Prolog

programming for artificial intelligence

Third edition

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Third edition

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I dedicate the third edition of this book
to my mother, the kindest person I know
and to my father, who, during world war II
escaped from a concentration camp by
digging an underground tunnel, which he
described in his novel, *The Telescope*

Contents

From Patrick Winston's Foreword to the Second Edition

I can never forget my excitement when I saw my first Prolog-style program in action. It was part of Terry Winograd's famous Shrdlu system, whose blocks-world problem solver arranged for a simulated robot arm to move blocks around a screen, solving intricate problems in response to human-specified goals.

Winograd's blocks-world problem solver was written in Microplanner, a language which we now recognize as a sort of Prolog. Nevertheless, in spite of the defects of Microplanner, the blocks-world problem solver was organized explicitly around goals, because a Prolog-style language encourages programmers to think in terms of goals. The goal-oriented procedures for grasping, clearing, getting rid of, moving, and ungrasping made it possible for a clear, transparent, concise program to seem amazingly intelligent.

Winograd's blocks-world problem solver permanently changed the way I think about programs. I even rewrote the blocks-world problem solver in Lisp for my Lisp textbook because that program unalterably impressed me with the power of the goal-oriented philosophy of programming and the fun of writing goal-oriented programs.

But learning about goal-oriented programming through Lisp programs is like reading Shakespeare in a language other than English. Some of the beauty comes through, but not as powerfully as in the original. Similarly, the best way to learn about goal-oriented programming is to read and write goal-oriented programs in Prolog, for goal-oriented programming is what Prolog is all about.

In broader terms, the evolution of computer languages is an evolution away from low-level languages, in which the programmer specifies how something is to be done, toward high-level languages, in which the programmer specifies simply *what* is to be done. With the development of Fortran, for example, programmers were no longer forced to speak to the computer in the procrustian low-level language of addresses and registers. Instead, Fortran programmers could speak in their own language, or nearly so, using a notation that made only moderate concessions to the one-dimensional, 80-column world.

Fortran and nearly all other languages are still how-type languages, however. In my view, modern Lisp is the champion of these languages, for Lisp in its Common Lisp form is enormously expressive, but how to do something is still what the Lisp programmer is allowed to be expressive about. Prolog, on the other hand, is a language that clearly breaks away from the how-type languages, encouraging the programmer to describe situations and problems, not the detailed means by which the problems are to be solved.

Consequently, an introduction to Prolog is important for all students of Computer Science, for there is no better way to see what the notion of what-type programming is all about.

In particular, the chapters of this book clearly illustrate the difference between how-type and what-type thinking. In the first chapter, for example, the difference is illustrated through problems dealing with family relations. The Prolog programmer straightforwardly describes the grandfather concept in explicit, natural terms: a grandfather is a father of a parent. Here is the Prolog notation:

```
grandfather( X, Z) :- father( X, Y), parent( Y, Z).
```

Once Prolog knows what a grandfather is, it is easy to ask a question: who are Patrick's grandfathers, for example. Here again is the Prolog notation, along with a typical answer:

```
?- grandfather( X, patrick).

X = james;
X = carl
```

It is Prolog's job to figure out how to solve the problem by combing through a database of known father and parent relations. The programmer specifies only what is known and what question is to be solved. The programmer is more concerned with knowledge and less concerned with algorithms that exploit the knowledge.

Given that it is important to learn Prolog, the next question is how. I believe that learning a programming language is like learning a natural language in many ways. For example, a reference manual is helpful in learning a programming language, just as a dictionary is helpful in learning a natural language. But no one learns a natural language with only a dictionary, for the words are only part of what must be learned. The student of a natural language must learn the conventions that govern how the words are put legally together, and later, the student should learn the art of those who put the words together with style.

Similarly, no one learns a programming language from only a reference manual, for a reference manual says little or nothing about the way the primitives of the language are put to use by those who use the language well. For this, a textbook is required, and the best textbooks offer copious examples, for good examples are distilled experience, and it is principally through experience that we learn.

In this book, the first example is on the first page, and the remaining pages constitute an example cornucopia, pouring forth Prolog programs written by a passionate Prolog programmer who is dedicated to the Prolog point of view. By carefully studying these examples, the reader acquires not only the mechanics of the language, but also a personal collection of precedents, ready to be taken apart, adapted, and reassembled together into new programs. With this acquisition of

precedent knowledge, the transition from novice to skilled programmer is already under way.

Of course, a beneficial side effect of good programming examples is that they expose a bit of interesting science as well as a lot about programming itself. The science behind the examples in this book is Artificial Intelligence. The reader learns about such problem-solving ideas as problem reduction, forward and backward chaining, 'how' and 'why' questioning, and various search techniques.

In fact, one of the great features of Prolog is that it is simple enough for students in introductory Artificial Intelligence subjects to learn to use immediately. I expect that many instructors will use this book as part of their artificial intelligence subjects so that their students can see abstract ideas immediately reduced to concrete, motivating form.

Among Prolog texts, I expect this book to be particularly popular, not only because of its examples, but also because of a number of other features:

- Careful summaries appear throughout.
- Numerous exercises reinforce all concepts.
- Structure selectors introduce the notion of data abstraction.
- Explicit discussions of programming style and technique occupy an entire chapter.
- There is honest attention to the problems to be faced in Prolog programming, as well as the joys.

Features like this make this a well done, enjoyable, and instructive book.

I keep the first edition of this textbook in my library on the outstanding-textbooks shelf, programming languages section, for as a textbook it exhibited all the strengths that set the outstanding textbooks apart from the others, including clear and direct writing, copious examples, careful summaries, and numerous exercises. And as a programming language textbook, I especially liked its attention to data abstraction, emphasis on programming style, and honest treatment of Prolog's problems as well as Prolog's advantages.

Preface

Prolog

Prolog is a programming language centred around a small set of basic mechanisms, including pattern matching, tree-based data structuring and automatic backtracking. This small set constitutes a surprisingly powerful and flexible programming framework. Prolog is especially well suited for problems that involve objects – in particular, structured objects – and relations between them. For example, it is an easy exercise in Prolog to express spatial relationships between objects, such as the blue sphere is behind the green one. It is also easy to state a more general rule: if object X is closer to the observer than object Y, and Y is closer than Z, then X must be closer than Z. Prolog can now reason about the spatial relationships and their consistency with respect to the general rule. Features like this make Prolog a powerful language for artificial intelligence (AI) and non-numerical programming in general. There are well-known examples of symbolic computation whose implementation in other standard languages took tens of pages of indigestible code. When the same algorithms were implemented in Prolog, the result was a crystal-clear program easily fitting on one page.

Development of Prolog

Prolog stands for *programming in logic* – an idea that emerged in the early 1970s to use logic as a programming language. The early developers of this idea included Robert Kowalski at Edinburgh (on the theoretical side), Maarten van Emden at Edinburgh (experimental demonstration) and Alain Colmerauer at Marseilles (implementation). David D.H. Warren's efficient implementation at Edinburgh in the mid-1970s greatly contributed to the popularity of Prolog. A more recent development is *constraint logic programming* (CLP), usually implemented as part of a Prolog system. CLP extends Prolog with constraint processing, which has proved in practice to be an exceptionally flexible tool for problems like scheduling and logistic planning. In 1996 the official ISO standard for Prolog was published.

Historical controversies about Prolog

There are some controversial views that historically accompanied Prolog. Prolog fast gained popularity in Europe as a practical programming tool. In Japan, Prolog was placed at the centre of the development of the fifth-generation computers. On the other hand, in the United States Prolog was generally accepted with some delay, due to several historical factors. One of these was an early American experience with the Microplanner language, also akin to the idea of logic programming, but inefficiently implemented. Some reservations also came in reaction to the early 'orthodox school' of logic programming, which insisted on the use of pure logic that should not be marred by adding practical facilities not related to logic. This led to some widespread misunderstandings about Prolog in the past. For example, some believed that only backward chaining reasoning can be programmed in Prolog. The truth is that Prolog is a general programming language and any algorithm can be programmed in it. The impractical 'orthodox school's' position was modified by Prolog practitioners who adopted a more pragmatic view, benefiting from combining the new, declarative approach with the traditional, procedural one.

Learning Prolog

Since Prolog has its roots in mathematical logic it is often introduced through logic. However, such a mathematically intensive introduction is not very useful if the aim is to teach Prolog as a practical programming tool. Therefore this book is not concerned with the mathematical aspects, but concentrates on the art of making the few basic mechanisms of Prolog solve interesting problems. Whereas conventional languages are procedurally oriented, Prolog introduces the descriptive, or *declarative*, view. This greatly alters the way of thinking about problems and makes learning to program in Prolog an exciting intellectual challenge. Many believe that every student of computer science should learn something about Prolog at some point because Prolog enforces a different problem-solving paradigm complementary to other programming languages.

Contents of the book

Part I of the book introduces the Prolog language and shows how Prolog programs are developed. Techniques to handle important data structures, such as trees and graphs, are also included because of their general importance. In Part II, Prolog is applied to a number of areas of AI, including problem solving and heuristic search, programming with constraints, knowledge representation and expert systems, planning, machine learning, qualitative reasoning, language processing and game playing. AI techniques are introduced and developed in depth towards their implementation in Prolog, resulting in complete programs. These can be used as building blocks for sophisticated applications. The concluding chapter, on meta-programming, shows how Prolog can be used to implement other languages and programming paradigms, including object-oriented programming, pattern-directed programming and writing interpreters for Prolog in Prolog. Throughout, the emphasis is on the clarity of programs; efficiency tricks that rely on implementation-dependent features are avoided.

Differences between the second and third edition

All the material has been revised and updated. There are new chapters on:

- constraint logic programming (CLP);
- inductive logic programming;
- qualitative reasoning.

Other major changes are:

- addition of belief networks (Bayes networks) in the chapter on knowledge representation and expert systems;
- addition of memory-efficient programs for best-first search (IDA*, RBFS) in the chapter on heuristic search;
- major updates in the chapter on machine learning;
- additional techniques for improving program efficiency in the chapter on programming style and technique.

Throughout, more attention is paid to the differences between Prolog implementations with specific references to the Prolog standard when appropriate (see also Appendix A).

Audience for the book

This book is for students of Prolog and AI. It can be used in a Prolog course or in an AI course in which the principles of AI are brought to life through Prolog. The reader is assumed to have a basic general knowledge of computers, but no knowledge of AI is necessary. No particular programming experience is required; in fact, plentiful experience and devotion to conventional procedural programming – for example in C or Pascal – might even be an impediment to the fresh way of thinking Prolog requires.

The book uses standard syntax

Among several Prolog dialects, the Edinburgh syntax, also known as DEC-10 syntax, is the most widespread, and is the basis of the ISO standard for Prolog. It is also used in this book. For compatibility with the various Prolog implementations, this book only uses a relatively small subset of the built-in features that are shared by many Prologs.

How to read the book

In Part I, the natural reading order corresponds to the order in the book. However, the part of Section 2.4 that describes the procedural meaning of Prolog in a more

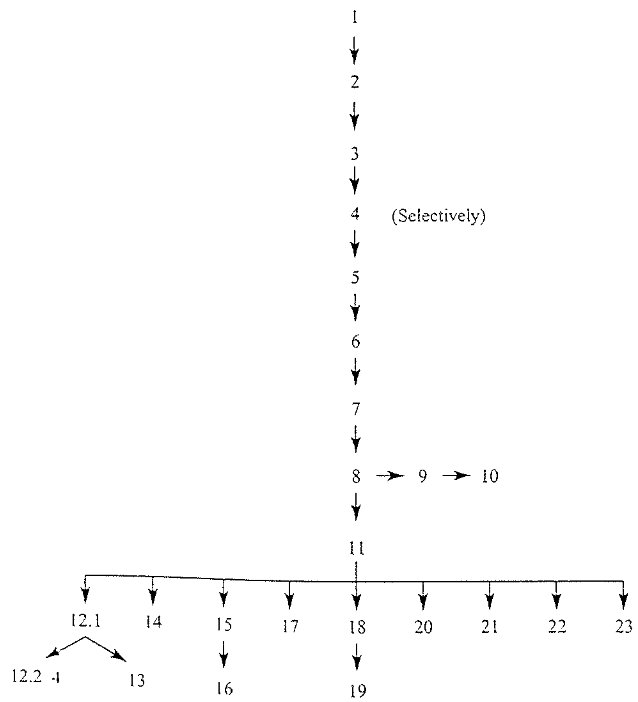

Figure P.1 Precedence constraints among the chapters.

formalized way can be skipped. Chapter 4 presents programming examples that can be read (or skipped) selectively. Chapter 10 on advanced tree representations can be skipped.

Part II allows more flexible reading strategies as most of the chapters are intended to be mutually independent. However, some topics will still naturally be covered before others, such as basic search strategies (Chapter 11). Figure P.1 summarizes the natural precedence constraints among the chapters.

Program code and course materials

Source code for all the programs in the book and relevant course materials are accessible from the companion web site (www.booksites.net/bratko).

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Ivan Bratko
January 2000

part I

The Prolog Language

chapter 1

Introduction to Prolog

This chapter reviews basic mechanisms of Prolog through an example program. Although the treatment is largely informal many important concepts are introduced such as: Prolog clauses, facts, rules and procedures. Prolog's built-in backtracking mechanism and the distinction between declarative and procedural meanings of a program are discussed.

1.1 Defining relations by facts

Prolog is a programming language for symbolic, non-numeric computation. It is specially well suited for solving problems that involve objects and relations between objects. Figure 1.1 shows an example: a family relation. The fact that Tom is a parent of Bob can be written in Prolog as:

```
parent( tom, bob).
```

Here we choose **parent** as the name of a relation; **tom** and **bob** are its arguments. For reasons that will become clear later we write names like **tom** with an initial lower-case letter. The whole family tree of Figure 1.1 is defined by the following Prolog program:

```
parent( pam, bob).
parent( tom, bob).
```

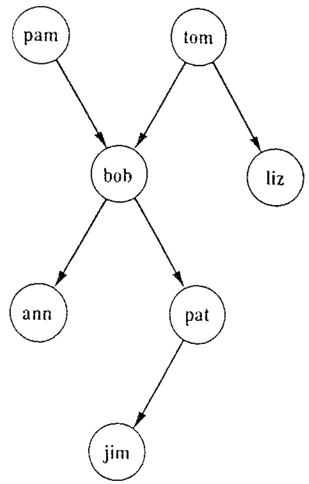


Figure 1.1 A family tree.

```
parent( tom, liz).
parent( bob, ann).
parent( bob, pat).
parent( pat, jim).
```

This program consists of six *clauses*. Each of these clauses declares one fact about the parent relation. For example, parent(tom, bob) is a particular *instance* of the parent relation. Such an instance is also called a *relationship*. In general, a relation is defined as the set of all its instances.

When this program has been communicated to the Prolog system, Prolog can be posed some questions about the parent relation. For example: Is Bob a parent of Pat? This question can be communicated to the Prolog system by typing into the terminal:

```
?- parent( bob, pat).
```

Having found this as an asserted fact in the program, Prolog will answer:

```
yes
```

A further query can be:

```
?- parent( liz, pat).
```

Prolog answers:

```
no
```


because the program does not mention anything about Liz being a parent of Pat. It also answers 'no' to the question:

```
?- parent( tom, ben).
```

because the program has not even heard of the name Ben.

More interesting questions can also be asked. For example: Who is Liz's parent?

```
?- parent( X, liz).
```

Prolog's answer will not be just 'yes' or 'no' this time. Prolog will tell us what is the value of X such that the above statement is true. So the answer is:

```
X = tom
```

The question Who are Bob's children? can be communicated to Prolog as:

```
?- parent( bob, X).
```

This time there is more than just one possible answer. Prolog first answers with one solution:

```
X = ann
```

We may now request another solution (by typing a semicolon), and Prolog will find:

```
X = pat
```

If we request more solutions again, Prolog will answer 'no' because all the solutions have been exhausted.

Our program can be asked an even broader question: Who is a parent of whom? Another formulation of this question is:

Find X and Y such that X is a parent of Y.

This is expressed in Prolog by:

```
?- parent( X, Y).
```

Prolog now finds all the parent-child pairs one after another. The solutions will be displayed one at a time as long as we tell Prolog we want more solutions, until all the solutions have been found. The answers are output as:

```
X = pam
Y = bob;

X = tom
Y = bob;

X = tom
Y = liz;

...
```

We can stop the stream of solutions by typing a return instead of a semicolon.

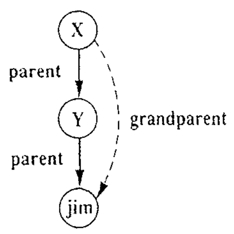


Figure 1.2 The **grandparent** relation expressed as a composition of two **parent** relations.

Our example program can be asked still more complicated questions like: Who is a grandparent of Jim? As our program does not directly know the **grandparent** relation this query has to be broken down into two steps, as illustrated by Figure 1.2.

(1) Who is a parent of Jim? Assume that this is some Y.

(2) Who is a parent of Y? Assume that this is some X.

Such a composed query is written in Prolog as a sequence of two simple ones:

```
?- parent( Y, jim), parent( X, Y).
```

The answer will be:

```
X = bob
Y = pat
```

Our composed query can be read: Find such X and Y that satisfy the following two requirements:

```
parent( Y, jim)   and   parent( X, Y)
```

If we change the order of the two requirements the logical meaning remains the same:

```
parent( X, Y)   and   parent( Y, jim)
```

We can indeed do this in our Prolog program, and the query:

```
?- parent( X, Y), parent( Y, jim).
```

will produce the same result.

In a similar way we can ask: Who are Tom's grandchildren?

```
?- parent( tom, X), parent( X, Y).
```

Prolog's answers are:

```
X = bob
Y = ann;
```



```
X = bob
Y = pat
```

Yet another question could be: Do Ann and Pat have a common parent? This can be expressed again in two steps:

(1) Who is a parent, X, of Ann?

(2) Is (this same) X a parent of Pat?

The corresponding question to Prolog is then:

```
?- parent( X, ann), parent( X, pat).
```

The answer is:

```
X = bob
```

Our example program has helped to illustrate some important points:

- It is easy in Prolog to define a relation, such as the **parent** relation, by stating the n-tuples of objects that satisfy the relation.
- The user can easily query the Prolog system about relations defined in the program.
- A Prolog program consists of *clauses*. Each clause terminates with a full stop.
- The arguments of relations can (among other things) be: concrete objects, or constants (such as **tom** and **ann**), or general objects such as X and Y. Objects of the first kind in our program are called *atoms*. Objects of the second kind are called *variables*.
- Questions to the system consist of one or more *goals*. A sequence of goals, such as:

  ```
  parent( X, ann), parent( X, pat)
  ```

  means the conjunction of the goals:

  X is a parent of Ann, and
  X is a parent of Pat.

  The word 'goals' is used because Prolog accepts questions as goals that are to be satisfied.
- An answer to a question can be either positive or negative, depending on whether the corresponding goal can be satisfied or not. In the case of a positive answer we say that the corresponding goal was *satisfiable* and that the goal *succeeded*. Otherwise the goal was *unsatisfiable* and it *failed*.
- If several answers satisfy the question then Prolog will find as many of them as desired by the user.

Exercises

1.1 Assuming the parent relation as defined in this section (see Figure 1.1), what will be Prolog's answers to the following questions?

(a) ?- parent(jim, X).

(b) ?- parent(X, jim).

(c) ?- parent(pam, X), parent(X, pat).

(d) ?- parent(pam, X), parent(X, Y), parent(Y, jim).

1.2 Formulate in Prolog the following questions about the parent relation:

(a) Who is Pat's parent?

(b) Does Liz have a child?

(c) Who is Pat's grandparent?

1.2 Defining relations by rules

Our example program can be easily extended in many interesting ways. Let us first add the information on the sex of the people that occur in the **parent** relation. This can be done by simply adding the following facts to our program:

```
female( pam).
male( tom).
male( bob).
female( liz).
female( pat).
female( ann).
male( jim).
```

The relations introduced here are **male** and **female**. These relations are unary (or one-place) relations. A binary relation like **parent** defines a relation between *pairs* of objects; on the other hand, unary relations can be used to declare simple yes/no properties of objects. The first unary clause above can be read: Pam is a female. We could convey the same information declared in the two unary relations with one binary relation, **sex**, instead. An alternative piece of program would then be:

```
sex( pam, feminine).
sex( tom, masculine).
sex( bob, masculine).
...
```

As our next extension to the program let us introduce the **offspring** relation as the inverse of the **parent** relation. We could define **offspring** in a similar way as the

parent relation; that is, by simply providing a list of simple facts about the **offspring** relation, each fact mentioning one pair of people such that one is an offspring of the other. For example:

```
offspring( liz, tom).
```

However, the offspring relation can be defined much more elegantly by making use of the fact that it is the inverse of **parent**, and that **parent** has already been defined. This alternative way can be based on the following logical statement:

For all X and Y,
  Y is an offspring of X if
    X is a parent of Y.

This formulation is already close to the formalism of Prolog. The corresponding Prolog clause which has the same meaning is:

```
offspring( Y, X) :- parent( X, Y).
```

This clause can also be read as:

For all X and Y,
  if X is a parent of Y then
  Y is an offspring of X.

Prolog clauses such as:

```
offspring( Y, X) :- parent( X, Y).
```

are called *rules*. There is an important difference between facts and rules. A fact like:

```
parent( tom, liz).
```

is something that is always, unconditionally, true. On the other hand, rules specify things that are true if some condition is satisfied. Therefore we say that rules have:

- a condition part (the right-hand side of the rule) and
- a conclusion part (the left-hand side of the rule).

The conclusion part is also called the *head* of a clause and the condition part the *body* of a clause. For example:

$$\underbrace{\textbf{offspring(Y, X)}}_{\text{head}} \text{ :- } \underbrace{\textbf{parent(X, Y).}}_{\text{body}}$$

If the condition **parent(X, Y)** is true then a logical consequence of this is **offspring(Y, X)**.

How rules are actually used by Prolog is illustrated by the following example. Let us ask our program whether Liz is an offspring of Tom:

```
?- offspring( liz, tom).
```

There is no fact about offsprings in the program, therefore the only way to consider this question is to apply the rule about offsprings. The rule is general in the sense that it is applicable to any objects X and Y; therefore it can also be applied to such particular objects as liz and tom. To apply the rule to liz and tom, Y has to be substituted with liz, and X with tom. We say that the variables X and Y become instantiated to:

X = tom and Y = liz

After the instantiation we have obtained a special case of our general rule. The special case is:

```
offspring( liz, tom) :- parent( tom, liz).
```

The condition part has become:

```
parent( tom, liz)
```

Now Prolog tries to find out whether the condition part is true. So the initial goal:

```
offspring( liz, tom)
```

has been replaced with the subgoal:

```
parent( tom, liz)
```

This (new) goal happens to be trivial as it can be found as a fact in our program. This means that the conclusion part of the rule is also true, and Prolog will answer the question with yes.

Let us now add more family relations to our example program. The specification of the mother relation can be based on the following logical statement:

For all X and Y,
  X is the mother of Y if
  X is a parent of Y and
  X is a female.

This is translated into Prolog as the following rule:

```
mother( X, Y) :- parent( X, Y), female( X).
```

A comma between two conditions indicates the conjunction of the conditions, meaning that *both* conditions have to be true.

Relations such as parent, offspring and mother can be illustrated by diagrams such as those in Figure 1.3. These diagrams conform to the following conventions. Nodes in the graphs correspond to objects – that is, arguments of relations. Arcs between nodes correspond to binary (or two-place) relations. The arcs are oriented so as to point from the first argument of the relation to the second argument. Unary relations are indicated in the diagrams by simply marking the corresponding objects

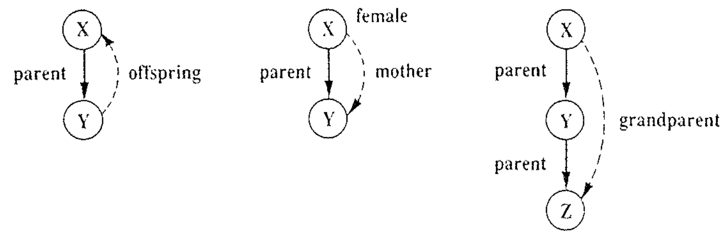


Figure 1.3 Definition graphs for the relations offspring, mother and grandparent in terms of other relations.

with the name of the relation. The relations that are being defined are represented by dashed arcs. So each diagram should be understood as follows: if the relations shown by solid arcs hold, then the relation shown by a dashed arc also holds. The grandparent relation can be, according to Figure 1.3, immediately written in Prolog as:

```
grandparent( X, Z) :- parent( X, Y), parent( Y, Z).
```

At this point it will be useful to make a comment on the layout of our programs. Prolog gives us almost full freedom in choosing the layout of the program. So we can insert spaces and new lines as it best suits our taste. In general we want to make our programs look nice and tidy, and, above all, easy to read. To this end we will often choose to write the head of a clause and each goal of the body on a separate line. When doing this, we will indent goals in order to make the difference between the head and the goals more visible. For example, the grandparent rule would be, according to this convention, written as follows:

```
grandparent( X, Z) :-
  parent( X, Y),
  parent( Y, Z).
```

Figure 1.4 illustrates the sister relation:

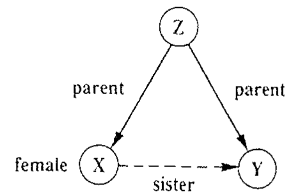


Figure 1.4 Defining the sister relation.

For any X and Y,
  X is a sister of Y if
  (1) both X and Y have the same parent, and
  (2) X is a female.

The graph in Figure 1.4 can be translated into Prolog as:

```
sister( X, Y) :-
  parent( Z, X),
  parent( Z, Y),
  female( X).
```

Notice the way in which the requirement 'both X and Y have the same parent' has been expressed. The following logical formulation was used: some Z must be a parent of X, and this *same* Z must be a parent of Y. An alternative, but less elegant way would be to say: Z1 is a parent of X, and Z2 is a parent of Y, and Z1 is equal to Z2.

We can now ask:

```
?- sister( ann, pat).
```

The answer will be 'yes', as expected (see Figure 1.1). Therefore we might conclude that the sister relation, as defined, works correctly. There is, however, a rather subtle flaw in our program, which is revealed if we ask the question Who is Pat's sister?:

```
?- sister( X, pat).
```

Prolog will find two answers, one of which may come as a surprise:

```
X = ann;
X = pat
```

So, Pat is a sister to herself?! This is probably not what we had in mind when defining the sister relation. However, according to our rule about sisters Prolog's answer is perfectly logical. Our rule about sisters does not mention that X and Y must not be the same if X is to be a sister of Y. As this is not required Prolog (rightfully) assumes that X and Y can be the same, and will as a consequence find that any female who has a parent is a sister of herself.

To correct our rule about sisters we have to add that X and Y must be different. We will see in later chapters how this can be done in several ways, but for the moment we will assume that a relation **different** is already known to Prolog, and that:

```
different( X, Y)
```

is satisfied if and only if X and Y are not equal. An improved rule for the sister relation can then be:

```
sister( X, Y) :-
  parent( Z, X),
  parent( Z, Y),
  female( X),
  different( X, Y).
```


Some important points of this section are:

- Prolog programs can be extended by simply adding new clauses.
- Prolog clauses are of three types: *facts*, *rules* and *questions*.
- *Facts* declare things that are always, unconditionally true.
- *Rules* declare things that are true depending on a given condition.
- By means of *questions* the user can ask the program what things are true.
- Prolog clauses consist of the *head* and the *body*. The body is a list of *goals* separated by commas. Commas are understood as conjunctions.
- Facts are clauses that have a head and the empty body. Questions only have the body. Rules have the head and the (non-empty) body.
- In the course of computation, a variable can be substituted by another object. We say that a variable becomes *instantiated*.
- Variables are assumed to be universally quantified and are read as 'for all'. Alternative readings are, however, possible for variables that appear only in the body. For example:

  ```
  hasachild( X) :- parent( X, Y).
  ```

  can be read in two ways:

  (a) *For all* X and Y,
  if X is a parent of Y then
  X has a child.

  (b) *For all* X,
  X has a child if
  there is *some* Y such that X is a parent of Y.

Exercises

1.3 Translate the following statements into Prolog rules:

(a) Everybody who has a child is happy (introduce a one-argument relation **happy**).

(b) For all X, if X has a child who has a sister then X has two children (introduce new relation **hastwochildren**).

1.4 Define the relation **grandchild** using the **parent** relation. Hint: It will be similar to the **grandparent** relation (see Figure 1.3).

1.5 Define the relation **aunt(X, Y)** in terms of the relations **parent** and **sister**. As an aid you can first draw a diagram in the style of Figure 1.3 for the **aunt** relation.

1.3 Recursive rules

Let us add one more relation to our family program, the predecessor relation. This relation will be defined in terms of the parent relation. The whole definition can be expressed with two rules. The first rule will define the direct (immediate) predecessors and the second rule the indirect predecessors. We say that some X is an indirect predecessor of some Z if there is a parentship chain of people between X and Z, as illustrated in Figure 1.5. In our example of Figure 1.1, Tom is a direct predecessor of Liz and an indirect predecessor of Pat.

The first rule is simple and can be formulated as:

For all X and Z,
  X is a predecessor of Z if
  X is a parent of Z.

This is straightforwardly translated into Prolog as:

```
predecessor( X, Z) :-
  parent( X, Z).
```

The second rule, on the other hand, is more complicated because the chain of parents may present some problems. One attempt to define indirect predecessors could be as shown in Figure 1.6. According to this, the predecessor relation would be defined by a set of clauses as follows:

```
predecessor( X, Z) :-
  parent( X, Z).

predecessor( X, Z) :-
  parent( X, Y),
  parent( Y, Z).
```

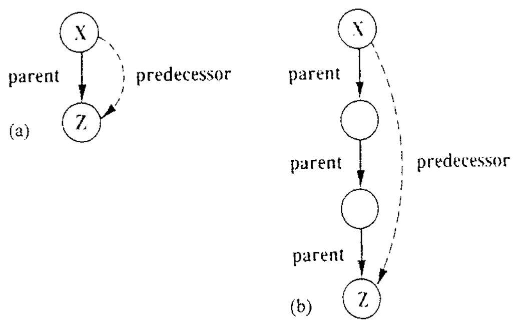


Figure 1.5 Examples of the predecessor relation: (a) X is a direct predecessor of Z; (b) X is an indirect predecessor of Z.

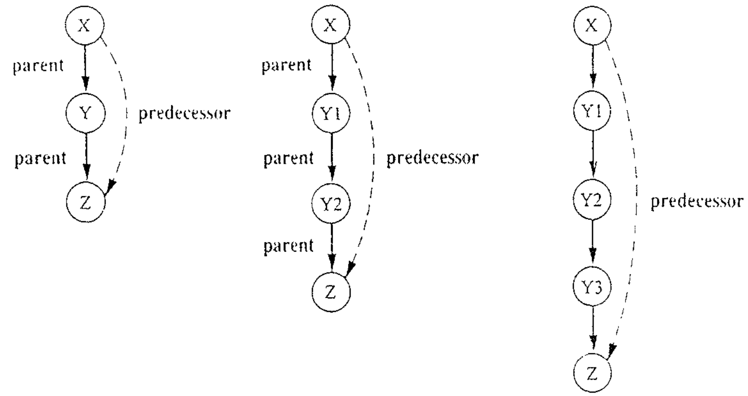


Figure 1.6 Predecessor-successor pairs at various distances.

```
predecessor( X, Z) :-
  parent( X, Y1),
  parent( Y1, Y2),
  parent( Y2, Z).

predecessor( X, Z) :-
  parent( X, Y1),
  parent( Y1, Y2),
  parent( Y2, Y3),
  parent( Y3, Z).

...
```

This program is lengthy and, more importantly, it only works to some extent. It would only discover predecessors to a certain depth in a family tree because the length of the chain of people between the predecessor and the successor would be limited according to the length of our predecessor clauses.

There is, however, an elegant and correct formulation of the predecessor relation: it will be correct in the sense that it will work for predecessors at any depth. The key idea is to define the predecessor relation in terms of itself. Figure 1.7 illustrates the idea:

For all X and Z,
  X is a predecessor of Z if
  there is a Y such that
  (1) X is a parent of Y and
  (2) Y is a predecessor of Z.

A Prolog clause with the above meaning is:

```
predecessor( X, Z) :-
  parent( X, Y),
  predecessor( Y, Z).
```

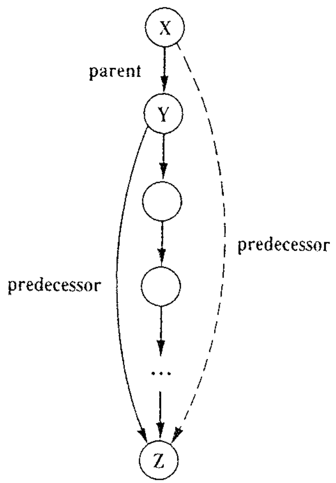


Figure 1.7 Recursive formulation of the predecessor relation.

We have thus constructed a complete program for the predecessor relation, which consists of two rules: one for direct predecessors and one for indirect predecessors. Both rules are rewritten together here:

```
predecessor( X, Z) :-
  parent( X, Z).

predecessor( X, Z) :-
  parent( X, Y),
  predecessor( Y, Z).
```

The key to this formulation was the use of predecessor itself in its definition. Such a definition may look surprising in view of the question: When defining something, can we use this same thing that has not yet been completely defined? Such definitions are, in general, called *recursive* definitions. Logically, they are perfectly correct and understandable, which is also intuitively obvious if we look at Figure 1.7. But will the Prolog system be able to use recursive rules? It turns out that Prolog can indeed very easily use recursive definitions. Recursive programming is, in fact, one of the fundamental principles of programming in Prolog. It is not possible to solve tasks of any significant complexity in Prolog without the use of recursion.

Going back to our program, we can ask Prolog: Who are Pam's successors? That is: Who is a person that has Pam as his or her predecessor?

```
?- predecessor( pam, X).

X = bob;

X = ann;

X = pat;

X = jim
```


Prolog's answers are, of course, correct and they logically follow from our definition of the predecessor and the parent relation. There is, however, a rather important question: *How* did Prolog actually use the program to find these answers?

An informal explanation of how Prolog does this is given in the next section. But first let us put together all the pieces of our family program, which was extended gradually by adding new facts and rules. The final form of the program is shown in Figure 1.8. Looking at Figure 1.8, two further points are in order here: the

```
parent( pam, bob).                 % Pam is a parent of Bob
parent( tom, bob).
parent( tom, liz).
parent( bob, ann).
parent( bob, pat).
parent( pat, jim).

female( pam).                      % Pam is female
male( tom).                        % Tom is male
male( bob).
female( liz).
female( ann).
female( pat).
male( jim).

offspring( Y, X) :-                % Y is an offspring of X if
  parent( X, Y).                   % X is a parent of Y

mother( X, Y) :-                   % X is the mother of Y if
  parent( X, Y),                   % X is a parent of Y and
  female( X).                      % X is female

grandparent( X, Z) :-              % X is a grandparent of Z if
  parent( X, Y),                   % X is a parent of Y and
  parent( Y, Z).                   % Y is a parent of Z

sister( X, Y) :-                   % X is a sister of Y if
  parent( Z, X),
  parent( Z, Y),                   % X and Y have the same parent and
  female( X),                      % X is female and
  different( X, Y).                % X and Y are different

predecessor( X, Z) :-              % Rule prl: X is a predecessor of Z
  parent( X, Z).

predecessor( X, Z) :-              % Rule pr2: X is a predecessor of Z
  parent( X, Y),
  predecessor( Y, Z).
```

Figure 1.8 The family program.

first will introduce the term 'procedure', the second will be about comments in programs.

The program in Figure 1.8 defines several relations – parent, male, female, predecessor, etc. The predecessor relation, for example, is defined by two clauses. We say that these two clauses are *about* the predecessor relation. Sometimes it is convenient to consider the whole set of clauses about the same relation. Such a set of clauses is called a *procedure*.

In Figure 1.8, the two rules about the predecessor relation have been distinguished by the names 'pr1' and 'pr2', added as *comments* to the program. These names will be used later as references to these rules. Comments are, in general, ignored by the Prolog system. They only serve as a further clarification to the person who reads the program. Comments are distinguished in Prolog from the rest of the program by being enclosed in special brackets '/*' and '*/'. Thus comments in Prolog look like this:

```
/* This is a comment */
```

Another method, more practical for short comments, uses the percent character '%'. Everything between '%' and the end of the line is interpreted as a comment:

```
% This is also a comment
```

Exercise

1.6 Consider the following alternative definition of the predecessor relation:

```
predecessor( X, Z) :-
  parent( X, Z).

predecessor( X, Z) :-
  parent( Y, Z),
  predecessor( X, Y).
```

Does this also seem to be a correct definition of predecessors? Can you modify the diagram of Figure 1.7 so that it would correspond to this new definition?

1.4 How Prolog answers questions

This section gives an informal explanation of *how* Prolog answers questions. A question to Prolog is always a sequence of one or more goals. To answer a question, Prolog tries to satisfy all the goals. What does it mean to *satisfy* a goal? To satisfy a goal means to demonstrate that the goal is true, assuming that the relations in the program are true. In other words, to satisfy a goal means to demonstrate that the goal *logically follows* from the facts and rules in the program. If the question contains variables, Prolog also has to find what are the particular objects (in place of variables) for which the goals are satisfied. The particular instantiation of variables to these objects is displayed to the user. If Prolog cannot demonstrate for some instantiation of variables that the goals logically follow from the program, then Prolog's answer to the question will be 'no'.

An appropriate view of the interpretation of a Prolog program in mathematical terms is then as follows: Prolog accepts facts and rules as a set of axioms, and the user's question as a *conjectured theorem*; then it tries to prove this theorem – that is, to demonstrate that it can be logically derived from the axioms.

We will illustrate this view by a classical example. Let the axioms be:

All men are fallible.
Socrates is a man.

A theorem that logically follows from these two axioms is:

Socrates is fallible.

The first axiom above can be rewritten as:

For all X, if X is a man then X is fallible.

Accordingly, the example can be translated into Prolog as follows:

```
fallible( X) :- man( X).        % All men are fallible
man( socrates).                 % Socrates is a man
?- fallible( socrates).         % Socrates is fallible?
yes
```

A more complicated example from the family program of Figure 1.8 is:

```
?- predecessor( tom, pat).
```

We know that parent(bob, pat) is a fact. Using this fact and rule *pr1* we can conclude predecessor(bob, pat). This is a *derived* fact: it cannot be found explicitly in our program, but it can be derived from facts and rules in the program. An inference step, such as this, can be written in a more compact form as:

parent(bob, pat) ==> predecessor(bob, pat)

This can be read: from parent(bob, pat) it follows that predecessor(bob, pat), by rule *pr1*. Further, we know that parent(tom, bob) is a fact. Using this fact and the derived fact predecessor(bob, pat) we can conclude predecessor(tom, pat), by rule *pr2*. We have thus shown that our goal statement predecessor(tom, pat) is true. This whole inference process of two steps can be written as:

parent(bob, pat) ==> predecessor(bob, pat)

parent(tom, bob) *and* predecessor(bob, pat) ==> predecessor(tom, pat)

We have thus shown *what* can be a sequence of steps that satisfy a goal – that is, make it clear that the goal is true. Let us call this a proof sequence. We have not, however, shown *how* the Prolog system actually finds such a proof sequence.

Prolog finds the proof sequence in the inverse order to that which we have just used. Instead of starting with simple facts given in the program, Prolog starts with the goals and, using rules, substitutes the current goals with new goals, until new goals happen to be simple facts. Given the question:

```
?- predecessor( tom, pat).
```

Prolog will try to satisfy this goal. In order to do so it will try to find a clause in the program from which the above goal could immediately follow. Obviously, the only clauses relevant to this end are *pr1* and *pr2*. These are the rules about the predecessor relation. We say that the heads of these rules *match* the goal.

The two clauses, *pr1* and *pr2*, represent two alternative ways for Prolog to proceed. Prolog first tries that clause which appears first in the program:

```
predecessor( X, Z) :- parent( X, Z).
```

Since the goal is predecessor(tom, pat), the variables in the rule must be instantiated as follows:

X = tom, Z = pat

The original goal predecessor(tom, pat) is then replaced by a new goal:

```
parent( tom, pat)
```

This step of using a rule to transform a goal into another goal, as above, is graphically illustrated in Figure 1.9. There is no clause in the program whose head matches the goal parent(tom, pat), therefore this goal fails. Now Prolog *backtracks* to the original goal in order to try an alternative way to derive the top goal predecessor(tom, pat). The rule *pr2* is thus tried:

```
predecessor( X, Z) :-
  parent( X, Y),
  predecessor( Y, Z).
```

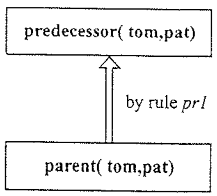


Figure 1.9 The first step of the execution. The top goal is true if the bottom goal is true.

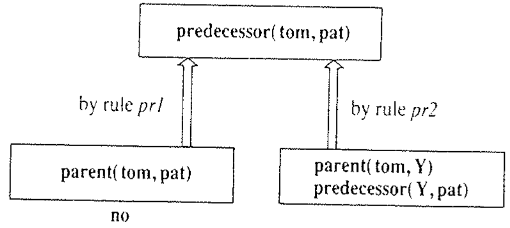


Figure 1.10 Execution trace continued from Figure 1.9.

As before, the variables X and Z become instantiated as:

X = tom, Z = pat

But Y is not instantiated yet. The top goal predecessor(tom, pat) is replaced by two goals:

```
parent( tom, Y),
predecessor( Y, pat)
```

This executional step is shown in Figure 1.10, which is an extension to the situation we had in Figure 1.9.

Being now faced with *two* goals, Prolog tries to satisfy them in the order in which they are written. The first one is easy as it matches one of the facts in the program. The matching forces Y to become instantiated to bob. Thus the first goal has been satisfied, and the remaining goal has become:

```
predecessor( bob, pat)
```

To satisfy this goal the rule *pr1* is used again. Note that this (second) application of the same rule has nothing to do with its previous application. Therefore, Prolog uses a new set of variables in the rule each time the rule is applied. To indicate this we shall rename the variables in rule *pr1* for this application as follows:

```
predecessor( X', Z') :-
  parent( X', Z').
```

The head has to match our current goal predecessor(bob, pat). Therefore:

X′ = bob, Z′ = pat

The current goal is replaced by:

```
parent( bob, pat)
```

This goal is immediately satisfied because it appears in the program as a fact. This completes the execution trace, which is graphically shown in Figure 1.11.

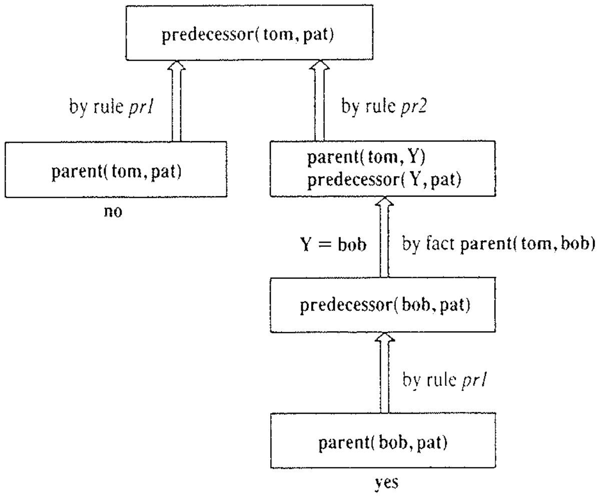


Figure 1.11 The complete execution trace to satisfy the goal **predecessor(tom, pat)**. The right-hand branch proves the goal is satisfiable.

The graphical illustration of the execution trace in Figure 1.11 has the form of a tree. The nodes of the tree correspond to goals, or to lists of goals that are to be satisfied. The arcs between the nodes correspond to the application of (alternative) program clauses that transform the goals at one node into the goals at another node. The top goal is satisfied when a path is found from the root node (top goal) to a leaf node labelled 'yes'. A leaf is labelled 'yes' if it is a simple fact. The execution of Prolog programs is the searching for such paths. During the search Prolog may enter an unsuccessful branch. When Prolog discovers that a branch fails it automatically *backtracks* to the previous node and tries to apply an alternative clause at that node.

Exercise

1.7 Try to understand how Prolog derives answers to the following questions, using the program of Figure 1.8. Try to draw the corresponding derivation diagrams in the style of Figures 1.9 to 1.11. Will any backtracking occur at particular questions?

(a) ?- parent(pam, bob).

(b) ?- mother(pam, bob).

(c) ?- grandparent(pam, ann).

(d) ?- grandparent(bob, jim).

1.5 Declarative and procedural meaning of programs

In our examples so far it has always been possible to understand the results of the program without exactly knowing *how* the system actually found the results. It therefore makes sense to distinguish between two levels of meaning of Prolog programs; namely,

- the *declarative meaning* and
- the *procedural meaning*.

The declarative meaning is concerned only with the *relations* defined by the program. The declarative meaning thus determines *what* will be the output of the program. On the other hand, the procedural meaning also determines *how* this output is obtained; that is, how the relations are actually evaluated by the Prolog system.

The ability of Prolog to work out many procedural details on its own is considered to be one of its specific advantages. It encourages the programmer to consider the declarative meaning of programs relatively independently of their procedural meaning. Since the results of the program are, in principle, determined by its declarative meaning, this should be (in principle) sufficient for writing programs. This is of practical importance because the declarative aspects of programs are usually easier to understand than the procedural details. To take full advantage of this, the programmer should concentrate mainly on the declarative meaning and, whenever possible, avoid being distracted by the executional details. These should be left to the greatest possible extent to the Prolog system itself.

This declarative approach indeed often makes programming in Prolog easier than in typical procedurally oriented programming languages such as C or Pascal. Unfortunately, however, the declarative approach is not always sufficient. It will later become clear that, especially in large programs, the procedural aspects cannot be completely ignored by the programmer for practical reasons of executional efficiency. Nevertheless, the declarative style of thinking about Prolog programs should be encouraged and the procedural aspects ignored to the extent that is permitted by practical constraints.

Summary

- Prolog programming consists of defining relations and querying about relations.
- A program consists of *clauses*. These are of three types: *facts*, *rules* and *questions*.
- A relation can be specified by *facts*, simply stating the n-tuples of objects that satisfy the relation, or by stating *rules* about the relation.

- A *procedure* is a set of clauses about the same relation.
- Querying about relations, by means of *questions*, resembles querying a database. Prolog's answer to a question consists of a set of objects that satisfy the question.
- In Prolog, to establish whether an object satisfies a query is often a complicated process that involves logical inference, exploring among alternatives and possibly *backtracking*. All this is done automatically by the Prolog system and is, in principle, hidden from the user.
- Two types of meaning of Prolog programs are distinguished: declarative and procedural. The declarative view is advantageous from the programming point of view. Nevertheless, the procedural details often have to be considered by the programmer as well.
- The following concepts have been introduced in this chapter:

  clause, fact, rule, question
  the head of a clause, the body of a clause
  recursive rule, recursive definition
  procedure
  atom, variable
  instantiation of a variable
  goal
  goal is satisfiable, goal succeeds
  goal is unsatisfiable, goal fails
  backtracking
  declarative meaning, procedural meaning

References

Various implementations of Prolog use different syntactic conventions. However, most of them follow the tradition of the so-called Edinburgh syntax (also called DEC-10 syntax, established by the historically influential implementation of Prolog for the DEC-10 computer; Pereira *et al.* 1978; Bowen 1981). The Edinburgh syntax also forms the basis of the ISO international standard for Prolog ISO/IEC 13211-1 (Deransart *et al.* 1996). Major Prolog implementations now largely comply with the standard. In this book we use a subset of the standard syntax, with some small and insignificant differences. In rare cases of such differences, there is a note to this effect at an appropriate place.

Bowen, D.L. (1981) *DECsystem-10 Prolog User's Manual*. University of Edinburgh: Department of Artificial Intelligence.

Deransart, P., Ed-Bdali, A. and Ceroni, L. (1996) *Prolog: The Standard*. Berlin: Springer-Verlag.

Pereira, L.M., Pereira, F. and Warren, D.H.D. (1978) *User's Guide to DECsystem-10 Prolog*. University of Edinburgh: Department of Artificial Intelligence.

chapter 2

Syntax and Meaning of Prolog Programs

This chapter gives a systematic treatment of the syntax and semantics of basic concepts of Prolog, and introduces structured data objects. The topics included are:

- simple data objects (atoms, numbers, variables)
- structured objects
- matching as the fundamental operation on objects
- declarative (or non-procedural) meaning of a program
- procedural meaning of a program
- relation between the declarative and procedural meanings of a program
- altering the procedural meaning by reordering clauses and goals.

Most of these topics have already been reviewed in Chapter 1. Here the treatment will become more formal and detailed.

2.1 Data objects

Figure 2.1 shows a classification of data objects in Prolog. The Prolog system recognizes the type of an object in the program by its syntactic form. This is possible because the syntax of Prolog specifies different forms for each type of data object. We have already seen a method for distinguishing between atoms and variables in Chapter 1: variables start with upper-case letters whereas atoms start with lower-case letters. No additional information (such as data-type declaration) has to be communicated to Prolog in order to recognize the type of an object.

2.1.1 Atoms and numbers

In Chapter 1 we have seen some simple examples of atoms and variables. In general, however, they can take more complicated forms – that is, strings of the following characters:

- upper-case letters A, B, . . . , Z
- lower-case letters a, b, . . . , z
- digits 0, 1, 2, . . . , 9
- special characters such as + – * / < > = : . & _ ~

Atoms can be constructed in three ways:

(1) Strings of letters, digits and the underscore character, '_', starting with a lower-case letter:

```
anna
nil
x25
x_25
x_25AB
x_
x___y
alpha_beta_procedure
miss_Jones
sarah_jones
```

(2) Strings of special characters:

```
<--->
======>
...
.:.
::=
```

When using atoms of this form, some care is necessary because some strings of special characters already have a predefined meaning; an example is ':-'.

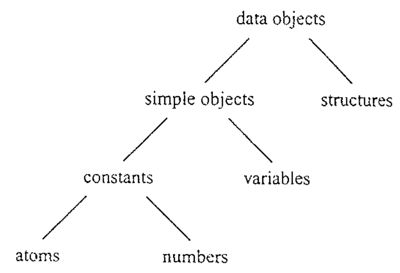


Figure 2.1 Data objects in Prolog.

(3) Strings of characters enclosed in single quotes. This is useful if we want, for example, to have an atom that starts with a capital letter. By enclosing it in quotes we make it distinguishable from variables:

```
'Tom'
'South_America'
'Sarah Jones'
```

Numbers used in Prolog include integer numbers and real numbers. The syntax of integers is simple, as illustrated by the following examples:

1   1313   0   –97

Not all integer numbers can be represented in a computer, therefore the range of integers is limited to an interval between some smallest and some largest number permitted by a particular Prolog implementation.

We will assume the simple syntax of real numbers, as shown by the following examples:

3.14   –0.0035   100.2

Real numbers are not very heavily used in typical Prolog programming. The reason for this is that Prolog is primarily a language for symbolic, non-numeric computation. In symbolic computation, integers are often used, for example, to count the number of items in a list; but there is typically less need for real numbers.

Apart from this lack of necessity to use real numbers in typical Prolog applications, there is another reason for avoiding real numbers. In general, we want to keep the meaning of programs as neat as possible. The introduction of real numbers somewhat impairs this neatness because of numerical errors that arise due to rounding when doing arithmetic. For example, the evaluation of the expression

$$10000 + 0.0001 - 10000$$

may result in 0 instead of the correct result 0.0001.

2.1.2 Variables

Variables are strings of letters, digits and underscore characters. They start with an upper-case letter or an underscore character:

```
X
Result
Object2
Participant_list
ShoppingList
_x23
_23
```

When a variable appears in a clause once only, we do not have to invent a name for it. We can use the so-called 'anonymous' variable, which is written as a single underscore character. For example, let us consider the following rule:

```
hasachild( X) :- parent( X, Y).
```

This rule says: for all X, X has a child if X is a parent of some Y. We are defining the property hasachild which, as it is meant here, does not depend on the name of the child. Thus, this is a proper place in which to use an anonymous variable. The clause above can thus be rewritten:

```
hasachild( X) :- parent( X, _).
```

Each time a single underscore character occurs in a clause it represents a new anonymous variable. For example, we can say that there is somebody who has a child if there are two objects such that one is a parent of the other:

```
somebody_has_child :- parent( _, _).
```

This is equivalent to:

```
somebody_has_child :- parent( X, Y).
```

But this is, of course, quite different from:

```
somebody_has_child :- parent( X, X).
```

If the anonymous variable appears in a question clause then its value is not output when Prolog answers the question. If we are interested in people who have children, but not in the names of the children, then we can simply ask:

```
?- parent( X, _).
```

The *lexical scope* of variable names is one clause. This means that, for example, if the name X15 occurs in two clauses, then it signifies two different variables. But each occurrence of X15 within the same clause means the same variable. The situation is different for constants: the same atom always means the same object in any clause – that is, throughout the whole program.

2.1.3 Structures

Structured objects (or simply *structures*) are objects that have several components. The components themselves can, in turn, be structures. For example, the date can be viewed as a structure with three components: day, month, year. Although composed of several components, structures are treated in the program as single objects. In order to combine the components into a single object we have to choose a *functor*. A suitable functor for our example is date. Then the date 1 May 2001 can be written as:

```
date( 1, may, 2001)
```

(see Figure 2.2).

All the components in this example are constants (two integers and one atom). Components can also be variables or other structures. Any day in May can be represented by the structure:

```
date( Day, may, 2001)
```

Note that Day is a variable and can be instantiated to any object at some later point in the execution.

This method for data structuring is simple and powerful. It is one of the reasons why Prolog is so naturally applied to problems that involve symbolic manipulation.

Syntactically, all data objects in Prolog are *terms*. For example,

```
may
```

and

```
date( 1, may, 2001)
```

are terms.

All structured objects can be pictured as trees (see Figure 2.2 for an example). The root of the tree is the functor, and the offsprings of the root are the components. If a component is also a structure then it is a subtree of the tree that corresponds to the whole structured object.

Our next example will show how structures can be used to represent some simple geometric objects (see Figure 2.3). A point in two-dimensional space is defined by its

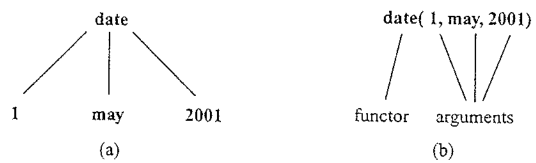


Figure 2.2 Date is an example of a structured object: (a) as it is represented as a tree; (b) as it is written in Prolog.

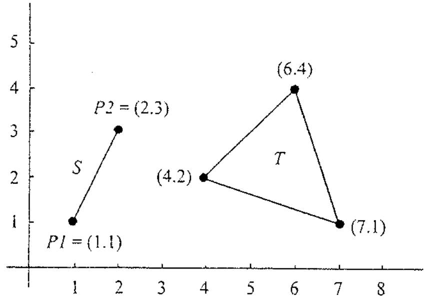


Figure 2.3 Some simple geometric objects.

two coordinates; a line segment is defined by two points; and a triangle can be defined by three points. Let us choose the following functors:

point	for points,
seg	for line segments, and
triangle	for triangles.

Then the objects in Figure 2.3 can be represented as follows:

```
P1 = point(1,1)
P2 = point(2,3)
S = seg( P1, P2) = seg( point(1,1), point(2,3) )
T = triangle( point(4,2), point(6,4), point(7,1) )
```

The corresponding tree representation of these objects is shown in Figure 2.4. In general, the functor at the root of the tree is called the *principal functor* of the term.

If in the same program we also had points in three-dimensional space then we could use another functor, point3, say, for their representation:

```
point3( X, Y, Z)
```

We can, however, use the same name, point, for points in both two and three dimensions, and write for example:

```
point( X1, Y1)   and   point( X, Y, Z)
```

If the same name appears in the program in two different roles, as is the case for point above, the Prolog system will recognize the difference by the number of arguments, and will interpret this name as two functors: one of them with two arguments and the other one with three arguments. This is so because each functor is defined by two things:

(1) the name, whose syntax is that of atoms;

(2) the *arity* – that is, the number of arguments.

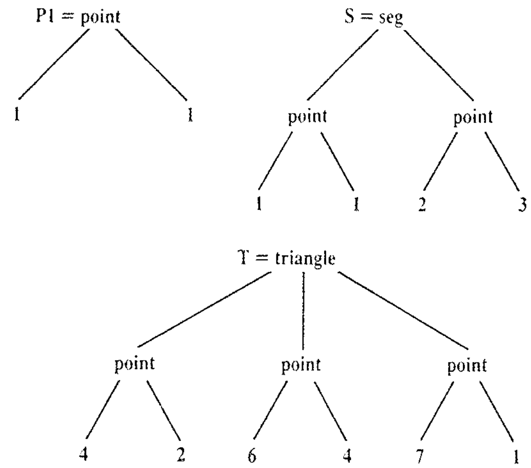


Figure 2.4 Tree representation of the objects in Figure 2.3.

As already explained, all structured objects in Prolog are trees, represented in the program by terms. We will study two more examples to illustrate how naturally complicated data objects can be represented by Prolog terms. Figure 2.5 shows the tree structure that corresponds to the arithmetic expression:

$$(a + b) * (c - 5)$$

According to the syntax of terms introduced so far this can be written, using the symbols '*', '+' and '−' as functors, as follows:

```
*( +( a, b), -( c, 5) )
```

This is, of course, a legal Prolog term; but this is not the form that we would normally like to have. We would normally prefer the usual, infix notation as used in mathematics. In fact, Prolog also allows us to use the infix notation so that the symbols '*', '+' and '−' are written as infix operators. Details of how the programmer can define his or her own operators will be discussed in Chapter 3.

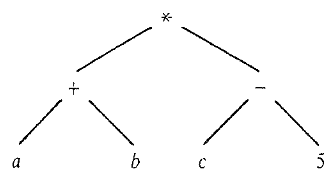


Figure 2.5 A tree structure that corresponds to the arithmetic expression $(a + b) * (c - 5)$.

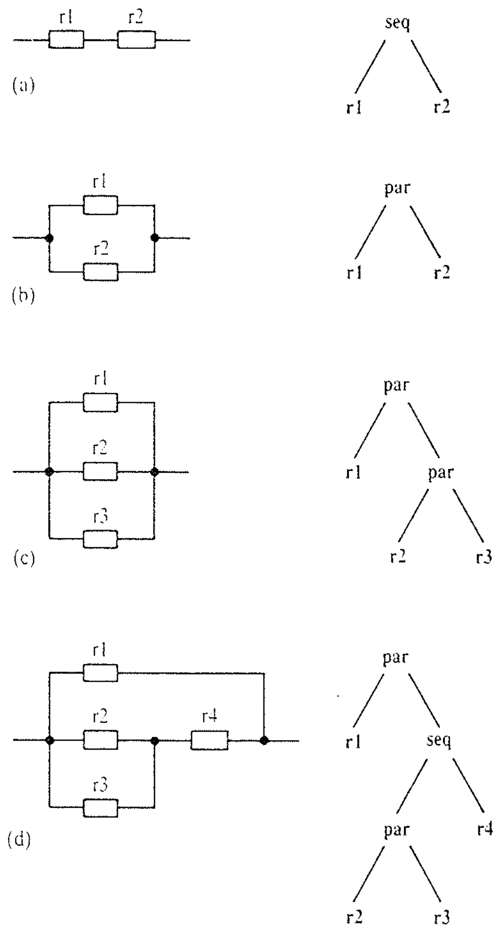


Figure 2.6 Some simple electric circuits and their tree representations: (a) sequential composition of resistors r1 and r2; (b) parallel composition of two resistors; (c) parallel composition of three resistors; (d) parallel composition of r1 and another circuit.

As the last example we consider some simple electric circuits shown in Figure 2.6. The right-hand side of the figure shows the tree representation of these circuits. The atoms r1, r2, r3 and r4 are the names of the resistors. The functors par and seq denote the parallel and the sequential compositions of resistors respectively. The corresponding Prolog terms are:


```
seq( r1, r2)
par( r1, r2)
par( r1, par( r2, r3) )
par( r1, seq( par( r2, r3), r4) )
```

Exercises

2.1 Which of the following are syntactically correct Prolog objects? What kinds of object are they (atom, number, variable, structure)?

(a) Diana
(b) diana
(c) 'Diana'
(d) _diana
(e) 'Diana goes south'
(f) goes(diana, south)
(g) 45
(h) 5(X, Y)
(i) +(north, west)
(j) three(Black(Cats))

2.2 Suggest a representation for rectangles, squares and circles as structured Prolog objects. Use an approach similar to that in Figure 2.4. For example, a rectangle can be represented by four points (or maybe three points only). Write some example terms that represent some concrete objects of these types using the suggested representation.

2.2 Matching

In the previous section we have seen how terms can be used to represent complex data objects. The most important operation on terms is *matching*. Matching alone can produce some interesting computation.

Given two terms, we say that they *match* if:

(1) they are identical, or
(2) the variables in both terms can be instantiated to objects in such a way that after the substitution of variables by these objects the terms become identical.

For example, the terms date(D, M, 2001) and date(D1, may, Y1) match. One instantiation that makes both terms identical is:

- D is instantiated to D1
- M is instantiated to may
- Y1 is instantiated to 2001

This instantiation is more compactly written in the familiar form in which Prolog outputs results:

```
D = D1
M = may
Y1 = 2001
```

On the other hand, the terms date(D, M, 2001) and date(D1, M1, 1444) do not match, nor do the terms date(X, Y, Z) and point(X, Y, Z).

Matching is a process that takes as input two terms and checks whether they match. If the terms do not match we say that this process *fails*. If they do match then the process *succeeds* and it also instantiates the variables in both terms to such values that the terms become identical.

Let us consider again the matching of the two dates. The request for this operation can be communicated to the Prolog system by the following question, using the operator '=':

```
?- date( D, M, 2001) = date( D1, may, Y1).
```

We have already mentioned the instantiation D = D1, M = may, Y1 = 2001, which achieves the match. There are, however, other instantiations that also make both terms identical. Two of them are as follows:

```
D = 1
D1 = 1
M = may
Y1 = 2001

D = third
D1 = third
M = may
Y1 = 2001
```

These two instantiations are said to be *less general* than the first one because they constrain the values of the variables D and D1 more strongly than necessary. For making both terms in our example identical, it is only important that D and D1 have the same value, although this value can be anything. Matching in Prolog always results in the *most general* instantiation. This is the instantiation that commits the variables to the least possible extent, thus leaving the greatest possible freedom for further instantiations if further matching is required. As an example consider the following question:

```
?- date( D, M, 2001) = date( D1, may, Y1),
   date( D, M, 2001) = date( 15, M, Y).
```


To satisfy the first goal, Prolog instantiates the variables as follows:

```
D = D1
M = may
Y1 = 2001
```

After having satisfied the second goal, the instantiation becomes more specific as follows:

```
D = 15
D1 = 15
M = may
Y1 = 2001
Y = 2001
```

This example also shows that variables, during the execution of consecutive goals, typically become instantiated to increasingly more specific values.

The general rules to decide whether two terms, S and T, match are as follows:

(1) If S and T are constants then S and T match only if they are the same object.

(2) If S is a variable and T is anything, then they match, and S is instantiated to T. Conversely, if T is a variable then T is instantiated to S.

(3) If S and T are structures then they match only if

    (a) S and T have the same principal functor, and

    (b) all their corresponding components match.

    The resulting instantiation is determined by the matching of the components.

The last of these rules can be visualized by considering the tree representation of terms, as in the example of Figure 2.7. The matching process starts at the root (the principal functors). As both functors match, the process proceeds to the arguments where matching of the pairs of corresponding arguments occurs. So the whole matching process can be thought of as consisting of the following sequence of (simpler) matching operations:

```
triangle = triangle,
point(1,1) = X,
A = point(4,Y),
point(2,3) = point(2,Z).
```

The whole matching process succeeds because all the matchings in the sequence succeed. The resulting instantiation is:

```
X = point(1,1)
A = point(4,Y)
Z = 3
```

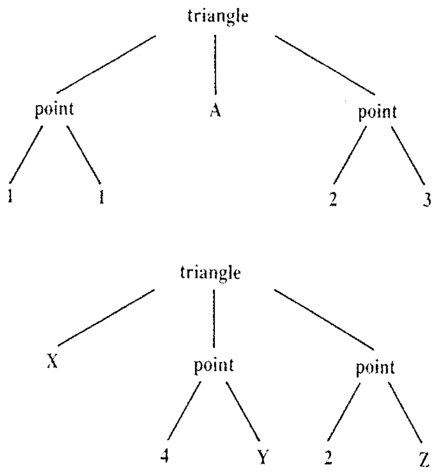


Figure 2.7 Matching **triangle(point(1,1), A, point(2,3)) = triangle(X, point(4,Y), point(2,Z))**.

The following example will illustrate how matching alone can be used for interesting computation. Let us return to the simple geometric objects of Figure 2.4, and define a piece of program for recognizing horizontal and vertical line segments. 'Vertical' is a property of segments, so it can be formalized in Prolog as a unary relation. Figure 2.8 helps to formulate this relation. A segment is vertical if the x-coordinates of its end-points are equal, otherwise there is no other restriction on the segment. The property 'horizontal' is similarly formulated, with only x and y interchanged. The following program, consisting of two facts, does the job:

```
vertical( seg( point(X,Y), point(X,Y1) ) ).
horizontal( seg( point(X,Y), point(X1,Y) ) ).
```

The following conversation is possible with this program:

```
?- vertical( seg( point(1,1), point(1,2) ) ).
yes
?- vertical( seg( point(1,1), point(2,Y) ) ).
no
?- horizontal( seg( point(1,1), point(2,Y) ) ).
Y = 1
```

The first question was answered 'yes' because the goal in the question matched one of the facts in the program. For the second question no match was possible. In the third question, Y was forced to become 1 by matching the fact about horizontal segments.

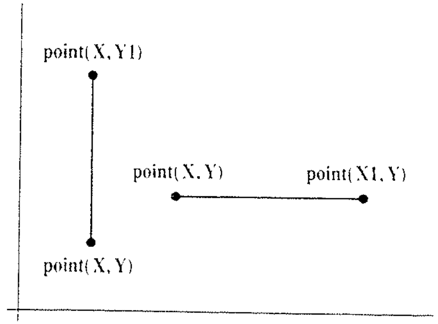


Figure 2.8 Illustration of vertical and horizontal line segments.

A more general question to the program is: Are there any vertical segments that start at the point (2,3)?

```
?- vertical( seg( point(2,3), P) ).
P = point(2,Y)
```

This answer means: Yes, any segment that ends at any point (2,Y), which means anywhere on the vertical line $x = 2$. It should be noted that Prolog's actual answer would probably not look as neat as above, but (depending on the Prolog implementation used) something like this:

```
P = point(2,_136)
```

This is, however, only a cosmetic difference. Here _136 is a variable that has not been instantiated. _136 is a legal variable name that the system has constructed during the execution. The system has to generate new names in order to rename the user's variables in the program. This is necessary for two reasons: first, because the same name in different clauses signifies different variables, and second, in successive applications of the same clause, its 'copy' with a new set of variables is used each time.

Another interesting question to our program is: Is there a segment that is both vertical and horizontal?

```
?- vertical( S), horizontal( S).
S = seg( point(X,Y), point(X,Y) )
```

This answer by Prolog says: Yes, any segment that is degenerated to a point has the property of being vertical and horizontal at the same time. The answer was, again, derived simply by matching. As before, some internally generated names may appear in the answer, instead of the variable names X and Y.

Will the following matching operations succeed or fail? If they succeed, what are the resulting instantiations of variables?

(a) point(A, B) = point(1, 2)

(b) point(A, B) = point(X, Y, Z)

(c) plus(2, 2) = 4

(d) +(2, D) = +(E, 2)

(e) triangle(point(−1,0), P2, P3) = triangle(P1, point(1,0), point(0,Y))

The resulting instantiation defines a family of triangles. How would you describe this family?

2.4 Using the representation for line segments as described in this section, write a term that represents any vertical line segment at $x = 5$.

2.5 Assume that a rectangle is represented by the term rectangle(P1, P2, P3, P4) where the P's are the vertices of the rectangle positively ordered. Define the relation:

```
regular( R)
```

which is true if R is a rectangle whose sides are vertical and horizontal.

2.3 Declarative meaning of Prolog programs

We have already seen in Chapter 1 that Prolog programs can be understood in two ways: declaratively and procedurally. In this and the next section we will consider a more formal definition of the declarative and procedural meanings of programs in basic Prolog. But first let us look at the difference between these two meanings again.

Consider a clause:

```
P :- Q, R.
```

where P, Q and R have the syntax of terms. Some alternative declarative readings of this clause are:

P is true if Q and R are true.
From Q and R follows P.

Two alternative procedural readings of this clause are:

To solve problem P, *first* solve the subproblem Q and *then* the subproblem R.
To satisfy P, *first* satisfy Q and *then* R.

Thus the difference between the declarative readings and the procedural ones is that the latter do not only define the logical relations between the head of the clause and the goals in the body, but also the *order* in which the goals are processed.

Let us now formalize the declarative meaning.

The declarative meaning of programs determines whether a given goal is true, and if so, for what values of variables it is true. To precisely define the declarative meaning we need to introduce the concept of *instance* of a clause. An instance of a clause C is the clause C with each of its variables substituted by some term. A *variant* of a clause C is such an instance of the clause C where each variable is substituted by another variable. For example, consider the clause:

```
hasachild( X) :- parent( X, Y).
```

Two variants of this clause are:

```
hasachild( A) :- parent( A, B).
hasachild( X1) :- parent( X1, X2).
```

Instances of this clause are:

```
hasachild( peter) :- parent( peter, Z).
hasachild( barry) :- parent( barry, small(caroline) ).
```

Given a program and a goal G, the declarative meaning says:

A goal G is true (that is, satisfiable, or logically follows from the program) if and only if:

(1) there is a clause C in the program such that

(2) there is a clause instance I of C such that

   (a) the head of I is identical to G, and

   (b) all the goals in the body of I are true.

This definition extends to Prolog questions as follows. In general, a question to the Prolog system is a *list* of goals separated by commas. A list of goals is true if *all* the goals in the list are true for the *same* instantiation of variables. The values of the variables result from the most general instantiation.

A comma between goals thus denotes the *conjunction* of goals: they *all* have to be true. But Prolog also accepts the *disjunction* of goals: *any one* of the goals in a disjunction has to be true. Disjunction is indicated by a semicolon. For example,

```
P :- Q; R.
```

is read: P is true if Q is true *or* R is true. The meaning of this clause is thus the same as the meaning of the following two clauses together:

```
P :- Q.
P :- R.
```

The comma binds stronger than the semicolon. So the clause:

```
P :- Q, R; S, T, U.
```

is understood as:

```
P :- ( Q, R); ( S, T, U).
```

and means the same as the clauses:

```
P :- Q, R.
P :- S, T, U.
```

Exercises

2.6 Consider the following program:

```
f( 1, one).

f( s(1), two).

f( s(s(1)), three).

f( s(s(s(X))), N) :-
  f( X, N).
```

How will Prolog answer the following questions? Whenever several answers are possible, give at least two.

(a) ?- f(s(1), A).

(b) ?- f(s(s(1)), two).

(c) ?- f(s(s(s(s(s(s(1)))))), C).

(d) ?- f(D, three).

2.7 The following program says that two people are relatives if

(a) one is a predecessor of the other, or

(b) they have a common predecessor, or

(c) they have a common successor:

```
relatives( X, Y) :-
  predecessor( X, Y).

relatives( X, Y) :-
  predecessor( Y, X).

relatives( X, Y) :-                 % X and Y have a common predecessor
  predecessor( Z, X),
  predecessor( Z, Y).

relatives( X, Y) :-                 % X and Y have a common successor
  predecessor( X, Z),
  predecessor( Y, Z).
```

Can you shorten this program by using the semicolon notation?

2.8 Rewrite the following program without using the semicolon notation.

```
translate( Number, Word) :-
  Number = 1, Word = one;
  Number = 2, Word = two;
  Number = 3, Word = three.
```


2.4 Procedural meaning

The procedural meaning specifies *how* Prolog answers questions. To answer a question means to try to satisfy a list of goals. They can be satisfied if the variables that occur in the goals can be instantiated in such a way that the goals logically follow from the program. Thus the procedural meaning of Prolog is a procedure for executing a list of goals with respect to a given program. To 'execute goals' means: try to satisfy them.

Let us call this procedure execute. As shown in Figure 2.9, the inputs to and the outputs from this procedure are:

input: a program and a goal list
output: a success/failure indicator and an instantiation of variables

The meaning of the two output results is as follows:

(1) The success/failure indicator is 'yes' if the goals are satisfiable and 'no' otherwise. We say that 'yes' signals a *successful* termination and 'no' a *failure*.

(2) An instantiation of variables is only produced in the case of a successful termination; in the case of failure there is no instantiation.

In Chapter 1, we have in effect already discussed informally what procedure execute does, under the heading 'How Prolog answers questions'. What follows in the rest of this section is just a more formal and systematic description of this

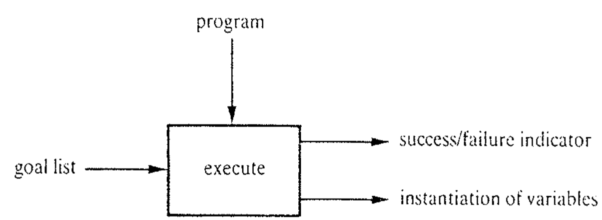


Figure 2.9 Input/output view of the procedure that executes a list of goals.

process, and can be skipped without seriously affecting the understanding of the rest of the book.

Particular operations in the goal execution process are illustrated by the example in Figure 2.10. It may be helpful to study Figure 2.10 before reading the following general description.

To execute a list of goals:

G1, G2, ..., Gm

the procedure execute does the following:

- If the goal list is empty then terminate with *success*.
- If the goal list is not empty then continue with (the following) operation called 'SCANNING'.
- *SCANNING*: Scan through the clauses in the program from top to bottom until the first clause, C, is found such that the head of C matches the first goal G1. If there is no such clause then terminate with *failure*.

  If there is such a clause C of the form

  H :- B1, ..., Bn.

  then rename the variables in C to obtain a variant C′ of C, such that C′ and the list G1, ..., Gm have no common variables. Let C′ be

  H′ :- B1′, ..., Bn′.

  Match G1 and H′; let the resulting instantiation of variables be S.

  In the goal list G1, G2, ..., Gm, replace G1 with the list B1′, ..., Bn′, obtaining a new goal list

  B1′, ..., Bn′, G2, ..., Gm

  (Note that if C is a fact then $n = 0$ and the new goal list is shorter than the original one; such shrinking of the goal list may eventually lead to the empty list and thereby a successful termination.)

  Substitute the variables in this new goal list with new values as specified in the instantiation S, obtaining another goal list

  B1″, ..., Bn″, G2′, ..., Gm′
- Execute (recursively with this same procedure) this new goal list. If the execution of this new goal list terminates with success then terminate the execution of the original goal list also with success. If the execution of the new goal list is not successful then abandon this new goal list and go back to SCANNING through the program. Continue the scanning with the clause that immediately follows the clause C (C is the clause that was last used) and try to find a successful termination using some other clause.

PROGRAM

```
big( bear).              % Clause 1
big( elephant).          % Clause 2
small( cat).             % Clause 3

brown( bear).            % Clause 4
black( cat).             % Clause 5
gray( elephant).         % Clause 6

dark( Z) :-              % Clause 7: Anything black is dark
  black( Z).

dark( Z) :-              % Clause 8: Anything brown is dark
  brown( Z).
```

QUESTION

```
?- dark( X), big( X).    % Who is dark and big?
```

EXECUTION TRACE

(1) Initial goal list: **dark(X), big(X)**.

(2) Scan the program from top to bottom looking for a clause whose head matches the first goal **dark(X)**. Clause 7 found:

    dark(Z) :- black(Z).

Replace the first goal by the instantiated body of clause 7, giving a new goal list:

    black(X), big(X)

(3) Scan the program to find a match with **black(X)**. Clause 5 found: **black(cat)**. This clause has no body, so the goal list, properly instantiated, shrinks to:

    big(cat)

(4) Scan the program for the goal **big(cat)**. No clause found. Therefore backtrack to step (3) and undo the instantiation X = cat. Now the goal list is again:

    black(X), big(X)

Continue scanning the program below clause 5. No clause found. Therefore backtrack to step (2) and continue scanning below clause 7. Clause 8 is found:

    dark(Z) :- brown(Z).

Replace the first goal in the goal list by **brown(X)**, giving:

    brown(X), big(X)

(5) Scan the program to match **brown(X)**, finding **brown(bear)**. This clause has no body, so the goal list shrinks to:

    big(bear)

(6) Scan the program and find clause **big(bear)**. It has no body so the goal list shrinks to empty. This indicates successful termination, and the corresponding variable instantiation is:

    X = bear

Figure 2.10 An example to illustrate the procedural meaning of Prolog: a sample trace of the procedure **execute**.

This procedure is more compactly written in a Pascal-like notation in Figure 2.11.

Several additional remarks are in order here regarding the procedure execute as presented. First, it was not explicitly described how the final resulting instantiation of variables is produced. It is the instantiation S which led to a successful termination, and was possibly further refined by additional instantiations that were done in the nested recursive calls to execute.

Whenever a recursive call to execute fails, the execution returns to SCANNING, continuing at the program clause C that had been last used before. As the application of the clause C did not lead to a successful termination Prolog has to try an alternative clause to proceed. What effectively happens is that Prolog abandons this whole part of the unsuccessful execution and backtracks to the point (clause C) where this failed branch of the execution was started. When the procedure backtracks to a certain point, all the variable instantiations that were done after that point are undone. This ensures that Prolog systematically examines all the possible alternative paths of execution until one is found that eventually succeeds, or until all of them have been shown to fail.

We have already seen that even after a successful termination the user can force the system to backtrack to search for more solutions. In our description of execute this detail was left out.

Of course, in actual implementations of Prolog, several other refinements have to be added to execute. One of them is to reduce the amount of scanning through the program clauses to improve efficiency. So a practical Prolog implementation will not scan through all the clauses of the program, but will only consider the clauses about the relation in the current goal.

onsider the program in Figure 2.10 and simulate, in the style of Figure 2.10, olog's execution of the question:

```
?- big( X), dark( X).
```

ıpare your execution trace with that of Figure 2.10 when the question was ıtially the same, but with the goals in the order:

```
dark( X), big( X).
```

ch of the two cases does Prolog have to do more work before the answer is


```
procedure execute (Program, GoalList, Success);
Input arguments:
  Program: list of clauses
  GoalList: list of goals
Output argument:
  Success: truth value; Success will become true if GoalList is true with respect to Program
Local variables:
  Goal: goal
  OtherGoals: list of goals
  Satisfied: truth value
  MatchOK: truth value
  Instant: instantiation of variables
  H, H', B1, B1', ..., Bn, Bn': goals
Auxiliary functions:
  empty(L): returns true if L is the empty list
  head(L): returns the first element of list L
  tail(L): returns the rest of L
  append(L1,L2): appends list L2 at the end of list L1
  match(T1,T2,MatchOK,Instant): tries to match terms T1 and T2; if
    succeeds then MatchOK is true and Instant is the corresponding instantiation of variables
  substitute(Instant,Goals): substitutes variables in Goals according to instantiation Instant

begin
  if empty(GoalList) then Success := true
  else
    begin
      Goal := head(GoalList);
      OtherGoals := tail(GoalList);
      Satisfied := false;
      while not Satisfied and "more clauses in program" do
        begin
          Let next clause in Program be
            H :- B1, ..., Bn.
          Construct a variant of this clause
            H' :- B1', ..., Bn'.
          match(Goal,H',MatchOK,Instant);
          if MatchOK then
            begin
              NewGoals := append([B1',...,Bn'], OtherGoals);
              NewGoals := substitute(Instant,NewGoals);
              execute(Program,NewGoals,Satisfied)
            end
        end;
      Success := Satisfied
    end
end;
```

Figure 2.11 Executing Prolog goals.

2.5 Example: monkey and banana

The monkey and banana problem is used as a simple example of problem solving. Our Prolog program for this problem will show how the mechanisms of matching and backtracking can be used in such exercises. We will develop the program in the non-procedural way, and then study its procedural behaviour in detail. The program will be compact and illustrative.

We will use the following variation of the problem. There is a monkey at the door into a room. In the middle of the room a banana is hanging from the ceiling. The monkey is hungry and wants to get the banana, but he cannot stretch high enough from the floor. At the window of the room there is a box the monkey may use. The monkey can perform the following actions: walk on the floor, climb the box, push the box around (if it is already at the box) and grasp the banana if standing on the box directly under the banana. Can the monkey get the banana?

One important task in programming is that of finding a representation of the problem in terms of the programming language used. In our case we can think of the 'monkey world' as always being in some *state* that can change in time. The current state is determined by the positions of the objects. For example, the initial state of the world is determined by:

(1) Monkey is at door.

(2) Monkey is on floor.

(3) Box is at window.

(4) Monkey does not have banana.

It is convenient to combine all of these four pieces of information into one structured object. Let us choose the word 'state' as the functor to hold the four components together. Figure 2.12 shows the initial state represented as a structured object.

Our problem can be viewed as a one-person game. Let us now formalize the rules of the game. First, the goal of the game is a situation in which the monkey has the banana; that is, any state in which the last component is 'has':

```
state( _, _, _, has)
```

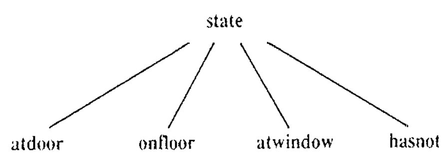


Figure 2.12 The initial state of the monkey world represented as a structured object. The four components are: horizontal position of monkey, vertical position of monkey, position of box, monkey has or has not banana.

Second, what are the allowed moves that change the world from one state to another? There are four types of moves:

(1) grasp banana,

(2) climb box,

(3) push box,

(4) walk around.

Not all moves are possible in every possible state of the world. For example, the move 'grasp' is only possible if the monkey is standing on the box directly under the banana (which is in the middle of the room) and does not have the banana yet. Such rules can be formalized in Prolog as a three-place relation named move:

```
move( State1, Move, State2)
```

The three arguments of the relation specify a move thus:

$$\text{State1} \xrightarrow[\text{Move}]{} \text{State2}$$

State1 is the state before the move, Move is the move executed and State2 is the state after the move.

The move 'grasp', with its necessary precondition on the state before the move, can be defined by the clause:

```
move( state( middle, onbox, middle, hasnot),     % Before move
      grasp,                                     % Move
      state( middle, onbox, middle, has) ).      % After move
```

This fact says that after the move the monkey has the banana, and he has remained on the box in the middle of the room.

In a similar way we can express the fact that the monkey on the floor can walk from any horizontal position Pos1 to any position Pos2. The monkey can do this regardless of the position of the box and whether it has the banana or not. All this can be defined by the following Prolog fact:

```
move( state( Pos1, onfloor, Box, Has),
      walk( Pos1, Pos2),                         % Walk from Pos1 to Pos2
      state( Pos2, onfloor, Box, Has) ).
```

Note that this clause says many things, including, for example:

- the move executed was 'walk from some position Pos1 to some position Pos2';
- the monkey is on the floor before and after the move;

- the box is at some point Box which remained the same after the move;
- the 'has banana' status Has remains the same after the move.

The clause actually specifies a whole set of possible moves because it is applicable to any situation that matches the specified state before the move. Such a specification is therefore sometimes also called a move *schema*. Using Prolog variables, such schemas can be easily programmed in Prolog.

The other two types of moves, 'push' and 'climb', can be similarly specified.

The main kind of question that our program will have to answer is: Can the monkey in some initial state State get the banana? This can be formulated as a predicate

```
canget( State)
```

where the argument State is a state of the monkey world. The program for canget can be based on two observations:

(1) For any state in which the monkey already has the banana, the predicate canget must certainly be true; no move is needed in this case. This corresponds to the Prolog fact:

```
canget( state( _, _, _, has) ).
```

(2) In other cases one or more moves are necessary. The monkey can get the banana in any state State1 if there is some move Move from State1 to some state State2, such that the monkey can then get the banana in state State2 (in zero or more moves). This principle is illustrated in Figure 2.13. A Prolog clause that corresponds to this rule is:

```
canget( State1) :-
  move( State1, Move, State2),
  canget( State2).
```

This completes our program, which is shown in Figure 2.14.

The formulation of canget is recursive and is similar to that of the predecessor relation of Chapter 1 (compare Figures 2.13 and 1.7). This principle is used in Prolog again and again.

We have developed our monkey and banana program in the non-procedural way. Let us now study its *procedural* behaviour by considering the following question

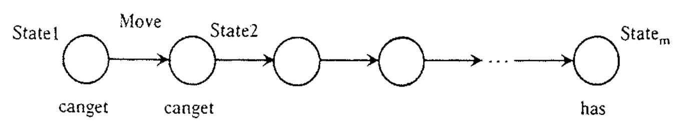


Figure 2.13 Recursive formulation of **canget**.


```
% move( State1, Move, State2): making Move in State1 results in State2;
%   a state is represented by a term:
%   state( MonkeyHorizontal, MonkeyVertical, BoxPosition, HasBanana)

move( state( middle, onbox, middle, hasnot),     % Before move
      grasp,                                      % Grasp banana
      state( middle, onbox, middle, has) ).       % After move

move( state( P, onfloor, P, H),
      climb,                                      % Climb box
      state( P, onbox, P, H) ).

move( state( P1, onfloor, P1, H),
      push( P1, P2),                              % Push box from P1 to P2
      state( P2, onfloor, P2, H) ).

move( state( P1, onfloor, B, H),
      walk( P1, P2),                              % Walk from P1 to P2
      state( P2, onfloor, B, H) ).

% canget( State): monkey can get banana in State

canget( state( _, _, _, has) ).                   % can 1: Monkey already has it

canget( State1) :-                                % can 2: Do some work to get it
      move( State1, Move, State2),                % Do something
      canget( State2).                            % Get it now
```

Figure 2.14 A program for the monkey and banana problem.

to the program:

```
?- canget( state( atdoor, onfloor, atwindow, hasnot) ).
```

Prolog's answer is 'yes'. The process carried out by Prolog to reach this answer proceeds, according to the procedural semantics of Prolog, through a sequence of goal lists. It involves some search for the right moves among the possible alternative moves. At some point this search will take a wrong move leading to a dead branch. At this stage, backtracking will help it to recover. Figure 2.15 illustrates this search process.

To answer the question Prolog had to backtrack once only. A right sequence of moves was found almost straight away. The reason for this efficiency of the program was the order in which the clauses about the move relation occurred in the program. The order in our case (luckily) turned out to be quite suitable. However, less lucky orderings are possible. According to the rules of the game, the monkey could just as easily try to walk here or there without ever touching the box, or aimlessly push the box around. A more thorough investigation will reveal, as shown in the following section, that the ordering of clauses is, in the case of our program, in fact critical.

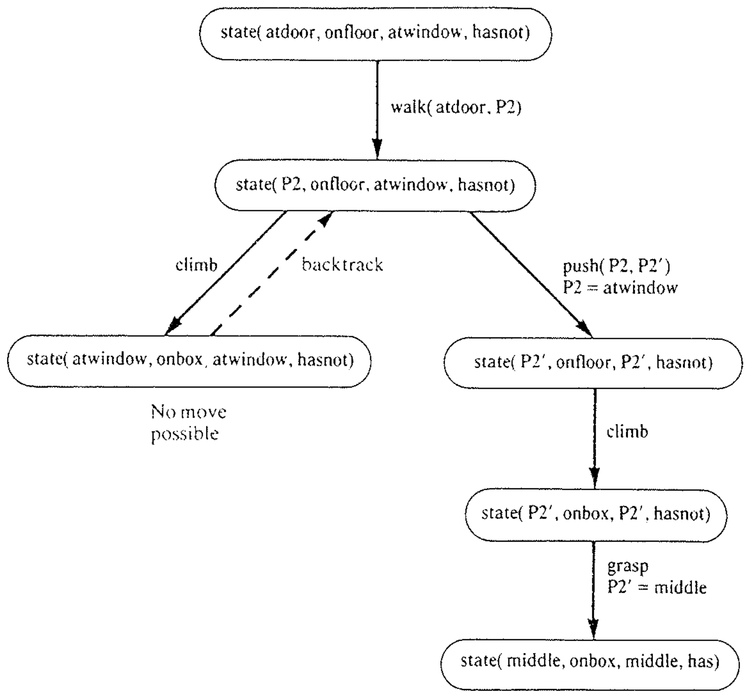


Figure 2.15 The monkey's search for the banana. The search starts at the top node and proceeds downwards, as indicated. Alternative moves are tried in the left-to-right order. Backtracking occurred once only.

2.6 Order of clauses and goals

2.6.1 Danger of indefinite looping

Consider the following clause:

```
p :- p.
```

This says that 'p is true if p is true'. This is declaratively perfectly correct, but procedurally is quite useless. In fact, such a clause can cause problems to Prolog. Consider the question:

```
?- p.
```


Using the clause above, the goal p is replaced by the same goal p; this will be in turn replaced by p, etc. In such a case Prolog will enter an infinite loop not noticing that no progress is being made.

This example is a simple way of getting Prolog to loop indefinitely. However, similar looping could have occurred in some of our previous example programs if we changed the order of clauses, or the order of goals in the clauses. It will be instructive to consider some examples.

In the monkey and banana program, the clauses about the move relation were ordered thus: grasp, climb, push, walk (perhaps 'unclimb' should be added for completeness). These clauses say that grasping is possible, climbing is possible, etc. According to the procedural semantics of Prolog, the order of clauses indicates that the monkey prefers grasping to climbing, climbing to pushing, etc. This order of preferences in fact helps the monkey to solve the problem. But what could happen if the order was different? Let us assume that the 'walk' clause appears first. The execution of our original goal of the previous section

```
?- canget( state( atdoor, onfloor, atwindow, hasnot) ).
```

would this time produce the following trace. The first four goal lists (with variables appropriately renamed) are the same as before:

(1) canget(state(atdoor, onfloor, atwindow, hasnot))

The second clause of canget ('can2') is applied, producing:

(2) move(state(atdoor, onfloor, atwindow, hasnot), M', S2'),
canget(S2')

By the move walk(atdoor, P2') we get:

(3) canget(state(P2', onfloor, atwindow, hasnot))

Using the clause 'can2' again the goal list becomes:

(4) move(state(P2', onfloor, atwindow, hasnot), M'', S2''),
canget(S2'')

Now the difference occurs. The first clause whose head matches the first goal above is now 'walk' (and not 'climb' as before). The instantiation is

S2'' = state(P2'', onfloor, atwindow, hasnot)

Therefore the goal list becomes:

(5) canget(state(P2'', onfloor, atwindow, hasnot))

Applying the clause 'can2' we obtain:

(6) move(state(P2'', onfloor, atwindow, hasnot), M''', S2'''),
canget(S2''')

Again, 'walk' is now tried first, producing:

```
(7)   canget( state( P2''', onfloor, atwindow, hasnot) )
```

Let us now compare the goals (3), (5) and (7). They are the same apart from one variable; this variable is, in turn, P', P'' and P'''. As we know, the success of a goal does not depend on particular names of variables in the goal. This means that from goal list (3) the execution trace shows no progress. We can see, in fact, that the same two clauses, 'can2' and 'walk', are used repetitively. The monkey walks around without ever trying to use the box. As there is no progress made this will (theoretically) go on for ever: Prolog will not realize that there is no point in continuing along this line.

This example shows Prolog trying to solve a problem in such a way that a solution is never reached, although a solution exists. Such situations are not unusual in Prolog programming. Infinite loops are, also, not unusual in other programming languages. What *is* unusual in comparison with other languages is that a Prolog program may be declaratively correct, but at the same time be procedurally incorrect in that it is not able to produce an answer to a question. In such cases Prolog may not be able to satisfy a goal because it tries to reach an answer by choosing a wrong path.

A natural question to ask at this point is: Can we not make some more substantial change to our program so as to drastically prevent any danger of looping? Or shall we always have to rely just on a suitable ordering of clauses and goals? As it turns out programs, especially large ones, would be too fragile if they just had to rely on some suitable ordering. There are several other methods that preclude infinite loops, and these are much more general and robust than the ordering method itself. These techniques will be used regularly later in the book, especially in those chapters that deal with path finding, problem solving and search.

