# A Predictive Methodology for Truthful Double Spectrum Auctions in Cognitive Radio Networks

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***Abstract*—Auction is often applied in cognitive radio networks due to its efficiency and fairness properties. An important issue in designing an auction mechanism is how to utilize the limited spectrum resource in an efficient manner. In order to achieve this goal, we propose a predictive double spectrum auction model in this paper. Our auction model first obtains the bidding range from statistical analysis, and then separates the interval into independent states and employees a Markovian prediction based algorithm to generate guidelines for the bidding range of primary and secondary users, respectively. Comparing with existing approaches, our proposed auction model is more efficient in spectrum utilization and satisfies the economic properties. Extensive simulation results show that our work achieves an utilization ratio up to $91\%$.**

## I. Introduction

Due to the multiformity of wireless communication services and the propagation of various wireless network technologies, the radio spectrum becomes a remarkable and limited resource. However, former studies [1] and report from FCC [2] indicate that there exist a large number of unused spectrum opportunities. Meanwhile, the licence policy exacerbates the current spectrum issue. In order to overcome the low utilization of radio spectrum in a time-spatial view, cognitive radio (CR) networks were proposed as a promising solution to efficiently utilize the limited spectrum resources [3].

In CR networks, the operators who possess the licensed spectrum resources are named primary users (PUs), and the users who ask to access spectrum resources from PUs are called secondary users (SUs) [4]. How to distribute the remanent spectrum resources among the SUs is a significant challenge. Considering the fairness and allocation efficiency, auctions are amid the best-known economical tools to address this problem [5].

In the auction model, the PUs are regarded as sellers and SUs are buyers, respectively. In general, the auction mechanisms can be divided into two categories [6] depending on the number of sellers and buyers, i.e., the single-sided spectrum auctions and double spectrum auctions. In single-side spectrum auctions [7], there is a single operator which performs as the seller on behalf of all primary users. The primary users grant all their rights on the spectrum resource to this operator. However, primary users are usually selfish and rational in reality, and they prefer participating in the market on their own perspectives. In order to cope with this demand,

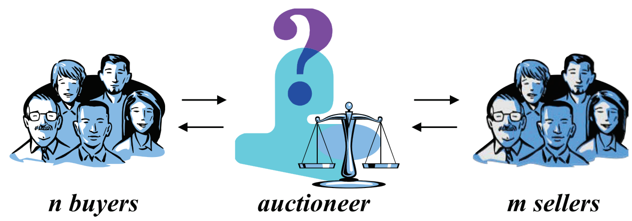


Fig. 1: Auction Market

researchers proposed the *double auction* mechanism which is illustrated in Figure 1. Due to its flexibility and reality, double auction is obviously easier to be accepted in the market, and it thus has become the main stream of the research interests as well as market reality.

In order to better utilize the spectrum market, researchers have been considering the *spectrum reuse* technology, i.e., multiple non-conflicting SUs share the same spectrum channels. Although proven to be efficient, previous studies using double auction (e.g. [8], [9], [10] and [11]) have been rarely considered reusability. In their recent work, Zhou and Zheng [12] first proposed a truthful double auction framework, *TRUST*, with consideration of the spectrum reuse issue. However, it fails to achieve a high spectrum utilization. In this paper, we extend *TRUST* with the correlation in a view of the time dimension, which significantly improves the spectrum utilization. To our best knowledge, this is the first work that satisfies truthfulness, spectrum reusability and high utilization properties under the double spectrum auction method.

