow A\langle v, \rho_\kappa, b, u \rangle \\
\sigma[b] \triangleq v & \text{where } b \triangleq alloc(\varsigma, 0) \\
& u \triangleq tick(\varsigma, 1) \\
& \sigma[b] \triangleq \kappa_\lambda \\
& \mathbf{appk}(done, e_h :: e_t, \rho_\kappa, c) \\
& \rightsquigarrow E\langle e_h, \rho_\kappa, b, u \rangle \\
& \text{where } b \triangleq alloc(\varsigma, 0) \\
& u \triangleq tick(\varsigma, 1) \\
& \sigma[b] \triangleq \mathbf{appk}(done \# [v], e_t, \rho_\kappa, c)
\end{array}$$

2 Abstract Semantics of Scheme CESKt*

Tick/Alloc:

$$\widehat{tick} : \hat{\Sigma} \times Kont \rightarrow Time$$

$$\widehat{tick}(\hat{\varsigma}, \kappa) \triangleq 0$$

$$\widehat{alloc} : \hat{\Sigma} \times Kont \rightarrow Addr$$

$$\widehat{alloc}(\hat{\varsigma}, \kappa) \triangleq 0$$

Abstract Eval Rules
Rules for when the control is an expression

$$E\langle \mathfrak{a}, \rho, a, t \rangle \rightsquigarrow A\langle v, \rho, a, t \rangle$$

where $v \triangleq \hat{\mathcal{A}}(\hat{\varsigma}, \sigma)$

$E\langle (\text{if } e_c \text{ } e_t \text{ } e_f), \rho, a, t \rangle \rightsquigarrow E\langle e_c, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{ifk}(e_t, e_f, \rho, a)$</p>	$E\langle (\text{prim } op), \rho, a, t \rangle \rightsquigarrow A\langle v, \rho, a, t \rangle$ <p style="text-align: center;">where $v = op$ applied to 0 arguments</p>
$E\langle (\text{let } () \text{ } e), \rho, a, t \rangle \rightsquigarrow E\langle e, \rho, a, t \rangle$	$E\langle (\text{prim } op \text{ } e_0 \text{ } e_s \dots), \rho, a, t \rangle$ <p style="text-align: center;">$\rightsquigarrow E\langle e_0, \rho, b, u \rangle$</p> <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{primk}(op, [], e_s, \rho, a)$</p>
$E\langle (\text{let } ([x_0 \text{ } e_0] [x_s \text{ } e_s] \dots) \text{ } e_b), \rho, a, t \rangle$ <p style="text-align: center;">$\rightsquigarrow E\langle e_0, \rho, b, u \rangle$</p> <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{letk}(x_0 :: x_s, [], e_s, e_b, \rho, a)$</p>	$E\langle (\text{apply-prim } op \text{ } e), \rho, a, t \rangle$ <p style="text-align: center;">$\rightsquigarrow \langle e, \rho, b, u \rangle$</p> <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{appprimk}(op, a)$</p>
$E\langle (\text{call/cc } e), \rho, a, t \rangle \rightsquigarrow E\langle e, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{calccck}(\rho, a)$</p>	$E\langle (\text{apply } e_f \text{ } e_x), \rho, a, t \rangle \rightsquigarrow E\langle e_f, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{appappk}(\emptyset, e_x, \rho, a)$</p>
$E\langle (\text{set! } x \text{ } e), \rho, a, t \rangle \rightsquigarrow E\langle e, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{setk}(x, a)$</p>	$E\langle (e_f \text{ } e_s \dots), \rho, a, t \rangle \rightsquigarrow E\langle e_f, \rho, b, u \rangle$ <p style="text-align: center;">where $b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$ $u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$ $\hat{\sigma}[b] \sqcup \text{appk}([], e_s, \rho, a)$</p>

Apply Rules

Rules for when the control is a value

$$\hat{\varsigma} = A\langle v, \rho, a, t \rangle$$

$$\kappa \in \hat{\sigma}(a)$$

Proceed by matching on κ

$$\mathbf{mt} \rightsquigarrow \hat{\varsigma}$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, c) \rightsquigarrow E\langle e_f, \rho_\kappa, c, t \rangle$$

$$\text{where } v = \#f$$

$$\mathbf{ifk}(e_t, e_f, \rho_\kappa, c) \rightsquigarrow E\langle e_t, \rho_\kappa, c, t \rangle$$

$$\text{where } v \neq \#f$$

$$\mathbf{letk}(vars, done, [], e_b, \rho_\kappa, c)$$

$$\rightsquigarrow \langle e_b, \rho'_\kappa, c, u \rangle$$

$$\text{where } b_i \triangleq \widehat{alloc}(\hat{\varsigma}, i, \kappa)$$

$$u \triangleq \widehat{tick}(\hat{\varsigma}, n, \kappa)$$

$$\rho'_\kappa \triangleq \rho_\kappa[vars_0 \mapsto b_0 \dots$$

$$vars_{n-1} \mapsto b_{n-1}]$$

$$done' \triangleq done \# [v]$$

$$\hat{\sigma}[b_0 \dots b_{n-1}] \sqcup done'_0 \dots done'_{n-1}$$

$$\mathbf{letk}(vars, done, e_h :: e_t e_b, \rho_\kappa, c)$$

$$\rightsquigarrow \langle e_h, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$$