2.6.2 Program variations through reordering of clauses and goals

Already in the example programs of Chapter 1 there was a latent danger of producing a cycling behaviour. Our program to specify the **predecessor** relation in Chapter 1 was:

```
predecessor( Parent, Child) :-
  parent( Parent, Child).

predecessor( Predecessor, Successor) :-
  parent( Predecessor, Child),
  predecessor( Child, Successor).
```

Let us analyze some variations of this program. All the variations will clearly have the same declarative meaning, but not the same procedural meaning. According to the declarative semantics of Prolog we can, without affecting the declarative meaning, change:

(1) the order of clauses in the program, and

(2) the order of goals in the bodies of clauses.

The **predecessor** procedure consists of two clauses, and one of them has two goals in the body. There are, therefore, four variations of this program, all with the same declarative meaning. The four variations are obtained by:

(1) swapping both clauses, and

(2) swapping the goals for each order of clauses.

The corresponding four procedures, called **pred1**, **pred2**, **pred3** and **pred4**, are shown in Figure 2.16.

```
% Four versions of the predecessor program

% The original version

pred1( X, Z) :-
  parent( X, Z).

pred1( X, Z) :-
  parent( X, Y),
  pred1( Y, Z).

% Variation a: swap clauses of the original version

pred2( X, Z) :-
  parent( X, Y),
  pred2( Y, Z).

pred2( X, Z) :-
  parent( X, Z).

% Variation b: swap goals in second clause of the original version

pred3( X, Z) :-
  parent( X, Z).

pred3( X, Z) :-
  pred3( X, Y),
  parent( Y, Z).

% Variation c: swap goals and clauses of the original version

pred4( X, Z) :-
  pred4( X, Y),
  parent( Y, Z).

pred4( X, Z) :-
  parent( X, Z).
```

Figure 2.16 Four versions of the **predecessor** program.

There are important differences in the behaviour of these four declaratively equivalent procedures. To demonstrate these, consider the parent relation as shown in Figure 1.1 of Chapter 1. Now, what happens if we ask whether Tom is a predecessor of Pat using the four variations of the predecessor relation:

```
?- pred1( tom, pat).
yes
?- pred2( tom, pat).
yes
?- pred3( tom, pat).
yes
?- pred4( tom, pat).
```

In the last case Prolog cannot find the answer. This is manifested on the terminal by a Prolog message such as 'More core needed' or 'Stack overflow'.

Figure 1.11 in Chapter 1 showed the trace of pred1 (in Chapter 1 called predecessor) produced for the above question. Figure 2.17 shows the corresponding traces for pred2, pred3 and pred4. Figure 2.17(c) clearly shows that pred4 is hopeless, and Figure 2.17(a) indicates that pred2 is rather inefficient compared to pred1: pred2 does much more searching and backtracking in the family tree.

This comparison should remind us of a general practical heuristic in problem solving: it is usually best to try the simplest idea first. In our case, all the versions of the predecessor relation are based on two ideas:

(1) the simpler idea is to check whether the two arguments of the predecessor relation satisfy the parent relation;

(2) the more complicated idea is to find somebody 'between' both people (somebody who is related to them by the parent and predecessor relations).

Of the four variations of the predecessor relation, pred1 does simplest things first. On the contrary, pred4 always tries complicated things first. pred2 and pred3 are in between the two extremes. Even without a detailed study of the execution traces, pred1 should be preferred merely on the grounds of the rule 'try simple things first'. This rule will be in general a useful guide in programming.

Our four variations of the predecessor procedure can be further compared by considering the question: What types of questions can particular variations answer, and what types can they not answer? It turns out that pred1 and pred2 are both able to reach an answer for any type of question about predecessors; pred4 can never reach an answer; and pred3 sometimes can and sometimes cannot. One example in which pred3 fails is:

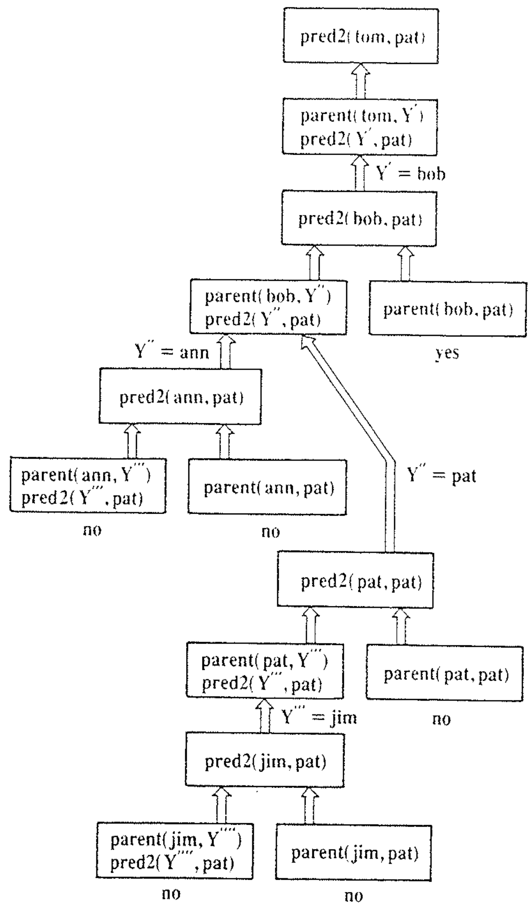
```
?- pred3( liz, jim).
```


This question again brings the system into an infinite sequence of recursive calls. Thus pred3 also cannot be considered procedurally correct.

2.6.3 Combining declarative and procedural views

The foregoing section has shown that the order of goals and clauses does matter. Furthermore, there are programs that are declaratively correct, but do not work in practice. Such discrepancies between the declarative and procedural meaning may

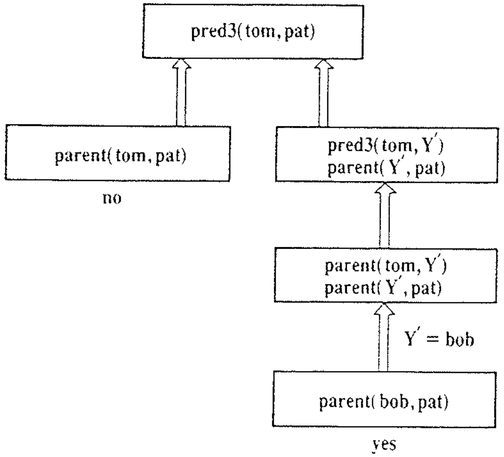
```
pred2( X, Z) :-
  parent( X, Y),
  pred2( Y, Z).
pred2( X, Z) :-
  parent( X, Z).
```



(a)

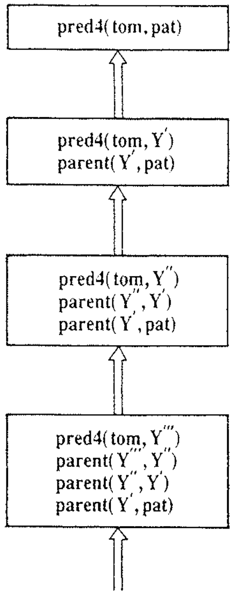
Figure 2.17 The behaviour of three formulations of the predecessor relation on the question: Is Tom a predecessor of Pat?

```
pred3( X, Z) :-
  parent( X, Z).
pred3( X, Z) :-
  pred3( X, Y),
  parent( Y, Z).
```



(b)

```
pred4( X, Z) :-
  pred4( X, Y),
  parent( Y, Z).
pred4( X, Z) :-
  parent( X, Z).
```



(c)

Figure 2.17 *contd*

appear annoying. One may argue: Why not simply forget about the declarative meaning? This argument can be brought to an extreme with a clause such as:

```
predecessor( X, Z) :- predecessor( X, Z).
```

which is declaratively correct, but is completely useless as a working program.

The reason why we should not forget about the declarative meaning is that progress in programming technology is achieved by moving away from procedural details toward declarative aspects, which are normally easier to formulate and understand. The system itself, not the programmer, should carry the burden of filling in the procedural details. Prolog does help toward this end, although, as we have seen in this section, it only helps partially: sometimes it does work out the procedural details itself properly, and sometimes it does not. The philosophy adopted by many is that it is better to have at least *some* declarative meaning rather than *none* ('none' is the case in most other programming languages). The practical aspect of this view is that it is often rather easy to get a working program once we have a program that is declaratively correct. Consequently, a useful practical approach that often works is to concentrate on the declarative aspects of the problem, then test the resulting program, and if it fails procedurally try to rearrange the clauses and goals into a suitable order.

2.7 The relation between Prolog and logic

Prolog is related to mathematical logic, so its syntax and meaning can be specified most concisely with references to logic. Prolog is indeed often defined that way. However, such an introduction to Prolog assumes that the reader is familiar with certain concepts of mathematical logic. These concepts are, on the other hand, certainly not necessary for understanding and using Prolog as a programming tool, which is the aim of this book. For the reader who is especially interested in the relation between Prolog and logic, the following are some basic links to mathematical logic, together with some appropriate references.

Prolog's syntax is that of the *first-order predicate logic* formulas written in the so-called *clause form* (a conjunctive normal form in which quantifiers are not explicitly written), and further restricted to Horn clauses only (clauses that have at most one positive literal). Clocksin and Mellish (1987) give a Prolog program that transforms a first-order predicate calculus formula into the clause form. The procedural meaning of Prolog is based on the *resolution principle* for mechanical theorem proving introduced by Robinson in his classic paper (1965). Prolog uses a special strategy for resolution theorem proving called SLD. An introduction to the first-order predicate calculus and resolution-based theorem proving can be found in several general books on artificial intelligence (Genesereth and Nilsson 1987; Ginsberg 1993; Poole *et al.* 1998; Russell and Norvig 1995; see also Flach 1994). Mathematical questions regarding the properties of Prolog's procedural meaning with respect to logic are analyzed by Lloyd (1991).

Matching in Prolog corresponds to what is called *unification* in logic. However, we avoid the word unification because matching, for efficiency reasons in most Prolog systems, is implemented in a way that does not exactly correspond to unification (see Exercise 2.10). But from the practical point of view this approximation to unification is quite adequate. Proper unification requires the so-called *occurs check*: does a given variable occur in a given term? The occurs check would make matching inefficient.

Exercise

2.10 What happens if we ask Prolog:

```
?-   X = f( X).
```

Should this request for matching succeed or fail? According to the definition of unification in logic this should fail, but what happens according to our definition of matching in Section 2.2? Try to explain why many Prolog implementations answer the question above with:

```
X = f(f(f(f(f(f(f(f(f(f(f(f(f(f( ...
```

Summary

So far we have covered a kind of basic Prolog, also called 'pure Prolog'. It is 'pure' because it corresponds closely to formal logic. Extensions whose aim is to tailor the language toward some practical needs will be covered later in the book (Chapters 3, 5, 6, 7). Important points of this chapter are:

- Simple objects in Prolog are *atoms*, *variables* and *numbers*. Structured objects, or *structures*, are used to represent objects that have several components.
- Structures are constructed by means of *functors*. Each functor is defined by its name and arity.
- The type of object is recognized entirely by its syntactic form.
- The *lexical* scope of variables is one clause. Thus the same variable name in two clauses means two different variables.
- Structures can be naturally pictured as trees. Prolog can be viewed as a language for processing trees.
- The *matching* operation takes two terms and tries to make them identical by instantiating the variables in both terms.

- Matching, if it succeeds, results in the *most general* instantiation of variables.
- The *declarative semantics* of Prolog defines whether a goal is true with respect to a given program, and if it is true, for what instantiation of variables it is true.
- A comma between goals means the conjunction of goals. A semicolon between goals means the disjunction of goals.
- The *procedural semantics* of Prolog is a procedure for satisfying a list of goals in the context of a given program. The procedure outputs the truth or falsity of the goal list and the corresponding instantiations of variables. The procedure automatically backtracks to examine alternatives.
- The declarative meaning of programs in 'pure Prolog' does not depend on the order of clauses and the order of goals in clauses.
- The procedural meaning does depend on the order of goals and clauses. Thus the order can affect the efficiency of the program; an unsuitable order may even lead to infinite recursive calls.
- Given a declaratively correct program, changing the order of clauses and goals can improve the program's efficiency while retaining its declarative correctness. Reordering is one method of preventing indefinite looping.
- There are other more general techniques, apart from reordering, to prevent indefinite looping and thereby make programs procedurally robust.
- Concepts discussed in this chapter are:

  data objects: atom, number, variable, structure
  term
  functor, arity of a functor
  principal functor of a term
  matching of terms
  most general instantiation
  declarative semantics
  instance of a clause, variant of a clause
  procedural semantics
  executing goals

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chapter 3

Lists, Operators, Arithmetic

In this chapter we will study a special notation for lists, one of the simplest and most useful structures, and some programs for typical operations on lists. We will also look at simple arithmetic and the operator notation, which often improves the readability of programs. Basic Prolog of Chapter 2, extended with these three additions, becomes a convenient framework for writing interesting programs.

3.1 Representation of lists

The *list* is a simple data structure widely used in non-numeric programming. A list is a sequence of any number of items, such as ann, tennis, tom, skiing. Such a list can be written in Prolog as:

```
[ ann, tennis, tom, skiing]
```

This is, however, only the external appearance of lists. As we have already seen in Chapter 2, all structured objects in Prolog are trees. Lists are no exception to this.

How can a list be represented as a standard Prolog object? We have to consider two cases: the list is either empty or non-empty. In the first case, the list is simply written as a Prolog atom, []. In the second case, the list can be viewed as consisting of two things:

(1) the first item, called the *head* of the list;

(2) the remaining part of the list, called the *tail*.

For our example list,

```
[ ann, tennis, tom, skiing]
```

the head is ann and the tail is the list:

```
[ tennis, tom, skiing]
```

In general, the head can be anything (any Prolog object, for example, a tree or a variable); the tail has to be a list. The head and the tail are then combined into a structure by a special functor,

```
.( Head, Tail)
```

Since Tail is in turn a list, it is either empty or it has its own head and tail. Therefore, to represent lists of any length no additional principle is needed. Our example list is then represented as the term:

```
.( ann, .( tennis, .( tom, .( skiing, [ ] ) ) ) )
```

Figure 3.1 shows the corresponding tree structure. Note that the empty list appears in our term. This is because the one but last tail is a single item list:

```
[ skiing]
```

This list has the empty list as its tail:

```
[ skiing] = .( skiing, [ ] )
```

This example shows how the general principle for structuring data objects in Prolog also applies to lists of any length. As our example also shows, the straightforward notation with dots and possibly deep nesting of subterms in the tail part can produce rather confusing expressions. This is the reason why Prolog provides the neater notation for lists, so that they can be written as sequences of items enclosed in square brackets. A programmer can use both notations, but the square bracket notation is, of course, normally preferred. We will be aware, however, that this is

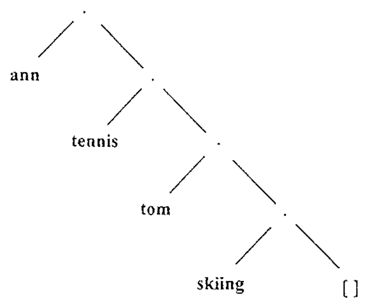

Figure 3.1 Tree representation of the list **[ann, tennis, tom, skiing]**.

only a cosmetic improvement and that our lists will be internally represented as binary trees. When such terms are output they will be automatically converted into their neater form. Thus the following conversation with Prolog is possible:

```
?- List1 = [a,b,c],
   List2 = .( a, .( b, .( c, [ ] ) ) ).

List1 = [a,b,c]
List2 = [a,b,c]

?- Hobbies1 = .( tennis, .( music, [ ] ) ),
   Hobbies2 = [ skiing, food],
   L = [ ann, Hobbies1, tom, Hobbies2].

Hobbies1 = [ tennis, music]
Hobbies2 = [ skiing, food]
L = [ ann, [tennis,music], tom, [skiing,food] ]
```

This example also reminds us that the elements of a list can be objects of any kind; in particular they can also be lists.

It is often practical to treat the whole tail as a single object. For example, let:

```
L = [a,b,c]
```

Then we could write:

```
Tail = [b,c]   and   L = .( a, Tail)
```

To express this in the square bracket notation for lists, Prolog provides another notational extension, the vertical bar, which separates the head and the tail:

```
L = [ a | Tail]
```

The vertical bar notation is in fact more general: we can list any number of elements followed by '|' and the list of remaining items. Thus alternative ways of writing the above list are:

```
[a,b,c] = [a | [b,c] ] = [a,b | [c] ] = [a,b,c | [ ] ]
```

To summarize:

- A list is a data structure that is either empty or consists of two parts: a *head* and a *tail*. The tail itself has to be a list.
- Lists are handled in Prolog as a special case of binary trees. For improved readability Prolog provides a special notation for lists, thus accepting lists written as:

  ```
  [ Item1, Item2, ...]
  ```

  or

  ```
  [ Head | Tail]
  ```

  or

  ```
  [ Item1, Item2, ... | Others]
  ```

3.2 Some operations on lists

Lists can be used to represent sets, although there is a difference: the order of elements in a set does not matter while the order of items in a list does; also, the same object can occur repeatedly in a list. Still, the most common operations on lists are similar to those on sets. Among them are:

- checking whether some object is an element of a list, which corresponds to checking for the set membership;
- concatenation of two lists, obtaining a third list, which may correspond to the union of sets;
- adding a new object to a list, or deleting some object from it.

In the remainder of this section we give programs for these and some other operations on lists.

3.2.1 Membership

Let us implement the membership relation as:

```
member( X, L)
```

where X is an object and L is a list. The goal member(X, L) is true if X occurs in L. For example,

```
member( b, [a,b,c] )
```

is true,

```
member( b, [a,[b,c]] )
```

is not true, but

```
member( [b,c], [a,[b,c]] )
```

is true. The program for the membership relation can be based on the following observation:

X is a member of L if either:
(1) X is the head of L, or
(2) X is a member of the tail of L.

This can be written in two clauses; the first is a simple fact and the second is a rule:

```
member( X, [X | Tail] ).

member( X, [Head | Tail] ) :-
  member( X, Tail).
```


3.2.2 Concatenation

For concatenating lists we will define the relation:

```
conc( L1, L2, L3)
```

Here L1 and L2 are two lists, and L3 is their concatenation. For example,

```
conc( [a,b], [c,d], [a,b,c,d] )
```

is true, but

```
conc( [a,b], [c,d], [a,b,a,c,d] )
```

is false. In the definition of conc we will have again two cases, depending on the first argument, L1:

(1) If the first argument is the empty list then the second and the third arguments must be the same list (call it L); this is expressed by the following Prolog fact:

```
conc( [ ], L, L).
```

(2) If the first argument of conc is a non-empty list then it has a head and a tail and must look like this:

```
[X | L1]
```

Figure 3.2 illustrates the concatenation of [X | L1] and some list L2. The result of the concatenation is the list [X | L3] where L3 is the concatenation of L1 and L2. In Prolog this is written as:

```
conc( [X | L1], L2, [X | L3] ) :-
  conc( L1, L2, L3).
```

This program can now be used for concatenating given lists, for example:

```
?- conc( [a,b,c], [1,2,3], L).
L = [a,b,c,1,2,3]
```

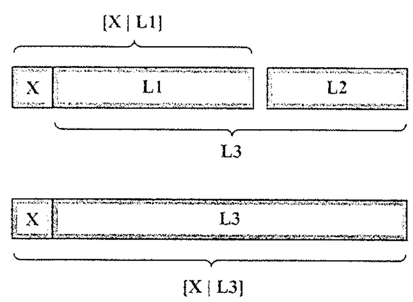

Figure 3.2 Concatenation of lists.

```
?- conc( [a,[b,c],d], [a,[ ],b], L).
L = [a, [b,c], d, a, [ ], b]
```

Although the conc program looks rather simple it can be used flexibly in many other ways. For example, we can use conc in the inverse direction for *decomposing* a given list into two lists, as follows:

```
?- conc( L1, L2, [a,b,c] ).

L1 = [ ]
L2 = [a,b,c];

L1 = [a]
L2 = [b,c];

L1 = [a,b]
L2 = [c];

L1 = [a,b,c]
L2 = [ ];

no
```

It is possible to decompose the list [a,b,c] in four ways, all of which were found by our program through backtracking.

We can also use our program to look for a certain pattern in a list. For example, we can find the months that precede and the months that follow a given month, as in the following goal:

```
?- conc( Before, [may | After],
         [jan,feb,mar,apr,may,jun,jul,aug,sep,oct,nov,dec] ).

Before = [jan,feb,mar,apr]
After = [jun,jul,aug,sep,oct,nov,dec].
```

Further we can find the immediate predecessor and the immediate successor of May by asking:

```
?- conc( _, [Month1,may,Month2 | _],
         [jan,feb,mar,apr,may,jun,jul,aug,sep,oct,nov,dec] ).

Month1 = apr
Month2 = jun
```

Further still, we can, for example, delete from some list, L1, everything that follows three successive occurrences of z in L1 together with the three z's. For example:

```
?- L1 = [a,b,z,z,c,z,z,z,d,e],
conc( L2, [z,z,z | _], L1).

L1 = [a,b,z,z,c,z,z,z,d,e]
L2 = [a,b,z,z,c]
```

We have already programmed the membership relation. Using conc, however, the membership relation could be elegantly programmed by the clause:


```
member1( X, L) :-
  conc( L1, [X | L2], L).
```

This clause says: X is a member of list L if L can be decomposed into two lists so that the second one has X as its head. Of course, member1 defines the same relation as member. We have just used a different name to distinguish between the two implementations. Note that the above clause can be written using anonymous variables as:

```
member1( X, L) :-
  conc( _, [X | _], L).
```

It is interesting to compare both implementations of the membership relation, member and member1. member has a rather straightforward procedural meaning, which is as follows:

To check whether some X is a member of some list L:
(1) first check whether the head of L is equal to X, and then
(2) check whether X is a member of the tail of L.

On the other hand, the declarative reading of member1 is straightforward, but its procedural meaning is not so obvious. An interesting exercise is to find how member1 actually computes something. An example execution trace will give some idea: let us consider the question:

```
?- member1( b, [a,b,c] ).
```

Figure 3.3 shows the execution trace. From the trace we can infer that member1 behaves similarly to member. It scans the list, element by element, until the item in question is found or the list is exhausted.

Exercises

3.1 (a) Write a goal, using conc, to delete the last three elements from a list L producing another list L1. Hint: L is the concatenation of L1 and a three-element list.

(b) Write a goal to delete the first three elements and the last three elements from a list L producing list L2.

3.2 Define the relation

last(Item, List)

so that **Item** is the last element of a list **List**. Write two versions: (a) using the conc relation, (b) without conc.

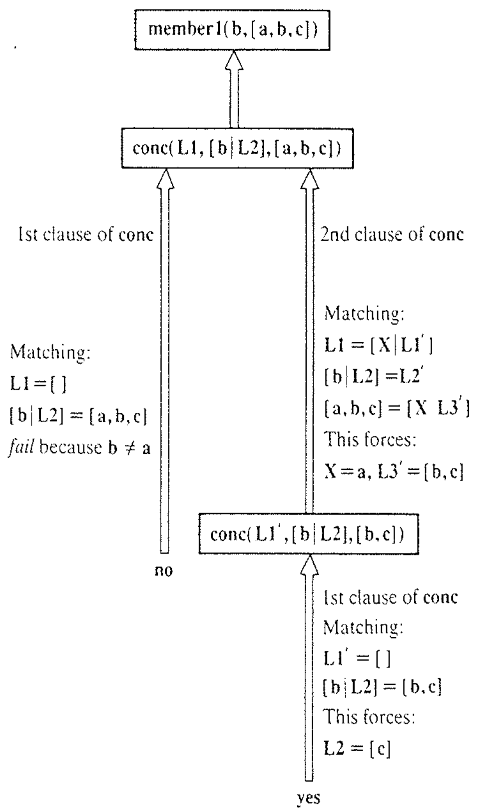

Figure 3.3 Procedure **member1** finds an item in a given list by sequentially searching the list.

3.2.3 Adding an item

To add an item to a list, it is easiest to put the new item in front of the list so that it becomes the new head. If X is the new item and the list to which X is added is L then the resulting list is simply:

```
[X | L]
```

So we actually need no procedure for adding a new element in front of the list. Nevertheless, if we want to define such a procedure explicitly, it can be written as the fact:

```
add( X, L, [X | L] ).
```


3.2.4 Deleting an item

Deleting an item, X, from a list, L, can be programmed as a relation

```
del( X, L, L1)
```

where L1 is equal to the list L with the item X removed. The **del** relation can be defined similarly to the membership relation. We have, again, two cases:

(1) If X is the head of the list then the result after the deletion is the tail of the list.

(2) If X is in the tail then it is deleted from there.

```
del( X, [X | Tail], Tail).

del( X, [Y | Tail], [Y | Tail1] ) :-
  del( X, Tail, Tail1).
```

Like **member**, **del** is also non-deterministic. If there are several occurrences of X in the list then **del** will be able to delete any one of them by backtracking. Of course, each alternative execution will only delete one occurrence of X, leaving the others untouched. For example:

```
?- del( a, [a,b,a,a], L).
L = [b,a,a];
L = [a,b,a];
L = [a,b,a];
no
```

del will fail if the list does not contain the item to be deleted.

del can also be used in the inverse direction, to add an item to a list by inserting the new item anywhere in the list. For example, if we want to insert a at any place in the list [1,2,3] then we can do this by asking the question: What is L such that after deleting a from L we obtain [1,2,3]?

```
?- del( a, L, [1,2,3] ).
L = [a,1,2,3];
L = [1,a,2,3];
L = [1,2,a,3];
L = [1,2,3,a];
no
```

In general, the operation of inserting X at any place in some list **List** giving **BiggerList** can be defined by the clause:

```
insert( X, List, BiggerList) :-
  del( X, BiggerList, List).
```

In member1 we elegantly implemented the membership relation by using conc. We can also use del to test for membership. The idea is simple: some X is a member of List if X can be deleted from List:

```
member2( X, List) :-
  del( X, List, _).
```

3.2.5 Sublist

Let us now consider the sublist relation. This relation has two arguments, a list L and a list S such that S occurs within L as its sublist. So,

```
sublist( [c,d,e], [a,b,c,d,e,f] )
```

is true, but

```
sublist( [c,e], [a,b,c,d,e,f] )
```

is not. The Prolog program for sublist can be based on the same idea as member1, only this time the relation is more general (see Figure 3.4). Accordingly, the relation can be formulated as:

S is a sublist of L if:
(1) L can be decomposed into two lists, L1 and L2, and
(2) L2 can be decomposed into two lists, S and some L3.

As we have seen before, the conc relation can be used for decomposing lists. So the above formulation can be expressed in Prolog as:

```
sublist( S, L) :-
  conc( L1, L2, L),
  conc( S, L3, L2).
```

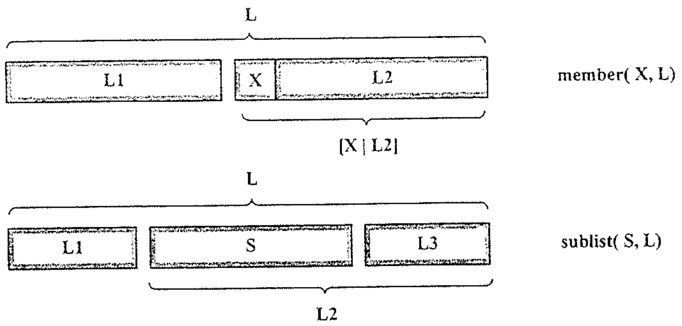

Figure 3.4 The **member** and **sublist** relations.

Of course, the sublist procedure can be used flexibly in several ways. Although it was designed to check if some list occurs as a sublist within another list it can also be used, for example, to find all sublists of a given list:

```
?- sublist( S, [a,b,c] ).
S = [ ];
S = [a];
S = [a,b];
S = [a,b,c];
S = [ ];
S = [b];
...
```

3.2.6 Permutations

Sometimes it is useful to generate permutations of a given list. To this end, we will define the permutation relation with two arguments. The arguments are two lists such that one is a permutation of the other. The intention is to generate permutations of a list through backtracking using the permutation procedure, as in the following example:

```
?- permutation( [a,b,c], P).
P = [a,b,c];
P = [a,c,b];
P = [b,a,c];
...
```

The program for permutation can be, again, based on the consideration of two cases, depending on the first list:

(1) If the first list is empty then the second list must also be empty.
(2) If the first list is not empty then it has the form [X | L], and a permutation of such a list can be constructed as shown in Figure 3.5: first permute L obtaining L1 and then insert X at any position into L1.

Two Prolog clauses that correspond to these two cases are:

```
permutation( [ ], [ ] ).

permutation( [X | L], P) :-
  permutation( L, L1),
  insert( X, L1, P).
```

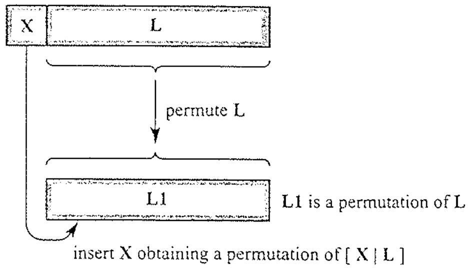

Figure 3.5 One way of constructing a permutation of the list [X | L].

One alternative to this program would be to delete an element, X, from the first list, permute the rest of it obtaining a list P, and then add X in front of P. The corresponding program is:

```
permutation2( [], [] ).

permutation2( L, [X | P] ) :-
  del( X, L, L1),
  permutation2( L1, P).
```

It is instructive to do some experiments with our permutation programs. Its normal use would be something like this:

```
?- permutation( [red,blue,green], P).
```

This would result in all six permutations, as intended:

```
P = [ red, blue, green];
P = [ red, green, blue];
P = [ blue, red, green];
P = [ blue, green, red];
P = [ green, red, blue];
P = [ green, blue, red];
no
```

Another attempt to use **permutation** is:

```
?- permutation( L, [a,b,c] ).
```

Our first version, **permutation**, will now instantiate L successfully to all six permutations. If the user then requests more solutions, the program would never answer 'no' because it would get into an infinite loop trying to find another permutation when there is none. Our second version, **permutation2**, will in this case find only the first (identical) permutation and then immediately get into an infinite loop. Thus, some care is necessary when using these permutation programs.

Exercises

3.3 Define two predicates

evenlength(List) and oddlength(List)

so that they are true if their argument is a list of even or odd length respectively. For example, the list [a,b,c,d] is 'evenlength' and [a,b,c] is 'oddlength'.

3.4 Define the relation

reverse(List, ReversedList)

that reverses lists. For example, reverse([a,b,c,d], [d,c,b,a]).

3.5 Define the predicate palindrome(List). A list is a palindrome if it reads the same in the forward and in the backward direction. For example, [m,a,d,a,m].

3.6 Define the relation

shift(List1, List2)

so that List2 is List1 'shifted rotationally' by one element to the left. For example,

```
?- shift( [1,2,3,4,5], L1),
   shift( L1, L2).
```

produces:

```
L1 = [2,3,4,5,1]
L2 = [3,4,5,1,2]
```

3.7 Define the relation

translate(List1, List2)

to translate a list of numbers between 0 and 9 to a list of the corresponding words. For example:

translate([3,5,1,3], [three,five,one,three])

Use the following as an auxiliary relation:

means(0, zero). means(1, one). means(2, two). ...

3.8 Define the relation

subset(Set, Subset)

where Set and Subset are two lists representing two sets. We would like to be able to use this relation not only to check for the subset relation, but also to generate all possible subsets of a given set. For example:

```
?- subset( [a,b,c], S).
S = [a,b,c];
S = [a,b];
S = [a,c];
S = [a];
S = [b,c];
S = [b];
...
```

3.9 Define the relation

```
dividelist( List, List1, List2)
```

so that the elements of List are partitioned between List1 and List2, and List1 and List2 are of approximately the same length. For example, dividelist([a,b,c,d,e], [a,c,e], [b,d]).

3.10 Rewrite the monkey and banana program of Chapter 2 as the relation

```
canget( State, Actions)
```

to answer not just 'yes' or 'no', but to produce a sequence of monkey's actions represented as a list of moves. For example:

```
Actions = [ walk(door,window), push(window,middle), climb, grasp]
```

3.11 Define the relation

```
flatten( List, FlatList)
```

where List can be a list of lists, and FlatList is List 'flattened' so that the elements of List's sublists (or sub-sublists) are reorganized as one plain list. For example:

```
?- flatten( [a,b,[c,d],[ ],[[[e]]],f], L).
L = [a,b,c,d,e,f]
```

3.3 Operator notation

In mathematics we are used to writing expressions like

$$2*a+b*c$$

where $+$ and $*$ are operators, and 2, *a*, *b*, are arguments. In particular, $+$ and $*$ are said to be *infix* operators because they appear *between* the two arguments. Such expressions can be represented as trees, as in Figure 3.6, and can be written as Prolog terms with $+$ and $*$ as functors:

```
+( *(2,a), *(b,c) )
```


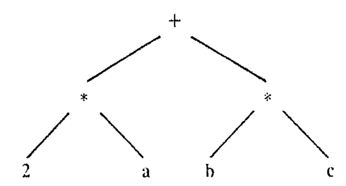

Figure 3.6 Tree representation of the expression 2*a + b*c.

Since we would normally prefer to have such expressions written in the usual, infix style with operators, Prolog caters for this notational convenience. Prolog will therefore accept our expression written simply as:

```
2*a + b*c
```

This will be, however, only the external representation of this object, which will be automatically converted into the usual form of Prolog terms. Such a term will be output for the user, again, in its external, infix form.

Thus operators in Prolog are merely a notational extension. If we write a + b, Prolog will handle it exactly as if it had been written +(a,b). In order that Prolog properly understands expressions such as a + b*c, Prolog has to know that * binds stronger than +. We say that + has higher precedence than *. So the precedence of operators decides what is the correct interpretation of expressions. For example, the expression a + b*c can be, in principle, understood either as

```
+( a, *(b,c) )
```

or as

```
*( +(a,b), c)
```

The general rule is that the operator with the highest precedence is the principal functor of the term. If expressions containing + and * are to be understood according to our normal conventions, then + has to have a higher precedence than *. Then the expression a + b*c means the same as a + (b*c). If another interpretation is intended, then it has to be explicitly indicated by parentheses – for example, (a + b)*c.

A programmer can define his or her own operators. So, for example, we can define the atoms **has** and **supports** as infix operators and then write in the program facts like:

```
peter has information.
floor supports table.
```

These facts are exactly equivalent to:

```
has( peter, information).
supports( floor, table).
```

A programmer can define new operators by inserting into the program special kinds of clauses, sometimes called *directives*, which act as operator definitions. An operator definition must appear in the program before any expression containing that operator. For our example, the operator has can be properly defined by the directive:

```
:- op( 600, xfx, has).
```

This tells Prolog that we want to use 'has' as an operator, whose precedence is 600 and its type is 'xfx', which is a kind of infix operator. The form of the specifier 'xfx' suggests that the operator, denoted by 'f', is between the two arguments denoted by 'x'.

Notice that operator definitions do not specify any operation or action. In principle, *no operation on data is associated with an operator* (except in very special cases). Operators are normally used, as functors, only to combine objects into structures and not to invoke actions on data, although the word 'operator' appears to suggest an action.

Operator names are atoms. An operator's precedence must be in some range which depends on the implementation. We will assume that the range is between 1 and 1200.

There are three groups of operator types which are indicated by type specifiers such as xfx. The three groups are:

(1) infix operators of three types:

    xfx xfy yfx

(2) prefix operators of two types:

    fx fy

(3) postfix operators of two types:

    xf yf

The specifiers are chosen so as to reflect the structure of the expression where 'f' represents the operator and 'x' and 'y' represent arguments. An 'f' appearing between the arguments indicates that the operator is infix. The prefix and postfix specifiers have only one argument, which follows or precedes the operator respectively.

There is a difference between 'x' and 'y'. To explain this we need to introduce the notion of the *precedence of argument*. If an argument is enclosed in parentheses or it is an unstructured object then its precedence is 0; if an argument is a structure then its precedence is equal to the precedence of its principal functor. 'x' represents an argument whose precedence must be strictly lower than that of the operator. 'y' represents an argument whose precedence is lower or equal to that of the operator.

These rules help to disambiguate expressions with several operators of the same precedence. For example, the expression

```
a - b - c
```


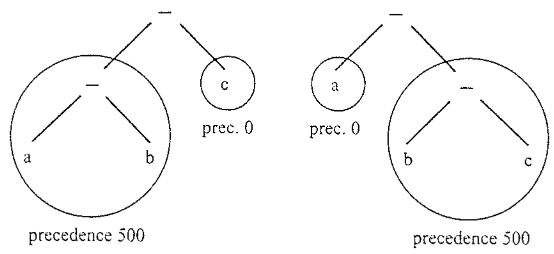

Figure 3.7 Two interpretations of the expression a – b – c assuming that '–' has precedence 500. If '–' is of type yfx, then interpretation 2 is invalid because the precedence of b – c is not less than the precedence of '–'.

is normally understood as (a – b) – c, and not as a – (b – c). To achieve the normal interpretation the operator '–' has to be defined as yfx. Figure 3.7 shows why the second interpretation is then ruled out.

As another example consider the prefix operator **not**. If **not** is defined as fy then the expression

```
not not p
```

is legal; but if **not** is defined as fx then this expression is illegal because the argument to the first **not** is **not p**, which has the same precedence as **not** itself. In this case the expression has to be written with parentheses:

```
not( not p)
```

For convenience, some operators are predefined in the Prolog system so that they can be readily used, and no definition is needed for them. What these operators are and what their precedences are depends on the implementation of Prolog. We will assume that this set of 'standard' operators is as if defined by the clauses in Figure 3.8. The operators in this figure are a subset of those defined in the Prolog standard, plus the operator **not**. As Figure 3.8 also shows, several operators can be declared by one clause if they all have the same precedence and if they are all of the same type. In this case the operators' names are written as a list.

The use of operators can greatly improve the readability of programs. As an example let us assume that we are writing a program for manipulating Boolean expressions. In such a program we may want to state, for example, one of de Morgan's equivalence theorems, which can in mathematics be written as:

~(A & B) <===> ~A v ~B

One way to state this in Prolog is by the clause:

```
equivalence( not( and( A, B)), or( not( A), not( B) ) ).
```

```
:- op( 1200, xfx, [ :-, -->] ).
:- op( 1200, fx [ :-, ?-] ).
:- op( 1100, xfy, ';' ).
:- op( 1050, xfy, -> ).
:- op( 1000, xfy, ',' ).
:- op( 900, fy, [ not, '\+'] ).
:- op( 700, xfx, [ =, \=, ==, \==, =.. ] ).
:- op( 700, xfx, [ is, =:=, =\=, < , =< , > , >=, @<, @=<, @>, @>=] ).
:- op( 500, yfx, [ + , - ] ).
:- op( 400, yfx, [ *, /, //, mod] ).
:- op( 200, xfx, **).
:- op( 200, xfy, ^).
:- op( 200, fy, - ).
```

Figure 3.8 A set of predefined operators.

However, it is in general a good programming practice to try to retain as much resemblance as possible between the original problem notation and the notation used in the program. In our example, this can be achieved almost completely by using operators. A suitable set of operators for our purpose can be defined as:

```
:- op( 800, xfx, <===>).
:- op( 700, xfy, v).
:- op( 600, xfy, &).
:- op( 500, fy, ~).
```

Now the de Morgan's theorem can be written as the fact:

```
~(A & B) <===> ~A v ~B.
```

According to our specification of operators above, this term is understood as shown in Figure 3.9.

To summarize:

- The readability of programs can be often improved by using the operator notation. Operators can be infix, prefix or postfix.
- In principle, no operation on data is associated with an operator except in special cases. Operator definitions do not define any action, they only introduce new notation. Operators, as functors, only hold together components of structures.
- A programmer can define his or her own operators. Each operator is defined by its name, precedence and type.
- The precedence is an integer within some range, usually between 1 and 1200. The operator with the highest precedence in the expression is the principal functor of the expression. Operators with lowest precedence bind strongest.

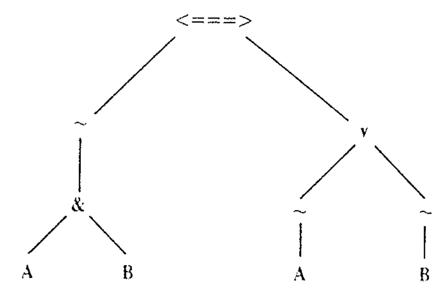

Figure 3.9 Interpretation of the term ~(A & B) <===> ~A v ~B.

- The type of an operator depends on two things: (1) the position of the operator with respect to the arguments, and (2) the precedence of the arguments compared to the precedence of the operator itself. In a specifier like xfy, x indicates an argument whose precedence is strictly lower than that of the operator; y indicates an argument whose precedence is less than or equal to that of the operator.

Exercises

3.12 Assuming the operator definitions

```
:- op( 300, xfx, plays).
:- op( 200, xfy, and).
```

then the following two terms are syntactically legal objects:

```
Term1 = jimmy plays football and squash
Term2 = susan plays tennis and basketball and volleyball
```

How are these terms understood by Prolog? What are their principal functors and what is their structure?

3.13 Suggest an appropriate definition of operators ('was', 'of', 'the') to be able to write clauses like

```
diana was the secretary of the department.
```

and then ask Prolog:

```
?- Who was the secretary of the department.
Who = diana

?- diana was What.
What = the secretary of the department
```

3.14 Consider the program:

```
t( 0+1, 1+0).
t( X+0+1, X+1+0).
t( X+1+1, Z) :-
  t( X+1, X1),
  t( X1+1, Z).
```

How will this program answer the following questions if '+' is an infix operator of type yfx (as usual):

(a) ?- t(0+1, A).

(b) ?- t(0+1+1, B).

(c) ?- t(1+0+1+1+1, C).

(d) ?- t(D, 1+1+1+0).

3.15 In the previous section, relations involving lists were written as:

```
member( Element, List),
conc( List1, List2, List3),
del( Element, List, NewList), ...
```

Suppose that we would prefer to write these relations as:

```
Element in List,
concatenating List1 and List2 gives List3,
deleting Element from List gives NewList, ...
```

Define 'in', 'concatenating', 'and', etc. as operators to make this possible. Also, redefine the corresponding procedures.

3.4 Arithmetic

Some of the predefined operators can be used for basic arithmetic operations. These are:

+	addition
−	subtraction
*	multiplication
/	division
**	power
//	integer division
mod	modulo, the remainder of integer division

Notice that this is an exceptional case in which an operator may in fact invoke an operation. But even in such cases an additional indication to perform arithmetic will be necessary. The following question is a naive attempt to request arithmetic computation:

```
?- X = 1 + 2.
```

Prolog will 'quietly' answer

```
X = 1 + 2
```

and not X = 3 as we might possibly expect. The reason is simple: the expression 1 + 2 merely denotes a Prolog term where + is the functor and 1 and 2 are its arguments. There is nothing in the above goal to force Prolog to actually activate the addition operation. A special predefined operator, is, is provided to circumvent this problem. The is operator will force evaluation. So the right way to invoke arithmetic is:

```
?- X is 1 + 2.
```

Now the answer will be:

```
X = 3
```

The addition here was carried out by a special procedure that is associated with the operator is. We call such procedures *built-in procedures*.

Different implementations of Prolog may use somewhat different notations for arithmetics. For example, the '/' operator may denote integer division or real division. In this book, '/' denotes real division, the operator // denotes integer division, and mod denotes the remainder. Accordingly, the question:

```
?- X is 5/2,
   Y is 5//2,
   Z is 5 mod 2.
```

is answered by:

```
X = 2.5
Y = 2
Z = 1
```

The left argument of the is operator is a simple object. The right argument is an arithmetic expression composed of arithmetic operators, numbers and variables. Since the is operator will force the evaluation, all the variables in the expression must already be instantiated to numbers at the time of execution of this goal. The precedence of the predefined arithmetic operators (see Figure 3.8) is such that the associativity of arguments with operators is the same as normally in mathematics. Parentheses can be used to indicate different associations. Note that +, −, *, / and div are defined as yfx, which means that evaluation is carried out from left to right. For example,

```
X is 5 - 2 - 1
```

is interpreted as:

```
X is (5 - 2) - 1
```

Prolog implementations usually also provide standard functions such as sin(X), cos(X), atan(X), log(X), exp(X), etc. These functions can appear to the right of operator is.

Arithmetic is also involved when *comparing* numerical values. We can, for example, test whether the product of 277 and 37 is greater than 10000 by the goal:

```
?- 277 * 37 > 10000.

yes
```

Note that, similarly to is, the '>' operator also forces the evaluation.

Suppose that we have in the program a relation born that relates the names of people with their birth years. Then we can retrieve the names of people born between 1980 and 1990 inclusive with the following question:

```
?- born( Name, Year),
   Year >= 1980,
   Year =< 1990.
```

The comparison operators are as follows:

X > Y	X is greater than Y
X < Y	X is less than Y
X >= Y	X is greater than or equal to Y
X =< Y	X is less than or equal to Y
X =:= Y	the values of X and Y are equal
X =\= Y	the values of X and Y are not equal

Notice the difference between the matching operator '=' and '=:='; for example, in the goals X = Y and X =:= Y. The first goal will cause the matching of the objects X and Y, and will, if X and Y match, possibly instantiate some variables in X and Y. There will be no evaluation. On the other hand, X =:= Y causes the arithmetic evaluation and cannot cause any instantiation of variables. These differences are illustrated by the following examples:

```
?- 1 + 2 =:= 2 + 1.

yes

?- 1 + 2 = 2 + 1.

no

?- 1 + A = B + 2.

A = 2
B = 1
```


Let us further illustrate the use of arithmetic operations by two simple examples. The first is computing the greatest common divisor; the second, counting the items in a list.

Given two positive integers, X and Y, their greatest common divisor, D, can be found according to three cases:

(1) If X and Y are equal then D is equal to X.

(2) If X < Y then D is equal to the greatest common divisor of X and the difference Y − X.

(3) If Y < X then do the same as in case (2) with X and Y interchanged.

It can be easily shown by an example that these three rules actually work. Choosing, for example, X = 20 and Y = 25, the above rules would give D = 5 after a sequence of subtractions.

These rules can be formulated into a Prolog program by defining a three-argument relation, say:

```
gcd( X, Y, D)
```

The three rules are then expressed as three clauses, as follows:

```
gcd( X, X, X).

gcd( X, Y, D) :-
  X < Y,
  Y1 is Y - X,
  gcd( X, Y1, D).

gcd( X, Y, D) :-
  Y < X,
  gcd( Y, X, D).
```

Of course, the last goal in the third clause could be equivalently replaced by the two goals:

```
X1 is X - Y,
gcd( X1, Y, D)
```

Our next example involves counting, which usually requires some arithmetic. An example of such a task is to establish the length of a list; that is, we have to count the items in the list. Let us define the procedure:

```
length( List, N)
```

which will count the elements in a list List and instantiate N to their number. As was the case with our previous relations involving lists, it is useful to consider two cases:

(1) If the list is empty then its length is 0.

(2) If the list is not empty then List = [Head | Tail]; then its length is equal to 1 plus the length of the tail Tail.

These two cases correspond to the following program:

```
length( [], 0).

length( [_ | Tail], N) :-
  length( Tail, N1),
  N is 1 + N1.
```

An application of **length** can be:

```
?- length( [a,b,[c,d],e], N).

N = 4
```

Note that in the second clause of **length**, the two goals of the body cannot be swapped. The reason for this is that N1 has to be instantiated before the goal:

```
N is 1 + N1
```

can be processed. With the built-in procedure **is**, a relation has been introduced that is sensitive to the order of processing and therefore the procedural considerations have become vital.

It is interesting to see what happens if we try to program the **length** relation without the use of **is**. Such an attempt can be:

```
length1( [], 0).

length1( [_ | Tail], N) :-
  length1( Tail, N1),
  N = 1 + N1.
```

Now the goal

```
?- length1( [a,b,[c,d],e], N).
```

will produce the answer:

```
N = 1+(1+(1+(1+0))).
```

The addition was never explicitly forced and was therefore not carried out at all. But in **length1** we can, unlike in **length**, swap the goals in the second clause:

```
length1( [_ | Tail], N) :-
  N = 1 + N1,
  length1( Tail, N1).
```

This version of **length1** will produce the same result as the original version. It can also be written shorter, as follows,

```
length1( [_ | Tail], 1 + N) :-
  length1( Tail, N).
```

still producing the same result. We can, however, use **length1** to find the number of elements in a list as follows:

```
?- length1( [a,b,c], N), Length is N.

N = 1+(1+(1+0))
Length = 3
```


Finally we note that the predicate **length** is often provided as a built-in predicate.

To summarize:

- Built-in procedures can be used for doing arithmetic.
- Arithmetic operations have to be explicitly requested by the built-in procedure **is**. There are built-in procedures associated with the predefined operators +, −, *, /, **div** and **mod**.
- At the time that evaluation is carried out, all arguments must be already instantiated to numbers.
- The values of arithmetic expressions can be compared by operators such as <, =<, etc. These operators force the evaluation of their arguments.

Exercises

3.16 Define the relation

```
max( X, Y, Max)
```

so that **Max** is the greater of two numbers X and Y.

3.17 Define the predicate

```
maxlist( List, Max)
```

so that **Max** is the greatest number in the list of numbers **List**.

3.18 Define the predicate

```
sumlist( List, Sum)
```

so that **Sum** is the sum of a given list of numbers **List**.

3.19 Define the predicate

```
ordered( List)
```

which is true if **List** is an ordered list of numbers. For example,

```
ordered( [1,5,6,6,9,12] ).
```

3.20 Define the predicate

```
subsum( Set, Sum, SubSet)
```

so that **Set** is a list of numbers, **SubSet** is a subset of these numbers, and the sum of the numbers in **SubSet** is **Sum**. For example:

```
?- subsum( [1,2,5,3,2], 5, Sub).
Sub = [1,2,2];
Sub = [2,3];
Sub = [5];
...
```

3.21 Define the procedure

```
between( N1, N2, X)
```

which, for two given integers N1 and N2, generates through backtracking all the integers X that satisfy the constraint N1 ≦ X ≦ N2.

3.22 Define the operators 'if', 'then', 'else' and ':=' so that the following becomes a legal term:

```
if X > Y then Z := X else Z := Y
```

Choose the precedences so that 'if' will be the principal functor. Then define the relation 'if' as a small interpreter for a kind of 'if-then-else' statement of the form

```
if Val1 > Val2 then Var := Val3 else Var := Val4
```

where Val1, Val2, Val3 and Val4 are numbers (or variables instantiated to numbers) and Var is a variable. The meaning of the 'if' relation should be: if the value of Val1 is greater than the value of Val2 then Var is instantiated to Val3, otherwise to Val4. Here is an example of the use of this interpreter:

```
?- X = 2, Y = 3,
   Val2 is 2*X,
   Val4 is 4*X,
   if Y > Val2 then Z := Y else Z := Val4,
   if Z > 5 then W := 1 else W := 0.

X = 2
Y = 3
Z = 8
W = 1
Val2 = 4
Val4 = 8
```

Summary

- The list is a frequently used structure. It is either empty or consists of a *head* and a *tail* which is a list as well. Prolog provides a special notation for lists.
- Common operations on lists, programmed in this chapter, are: list membership, concatenation, adding an item, deleting an item, sublist.

- The *operator notation* allows the programmer to tailor the syntax of programs toward particular needs. Using operators the readability of programs can be greatly improved.
- New operators are defined by the directive op, stating the name of an operator, its type and precedence.
- In principle, there is no operation associated with an operator; operators are merely a syntactic device providing an alternative syntax for terms.
- Arithmetic is done by built-in procedures. Evaluation of an arithmetic expression is forced by the procedure is and by the comparison predicates <, =<, etc.
- Concepts introduced in this chapter are:

  list, head of list, tail of list
  list notation
  operators, operator notation
  infix, prefix and suffix operators
  precedence of an operator
  arithmetic built-in procedures

chapter 4

Using Structures: Example Programs

Data structures, with matching, backtracking and arithmetic, are a powerful programming tool. In this chapter we will develop the skill of using this tool through programming examples: retrieving structured information from a database, simulating a non-deterministic automaton, travel planning, and eight queens on the chessboard. We will also see how the principle of data abstraction can be carried out in Prolog. The programming examples in this chapter can be read selectively.

4.1 Retrieving structured information from a database

This exercise develops techniques of representing and manipulating structured data objects. It also illustrates Prolog as a natural database query language.

A database can be naturally represented in Prolog as a set of facts. For example, a database about families can be represented so that each family is described by one clause. Figure 4.1 shows how the information about each family can be structured. Each family has three components: husband, wife and children. As the number of children varies from family to family the children are represented by a list that is capable of accommodating any number of items. Each person is, in turn, represented by a structure of four components: name, surname, date of birth, job. The job information is 'unemployed', or it specifies the working organization and salary. The family of Figure 4.1 can be stored in the database by the clause:

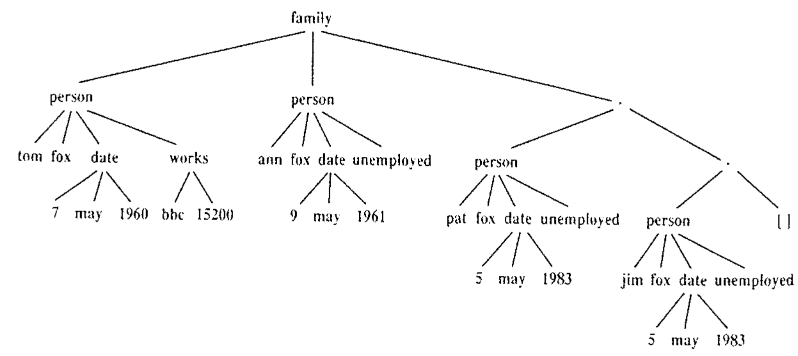

Figure 4.1 Structuring information about the family.

```
family(
  person( tom, fox, date(7,may,1960), works(bbc,15200) ),
  person( ann, fox, date(9,may,1961), unemployed),
  [ person( pat, fox, date(5,may,1983), unemployed),
    person( jim, fox, date(5,may,1983), unemployed) ] ).
```

Our database would then be comprised of a sequence of facts like this describing all families that are of interest to our program.

Prolog is, in fact, a very suitable language for retrieving the desired information from such a database. One nice thing about Prolog is that we can refer to objects without actually specifying all the components of these objects. We can merely indicate the *structure* of objects that we are interested in, and leave the particular components in the structures unspecified or only partially specified. Figure 4.2 shows some examples. So we can refer to all Armstrong families by:

```
family( person( _, armstrong, _, _), _, _)
```

The underscore characters denote different anonymous variables; we do not care about their values. Further, we can refer to all families with three children by the term:

```
family( _, _, [_, _, _])
```

To find all married women that have at least three children we can pose the question:

```
?- family( _, person( Name, Surname, _, _), [_, _, _ | _]).
```

The point of these examples is that we can specify objects of interest not by their content, but by their structure. We only indicate their structure and leave their arguments as unspecified slots.

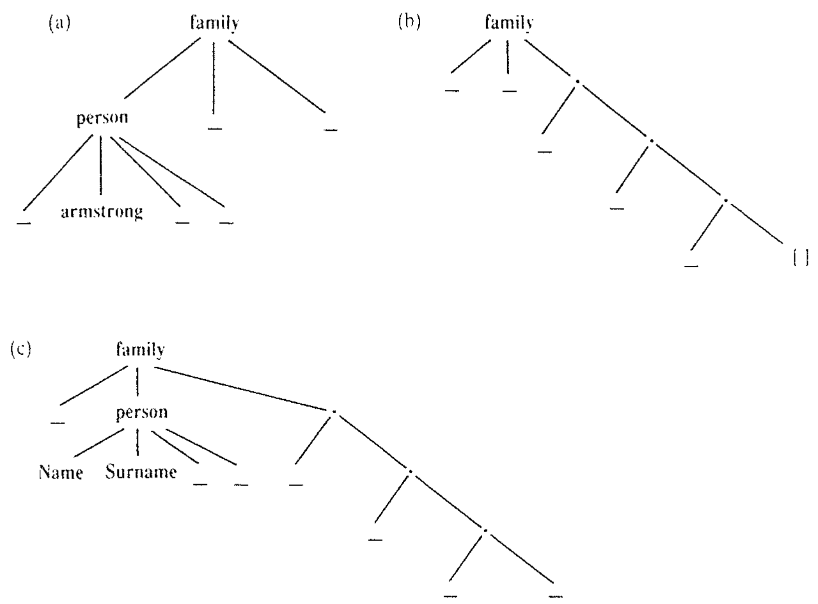

Figure 4.2 Specifying objects by their structural properties: (a) any Armstrong family; (b) any family with exactly three children; (c) any family with at least three children. Structure (c) makes provision for retrieving the wife's name through the instantiation of the variables Name and Surname.

We can provide a set of procedures that can serve as a utility to make the interaction with the database more comfortable. Such utility procedures could be part of the user interface. Some useful utility procedures for our database are:

```
husband( X) :-                     % X is a husband
  family( X, _, _).

wife( X) :-                        % X is a wife
  family( _, X, _).

child( X) :-                       % X is a child
  family( _, _, Children),
  member( X, Children).            % X in list Children

exists( Person) :-                 % Any person in the database
  husband( Person)
  ;
  wife( Person)
  ;
  child( Person).
```



```
dateofbirth( person( _, _, Date, _), Date).

salary( person( _, _, _, works( _, S) ), S).      % Salary of working person

salary( person( _, _, _, unemployed), 0).         % Salary of unemployed
```

We can use these utilities, for example, in the following queries to the database:

- Find the names of all the people in the database:

```
?- exists( person( Name, Surname, _, _) ).
```

- Find all children born in 2000:

```
?- child( X),
   dateofbirth( X, date( _, _, 2000) ).
```

- Find all employed wives:

```
?- wife( person( Name, Surname, _, works( _, _) ) ).
```

- Find the names of unemployed people who were born before 1973:

```
?- exists( person( Name, Surname, date( _, _, Year), unemployed) ),
   Year < 1973.
```

- Find people born before 1960 whose salary is less than 8000:

```
?- exists( Person),
   dateofbirth( Person, date( _, _, Year) ),
   Year < 1960,
   salary( Person, Salary),
   Salary < 8000.
```

- Find the names of families with at least three children:

```
?- family( person( _, Name, _, _), _, [_, _, _ | _] ).
```

To calculate the total income of a family it is useful to define the sum of salaries of a list of people as a two-argument relation:

```
total( List_of_people, Sum_of_their_salaries)
```

This relation can be programmed as:

```
total( [], 0).                          % Empty list of people

total( [Person | List], Sum) :-
  salary( Person, S),                   % S: salary of first person
  total( List, Rest),                   % Rest: sum of salaries of others
  Sum is S + Rest.
```

The total income of families can then be found by the question:

```
?- family( Husband, Wife, Children),
   total( [Husband, Wife | Children], Income).
```

Let the length relation count the number of elements of a list, as defined in Section 3.4. Then we can specify all families that have an income per family member of less than 2000 by:

```
?- family( Husband, Wife, Children),
   total( [Husband, Wife | Children], Income),
   length( [Husband, Wife | Children], N),        % N: size of family
   Income/N < 2000.
```

Exercises

4.1 Write queries to find the following from the family database:

(a) names of families without children;

(b) all employed children;

(c) names of families with employed wives and unemployed husbands;

(d) all the children whose parents differ in age by at least 15 years.

4.2 Define the relation

```
twins( Child1, Child2)
```

to find twins in the family database.

4.2 Doing data abstraction

Data abstraction can be viewed as a process of organizing various pieces of information into natural units (possibly hierarchically), thus structuring the information into some conceptually meaningful form. Each such unit of information should be easily accessible in the program. Ideally, all the details of implementing such a structure should be invisible to the user of the structure – the programmer can then just concentrate on objects and relations between them. The point of the process is to make the use of information possible without the programmer having to think about the details of how the information is actually represented.

Let us discuss one way of carrying out this principle in Prolog. Consider our family example of the previous section again. Each family is a collection of pieces of information. These pieces are all clustered into natural units such as a person or a family, so they can be treated as single objects. Assume again that the family information is structured as in Figure 4.1. In the previous section, each family was represented by a Prolog clause. Here, a family will be represented as a structured object, for example:

```
FoxFamily = family( person( tom, fox, _, _), _, _)
```


Let us now define some relations through which the user can access particular components of a family without knowing the details of Figure 4.1. Such relations can be called *selectors* as they select particular components. The name of such a selector relation will be the name of the component to be selected. The relation will have two arguments: first, the object that contains the component, and second, the component itself:

```
selector_relation( Object, Component_selected)
```

Here are some selectors for the family structure:

```
husband( family( Husband, _, _), Husband).

wife( family( _, Wife, _), Wife).

children( family( _, _, ChildList), ChildList).
```

We can also define selectors for particular children:

```
firstchild( Family, First) :-
  children( Family, [First | _] ).

secondchild( Family, Second) :-
  children( Family, [_, Second | _] ).

...
```

We can generalize this to selecting the Nth child:

```
nthchild( N, Family, Child) :-
  children( Family, ChildList),
  nth_member( N, ChildList, Child).        % Nth element of a list
```

Another interesting object is a person. Some related selectors according to Figure 4.1 are:

```
firstname( person( Name, _, _, _), Name).

surname( person( _, Surname, _, _), Surname).

born( person( _, _, Date, _), Date).
```

How can we benefit from selector relations? Having defined them, we can now forget about the particular way that structured information is represented. To create and manipulate this information, we just have to know the names of the selector relations and use these in the rest of the program. In the case of complicated representations, this is easier than always referring to the representation explicitly. In our family example in particular, the user does not have to know that the children are represented as a list. For example, assume that we want to say that Tom Fox and Jim Fox belong to the same family and that Jim is the second child of Tom. Using the selector relations above, we can define two persons, call them Person1 and Person2, and the family. The following list of goals does this:

```
firstname( Person1, tom), surname( Person1, fox),     % Person1 is Tom Fox
firstname( Person2, jim), surname( Person2, fox),     % Person2 is Jim Fox
husband( Family, Person1),
secondchild( Family, Person2)
```

As a result, the variables Person1, Person2 and Family are instantiated as:

```
Person1 = person( tom, fox, _, _)
Person2 = person( jim, fox, _, _)
Family = family( person( tom, fox, _, _), _, [ _, person( jim, fox) | _])
```

The use of selector relations also makes programs easier to modify. Imagine that we would like to improve the efficiency of a program by changing the representation of data. All we have to do is to change the definitions of the selector relations, and the rest of the program will work unchanged with the new representation.

Exercise

4.3 Complete the definition of nthchild by defining the relation

```
nth_member( N, List, X)
```

which is true if X is the Nth member of List.

4.3 Simulating a non-deterministic automaton

This exercise shows how an abstract mathematical construct can be translated into Prolog. In addition, our resulting program will turn out to be much more flexible than initially intended.

A *non-deterministic finite automaton* is an abstract machine that reads as input a string of symbols and decides whether to *accept* or to *reject* the input string. An automaton has a number of *states* and it is always in one of the states. It can change its state by moving from the current state to another state. The internal structure of the automaton can be represented by a transition graph such as that in Figure 4.3. In this example, s_1, s_2, s_3 and s_4 are the *states* of the automaton. Starting from the initial state (s_1 in our example), the automaton moves from state to state while reading the input string. Transitions depend on the current input symbol, as indicated by the arc labels in the transition graph.

A transition occurs each time an input symbol is read. Note that transitions can be non-deterministic. In Figure 4.3, if the automaton is in state s_1 and the current input symbol is *a* then it can transit into s_1 or s_2. Some arcs are labelled *null* denoting the 'null symbol'. These arcs correspond to 'silent moves' of the automaton. Such a

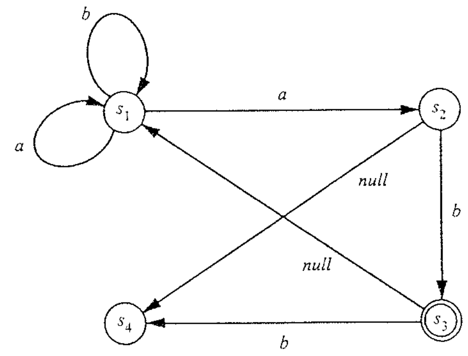

Figure 4.3 An example of a non-deterministic finite automaton.

move is said to be *silent* because it occurs without any reading of input, and the observer, viewing the automaton as a black box, will not be able to notice that any transition has occurred.

The state s_3 is double circled, which indicates that it is a *final state*. The automaton is said to *accept* the input string if there is a transition path in the graph such that

(1) it starts with the initial state,

(2) it ends with a final state, and

(3) the arc labels along the path correspond to the complete input string.

It is entirely up to the automaton to decide which of the possible moves to execute at any time. In particular, the automaton may choose to make or not to make a silent move, if it is available in the current state. But abstract non-deterministic machines of this kind have a magic property: if there is a choice then they always choose a 'right' move; that is, a move that leads to the acceptance of the input string, if such a move exists. The automaton in Figure 4.3 will, for example, accept the strings *ab* and *aabaab*, but it will reject the strings *abb* and *abba*. It is easy to see that this automaton accepts any string that terminates with *ab*, and rejects all others.

In Prolog, an automaton can be specified by three relations:

(1) a unary relation **final** which defines the final states of the automaton;

(2) a three-argument relation **trans** which defines the state transitions so that

```
trans( S1, X, S2)
```

means that a transition from a state S1 to S2 is possible when the current input symbol X is read;

(3) a binary relation

```
silent( S1, S2)
```

meaning that a silent move is possible from S1 to S2.

For the automaton in Figure 4.3 these three relations are:

```
final( s3).

trans( s1, a, s1).
trans( s1, a, s2).
trans( s1, b, s1).
trans( s2, b, s3).
trans( s3, b, s4).

silent( s2, s4).
silent( s3, s1).
```

We will represent input strings as Prolog lists. So the string *aab* will be represented by [a,a,b]. Given the description of the automaton, the simulator will process a given input string and decide whether the string is accepted or rejected. By definition, the non-deterministic automaton accepts a given string if (starting from an initial state), after having read the whole input string, the automaton can (possibly) be in its final state. The simulator is programmed as a binary relation, accepts, which defines the acceptance of a string from a given state. So

```
accepts( State, String)
```

is true if the automaton, starting from the state State as initial state, accepts the string String. The accepts relation can be defined by three clauses. They correspond to the following three cases:

(1) The empty string, [], is accepted from a state State if State is a final state.

(2) A non-empty string is accepted from State if reading the first symbol in the string can bring the automaton into some state State1, and the rest of the string is accepted from State1. Figure 4.4(a) illustrates.

(3) A string is accepted from State if the automaton can make a silent move from State to State1 and then accept the (whole) input string from State1. Figure 4.4(b) illustrates.

These rules can be translated into Prolog as:

```
accepts( State, [] ) :-                 % Accept empty string
  final( State).

accepts( State, [X | Rest] ) :-         % Accept by reading first symbol
  trans( State, X, State1),
  accepts( State1, Rest).

accepts( State, String) :-              % Accept by making silent move
  silent( State, State1),
  accepts( State1, String).
```


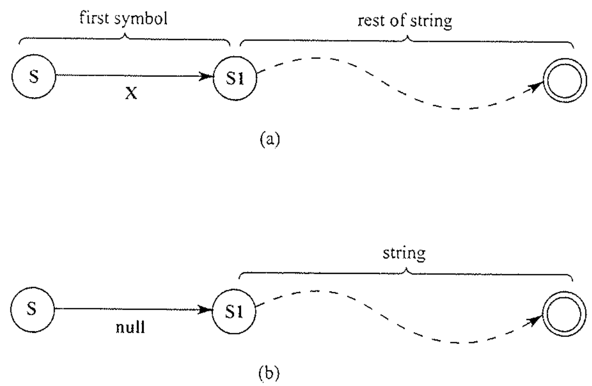

Figure 4.4 Accepting a string: (a) by reading its first symbol X; (b) by making a silent move.

The program can be asked, for example, about the acceptance of the string *aaab* by:

```
?- accepts( s1, [a,a,a,b] ).

yes
```

As we have already seen, Prolog programs are often able to solve more general problems than problems for which they were originally developed. In our case, we can also ask the simulator which state our automaton can be in initially so that it will accept the string *ab*:

```
?- accepts( S, [a,b] ).

S = s1;

S = s3
```

Amusingly, we can also ask: What are all the strings of length 3 that are accepted from state s_1?

```
?- accepts( s1, [X1,X2,X3] ).

X1 = a
X2 = a
X3 = b;

X1 = b
X2 = a
X3 = b;

no
```

If we prefer the acceptable input strings to be typed out as lists then we can formulate the question as:

```
?- String = [_, _, _], accepts( s1, String).
String = [a,a,b];
String = [b,a,b];
no
```

We can make further experiments asking even more general questions, such as: From what states will the automaton accept input strings of length 7?

Further experimentation could involve modifications in the structure of the automaton by changing the relations **final**, **trans** and **silent**. The automaton in Figure 4.3 does not contain any cyclic 'silent path' (a path that consists only of silent moves). If in Figure 4.3 a new transition

```
silent( s1, s3)
```

is added then a 'silent cycle' is created. But our simulator may now get into trouble. For example, the question

```
?- accepts( s1, [a] ).
```

would induce the simulator to cycle in state s_1 indefinitely, all the time hoping to find some way to the final state.

Exercises

4.4 Why could cycling not occur in the simulation of the original automaton in Figure 4.3, when there was no 'silent cycle' in the transition graph?

4.5 Cycling in the execution of accepts can be prevented, for example, by counting the number of moves made so far. The simulator would then be requested to search only for paths of some limited length. Modify the accepts relation this way. Hint: Add a third argument: the maximum number of moves allowed:

```
accepts( State, String, MaxMoves)
```

4.4 Travel agent

In this section we will construct a program that gives advice on planning air travel. The program will be a rather simple advisor, yet it will be able to answer some useful questions, such as:

- What days of the week is there a direct evening flight from Ljubljana to London?
- How can I get from Ljubljana to Edinburgh on Thursday?

- I have to visit Milan, Ljubljana and Zurich, starting from London on Tuesday and returning to London on Friday. In what sequence should I visit these cities so that I have no more than one flight each day of the tour?

The program will be centred around a database holding the flight information. This will be represented as a three-argument relation:

```
timetable( Place1, Place2, ListOfFlights)
```

where ListOfFlights is a list of structured items of the form:

```
DepartureTime / ArrivalTime / FlightNumber / ListOfDays
```

Here the operator '/' only holds together the components of the structure, and of course does not mean arithmetic division. ListOfDays is either a list of weekdays or the atom alldays. One clause of the timetable relation can be, for example:

```
timetable( london, edinburgh,
           [ 9:40 / 10:50 / ba4733 / alldays,
           19:40 / 20:50 / ba4833 / [mo,tu,we,th,fr,su] ] ).
```

The times are represented as structured objects with two components, hours and minutes, combined by the operator ':'.

The main problem is to find exact routes between two given cities on a given day of the week. This will be programmed as a four-argument relation:

```
route( Place1, Place2, Day, Route)
```

Here Route is a sequence of flights that satisfies the following criteria:

(1) the start point of the route is Place1;

(2) the end point is Place2;

(3) all the flights are on the same day of the week, Day;

(4) all the flights in Route are in the timetable relation;

(5) there is enough time for transfer between flights.

The route is represented as a list of structured objects of the form:

```
From / To / FlightNumber / Departure_time
```

We will also use the following auxiliary predicates:

(1) flight(Place1, Place2, Day, FlightNum, DepTime, ArrTime)

This says that there is a flight, FlightNum, between Place1 and Place2 on the day of the week Day with the specified departure and arrival times.

(2) deptime(Route, Time)

Departure time of Route is Time.

(3) transfer(Time1, Time2)

There is at least 40 minutes between Time1 and Time2, which should be sufficient for transfer between two flights.

The problem of finding a route is reminiscent of the simulation of the non-deterministic automaton of the previous section. The similarities of both problems are as follows:

- The states of the automaton correspond to the cities.
- A transition between two states corresponds to a flight between two cities.
- The transition relation of the automaton corresponds to the timetable relation.
- The automaton simulator finds a path in the transition graph between the initial state and a final state; the travel planner finds a route between the start city and the end city of the tour.

Not surprisingly, therefore, the route relation can be defined similarly to the accepts relation, with the exception that here we have no 'silent moves'. We have two cases:

(1) Direct flight connection: if there is a direct flight between places Place1 and Place2 then the route consists of this flight only:

```
route( Place1, Place2, Day, [ Place1 / Place2 / Fnum / Dep ] ) :-
  flight( Place1, Place2, Day, Fnum, Dep, Arr).
```

(2) Indirect flight connection: the route between places P1 and P2 consists of the first flight, from P1 to some intermediate place P3, followed by a route between P3 to P2. In addition, there must be enough time between the arrival of the first flight and the departure of the second flight for transfer.

```
route( P1, P2, Day, [P1 / P3 / Fnum1 / Dep1 | RestRoute] ) :-
  route( P3, P2, Day, RestRoute),
  flight( P1, P3, Day, Fnum1, Dep1, Arr1),
  deptime( RestRoute, Dep2),
  transfer( Arr1, Dep2).
```

The auxiliary relations flight, transfer and deptime are easily programmed and are included in the complete travel planning program in Figure 4.5. Also included is an example timetable database.

Our route planner is extremely simple and may examine paths that obviously lead nowhere. Yet it will suffice if the flight database is not large. A really large database would require more intelligent planning to cope with the large number of potential candidate paths.

Some example questions to the program are as follows:


```
% A FLIGHT ROUTE PLANNER

:- op( 50, xfy, :).

% route( Place1, Place2, Day, Route):
%   Route is a sequence of flights on Day, starting at Place1, ending at Place2

route( P1, P2, Day, [ P1 / P2 / Fnum / Deptime ] ) :-          % Direct flight
  flight( P1, P2, Day, Fnum, Deptime, _).

route( P1, P2, Day, [ (P1/P3/Fnum1/Dep1) | RestRoute] ) :-     % Indirect connection
  route( P3, P2, Day, RestRoute),
  flight( P1, P3, Day, Fnum1, Dep1, Arr1),
  deptime( RestRoute, Dep2),                                   % Departure time of Route
  transfer( Arr1, Dep2).                                       % Enough time for transfer

flight( Place1, Place2, Day, Fnum, Deptime, Arrtime) :-
  timetable( Place1, Place2, Flightlist),
  member( Deptime / Arrtime / Fnum / Daylist , Flightlist),
  flyday( Day, Daylist).

flyday( Day, Daylist) :-
  member( Day, Daylist).

flyday( Day, alldays) :-
  member( Day, [mo,tu,we,th,fr,sa,su] ).

deptime( [ P1 / P2 / Fnum / Dep | _], Dep).

transfer( Hours1:Mins1, Hours2:Mins2) :-
  60 * (Hours2 - Hours1) + Mins2 - Mins1 >= 40.

member( X, [X | L] ).

member( X, [Y | L] ) :-
  member( X, L).

% A FLIGHT DATABASE

timetable( edinburgh, london,
           [  9:40 / 10:50 / ba4733 / alldays,
             13:40 / 14:50 / ba4773 / alldays,
             19:40 / 20:50 / ba4833 / [mo,tu,we,th,fr,su] ] ).

timetable( london, edinburgh,
           [  9:40 / 10:50 / ba4732 / alldays,
             11:40 / 12:50 / ba4752 / alldays,
             18:40 / 19:50 / ba4822 / [mo,tu,we,th,fr] ] ).

timetable( london, ljubljana,
           [ 13:20 / 16:20 / jp212 / [mo,tu,we,fr,su],
             16:30 / 19:30 / ba473 / [mo,we,th,sa] ] ).

timetable( london, zurich,
           [  9:10 / 11:45 / ba614 / alldays,
             14:45 / 17:20 / sr805 / alldays ] ).
```

Figure 4.5 A flight route planner and an imaginary flight timetable.

Figure 4.5 *contd*

```
timetable( london, milan,
        [  8:30 / 11:20 / ba510 / alldays,
          11:00 / 13:50 / az459 / alldays ] ).
timetable( ljubljana, zurich,
        [ 11:30 / 12:40 / jp322 / [tu,th] ] ).
timetable( ljubljana, london,
        [ 11:10 / 12:20 / jp211 / [mo,tu,we,fr,su],
          20:30 / 21:30 / ba472 / [mo,we,th,sa] ] ).
timetable( milan, london,
        [  9:10 / 10:00 / az458 / alldays,
          12:20 / 13:10 / ba511 / alldays ] ).
timetable( milan, zurich,
        [  9:25 / 10:15 / sr621 / alldays,
          12:45 / 13:35 / sr623 / alldays ] ).
timetable( zurich, ljubljana,
        [ 13:30 / 14:40 / jp323 / [tu,th] ] ).
timetable( zurich, london,
        [  9:00 /  9:40 / ba613 / [mo,tu,we,th,fr,sa],
          16:10 / 16:55 / sr806 / [mo,tu,we,th,fr,su] ] ).
timetable( zurich, milan,
        [  7:55 /  8:45 / sr620 / alldays ] ).
```

- What days of the week is there a direct evening flight from Ljubljana to London?

```
?- flight( ljubljana, london, Day, _, DeptHour: _, _), DeptHour >= 18.

Day = mo;
Day = we;
...
```

- How can I get from Ljubljana to Edinburgh on Thursday?

```
?- route( ljubljana, edinburgh, th, R).

R = [ ljubljana / zurich / jp322 / 11:30, zurich / london / sr806 / 16:10,
      london / edinburgh / ba4822 / 18:40]
```

- How can I visit Milan, Ljubljana and Zurich, starting from London on Tuesday and returning to London on Friday, with no more than one flight each day of the tour? This question is somewhat trickier. It can be formulated by using the permutation relation, programmed in Chapter 3. We are asking for a permutation of the cities Milan, Ljubljana and Zurich such that the corresponding flights are possible on successive days:


```
?- permutation( [milan, ljubljana, zurich], [City1, City2, City3] ),
flight( london, City1, tu, FN1, _, _),
flight( City1, City2, we, FN2, _, _),
flight( City2, City3, th, FN3, _, _),
flight( City3, london, fr, FN4, _, _).

City1 = milan
City2 = zurich
City3 = ljubljana
FN1 = ba510
FN2 = sr621
FN3 = jp323
FN4 = jp211
```

Finally let us note that this program is susceptible to indefinite loops, which happens for example if we ask it to find a route not in the timetable:

```
?- route ( moscow, edinburgh, mo, R).
```

It is better therefore to keep questions safe by limiting the length of the route. We can use the usual trick with conc:

```
?- conc( R, _, [_,_,_,_]), route( moscow, edinburgh, mo, R).

no
```

The conc goal limits the list R to length 4 and also forces the search to consider shortest routes first.

4.5 The eight queens problem

The problem here is to place eight queens on the empty chessboard in such a way that no queen attacks any other queen. The solution will be programmed as a unary predicate

```
solution( Pos)
```

which is true if and only if Pos represents a position with eight queens that do not attack each other. It will be interesting to compare various ideas for programming this problem. Therefore we will present three programs based on somewhat different representations of the problem.

4.5.1 Program 1

First we have to choose a representation of the board position. One natural choice is to represent the position by a list of eight items, each of them corresponding to one queen. Each item in the list will specify a square of the board on which the

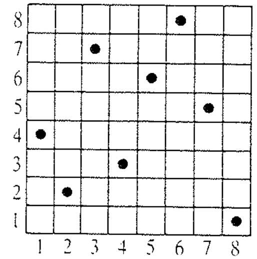

Figure 4.6 A solution to the eight queens problem. This position can be specified by the list [1/4, 2/2, 3/7, 4/3, 5/6, 6/8, 7/5, 8/1].

corresponding queen is sitting. Further, each square can be specified by a pair of coordinates (X and Y) on the board, where each coordinate is an integer between 1 and 8. In the program we can write such a pair as:

```
X/Y
```

where, of course, the '/' operator is not meant to indicate division, but simply combines both coordinates together into a square. Figure 4.6 shows one solution of the eight queens problem and its list representation.

Having chosen this representation, the problem is to find such a list of the form:

```
[ X1/Y1, X2/Y2, X3/Y3, ..., X8/Y8]
```

which satisfies the no-attack requirement. Our procedure solution will have to search for a proper instantiation of the variables X1, Y1, X2, Y2, ..., X8, Y8. As we know that all the queens will have to be in different columns to prevent vertical attacks, we can immediately constrain the choice and so make the search task easier. We can thus fix the X-coordinates so that the solution list will fit the following, more specific template:

```
[ 1/Y1, 2/Y2, 3/Y3, ..., 8/Y8]
```

We are interested in the solution on a board of size 8 by 8. However, in programming, the key to the solution is often in considering a more general problem. Paradoxically, it is often the case that the solution for the more general problem is easier to formulate than that for the more specific, original problem. The original problem is then simply solved as a special case of the more general problem.

The creative part of the problem is to find the correct generalization of the original problem. In our case, a good idea is to generalize the number of queens (the number of columns in the list) from 8 to any number, including zero. The solution relation can then be formulated by considering two cases:

Case 1 The list of queens is empty: the empty list is certainly a solution because there is no attack.

Case 2 The list of queens is non-empty: then it looks like this:

```
[ X/Y | Others]
```

In case 2, the first queen is at some square X/Y and the other queens are at squares specified by the list Others. If this is to be a solution then the following conditions must hold:

(1) There must be no attack between the queens in the list Others; that is, Others itself must also be a solution.

(2) X and Y must be integers between 1 and 8.

(3) A queen at square X/Y must not attack any of the queens in the list Others.

To program the first condition we can simply use the solution relation itself. The second condition can be specified as follows: Y will have to be a member of the list of integers between 1 and 8 – that is, [1,2,3,4,5,6,7,8]. On the other hand, we do not have to worry about X since the solution list will have to match the template in which the X-coordinates are already specified. So X will be guaranteed to have a proper value between 1 and 8. We can implement the third condition as another relation, noattack. All this can then be written in Prolog as follows:

```
solution( [X/Y | Others] ) :-
  solution( Others),
  member( Y, [1,2,3,4,5,6,7,8] ),
  noattack( X/Y, Others).
```

It now remains to define the noattack relation:

```
noattack( Q, Qlist)
```

Again, this can be broken down into two cases:

(1) If the list Qlist is empty then the relation is certainly true because there is no queen to be attacked.

(2) If Qlist is not empty then it has the form [Q1 | Qlist1] and two conditions must be satisfied:

  (a) the queen at Q must not attack the queen at Q1, and

  (b) the queen at Q must not attack any of the queens in Qlist1.

To specify that a queen at some square does not attack another square is easy: the two squares must not be in the same row, the same column or the same diagonal. Our solution template guarantees that all the queens are in different columns, so it only remains to specify explicitly that:

- the Y-coordinates of the queens are different, and
- they are not in the same diagonal, either upward or downward; that is, the distance between the squares in the X-direction must not be equal to that in the Y-direction.

```
% solution( BoardPosition) if BoardPosition is a list of non-attacking queens

solution( [ ] ).

solution( [X/Y | Others] ) :-            % First queen at X/Y, other queens at Others
  solution( Others),
  member( Y, [1,2,3,4,5,6,7,8] ),
  noattack( X/Y, Others).                % First queen does not attack others

noattack( _, [ ] ).                      % Nothing to attack

noattack( X/Y, [X1/Y1 | Others] ) :-
  Y =\= Y1,                              % Different Y-coordinates
  Y1 - Y =\= X1 - X,                     % Different diagonals
  Y1 - Y =\= X - X1,
  noattack( X/Y, Others).

member( Item, [Item | Rest] ).

member( Item, [First | Rest] ) :-
  member( Item, Rest).

% A solution template

template( [1/Y1,2/Y2,3/Y3,4/Y4,5/Y5,6/Y6,7/Y7,8/Y8] ).
```

Figure 4.7 Program 1 for the eight queens problem.

Figure 4.7 shows the complete program. To alleviate its use a template list has been added. This list can be retrieved in a question for generating solutions. So we can now ask:

```
?- template( S), solution( S).
```

and the program will generate solutions as follows:

```
S = [ 1/4, 2/2, 3/7, 4/3, 5/6, 6/8, 7/5, 8/1];
S = [ 1/5, 2/2, 3/4, 4/7, 5/3, 6/8, 7/6, 8/1];
S = [ 1/3, 2/5, 3/2, 4/8, 5/6, 6/4, 7/7, 8/1];
...
```

Exercise

4.6 When searching for a solution, the program of Figure 4.7 explores alternative values for the Y-coordinates of the queens. At which place in the program is the order of alternatives defined? How can we easily modify the program to change the order? Experiment with different orders with the view of studying the time efficiency of the program.

4.5.2 Program 2

In the board representation of program 1, each solution had the form

[1/Y1, 2/Y2, 3/Y3, ..., 8/Y8]

because the queens were simply placed in consecutive columns. No information is lost if the X-coordinates were omitted. So a more economical representation of the board position can be used, retaining only the Y-coordinates of the queens:

[Y1, Y2, Y3, ..., Y8]

To prevent the horizontal attacks, no two queens can be in the same row. This imposes a constraint on the Y-coordinates. The queens have to occupy all the rows 1, 2, ..., 8. The choice that remains is the *order* of these eight numbers. Each solution is therefore represented by a permutation of the list

[1,2,3,4,5,6,7,8]

Such a permutation, S, is a solution if all the queens are safe. So we can write:

```
solution( S) :-
  permutation( [1,2,3,4,5,6,7,8], S),
  safe( S).
```

We have already programmed the **permutation** relation in Chapter 3, but the **safe** relation remains to be specified. We can split its definition into two cases:

(1) S is the empty list: this is certainly safe as there is nothing to be attacked.

(2) S is a non-empty list of the form [Queen | Others]. This is safe if the list Others is safe, and Queen does not attack any queen in the list Others.

In Prolog, this is:

```
safe( [ ] ).

safe( [Queen | Others] ) :-
  safe( Others),
  noattack( Queen, Others).
```

The **noattack** relation here is slightly trickier. The difficulty is that the queens' positions are only defined by their Y-coordinates, and the X-coordinates are not explicitly present. This problem can be circumvented by a small generalization of the **noattack** relation, as illustrated in Figure 4.8. The goal

```
noattack( Queen,Others)
```

is meant to ensure that **Queen** does not attack **Others** when the X-distance between **Queen** and **Others** is equal to 1. What is needed is the generalization of the X-distance between **Queen** and **Others**. So we add this distance as the third argument of the **noattack** relation:

```
noattack( Queen, Others, Xdist)
```

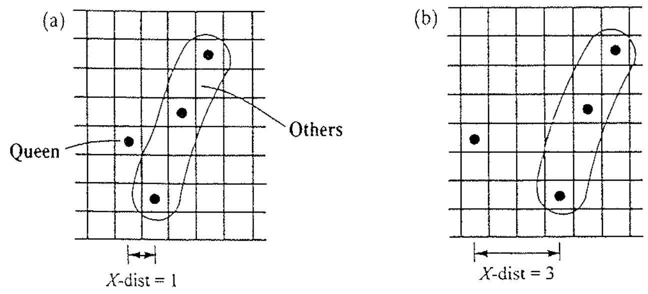

Figure 4.8 (a) X-distance between **Queen** and **Others** is 1. (b) X-distance between **Queen** and **Others** is 3.

Accordingly, the noattack goal in the safe relation has to be modified to

```
noattack( Queen, Others, 1)
```

The noattack relation can now be formulated according to two cases, depending on the list Others: if Others is empty then there is no target and certainly no attack; if Others is non-empty then Queen must not attack the first queen in Others (which is Xdist columns from Queen) and also the tail of Others at Xdist + 1. This leads to the program shown in Figure 4.9.

4.5.3 Program 3

Our third program for the eight queens problem will be based on the following reasoning. Each queen has to be placed on some square; that is, into some column, some row, some upward diagonal and some downward diagonal. To make sure that all the queens are safe, each queen must be placed in a different column, a different row, a different upward and a different downward diagonal. It is thus natural to consider a richer representation with four coordinates:

- x columns
- y rows
- u upward diagonals
- v downward diagonals

The coordinates are not independent: given x and y, u and v are determined (Figure 4.10 illustrates). For example, as:

$$u = x - y$$
$$v = x + y$$


```
% solution( Queens) if Queens is a list of Y-coordinates of eight non-attacking queens

solution( Queens) :-
  permutation( [1,2,3,4,5,6,7,8], Queens),
  safe( Queens).

permutation( [], [] ).

permutation( [Head | Tail], PermList) :-
  permutation( Tail, PermTail),
  del( Head, PermList, PermTail).                % Insert Head in permuted Tail

% del( Item, List, NewList): deleting Item from List gives NewList

del( Item, [Item | List], List).

del( Item, [First | List], [First | List1] ) :-
  del( Item, List, List1).

% safe( Queens) if Queens is a list of Y-coordinates of non-attacking queens

safe( [] ).

safe( [Queen | Others] ) :-
  safe( Others),
  noattack( Queen, Others, 1).

noattack( _, [], _).

noattack( Y, [Y1 | Ylist], Xdist) :-
  Y1 - Y =\= Xdist,
  Y - Y1 =\= Xdist,
  Dist1 is Xdist + 1,
  noattack( Y, Ylist, Dist1).
```

Figure 4.9 Program 2 for the eight queens problem.

The domains for all four dimensions are:

Dx = [1,2,3,4,5,6,7,8]

Dy = [1,2,3,4,5,6,7,8]

Du = [−7,−6,−5,−4,−3,−2,−1,0,1,2,3,4,5,6,7]

Dv = [2,3,4,5,6,7,8,9,10,11,12,13,14,15,16]

The eight queens problem can now be stated as follows: select eight 4-tuples (X,Y,U,V) from the domains (X from Dx, Y from Dy, etc.), never using the same element twice from any of the domains. Of course, once X and Y are chosen, U and V are determined. The solution can then be, roughly speaking, as follows: given all four domains, select the position of the first queen, delete the corresponding items from the four domains, and then use the rest of the domains for placing the rest of the queens. A program based on this idea is shown in Figure 4.11. The board

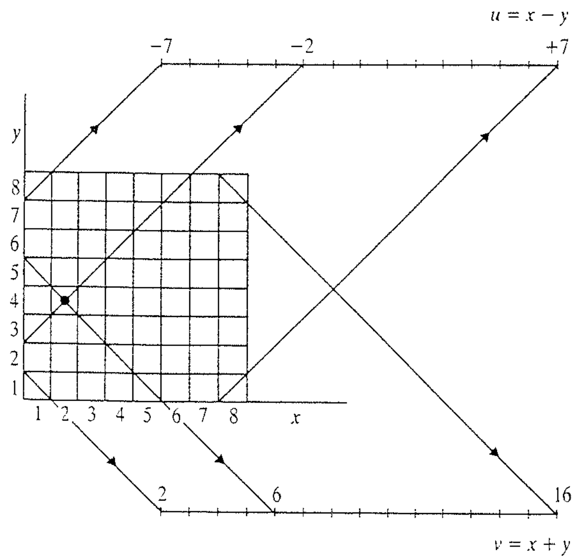

Figure 4.10 The relation between columns, rows, upward and downward diagonals. The indicated square has coordinates: $x = 2$, $y = 4$, $u = 2 - 4 = -2$, $v = 2 + 4 = 6$.

```
% solution( Ylist) if Ylist is a list of Y-coordinates of eight non-attacking queens

solution( Ylist) :-
  sol( Ylist,                                % Y-coordinates of queens
    [1,2,3 4,5,6,7,8],                       % Domain for X-coordinates
    [1,2,3,4,5,6,7,8],                       % Domain for Y-coordinates
    [-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7],  % Upward diagonals
    [2,3,4,5,6,7,8,9,10,11,12,13,14,15,16] ). % Downward diagonals

sol( [], [], Dy, Du, Dv).

sol( [Y | Ylist], [X | Dx1], Dy, Du, Dv) :-
  del( Y, Dy, Dy1),                          % Choose a Y-coordinate
  U is X-Y,                                  % Corresponding upward diagonal
  del( U, Du, Du1),                          % Remove it
  V is X+Y,                                  % Corresponding downward diagonal
  del( V, Dv, Dv1),                          % Remove it
  sol( Ylist, Dx1, Dy1, Du1, Dv1).           % Use remaining values

del( Item, [Item | List], List).

del( Item, [First | List], [First | List1] ) :-
  del( Item, List, List1).
```

Figure 4.11 Program 3 for the eight queens problem.

position is, again, represented by a list of Y-coordinates. The key relation in this program is

```
sol( Ylist, Dx, Dy, Du, Dv)
```

which instantiates the Y-coordinates (in Ylist) of the queens, assuming that they are placed in consecutive columns taken from Dx. All Y-coordinates and the corresponding U and V-coordinates are taken from the lists Dy, Du and Dv. The top procedure, solution, can be invoked by the question:

```
?- solution( S).
```

This will cause the invocation of sol with the complete domains that correspond to the problem space of eight queens.

The sol procedure is general in the sense that it can be used for solving the N-queens problem (on a chessboard of size N by N). It is only necessary to properly set up the domains Dx, Dy, etc.