In this paper, we propose a predictive double spectrum auction model, introducing a Markovian approach and enabling spectrum reemployment among multiple non-conflicting buyers. In our model, a central auctioneer assembles all the bids from sellers and buyers periodically. At the initial phase, we obtain the range of bidding based on former auction rounds. Based on the historical data, the auctioneer calculates rational state transition matrices for both sides and sends them to sellers and buyer groups respectively. These matrices guide players to offer bids in a reasonable range. By this means, the whole auction procedure is operated in a supervised manner, and this specific design optimizes the auction performances. The proposed auction model considers the spectrum reuse, im-

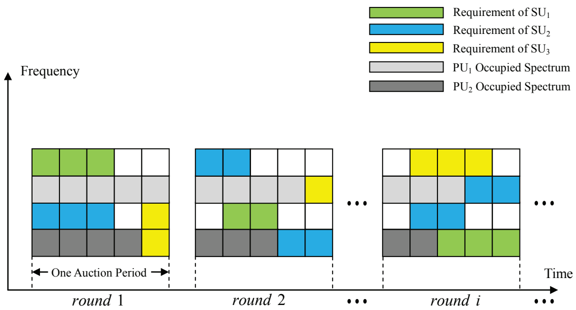


Fig. 2: Spectrum Resource Model

proves the spectrum utilization and prevents malicious actions in the market. Therefore, our predictive model leads the market to a stable, fair and highly efficient state. Our contributions are:

- Considering the correlation in the time dimension, we propose a novel double spectrum auction model. What's more, our model addresses the requirement of SUs flexibility and resource reuse at the same time.
- We integrate economic tools with a fashionable methodology, namely a Markovian prediction based mechanism. Employing a probability way makes the bids more rational and authentic. Proposed mechanism leads to a stable market circumstance.
- In addition to truthfulness, individual rationality and ex-post budget balance are the other two critical properties demanded. We prove that our model fits these properties and thus leads to the economic-robustness.

The rest of the paper is organized as follows. In Section II, we present our spectrum allocation problem and describe several related topics. Section III reveals our predictive auction algorithm cooperated with the certification about economic-robust. Performance of our algorithm is evaluated in Section IV. Section V concludes and states our future work.

## II. Preliminaries

This section discusses a two-dimensional spectrum resource model for an outline, as well as critical economic properties which are needed in the auction.

### A. Spectrum Resource Model

We formulate the spectrum resource model in Figure 2. Spectrum resources are divided in a time-spatial view, and this segmentation is made by an auctioneer. Each colored piece means a spectrum slot, and we consider spectrum slots are homogenous goods to different buyers. The allocation scheme is unstable in different round. It is noted that each seller will lease at most one spectrum slot per auction round. Considering multiple sellers and multiple buyers, the problem here is referred to a double auction issue.

Now we formally define the algebraic form of our auction model. The auction consists of a set of sellers $\mathbb{P} = \{p_1, p_2, \cdots, p_m\}$. Each of them holds the right of one spectrum channel in one auction period, i.e., the discrete time interval $[t_0, t_0 + T]$. For each seller $p_j$ $(j = 1, 2, \cdots, m)$ at round $t$, we denote $C_j(t)$ as the cost valuation of its one spectrum slot, $B_j^s(t)$ is the minimum payment needed to sell a spectrum, and $I_j^s(t)$ is the payment when he actually wins. Hence, the revenue of seller $p_j$ is $W_j^s(t) = I_j^s(t) - B_j^s(t)$. Besides, the set of buyers is $\mathbb{S} = \{s_1, s_2, \cdots, s_n\}$. For each buyer $s_i$ $(i = 1, 2, \cdots, n)$ at round $t$, we denote $V_i(t)$ as its expected valuation of one spectrum slot, $B_i^b(t)$ is the maximum price he wants to pay, and $I_i^b(t)$ is the charging price when he actually wins. Thus the revenue of buyer $s_i$ is $W_i^b(t) = B_i^b(t) - I_i^b(t)$. The utility functions of each seller $p_j$ and buyer $s_i$ are:

$$U_j^s(t) = \begin{cases} I_j^s(t) - C_j(t) & \text{seller } p_j \text{ wins,} \\ 0 & \text{otherwise;} \end{cases}$$
$$U_i^b(t) = \begin{cases} V_i(t) - I_i^b(t) & \text{buyer } s_i \text{ wins,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Note that the auction may have more than one pair of seller-buyer per round, and $C_j(t)$ and $V_i(t)$ are private to their own players, respectively.

