Machine Learning 2 - HW1

Submission date: 09/12/2022

Theoretical Part (20 points):

1. The Softmax function is used to normalize the output of a neural network $f(\cdot, w)$ to a probability distribution over predicted output classes.
   For n classes, denote by $f(\cdot, w)_i$ the unnormalized output corresponding to the ith class:

$$\hat{y}_i = \text{Softmax}(x)_i = \frac{\exp^{f(x;w)_i}}{\sum_{j=0}^{n} \exp^{f(x;w)_j}}.$$

   Denote $by\ L(yy)$ to be the loss function.

   Show the derivative of the loss $w.r.t$ the weights. I.e., $\frac{\partial L(\hat{y}, y)}{\partial w}$.

Practical Part:

1. (40 points) In the following exercise, you will create a classifier for the MNIST dataset. You shouldwrite your own training and evaluation code and meet the following constraints:

   - You are only allowed to use torch tensor manipulations.
   - You are NOT allowed to use:
     - Auto-differentiation - *backward()*
     - Built-in loss functions
     - Built-in activations
     - Built-in optimization
     - Built-in layers (torch.nn)

   The neural network you build should:

   - Have at least one hidden layer
   - Obtain at least 75% accuracy on the test set

2. (40 points) In this exercise, we will demonstrate overfitting to random labels.The settings are the following:
   - Use the MNIST dataset.
   - Work on the first 128 samples from the training dataset.
   - Fix the following parameters:
     - Shuffle to False.
     - Batch size to 128.
   - Generate random labels from Bernoulli distribution with a probability of ½. I.e., each sample is assigned a random label which is zero or one.

   Show that using the network (architecture) from Ex.1 and cross-entropy loss you are able to achieve a loss value of ~0 (the lower the better).

   Plot the loss convergence for this data and the test data as a function of epochs.

What is the mean loss value of the test data? Explain.