It is practical to mechanize the generation of the domains. For that we need a procedure

```
gen( N1, N2, List)
```

which will, for two given integers N1 and N2, produce the list:

```
List = [ N1, N1 + 1, N1 + 2, ..., N2 - 1, N2]
```

Such a procedure is:

```
gen( N, N, [N] ).

gen( N1, N2, [N1 | List] ) :-
  N1 < N2,
  M is N1 + 1,
  gen( M, N2, List).
```

The top level relation, solution, has to be accordingly generalized to

```
solution( N, S)
```

where N is the size of the board and S is a solution represented as a list of Y-coordinates of N queens. The generalized solution relation is:

```
solution( N, S) :-
  gen( 1, N, Dxy),              % Dxy - domain for X and Y
  Nu1 is 1 - N, Nu2 is N - 1,
  gen( Nu1, Nu2, Du),
  Nv2 is N + N,
  gen( 2, Nv2, Dv),
  sol( S, Dxy, Dxy, Du, Dv).
```

For example, a solution to the 12-queens problem would be generated by:

```
?- solution( 12, S).
S = [1,3,5,8,10,12,6,11,2,7,9,4]
```

4.5.4 Concluding remarks

The three solutions to the eight queens problem show how the same problem can be approached in different ways. We also varied the representation of data. Sometimes the representation was more economical, sometimes it was more explicit and partially redundant. The drawback of the more economical representation is that some information always has to be recomputed when it is required.

At several points, the key step toward the solution was to generalize the problem. Paradoxically, by considering a more general problem, the solution became easier to formulate. This generalization principle is a kind of standard technique that can often be applied.

Of the three programs, the third one illustrates best how to approach general problems of constructing under constraints a structure from a given set of elements.

A natural question is: Which of the three programs is most efficient? In this respect, program 2 is far inferior while the other two programs are similar. The reason is that permutation-based program 2 constructs complete permutations while the other two programs are able to recognize and reject unsafe permutations when they are only partially constructed. Program 3 avoids some of the arithmetic computation that is essentially captured in the redundant board representation this program uses.

Exercise

4.7 Let the squares of the chessboard be represented by pairs of their coordinates of the form X/Y, where both X and Y are between 1 and 8.

(a) Define the relation **jump(Square1, Square2)** according to the knight jump on the chessboard. Assume that **Square1** is always instantiated to a square while **Square2** can be uninstantiated. For example:

```
?- jump( 1/1, S).

S = 3/2;

S = 2/3;

no
```

(b) Define the relation **knightpath(Path)** where **Path** is a list of squares that represent a legal path of a knight on the empty chessboard.

(c) Using this **knightpath** relation, write a question to find any knight's path of length 4 moves from square 2/1 to the opposite edge of the board (Y = 8) that goes through square 5/4 after the second move.

Summary

The examples of this chapter illustrate some strong points and characteristic features of Prolog programming:

- A database can be naturally represented as a set of Prolog facts.
- Prolog's mechanisms of querying and matching can be flexibly used for retrieving structured information from a database. In addition, utility procedures can be easily defined to further alleviate the interaction with a particular database.
- *Data abstraction* can be viewed as a programming technique that makes the use of complex data structures easier, and contributes to the clarity of programs. It is easy in Prolog to carry out the essential principles of data abstraction.
- Abstract mathematical constructs, such as automata, can often be readily translated into executable Prolog definitions.
- As in the case of eight queens, the same problem can be approached in different ways by varying the representation of the problem. Often, introducing redundancy into the representation saves computation. This entails trading space for time.
- Often, the key step toward a solution is to generalize the problem. Paradoxically, by considering a more general problem the solution may become easier to formulate.

chapter 5

Controlling Backtracking

We have already seen that a programmer can control the execution of a program through the ordering of clauses and goals. In this chapter we will look at another control facility, called 'cut', for preventing backtracking. The cut also extends the expressive power of Prolog and enables the definition of a kind of negation, called 'negation as failure' and associated with the 'closed world assumption'.

5.1 Preventing backtracking

Prolog will automatically backtrack if this is necessary for satisfying a goal. Automatic backtracking is a useful programming concept because it relieves the programmer of the burden of programming backtracking explicitly. On the other hand, uncontrolled backtracking may cause inefficiency in a program. Therefore we sometimes want to control, or to prevent, backtracking. We can do this in Prolog by using the 'cut' facility.

Let us first study the behaviour of a simple example program whose execution involves some unnecessary backtracking. We will identify those points at which the backtracking is useless and leads to inefficiency.

Consider the double-step function shown in Figure 5.1. The relation between X and Y can be specified by three rules:

Rule 1: if $X < 3$ then $Y = 0$

Rule 2: if $3 \leq X$ and $X < 6$ then $Y = 2$

Rule 3: if $6 \leq X$ then $Y = 4$

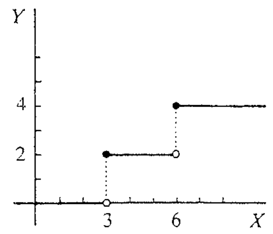

Figure 5.1 A double-step function.

This can be written in Prolog as a binary relation:

```
f( X, Y)
```

as follows:

```
f( X, 0) :- X < 3.            % Rule 1
f( X, 2) :- 3 =< X, X < 6.    % Rule 2
f( X, 4) :- 6 =< X.           % Rule 3
```

This program, of course, assumes that before f(X, Y) is executed X is already instantiated to a number, as this is required by the comparison operators.

We will make two experiments with this program. Each experiment will reveal some source of inefficiency in the program, and we will remove each source in turn by using the cut mechanism.

5.1.1 Experiment 1

Let us analyze what happens when the following question is posed:

```
?- f( 1, Y), 2 < Y.
```

When executing the first goal, f(1, Y), Y becomes instantiated to 0. So the second goal becomes

```
2 < 0
```

which fails, and so does the whole goal list. This is straightforward, but before admitting that the goal list is not satisfiable, Prolog tries, through backtracking, two useless alternatives. The detailed trace is shown in Figure 5.2.

The three rules about the f relation are mutually exclusive so that one of them at most will succeed. Therefore we, not Prolog, know that as soon as one rule succeeds there is no point in trying to use the others, as they are bound to fail. In the example of Figure 5.2, rule 1 has become known to succeed at the point indicated by 'CUT'. In order to prevent futile backtracking at this point we have to tell Prolog explicitly

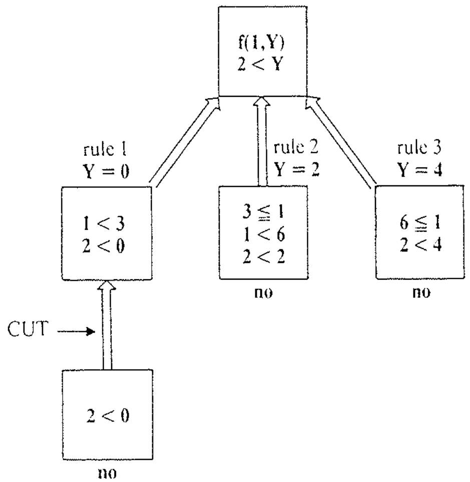

Figure 5.2 At the point marked 'CUT' we already know that the rules 2 and 3 are bound to fail.

not to backtrack. We can do this by using the cut mechanism. The 'cut' is written as ! and is inserted between goals as a kind of pseudo-goal. Our program, rewritten with cuts, is:

```
f( X, 0) :- X < 3, !.
f( X, 2) :- 3 =< X, X < 6, !.
f( X, 4) :- 6 =< X.
```

The ! symbol will now prevent backtracking at the points at which it appears in the program. If we now ask:

```
?- f( 1, Y), 2 < Y.
```

Prolog will produce the same left-hand branch as in Figure 5.2. This branch will fail at the goal $2 < 0$. Now Prolog will try to backtrack, but not beyond the point marked ! in the program. The alternative branches that correspond to 'rule 2' and 'rule 3' will not be generated.

The new program, equipped with cuts, is in general more efficient than the original version without cuts. When the execution fails, the new program will in general recognize this sooner than the original program.

To conclude, we have improved the efficiency by adding cuts. If the cuts are now removed in this example, the program will still produce the same result; it will perhaps only spend more time. In our case, by introducing the cut we only changed the procedural meaning of the program; that is, the results of the program were not affected. We will see later that using a cut may affect the results as well.

5.1.2 Experiment 2

Let us now perform a second experiment with the second version of our program. Suppose we ask:

```
?- f( 7, Y).
Y = 4
```

Let us analyze what has happened. All three rules were tried before the answer was obtained. This produced the following sequence of goals:

Try rule 1: $7 < 3$ fails, backtrack and try rule 2 (cut was not reached)

Try rule 2: $3 \leqq 7$ succeeds, but then $7 < 6$ fails, backtrack and try rule 3 (cut was not reached)

Try rule 3: $6 \leqq 7$ succeeds

This trace reveals another source of inefficiency. First it is established that X < 3 is not true (7 < 3 fails). The next goal is 3 =< X (3 =< 7 succeeds). But we know that once the first test has failed the second test is bound to succeed as it is the negation of the first. Therefore the second test is redundant and the corresponding goal can be omitted. The same is true about the goal 6 =< X in rule 3. This leads to the following, more economical formulation of the three rules:

if $X < 3$ then $Y = 0$,
otherwise if $X < 6$ then $Y = 2$,
otherwise $Y = 4$.

We can now omit the conditions in the program that are guaranteed to be true whenever they are executed. This leads to the third version of the program:

```
f( X, 0) :- X < 3, !.
f( X, 2) :- X < 6, !.
f( X, 4).
```

This program produces the same results as our original version, but is more efficient than both previous versions. But what happens if we *now* remove the cuts? The program becomes:

```
f( X, 0) :- X < 3.
f( X, 2) :- X < 6.
f( X, 4).
```

This may produce multiple solutions, some of which are not correct. For example:

```
?- f( 1, Y).
Y = 0;
```

```
Y = 2;
Y = 4;
no
```

It is important to notice that, in contrast to the second version of the program, this time the cuts do not only affect the procedural behaviour, but also change the results of the program.

A more precise meaning of the cut mechanism is as follows:

Let us call the 'parent goal' the goal that matched the head of the clause containing the cut. When the cut is encountered as a goal it succeeds immediately, but it commits the system to all choices made between the time the 'parent goal' was invoked and the time the cut was encountered. All the remaining alternatives between the parent goal and the cut are discarded.

To clarify this definition consider a clause of the form:

```
H :- B1, B2, ..., Bm, !, ..., Bn.
```

Let us assume that this clause was invoked by a goal G that matched H. Then G is the parent goal. At the moment that the cut is encountered, the system has already found some solution of the goals B1, ..., Bm. When the cut is executed, this (current) solution of B1, ..., Bm becomes frozen and all possible remaining alternatives are discarded. Also, the goal G now becomes committed to this clause: any attempt to match G with the head of some other clause is precluded.

Let us apply these rules to the following example:

```
C :- P, Q, R, !, S, T, U.
C :- V.
A :- B, C, D.
?- A.
```

Here A, B, C, D, P, etc. have the syntax of terms. The cut will affect the execution of the goal C as illustrated by Figure 5.3. Backtracking will be possible within the goal list P, Q, R; however, as soon as the cut is reached, all alternative solutions of the goal list P, Q, R are suppressed. The alternative clause about C,

```
C :- V.
```

will also be discarded. However, backtracking will still be possible within the goal list S, T, U. The 'parent goal' of the clause containing the cut is the goal C in the clause:

```
A :- B, C, D.
```


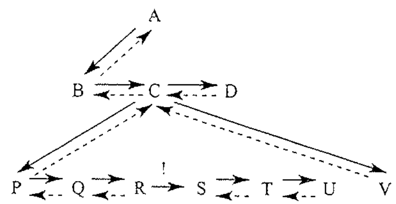

Figure 5.3 The effect of the cut on the execution. Starting with A, the solid arrows indicate the sequence of calls; the dashed arrows indicate backtracking. There is 'one way traffic' between R and S.

Therefore the cut will only affect the execution of the goal C. On the other hand, it will be 'invisible' from goal A. So automatic backtracking within the goal list B, C, D will remain active regardless of the cut within the clause used for satisfying C.

5.2 Examples using cut

5.2.1 Computing maximum

The procedure for finding the larger of two numbers can be programmed as a relation

```
max( X, Y, Max)
```

where Max = X if X is greater than or equal to Y, and Max is Y if X is less than Y. This corresponds to the following two clauses:

```
max( X, Y, X) :- X >= Y.
max( X, Y, Y) :- X < Y.
```

These two rules are mutually exclusive. If the first one succeeds then the second one will fail. If the first one fails then the second must succeed. Therefore a more economical formulation, with 'otherwise', is possible:

If X ≧ Y then Max = X,
otherwise Max = Y.

This is written in Prolog using a cut as:

```
max( X, Y, X) :- X >= Y, !.
max( X, Y, Y).
```

It should be noted that the use of this procedure requires care. It is safe if in the goal max(X,Y,Max) the argument Max is not instantiated. The following example of incorrect use illustrates the problem:

```
?- max( 3, 1, 1).

yes
```

The following reformulation of max overcomes this limitation:

```
max( X, Y, Max) :-
  X >= Y, !, Max = X
  ;
  Max = Y.
```

5.2.2 Single-solution membership

We have been using the relation

```
member( X, L)
```

for establishing whether X is in list L. The program was:

```
member( X, [X | L] ).

member( X, [Y | L] ) :- member( X, L).
```

This is non-deterministic: if X occurs several times then any occurrence can be found. Let us now change member into a deterministic procedure which will find only the first occurrence. The change is simple: we only have to prevent backtracking as soon as X is found, which happens when the first clause succeeds. The modified program is:

```
member( X, [X | L] ) :- !.

member( X, [Y | L] ) :- member( X, L).
```

This program will generate just one solution. For example:

```
?- member( X, [a,b,c] ).

X = a;

no
```

5.2.3 Adding an element to a list without duplication

Often we want to add an item X to a list L so that X is added only if X is not yet in L. If X is already in L then L remains the same because we do not want to have redundant duplicates in L. The add relation has three arguments:

```
add( X, L, L1)
```


where X is the item to be added, L is the list to which X is to be added and L1 is the resulting new list. Our rule for adding can be formulated as:

If X is a member of list L then L1 = L,
otherwise L1 is equal to L with X inserted.

It is easiest to insert X in front of L so that X becomes the head of L1. This is then programmed as follows:

```
add( X, L, L) :- member( X, L), !.

add( X, L, [X | L] ).
```

The behaviour of this procedure is illustrated by the following example:

```
?- add( a, [b,c], L).

L = [a,b,c]

?- add( X, [b,c], L).

L = [b,c]
X = b

?- add( a, [b,c,X], L).

L = [b,c,a]
X = a
```

Similar to the foregoing example with max, add(X, L1, L2) is intended to be called with L2 uninstantiated. Otherwise the result may be unexpected: for example add(a, [a], [a,a]) succeeds.

This example is instructive because we cannot easily program the 'non-duplicate add' without the use of cut or another construct derived from the cut. If we omit the cut in the foregoing program then the add relation will also add duplicate items. For example:

```
?- add( a, [a,b,c], L).

L = [a,b,c];

L = [a,a,b,c]
```

So the cut is necessary here to specify the intended relation, and not only to improve efficiency. The next example also illustrates this point.

5.2.4 Classification into categories

Assume we have a database of results of tennis games played by members of a club. The pairings were not arranged in any systematic way, so each player just played some other players. The results are in the program represented as facts like:

```
beat( tom, jim).
beat( ann, tom).
beat( pat, jim).
```

We want to define a relation

```
class( Player, Category)
```

that ranks the players into categories. We have just three categories:

winner: every player who won all his or her games is a winner
fighter: any player that won some games and lost some
sportsman: any player who lost all his or her games

For example, if all the results available are just those above then Ann and Pat are winners, Tom is a fighter and Jim is a sportsman.

It is easy to specify the rule for a fighter:

X is a fighter if
  there is some Y such that X beat Y and
  there is some Z such that Z beat X.

Now a rule for a winner:

X is a winner if
  X beat some Y and
  X was not beaten by anybody.

This formulation contains 'not' which cannot be directly expressed with our present Prolog facilities. So the formulation of **winner** appears trickier. The same problem occurs with **sportsman**. The problem can be circumvented by combining the definition of **winner** with that of **fighter**, and using the 'otherwise' connective. Such a formulation is:

If X beat somebody and X was beaten by somebody
  then X is a fighter,
  otherwise if X beat somebody
    then X is a winner,
    otherwise if X got beaten by somebody
      then X is a sportsman.

This formulation can be readily translated into Prolog. The mutual exclusion of the three alternative categories is indicated by the cuts:

```
class( X, fighter) :-
  beat( X, _),
  beat( _, X), !.

class( X, winner) :-
  beat( X, _), !.

class( X, sportsman) :-
  beat( _, X).
```


Notice that the cut in the clause for **winner** is not necessary. Care is needed when using such procedures containing cuts. Here is what can happen:

```
?- class( tom, C).
C = fighter;                  % As intended
no

?- class( tom, sportsman).
yes                           % Not as intended
```

The call of **class** is safe if the second argument is not instantiated. Otherwise we may get an unintended result.

Exercises

5.1 Let a program be:

```
p( 1).
p( 2) :- !.
p( 3).
```

Write all Prolog's answers to the following questions:

(a) ?- p(X).
(b) ?- p(X), p(Y).
(c) ?- p(X), !, p(Y).

5.2 The following relation classifies numbers into three classes: positive, zero and negative:

```
class( Number, positive) :- Number > 0.
class( 0, zero).
class( Number, negative) :- Number < 0.
```

Define this procedure in a more efficient way using cuts.

5.3 Define the procedure

```
split( Numbers, Positives, Negatives)
```

which splits a list of numbers into two lists: positive ones (including zero) and negative ones. For example:

```
split( [3,-1,0,5,-2], [3,0,5], [-1,-2] )
```

Propose two versions: one with a cut and one without.

5.3 Negation as failure

'Mary likes all animals but snakes'. How can we say this in Prolog? It is easy to express one part of this statement: Mary likes any X if X is an animal. This is in Prolog:

```
likes( mary, X) :- animal( X).
```

But we have to exclude snakes. This can be done by using a different formulation:

> If X is a snake then 'Mary likes X' is not true,
> otherwise if X is an animal then Mary likes X.

That something is not true can be said in Prolog by using a special goal, **fail**, which always fails, thus forcing the parent goal to fail. The above formulation is translated into Prolog, using **fail**, as follows:

```
likes( mary, X) :-
  snake( X), !, fail.

likes( mary, X) :-
  animal( X).
```

The first rule here will take care of snakes: if X is a snake then the cut will prevent backtracking (thus excluding the second rule) and **fail** will cause the failure. These two clauses can be written more compactly as one clause:

```
likes( mary, X) :-
  snake( X), !, fail
  ;
  animal( X).
```

We can use the same idea to define the relation

```
different( X, Y)
```

which is true if X and Y are different. We have to be more precise, however, because 'different' can be understood in several ways:

- X and Y are not literally the same;
- X and Y do not match;
- the values of arithmetic expressions X and Y are not equal.

Let us choose here that X and Y are different if they do not match. The key to saying this in Prolog is:

> If X and Y match then **different(X, Y)** fails,
> otherwise **different(X, Y)** succeeds.

We again use the cut and **fail** combination:

```
different( X, X) :- !, fail.

different( X, Y).
```

This can also be written as one clause:

```
different( X, Y) :-
  X = Y, !, fail
  ;
  true.
```

true is a goal that always succeeds.

These examples indicate that it would be useful to have a unary predicate 'not' such that

```
not( Goal)
```

is true if **Goal** is not true. We will now define the **not** relation as follows:

> If **Goal** succeeds then **not(Goal)** fails,
> otherwise **not(Goal)** succeeds.

This definition can be written in Prolog as:

```
not( P) :-
  P, !, fail
  ;
  true.
```

Henceforth, we will assume that **not** is a built-in Prolog procedure that behaves as defined here. We will also assume that **not** is defined as a prefix operator, so that we can also write the goal

```
not( snake(X) )
```

as:

```
not snake( X)
```

Some Prolog implementations, in fact, support this notation. If not, then we can always define **not** ourselves. Alternatively, **not Goal** is written as **\+ Goal**. This more mysterious notation is also recommended in the Prolog standard for the following reason. **not** defined as failure, as here, does not exactly correspond to negation in mathematical logic. This difference can cause unexpected behaviour if **not** is used without care. This will be discussed later in the chapter.

Nevertheless, **not** is a useful facility and can often be used advantageously in place of cut. Our two examples can be rewritten with **not** as:

```
likes( mary, X) :-
  animal( X),
  not snake( X).
```

```
solution( [] ).

solution( [X/Y | Others] ) :-
  solution( Others),
  member( Y, [1,2,3,4,5,6,7,8] ),              % Usual member predicate
  not attacks( X/Y, Others).

attacks( X/Y, Others) :-
  member( X1/Y1, Others),
  ( Y1 = Y;
   Y1 is Y + X1 - X;
   Y1 is Y - X1 + X ).
```

Figure 5.4 Another eight queens program.

```
different( X, Y) :-
  not( X = Y).
```

This certainly looks better than our original formulations. It is more natural and is easier to read.

Our tennis classification program of the previous section can also be rewritten, using not, in a way that is closer to the initial definition of the three categories:

```
class( X, fighter) :-
  beat( X, _),
  beat( _, X).

class( X, winner) :-
  beat( X, _),
  not beat( _, X).

class( X, sportsman) :-
  beat( _, X),
  not beat( X, _).
```

As another example of the use of not let us reconsider program 1 for the eight queens problem of the previous chapter (Figure 4.7). We specified the no_attack relation between a queen and other queens. This relation can be formulated also as the negation of the attack relation. Figure 5.4 shows a program modified accordingly.

Exercises

5.4 Given two lists, Candidates and RuledOut, write a sequence of goals (using member and not) that will through backtracking find all the items in Candidates that are not in RuledOut.

5.5 Define the set subtraction relation

```
set_difference( Set1, Set2, SetDifference)
```

where all the three sets are represented as lists. For example:

```
set_difference( [a,b,c,d], [b,d,e,f], [a,c] )
```

5.6 Define the predicate

```
unifiable( List1, Term, List2)
```

where List2 is the list of all the members of List1 that match Term, but are not instantiated by this matching. For example:

```
?- unifiable( [X, b, t(Y)], t(a), List).
List = [X, t(Y)]
```

Note that X and Y have to remain uninstantiated although the matching with t(a) does cause their instantiation. Hint: Use not(Term1 = Term2). If Term1 = Term2 succeeds then not(Term1 = Term2) fails and the resulting instantiation is undone!

5.4 Problems with cut and negation

Using the cut facility we get something, but not for nothing. The advantages and disadvantages of using cut were illustrated by examples in the previous sections. Let us summarize, first the advantages:

(1) With cut we can often improve the efficiency of the program. The idea is to explicitly tell Prolog: do not try other alternatives because they are bound to fail.

(2) Using cut we can specify mutually exclusive rules; so we can express rules of the form:

if condition P *then* conclusion Q,
otherwise conclusion R

In this way, cut enhances the expressive power of the language.

The reservations against the use of cut stem from the fact that we can lose the valuable correspondence between the declarative and procedural meaning of programs. If there is no cut in the program we can change the order of clauses and goals, and this will only affect the efficiency or termination of the program, not the declarative meaning. On the other hand, in programs with cuts, a change in the order of clauses may affect the declarative meaning. This means that we can get different results. The following example illustrates:

```
p :- a, b.
p :- c.
```

The declarative meaning of this program is: p is true if and only if a and b are both true or c is true. This can be written as a logic formula:

$$p \iff (a \,\&\, b) \lor c$$

We can change the order of the two clauses and the declarative meaning remains the same. Let us now insert a cut:

```
p :- a, !, b.
p :- c.
```

The declarative meaning is now:

$$p \iff (a \,\&\, b) \lor (\sim a \,\&\, c)$$

If we swap the clauses,

```
p :- c.
p :- a, !, b.
```

then the meaning becomes:

$$p \iff c \lor (a \,\&\, b)$$

The important point is that when we use the cut facility we have to pay more attention to the procedural aspects. Unfortunately, this additional difficulty increases the probability of a programming error.

In our examples in the previous sections we have seen that sometimes the removal of a cut from the program can change the declarative meaning of the program. But there were also cases in which the cut had no effect on the declarative meaning. The use of cuts of the latter type is less delicate, and therefore cuts of this kind are sometimes called 'green cuts'. From the point of view of readability of programs, green cuts are 'innocent' and their use is quite acceptable. When reading a program, green cuts can simply be ignored.

On the contrary, cuts that do affect the declarative meaning are called 'red cuts'. Red cuts are the ones that make programs hard to understand, and they should be used with special care.

Cut is often used in combination with a special goal, **fail**. In particular, we defined the negation of a goal (**not**) as the failure of the goal. The negation, so defined, is just a special, more restricted way of using cut. For reasons of clarity we will prefer to use **not** instead of the *cut–fail* combination (whenever possible), because the negation is intuitively clearer than the *cut–fail* combination.

It should be noted that **not** may also cause problems, and so should also be used with care. The problem is that **not**, as defined here, does not correspond exactly to negation in mathematics. If we ask Prolog:


```
?- not human( mary).
```

Prolog will probably answer 'yes'. But this should not be understood as Prolog saying 'Mary is not human'. What Prolog really means to say is: 'There is not enough information in the program to prove that Mary is human'. This arises because when processing a **not** goal, Prolog does not try to prove this goal directly. Instead, it tries to prove the opposite, and if the opposite cannot be proved then Prolog assumes that the **not** goal succeeds.

Such reasoning is based on the so-called *closed world assumption*. According to this assumption *the world is closed* in the sense that everything that exists is stated in the program or can be derived from the program. Accordingly then, if something is not in the program (or cannot be derived from it) then it is not true and consequently its negation is true. This deserves special care because we do not normally assume that 'the world is closed'. When we do not explicitly enter the clause

```
human( mary).
```

into our program, we do not mean to imply that Mary is not human.

To further study the special care that **not** requires, consider the following example about restaurants:

```
good_standard( jeanluis).
expensive( jeanluis).
good_standard( francesco).
reasonable( Restaurant) :-
  not expensive( Restaurant).
```

If we ask:

```
?- good_standard( X), reasonable( X).
```

Prolog will answer:

```
X = francesco
```

If we ask apparently the same question

```
?- reasonable( X), good_standard( X).
```

then Prolog will answer:

```
no
```

The reader is invited to trace the program to understand why we get different answers. The key difference between both questions is that the variable X is, in the first case, already instantiated when **reasonable(X)** is executed, whereas X is not yet instantiated in the second case. The general hint is: **not Goal** works safely if the variables in **Goal** are instantiated at the time **not Goal** is called. Otherwise we may get unexpected results due to reasons explained in the sequel.

The problem with uninstantiated negated goals arises from unfortunate change of the quantification of variables in negation as failure. In the usual interpretation in Prolog, the question:

```
?- expensive( X).
```

means: Does there *exist* X such that **expensive(X)** is true? If yes, what is X? So X is *existentially* quantified. Accordingly Prolog answers X = **jeanluis**. But the question:

```
?- not expensive( X).
```

is not interpreted as: Does there exist X such that **not expensive(X)**? The expected answer would be X = **francesco**. But Prolog answers 'no' because negation as failure changes the quantification to universal. The question **not expensive(X)** is interpreted as:

not(exists X such that **expensive(X)**)

This is equivalent to:

For *all* X: **not expensive(X)**

We have discussed problems with cut, which also indirectly occur in **not**, in detail. The intention has been to warn users about the necessary care, not to definitely discourage the use of cut. Cut is useful and often necessary. And after all, the kind of complications that are incurred by cut in Prolog commonly occur when programming in other languages as well.

Summary

- The cut facility prevents backtracking. It is used both to improve the efficiency of programs and to enhance the expressive power of the language.
- Efficiency is improved by explicitly telling Prolog (with cut) not to explore alternatives that we know are bound to fail.
- Cut makes it possible to formulate mutually exclusive conclusions through rules of the form:

  if **Condition** *then* **Conclusion1** *otherwise* **Conclusion2**
- Cut makes it possible to introduce *negation as failure*: **not Goal** is defined through the failure of **Goal**.
- Two special goals are sometimes useful: **true** always succeeds, **fail** always fails.
- There are also some reservations against cut: inserting a cut may destroy the correspondence between the declarative and procedural meaning of a program. Therefore, it is part of good programming style to use cut with care and not to use it without reason.

- **not** defined through failure does not exactly correspond to negation in mathematical logic. Therefore, the use of **not** also requires special care.

References

The distinction between 'green cuts' and 'red cuts' was proposed by van Emden (1982). Le (1993) proposes a different negation for Prolog which is mathematically advantageous, but computationally more expensive.

Le, T.V. (1993) *Techniques of Prolog Programming*. John Wiley & Sons.
van Emden, M. (1982) Red and green cuts. *Logic Programming Newsletter*: 2.

chapter 6

Input and Output

In this chapter we will investigate some built-in facilities for reading data from computer files and for outputting data to files. These procedures can also be used for formatting data objects in the program to achieve a desired output representation of these objects. We will also look at facilities for reading programs and for constructing and decomposing atoms.

6.1 Communication with files

The method of communication between the user and the program that we have been using up to now consists of user questions to the program and program answers in terms of instantiations of variables. This method of communication is simple and suffices to get the information in and out. However, it is often not quite sufficient because it is too rigid. Extensions to this basic communication method are needed in the following areas:

- input of data in forms other than questions – for example, in the form of English sentences,
- output of information in any format desired, and
- input from and output to any computer file or device and not just the user terminal.

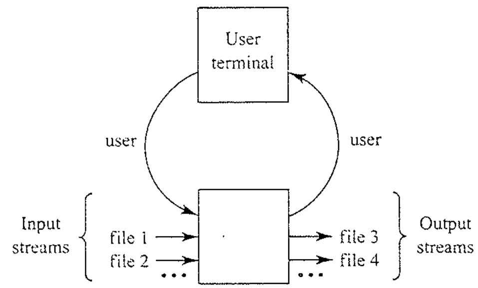

Figure 6.1 Communication between a Prolog program and several files.

Built-in predicates aimed at these extensions depend on the implementation of Prolog. We will study here a simple and handy repertoire of such predicates, which is part of many Prolog implementations. However, the implementation manual should be consulted for details and specificities. Many Prolog implementations provide various additional facilities not covered here. Such extra facilities handle windows, provide graphics primitives for drawing on the screen, input information from the mouse, and so on.

We will first consider the question of directing input and output to files, and then how data can be input and output in different forms.

Figure 6.1 shows a general situation in which a Prolog program communicates with several files. The program can, in principle, read data from several input files, also called *input streams*, and output data to several output files, also called *output streams*. Data coming from the user's terminal is treated as just another input stream. Data output to the terminal is, analogously, treated as another output stream. Both of these 'pseudo-files' are referred to by the name **user**. The names of other files can be chosen by the programmer according to the rules for naming files in the computer system used.

At any time during the execution of a Prolog program, only two files are 'active': one for input and one for output. These two files are called the *current input stream* and the *current output stream* respectively. At the beginning of execution these two streams correspond to the user's terminal. The current input stream can be changed to another file, **Filename**, by the goal:

```
see( Filename)
```

Such a goal succeeds (unless there is something wrong with **Filename**) and causes, as a side effect, that input is switched from the previous input stream to **Filename**. So a typical example of using the **see** predicate is the following sequence of goals, which reads something from **file1** and then switches back to the terminal:

```
...
see( file1),
read_from_file( Information),
see( user),
...
```

The current output stream can be changed by a goal of the form:

```
tell( Filename)
```

A sequence of goals to output some information to file3, and then redirect succeeding output back to the terminal, is:

```
...
tell( file3),
write_on_file( Information),
tell( user),
...
```

The goal

```
seen
```

closes the current input file. The goal

```
told
```

closes the current output file.

We will assume here that files can only be processed sequentially although many Prolog implementations also handle files with random access. Sequential files behave in the same way as the terminal. Each request to read something from an input file will cause reading at the current position in the current input stream. After the reading, the current position will be, of course, moved to the next unread item. So the next request for reading will start reading at this new current position. If a request for reading is made at the end of a file, then the information returned by such a request is the atom **end_of_file**.

Writing is similar; each request to output information will append this information at the end of the current output stream. It is not possible to move backward and to overwrite part of the file.

We will here only consider 'text-files' – that is, files of characters. Characters are letters, digits and special characters. Some of them are said to be non-printable because when they are output on the terminal they do not appear on the screen. They may, however, have other effects, such as spacing between columns and lines.

There are two main ways in which files can be viewed in Prolog, depending on the form of information. One way is to consider the character as the basic element of the file. Accordingly, one input or output request will cause a single character to be read or written. We assume the built-in predicates for this are **get**, **get0** and **put**.

The other way of viewing a file is to consider bigger units of information as basic building blocks of the file. Such a natural bigger unit is the Prolog term. So each input/output request of this type would transfer a whole term from the current input stream or to the current output stream respectively. Predicates for transfer of terms are **read** and **write**. Of course, in this case, the information in the file has to be in a form that is consistent with the syntax of terms.

What kind of file organization is chosen will, of course, depend on the problem. Whenever the problem specification will allow the information to be naturally squeezed into the syntax of terms, we will prefer to use a file of terms. It will then be possible to transfer a whole meaningful piece of information with a single request. On the other hand, there are problems whose nature dictates some other organization of files. An example is the processing of natural language sentences, say, to generate a dialogue in English between the system and the user. In such cases, files will have to be viewed as sequences of characters that cannot be parsed into terms.

6.2 Processing files of terms

6.2.1 *read* and *write*

The built-in predicate **read** is used for reading terms from the current input stream. The goal

```
read( X)
```

will cause the next term, T, to be read, and this term will be matched with X. If X is a variable then, as a result, X will become instantiated to T. If matching does not succeed then the goal **read(X)** fails. The predicate **read** is deterministic, so in the case of failure there will be no backtracking to input another term. Each term in the input file must be followed by a full stop and a space or carriage-return.

If **read(X)** is executed when the end of the current input file has been reached then X will become instantiated to the atom **end_of_file**.

The built-in predicate **write** outputs a term. So the goal

```
write( X)
```

will output the term X on the current output file. X will be output in the same standard syntactic form in which Prolog normally displays values of variables. A useful feature of Prolog is that the **write** procedure 'knows' to display any term no matter how complicated it may be.

Typically, there are additional built-in predicates for formatting the output. They insert spaces and new lines into the output stream. The goal

```
tab( N)
```

causes N spaces to be output. The predicate **nl** (which has no arguments) causes the start of a new line at output.

The following examples will illustrate the use of these procedures.

Let us assume that we have a procedure that computes the cube of a number:

```
cube( N, C) :-
  C is N * N * N.
```

Suppose we want to use this for calculating the cubes of a sequence of numbers. We could do this by a sequence of questions:

```
?- cube( 2, X).

X = 8

?- cube( 5, Y).

Y = 125

?- cube( 12, Z).

Z = 1728
```

For each number, we had to type in the corresponding goal. Let us now modify this program so that the cube procedure will read the data itself. Now the program will keep reading data and outputting their cubes until the atom stop is read:

```
cube :-
  read( X),
  process( X).

process( stop) :- !.

process( N) :-
  C is N * N * N,
  write( C),
  cube.
```

This is an example of a program whose declarative meaning is awkward to formulate. However, its procedural meaning is straightforward: to execute cube, first read X and then process it; if X = stop then everything has been done, otherwise write the cube of X and recursively call the cube procedure to process further data. A table of the cubes of numbers can be produced using this new procedure as follows:

```
?- cube.
2.
8
5.
125
12.
1728
stop.
yes
```


The numbers 2, 5 and 12 were typed in by the user on the terminal; the other numbers were output by the program. Note that each number entered by the user had to be followed by a full stop, which signals the end of a term.

It may appear that the above cube procedure could be simplified. However, the following attempt to simplify is not correct:

```
cube :-
  read( stop), !.

cube :-
  read( N),
  C is N * N * N,
  write( C),
  cube.
```

The reason why this is wrong can be seen easily if we trace the program with input data 5, say. The goal read(stop) will fail when the number is read, and this number will be lost for ever. The next read goal will input the next term. On the other hand, it could happen that the stop signal is read by the goal read(N), which would then cause a request to multiply non-numeric data.

The cube procedure conducts interaction between the user and the program. In such cases it is usually desirable that the program, before reading new data from the terminal, signals to the user that it is ready to accept the information, and perhaps also says what kind of information it is expecting. This is usually done by sending a 'prompt' signal to the user before reading. Our cube procedure would be accordingly modified, for example, as follows:

```
cube :-
  write( 'Next item, please: '),
  read( X),
  process( X).

process( stop) :- !.

process( N) :-
  C is N * N * N,
  write( 'Cube of '), write( N), write( ' is '),
  write( C), nl,
  cube.
```

A conversation with this new version of cube would then be, for example, as follows:

```
?- cube.
Next item, please: 5.
Cube of 5 is 125
Next item, please: 12.
Cube of 12 is 1728
Next item, please: stop.
yes
```

Depending on the implementation, an extra request (like **ttyflush**, say) after writing the prompt might be necessary in order to force the prompt to actually appear on the screen before reading.

In the following sections we will look at some typical examples of operations that involve reading and writing.

6.2.2 Displaying lists

Besides the standard Prolog format for lists, there are several other natural forms for displaying lists which have advantages in some situations. The following procedure

```
writelist( L)
```

outputs a list L so that each element of L is written on a separate line:

```
writelist( [ ] ).

writelist( [X | L] ) :-
  write( X), nl,
  writelist( L).
```

If we have a list of lists, one natural output form is to write the elements of each list in one line. To this end, we will define the procedure writelist2. An example of its use is:

```
?- writelist2( [ [a,b,c], [d,e,f], [g,h,i] ] ).

a b c
d e f
g h i
```

A procedure that accomplishes this is:

```
writelist2( [ ] ).

writelist2( [L | LL] ) :-
  doline( L), nl,
  writelist2( LL).

doline( [ ] ).

doline( [X | L] ) :-
  write( X), tab( 1),
  doline( L).
```

A list of integer numbers can be sometimes conveniently shown as a bar graph. The following procedure, **bars**, will display a list in this form, assuming that the numbers in the list are sufficiently small. An example of using **bars** is:

```
?- bars( [3,4,6,5] ).
***
****
******
*****
```


The **bars** procedure can be defined as follows:

```
bars( [ ] ).

bars( [N | L] ) :-
  stars( N), nl,
  bars( L).

stars( N) :-
  N > 0,
  write( *),
  N1 is N - 1,
  stars( N1).

stars( N) :-
  N =< 0.
```

6.2.3 Processing a file of terms

A typical sequence of goals to process a whole file, F, would look something like this:

```
..., see( F), processfile, see( user), ...
```

Here **processfile** is a procedure to read and process each term in F, one after another, until the end of the file is encountered. A typical schema for **processfile** is:

```
processfile :-
  read( Term),                    % Assuming Term not a variable
  process( Term).

process( end_of_file) :- !.       % All done

process( Term) :-
  treat( Term),                   % Process current item
  processfile.                    % Process rest of file
```

Here **treat(Term)** represents whatever is to be done with each term. An example would be a procedure to display on the terminal each term together with its consecutive number. Let us call this procedure **showfile**. It has to have an additional argument to count the terms read:

```
showfile( N) :-
  read( Term),
  show( Term, N).

show( end_of_file, _) :- !.

show( Term, N) :-
  write( N), tab( 2), write( Term), nl,
  N1 is N + 1,
  showfile( N1).
```

Exercises

6.1 Let f be a file of terms. Define a procedure

```
findterm( Term)
```

that displays on the terminal the first term in f that matches Term.

6.2 Let f be a file of terms. Write a procedure

```
findallterms( Term)
```

that displays on the terminal all the terms in f that match Term. Make sure that Term is not instantiated in the process (which could prevent its match with terms that occur later in the file).

6.3 Manipulating characters

A character is written on the current output stream with the goal

```
put( C)
```

where C is the ASCII code (a number between 0 and 127) of the character to be output. For example, the question

```
?- put( 65), put( 66), put( 67).
```

would cause the following output:

```
ABC
```

65 is the ASCII code of 'A', 66 of 'B', 67 of 'C'.

A single character can be read from the current input stream by the goal

```
get0( C)
```

This causes the current character to be read from the input stream, and the variable C becomes instantiated to the ASCII code of this character. A variation of the predicate get0 is get, which is used for reading non-blank characters. So the goal

```
get( C)
```

will cause the skipping over of all non-printable characters (blanks in particular) from the current input position in the input stream up to the first printable character. This character is then also read and C is instantiated to its ASCII code.

As an example of using predicates that transfer single characters let us define a procedure, squeeze, to do the following: read a sentence from the current input stream, and output the same sentence reformatted so that multiple blanks between words are replaced by single blanks. For simplicity we will assume that any input

sentence processed by squeeze ends with a full stop and that words are separated simply by one or more blanks, but no other character. An acceptable input is then:

```
The       robot tried      to pour wine out      of the       bottle.
```

The goal squeeze would output this in the form:

```
The robot tried to pour wine out of the bottle.
```

The squeeze procedure will have a similar structure to the procedures for processing files in the previous section. First it will read the first character, output this character, and then complete the processing depending on this character. There are three alternatives that correspond to the following cases: the character is either a full stop, a blank or a letter. The mutual exclusion of the three alternatives is achieved in the program by cuts:

```
squeeze :-
  get0( C),
  put( C),
  dorest( C).

dorest( 46) :- !.                 % 46 is ASCII for full stop, all done

dorest( 32) :- !,                 % 32 is ASCII for blank
  get( C),                        % Skip other blanks
  put( C),
  dorest( C).

dorest( Letter) :-
  squeeze.
```

Exercise

6.3 Generalize the squeeze procedure to handle commas as well. All blanks immediately preceding a comma are to be removed, and we want to have one blank after each comma.

6.4 Constructing and decomposing atoms

It is often desirable to have information, read as a sequence of characters, represented in the program as an atom. There is a built-in predicate, name, which can be used to this end. name relates atoms and their ASCII encodings. Thus,

```
name( A, L)
```

is true if L is the list of ASCII codes of the characters in A. For example,

```
name( zx232, [122,120,50,51,50] )
```

is true. There are two typical uses of name:

(1) given an atom, break it down into single characters;

(2) given a list of characters, combine them into an atom.

An example of the first kind of application would be a program that deals with orders, taxis and drivers. These would be, in the program, represented by atoms such as:

```
order1, order2, driver1, driver2, taxia1, taxilux
```

The following predicate:

```
taxi( X)
```

tests whether an atom X represents a taxi:

```
taxi( X) :-
  name( X, Xlist),
  name( taxi, Tlist),
  conc( Tlist, _, Xlist).          % Is word 'taxi' prefix of X?
```

Predicates order and driver can be defined analogously.

The next example illustrates the use of combining characters into atoms. We will define a predicate:

```
getsentence( Wordlist)
```

that reads a free-form natural language sentence and instantiates Wordlist to some internal representation of the sentence. A natural choice for the internal representation, which would enable further processing of the sentence, is this: each word of the input sentence is represented as a Prolog atom; the whole sentence is represented as a list of atoms. For example, if the current input stream is:

Mary was pleased to see the robot fail.

then the goal getsentence(Sentence) will cause the instantiation:

```
Sentence = [ 'Mary', was, pleased, to, see, the, robot, fail]
```

For simplicity, we will assume that each sentence terminates with a full stop and that there are no punctuation symbols within the sentence.

The program is shown in Figure 6.2. The procedure getsentence first reads the current input character, Char, and then supplies this character to the procedure getrest to complete the job. getrest has to react properly according to three cases:

(1) Char is the full stop: then everything has been read.

(2) Char is the blank: ignore it, getsentence from rest of input.


```
/*
  Procedure getsentence reads in a sentence and combines the words into a list of atoms.
  For example
    getsentence( Wordlist)
  produces
    Wordlist = [ 'Mary', was, pleased, to, see, the, robot, fail]
  if the input sentence is:
    Mary was pleased to see the robot fail.
*/

getsentence( Wordlist) :-
  get0( Char),
  getrest( Char, Wordlist).

getrest( 46, [] ) :- !.                       % End of sentence: 46 = ASCII for '.'

getrest( 32, Wordlist) :- !,                  % 32 = ASCII for blank
  getsentence( Wordlist).                     % Skip the blank

getrest( Letter, [Word | Wordlist] ) :-
  getletters( Letter, Letters, Nextchar),     % Read letters of current word
  name( Word, Letters),
  getrest( Nextchar, Wordlist).

getletters( 46, [], 46) :- !.                 % End of word: 46 = full stop

getletters( 32, [], 32) :- !.                 % End of word: 32 = blank

getletters( Let, [Let | Letters], Nextchar) :-
  get0( Char),
  getletters( Char, Letters, Nextchar).
```

Figure 6.2 A procedure to transform a sentence into a list of atoms.

(3) Char is a letter: first read the word, Word, which begins with Char, and then use getsentence to read the rest of the sentence, producing Wordlist. The cumulative result is the list [Word | Wordlist].

The procedure that reads the characters of one word is:

```
getletters( Letter, Letters, Nextchar)
```

The three arguments are:

(1) Letter is the current letter (already read) of the word being read.

(2) Letters is the list of letters (starting with Letter) up to the end of the word.

(3) Nextchar is the input character that immediately follows the word read. Nextchar must be a non-letter character.

We conclude this example with a comment on the possible use of the getsentence procedure. It can be used in a program to process text in natural language. Sentences represented as lists of words are in a form that is suitable for further processing in Prolog. A simple example is to look for certain keywords in input sentences. A much more difficult task would be to understand the sentence; that is, to extract from the sentence its meaning, represented in some chosen formalism. This is an important research area of Artificial Intelligence, and is introduced in Chapter 21.

Exercises

6.4 Define the relation

```
starts( Atom, Character)
```

to check whether Atom starts with Character.

6.5 Define the procedure plural that will convert nouns into their plural form. For example:

```
?- plural( table, X).
X = tables
```

6.6 Write the procedure

```
search( KeyWord, Sentence)
```

that will, each time it is called, find a sentence in the current input file that contains the given KeyWord. Sentence should be in its original form, represented as a sequence of characters or as an atom (procedure getsentence of this section can be accordingly modified).

6.5 Reading programs

We can communicate our programs to the Prolog system by means of built-in predicates that *consult* or *compile* files with programs. The details of 'consulting' and compiling files depend on the implementation of Prolog. Here we look at some basic facilities that are available in many Prologs.

We tell Prolog to read a program from a file F with a goal of the form consult(F), for example:

```
?- consult( program3).
```

Depending on the implementation, the file name program3 will possibly have to have an extension indicating that it is a Prolog program file. The effect of this goal

will be that all the clauses in file program3 are read and loaded into the memory. So they will be used by Prolog when answering further questions from the user. Another file may be 'consulted' at some later time during the same session. Basically, the effect is again that the clauses from this new file are added into the memory. However, details depend on the implementation and other circumstances. If the new file contains clauses about a procedure defined in the previously consulted file, then the new clauses may be simply added at the end of the current set of clauses, or the previous definition of this procedure may be entirely replaced by the new one.

Several files may be consulted by the same consult goal, for example:

```
?- consult( [ program3, program4, queens]).
```

Such a question can also be written more simply as:

```
?- [ program3, program4, queens].
```

Consulted programs are used by a Prolog *interpreter*. If a Prolog implementation also features a *compiler*, then programs can be loaded in a compiled form. This enables more efficient execution with a typical speed-up factor of 5 or 10 between the interpreted and compiled code. Programs are loaded into memory in the compiled form by the built-in predicate compile, for example:

```
?- compile( program3).
```

or

```
?- compile( [ program4, queens, program6]).
```

Compiled programs are more efficiently executed, but interpreted programs are easier to debug because they can be inspected and traced by Prolog's debugging facilities. Therefore an interpreter is typically used in the program development phase, and a compiler is used with the final program.

It should be noted, again, that the details of consulting and compiling files depend on the implementation of Prolog. Usually a Prolog implementation also allows the user to enter and edit the program interactively.

Summary

- Input and output (other than that associated with querying the program) is done using built-in procedures. This chapter introduced a simple and practical repertoire of such procedures that can be found in many Prolog implementations.
- This repertoire assumes that files are sequential. There is the *current input stream* and the *current output stream*. The user terminal is treated as a file called user.

- Switching between streams is done by:

see(File)	File becomes the current input stream
tell(File)	File becomes the current output stream
seen	close the current input stream
told	close the current output stream

- Files are read and written in two ways:

  as sequences of characters
  as sequences of terms

  Built-in procedures for reading and writing characters and terms are:

read(Term)	input next term
write(Term)	output Term
put(CharCode)	output character with the given ASCII code
get0(CharCode)	input next character
get(CharCode)	input next 'printable' character

- Two procedures help formatting:

nl	output new line
tab(N)	output N blanks

- The procedure name(Atom, CodeList) decomposes and constructs atoms. CodeList is the list of ASCII codes of the characters in Atom.
- Many Prolog implementations provide additional facilities to handle non-sequential files, windows, provide graphics primitives, input information from the mouse, etc.

Reference to Prolog standard

For some of the predicates mentioned in this chapter, ISO standard for Prolog (Deransart *et al.* 1996) recommends different names from those used in most Prolog implementations. However, the predicates are conceptually the same, so compatibility is only a matter of renaming. The concerned predicates in this chapter are: see(Filename), tell(Filename), get(Code), put(Code), name(Atom,CodeList). The corresponding predicate names in the standard are: set_input(Filename), set_output(Filename), get_code(Code), put_code(Code), atom_codes(Atom,CodeList).

Deransart, P., Ed-Bdali, A. and Ceroni, L. (1996) *Prolog: The Standard*. Berlin: Springer-Verlag.

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chapter 7

More Built-in Predicates

In this chapter we will examine some more built-in predicates for advanced Prolog programming. These features enable the programming of operations that are not possible using only the features introduced so far. One set of such predicates manipulate terms: testing whether some variable has been instantiated to an integer, taking terms apart, constructing new terms, etc. Another useful set of procedures manipulates the 'database': they add new clauses to the program or remove existing ones.

The built-in predicates largely depend on the implementation of Prolog. However, the predicates discussed in this chapter are provided by many Prolog implementations. Various implementations may provide additional features.

7.1 Testing the type of terms

7.1.1 Predicates *var*, *nonvar*, *atom*, *integer*, *float*, *number*, *atomic*, *compound*

Terms may be of different types: variable, integer, atom, etc. If a term is a variable then it can be, at some point during the execution of the program, instantiated or uninstantiated. Further, if it is instantiated, its value can be an atom, a structure, etc. It is sometimes useful to know what the type of this value is. For example, we may want to add the values of two variables, X and Y, by:

```
Z is X + Y
```

Before this goal is executed, X and Y have to be instantiated to numbers. If we are not sure that X and Y will indeed be instantiated to numbers at this point then we should check this in the program before arithmetic is done.

To this end we can use the built-in predicate **number**. number(X) is true if X is a number or if it is a variable whose value is a number. We say that X must 'currently stand for' a number. The goal of adding X and Y can then be protected by the following test on X and Y:

```
..., number( X), number( Y), Z is X + Y, ...
```

If X and Y are not both numbers then no arithmetic will be attempted. So the number goals 'guard' the goal Z is X + Y before meaningless execution.

Built-in predicates of this sort are: **var, nonvar, atom, integer, float, number, atomic, compound**. Their meaning is as follows:

var(X)	succeeds if X is currently an uninstantiated variable
nonvar(X)	succeeds if X is not a variable, or X is an already instantiated variable
atom(X)	is true if X currently stands for an atom
integer(X)	is true if X currently stands for an integer
float(X)	is true if X currently stands for a real number
number(X)	is true if X currently stands for a number
atomic(X)	is true if X currently stands for a number or an atom
compound(X)	is true if X currently stands for a compound term (a structure)

The following example questions to Prolog illustrate the use of these built-in predicates:

```
?- var( Z), Z = 2.

Z = 2

?- Z = 2, var( Z).

no

?- integer( Z), Z = 2.

no

?- Z = 2, integer( Z), nonvar( Z).

Z = 2

?- atom( 3.14).

no
```



```
?- atomic( 3.14).

yes

?- atom( ==> ).

yes

?- atom( p(1) ).

no

?- compound ( 2 + X)

yes
```

We will illustrate the need for **atom** by an example. We would like to count how many times a given atom occurs in a given list of objects. To this purpose we will define a procedure:

```
count( A, L, N)
```

where A is the atom, L is the list and N is the number of occurrences. The first attempt to define **count** could be:

```
count( _, [], 0).

count( A, [A | L], N) :- !,
  count( A, L, N1),                % N1 = number of occurrences in tail
  N is N1 + 1.

count( A, [_ | L], N) :-
  count( A, L, N).
```

Now let us try to use this procedure on some examples:

```
?- count( a, [a,b,a,a], N).

N = 3

?- count( a, [a,b,X,Y], Na).

Na = 3

...

?- count( b, [a,b,X,Y], Nb).

Nb = 3

...

?- L = [a, b, X, Y], count( a, L, Na), count( b, L, Nb).

Na = 3
Nb = 1
X = a
Y = a
...
```

In the last example, X and Y both became instantiated to a and therefore we only got Nb = 1; but this is not what we had in mind. We are interested in the number of real occurrences of the given *atom*, and not in the number of terms that *match* this atom. According to this more precise definition of the count relation we have to check whether the head of the list is an atom. The modified program is as follows:

```
count( _, [ ], 0).

count( A, [B | L], N) :-
  atom( B), A = B, !,          % B is atom A?
  count( A, L, N1),            % Count in tail
  N is N1 + 1
  ;
  count( A, L, N).             % Otherwise just count the tail
```

The following, more complex programming exercise in solving cryptarithmetic puzzles makes use of the nonvar predicate.

7.1.2 A cryptarithmetic puzzle using *nonvar*

A popular example of a cryptarithmetic puzzle is

```
  D O N A L D
+ G E R A L D
  -----------
  R O B E R T
```

The problem here is to assign decimal digits to the letters D, O, N, etc., so that the above sum is valid. All letters have to be assigned different digits, otherwise trivial solutions are possible – for example, all letters equal zero.

We will define a relation

```
sum( N1, N2, N)
```

where N1, N2 and N represent the three numbers of a given cryptarithmetic puzzle. The goal sum(N1, N2, N) is true if there is an assignment of digits to letters such that N1 + N2 = N.

The first step toward a solution is to decide how to represent the numbers N1, N2 and N in the program. One way of doing this is to represent each number as a list of decimal digits. For example, the number 225 would be represented by the list [2,2,5]. As these digits are not known in advance, an uninstantiated variable will stand for each digit. Using this representation, the problem can be depicted as:

```
  [ D, O, N, A, L, D ]
+ [ G, E, R, A, L, D ]
= [ R, O, B, E, R, T ]
```


Number1 = $[D_{11}, D_{12}, \ldots, D_{1i}, \ldots]$
Number2 = $[D_{21}, D_{22}, \ldots, D_{2i}, \ldots]$
Number3 = $[D_{31}, D_{32}, \ldots, D_{3i}, \ldots]$

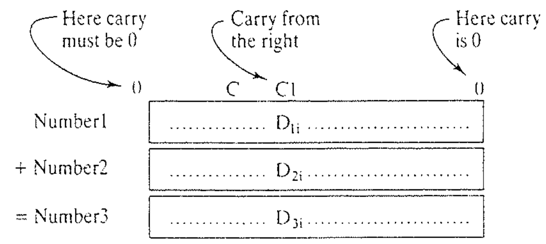

Figure 7.1 Digit by digit summation. The relations at the indicated ith digit position are: $D_{3i} = (C1 + D_{1i} + D_{2i}) \bmod 10$; $C = (C1 + D_{1i} + D_{2i}) \operatorname{div} 10$.

The task is to find such an instantiation of the variables D, O, N, etc., for which the sum is valid. When the sum relation has been programmed, the puzzle can be stated to Prolog by the question:

```
?- sum( [D,O,N,A,L,D], [G,E,R,A,L,D], [R,O,B,E,R,T] ).
```

To define the sum relation on lists of digits, we have to implement the actual rules for doing summation in the decimal number system. The summation is done digit by digit, starting with the right-most digits, continuing toward the left, always taking into account the carry digit from the right. It is also necessary to maintain a set of available digits; that is, digits that have not yet been used for instantiating variables already encountered. So, in general, besides the three numbers N1, N2 and N, some additional information is involved, as illustrated in Figure 7.1:

- carry digit before the summation of the numbers;
- carry digit after the summation;
- set of digits available before the summation;
- remaining digits, not used in the summation.

To formulate the sum relation we will use, once again, the principle of generalization of the problem: we will introduce an auxiliary, more general relation, sum1. sum1 has some extra arguments, which correspond to the foregoing additional information:

```
sum1( N1, N2, N, C1, C, Digits1, Digits)
```

N1, N2 and N are our three numbers, as in the sum relation, C1 is carry from the right (before summation of N1 and N2), and C is carry to the left (after the summation). The following example illustrates:

```
?- sum1( [H,E], [6,E], [U,S], 1, 1, [1,3,4,7,8,9], Digits).

H = 8
E = 3
S = 7
U = 4
Digits = [1,9]
```

This corresponds to the following summation:

```
1 ←   ← 1
   8 3
   6 3
   ---
   4 7
```

As Figure 7.1 shows, C1 and C have to be 0 if N1, N2 and N are to satisfy the sum relation. Digits1 is the list of available digits for instantiating the variables in N1, N2 and N; Digits is the list of digits that were not used in the instantiation of these variables. Since we allow the use of any decimal digit in satisfying the sum relation, the definition of sum in terms of sum1 is as follows:

```
sum( N1, N2, N) :-
  sum1( N1, N2, N, 0, 0, [0,1,2,3,4,5,6,7,8,9], _).
```

The burden of the problem has now shifted to the sum1 relation. This relation is, however, general enough that it can be defined recursively. We will assume, without loss of generality, that the three lists representing the three numbers are of equal length. Our example problem, of course, satisfies this constraint; if not, a 'shorter' number can be prefixed by zeros.

The definition of sum1 can be divided into two cases:

(1) The three numbers are represented by empty lists. Then:

```
sum1( [ ], [ ], [ ], C, C, Digs, Digs).
```

(2) All three numbers have some left-most digit and the remaining digits on their right. So they are of the form:

```
[D1 | N1],    [D2 | N2],    [D | N]
```

In this case two conditions must be satisfied:

(a) The three numbers N1, N2 and N have to satisfy the sum1 relation, giving some carry digit, C2, to the left, and leaving some unused subset of decimal digits, Digs2.

(b) The left-most digits D1, D2 and D, and the carry digit C2 have to satisfy the relation indicated in Figure 7.1: C2, D1 and D2 are added giving D and a carry to the left. This condition will be formulated in our program as a relation digitsum.

Translating this case into Prolog we have:

```
sum1( [D1 | N1], [D2 | N2], [D | N], C1, C, Digs1, Digs) :-
  sum1( N1, N2, N, C1, C2, Digs1, Digs2),
  digitsum( D1, D2, C2, D, C, Digs2, Digs).
```

It only remains to define the digitsum relation in Prolog. There is one subtle detail that involves the use of the metalogical predicate nonvar. D1, D2 and D have to be decimal digits. If any of them is not yet instantiated then it has to become instantiated to one of the digits in the list Digs2. This digit has to be deleted from the set of available digits. If D1, D2 or D is already instantiated then, of course, none

```
% Solving cryptarithmetic puzzles

sum( N1, N2, N) :-                          % Numbers represented as lists of digits
  sum1( N1, N2, N,
        0, 0,                               % Carries from right and to left both 0
        [0,1,2,3,4,5,6,7,8,9], _).          % All digits available

sum1( [ ], [ ], [ ], C, C, Digits, Digits).

sum1( [D1 | N1], [D2 | N2], [D | N], C1, C, Digs1, Digs) :-
  sum1( N1, N2, N, C1, C2, Digs1, Digs2),
  digitsum( D1, D2, C2, D, C, Digs2, Digs).

digitsum( D1, D2, C1, D, C, Digs1, Digs) :-
  del_var( D1, Digs1, Digs2),               % Select an available digit for D1
  del_var( D2, Digs2, Digs3),               % Select an available digit for D2
  del_var( D, Digs3, Digs),                 % Select an available digit for D
  S is D1 + D2 + C1,
  D is S mod 10,                            % Remainder
  C is S // 10.                             % Integer division

del_var( A, L, L) :-
  nonvar( A), !.                            % A already instantiated

del_var( A, [A | L], L).                    % Delete the head

del_var( A, [B | L], [B | L1] ) :-
  del_var( A, L, L1).                       % Delete from tail

% Some puzzles

puzzle1( [D,O,N,A,L,D],
         [G,E,R,A,L,D],
         [R,O,B,E,R,T] ).

puzzle2( [0,S,E,N,D],
         [0,M,O,R,E],
         [M,O,N,E,Y] ).
```

Figure 7.2 A program for cryptarithmetic puzzles.

of the available digits will be spent. This is realized in the program as a non-deterministic deletion of an item from a list. If this item is non-variable then nothing is deleted (no instantiation occurs). This is programmed as:

```
del_var( Item, List, List) :-
  nonvar( Item), !.                     % Item already instantiated

del_var( Item, [Item | List], List).    % Delete the head

del_var( Item, [A | List], [A | List1] ) :-
  del_var( Item, List, List1).          % Delete Item from tail
```

A complete program for cryptarithmetic puzzles is shown in Figure 7.2. The program also includes the definition of two puzzles. The question to Prolog about DONALD, GERALD and ROBERT, using this program, would be:

```
?- puzzle1( N1, N2, N), sum( N1, N2, N).
```

Sometimes this puzzle is made easier by providing part of the solution as an additional constraint that D be equal 5. The puzzle in this form could be communicated to Prolog using sum1:

```
?- sum1( [5,O,N,A,L,5],
         [G,E,R,A,L,5],
         [R,O,B,E,R,T],
         0, 0, [0,1,2,3,4,6,7,8,9], _).
```

It is interesting that in both cases there is only one solution. That is, there is only one way of assigning digits to letters.

Exercises

7.1 Write a procedure simplify to symbolically simplify summation expressions with numbers and symbols (lower-case letters). Let the procedure rearrange the expressions so that all the symbols precede numbers. These are examples of its use:

```
?- simplify( 1 + 1 + a, E).
E = a + 2

?- simplify( 1 + a + 4 + 2 + b + c, E).
E = a + b + c + 7

?- simplify( 3 + x + x, E).
E = 2*x + 3
```

7.2 Define the procedure:

```
add_to_tail( Item, List)
```


to store a new element into a list. Assume that all of the elements that can be stored are non-variables. List contains all the stored elements followed by a tail that is not instantiated and can thus accommodate new elements. For example, let the existing elements stored be a, b and c. Then

```
List = [a, b, c | Tail]
```

where Tail is a variable. The goal

```
add_to_tail( d, List)
```

will cause the instantiation

```
Tail = [d | NewTail]   and   List = [a, b, c, d | NewTail]
```

Thus the structure can, in effect, grow by accepting new items. Define also the corresponding membership relation.

7.2 Constructing and decomposing terms: =.., *functor*, *arg*, *name*

There are three built-in predicates for decomposing terms and constructing new terms: functor, arg and '=..'. We will first look at =.., which is written as an infix operator and reads as 'univ'. The goal

```
Term =.. L
```

is true if L is a list that contains the principal functor of Term, followed by its arguments. The following examples illustrate:

```
?- f( a, b) =.. L.
L = [f, a, b]

?- T =.. [rectangle, 3, 5].
T = rectangle( 3, 5)

?- Z =.. [p, X, f(X,Y)].
Z = p( X, f(X,Y) )
```

Why would we want to decompose a term into its components – its functor and its arguments? Why construct a new term from a given functor and arguments? The following example illustrates the need for this.

Let us consider a program that manipulates geometric figures. Figures are squares, rectangles, triangles, circles, etc. They can, in the program, be represented as terms such that the functor indicates the type of figure, and the arguments specify the size of the figure, as follows:

```
square( Side)
triangle( Side1, Side2, Side3)
circle( R)
```

One operation on such figures can be enlargement. We can implement this as a three-argument relation

```
enlarge( Fig, Factor, Fig1)
```

where Fig and Fig1 are geometric figures of the same type (same functor), and the parameters of Fig1 are those of Fig multiplicatively enlarged by Factor. For simplicity, we will assume that all the parameters of Fig are already known; that is, instantiated to numbers, and so is Factor. One way of programming the enlarge relation is:

```
enlarge( square(A), F, square(A1) ) :-
  A1 is F*A.

enlarge( circle(R), F, circle(R1) ) :-
  R1 is F*R1.

enlarge( rectangle(A,B), F, rectangle(A1,B1) ) :-
  A1 is F*A, B1 is F*B.

...
```

This works, but it is awkward when there are many different figure types. We have to foresee all types that may possibly occur. Thus, we need an extra clause for each type although each clause says essentially the same thing: take the parameters of the original figure, multiply all the parameters by the factor, and make a figure of the same type with new parameters.

One (unsuccessful) attempt to handle, at least, all one-parameter figures with one clause could be:

```
enlarge( Type(Par), F, Type(Par1) ) :-
  Par1 is F*Par.
```

However, this is normally not allowed in Prolog because the functor has to be an atom; so the variable Type would not be accepted syntactically as a functor. The correct method is to use the predicate '=..'. Then the enlarge procedure can be stated completely generally, for any type of object, as follows:

```
enlarge( Fig, F, Fig1) :-
  Fig =.. [Type | Parameters],
  multiplylist( Parameters, F, Parameters1),
  Fig1 =.. [Type | Parameters1].

multiplylist( [], _, [] ).

multiplylist( [X | L], F, [X1 | L1] ) :-
  X1 is F*X, multiplylist( L, F, L1).
```

Our next example of using the '=..' predicate comes from symbolic manipulation of formulas where a frequent operation is to substitute some subexpression by another expression. We will define the relation

```
substitute( Subterm, Term, Subterm1, Term1)
```


as follows: if all occurrences of Subterm in Term are substituted by Subterm1 then we get Term1. For example:

```
?- substitute( sin(x), 2*sin(x)*f(sin(x)), t, F).

F = 2*t*f(t)
```

By 'occurrence' of Subterm in Term we will mean something in Term that *matches* Subterm. We will look for occurrences from top to bottom. So the goal

```
?- substitute( a+b, f( a, A+B), v, F).
```

will produce

```
F = f( a, v)                F = f( a, v+v)
A = a          and not      A = a+b
B = b                       B = a+b
```

In defining the substitute relation we have to consider the following decisions depending on the case:

If Subterm = Term then Term1 = Subterm1;
otherwise if Term is 'atomic' (not a structure)
then Term1 = Term (nothing to be substituted),
otherwise the substitution is to be carried out on the arguments of Term.

These rules can be converted into a Prolog program, shown in Figure 7.3.

```
% substitute( Subterm, Term, Subterm1, Term1):
%   if all occurrences of Subterm in Term are substituted with Subterm1 then we get Term1.

% Case 1: Substitute whole term

substitute( Term, Term, Term1, Term1) :- !.

% Case 2: Nothing to substitute if Term atomic

substitute( _, Term, _, Term) :-
  atomic( Term), !.

% Case 3: Do substitution on arguments

substitute( Sub, Term, Sub1, Term1) :-
  Term =.. [F | Args],                          % Get arguments
  substlist( Sub, Args, Sub1, Args1),           % Perform substitution on them
  Term1 =.. [F | Args1].

substlist( _, [], _, [] ).

substlist( Sub, [Term | Terms], Sub1, [Term1 | Terms1] ) :-
  substitute( Sub, Term, Sub1, Term1),
  substlist( Sub, Terms, Sub1, Terms1).
```

Figure 7.3 A procedure for substituting a subterm of a term by another subterm.

Terms that are constructed by the '=..' predicate can, of course, be also used as goals. The advantage of this is that the program itself can, during execution, generate and execute goals of forms that were not necessarily foreseen at the time of writing the program. A sequence of goals illustrating this effect would be something like the following:

```
obtain( Functor),
compute( Arglist),
Goal =.. [Functor | Arglist],
Goal
```

Here, **obtain** and **compute** are some user-defined procedures for getting the components of the goal to be constructed. The goal is then constructed by '=..', and invoked for execution by simply stating its name, Goal.

Some implementations of Prolog may require that all the goals, as they appear in the program, are *syntactically* either atoms or structures with an atom as the principal functor. Thus a variable, regardless of its eventual instantiation, in such a case may not be syntactically acceptable as a goal. This problem is circumvented by another built-in predicate, **call**, whose argument is the goal to be executed. Accordingly, the above example would be rewritten as:

```
...
Goal =.. [Functor | Arglist],
call( Goal)
```

Sometimes we may want to extract from a term just its principal functor or one of its arguments. We can, of course, use the '=..' relation. But it can be neater, and also more efficient, to use one of the other two built-in procedures for manipulating terms: **functor** and **arg**. Their meaning is as follows: a goal

```
functor( Term, F, N)
```

is true if F is the principal functor of Term and N is the arity of F. A goal

```
arg( N, Term, A)
```

is true if A is the Nth argument in Term, assuming that arguments are numbered from left to right starting with 1. The following examples illustrate:

```
?- functor( t( f(X), X, t), Fun, Arity).

Fun = t
Arity = 3

?- arg( 2, f( X, t(a), t(b)), Y).

Y = t(a)

?- functor( D, date, 3),
   arg( 1, D, 29),
   arg( 2, D, june),
   arg( 3, D, 1982).

D = date( 29, june, 1982)
```


The last example shows a special application of the **functor** predicate. The goal **functor(D, date, 3)** generates a 'general' term whose principal functor is **date** with three arguments. The term is general in that the three arguments are uninstantiated variables whose names are generated by Prolog. For example:

```
D = date( _5, _6, _7)
```

These three variables are then instantiated in the example above by the three arg goals.

Related to this set of built-in predicates is the predicate **name** for constructing/decomposing atoms, introduced in Chapter 6. We will repeat its meaning here for completeness.

```
name( A, L)
```

is true if L is the list of ASCII codes of the characters in atom A.

Exercises

7.3 Define the predicate **ground(Term)** so that it is true if **Term** does not contain any uninstantiated variables.

7.4 The **substitute** procedure of this section only produces the 'outer-most' substitution when there are alternatives. Modify the procedure so that all possible alternative substitutions are produced through backtracking. For example:

```
?- substitute( a+b, f(A+B), new, NewTerm).

A = a
B = b
NewTerm = f( new);

A = a+b
B = a+b
NewTerm = f( new+new)
```

Our original version only finds the first answer.

7.5 Define the relation

```
subsumes( Term1, Term2)
```

so that **Term1** is more general than **Term2**. For example:

```
?- subsumes( X, c).

yes

?- subsumes( g(X), g(t(Y)) ).

yes

?- subsumes( f(X,X), f(a,b) ).

no
```

7.3 Various kinds of equality and comparison

When do we consider two terms to be equal? Until now we have introduced three kinds of equality in Prolog. The first was based on matching, written as:

```
X = Y
```

This is true if X and Y match. Another type of equality was written as:

```
X is E
```

This is true if X matches the value of the arithmetic expression E. We also had:

```
E1 =:= E2
```

This is true if the values of the arithmetic expressions E1 and E2 are equal. In contrast, when the values of two arithmetic expressions are not equal, we write:

```
E1 =\= E2
```

Sometimes we are interested in a stricter kind of equality: the *literal equality* of two terms. This kind of equality is implemented as another built-in predicate written as an infix operator '==':

```
T1 == T2
```

This is true if terms T1 and T2 are identical; that is, they have exactly the same structure and all the corresponding components are the same. In particular, the names of the variables also have to be the same. The complementary relation is 'not identical', written as:

```
T1 \== T2
```

Here are some examples:

```
?- f( a, b) == f( a, b).
yes

?- f( a, b) == f( a, X).
no

?- f( a, X) == f( a, Y).
no

?- X \== Y.
yes

?- t( X, f(a,Y) ) == t( X, f(a,Y) ).
yes
```

As an example, let us redefine the relation

count(Term, List, N)

from Section 7.1. This time let N be the number of literal occurrences of the term Term in a list List:

```
count( _, [], 0).

count( Term, [Head | L ], N) :-
  Term == Head, !,
  count( Term, L, N1),
  N is N1 + 1
  ;
  count( Term, L, N).
```

We have already seen predicates that compare terms arithmetically, for example X+2 < 5. Another set of built-in predicates compare terms alphabetically and thus define an ordering relation on terms. For example, the goal

```
X @< Y
```

is read: term X precedes term Y. The precedence between simple terms is determined by alphabetical or numerical ordering. The precedence between structures is determined by the precedence of their principal functors. If the principal functors are equal, then the precedence between the top-most, left-most functors in the subterms in X and Y decides. Examples are:

```
?- paul @< peter.
yes

?- f(2) @< f(3).
yes

?- g(2) @< f(3).
no

?- g(2) @>= f(3).
yes

?- f( a, g(b), c) @< f( a, h(a), a).
yes
```

All the built-in predicates in this family are @<, @=<, @>, @>= with their obvious meanings.

7.4 Database manipulation

According to the relational model of databases, a database is a specification of a set of relations. A Prolog program can be viewed as such a database: the specification of relations is partly explicit (facts) and partly implicit (rules). Some built-in predicates make it possible to update this database during the execution of the program. This is done by adding (during execution) new clauses to the program or by deleting existing clauses. Predicates that serve these purposes are assert, asserta, assertz and retract.

A goal

```
assert( C)
```

always succeeds and, as its side effect, causes a clause C to be 'asserted' – that is, added to the database. A goal

```
retract( C)
```

does the opposite: it deletes a clause that matches C. The following conversation with Prolog illustrates:

```
?- crisis.
no
?- assert( crisis).
yes
?- crisis.
yes
?- retract( crisis).
yes
?- crisis.
no
```

Clauses thus asserted act exactly as part of the 'original' program. The following example shows the use of assert and retract as one method of handling changing situations. Let us assume that we have the following program about weather:

```
nice :-
  sunshine, not raining.

funny :-
  sunshine, raining.

disgusting :-
  raining, fog.

raining.

fog.
```

The following conversation with this program will gradually update the database:

```
?- nice.
no
```



```
?- disgusting.
yes
?- retract( fog).
yes
?- disgusting.
no
?- assert( sunshine).
yes
?- funny.
yes
?- retract( raining).
yes
?- nice.
yes
```

Clauses of any form can be asserted or retracted. However, depending on the implementation of Prolog, it may be required that predicates manipulated through assert/retract be declared as *dynamic*, using the directive dynamic(PredicateIndicator). Predicates that are only brought in by assert, and not by consult, are automatically assumed as dynamic.

The next example illustrates that retract is also non-deterministic: a whole set of clauses can, through backtracking, be removed by a single retract goal. Let us assume that we have the following facts in the 'consulted' program:

```
fast( ann).
slow( tom).
slow( pat).
```

We can add a rule to this program, as follows:

```
?- assert(
        ( faster(X,Y) :-
             fast(X), slow(Y) ) ).

yes

?- faster( A, B).

A = ann
B = tom

?- retract( slow(X) ).

X = tom;
X = pat;
no
```

```
?- faster( ann, _).
no
```

Notice that when a rule is asserted, the syntax requires that the rule (as an argument to assert) be enclosed in parentheses.

When asserting a clause, we may want to specify the position at which the new clause is inserted to the database. The predicates asserta and assertz enable us to control the position of insertion. The goal

```
asserta( C)
```

adds C at the beginning of the database. The goal

```
assertz( C)
```

adds C at the end of the database. We will assume that assert is equivalent to assertz, as usual in Prolog implementations. The following example illustrates these effects:

```
?- assert( p(b) ), assertz( p(c) ), assert( p(d) ), asserta( p(a) ).
yes

?- p( X).
X = a;
X = b;
X = c;
X = d
```

There is a relation between consult and assertz. Consulting a file can be defined in terms of assertz as follows: to consult a file, read each term (clause) in the file and assert it at the end of the database.

One useful application of asserta is to store already computed answers to questions. For example, let there be a predicate

```
solve( Problem, Solution)
```

defined in the program. We may now ask some question and request that the answer be remembered for future questions.

```
?- solve( problem1, Solution),
   asserta( solve( problem1, Solution) ).
```

If the first goal above succeeds then the answer (Solution) is stored and used, as any other clause, in answering further questions. The advantage of such a 'memoization' of answers is that a further question that matches the asserted fact will normally be answered much quicker than the first one. The result now will be simply retrieved as a fact, and not computed through a possibly time-consuming process. This technique of storing derived solutions is also called 'caching'.

An extension of this idea is to use asserting for generating all solutions in the form of a table of facts. For example, we can generate a table of products of all pairs

of integers between 0 and 9 as follows: generate a pair of integers X and Y, compute Z is X*Y, assert the three numbers as one line of the product table, and then force the failure. The failure will cause, through backtracking, another pair of integers to be found and so another line tabulated, etc. The following procedure maketable implements this idea:

```
maketable :-
  L = [0,1,2,3,4,5,6,7,8,9],
  member( X, L),                % Choose first factor
  member( Y, L),                % Choose second factor
  Z is X*Y,
  assert( product(X,Y,Z) ),
  fail.
```

The question

```
?- maketable.
```

will, of course, not succeed, but it will, as a side effect, add the whole product table to the database. After that we can ask, for example, what pairs give the product 8:

```
?- product( A, B, 8).
A = 1
B = 8;

A = 2
B = 4;

...
```

A remark on the style of programming should be made at this stage. The foregoing examples illustrate some obviously useful applications of assert and retract. However, their use requires special care. Excessive and careless use of these facilities cannot be recommended as good programming style. By asserting and retracting we, in fact, modify the program. Therefore relations that hold at some point will not be true at some other time. At different times the same questions receive different answers. A lot of asserting and retracting may thus obscure the meaning of the program. The resulting behaviour of the program may become difficult to understand, difficult to explain and to trust.

Exercises

7.6 (a) Write a Prolog question to remove the whole product table from the database.

(b) Modify the question so that it only removes those entries where the product is 0.

7.7 Define the relation

```
copy_term( Term, Copy)
```

which will produce a copy of Term so that Copy is Term with all its variables renamed. This can be easily programmed by using asserta and retract. In some Prologs copy_term is provided as a built-in predicate.

7.5 Control facilities

So far we have covered most of the extra control facilities except repeat. For completeness the complete set is presented here.

- *cut*, written as '!', prevents backtracking. It was introduced in Chapter 5. A useful predicate is once(P) defined in terms of cut as:

```
once( P) :- P, !.
```

  once(P) produces one solution only. The cut, nested in once, does not prevent backtracking in other goals.
- fail is a goal that always fails.
- true is a goal that always succeeds.
- not(P) is negation as failure that behaves exactly as if defined as:

```
not( P) :- P, !, fail; true.
```

  Some problems with cut and not were discussed in detail in Chapter 5.
- call(P) invokes a goal P. It succeeds if P succeeds.
- repeat is a goal that always succeeds. Its special property is that it is non-deterministic; therefore, each time it is reached by backtracking it generates another alternative execution branch. repeat behaves as if defined by:

```
repeat.
repeat :- repeat.
```

  A typical way of using repeat is illustrated by the following procedure dosquares which reads a sequence of numbers and outputs their squares. The sequence is concluded with the atom stop, which serves as a signal for the procedure to terminate.

```
dosquares :-
  repeat,
  read( X),
  ( X = stop, !
   ;
   Y is X*X, write(Y),
   fail
  ).
```


7.6 *bagof*, *setof* and *findall*

We can generate, by backtracking, all the objects, one by one, that satisfy some goal. Each time a new solution is generated, the previous one disappears and is not accessible any more. However, sometimes we would prefer to have all the generated objects available together – for example, collected into a list. The built-in predicates bagof, setof and findall serve this purpose.

The goal

```
bagof( X, P, L)
```

will produce the list L of all the objects X such that a goal P is satisfied. Of course, this usually makes sense only if X and P have some common variables. For example, let us have these facts in the program:

```
age( peter, 7).
age( ann, 5).
age( pat, 8).
age( tom, 5).
```

Then we can obtain the list of all the children of age 5 by the goal:

```
?- bagof( Child, age( Child, 5), List).

List = [ ann, tom]
```

If, in the above goal, we leave the age unspecified, then we get, through backtracking, three lists of children, corresponding to the three age values:

```
?- bagof( Child, age( Child, Age), List).

Age = 7
List = [ peter];

Age = 5
List = [ ann, tom];

Age = 8
List = [ pat];

no
```

We may prefer to have all of the children in one list regardless of their age. This can be achieved by explicitly stating in the call of bagof that we do not care about the value of Age as long as such a value exists. This is stated as:

```
?- bagof( Child, Age ^ age( Child, Age), List).

List = [ peter, ann, pat, tom]
```

Syntactically, '^' is a predefined infix operator of type xfy.

If there is no solution for P in the goal bagof(X, P, L), then the bagof goal simply fails. If the same object X is found repeatedly, then all of its occurrences will appear in L, which leads to duplicate items in L.

The predicate setof is similar to bagof. The goal

```
setof( X, P, L)
```

will again produce a list L of objects X that satisfy P. Only this time the list L will be ordered, and duplicate items, if there are any, will be eliminated. The ordering of the objects is according to built-in predicate @<, which defines the precedence among terms. For example:

```
?- setof( Child, Age ^ age( Child, Age), ChildList),
   setof( Age, Child ^ age( Child, Age), AgeList).

ChildList = [ ann, pat, peter, tom]
AgeList = [ 5, 7, 8]
```

There is no restriction on the kind of objects that are collected. So we can, for example, construct the list of children ordered by their age, by collecting pairs of the form Age/Child:

```
?- setof( Age/Child, age( Child, Age), List).

List = [ 5/ann, 5/tom, 7/peter, 8/pat]
```

Another predicate of this family, similar to bagof, is findall.

```
findall( X, P, L)
```

produces, again, a list of objects that satisfy P. The difference with respect to bagof is that *all* of the objects X are collected regardless of (possibly) different solutions for variables in P that are not shared with X. This difference is shown in the following example:

```
?- findall( Child, age( Child, Age), List).

List = [ peter, ann, pat, tom]
```

If there is no object X that satisfies P then findall will succeed with L = [ ].

If findall is not available as a built-in predicate in the implementation used then it can be easily programmed as follows. All solutions for P are generated by forced backtracking. Each solution is, when generated, immediately asserted into the database so that it is not lost when the next solution is found. After all the solutions have been generated and asserted, they have to be collected into a list and retracted from the database. This whole process can be imagined as all the solutions generated forming a queue. Each newly generated solution is, by assertion, added to the end of this queue. When the solutions are collected the queue dissolves. Note, in addition, that the end of this queue has to be marked, for example, by the atom 'bottom' (which, of course, should be different from any solution that is possibly expected). An implementation of findall along these lines is shown as Figure 7.4.


```
findall( X, Goal, Xlist) :-
  call( Goal),                          % Find a solution
  assertz( queue(X) ),                  % Assert it
  fail;                                 % Try to find more solutions
  assertz( queue(bottom) ),             % Mark end of solutions
  collect( Xlist).                      % Collect the solutions

collect( L) :-
  retract( queue(X) ), !,               % Retract next solution
  ( X == bottom, !, L = []              % End of solutions?
    ;
    L = [X | Rest], collect( Rest) ).   % Otherwise collect the rest
```

Figure 7.4 An implementation of the **findall** relation.

Exercises

7.8 Use bagof to define the relation powerset(Set, Subsets) to compute the set of all subsets of a given set (all sets represented as lists).

7.9 Use bagof to define the relation

```
copy_term( Term, Copy)
```

such that Copy is Term with all its variables renamed.

Summary

- A Prolog implementation normally provides a set of built-in procedures to accomplish several useful operations that are not possible in pure Prolog. In this chapter, such a set of predicates available in many Prolog implementations was introduced.
- The type of a term can be tested by the following predicates:

var(X)	X is a (non-instantiated) variable
nonvar(X)	X is not a variable
atom(X)	X is an atom
integer(X)	X is an integer
float(X)	X is a real number
atomic(X)	X is either an atom or a number
compound(X)	X is a structure

- Terms can be constructed or decomposed:

```
Term =.. [ Functor | ArgumentList]
functor( Term, Functor, Arity)
arg( N, Term, Argument)
name( Atom, CharacterCodes)
```

- Terms can be compared:

X = Y	X and Y match
X == Y	X and Y are identical
X \== Y	X and Y are not identical
X =:= Y	X and Y are arithmetically equal
X =\= Y	X and Y are not arithmetically equal
X < Y	arithmetic value of X is less than Y (related: =<, >, >=)
X @< Y	term X precedes term Y (related: @=<, @>, @>=)

- A Prolog program can be viewed as a relational database that can be updated by the following procedures:

assert(Clause)	add Clause to the program
asserta(Clause)	add at the beginning
assertz(Clause)	add at the end
retract(Clause)	remove a clause that matches Clause

- All the objects that satisfy a given condition can be collected into a list by the predicates:

bagof(X, P, L)	L is the list of all X that satisfy condition P
setof(X, P, L)	L is the sorted list of all X that satisfy condition P
findall(X, P, L)	similar to bagof

- repeat is a control facility that generates an unlimited number of alternatives for backtracking.

chapter 8

Programming Style and Technique

In this chapter we will review some general principles of good programming and discuss the following questions in particular: How to think about Prolog programs? What are elements of good programming style in Prolog? How to debug Prolog programs? How to make Prolog programs more efficient?

8.1 General principles of good programming

What is a good program? Answering this question is not trivial as there are several criteria for judging how good a program is. Generally accepted criteria include the following:

- *Correctness* Above all, a good program should be correct. That is, it should do what it is supposed to do. This may seem a trivial, self-explanatory requirement. However, in the case of complex programs, correctness is often not attained. A common mistake when writing programs is to neglect this obvious criterion and pay more attention to other criteria, such as efficiency or external glamour of the program.
- *User-friendliness* A good program should be easy to use and interact with.
- *Efficiency* A good program should not needlessly waste computer time and memory space.

- *Readability* A good program should be easy to read and easy to understand. It should not be more complicated than necessary. Clever programming tricks that obscure the meaning of the program should be avoided. The general organization of the program and its layout help its readability.
- *Modifiability* A good program should be easy to modify and to extend. Transparency and modular organization of the program help modifiability.
- *Robustness* A good program should be robust. It should not crash immediately when the user enters some incorrect or unexpected data. The program should, in the case of such errors, stay 'alive' and behave reasonably (should report errors).
- *Documentation* A good program should be properly documented. The minimal documentation is the program's listing including sufficient program comments.

The importance of particular criteria depends on the problem and on the circumstances in which the program is written, and on the environment in which it is used. There is no doubt that correctness has the highest priority. The issues of readability, user-friendliness, modifiability, robustness and documentation are usually given, at least, as much priority as the issue of efficiency.

There are some general guidelines for practically achieving the above criteria. One important rule is to first *think* about the problem to be solved, and only then to start writing the actual code in the programming language used. Once we have developed a good understanding of the problem and the solution is well thought through, the actual coding will be fast and easy, and there is a good chance that we will soon get a correct program.

A common mistake is to start writing the code even before the full definition of the problem has been understood. A fundamental reason why early coding is bad practice is that the thinking about the problem and the ideas for a solution should be done in terms that are most relevant to the problem. These terms are usually far from the syntax of the programming language used, and they may include natural language statements and pictorial representation of ideas.

Such a formulation of the solution will have to be transformed into the programming language, but this transformation process may not be easy. A good approach is to use the principle of *stepwise refinement*. The initial formulation of the solution is referred to as the 'top-level solution', and the final program as the 'bottom-level solution'.

According to the principle of stepwise refinement, the final program is developed through a sequence of transformations, or 'refinements', of the solution. We start with the first, top-level solution and then proceed through a sequence of solutions; these are all equivalent, but each solution in the sequence is expressed in more detail. In each refinement step, concepts used in previous formulations are elaborated to greater detail and their representation gets closer to the programming language. It should be realized that refinement applies both to procedure definitions and to data structures. In the initial stages we normally work with more abstract, bulky units of information whose structure is refined later.

Such a strategy of top-down stepwise refinement has the following advantages:

- it allows for formulation of rough solutions in terms that are most relevant to the problem;
- in terms of such powerful concepts, the solution should be succinct and simple, and therefore likely to be correct;
- each refinement step should be small enough so that it is intellectually manageable; if so, the transformation of a solution into a new, more detailed representation is likely to be correct, and so is the resulting solution at the next level of detail.

In the case of Prolog we may talk about the stepwise refinement of *relations*. If the problem suggests thinking in algorithmic terms, then we can also talk about refinement of *algorithms*, adopting the procedural point of view in Prolog.

In order to properly refine a solution at some level of detail, and to introduce useful concepts at the next lower level, we need ideas. Therefore programming is creative, especially so for beginners. With experience, programming gradually becomes less of an art and more of a craft. But, nevertheless, a major question is: How do we get ideas? Most ideas come from experience, from similar problems whose solutions we know. If we do not know a direct programming solution, another similar problem could be helpful. Another source of ideas is everyday life. For example, if the problem is to write a program to sort a list of items we may get an idea from considering the question: How would I myself sort a set of exam papers according to the alphabetical order of students?

General principles of good programming outlined in this section basically apply to Prolog as well. We will discuss some details with particular reference to Prolog in the following sections.

8.2 How to think about Prolog programs

One characteristic feature of Prolog is that it allows for both the procedural and declarative way of thinking about programs. The two approaches have been discussed in detail in Chapter 2, and illustrated by examples throughout the text. Which approach will be more efficient and practical depends on the problem. Declarative solutions are usually easier to develop, but may lead to an inefficient program.

During the process of developing a solution we have to find ideas for reducing problems to one or more easier subproblems. An important question is: How do we find proper subproblems? There are several general principles that often work in Prolog programming. These will be discussed in the following sections.

8.2.1 Use of recursion

The principle here is to split the problem into cases belonging to two groups:

(1) trivial, or 'boundary' cases;

(2) 'general' cases where the solution is constructed from solutions of (simpler) versions of the original problem itself.

In Prolog we use this technique all the time. Let us look at one more example: processing a list of items so that each item is transformed by the same transformation rule. Let this procedure be

```
maplist( List, F, NewList)
```

where List is an original list, F is a transformation rule (a binary relation) and NewList is the list of all transformed items. The problem of transforming List can be split into two cases:

(1) Boundary case: List = [ ]

if List = [ ] then NewList = [ ], regardless of F

(2) General case: List = [X | Tail]

To transform a list of the form [X | Tail], do:
transform the item X by rule F obtaining NewX, and
transform the list Tail obtaining NewTail;
the whole transformed list is [NewX | NewTail].

In Prolog:

```
maplist( [], _, [] ).

maplist( [X | Tail], F, [NewX | NewTail] ) :-
  G =.. [F, X, NewX],
  call( G),
  maplist( Tail, F, NewTail).
```

Suppose we have a list of numbers and want to compute the list of their squares. maplist can be used for this as follows:

```
square( X, Y) :-
  Y is X*X.

?- maplist( [2, 6, 5], square, Squares).
Squares = [ 4, 36, 25]
```


One reason why recursion so naturally applies to defining relations in Prolog is that data objects themselves often have recursive structure. Lists and trees are such objects. A list is either empty (boundary case) or has a head and a tail that is itself a list (general case). A binary tree is either empty (boundary case) or it has a root and two subtrees that are themselves binary trees (general case). Therefore, to process a whole non-empty tree, we must do something with the root, and process the subtrees.

8.2.2 Generalization

It is often a good idea to generalize the original problem, so that the solution to the generalized problem can be formulated recursively. The original problem is then solved as a special case of its more general version. Generalization of a relation typically involves the introduction of one or more extra arguments. A major problem, which may require deeper insight into the problem, is how to find the right generalization.

As an example let us revisit the eight queens problem. The original problem was to place eight queens on the chessboard so that they do not attack each other. Let us call the corresponding relation:

```
eightqueens( Pos)
```

This is true if Pos is a position with eight non-attacking queens. A good idea in this case is to generalize the number of queens from eight to N. The number of queens now becomes the additional argument:

```
nqueens( Pos, N)
```

The advantage of this generalization is that there is an immediate recursive formulation of the nqueens relation:

(1) Boundary case: $N = 0$

To safely place zero queens is trivial.

(2) General case: $N > 0$

To safely place N queens on the board, satisfy the following:

- achieve a safe configuration of $(N - 1)$ queens; and
- add the remaining queen so that she does not attack any other queen.

Once the generalized problem has been solved, the original problem is easy:

```
eightqueens( Pos) :- nqueens( Pos, 8).
```

8.2.3 Using pictures

When searching for ideas about a problem, it is often useful to introduce some graphical representation of the problem. A picture may help us to perceive some essential relations in the problem. Then we just have to describe what we *see* in the picture in the programming language.