### B. Economic Properties

The objective of our proposed auction model is to satisfy critical economic properties, i.e., to guarantee truthfulness, individual rationality and ex-post budget balance. These three economic properties are discussed as follows.

1) Truthfulness. It means that no seller $p_j$ or buyer $s_i$ can improve its own utility by offering untruthful bids, while neglecting other players' bids. Bidding untruthfully refers to the status that $B_j^s(t) \neq C_j(t)$ and $B_i^b(t) \neq V_i(t)$.
   The players in the auction behave selfishly, so they can manipulate their bids and beat others in untruthful auctions. On the contrary, truthful auctions can avoid market manipulation. Therefore, truthfulness is fundamental to resist market cheating and leads to a good economic environment.
2) Individual Rationality. Individual rationality in double auctions means that the winning bids of sellers and buyers are within a rational range. Individual rationality is formulated as:
$$I_j^s(t) \geq B_j^s(t), I_i^b(t) \leq B_i^b(t), \forall p_j, s_i. \tag{2}$$
   Based on this property, players are willing to participate the auction.
3) Ex-post Budget Balance. For the auctioneer, the profit is the difference between the income from buyers and outcome to sellers. Ex-post budget balance means that the profit of the auctioneer $\Psi(t) \geq 0$ and is shown as:
$$\Psi(t) = \sum_{i=1}^{n} I_i^b(t) - \sum_{j=1}^{m} I_j^s(t) \geq 0. \tag{3}$$
   This property provides the inspiration for the auctioneer to operate the auction.

## III. A Predictive Algorithm for Double Spectrum Auctions

In this section, we first construct the structure of our predictive algorithm. Afterwards, we present the static winning determination and pricing strategy. Finally, we prove that our algorithm satisfies economic properties in previous section.

### A. Markovian Prediction Based Algorithm

This part represents an algorithm running in a circulation pattern. At the initial phase, we obtain the bidding range based on former auction rounds. Then this range interval is divided and each part is an independent state in a Markov chain. Afterwards, a Markovian prediction based algorithm is exploited by the central auctioneer to construct predicting matrices, which act as guidelines for players' bidding. The whole process consists of three stages, i.e., the division of valuation interval, construction of state transition matrix and guidelines for game players.

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**Algorithm 1:** RangeDividing $divIn((MIN, MAX), k, \varepsilon)$

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**Input**: valuation range $(MIN, MAX)$, index $k$ and relaxing faction $\varepsilon$
**Output**: divided intervals $\{O_p | p = 1, 2, \dots, k\}$

```
if (MIN, MAX) is a legal range then
    τ = MAX − MIN
    for i = 1 to k − 1 do
        O_i = (MIN + (i−1)τ/k, MIN + iτ/k]
    end
    O_k = (MAX − τ/k, MAX + ε)
end
return {O_p | p = 1, 2, ..., k}
```

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*1) **Division of Valuation Interval**:* After a certain amount of auctions, the auctioneer gets a range of $I_j^s(t)$ and $I_i^b(t)$. We apply a same division method to both sides as follows. Denote the valuation range as $(MIN, MAX)$, and we divide the range into $k$ parts as $\{O_p | p = 1, 2, \dots, k\}$ , where $O_p \bigcap O_q = \emptyset$ and $\bigcup_p O_p = (MIN, MAX)$. Each divided part is an independent state in a Markov chain. Without loss of generality, we introduce a relaxing faction $\varepsilon$ for the accuracy of prediction. The detailed process is shown as $Algorithm$ 1.