$$u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$$

$$\hat{\sigma}[b] \sqcup \mathbf{letk}(vars, done \# [v],$$

$$e_t, e_b, \rho_\kappa, c)$$

$$\mathbf{callocck}(-, c) \rightsquigarrow E\langle e, \rho'_\lambda, c, t \rangle$$

$$\text{where } v = ((\lambda (x) e), \rho_\lambda)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto c]$$

$$\mathbf{callocck}(\rho_\kappa, -) \rightsquigarrow A\langle \kappa, \rho_\kappa, b, u \rangle$$

$$\text{where } v = \kappa'$$

$$b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$$

$$u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$$

$$\hat{\sigma}[b] \sqcup \kappa'$$

$$\mathbf{setk}(x, \rho_\kappa, c) \rightsquigarrow A\langle \text{Void}, \rho_\kappa, c, t \rangle$$

$$\text{where } \hat{\sigma}[\rho_\kappa(x)] \triangleq v$$

$$\mathbf{appprimk}(op, \rho_\kappa, c) \rightsquigarrow A\langle v', \rho_\kappa, c, t \rangle$$

$$\text{where } v' \triangleq op \text{ applied to } v$$

$$\mathbf{primk}(op, done, [], \rho_\kappa, c)$$

$$\rightsquigarrow A\langle v', \rho_\kappa, c, t \rangle$$

$$\text{where } v' \triangleq op \text{ applied to } (done \# [v])$$

$$\mathbf{primk}(op, done, e_h :: e_t, \rho_\kappa, c)$$

$$\rightsquigarrow E\langle e_h, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq \widehat{alloc}(\hat{\varsigma}, 0, \kappa)$$

$$u \triangleq \widehat{tick}(\hat{\varsigma}, 1, \kappa)$$

$$\hat{\sigma}[b] \sqcup \mathbf{primk}(op, done \# [v],$$

$$e_t, \rho_\kappa, c)$$

More Apply Rules

Rules for when the control is a value

$$\mathbf{appappk}(\emptyset, e, \rho_\kappa, c) \rightsquigarrow E\langle e, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq \widehat{alloc}(\varsigma, 0)$$

$$u \triangleq \widehat{tick}(\varsigma, 1)$$

$$\hat{\sigma}[b] \sqcup \mathbf{appappk}(v, e, \rho_\kappa, c)$$

$$\mathbf{appappk}(v_f, -, -, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle$$

$$\text{where } v_f = ((\lambda (x_s \dots) e_b), \rho_\lambda)$$

$$b_i \triangleq \widehat{alloc}(\varsigma, i)$$

$$u \triangleq \widehat{tick}(\varsigma, n)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto b_0 \dots$$

$$x_{n-1} \mapsto b_{n-1}]$$

$$\hat{\sigma}[b_0 \dots b_{n-1}] \sqcup v_0 \dots v_{n-1}$$

$$\mathbf{appappk}(v_f, -, -, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle$$

$$\text{where } v_f = ((\lambda x e_b), \rho_\lambda)$$

$$b \triangleq \widehat{alloc}(\varsigma, 0)$$

$$u \triangleq \widehat{tick}(\varsigma, 1)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto b]$$

$$\hat{\sigma}[b] \sqcup v$$

$$\mathbf{appk}(done, [], -, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle$$

$$\text{where } done = ((\lambda (x_s \dots) e_b), \rho_\lambda) :: v_s$$

$$b_i \triangleq \widehat{alloc}(\varsigma, i)$$

$$u \triangleq \widehat{tick}(\varsigma, n)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x_0 \mapsto b_0 \dots$$

$$x_{n-1} \mapsto b_{n-1}]$$

$$v'_s \triangleq v_s \# [v]$$

$$\hat{\sigma}[b_0] \sqcup v'_0 \dots b_{n-1} \mapsto v'_{n-1}$$

$$\mathbf{appk}(done, [], -, c) \rightsquigarrow E\langle e_b, \rho'_\lambda, c, u \rangle$$

$$\text{where } done = ((\lambda x e_b), \rho_\lambda) :: v_s$$

$$b \triangleq \widehat{alloc}(\varsigma, 0)$$

$$u \triangleq \widehat{tick}(\varsigma, 1)$$

$$\rho'_\lambda \triangleq \rho_\lambda[x \mapsto b]$$

$$v'_s \triangleq (v_s \# [v])$$

$$\hat{\sigma}[b] \sqcup v'_s$$

$$\mathbf{appk}([\kappa_\lambda], [], \rho_\kappa, c) \rightsquigarrow A\langle v, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq \widehat{alloc}(\varsigma, 0)$$

$$u \triangleq \widehat{tick}(\varsigma, 1)$$

$$\hat{\sigma}[b] \sqcup \kappa_\lambda$$

$$\mathbf{appk}(done, e_h :: e_t, \rho_\kappa, c)$$

$$\rightsquigarrow E\langle e_h, \rho_\kappa, b, u \rangle$$

$$\text{where } b \triangleq \widehat{alloc}(\varsigma, 0)$$

$$u \triangleq \widehat{tick}(\varsigma, 1)$$

$$\hat{\sigma}[b] \sqcup \mathbf{appk}(done \# [v], e_t, \rho_\kappa, c)$$