The use of pictorial representations is often useful in problem solving in general; it seems, however, that it works with Prolog particularly well. The following arguments explain why:

(1) Prolog is particularly suitable for problems that involve objects and relations between objects. Often, such problems can be naturally illustrated by graphs in which nodes correspond to objects and arcs correspond to relations.

(2) Structured data objects in Prolog are naturally pictured as trees.

(3) The declarative meaning of Prolog facilitates the translation of pictorial representations into Prolog because, in principle, the order in which the picture is described does not matter. We just put what we see into the program in any order. (For practical reasons of the program's efficiency this order will possibly have to be polished later.)

8.3 Programming style

The purpose of conforming to some stylistic conventions is:

- to reduce the danger of programming errors; and
- to produce programs that are readable and easy to understand, easy to debug and to modify.

We will review here some ingredients of good programming style in Prolog: some general rules of good style, tabular organization of long procedures and commenting.

8.3.1 Some rules of good style

- Program clauses should be short. Their body should typically contain no more than a few goals.
- Procedures should be short because long procedures are hard to understand. However, long procedures are acceptable if they have some uniform structure (this will be discussed later in this section).
- Mnemonic names for procedures and variables should be used. Names should indicate the meaning of relations and the role of data objects.

- The layout of programs is important. Spacing, blank lines and indentation should be consistently used for the sake of readability. Clauses about the same procedure should be clustered together; there should be blank lines between clauses (unless, perhaps, there are numerous facts about the same relation); each goal can be placed on a separate line. Prolog programs sometimes resemble poems for the aesthetic appeal of ideas and form.
- Stylistic conventions of this kind may vary from program to program as they depend on the problem and personal taste. It is important, however, that the same conventions are used consistently throughout the whole program.
- The cut operator should be used with care. Cut should not be used if it can be easily avoided. It is better to use, where possible, 'green cuts' rather than 'red cuts'. As discussed in Chapter 5, a cut is called 'green' if it can be removed without altering the declarative meaning of the clause. The use of 'red cuts' should be restricted to clearly defined constructs such as not or the selection between alternatives. An example of the latter construct is:

  if Condition *then* Goal1 *else* Goal2

  This translates into Prolog, using cut, as:

```
Condition, !,        % Condition true?
Goal1                % If yes then Goal1
;
Goal2                % Otherwise Goal2
```

- The not procedure can also lead to surprising behaviour, as it is related to cut. We have to be well aware of how not is defined in Prolog. However, if there is a dilemma between not and cut, the former is perhaps better than some obscure construct with cut.
- Program modification by assert and retract can grossly degrade the transparency of the program's behaviour. In particular, the same program will answer the same question differently at different times. In such cases, if we want to reproduce the same behaviour we have to make sure that the whole previous state, which was modified by assertions and retractions, is completely restored.
- The use of a semicolon may obscure the meaning of a clause. The readability can sometimes be improved by splitting the clause containing the semicolon into more clauses; but this will, possibly, be at the expense of the length of the program and its efficiency.

To illustrate some points of this section consider the relation

```
merge( List1, List2, List3)
```

where List1 and List2 are ordered lists that merge into List3. For example:

```
merge( [2,4,7], [1,3,4,8], [1,2,3,4,4,7,8] )
```

The following is an implementation of merge in bad style:

```
merge( List1, List2, List3) :-
  List1 = [ ], !, List3 = List2;         % First list empty
  List2 = [ ], !, List3 = List1;         % Second list empty
  List1 = [X | Rest1],
  List2 = [Y | Rest2],
  ( X < Y, !,
    Z = X,                               % Z is head of List3
    merge( Rest1, List2, Rest3);
    Z = Y,
    merge( List1, Rest2, Rest3) ),
  List3 = [Z | Rest3].
```

Here is a better version which avoids semicolons:

```
merge( [ ], List, List) :-
  !.                                     % Prevent redundant solutions

merge( List, [ ], List).

merge( [X | Rest1], [Y | Rest2], [X | Rest3] ) :-
  X < Y, !,
  merge( Rest1, [Y | Rest2], Rest3).

merge( List1, [Y | Rest2], [Y | Rest3] ) :-
  merge( List1, Rest2, Rest3).
```

8.3.2 Tabular organization of long procedures

Long procedures are acceptable if they have some uniform structure. Typically, such a form is a set of facts when a relation is effectively defined in the tabular form. Advantages of such an organization of a long procedure are:

- Its structure is easily understood.
- Incrementability: it can be refined by simply adding new facts.
- It is easy to check and correct or modify (by simply replacing some fact independently of other facts).

8.3.3 Commenting

Program comments should explain in the first place what the program is about and how to use it, and only then the details of the solution method used and other programming details. The main purpose of comments is to enable the user to use the program, to understand it and to possibly modify it. Comments should describe, in the shortest form possible, everything that is essential to these ends.

Undercommenting is a usual fault, but a program can also be overcommented. Explanation of details that are obvious from the program code itself is only a needless burden to the program.

Long passages of comments should precede the code they refer to, while short comments should be interspersed with the code itself. Information that should, in general, be included in comments comprises the following:

- What the program does, how it is used (for example, what goal is to be invoked and what are the expected results), examples of using the program.
- What are top-level predicates?
- How are main concepts (objects) represented?
- Execution time and memory requirements of the program.
- What are the program's limitations?
- Are there any special system-dependent features used?
- What is the meaning of the predicates in the program? What are their arguments? Which arguments are 'input' and which are 'output', if known? (Input arguments have fully specified values, without uninstantiated variables, when the predicate is called.)
- Algorithmic and implementation details.
- The following conventions are often used when describing predicates. References to a predicate are made by stating the predicate's name and its arity, written as:

  PredicateName / Arity

  For example, merge(List1, List2, List3) would be referred to as merge/3. The input/output modes of the arguments are indicated by prefixing arguments' names by '+' (input) or '-' (output). For example, merge(+List1, +List2, -List3) indicates that the first two arguments of merge are input, and the third one is output.

8.4 Debugging

When a program does not do what it is expected to do the main problem is to locate the error(s). It is easier to locate an error in a part of the program (or a module) than in the program as a whole. Therefore, a good principle of debugging is to start by testing smaller units of the program, and when these can be trusted, to test bigger modules or the whole program.

Debugging in Prolog is facilitated by two things: first, Prolog is an interactive language so any part of the program can be directly invoked by a proper question to the Prolog system; second, Prolog implementations usually provide special debugging aids. As a result of these two features, debugging of Prolog programs can, in general, be done more efficiently than in most other programming languages.

The basis for debugging aids is *tracing*. 'Tracing a goal' means that the information regarding the goal's satisfaction is displayed during execution. This information includes:

- Entry information: the predicate name and the values of arguments when the goal is invoked.
- Exit information: in the case of success, the values of arguments that satisfy the goal; otherwise an indication of failure.
- Re-entry information: invocation of the same goal caused by backtracking.

Between entry and exit, the trace information for all the subgoals of this goal can be obtained. So we can trace the execution of our question all the way down to the lowest level goals until facts are encountered. Such detailed tracing may turn out to be impractical due to the excessive amount of tracing information; therefore, the user can specify selective tracing. There are two selection mechanisms: first, suppress tracing information beyond a certain level; second, trace only some specified subset of predicates, and not all of them.

Such debugging aids are activated by system-dependent built-in predicates. A typical subset of such predicates is as follows:

```
trace
```

triggers exhaustive tracing of goals that follow.

```
notrace
```

stops further tracing.

```
spy( P)
```

specifies that a predicate P be traced. This is used when we are particularly interested in the named predicate and want to avoid tracing information from other goals (either above or below the level of a call of P). Several predicates can be simultaneously active for 'spying'.

```
nospy( P)
```

stops 'spying' P.

Tracing beyond a certain depth can be suppressed by special commands during execution. There may be several other debugging commands available, such as returning to a previous point of execution. After such a return we can, for example, repeat the execution at a greater detail of tracing.

8.5 Improving efficiency

There are several aspects of efficiency, including the most common ones, execution time and space requirements of a program. Another aspect is the time needed by the programmer to develop the program.

The traditional computer architecture is not particularly suitable for the Prolog style of program execution – that is, satisfying a list of goals. Therefore, the limitations of time and space may be experienced earlier in Prolog than in many other programming languages. Whether this will cause difficulties in a practical application depends on the problem. The issue of time efficiency is practically meaningless if a Prolog program that is run a few times per day takes 1 second of CPU time and a corresponding program in some other language, say Fortran, takes 0.1 seconds. The difference in efficiency will perhaps matter if the two programs take 50 minutes and 5 minutes respectively.

On the other hand, in many areas of application Prolog will greatly reduce the program development time. Prolog programs will, in general, be easier to write, to understand and to debug than in traditional languages. Problems that gravitate toward the 'Prolog domain' involve symbolic, non-numeric processing, structured data objects and relations between them. In particular, Prolog has been successfully applied in areas such as symbolic solving of equations, planning, databases, general problem solving, prototyping, implementation of programming languages, discrete and qualitative simulation, architectural design, machine learning, natural language understanding, expert systems, and other areas of artificial intelligence. On the other hand, numerical mathematics is an area for which Prolog is not a natural candidate.

With respect to the execution efficiency, executing a *compiled* program is generally more efficient than *interpreting* the program. Therefore, if the Prolog system contains both an interpreter and a compiler, then the compiler should be used if efficiency is critical.

If a program suffers from inefficiency then it can often be radically improved by improving the algorithm itself. However, to do this, the procedural aspects of the program have to be studied. A simple way of improving the executional efficiency is to find a better ordering of clauses of procedures, and of goals in the bodies of procedures. Another relatively simple method is to provide guidance to the Prolog system by means of cuts.

Ideas for improving the efficiency of a program usually come from a deeper understanding of the problem. A more efficient algorithm can, in general, result from improvements of two kinds:

- Improving search efficiency by avoiding unnecessary backtracking and stopping the execution of useless alternatives as soon as possible.
- Using more suitable data structures to represent objects in the program, so that operations on objects can be implemented more efficiently.

We will study both kinds of improvements by looking at examples. Yet another technique of improving efficiency will be illustrated by an example. This technique, called caching, is based on asserting into the database intermediate results that are likely to be needed again in the future computation. Instead of repeating the computation, such results are simply retrieved as already known facts.

8.5.1 Improving the efficiency of an eight queens program

As a simple example of improving the search efficiency let us revisit the eight queens problem (see Figure 4.7). In this program, the Y-coordinates of the queens are found by successively trying, for each queen, the integers between 1 and 8. This was programmed as the goal:

```
member( Y, [1,2,3,4,5,6,7,8] )
```

The way that member works is that Y = 1 is tried first, and then Y = 2, Y = 3, etc. As the queens are placed one after another in adjacent columns on the board, it is obvious that this order of trials is not the most appropriate. The reason for this is that the queens in adjacent columns will attack each other if they are not placed at least two squares apart in the vertical direction. According to this observation, a simple attempt to improve the efficiency is to rearrange the candidate coordinate values. For example:

```
member( Y, [1,5,2,6,3,7,4,8] )
```

This minor change will reduce the time needed to find the first solution by a factor of 3 or 4.

In the next example, a similarly simple idea of reordering will convert a practically unacceptable time complexity into a trivial one.

8.5.2 Improving the efficiency in a map colouring program

The map colouring problem is to assign each country in a given map one of four given colours in such a way that no two neighbouring countries are painted with the same colour. There is a theorem which guarantees that this is always possible.

Let us assume that a map is specified by the neighbour relation

```
ngb( Country, Neighbours)
```

where Neighbours is the list of countries bordering on Country. So the map of Europe, with 30 countries, would be specified (in alphabetical order) as:

```
ngb( albania, [greece, macedonia, yugoslavia] ).
ngb( andorra, [france, spain] ).
```



```
ngb( austria, [czech_republic, germany, hungary, italy, liechtenstein,
               slovakia, slovenia, switzerland] ).
...
```

Let a solution be represented as a list of pairs of the form

```
Country/Colour
```

which specifies a colour for each country in a given map. For the given map, the names of countries are fixed in advance, and the problem is to find the values for the colours. Thus, for Europe, the problem is to find a proper instantiation of variables C1, C2, C3, etc. in the list:

```
[ albania/C1, andorra/C2, austria/C3, ... ]
```

Now let us define the predicate

```
colours( Country_colour_list)
```

which is true if the Country_colour_list satisfies the map colouring constraint with respect to a given ngb relation. Let the four colours be yellow, blue, red and green. The condition that no two neighbouring countries are of the same colour can be formulated in Prolog as follows:

```
colours( [ ] ).

colours( [Country/Colour | Rest] ) :-
  colours( Rest),
  member( Colour, [yellow, blue, red, green] ),
  not( member( Country1/Colour, Rest), neighbour( Country, Country1) ).

neighbour( Country, Country1) :-
  ngb( Country, Neighbours),
  member( Country1, Neighbours).
```

Here, member(X,L) is, as usual, the list membership relation. This will work well for simple maps, with a small number of countries. Europe might be problematic, however. Assuming that the built-in predicate setof is available, one attempt to colour Europe could be as follows. First, let us define an auxiliary relation:

```
country( C) :- ngb( C, _).
```

Then the question for colouring Europe can be formulated as:

```
?- setof( Cntry/Colour, country( Cntry), CountryColourList),
   colours( CountryColourList).
```

The setof goal will construct a template country/colour list for Europe in which uninstantiated variables stand for colours. Then the colours goal is supposed to instantiate the colours. However, this attempt will probably fail because of inefficiency.

A detailed study of the way Prolog tries to satisfy the colours goal reveals the source of inefficiency. Countries in the country/colour list are arranged in alphabetical order, and this has nothing to do with their geographical arrangement. The order in which the countries are assigned colours corresponds to the order in the list (starting at the end), which is in our case independent of the ngb relation. So the colouring process starts at some end of the map, continues at some other end, etc., moving around more or less randomly. This may easily lead to a situation in which a country that is to be coloured is surrounded by many other countries, already painted with all four available colours. Then backtracking is necessary, which leads to inefficiency.

It is clear, then, that the efficiency depends on the order in which the countries are coloured. Intuition suggests a simple colouring strategy that should be better than random: start with some country that has many neighbours, and then proceed to the neighbours, then to the neighbours of neighbours, etc. For Europe, then, Germany (having most neighbours) is a good candidate to start with. Of course, when the template country/colour list is constructed, Germany has to be put at the end of the list and other countries have to be added at the front of the list. In this way the colouring algorithm, which starts at the rear end, will commence with Germany and proceed from there from neighbour to neighbour.

Such a country/colour template dramatically improves the efficiency with respect to the original, alphabetical order, and possible colourings for the map of Europe will be now produced without difficulty.

We can construct a properly ordered list of countries manually, but we do not have to. The following procedure, makelist, does it. It starts the construction with some specified country (Germany in our case) and collects the countries into a list called Closed. Each country is first put into another list, called Open, before it is transferred to Closed. Each time that a country is transferred from Open to Closed, its neighbours are added to Open.

```
makelist( List) :-
  collect( [germany], [ ], List).

collect( [ ], Closed, Closed).           % No more candidates for Closed

collect( [X | Open], Closed, List) :-
  member( X, Closed), !,                 % X has already been collected?
  collect( Open, Closed, List).          % Discard X

collect( [X | Open], Closed, List) :-
  ngb( X, Ngbs),                         % Find X's neighbours
  conc( Ngbs, Open, Open1),              % Put them to Open1
  collect( Open1, [X | Closed], List).   % Collect the Rest
```

The conc relation is, as usual, the list concatenation relation.

8.5.3 Improving efficiency of list concatenation by difference lists

In our programs so far, the concatenation of lists has been programmed as:

```
conc( [ ], L, L).

conc( [X | L1], L2, [X | L3] ) :-
  conc( L1, L2, L3).
```

This is inefficient when the first list is long. The following example explains why:

```
?- conc( [a,b,c], [d,e], L).
```

This produces the following sequence of goals:

```
conc( [a,b,c], [d,e], L)
  conc( [b,c], [d,e], L')          where L = [a | L']
    conc( [c], [d,e], L'')           where L' = [b | L'']
      conc( [ ], [d,e], L''')          where L'' = [c | L''']
        true                             where L''' = [d,e]
```

From this it is clear that the program in effect scans all of the first list, until the empty list is encountered.

But could we not simply skip the whole of the first list in a single step and append the second list, instead of gradually working down the first list? To do this, we need to know where the end of a list is; that is, we need another representation of lists. One solution is the data structure called *difference lists*. So a list is represented by a pair of lists. For example, the list

```
[a,b,c]
```

can be represented by the two lists:

```
L1 = [a,b,c,d,e]
L2 = [d,e]
```

Such a pair of lists, which we will for brevity choose to write as L1-L2, represents the 'difference' between L1 and L2. This of course only works under the condition that L2 is a suffix of L1. Note that the same list can be represented by several 'difference pairs'. So the list [a,b,c] can be represented, for example, by

```
[a,b,c] - [ ]
```

or

```
[a,b,c,d,e] - [d,e]
```

or

```
[a,b,c,d,e | T] - [d,e | T]
```

or

```
[a,b,c | T] - T
```

where T is any list. The empty list is represented by any pair of the form **L-L**.

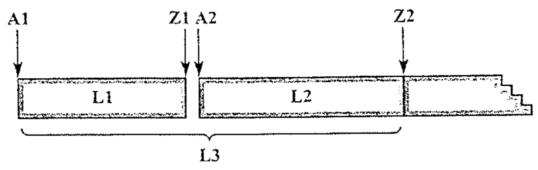

Figure 8.1 Concatenation of lists represented by difference pairs. L1 is represented by A1-Z1, L2 by A2-Z2, and the result L3 by A1-Z2 when Z1 = A2 must be true.

As the second member of the pair indicates the end of the list, the end is directly accessible. This can be used for an efficient implementation of concatenation. The method is illustrated in Figure 8.1. The corresponding concatenation relation translates into Prolog as the fact:

```
concat( A1 - Z1, Z1 - Z2, A1 - Z2).
```

Let us use concat to concatenate the lists [a,b,c], represented by the pair [a,b,c | T1]-T1, and the list [d,e], represented by [d,e | T2]-T2:

```
?- concat( [a,b,c | T1] - T1, [d,e | T2] - T2, L).
```

The concatenation is done just by matching this goal with the clause about concat, giving:

```
T1 = [d,e | T2]
L  = [a,b,c,d,e | T2] - T2
```

Due to its efficiency, this *difference lists* technique for list concatenation is very popular, although it cannot be used as flexibly as our usual conc procedure.

8.5.4 Last call optimization and accumulators

Recursive calls normally take up memory space, which is only freed after the return from the call. A large number of nested recursive calls may lead to shortage of memory. In special cases, however, it is possible to execute nested recursive calls without requiring extra memory. In such a case a recursive procedure has a special form, called *tail recursion*. A tail-recursive procedure only has one recursive call, and this call appears as the *last goal* of the *last clause* in the procedure. In addition, the goals preceding the recursive call must be deterministic, so that no backtracking occurs after this last call. We can force this determinism by placing a cut just before the recursive call. Typically a tail-recursive procedure looks like this:

```
p(...) :- ....          % No recursive call in the body of this clause
p(...) :- ....          % No recursive call in the body of this clause
...
```



```
p(...) :-
  ..., !,               % The cut ensures no backtracking
  p(...).               % Tail-recursive call
```

In the cases of such tail-recursive procedures, no information is needed upon the return from a call. Therefore such recursion can be carried out simply as iteration in which a next cycle in the loop does *not* require additional memory. A Prolog system will typically notice such an opportunity of saving memory and realize tail recursion as iteration. This is called *tail recursion optimization*, or *last call optimization*.

When memory efficiency is critical, tail-recursive formulations of procedures help. Often it is indeed possible to re-formulate a recursive procedure into a tail-recursive one. Let us consider the predicate for computing the sum of a list of numbers:

```
sumlist( List, Sum)
```

Here is a simple first definition:

```
sumlist( [ ], 0).

sumlist( [ First | Rest], Sum) :-
  sumlist( Rest, Sum0),
  Sum is X + Sum0.
```

This is not tail recursive, so the summation over a very long list will require many recursive calls and therefore a lot of memory. However, we know that in a typical procedural language such summation can be carried out as a simple iterative loop. How can we make sumlist tail recursive and enable Prolog too to carry it out as iteration? Unfortunately we cannot simply swap the goals in the body of the second clause, because the is goal can only be executed *after* Sum0 has been computed. But the following is a common trick that does it:

```
sumlist( List, Sum) :-
  sumlist( List, 0, Sum).              % Call auxiliary predicate

% sumlist( List, PartialSum, TotalSum):
%   TotalSum = PartialSum + sum over List

sumlist( [ ], Sum, Sum).               % Total sum = partial sum

sumlist( [ First | Rest ], PartialSum, TotalSum) :-
  NewPartialSum is PartialSum + First,
  sumlist( Rest, NewPartialSum, TotalSum).
```

This is now tail recursive and Prolog can benefit from last call optimization.

The technique of making our sumlist procedure tail recursive as above is frequently used. To define our target predicate sumlist/2, we introduced an auxiliary predicate sumlist/3. The additional argument, PartialSum, enabled a tail-recursive formulation. Such extra arguments are common and they are called *accumulators*. The final result is gradually accumulated in such an accumulator during successive recursion calls.

Here is another famous example of tail-recursion formulation through introducing an accumulator argument:

```
reverse( List, ReversedList)
```

ReversedList has the same elements as List, but in the reverse order. The following is a first, straightforward attempt:

```
reverse( [ ], [ ]).

reverse( [X | Rest], Reversed) :-
  reverse( Rest, RevRest),
  conc( RevRest, [X], Reversed).        % Append X at end
```

This is not tail recursive. Apart from that, it is also very inefficient because of the goal conc(RevRest, [X], Reversed), which requires time proportional to the length of RevRest. Therefore, to reverse a list of length n, the procedure above will require time proportional to n^2. But, of course, a list can be reversed in linear time. Therefore, due to its inefficiency, the procedure above is also known as 'naive reverse'. A much more efficient version below introduces an accumulator:

```
reverse( List, Reversed) :-
  reverse( List, [ ], Reversed).

% reverse( List, PartReversed, Reversed):
%   Reversed is obtained by adding the elements of List in reverse order
%   to PartReversed

reverse( [ ], Reversed, Reversed).

reverse( [X | Rest], PartReversed, TotalReversed) :-
  reverse( Rest, [X | PartReversed], TotalReversed).      % Add X to accumulator
```

This is efficient (time is linear in the length of list) and tail recursive.

8.5.5 Simulating arrays with *arg*

The list structure is the easiest representation for sets in Prolog. However, accessing an item in a list is done by scanning the list. This takes time proportional to the length of the list. For long lists this is very inefficient. Tree structures, discussed in Chapters 9 and 10, offer much more efficient access. However, often it is possible to access an element of a structure through the element's *index*. In such cases, *array* structures, provided in other programming languages, are the most effective because they enable direct access to a required element.

There is no array facility in Prolog, but arrays can be simulated to some extent by using the built-in predicates arg and functor. Here is an example. The goal:

```
functor( A, f, 100)
```

makes a structure with 100 elements:

```
A = f( _, _, _, ...)
```


In other languages, a typical example statement that involves direct access to an element of an array is:

```
A[60] := 1
```

This initializes the value of the 60th element of array A to 1. We can achieve analogous effect in Prolog by the goal:

```
arg( 60, A, 1)
```

This directly accesses the 60th component of the compound term A, which as the result gets instantiated to:

```
A = f( _, ..., _, 1, _, ..., _)        % 60th component equal 1
```

The point is that time needed to access the Nth component of a structure does not depend on N. Another typical example statement from other programming languages is:

```
X := A[60]
```

This translates into our simulated array in Prolog as:

```
arg( 60, A, X)
```

This is much more efficient than having a list of 100 elements and accessing the 60th element by nested recursion down the list. However, the updating of the value of an element in a simulated array is awkward. Once the values in an array have been initialized, they can be changed, for example:

```
A[60] := A[60] + 1
```

A straightforward way to simulate such update of a *single* value in an array in Prolog would be as follows: build a *whole* new structure with 100 components using functor, insert the new value at the appropriate place in the structure, and fill all the other components by the corresponding components of the previous structure. All this is awkward and very inefficient. One idea to improve this is to provide uninstantiated 'holes' in the components of the structure, so that future values of array elements can be accommodated in these holes. So we can, for example, store successive update values in a list in which the rest of the list is an uninstantiated variable – a 'hole' for future values. As an example consider the following updates of the value of variable X in a procedural language:

```
X := 1;     X := 2;     X := 3
```

These updates can be simulated in Prolog with the 'holes' technique as follows:

```
X = [ 1 | Rest1]              % Corresponds to X := 1, Rest1 is hole for future values
Rest1 = [ 2 | Rest2]          % Corresponds to X := 2, Rest 2 is hole for future values
Rest2 = [ 3 | Rest3]          % Corresponds to X := 3
```

At this point X =[1, 2, 3 | Rest3]. Obviously the whole history of the values of X is maintained, and the current value is the one just preceding the 'hole'. If there are many successive updates, the 'current' value gets nested deep, and the technique becomes inefficient again. A further idea, to overcome this source of inefficiency, is to throw away the previous values at the moment when a list gets too long, and start again with a list consisting of just a head and an uninstantiated tail.

In spite of these potential complications, in many cases the simulation of arrays with arg is simple and works well. One such example is our solution 3 for the eight queens problem in Chapter 4 (Figure 4.11). This program places a next queen into a currently free column (X-coordinate), row (Y-coordinate), upward diagonal (U-coordinate) and downward diagonal (V-coordinate). The sets of currently free coordinates are maintained, and when a new queen is placed the corresponding occupied coordinates are deleted from these sets. The deletion of U and V coordinates in Figure 4.11 involves scanning the corresponding lists, which is inefficent. Efficiency can easily be improved by simulated arrays. So the set of all 15 upward diagonals can be represented by the following term with 15 components:

```
Du = u( _, _, _, _, _, _, _, _, _, _, _, _, _, _, _)
```

Consider placing a queen at the square (X,Y) = (1,1). This square lies on the 8th upward diagonal. The fact that this diagonal is now occupied can be marked by instantiating the 8th component of Du to 1 (that is, the corresponding X-coordinate):

```
arg( 8, Du, 1)
```

Now Du becomes:

```
Du = u( _, _, _, _, _, _, _, 1, _, _, _, _, _, _, _)
```

If later a queen is attempted to be placed at (X,Y) = (3,3), also lying on the 8th diagonal, this would require:

```
arg( 8, Du, 3)                    % Here X = 3
```

This will fail because the 8th component of Du is already 1. So the program will not allow another queen to be placed on the same diagonal. This implementation of the sets of upward and downward diagonals leads to a considerably more efficient program than the one in Figure 4.11.

8.5.6 Improving efficiency by asserting derived facts

Sometimes during computation the same goal has to be satisfied again and again. As Prolog has no special mechanism to discover such situations whole computation sequences are repeated.

As an example consider a program to compute the Nth Fibonacci number for a given N. The Fibonacci sequence is:

1, 1, 2, 3, 5, 8, 13, ...

Each number in the sequence, except for the first two, is the sum of the previous two numbers. We will define a predicate

```
fib( N, F)
```

to compute, for a given N, the Nth Fibonacci number, F. We count the numbers in the sequence starting with N = 1. The following fib program deals first with the first two Fibonacci numbers as two special cases, and then specifies the general rule about the Fibonacci sequence:

```
fib( 1, 1).                     % 1st Fibonacci number

fib( 2, 1).                     % 2nd Fibonacci number

fib( N, F) :-                   % Nth Fib. number, N > 2
  N > 2,
  N1 is N - 1, fib( N1, F1),
  N2 is N - 2, fib( N2, F2),
  F is F1 + F2.                 % Nth number is the sum of its two predecessors
```

This program tends to redo parts of the computation. This is easily seen if we trace the execution of the following goal:

```
?- fib( 6, F).
```

Figure 8.2 illustrates the essence of this computational process. For example, the third Fibonacci number, *f(3)*, is needed in three places and the same computation is repeated each time.

This can be easily avoided by remembering each newly computed Fibonacci number. The idea is to use the built-in procedure asserta and to add these (intermediate) results as facts to the database. These facts have to precede other clauses about fib to prevent the use of the general rule in cases where the result is already known. The modified procedure, fib2, differs from fib only in this assertion:

```
fib2( 1, 1).                    % 1st Fibonacci number

fib2( 2, 1).                    % 2nd Fibonacci number

fib2( N, F) :-                  % Nth Fib. number, N > 2
  N > 2,
  N1 is N - 1, fib2( N1, F1),
  N2 is N - 2, fib2( N2, F2),
  F is F1 + F2,
  asserta( fib2( N, F) ).       % Remember Nth number
```

This program will try to answer any fib2 goal by first looking at stored facts about this relation, and only then resort to the general rule. As a result, when a goal fib2(N, F) is executed all Fibonacci numbers, up to the Nth number, will get

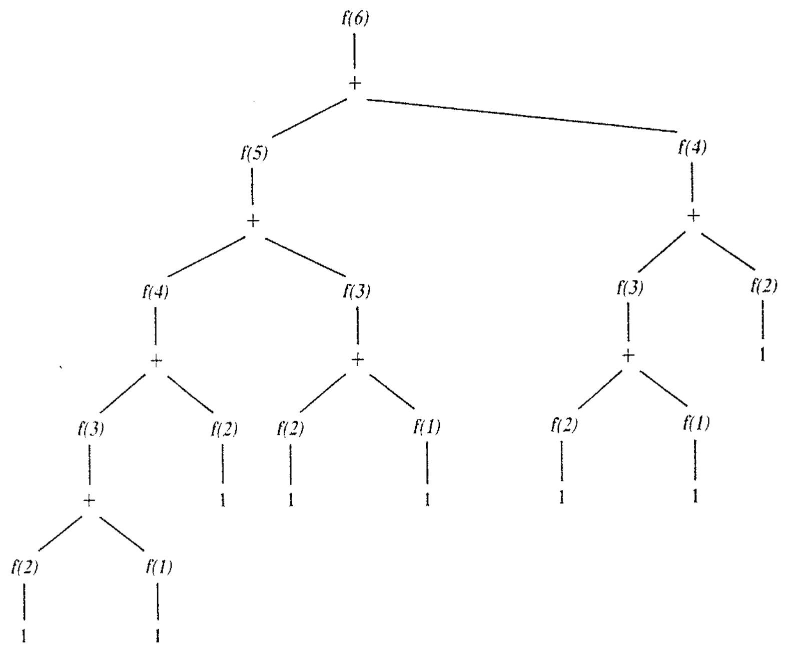

Figure 8.2 Computation of the 6th Fibonacci number by procedure **fib**.

tabulated. Figure 8.3 illustrates the computation of the 6th Fibonacci number by **fib2**. A comparison with Figure 8.2 shows the saving in the computational complexity. For greater N, the savings would be much more substantial.

Asserting intermediate results, also called *caching*, is a standard technique for avoiding repeated computations. It should be noted, however, that in the case of Fibonacci numbers we can preferably avoid repeated computation by using another algorithm, rather than by asserting intermediate results. This other algorithm will lead to a program that is more difficult to understand, but more efficient to execute. The idea this time is not to define the Nth Fibonacci number simply as the sum of its two predecessors and leave the recursive calls to unfold the whole computation 'downwards' to the two initial Fibonacci numbers. Instead, we can work 'upwards', starting with the initial two numbers, and compute the numbers in the sequence one by one in the forward direction. We have to stop when we have computed the Nth number. Most of the work in such a program is done by the procedure:

```
forwardfib( M, N, F1, F2, F)
```

Here, F1 and F2 are the (M − 1)st and Mth Fibonacci numbers, and F is the Nth Fibonacci number. Figure 8.4 helps to understand the **forwardfib** relation. According

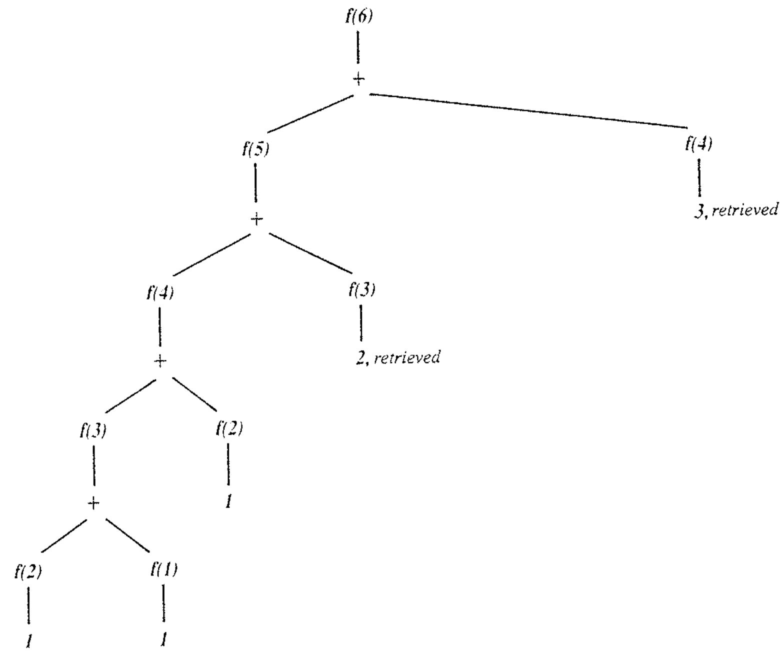

Figure 8.3 Computation of the 6th Fibonacci number by procedure **fib2**, which remembers previous results. This saves some computation in comparison with **fib**, see Figure 8.2.

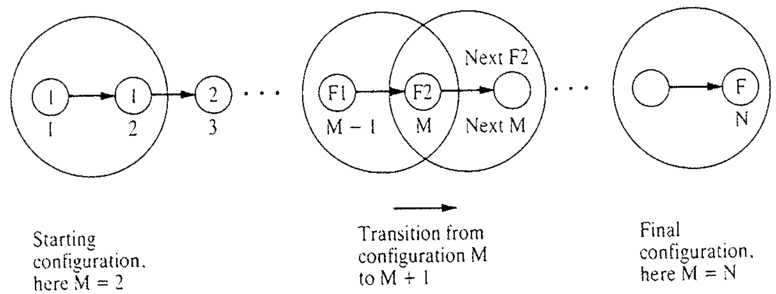

Figure 8.4 Relations in the Fibonacci sequence. A 'configuration', depicted by a large circle, is defined by three things: an index M and two consecutive Fibonacci numbers $f(M-1)$ and $f(M)$.

to this figure, forwardfib finds a sequence of transformations to reach a final configuration (when M = N) from a given starting configuration. When forwardfib is invoked, all the arguments except F have to be instantiated, and M has to be less than or equal to N. The program is:

```
fib3( N, F) :-
  forwardfib( 2, N, 1, 1, F).           % The first two Fib. numbers are 1

forwardfib( M, N, F1, F2, F2) :-
  M >= N.                               % Nth Fibonacci number reached

forwardfib( M, N, F1, F2, F) :-
  M < N,                                % Nth number not yet reached
  NextM is M + 1,
  NextF2 is F1 + F2,
  forwardfib( NextM, N, F2, NextF2, F).
```

Notice that forwardfib is tail recursive, and M, F1 and F2 are accumulator arguments.

Exercises

8.1 Procedures sub1, sub2 and sub3, shown below, all implement the sublist relation. sub1 is a more procedural definition whereas sub2 and sub3 are written in a more declarative style. Study the behaviour, with reference to efficiency, of these three procedures on some sample lists. Two of them behave nearly equivalently and have similar efficiency. Which two? Why is the remaining one less efficient?

```
sub1( List, Sublist) :-
  prefix( List, Sublist).

sub1( [ _ | Tail], Sublist) :-
  sub1( Tail, Sublist).                 % Sublist is sublist of Tail

prefix( _, [ ] ).

prefix( [X | List1], [X | List2] ) :-
  prefix( List1, List2).

sub2( List, Sublist) :-
  conc( List1, List2, List),
  conc( List3, Sublist, List1).

sub3( List, Sublist) :-
  conc( List1, List2, List),
  conc( Sublist, _, List2).
```

8.2 Define the relation

```
add_at_end( List, Item, NewList)
```

to add Item at the end of List producing NewList. Let both lists be represented by difference pairs.

8.3 Define the relation

```
reverse( List, ReversedList)
```

where both lists are represented by difference pairs.

8.4 Rewrite the collect procedure of Section 8.5.2 using difference pair representation for lists so that the concatenation can be done more efficiently.

8.5 The following procedure computes the maximum value in a list of numbers:

```
max( [X], X).

max( [X | Rest], Max) :-
  max( Rest, MaxRest),
  ( MaxRest >= X, !, Max = MaxRest
   ;
   Max = X ).
```

Transform this into a tail-recursive procedure. Hint: Introduce accumulator argument MaxSoFar.

8.6 Rewrite program 3 for eight queens (Figure 4.11) using simulated array with arg to represent the sets of free diagonals, as discussed in Section 8.5.5. Measure the improvement in efficiency.

8.7 Implement the updating of the value of an element of an array simulated by functor and arg, using 'holes' for future values along the lines discussed in Section 5.5.5.

Summary

- There are several criteria for evaluating programs:

  correctness
  user-friendliness
  efficiency
  readability
  modifiability
  robustness
  documentation

- The principle of *stepwise refinement* is a good way of organizing the program development process. Stepwise refinement applies to relations, algorithms and data structures.
- In Prolog, the following techniques often help to find ideas for refinements:

  Using recursion: identify boundary and general cases of a recursive definition.

  Generalization: consider a more general problem that may be easier to solve than the original one.

Using pictures: graphical representation may help to identify important relations.

- It is useful to conform to some stylistic conventions to reduce the danger of programming errors, make programs easier to read, debug and modify.
- Prolog systems usually provide program debugging aids. Trace facilities are most useful.
- There are many ways of improving the efficiency of a program. Simple techniques include:

  reordering of goals and clauses
  controlling backtracking by inserting cuts
  remembering (by asserta) solutions that would otherwise be computed again
- More sophisticated techniques aim at better algorithms (improving search efficiency in particular) and better data structures. Frequently used programming techniques of this kind are:

  difference lists
  tail recursion, last call optimization
  accumulator arguments
  simulating arrays with functor and arg

References

Ross (1989) and O'Keefe (1990) explore in depth the efficiency issues, program design and programming style in Prolog. Sterling (1990) edited a collection of papers describing the design of large Prolog programs for a number of practical applications.

O'Keefe, R.A. (1990) *The Craft of Prolog*. Cambridge, MA: MIT Press.
Ross, P. (1989) *Advanced Prolog: Techniques and Examples*. Harlow: Addison-Wesley.
Sterling, L. (1990) *The Practice of Prolog*. Cambridge, MA: MIT Press.

chapter 9

Operations on Data Structures

One fundamental question in programming is how to represent complex data objects, such as sets, and efficiently implement operations on such objects. The theme of this chapter is some frequently used data structures that belong to three big families: lists, trees and graphs. We will examine ways of representing these structures in Prolog, and develop programs for some operations on these structures, such as sorting a list, representing data sets by tree structures, storing data in trees and retrieving data from trees, path finding in graphs, etc. We will study several examples because these operations are extremely instructive for programming in Prolog.

9.1 Sorting lists

A list can be sorted if there is an ordering relation between the items in the list. We will for the purpose of this discussion assume that there is an ordering relation

```
gt( X, Y)
```

meaning that X is *greater than* Y, whatever 'greater than' means. If our items are numbers then the gt relation will perhaps be defined as:

```
gt( X, Y) :- X > Y.
```

If the items are atoms then the gt relation can correspond to the alphabetical order, for example defined by:

```
gt( X, Y) :- X @> Y.
```

Remember that this relation also orders compound terms.

Let

```
sort( List, Sorted)
```

denote a relation where List is a list of items and Sorted is a list of the same items sorted in the ascending order according to the gt relation. We will develop three definitions of this relation in Prolog, based on different ideas for sorting a list. The first idea is as follows:

To sort a list, List:

- Find two adjacent elements, X and Y, in List such that gt(X, Y) and swap X and Y in List, obtaining List1; then sort List1.
- If there is no pair of adjacent elements, X and Y, in List such that gt(X, Y), then List is already sorted.

The purpose of swapping two elements, X and Y, that occur out of order, is that after the swapping the new list is closer to a sorted list. After a sufficient amount of swapping we should end up with all the elements in order. This principle of sorting is known as *bubble sort*. The corresponding Prolog procedure will be therefore called bubblesort:

```
bubblesort( List, Sorted) :-
  swap( List, List1), !,              % A useful swap in List?
  bubblesort( List1, Sorted).

bubblesort( Sorted, Sorted).          % Otherwise list is already sorted

swap( [X, Y | Rest], [Y, X | Rest] ) :-  % Swap first two elements
  gt( X, Y).

swap( [Z | Rest], [Z | Rest1] ) :-    % Swap elements in tail
  swap( Rest, Rest1).
```

Another simple sorting algorithm is *insertion sort*, which is based on the following idea:

To sort a non-empty list, L = [X | T]:

(1) Sort the tail T of L.

(2) Insert the head, X, of L into the sorted tail at such a position that the resulting list is sorted. The result is the whole sorted list.

This translates into Prolog as the following insertsort procedure:

```
insertsort( [], [] ).

insertsort( [X | Tail], Sorted) :-
  insertsort( Tail, SortedTail),      % Sort the tail
  insert( X, SortedTail, Sorted).     % Insert X at proper place

insert( X, [Y | Sorted], [Y | Sorted1] ) :-
  gt( X, Y), !,
  insert( X, Sorted, Sorted1).

insert( X, Sorted, [X | Sorted] ).
```

The sorting procedures bubblesort and insertsort are simple, but inefficient. Of the two procedures, insertion sort is the more efficient one. However, the average time that insertsort requires for sorting a list of length n grows proportionally to n^2. For long lists, therefore, a much better sorting algorithm is *quicksort*. This is based on the following idea, which is illustrated in Figure 9.1.

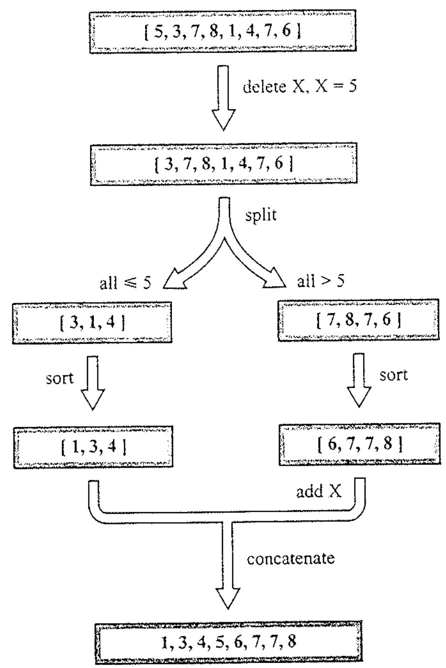

Figure 9.1 Sorting a list by *quicksort*.

To sort a non-empty list, L:

(1) Delete some element X from L and split the rest of L into two lists, called **Small** and **Big**, as follows: all elements in L that are greater than X belong to **Big**, and all others to **Small**.

(2) Sort **Small** obtaining **SortedSmall**.

(3) Sort **Big** obtaining **SortedBig**.

(4) The whole sorted list is the concatenation of **SortedSmall** and **[X | SortedBig]**.

If the list to be sorted is empty then the result of sorting is also the empty list. A Prolog implementation of quicksort is shown in Figure 9.2. A particular detail of this implementation is that the element, X, that is deleted from L is always simply the head of L. The splitting is programmed as a four-argument relation:

```
split( X, L, Small, Big)
```

The time complexity of this algorithm depends on how lucky we are when splitting the list to be sorted. If the list is split into two lists of approximately equal lengths then the time complexity of this sorting procedure is of the order $n \log n$ where n is the length of the list to be sorted. If, on the contrary, splitting always results in one list far bigger than the other, then the complexity is in the order of n^2. Analysis would show that the average performance of quicksort is, fortunately, closer to the best case than to the worst case.

The program in Figure 9.2 can be further improved by a better implementation of the concatenation operation. Using the difference-pair representation of lists, introduced in Chapter 8, concatenation is reduced to triviality. To use this idea in

```
% quicksort( List, SortedList): sort List by the quicksort algorithm

quicksort( [ ], [ ] ).

quicksort( [X | Tail], Sorted) :-
  split( X, Tail, Small, Big),
  quicksort( Small, SortedSmall),
  quicksort( Big, SortedBig),
  conc( SortedSmall, [X | SortedBig], Sorted).

split( X, [ ], [ ], [ ] ).

split( X, [Y | Tail], [Y | Small], Big) :-
  gt( X, Y), !,
  split( X, Tail, Small, Big).

split( X, [Y | Tail], Small, [Y | Big] ) :-
  split( X, Tail, Small, Big).
```

Figure 9.2 Quicksort.


```
% quicksort( List, SortedList): sort List with the quicksort algorithm

quicksort( List, Sorted) :-
  quicksort2( List, Sorted - [ ] ).

% quicksort2( List, SortedDiffList): sort List, result is represented as difference list

quicksort2( [ ], Z - Z).

quicksort2( [X | Tail], A1 - Z2) :-
  split( X, Tail, Small, Big),
  quicksort2( Small, A1 - [X | A2] ),
  quicksort2( Big, A2 - Z2).
```

Figure 9.3 A more efficient implementation of **quicksort** using difference-pair representation for lists. Relation **split(X, List, Small, Big)** is as defined in Figure 9.2.

our sorting procedure, the lists in the program of Figure 9.2 can be represented by pairs of lists of the form A-Z as follows:

SortedSmall	is represented by	A1 - Z1
SortedBig	is represented by	A2 - Z2

Then the concatenation of the lists SortedSmall and [X | SortedBig] corresponds to the concatenation of pairs:

A1 - Z1 and [X | A2] - Z2

The resulting concatenated list is represented by:

A1 - Z2 where Z1 = [X | A2]

The empty list is represented by any pair Z-Z. Introducing these changes systematically into the program of Figure 9.2 we get a more efficient implementation of quicksort, programmed as **quicksort2** in Figure 9.3. The procedure **quicksort** still uses the usual representation of lists, but the actual sorting is done by the more efficient **quicksort2**, which uses the difference-pair representation. The relation between the two procedures is:

```
quicksort( L, S) :-
  quicksort2( L, S - [ ] ).
```

Exercises

9.1 Write a procedure to merge two sorted lists producing a third list. For example:

```
?- merge( [2,5,6,6,8], [1,3,5,9], L).
L = [1,2,3,5,5,6,6,8,9]
```

9.2 The difference between the sorting programs of Figures 9.2 and 9.3 is in the representation of lists – the latter uses difference-lists. Transformation between plain lists and difference-lists is straightforward and could be mechanized. Carry out the corresponding changes systematically in the program of Figure 9.2 to transform it into the program of Figure 9.3.

9.3 Our quicksort program performs badly when the list to be sorted is already sorted or almost sorted. Analyze why.

9.4 Another good idea for sorting a list that avoids the weakness of quicksort is based on dividing the list, then sorting smaller lists, and then merging these sorted smaller lists. Accordingly, to sort a list L:

- divide L into two lists, L1 and L2, of approximately equal length;
- sort L1 and L2 giving S1 and S2;
- merge S1 and S2 giving L sorted.

This is known as the merge-sort algorithm. Implement merge-sort and compare its efficiency with the quicksort program.

9.2 Representing sets by binary trees

One usual application of lists is to represent sets of objects. A disadvantage of using a list for representing a set is that the set membership testing is relatively inefficient. The predicate member(X, L) to test whether X is a member of a list L is usually programmed as:

```
member( X, [X | L] ).

member( X, [Y | L] ) :-
  member( X, L).
```

To find X in a list L, this procedure scans the list element by element until X is found or the end of the list is encountered. This is very inefficient in the case of long lists.

For representing sets, there are various tree structures that facilitate more efficient implementation of the set membership relation. We will here consider binary trees.

A binary tree is either empty or it consists of three things:

- a root;
- a left subtree;
- a right subtree.

The root can be anything, but the subtrees have to be binary trees again. Figure 9.4 shows an example. This tree represents the set {a, b, c, d}. The elements of the set are

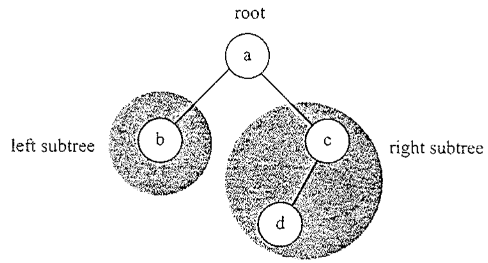

Figure 9.4 A binary tree.

stored as nodes of the tree. In Figure 9.4, the empty subtrees are not pictured; for example, the node b has two subtrees that are both empty.

There are many ways to represent a binary tree by a Prolog term. One simple possibility is to make the root of a binary tree the principal functor of the term, and the subtrees its arguments. Accordingly, the example tree of Figure 9.4 would be represented by:

```
a( b, c(d) )
```

Among other disadvantages, this representation requires another functor for each node of the tree. This can lead to troubles if nodes themselves are structured objects.

A better and more usual way to represent binary trees is as follows: we need a special symbol to represent the empty tree, and we need a functor to construct a non-empty tree from its three components (the root and the two subtrees). We will make the following choice regarding the functor and the special symbol:

- Let the atom nil represent the empty tree.
- Let the functor be t so the tree that has a root X, a left subtree L and a right subtree R is represented by the term t(L, X, R) (see Figure 9.5).

In this representation, the example tree of Figure 9.4 is represented by the term:

```
t( t( nil, b, nil), a, t( t( nil, d, nil), c, nil) )
```

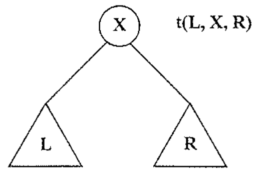

Figure 9.5 A representation of binary trees.

Let us now consider the set membership relation, here named in. A goal

```
in( X, T)
```

is true if X is a node in a tree T. The in relation can be defined by the following rules:

X is in a tree T if:

- the root of T is X, or
- X is in the left subtree of T, or
- X is in the right subtree of T.

These rules directly translate into Prolog:

```
in( X, t( _, X, _) ).

in( X, t( L, _, _) ) :-
  in( X, L).

in( X, t( _, _, R) ) :-
  in( X, R).
```

Obviously, the goal

```
in( X, nil)
```

will fail for any X.

Let us investigate the behaviour of this procedure. In the following examples, T is the tree of Figure 9.4. The goal

```
in( X, T)
```

will, through backtracking, find all the data in the set in the following order:

X = a;   X = b;   X = c;   X = d

Now let us consider efficiency. The goal

```
in( a, T)
```

succeeds immediately by the first clause of the procedure in. On the other hand, the goal

```
in( d, T)
```

will cause several recursive calls of in before d is eventually found. Similarly, the goal

```
in( e, T)
```

will fail only after the whole tree has been searched by recursive calls of in on all the subtrees of T.

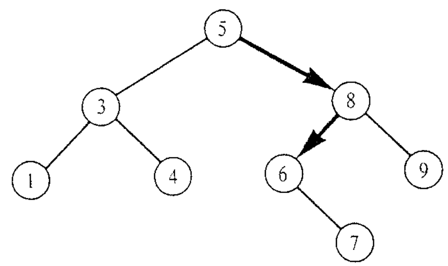

Figure 9.6 A binary dictionary. Item 6 is reached by following the indicated path 5 → 8 → 6.

This is, then, as inefficient as simply representing a set by a list. A major improvement can, however, be achieved if there is an ordering relation between the data in the set. Then the data in the tree can be ordered from left to right according to this relation. We say that a non-empty tree t(Left, X, Right) is ordered from left to right if:

(1) all the nodes in the left subtree, Left, are less than X; and

(2) all the nodes in the right subtree, Right, are greater than X; and

(3) both subtrees are also ordered.

Such a binary tree will be called a *binary dictionary*. Figure 9.6 shows an example.

The advantage of ordering is that, to search for an object in a binary dictionary, it is always sufficient to search at most one subtree. The key to this economization when searching for X is that we can by comparing X and the root immediately discard at least one of the subtrees. For example, let us search for the item 6 in the tree of Figure 9.6. We start at the root, 5, compare 6 and 5, and establish 6 > 5. As all the data in the left subtree must be less than 5, the only remaining possibility to find 6 is the right subtree. So we continue the search in the right subtree, moving to node 8, etc.

The general method for searching in the binary dictionary is:

To find an item X in a dictionary D:

- if X is the root of D then X has been found, otherwise
- if X is less than the root of D then search for X in the left subtree of D, otherwise
- search for X in the right subtree of D;
- if D is empty the search fails.

```
% in( X, Tree): X in binary dictionary Tree

in( X, t( _, X, _ ) ).

in( X, t( Left, Root, Right) ) :-
  gt( Root, X),                    % Root greater than X
  in( X, Left).                    % Search left subtree

in( X, t( Left, Root, Right) ) :-
  gt( X, Root),                    % X greater than Root
  in( X, Right).                   % Search right subtree
```

Figure 9.7 Finding an item X in a binary dictionary.

These rules are programmed as the procedure in in Figure 9.7. The relation gt(X, Y) means: X is greater than Y. If the items stored in the tree are numbers then this relation is simply X > Y.

In a way, the in procedure itself can be also used for *constructing* a binary dictionary. For example, the following sequence of goals will construct a dictionary D that contains the elements 5, 3, 8:

```
?- in( 5, D), in( 3, D), in( 8, D).

D = t( t( D1, 3, D2), 5, t( D3, 8, D4) ).
```

The variables D1, D2, D3 and D4 are four unspecified subtrees. They can be anything and D will still contain the given items 3, 5 and 8. The dictionary that is constructed depends on the order of the goals in the question (Figure 9.8).

A comment is in order here on the search efficiency in dictionaries. Generally speaking, the search for an item in a dictionary is more efficient than searching in a

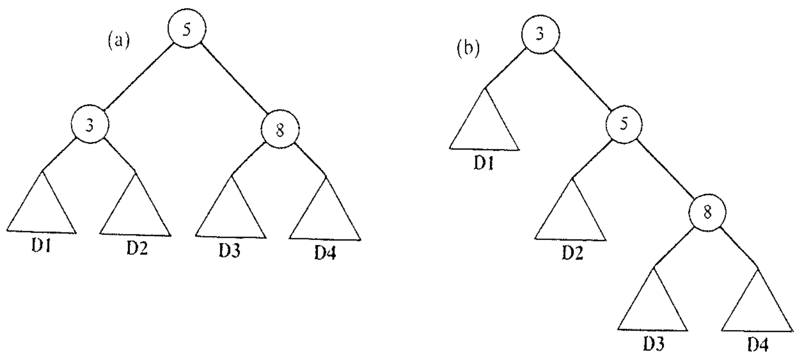

Figure 9.8 (a) Tree D that results from the sequence of goals: in(5, D), in(3, D), in(8, D). (b) Tree resulting from: in(3, D), in(5, D), in(8, D).

list. What is the improvement? Let n be the number of items in our data set. If the set is represented by a list then the expected search time will be proportional to its length n. On average, we have to scan the list up to something like half-way through it. If the set is represented by a binary dictionary, the search time will be roughly proportional to the height of the tree. The height of a tree is the length of a longest path between the root and a leaf in the tree. The height, however, depends on the shape of the tree.

We say that a tree is (approximately) *balanced* if, for each node in the tree, its two subtrees accommodate an approximately equal number of items. If a dictionary with n nodes is nicely balanced then its height is proportional to $\log n$. We say that a balanced tree has *logarithmic complexity*. The difference between n and $\log n$ is the improvement of a balanced dictionary over a list. This holds, unfortunately, only when a tree is approximately balanced. If the tree gets out of balance its performance will degrade. In extreme cases of totally unbalanced trees, a tree is in effect reduced to a list. In such a case the tree's height is n, and the tree's performance is equally poor as that of a list. Therefore we are always interested in balanced dictionaries. Methods of achieving this objective will be discussed in Chapter 10.

Exercises

9.5 Define the predicates

(a) binarytree(Object)

(b) dictionary(Object)

to recognize whether Object is a binary tree or a binary dictionary respectively, written in the notation of this section.

9.6 Define the procedure

height(BinaryTree, Height)

to compute the height of a binary tree. Assume that the height of the empty tree is 0, and that of a one-element tree is 1.

9.7 Define the relation

linearize(Tree, List)

to collect all the nodes in Tree into a list.

9.8 Define the relation

maxelement(D, Item)

so that Item is the largest element stored in the binary dictionary D.

9.9 Modify the procedure

```
in( Item, BinaryDictionary)
```

by adding the third argument, Path, so that Path is the path between the root of the dictionary and Item.

9.3 Insertion and deletion in a binary dictionary

When maintaining a dynamic set of data we may want to insert new items into the set and also delete some old items from the set. So one common repertoire of operations on a set of data, S, is:

in(X, S)	X is a member of S
add(S, X, S1)	Add X to S giving S1
del(S, X, S1)	Delete X from S giving S1

Let us now define the *add* relation. It is easiest to insert new data at the bottom level of the tree, so that a new item becomes a leaf of the tree at such a position that the ordering of the tree is preserved. Figure 9.9 shows changes in a tree during a sequence of insertions. Let us call this kind of insertion addleaf(D, X, D1).

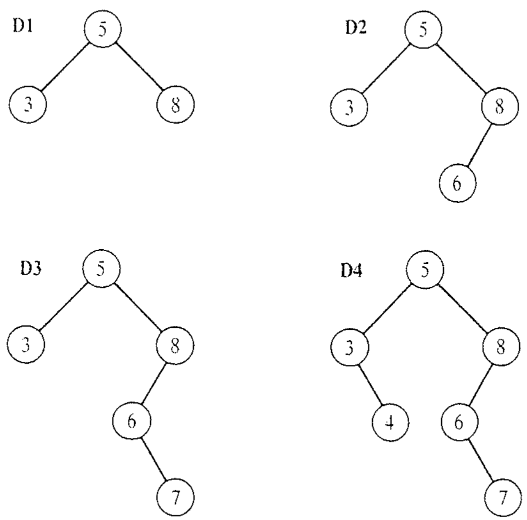

Figure 9.9 Insertion into a binary dictionary at the leaf level. The trees correspond to the following sequence of insertions: add(D1, 6, D2), add(D2, 7, D3), add(D3, 4, D4).


```
% addleaf( Tree, X, NewTree):
%   inserting X as a leaf into binary dictionary Tree gives NewTree

addleaf( nil, X, t( nil, X, nil) ).

addleaf( t( Left, X, Right), X, t( Left, X, Right) ).

addleaf( t( Left, Root, Right), X, t( Left1, Root, Right) ) :-
  gt( Root, X),
  addleaf( Left, X, Left1).

addleaf( t( Left, Root, Right), X, t( Left, Root, Right1) ) :-
  gt( X, Root),
  addleaf( Right, X, Right1).
```

Figure 9.10 Inserting an item as a leaf into the binary dictionary.

Rules for adding at the leaf level are:

- The result of adding X to the empty tree is the tree t(nil, X, nil).
- If X is the root of D then D1 = D (no duplicate item gets inserted).
- If the root of D is greater than X then insert X into the left subtree of D; if the root of D is less than X then insert X into the right subtree.

Figure 9.10 shows a corresponding program.

Let us now consider the *delete* operation. It is easy to delete a leaf, but deleting an internal node is more complicated. The deletion of a leaf can be in fact defined as the inverse operation of inserting at the leaf level:

```
delleaf( D1, X, D2) :-
  addleaf( D2, X, D1).
```

Unfortunately, if X is an internal node then this does not work because of the problem illustrated in Figure 9.11. X has two subtrees, Left and Right. After X is

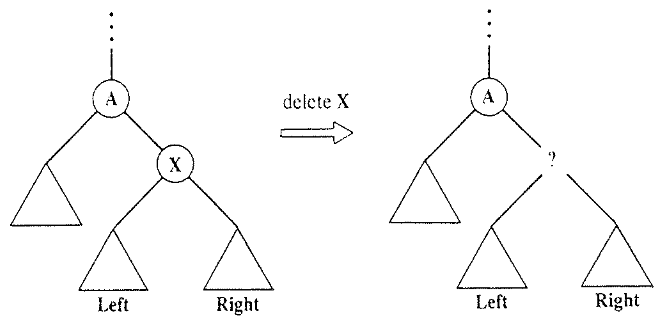

Figure 9.11 Deleting X from a binary dictionary. The problem is how to patch up the tree after X is removed.

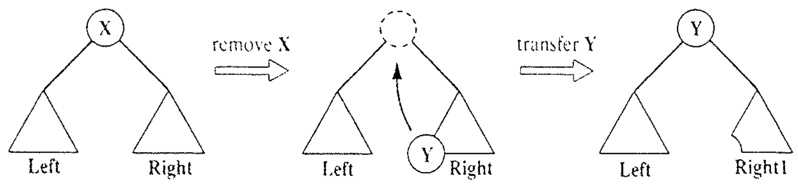

Figure 9.12 Filling the gap after removal of X.

removed, we have a hole in the tree and Left and Right are no longer connected to the rest of the tree. They cannot both be directly connected to the father of X, A, because A can accommodate only one of them.

If one of the subtrees Left and Right is empty then the solution is simple: the non-empty subtree is connected to A. If they are both non-empty then one idea is as shown in Figure 9.12. The left-most node of Right, Y, is transferred from its current position upwards to fill the gap after X. After this transfer, the tree remains ordered. Of course, the same idea works symmetrically, with the transfer of the right-most node of Left.

According to these considerations, the operation to delete an item from the binary dictionary is programmed in Figure 9.13. The transfer of the left-most node of the right subtree is accomplished by the relation

    delmin(Tree, Y, Tree1)

where Y is the minimal (that is, the left-most) node of Tree, and Tree1 is Tree with Y deleted.

There is another elegant solution to *add* and *delete*. The *add* relation can be defined non-deterministically so that a new item is inserted at any level of the tree, not just at the leaf level. The rules are:

To add X to a binary dictionary D either:

- add X at the root of D (so that X becomes the new root), or
- if the root of D is greater than X then insert X into the left subtree of D, otherwise insert X into the right subtree of D.

The difficult part of this is the insertion at the root of D. Let us formulate this operation as a relation

    addroot(D, X, D1)

where X is the item to be inserted at the root of D and D1 is the resulting dictionary with X as its root. Figure 9.14 illustrates the relations between X, D and D1. The


```
% del( Tree, X, NewTree):
%   deleting X from binary dictionary Tree gives NewTree

del( t( nil, X, Right), X, Right).

del( t( Left, X, nil), X, Left).

del( t( Left, X, Right), X, t( Left, Y, Right1) ) :-
  delmin( Right, Y, Right1).

del( t( Left, Root, Right), X, t( Left1, Root, Right) ) :-
  gt( Root, X),
  del( Left, X, Left1).

del( t( Left, Root, Right), X, t( Left, Root, Right1) ) :-
  gt( X, Root),
  del( Right, X, Right1).

% delmin( Tree, Y, NewTree):
%   delete minimal item Y in binary dictionary Tree producing NewTree

delmin( t( nil, Y, Right), Y, Right).

delmin( t( Left, Root, Right), Y, t( Left1, Root, Right) ) :-
  delmin( Left, Y, Left1).
```

Figure 9.13 Deleting from the binary dictionary.

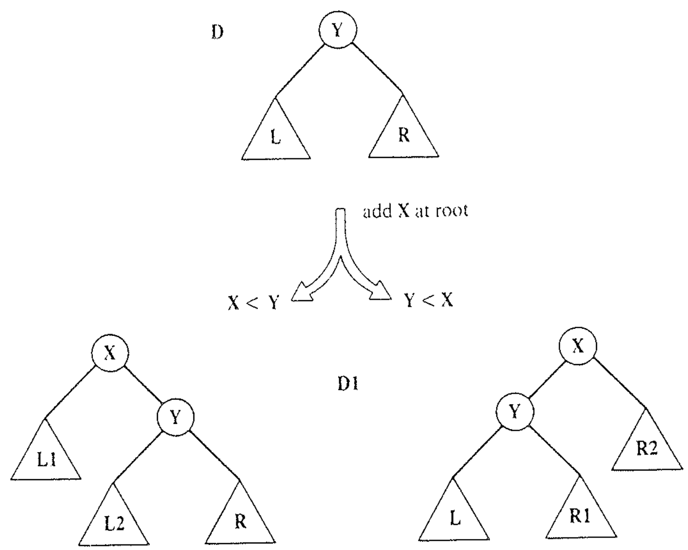

Figure 9.14 Inserting X at the root of a binary dictionary.

remaining question is now: What are the subtrees L1 and L2 in Figure 9.14 (or R1 and R2 alternatively)? The answer can be derived from the following constraints:

- L1 and L2 must be binary dictionaries;
- the set of nodes in L1 and L2 is equal to the set of nodes in L;
- all the nodes in L1 are less than X, and all the nodes in L2 are greater than X.

The relation that imposes all these constraints is just our addroot relation. Namely, if X were added as the root into L, then the subtrees of the resulting tree would be just L1 and L2. In Prolog, L1 and L2 must satisfy the goal:

```
addroot( L, X, t( L1, X, L2) )
```

The same constraints apply to R1 and R2:

```
addroot( R, X, t( R1, X, R2) )
```

Figure 9.15 shows a complete program for the 'non-deterministic' insertion into the binary dictionary.

The nice thing about this insertion procedure is that there is no restriction on the level of insertion. Therefore *add* can be used in the inverse direction in order to delete an item from the dictionary. For example, the following goal list constructs

```
% add( Tree, X, NewTree):
%   inserting X at any level of binary dictionary Tree gives NewTree

add( Tree, X, NewTree) :-
  addroot( Tree, X, NewTree).                 % Add X as new root

add( t( L, Y, R), X, t( L1, Y, R) ) :-        % Insert X into left subtree
  gt( Y, X),
  add( L, X, L1).

add( t( L, Y, R), X, t( L, Y, R1) ) :-        % Insert X into right subtree
  gt( X, Y),
  add( R, X, R1).

% addroot( Tree, X, NewTree): inserting X as the root into Tree gives NewTree

addroot( nil, X, t( nil, X, nil) ).           % Insert into empty tree

addroot( t( L, Y, R), X, t( L1, X, t( L2, Y, R) ) ) :-
  gt( Y, X),
  addroot( L, X, t( L1, X, L2) ).

addroot( t( L, Y, R), X, t( t( L, Y, R1), X, R2) ) :-
  gt( X, Y),
  addroot( R, X, t( R1, X, R2) ).
```

Figure 9.15 Insertion into the binary dictionary at any level of the tree.

a dictionary D containing the items 3, 5, 1, 6, and then deletes 5 yielding a dictionary DD:

```
add( nil, 3, D1), add( D1, 5, D2), add( D2, 1, D3),
add( D3, 6, D), add( DD, 5, D)
```

9.4 Displaying trees

Like all data objects in Prolog, a binary tree, T, can be directly output by the built-in procedure write. However, the goal

```
write( T)
```

will only output all the information, but will not graphically indicate the actual tree structure. It can be rather tiring to imagine the actual tree structure from a Prolog term that represents that tree. Therefore it is often desirable to have a tree typed out in a way that graphically indicates its structure.

There is a relatively simple method for displaying trees in such a form. The trick is to display a tree growing from left to right, and not from top to bottom as trees are usually pictured. The tree is rotated to the left so that the root becomes the left-most element, and the leaves are moved to the right. Figure 9.16 illustrates.

Let us define a procedure

```
show( T)
```

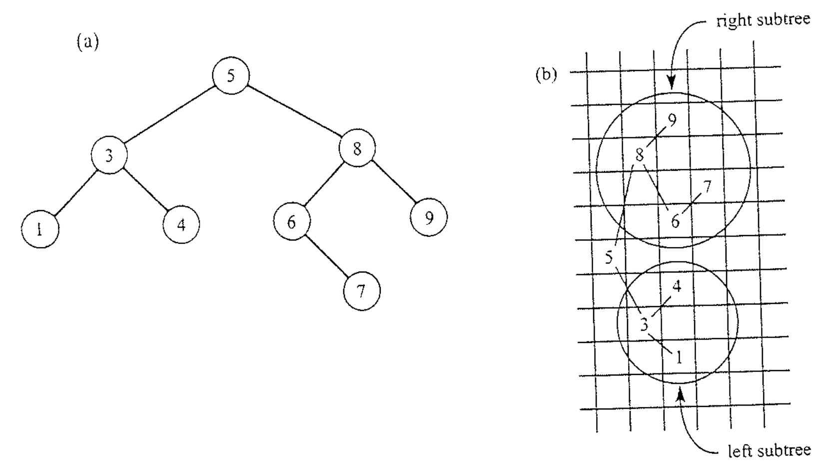

Figure 9.16 (a) A tree as normally pictured. (b) The same tree as typed out by the procedure show (arcs are added for clarity).

```
% show( Tree): display binary tree

show( Tree) :-
  show2( Tree, 0).

% show2( Tree, Indent): display Tree indented by Indent

show2( nil, _).

show2( t( Left, X, Right), Indent) :-
  Ind2 is Indent + 2,                    % Indentation of subtrees
  show2( Right, Ind2),                   % Display right subtree
  tab( Indent), write( X), nl,           % Write root
  show2( Left, Ind2).                    % Display left subtree
```

Figure 9.17 Displaying a binary tree.

to display a tree T in the form indicated in Figure 9.16. The principle is:

To show a non-empty tree, T:

(1) show the right subtree of T, indented by some distance, H, to the right;

(2) write the root of T;

(3) show the left subtree of T indented by distance H to the right.

The indentation distance H, which can be appropriately chosen, is an additional parameter for displaying trees. Introducing H we have the procedure

```
show2( T, H)
```

to display T indented H spaces from the left margin. The relation between the procedures show and show2 is:

```
show( T) :- show2( T, 0).
```

The complete program, which indents by 2, is shown in Figure 9.17. The principle of achieving such an output format can be easily adopted for displaying other types of trees.

Exercise

9.10 Our procedure for displaying trees shows a tree in an unusual orientation, so that the root is on the left and the leaves of the tree are on the right. Write a (more difficult) procedure to display a tree in the usual orientation with the root at the top and the leaves at the bottom.

9.5 Graphs

9.5.1 Representing graphs

Graph structures are used in many applications, such as representing relations, situations or problems. A graph is defined by a set of *nodes* and a set of *edges*, where each edge is a pair of nodes. When the edges are directed they are also called *arcs*. Arcs are represented by *ordered* pairs. Such a graph is a *directed* graph. The edges can be attached costs, names, or any kind of labels, depending on the application. Figure 9.18 shows examples.

Graphs can be represented in Prolog in several ways. One method is to represent each edge or arc separately as one clause. The graphs in Figure 9.18 can be thus represented by sets of clauses, for example:

```
connected( a, b).
connected( b, c).
...
arc( s, t, 3).
arc( t, v, 1).
arc( u, t, 2).
...
```

Another method is to represent a whole graph as one data object. A graph can be thus represented as a pair of two sets: nodes and edges. Each set can be represented as a list; each edge is a pair of nodes. Let us choose the functor graph to combine both sets into a pair, and the functor e for edges. Then one way to represent the (undirected) graph in Figure 9.18 is:

```
G1 = graph( [a,b,c,d], [e(a,b), e(b,d), e(b,c), e(c,d)] )
```

To represent a directed graph we can choose the functors digraph and a (for arcs). The directed graph of Figure 9.18 is then:

```
G2 = digraph( [s,t,u,v], [a(s,t,3), a(t,v,1), a(t,u,5), a(u,t,2), a(v,u,2)] )
```

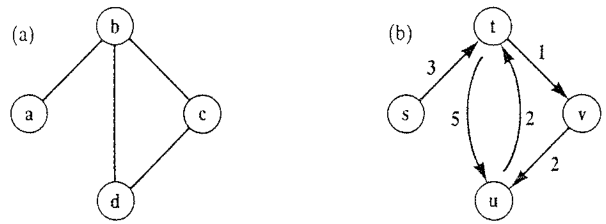

Figure 9.18 (a) A graph. (b) A directed graph with costs attached to the arcs.

If each node is connected to at least one other node then we can omit the list of nodes from the representation as the set of nodes is then implicitly specified by the list of edges.

Yet another method is to associate with each node a list of nodes that are adjacent to that node. Then a graph is a list of pairs consisting of a node plus its adjacency list. Our example graphs can then, for example, be represented by:

```
G1 = [ a -> [b], b -> [a,c,d], c -> [b,d], d -> [b,c] ]
G2 = [ s -> [t/3], t -> [u/5, v/1], u -> [t/2], v -> [u/2] ]
```

The symbols '->' and '/' above are, of course, infix operators.

What will be the most suitable representation will depend on the application and on operations to be performed on graphs. Two typical operations are:

- find a path between two given nodes;
- find a subgraph, with some specified properties, of a graph.

Finding a spanning tree of a graph is an example of the latter operation. In the following sections we will look at some simple programs for finding a path and for finding a spanning tree.

9.5.2 Finding a path

Let G be a graph, and A and Z two nodes in G. Let us define the relation

```
path( A, Z, G, P)
```

where P is an acyclic path between A and Z in G. P is represented as a list of nodes on the path. If G is the graph in the left-hand side of Figure 9.18 then:

```
path( a, d, G, [a,b,d] )
path( a, d, G, [a,b,c,d] )
```

Since a path must not contain any cycle, a node can appear in the path at most once. One method to find a path is:

To find an acyclic path, P, between A and Z in a graph, G:

If A = Z then P = [A], otherwise
find an acyclic path, P1, from some node Y to Z, and find a path from A to Y avoiding the nodes in P1.

This formulation implies another relation: find a path under the restriction of avoiding some subset of nodes (P1 above). We will, accordingly, define another procedure:

```
path1( A, Path1, G, Path)
```


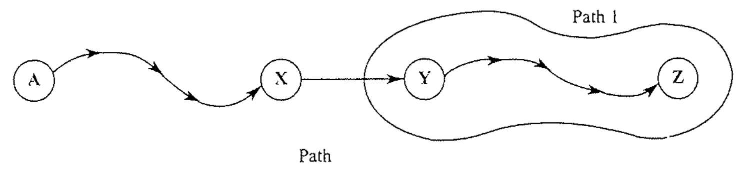

Figure 9.19 The **path1** relation: **Path** is a path between A and Z; the last part of **Path** overlaps with **Path1**.

As illustrated in Figure 9.19, the arguments are:

- A is a node,
- G is a graph,
- Path1 is a path in G,
- Path is an acyclic path in G that goes from A to the beginning of Path1 and continues along Path1 up to its end.

The relation between **path** and **path1** is:

```
path( A, Z, G, Path) :- path1( A, [Z], G, Path).
```

Figure 9.19 suggests a recursive definition of **path1**. The boundary case arises when the start node of Path1 (Y in Figure 9.19) coincides with the start node of Path, A. If the start nodes do not coincide then there must be a node, X, such that:

(1) Y is adjacent to X, and
(2) X is not in Path1, and
(3) Path must satisfy the relation path1(A, [X | Path1], G, Path).

A complete program is shown in Figure 9.20. In this program, **member** is the list membership relation. The relation

```
adjacent( X, Y, G)
```

```
% path( A, Z, Graph, Path): Path is an acyclic path from A to Z in Graph

path( A, Z, Graph, Path) :-
  path1( A, [Z], Graph, Path).

path1( A, [A | Path1], _, [A | Path1] ).

path1( A, [Y | Path1], Graph, Path) :-
  adjacent( X, Y, Graph),
  not member( X, Path1),                    % No-cycle condition
  path1( A, [X, Y | Path1], Graph, Path).
```

Figure 9.20 Finding an acyclic path, **Path**, from A to Z in **Graph**.

means that there is an arc from X to Y in graph G. The definition of this relation depends on the representation of graphs. If G is represented as a pair of sets (nodes and edges),

```
G = graph( Nodes, Edges)
```

then:

```
adjacent( X, Y, graph( Nodes, Edges) ) :-
  member( e(X,Y), Edges)
  ;
  member( e(Y,X), Edges).
```

A classical problem on graphs is to find a Hamiltonian path; that is, an acyclic path comprising all the nodes in the graph. Using path this can be done as follows:

```
hamiltonian( Graph, Path) :-
  path( _, _, Graph, Path),
  covers( Path, Graph).

covers( Path, Graph) :-
  not ( node( N, Graph), not member( N, Path) ).
```

Here, node(N, Graph) means: N is a node in Graph.

We can attach costs to paths. The cost of a path is the sum of the costs of the arcs in the path. If there are no costs attached to the arcs then we can talk about the length instead, counting 1 for each arc in the path. Our path and path1 relations can be modified to handle costs by introducing an additional argument, the cost, for each path:

```
path( A, Z, G, P, C)

path1( A, P1, C1, G, P, C)
```

Here, C is the cost of P and C1 is the cost of P1. The relation adjacent now also has an extra argument, the cost of an arc. Figure 9.21 shows a path-finding program that computes a path and its cost.

This procedure can be used for finding a minimum cost path. We can find such a path between two nodes, node1 and node2, in some graph Graph by the goals:

```
path( node1, node2, Graph, MinPath, MinCost),
not ( path( node1, node2, Graph, _, Cost), Cost < MinCost)
```

We can also find a maximum cost path between any pair of nodes in a graph Graph by the goals:

```
path( _, _, Graph, MaxPath, MaxCost),
not ( path( _, _, Graph, _, Cost), Cost > MaxCost)
```

It should be noted that this is a very inefficient way for finding minimal or maximal paths. This method unselectively investigates possible paths and is completely unsuitable for large graphs because of its high time complexity. The path-finding


```
% path( A, Z, Graph, Path, Cost):
%   Path is an acyclic path with cost Cost from A to Z in Graph

path( A, Z, Graph, Path, Cost) :-
  path1( A, [Z], 0, Graph, Path, Cost).

path1( A, [A | Path1], Cost1, Graph, [A | Path1], Cost1).

path1( A, [Y | Path1], Cost1, Graph, Path, Cost) :-
  adjacent( X, Y, CostXY, Graph),
  not member( X, Path1),
  Cost2 is Cost1 + CostXY,
  path1( A, [X, Y | Path1], Cost2, Graph, Path, Cost).
```

Figure 9.21 Path-finding in a graph: Path is an acyclic path with cost Cost from A to Z in Graph.

problem frequently arises in Artificial Intelligence. We will study more sophisticated methods for finding optimal paths in Chapters 11 and 12.

9.5.3 Finding a spanning tree of a graph

A graph is said to be *connected* if there is a path from any node to any other node. Let G = (V, E) be a connected graph with the set of nodes V and the set of edges E. A *spanning tree* of G is a connected graph T = (V, E′) where E′ is a subset of E such that:

(1) T is connected, and

(2) there is no cycle in T.

These two conditions guarantee that T is a tree. For the left-hand side graph of Figure 9.18, there are three spanning trees, which correspond to three lists of edges:

```
Tree1 = [a-b, b-c, c-d]

Tree2 = [a-b, b-d, d-c]

Tree3 = [a-b, b-d, b-c]
```

Here each term of the form X-Y denotes an edge between nodes X and Y. We can pick any node in such a list as the root of a tree. Spanning trees are of interest, for example, in communication problems because they provide, with the minimum number of communication lines, a path between any pair of nodes.

We will define a procedure

```
stree( G, T)
```

where T is a spanning tree of G. We will assume that G is connected. We can imagine constructing a spanning tree algorithmically as follows: Start with the empty set of edges and gradually add new edges from G, taking care that a cycle is never created, until no more edge can be added because it would create a cycle. The resulting set of edges defines a spanning tree. The no-cycle condition can be maintained by a simple rule: an edge can be added only if one of its nodes is already in the growing tree, and the other node is not yet in the tree. A program that implements this idea is shown in Figure 9.22. The key relation in this program is:

spread(Tree1, Tree, G)

All the three arguments are sets of edges. G is a connected graph; Tree1 and Tree are subsets of G such that they both represent trees. Tree is a spanning tree of G obtained by adding zero or more edges of G to Tree1. We can say that 'Tree1 gets spread to Tree'.

It is interesting that we can also develop a working program for constructing a spanning tree in another, completely declarative way, by simply stating

```
% Finding a spanning tree of a graph
%
% Trees and graphs are represented by lists of their edges.
% For example: Graph = [a-b, b-c, b-d, c-d]

% stree( Graph, Tree): Tree is a spanning tree of Graph

stree( Graph, Tree) :-
  member( Edge, Graph),
  spread( [Edge], Tree, Graph).

% spread( Tree1, Tree, Graph): Tree1 'spreads to' spanning tree Tree of Graph

spread( Tree1, Tree, Graph) :-
  addedge( Tree1, Tree2, Graph),
  spread( Tree2, Tree, Graph).

spread( Tree, Tree, Graph) :-
  not addedge( Tree, _, Graph).          % No edge can be added without creating a cycle

% addedge( Tree, NewTree, Graph):
%   add an edge from Graph to Tree without creating a cycle

addedge( Tree, [A-B | Tree], Graph) :-
  adjacent( A, B, Graph),                % Nodes A and B adjacent in Graph
  node( A, Tree),                        % A in Tree
  not node( B, Tree).                    % A-B doesn't create a cycle in Tree

adjacent( Node1, Node2, Graph) :-
  member( Node1-Node2, Graph)
  ;
  member( Node2-Node1, Graph).

node( Node, Graph) :-                    % Node is a node in Graph if
  adjacent( Node, _, Graph).             % Node is adjacent to anything in Graph
```

Figure 9.22 Finding a spanning tree of a graph: an 'algorithmic' program. The program assumes that the graph is connected.

mathematical definitions. We will assume that both graphs and trees are represented by lists of their edges, as in the program of Figure 9.22. The definitions we need are:

(1) T is a spanning tree of G if:
- T is a subset of G, and
- T is a tree, and
- T 'covers' G; that is, each node of G is also in T.

(2) A set of edges T is a tree if:
- T is connected, and
- T has no cycle.

```
% Finding a spanning tree
% Graphs and trees are represented as lists of edges.

% stree( Graph, Tree): Tree is a spanning tree of Graph

stree( Graph, Tree) :-
  subset( Graph, Tree),
  tree( Tree),
  covers( Tree, Graph).

tree( Tree) :-
  connected( Tree),
  not hasacycle( Tree).

% connected( Graph): there is a path between any two nodes in Graph

connected( Graph) :-
  not ( node( A, Graph), node( B, Graph), not path( A, B, Graph, _) ).

hasacycle( Graph) :-
  adjacent( Node1, Node2, Graph),
  path( Node1, Node2, Graph, [Node1, X, Y | _] ).     % Path of length > 1

% covers( Tree, Graph): every node of Graph is in Tree

covers( Tree, Graph) :-
  not ( node( Node, Graph), not node( Node, Tree) ).

% subset( List1, List2): List2 represents a subset of List1

subset( [], [] ).

subset( [X | Set], Subset) :-                          % X not in subset
  subset( Set, Subset).

subset( [X | Set], [X | Subset]) :-                    % X included in subset
  subset( Set, Subset).
```

Figure 9.23 Finding a spanning tree of a graph: a 'declarative' program. Relations **node** and **adjacent** are as in Figure 9.22.

Using our path program of the previous section, these definitions can be stated in Prolog as shown in Figure 9.23. It should be noted, however, that this program is, in this form, of little practical interest because of its inefficiency.

Exercises

9.11 Consider spanning trees of graphs that have costs attached to edges. Let the *cost* of a spanning tree be defined as the sum of the costs of all the edges in the tree. Write a program to find a minimum-cost spanning tree of a given graph.

9.12 Experiment with the spanning tree programs in Figures 9.22 and 9.23, and measure their execution times. Identify the sources of inefficiency in the second program.

Summary

In this chapter we studied Prolog implementations of some frequently used data structures and associated operations on them. These include:

- Lists:
  - sorting lists:
    - bubble sort
    - insertion sort
    - quicksort
  - efficiency of these procedures
- Representing sets as binary trees and binary dictionaries:
  - searching for an item in a tree
  - adding an item
  - deleting an item
  - adding as a leaf, adding as the root
  - the balance of trees, how balance affects the efficiency of these operations
  - displaying trees
- Graphs:
  - representing graphs
  - finding a path in a graph
  - finding a spanning tree of a graph

References

In this chapter we have tackled in Prolog classical topics of sorting and of maintaining data structures for representing sets. These topics are covered in general books on algorithms and data structures, for example, Aho, Hopcroft and Ullman (1974, 1983), Cormen, Leiserson and

Rivest (1990), Gonnet and Baeza-Yates (1991) and Kingston (1998). The Prolog program for insertion at any level of the binary tree (Section 9.3) was first shown to the author by M. van Emden (personal communication).

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chapter 10

Advanced Tree Representations

In this chapter we look at advanced techniques for representing data sets by trees. The key idea is to keep the tree balanced, or approximately balanced, in order to prevent the tree from degenerating toward a list. Such tree-balancing schemes guarantee relatively fast, logarithmic-time data-access even in the worst case. Two such schemes are presented in this chapter: 2-3 trees and AVL-trees. (The knowledge of this chapter is not a prerequisite to any other chapter.)

10.1 The 2-3 dictionary

A binary tree is said to be well balanced if both its subtrees are of approximately equal height (or size) and they are also balanced. The height of a balanced tree is approximately $\log n$ where n is the number of nodes in the tree. The time needed to evaluate the relations in, add and delete on binary dictionaries grows proportionally with the height of the tree. On balanced dictionaries, then, all these operations can be done in time that is in the order of $\log n$. The logarithmic growth of the complexity of the set membership testing is a definite improvement over the list representation of sets, where the complexity grows linearly with the size of the data set. However, poor balance of a tree will degrade the performance of the dictionary. In extreme cases, the binary dictionary degenerates into a list, as shown in Figure 10.1. The form of the dictionary depends on the sequence in which the data is inserted. In the best case we get a good balance with performance in the order $\log n$, and in the worst case the performance is in the order n. Analysis shows that on average, assuming that any sequence of data is equally likely, the complexity of in, add and delete is still in the order $\log n$. So the average performance is, fortunately, closer to the best case than to the worst case. There are, however, several rather

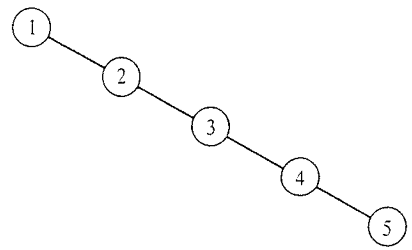

Figure 10.1 A totally unbalanced binary dictionary. Its performance is reduced to that of a list.

simple schemes for keeping good balance of the tree regardless of the data sequence. Such schemes guarantee the *worst case* performance of in, add and delete in the order $\log n$. One of them is the 2-3 tree; another scheme is the AVL-tree.

The *2-3* tree is defined as follows: it is either empty, or it consists of a single node, or it is a tree that satisfies the following conditions:

- each internal node has two or three children, and
- all the leaves are at the same level.

A 2-3 dictionary is a 2-3 tree in which the data items are stored in the leaves, ordered from left to right. Figure 10.2 shows an example. The internal nodes contain labels that specify the minimal elements of the subtrees as follows:

- if an internal node has two subtrees, this internal node contains the minimal element of the second subtree;
- if an internal node has three subtrees then this node contains the minimal elements of the second and of the third subtree.

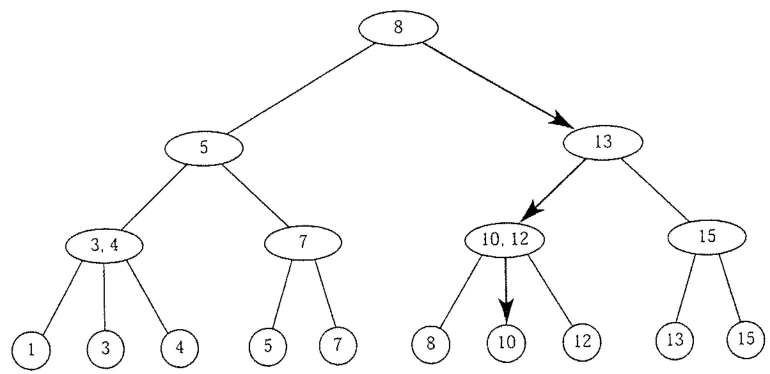

Figure 10.2 A 2-3 dictionary. The indicated path corresponds to searching for the item 10.

To search for an item, X, in a 2-3 dictionary we start at the root and move toward the bottom level according to the labels in the internal nodes. Let the root contain the labels M1 and M2. Then:

- if X < M1 then continue the search in the left subtree, otherwise
- if X < M2 then continue the search in the middle subtree, otherwise
- continue the search in the right subtree.

If the root only contains one label, M, then proceed to the left subtree if X < M, and to the right subtree otherwise. This is repeated until the leaf level is reached, and at this point X is either successfully found or the search fails.

As all the leaves are at the same level, the 2-3 tree is perfectly balanced with respect to the heights of the subtrees. All search paths from the root to a leaf are of the same length which is of the order $\log n$, where n is the number of items stored in the tree.

When inserting new data, the 2-3 tree can also grow in breadth, not only in depth. Each internal node that has two children can accommodate an additional child, which results in the breadth-wise growth. If, on the other hand, a node with three children accepts another child then this node is split into two nodes, each of them taking over two of the total of four children. The so-generated new internal node gets incorporated further up in the tree. If this happens at the top level then the tree is forced to grow upwards. Figure 10.3 illustrates these principles.

Insertion into the 2-3 dictionary will be programmed as the relation

```
add23( Tree, X, NewTree)
```

where NewTree is obtained by inserting X into Tree. The main burden of insertion will be transferred to two auxiliary relations, both called ins. The first one has three arguments:

```
ins( Tree, X, NewTree)
```

where NewTree is the result of inserting X into Tree. Tree and NewTree have the *same height*. But, of course, it is not always possible to preserve the same height after

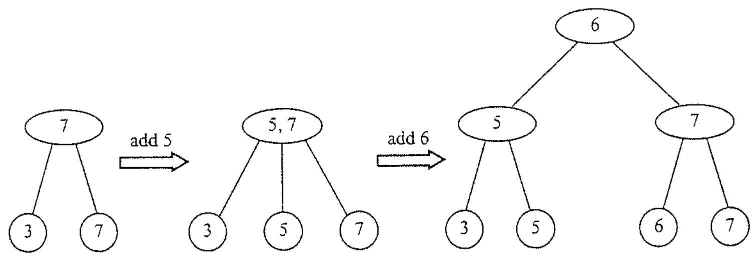

Figure 10.3 Inserting into a 2-3 dictionary. The tree first grows in breadth and then upwards.

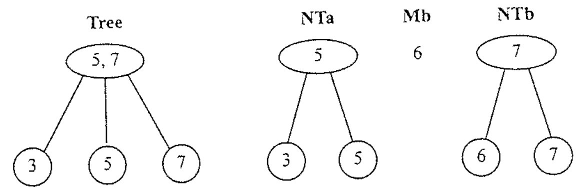

Figure 10.4 The objects in the figure satisfy the relation ins(Tree, 6, NTa, Mb, NTb).

insertion. Therefore we have another ins relation, with five arguments, to cater for this case:

```
ins( Tree, X, NTa, Mb, NTb)
```

Here, when inserting X into Tree, Tree is split into two trees: NTa and NTb. Both NTa and NTb have the same height as Tree. Mb is the minimal element of NTb. Figure 10.4 shows an example.

In the program, a 2-3 tree will be represented, depending on its form, as follows:

- nil represents the empty tree.
- l(X) represents a single node tree, a leaf with item X.
- n2(T1, M, T2) represents a tree with two subtrees, T1 and T2; M is the minimal element of T2.
- n3(T1, M2, T2, M3, T3) represents a tree with three subtrees, T1, T2 and T3; M2 is the minimal element of T2, and M3 is the minimal element of T3.

T1, T2 and T3 are all 2-3 trees.