*2) **Construction of State Transition Matrix**:* We express a variable $x_t$, and $x_t = p$ means that the value of $x$ is belong to range $O_p$ in round $t$. In other words, we map the variable to the Markov state. Thus we have $x_t = 1, 2, \dots, k$. Considering the temporal correlation, we construct state transition matrix $(F_{ij})_{k\times k}$ between $x_t$ and $x_{t+1}$. Each element $f_{ij}$ describes the frequency of state $x_t = i$ to $x_{t+1} = j$. Afterwards, we define a random variable $\zeta = 2\sum\limits_i \sum\limits_j F'_{ij} |\log \frac{F'_{ij}}{F'_{\cdot j}}|$, which satisfies a chi-square distribution with a $(k-1)^2$ freedom degree, i.e., $\chi^2[(k-1)^2]$. In the following part, we will verify whether this variable fits the Markov property. The Markov chain plays an important role in predicting business strategy, and this statistic-based methodology is widely used in various aspects [13] [14]. These above procedures are shown as $Algorithm$ 2.

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**Algorithm 2:** Dealing with State Transition Matrix $conMtx(\{f_{ij} | i, j = 1, 2, \dots, k\}, k)$

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**Input**: frequency set $\{f_{ij} | i, j = 1, 2, \dots, k\}$, index $k$
**Output**: $(F'_{ij})_{k\times k}, (F'_{\cdot j}), \zeta$

1.
$$(F_{ij})_{k\times k} = \begin{pmatrix} f_{11} & f_{12} & \dots & f_{1k} \\ f_{21} & f_{22} & \dots & f_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ f_{k1} & f_{k2} & \dots & f_{kk} \end{pmatrix}$$
2. $(F'_{ij})_{k\times k} = \frac{f_{ij}}{\sum\limits_i \sum\limits_j f_{ij}}$
3. $(F'_{\cdot j}) = \sum\limits_i F'_{ij} = \frac{\sum\limits_i f_{ij}}{\sum\limits_i \sum\limits_j f_{ij}}, \forall j = 1, 2, \dots, k$
4. $\zeta = 2\sum\limits_i \sum\limits_j F'_{ij} |\log \frac{F'_{ij}}{F'_{\cdot j}}|$
5. return $(F'_{ij})_{k\times k}, (F'_{\cdot j}), \zeta$

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*3) **Guidelines for Game Players**:* Now we show guidelines for game players with an overall perspective of our algorithm, which is shown as $Algorithm$ 3. The spectrum buyers follow a group manner, which will be illustrated in Section III-B, as well as the particular item "modified McAfee double auction procedure".

The algorithm consists of a random part and a guided part. In the former part, players randomly bid and the auctioneer executes the modified McAfee double auction algorithm within every $\Delta$ auction rounds. Getting the divided intervals with $Algorithm$ 1, we regard each interval segment as an independent state in a Markov chain, and calculate state transition frequency $\{f_{ij}\}$ combining with historical probability $\{P_{ij}\}$ for buyer groups (or $\{Q_{ij}\}$ for sellers). Then we construct state transition matrices ($Algorithm$ 2, for buyer groups and sellers), and verify if both of statistic-based variables (i.e., $\zeta$ and $\vartheta$) fit the Markov property or not. A positive answer will turn the algorithm to the guided part. Otherwise, we repeat the above steps in next $\Delta$ rounds. With simplicity and generality, we set $MIN = 0$ and only look for the maximum bids in the interval division procedure.

In the guided part, we calculate the $x^*_{t+1}$ and $y^*_{t+1}$ for maximizing the expectation of individual utility, which can be regarded as guidelines for players' bidding. The auctioneer transmits $x^*_{t+1}$ to buyer group $i$, and $y^*_{t+1}$ to seller $j$. Then the players update their own trading strategies. Group $i$ bids as $B_i^{bg}(t+1) = \min\{\mathcal{F}(x^*_{t+1}), V_i(t+1)\}$, and seller $j$ bids as $B_j^s(t+1) = \max\{\mathcal{H}(y^*_{t+1}), C_j(t+1)\}$ [1]. Therefore, the auctioneer executes the modified McAfee algorithm in a guided method. Ultimately, all game players can offer their biddings within a rational range, and the spectrum market gets a high utilization with a well trade-off of social welfare.