The relation between add23 and ins is: if after insertion the tree does not grow upwards then simply:

```
add23( Tree, X, NewTree) :-
  ins( Tree, X, NewTree).
```

If, however, the height after insertion increases, then ins determines the two subtrees, T1 and T2, which are then combined into a bigger tree:

```
add23( Tree, X, n2( T1, M, T2) ) :-
  ins( Tree, X, T1, M, T2).
```

The ins relation is more complicated because it has to deal with many cases: inserting into the empty tree, a single node tree, a tree of type n2 or n3. Additional subcases arise from insertion into the first, second or third subtree. Accordingly, ins will be defined by a set of rules so that each clause about ins will deal with one of the cases. Figure 10.5 illustrates some of these cases. The cases in this figure translate into Prolog as follows:

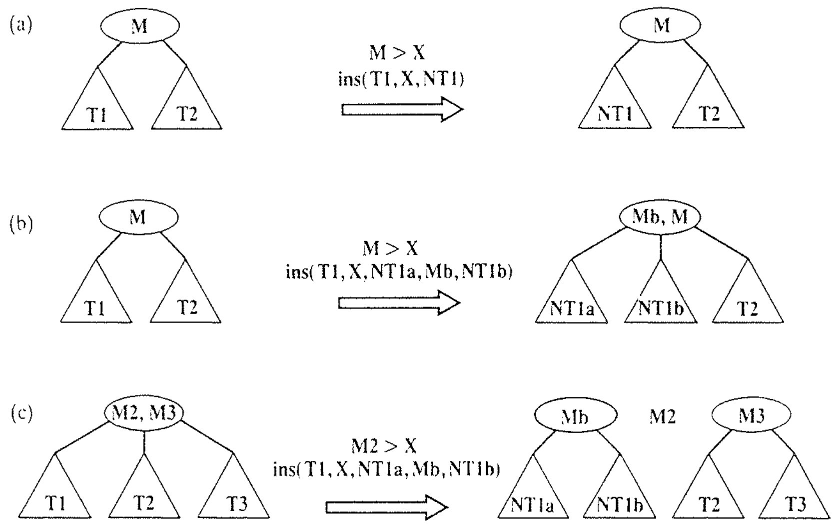

Figure 10.5 Some cases of the ins relation.
(a) ins(n2(T1, M, T2), X, n2(NT1, M, T2));
(b) ins(n2(T1, M, T2), X, n3(NT1a, Mb, NT1b, M, T2));
(c) ins(n3(T1, M2, T2, M3, T3), X, n2(NT1a, Mb, NT1b), M2, n2(T2, M3, T3))

Case a

```
ins( n2( T1, M, T2), X, n2( NT1, M, T2) ) :-
  gt( M, X),                              % M greater than X
  ins( T1, X, NT1).
```

Case b

```
ins( n2( T1, M, T2), X, n3( NT1a, Mb, NT1b, M, T2) ) :-
  gt( M, X),
  ins( T1, X, NT1a, Mb, NT1b).
```

Case c

```
ins( n3( T1, M2, T2, M3, T3), X, n2( NT1a, Mb, NT1b), M2, n2( T2, M3, T3) ) :-
  gt( M2, X),
  ins( T1, X, NT1a, Mb, NT1b).
```

Figure 10.6 shows the complete program for inserting into the 2-3 dictionary. Figure 10.7 shows a program for displaying 2-3 trees.

Our program occasionally does some unnecessary backtracking. If the three-argument ins fails then the five-argument ins is called, which redoes part of the


```
% Insertion in the 2-3 dictionary

add23( Tree, X, Tree1) :-                    % Add X to Tree giving Tree1
  ins( Tree, X, Tree1).                      % Tree grows in breadth
add23( Tree, X, n2( T1, M2, T2) ) :-         % Tree grows upwards
  ins( Tree, X, T1, M2, T2).
add23( nil, X, l(X) ).
ins( l(A), X, l(A), X, l(X) ) :-
  gt( X, A).
ins( l(A), X, l(X), A, l(A) ) :-
  gt( A, X).
ins( n2( T1, M, T2), X, n2( NT1, M, T2) ) :-
  gt( M, X),
  ins( T1, X, NT1).
ins( n2( T1, M, T2), X, n3( NT1a, Mb, NT1b, M, T2) ) :-
  gt( M, X),
  ins( T1, X, NT1a, Mb, NT1b).
ins( n2( T1, M, T2), X, n2( T1, M, NT2) ) :-
  gt( X, M),
  ins( T2, X, NT2).
ins( n2( T1, M, T2), X, n3( T1, M, NT2a, Mb, NT2b) ) :-
  gt( X, M),
  ins( T2, X, NT2a, Mb, NT2b).
ins( n3( T1, M2, T2, M3, T3), X, n3( NT1, M2, T2, M3, T3) ) :-
  gt( M2, X),
  ins( T1, X, NT1).
ins( n3( T1, M2, T2, M3, T3), X, n2( NT1a, Mb, NT1b), M2, n2( T2, M3, T3) ) :-
  gt( M2, X),
  ins( T1, X, NT1a, Mb, NT1b).
ins( n3( T1, M2, T2, M3, T3), X, n3( T1, M2, NT2, M3, T3) ) :-
  gt( X, M2), gt( M3, X),
  ins( T2, X, NT2).
ins( n3( T1, M2, T2, M3, T3), X, n2( T1, M2, NT2a), Mb, n2( NT2b, M3, T3) ) :-
  gt( X, M2), gt( M3, X),
  ins( T2, X, NT2a, Mb, NT2b).
ins( n3( T1, M2, T2, M3, T3), X, n3( T1, M2, T2, M3, NT3) ) :-
  gt( X, M3),
  ins( T3, X, NT3).
ins( n3( T1, M2, T2, M3, T3), X, n2( T1, M2, T2), M3, n2( NT3a, Mb, NT3b) ) :-
  gt( X, M3),
  ins( T3, X, NT3a, Mb, NT3b).
```

Figure 10.6 Inserting in the 2-3 dictionary. In this program, an attempt to insert a duplicate item will fail.

```
% Displaying 2-3 dictionary

show( T) :-
  show( T, 0).

show( nil, _).

show( l(A), H) :-
  tab( H), write( A), nl.

show( n2( T1, M, T2), H) :-
  H1 is H + 5,
  show( T2, H1),
  tab( H), write( --), nl,
  tab( H), write( M), nl,
  tab( H), write( --), nl,
  show( T1, H1).

show( n3( T1, M2, T2, M3, T3), H) :-
  H1 is H + 5,
  show( T3, H1),
  tab( H), write( --), nl,
  tab( H), write( M3), nl,
  show( T2, H1),
  tab( H), write( M2), nl,
  tab( H), write( --), nl,
  show( T1, H1).
```

```
               15
          --
          15
          --
               13
     --
     13
     --
               12
          --
          12
               10
          10
          --
               8
--
8
--
               7
          --
          7
          --
               5
     --
     5
     --
               4
          --
          4
               3
          3
          --
               1
```

Figure 10.7 Left: a program to display a 2-3 dictionary. Right: the dictionary of Figure 10.2 as displayed by this program.

work. This source of inefficiency can easily be eliminated by, for example, redefining ins as:

```
ins2( Tree, X, NewTrees)
```

NewTrees is a list of length 1 or 3, as follows:

```
NewTrees = [ NewTree]  if  ins( Tree, X, NewTree)
NewTrees = [ NTa, Mb, NTb]  if  ins( Tree, X, NTa, Mb, NTb)
```


The add23 relation would be, accordingly, redefined as:

```
add23( T, X, T1) :-
  ins2( T, X, Trees),
  combine( Trees, T1).
```

The combine relation has to produce a single tree, T1, from the list Trees.

Exercises

10.1 Define the relation

```
in( Item, Tree)
```

to search for Item in a 2-3 dictionary Tree.

10.2 Modify the program of Figure 10.6 to avoid backtracking (define relations ins2 and combine).

10.2 AVL-tree: an approximately balanced tree

AVL-tree is a binary tree that has the following properties:

(1) Its left subtree and right subtree differ in height by 1 at the most.

(2) Both subtrees themselves are also AVL-trees.

This definition allows for trees that are slightly out of balance. It can be shown that the height of an AVL-tree is always, even in the worst case, roughly proportional to $\log n$ where n is the number of nodes in the tree. This guarantees the logarithmic performance for the operations in, add and del.

Operations on the AVL-dictionary are essentially the same as on binary dictionaries, with some additions to maintain approximate balance of the tree. If the tree gets out of approximate balance after an insertion or deletion then some additional mechanism will get it back into the required degree of balance. To implement this mechanism efficiently, we have to maintain some additional information about the balance of the tree. Essentially we only need the difference between the heights of its subtrees, which is either -1, 0 or $+1$. For the sake of simplicity of the operations involved we will, however, prefer to maintain the complete heights of trees and not only the differences.

We will define the insertion relation as:

```
addavl( Tree, X, NewTree)
```

where both Tree and NewTree are AVL-dictionaries such that NewTree is Tree with X inserted. AVL-trees will be represented by terms of the form:

```
t( Left, A, Right) / Height
```

where A is the root, Left and Right are the subtrees, and Height is the height of the tree. The empty tree is represented by nil/0. Now let us consider the insertion of X into a non-empty AVL-dictionary:

```
Tree = t( L, A, R)/H
```

We will start our discussion by only considering the case where X is greater than A. Then X is to be inserted into R and we have the following relation:

```
addavl( R, X, t( R1, B, R2)/Hb)
```

Figure 10.8 illustrates the following ingredients from which NewTree is to be constructed:

```
L, A, R1, B, R2
```

What can be the heights of L, R, R1 and R2? L and R can only differ in height by 1 at the most. Figure 10.8 shows what the heights of R1 and R2 can be. As only one item, X, has been inserted into R, at most one of the subtrees R1 and R2 can have the height $h + 1$.

In the case that X is less than A then the situation is analogous with left and right subtrees interchanged. Therefore, in any case, we have to construct NewTree from three trees (let us call them Tree1, Tree2 and Tree3), and two single items, A and B. Let us now consider the question: How can we combine these five ingredients to make

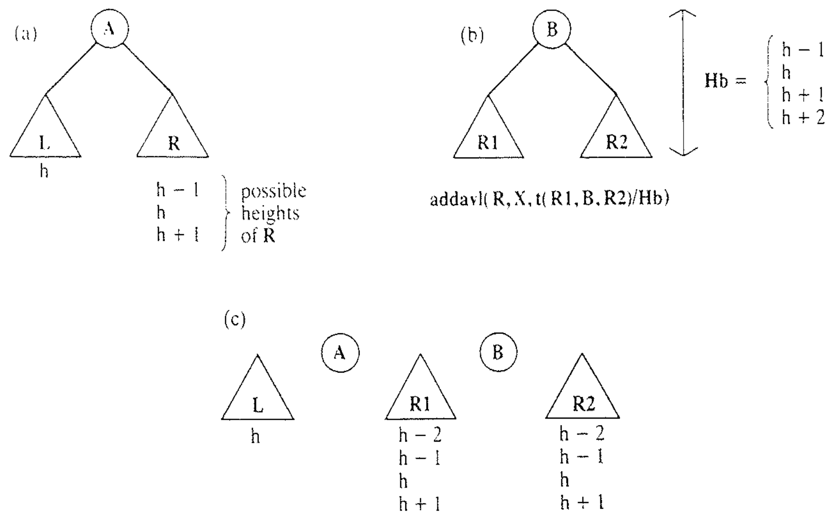

Figure 10.8 The problem of AVL insertion: (a) AVL-tree before inserting X, X > A; (b) AVL-tree after inserting X into R; (c) ingredients from which the new tree is to be constructed.

NewTree so that NewTree is an AVL-dictionary? The order from left to right in NewTree has to be:

```
Tree1, A, Tree2, B, Tree3
```

We have to consider three cases:

(1) The middle tree, Tree2, is taller than both other trees.

(2) Tree1 is at least as tall as Tree2 and Tree3.

(3) Tree3 is at least as tall as Tree2 and Tree1.

Figure 10.9 shows how NewTree can be constructed in each of these cases. In case 1, the middle tree Tree2 has to be decomposed and its parts incorporated into NewTree. The three rules of Figure 10.9 are easily translated into Prolog as a relation:

```
combine( Tree1, A, Tree2, B, Tree3, NewTree)
```

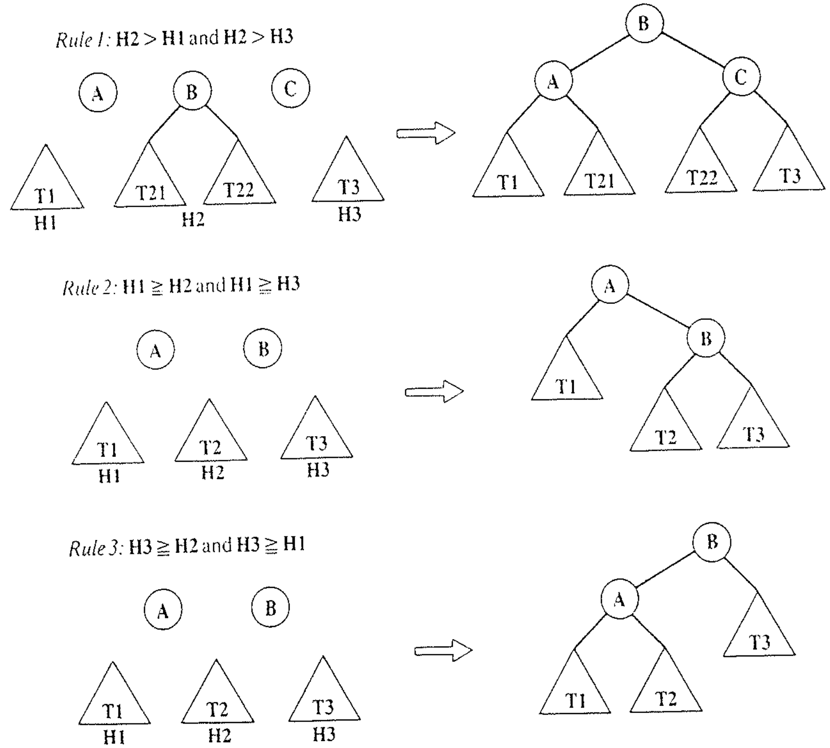

Figure 10.9 Three combination rules for AVL-trees.

The last argument, NewTree, is an AVL-tree constructed from five ingredients, the first five arguments. Rule 1, for example, becomes:

```
combine(
  T1/H1, A, t(T21,B,T22) / H2, C, T3/H3,            % Five ingredients
  t( t(T1/H1,A,T21)/Ha, B, t(T22,C,T3/H3)/Hc)/Hb) :-  % Their combination
  H2 > H1, H2 > H3,                                 % Middle tree is tallest
  Ha is H1 + 1,                                     % Height of left subtree
  Hc is H3 + 1,                                     % Height of right subtree
  Hb is Ha + 1.                                     % Height of the whole tree
```

```
% addavl( Tree, X, NewTree): insertion into AVL-dictionary
%   Tree = t( Left, Root, Right)/HeightOfTree
%   Empty tree = nil/0

addavl( nil/0, X, t( nil/0, X, nil/0)/1).          % Add X to empty tree

addavl( t( L, Y, R)/Hy, X, NewTree) :-             % Add X to non-empty tree
  gt( Y, X),
  addavl( L, X, t( L1, Z, L2)/_ ),                 % Add into left subtree
  combine( L1, Z, L2, Y, R, NewTree).              % Combine ingredients of NewTree

addavl( t( L, Y, R)/Hy, X, NewTree) :-
  gt( X, Y),
  addavl( R, X, t( R1, Z, R2)/_ ),                 % Add into right subtree
  combine( L, Y, R1, Z, R2, NewTree).

% combine( Tree1, A, Tree2, B, Tree3, NewTree):
%   combine Tree1, Tree2, Tree3 and nodes A and B into an AVL-tree

combine( T1/H1, A, t( T21, B, T22)/H2, C, T3/H3,
        t( t(T1/H1,A,T21)/Ha, B, t(T22,C,T3/H3)/Hc)/Hb) :-
  H2 > H1, H2 > H3,                                % Middle subtree tallest
  Ha is H1 + 1,
  Hc is H3 + 1,
  Hb is Ha + 1.

combine( T1/H1, A, T2/H2, C, T3/H3, t( T1/H1, A, t(T2/H2,C,T3/H3)/Hc)/Ha) :-
  H1 >= H2, H1 >= H3,                              % Tall left subtree
  max1( H2, H3, Hc),
  max1( H1, Hc, Ha).

combine( T1/H1, A, T2/H2, C, T3/H3, t( t(T1/H1,A,T2/H2)/Ha, C, T3/H3)/Hc) :-
  H3 >= H2, H3 >= H1,                              % Tall right subtree
  max1( H1, H2, Ha),
  max1( Ha, H3, Hc).

max1( U, V, M) :-                                  % M is 1 + max. of U and V
  U > V, !, M is U + 1
  ;
  M is V + 1.
```

Figure 10.10 AVL-dictionary insertion. In this program, an attempt to insert a duplicate will fail. See Figure 10.9 for **combine**.

A complete **addavl** program, which also computes the heights of the tree and the subtrees, is shown as Figure 10.10.

Our program works with the heights of trees. A more economical representation is, as said earlier, possible. In fact, we only need the balance, which can only be -1, 0 or $+1$. The disadvantage of such economization would be, however, somewhat more complicated combination rules.

Exercises

10.3 Define the relation

avl(Tree)

to test whether a binary tree Tree is an AVL-tree; that is, all the sibling subtrees may differ in their heights by 1 at the most. Let binary trees be represented by terms of the form t(Left, Root, Right) or nil.

10.4 Trace the execution of the AVL insertion algorithm, starting with the empty tree and successively inserting 5, 8, 9, 3, 1, 6, 7. How is the root item changing during this process?

Summary

- 2-3 trees and AVL-trees, implemented in this chapter, are types of balanced trees.
- Balanced, or approximately balanced, trees guarantee efficient execution of the three basic operations on trees: looking for an item, adding or deleting an item. All these operations can be done in time proportional to $\log n$, where n is the number of nodes in the tree.

References

2-3 trees are described in detail by, for example, Aho, Hopcroft and Ullman (1974, 1983). In their 1983 book an implementation in Pascal is also given. Wirth (1976) gives a Pascal program to handle AVL-trees. 2-3 trees are a special case of more general B-trees. This and several other variations or data structures related to 2-3 trees and AVL-trees are covered, among others, by Gonnet and Baeza-Yates (1991) together with various results on the behaviour of these structures. A program for AVL-tree insertion that only uses tree-bias information (that is, the difference between the heights of the subtrees -1, 0 or $+1$, and not the complete height) was published by van Emden (1981).

Aho, A.V., Hopcroft, J.E. and Ullman, J.D. (1974) *The Design and Analysis of Computer Algorithms*. Addison-Wesley.

Aho, A.V., Hopcroft, J.E. and Ullman, J.D. (1983) *Data Structures and Algorithms*. Addison-Wesley.

Gonnet, G.H. and Baeza-Yates, R. (1991) *Handbook of Algorithms and Data Structures in Pascal and C*. Addison-Wesley.

van Emden, M. (1981) *Logic Programming Newsletter* 2.

Wirth, N. (1976) *Algorithms + Data Structures = Programs*. Englewood Cliffs, NJ: Prentice Hall.

part II

Prolog in Artificial Intelligence

chapter 11

Basic Problem-Solving Strategies

This chapter is centred around a general scheme, called *state space*, for representing problems. A state space is a graph whose nodes correspond to problem situations, and a given problem is reduced to finding a path in this graph. We will study examples of formulating problems using the state-space approach, and discuss general methods for solving problems represented in this formalism. Problem solving involves graph searching and exploring alternatives. The basic strategies for exploring alternatives, presented in this chapter, are the depth-first search, breadth-first search and iterative deepening.

11.1 Introductory concepts and examples

Let us consider the example in Figure 11.1. The problem is to find a plan for rearranging a stack of blocks as shown in the figure. We are only allowed to move one block at a time. A block can be grasped only when its top is clear. A block can be put on the table or on some other block. To find a required plan, we have to find a sequence of moves that accomplish the given transformation.

We can think of this problem as a problem of exploring among possible alternatives. In the initial problem situation we are only allowed one alternative: put block C on the table. After C has been put on the table, we have three alternatives:

- put A on table, or
- put A on C, or
- put C on A.

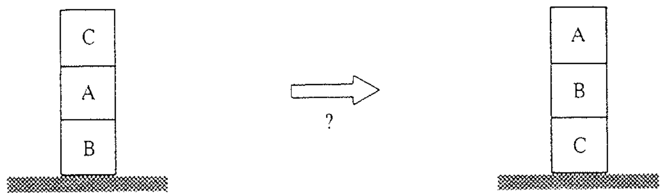

Figure 11.1 A blocks rearrangement problem.

We will not seriously consider putting C on the table as this clearly has no effect on the situation.

As this example illustrates, we have, in such a problem, two types of concept:

(1) Problem situations.

(2) Legal moves, or actions, that transform problem situations into other situations.

Problem situations and possible moves form a directed graph, called a *state space*. A state space for our example problem is shown in Figure 11.2. The nodes of the graph correspond to problem situations, and the arcs correspond to legal transitions between states. The problem of finding a solution plan is equivalent to finding a path between the given initial situation (the start node) and some specified final situation, also called a *goal node*.

Figure 11.3 shows another example problem: an eight puzzle and its representation as a path-finding problem. The puzzle consists of eight sliding tiles, numbered by digits from 1 to 8, and arranged in a 3 by 3 array of nine cells. One of the cells is always empty, and any adjacent tile can be moved into the empty cell. We can say that the empty cell is allowed to move around, swapping its place with any of the adjacent tiles. The final situation is some special arrangement of tiles, as shown for example in Figure 11.3.

It is easy to construct similar graph representations for other popular puzzles. Straightforward examples are the Tower of Hanoi, or getting fox, goose and grain across the river. In the latter problem, the boat can only hold the farmer and one other object, and the farmer has to protect the goose from the fox, and the grain from the goose. Many practical problems also naturally fit this paradigm. Among them is the travelling salesman problem, which is the formal model of many practical optimization problems. The problem is defined by a map with n cities and road distances between the cities. The task is to find a shortest route from some starting city, visiting all the cities and ending in the starting city. No city, with the exception of the starting one, may appear in the tour twice.

Let us summarize the concepts introduced by these examples. The state space of a given problem specifies the 'rules of the game': nodes in the state space correspond

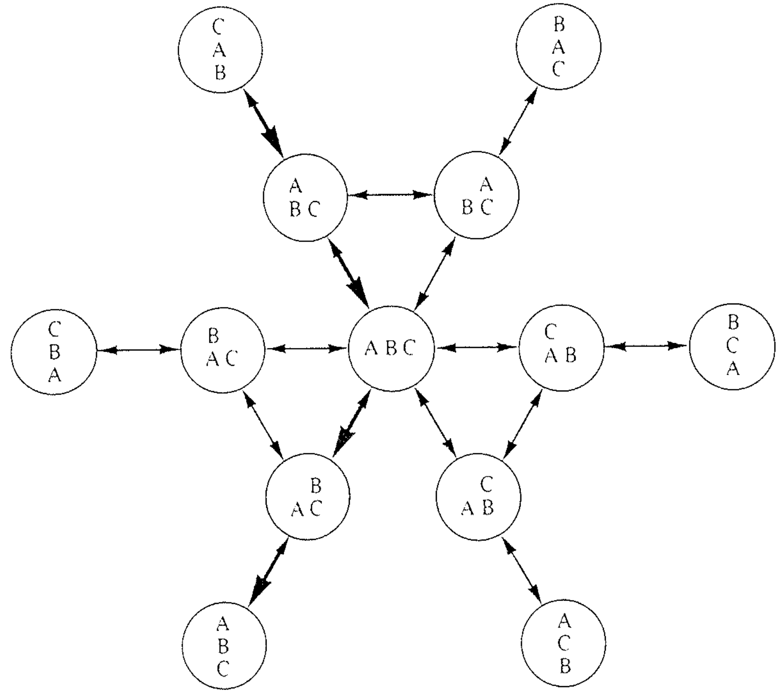

Figure 11.2 A state-space representation of the block manipulation problem. The indicated path is a solution to the problem in Figure 11.1.

to situations, and arcs correspond to 'legal moves', or actions, or solution steps. A particular problem is defined by:

- a state space,
- a start node,
- a goal condition (a condition to be reached); 'goal nodes' are those nodes that satisfy this condition.

We can attach costs to legal moves or actions. For example, costs attached to moving blocks in the block manipulation problem would indicate that some blocks are harder to move than others. In the travelling salesman problem, moves correspond to direct city-to-city journeys. Naturally, the costs of such moves are the distances between the cities.

In cases where costs are attached to moves, we are normally interested in minimum cost solutions. The cost of a solution is the sum of the costs of the arcs along the solution path. Even if no costs are given we may have an optimization problem: we may be interested in shortest solutions.

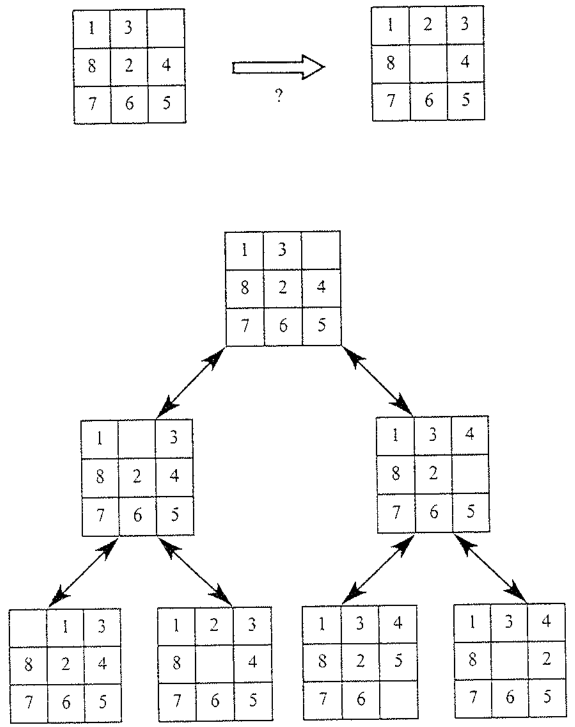

Figure 11.3 An eight puzzle and a corresponding state-space representation.

Before presenting some programs that implement classical algorithms for searching state spaces, let us first discuss how a state space can be represented in a Prolog program.

We will represent a state space by a relation

```
s( X, Y)
```

which is true if there is a legal move in the state space from a node X to a node Y. We will say that Y is a *successor* of X. If there are costs associated with moves then we will add a third argument, the cost of the move:

```
s( X, Y, Cost)
```

This relation can be represented in the program explicitly by a set of facts. For typical state spaces of any significant complexity this would be, however, impractical or impossible. Therefore the successor relation, s, is usually defined implicitly by stating the rules for computing successor nodes of a given node.

Another question of general importance is, how to represent problem situations, that is nodes themselves. The representation should be compact, but it should also enable efficient execution of operations required; in particular, the evaluation of the successor relation, and possibly the associated costs.

As an example, let us consider the block manipulation problem of Figure 11.1. We will consider a more general case, so that there are altogether any number of blocks that are arranged in one or more stacks. The number of stacks will be limited to some given maximum to make the problem more interesting. This may also be a realistic constraint because a robot that manipulates blocks may be only given a limited working space on the table.

A problem situation can be represented as a list of stacks. Each stack can be, in turn, represented by a list of blocks in that stack ordered so that the top block in the stack is the head of the list. Empty stacks are represented by empty lists. The initial situation of the problem in Figure 11.1 can be thus represented by:

```
[ [c,a,b], [ ], [ ] ]
```

A goal situation is any arrangement with the ordered stack of all the blocks. There are three such situations:

```
[ [a,b,c], [ ], [ ] ]
[ [ ], [a,b,c], [ ] ]
[ [ ], [ ], [a,b,c] ]
```

The successor relation can be programmed according to the following rule: Situation2 is a successor of Situation1 if there are two stacks, Stack1 and Stack2, in Situation1, and the top block of Stack1 can be moved to Stack2. As all situations are represented as lists of stacks, this is translated into Prolog as:

```
s( Stacks, [Stack1, [Top1 | Stack2] | OtherStacks] ) :-   % Move Top1 to Stack2
  del( [Top1 | Stack1], Stacks, Stacks1),                  % Find first stack
  del( Stack2, Stacks1, OtherStacks).                      % Find second stack

del( X, [X | L], L).

del( X, [Y | L], [Y | L1] ) :-
  del( X, L, L1).
```

The goal condition for our example problem is:

```
goal( Situation) :-
  member( [a,b,c], Situation).
```

We will program search algorithms as a relation

```
solve( Start, Solution)
```

where Start is the start node in the state space, and Solution is a path between Start and any goal node. For our block manipulation problem the corresponding call can be:

```
?- solve( [ [c,a,b], [ ], [ ] ], Solution).
```

As the result of a successful search, **Solution** is instantiated to a list of block arrangements. This list represents a plan for transforming the initial state into a state in which all the three blocks are in one stack arranged as [a,b,c].

11.2 Depth-first search and iterative deepening

Given a state-space formulation of a problem, there are many approaches to finding a solution path. Two basic search strategies are: *depth-first* search and *breadth-first* search. In this section we will implement depth-first search and its variation called *iterative deepening*.

We will start the development of this algorithm and its variations with a simple idea:

To find a solution path, Sol, from a given node, N, to some goal node:

- if N is a goal node then Sol = [N], or
- if there is a successor node, N1, of N, such that there is a path Sol1 from N1 to a goal node, then Sol = [N | Sol1].

This translates into Prolog as:

```
solve( N, [N] ) :-
  goal( N).

solve( N, [ N | Sol1] ) :-
  s( N, N1),
  solve( N1, Sol1).
```

This program is in fact an implementation of the depth-first strategy. It is called 'depth-first' because of the order in which the alternatives in the state space are explored. Whenever the depth-first algorithm is given a choice of continuing the search from several nodes it always decides to choose a deepest one. A 'deepest' node is one that is farthest from the start node. Figure 11.4 illustrates the order in which the nodes are visited. This order corresponds to the Prolog trace when answering the question:

```
?- solve( a, Sol).
```

The depth-first search is most amenable to the recursive style of programming in Prolog. The reason for this is that Prolog itself, when executing goals, explores alternatives in the depth-first fashion.

The depth-first search is simple and easy to program, and may work well in certain cases. The eight queens programs of Chapter 4 were, in fact, examples of

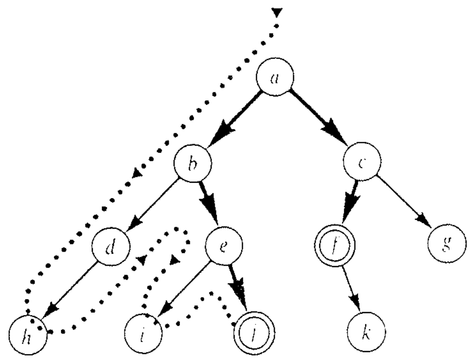

Figure 11.4 A simple state space: *a* is the start node, *f* and *j* are goal nodes. The order in which the depth-first strategy visits the nodes in this state space is: *a*, *b*, *d*, *h*, *e*, *i*, *j*. The solution found is: [a,b,e,j]. On backtracking, the other solution is discovered: [a,c,f].

depth-first search. A state-space formulation of the eight queens problem that could be used by the solve procedure above can be as follows:

- nodes are board positions with zero or more queens placed in consecutive files of the board;
- a successor node is obtained by placing another queen into the next file so that she does not attack any of the existing queens;
- the start node is the empty board represented by the empty list;
- a goal node is any position with eight queens (the successor rule guarantees that the queens do not attack each other).

Representing the board position as a list of Y-coordinates of the queens, this can be programmed as:

```
s( Queens, [Queen | Queens] ) :-
  member( Queen, [1,2,3,4,5,6,7,8] ),    % Place Queen into any row
  noattack( Queen, Queens).

goal( [_,_,_,_,_,_,_,_] ).               % Position with 8 queens
```

The noattack relation requires that Queen does not attack any of the Queens; it can be easily programmed as in Chapter 4. The question

```
?- solve( [ ], Solution).
```

will produce a list of board positions with increasing number of queens. The list will end with a safe configuration of eight queens. It will also find alternative solutions through backtracking.

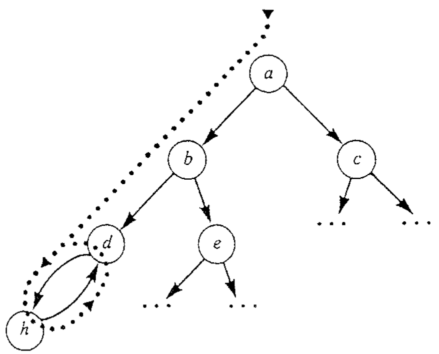

Figure 11.5 Starting at *a*, the depth-first search ends in cycling between *d* and *h*: *a*, *b*, *d*, *h*, *d*, *h*, *d* . . . .

The depth-first search often works well, as in this example, but there are many ways in which our simple solve procedure can run into trouble. Whether this will actually happen or not depends on the state space. To embarrass our solve procedure with the problem of Figure 11.4, a slight modification of this problem is sufficient: add an arc from *h* to *d*, thus creating a cycle (Figure 11.5). The search would in this case proceed as follows: start at *a* and descend to *h* following the left-most branch of the graph. At this point, in contrast with Figure 11.4, *h* has a successor, *d*. Therefore the execution will *not backtrack* from *h*, but *proceed* to *d* instead. Then the successor of *d*, *h*, will be found, etc., resulting in cycling between *d* and *h*.

An obvious improvement of our depth-first program is to add a cycle-detection mechanism. Accordingly, any node that is already in the path from the start node to the current node should not be considered again. We can formulate this as a relation:

depthfirst(Path, Node, Solution)

As illustrated in Figure 11.6, Node is the state from which a path to a goal state is to be found; Path is a path (a list of nodes) between the start node and Node; Solution is Path extended via Node to a goal node.

For the sake of ease of programming, paths will be in our program represented by lists in the *inverse* order. The argument Path can be used for two purposes:

(1) to prevent the algorithm from considering those successors of Node that have already been encountered (cycle detection);

(2) to construct a solution path Solution.

A corresponding depth-first search program is shown in Figure 11.7.

With the cycle-detection mechanism, our depth-first procedure will find solution paths in state spaces such as that in Figure 11.5. There are, however, state spaces in which this program will still easily get lost. Many state spaces are infinite. In such a space, the depth-first algorithm may miss a goal node, proceeding along an infinite branch of the graph. The program may then indefinitely explore this infinite part of the space never getting closer to a goal. The eight queens state space, as defined in this section, may seem to be susceptible to this kind of trap. However, this space is, incidentally, finite, because by the limited choice of Y-coordinates eight queens at most can be placed safely.

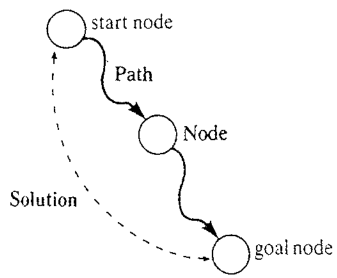

Figure 11.6 Relation depthfirst(Path, Node, Solution).

To avoid aimless infinite (non-cyclic) branches, we can add another refinement to the basic depth-first search procedure: limiting the depth of search. We then have the following arguments for the depth-first search procedure:

depthfirst2(Node, Solution, Maxdepth)

The search is not allowed to go in depth beyond Maxdepth. This constraint can be programmed by decreasing the depth limit at each recursive call, and not allowing this limit to become negative. The resulting program is shown in Figure 11.8.

```
% solve( Node, Solution):
%   Solution is an acyclic path (in reverse order) between Node and a goal

solve( Node, Solution) :-
  depthfirst( [ ], Node, Solution).

% depthfirst( Path, Node, Solution):
%   extending the path [Node | Path] to a goal gives Solution

depthfirst( Path, Node, [Node | Path] ) :-
  goal( Node).

depthfirst( Path, Node, Sol) :-
  s( Node, Node1),
  not member( Node1, Path),                  % Prevent a cycle
  depthfirst( [Node | Path], Node1, Sol).
```

Figure 11.7 A depth-first search program that avoids cycling.

```
% depthfirst2( Node, Solution, Maxdepth):
%   Solution is a path, not longer than Maxdepth, from Node to a goal

depthfirst2( Node, [Node], _) :-
  goal( Node).

depthfirst2( Node, [Node | Sol], Maxdepth) :-
  Maxdepth > 0,
  s( Node, Node1),
  Max1 is Maxdepth - 1,
  depthfirst2( Node1, Sol, Max1).
```

Figure 11.8 A depth-limited, depth-first search program.

A difficulty with the depth-limited program in Figure 11.8 is that we have to guess a suitable limit in advance. If we set the limit too low – that is, less than any solution path – then the search will fail. If we set the limit too high, the search will become too complex. To circumvent this difficulty, we can execute the depth-limited search iteratively, varying the depth limit: start with a very low depth limit and gradually increase the limit until a solution is found. This technique is called *iterative deepening*. It can be implemented by modifying the program of Figure 11.8 in the following way. The depthfirst2 procedure can be called from another procedure which would, on each recursive call, increase the limit by 1.

There is, however, a more elegant implementation based on a procedure

```
path( Node1, Node2, Path)
```

where Path is an acyclic path, in reverse order, between nodes Node1 and Node2 in the state space. Let the path be represented as a list of nodes in the inverse order. Then path can be written as:

```
path( Node, Node, [Node] ).                  % Single node path

path( FirstNode, LastNode, [LastNode | Path] ) :-
  path( FirstNode, OneButLast, Path),        % Path up to one-but-last node
  s( OneButLast, LastNode),                  % Last step
  not member( LastNode, Path).               % No cycle
```

Let us find some paths starting with node a in the state space of Figure 11.4:

```
?- path( a, Last, Path).

Last = a
Path = [a];

Last = b
Path = [b,a];

Last = c
Path = [c,a];

Last = d
Path = [d,b,a];

...
```


The path procedure generates, for the given initial node, all possible acyclic paths of increasing length. This is exactly what we need in the iterative deepening approach: generate paths of increasing length until a path is generated that ends with a goal node. This immediately gives a depth-first iterative deepening search program:

```
depth_first_iterative_deepening( Node, Solution) :-
  path( Node, GoalNode, Solution),
  goal( GoalNode).
```

This procedure is in fact quite useful in practice, as long as the combinatorial complexity of the problem does not require the use of problem-specific heuristics. The procedure is simple and, even if it does not do anything very clever, it does not waste much time or space. In comparison with some other search strategies, such as breadth first, the main advantage of iterative deepening is that it requires relatively little memory space. At any point of execution, the space requirements are basically reduced to *one path* between the start node of the search and the current node. Paths are generated, checked and forgotten, which is in contrast to some other search procedures (like breadth-first search) that, during search, keep many candidate paths at the same time. A disadvantage of iterative deepening is the consequence of its main strength: on each iteration, when the depth limit is increased, the paths previously computed have to be recomputed and extended to the new limit. In typical search problems, however, this recomputation does not critically affect the overall computation time. Typically, most computation is done at the deepest level of search; therefore, repeated computation at upper levels adds relatively little to the total time.

Exercises

11.1 Write a depth-first search procedure (with cycle detection)

```
depthfirst1( CandidatePath, Solution)
```

to find a solution path Solution as an extension of CandidatePath. Let both paths be represented as lists of nodes in the inverse order, so that the goal node is the head of Solution.

11.2 Write a depth-first procedure that combines both the cycle-detection and the depth-limiting mechanisms of the procedures in Figures 11.7 and 11.8.

11.3 The procedure depth_first_iterative_deepening/2 in this section may get into an indefinite loop if there is no solution path in the state space. It keeps searching for longer solution paths even when it is obvious that there do not exist any longer paths than those already searched. The same problem may occur when the user requests alternative solutions after all the solutions have already been found. Write an iterative deepening search program that will look for paths of length $i+1$ only if there was at least one path found of length i.

11.4 Experiment with the depth-first programs of this section in the blocks world planning problem of Figure 11.1.

11.5 Write a procedure

```
show( Situation)
```

to display a problem state, Situation, in the blocks world. Let Situation be a list of stacks, and a stack in turn a list of blocks. The goal

```
show( [ [a], [e,d], [c,b] ] )
```

should display the corresponding situation; for example, as:

```
    e  c
 a  d  b
===========
```

11.3 Breadth-first search

In contrast to the depth-first search strategy, the breadth-first search strategy chooses to first visit those nodes that are closest to the start node. This results in a search process that tends to develop more into breadth than into depth, as illustrated by Figure 11.9.

The breadth-first search is not so easy to program as the depth-first search. The reason for this difficulty is that we have to maintain a *set* of alternative candidate nodes, not just one as in depth-first search. This set of candidates is the whole growing bottom edge of the search tree. However, even this set of nodes is not sufficient if we also want to extract a solution path from the search process. Therefore, instead of maintaining a set of candidate nodes, we maintain a set of candidate *paths*. Then,

```
breadthfirst( Paths, Solution)
```

is true if some path from a candidate set Paths can be extended to a goal node. Solution is such an extended path.

We will use the following representation for the set of candidate paths. The set will be represented as a list of paths, and each path will be a list of nodes in the inverse order; that is, the head will be the most recently generated node, and the last element of the list will be the start node of the search. The search is initiated with a single element candidate set:

```
[ [StartNode] ]
```


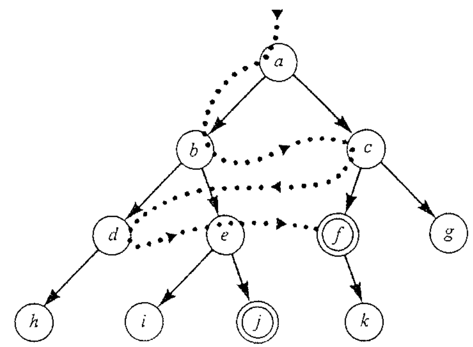

Figure 11.9 A simple state space: *a* is the start node, *f* and *j* are goal nodes. The order in which the breadth-first strategy visits the nodes in this state space is: *a*, *b*, *c*, *d*, *e*, *f*. The shorter solution [a,c,f] is found before the longer one [a,b,e,j].

An outline for breadth-first search is:

To do the breadth-first search when given a set of candidate paths:

- if the head of the first path is a goal node then this path is a solution of the problem, otherwise
- remove the first path from the candidate set and generate the set of all possible one-step extensions of this path, add this set of extensions at the end of the candidate set, and execute breadth-first search on this updated set.

For our example problem of Figure 11.9, this process develops as follows:

(1) Start with the initial candidate set:

[[a]]

(2) Generate extensions of [a]:

[[b,a], [c,a]]

(Note that all paths are represented in the inverse order.)

(3) Remove the first candidate path, [b,a], from the set and generate extensions of this path:

[[d,b,a], [e,b,a]]

Add the list of extensions to the end of the candidate set:

[[c,a], [d,b,a], [e,b,a]]

(4) Remove [c,a] and add its extensions to the end of the candidate set, producing:

[[d,b,a], [e,b,a], [f,c,a], [g,c,a]]

In further steps, [d,b,a] and [e,b,a] are extended and the modified candidate set becomes:

[[f,c,a], [g,c,a], [h,d,b,a], [i,e,b,a], [j,e,b,a]]

Now the search process encounters [f,c,a], which contains a goal node, f. Therefore this path is returned as a solution.

A program that carries out this process is shown in Figure 11.10. In this program all one-step extensions are generated by using the built-in procedure bagof. A test to prevent the generation of cyclic paths is also made. Note that in the case that no extension is possible, bagof fails and therefore an alternative call to breadthfirst is provided. member and conc are the list membership and list concatenation relations respectively.

A drawback of this program is the inefficiency of the conc operation. This can be rectified by using the difference-pair representation of lists introduced in Chapter 8. The set of candidate paths would then be represented by a pair of lists, Paths and Z, written as:

Paths - Z

Introducing this representation into the program of Figure 11.10, it can be systematically transformed into the program shown in Figure 11.11. This transformation is left as an exercise for the reader.

```
% solve( Start, Solution):
%    Solution is a path (in reverse order) from Start to a goal

solve( Start, Solution) :-
  breadthfirst( [ [Start] ], Solution).

% breadthfirst( [ Path1, Path2, ...], Solution):
%    Solution is an extension to a goal of one of paths

breadthfirst( [ [Node | Path] | _], [Node | Path]) :-
  goal( Node).

breadthfirst( [Path | Paths], Solution) :-
  extend( Path, NewPaths),
  conc( Paths, NewPaths, Paths1),
  breadthfirst( Paths1, Solution).

extend( [Node | Path], NewPaths) :-
  bagof( [NewNode, Node | Path],
         ( s( Node, NewNode), not member( NewNode, [Node | Path] ) ),
         NewPaths),
  !.

extend( Path, [] ).                    % bagof failed: Node has no successor
```

Figure 11.10 An implementation of breadth-first search.


```
% solve( Start, Solution):
%    Solution is a path (in reverse order) from Start to a goal

solve( Start, Solution) :-
  breadthfirst( [ [Start] | Z] - Z, Solution).

breadthfirst( [ [Node | Path] | _] - _, [Node | Path] ) :-
  goal( Node).

breadthfirst( [Path | Paths] - Z, Solution) :-
  extend( Path, NewPaths),
  conc( NewPaths, Z1, Z),                % Add NewPaths at end
  Paths \== Z1,                          % Set of candidates not empty
  breadthfirst( Paths - Z1, Solution).
```

Figure 11.11 A more efficient program than that of Figure 11.10 for the breadth-first search. The improvement is based on using the difference-pair representation for the list of candidate paths. Procedure **extend** is as in Figure 11.10.

Exercises

11.6 Let the state space be a tree with uniform branching b, and let the solution length be d. For the special case $b = 2$ and $d = 3$, how many nodes are generated in the worst case by breadth-first search and by iterative deepening (counting regenerated nodes as well)? Denote by $N(b, d)$ the number of nodes generated by iterative deepening in the general case. Find a recursive formula giving $N(b, d)$ in terms of $N(b, d - 1)$.

11.7 Rewrite the breadth-first program of Figure 11.10 using the difference-pair representation for the list of candidate paths, and show that the result can be the program in Figure 11.11. In Figure 11.11, what is the purpose of the goal:

Paths \== Z1

Test what happens if this goal is omitted; use the state space of Figure 11.9. The difference should only show when trying to find more solutions when there are none left.

11.8 How can the search programs of this section be used for searching from a *starting set* of nodes instead of a single start node?

11.9 How can the search programs of this chapter be used to search in the backward direction; that is, starting from a goal node and progressing toward the start node (or a start node in the case of multiple start nodes)? Hint: redefine the s relation. In what situations would the backward search be advantageous over the forward search?

11.10 Sometimes it is beneficial to search *bidirectionally*; that is, to work from both ends, the start and the goal. The search ends when both ends come together. Define the search space (relation s) and the goal relation for a given graph so that our search procedures would, in effect, perform bidirectional search.

11.11 Three search procedures find1, find2 and find3 defined below use different search strategies. Identify these strategies.

```
find1( Node, [Node]) :-
  goal( Node).

find1( Node, [Node | Path]) :-
  s( Node, Node1),
  find1( Node1, Path).

find2( Node, Path) :-
  conc( Path, _, _),              % Usual conc/3 for list concatenation
  find1( Node, Path).

find3( Node, Path) :-
  goal( Goal),
  find3( Node, [Goal], Path).

find3( Node, [Node | Path], [Node | Path]).

find3( Node, [Node2 | Path2], Path) :-
  s( Node1, Node2),
  find3( Node, [Node1, Node2 | Path2], Path).
```

11.12 Study the following search program and describe its search strategy:

```
% search( Start, Path1 - Path2): Find path from start node S to a goal node
% Solution path is represented by two lists Path1 and Path2

search( S, P1 - P2) :-
  similar_length( P1, P2),          % Lists of approximately equal lengths
  goal( G),
  path2( G, P2, N),
  path1( S, P1, N).

path1( N, [N], N).

path1( First, [First | Rest], Last) :-
  s( First, Second),
  path1( Second, Rest, Last).

path2( N, [N], N).

path2( First, [First | Rest], Last) :-
  s( Second, First),
  path2( Second, Rest, Last).

similar_length( List1, List2) :-    % Lists of similar length
  equal_length( List2, List),       % Lists of equal length
  ( List1 = List; List1 = [_ | List]).

equal_length( [], []).

equal_length( [X1 | L1], [X2 | L2]) :-
  equal_length( L1, L2).
```

11.13 Experiment with various search techniques in the blocks world planning problem.

11.14 The breadth-first programs of this chapter only check for repeated nodes that appear in the same candidate path. In graphs, a node can be reached by different paths. This is not detected by our programs, which, therefore, duplicate the search below such nodes. Modify the programs of Figures 11.10 and 11.11 to prevent this unnecessary work.

11.4 Analysis of basic search techniques

We will now analyze and compare the basic search techniques. First we will consider their application to searching graphs, then comment on the optimality of solutions they produce. Finally we will analyze their time and space complexity.

Examples so far might have made the wrong impression that our search programs only work for state spaces that are trees and not general graphs. However, when a graph is searched it, in effect, unfolds into a tree so that some paths are possibly copied in other parts of the tree. Figure 11.12 illustrates this. So our programs work on graphs as well, although they may unnecessarily duplicate some work in cases where a node is reached by various paths. This can be prevented by checking for repetition of a node in *all* the candidate paths, and not only in the path in which the node was generated. Of course, such checking is only possible in our breadth-first programs where alternative paths are available for checking.

Our breadth-first search programs generate solution paths, one after another, ordered according to their lengths: shortest solutions come first. This is important if

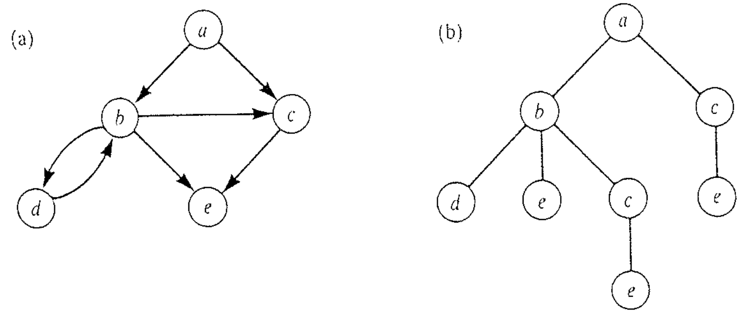

Figure 11.12 (a) A state space: *a* is the start node. (b) The tree of all possible non-cyclic paths from *a* as effectively developed by the breadth-first search program of Figure 11.10.

optimality (with respect to length) is a concern. The breadth-first strategy is guaranteed to produce a shortest solution first. This is, of course, not true for the depth-first strategy. However, depth-first iterative deepening performs depth-first search to increasing depth limits and is thus bound to find the shortest solutions first. So iterative deepening in a way simulates breadth-first search.

Our programs do not, however, take into account any costs associated with the arcs in the state space. If the minimal cost of the solution path is the optimization criterion (and not its length) then the breadth-first search is not sufficient. The best-first search of Chapter 12 will aspire to optimize the cost.

The typical problem associated with search is the *combinatorial complexity*. For non-trivial problem domains the number of alternatives to be explored is so high that the complexity becomes most critical. It is easy to see how this happens. To simplify the analysis let us assume the state space is a tree with uniform branching b. That is, each node in the tree, except the leaves, has exactly b successors. Assume a shortest solution path has length d, and there are no leaves in the tree at depth d or less. The number of alternative paths of length d from the start node is b^d. Breadth-first search will explore the number of paths of order b^d, $O(b^d)$. The number of candidate paths grows very fast with their length, which leads to what is called *combinatorial explosion*.

Let us now compare the complexity of the basic search algorithms. The time complexity is usually measured as the number of nodes generated by a search algorithm. The space complexity is usually measured as the maximum number of nodes that have to be stored in memory during search.

Consider breadth-first search in a tree with branching factor b and a shortest solution path of length d. The number of nodes at consecutive levels in the tree grows exponentially with depth, so the number of nodes generated by breadth-first search is:

$$1 + b + b^2 + b^3 + \dots$$

The total number of nodes up to the solution depth d is $O(b^d)$. So the time complexity of breadth-first search is $O(b^d)$. Breadth-first search maintains all the candidate paths in memory, so its space complexity is also $O(b^d)$.

Analysis of unlimited depth-first search is less clear because it may completely miss the solution path of length d and get lost in an infinite subtree. To facilitate the analysis let us consider depth-first search limited to a maximum depth d_{max} so that $d \leqslant d_{max}$. Time complexity of this is $O(b^{d_{max}})$. Space complexity is however only $O(d_{max})$. Depth-first search essentially only maintains the currently explored path between the start node and the current node of the search. Compared to breadth-first search, depth-first search has the advantage of much lower space complexity, and the disadvantage of no guarantee regarding the optimality.

Iterative deepening performs $(d + 1)$ depth-first searches to increasing depths: 0, 1, ..., d. So its space complexity is $O(d)$. It visits the start node $(d + 1)$ times, the children of the start node d times, etc. In the worst case the number of nodes generated is:

$$(d+1)*1 + d*b + (d-1)*b^2 + \dots + 1*d^b$$

This is also $O(b^d)$. In fact, the overhead, in comparison with breadth-first search, of regenerating shallow nodes, is surprisingly small. It can be shown that the ratio between the number of nodes generated by iterative deepening and those generated by breadth-first search is approximately $b/(b-1)$. For $b \geqslant 2$ this overhead of iterative deepening is relatively small in view of the enormous space advantage over breadth-first search. In this sense iterative deepening combines the best properties of breadth-first search (optimality guarantee) and depth-first search (space economy), and it is therefore in practice often the best choice among the basic search methods.

Let us also consider bidirectional search (Exercises 11.10–12). In cases when it is applicable (goal node known) it may result in considerable savings. Assume a search graph with uniform branching b in both directions, and the bidirectional search is realized as breadth-first search in both directions. Let a shortest solution path have length d, so the bidirectional search will stop when both breadth-first searches meet, that is when they both get to half-way between the start and the goal node. That is when each of them has progressed to depth about $d/2$ from their corresponding ends. The complexity of each of them is thus roughly $b^{d/2}$. Under these favourable circumstances bidirectional search succeeds to find a solution of length d needing approximately equal resources as breadth-first search would need to solve a simpler problem of length $d/2$. Table 11.1 summarizes the comparison of the basic search techniques.

The basic search techniques do not do anything clever about the combinatorial explosion. They treat all the candidates as equally promising, and do not use any problem-specific information to guide the search in a more promising direction. They are *uninformed* in this sense. Therefore, basic search techniques are not sufficient for solving large-scale problems. For such problems, problem-specific

Table 11.1 Approximate complexities of the basic search techniques. b is the branching factor, d is the shortest solution length, d_{max} is the depth limit for depth-first search, $d \leqslant d_{max}$.

	Time	*Space*	*Shortest solution guaranteed*
Breadth-first	b^d	b^d	yes
Depth-first	$b^{d_{max}}$	d_{max}	no
Iterative deepening	b^d	d	yes
Bidirectional, if applicable	$b^{d/2}$	$b^{d/2}$	yes

information has to be used to guide the search. Such guiding information is called *heuristic*. Algorithms that use heuristics perform *heuristic search*. The next chapter presents such a search method.

Summary

- *State space* is a formalism for representing problems.
- State space is a directed graph whose nodes correspond to problem situations and arcs to possible moves. A particular problem is defined by a *start node* and a *goal condition*. A solution of the problem then corresponds to a path in the graph. Thus problem solving is reduced to searching for a path in a graph.
- Optimization problems can be modelled by attaching costs to the arcs of a state space.
- Three basic search strategies that systematically explore a state space are *depth first*, *breadth first*, and *iterative deepening*.
- The depth-first search is easiest to program, but is susceptible to cycling. Two simple methods to prevent cycling are: limit the depth of search; test for repeated nodes.
- Implementation of the breadth-first strategy is more complicated as it requires maintaining the set of candidates. This can be most easily represented as a list of lists.
- The breadth-first search always finds a shortest solution path first, but this is not the case with the depth-first strategy.
- Breadth-first search requires more space than depth-first search. In practice, space is often the critical limitation.
- Depth-first iterative deepening combines the desirable properties of depth-first and breadth-first search.
- In the case of large state spaces there is the danger of *combinatorial explosion*. The basic search strategies are poor tools for combating this difficulty. Heuristic guidance is required in such cases.
- Concepts introduced in this chapter are:

  state space
  start node, goal condition, solution path
  search strategy
  depth-first search
  breadth-first search
  iterative deepening search
  bidirectional search
  heuristic search

References

The basic search strategies are described in any general text on artificial intelligence, for example Russell and Norvig (1995) and Winston (1992). Kowalski (1980) showed how logic can be used for implementing these principles. Korf (1985) analyzed the comparative advantages of iterative deepening.

Korf, R.E. (1985) Depth-first iterative deepening: an optimal admissible tree search. *Artificial Intelligence*, 27: 97–109.
Kowalski, R. (1980) *Logic for Problem Solving*. North-Holland.
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chapter 12

Best-First Heuristic Search

Graph searching in problem solving typically leads to the problem of combinatorial complexity due to the proliferation of alternatives. Heuristic search aspires to fight this problem efficiently. One way of using heuristic information about a problem is to compute numerical *heuristic estimates* for the nodes in the state space. Such an estimate of a node indicates how promising a node is with respect to reaching a goal node. The idea is to continue the search always from the most promising node in the candidate set. The best-first search programs of this chapter are based on this principle.

12.1 Best-first search

A best-first search program can be derived as a refinement of a breadth-first search program. The best-first search also starts at the start node and maintains the set of candidate paths. The breadth-first search always chooses for expansion a shortest candidate path (that is, shallowest tip nodes of the search). The best-first search refines this principle by computing a heuristic estimate for each candidate and chooses for expansion the best candidate according to this estimate.

We will from now on assume that a cost function is defined for the arcs of the state space. So $c(n,n')$ is the cost of moving from a node n to its successor n' in the state space.

Let the heuristic estimator be a function f, such that for each node n of the space, $f(n)$ estimates the 'difficulty' of n. Accordingly, the most promising current

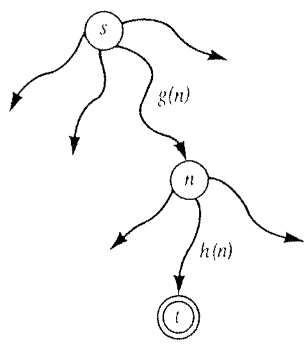


Figure 12.1 Construction of a heuristic estimate $f(n)$ of the cost of the cheapest path from s to t via n: $f(n) = g(n) + h(n)$.

candidate node is the one that minimizes f. We will use here a specially constructed function f which leads to the well-known A* algorithm. $f(n)$ will be constructed so as to estimate the cost of a best solution path from the start node, s, to a goal node, under the constraint that this path goes through n. Let us suppose that there is such a path and that a goal node that minimizes its cost is t. Then the estimate $f(n)$ can be constructed as the sum of two terms, as illustrated in Figure 12.1:

$$f(n) = g(n) + h(n)$$

$g(n)$ is an estimate of the cost of an optimal path from s to n; $h(n)$ is an estimate of the cost of an optimal path from n to t.

When a node n is encountered by the search process we have the following situation: a path from s to n must have already been found, and its cost can be computed as the sum of the arc costs on the path. This path is not necessarily an optimal path from s to n (there may be a better path from s to n, not yet found by the search), but its cost can serve as an estimate $g(n)$ of the minimal cost from s to n. The other term, $h(n)$, is more problematic because the 'world' between n and t has not been explored by the search until this point. Therefore, $h(n)$ is typically a real heuristic guess, based on the algorithm's general knowledge about the particular problem. As h depends on the problem domain there is no universal method for constructing h. Concrete examples of how such a heuristic guess can be made will be shown later. But let us assume for now that a function h is given, and concentrate on details of our best-first program.

We can imagine the best-first search to work as follows. The search process consists of a number of competing subprocesses, each of them exploring its own alternative; that is, exploring its own subtree. Subtrees have subtrees: these are explored by subprocesses of subprocesses, etc. Among all these competing processes, only one is active at each time: the one that deals with the currently most promising alternative; that is, the alternative with the lowest f-value. The remaining processes have to wait quietly until the current f-estimates change so that some other alternative becomes more promising. Then the activity is switched to this alternative. We can imagine this activate–deactivate mechanism as functioning as follows: the process working on the currently top-priority alternative is given some budget and the process is active until this budget is exhausted. During this activity, the process keeps expanding its subtree and reports a solution if a goal node was encountered. The budget for this run is defined by the heuristic estimate of the closest competing alternative.

Figure 12.2 shows an example of such behaviour. Given a map, the task is to find the shortest route between the start city s and the goal city t. In estimating the cost of the remaining route distance from a city X to the goal we simply use the straight-line distance denoted by $dist(X, t)$. So:

$$f(X) = g(X) + h(X) = g(X) + dist(X, t)$$

In this example, we can imagine the best-first search as consisting of two processes, each of them exploring one of the two alternative paths: Process 1 the path via a, Process 2 the path via e. In initial stages, Process 1 is more active because f-values along its path are lower than along the other path. At the moment that Process 1 is at c and Process 2 still at e, the situation changes:

$$f(c) = g(c) + h(c) = 6 + 4 = 10$$
$$f(e) = g(e) + h(e) = 2 + 7 = 9$$

So $f(e) < f(c)$, and now Process 2 proceeds to f and Process 1 waits. Here, however,

$$f(f) = 7 + 4 = 11$$
$$f(c) = 10$$
$$f(c) < f(f)$$

Therefore Process 2 is stopped and Process 1 is allowed to proceed, but only to d when $f(d) = 12 > 11$. Process 2, invoked at this point, now runs smoothly up to the goal t.

The search thus outlined, starting with the start node, keeps generating new successor nodes, always expanding in the most promising direction according to the f-values. During this process, a search tree is generated whose root is the start node of the search. Our best-first search program will thus keep expanding this search tree until a solution is found. This tree will be represented in the program by terms of two forms:

(1) l(N, F/G) represents a single node tree (a leaf); N is a node in the state space, G is g(N) (cost of the path found from the start node to N); F is f(N) = G + h(N).

(2) t(N, F/G, Subs) represents a tree with non-empty subtrees; N is the root of the tree, Subs is a list of its subtrees; G is g(N); F is the *updated* f-value of N – that is, the f-value of the most promising successor of N; the list Subs is ordered according to increasing f-values of the subtrees.

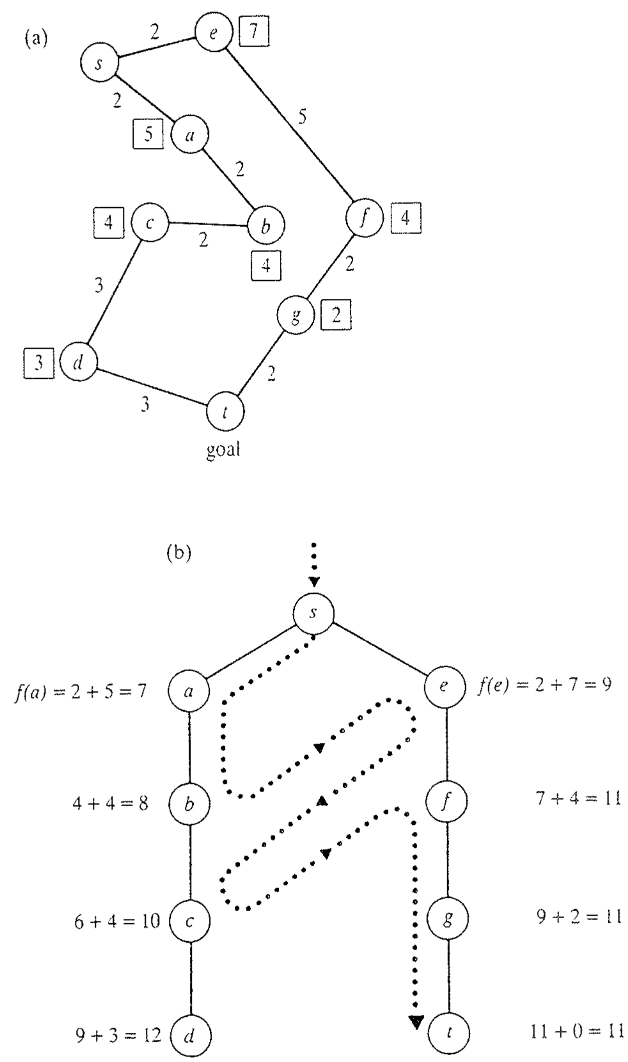


Figure 12.2 Finding the shortest route from s to t in a map. (a) The map with links labelled by their lengths; the numbers in the boxes are straight-line distances to t. (b) The order in which the map is explored by a best-first search. Heuristic estimates are based on straight-line distances. The dotted line indicates the switching of activity between alternate paths. The line shows the ordering of nodes according to their f-values; that is, the order in which the nodes are *expanded* (not the order in which they are generated).

For example, consider the search in Figure 12.2 again. At the moment that the node s has been expanded, the search tree consists of three nodes: the root s and its children, a and e. In our program this search tree will be represented by the term:

```
t( s, 7/0, [l(a,7/2), l(e,9/2)] )
```

The f-value of the root s is equal to 7, that is the f-value of the root's most promising successor a. The search tree is now expanded by expanding the most promising subtree a. The closest competitor to a is e whose f-value is 9. Therefore, a is allowed to expand as long as the f-value of a does not exceed 9. Thus, the nodes b and c are generated. $f(c) = 10$, so the bound for expansion has been exceeded and alternative a is no longer allowed to grow. At this moment, the search tree is:

```
t( s, 9/0, [l(e,9/2), t(a,10/2, [t(b,10/4, [l(c,10/6)] ) ] ) ] )
```

Notice that now the f-value of node a is 10 while that of node s is 9. They have been updated because new nodes, b and c, have been generated. Now the most promising successor of s is e, whose f-value is 9.

The updating of the f-values is necessary to enable the program to recognize the most promising subtree at each level of the search tree (that is, the tree that contains the most promising tip node). This modification of f-estimates leads, in fact, to a generalization of the definition of f. The generalization extends the definition of the function f from nodes to trees. For a single node tree (a leaf), n, we have the original definition:

$$f(n) = g(n) + h(n)$$

For a tree, T, whose root is n, and n's subtrees are S_1, S_2, etc.,

$$f(T) = \min_i f(S_i)$$

A best-first program along these lines is shown as Figure 12.3. Some more explanation of this program follows.

The key procedure is **expand**, which has six arguments:

expand(P, Tree, Bound, Tree1, Solved, Solution)

It expands a current (sub)tree as long as the f-value of this tree remains less or equal to **Bound**. The arguments of **expand** are:

P	Path between the start node and **Tree**.
Tree	Current search (sub)tree.
Bound	f-limit for expansion of **Tree**.
Tree1	**Tree** expanded within **Bound**; consequently, the f-value of **Tree1** is greater than **Bound** (unless a goal node has been found during the expansion).
Solved	Indicator whose value is 'yes', 'no' or 'never'.
Solution	A solution path from the start node 'through **Tree1**' to a goal node within **Bound** (if such a goal node exists).


```
% bestfirst( Start, Solution): Solution is a path from Start to a goal

bestfirst( Start, Solution) :-
  expand( [ ], l( Start, 0/0), 9999, _, yes, Solution).      % Assume 9999 is > any f-value

% expand( Path, Tree, Bound, Tree1, Solved, Solution):
%    Path is path between start node of search and subtree Tree,
%    Tree1 is Tree expanded within Bound,
%    if goal found then Solution is solution path and Solved = yes

% Case 1: goal leaf-node, construct a solution path

expand( P, l( N, _), _, _, yes, [N | P] ) :-
  goal(N).

% Case 2: leaf-node, f-value less than Bound
% Generate successors and expand them within Bound

expand( P, l(N, F/G), Bound, Tree1, Solved, Sol) :-
  F =< Bound,
  ( bagof( M/C, ( s(N, M, C), not member(M, P) ), Succ),
    !,                                                      % Node N has successors
    succlist( G, Succ, Ts),                                 % Make subtrees Ts
    bestf( Ts, F1),                                         % f-value of best successor
    expand( P, t(N, F1/G, Ts), Bound, Tree1, Solved, Sol)
    ;
    Solved = never                                          % N has no successors – dead end
  ).

% Case 3: non-leaf, f-value less than Bound
% Expand the most promising subtree; depending on
% results, procedure continue will decide how to proceed

expand( P, t(N, F/G, [T | Ts]), Bound, Tree1, Solved, Sol) :-
  F =< Bound,
  bestf( Ts, BF), min( Bound, BF, Bound1),                  % Bound1 = min( Bound, BF)
  expand( [N | P], T, Bound1, T1, Solved1, Sol),
  continue( P, t(N, F/G, [T1 | Ts]), Bound, Tree1, Solved1, Solved, Sol).

% Case 4: non-leaf with empty subtrees
% This is a dead end which will never be solved

expand( _, t(_, _, [ ]), _, _, never, _) :- !.

% Case 5: value greater than Bound
% Tree may not grow

expand( _, Tree, Bound, Tree, no, _) :-
  f( Tree, F), F > Bound.
```

Figure 12.3 A best-first search program.

Figure 12.3 *contd*

```
% continue( Path, Tree, Bound, NewTree, SubtreeSolved, TreeSolved, Solution)

continue( _, _, _, _, yes, yes, Sol).

continue( P, t(N, F/G, [T1 | Ts]), Bound, Tree1, no, Solved, Sol) :-
  insert( T1, Ts, NTs),
  bestf( NTs, F1),
  expand( P, t(N, F1/G, NTs), Bound, Tree1, Solved, Sol).

continue( P, t(N, F/G, [_ | Ts]), Bound, Tree1, never, Solved, Sol) :-
  bestf( Ts, F1),
  expand( P, t(N, F1/G, Ts), Bound, Tree1, Solved, Sol).

% succlist( G0, [Node1/Cost1, ...], [l(BestNode, BestF/G), ...] ):
%   make list of search leaves ordered by their f-values

succlist( _, [], [] ).

succlist( G0, [N/C | NCs], Ts) :-
  G is G0 + C,
  h( N, H),                                % Heuristic term h(N)
  F is G + H,
  succlist( G0, NCs, Ts1),
  insert( l(N, F/G), Ts1, Ts).

% Insert T into list of trees Ts preserving order with respect to f-values

insert( T, Ts, [T | Ts] ) :-
  f( T, F), bestf( Ts, F1),
  F =< F1, !.

insert( T, [T1 | Ts], [T1 | Ts1] ) :-
  insert( T, Ts, Ts1).

% Extract f-value

f( l(_, F/_), F).                          % f-value of a leaf

f( t(_, F/_, _), F).                       % f-value of a tree

bestf( [T | _], F) :-                      % Best f-value of a list of trees
  f( T, F).

bestf( [], 9999).                          % No trees: bad f-value
```

P, Tree and Bound are 'input' parameters to expand; that is, they are already instantiated whenever expand is called. expand produces three kinds of results, which is indicated by the value of the argument Solved as follows:

(1) **Solved** = yes.
Solution = a solution path found by expanding **Tree** within **Bound**.
Tree1 = uninstantiated.

(2) **Solved** = no.
Tree1 = **Tree** expanded so that its *f*-value exceeds **Bound** (Figure 12.4 illustrates)
Solution = uninstantiated.

(3) **Solved** = never.
Tree1 and **Solution** = uninstantiated.

The last case indicates that Tree is a 'dead' alternative and should never be given another chance by reactivating its exploration. This case arises when the *f*-value of Tree is less or equal to Bound, but the tree cannot grow because no leaf in it has any successor at all, or such a successor would create a cycle.

Some clauses about expand deserve explanation. The clause that deals with the most complicated case when Tree has subtrees – that is,

Tree = t(N, F/G, [T | Ts])

says the following. First, the most promising subtree, T, is expanded. This expansion is not given the bound Bound, but possibly some lower value, depending on the *f*-values of the other competing subtrees, Ts. This ensures that the currently growing subtree is always the most promising subtree. The expansion process then switches between the subtrees according to their *f*-values. After the best candidate has been expanded, an auxiliary procedure continue decides what to do next; this depends on

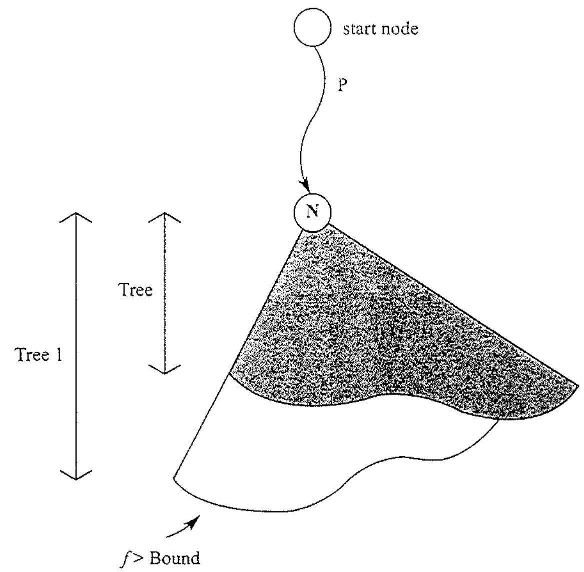


Figure 12.4 The **expand** relation: expanding **Tree** until the *f*-value exceeds **Bound** results in **Tree1**.

the type of result produced by this expansion. If a solution was found then this is returned, otherwise expansion continues.

The clause that deals with the case

Tree = l(N, F/G)

generates successor nodes of N together with the costs of the arcs between N and successor nodes. Procedure succlist makes a list of subtrees from these successor nodes, also computing their g-values and f-values as shown in Figure 12.5. The resulting tree is then further expanded as far as Bound permits. If, on the other hand, there were no successors, then this leaf is abandoned for ever by instantiating Solved = 'never'.

Other relations are:

s(N, M, C) M is a successor node of N in the state space; C is the cost of the arc from N to M.

h(N, H) H is a heuristic estimate of the cost of the best path from node N to a goal node.

Application of this best-first search program to some example problems will be shown in the next section. But first some general, concluding comments on this program. It is a variation of a heuristic algorithm known in the literature as the A* algorithm (see references at the end of the chapter). A* has attracted a great deal of attention. It is one of the fundamental algorithms of artificial intelligence. We will mention here an important result from the mathematical analysis of A*:

A search algorithm is said to be *admissible* if it always produces an optimal solution (that is, a minimum-cost path) provided that a solution exists at all. Our implementation, which produces all solutions through backtracking, can be considered admissible if the *first* solution found is optimal. Let, for each node n in the state space, $h^*(n)$ denote the cost of an optimal path from n to a goal node. A theorem about the admissibility of A* says: an A* algorithm that uses a heuristic function h such that for all nodes n in the state space

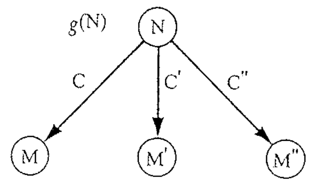
$$h(n) \leqslant h^*(n)$$

is admissible.

This result is of great practical value. Even if we do not know the exact value of h^* we just have to find a lower bound of h^* and use it as h in A*. This is sufficient guarantee that A* will produce an optimal solution.

There is a trivial lower bound, namely:

$$h(n) = 0, \quad \text{for all } n \text{ in the state space}$$



$g(\mathrm{M}) = g(\mathrm{N}) + \mathrm{C}$

$f(\mathrm{M}) = g(\mathrm{M}) + h(\mathrm{M})$

Figure 12.5 Relation between the g-value of node N, and the f- and g-values of its children in the search space.

This indeed guarantees admissibility. The disadvantage of $h = 0$ is, however, that it has no heuristic power and does not provide any guidance for the search. A* using $h = 0$ behaves similarly to the breadth-first search. It in fact reduces to the breadth-first search in the case that the arc-cost function $c(n, n') = 1$ for all arcs (n, n') in the state space. The lack of heuristic power results in high complexity. We would therefore like to have h, which is a lower bound of h^* (to ensure admissibility), and which is also as close as possible to h^* (to ensure efficiency). Ideally, if we knew h^*, we would use h^* itself: A* using h^* finds an optimal solution directly, without any backtracking at all.

Exercises

12.1 Define the problem-specific relations s, goal and h for the route-finding problem of Figure 12.2. Inspect the behaviour of our A* program on this problem.

12.2 The following statement resembles the admissibility theorem: 'For every search problem, if A* finds an optimal solution then $h(n) \leqslant h^*(n)$ for all nodes n in the state space.' Is this correct?

12.3 Let h_1, h_2 and h_3 be three admissible heuristic functions ($h_i \leqslant h^*$) alternatively used by A* on the same state space. Combine these three functions into another heuristic function h which will also be admissible and guide the search at least as well as any of the three functions h_i alone.

12.4 A mobile robot moves in the x–y plane among obstacles. All the obstacles are rectangles aligned with the x and y axes. The robot can only move in the directions x and y, and is so small that it can be approximated by a point. The robot has to plan collision-free paths between its current position to some given goal position. The robot aims at minimizing the path length and the changes of the direction of movement (let the cost of one change of direction be equal to one unit of length travelled). The robot uses the A* algorithm to find optimal paths. Define the predicates s(State,NewState,Cost) and h(State,H) (preferably admissible) to be used by the A* program for this search problem. Assume that the goal position for the robot is defined by the predicate goal(Xg/Yg) where Xg and Yg are the *x* and *y* coordinates of the goal point. The obstacles are represented by the predicate

    obstacle(Xmin/Ymin, Xmax/Ymax)

where Xmin/Ymin is the bottom left corner of the obstacle, and Xmax/Ymax is its top right corner.

12.2 Best-first search applied to the eight puzzle

If we want to apply the best-first search program of Figure 12.3 to some particular problem we have to add problem-specific relations. These relations define the particular problem ('rules of the game') and also convey heuristic information about how to solve that problem. This heuristic information is supplied in the form of a heuristic function.

Problem-specific predicates are:

    s(Node, Node1, Cost)

This is true if there is an arc, costing Cost, between Node and Node1 in the state space.

    goal(Node)

is true if Node is a goal node in the state space.

    h(Node, H)

H is a heuristic estimate of the cost of a cheapest path from Node to a goal node.

In this and the following sections we will define these relations for two example problem domains: the eight puzzle (described in Section 11.1) and the task-scheduling problem.

Problem-specific relations for the eight puzzle are shown in Figure 12.6. A node in the state space is some configuration of the tiles on the board. In the program, this is represented by a list of the current positions of the tiles. Each position is specified by a pair of coordinates: X/Y. The order of items in the list is as follows:

(1) the current position of the empty square,
(2) the current position of tile 1,
(3) the current position of tile 2,
...

The goal situation (see Figure 11.3) is defined by the clause:

    goal([2/2,1/3,2/3,3/3,3/2,3/1,2/1,1/1,1/2]).


```
/* Problem-specific procedures for the eight puzzle

Current situation is represented as a list of positions of the tiles, with first item in the list
corresponding to the empty square.

Example:
                          This position is represented by:
3   1 2 3
2   8   4                 [2/2, 1/3, 2/3, 3/3, 3/2, 3/1, 2/1, 1/1, 1/2]
1   7 6 5
    1 2 3

'Empty' can move to any of its neighbours, which means that 'empty' and its neighbour
interchange their positions.
*/

% s( Node, SuccessorNode, Cost)

s( [Empty | Tiles], [Tile | Tiles1], 1) :-          % All arc costs are 1
  swap( Empty, Tile, Tiles, Tiles1).                % Swap Empty and Tile in Tiles

swap( Empty, Tile, [Tile | Ts], [Empty | Ts] ) :-
  mandist( Empty, Tile, 1).                         % Manhattan distance = 1

swap( Empty, Tile, [T1 | Ts], [T1 | Ts1] ) :-
  swap( Empty, Tile, Ts, Ts1).

mandist( X/Y, X1/Y1, D) :-                          % D is Manh. dist. between two squares
  dif( X, X1, Dx),
  dif( Y, Y1, Dy),
  D is Dx + Dy.

dif( A, B, D) :-                                    % D is | A-B |
  D is A-B, D >= 0, !
  ;
  D is B-A.

% Heuristic estimate h is the sum of distances of each tile
% from its 'home' square plus 3 times 'sequence' score

h( [Empty | Tiles], H) :-
  goal( [Empty1 | GoalSquares] ),
  totdist( Tiles, GoalSquares, D),                  % Total distance from home squares
  seq( Tiles, S),                                   % Sequence score
  H is D + 3*S.

totdist( [], [], 0).
```

Figure 12.6 Problem-specific procedures for the eight puzzle, to be used in best-first search of Figure 12.3.

Figure 12.6 *contd*

```
totdist( [Tile | Tiles], [Square | Squares], D) :-
  mandist( Tile, Square, D1),
  totdist( Tiles, Squares, D2),
  D is D1 + D2.

% seq( TilePositions, Score): sequence score

seq( [First | OtherTiles], S) :-
  seq( [First | OtherTiles ], First, S).

seq( [Tile1, Tile2 | Tiles], First, S) :-
  score( Tile1, Tile2, S1),
  seq( [Tile2 | Tiles], First, S2),
  S is S1 + S2.

seq( [Last], First, S) :-
  score( Last, First, S).

score( 2/2, _, 1) :- !.                      % Tile in centre scores 1

score( 1/3, 2/3, 0) :- !.                    % Proper successor scores 0
score( 2/3, 3/3, 0) :- !.
score( 3/3, 3/2, 0) :- !.
score( 3/2, 3/1, 0) :- !.
score( 3/1, 2/1, 0) :- !.
score( 2/1, 1/1, 0) :- !.
score( 1/1, 1/2, 0) :- !.
score( 1/2, 1/3, 0) :- !.

score( _, _, 2).                             % Tiles out of sequence score 2

goal( [2/2,1/3,2/3,3/3,3/2,3/1,2/1,1/1,1/2] ).   % Goal squares for tiles

% Display a solution path as a list of board positions

showsol( [] ).

showsol( [P | L] ) :-
  showsol( L),
  nl, write( '---'),
  showpos( P).

% Display a board position

showpos( [S0,S1,S2,S3,S4,S5,S6,S7,S8] ) :-
  member( Y, [3,2,1] ),                      % Order of Y-coordinates
  nl, member( X, [1,2,3] ),                  % Order of X-coordinates
  member( Tile-X/Y,                          % Tile on square X/Y
         [' '-S0,1-S1,2-S2,3-S3,4-S4,5-S5,6-S6,7-S7,8-S8] ),
  write( Tile),
  fail                                       % Backtrack to next square
  ;
  true.                                      % All squares done
```



```
% Starting positions for some puzzles

start1( [2/2,1/3,3/2,2/3,3/3,3/1,2/1,1/1,1/2] ).      % Requires 4 steps
start2( [2/1,1/2,1/3,3/3,3/2,3/1,2/2,1/1,2/3] ).      % Requires 5 steps
start3( [2/2,2/3,1/3,3/1,1/2,2/1,3/3,1/1,3/2] ).      % Requires 18 steps

% An example query: ?- start1( Pos), bestfirst( Pos, Sol), showsol( Sol).
```

An auxiliary relation is:

mandist(S1, S2, D)

D is the 'Manhattan distance' between squares S1 and S2; that is, the distance between S1 and S2 in the horizontal direction plus the distance between S1 and S2 in the vertical direction.

We want to minimize the *length* of solutions. Therefore, we define the cost of all the arcs in the state space to equal 1. In the program of Figure 12.6, three example starting positions from Figure 12.7 are also defined.

The heuristic function, *h*, is programmed as:

h(Pos, H)

Pos is a board position; H is a combination of two measures:

(1) totdist: the 'total distance' of the eight tiles in Pos from their 'home squares'. For example, in the starting position of the puzzle in Figure 12.7(a), totdist = 4.

(2) seq: the 'sequence score' that measures the degree to which the tiles are already ordered in the current position with respect to the order required in the goal configuration. seq is computed as the sum of scores for each tile according to the following rules:
- a tile in the centre scores 1;
- a tile on a non-central square scores 0 if the tile is, in the clockwise direction, followed by its proper successor;
- such a tile scores 2 if it is not followed by its proper successor.

For example, for the starting position of the puzzle in Figure 12.7(a), seq = 6.

1	3	4
8		2
7	6	5

(a)

2	8	3
1	6	4
7		5

(b)

2	1	6
4		8
7	5	3

(c)

Figure 12.7 Three starting positions for the eight puzzle: (a) requires four steps; (b) requires five steps; (c) requires 18 steps.

The heuristic estimate, H, is computed as:

H = totdist + 3 * seq

This heuristic function works well in the sense that it very efficiently directs the search toward the goal. For example, when solving the puzzles of Figure 12.7(a) and (b), no node outside the shortest solution path is ever expanded before the first solution is found. This means that the shortest solutions are found directly in these cases without any backtracking. Even the difficult puzzle of Figure 12.7(c) is solved almost directly. A drawback of this heuristic is, however, that it is not admissible: it does not guarantee that the shortest solution path will always be found before any longer solution. The h function used does not satisfy the admissibility condition: $h \leqq h^*$ for all the nodes. For example, for the initial position in Figure 12.7(a),

$$h = 4 + 3 * 6 = 22, \quad h^* = 4$$

On the other hand, the 'total distance' measure itself is admissible: for all positions:

totdist $\leqq h^*$

This relation can be easily proved by the following argument: if we relaxed the problem by allowing the tiles to climb on top of each other, then each tile could travel to its home square along a trajectory whose length is exactly the Manhattan distance between the tile's initial square and its home square. So the optimal solution in the relaxed puzzle would be exactly of length **totdist**. In the original problem, however, there is interaction between the tiles and they are in each other's way. This can prevent the tiles from moving along the shortest trajectories, which ensues our optimal solution's length be equal or greater than **totdist**.

Exercise

12.5 Modify the best-first search program of Figure 12.3 to count the number of nodes generated in the search. One easy way is to keep the current number of nodes asserted as a fact, and update it by **retract** and **assert** whenever new nodes are generated. Experiment with various heuristic functions for the eight puzzle with respect to their heuristic power, which is reflected in the number of nodes generated.

12.3 Best-first search applied to scheduling

Let us consider the following task-scheduling problem. We are given a collection of *tasks*, $t_1, t_2, \ldots$, with their execution times $D_1, D_2, \ldots$ respectively. The tasks are to be executed on a set of m identical *processors*. Any task can be executed on any processor, but each processor can only execute one task at a time. There is a precedence relation between tasks which tells what tasks, if any, have to be completed before some other task can be started. The scheduling problem is to assign tasks to processors so that the precedence relation is not violated and that all the tasks together are processed in the shortest possible time. The time that the last task in a schedule is completed is called the *finishing time* of the schedule. We want to minimize the finishing time over all permissible schedules.

Figure 12.8 shows such a task-scheduling problem and two permissible schedules, one of which is optimal. This example shows an interesting property of optimal schedules; namely, that they may include 'idle time' for processors. In the optimal schedule of Figure 12.8, processor 2 after having executed task t_2 waits for two time units although it could start executing task t_7.

One way to construct a schedule is roughly as follows. We start with the empty schedule (with void time slots for each processor) and gradually insert tasks one by one into the schedule until all the tasks have been inserted. Usually there are

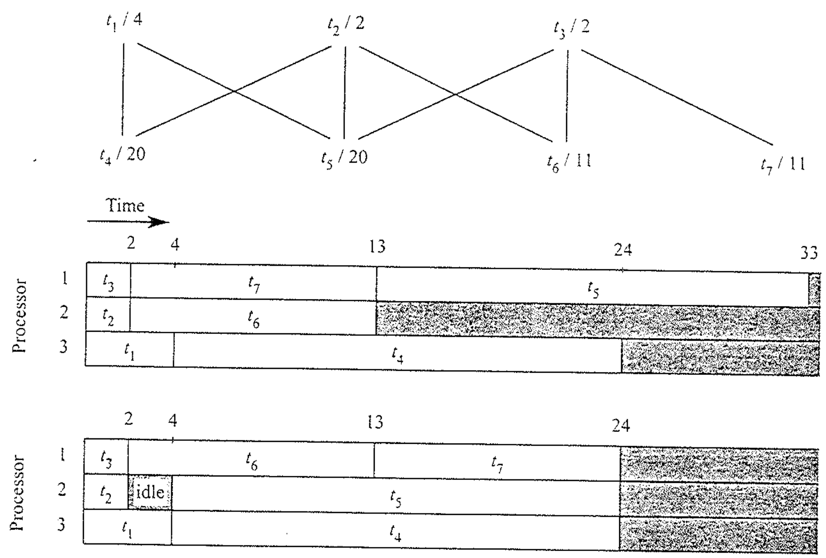


Figure 12.8 A task-scheduling problem with seven tasks and three processors. The top part of the diagram shows the task precedence relation and the duration of the tasks. Task t_5, for example, requires 20 time units, and its execution can only start after three other tasks, t_1, t_2 and t_3, have been completed. Two permissible schedules are shown; an optimal one with the finishing time 24, and a suboptimal one with the finishing time 33. In this problem any optimal schedule has to include idle time. Coffman/Denning, *Operating Systems Theory*, © 1973, p. 86. Adapted by permission of Prentice Hall, Englewood Cliffs, New Jersey.

alternatives at any such insertion step because there are several candidate tasks waiting to be processed. Therefore, the scheduling problem is one of search. Accordingly, we can formulate the scheduling problem as a state-space search problem as follows:

- states are partial schedules;
- a successor state of some partial schedule is obtained by adding a not yet scheduled task to this schedule; another possibility is to leave a processor that has completed its current task idle;
- the start state is the empty schedule;
- any schedule that includes all the tasks in the problem is a goal state;
- the cost of a solution (which is to be minimized) is the finishing time of a goal schedule;
- accordingly, the cost of a transition between two (partial) schedules whose finishing times are F_1 and F_2 respectively is the difference $F_2 - F_1$.

Some refinements are needed to this rough scenario. First, we decide to fill the schedule according to increasing times so that tasks are inserted into the schedule from left to right. Also, each time a task is added, the precedence constraint has to be checked. Further, there is no point in leaving a processor idle indefinitely if there are still some candidate tasks waiting. So we decide to leave a processor idle only until some other processor finishes its current task, and then consider again assigning a task to it.

Now let us decide on the representation of problem situations – that is, partial schedules. We need the following information:

(1) list of waiting tasks and their execution times,

(2) current engagements of the processors;

We will also add for convenience:

(3) the finishing time of the (partial) schedule; that is, the latest end-time of the current engagements of the processors.

The list of waiting tasks and their execution times will be represented in the program as a list of the form:

```
[ Task1/D1, Task2/D2, ...]
```

The current engagements of the processors will be represented by a list of tasks currently being processed; that is, pairs of the form:

```
Task/FinishingTime
```


There are m such pairs in the list, one for each processor. We will always add a new task to a schedule at the moment that the first current execution is completed. To this end, the list of current engagements will be kept ordered according to increasing finishing times. The three components of a partial schedule (waiting tasks, current engagements and finishing time) will be combined in the program into a single expression of the form:

```
WaitingList * ActiveTasks * FinishingTime
```

In addition to this information we have the precedence constraint, which will be specified in the program as a relation:

```
prec( TaskX, TaskY)
```

Now let us consider a heuristic estimate. We will use a rather straightforward heuristic function, which will not provide a very efficient guidance to the search algorithm. The function will be admissible and will hence guarantee an optimal schedule. It should be noted, however, that a much more powerful heuristic would be needed for large scheduling problems.

Our heuristic function will be an optimistic estimate of the finishing time of a partial schedule completed with all currently waiting tasks. This optimistic estimate will be computed under the assumption that two constraints on the actual schedule be relaxed:

(1) remove the precedence constraint;

(2) allow (unrealistically) that a task can be executed in a distributed fashion on several processors, and that the sum of the execution times of this task over all these processors is equal to the originally specified execution time of this task on a single processor.

Let the execution times of the currently waiting tasks be D_1, D_2, ..., and the finishing times of the current processors engagements be F_1, F_2, ... Such an optimistically estimated finishing time, *Finall*, to complete all the currently active and all the waiting tasks, is:

$$Finall = (\sum_i D_i + \sum_j F_j)/m$$

where m is the number of processors. Let the finishing time of the current partial schedule be:

$$Fin = \max_j(F_j)$$

Then the heuristic estimate H (an extra time needed to complete the partial schedule with the waiting tasks) is:

if $Finall > Fin$ then $H = Finall - Fin$ else $H = 0$

A complete program that defines the state-space relations for task scheduling as outlined above is shown in Figure 12.9. The figure also includes a specification of the particular scheduling problem of Figure 12.8. These definitions can now be used by the best-first search program of Figure 12.3. One of the optimal solutions produced by best-first search in the thus specified problem space is an optimal schedule of Figure 12.8.

```
/* Problem-specific relations for task scheduling

Nodes in the state space are partial schedules specified by:

[ WaitingTask1/D1, WaitingTask2/D2, ...] * [ Task1/F1, Task2/F2, ...] * FinTime

The first list specifies the waiting tasks and their durations; the second list specifies the
currently executed tasks and their finishing times, ordered so that F1 ≦ F2, F2 ≦ F3 ....
Fintime is the latest completion time of current engagements of the processors.
*/

% s( Node, SuccessorNode, Cost)

s( Tasks1 * [_/F | Active1] * Fin1, Tasks2 * Active2 * Fin2, Cost) :-
  del( Task/D, Tasks1, Tasks2),                              % Pick a waiting task
  not ( member( T/_, Tasks2), before( T, Task) ),            % Check precedence
  not ( member( T1/F1, Active1), F < F1, before( T1, Task) ), %Active tasks too
  Time is F + D,                                    % Finishing time of activated task
  insert( Task/Time, Active1, Active2, Fin1, Fin2),
  Cost is Fin2 - Fin1.

s( Tasks * [_/F | Active1] * Fin, Tasks * Active2 * Fin, 0) :-
  insertidle( F, Active1, Active2).                          % Leave processor idle

before( T1, T2) :-                                           % Task T1 before T2
  prec( T1, T2).                                             % according to precedence

before( T1, T2) :-
  prec( T, T2),
  before( T1, T).

insert( S/A, [T/B | L], [S/A, T/B | L], F, F) :-             % Task lists are ordered
  A =< B, !.

insert( S/A, [T/B | L], [T/B | L1], F1, F2) :-
  insert( S/A, L, L1, F1, F2).

insert( S/A, [ ], [S/A], _, A).

insertidle( A, [T/B | L], [idle/B, T/B | L] ) :-             % Leave processor idle
  A < B, !.                                     % until first greater finishing time

insertidle( A, [T/B | L], [T/B | L1] ) :-
  insertidle( A, L, L1).

del( A, [A | L], L).                                         % Delete item from list
```



```
del( A, [B | L], [B | L1] ) :-
  del( A, L, L1).

goal( [ ] * _ * _).                                % Goal state: no task waiting

% Heuristic estimate of a partial schedule is based on an
% optimistic estimate of the final finishing time of this
% partial schedule extended by all the remaining waiting tasks.

h( Tasks * Processors * Fin, H) :-
  totaltime( Tasks, Tottime),                      % Total duration of waiting tasks
  sumnum( Processors, Ftime, N),                   % Ftime is sum of finishing times
                                                   % of processors, N is their number
  Finall is ( Tottime + Ftime)/N,
  ( Finall > Fin, !, H is Finall - Fin
   ;
   H = 0
  ).

totaltime( [ ], 0).

totaltime( [_/D | Tasks], T) :-
  totaltime( Tasks, T1),
  T is T1 + D.

sumnum( [ ], 0, 0).

sumnum( [_/T | Procs], FT, N) :-
  sumnum( Procs, FT1, N1),
  N is N1 + 1,
  FT is FT1 + T.

% A task-precedence graph

prec( t1, t4).   prec( t1, t5).   prec( t2, t4).   prec( t2, t5).

prec( t3, t5).   prec( t3, t6).   prec( t3, t7).

% A start node

start( [t1/4, t2/2, t3/2, t4/20, t5/20, t6/11, t7/11] * [idle/0, idle/0, idle/0] * 0).

% An example query: ?- start( Problem), bestfirst( Problem, Sol).
```

Figure 12.9 Problem-specific relations for the task-scheduling problem. The particular scheduling problem of Figure 12.8 is also defined by its precedence graph and an initial (empty) schedule as a start node for search.

Project

In general, scheduling problems are known to be combinatorially difficult. Our simple heuristic function does not provide very powerful guidance. Propose other functions and experiment with them.

12.4 Space-saving techniques for best-first search

12.4.1 Time and space complexity of A*

Heuristic guidance in best-first search typically reduces the search to visit only a small part of the problem space. This can be viewed as the reduction of effective branching of search. If the 'average' branching in the problem space is b, then heuristic guidance effectively results in average branching b' where b' is typically substantially less than b.

In spite of such a reduction of search effort, the order of the complexity of A* (and our implementation of it in Figure 12.3) is still exponential in the depth of search. This holds for both time and space complexity because the algorithm maintains all the generated nodes in the memory. In practical applications, it depends on the particular circumstances which of the two resources is more critical: time or space. However, in most practical situations space is more critical. A* may use up all the available memory in a matter of minutes. After that the search practically cannot proceed although the user would find it acceptable that the algorithm would run for hours or even days.

Several variations of the A* algorithm have been developed that save space, at the expense of time. The basic idea is similar to depth-first iterative deepening discussed in Chapter 11. Space requirements are reduced from exponential to linear in the depth of search. The price is the regeneration of already generated nodes in the search space.

In the following sections we will look at two space-saving techniques in the context of best-first search. The first of them is called IDA* (iterative deepening A*); the second one appears under the name RBFS (recursive best-first search).

12.4.2 IDA* – iterative deepening A*

IDA* is similar to depth-first iterative deepening, with the following difference. In iterative deepening, depth-first searches are performed to increasing depth limits. On each iteration the depth-first search is bounded by the current depth limit. In IDA* however the successive depth-first searches are bounded by the current limit in the *values* of the nodes (heuristic f-values of the nodes). So the basic search mechanism in IDA* is again depth-first, which has very low space complexity. IDA* can be stated as follows:

```
Bound := f(StartNode);
Repeat
  perform depth-first search starting with StartNode, subject to
    condition that a node N is expanded only if f(N) ≤ Bound;
  if this depth-first search encounters a goal node
    then signal 'solution found',
  else
    compute NewBound as the minimum of the f-values of
    the nodes reached just outside Bound:
      NewBound = min{ f(N) | N generated by this search, f(N) > Bound}
    Bound := NewBound
until solution found.
```


To illustrate this algorithm, consider A* applied to the route-finding problem (Figure 12.2b; let $f(s) = 6$). IDA* proceeds as follows:

Bound = $f(s) = 6$
Perform depth-first search limited by $f \leqslant 6$. This search expands s, generating a and e, and finds:
$f(a) = 7 >$ Bound
$f(e) = 9 >$ Bound
NewBound = min{ 7, 9} = 7
Perform depth-first search limited by $f \leqslant 7$, starting with node s.
The nodes just outside this bound are b and e.
NewBound = min { $f(b)$, $f(e)$} = min{ 8, 9} = 8
From now on, the bound changes as follows: 9 ($f(e)$), 10 ($f(c)$), 11 ($f(f)$). For each of these values, depth-first search is performed.
When depth-first search is performed with Bound = 11, solution is found.

This example is only intended to illustrate how IDA* works, although the example makes the algorithm look awkward because of repeated depth-first searches. However, in large search problems IDA*'s space economy could be very beneficial while the overheads of repeated searches could be quite acceptable. How bad these overheads are depends on the properties of the search space, in particular on the properties of the evaluation function f. The favourable cases are when many nodes have equal f-values. In such cases each successive depth-first search explores many new nodes, more than the number of regenerated nodes. So the overhead is comparatively small. Unfavourable cases are those when the f-values tend not to be shared among many nodes. In the extreme case each node has a different f-value. Then many successive f-bounds are needed, and each new depth-first search will only generate one new node whereas all the rest will be the regeneration of the already generated (and forgotten) nodes. In such extreme cases the overheads of IDA* are of course unacceptable.

Another property of interest of IDA* regards the admissibility. Let $f(N)$ be defined as $g(N) + h(N)$ for all nodes N. If h is admissible ($h(N) \leqslant h^*(N)$ for all N) then IDA* is guaranteed to find an optimal solution.

A possible drawback of IDA* is that it does not guarantee that the nodes are explored in the best-first order (i.e. the order of increasing f-values). Suppose now that f is an evaluation function not necessarily of the form $f = g + h$. If function f is not monotonic then the best-first order is not guaranteed. Function f is said to be *monotonic* if its value monotonically increases along the paths in the state space. That is: f is monotonic if for all pairs of nodes N and N′: if $s(N, N')$ then $f(N) \leqslant f(N')$. The reason for a non best-first order is that with non-monotonic f, the f-bound may become so large that nodes with different f-values will be expanded for the first time by this depth-first search. This depth-first search will keep expanding nodes as long as they are within the f-bound, and will not care about the order in which they are expanded. In principle we are interested in best-first order because we expect that the function f reflects the quality of solutions.

One easy way of implementing IDA* in Prolog is shown in Figure 12.10. This program largely exploits Prolog's backtracking mechanism. The f-bound is maintained as a fact of the form

```
next_bound( Bound)
```

which is updated through assert and retract. On each iteration, the bound for depth-first search is retrieved from this fact. Then (through retract and assert on this fact), the bound for the next iteration is initialized to 99999. That is a large value assumed greater than any possible f-value. The depth-first search is programmed to only allow the expanding of a node N if $f(N) \leqslant$ Bound. If, on the other hand, $f(N) >$ Bound then the value $f(N)$ is compared to NextBound (stored under next_bound(NextBound)). If $f(N) <$ NextBound then the next_bound fact is updated to store $f(N)$.

Exercises

12.6 Construct an example state space and the function f for which IDA* would not expand nodes in the best-first order.

12.7 Apply the IDA* program of Figure 12.10 to the eight puzzle using the definitions of the successor relation s/3 and totdist/3 as in Figure 12.6. Use just total distance (totdist/3) as the heuristic function h (to ensure admissibility). Define also the predicate f/2 (needed by the IDA* program) so that $f(N) = g(N) + h(N)$. To enable the computation of $g(N)$, keep the g-value of a node explicitly as part of the representation of the node (e.g. N = G:TilePositions). Experiment with the eight puzzles defined in Figure 12.6. Try also the start state [1/2,3/3,3/1,1/3,3/2,1/1,2/3, 2/1,2/2]. Compare the execution times of and lengths of solutions found by A* (using heuristic function of Figure 12.6) and IDA* (using just total distance).

12.4.3 RBFS – recursive best-first search

IDA* is a valuable idea and very easy to implement, but in unfavourable situations the overheads of regenerating nodes become unacceptable. Therefore a better,


```
% idastar( Start, Solution):
%   Perform IDA* search; Start is the start node, Solution is solution path

idastar( Start, Solution) :-
  retract( next_bound(_)), fail                  % Clear next_bound
  ;
  asserta( next_bound( 0)),                      % Initialize bound
  idastar0( Start, Solution).

idastar0( Start, Sol) :-
  retract( next_bound( Bound)),                  % Current bound
  asserta( next_bound( 99999)),                  % Initialize next bound
  f( Start, F),                                  % f-value of start node
  df( [Start], F, Bound, Sol)                    % Find solution; if not, change bound
  ;
  next_bound( NextBound),
  NextBound < 99999,                             % Bound finite
  idastar0( Start, Sol).                         % Try with new bound

% df( Path, F, Bound, Sol):
%   Perform depth-first search within Bound
%   Path is the path from start node so far (in reverse order)
%   F is the f-value of the current node, i.e. the head of Path

df( [N | Ns], F, Bound, [N | Ns]) :-
  F =< Bound,
  goal( N).                                      % Succeed: solution found

df( [N | Ns], F, Bound, Sol) :-
  F =< Bound,                                    % Node N within f-bound
  s( N, N1), not member( N1, Ns),                % Expand N
  f( N1, F1),
  df( [N1,N | Ns], F1, Bound, Sol).

df( _, F, Bound, _) :-
  F > Bound,                                     % Beyond Bound
  update_next_bound( F),                         % Just update next bound
  fail.                                          % and fail

update_next_bound( F) :-
  next_bound( Bound),
  Bound =< F, !                                  % Do not change next bound
  ;
  retract( next_bound( Bound)), !,               % Lower next bound
  asserta( next_bound( F)).
```

Figure 12.10 An implementation of the IDA* algorithm.

although more complicated space-saving technique is the so-called RBFS ('recursive best-first search'). RBFS is very similar to our A* program of Figure 12.3 (which also is recursive in the same sense as RBFS!). The difference between our A* program and RBFS is that A* keeps in memory *all* the already generated nodes whereas RBFS only keeps the current search path and the sibling nodes along this path. When RBFS temporarily suspends the search of a subtree (because it no longer looks the best), it 'forgets' that subtree to save space. So RBFS's space complexity is (as in IDA*) only linear in the depth of search. The only thing that RBFS remembers of such an abandoned subtree is the updated f-value of the root of the subtree. The f-values are updated through backing-up the f-values in the same way as in the A* program. To distinguish between the 'static' evaluation function f and these backed-up values, we write (for a node N):

$f(\mathrm{N})$ = value of node N returned by the evaluation function (always the same during search)

$F(\mathrm{N})$ = backed-up f-value (changes during search because it depends on the descendent nodes of N)

$F(\mathrm{N})$ is defined as follows:

$F(\mathrm{N}) = f(\mathrm{N})$ if N has (never) been expanded by the search
$F(\mathrm{N}) = \min\{ F(\mathrm{N}_i) \mid \mathrm{N}_i$ is a child of N$\}$

As the A* program, RBFS also explores subtrees within a given f-bound. The bound is determined by the F-values of the siblings along the current search path (the smallest F-value of the siblings, that is the F-value of the closest competitor to the current node). Suppose that a node N is currently the best node in the search tree (i.e. has the lowest F-value). Then N is expanded and N's children are explored up to some f-bound Bound. When this bound is exceeded (manifested by $F(\mathrm{N}) >$ Bound) then all the nodes generated below N are 'forgotten'. However, the updated value $F(\mathrm{N})$ is retained and is used in deciding about how to continue search.

The F-values are not only determined by backing-up the values from a node's children, but can also be *inherited* from the node's parents. Such inheritance occurs as follows. Let there be a node N which is about to be expanded by the search. If $F(\mathrm{N}) > f(\mathrm{N})$ then we know that N must have already been expanded earlier and $F(\mathrm{N})$ was determined from N's children, but the children have then been removed from the memory. Now suppose a child N_i of N is generated again and N_i's static value $f(\mathrm{N}_i)$ is also computed again. Now $F(\mathrm{N}_i)$ is determined as follows:

if $f(\mathrm{N}_i) < F(\mathrm{N})$ then $F(\mathrm{N}_i) = F(\mathrm{N})$ else $F(\mathrm{N}_i) = f(\mathrm{N}_i)$

This can be written shorter as:

$$F(\mathrm{N}_i) = \max\{F(\mathrm{N}), f(\mathrm{N}_i)\}$$

Thus in the case $f(\mathrm{N}_i) < F(\mathrm{N})$, N_i's F-value is inherited from N_i's parent N. This is justified by the following argument: when N_i was generated (and removed) earlier,

the value $F(\mathrm{N}_i)$ was necessarily $\geqslant F(\mathrm{N})$. Otherwise, by the back-up rule, $F(\mathrm{N})$ would have been smaller.

To illustrate how RBFS works, consider the route-finding problem of Figure 12.2. Figure 12.11 shows selected snapshots of the current path (including the siblings along the path) kept in the memory. The search keeps switching (as in A*) between the alternative paths. However, when such a switch occurs, the previous path is removed from the memory to save space. In Figure 12.11, the numbers written next to the nodes are the nodes' F-values. At snapshot (A), node a is the best candidate node ($F(a) < F(e)$). Therefore the subtree below a is searched with Bound = 9

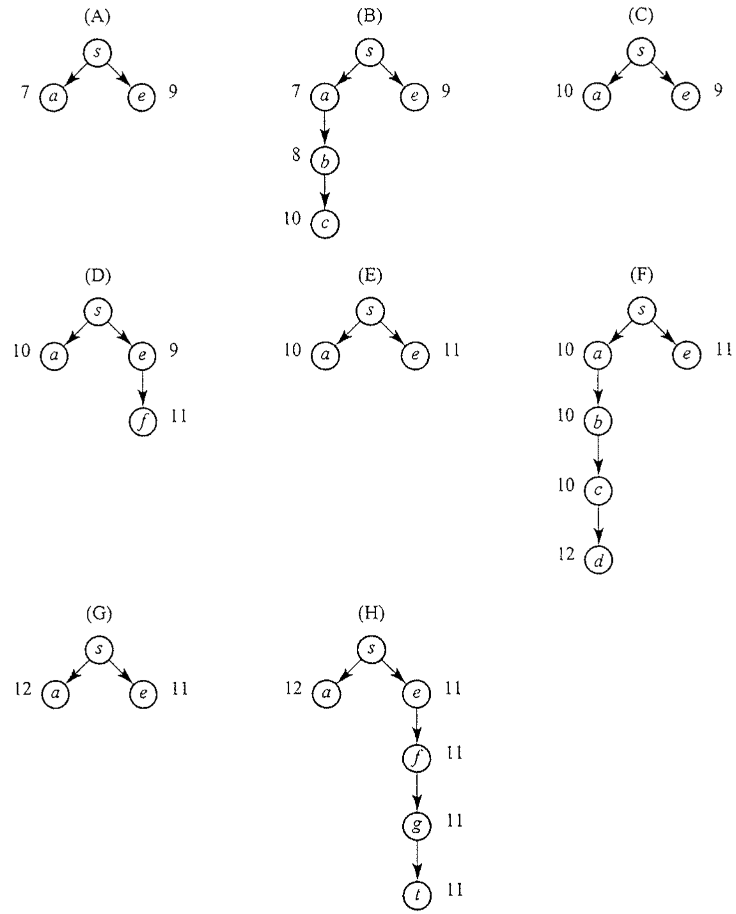


Figure 12.11 The trace of the RBFS algorithm on the route problem of Figure 12.2. The figures next to the nodes are the nodes' F-values (which change during search).

(i.e. $F(e)$, the closest – the only competitor). When this search reaches node c (snapshot B), it is found that $F(c) = 10 >$ Bound. Therefore this path is (temporarily) abandoned, the nodes c and b removed from the memory and the value $F(c) = 10$ backed-up to node a, so $F(a)$ becomes 10 (snapshot C). Now e is the best competitor ($F(e) = 9 < 10 = F(a)$), and its subtree is searched with Bound $= 10 = F(a)$. This search stops at node f because $F(f) = 11 >$ Bound (snapshot D). Node e is removed, and $F(e)$ becomes 11 (snapshot E). Now the search switches to a again with Bound $= 11$. When b is regenerated, it is found that $f(b) = 8 < F(a)$. Therefore node b *inherits* its F-value from node a, so $F(b)$ becomes 10. Next c is regenerated and c also inherits its F-value from b, so $F(c)$ becomes 10. The Bound $= 11$ is exceeded at snapshot F, the nodes d, c and b are removed and $F(d) = 12$ is backed-up to node a (snapshot G). Now the search switches to node e and runs smoothly to the goal t.

Let us now formulate the RBFS algorithm more formally. The algorithm is centred around the updating of the F-values of nodes. So a good way to formulate the algorithm is by defining a function:

NewF(N, F(N), *Bound*)

where N is a node whose current F-value is F(N). The function carries out the search within Bound, starting with node N, and computes the new F-value of N, *NewF*, resulting from this search. The new value is determined at the moment that the bound is exceeded. If, however, a goal is found before this, then the function terminates, signalling success as a side effect. The function *NewF* is defined in Figure 12.12.

```
function NewF( N, F(N), Bound)
  begin
    if F(N) > Bound then NewF := F(N)
    else if goal(N) then exit search with success
    else if N has no children then NewF := infinity (dead end)
    else
      begin
        for each child Ni of N do
          if f(N) < F(N) then F(Ni) := max( F(N), f(Ni))
          else F(Ni) := f(Ni);
        sort children Ni in increasing order of their F-values;
        while F(N1) ≤ Bound and F(N1) < infinity do
          begin
            Bound1 := min( Bound, F-value of best sibling of N1);
            F(N1) := NewF( N1, F(N1), Bound1);
            reorder nodes N1, N2, ... according to new F(N1)
          end
        NewF := F(N1)
  end
```

Figure 12.12 The updating of F-values in RBFS search.

A Prolog implementation of RBFS is shown in Figure 12.13. This program is similar to the A* program of Figure 12.3. The heart of the program is the procedure

rbfs(Path, SiblingNodes, Bound, NewBestFF, Solved, Solution)

which carries out the RBFS algorithm. The arguments are:

Path	Path so far from start node of search in reverse order
SiblingNodes	The children of the last node in the path so far, i.e. the head of Path
Bound	Upper bound on F-values for extending the search from SiblingNodes.
NewBestFF	The best F-value after extending search just beyond Bound.
Solved	Indicates the success of search below SiblingNodes (Solved = yes if a goal was found, no if search went just beyond Bound, never if this is a dead end).
Solution	Solution path if Solve = yes, otherwise undefined.

The representation of the nodes includes, besides a state in the state space also the path costs, f-values and F-values as follows:

Node = (State, G/F/FF)

```
% Linear-space best-first search; the RBFS algorithm
% The program assumes 99999 is greater than any f-value

% bestfirst( Start, Solution): Solution is a path from Start to a goal

bestfirst( Start, Solution) :-
  rbfs( [], [ (Start, 0/0/0) ], 99999, _, yes, Solution).

% rbfs( Path, SiblingNodes, Bound, NewBestFF, Solved, Solution):
%   Path = path so far in reverse order
%   SiblingNodes = children of head of Path
%   Bound = upper bound on F-value for search from SiblingNodes
%   NewBestFF = best f-value after searching just beyond Bound
%   Solved = yes, no, or never
%   Solution = solution path if Solve = yes
%
%   Representation of nodes: Node = ( State, G/F/FF)
%   where G is cost till State, F is static f-value of State,
%   FF is backed-up value of State

rbfs( Path, [ (Node, G/F/FF) | Nodes], Bound, FF, no, _) :-
  FF > Bound, !.
```

Figure 12.13 A best-first search program that only requires space linear in the depth of search (RBFS algorithm).

Figure 12.13 *contd*

```
rbfs( Path, [ (Node, G/F/FF) | _], _, _, yes, [Node | Path]) :-
  F = FF,                          % Only report solution once, when first reached; then F=FF
  goal( Node).

rbfs( _, [], _, _, never, _) :- !.                 % No candidates, dead end!

rbfs( Path, [ (Node, G/F/FF) | Ns], Bound, NewFF, Solved, Sol) :-
  FF =< Bound,                                     % Within Bound: generate children
  findall( Child/Cost,
           ( s( Node, Child, Cost), not member( Child, Path)),
           Children),
  inherit( F, FF, InheritedFF),                    % Children may inherit FF
  succlist( G, InheritedFF, Children, SuccNodes),  % Order children
  bestff( Ns, NextBestFF),                         % Closest competitor FF among siblings
  min( Bound, NextBestFF, Bound2), !,
  rbfs( [Node | Path], SuccNodes, Bound2, NewFF2, Solved2, Sol),
  continue(Path, [(Node,G/F/NewFF2)|Ns], Bound, NewFF, Solved2, Solved, Sol).

% continue( Path, Nodes, Bound, NewFF, ChildSolved, Solved, Solution)

continue( Path, [N | Ns], Bound, NewFF, never, Solved, Sol) :- !,
  rbfs( Path, Ns, Bound, NewFF, Solved, Sol).      % Node N a dead end

continue( _, _, _, _, yes, yes, Sol).

continue( Path, [ N | Ns], Bound, NewFF, no, Solved, Sol) :-
  insert( N, Ns, NewNs), !,                        % Ensure siblings are ordered by values
  rbfs( Path, NewNs, Bound, NewFF, Solved, Sol).

succlist( _, _, [], []).

succlist( G0, InheritedFF, [Node/C | NCs], Nodes) :-
  G is G0 + C,
  h( Node, H),
  F is G + H,
  max( F, InheritedFF, FF),
  succlist( G0, InheritedFF, NCs, Nodes2),
  insert( (Node, G/F/FF), Nodes2, Nodes).

inherit( F, FF, FF) :-                             % Child inherits father's FF if
  FF > F, !.                                       % father's FF greater than father's F

inherit( F, FF, 0).

insert( (N, G/F/FF), Nodes, [ (N, G/F/FF) | Nodes]) :-
  bestff( Nodes, FF2),
  FF =< FF2, !.

insert( N, [N1 | Ns], [N1 | Ns1]) :-
  insert( N, Ns, Ns1).

bestff( [ (N, F/G/FF) | Ns], FF).                  % First node - best FF

bestff( [], 99999).                                % No nodes - FF = 'infinite'
```


where G is the cost of the path from the start state to State, F is static value f(State), and FF is the current backed-up value F(State). It should be noted that variable F in the program denotes an f-value, and FF in the program denotes an F-value. The procedure rbfs carries out the search below SiblingNodes within Bound, and computes NewBestFF according to the function *NewF* in Figure 12.12.

Let us summarize the important properties of the RBFS algorithm. First, its space complexity is linear in the depth of search. The price for this is the time needed to regenerate already generated nodes. However, these overheads are substantially smaller than in IDA*. Second, like A* and unlike IDA*, RBFS expands the nodes in the best-first order even in the case of a non-monotonic f-function.

Exercises

12.8 Consider the route-finding problem of Figure 12.2. How many nodes are generated by A*, IDA* and RBFS on this problem (counting also the regenerated nodes)?

12.9 Consider the state space in Figure 12.14. Let a be the start node and l the goal node. Give the order in which nodes are generated (including regeneration) by the RBFS algorithm. How are the backed-up values $F(b)$ and $F(c)$ changing during this process?

12.10 Inheritance of F-values in RBFS saves time (prevents unnecessary regeneration of nodes). Explain how. Hint: Consider the search by RBFS of the binary tree in which for each node its f-value is equal to the depth of the node in the tree. Study the execution of RBFS with and without inheritance of F-values on this state space up to depth 8.

12.11 Compare the A*, IDA* and RBFS programs on the eight puzzle problems. Measure execution times and the number of nodes generated during search, including the regenerated nodes (add node counters to the programs).

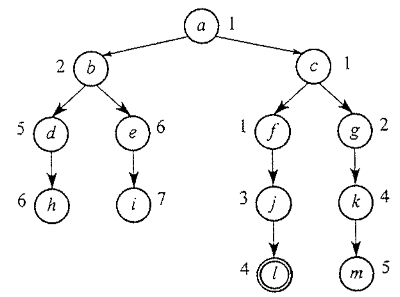


Figure 12.14 A state space; the numbers next to the nodes are the nodes' f-values; l is the goal node.

Summary

- Heuristic information can be used to estimate how far a node is from a nearest goal node in the state space. In this chapter we considered the use of numerical heuristic estimates.
- The *best-first* heuristic principle guides the search process so as to always expand the node that is currently the most promising according to the heuristic estimates. The well-known A* algorithm that uses this principle was programmed in this chapter.
- To use A* for solving a concrete problem, a state space, a goal predicate, and a heuristic function have to be defined. For complex problems, the difficult part is to find a good heuristic function.
- The *admissibility* theorem helps to establish whether A*, using a particular heuristic function, will always find an optimal solution.
- The time and space requirements of A* typically grow exponentially with solution length. In practical applications, space is often more critical than time. Special techniques for best-first search aim at saving space at the expense of time.
- IDA* is a simple space-efficient best-first search algorithm based on a similar idea as iterative deepening. Overheads due to node regeneration in IDA* are acceptable in cases when many nodes in the state space have equal *f*-values. When the nodes tend not to share *f*-values the overheads become unacceptable.
- RBFS is a more sophisticated space-efficient best-first search algorithm that generally regenerates less nodes than IDA*.
- The space requirements of both IDA* and RBFS are very modest. They only grow linearly with the depth of search.
- In this chapter best-first search was applied to the eight puzzle problem and a task-scheduling problem.
- Concepts discussed in this chapter are:

  heuristic estimates
  heuristic search
  best-first search
  algorithms A*, IDA*, RBFS
  admissibility of search algorithms, admissibility theorem
  space-efficiency of best-first search
  monotonicity of evaluation function

References

The best-first search program of this chapter is a variation of many similar algorithms of which A* is the most popular. Descriptions of A* can be found in general text books on AI, such as Nilsson (1980), Winston (1992) and Russell and Norvig (1995). Doran and Michie (1966) originated the best-first search guided by distance-to-goal estimate. The admissibility theorem was discovered by Hart, Nilsson and Raphael (1968). In the literature, the property $h \leqslant h^*$ is often included in the definition of A*. An excellent and rigorous treatment of many variations of best-first search algorithms and related mathematical results is provided by Pearl (1984). Kanal and Kumar (1988) edited a collection of interesting papers dealing with various advanced aspects of search. An early description of IDA* appeared in Korf (1985). The RBFS algorithm was introduced and analyzed by Korf (1993). Russell and Norvig (1995) discuss further ideas for space-bounded search.

The eight puzzle was used in artificial intelligence as a test problem for studying heuristic principles by several researchers – for example, Doran and Michie (1966), Michie and Ross (1970), and Gaschnig (1979).

Our task-scheduling problem and its variations arise in numerous applications in which servicing of requests for resources is to be planned. Our example task-scheduling problem in Section 12.3 is borrowed from Coffman and Denning (1973).

Finding good heuristics is important and difficult, therefore the study of heuristics is one of the central themes of artificial intelligence. There are, however, also some limitations on how far we can get in the refinement of heuristics. It may appear that to solve any combinatorial problem efficiently we only have to find a powerful heuristic. However, there are problems (including many scheduling problems) for which no general heuristic exists that would guarantee both efficiency and admissibility in all cases. Many theoretical results that pertain to this limitation issue are collected in Garey and Johnson (1979).

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chapter 13

Problem Decomposition and AND/OR Graphs

AND/OR graphs are a suitable representation for problems that can be naturally decomposed into mutually independent subproblems. Examples of such problems include route finding, design, symbolic integration, game playing, theorem proving, etc. In this chapter we will develop programs for searching AND/OR graphs, including a heuristically guided best-first AND/OR search.

13.1 AND/OR graph representation of problems

In Chapters 11 and 12, problem solving was centred around the state-space representation of problems. Accordingly, problem solving was reduced to finding a path in a state-space graph. Another representation, the AND/OR graph representation, more naturally suits certain categories of problems. This representation relies on the decomposition of problems into subproblems. Decomposition into subproblems is advantageous if the subproblems are mutually independent, and can therefore be solved independently of each other.

Let us illustrate this with an example. Consider the problem of finding a route in a road map between two given cities, as shown in Figure 13.1. We will disregard path lengths for the moment. The problem could, of course, be formulated as path finding in a state space. The corresponding state space would look just like the map: the nodes in the state space correspond to cities, the arcs correspond to direct

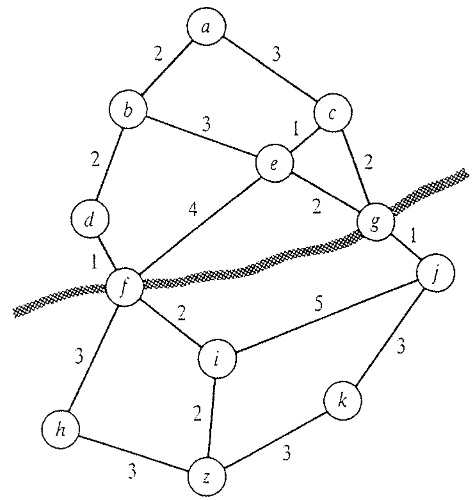

Figure 13.1 Finding a route from a to z in a road map. The river has to be crossed at f or g. An AND/OR representation of this problem is shown in Figure 13.2.

connections between cities, arc costs correspond to distances between cities. However, let us construct another representation of this problem, based on a natural decomposition of the problem.

In the map of Figure 13.1, there is also a river. Let us assume that there are only two bridges at which the river can be crossed, one bridge at city *f* and the other at city *g*. Obviously, our route will have to include one of the bridges; so it will have to go through *f* or through *g*. We have, then, two major alternatives:

To find a path between *a* and *z*, find *either*
(1) a path from *a* to *z* via *f*, *or*
(2) a path from *a* to *z* via *g*.

Each of these two alternative problems can now be decomposed as follows:

(1) To find a path from *a* to *z* via *f*:
  1.1 find a path from *a* to *f*, *and*
  1.2 find a path from *f* to *z*.
(2) To find a path from *a* to *z* via *g*:
  2.1 find a path from *a* to *g*, *and*
  2.2 find a path from *g* to *z*.

To summarize, we have two main alternatives for solving the original problem: (1) via *f* or (2) via *g*. Further, each of these alternatives can be *decomposed* into two subproblems (1.1 and 1.2, or 2.1 and 2.2 respectively). What is important here is that (in both alternatives) each of the subproblems can be solved independently of the

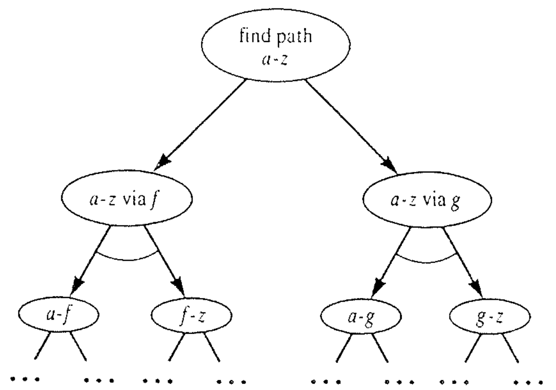

Figure 13.2 An AND/OR representation of the route-finding problem of Figure 13.1. Nodes correspond to problems or subproblems, and curved arcs indicate that all (both) subproblems have to be solved.

other. Such a decomposition can be pictured as an *AND/OR graph* (Figure 13.2). Notice the curved arcs which indicate the AND relationship between subproblems. Of course, the graph in Figure 13.2 is only the top part of a corresponding AND/OR tree. Further decomposition of subproblems could be based on the introduction of additional intermediate cities.

What are goal nodes in such an AND/OR graph? Goal nodes correspond to subproblems that are trivial or 'primitive'. In our example, such a subproblem would be 'find a route from *a* to *c*', for there is a direct connection between cities *a* and *c* in the road map.

Some important concepts have been introduced in this example. An AND/OR graph is a directed graph in which nodes correspond to problems, and arcs indicate relations between problems. There are also relations among arcs themselves. These relations are AND and OR, depending on whether we have to solve just one of the successor problems or several (see Figure 13.3). In principle, a node can issue both AND-related arcs and OR-related arcs. We will, however, assume that each node has

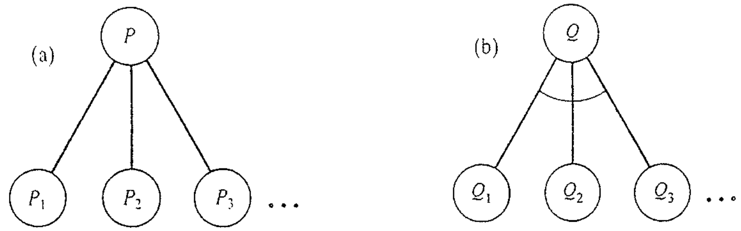

Figure 13.3 (a) To solve P solve any of P_1 or P_2 or . . . . (b) To solve Q solve all Q_1 and Q_2 . . . .

either only AND successors or only OR successors. Each AND/OR graph can be transformed into this form by introducing auxiliary OR nodes if necessary. Then, a node that only issues AND arcs is called an AND node; a node that only issues OR arcs is called an OR node.

In the state-space representation, a solution to the problem was a path in the state space. What is a solution in the AND/OR representation? A solution, of course, has to include all the subproblems of an AND node. Therefore, a solution is not a path any more, but it is a tree. Such a solution tree, T, is defined as follows:

- the original problem, P, is the root node of T;
- if P is an OR node then exactly one of its successors (in the AND/OR graph), together with its own solution tree, is in T;
- if P is an AND node then all of its successors (in the AND/OR graph), together with their solution trees, are in T.

Figure 13.4 illustrates this definition. In this figure, there are costs attached to arcs. Using costs we can formulate an optimization criterion. We can, for example, define

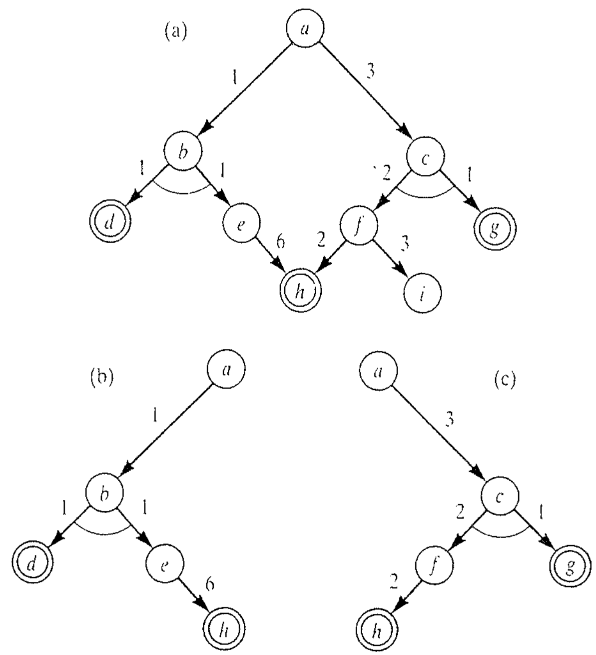

Figure 13.4 (a) An AND/OR graph: d, g and h are goal nodes; a is the problem to be solved. (b) and (c) Two solution trees whose costs are 9 and 8 respectively. The cost of a solution tree is here defined as the sum of all the arc costs in the solution tree.

the cost of a solution graph as the sum of all the arc costs in the graph. As we are normally interested in the minimum cost, the solution graph in Figure 13.4(c) will be preferred.

But we do not have to base our optimization measure on the costs of arcs. Sometimes it is more natural to associate costs with nodes rather than arcs, or with both arcs and nodes.

To summarize:

- AND/OR representation is based on the philosophy of decomposing a problem into subproblems.
- Nodes in an AND/OR graph correspond to problems; links between nodes indicate relations between problems.
- A node that issues OR links is an OR node. To solve an OR node, one of its successor nodes has to be solved.
- A node that issues AND links is an AND node. To solve an AND node, all of its successors have to be solved.
- For a given AND/OR graph, a particular problem is specified by two things:

  a start node, and
  a goal condition for recognizing goal nodes.
- *Goal nodes* (or 'terminal' nodes) correspond to trivial (or 'primitive') problems.
- A solution is represented by a *solution graph*, a subgraph of the AND/OR graph.
- The state-space representation can be viewed as a special case of the AND/OR representation in which all the nodes are OR nodes.
- To benefit from the AND/OR representation, AND-related nodes should represent subproblems that can be solved independently of each other. The independency criterion can be somewhat relaxed, as follows: there must exist an ordering of AND subproblems so that solutions of subproblems that come earlier in this ordering are not destroyed when solving later subproblems.
- Costs can be attached to arcs or nodes or both in order to formulate an optimization criterion.

13.2 Examples of AND/OR representation

13.2.1 AND/OR representation of route finding

For the shortest route problem of Figure 13.1, an AND/OR graph including a cost function can be defined as follows:

- OR nodes are of the form X-Z, meaning: find a shortest path from X to Z.

- AND nodes are of the form

  X-Z via Y

  meaning: find a shortest path from X to Z under the constraint that the path goes through Y.
- A node X-Z is a goal node (primitive problem) if X and Z are directly connected in the map.
- The cost of each goal node X-Z is the given road distance between X and Z.
- The costs of all other (non-terminal) nodes are 0.

The cost of a solution graph is the sum of the costs of all the nodes in the solution graph (in our case, this is just the sum over the terminal nodes). For the problem of Figure 13.1, the start node is *a-z*. Figure 13.5 shows a solution tree of cost 9. This tree corresponds to the path [a,b,d,f,i,z]. This path can be reconstructed from the solution tree by visiting all the leaves in this tree in the left-to-right order.

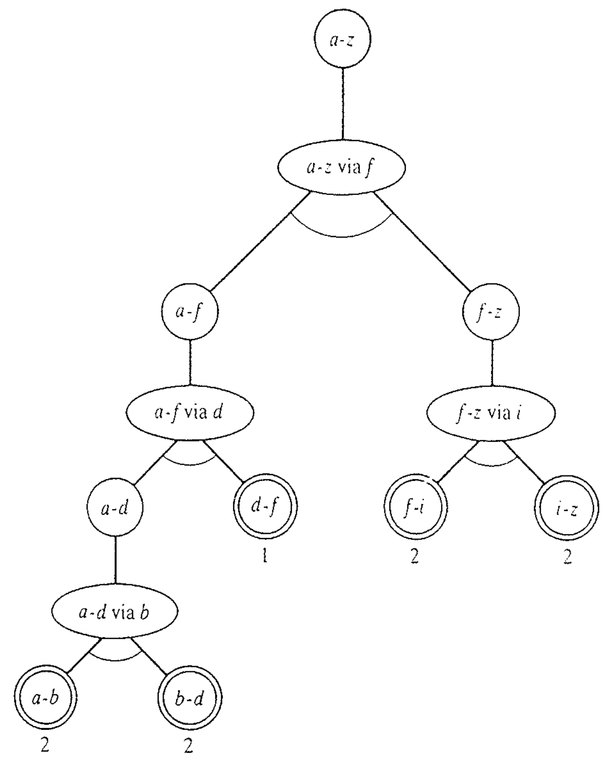

Figure 13.5 The cheapest solution tree for the route problem of Figure 13.1 formulated as an AND/OR graph.

13.2.2 The Tower of Hanoi problem

The Tower of Hanoi problem, shown in Figure 13.6, is another, classical example of effective application of the AND/OR decomposition scheme. For simplicity, we will consider a simple version of this problem, containing three disks only:

> There are three pegs, 1, 2 and 3, and three disks, *a*, *b* and *c* (*a* being the smallest and *c* being the biggest). Initially, all the disks are stacked on peg 1. The problem is to transfer them all on to peg 3. Only one disk can be moved at a time, and no disk can ever be placed on top of a smaller disk.

This problem can be viewed as the problem of achieving the following set of goals:

(1) Disk *a* on peg 3.

(2) Disk *b* on peg 3.

(3) Disk *c* on peg 3.

These goals are, unfortunately, not independent. For example, disk *a* can immediately be placed on peg 3, satisfying the first goal. This will, however, prevent the fulfilment of the other two goals (unless we undo the first goal again). Fortunately, there is a convenient ordering of these goals so that a solution can easily be derived from this ordering. The ordering can be found by the following reasoning: goal 3 (disk *c* on peg 3) is the hardest because moving disk *c* is subject to most constraints. A good idea that often works in such situations is: try to achieve the hardest goal first. The logic behind this principle is: as other goals are easier (not as constrained as the hardest) they can hopefully be achieved without the necessity of undoing this hardest goal.

The problem-solving strategy that results from this principle in our task is:

> First satisfy the goal 'disk *c* on peg 3',
> then satisfy the remaining goals.

But the first goal cannot immediately be achieved: disk *c* cannot move in the initial situation. Therefore, we first have to prepare this move and our strategy is refined to:

(1) Enable moving disk *c* from 1 to 3.

(2) Move disk *c* from 1 to 3.

(3) Achieve remaining goals: *a* on 3, and *b* on 3.

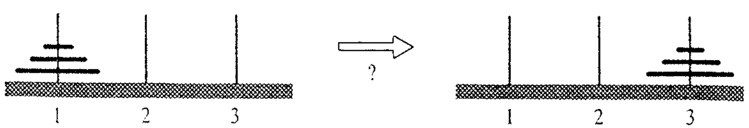

Figure 13.6 The Tower of Hanoi problem.

Disk *c* can only move from 1 to 3 if both *a* and *b* are stacked on peg 2. Then, our initial problem of moving *a*, *b* and *c* from peg 1 to peg 3 is reduced to three subproblems:

> To move *a*, *b* and *c* from 1 to 3:
> (1) move *a* and *b* from 1 to 2, *and*
> (2) move *c* from 1 to 3, *and*
> (3) move *a* and *b* from 2 to 3.

Problem 2 is trivial (one-step solution). The other two subproblems can be solved independently of problem 2 because disks *a* and *b* can be moved regardless of the position of disk *c*. To solve problems 1 and 3, the same decomposition principle can be applied (disk *b* is the hardest this time). Accordingly, problem 1 is reduced to three trivial subproblems:

> To move *a* and *b* from 1 to 2:
> (1) move *a* from 1 to 3, *and*
> (2) move *b* from 1 to 2, *and*
> (3) move *a* from 3 to 2.

13.2.3 AND/OR formulation of game playing

Games like chess and checkers can naturally be viewed as problems, represented by AND/OR graphs. Such games are called two-person, perfect-information games, and we will assume here that there are only two possible outcomes: WIN or LOSS. (We can think of games with three outcomes – WIN, LOSS and DRAW – as also having just two outcomes: WIN and NO-WIN.) As the two players move in turn we have two kinds of positions, depending on who is to move. Let us call the two players 'us' and 'them', so the two kinds of positions are: 'us-to-move' positions and 'them-to-move' positions. Assume that the game starts in an us-to-move position P. Each alternative us-move in this position leads to one of them-to-move positions Q_1, Q_2, ... (Figure 13.7). Further, each alternative them-move in Q_1 leads to one of the positions R_{11}, R_{12}, ... In the AND/OR tree of Figure 13.7, nodes correspond to positions, and arcs correspond to possible moves. Us-to-move levels alternate with them-to-move levels. To win in the initial position, P, we have to find a move from P to Q_i, for some i, so that the position Q_i is won. Thus, P is won if Q_1 or Q_2 or ... is won. Therefore position P is an OR node. For all i, position Q_i is a them-to-move position, so if it is to be won for us it has to be won after *each* them-move. Thus Q_i is won if all R_{i1} *and* R_{i2} *and* ... are won. Accordingly, all them-to-move positions are AND nodes. Goal nodes are those positions that are won by the rules of the game; for example, their king checkmated in chess. Those positions that are lost by the rules of the game correspond to unsolvable problems. To solve the game we have to find a solution tree that guarantees our victory regardless of the opponent's replies. Such

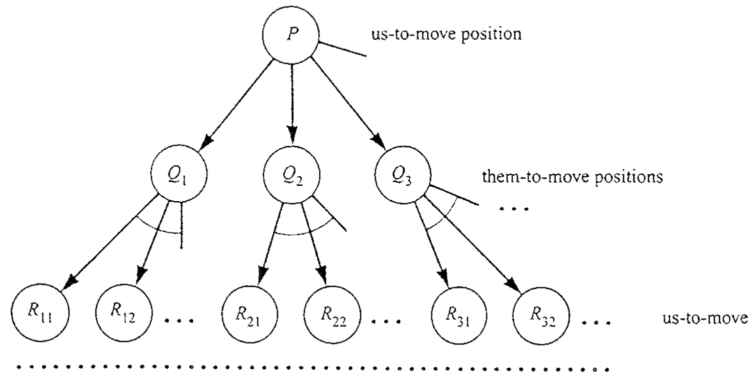

Figure 13.7 An AND/OR formulation of a two-person game; the players are 'us' and 'them'.

a solution tree, then, is a complete strategy for winning the game: for each possible continuation that can be chosen by the opponent, there is a move in such a strategy tree that forces a win.

13.3 Basic AND/OR search procedures

In this section we will only be interested in finding *some* solution of the problem, regardless of its cost. So for the purposes of this section we can ignore the costs of links or nodes in an AND/OR graph.

The simplest way of searching AND/OR graphs in Prolog is to use Prolog's own search mechanism. This happens to be trivial as Prolog's procedural meaning itself is nothing but a procedure for searching AND/OR graphs. For example, the AND/OR graph of Figure 13.4 (ignoring the arc costs) can be specified by the following clauses:

```
a :- b.          % a is an OR node with two successors, b and c
a :- c.
b :- d, e.       % b is an AND node with two successors, d and e
e :- h.
c :- f, g.
f :- h, i.
d.   g.   h.     % d, g and h are goal nodes
```

To ask whether problem a can be solved we can simply ask:

```
?- a.
```


Now Prolog will effectively search the graph of Figure 13.4 in the depth-first fashion and answer 'yes', after having visited that part of the search graph corresponding to the solution tree in Figure 13.4(b).

The advantage of this approach to programming AND/OR search is its simplicity. There are disadvantages, however:

- We only get an answer 'yes' or 'no', not a solution tree as well. We could reconstruct the solution tree from the program's trace, but this can be awkward and insufficient if we want a solution tree explicitly accessible as an object in the program.
- This program is hard to extend so as to be able to handle costs as well.
- If our AND/OR graph were a general graph, containing cycles, then Prolog with its depth-first strategy could enter an indefinite recursive loop.

Let us rectify these deficiencies gradually. We will first define our own depth-first search procedure for AND/OR graphs.

To begin with, we will change the representation of AND/OR graphs in the program. For that we will introduce a binary relation represented in the infix notation with the operator '--->'. For example, node *a* linked to its two OR successors will be represented by the clause:

```
a ---> or : [b,c].
```

The symbols '--->' and ':' are both infix operators that can be defined by:

```
:- op( 600, xfx, --->).
:- op( 500, xfx, :).
```

The complete AND/OR graph of Figure 13.4 is thus specified by the clauses:

```
a ---> or : [b,c].
b ---> and : [d,e].
c ---> and : [f,g].
e ---> or : [h].
f ---> or : [h,i].

goal( d).    goal( g).    goal( h).
```

The depth-first AND/OR procedure can be constructed from the following principles:

To solve a node, N, use the following rules:

(1) If N is a goal node then it is trivially solved.

(2) If N has OR successors then solve one of them (attempt them one after another until a solvable one is found).

(3) If N has AND successors then solve all of them (attempt them one after another until they have all been solved).

If the above rules do not produce a solution then assume the problem cannot be solved.

A corresponding program can be as follows:

```
solve( Node) :-
  goal( Node).

solve( Node) :-
  Node ---> or : Nodes,            % Node is an OR node
  member( Node1, Nodes),           % Select a successor Node1 of Node
  solve( Node1).

solve( Node) :-
  Node ---> and : Nodes,           % Node is an AND node
  solveall( Nodes).                % Solve all Node's successors

solveall( [] ).

solveall( [Node | Nodes] ) :-
  solve( Node),
  solveall( Nodes).
```

member is the usual list membership relation.

This program still has the following disadvantages:

- it does not produce a solution tree, and
- it is susceptible to infinite loops, depending on the properties of the AND/OR graph (cycles).

The program can easily be modified to produce a solution tree. We have to modify the solve relation so that it has two arguments:

```
solve( Node, SolutionTree)
```

Let us represent a solution tree as follows. We have three cases:

(1) If Node is a goal node then the corresponding solution tree is Node itself.

(2) If Node is an OR node then its solution tree has the form:

```
Node ---> Subtree
```

where Subtree is a solution tree for one of the successors of Node.

(3) If Node is an AND node then its solution tree has the form:

```
Node ---> and : Subtrees
```

where Subtrees is the list of solution trees of all of the successors of Node.

For example, in the AND/OR graph of Figure 13.4, the first solution of the top node *a* is represented by:

```
a ---> b ---> and : [d, e ---> h]
```

The three forms of a solution tree correspond to the three clauses about our solve relation. So our initial solve procedure can be altered by simply modifying each of the three clauses; that is, by just adding solution tree as the second argument to solve. The resulting program is shown as Figure 13.8. An additional procedure in this program is show for displaying solution trees. For example, the solution tree of Figure 13.4 is displayed by show in the following form:

```
a ---> b ---> d
              e ---> h
```

The program of Figure 13.8 is still prone to infinite loops. One simple way to prevent infinite loops is to keep trace of the current depth of the search and prevent the program from searching beyond some depth limit. We can do this by simply introducing another argument to the solve relation:

```
solve( Node, SolutionTree, MaxDepth)
```

As before, Node represents a problem to be solved, and SolutionTree is a solution not deeper than MaxDepth. MaxDepth is the allowed depth of search in the graph. In the case that MaxDepth = 0 no further expansion is allowed; otherwise, if MaxDepth > 0 then Node can be expanded and its successors are attempted with a lower depth limit MaxDepth − 1. This can easily be incorporated into the program of Figure 13.8. For example, the second clause about solve becomes:

```
solve( Node, Node ---> Tree, MaxDepth) :-
  MaxDepth > 0,
  Node ---> or : Nodes,            % Node is an OR node
  member( Node1, Nodes),           % Select a successor Node1 of Node
  Depth1 is MaxDepth - 1,          % New depth limit
  solve( Node1, Tree, Depth1).     % Solve successor with lower limit
```

This depth-limited, depth-first procedure can also be used in the iterative deepening regime, thereby simulating the breadth-first search. The idea is to do the depth-first search repetitively, each time with a greater depth limit, until a solution is found. That is, try to solve the problem with depth limit 0, then with 1, then with 2, etc. Such a program is:

```
iterative_deepening( Node, SolTree) :-
  trydepths( Node, SolTree, 0).    % Try search with increasing depth limit, start with 0

trydepths( Node, SolTree, Depth) :-
  solve( Node, SolTree, Depth)
  ;
  Depth1 is Depth + 1,             % Get new depth limit
  trydepths( Node, SolTree, Depth1).   % Try higher depth limit
```

```
% Depth-first AND/OR search

% solve( Node, SolutionTree):
%   find a solution tree for Node in an AND/OR graph

solve( Node, Node) :-                     % Solution tree of goal node is Node itself
  goal( Node).

solve( Node, Node ---> Tree) :-
  Node ---> or : Nodes,                   % Node is an OR node
  member( Node1, Nodes),                  % Select a successor Node1 of Node
  solve( Node1, Tree).

solve( Node, Node ---> and : Trees) :-
  Node ---> and : Nodes,                  % Node is an AND node
  solveall( Nodes, Trees).                % Solve all Node's successors

% solveall( [Node1, Node2, ...], [SolutionTree1, SolutionTree2, ...] )

solveall( [], [] ).

solveall( [Node | Nodes], [Tree | Trees] ) :-
  solve( Node, Tree),
  solveall( Nodes, Trees).

show( Tree) :-                            % Display solution tree
  show( Tree, 0).                         % Indented by 0

% show( Tree, H): display solution tree indented by H

show( Node ---> Tree, H) :- !,
  write( Node), write( ' ---> '),
  H1 is H + 7,
  show( Tree, H1).

show( and : [T], H) :- !,                 % Display single AND tree
  show( T).

show( and : [T | Ts], H) :- !,            % Display AND list of solution trees
  show( T, H),
  tab( H),
  show( and : Ts, H).

show( Node, H) :-
  write( Node), nl.
```

Figure 13.8 Depth-first search for AND/OR graphs. This program does not avoid infinite cycling. Procedure **solve** finds a solution tree and procedure **show** displays such a tree. **show** assumes that each node only takes one character on output.

As with iterative deepening in state space (see Chapter 11), a disadvantage of this breadth-first simulation is that the program researches top parts of the search space each time that the depth limit is increased. On the other hand, the important advantage as compared with genuine breadth-first search is space economy.

Exercises

13.1 Complete the depth-limited, depth-first AND/OR search program according to the procedure outlined in this section.

13.2 Define in Prolog an AND/OR space for the Tower of Hanoi problem and use this definition with the search procedures of this section.

13.3 Consider some simple two-person, perfect-information game without chance and define its AND/OR representation. Use a depth-first AND/OR search program to find winning strategies in the form of AND/OR trees.

13.4 Best-first AND/OR search

13.4.1 Heuristic estimates and the search algorithm

The basic search procedures of the previous section search AND/OR graphs systematically and exhaustively, without any heuristic guidance. For complex problems such procedures are too inefficient due to the combinatorial complexity of the search space. Heuristic guidance that aims to reduce the complexity by avoiding useless alternatives becomes necessary. The heuristic guidance introduced in this section will be based on numerical heuristic estimates of the difficulty of problems in the AND/OR graph. The program that we shall develop is an implementation of the algorithm known as AO*. It can be viewed as a generalization of the A* best-first search program for the state-space representation of Chapter 12.

Let us begin by introducing an optimization criterion based on the costs of arcs in the AND/OR graph. First, we extend our representation of AND/OR graphs to include arc costs. For example, the AND/OR graph of Figure 13.4 can be represented by the following clauses:

```
a ---> or : [b/1, c/3].
b ---> and : [d/1, e/1].
c ---> and : [f/2, g/1].
e ---> or : [h/6].
f ---> or : [h/2, i/3].

goal( d).    goal( g).    goal( h).
```

We shall define the cost of a solution tree as the sum of all the arc costs in the tree. The optimization objective is to find a minimum-cost solution-tree. For illustration, see Figure 13.4 again.

It is useful to define the *cost of a node* in the AND/OR graph as the cost of the node's optimal solution tree. So defined, the cost of a node corresponds to the difficulty of the node.

We shall now assume that we can estimate the costs of nodes (without knowing their solution trees) in the AND/OR graph with some heuristic function h. Such estimates will be used for guiding the search. Our heuristic search program will begin the search with the start node and, by expanding already visited nodes, gradually grow a search tree. This process will grow a *tree* even in cases where the AND/OR graph itself is not a tree; in such a case the graph unfolds into a tree by duplicating parts of the graph.

The search process will at any time of the search select the 'most promising' candidate solution tree for the next expansion. Now, how is the function h used to estimate how promising a candidate solution tree is? Or, how promising a node (the root of a candidate solution tree) is?

For a node N in the search tree, $H(N)$ will denote its estimated difficulty. For a tip node N of the current search tree, $H(N)$ is simply $h(N)$. On the other hand, for an interior node of the search tree we do not have to use function h directly because we already have some additional information about such a node; that is, we already know its successors. Therefore, as Figure 13.9 shows, for an interior OR node N we approximate its difficulty as:

$$H(N) = \min_i(cost(N, N_i) + H(N_i))$$

where $cost(N, N_i)$ is the cost of the arc from N to N_i. The minimization rule in this

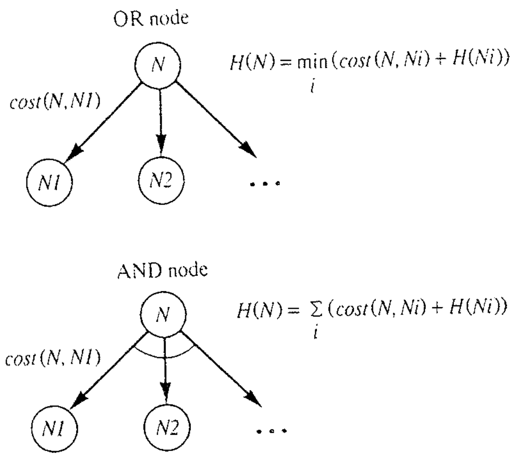

Figure 13.9 Estimating the difficulty, H, of problems in the AND/OR graph.

formula is justified by the fact that, to solve N, we just have to solve one of its successors.

The difficulty of an AND node N is approximated by:

$$H(N) = \sum_i (cost(N, N_i) + H(N_i))$$

We say that the H-value of an interior node is a 'backed-up' estimate.

In our search program, it will be more practical to use (instead of the H-values) another measure, F, defined in terms of H, as follows. Let a node M be the predecessor of N in the search tree, and the cost of the arc from M to N be $cost(M, N)$, then we define:

$$F(N) = cost(M, N) + H(N)$$

Accordingly, if M is the parent node of N, and $N_1, N_2, \ldots$ are N's children, then:

$$F(N) = cost(M, N) + \min_i F(N_i), \quad \text{if } N \text{ is an OR node}$$

$$F(N) = cost(M, N) + \sum_i F(N_i), \quad \text{if } N \text{ is an AND node}$$

The start node S of the search has no predecessor, but let us choose the cost of its (virtual) incoming arc as 0. Now, if h for all goal nodes in the AND/OR graph is 0, and an optimal solution tree has been found, then $F(S)$ is just the cost of this solution tree (that is, the sum of all the costs of its arcs).

At any stage of the search, each successor of an OR node represents an alternative candidate solution subtree. The search process will always decide to continue the exploration at that successor whose F-value is minimal. Let us return to Figure 13.4 again and trace such a search process when searching the AND/OR graph of this figure. Initially, the search tree is just the start node a, and then the tree grows until a solution tree is found. Figure 13.10 shows some snapshots taken during the growth of the search tree. We shall assume for simplicity that $h = 0$ for all the nodes. Numbers attached to nodes in Figure 13.10 are the F-values of the nodes (of course, these change during the search as more information is accumulated). Here are some explanatory remarks to Figure 13.10.

Expanding the initial search tree (snapshot A) produces tree B. Node a is an OR node, so we now have two candidate solution subtrees: b and c. As $F(b) = 1 < 3 = F(c)$, alternative b is selected for expansion. Now, how far can alternative b be expanded? The expansion can proceed until either:

(1) the F-value of node b has become greater than that of its competitor c, or

(2) it has become clear that a solution tree has been found.

So candidate b starts to grow with the upper bound for $F(b)$: $F(b) \leqslant 3 = F(c)$. First, b's successors d and e are generated (snapshot C) and the F-value of b is increased to 3.

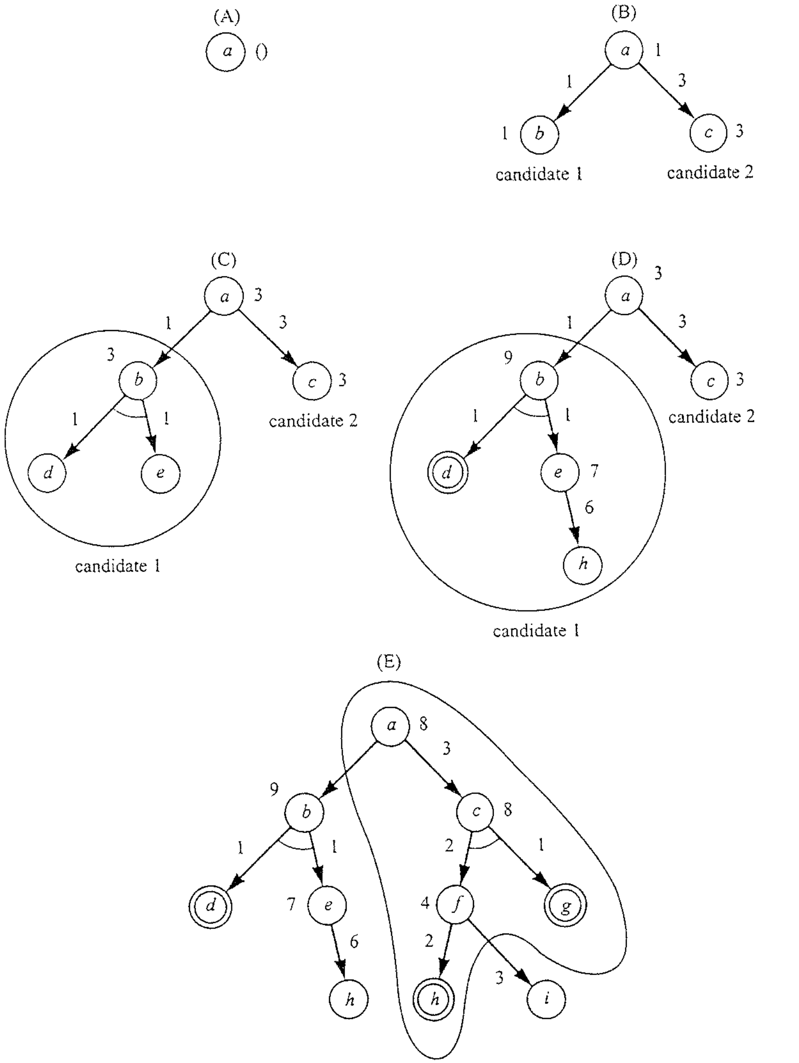

Figure 13.10 A trace of a best-first AND/OR search (using $h = 0$) solving the problem of Figure 13.4.

As this does not exceed the upper bound, the candidate tree rooted in b continues to expand. Node d is found to be a goal node, and then node e is expanded, resulting in snapshot D. At this point $F(b) = 9 > 3$, which stops the expansion of alternative b. This prevents the process from realizing that h is also a goal node and that a solution tree has already been generated. Instead, the activity now switches to the competing alternative c. The bound on $F(c)$ for expanding this alternative is set to 9, since at this point $F(b) = 9$. Within this bound the candidate tree rooted in c is expanded until the situation of snapshot E is reached. Now the process realizes that a solution tree (which includes goal nodes h and g) has been found, and the whole process terminates. Notice that the cheaper of the two possible solution trees was reported a solution by this process – that is, the solution tree in Figure 13.4(c).

13.4.2 Search program

A program that implements the ideas of the previous section is given in Figure 13.12. Before explaining some details of this program, let us consider the representation of the search tree that this program uses.

There are several cases, as shown in Figure 13.11. The different forms of the search tree arise from combining the following possibilities with respect to the tree's size and 'solution status':

- Size:
  (1) the tree is either a single node tree (a leaf), or
  (2) it has a root and (non-empty) subtrees.
- Solution status:
  (1) the tree has already been discovered to be solved (the tree is a solution tree), or
  (2) it is still just a *candidate* solution tree.

The principal functor used to represent the tree indicates a combination of these possibilities. This can be one of the following:

leaf **solvedleaf** **tree** **solvedtree**

Further, the representation comprises some or all of the following information:

- root node of the tree,
- F-value of the tree,
- the cost C of the arc in the AND/OR graph pointing to the tree,
- list of subtrees,
- relation among subtrees (AND or OR).

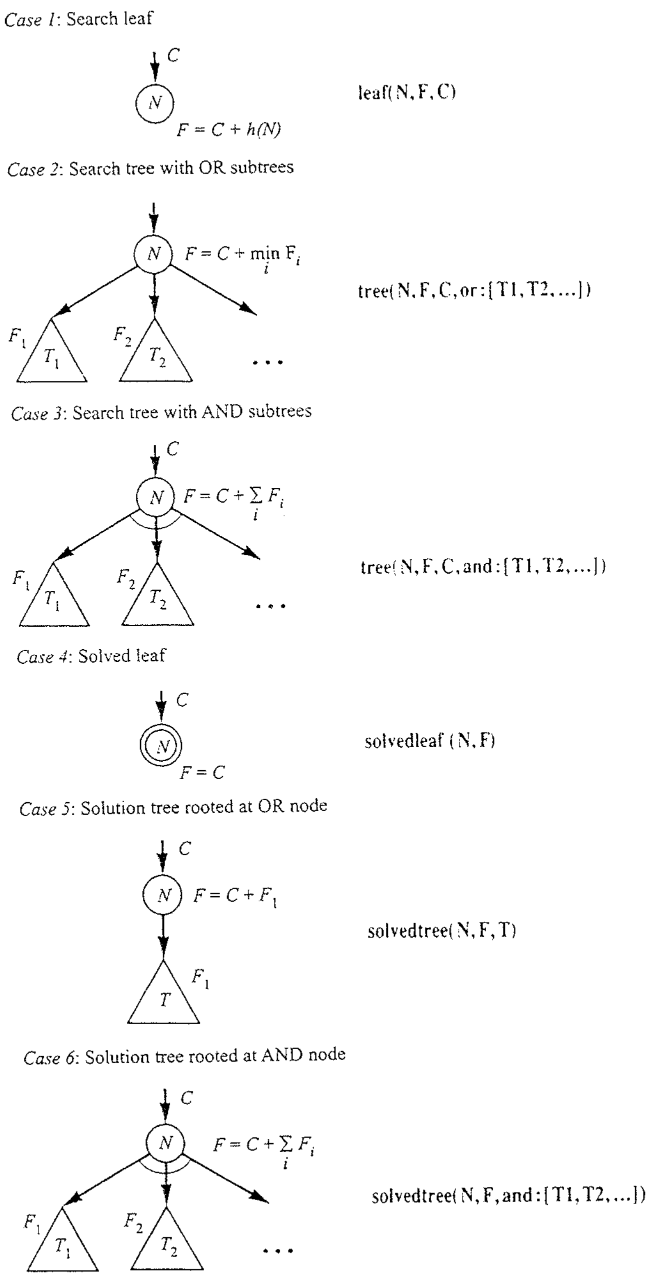

Figure 13.11 Representation of the search tree.

The list of subtrees is always ordered according to increasing F-values. A subtree can already be solved. Such subtrees are accommodated at the end of the list.

Now to the program of Figure 13.12. The top-level relation is

andor(Node, SolutionTree)

where Node is the start node of the search. The program produces a solution tree (if one exists) with the aspiration that this will be an optimal solution. Whether it will really be a cheapest solution depends on the heuristic function h used by the algorithm. There is a theorem that talks about this dependence on h. The theorem is similar to the admissibility theorem about the state-space, best-first search of Chapter 12 (algorithm A*). Let $COST(N)$ denote the cost of a cheapest solution tree of a node N. If for each node N in the AND/OR graph the heuristic estimate $h(N) \leqslant COST(N)$ then andor is guaranteed to find an optimal solution. If h does not satisfy this condition then the solution found may be suboptimal. A trivial heuristic function that satisfies the admissibility condition is $h = 0$ for all the nodes. The disadvantage of this function is, of course, lack of heuristic power.

```
/* BEST-FIRST AND/OR SEARCH

This program only generates one solution. This solution is guaranteed to be a cheapest one if
the heuristic function used is a lower bound of the actual costs of solution trees.

Search tree is either:

tree( Node, F, C, SubTrees)              tree of candidate solutions

leaf( Node, F, C)                        leaf of a search tree

solvedtree( Node, F, SubTrees)           solution tree

solvedleaf( Node, F)                     leaf of solution tree

C is the cost of the arc pointing to Node

F = C + H, where H is the heuristic estimate of an optimal solution subtree rooted in Node

SubTrees are always ordered so that:
(1) all solved subtrees are at the end of a list;
(2) other (unsolved subtrees) are ordered according to ascending F-values.
*/

:- op( 500, xfx, :).

:- op( 600, xfx, --->).

andor( Node, SolutionTree) :-
  expand( leaf( Node, 0, 0), 9999, SolutionTree, yes).     % Assuming 9999 > any F-value
```

Figure 13.12 Best-first AND/OR search program.

Figure 13.12 *contd*

```
% Procedure expand( Tree, Bound, NewTree, Solved)
% expands Tree with Bound producing NewTree whose
% 'solution-status' is Solved

% Case 1: bound exceeded

expand( Tree, Bound, Tree, no) :-
  f( Tree, F), F > Bound, !.                % Bound exceeded

% In all remaining cases F ≦ Bound

% Case 2: goal encountered

expand( leaf( Node, F, C), _, solvedleaf( Node, F), yes) :-
  goal( Node), !.

% Case 3: expanding a leaf

expand( leaf( Node, F, C), Bound, NewTree, Solved) :-
  expandnode( Node, C, Tree1), !,
  expand( Tree1, Bound, NewTree, Solved)
  ;
  Solved = never, !.                        % No successors, dead end

% Case 4: expanding a tree

expand( tree( Node, F, C, SubTrees), Bound, NewTree, Solved) :-
  Bound1 is Bound - C,
  expandlist( SubTrees, Bound1, NewSubs, Solved1),
  continue( Solved1, Node, C, NewSubs, Bound, NewTree, Solved).

% expandlist( Trees, Bound, NewTrees, Solved)
% expands tree list Trees with Bound producing
% NewTrees whose 'solved-status' is Solved

expandlist( Trees, Bound, NewTrees, Solved) :-
  selecttree( Trees, Tree, OtherTrees, Bound, Bound1),
  expand( Tree, Bound1, NewTree, Solved1),
  combine( OtherTrees, NewTree, Solved1, NewTrees, Solved).

% 'continue' decides how to continue after expanding a tree list

continue( yes, Node, C, SubTrees, _, solvedtree( Node, F, SubTrees), yes) :-
  backup( SubTrees, H), F is C + H, !.

continue( never, _, _, _, _, _, never) :- !.

continue( no, Node, C, SubTrees, Bound, NewTree, Solved) :-
  backup( SubTrees, H), F is C + H, !,
  expand( tree( Node, F, C, SubTrees), Bound, NewTree, Solved).

% 'combine' combines results of expanding a tree and a tree list

combine( or : _, Tree, yes, Tree, yes) :- !.        % OR list solved
```



```
combine( or : Trees, Tree, no, or : NewTrees, no) :-
  insert( Tree, Trees, NewTrees), !.                % OR list still unsolved

combine( or : [], _, never, _, never) :- !.        % No more candidates

combine( or : Trees, _, never, or : Trees, no) :- !.   % There are more candidates

combine( and : Trees, Tree, yes, and : [Tree | Trees], yes) :-
  allsolved( Trees), !.                             % AND list solved

combine( and : _, _, never, _, never) :- !.         % AND list unsolvable

combine( and : Trees, Tree, YesNo, and : NewTrees, no) :-
  insert( Tree, Trees, NewTrees), !.                % AND list still unsolved

% 'expandnode' makes a tree of a node and its successors

expandnode( Node, C, tree( Node, F, C, Op : SubTrees) ) :-
  Node ---> Op : Successors,
  evaluate( Successors, SubTrees),
  backup( Op : SubTrees, H), F is C + H.

evaluate( [], [] ).

evaluate( [Node/C | NodesCosts], Trees) :-
  h( Node, H), F is C + H,
  evaluate( NodesCosts, Trees1),
  insert( leaf( Node, F, C), Trees1, Trees).

% 'allsolved' checks whether all trees in a tree list are solved

allsolved( [] ).

allsolved( [Tree | Trees] ) :-
  solved( Tree),
  allsolved( Trees).

solved( solvedtree( _, _, _) ).

solved( solvedleaf( _, _) ).

f( Tree, F) :-                                      % Extract F-value of a tree
  arg( 2, Tree, F), !.                              % F is the 2nd argument in Tree

% insert( Tree, Trees, NewTrees) inserts Tree into
% tree list Trees producing NewTrees

insert( T, [], [T] ) :- !.

insert( T, [T1 | Ts], [T, T1 | Ts] ) :-
  solved( T1), !.

insert( T, [T1 | Ts], [T1 | Ts1] ) :-
  solved( T),
  insert( T, Ts, Ts1), !.

insert( T, [T1 | Ts], [T, T1 | Ts ] ) :-
  f( T, F), f( T1, F1), F =< F1, !.

insert( T, [T1 | Ts], [T1 | Ts1] ) :-
  insert( T, Ts, Ts1).
```

Figure 13.12 *contd*

```
% 'backup' finds the backed-up F-value of AND/OR tree list

backup( or : [Tree | _], F) :-                    % First tree in OR list is best
  f( Tree, F), !.

backup( and : [ ], 0) :- !.

backup( and : [Tree1 | Trees], F) :-
  f( Tree1, F1),
  backup( and : Trees, F2),
  F is F1 + F2, !.

backup( Tree, F) :-
  f( Tree, F).

% Relation selecttree( Trees, BestTree, OtherTrees, Bound, Bound1):
% OtherTrees is an AND/OR list Trees without its best member
% BestTree; Bound is expansion bound for Trees, Bound1 is
% expansion bound for BestTree

selecttree( Op : [Tree], Tree, Op : [ ], Bound, Bound) :- !.    % The only candidate

selecttree( Op : [Tree | Trees], Tree, Op : Trees, Bound, Bound1) :-
  backup( Op : Trees, F),
  ( Op = or, !, min( Bound, F, Bound1);
    Op = and, Bound1 is Bound - F).

min( A, B, A) :- A < B, !.

min( A, B, B).
```

The key relation in the program of Figure 13.12 is:

expand(Tree, Bound, Tree1, Solved)

Tree and Bound are 'input' arguments, and Tree1 and Solved are 'output' arguments. Their meaning is:

Tree	is a search tree that is to be expanded.
Bound	is a limit for the F-value within which Tree is allowed to expand.
Solved	is an indicator whose value indicates one of the following three cases:

(1) Solved = yes: Tree can be expanded within bound so as to comprise a solution tree Tree1;
(2) Solved = no: Tree can be expanded to Tree1 so that the F-value of Tree1 exceeds Bound, and there was no solution subtree before the F-value overstepped Bound;
(3) Solved = never: Tree is unsolvable.

Tree1 is, depending on the cases above, either a solution tree, an extension of Tree just beyond Bound, or uninstantiated in the case Solved = never.

Procedure

expandlist(Trees, Bound, Trees1, Solved)

is similar to expand. As in expand, Bound is a limit of the expansion of a tree, and Solved is an indicator of what happened during the expansion ('yes', 'no' or 'never'). The first argument is, however, a *list* of trees (an AND list or an OR list):

Trees = or : [T1, T2, ...]   or   Trees = and : [T1, T2, ...]

expandlist selects the most promising tree T (according to F-values) in Trees. Due to the ordering of the subtrees this is always the first tree in Trees. This most promising subtree is expanded with a new bound Bound1. Bound1 depends on Bound and also on the other trees in Trees. If Trees is an OR list then Bound1 is the lower of Bound and the F-value of the next best tree in Trees. If Trees is an AND list then Bound1 is Bound minus the sum of the F-values of the remaining trees in Trees. Trees1 depends on the case indicated by Solved. In the case Solved = no, Trees1 is the list Trees with the most promising tree in Trees expanded with Bound1. In the case Solved = yes, Trees1 is a solution of the list Trees (found within Bound). If Solved = never, Trees1 is uninstantiated.

The procedure continue, which is called after expanding a tree list, decides what to do next, depending on the results of expandlist. It either constructs a solution tree, or updates the search tree and continues its expansion, or signals 'never' in the case that the tree list was found unsolvable.

Another procedure,

combine(OtherTrees, NewTree, Solved1, NewTrees, Solved)

relates several objects dealt with in expandlist. NewTree is the expanded tree in the tree list of expandlist, OtherTrees are the remaining, unchanged trees in the tree list, and Solved1 indicates the 'solution-status' of NewTree. combine handles several cases, depending on Solved1 and on whether the tree list is an AND list or an OR list. For example, the clause

combine(or : _, Tree, yes, Tree, yes).

says: in the case that the tree list is an OR list, and the just expanded tree was solved, and its solution tree is Tree, then the whole list has also been solved, and its solution is Tree itself. Other cases are best understood from the code of combine itself.

For displaying a solution tree, a procedure similar to show of Figure 13.8 can be defined. This procedure is left as an exercise for the reader.

13.4.3 Example of problem-defining relations: route finding

Let us now formulate the route-finding problem as an AND/OR search so that this formulation can be directly used by our andor procedure of Figure 13.12. We shall assume that the road map is represented by a relation

```
s( City1, City2, D)
```

meaning that there is a direct connection between City1 and City2 of distance D. Further, we shall assume there is a relation

```
key( City1 - City2, City3)
```

meaning: to find a route from City1 to City2 we should consider paths that go through City3 (City3 is a 'key point' between City1 and City2). For example, in the map of Figure 13.1, *f* and *g* are key points between *a* and *z*:

```
key( a-z, f).     key( a-z, g).
```

We shall implement the following principles of route finding:

To find a route between two cities X and Z:

(1) if there are key points Y1, Y2, ... between X and Z then find either
- route from A to Z via Y1, or
- route from A to Z via Y2, or

...

(2) if there is no key point between X and Z then simply find some neighbour city Y of X such that there is a route from Y to Z.

We have, then, two kinds of problems that will be represented as:

(1) X-Z find a route from X to Z

(2) X-Z via Y find a route from X to Z through Y

Here 'via' is an infix operator with precedence higher than that of '-' and lower than that of '--->'. The corresponding AND/OR graph can now be implicitly defined by the following piece of program:

```
:- op( 560, xfx, via).

% Expansion rule for problem X-Z when
% there are key points between X and Z,
% costs of all arcs are equal 0

X-Z ---> or : ProblemList
  :- bagof( ( X-Z via Y)/0, key( X-Z, Y), ProblemList), !.

% Expansion rule for problem X-Z with no key points

X-Z ---> or : ProblemList
  :- bagof( ( Y-Z)/D, s( X, Y, D), ProblemList).

% Reduce a 'via problem' to two AND-related subproblems

X-Z via Y ---> and : [( X-Y)/0, ( Y-Z)/0].

goal( X-X).                        % To go from X to X is trivial
```

The function *h* could be defined, for example, as the air distance between cities.

Exercises

13.4 Let an AND/OR graph be defined by:

```
a ---> or:[b/1,c/2].
b ---> and:[d/1,e/1].
c ---> and:[e/1,f/1].
d ---> or:[h/6].
e ---> or:[h/2].
f ---> or:[h/4,i/2].

goal(h).   goal(i).
```

Draw all the solution trees and calculate their costs. Suppose this graph is searched by the AO* algorithm, where the start node is *a*, and the heuristic *h*-values of the nodes *b*, *d*, *e*, *h* and *i* are all 0. Give the intervals for the values $h(c)$ and $h(f)$ that allow AO* to find the optimal solution.

13.5 Write a procedure

```
show2( SolutionTree)
```

to display a solution tree found by the andor program of Figure 13.12. Let the display format be similar to that of the show procedure of Figure 13.8, so that show2 can be written as a modification of show, using a different tree representation. Another useful modification would be to replace the goal write(Node) in show by a user-defined procedure

```
writenode( Node, H)
```

which outputs Node in some suitable form, and instantiates H to the number of characters that Node takes if output in this form. H is then used for proper indentation of subtrees.

Summary

- AND/OR graph is a formalism for representing problems. It naturally suits problems that are decomposable into independent subproblems. Game playing is an example of such problems.

- Nodes of an AND/OR graph are of two types: AND nodes and OR nodes.
- A concrete problem is defined by a start node and a goal condition. A solution of a problem is represented by a solution graph.
- Costs of arcs and nodes can be introduced into an AND/OR graph to model optimization problems.
- Solving a problem, represented by an AND/OR graph, involves searching the graph. The depth-first strategy searches the graph systematically and is easy to program. However, it may suffer from inefficiency due to combinatorial explosion.
- Heuristic estimates can be introduced to estimate the difficulty of problems, and the best-first heuristic principle can be used to guide the search. Implementing this strategy is more difficult.
- Prolog programs for depth-first search, depth-first iterative deepening and best-first search of AND/OR graphs were developed in this chapter.
- Concepts introduced in this chapter are:

    AND/OR graphs
    AND arcs, OR arcs
    AND nodes, OR nodes
    solution graph, solution tree
    arc costs, node costs
    heuristic estimates in AND/OR graphs, backed-up estimates
    depth-first search in AND/OR graphs
    iterative deepening in AND/OR graphs
    best-first search in AND/OR graphs

References

AND/OR graphs and related search algorithms are part of the classical artificial intelligence problem-solving and game-playing machinery. An early example of their application is a symbolic integration program (Slagle 1963). Prolog itself does AND/OR search. A general treatment of AND/OR graphs and the best-first AND/OR search algorithm can be found in general books on artificial intelligence (Nilsson 1971; Nilsson 1980). Our best-first AND/OR program is a variation of an algorithm known as AO*. Formal properties of AO* (including its admissibility) have been studied by several authors; Pearl (1984) gives a comprehensive account of these results.

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Pearl, J. (1984) *Heuristics: Intelligent Search Strategies for Computer Problem Solving*. Addison-Wesley.
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chapter 14

Constraint Logic Programming

Constraint programming is a useful paradigm for formulating and solving problems that can be naturally defined in terms of constraints among a set of variables. Solving such problems consists of finding such combinations of values of the variables that satisfy the constraints. This is called *constraint satisfaction*. *Constraint logic programming* (CLP) combines the constraint approach with logic programming. In CLP, constraint satisfaction is embedded into a logic programming language, such as Prolog. In this chapter we introduce CLP and look at some example applications, including scheduling and simulation.

14.1 Constraint satisfaction and logic programming

14.1.1 Constraint satisfaction

A constraint satisfaction problem is stated as follows:

Given:
(1) a set of *variables*,
(2) the *domains* from which the variables can take values, and
(3) *constraints* that the variables have to satisfy.

Find:
An assignment of values to the variables, so that these values satisfy all the given constraints.

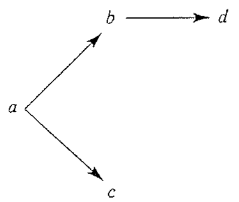


Figure 14.1 Precedence constraints among tasks *a*, *b*, *c*, *d*.

Often there are several assignments that satisfy the constraints. In optimization problems, we can specify a criterion to choose among the assignments that satisfy the constraints.

The constraint satisfaction approach, and in particular its combination with logic programming, has proved to be a very successful tool for a large variety of problems. Typical examples come from scheduling, logistics, and resource management in production, transportation and placement. These problems involve assigning resources to activities, like machines to jobs, people to rosters, crew to trains or planes, doctors and nurses to duties and wards, etc.

Let us look at a typical example from scheduling. Let there be four tasks *a*, *b*, *c* and *d* whose durations are 2, 3, 5 and 4 hours respectively. Let there be these precedence constraints among the tasks: task *a* has to precede tasks *b* and *c*, and *b* has to precede *d* (Figure 14.1). The problem is to find the start times Ta, Tb, Tc and Td of the corresponding tasks so that the finishing time Tf of the schedule is minimal. Assume the earliest start time is 0.

The corresponding constraint satisfaction problem can be formally stated as follows:

Variables: Ta, Tb, Tc, Td, Tf
Domains: All the variables are non-negative real numbers
Constraints:

$Ta + 2 \leqslant Tb$	(task *a* which takes 2 hours precedes *b*)
$Ta + 2 \leqslant Tc$	(*a* precedes *c*)
$Tb + 3 \leqslant Td$	(*b* precedes *d*)
$Tc + 5 \leqslant Tf$	(*c* finished by Tf)
$Td + 4 \leqslant Tf$	(*d* finished by Tf)

Criterion: minimize Tf

This constraint satisfaction problem has a set of solutions which all minimize the finishing time. This set of solutions can be stated as:

$Ta = 0$
$Tb = 2$
$2 \leqslant Tc \leqslant 4$
$Td = 5$
$Tf = 9$

All the starting times are determined except for task *c*, which may start at any time in the interval between 2 and 4.

14.1.2 Satisfying constraints

Constraint satisfaction problems are often depicted as graphs, called *constraint networks*. The nodes in the graph correspond to the variables, and the arcs correspond to the constraints. For each binary constraint $p(X,Y)$ between the variables X and Y, there are two directed arcs (X,Y) and (Y,X) in the graph. To find a solution of a constraint satisfaction problem, various *consistency algorithms* can be used. These algorithms are best viewed as operating over constraint networks. They check the consistency of the domains of variables with respect to constraints. We show the main ideas of such algorithms below. It should be noted that we only present a consistency technique operating on binary constraints, although constraints can in general be of any arity.

Consider variables X and Y whose domains are Dx and Dy respectively. Let there be a binary constraint $p(X,Y)$ between X and Y. The arc (X,Y) is said to be *arc consistent* if for each value of X in Dx, there is some value for Y in Dy satisfying the constraint $p(X,Y)$. If (X,Y) is not arc consistent, then all the values in Dx for which there is no corresponding value in Dy may be deleted from Dx. This makes (X,Y) arc consistent.

For example, consider the variables X and Y whose domains are sets of all integers between 0 and 10 inclusive, written as:

$$Dx = 0\,..\,10, \quad Dy = 0\,..\,10$$

Let there be the constraint $p(X,Y)$: $X + 4 \leqslant Y$. Now, the arc (X,Y) is not arc consistent. For example, for $X = 7$ there is no value of Y in Dy satisfying $p(7,Y)$. To make the arc (X,Y) consistent, the domain Dx is reduced to $Dx = 0\,..\,6$. Similarly, the arc (Y,X) can be made consistent by reducing Dy: $Dy = 4\,..\,10$. By such a reduction of Dx and Dy we do not possibly lose any solution of the constraint problem because the discarded values clearly cannot be part of any solution.

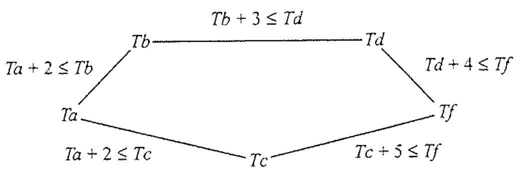
After reducing domain Dx, some other arc may become inconsistent. For an arc of the form (Z,X) there may be values of Z for which, after reducing Dx, there may no longer be any corresponding value in Dx. Then in turn the arc (Z,X) can be made consistent by correspondingly reducing the domain Dz. So this effect may propagate through the network, possibly cyclically, for some time until either all the arcs become consistent, or some domain becomes empty. In the latter case the constraints are clearly not satisfiable. In the case where all the arcs are consistent, there are again two cases:

(1) Each domain has a single value; this means we have a single solution to the constraint problem.

(2) All the domains are non-empty, and at least one domain has multiple values.

For case 2, the arc consistency, of course, does not guarantee that all possible combinations of domain values are solutions to the constraint problem. It may even be that no combination of values actually satisfies all the constraints. Therefore some combinatorial search is needed over the reduced domains to find a solution. One possibility is to choose one of the multi-valued domains and try to assign repeatedly its values to the corresponding variable. Assigning a particular value to the variable means reducing the variable's domain, possibly causing inconsistent arcs to appear again. So the consistency algorithm can be applied again to further reduce the domains of variables, etc. If the domains are finite, this will eventually result in either an empty domain, or all the domains single-valued. The search can be done differently, not necessarily by choosing a single value from a domain. An alternative policy may be to choose a non-singleton domain and split it into two approximately equal size subsets. The algorithm is then applied to both subsets.

For illustration let us consider how this algorithm may work on our scheduling example. Let the domains of all the variables be integers between 0 and 10. Figure 14.2 shows the constraint network and the trace of a constraint satisfaction algorithm. Initially, at step 'Start', all the domains are 0..10. In each execution step, one of the arcs in the network is made consistent. In step 1, arc (*Tb*,*Ta*) is considered, which reduces the domain of *Tb* to 2..10. Next, arc (*Td*,*Tb*) is considered, which reduces the domain of *Td* to 5..10, etc. After step 8 all the arcs are consistent and all the reduced domains are multi-valued. Being interested in the minimal finishing time, assigning $Tf = 9$ may now be tried. Arc consistency is then



Step	*Arc*	*Ta*	*Tb*	*Tc*	*Td*	*Tf*
Start		0..10	0..10	0..10	0..10	0..10
1	(*Tb*,*Ta*)		2..10			
2	(*Td*,*Tb*)				5..10	
3	(*Tf*,*Td*)					9..10
4	(*Td*,*Tf*)				5..6	
5	(*Tb*,*Td*)		2..3			
6	(*Ta*,*Tb*)	0..1				
7	(*Tc*,*Ta*)			2..10		
8	(*Tc*,*Tf*)			2..5		

Figure 14.2 Top: constraint network for the scheduling problem. Bottom: an arc consistency execution trace.

executed again, reducing all the domains to singletons except the domain of *Tc* to 2..4.

Notice how consistency techniques exploit the constraints to reduce the domains of variables as soon as new information is available. New information triggers the related constraints, which results in reduced domains of the concerned variables. Such execution can be viewed as data-driven. Constraints are active in the sense that they do not wait to be explicitly called by the programmer, but activate automatically when relevant information appears. This idea of data-driven computation is further discussed in Chapter 23 under 'Pattern-directed programming'.

Exercises

14.1 Try to execute the arc consistency algorithm with different orders of the arcs. What happens?

14.2 Execute the arc consistency algorithm on the final state of the trace in Figure 14.2, after *Tf* is assigned value 9.

14.1.3 Extending Prolog to constraint logic programming

Let us now consider the relation between Prolog and constraint satisfaction. Pure Prolog itself can be viewed as a rather specific constraint satisfaction language where all the constraints are of very limited form. They are just equalities between terms. These equality constraints are checked by Prolog's matching of terms. Although constraints among the arguments of a predicate are also stated in terms of other predicates, these predicate calls eventually unfold to matching. Prolog can be extended to a 'real' CLP language by introducing other types of constraints in addition to matching. Of course, the Prolog interpreter has to be enhanced so that it can handle these other types of constraints. A CLP system that can handle arithmetic equality and inequality constraints can directly solve our example scheduling problem as stated above.

A program with constraints is interpreted roughly as follows. During the execution of a list of goals, a set CurrConstr of current constraints is maintained. Initially this set is empty. The goals in the list of goals are executed one by one in the usual order. Normal Prolog goals are processed as usual. When a goal with constraints Constr is processed, the constraint sets Constr and CurrConstr are merged, giving NewConstr. The domain-specialized constraint solver then tries to satisfy NewConstr. Two basic outcomes are possible: (a) NewConstr is found to be unsatisfiable, which corresponds to the goal's failure and causes backtracking; (b) NewConstr is not found to be unsatisfiable, and constraints in NewConstr are simplified as much as possible by the constraint solver. For example, two constraints $X \leqslant 3$ and $X \leqslant 2$

are simplified into the single constraint $X \leqslant 2$. The extent of simplification depends on the current state of the information about the variables, as well as on the abilities of the particular constraint solver. The remaining goals in the list are executed with the so updated set of current constraints.

CLP systems differ in the domains and types of constraints they can process. Families of CLP techniques appear under names of the form CLP(X) where X stands for the domain. For example, in CLP(R) the domains of the variables are real numbers, and constraints are arithmetic equalities, inequalities and disequalities over real numbers. CLP(X) systems over other domains include: CLP(Z) (integers), CLP(Q) (rational numbers), CLP(B) (Boolean domains), and CLP(FD) (user-defined finite domains). Available domains and types of constraints in actual implementations largely depend on the available techniques for solving particular types of constraints. In CLP(R), for example, linear equalities and inequalities are typically available because efficient techniques exist for handling these types of constraints. On the other hand, the use of non-linear constraints is very limited.

In the remainder of this chapter we will look in more detail at CLP(R), CLP(Q) and CLP(FD) using the syntactic conventions for CLP in SICStus Prolog (see reference at the end of the chapter).