[1] Here, $\mathcal{F}$ and $\mathcal{H}$ denote the bidding distribution function of buyers and sellers, respectively.

**Algorithm 3:** Markov Chain Based Algorithm

**Input**: $\delta_1 = 0$, $\delta_2 = \Delta$, $MAX_1 = 0$, $MAX_2 = 0$, $(M_{ij})_{k_1 \times k_1} = \mathbf{0}$, $(N_{ij})_{k_2 \times k_2} = \mathbf{0}$, $k_1, k_2, \varepsilon_1, \varepsilon_2, \alpha$
**Output**: $x^*_{t+1}$, $y^*_{t+1}$, $\xi$

**while** *1* **do**
  **for** $t = \delta_1$ *to* $\delta_2$ **do**
    Modified McAfee double auction procedure
  **end**
  $MAX_1 = max\{B_i^{bg}(\delta_1), \dots, B_i^{bg}(\delta_2)\}$
  $MAX_2 = max\{B_j^{s}(\delta_1), \dots, B_j^{s}(\delta_2)\}$
  $\{A_i | i = 1, 2, \dots, k_1\} = divIn((0, MAX_1), k_1, \varepsilon_1)$
  $\{B_j | j = 1, 2, \dots, k_2\} = divIn((0, MAX_2), k_2, \varepsilon_2)$
  Calculate frequency $\{f_{ij}^A\}$ and $\{f_{ij}^B\}$
  $\{P_{ij}\} = \{P_{ij}\} + \{f_{ij}^A\}$
  $\{Q_{ij}\} = \{Q_{ij}\} + \{f_{ij}^B\}$
  $\{(P'_{ij})_{k_1 \times k_1}, (P'_{\cdot j}), \zeta\} = conMtx(\{P_{ij}\}, k_1)$
  $\{(Q'_{ij})_{k_2 \times k_2}, (Q'_{\cdot j}), \vartheta\} = conMtx(\{Q_{ij}\}, k_2)$
  **if** $\zeta > \chi^2_\alpha[(k_1 - 1)^2]$ *and* $\vartheta > \chi^2_\alpha[(k_2 - 1)^2]$ **then**
    $x^*_{t+1} \in \arg\max_{x_{t+1}}\{\sum[V_i(t) - E(x_{t+1})] \cdot P'_{x_t x_{t+1}}\}$
    $y^*_{t+1} \in \arg\max_{y_{t+1}}\{\sum[E(y_{t+1}) - C_j(t)] \cdot Q'_{y_t y_{t+1}}\}$
    $\xi = \delta_2$
    break
  **end**
  **else**
    $\delta_1 = \delta_1 + \Delta, \delta_2 = \delta_2 + \Delta$
  **end**
**end**
return $x^*_{t+1}$, $y^*_{t+1}$, $\xi$

### *B. Modified McAfee Double Auction*

McAfee double auction [15] is a primary strategy in auction market. This strategy matches sellers and buyers one by one via a series of conditions, and discards the last pair who meets the conditions. McAfee strategy satisfies the economic properties above. Another designing algorithm, *TRUST* [12], updates the McAfee strategy to achieve spectrum reusability.

Before the actual auction, buyers are divided into groups in a non-conflicting manner. Buyers in the same group are assigned to the same spectrum resources. It is also worthy to note that the group information is private to the auctioneer. Graph theory [16], interference model [17], etc. can be used to assign buyer groups, however, this topic is beyond the scope of this paper. Assume that the formulation of groups are fixed in the auction procedure, which is in a view of reality.

We extend *TRUST* to a time dimension form here. Denote buyer groups as $\mathbb{G} = \{g_1, g_2, \cdots, g_q\}$, and for group $g_c$ in round $t$, the group bid $B_c^{bg}(t)$ is

$$B_c^{bg}(t) = \mathit{min}\{B_i^b(t) | i \in g_c\} \cdot |g_c|. \tag{4}$$

The auctioneer arranges the seller and buyer group bids as

$$seller: B_1^s(t) \leq B_2^s(t) \leq \ldots \leq B_m^s(t),$$

$$buyer: B_1^{bg}(t) \geq B_2^{bg}(t) \geq \ldots \geq B_q^{bg}(t). \tag{5}$$

Define a winning index as $k(t)$:

$$k(t) = \underset{k \leq \min\{m,q\}}{\arg\max}\; B_k^{bg}(t) \geq B_k^s(t). \tag{6}$$

Thus the auction winners are the first $(k-1)$ sellers and $(k-1)$ buyer groups. For winning sellers, the auctioneer pays each player $j$ by $I_j^s(t) = B_k^s(t)$. Meanwhile, the auctioneer charges each buyer in winning group $g_w$ as

$$I_i^b(t) = \frac{B_k^{bg}(t)}{|g_w|}, \forall i \in g_w, w = 1, 2, \ldots, k-1. \tag{7}$$

### *C. Proof of Economic Properties*

This part gives the proof of three economic properties described in Section II. The proof contains two parts: one for the random stage and the other for the guided stage.

**Theorem 1.** *Proposed algorithm satisfies economic properties in the random stage.*

*Proof.* Modified McAfee double auction algorithm is applied to every round of random stage. Obviously, truthfulness, ex-post budget balance and individual rationality are confirmed as a time form of *TRUST*. And therefore, proposed algorithm satisfies economic properties in the random stage. $\square$

**Theorem 2.** *Proposed algorithm satisfies economic properties in the guided stage.*

*Proof.* In the guided stage, the pricing strategy is different from Theorem 1. Firstly, we prove the **truthfulness** holds in our algorithm. Depending on a player bidding truthfully or not, we obtain four cases in the guided auction stage.

- Case 1: A player loses when he bids truthfully or not.
  For both buyers and sellers, the utility is always 0, according to Equation 1. That means there is no increase to their utility.
- Case 2: A player wins when he bids truthfully or not.
  For winning buyer $s_i$, the charging price stays constant whatever he bids (see Equation 1 and 7). This leads to a same utility for the winning buyer. The situation of winning seller $p_j$ is same as $s_i$.
- Case 3: A player wins when he bids truthfully, and he loses with untruthful bidding.
  For the winning situation, the buyer $s_i$ bids truthfully, namely $B_i^b(t) = \min\{\mathcal{F}(x_t^*), V_i(t)\} \leq V_i(t)$. The result is a positive utility of that. Untruthful situation leads to zero utility since he loses the bid. The sellers holds the same circumstance.
- Case 4: A player wins when he bids untruthfully, while he loses with truthful bidding.
  For buyer $s_i \in g_c$, we can derive Case 4 to $B_i(t)^b = \min\{\mathcal{H}(x_t^*), V_i(t)\} > V_i(t)$. Since the group bid $B_c^{bg}(t) = \min\{B_i^b(t) | i \in g_c\} \cdot |g_c|$ increases, the charging price $I_i^b(t)$ will be also added. Equation 1 tells us the utility is a negative value at this time. The similar process happens at the seller side.