14.2 CLP over real numbers: CLP(R)

Consider the query:

```
?- 1 + X = 5.
```

In Prolog this matching fails, so Prolog's answer is 'no'. However, if the user's intention is that X is a number and '+' is arithmetic addition, then the answer X = 4 would be more appropriate. Using the built-in predicate is instead of '=' does not quite achieve this interpretation, but a CLP(R) system does. In our syntactic convention, this CLP(R) query will be written as:

```
?- { 1 + X = 5 }.          % Numerical constraint
X = 4
```

The constraint is handled by a specialized constraint solver that 'understands' operations on real numbers, and can typically solve sets of equations or inequations of certain types. According to our syntactic convention, a set of constraints is inserted into a Prolog clause as a goal enclosed in curly brackets. Individual constraints are separated by commas and semicolons. As in Prolog, a comma means conjunction, and a semicolon means disjunction. So the conjunction of constraints C1, C2 and C3 is written as:

```
{ C1, C2, C3}
```


Each constraint is of the form:

```
Expr1  Operator  Expr2
```

Both Expr1 and Expr2 are usual arithmetic expressions. They may, depending on the particular CLP(R) system, also include calls to some standard functions, such as sin(X). The operator can be one of the following, depending on the type of constraint:

=	for equations
=\=	for disequations
<, =<, >, >=	for inequations

Let us now look at some simple examples of using these constraints, and study in particular the flexibility they offer in comparison with the usual built-in numerical facilities in Prolog.

To convert temperature from Centigrade into Fahrenheit, we may in Prolog use the built-in predicate 'is'. For example:

```
convert( Centigrade, Fahrenheit) :-
  Centigrade is (Fahrenheit - 32)*5/9.
```

This can be used to convert 35 degrees Centigrade into Fahrenheit, but not to convert from Fahrenheit to Centigrade because the built-in predicate is expects everything on the right-hand side instantiated. To make the procedure work in both directions, we can test which of the arguments is instantiated to a number, and then use the conversion formula properly rewritten for each case. All this is much more elegant in CLP(R), where the same formula, interpreted as a numerical constraint, works in both directions.

```
convert( Centigrade, Fahrenheit) :-
  { Centigrade = (Fahrenheit - 32)*5/9 }.

?- convert( 35, F).
F = 95

?- convert( C, 95).
C = 35
```

Our CLP(R) program even works when neither of the two arguments is instantiated:

```
?- convert( C, F).
{ F = 32.0 + 1.8*C }
```

As the calculation in this case is not possible, the answer is a formula, meaning: the solution is a set of all F and C that satisfy this formula. Notice that this formula, produced by the CLP system, is a simplification of the constraint in our convert program.

A typical constraint solver can handle sets of linear equations, inequations and disequations. Here are some examples:

```
?- { 3*X - 2*Y = 6, 2*Y = X}.
X = 3.0
Y = 1.5

?- { Z =< X-2, Z =< 6-X, Z+1 = 2}.
Z = 1.0
{X >= 3.0}
{X =< 5.0}
```

A CLP(R) solver also includes a linear optimization facility. This finds the extreme value of a given linear expression inside the region that satisfies the given linear constraints. The built-in CLP(R) predicates for this are:

```
minimize( Expr)
maximize( Expr)
```

In these two predicates, Expr is a linear expression in terms of variables that appear in linear constraints. The predicates find the variable values that satisfy these constraints and respectively minimize or maximize the value of the expression. For example:

```
?- { X =< 5}, maximize(X).
X = 5.0

?- { X =< 5, 2 =< X}, minimize( 2*X + 3).
X = 2.0

?- { X >=2, Y >=2, Y =< X+1, 2*Y =< 8-X, Z = 2*X + 3*Y}, maximize(Z).
X = 4.0
Y = 2.0
Z = 14.0

?- { X =< 5}, minimize( X).
no
```

In the last example, X was not bounded downwards, therefore the minimization goal failed.

The following CLP(R) predicates find the supremum (least upper bound) or infimum (greatest lower bound) of an expression:

```
sup( Expr, MaxVal)
inf( Expr, MinVal)
```

Expr is a linear expression in terms of linearly constrained variables. MaxVal and MinVal are the maximum and the minimum values that this expression takes within the region where the constraints are satisfied. Unlike with maximize/1 and minimize/1, the variables in Expr do not get instantiated to the extreme points. For example:


```
?- { 2 =< X, X =< 5}, inf( X, Min), sup( X, Max).
Max = 5.0
Min = 2.0
{X >= 2.0}
{X =< 5.0}

?- {X >=2, Y >=2, Y =< X+1, 2*Y =< 8-X, Z = 2*X + 3*Y},
   sup(Z,Max), inf(Z,Min), maximize(Z).
X = 4.0
Y = 2.0
Z = 14.0
Max = 14.0
Min = 10.0
```

Our initial simple scheduling example with four tasks *a*, *b*, *c* and *d* can be stated in CLP(R) in a straightforward way:

```
?- { Ta + 2 =< Tb,          % a precedes b
     Ta + 2 =< Tc,          % a precedes c
     Tb + 3 =< Td,          % b precedes d
     Tc + 5 =< Tf,          % c finished by finishing time Tf
     Td + 4 =< Tf},         % d finished by Tf
     minimize( Tf).
Ta = 0.0
Tb = 2.0
Td = 5.0
Tf = 9.0
{Tc =< 4.0}
{Tc >= 2.0}
```

The next example further illustrates the flexibility of constraints in comparison with Prolog standard arithmetic facilities. Consider the predicate fib(N,F) for computing F as the Nth Fibonacci number: F(0)=1, F(1)=1, F(2)=2, F(3)=3, F(4)=5, etc. In general, for N>1, F(N)=F(N−1)+F(N−2). Here is a Prolog definition of fib/2:

```
fib( N, F) :-
  N=0, F=1
  ;
  N=1, F=1
  ;
  N>1,
  N1 is N-1, fib(N1,F1),
  N2 is N-2, fib(N2,F2),
  F is F1 + F2.
```

An intended use of this program is:

```
?- fib( 6,F).
F=13
```

Consider, however, a question in the opposite direction:

```
?- fib( N, 13).
```

This produces an error because the goal N > 1 is executed with N uninstantiated. However, this program rewritten in CLP(R) can be used more flexibly:

```
fib( N, F) :-
  { N = 0, F = 1}
  ;
  { N = 1, F = 1}
  ;
  { N > 1, F = F1+F2, N1 = N-1, N2 = N-2} ,
  fib( N1, F1),
  fib( N2, F2).
```

This can be executed in the opposite direction: given a Fibonacci number F, what is its index N?

```
?- fib( N, 13).
N = 6
```

However, this program still gets into trouble when asked an unsatisfiable question:

```
?- fib( N, 4).
```

The program keeps trying to find two Fibonacci numbers F1 and F2 such that F1 + F2 = 4. It keeps generating larger and larger solutions for F1 and F2, all the time hoping that eventually their sum will be equal 4. It does not realize that once their sum has exceeded 4, it will only be increasing and so can never become equal to 4. Finally this hopeless search ends in a stack overflow. It is instructive to see how this can be fixed by adding some obvious extra constraints to the **fib** procedure. It is easy to realize that for all N: F(N) $\geqslant$ N. Therefore the variables N1, F1, N2 and F2 in our program must always satisfy the constraints: F1 >= N1, F2 >= N2. These extra constraints can be added to the constraints in the third disjunct of the body of the fib clause. This then becomes:

```
fib( N, F) :-
  ...
  ;
  { N > 1, F = F1+F2, N1 = N-1, N2 = N-2,
   F1 >= N1, F2 >= N2},                % Extra constraints
  fib( N1, F1),
  fib( N2, F2).
```

The extra constraints enable the program to realize that the above question fails:

```
?- fib( N, 4).
no
```

The recursive calls of fib in effect expand the expression for F in the condition F = 4:


```
4 = F = F1 + F2 =
F1' + F2' + F2 =
F1" + F2" + F2' + F2
```

Each time this expression is expanded, new constraints are added to the previous constraints. At the time that the four-term sum expression is obtained, the constraint solver finds out that the accumulated constraints are a contradiction that can never be satisfied.

Let us briefly mention CLP(Q), a close relative to CLP(R). The difference is that in CLP(R) real numbers are approximated by floating point numbers, whereas the domain Q is rational numbers, that is quotients between two integers. They can be used as another approximation to real numbers. The domain Q may have the advantage over the domain R (represented as floating point numbers) in that the solutions of some arithmetic constraints can be stated *exactly* as quotients of integers, whereas floating point numbers are only approximations. Here is an example:

```
?- { X = 2*Y, Y = 1-X }.
```

A CLP(Q) solver gives: X = 2/3, Y = 1/3

A CLP(R) solver gives something like: X = 0.666666666, Y = 0.333333333

Exercise

14.3 Demonstrate how in the example query above '?- fib(N,4).' the constraint solver may realize that the accumulated constraints are not satisfiable.

14.3 Scheduling with CLP

Scheduling problems considered in this section comprise the following elements:

- A set of tasks $T_1, \ldots, T_n$.
- Durations $D_1, \ldots, D_n$ of the tasks.
- Precedence constraints given as the relation

  prec(T_i, T_j)

  which says that the task T_i has to be completed by the time T_j can start.
- A set of m processors that are available for executing the tasks.
- Resource constraints: which tasks may be executed by which processors (which processors are suitable for a particular task).

The problem is to find a schedule whose finishing time is minimal. A schedule assigns a processor to each task, and states the starting time for each task. Of course, a schedule has to satisfy all the precedence and resource constraints: each task has to be executed by a suitable processor, and no processor can execute two tasks at the same time. The variables in the corresponding CLP formulation of the scheduling problem are: start times S1, ..., Sn, and names P1, ..., Pn of the processors assigned to the each task.

An easy special case of this scheduling problem is when there is no constraint on resources. In this case resources are assumed unlimited, so there is a free processor always available to do any task at any time. Therefore, in this case only the constraints corresponding to the precedences among the tasks have to be satisfied. As already shown in the introductory section to this chapter, these can be stated in a straightforward way. Suppose we have a precedence constraint regarding tasks a and b: prec(a,b). Let the duration of task a be Da, and the starting times of a and b be Sa and Sb. To satisfy the precedence constraint, Sa and Sb have to satisfy the constraint:

```
{ Sa + Da =< Sb }
```

In addition we require that no task Ti may start before time 0, and all the tasks must be completed by the finishing time FinTime of the schedule:

```
{ Si >= 0, Si + Di =< FinTime }
```

We will specify a particular scheduling task by the following predicates:

```
tasks( [Task1/Duration1, Task2/Duration2, ...])
```

This gives the list of all the task names and their durations.

```
prec( Task1, Task2)
```

This specifies the precedence between tasks Task1 and Task2.

```
resource( Task, [ Proc1, Proc2, ...])
```

This states that Task can be carried out by any of the processors Proc1, Proc2, ... Notice that this is a similar scheduling problem specification as used in Chapter 12 (Section 12.3), where a best-first heuristic search was applied to find a best schedule. The formulation here is in fact a little more general. In Chapter 12 resource constraints only consisted in the limited number of processors, but they were all suitable for any task.

We will first consider the simple case with no resource constraints. One way of solving this kind of scheduling problem was shown in Section 14.1. The program in Figure 14.3 is a more general realization of the same idea. This program assumes a definition of a concrete scheduling problem using the representation described above. The main predicate is:

```
schedule( Schedule, FinTime)
```



```
% Scheduling with CLP with unlimited resources

schedule( Schedule, FinTime) :-
  tasks( TasksDurs),
  precedence_constr( TasksDurs, Schedule, FinTime),   % Construct precedence constraints
  minimize( FinTime).

precedence_constr( [], [], FinTime).

precedence_constr( [T/D | TDs], [T/Start/D | Rest], FinTime) :-
  { Start >= 0,                                        % Earliest start at 0
    Start + D =< FinTime},                             % Must finish by FinTime
  precedence_constr( TDs, Rest, FinTime),
  prec_constr( T/Start/D, Rest).

prec_constr( _, []).

prec_constr( T/S/D, [T1/S1/D1 | Rest]) :-
  ( prec( T, T1), !, { S+D =< S1}
   ;
   prec( T1, T), !, { S1+D1 =< S}
   ;
   true ),
  prec_constr( T/S/D, Rest).

% List of tasks to be scheduled

tasks( [ t1/5, t2/7, t3/10, t4/2, t5/9]).

% Precedence constraints

prec( t1, t2).   prec( t1, t4).   prec( t2, t3).   prec( t4, t5).
```

Figure 14.3 Scheduling with precedence constraints and no resource constraints.

where Schedule is a best schedule for the problem specified by the predicates tasks and prec. FinTime is the finishing time of this schedule. The representation of a schedule is:

```
Schedule = [ Task1/Start1/Duration1, Task2/Start2/Duration2, ...]
```

The procedure schedule basically does the following:

(1) Construct the inequality constraints between the starting times in the schedule, corresponding to the precedences among the tasks.

(2) Minimize the finishing time within the constructed inequality constraints.

As all the constraints are linear inequalities, this amounts to a linear optimization problem – a built-in facility in CLP(R).

The predicate

```
precedence_constr( TasksDurations, Schedule, FinTime)
```

constructs the constraints among the start times of the tasks in Schedule, and the schedule's finishing time FinTime. The predicate

```
prec_constr( Task/Start/Duration, RestOfSchedule)
```

constructs the constraints between the start time Start of Task and the start times in RestOfSchedule, so that these constraints on start times correspond to the precedence constraints among the tasks.

The program in Figure 14.3 also comprises the definition of a simple scheduling problem with five tasks. The scheduler is executed by the question:

```
?- schedule( Schedule, FinTime).
FinTime = 22,
Schedule = [t1/0/5,t2/5/7,t3/12/10,t4/S4/2,t5/S5/9],
{S5 =< 13}
{S4 >= 5}
{S4-S5 =< -2}
```

For tasks t4 and t5 there is some freedom regarding their start times. All the start times S4 and S5 for t4 and t5 within the indicated intervals achieve optimal finishing time of the schedule. The other three tasks (t1, t2 and t3) are on the *critical path* and their starting times cannot be moved.

Now let us consider the more difficult type of scheduling problems, those with resource constraints. For example, in the scheduling problem in Chapter 12 (Section 12.3, Figure 12.8), there are three processors altogether. Although any of the processors can execute any of the tasks, this constraint means that three tasks at the most may be processed at the same time.

This time we have to deal with two types of constraints:

(1) Precedences between tasks.

(2) Resource constraints.

The precedence constraints can be handled in the same way as in the program of Figure 14.3. Now consider the resource constraints. Handling resource constraints involves assigning some processor to each task. This can be done by introducing, for each task Ti, another variable Pi whose possible values are the names of the processors. Correspondingly we will slightly extend the representation of a schedule to:

```
Schedule = [ Task1/Proc1/Start1/Dur1, Task2/Proc2/Start2/Dur2, ... ]
```

where Proc1 is the processor assigned to Task1, Start1 is the starting time of Task1 and Dur1 is its duration. Using this representation of a schedule, we will now develop the program in Figure 14.4 as an extension of the program of Figure 14.3. Our main predicate is again:

```
schedule( BestSchedule, BestTime)
```



```
% Scheduling with CLP with limited resources

schedule( BestSchedule, BestTime) :-
  tasks( TasksDurs),
  precedence_constr( TasksDurs, Schedule, FinTime),   % Set up precedence inequalities
  initialize_bound,                                   % Initialize bound on finishing time
  assign_processors( Schedule, FinTime),              % Assign processors to tasks
  minimize( FinTime),
  update_bound( Schedule, FinTime),
  fail                                                % Backtrack to find more schedules
  ;
  bestsofar( BestSchedule, BestTime).                 % Final best

% precedence_constr( TasksDurs, Schedule, FinTime):
%   For given tasks and their durations, construct a structure Schedule
%   comprising start time variables. These variables and finishing time FinTime
%   are constrained by inequalities due to precedences.

precedence_constr( [ ], [ ], FinTime).

precedence_constr( [T/D | TDs], [T/Proc/Start/D | Rest], FinTime) :-
  { Start >= 0,                                       % Earliest start at 0
    Start + D =< FinTime },                           % Must finish by FinTime
  precedence_constr( TDs, Rest, FinTime),
  prec_constr( T/Proc/Start/D, Rest).

prec_constr( _, [ ]).

prec_constr( T/P/S/D, [T1/P1/S1/D1 | Rest]) :-
  ( prec( T, T1), !, { S+D =< S1}
    ;
    prec( T1, T), !, { S1+D1 =< S}
    ;
    true ),
  prec_constr( T/P/S/D, Rest).

% assign_processors( Schedule, FinTime):
%   Assign processors to tasks in Schedule

assign_processors( [ ], FinTime).

assign_processors( [T/P/S/D | Rest], FinTime) :-
  assign_processors( Rest, FinTime),
  resource( T, Processors),                           % T can be executed by any of Processors
  member( P, Processors),                             % Choose one of suitable processors
  resource_constr( T/P/S/D, Rest),                    % Impose resource constraints
  bestsofar( _, BestTimeSoFar),
  { FinTime < BestTimeSoFar }.                        % New schedule better than best so far
```

Figure 14.4 A CLP(R) scheduling program for problems with precedence and resource constraints.

Figure 14.4 *contd*

```
% resource_constr( ScheduledTask, TaskList):
%    Construct constraints to ensure no resource conflict
%    between ScheduledTask and TaskList

resource_constr( _, [ ]).

resource_constr( Task, [Task1 | Rest]) :-
  no_conflict( Task, Task1),
  resource_constr( Task, Rest).

no_conflict( T/P/S/D, T1/P1/S1/D1) :-
  P \== P1, !                                 % Different processors
  ;
  prec( T, T1), !                             % Already constrained
  ;
  prec( T1, T), !                             % Already constrained
  ;
  { S+D =< S1                                 % Same processor, no time overlap
    ;
    S1+D1 =< S}.

initialize_bound :-
  retract(bestsofar(_,_)), fail
  ;
  assert( bestsofar( dummy_schedule, 9999)).  % Assuming 9999 > any finishing time

% update_bound( Schedule, FinTime):
%    update best schedule and time

update_bound( Schedule, FinTime) :-
  retract( bestsofar( _, _)), !,
  assert( bestsofar( Schedule, FinTime)).

% List of tasks to be scheduled

tasks( [t1/4,t2/2,t3/2, t4/20, t5/20, t6/11, t7/11]).

% Precedence constraints

prec( t1, t4).  prec( t1, t5).  prec( t2, t4).  prec( t2, t5).
prec( t2, t6).  prec( t3, t5).  prec( t3, t6).  prec( t3, t7).

% resource( Task, Processors):
%    Any processor in Processors suitable for Task

resource( _, [1,2,3]).                  % Three processors, all suitable for any task
```

which finds a schedule with the minimum finishing time BestTime. Inequality constraints on start times due to precedences between tasks are again constructed by the predicate

```
precedence_constr( TasksDurations, Schedule, FinTime)
```


This is almost the same as in Figure 14.3. The only slight difference is due to the different representation of a schedule.

Let us now deal with resource constraints. This cannot be done so efficiently as with precedence constraints. To satisfy resource constraints, an optimal assignment of processors to tasks is needed. This requires a search among possible assignments, and there is no general way to do this in polynomial time. In the program of Figure 14.4, this search is done according to the branch-and-bound method, roughly as follows. Alternative schedules are non-deterministically constructed one by one (generate a schedule and fail). The best schedule so far is asserted in the database as a fact. Each time a new schedule is constructed, the best-so-far is updated. When a new schedule is being constructed, the best-so-far finishing time is used as an upper bound on the finishing time of the new schedule. As soon as it is found that a new, partially built schedule cannot possibly better the best-so-far time, it is abandoned.

This is implemented in Figure 14.4 as follows. Procedure assign_processors non-deterministically assigns suitable processors to tasks, one at a time. Assigning a processor to a task results in additional constraints on start times, to ensure that there is no time overlap between tasks assigned to the same processor. So each time a processor is assigned to a task, the partial schedule is refined. Each time a partial schedule is so refined, it is checked whether it has any chance of bettering the best schedule so far. For the current partial schedule to have any such chance, FinTime has to be less than the best time so far. In the program this is done by constraining:

```
{ FinTime < BestTimeSoFar }
```

If this constraint is incompatible with the other current constraints, then this partial schedule has no hope. As more resource constraints will have to be satisfied to complete the schedule, the actual finishing time may eventually only become worse. So if FinTime < BestTimeSoFar is not satisfiable, then the partial schedule is abandoned; otherwise another task is assigned a processor, etc. Whenever a complete schedule is built, its finishing time is guaranteed to be less than the best finishing time found so far. Therefore the best-so-far is updated. Finally, when the best-so-far cannot be bettered, the search stops. Of course, this algorithm only produces one of possibly many best schedules.

It should be noted that this process is combinatorially complex due to the exponential number of possible assignments of processors to tasks. Bounding the current partial schedule by BestTimeSoFar leads to abandoning whole sets of bad schedules before they are completely built. How much computation time is saved by this depends on how good the upper bound is. If the upper bound is tight, bad schedules will be recognized and abandoned at an early stage thus saving more time. So the sooner some good schedule is found, the sooner a tight upper bound is applied and more search space is pruned away.

Figure 14.4 also includes the specification, according to our representation convention, of the scheduling problem of Figure 12.8. The question to schedule this problem is:

```
?- schedule( Schedule, FinTime).
FinTime = 24
Schedule = [ t1/3/0/4, t2/2/0/2, t3/1/0/2, t4/3/4/20,
             t5/2/4/20, t6/1/2/11, t7/1/13/11]
```

Task t1 is executed by processor 3 starting at time 0, task t2 by processor 2 starting at 0, etc. There is one more point of interest in this scheduling problem. All the three available processors are equivalent, so permuting the assignments of the processors to the tasks has no effect. Therefore it makes no sense to search through all the permutations as they should give the same results. We could avoid these useless permutations by, for example, fixing processor 1 to task t7, and limiting the choice for task t6 to processors 1 and 2 only. This can be done easily by changing the predicate resource. Although this is in general a good idea, it turns out that it is not worth doing in this particular exercise. Although this would reduce the number of possible assignments by a factor of 6, the time saving is, possibly surprisingly, insignificant. The reason is that, once an optimal schedule is found, this gives a tight upper bound on the finishing time, and then other possible processor assignments are abandoned very quickly.

Exercises

14.4 Experiment with the program in Figure 14.3. Try different resource specifications, aiming at removing useless permutations, and timing the program's running times. What are the improvements?

14.5 The program in Figure 14.4 initializes the upper bound on the finishing times (bestsofar) to a high value that is obviously a gross overestimate. This makes sure that an optimal schedule is within the bound, and will be found by the program. This policy, although safe, is inefficient because such a loose upper bound does not constrain the search well. Investigate different policies of initializing and changing the upper bound, for example, starting with a very low bound and increasing it if necessary (if no schedule exists within this bound). Compare the run times of various policies. Measure the run time for the case when the bound is immediately set to the true minimal finishing time.

14.4 A simulation program with constraints

Numerical simulation can be sometimes done very elegantly with CLP(R). It is particularly appropriate when a simulated system can be viewed as consisting of a number of components and connections among the components. Electrical circuits are examples of such systems. Resistors and capacitors are examples of components.

Real-valued parameters and variables are associated with components, such as electrical resistances, voltages and currents. Such a setting fits well the style of constraint programming. The laws of physics impose constraints on the variables associated with components. Connections between components impose additional constraints. So to carry out numerical simulation with CLP(R) for a family of systems, such as electrical networks, we have to define the laws for the types of components in the domain, and the laws for connecting components. These laws are stated as constraints on variables. To simulate a concrete system from such a family, we then have to specify the concrete components and connections in the system. This will cause the CLP interpreter to set up the constraints for the complete system, and carry out the simulation by satisfying the constraints. Of course, this approach is effective if the type of constraints in the simulated domain are handled efficiently by our particular CLP system.

In this section we will apply this approach to the simulation of electrical circuits consisting of resistors, diodes and batteries. The relations between voltages and currents in such circuits are piecewise linear. Given that our CLP(R) system efficiently handles linear equations and inequations, it is a suitable tool for simulating such circuits.

Figure 14.5 shows our components and connections, and the corresponding constraints enforced by the components and connections. We can define these elements in a CLP(R) program as follows. A resistor has some resistance R and two terminals T1 and T2. The variables associated with each terminal are the electrical potential V and the current I (directed into the resistor). So a terminal T is a pair (V,I). The lawful behaviour of the resistor can then be defined by the predicate:

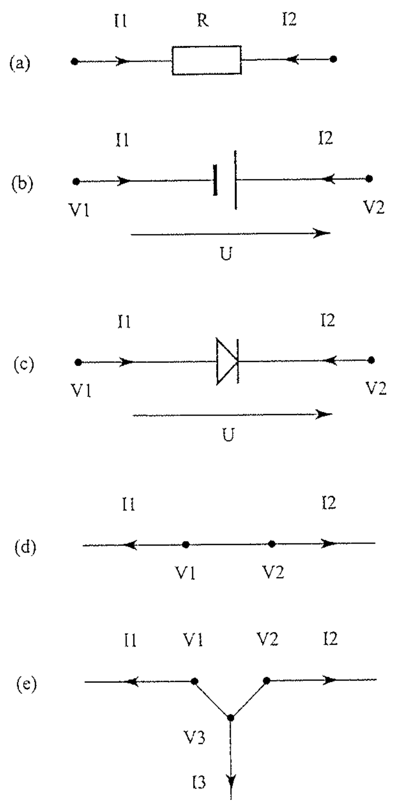
```
resistor( (V1,I1), (V2,I2), R) :-
  { I1 = -I2, V1-V2 = I1*R }.
```

The behaviour of the battery can be defined similarly, as shown in the program of Figure 14.6. The figure also gives a definition of the diode. For connections, it is best to define the general case when any number of terminals are connected:

```
conn( [ Terminal1, Terminal2, ... ])
```

The voltages at all the terminals must be equal, and the sum of the currents into all of the terminals must be equal to 0.

It is now easy to compose circuits to be simulated. Figure 14.7 shows some circuits. The figure also gives definitions of these circuits executable by our simulator in CLP(R). Consider circuit (a). The following example illustrates that our simulator can, to some extent, also be used for design, not only for simulation. In the definition of predicate circuit_a in Figure 14.7, we have chosen to make the terminal T21 one of the arguments of this predicate. This makes it possible to 'read' the voltage and current at this point in the circuit. The potential at terminal T2 is fixed to 0, the battery has 10 V, but the resistors are left unspecified (they also are arguments of circuit_a).



(a) $V1 - V2 = I1 * R$
$I1 = -I2$

(b) $V1 - V2 = U$
$I1 = -I2$

(c) $I1 + I2 = 0$
$I1 > 0 \Rightarrow V1 = V2$
$I1 = 0 \Rightarrow V1 \leq V2$

(d) $V1 = V2$
$I1 + I2 = 0$

(e) $V1 = V2 = V3$
$I1 + I2 + I3 = 0$

Figure 14.5 Components and connections for electrical circuits, and the corresponding constraints; (a) resistor; (b) battery; (c) diode; (d, e) connection between two or three terminals.

```
% Electric circuit simulator in CLP(R)

% resistor( T1, T2, R):
%   R=resistance; T1, T2 its terminals

resistor( (V1,I1), (V2,I2), R) :-
  { I1 = -I2, V1-V2 = I1*R }.

% diode( T1, T2):
%   T1, T2 terminals of a diode
%   Diode open in direction from T1 to T2

diode( (V1,I1), (V2,I2) ) :-
  { I1 + I2 = 0 },
  { I1 > 0, V1 = V2
    ;
    I1 = 0, V1 =< V2 }.

battery( (V1,I1), (V2,I2), Voltage) :-
  { I1 + I2 = 0, Voltage = V1 - V2 }.

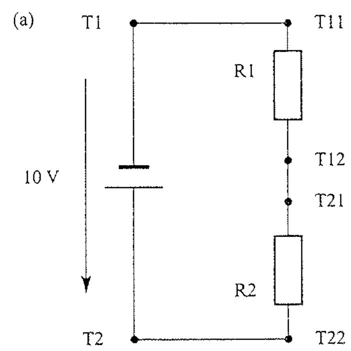
% conn( [T1,T2,...]):
%   Terminals T1, T2, ... connected
%   Therefore all electrical potentials equal, sum of currents = 0

conn( Terminals) :-
  conn( Terminals, 0).

conn( [ (V,I) ], Sum) :-
  { Sum + I = 0 }.

conn( [ (V1,I1), (V2,I2) | Rest], Sum) :-
  { V1 = V2, Sum1 = Sum + I1},
  conn( [ (V2, I2) | Rest], Sum1).
```

Figure 14.6 Constraints for some electrical components and connections.

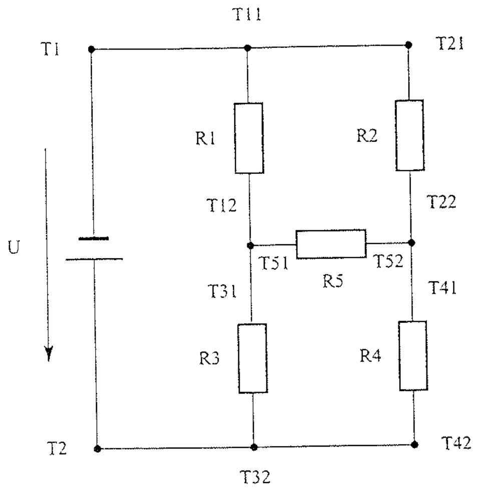



```
circuit_a( R1, R2, T21) :-
  T2 = (0,_),                   % Terminal T2 at potential 0
  battery( T1, T2, 10),         % Battery 10 V
  resistor( T11, T12, R1),
  resistor( T21, T22, R2),
  conn( [ T1, T11]),
  conn( [ T12, T21]),
  conn( [ T2, T22]).
```

Figure 14.7 Two electrical circuits.

Figure 14.7 *contd*

(b)




```
circuit_b( U, T11, T21, T31, T41, T51, T52) :-
  T2 = ( 0, _),                         % Terminal T2 at potential 0
  battery( T1, T2, U),
  resistor( T11, T12, 5),               % R1 = 5
  resistor( T21, T22, 10),              % R2 = 10
  resistor( T31, T32, 15),              % R3 = 15
  resistor( T41, T42, 10),              % R4 = 10
  resistor( T51, T52, 50),              % R5 = 50
  conn( [T1, T11, T21]),
  conn( [T12, T31, T51]),
  conn( [T22, T41, T52]),
  conn( [T2, T32, T42]).
```

Consider the question: What should the resistors be so that the voltage at terminal T21 is 6 V and the current is 1 A?

```
?- circuit_a( R1, R2, (6,1)).
  R1 = 4.0
  R2 = 6.0
```

Let us now consider the more complex circuit (b). A question may be: Given the battery voltage 10 V, what are the electrical potentials and the current at the 'middle' resistor R5?

```
?- circuit_b(10, _, _, _, _, T51, T52).
T51 = ( 7.340425531914894, 0.0425531914893617)
T52 = ( 5.212765957446809, -0.0425531914893617)
```


So the potentials at the terminals of R5 are 7.340 V and 5.213 V respectively, and the current is 0.04255 A.

Exercise

14.6 Experiment with the program in Figure 14.7. Define other circuits. For example, extend the circuit of Figure 14.7(b) by adding a diode in series with resistor R5. How does this affect the voltage at T51? Try also the opposite orientation of the diode.

14.5 CLP over finite domains: CLP(FD)

CLP over finite domains comprises some specific mechanisms. We will here illustrate these by some typical examples, using the syntax and some of the predicates from the SICStus Prolog CLP(FD) library. We will here only introduce a subset of these predicates, those needed for our examples.

Domains of variables will be sets of integers. The predicate in/2 is used to state the domain of a variable, written as:

```
X in Set
```

where Set can be:

{Integer1, Integer2, ...}	set of given integers
Term1..Term2	set of integers between Term1 and Term2
Set1 \/ Set2	union of Set1 and Set2
Set1 /\ Set2	intersection of Set1 and Set2
\ Set1	complement of Set1

Arithmetic constraints have the form:

```
Exp1 Relation Exp2
```

where Exp1 and Exp2 are arithmetic expressions, and Relation can be:

#=	equal
#\=	not equal
#<	less than
#>	greater than
#=<	less or equal
#>=	greater or equal

For example:

```
?- X in 1..5, Y in 0..4,
   X #< Y, Z #= X+Y+1.

X in 1..3
Y in 2..4
Z in 3..7

?- X in 1..5, X in \{4}.

X in (1..3) \/ {5}
```

Predicate indomain(X) assigns through backtracking possible values to X, for example:

```
?- X in 1..3, indomain(X).

X = 1;
X = 2;
X = 3
```

There are some useful predicates on lists of variables:

```
domain( L, Min, Max)
```

meaning: all the variables in list L have domains Min . . Max.

```
all_different( L)
```

meaning: all the variables in L must have different values.

```
labeling( Options, L)
```

generates concrete possible values of the variables in list L. Options is a list of options regarding the order in which the variables in L are 'labelled'. If Options = [] then by default the variables are labelled from left to right, taking the possible values one by one from the smallest to the largest. This simplest labelling strategy will satisfy for all the examples in this chapter, although it will not always be the most efficient. Here are some examples:

```
?- domain( [X,Y], 1, 2), labeling( [], [X,Y]).
X=1, Y=1;
X=1, Y=2;
X=2, Y=1;
X=2, Y=2

?- L = [X,Y], domain( L, 1, 2), all_different( L), labeling( [], L).
L = [1,2], X = 1, Y = 2;
L = [2,1], X = 2, Y = 1
```

It is easy to solve cryptarithmetic puzzles using these primitives. Consider the puzzle:


```
% Cryptarithmetic puzzle DONALD+GERALD=ROBERT in CLP(FD)

solve( [D,O,N,A,L,D], [G,E,R,A,L,D], [R,O,B,E,R,T]) :-
  Vars = [D,O,N,A,L,G,E,R,B,T],                    % All variables in the puzzle
  domain( Vars, 0, 9),                             % They are all decimal digits
  all_different( Vars),                            % They are all different
  100000*D + 10000*O + 1000*N + 100*A + 10*L + D +
  100000*G + 10000*E + 1000*R + 100*A + 10*L + D #=
  100000*R + 10000*O + 1000*B + 100*E + 10*R + T,
  labeling( [], Vars).
```

Figure 14.8 A cryptarithmetic puzzle in CLP(FD).

```
  D O N A L D
+ G E R A L D
  -----------
  R O B E R T
```

Each letter above is to be substituted by a different decimal digit so that the numbers add up. Figure 14.8 shows a CLP(FD) program to solve this puzzle. The query to this program is:

```
?- solve( N1, N2, N3).
N1 = [5,2,6,4,8,5]
N2 = [1,9,7,4,8,5]
N3 = [7,2,3,9,7,0]
```

Figure 14.9 shows a CLP(FD) program for the familiar eight queens problem. It is instructive to note that both programs, cryptarithmetic and eight queens, have the following basic structure: first, the domains of variables are defined, then constraints are imposed, and finally the labelling finds concrete solutions. This is a usual structure of CLP(FD) programs.

Finally, let us consider optimization problems with CLP(FD), such as the minimization of the finishing time in scheduling. Built-in CLP(FD) predicates useful for optimization are:

```
minimize( Goal, X)   and   maximize( Goal, X)
```

which find such a solution of Goal that minimizes (maximizes) the value of X. Typically Goal is an 'indomain' or a labelling goal. For example:

```
?- X in 1..20, V #= X*(20-X), maximize( indomain(X), V).
X = 10, V = 100
```

Our simple scheduling problem of Figure 14.1 can be solved by the following query:

```
% 8 queens in CLP(FD)

solution( Ys) :-                    % Ys is list of Y-coordinates of queens
  Ys = [_,_,_,_,_,_,_,_],           % There are 8 queens
  domain( Ys, 1, 8),                % All the coordinates have domains 1..8
  all_different( Ys),               % All different to avoid horizontal attacks
  safe( Ys),                        % Constrain to prevent diagonal attacks
  labeling( [ ], Ys).               % Find concrete values for Ys

safe( [ ]).

safe( [Y | Ys]) :-
  no_attack( Y, Ys, 1),             % 1 = horizontal distance between queen Y and Ys
  safe( Ys).

% no_attack( Y, Ys, D):
%   queen at Y doesn't attack any queen at Ys;
%   D is column distance between first queen and other queens

no_attack( Y, [ ], _).

no_attack( Y1, [Y2 | Ys], D) :-
  D #\= Y1-Y2,
  D #\= Y2-Y1,
  D1 is D+1,
  no_attack( Y1, Ys, D1).
```

Figure 14.9 A CLP(FD) program for eight queens.

```
?- StartTimes = [Ta,Tb,Tc,Td,Tf],     % Tf finishing time
   domain( StartTimes, 0, 20),
   Ta #>= 0,
   Ta + 2 #=< Tb,                     % a precedes b
   Ta + 2 #=< Tc,                     % a precedes c
   Tb + 3 #=< Td,                     % b precedes d
   Tc + 5 #=< Tf,                     % c finished by Tf
   Td + 4 #=< Tf,
   minimize( labeling([ ],StartTimes), Tf).
StartTimes = [0,2,2,5,9]
```

Here only one optimal solution is generated.

Exercises

14.7 Measure the time needed by the program in Figure 14.8 to solve the puzzle. Then replace the labelling goal by:

```
labeling( [ff], Vars)
```


The labelling option 'ff' stands for 'first fail'. That is, the variable with currently the smallest domain will be assigned a value first. Having the smallest domain, this variable is generally the most likely one to cause a failure. This labelling strategy aims at discovering inconsistency as soon as possible, thus avoiding futile search through inconsistent alternatives. Measure the execution time of the modified program.

14.8 Generalize the eight-queens CLP(FD) program to an N queens program. For large N, a good labelling strategy for N queens is 'middle-out', which starts in the middle of the domain and then continues with values further and further away from the middle. Implement this labelling strategy and compare its efficiency experimentally with the straight labelling (as in Figure 14.9).

Summary

- Constraint satisfaction problems are stated in terms of variables, domains of variables, and constraints among the variables.
- Constraint satisfaction problems are often represented as constraint networks.
- Consistency algorithms operate on constraint networks and reduce the domains of variables.
- Constraint logic programming (CLP) combines the constraint satisfaction approach and logic programming.
- CLP systems differ in types of domains and constraints. CLP systems include CLP(R) (over real numbers), CLP(Q) (over rational numbers), CLP(Z) (over integers), CLP(FD) (over finite domains), CLP(B) (over Booleans).
- The power of a CLP system mainly depends on the power of specialized solvers that can solve systems of specific types of constraints, and possibly optimize a given measure within the constraints.
- One aspect in CLP programming is specifying constraints that are as strong as possible. The stronger the constraints, the more they reduce the search space and thus contribute to efficiency. Even adding redundant constraints may improve efficiency.
- Typical practical application areas for CLP include scheduling, logistics and resource management.
- In this chapter, CLP programs were presented for scheduling, simulation of electrical circuits, a cryptarithmetic puzzle and eight queens.
- Concepts discussed in this chapter are:

  constraint satisfaction problems
  constraint satisfaction

constraint networks
arc consistency algorithms
constraint logic programming (CLP)
CLP(R), CLP(Q), CLP(FD)
branch-and-bound method

References

Marriott and Stuckey (1998) is an excellent introduction to techniques of constraint satisfaction and CLP programming. Van Hentenryck (1989) is a well-known discussion of various programming techniques in CLP. Jaffar and Maher (1994), and Mackworth (1992) survey constraint solving techniques. Ongoing research in this area appears in the specialized journal *Constraints* (published by Kluwer Academic), as well as in the *Journal of Logic Programming* and the *Artificial Intelligence Journal*. The Fibonacci example in this chapter is similar to the one given by Cohen (1990).

The syntax for constraints in this chapter is as used in the SICStus Prolog (SICStus 1999).

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chapter 15

Knowledge Representation and Expert Systems

An expert system is a program that behaves like an expert for some problem domain. It should be capable of *explaining* its decisions and the underlying reasoning. Often an expert system is expected to be able to deal with uncertain and incomplete information. In this chapter we will review basic concepts in representing knowledge and building expert systems.

15.1 Functions and structure of an expert system

An *expert system* is a program that behaves like an expert in some, usually narrow, domain of application. Typical applications include tasks such as medical diagnosis, locating equipment failures, or interpreting measurement data. Expert systems have to be capable of solving problems that require expert knowledge in a particular domain. They should possess that knowledge in some form. Therefore they are also called *knowledge-based systems*. However, not every knowledge-based system can be considered an expert system. We will take the view that an expert system also has to be capable, in some way, of *explaining* its behaviour and its decisions to the user, as human experts do. Such an explanation feature is especially necessary in uncertain domains (like medical diagnosis) to enhance the user's confidence in the system's advice, or to enable the user to detect a possible flaw in the system's reasoning. Therefore, expert systems have to have a friendly user-interaction capability that will make the system's reasoning transparent to the user.

An additional feature that is often required of an expert system is the ability to deal with uncertainty and incompleteness. Information about the problem to be solved can be incomplete or unreliable; relations in the problem domain can be approximate. For example, we may not be quite sure that some symptom is present in the patient, or that some measurement data is absolutely correct; some drug *may* cause some problem, but *usually* does not. All this requires reasoning with uncertainty.

To build an expert system we have, in general, to develop the following functions:

- *problem-solving* function capable of using domain-specific knowledge – this may require *dealing with uncertainty;*
- *user-interaction* function, which includes explanation of the system's intentions and decisions during and after the problem-solving process.

Each of these functions can be very complicated, and can depend on the domain of application and practical requirements. Various intricate problems may arise in the design and implementation. This involves the representation of knowledge and associated reasoning. In this chapter we will develop a framework of basic ideas that can be further refined. In Chapter 16 we will implement a complete rule-based expert system shell. The main difficulty there will be maintaining smooth interaction with the user during the reasoning process.

It is convenient to divide the development of an expert system into three main modules, as illustrated in Figure 15.1:

(1) a knowledge base,

(2) an inference engine,

(3) a user interface.

A *knowledge base* comprises the knowledge that is specific to the domain of application, including such things as simple facts about the domain, rules or constraints that describe relations or phenomena in the domain, and possibly also methods, heuristics and ideas for solving problems in this domain. An *inference engine* knows how to actively use the knowledge in the base. A *user interface* caters for smooth communication between the user and the system, also providing the user with an insight into the problem-solving process carried out by the inference

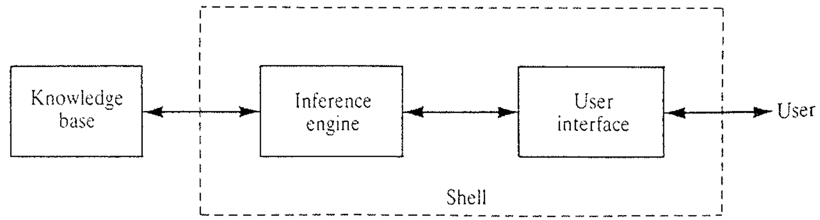


Figure 15.1 The structure of expert systems.

engine. It is convenient to view the inference engine and the interface as one module, usually called an *expert system shell*, or simply a *shell* for brevity.

The foregoing scheme separates knowledge from algorithms that use the knowledge. This division is suitable for the following reasons: the knowledge base clearly depends on the application. On the other hand, the shell is, in principle at least, domain independent. Thus a rational way of developing expert systems for several applications consists of developing a shell that can be used universally, and then to plug in a new knowledge base for each application. Of course, all the knowledge bases will have to conform to the same formalism that is 'understood' by the shell. According to practical experience in complex expert systems the scenario with one shell and many knowledge bases will not work quite so smoothly unless the application domains are indeed very similar. Nevertheless, even if modifications of the shell from one domain to another are necessary, at least the main principles can be retained.

This chapter presents some basic expert systems techniques. In particular, we will look at representing knowledge with if-then rules, basic inference mechanisms in rule-based expert systems (such as forward or backward chaining), enhancing rule-based representation with uncertainty, belief networks, semantic networks and frame-based representation of knowledge.

15.2 Representing knowledge with if-then rules

In principle, any consistent formalism in which we can express knowledge about some problem domain can be considered for use in an expert system. However, the language of *if-then* rules, also called *production* rules, is by far the most popular formalism for representing knowledge. In general, such rules are conditional statements, but they can have various interpretations. Examples are:

- *if* precondition P *then* conclusion C;
- *if* situation S *then* action A;
- *if* conditions C1 and C2 hold *then* condition C does not hold.

If-then rules usually turn out to be a natural form of expressing knowledge, and have the following additional desirable features:

- *Modularity*: each rule defines a small, relatively independent piece of knowledge.
- *Incrementability*: new rules can be added to the knowledge base relatively independently of other rules.
- *Modifiability* (as a consequence of modularity): old rules can be changed relatively independent of other rules.
- Support system's *transparency*.

This last property is an important and distinguishing feature of expert systems. By transparency of the system we mean the system's ability to explain its decisions and solutions. If-then rules facilitate answering the following basic types of user's questions:

(1) 'How' questions: *How* did you reach this conclusion?
(2) 'Why' questions: *Why* are you interested in this information?

Mechanisms, based on if-then rules, for answering such questions will be discussed later.

If-then rules often define logical relations between concepts of the problem domain. Purely logical relations can be characterized as belonging to 'categorical knowledge', 'categorical' because they are always meant to be absolutely true. However, in some domains, such as medical diagnosis, 'soft' or probabilistic knowledge prevails. It is 'soft' in the sense that empirical regularities are usually only valid to a certain degree (often but not always). In such cases if-then rules may be modified by adding a likelihood qualification to their logical interpretation. For example:

if condition A *then* conclusion B follows *with certainty* F

Figures 15.2, 15.3 and 15.4 give an idea of the variety of ways of expressing knowledge by if-then rules. They show example rules from three different

---

if
  1 the infection is primary bacteremia, and
  2 the site of the culture is one of the sterilesites, and
  3 the suspected portal of entry of the organism is the gastrointestinal tract
then
  there is suggestive evidence (0.7) that the identity of the organism is bacteroides.

Figure 15.2 An if-then rule from the MYCIN system for medical consultation (Shortliffe 1976). The parameter 0.7 says to what degree the rule can be trusted.

---

if
  the pressure in V-01 reached relief valve lift pressure
then
  the relief valve on V-01 has lifted [$N = 0.005$, $S = 400$]
if
  NOT the pressure in V-01 reached relief valve lift pressure, and the relief valve on V-01 has lifted
then
  the V-01 relief valve opened early (the set pressure has drifted) [$N = 0.001$, $S = 2000$]

Figure 15.3 Two rules from an AL/X demonstration knowledge base for fault diagnosis (Reiter 1980). *N* and *S* are the 'necessity' and 'sufficiency' measures. *S* estimates to what degree the condition part of a rule *suffices* to infer the conclusion part. *N* estimates to what degree the truth of the condition part is *necessary* for the conclusion to be true.

knowledge-based systems: MYCIN for medical consultation, AL/X for diagnosing equipment failures and AL3 for problem solving in chess.

In general, if you want to develop a serious expert system for some chosen domain then you have to consult actual experts for that domain and learn a great deal about it yourself. Extracting some understanding of the domain from experts and literature, and moulding this understanding into a chosen knowledge-representation formalism is called *knowledge elicitation*. This is, as a rule, a complex effort that we cannot go into here. But we do need some domain and a small knowledge base as material to carry out our examples in this chapter. Consider the toy knowledge base shown in Figure 15.5. It is concerned with diagnosing the problem of water leaking in a flat. A problem can arise either in the bathroom or in the kitchen. In either case, the leakage also causes a problem (water on the floor) in the hall. Apart from its overall naivete, this knowledge base only assumes single faults; that is, the problem may be in the bathroom or the kitchen, but not in both

---

if
  1 there is a hypothesis, H, that a plan P succeeds, and
  2 there are two hypotheses,
    H_1, that a plan R_1 refutes plan P, and
    H_2, that a plan R_2 refutes plan P, and
  3 there are facts: H_1 is *false*, and H_2 is *false*
then
  1 generate the hypothesis, H_3, that the combined plan 'R_1 or R_2' refutes plan P, and
  2 generate the fact: H_3 *implies not*(H)

Figure 15.4 A rule for plan refinement in chess problem solving from the AL3 system (Bratko 1982).

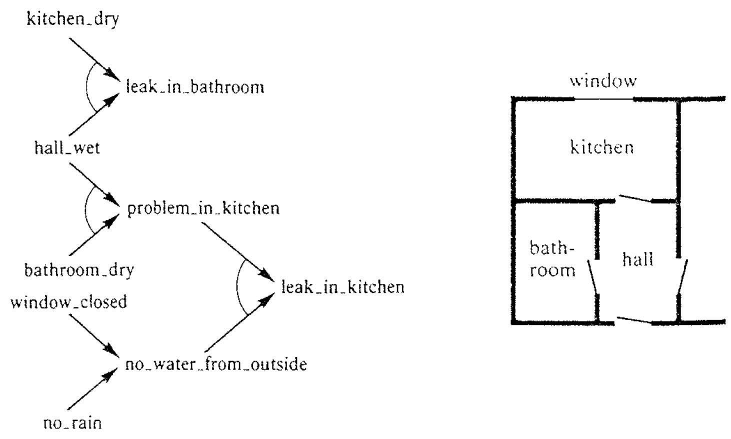


Figure 15.5 A toy knowledge base to diagnose leaks in water appliances in the flat shown.

of them at the same time. This knowledge base is shown in Figure 15.5 as an *inference network*. Nodes in the network correspond to propositions and links correspond to rules in the knowledge base. Arcs that connect some of the links indicate the conjunctive connection between the corresponding propositions. Accordingly, the rule about a problem in the kitchen in this network is:

if hall_wet *and* bathroom_dry *then* problem_in_kitchen

The network representation of Figure 15.5 is, in fact, an AND/OR graph as discussed in Chapter 13. This indicates the relevance of AND/OR representation of problems in the context of rule-based expert systems.

15.3 Forward and backward chaining in rule-based systems

Once knowledge is represented in some form, we need a reasoning procedure to draw conclusions from the knowledge base. There are two basic ways of reasoning with if-then rules:

- backward chaining, and
- forward chaining.

Both of these procedures are very easily implemented in Prolog. In fact, procedures for searching AND/OR graphs, presented in Chapter 13, can be used here. The more advanced search techniques for AND/OR graphs are in a way even more sophisticated than those usually used in expert systems. On the other hand, there is often an

emphasis in expert systems on the style of search, from its semantic point of view, in relation to the human reasoning; that is, it is desirable to sequence the reasoning in ways that humans find natural in the domain of application. This is important when interaction with the user occurs during the reasoning process and we want to make this process transparent to the user. This section sketches, in Prolog, the basic reasoning procedures as they appear in the context of expert systems, although they are similar to searching AND/OR graphs.

15.3.1 Backward chaining

With our example knowledge base of Figure 15.5, reasoning in the backward chaining style may proceed as follows. We start with a hypothesis – for example, leak_in_kitchen – then we reason backwards in the inference network. To confirm the hypothesis, we need problem_in_kitchen and no_water_from_outside to be true. The former is confirmed if we find that the hall is wet and the bathroom is dry. The latter is confirmed, for example, if we find that the window was closed.

This style of reasoning is called backward chaining because we follow a chain of rules backwards from the hypothesis (leak_in_kitchen) to the pieces of evidence (hall_wet, etc.). This is trivially programmed in Prolog – this is, in fact, Prolog's own built-in style of reasoning. The straightforward way is to state the rules in the knowledge base as Prolog rules:

```
leak_in_bathroom :-
  hall_wet,
  kitchen_dry.

problem_in_kitchen :-
  hall_wet,
  bathroom_dry.

no_water_from_outside :-
  window_closed
  ;
  no_rain.

leak_in_kitchen :-
  problem_in_kitchen,
  no_water_from_outside.
```

The observed pieces of evidence can be stated as Prolog facts:

```
hall_wet.
bathroom_dry.
window_closed.
```

The hypothesis can now be checked by:

```
?- leak_in_kitchen.
yes
```

Using Prolog's own syntax for rules, as in the foregoing, has certain disadvantages however:

(1) This syntax may not be the most suitable for a user unfamiliar with Prolog; for example, the domain expert should be able to read the rules, specify new rules and modify them.

(2) The knowledge base is, therefore, not syntactically distinguishable from the rest of the program; a more explicit distinction between the knowledge base and the rest of the program may be desirable.

It is easiest to tailor the syntax of expert rules to our taste by using Prolog operator notation. For example, we can choose to use 'if', 'then', 'and' and 'or' as operators, appropriately declared as:

```
:- op( 800, fx, if).
:- op( 700, xfx, then).
:- op( 300, xfy, or).
:- op( 200, xfy, and).
```

This suffices to write our example rules of Figure 15.5 as:

```
if
  hall_wet and kitchen_dry
then
  leak_in_bathroom.

if
  hall_wet and bathroom_dry
then
  problem_in_kitchen.

if
  window_closed or no_rain
then
  no_water_from_outside.
...
```

Let the observable findings be stated as a procedure **fact**:

```
fact( hall_wet).
fact( bathroom_dry).
fact( window_closed).
```

Of course, we now need a new interpreter for rules in the new syntax. Such an interpreter can be defined as the procedure

```
is_true( P)
```

where proposition P is either given in procedure **fact** or can be derived using rules. The new rule interpreter is given in Figure 15.6. Note that it still does backward chaining in the depth-first manner. The interpreter can now be called by the question:

```
?- is_true( leak_in_kitchen).
yes
```

A major practical disadvantage of the simple inference procedures in this section is that the user has to state all the relevant information as facts in advance, before the reasoning process is started. So the user may state too much or too little. Therefore, it would be better for the information to be provided by the user interactively in a dialogue when it is needed. Such a dialogue facility will be programmed in Chapter 16.


```
% A simple backward chaining rule interpreter

:- op( 800, fx, if).
:- op( 700, xfx, then).
:- op( 300, xfy, or).
:- op( 200, xfy, and).

is_true( P) :-
  fact( P).

is_true( P) :-
  if Condition then P,                % A relevant rule
  is_true( Condition).                % whose condition is true

is_true( P1 and P2) :-
  is_true( P1),
  is_true( P2).

is_true( P1 or P2) :-
  is_true( P1)
  ;
  is_true( P2).
```

Figure 15.6 A backward chaining interpreter for if-then rules.

15.3.2 Forward chaining

In backward chaining we start with a hypothesis (such as leak in the kitchen) and work backwards, according to the rules in the knowledge base, toward easily confirmed findings (such as the hall is wet). Sometimes it is more natural to reason in the opposite direction, from the 'if' part to the 'then' part. Forward chaining does not start with a hypothesis, but with some confirmed findings. Once we have observed that the hall is wet and the bathroom is dry, we conclude that there is a problem in the kitchen; also, having noticed the kitchen window is closed, we infer that no water came from the outside; this leads to the final conclusion that there is a leak in the kitchen.

Programming simple forward chaining in Prolog is still easy if not exactly as trivial as backward chaining. Figure 15.7 shows a forward chaining interpreter, assuming that rules are, as before, in the form:

```
if Condition then Conclusion
```

where Condition can be an AND/OR expression. For simplicity we assume throughout this chapter that rules do not contain variables. This interpreter starts with what is already known (stated in the fact relation), derives all conclusions that follow from this and adds (using assert) the conclusions to the fact relation. Our example knowledge base is run by this interpreter thus:

```
?- forward.

Derived: problem_in_kitchen
Derived: no_water_from_outside
Derived: leak_in_kitchen
No more facts
```

```
% Simple forward chaining in Prolog

forward :-
  new_derived_fact( P),                    % A new fact
  !,
  write( 'Derived: '), write( P), nl,
  assert( fact( P)),
  forward                                  % Continue
  ;
  write( 'No more facts').                 % All facts derived

new_derived_fact( Concl) :-
  if Cond then Concl,                      % A rule
  not fact( Concl),                        % Rule's conclusion not yet a fact
  composed_fact( Cond).                    % Condition true?

composed_fact( Cond) :-
  fact( Cond).                             % Simple fact

composed_fact( Cond1 and Cond2) :-
  composed_fact( Cond1),
  composed_fact( Cond2).                   % Both conjuncts true

composed_fact( Cond1 or Cond2) :-
  composed_fact( Cond1)
  ;
  composed_fact( Cond2).
```

Figure 15.7 A forward chaining rule interpreter.

15.3.3 Forward chaining vs backward chaining

If-then rules form chains that, in Figure 15.5, go from left to right. The elements on the left-hand side of these chains are input information, while those on the right-hand side are derived information:

input information → ... → derived information

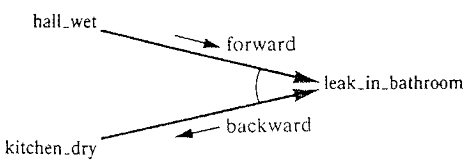
These two kinds of information have a variety of names, depending on the context in which they are used. Input information can be called *data* (for example, measurement data) or *findings* or *manifestations*. Derived information can be called *hypotheses* to be proved, or *causes* of manifestations, or *diagnoses*, or *explanations* that explain findings. So chains of inference steps connect various types of information, such as:

data → ... → goals
evidence → ... → hypotheses
findings, observations → ... → explanations, diagnoses
manifestations → ... → diagnoses, causes

Both forward and backward chaining involve search, but they differ in the direction of search. Backward chaining searches from goals to data, from diagnoses to findings, etc. In contrast, forward chaining searches from data to goals, from findings to explanations or diagnoses, etc. As backward chaining starts with goals we say that it is *goal driven*. Similarly, since forward chaining starts with data we say that it is *data driven*.

An obvious question is: Which is better, forward or backward chaining? This question is similar to the dilemma between forward and backward search in state space (see Chapter 11). As there, the answer here also depends on the problem. If we want to check whether a particular hypothesis is true then it is more natural to chain backward, starting with the hypothesis in question. On the other hand, if there are many competing hypotheses, and there is no reason to start with one rather than another, it may be better to chain forward. In particular, forward chaining is more natural in monitoring tasks where the data are acquired continuously and the system has to detect whether an anomalous situation has arisen; a change in the input data can be propagated in the forward chaining fashion to see whether this change indicates some fault in the monitored process or a change in the performance level. In choosing between forward or backward chaining, simply the shape of the rule network can also help. If there are a few data nodes (the left flank of the network) and many goal nodes (right flank) then forward chaining looks more appropriate; if there are few goal nodes and many data nodes then vice versa.

Expert tasks are usually more intricate and call for a combination of chaining in both directions. In medicine, for example, some initial observations in the patient typically trigger doctor's reasoning in the forward direction to generate some initial diagnostic hypothesis. This initial hypothesis has to be confirmed or rejected by additional evidence, which is done in the backward chaining style. In our example of Figure 15.5, observing the hall wet may trigger the following inference steps:



Exercise

15.1 Write a program that combines forward and backward chaining in the style discussed in this section.

15.4 Generating explanation

There are standard ways of generating explanation in rule-based systems. Two usual types of explanation are called 'how' and 'why' explanation. Let us consider first the 'how' explanation. When the system comes up with an answer, the user may ask: *How* did you find this answer? The typical explanation consists of presenting the user with the trace of *how* the answer was derived. Suppose the system has just found that there is a leak in the kitchen and the user is asking 'How?'. The explanation can be along the following line:

Because:

(1) there is a problem in the kitchen, which was concluded from hall wet and bathroom dry, and
(2) no water came from outside, which was concluded from window closed.

Such an explanation is in fact a *proof tree* of how the final conclusion follows from rules and facts in the knowledge base. Let '<=' be defined as an infix operator. Then we can choose to represent the proof tree of a proposition P in one of the following forms, depending on the case:

(1) If P is a fact then the proof tree is P.

(2) If P was derived using a rule

    if Cond then P

then the proof tree is

    P <= ProofCond

where ProofCond is a proof tree of Cond.

(3) Let P1 and P2 be propositions whose proof trees are Proof1 and Proof2. If P is P1 and P2 then the proof tree is Proof1 and Proof2. If P is P1 or P2 then the proof tree is either Proof1 or Proof2.

Constructing proof trees in Prolog is straightforward and can be achieved by modifying the predicate is_true of Figure 15.6 according to the three cases above. Figure 15.8 shows such a modified is_true predicate. Notice that proof trees of this kind are essentially the same as solution trees for problems represented by AND/OR graphs. Displaying proof trees in some user-friendly format can be programmed similarly to displaying AND/OR solution trees. More sophisticated output formatting of proof trees is part of the shell program in Chapter 16.

In contrast, the 'why' explanation is required *during* the reasoning process, not at the end of it. This requires user interaction with the reasoning process. The system asks the user for information at the moment that the information is needed. When asked, the user may answer 'Why?', thus triggering the 'why' explanation. This kind of interaction and 'why' explanation is programmed as part of the shell in Chapter 16.

```
% is_true( P, Proof) Proof is a proof that P is true

:- op( 800, xfx, <=).

is_true( P, P) :-
  fact( P).

is_true( P, P <= CondProof) :-
  if Cond then P,
  is_true( Cond, CondProof).

is_true( P1 and P2, Proof1 and Proof2) :-
  is_true( P1, Proof1),
  is_true( P2, Proof2).

is_true( P1 or P2, Proof) :-
  is_true( P1, Proof)
  ;
  is_true( P2, Proof).
```

Figure 15.8 Generating proof trees.

15.5 Introducing uncertainty

15.5.1 A simple uncertainty scheme

In the foregoing discussion, the representation assumes problem domains that are *categorical*; that is, answers to all questions are either true or false, not somewhere between. As data, rules were also categorical: 'categorical implications'. However, many expert domains are not categorical. Typical expert behaviour is full of guesses (although highly articulated) that are usually true, but there can be exceptions. Both data about a particular problem and general rules can be less than certain. We can model uncertainty by assigning some qualification, other than just true or false, to assertions. Such qualification can be expressed by descriptors – for example, *true*, *highly likely*, *likely*, *unlikely*, *impossible*. Alternatively, the degree of belief can be expressed by a real number in some interval – for example, between 0 and 1 or −5 and +5. Such numbers are known by a variety of names, such as 'certainty factor', 'measure of belief' or 'subjective probability'. The most principled possibility is to use probabilities because of their solid mathematical foundation. However, correct reasoning with probabilities according to the probability calculus is typically more demanding than the reasoning in simpler, *ad hoc* uncertainty schemes.

We will discuss the use of probabilities in association with belief networks in the next section. In this section we will extend our rule-based representation with a simpler uncertainty scheme that only very roughly approximates probabilities. Each proposition will be added a number between 0 and 1 as a certainty qualification. We will choose to represent this as a pair of the form:

```
Proposition : CertaintyFactor
```

This notation applies to rules as well. So the following form defines a rule and the degree of certainty to which the rule is valid:

```
if Condition then Conclusion : Certainty.
```

In any representation with uncertainty we need a way of combining the certainties of propositions and rules. For example, let there be two propositions $P1$ and $P2$ whose certainties are $C1$ and $C2$. What is the certainty of logical combinations $P1$ *and* $P2$, $P1$ *or* $P2$? The following is one simple scheme for combining certainties. Let $P1$ and $P2$ be propositions, and $c(P1)$ and $c(P2)$ denote their certainties. Then:

$$c(P1 \text{ and } P2) = min(c(P1), c(P2))$$
$$c(P1 \text{ or } P2) = max(c(P1), c(P2))$$

If there is a rule

```
if P1 then P2 : C
```

then

$$c(P2) = c(P1) * C$$

For simplicity we assume that no more than one rule ever bears on the same assertion. If there were two rules bearing on the same assertion in the knowledge base, they could be transformed using or into equivalent rules that satisfy this assumption. Figure 15.9 shows a rule interpreter for this uncertainty scheme. The interpreter assumes that the user specifies the certainty estimates for the observables (left-most nodes in the rule network) by the relation

```
given( Proposition, Certainty)
```

Now we can 'soften' some rule in our knowledge base of Figure 15.5. For example:

```
if
  hall_wet and bathroom_dry
then
  problem_in_kitchen : 0.9.
```

A situation in which the hall is wet, the bathroom is dry, the kitchen is not dry, the window is not closed and we think that there was no rain but are not quite sure can be specified as:

```
given( hall_wet, 1).              % Hall is wet
given( bathroom_dry, 1).          % Bathroom is dry
given( kitchen_dry, 0).           % Kitchen is not dry
given( no_rain, 0.8).             % Probably no rain, but not sure
given( window_closed, 0).         % Window not closed
```

```
% Rule interpreter with certainties

% certainty( Proposition, Certainty)

certainty( P, Cert) :-
  given( P, Cert).

certainty( Cond1 and Cond2, Cert) :-
  certainty( Cond1, Cert1),
  certainty( Cond2, Cert2),
  min( Cert1, Cert2, Cert).

certainty( Cond1 or Cond2, Cert) :-
  certainty( Cond1, Cert1),
  certainty( Cond2, Cert2),
  max( Cert1, Cert2, Cert).

certainty( P, Cert) :-
  if Cond then P : C1,
  certainty( Cond, C2),
  Cert is C1 * C2.
```

Figure 15.9 An interpreter for rules with certainties.

Now we can ask about a leak in the kitchen:

```
?- certainty( leak_in_kitchen, C).
C = 0.8
```

This is obtained as follows. The facts that the hall is wet and the bathroom is dry indicate a problem in the kitchen with certainty 0.9. Since there was some possibility of rain, the certainty of no_water_from_outside is 0.8. Finally, the certainty of leak_in_kitchen is min(0.8,0.9) = 0.8.

15.5.2 Difficulties in handling uncertainty

The question of handling uncertain knowledge has been much researched and debated. Typical controversial issues were the usefulness of probability theory in handling uncertainty in expert systems on the one hand and drawbacks of *ad hoc* uncertainty schemes on the other. Our ultra-simple approach in Section 15.5.1 belongs to the latter, and can be easily criticized. For example, suppose the certainty of *a* is 0.5 and that of *b* is 0. Then in our scheme the certainty of *a or b* is 0.5. Now suppose that the certainty of *b* increases to 0.5. In our scheme this change will not affect the certainty of *a or b* at all, which is counter-intuitive.

Many schemes for handling uncertainty have been proposed, used and investigated. The most common problem in such schemes typically stems from ignoring some dependences between propositions. For example, let there be a rule:

if a or b then c

The certainty of *c* should not only depend on the certainty of *a* and *b*, but also on any correlation between *a* and *b*; that is, whether they tend to occur together or they depend on each other in some other way. Completely correct treatment of these dependencies is more complicated than it is often considered acceptable and may require information that is not normally available. The difficulties are therefore often dodged by making the assumption of *independence* of events, such as *a* and *b* in the rule above. Unfortunately, this assumption is not generally justifiable. In practice it is often simply not true, and may therefore lead to incorrect and counter-intuitive results. It has often been admitted that such departures from mathematically sound handling of uncertainty may be unsafe in general, but it has also been argued that they are solutions that work in practice. Along with this, it has been argued that probability theory, although mathematically sound, is impractical and not really appropriate for the following reasons:

- Human experts seem to have trouble thinking in terms of actual probabilities; their likelihood estimates do not quite correspond to probabilities as defined mathematically.

- Mathematically correct probabilistic treatment requires either information that is not available or some simplification assumptions that are not really quite justified in a practical application. In the latter case, the treatment would become mathematically unsound again.

Conversely, there have been equally eager arguments in favour of mathematically well-justified approaches based on the probability theory. Both of the foregoing objections regarding probability have been convincingly answered in favour of probability theory. In *ad hoc* schemes that 'work in practice', dangers clearly arise from simplifications that involve unsafe assumptions. In the next section we introduce *belief networks* – a representation that allows correct treatment of probability and at the same time enables relatively economical treatment of dependences.

Exercise

15.2 Let an expert system approximate the probability of the union of two events by the same formula as in our simple uncertainty scheme:

$$p(A \text{ or } B) \approx \max(p(A), p(B))$$

Under what condition does this formula give probabilistically correct results? In what situation does the formula make the greatest error, what is the error?

15.6 Belief networks

15.6.1 Probabilities, beliefs, and belief networks

The main question addressed by *belief networks*, also called *Bayesian networks*, is: How to handle uncertainty correctly, in a principled way that is also practical at the same time? We will show that these two goals of correctness and practicality are hard to achieve together, but belief networks offer a good solution. But let us first define a framework for discussion.

We will be assuming that the world is defined by a vector of variables that randomly take values from their domains (sets of their possible values). We will in all our examples limit the discussion to Boolean random variables only, whose possible values are true or false. For example, 'burglary' and 'alarm' are such variables. Variable 'alarm' is true when the alarm is sounding, and 'burglary' is true when the house has been broken into. Otherwise these variables are false. A state of such a world at some time is completely specified by giving the values of all the variables at this time.

When variables are Boolean, it is natural to talk about *events*. For example, event 'alarm' happens when variable alarm = true.

An agent (a human or an expert system) usually cannot tell for sure whether such a variable is true or false. So instead the agent can only reason about the *probability* that a variable is true. Probabilities in this context are used as a measure of the agent's beliefs. The agent's beliefs, of course, depend on how much the agent knows about the world. Therefore such beliefs are also called *subjective probabilities*, meaning that they 'belong to the subject'. 'Subjective' here does not mean 'arbitrary'. Although these probabilities model an agent's subjective beliefs, they still conform to the calculus of probability theory.

Let us introduce some notation. Let X and Y be propositions, then:

$X \wedge Y$ is the conjunction of X and Y
$X \vee Y$ is the disjunction of X and Y
$\sim X$ is the negation of X

$p(X)$ denotes the probability that proposition X is true. $p(X|Y)$ denotes the *conditional* probability that X is true given that Y is true.

A typical question about the world is: Given that the values of some variables have been observed, what are the probabilities of some of the remaining variables? Or: Given that some events have been observed, what are the probabilities of some other events? For example, alarm sounding has been observed, what is the probability that burglary has occurred?

The main difficulty is how to handle dependences among variables in the problem. Let there be n binary variables in the problem, then $2^n - 1$ numbers are needed to define the complete probability distribution among the 2^n possible states of the world. This is usually too many! It is not only impractical and computationally expensive. It is usually impossible to make reasonable estimates of all these probabilities because there is not enough information available.

In fact, usually not all these probabilities are necessary. The complete probability distribution does not make any assumptions regarding independence among the variables. But it is usually unnecessary to be so cautious. Fortunately, some things *are* independent after all.

Therefore, to make the probabilistic approach practical, we have to exploit these independences. We need economical means of stating dependences among the variables, and at the same time benefit (in terms of complexity) from those things that actually are independent.

Belief networks provide an elegant way of declaring how things depend on each other, and what things are independent of each other. In belief networks this can be stated in a natural and intuitive way.

Figure 15.10 shows an example belief network about a burglary alarm system. The sensor may be triggered by a burglar when the house is broken into, or by strong lightning. The sensor is supposed to trigger a sound alarm and a warning phone call. A typical question that such a belief network helps to answer is something like:

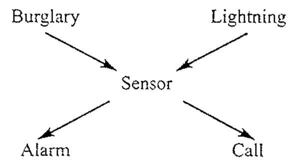


Figure 15.10 A belief network. When a burglar breaks into the house, he is likely to trigger the sensor. The sensor is in turn supposed to trigger a sound alarm, and start an automatic phone call with a warning. A storm with strong lightning may also trigger the sensor.

Suppose the weather is fine and we hear the alarm. Given these two facts, what is the probability of burglary?

The structure of this belief network indicates some probabilistic dependences, as well as independences. It says, for example, that burglary is independent of lightning. If, however, it becomes known that alarm is true, then under this condition the probability of burglary is no longer independent of lightning.

It is intuitively obvious that links in the network indicate causality. Burglary is a cause of triggering the sensor. The sensor may in turn cause an alarm. So the structure of the network allows us to reason like this: if alarm is true then burglary becomes likely as it is one of the causes that explain the alarm. If then we learn there was a heavy storm, burglary becomes less likely. Alarm is explained by another cause, lightning, so the first possible cause becomes less likely.

In this example the reasoning was both *diagnostic* and *predictive*: knowing alarm is true (consequence, or symptom of burglary), we inferred diagnostically that it might have been caused by burglary. Then we learned about the storm, and inferred predictively that it might have caused the alarm.

Let us now define more formally what exactly is stated by the links in a belief network. What kind of probabilistic inferences can we make given a belief network?

First we have to define a node Z to be a *descendant* of node X if there is a path, according to directed links in the network, from X to Z.

Now suppose that Y_1, Y_2, ... are the parents of X in a belief network. By definition, the belief network implies the following useful relation of probabilistic independence: X is independent of X's non-descendants given its parents. So to compute the probability of X, it is sufficient to take into account X's descendants and X's parents Y_1, Y_2, etc. All the possible effects of other variables on X are accumulated through X's parents.

This meaning of the links in a belief network turns out to provide a practical means to (a) define probabilistic relations among the variables in a world, and (b) answer questions about this world.

To understand the way a belief network is used to represent the knowledge about a world, consider the example network of Figure 15.10 again. First, the structure of the network specifies the dependences and independences among the variables.

Second, the links also have a natural causal interpretation. To complete the representation, we have to specify some probabilities, that is give some actual numbers. For the nodes that have no parents ('root causes'), *a priori* probabilities are specified. In our case burglary and lightning are root causes. For other nodes X, we have to specify the conditional probabilities of the form:

$p(X \mid \text{State of } X\text{'s parents})$

Sensor has two parents: burglary and lightning. There are four possible combined states of the two parents: burglary and lightning, burglary and not lightning, etc. These states will be written as logical formulas burglary ∧ lightning, burglary ∧ ~lightning, etc. So the complete specification of this belief network can be:

$p(\text{burglary}) = 0.001$
$p(\text{lightning}) = 0.02$
$p(\text{sensor} \mid \text{burglary} \wedge \text{lightning}) = 0.9$
$p(\text{sensor} \mid \text{burglary} \wedge \sim\text{lightning}) = 0.9$
$p(\text{sensor} \mid \sim\text{burglary} \wedge \text{lightning}) = 0.1$
$p(\text{sensor} \mid \sim\text{burglary} \wedge \sim\text{lightning}) = 0.001$
$p(\text{alarm} \mid \text{sensor}) = 0.95$
$p(\text{alarm} \mid \sim\text{sensor}) = 0.001$
$p(\text{call} \mid \text{sensor}) = 0.9$
$p(\text{call} \mid \sim\text{sensor}) = 0.0$

This complete specification comprises ten probabilities. If the structure of the network (stating the independences) were not provided, the complete specification would require $2^5 - 1 = 31$ probabilities. There are 2^n possible states of a world comprising n Boolean variables. So the structure in this network saves 21 numbers. In networks with more nodes the savings would of course be much greater.

How much can be saved depends on the problem. If every variable in the problem depends on everything else, then of course no saving is possible. However, if the problem does permit savings, then the savings will depend on the structure of a belief network. Different belief networks can be drawn for the same problem, but some networks are better than others. The general rule is that good networks respect causality between the variables. So we should make a directed link from X to Y if X causes Y. For example, in the burglary domain, although it is possible to reason from alarm to burglary, it would lead to an awkward network if we started constructing the network with a link from alarm to burglary. That would require more links in the network. Also it would be more difficult to estimate the required probabilities in a 'non-causal' direction, such as $p(\text{burglary}|\text{alarm})$.

15.6.2 Some formulas from probability calculus

In the following we recall some formulas from the probability calculus that will be useful for reasoning in belief networks. Let X and Y be propositions. Then:

$p(\sim X) = 1 - p(X)$
$p(X \wedge Y) = p(X)\, p(Y|X) = p(Y)\, p(X|Y)$
$p(X \vee Y) = p(X) + p(Y) - p(X \wedge Y)$
$p(X) = p(X \wedge Y) + p(X \wedge \sim Y) = p(Y)\, p(X|Y) + p(\sim Y)\, p(X|\sim Y)$

Propositions X and Y are said to be *independent* if $p(X|Y) = p(X)$ and $p(Y|X) = p(Y)$. That is: knowing Y does not affect the belief in X and vice versa. If X and Y are independent then:

$$p(X \wedge Y) = p(X)\, p(Y)$$

Propositions X and Y are *disjoint* if they cannot both be true at the same time: $p(X \wedge Y) = 0$ and $p(X|Y) = 0$ and $p(Y|X) = 0$.

Let $X_1, \ldots, X_n$ be propositions; then:

$$p(X_1 \wedge \ldots \wedge X_n) = p(X_1)\, p(X_2|X_1)\, p(X_3|X_1 \wedge X_2) \ldots p(X_n|X_1 \wedge \ldots \wedge X_{n-1})$$

If all X_i are independent of each other then this simplifies into:

$$p(X_1 \wedge \ldots \wedge X_n) = p(X_1)\, p(X_2)\, p(X_3) \ldots p(X_n)$$

Finally, we will need Bayes' theorem:

$$p(X|Y) = p(X)\frac{p(Y|X)}{p(Y)}$$

This formula, which follows from the law for conjunction $p(X \wedge Y)$ above, is useful for reasoning between causes and effects. Considering burglary as a cause of alarm, it is natural to think in terms of what proportion of burglaries trigger alarm. That is $p(\text{alarm}|\text{burglary})$. But when we hear the alarm, we are interested in knowing the probability of its cause, that is: $p(\text{burglary}|\text{alarm})$. Bayes' formula helps:

$$p(\text{burglary} \mid \text{alarm}) = p(\text{burglary})\frac{p(\text{alarm} \mid \text{burglary})}{p(\text{alarm})}$$

A variant of Bayes' theorem takes into account background knowledge B. It allows us to reason about the probability of a hypothesis H, given evidence E, all in the presence of background knowledge B:

$$p(H \mid E \wedge B) = p(H \mid B)\frac{p(E \mid E \wedge B)}{p(E \mid B)}$$

15.6.3 Reasoning in belief networks

In this section we will implement a program that interprets belief networks. Given a belief network, we would like this interpreter to answer queries of the form: What is the probability of some propositions, given some other proposition? Example queries are:

$p($ burglary | alarm$) = ?$
$p($ burglary $\wedge$ lightning$) = ?$
$p($ burglary | alarm $\wedge \sim$lightning$) = ?$
$p($ alarm $\wedge \sim$call | burglary$) = ?$

The interpreter will derive an answer to any of these questions by recursively applying the following rules:

(1) Probability of conjunction:

$$p(X_1 \wedge X_2 \mid Cond) = p(X_1 \mid Cond) * p(X_2 \mid X_1 \wedge Cond)$$

(2) Probability of a certain event:

$$p(X \mid Y_1 \wedge \ldots \wedge X \wedge \ldots) = 1$$

(3) Probability of impossible event:

$$p(X \mid Y_1 \wedge \ldots \wedge \sim X \wedge \ldots) = 0$$

(4) Probability of negation:

$$p(\sim X \mid Cond) = 1 - p(X \mid Cond)$$

(5) If condition involves a descendant of X then use Bayes' theorem:

→ BG KNOWLEDGE

If $Cond_0 = Y \wedge Cond$ where Y is a descendant of X in the belief network then $p(X|Cond_0) = p(X|Cond) * p(Y|X \wedge Cond) \,/\, p(Y|Cond)$

(6) Cases when condition $Cond$ does not involve a descendant of X:

(a) If X has no parents then $p(X|Cond) = p(X)$, $p(X)$ given

(b) If X has parents $Parents$ then

$$p(X \mid Cond) = \sum_{S \in possible\_states(Parents)} p(X \mid S) p(S \mid Cond)$$

As an example consider the question: What is the probability of burglary given alarm?

$p($burglary|alarm$) = ?$

By rule 5 above:

$p($burglary|alarm$) = p($burglary$) * p($alarm|burglary$) \,/\, p($alarm$)$

By rule 6:

$p($alarm|burglary$) = p($alarm|sensor$)\, p($sensor|burglary$) +$
$p($alarm|$\sim$sensor$)\, p(\sim$sensor|burglary$)$

By rule 6:

$p($sensor|burglary$) =$
$p($sensor|burglary$\wedge$lightning$)\, p($burglary$\wedge$lightning|burglary$) +$
$p($sensor|$\sim$burglary$\wedge$lightning$)\, p(\sim$burglary$\wedge$lightning|burglary$) +$
$p($sensor|burglary$\wedge\sim$lightning$)\, p($burglary$\wedge\sim$lightning|burglary$) +$
$p($sensor|$\sim$burglary$\wedge\sim$lightning$)\, p(\sim$burglary$\wedge\sim$lightning|burglary$)$

Using rules 1, 2, 3 and 4 at various places, and the conditional probabilities given in the network, we have:

$p($sensor|burglary$) = 0.9 * 0.02 + 0 + 0.9 * 0.98 + 0 = 0.9$
$p($alarm|burglary$) = 0.95 * 0.9 + 0.001 * (1 - 0.9) = 0.8551$

Using rules 1, 4 and 6 several times we get:

$p($alarm$) = 0.00467929$

Finally:

$p($burglary|alarm$) = 0.001 * 0.8551 \,/\, 0.00467929 = 0.182741$

The reasoning along these lines is implemented in Figure 15.11. The conjunction $X_1 \wedge X_2 \wedge \ldots$ of propositions is represented by a list of the propositions [X1,X2, ...]. The negation ~X is represented by the Prolog term not X. The main predicate in this program is:

prob(Proposition, Cond, P)

where P is the conditional probability of **Proposition** given **Cond**. The program expects a belief network to be represented by the following relations:

parent(ParentNode, Node): defines the structure of the network
p(X, ParentsState, P): P is the conditional probability of X given the state of the parents **ParentsState**
p(X, P): X is a root node and P is its probability

Figure 15.12 defines the belief network of Figure 15.10 using these predicates. The following conversation with the belief network program of Figure 15.11 and the network of Figure 15.12 is possible. Suppose we receive a warning phone call, so we want to know about burglary:

```
?- prob( burglary, [call], P).
P = 0.232137
```

Now we learn there was a heavy storm, so:

```
?- prob( burglary, [call, lightning], P).
P = 0.00892857
```

```
% Reasoning in belief networks

% Belief network is represented by relations:
%   parent( ParentNode, Node)
%   p( Node, ParentStates, Prob)
%     where Prob is conditional probability of Node given
%     values of parent variables ParentStates, for example:
%     p( alarm, [ burglary, not earthquake], 0.99)
%   p( Node, Prob)
%     probability of node without parents

% prob( Event, Condition, P):
%   probability of Event, given Cond, is P;
%   Event is a variable, its negation, or a list
%   of simple events representing their conjunction

prob( [X | Xs], Cond, P) :- !,                % Probability of conjunction
  prob( X, Cond, Px),
  prob( Xs, [X | Cond], PRest),
  P is Px * PRest.

prob( [], _, 1) :- !.                         % Empty conjunction

prob( X, Cond, 1) :-
  member( X, Cond), !.                        % Cond implies X

prob( X, Cond, 0) :-
  member( not X, Cond), !.                    % Cond implies X is false

prob( not X, Cond, P) :- !,                   % Probability of negation
  prob( X, Cond, P0),
  P is 1 - P0.

% Use Bayes rule if condition involves a descendant of X

prob( X, Cond0, P) :-
  delete( Y, Cond0, Cond),
  predecessor( X, Y), !,                      % Y is a descendant of X
  prob( X, Cond, Px),
  prob( Y, [X | Cond], PyGivenX),
  prob( Y, Cond, Py),
  P is Px * PyGivenX / Py.                    % Assuming Py > 0

% Cases when condition does not involve a descendant

prob( X, Cond, P) :-
  p( X, P), !.                                % X a root cause - its probability given

prob( X, Cond, P) :- !,
  findall( (CONDi,Pi), p(X,CONDi,Pi), CPlist),   % Conditions on parents
  sum_probs( CPlist, Cond, P).
```

Figure 15.11 An interpreter for belief networks.


```
% sum_probs( CondsProbs, Cond, WeightedSum)
%   CondsProbs is a list of conditions and corresponding probabilities,
%   WeightedSum is weighted sum of probabilities of Conds given Cond

sum_probs( [], _, 0).

sum_probs( [ (COND1,P1) | CondsProbs], COND, P) :-
  prob( COND1, COND, PC1),
  sum_probs( CondsProbs, COND, PRest),
  P is P1 * PC1 + PRest.

predecessor( X, not Y) :- !,                  % Negated variable Y
  predecessor( X, Y).

predecessor( X, Y) :-
  parent( X, Y).

predecessor( X, Z) :-
  parent( X, Y),
  predecessor( Y, Z).

member( X, [X | _]).

member( X, [_ | L]) :-
  member( X, L).

delete( X, [X | L], L).

delete( X, [Y | L], [Y | L2]) :-
  delete( X, L, L2).
```

```
% Belief network 'sensor'

parent( burglary, sensor).                    % Burglary tends to trigger sensor
parent( lightning, sensor).                   % Strong lightning may trigger sensor
parent( sensor, alarm).
parent( sensor, call).

p( burglary, 0.001).
p( lightning, 0.02).
p( sensor, [ burglary, lightning], 0.9).
p( sensor, [ burglary, not lightning], 0.9).
p( sensor, [ not burglary, lightning], 0.1).
p( sensor, [ not burglary, not lightning], 0.001).
p( alarm, [ sensor], 0.95).
p( alarm, [ not sensor], 0.001).
p( call, [ sensor], 0.9).
p( call, [ not sensor], 0.0).
```

Figure 15.12 A specification of the belief network of Figure 15.10 as expected by the program of Figure 15.11.

As the warning call can be explained by strong lightning, burglary becomes much less likely. However, if the weather was fine then burglary becomes more likely:

```
?- prob( burglary, [call, not lightning], P).
P = 0.473934
```

It should be noted that our implementation of belief networks aimed at a short and clear program. As a result, the program is rather inefficient. This is no problem for small belief networks like the one defined by Figure 15.12, but it would be for a larger network. However, a more efficient implementation would be considerably more complicated.

15.7 Semantic networks and frames

In this section we look at two other frameworks for representing knowledge: *semantic networks* and *frames*. These differ from rule-based representations in that they are directed to representing, in a structured way, large sets of facts. The set of facts is structured and possibly compressed: facts can be abstracted away when they can be reconstructed through inference. Both semantic networks and frames use the mechanism of inheritance in a similar way as in object-oriented programming.

Semantic networks and frames can be easily implemented in Prolog. Essentially, this amounts to adopting, in a disciplined way, a particular style of programming and organizing a program.

15.7.1 Semantic networks

A semantic network consists of entities and relations between the entities. It is customary to represent a semantic network as a graph. There are various types of semantic network with various conventions, but usually nodes of the graph correspond to entities, while relations are shown as links labelled by the names of relations. Figure 15.13 shows such a network. The relation name isa stands for 'is a'. This network represents the following facts:

- A bird is a kind of animal.
- Flying is the normal moving method of birds.
- An albatross is a bird.
- Albert is an albatross, and so is Ross.

Notice that isa sometimes relates a class of objects with a superclass of the class (animal is a superclass of bird), and sometimes an *instance* of a class with the class itself (Albert is an albatross).

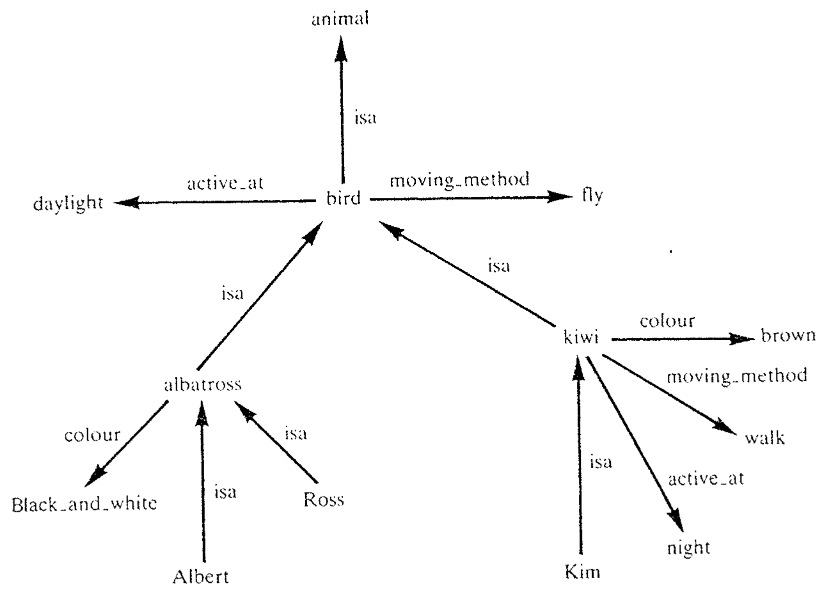


Figure 15.13 A semantic network.

A network of this kind is immediately translated into Prolog facts; for example, as:

```
isa( bird, animal).
isa( ross, albatross).
moving_method( bird, fly).
moving_method( kiwi, walk).
```

In addition to these facts, which are explicitly stated, some other facts can be inferred from the network. Ways of inferring other facts are built into a semantic network type representation as part of the representation. A typical built-in principle of inference is *inheritance*. So, in Figure 15.13, the fact 'albatross flies' is inherited from the fact 'birds fly'. Similarly, through inheritance, we have 'Ross flies' and 'Kim walks'. Facts are inherited through the isa relation. In Prolog, we can state that the method of moving is inherited as:

```
moving_method( X, Method) :-
  isa( X, SuperX),                    % Climb isa hierarchy
  moving_method( SuperX, Method).
```

It is a little awkward to state a separate inheritance rule for each relation that can be inherited. Therefore, it is better to state a more general rule about facts: facts can be either explicitly stated in the network or inherited:

```
fact( Fact) :-                    % Fact not a variable; Fact = Rel( Arg1, Arg2)
  Fact, !.                        % Fact explicit in network - do not inherit

fact( Fact) :-
  Fact =.. [ Rel, Arg1, Arg2],
  isa( Arg1, SuperArg),           % Climb isa hierarchy
  SuperFact =.. [ Rel, SuperArg, Arg2],
  fact( SuperFact).
```

Our semantic network can now be asked some questions:

```
?- fact( moving_method( kim, Method) ).
Method = walk
```

This was inherited from the explicitly given fact that kiwis walk. On the other hand:

```
?- fact( moving_method( albert, Method) ).
Method = fly
```

This was inherited from the class bird. Note that in climbing up the isa hierarchy, the inherited fact is the one encountered first.

15.7.2 Frames

In frame representation, facts are clustered around objects. 'Object' here means either a concrete physical object or a more abstract concept, such as a class of objects, or even a situation. Good candidates for representation by frames are, for example, the typical meeting situation or game conflict situation. Such situations have, in general, some common stereotype structure that can be filled with details of a particular situation.

A *frame* is a data structure whose components are called *slots*. Slots have names and accommodate information of various kinds. So, in slots, we can find simple values, references to other frames or even procedures that can compute the slot value from other information. A slot may also be left unfilled. Unfilled slots can be filled through inference. As in semantic networks, the most common principle of inference is inheritance. When a frame represents a class of objects (such as albatross) and another frame represents a superclass of this class (such as bird), then the class frame can inherit values from the superclass frame.

Some knowledge about birds can be put into frames as follows:

FRAME: bird
a_kind_of: animal
moving_method: fly
active_at: daylight

This frame stands for the class of all birds. Its three slots say that birds are a kind of animal (animal is a superclass of bird), that a typical bird flies and that a bird is

active during daylight. Here are the frames for two subclasses of bird – albatross and kiwi:

FRAME: albatross
a_kind_of: bird
colour: black_and_white
size: 115

FRAME: kiwi
a_kind_of: bird
moving_method: walk
active_at: night
colour: brown
size: 40

Albatross is a very typical bird and it inherits the flying ability and daylight activity from the frame bird. Therefore, nothing is stated about moving_method and active_at in the albatross frame. On the other hand, kiwi is a rather untypical bird, and the usual moving_method and active_at values for birds have to be overruled in the case of the kiwi. We can also have a particular *instance* of a class; for example, an albatross called Albert:

FRAME: Albert
instance_of: albatross
size: 120

Notice the difference between the two relations a_kind_of and instance_of. The former is the relation between a class and a superclass, while the latter is the relation between a member of a class and the class.

The information in our example frames can be represented in Prolog as a set of facts, one fact for each slot value. This can be done in various ways. We will choose the following format for these facts:

```
Frame_name( Slot, Value)
```

The advantage of this format is that now all the facts about a particular frame are collected together under the relation whose name is the name of the frame itself. Figure 15.14 gives some frames in this format.