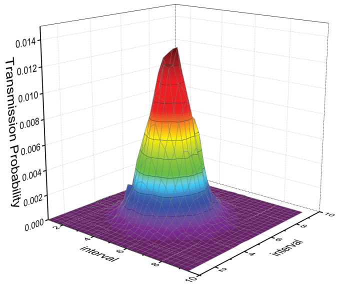

Fig. 3: Buyer Matrix

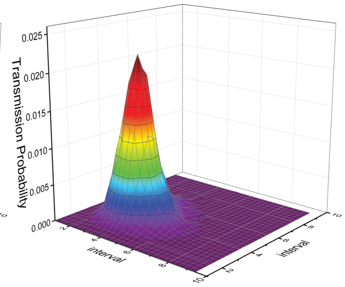

Fig. 4: Seller Matrix

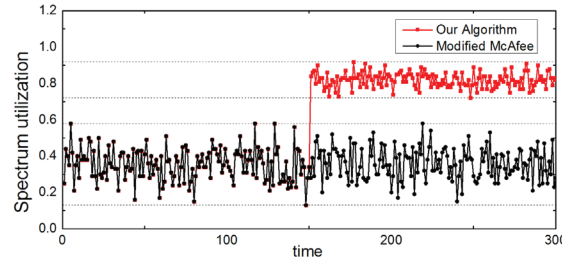


Fig. 5: Performance of Utilization

Secondly, we now prove that the **ex-post budget balance** property is satisfied. The profit of the central auctioneer is:

$$\begin{aligned}\Psi(t) &= \sum_{i=1}^{n} I_i^b(t) - \sum_{j=1}^{m} I_j^s(t) \\ &= \sum_{i=1}^{q} I_i^{bg}(t) - \sum_{j=1}^{m} I_j^s(t) \\ &= \sum_{i=1}^{k-1} I_i^{bg}(t) - \sum_{j=1}^{k-1} I_j^s(t) \\ &= \sum_{i=1}^{k-1} [B_k^{bg}(t) - B_k^s(t)] \geq 0.\end{aligned} \tag{8}$$

Profit $\Psi(t) \geq 0$ means it fits the ex-post budget balance. Note that we assemble individual buyer to a group view here.

At last, we give the proof of **individual rationality**. For each winning seller $p_j$, we have $I_j^s(t) = B_k^s(t) \geq B_j^s(t)$. On the other hand, for each winning buyer $s_i$, we also have $I_i^b(t) = \frac{B_k^{bg}(t)}{|g_c|} \leq \frac{B_c^{bg}(t)}{|g_c|} \leq B_i^b(t)$, where $i \in g_c$. Therefore, the players are individually rational.

Concluding above, we show that our proposed algorithm satisfies economic properties in guided stage. □

## IV. Performance Evaluation

In this section, we first show guideline matrices of sellers and buyer groups. Afterwards, we show the spectrum utilization of proposed algorithm. Finally, we discuss that how divided number affects the system performance.

### A. Simulation Setup

Here we apply the Max-IS algorithm [18] to classify buyer groups. Max-IS assigns buyer groups by finding the maximum independent set of the conflict graph. Meanwhile, we set 100 buyers randomly located in the network and the bids of buyer side are uniformly distributed over $(0, 1]$. Additionally, we set there are 10 sellers and the bids of seller side are uniformly distributed over $(0, 2]$ by default.

### B. Guideline Matrices

We set the parameter $\Delta = 30$ in the simulation process. Based on proposed algorithm, we calculate state transition matrices by the end of every $\Delta$ auction rounds. The calculation results are shown in Table I. Here we divide the range interval into 10 independent parts for both sides, and the confidence interval is set as $\alpha = 0.005$.

TABLE I: Verifying Procedure

| Timing | $\Delta$ | $2\Delta$ | $3\Delta$ | $4\Delta$ | $5\Delta$ |
|---|---|---|---|---|---|
| $\chi^2_\alpha[(n-1)^2]$ | | | 116.32 | | |
| buyer variable $\zeta$ | 41.33 | 59.32 | 75.15 | 104.21 | 128.99 |
| seller variable $\vartheta$ | 43.35 | 55.23 | 77.58 | 101.63 | 129.51 |

In Table I, $\zeta(5\Delta) > \chi^2_\alpha[(n-1)^2]$ and $\vartheta(5\Delta) > \chi^2_\alpha[(n-1)^2]$ at the same time. Therefore, at the round $5\Delta$, state transition matrices fit the Markov property and we obtain the predicting matrices. The guideline matrices of buyers and sellers are shown in Figure 3 and Figure 4 respectively. We can find that the elements in these two matrices have a high aggregation degree, which indicates that the bidding market becomes a regular and stable one in the proposed algorithm.

### C. Performance of Utilization

We give the comparison of utilization performance in Figure 5. In Figure 5, the abscissa axis is simulation time and the vertical axis is the ratio of spectrum utilization. We compare the performance of our algorithm with former *TRUST* in the same scenario.

Figure 5 depicts the comparison of the spectrum utilization. We can find out that our algorithm coincides with *TRUST* before $5\Delta$, while it raises to a higher level utilization of spectrum resource. In each steady state, the utilization ratio of *TRUST* is about $40\%$, and proposed algorithm is nearly $80\%$. The mutational point is around $5\Delta$. Attention that our algorithm can reach utilization ratio of $91\%$ while takes the spectrum reuse into consideration.

Stability is another criterion to evaluate the property of one algorithm. Good stability in the market indicates that the entire market works in a rational and logical manner. Denoted by the dotted line, we can find the range of utilization ratio in *TRUST* is $12\% \sim 59\%$. On the other side, the ratio range of our algorithm is $71\% \sim 91\%$, which is much better than the former one.

Overall, the proposed algorithm processes excellent utilization and stability. That is because we apply a supervised style to guide bidding for all game players. By proposing guiding matrices, buyers or sellers will bid the spectrum resource in a certain range. The reason above leads to the improved performance of utilization and stability.

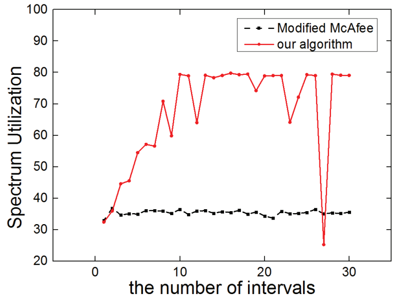

Fig. 6: Impact of Divided Intervals

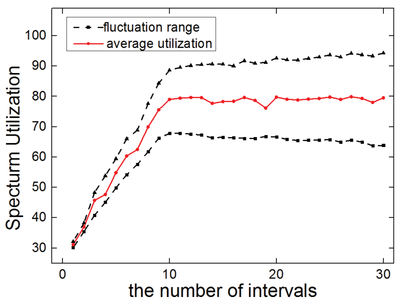

Fig. 7: Utilization Fluctuation

### *D. Impact of Divided Interval*

This part discusses the influence of divided segment quantity on system utilization. As our description before, the guiding matrix is closely related to the number of divided interval parts. Here we apply spectrum utilization as the system evaluation index, and the simulation scenario stays unchangeable. We examine the performance of spectrum utilization with divided number varying from 0 to 30.

Figure 6 shows the average spectrum utilization with the increase of divided number. Although performs more fluctuant than the modified McAfee model, our algorithm has a high-level spectrum utilization. We can see that the utilization continues to increase until the summit value of 80% before number 10. The performance is not always excellent since some severely fluctuation, especially at number 27. This phenomenon indicates that small divided parts may lead to a low utilization, and thus an intelligent method for division is eagerly needed.

Figure 7 shows the fluctuation level of our algorithm. The red line refers to the average ratio, and the other lines are fluctuation range. With the increase of divided number, spectrum utilization tends to be a gentle value of 80%, which is as same as the former result. Upper bound of the ratio is 94% and lower bound is 63%.

## V. Conclusion and Future Works

In this paper, we design an economic-robust double spectrum auction mechanism for cognitive radio networks. We distribute spectrum resources in a time-spatial view, and map the allocation problem to auction in the market. Then we propose a Markov prediction based algorithm to tackle the problems above. Meanwhile, we prove that our algorithm is truthful, individual rational and ex-post budget balanced. Via extensive simulation, we show that our algorithm processes a good performance on spectrum utilization. As for the future work, we will improve the spectrum utilization by changing the hard style of interval division to an adaptive mechanism.

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