To use such a set of frames, we need a procedure for retrieving facts about slot values. Let us implement such a fact retriever as a Prolog procedure

```
value( Frame, Slot, Value)
```

where Value is the value of slot Slot in frame Frame. If the slot is filled – that is, its value is explicitly stated in the frame – then this is the value; otherwise the value is obtained through inference – for example, inheritance. To find a value by

```
% A frame is represented as a set of Prolog facts:
%    frame_name( Slot, Value)
% where Value is either a simple value or a procedure

% Frame bird: the prototypical bird

bird( a_kind_of, animal).
bird( moving_method, fly).
bird( active_at, daylight).

% Frame albatross: albatross is a typical bird with some
% extra facts: it is black and white, and it is 115 cm long

albatross( a_kind_of, bird).
albatross( colour, black_and_white).
albatross( size, 115).

% Frame kiwi: kiwi is a rather untypical bird in that it
% walks instead of flies, and it is active at night

kiwi( a_kind_of, bird).
kiwi( moving_method, walk).
kiwi( active_at, night).
kiwi( size, 40).
kiwi( colour, brown).

% Frame albert: an instance of a big albatross

albert( instance_of, albatross).
albert( size, 120).

% Frame ross: an instance of a baby albatross

ross( instance_of, albatross).
ross( size, 40).

% Frame animal: slot relative_size obtains its value by
% executing procedure relative_size

animal( relative_size, execute( relative_size( Object, Value), Object, Value) ).
```

Figure 15.14 Some frames.

inheritance, we have to move from the current frame to a more general frame according to the a_kind_of relation or instance_of relation between frames. Such a move leads to a 'parent frame' and the value may be found in this frame explicitly, or through further inheritance. This direct retrieval or retrieval by inheritance can be stated in Prolog as:

```
value( Frame, Slot, Value) :-
  Query =.. [ Frame, Slot, Value],
  call( Query), !.                    % Value directly retrieved
```



```
value( Frame, Slot, Value) :-
  parent( Frame, ParentFrame),        % A more general frame
  value( ParentFrame, Slot, Value).

parent( Frame, ParentFrame) :-
  ( Query =.. [ Frame, a_kind_of, ParentFrame]
   ;
   Query =.. [ Frame, instance_of, ParentFrame] ),
  call( Query).
```

This is sufficient to find values from frames in Figure 15.14, as in the questions:

```
?- value( albert, active_at, AlbertTime).
AlbertTime = daylight

?- value( kiwi, active_at, KiwiTime).
KiwiTime = night
```

Let us now consider a more complicated case of inference when a procedure for *computing* the value is given in the slot instead of the value itself. For example, one slot for all animals could be their relative size with respect to the typical size of a grown-up instance of their species. Stated in percentages, for the two albatrosses in Figure 15.14, the relative sizes are: Albert 104 percent, Ross 35 percent. These figures are obtained as the ratio (in percentage) between the size of the particular individual and the typical size of the individual's class. For Ross, this is:

$$40/115 * 100\% = 34.78\%$$

Thus, the value for the slot relative_size is obtained by executing a procedure. This procedure is universal for all the animals, so it is rational to define the slot relative_size in the frame animal and fill this slot by the corresponding procedure. The frame interpreter value should now be able to answer the question:

```
?- value( ross, relative_size, R).
R = 34.78
```

The way this result is obtained should be something like the following. We start at the frame ross and, seeing no value for relative_size, climb up the chain of relations instance_of (to get to frame albatross), a_kind_of (to get to frame bird) and a_kind_of again, to get finally to frame animal. Here we find the procedure for computing relative size. This procedure needs the values in slot size of frame ross and frame albatross. These values can be obtained through inheritance by our existing value procedure. It remains to extend the procedure value to handle the cases where a procedure in a slot is to be executed. Before we do that, we need to consider how such a procedure can be (indirectly) represented as the content of a slot. Let the procedure for computing the relative size be implemented as a Prolog predicate:

```
relative_size( Object, RelativeSize) :-
  value( Object, size, ObjSize),
  value( Object, instance_of, ObjClass),
  value( ObjClass, size, ClassSize),
  RelativeSize is ObjSize/ClassSize * 100.      % Percentage of class size
```

We can now fill the slot relative_size in frame animal with the call of this procedure. In order to prevent the arguments Object and RelativeSize getting lost in communication between frames, we also have to state them as part of the relative_size slot information. The contents of this slot can all be put together as one Prolog term; for example, as:

```
execute( relative_size( Object, RelSize), Object, RelSize)
```

The relative_size slot of frame animal is then specified by:

```
animal( relative_size, execute( relative_size( Obj, Val), Obj, Val)).
```

Now we are ready to modify the procedure value to handle procedural slots. First, we have to realize that the information found in a slot can be a procedure call; therefore, it has to be further processed by carrying out this call. This call may need, as arguments, slot values of the original frame in question. Our old procedure value forgets about this frame while climbing up to more general frames. Therefore, we have to introduce the original frame as an additional argument. The following piece of program does this:

```
value( Frame, Slot, Value) :-
  value( Frame, Frame, Slot, Value).

% Directly retrieving information in slot of (super)frame

value( Frame, SuperFrame, Slot, Value) :-
  Query =.. [ SuperFrame, Slot, Information],
  call( Query),                                 % Value directly retrieved
  process( Information, Frame, Value), !.       % Information is either a value
                                                % or a procedure call

% Inferring value through inheritance

value( Frame, SuperFrame, Slot, Value) :-
  parent( SuperFrame, ParentSuperFrame),
  value( Frame, ParentSuperFrame, Slot, Value).

% process( Information, Frame, Value)

process( execute( Goal, Frame, Value), Frame, Value) :- !,
  call( Goal).

process( Value, _, Value).                      % A value, not procedure call
```

With this extension of our frame interpreter we have got close to the programming paradigm of object-oriented programming. Although the terminology in that paradigm is usually different, the computation is essentially based on triggering the execution of procedures that belong to various frames.

Of the many subtleties of inference among frames, we have not addressed the question of *multiple inheritance*. This problem arises when a frame has more than one 'parent' frame (according to the relation instance_of or a_kind_of). Then, an inherited slot value may potentially come from more than one parent frame and the question of which one to adopt arises. Our procedure value, as it stands, simply takes the first value encountered, found by the depth-first search among the frames that potentially can supply the value. However, other strategies or tie-breaking rules may be more appropriate.

Exercises

15.3 Trace the execution of the query

```
?- value( ross, relative_size, Value).
```

to make sure you understand clearly how information is passed throughout the frame network in our frame interpreter.

15.4 Let geometric figures be represented as frames. The following clauses represent a square s1 and a rectangle r2 and specify the method for computing the area of a figure:

```
s1( instance_of, square).
s1( side, 5).
r2( instance_of, rectangle).
r2( length, 6).
r2( width, 4).

square( a_kind_of, rectangle).
square( length, execute( value(Obj,side,L), Obj, L)).
square( width, execute( value(Obj,side,W), Obj, W)).

rectangle( area, execute( area(Obj,A), Obj, A)).

area( Obj, A) :-
  value( Obj, length, L), value( Obj, width, W),
  A is L*W.
```

How will the frame interpreter programmed in this section answer the question:

```
?- value( r2, length, A), value( s1, length, B), value( s1, area, C).
```

Summary

- Typical functions required of an expert system are: solving problems in a given domain, explaining the problem-solving process, and handling uncertain and incomplete information.

- It is convenient to view an expert system as consisting of two modules: a shell and a knowledge base. A shell, in turn, consists of an inference mechanism and a user interface.
- Building an expert system shell involves decisions regarding the knowledge representation formalism, the inference mechanism, the user interaction facility and the treatment of uncertainty.
- If-then rules, or production rules, are the most common form of representing knowledge in expert systems.
- Two basic ways of reasoning in rule-based systems are: backward and forward chaining.
- Two usual types of explanation are associated with user's questions 'How?' and 'Why?'. A proof tree can be used as a 'how' explanation.
- Reasoning with uncertainty can be incorporated into the basic rule representation, and forward or backward chaining schemes. However, such additions to rules typically make unjustified assumptions that simplify the probabilistic dependences between the variables in the domain.
- Belief networks, also called Bayesian networks, provide a way of using the probability calculus for handling uncertainty in knowledge representation. Belief networks enable: relatively economical handling of probabilistic dependencies, exploiting independences, and natural representation of causality.
- Other traditional knowledge representation schemes, suitable for representing large sets of facts in a structured way, are semantic networks and frames. Here facts can be directly retrieved or inferred through built-in mechanisms such as inheritance.
- In this chapter, stylized Prolog implementations were developed for backward and forward chaining, generating proof trees, interpreting rules with uncertainty, reasoning in belief networks, inheritance in semantic networks and frames.
- Concepts discussed in this chapter are:

  expert systems
  knowledge base, expert system shell, inference engine
  if-then rules, production systems
  backward chaining, forward chaining
  'how' explanation, 'why' explanation
  categorical knowledge, uncertain knowledge
  belief networks, Bayesian networks
  semantic networks
  frames
  inheritance

References

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chapter 16

An Expert System Shell

In this chapter we develop a complete rule-based expert system shell using the backward chaining approach introduced in Chapter 15. It incorporates proof construction along the lines introduced in Chapter 15 and some other features not covered there. We will allow variables in if-then rules and introduce a feature known as 'query the user', which prompts the user for information only as and when it is needed during the reasoning process.

16.1 Knowledge representation format

Our work plan for developing the shell will be as follows:

(1) Select and define details of the formalism for representing knowledge in the shell.

(2) Design details of an inference mechanism that suits this formalism.

(3) Add user-interaction facilities.

Let us start with item 1 – knowledge representation formalism. We will use if-then rules with a syntax similar to that in Chapter 15. However, we need some extra information in rules. For example, rules will have names. These extra details are given in Figure 16.1, which shows a knowledge base in the format accepted by our intended shell. This knowledge base consists of simple rules that help to identify animals from their basic characteristics, assuming that the identification problem is limited just to a small number of animals.

```
% A small knowledge base for identifying animals
:- op( 100, xfx, [has, gives, 'does not', eats, lays, isa] ).
:- op( 100, xf, [swims, flies] ).
rule1 :: if
            Animal has hair
            or
            Animal gives milk
        then
            Animal isa mammal.
rule2 :: if
            Animal has feathers
            or
            Animal flies and
            Animal lays eggs
        then
            Animal isa bird.
rule3 :: if
            Animal isa mammal and
            ( Animal eats meat
              or
              Animal has 'pointed teeth' and
              Animal has claws and
              Animal has 'forward pointing eyes')
        then
            Animal isa carnivore.
rule4 :: if
            Animal isa carnivore and
            Animal has 'tawny colour' and
            Animal has 'dark spots'
        then
            Animal isa cheetah.
rule5 :: if
            Animal isa carnivore and
            Animal has 'tawny colour' and
            Animal has 'black stripes'
        then
            Animal isa tiger.
rule6 :: if
            Animal isa bird and
            Animal 'does not' fly and
            Animal swims
        then
            Animal isa penguin.
```

Figure 16.1 A simple knowledge base for identifying animals. Adapted from Winston (1984). The relation 'askable' defines those things that can be asked of the user. The operators '::', **if**, **then**, **and**, **or** are declared as in Figure 16.6.


```
rule7 :: if
            Animal isa bird and
            Animal isa 'good flyer'
        then
            Animal isa albatross.
fact :: X isa animal :-
  member( X, [cheetah, tiger, penguin, albatross] ).
askable( _ gives _, 'Animal' gives 'What').
askable( _ flies, 'Animal' flies).
askable( _ lays eggs, 'Animal' lays eggs).
askable( _ eats _, 'Animal' eats 'What').
askable( _ has _, 'Animal' has 'Something').
askable( _ 'does not' _, 'Animal' 'does not' 'DoSomething').
askable( _ swims, 'Animal' swims).
askable( _ isa 'good flier', 'Animal' isa 'good flier').
```

Rules in this knowledge base are of the form:

RuleName :: if Condition then Conclusion.

where **Condition** is a set of simple assertions combined by the logical operators **and** and **or**, and **Conclusion** is a simple assertion that must not contain logical operators. We will also allow for the operator **not** to be used in the condition part of rules, although with some reservations. By an appropriate Prolog definition of operators (as in Figure 16.1) these rules are syntactically legal Prolog clauses. The operator **and** binds stronger than **or**, which is the normal convention.

Notice that assertions in rules can be terms that contain variables. Our next example shows why variables are useful and enormously enhance the power of the knowledge representation language. This example knowledge base helps to locate failures in a simple electric network that consists of some electric devices and fuses. Such a network is shown in Figure 16.2. One rule can be:

if
    light1 is on *and*
    light1 is not working *and*
    fuse1 is proved intact
then
    light1 is proved broken.

Another rule can be:

if
    heater is working
then
    fuse1 is proved intact.

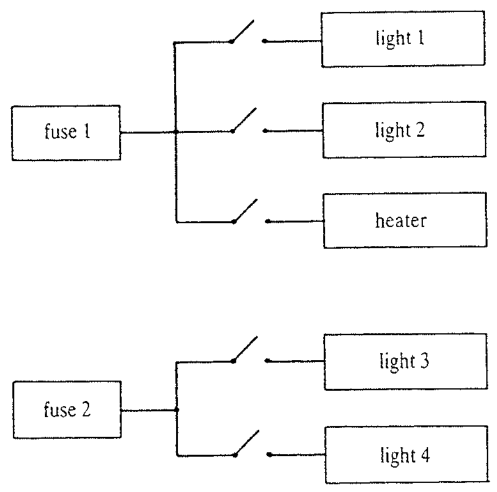

Figure 16.2 Connections between fuses and devices in a simple electric network.

```
% A small knowledge base for locating faults in an electric network

% If a device is on and not working and its fuse is intact
% then the device is broken

broken_rule ::
        if
            on( Device) and
            device( Device) and
            (not working( Device) ) and
            connected( Device, Fuse) and
            proved( intact( Fuse) )
        then
            proved( broken( Device) ).

% If a unit is working then its fuse is OK

fuse_ok_rule ::
        if
            connected( Device, Fuse) and
            working( Device)
        then
            proved( intact( Fuse) ).
```



```
% If two different devices are connected to a fuse and
% are both on and not working then the fuse has failed
% NOTE: This assumes that at most one device is broken!

fused_rule ::
        if
            connected( Device1, Fuse) and
            on( Device1) and
            (not working( Device1) ) and
            samefuse( Device2, Device1) and
            on( Device2) and
            (not working( Device2) )
        then
            proved( failed( Fuse) ).

same_fuse_rule ::
        if
            connected( Device1, Fuse) and
            connected( Device2, Fuse) and
            different( Device1, Device2)
        then
            samefuse( Device1, Device2).

fact :: different( X, Y) :- not ( X = Y).

fact :: device( heater).
fact :: device( light1).
fact :: device( light2).
fact :: device( light3).
fact :: device( light4).

fact :: connected( light1, fuse1).
fact :: connected( light2, fuse1).
fact :: connected( heater, fuse1).
fact :: connected( light3, fuse2).
fact :: connected( light4, fuse2).

askable( on( D), on( 'Device') ).
askable( working( D), working( 'Device') ).
```

Figure 16.3 A knowledge base for locating a fault in a network such as the one in Figure 16.2.

These two rules already rely on the facts (about our *particular* network) that *light1* is connected to *fuse1*, and that *light1* and *heater* share the same fuse. For another network we would need another set of rules. Therefore it is better to state rules more generally, using Prolog variables, so that they can be used for any network, and then add some extra information about a particular network. Thus one useful rule may be: if a device is on and not working and its fuse is intact then the device is broken. This translates into our rule formalism as:

```
broken_rule :: if   Device is on and
                    not ( Device is working) and
                    Device is connected to Fuse and
                    Fuse is proved intact
               then
                    Device is proved broken.
```

A knowledge base of this sort is shown in Figure 16.3.

16.2 Designing the inference engine

16.2.1 Target behaviour

As discussed in Chapter 15, if-then rules like those in Figures 16.1 and 16.3 can be rewritten as Prolog rules, and Prolog's own interpreter could be directly used as an expert system shell. However, the resulting behaviour is not quite satisfactory from the expert system point of view for at least two reasons:

(1) We cannot conveniently ask for explanation; for example, *how* was the answer derived?

(2) A complete set of data has to be input into the system (as Prolog facts) before any question can be asked. Much of this data may not be needed in the shell's inferencing about the case in hand. Thus, the user may do unnecessary work inputting irrelevant information. Worse still, the user may easily forget to provide all the relevant information, in which case the system will produce wrong answers.

To rectify these two drawbacks we need a better interaction between the user and the system during and after the reasoning process. Let us set as our goal that the system be capable of interacting with the user, as the following example conversation illustrates (user's responses are in boldface and Prolog's output is in italics):

Question, please:
peter isa tiger.

Is it true: peter has hair?
yes.

Is it true: peter eats meat?
no.

Is it true: peter has pointed teeth?
yes.

Is it true: peter has claws?
why.

To investigate, by rule3, peter isa carnivore
To investigate, by rule5, peter isa tiger
This was your question

Is it true: peter has claws?
yes.

Is it true: peter has forward pointing eyes?
yes.

Is it true: peter has tawny colour?
yes.

Is it true: peter has black stripes?
yes.

(peter isa tiger) is true

Would you like to see how?
yes.

```
peter isa tiger
  was derived by rule5 from
    peter isa carnivore
      was derived by rule3 from
        peter isa mammal
          was derived by rule1 from
            peter has hair
              was told
        and
        peter has pointed teeth
          was told
        and
        peter has claws
          was told
        and
        peter has forward pointing eyes
          was told
    and
    peter has tawny colour
      was told
    and
    peter has black stripes
      was told
```

As this conversation shows, the system asks questions of the user about 'primitive' information, such as:

Is it true: peter eats meat?

Such information cannot be found in the knowledge base or derived from other information. The user can respond to such queries in two ways:

(1) by supplying the relevant information as an answer to the query, or

(2) by asking the system *why* this information is needed.

The latter option is useful in order to enable the user to get insight into the system's current intentions. The user will ask 'why' if the system's query appears irrelevant, or if answering the query would require additional work on the part of the user. From the system's explanation the user will judge whether the information the system is asking for is worth the extra effort of obtaining that information. Suppose, for example, the system is asking 'Does the animal eat meat?' Then the user, not yet knowing the answer and not seeing the animal eating anything, may decide that it is not worth waiting to actually catch the animal at eating meat.

We might use Prolog's trace facility in order to obtain some insight into the system's reasoning process. But such a trace facility would normally prove to be too rigid for our purpose. So, instead of using Prolog's own interpreting mechanism, which falls short of this type of user interaction, we will build a special interpreter facility on top of Prolog. This new interpreter will include a user-interaction facility.

16.2.2 Outline of the reasoning process

An answer to a given question can be found in several ways, according to the following principles:

To find an answer *Answ* to a question Q use one of the following:

- if Q is found as a fact in the knowledge base then *Answ* is 'Q is true'.
- if there is a rule in the knowledge base of the form

  'if *Condition* then Q'

  then explore *Condition* and use the result to construct answer *Answ* to question Q.
- if Q is an 'askable' question then ask the user about Q.
- if Q is of the form *Q1 and Q2* then explore *Q1* and now:

  if *Q1* is false then *Answ* is 'Q is false', else explore *Q2* and appropriately combine answers to both *Q1* and *Q2* into *Answ*.
- if Q is of the form *Q1 or Q2* then explore *Q1* and now:

  if *Q1* is true then *Answ* is 'Q is true', or alternatively explore *Q2* and appropriately combine answers to both *Q1* and *Q2* into *Answ*.

Questions of the form:

not Q

are more problematic and will be discussed later.

16.2.3 Answering 'why' questions

A 'why' question occurs when the system asks the user for some information and the user wants to know *why* this information is needed. Suppose that the system has asked:

Is a true?

The user may reply:

Why?

An appropriate explanation can be along the following line:

Because:
I can use *a* to investigate *b* by rule R_a, and
I can use *b* to investigate *c* by rule R_b, and
...
I can use *y* to investigate *z* by rule R_y, and
z was your original question.

The explanation consists of showing the purpose of the information asked of the user. The purpose is shown in terms of a chain of rules and goals that connect this piece of information with the user's original question. We will call such a chain a *trace*. We can visualize a trace as a chain of rules that connects the currently explored goal and the top goal in an AND/OR tree of questions. Figure 16.4 illustrates. So, the answering of 'why' queries is accomplished by moving from the current goal upwards in the search tree toward the top goal. To be able to do that we have to maintain the trace explicitly during the backward chaining process.

16.2.4 Answering 'how' questions

Once the system has come up with an answer to the user's question, the user may like to see *how* this conclusion was reached. A proper way of answering such a 'how' question is to display the evidence: that is, rules and subgoals from which the conclusion was reached. For our rule language, such evidence consists of a proof tree. The generation of proof trees was programmed in a stylized form in Chapter 15. We will use the same principle in a more elaborate way. An effective way of

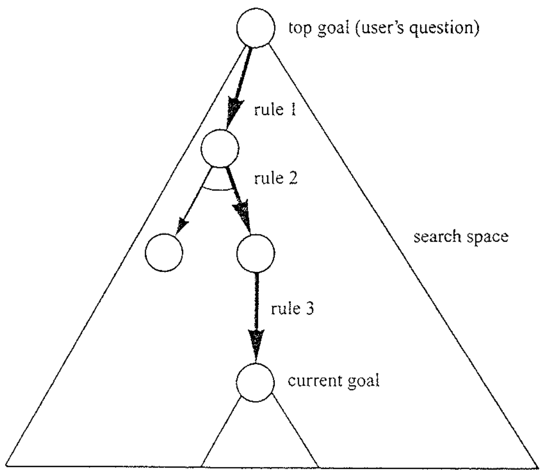

Figure 16.4 The 'why' explanation. The question 'Why are you interested in the current goal?' is explained by the chain of rules and goals between the current goal and the user's original question at the top. The chain is called a trace.

presenting proof trees as 'how' explanations is to use text indentation to convey the tree structure. For example:

peter isa carnivore
  was derived by rule3 from
    peter isa mammal
      was derived by rule1 from
        peter has hair
          was told
    and
    peter eats meat
      was told

16.3 Implementation

We will now implement our shell along the ideas developed in the previous section. Figure 16.5 illustrates the main objects manipulated by the shell. **Goal** is a question to be investigated; **Trace** is a chain of ancestor goals and rules between **Goal** and the top-level question; **Answer** is a proof tree for **Goal**.

The main procedures of the shell will be:

explore(Goal, Trace, Answer)

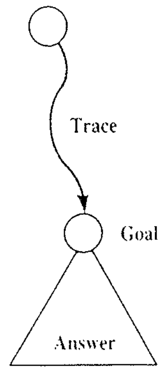

Figure 16.5 The relation **explore(Goal, Trace, Answer)**. **Answer** is a proof tree for **Goal**.

which finds an answer **Answer** to a question **Goal**;

useranswer(Goal, Trace, Answer)

generates solutions for an 'askable' **Goal** by asking the user about **Goal** and answers 'why' questions;

present(Answer)

displays the result and answers 'how' questions. These procedures are properly put into execution by the 'driver' procedure **expert**. These four main procedures are explained in the following sections and coded in Prolog in Figures 16.6 to 16.9. The programs in these figures are to be put together to form the complete shell program.

16.3.1 Procedure *explore*

The heart of the shell is the procedure:

explore(Goal, Trace, Answer)

which will find, in the backward chaining style, an answer **Answer** to a given question **Goal** by using the principles outlined in Section 16.2.2: either find **Goal** as a fact in the knowledge base, or apply a rule in the knowledge base, or ask the user, or treat **Goal** as an AND or OR combination of subgoals.

The meaning and the structure of the arguments are as follows:

Goal is a question to be investigated, represented as an AND/OR combination of simple assertions. For example:

(X has feathers) or (X flies) and (X lays eggs)

Trace is a chain of ancestor goals and rules between Goal and the original, top goal, represented as a list of items of the form:

Goal by Rule

This means that Goal is being investigated by means of rule Rule. For example, let the top goal be 'peter isa tiger', and the currently investigated goal be 'peter eats meat'. The corresponding trace, according to the knowledge base of Figure 16.1, is:

[(peter isa carnivore) by rule3, (peter isa tiger) by rule5]

This means the following:

I can use 'peter eats meat' in order
to investigate, by rule3, 'peter isa carnivore'.
Further, I can use 'peter isa carnivore' in order
to investigate, by rule5, 'peter isa tiger'.

Answer is a proof tree (that is, an AND/OR solution tree) for the question Goal. The general form for Answer is:

Conclusion was Found

where Found represents a justification for Conclusion. The following three example answers illustrate different possibilities:

```
(1)  ( connected( heater, fuse1) is true ) was 'found as a fact'
(2)  ( peter eats meat ) is false was told
(3)  ( peter isa carnivore ) is true was ( 'derived by' rule3 from
       ( peter isa mammal ) is true was ( 'derived by' rule1 from
         ( peter has hair ) is true was told ) and
       ( peter eats meat ) is true was told )
```

Figure 16.6 shows the Prolog code for explore. This code implements the principles of Section 16.2.2, using the data structures specified above.

16.3.2 Procedure *useranswer*

Procedure useranswer is an implementation of the 'query the user' facility. It queries the user for information when needed, and also explains to the user *why* it is needed. Before developing useranswer let us consider a useful auxiliary procedure:

getreply(Reply)

During conversation, the user is often expected to reply with 'yes', 'no' or 'why'. The purpose of getreply is to extract such an answer from the user and also to understand it properly if the user abbreviates ('y' or 'n') or makes a typing error. If the user's reply is unintelligible then getreply will request another reply from the user.


```
% Procedure
%
% explore( Goal, Trace, Answer)
%
% finds Answer to a given Goal. Trace is a chain of ancestor
% goals and rules. 'explore' tends to find a positive answer
% to a question. Answer is 'false' only when all the
% possibilities have been investigated and they all resulted
% in 'false'.

:- op( 900, xfx, ::).
:- op( 800, xfx, was).
:- op( 870, fx, if).
:- op( 880, xfx, then).
:- op( 550, xfy, or).
:- op( 540, xfy, and).
:- op( 300, fx, 'derived by').
:- op( 600, xfx, from).
:- op( 600, xfx, by).

% Program assumes: op( 700, xfx, is), op( 900, fx, not)

explore( Goal, Trace, Goal is true was 'found as a fact') :-
  fact :: Goal.

% Assume only one rule about each type of goal

explore( Goal, Trace,
        Goal is TruthValue was 'derived by' Rule from Answer) :-
  Rule :: if Condition then Goal,                    % Rule relevant to Goal
  explore( Condition, [Goal by Rule | Trace], Answer),
  truth( Answer, TruthValue).

explore( Goal1 and Goal2, Trace, Answer) :- !,
  explore( Goal1, Trace, Answer1),
  continue( Answer1, Goal1 and Goal2, Trace, Answer).

explore( Goal1 or Goal2, Trace, Answer) :-
  exploreyes( Goal1, Trace, Answer)                  % Positive answer to Goal1
  ;
  exploreyes( Goal2, Trace, Answer).                 % Positive answer to Goal2

explore( Goal1 or Goal2, Trace, Answer1 and Answer2) :- !,
  not exploreyes( Goal1, Trace, _),
  not exploreyes( Goal2, Trace, _),                  % No positive answer
  explore( Goal1, Trace, Answer1),                   % Answer1 must be negative
  explore( Goal2, Trace, Answer2).                   % Answer2 must be negative

explore( Goal, Trace, Goal is Answer was told) :-
  useranswer( Goal, Trace, Answer).                  % User-supplied answer
```

Figure 16.6 The core procedure of an expert system shell.

Figure 16.6 *contd*

```
exploreyes( Goal, Trace, Answer) :-
  explore( Goal, Trace, Answer),
  positive( Answer).

continue( Answer1, Goal1 and Goal2, Trace, Answer) :-
  positive( Answer1),
  explore( Goal2, Trace, Answer2),
  ( positive( Answer2),
    Answer = Answer1 and Answer2
    ;
    negative( Answer2),
    Answer = Answer2
  ).

continue( Answer1, Goal1 and Goal2, _, Answer1) :-
  negative( Answer1).

truth( Question is TruthValue was Found, TruthValue) :- !.

truth( Answer1 and Answer2, TruthValue) :-
  truth( Answer1, true),
  truth( Answer2, true), !,
  TruthValue = true
  ;
  TruthValue = false.

positive( Answer) :-
  truth( Answer, true).

negative( Answer) :-
  truth( Answer, false).
```

```
getreply( Reply) :-
  read( Answer),
  means( Answer, Reply), !                      % Answer means something?
  ;
  nl, write( 'Answer unknown, try again please'), nl,   % No
  getreply( Reply).                             % Try again

means( yes, yes).
means( y, yes).
means( no, no).
means( n, no).
means( why, why).
means( w, why).
```

Note that getreply should be used with care because it involves interaction with the user accomplished through read and write, and can therefore only be understood procedurally, and not declaratively. For example, if we call getreply with:

```
getreply( yes)
```


and the user types 'no', our procedure will respond with: 'Answer unknown, try again please.' Therefore, getreply should be called with its argument uninstantiated; for example, as:

```
getreply( Reply),
( Reply = yes, interpretyes( ...)
  ;
  Reply = no, interpretno( ...)
  ;
  ...)
```

The procedure:

```
useranswer( Goal, Trace, Answer)
```

asks the user about Goal. Answer is the result of this inquiry. Trace is used for explanation in the case that the user asks 'why'. useranswer should first check whether Goal is the kind of information that can be asked of the user. In our shell, such kinds of goal are called 'askable'. Suppose, to begin with, that we define what is askable by a relation:

```
askable( Goal)
```

This will be refined later. If Goal is 'askable' then Goal is displayed and the user will specify whether it is true or false. In the case where the user asks 'why', Trace will be displayed. If Goal is true then the user will also specify the values of variables in Goal (if there are any). This can be programmed as a first attempt as follows:

```
useranswer( Goal, Trace, Answer) :-
  askable( Goal),                        % Can Goal be asked of the user?
  ask( Goal, Trace, Answer).             % Ask user about Goal

ask( Goal, Trace, Answer) :-
  introduce( Goal),                      % Show question to user
  getreply( Reply),                      % Read user's reply
  process( Reply, Goal, Trace, Answer).  % Process the reply

process( why, Goal, Trace, Answer) :-    % User is asking 'why'
  showtrace( Trace),                     % Show why
  ask( Goal, Trace, Answer).             % Ask again

process( yes, Goal, Trace, Answer) :-    % User says Goal is true
  Answer = true,
  askvars( Goal)                         % Ask about variables
  ;
  ask( Goal, Trace, Answer).             % Ask for more solutions

process( no, Goal, Trace, false).        % User says Goal is false

introduce( Goal) :-
  nl, write( 'Is it true: '),
  write( Goal), write( ?), nl.
```

The call askvars(Goal) will ask the user to specify the value of each variable in Goal:

```
askvars( Term) :-
  var( Term), !,                        % A variable?
  nl, write( Term), write( ' = '),
  read( Term).                          % Read variable's value

askvars( Term) :-
  Term =.. [Functor | Args],            % Get arguments of a structure
  askarglist( Args).                    % Ask about variables in arguments

askarglist( [ ] ).

askarglist( [Term | Terms] ) :-
  askvars( Term),
  askarglist( Terms).
```

Let us make a few experiments with this useranswer procedure. For example, let the binary relation eats be declared as 'askable':

```
askable( X eats Y).
```

(In the following dialogue between Prolog and the user, the user-typed text is in boldface and Prolog's output is in italics.)

```
?- useranswer( peter eats meat, [ ], Answer).

Is it true: peter eats meat?            % Question to user
yes.                                    % User's reply

Answer = true
```

A more interesting example that involves variables may look like this:

```
?- useranswer( Who eats What, [ ], Answer).

Is it true: _17 eats _18?               % Prolog gives internal names to variables
yes.
_17 = peter.                            % Asking about variables
_18 = meat.

Answer = true

Who = peter

What = meat                             % Backtrack for more solutions

Is it true: _17 eats _18?
yes.
_17 = susan.
_18 = bananas.

Answer = true
Who = susan
What = bananas;

Is it true: _17 eats _18?
no.

Answer = false
```


16.3.3 Refining *useranswer*

One drawback of our useranswer procedure that shows in the foregoing conversation is the awkward appearance of Prolog-generated variable names in Prolog's output. Symbols like _17 should be replaced by some meaningful words when displayed to the user.

Another, more serious, defect of this version of useranswer is the following. If we subsequently use useranswer on the same goal, the user will have to repeat all the solutions. If our expert system would, during its reasoning process, come to explore the same 'askable' goal twice it would bore the user with exactly the same conversation again, instead of using the information previously supplied by the user.

Let us now rectify these two defects. First, an improvement of the external appearance of queries to the user can be based on introducing some standard format for each 'askable' goal. To this end, a second argument can be added to the relation askable to specify this format, as shown by the following example:

```
askable( X eats Y, 'Animal' eats 'Something').
```

In querying the user, each variable in the question should then be replaced by keywords in the question format. For example:

```
?- useranswer( X eats Y, [ ], Answer).

Is it true: Animal eats Something?
yes.
Animal = peter.
Something = meat.

Answer = true
X = peter
Y = meat
```

In an improved version of useranswer, shown in Figure 16.7, this formatting of queries is done by the procedure:

```
format( Goal, ExternFormat, Question, Vars0, Variables)
```

Goal is a goal to be formatted. ExternFormat specifies the external format for Goal, defined by:

```
askable( Goal, ExternFormat)
```

Question is Goal formatted according to ExternFormat. Variables is a list of variables that appear in Goal accompanied by their corresponding keywords (as specified in ExternFormat), added on a list Vars0. For example:

```
?- format( X gives documents to Y,
           'Who' gives 'What' to 'Whom',
           Question, [ ], Variables).

Question = 'Who' gives documents to 'Whom',
Variables = [X/'Who', Y/'Whom'].
```

```
% Procedure
%
% useranswer( Goal, Trace, Answer)
%
% generates, through backtracking, user-supplied solutions to Goal.
% Trace is a chain of ancestor goals and rules used for 'why' explanation.

useranswer( Goal, Trace, Answer) :-
  askable( Goal, _),                          % May be asked of the user
  freshcopy( Goal, Copy),                     % Variables in Goal renamed
  useranswer( Goal, Copy, Trace, Answer, 1).

% Do not ask again about an instantiated goal

useranswer( Goal, _, _, _, N) :-
  N > 1,                                      % Repeated question?
  instantiated( Goal), !,                     % No variables in Goal
  fail.                                       % Do not ask again

% Is Goal implied true or false for all instantiations?

useranswer( Goal, Copy, _, Answer, _) :-
  wastold( Copy, Answer, _),
  instance_of( Copy, Goal), !.                % Answer to Goal implied

% Retrieve known solutions, indexed from N on, for Goal

useranswer( Goal, _, _, true, N) :-
  wastold( Goal, true, M),
  M >= N.

% Has everything already been said about Goal?

useranswer( Goal, Copy, _, Answer, _) :-
  end_answers( Copy),
  instance_of( Copy, Goal), !,          % Everything was already said about Goal
  fail.

% Ask the user for (more) solutions

useranswer( Goal, _, Trace, Answer, N) :-
  askuser( Goal, Trace, Answer, N).

askuser( Goal, Trace, Answer, N) :-
  askable( Goal, ExternFormat),
  format( Goal, ExternFormat, Question, [], Variables),   % Get question format
  ask( Goal, Question, Variables, Trace, Answer, N).

ask( Goal, Question, Variables, Trace, Answer, N) :-
  nl,
```

Figure 16.7 Expert system shell: querying the user and answering 'why' questions.


```
  ( Variables = [], !,
    write( 'Is it true: ')                    % Introduce question
    ;
    write( 'Any (more) solution to: ')        % Introduce question
  ),
  write( Question), write( '? '),
  getreply( Reply), !,                        % Reply = yes/no/why
  process( Reply, Goal, Question, Variables, Trace, Answer, N).

process( why, Goal, Question, Variables, Trace, Answer, N) :-
  showtrace( Trace),
  ask( Goal, Question, Variables, Trace, Answer, N).

process( yes, Goal, _, Variables, Trace, true, N) :-
  nextindex( Next),                           % Get new free index for 'wastold'
  Next1 is Next + 1,
  ( askvars( Variables),
    assertz( wastold( Goal, true, Next) )     % Record solution
    ;
    freshcopy( Goal, Copy),                   % Copy of Goal
    useranswer( Goal, Copy, Trace, Answer, Next1)   % More answers?
  ).

process( no, Goal, _, _, _, false, N) :-
  freshcopy( Goal, Copy),
  wastold( Copy, true, _), !,                 % 'no' means: no more solutions
  assertz( end_answers( Goal) ),              % Mark end of answers
  fail
  ;
  nextindex( Next),                           % Next free index for 'wastold'
  assertz( wastold( Goal, false, Next) ).     % 'no' means: no solution

format( Var, Name, Name, Vars, [Var/Name | Vars] ) :-
  var( Var), !.

format( Atom, Name, Atom, Vars, Vars) :-
  atomic( Atom), !,
  atomic( Name).

format( Goal, Form, Question, Vars0, Vars) :-
  Goal =.. [Functor | Args1],
  Form =.. [Functor | Forms],
  formatall( Args1, Forms, Args2, Vars0, Vars),
  Question =.. [Functor | Args2].

formatall( [], [], [], Vars, Vars).

formatall( [X | XL], [F | FL], [Q | QL], Vars0, Vars) :-
  formatall( XL, FL, QL, Vars0, Vars1),
  format( X, F, Q, Vars1, Vars).

askvars( [] ).
```

Figure 16.7 *contd*

```
askvars( [Variable/Name | Variables] ) :-
  nl, write( Name), write( ' = '),
  read( Variable),
  askvars( Variables).

showtrace( [] ) :-
  nl, write( 'This was your question'), nl.

showtrace( [Goal by Rule | Trace] ) :-
  nl, write( 'To investigate, by '),
  write( Rule), write( ', '),
  write( Goal),
  showtrace( Trace).

instantiated( Term) :-
  numbervars( Term, 0, 0).                 % No variables in Term

% instance_of( T1, T2): instance of T1 is T2; that is,
% term T1 is more general than T2 or equally general as T2

instance_of( Term, Term1) :-               % Instance of Term is Term1
  freshcopy( Term1, Term2),                % Copy of Term1 with fresh set of variables
  numbervars( Term2, 0, _), !,
  Term = Term2.                            % This succeeds if Term1 is instance of Term

freshcopy( Term, FreshTerm) :-             % Make a copy of Term with variables renamed
  asserta( copy( Term) ),
  retract( copy( FreshTerm) ), !.

nextindex( Next) :-                        % Next free index for 'wastold'
  retract( lastindex( Last) ), !,
  Next is Last + 1,
  assert( lastindex( Next) ).

% Initialize dynamic procedures lastindex/1, wastold/3, end_answers/1

:- assertz( lastindex(0) ),
   assertz( wastold( dummy, false, 0) ),
   assertz( end_answers( dummy) ).
```

The other refinement, to avoid repeated questions to the user, will be more difficult. First, all user's answers should be remembered so that they can be retrieved at some later point. This can be accomplished by asserting user's answers as elements of a relation. For example:

```
assert( wastold( mary gives documents to friends, true) ).
```

In a situation where there are several user-supplied solutions to the same goal there will be several facts asserted about that goal. Here a complication arises. Suppose that variants of a goal (the goal with variables renamed) appear at several places. For example:

```
(X has Y) and                 % First occurrence – Goal1
...
(X1 has Y1) and               % Second occurrence – Goal2
...
```

Further suppose that the user will be asked (through backtracking) for several solutions to Goal1. After that the reasoning process will advance to Goal2. As we already have some solutions for Goal1 we want the system to apply them automatically to Goal2 as well (since they obviously satisfy Goal2). Now suppose that the system tries these solutions for Goal2, but none of them satisfies some further goal. So the system will backtrack to Goal2 and should ask the user for more solutions. If the user does supply more solutions then these will have to be remembered as well. In the case that the system later backtracks to Goal1 these new solutions will also have to be automatically applied to Goal1.

In order to properly use the information supplied by the user at different places we will index this information. So the asserted facts will have the form:

```
wastold( Goal, TruthValue, Index)
```

where Index is a counter of user-supplied answers. The procedure

```
useranswer( Goal, Trace, Answer)
```

will have to keep track of the number of solutions already produced through backtracking. This can be accomplished by means of another procedure, useranswer with four arguments,

```
useranswer( Goal, Trace, Answer, N)
```

where N is an integer. This call has to produce solutions to Goal indexed N or higher. A call

```
useranswer( Goal, Trace, Answer)
```

is meant to produce *all* solutions to Goal. Solutions will be indexed from 1 on, so we have the following relation:

```
useranswer( Goal, Trace, Answer) :-
  useranswer( Goal, Trace, Answer, 1).
```

An outline of

```
useranswer( Goal, Trace, Answer, N)
```

is: generate solutions to Goal by first retrieving known solutions indexed from N onwards. When these are exhausted then start querying the user about Goal and assert the thus obtained new solutions properly indexed by consecutive numbers. When the user says there are no more solutions, assert:

```
end_answers( Goal)
```

If the user says in the first place that there are no solutions at all then assert:

```
wastold( Goal, false, Index)
```

When retrieving solutions, useranswer will have to properly interpret such information.

However, there is a further complication. The user may also specify general solutions, leaving some variables uninstantiated. If a positive solution is retrieved which is more general than or as general as Goal, then there is of course no point in further asking about Goal since we already have the most general solution. If

```
wastold( Goal, false, _)
```

then an analogous decision is to be made.

The useranswer program in Figure 16.7 takes all this into account. Another argument, Copy (a copy of Goal), is added and used in several matchings in place of Goal so that the variables of Goal are not destroyed. The program also uses two auxiliary relations. One is:

```
instantiated( Term)
```

which is true if Term contains no variables. The other is:

```
instance_of( Term, Term1)
```

where Term1 is an instance of Term; that is, Term is at least as general as Term1. For example:

```
instance_of( X gives information to Y, mary gives information to Z)
```

These two procedures both rely on another procedure:

```
numbervars( Term, N, M)
```

This procedure 'numbers' the variables in Term by replacing each variable in Term by some newly generated term so that these 'numbering' terms correspond to integers between N and M – 1. For example, let these terms be of the form:

```
var/0, var/1, var/2, ...
```

Then

```
?- Term = f( X, t( a, Y, X) ), numbervars( Term, 5, M).
```

will result in:

```
Term = f( var/5, t( a, var/6, var/5) )
X  = var/5
Y  = var/6
M  = 7
```


Such a numbervars procedure is often supplied as a built-in predicate in a Prolog system. If not, it can be programmed as follows:

```
numbervars( Term, N, Nplus1)  :-
  var( Term), !,                      % Variable?
  Term = var/ N,
  Nplus1 is N + 1.

numbervars( Term, N, M)  :-
  Term =.. [Functor | Args],          % Structure or atomic
  numberargs( Args, N, M).            % Number variables in arguments

numberargs( [], N, N)  :-  !.

numberargs( [X | L], N, M)  :-
  numbervars( X, N, N1),
  numberargs( L, N1, M).
```

16.3.4 Procedure *present*

The procedure

```
present( Answer)
```

in Figure 16.8 displays the final result of a consultation session and generates the 'how' explanation. Answer includes both an answer to the user's question, and a proof tree showing *how* this conclusion was reached. Procedure present first presents the conclusion. If the user then wants to see *how* the conclusion was reached, then the proof tree is displayed in a suitable form which constitutes a 'how' explanation. This form was illustrated by an example in Sections 16.2.1 and 16.2.4.

16.3.5 Top-level driver

For a convenient access to the shell from the Prolog interpreter we need a 'driver' procedure, which may look like the procedure expert in Figure 16.9. expert starts the execution and coordinates the three main modules of the shell shown in Figures 16.6 to 16.8. For example:

```
?- expert.

Question, please:                         % Prompt the user
X isa animal and goliath isa X.           % User's question

Is it true: goliath has hair?
...
```

```
% Displaying the conclusion of a consultation and 'how' explanation

present( Answer) :-
  nl, showconclusion( Answer),
  nl, write( 'Would you like to see how? '),
  getreply( Reply),
  ( Reply = yes, !,
    show( Answer)                              % Show solution tree
    ;
    true
  ).

showconclusion( Answer1 and Answer2) :- !,
  showconclusion( Answer1), write( 'and '),
  showconclusion( Answer2).

showconclusion( Conclusion was Found) :-
  write( Conclusion).

% 'show' displays a complete solution tree

show( Solution) :-
  nl, show( Solution, 0), !.                   % Indent by 0

show( Answer1 and Answer2, H) :- !,            % Indent by H
  show( Answer1, H),
  tab( H), write( and), nl,
  show( Answer2, H).

show( Answer was Found, H) :-                  % Indent by H
  tab( H), writeans( Answer),                  % Show conclusion
  nl, tab( H),
  write( 'was '),
  show1( Found, H).                            % Show evidence

show1( Derived from Answer, H) :- !,
  write( Derived), write( 'from'),             % Show rule name
  nl, H1 is H + 4,
  show( Answer, H1).                           % Show antecedent

show1( Found, _) :-                            % Found = 'told' or 'found as fact'
  write( Found), nl.

writeans( Goal is true) :- !,
  write( Goal).                                % Omit 'is true' on output

writeans( Answer) :-                           % This is negative answer
  write( Answer).
```

Figure 16.8 Expert system shell: displaying final result and 'how' explanation.


```
% Top-level driving procedure

expert :-
  getquestion( Question),                      % Input user's question
  ( answeryes( Question)                       % Try to find positive answer
   ;
   answerno( Question)                         % If no positive answer then find negative
  ).

answeryes( Question) :-                        % Look for positive answers to Question
  markstatus( negative),                       % No positive answer yet
  explore( Question, [ ], Answer),             % Trace is empty
  positive( Answer),                           % Look for positive answers
  markstatus( positive),                       % Positive answer found
  present( Answer), nl,
  write( 'More solutions? '),
  getreply( Reply),                            % Read user's reply
  Reply = no.                                  % Otherwise backtrack to 'explore'

answerno( Question) :-                         % Look for negative answer to question
  retract( no_positive_answer_yet), !,         % Has there been no positive answer?
  explore( Question, [ ], Answer),
  negative( Answer),
  present( Answer), nl,
  write( 'More negative solutions? '),
  getreply( Reply),
  Reply = no.                                  % Otherwise backtrack to 'explore'

markstatus( negative) :-
  assert( no_positive_answer_yet).

markstatus( positive) :-
  retract( no_positive_answer_yet), !
  ;
  true.

getquestion( Question) :-
  nl, write( 'Question, please'), nl,
  read( Question).
```

Figure 16.9 Expert system shell: a 'driver'. The shell is called from Prolog through the procedure **expert**.

16.3.6 A comment on the shell program

Our shell program at some places appears to lack the declarative clarity that is typical of Prolog programs. The reason for this is that in such a shell we have to impose rather strict control over the execution because an expert system is expected not only to find an answer, but also to find it in a way that appears sensible to the user who keeps interacting with the system. Therefore, we have to implement a particular problem-solving *behaviour* and not just an input-output relation. Thus a resulting program is in fact more procedurally biased than usual. This is one example when we cannot rely on Prolog's own procedural engine, but have to specify the procedural behaviour in detail.

16.3.7 Negated goals

It seems natural to allow for negation in the left-hand sides of rules and hence also in questions that are investigated by explore. A straightforward attempt to deal with negated questions is as follows:

```
explore( not Goal, Trace, Answer) :- !,
  explore( Goal, Trace, Answer1),
  invert( Answer1, Answer).              % Invert truth value

invert( Goal is true was Found, ( not Goal) is false was Found).

invert( Goal is false was Found, ( not Goal) is true was Found).
```

This is fine if Goal is instantiated. If it is not, problems arise. Consider, for example:

```
?- expert.

Question, please:
not ( X eats meat).

Any (more) solution to: Animal eats meat?
yes.
Animal = tiger.
```

Now the system will come up with the answer:

not (tiger eats meat) is false

This is not satisfying. The problem stems from what we mean by a question such as:

not (X eats meat)

We in fact want to ask: Is there an X such that X does not eat meat? But the way this question is interpreted by explore (as defined) is as follows:

(1) Is there an X such that X eats meat?

(2) Yes, tiger eats meat.

Thus:

(3) not (tiger eats meat) is false.

In short, the interpretation is: Is it true that no X eats meat? So we will get a positive answer only in the case that *nobody* eats meat. Said another way, explore will answer the question as if X was *universally* quantified:

for *all* X: not (X eats meat)?

and not as if it was *existentially* quantified, which was our intention:

for *some* X: not (X eats meat)?

If the question explored is instantiated then this problem disappears.

Otherwise, proper treatment is more complicated. Some decisions can be as follows:

To explore *not Goal*, explore *Goal* and now:

- if *Goal* is false then (*not Goal*) is true;
- if *Goal'* is a solution of *Goal*
  and Goal' is as general as *Goal*
  then (*not Goal*) is false;
- if *Goal'* is a solution of *Goal* and
  Goal' is more specific than *Goal*
  then we cannot say anything definite about *not Goal*.

We can avoid these complications by only allowing instantiated negated goals. This can often be achieved by proper statement of rules in the knowledge base. In Figure 16.3 we achieved this in 'broken_rule':

```
broken_rule :: if
                  on( Device) and
                  device( Device) and              % Instantiate Device
                  not working( Device) and
                  connected( Device, Fuse) and
                  proved( intact( Fuse))
               then
                  proved( broken( Device)).
```

The condition

device(Device)

will 'protect' the subsequent condition

not working(Device)

from being evaluated uninstantiated.

Exercise

16.1 A knowledge base can in principle contain cycles. For example:

```
rule1:: if bottle_empty then john_drunk.
rule2:: if john_drunk then bottle_empty.
```

Using such a knowledge base, our explore procedure may start cycling between same goals. Modify explore to prevent such cycling. Trace can be used for this. However, some care is necessary: if the current goal *matches* a previous goal, this should not be considered a cycle if the current goal is more general than the previous one.

16.4 Concluding remarks

Our expert system shell can be elaborated in a number of ways. Several critical comments and suggestions for elaboration can be made at this point.

Our programs are a straightforward implementation of basic ideas, and do not pay much attention to the issue of efficiency. A more efficient implementation would require more sophisticated data structures, indexing or hierarchy of rules, etc.

Our explore procedure is susceptible to cycling if the rules in the knowledge base 'cyclicly' mention the same goal. This can be easily rectified by adding a cycle check in explore: test whether the current goal is an instance of another goal that is already on Trace.

Our 'how' explanation outputs a whole proof tree. In the case of a large proof tree it would be better to output just the top part of the tree and then let the user 'walk' through the rest of the tree as he or she wishes. The user would then inspect the proof tree selectively by using commands such as 'Move down branch 1', 'Move down branch 2', . . . , 'Move up', 'Enough'.

In the 'how' and 'why' explanations, our shell just mentions rules by their names, and does not show the rules explicitly. The user should be offered the option to request rules to be displayed explicitly during a consultation session.

Querying the user so that the dialogue looks natural proved to be complicated. Our solution works to some extent, but further problems may appear in several ways, for example:

Is it true: susan flies?
no.
Is it true: susan isa good flyer?

Of course not, if Susan cannot fly at all! Another example is:

Any (more) solution to: Somebody flies?
yes.
Somebody = bird.
Is it true: albatross flies?

To cope with such defects, additional relations between concepts dealt with by the expert system would have to be added. Typically, these new relations would specify hierarchical relations between objects and how properties are inherited. This can be done using semantic network or frame representations, as introduced in Chapter 15.

Another refinement of the user-querying procedure would involve the planning of an optimal querying strategy. The optimization objective would be to minimize the number of questions asked of the user before a conclusion is reached. There would be, of course, alternative strategies and which of them would eventually be the shortest would depend on user's answers. A decision of what alternative strategy to pursue can be based on some *a priori* probabilities to assess probabilistically the 'cost' of each alternative. This assessment might have to be updated after each user's answer.

There is another measure that can be optimized: the length of the derivation of a conclusion. This would tend to produce simple 'how' explanations. We can reduce the complexity of explanations also by selectively treating individual rules. Thus some rules would not be put into Trace and Answer in the explore procedure. In this case the knowledge base would have to specify which rules are 'traceable', and should therefore appear in explanations, and which should not.

An intelligent expert system should be probabilistically guided so that it concentrates on the currently most likely hypothesis among the competing ones. It should query the user about the information that discriminates best among the top hypotheses.

Our example expert systems were of classification, or 'analysis', type as opposed to the 'synthesis' type where the task is to *construct* something. The result can in the latter case be a plan of actions to accomplish some task – for example, a plan for a robot, a computer configuration that satisfies a given specification, or a forced combination in chess. Our fault diagnosis example can be naturally extended to involve actions; for example, if nothing can be inferred because devices are switched off the system may suggest 'Switch on light 3'. This would entail the problem of optimal plans: minimize the number of actions necessary to reach a conclusion.

Project

Consider critical comments and possible extensions to our expert system shell, as discussed, and design and implement corresponding improvements.

Summary

The shell, developed and programmed in this chapter,

interprets if-then rules,
provides 'how' and 'why' explanations, and
queries the user about the information needed.

References

The design of our expert system shell is to some degree similar to that described by Hammond (1984). Some of the examples used in the text are adapted from Winston (1984).

Hammond, P. (1984) Micro-PROLOG for Expert Systems. In: *Micro-PROLOG: Programming in Logic* (Clark, K.L. and McCabe, F.G., eds). Englewood Cliffs, NJ: Prentice Hall.
Winston, P.H. (1984) *Artificial Intelligence*, second edition. Reading, MA: Addison-Wesley.

chapter 17

Planning

Planning is a topic of traditional interest in artificial intelligence. It involves reasoning about the effects of actions and the sequencing of available actions to achieve a given cumulative effect. In this chapter we develop several simple planners that illustrate the principles of planning.

17.1 Representing actions

Figure 17.1 shows an example of a planning task. It can be solved by searching in a corresponding state space, as discussed in Chapter 11. However, here the problem will be represented in a way that makes it possible to reason explicitly about the effects of actions among which the planner can choose.

Actions change the current state of the planning world, thereby causing the transition to a new state. However, an action does not normally change everything in the current state, just some component(s) of the state. A good representation should therefore take into account this 'locality' of the effects of actions. To facilitate the reasoning about such local effects of actions, a state will be represented as a list of relationships that are currently true. Of course, we choose to mention

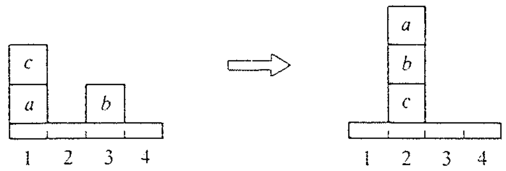

Figure 17.1 A planning problem in the blocks world: find a sequence of actions that achieve the goals: *a* on *b*, and *b* on *c*. These actions transform the initial state (on the left) into a final state (on the right).

only those relationships that pertain to the planning problem. For planning in the blocks world, such relationships are:

on(Block, Object)

and

clear(Object)

The latter expression says that there is nothing on the top of Object. In planning in the blocks world, such a relationship is important because a block to be moved must be clear, and a block or place to which the block is moved must also be clear. Object can be a block or a place. In our example, *a*, *b* and *c* are blocks, and 1, 2, 3 and 4 are places. The initial state of the world in Figure 17.1 can then be represented by the list of relationships:

[clear(2), clear(4), clear(b), clear(c), on(a, 1), on(b, 3), on(c, a)]

Notice that we distinguish between a relation and a relationship. For example, on(c, a) is a relationship; that is, a *particular instance* of the relation on. Relation on is, on the other hand, the set of all on relationships.

Each available action is defined in terms of its precondition and its effects. More precisely, each action is specified by three things:

(1) *precondition*: a condition that has to be satisfied in a situation for the action to be possible;

(2) *add-list*: a list of relationships that the action establishes in the situation – these conditions become true after the action is executed;

(3) *delete-list*: a list of relationships that the action destroys.

Preconditions will be defined by the procedure:

can(Action, Cond)

which says: Action can only be executed in a situation in which condition Cond holds.

The effects of an action will be defined by two procedures:

adds(Action, AddRels)
deletes(Action, DelRels)

Here AddRels is a list of relationships that Action establishes. After Action is executed in a state, AddRels is added to the state to obtain the new state. DelRels is a list of relationships that Action destroys. These relationships are removed from the state to which Action is applied.

For our blocks world the only kind of action will be:

move(Block, From, To)

where Block is the block to be moved, From is the block's current position and To is its new position. A complete definition of this action is given in Figure 17.2.

```
% Definition of action move( Block, From, To) in blocks world
% can( Action, Condition): Action possible if Condition true
can( move( Block, From, To), [ clear( Block), clear( To), on( Block, From)] ) :-
  block( Block),                    % Block to be moved
  object( To),                      % 'To' is a block or a place
  To \== Block,                     % Block cannot be moved to itself
  object( From),                    % 'From' is a block or a place
  From \== To,                      % Move to new position
  Block \== From.                   % Block not moved from itself
% adds( Action, Relationships): Action establishes Relationships
adds( move(X,From,To), [ on(X,To), clear(From)]).
% deletes( Action, Relationships): Action destroys Relationships
deletes( move(X,From,To), [ on(X,From), clear(To)]).
object( X) :-                       % X is an object if
  place( X)                         % X is a place
  ;                                 % or
  block( X).                        % X is a block
% A blocks world
block( a).
block( b).
block( c).
place( 1).
place( 2).
place( 3).
place( 4).
% A state in the blocks world
%
%        c
%        a   b
%       = = = =
% place 1 2 3 4
state1( [ clear(2), clear(4), clear(b), clear(c), on(a,1), on(b,3), on(c,a) ] ).
```

Figure 17.2 A definition of the planning space for the blocks world.

A definition of possible actions for a planning problem implicitly defines the space of all possible plans; therefore, such a definition will also be referred to as a *planning space*.

Planning in the blocks world is a traditional planning exercise, usually associated with robot programming when a robot is supposed to build structures from blocks. However, we can easily find many examples of planning in our everyday life. Figure 17.3 gives the definition of a planning space for manipulating a camera. This planning problem is concerned with getting a camera ready; that is, loading new film and replacing the battery if necessary. A plan to load a new film in this planning space is as follows: open the case to get the camera out, rewind old film, open the film slot, remove old film, insert new film, close the film slot. A state in which the camera is ready for taking pictures is:

```
[ slot_closed( battery), slot_closed( film), in( battery),
  ok( battery), in( film), film_at_start, film_unused]
```

The goal of a plan is stated in terms of relationships that are to be established. For the blocks world task in Figure 17.1, the goal can be stated as the list of relationships:

```
[ on( a, b), on( b, c)]
```

For the camera problem, a goal list that ensures the camera is ready is:

```
[ in( film), film_at_start, film_unused, in( battery),
  ok( battery), slot_closed( film), slot_closed( battery)]
```

In the next section we will show how such a representation can be used in a process known as 'means-ends analysis' by which a plan can be derived.

```
% Planning space for getting camera ready

% Opening the case

can( open_case, [camera_in_case]).
adds( open_case, [camera_outside_case]).
deletes( open_case, [camera_in_case]).

% Closing the case

can( close_case, [camera_outside_case, slot_closed( film), slot_closed( battery)]).
adds( close_case, [camera_in_case]).
deletes( close_case, [camera_outside_case]).

% Opening a slot (film slot or battery slot)

can( open_slot( X), [camera_outside_case, slot_closed( X)]).
adds( open_slot( X), [slot_open( X)]).
deletes( open_slot( X), [slot_closed( X)]).
```



```
% Closing a slot

can( close_slot( X), [camera_outside_case, slot_open( X)]).
adds( close_slot( X), [slot_closed( X)]).
deletes( close_slot( X), [slot_open( X)]).

% Rewinding film

can( rewind, [camera_outside_case, in( film), film_at_end]).
adds( rewind, [film_at_start]).
deletes( rewind, [film_at_end]).

% Removing battery or film

can( remove( battery), [slot_open( battery), in( battery)]).
can( remove( film), [slot_open( film), in( film), film_at_start]).
adds( remove( X), [slot_empty( X)]).
deletes( remove( X), [in( X)]).

% Inserting new battery or film

can( insert_new( X), [slot_open( X), slot_empty( X)]).
adds( insert_new( battery), [in( battery), ok( battery)]).
adds( insert_new( film), [in( film), film_at_start, film_unused]).
deletes( insert_new( X), [slot_empty( X)]).

% Taking pictures

can( take_pictures, [in( film), film_at_start, film_unused,
  in( battery), ok( battery), slot_closed( film), slot_closed( battery)]).
adds( take_pictures, [film_at_end]).
deletes( take_pictures, [film_at_start, film_unused]).

% A state with film used and battery weak (note: battery is
% assumed weak because ok( battery) is not included in the state)

state1( [camera_in_case, slot_closed( film), slot_closed( battery),
  in( film), film_at_end, in( battery)]).
```

Figure 17.3 A definition of planning space for manipulating camera.

Exercises

17.1 Extend the blocks world definition of Figure 17.2 to include other types of objects, such as pyramids, balls and boxes. Add the corresponding extra condition 'safe-to-stack' in the can relation. For example, it is not safe to stack a block on a pyramid; a ball can be safely put into a box, but not on a block (it might roll off).

17.2 Define the planning space for the monkey and banana problem introduced in Chapter 2 where the actions are 'walk', 'push', 'climb' and 'grasp'.

17.2 Deriving plans by means-ends analysis

Consider the initial state of the planning problem in Figure 17.1. Let the goal be: on(a, b). The planner's problem is to find a plan – that is, a sequence of actions – that achieves this goal. A typical planner would reason as follows:

(1) Find an action that achieves on(a, b). By looking at the adds relation, it is found that such an action has the form

    move(a, From, b)

for any From. Such an action will certainly have to be part of the plan, but we cannot execute it immediately in our initial state.

(2) Now enable the action move(a, From, b). Look at the can relation to find the action's precondition. This is:

    [clear(a), clear(b), on(a, From)]

In the initial state we already have clear(b) and on(a, From) (where From = 1), but not clear(a). Now the planner concentrates on clear(a) as the new goal to be achieved.

(3) Look at the adds relation again to find an action that achieves clear(a). This is any action of the form

    move(Block, a, To)

The precondition for this action is:

    [clear(Block), clear(To), on(Block, a)]

clear(c) clear(2) on(c, a)

This is satisfied in our initial situation if:

    Block = c and To = 2

So move(c, a, 2) can be executed in the initial state, resulting in a new state. This new state is obtained from the initial state by:

- removing from the initial state all the relationships that the action move(c, a, 2) deletes;
- adding to the resulting list all the relationships that the action adds.

This produces the list:

    [clear(a), clear(b), clear(c), clear(4), on(a, 1), on(b, 3), on(c, 2)]

Now the action move(a, 1, b) can be executed, which achieves the final goal on(a, b). The plan found can be written as the list:

    [move(c, a, 2), move(a, 1, b)]

This style of reasoning is called *means-ends analysis*. The means are the available actions, the ends are the goals to be achieved. Notice that in the foregoing example a correct plan was found immediately, without any backtracking. The example thus illustrates how the reasoning about goals and effects of actions directs the planning process in a proper direction. Unfortunately, it is not true that backtracking can always be avoided in this way. On the contrary, combinatorial complexity and search are typical of planning.

The principle of planning by means-ends analysis is illustrated in Figure 17.4. It can be stated as follows:

To solve a list of goals Goals in state State, leading to state FinalState, do:

If all the Goals are true in State then FinalState = State. Otherwise do the following steps:

(1) Select a still unsolved goal Goal in Goals.
(2) Find an action Action that adds Goal to the current state.
(3) Enable Action by solving the precondition Condition of Action, giving MidState1.
(4) Apply Action to MidState1, giving MidState2 (in MidState2, Goal is true).
(5) Solve Goals in MidState2, leading to FinalState.

This is programmed in Prolog in Figure 17.5 as the procedure:

    plan(State, Goals, Plan, FinalState)

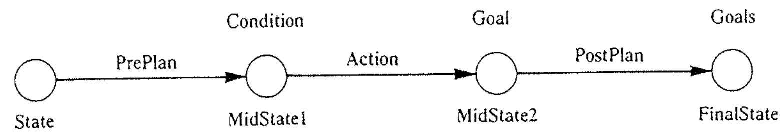

Figure 17.4 The principle of means-ends planning.

```
% A simple means-ends planner

% plan( State, Goals, Plan, FinalState)

plan( State, Goals, [], State) :-              % Plan empty
  satisfied( State, Goals).                    % Goals true in State

% The way plan is decomposed into stages by conc, the
% precondition plan (PrePlan) is found in breadth-first
% fashion. However, the length of the rest of plan is not
% restricted and goals are achieved in depth-first style.

plan( State, Goals, Plan, FinalState) :-
  conc( PrePlan, [Action | PostPlan], Plan),   % Divide plan
  select( State, Goals, Goal),                 % Select a goal
  achieves( Action, Goal),                     % Relevant action
  can( Action, Condition),
  plan( State, Condition, PrePlan, MidState1), % Enable Action
  apply( MidState1, Action, MidState2),        % Apply Action
  plan( MidState2, Goals, PostPlan, FinalState). % Achieve remaining goals

% satisfied( State, Goals): Goals are true in State

satisfied( State, []).

satisfied( State, [Goal | Goals]) :-
  member( Goal, State),
  satisfied( State, Goals).

select( State, Goals, Goal) :-
  member( Goal, Goals),
  not member( Goal, State).                    % Goal not satisfied already

% achieves( Action, Goal): Goal is add-list of Action

achieves( Action, Goal) :-
  adds( Action, Goals),
  member( Goal, Goals).

% apply( State, Action, NewState): Action executed in State produces NewState

apply( State, Action, NewState) :-
  deletes( Action, DelList),
  delete_all( State, DelList, State1), !,
  adds( Action, AddList),
  conc( AddList, State1, NewState).

% delete_all( L1, L2, Diff) if Diff is set-difference of L1 and L2

delete_all( [], _, []).

delete_all( [X | L1], L2, Diff) :-
  member( X, L2), !,
  delete_all( L1, L2, Diff).

delete_all( [X | L1], L2, [X | Diff]) :-
  delete_all( L1, L2, Diff).
```


Figure 17.5 A simple means-ends planner.

State and FinalState are the initial and final states of the plan respectively. Goals is the list of goals to be achieved and Plan is a list of actions that achieve the goals. It should be noted that this planning program assumes a definition of the planning space in which all the actions and goals are fully instantiated; that is, they do not contain any variables. Variables would require more complicated treatment. This will be discussed later.

The planner can now be used to find a plan for placing block *a* on *b*, starting in the initial state of Figure 17.1, as follows:

```
?- Start = [ clear( 2), clear( 4), clear( b), clear( c), on( a, 1), on( b, 3), on( c, a)],
   plan( Start, [ on( a, b)], Plan, FinState).

Plan = [ move( c, a, 2), move( a, 1, b)]
FinState = [ on( a, b), clear( 1), on( c, 2), clear( a), clear( 4), clear( c), on( b, 3)]
```

For the camera world, imagine an initial state with the battery weak and the film in the camera already used. Here are queries to find: a plan FixBattery to fix the battery; a plan FixCamera to get the camera ready for taking pictures:

```
?- Start = [ camera_in_case, slot_closed( film), slot_closed( battery), in( film),
            film_at_end, in( battery)],
   plan( Start, [ ok( battery)], FixBattery, _).

FixBattery = [ open_case, open_slot( battery), remove( battery), insert_new( battery)]

?- Start = [ camera_in_case, slot_closed( film), slot_closed( battery), in( film),
            film_at_end, in( battery)],
   can( take_pictures, CameraReady),   % Condition for taking pictures
   plan( Start, CameraReady, FixCamera, FinState).

CameraReady = [ in( film), film_at_start, film_unused, in( battery),
               ok( battery), slot_closed( film), slot_closed( battery)]

FixCamera = [ open_case, rewind, open_slot( film), remove( film),
             insert_new( film), open_slot( battery), remove( battery),
             insert_new( battery), close_slot( film), close_slot( battery)]

FinState = [ slot_closed( battery), slot_closed( film), in( battery), ok( battery),
            in( film), film_at_start, film_unused, camera_outside_case]
```

All this is very smooth: the shortest plans are found in all cases. However, further experiments with our planner would reveal some difficulties. We will analyze the defects and introduce improvements in the next section.

Exercise

17.3 Trace by hand the means-ends planning process for achieving on(a, 3) from the initial situation of Figure 17.1.

17.3 Protecting goals

If we try our planner on the blocks and camera worlds, it turns out that the blocks world is much more complex. This may appear surprising because the definition of the blocks world looks simpler than that of the camera world. The real reason for the greater difficulty of the blocks world lies in its higher combinatorial complexity. In the blocks world, the planner usually has more choice between several actions that all make sense with respect to the means-ends principle. More choice leads to higher combinatorial complexity. Experimenting with our planner on the blocks domain is thus more critical and can reveal several defects of the planner.

Let us try the task in Figure 17.1. Suppose that Start is a description of the initial state in Figure 17.1. Then the task of Figure 17.1 can be stated as the goal:

```
plan( Start, [ on( a, b), on( b, c)], Plan, _)
```

The plan found by the planner is:

```
Plan = [ move( b, 3, c),
         move( b, c, 3),
         move( c, a, 2),
         move( a, 1, b),
         move( a, b, 1),
         move( b, 3, c),
         move( a, 1, b)]
```

This plan containing seven steps is not exactly the most elegant! The shortest possible plan for this task only needs three steps. Let us analyze why our planner needs so many. The reason is that it pursues different goals at different stages of planning, as follows:

```
move( b, 3, c)   to achieve goal on( b, c)
move( b, c, 3)   to achieve clear( c) to enable next move
move( c, a, 2)   to achieve clear( a) to enable move( a, 1, b)
move( a, 1, b)   to achieve goal on( a, b)
move( a, b, 1)   to achieve clear( b) to enable move( b, 3, c)
move( b, 3, c)   to achieve goal on( b, c) (again)
move( a, 1, b)   to achieve goal on( a, b) (again)
```


The defect revealed here is that the planner sometimes destroys goals that have already been achieved. The planner easily achieved one of the two given goals, on(b, c), but then destroyed it immediately when it started to work on the other goal on(a, b). Then it attempted the goal on(b, c) again. This was reachieved in two moves, but on(a, b) was destroyed in the meantime. Luckily, on(a, b) was then reachieved without destroying on(b, c) again. This rather disorganized behaviour leads, even more drastically, to total failure in the next example:

```
plan( Start, [ clear( 2), clear( 3)], Plan, _)
```

The planner now indefinitely keeps extending the following sequence of moves:

```
move( b, 3, 2)   to achieve clear( 3)
move( b, 2, 3)   to achieve clear( 2)
move( b, 3, 2)   to achieve clear( 3)
move( b, 2, 3)   to achieve clear( 2)
...
```

Each move achieves one of the goals and at the same time destroys the other one. The planning space is unfortunately defined so that the places 2 and 3 are always considered first in moving *b* from its current position to a new position.

One idea that is obvious from the foregoing examples is that the planner should try to preserve the goals that have already been achieved. This can be done by maintaining the list of already achieved goals and, in the sequel, avoiding those actions that destroy goals in this list. Thus, we introduce a new argument into our plan relation:

```
plan( State, Goals, ProtectedGoals, Plan, FinalState)
```

Here ProtectedGoals is a list of goals that Plan 'protects'. That is, no action in Plan may delete any of the goals in ProtectedGoals. Once a new goal is achieved, it is added to the list of protected goals. The program in Figure 17.6 is a modification of the planner of Figure 17.5 with goal protection introduced. The task of clearing 2 and 3 is now solved by a two-step plan:

```
move( b, 3, 2)   achieving clear( 3)
move( b, 2, 4)   achieving clear( 2) while protecting clear( 3)
```

This is now clearly better than before, although still not optimal because only one move is really necessary: move(b, 3, 4).

Unnecessarily long plans result from the search strategy that our planner uses. To optimize the length of plans, the search behaviour of the planner should be studied. This will be done in the next section.

```
% A means-ends planner with goal protection

plan( InitialState, Goals, Plan, FinalState) :-
  plan( InitialState, Goals, [ ], Plan, FinalState).

% plan( InitialState, Goals, ProtectedGoals, Plan, FinalState):
%    Goals true in FinalState, ProtectedGoals never destroyed by Plan

plan( State, Goals, _, [ ], State) :-
  satisfied( State, Goals).                         % Goals true in State

plan( State, Goals, Protected, Plan, FinalState) :-
  conc( PrePlan, [Action | PostPlan], Plan),        % Divide plan
  select( State, Goals, Goal),                      % Select an unsatisfied goal
  achieves( Action, Goal),
  can( Action, Condition),
  preserves( Action, Protected),                    % Do not destroy protected goals
  plan( State, Condition, Protected, PrePlan, MidState1),
  apply( MidState1, Action, MidState2),
  plan( MidState2, Goals, [Goal | Protected], PostPlan, FinalState).

% preserves( Action, Goals): Action does not destroy any one of Goals

preserves( Action, Goals) :-                        % Action does not destroy Goals
  deletes( Action, Relations),
  not ( member( Goal, Relations),
        member( Goal, Goals) ).
```

Figure 17.6 A means-ends planner with goal protection. Predicates **satisfied**, **select**, **achieves** and **apply** are defined as in Figure 17.5.

17.4 Procedural aspects and breadth-first regime

The planners in Figures 17.5 and 17.6 use essentially depth-first search strategies, although not entirely. To get a clear insight into what is going on, we have to find in what order candidate plans are generated by the planner. The goal

```
conc( PrePlan, [Action | PostPlan], Plan)
```

in procedure plan is important in this respect. **Plan** is not yet instantiated at this point and conc generates, through backtracking, alternative candidates for **PrePlan** in the following order:

```
PrePlan = [ ];
PrePlan = [ _ ];
PrePlan = [ _, _ ];
PrePlan = [ _, _, _ ];
...
```


Short candidates for **PrePlan** come first. **PrePlan** establishes a precondition for **Action**. This entails the finding of an action whose precondition can be achieved by as short a plan as possible (in the iterative deepening fashion). On the other hand, the candidate list for **PostPlan** is completely uninstantianted, and thus its length is unlimited. Therefore, the resulting search behaviour is 'globally' depth first and 'locally' breadth first. It is depth-first search with respect to the forward chaining of actions that are appended to the emerging plan. Each action is enabled by a 'preplan'. This preplan is, on the other hand, found in the breadth-first fashion.

One way of minimizing the length of plans is to force the planner into the breadth-first regime so that *all* alternative short plans are considered before any longer one. We can impose this strategy by embedding our planner into a procedure that generates candidate plans in the order of increasing length. For example, the following results in iterative deepening:

```
breadth_first_plan( State, Goals, Plan, FinalState) :-
  candidate( Plan),                                 % Generate short plans first
  plan( State, Goals, Plan, FinalState).

candidate( [ ]).

candidate( [First | Rest]) :-
  candidate( Rest).
```

More elegantly, the same effect can be programmed by inserting a corresponding candidate plan generator directly into our **plan** procedure. Such a suitable generator is simply

```
conc( Plan, _, _)
```

which through backtracking generates lists of increasing length. Our planner of Figure 17.5 then becomes modified as follows:

```
plan( State, Goals, Plan, FinState) :-
  conc( Plan, _, _),                   % Generate short candidates first
  conc( PrePlan, [Action | PostPlan], Plan),
  ...
```

Similar modification also brings the goal-protecting planner of Figure 17.6 into the breadth-first regime.

Let us try our modified planners, working now in the breadth-first regime, on our two example tasks. Assuming that **Start** is the initial situation of Figure 17.1, the goal

```
plan( Start, [ clear( 2), clear( 3)], Plan, _)
```

produces:

```
Plan = [ move( b, 3, 4)]
```

This is now optimal. But the task of Figure 17.1 will still be somewhat problematical. The goal

```
plan( Start, [ on( a, b), on( b, c)], Plan, _ )
```

produces the plan:

```
move( c, a, 2)
move( b, 3, a)
move( b, a, c)
move( a, 1, b)
```

We get this result with both of the planners, with or without protection, working in the breadth-first regime. The second move above is superfluous and apparently makes no sense. Let us investigate how it came to be included in the plan at all and why even the breadth-first search resulted in a plan longer than optimal.

We have to answer two questions. First, what reasoning led the planner to construct the funny plan above? Second, why did the planner not find the optimal plan in which the mysterious move(b, 3, a) is not included? Let us start with the first question. The last move, move(a, 1, b), achieves the goal on(a, b). The first three moves achieve the precondition for move(a, 1, b), in particular the condition clear(a). The third move clears *a*, and part of the precondition for the third move is on(b, a). This is achieved by the second move, move(b, 3, a). The first move clears *a* to enable the second move. This explains the reasoning behind our awkward plan and also illustrates what sort of exotic ideas may appear during means-ends planning.

The second question is: Why after move(c, a, 2) did the planner not immediately consider move(b, 3, c), which leads to the optimal plan? The reason is that the planner was working on the goal on(a, b) all the time. The action move(b, 3, c) is completely superfluous to this goal and hence not tried. Our four-step plan achieves on(a, b) and, by chance, also on(b, c). So on(b, c) is a result of pure luck and not of any conscious effort by the planner. Blindly pursuing just the goal on(a, b) and relevant preconditions, the planner saw no reason for move(b, 3, c) before move(b, 3, a).

It follows from the above example that the means-ends mechanism of planning as implemented in our planners is *incomplete*. It does not suggest all relevant actions to the planning process. The reason for this lies in its *locality*. The planner only considers those actions that pertain to the current goal and disregards other goals until the current goal is achieved. Therefore, it does not (unless by accident) produce plans in which actions that pertain to different goals are interleaved. The key question to completeness, thereby ensuring that optimal plans *are* within the planning scheme, is to enable interaction between different goals. This will be done in the next section through the mechanism of *goal regression*.

Before concluding this section, a note is needed regarding the efficiency of the depth-first and breadth-first planners discussed here. Although the breadth-first regime produced a much shorter plan for our example problem (although still suboptimal!), the computation time needed for this shorter plan is much longer than the time needed by the depth-first planner to find the longer seven-step plan. So the depth-first planner should not be considered *a priori* inferior to the breadth-first planner, even if it is inferior with respect to the length of plans. It should also be noted that the breadth-first effect was in our planners achieved through the technique of iterative deepening, discussed in Chapter 11.

Exercise

17.4 The natural places where domain-specific planning knowledge can be introduced into our planner are the predicates select and achieves. They select the next goal to be attempted (determining the order in which goals are achieved) and the action to be tried. Redefine these two predicates for the blocks world, so that the goals and actions are more intelligently selected. For this purpose, it is useful to add the current state State as an extra argument to the predicate achieves.

17.5 Goal regression

Suppose we are interested in a list of goals Goals being true in some state S. Let the state just before S be S0 and the action in S0 be A. Now let us ask the question: What goals Goals0 have to be true in S0 to make Goals true in S?

$$\text{state S0: Goals0} \xrightarrow[\text{A}]{} \text{state S: Goals}$$

Goals0 must have the following properties:

(1) Action A must be possible in S0, therefore Goals0 must imply the precondition for A.

(2) For each goal G in Goals either:

- action A adds G, or
- G is in Goals0 and A does not delete G.

Determining Goals0 from given Goals and action A will be called *regressing* Goals *through* action A. Of course, we are only interested in those actions that add some goal G in Goals. The relations between various sets of goals and conditions are illustrated in Figure 17.7.

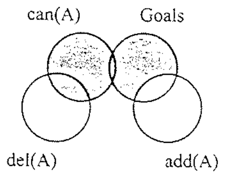

Figure 17.7 Relations between various sets of conditions in goal regression through action A. The shaded area represents the resulting regressed goals Goals0:
Goals0 = can(A) ∪ Goals – add(A). Notice that the intersection between Goals and the delete-list of A must be empty.

The mechanism of goal regression can be used for planning in the following way:

To achieve a list of goals Goals from some initial situation StartState, do:

If Goals are true in StartState then the empty plan suffices;

Otherwise select a goal G in Goals and an action A that adds G; then regress Goals through A obtaining NewGoals and find a plan for achieving NewGoals from StartState.

This can be improved by observing that some combinations of goals are impossible. For example, on(a, b) and clear(b) cannot be true at the same time. This can be formulated as the relation

```
impossible( Goal, Goals)
```

which says that Goal is not possible in combination with goals Goals; that is, Goal *and* Goals can never be achieved because they are incompatible. For our blocks world, these incompatible combinations can be defined as follows:

```
impossible( on( X, X), _ ).                  % Block cannot be on itself

impossible( on( X, Y), Goals) :-
  member( clear( Y), Goals)
  ;
  member( on( X, Y1), Goals), Y1 \== Y       % Block cannot be in two places
  ;
  member( on( X1, Y), Goals), X1 \== X.      % Two blocks cannot be in same place

impossible( clear( X), Goals) :-
  member( on( _, X), Goals).
```

A planner based on goal regression as outlined here is programmed in Figure 17.8. This program considers candidate plans in the breadth-first style when shorter plans are tried first. This is ensured by the goal

```
conc( PrePlan, [Action], Plan)
```


in the procedure plan. This planner finds the optimal, three-step plan for the problem of Figure 17.1.

Exercise

17.5 Trace the planning process, based on goal regression, for achieving on(a, b) from the initial situation of Figure 17.1. Suppose the plan is:

```
[ move( c, a, 2), move( a, 1, b)]
```

If the goal-list after the second action of the plan is [on(a, b)], what are the regressed goal-lists before the second move and before the first move?

```
% A means-ends planner with goal regression

% plan( State, Goals, Plan)

plan( State, Goals, [ ]) :-
  satisfied( State, Goals).                       % Goals true in State

plan( State, Goals, Plan) :-
  conc( PrePlan, [Action], Plan),                 % Divide plan achieving breadth-first effect
  select( State, Goals, Goal),                    % Select a goal
  achieves( Action, Goal),
  can( Action, Condition),                        % Ensure Action contains no variables
  preserves( Action, Goals),                      % Protect Goals
  regress( Goals, Action, RegressedGoals),        % Regress Goals through Action
  plan( State, RegressedGoals, PrePlan).

satisfied( State, Goals) :-
  delete_all( Goals, State, [ ]).                 % All Goals in State

select( State, Goals, Goal) :-                    % Select Goal from Goals
  member( Goal, Goals).                           % A simple selection principle

achieves( Action, Goal) :-
  adds( Action, Goals),
  member( Goal, Goals).

preserves( Action, Goals) :-                      % Action does not destroy Goals
  deletes( Action, Relations),
  not ( member( Goal, Relations),
        member( Goal, Goals) ).

regress( Goals, Action, RegressedGoals) :-        % Regress Goals through Action
  adds( Action, NewRelations),
  delete_all( Goals, NewRelations, RestGoals),
  can( Action, Condition),
  addnew( Condition, RestGoals, RegressedGoals).      % Add precond., check imposs.
```

Figure 17.8 A planner based on goal regression. This planner searches in breadth-first style.

Figure 17.8 *contd*

```
% addnew( NewGoals, OldGoals, AllGoals):
%   OldGoals is the union of NewGoals and OldGoals
%   NewGoals and OldGoals must be compatible

addnew( [ ], L, L).

addnew( [Goal | _], Goals, _) :-
  impossible( Goal, Goals),          % Goal incompatible with Goals
  !,
  fail.                              % Cannot be added

addnew( [X | L1], L2, L3) :-
  member( X, L2), !,                 % Ignore duplicate
  addnew( L1, L2, L3).

addnew( [X | L1], L2, [X | L3]) :-
  addnew( L1, L2, L3).

% delete_all( L1, L2, Diff): Diff is set-difference of lists L1 and L2

delete_all( [ ], _, [ ]).

delete_all( [X | L1], L2, Diff) :-
  member( X, L2), !,
  delete_all( L1, L2, Diff).

delete_all( [X | L1], L2, [X | Diff]) :-
  delete_all( L1, L2, Diff).
```

17.6 Combining means-ends planning with best-first heuristic

The planners developed thus far only use very basic search strategies: depth-first or breadth-first search (iterative deepening) or a combination of these two. These strategies are completely uninformed in the sense that they do not use any domain-specific knowledge in choosing among alternatives. Consequently, they are in principle very inefficient, except in some special cases. There are several ways of introducing heuristic guidance, based on domain knowledge, into our planners. Some obvious places where domain-specific planning knowledge can be introduced into the planners are as follows:

- relation select(State, Goals, Goal), which decides in what order the goals are attempted. For example, a piece of useful knowledge about building block structures is that, at any time, everything has to be properly supported, and therefore structures have to be built in the bottom-up order. A heuristic selection rule based on this would say: the 'top-most' on relations should be achieved last (that is, they should be selected first by the goal regression planner as it builds plans backwards). Another heuristic would suggest that the selection of those goals that are already true in the initial situation should be deferred.
- relation achieves(Action, Goal), which decides which alternative action will be tried to achieve the given goal. (In fact, our planners also generate alternatives when executing the can predicate, when actions become instantiated.) Some actions seem better, for example, because they achieve several goals simultaneously; alternatively, through experience, we may be able to tell that some action's precondition will be easier to satisfy than others.
- decision about which of the alternative regressed goal sets to consider next: continue working on the one that looks easiest, because that one will probably be accomplished by the shortest plan.

This last possibility shows how we can impose the best-first search regime on our planner. This involves heuristically estimating the difficulty of alternative goal sets and then continuing to expand the most promising alternative goal set.

To use our best-first search programs of Chapter 12, we have to specify the corresponding state space and a heuristic function; that is, we have to define the following:

(1) A successor relation between the nodes in the state space s(Node1, Node2, Cost).
(2) The goal nodes of the search by relation goal(Node).
(3) A heuristic function by relation h(Node, HeuristicEstimate).
(4) The start node of the search.

One way of formulating the state space is for goal sets to correspond to nodes in the state space. Then, in the state space there will be a link between two goal sets Goals1 and Goals2 if there is an action A such that

(1) A adds some goal in Goals1,
(2) A does not destroy any goal in Goals1, and
(3) Goals2 is a result of regressing Goals1 through action A, as already defined by relation regress in Figure 17.8:

```
regress( Goals1, A, Goals2)
```

For simplicity we will assume that all the actions have the same cost, and will accordingly assign cost 1 to all the links in the state space. Thus the successor relation will look like this:

```
s( Goals1, Goals2, 1) :-
  member( Goal, Goals1),             % Select a goal
  achieves( Action, Goal),           % A relevant action
  can( Action, Condition),
  preserves( Action, Goals1),
  regress( Goals1, Action, Goals2).
```

Any goal set that is true in the initial situation of the plan is a goal node of the state-space search. The start node for search is the list of goals to be achieved by the plan.

Although the representation above contains all the essential information, it has a small deficiency. This is due to the fact that our best-first search program finds a solution path as a sequence of states and does not include actions between states. For example, the sequence of states (goal-lists) for achieving on(a, b) in the initial situation in Figure 17.1 is:

```
[ [ clear( c), clear( 2), on( c, a), clear( b), on( a, 1)],     % True in initial situation
  [ clear( a), clear( b), on( a, 1)],                           % True after move( c, a, 2)
  [ on( a, b)] ]                                                % True after move( a, 1, b)
```

Notice that the best-first program returns the solution path in the inverse order. In our case this is in fact an advantage because plans are built in the backward direction, so the inverse sequence returned by the search program corresponds to the actual order in the plan. It is, however, awkward that the actions are not explicitly mentioned in the plan, although they could be reconstructed from the differences in the given goal-lists. But we can easily get actions explicitly into the solution path. We only have to add, to each state, the action that follows the state. So nodes of the state space become pairs of the form:

```
Goals -> Action
```

The state-space implementation in Figure 17.9 contains this representational detail. This implementation uses a very crude heuristic function. This is simply the number of yet unsatisfied goals in the goal-list.

```
% State space representation of means-ends planning with goal regression

:- op( 300, xfy, ->).

s( Goals -> NextAction, NewGoals -> Action, 1) :-    % All costs are 1
  member( Goal, Goals),
  achieves( Action, Goal),
  can( Action, Condition),
  preserves( Action, Goals),
  regress( Goals, Action, NewGoals).

goal( Goals -> Action) :-
  start( State),                                      % User-defined initial situation
  satisfied( State, Goals).                           % Goals true in initial situation

h( Goals -> Action, H) :-                             % Heuristic estimate
  start( State),
  delete_all( Goals, State, Unsatisfied),             % Unsatisfied goals
  length( Unsatisfied, H).                            % Number of unsatisfied goals
```

Figure 17.9 A state-space definition for means-ends planning based on goal regression. Relations **satisfied**, **achieves**, **preserves**, **regress**, **addnew** and **delete_all** are as defined in Figure 17.8.

The state-space definition in Figure 17.9 can now be used by the best-first programs of Chapter 12 as follows. We have to consult the planning problem definition in terms of relations adds, deletes and can (Figure 17.2 for the blocks world). We also have to supply the relation impossible and the relation start, which describes the initial situation of the plan. For the situation in Figure 17.1, this is:

```
start( [ on( a, 1), on( b, 3), on( c, a), clear( b), clear( c), clear( 2), clear( 4)].
```

To solve the task of Figure 17.1 by our means-ends best-first planner, we can now call the best-first procedure by:

```
?- bestfirst( [ on( a, b), on( b, c)] -> stop, Plan).
```

The null action stop is added here because in our representation each goal-list must be followed, at least syntactically, by an action. But [on(a, b), on(b, c)] is the final goal-list of the plan, which is not actually followed by any action. The solution is presented by the following list of goal-lists and actions between them:

```
Plan = [
 [ clear( 2), on( c, a), clear( c), on( b, 3), clear( b), on( a, 1)] -> move( c, a, 2),
 [ clear( c), on( b, 3), clear( a), clear( b), on( a, 1)] -> move( b, 3, c),
 [ clear( a), clear( b), on( a, 1), on( b, c)] -> move( a, 1, b),
 [ on( a, b), on( b, c)] -> stop]
```

Although this best-first planner uses the simple-minded heuristic estimates, it is fast compared with our other planners.

Exercises

17.6 Consider the simple-minded heuristic function for the blocks world of Figure 17.9. Does this function satisfy the condition of the admissibility theorem for best-first search? (See Chapter 12 for the admissibility theorem.)

17.7 The example heuristic function for the blocks world developed in this chapter simply counts the goals to be achieved. This is very crude as some goals are clearly more difficult than others. For example, it is trivial to achieve on(a, b) if blocks *a* and *b* are already clear, whereas it is difficult if *a* and *b* are buried under high stacks of other blocks. Therefore, a better heuristic function would try to estimate the difficulty of individual goals – for example, take into account the number of blocks to be removed before the block of interest could be moved. Propose such better heuristic functions and experiment with them.

17.8 Modify the planning state-space definition of Figure 17.9 to introduce the cost of actions:

```
s( State1, State2, Cost)
```

The cost can, for example, depend on the weight of the object moved and the distance by which it is moved. Use this definition to find minimal cost plans in the blocks world.

17.7 Uninstantiated actions and partial-order planning

The planners developed in this chapter are implemented in programs made as simple as possible to illustrate the principles. No thought has been given to their efficiency. By choosing better representations and corresponding data structures, significant improvements in efficiency are possible. Our planners can also be enhanced by two other essential mechanisms: allowing uninstantiated variables in goals and actions, and partial-order planning. They are briefly discussed in this section.

17.7.1 Uninstantiated actions and goals

All our algorithms were greatly simplified by the requirement that all the goals for the planner should always be completely instantiated. This requirement was attained by a proper definition of the planning space (relations adds, deletes and can). In Figure 17.2, for example, the complete instantiation of variables is forced by the can relation, defined as:

```
can( move( Block, From, To), [ clear( Block), clear( To), on( Block, From)] ) :-
  block( Block),
  object( To),
  ...
```

Goals like block(Block) above force the instantiation of variables. This may lead to the generation of numerous irrelevant alternative moves that are considered by the planner. For example, consider the situation in Figure 17.1 when the planner is asked to achieve the goal clear(a). The relation achieves proposes this general move to achieve clear(a):

```
move( Something, a, Somewhere)
```

Then the precondition is found by:

```
can( move( Something, a, Somewhere), Condition)
```

This forces, through backtracking, various alternative instantiations of Something and Somewhere. So all the following moves are considered before the one that works is found:


```
move( b, a, 1)
move( b, a, 2)
move( b, a, 3)
move( b, a, 4)
move( b, a, c)
move( c, a, 1)
move( c, a, 2)
```

A more powerful representation that avoids this inefficiency would allow uninstantiated variables in goals. For the blocks world, for example, one attempt to define such an alternative relation can would be simply:

```
can( move( Block, From, To), [ clear( Block), clear( To), on( Block, From)] ).
```

Now consider the situation in Figure 17.1 and the goal clear(a) again. Relation achieves again proposes the action:

```
move( Something, a, Somewhere)
```

This time, when can is evaluated, the variables remain uninstantiated, and the precondition list is:

```
[ clear( Something), clear( Somewhere), on( Something, a)]
```

Notice that this goal-list, which the planner now has to attempt, contains variables. The goal-list is satisfied immediately in the initial situation if:

```
Something = c
Somewhere = 2
```

The key to this improvement in efficiency, where the right move was found virtually without search, is that uninstantiated moves and goals stand for *sets* of alternative moves and goals. Their instantiation is deferred until later, when it becomes clear what their values should be. On the other hand, the specification in Figure 17.2 forces immediate instantiation of actions and thereby also goals in action preconditions.

The foregoing example demonstrates the power of representation with variables. However, this method is not without complications. First, the foregoing attempt to define can is inadequate because it allows, in the situation of Figure 17.1, the move:

```
move( c, a, c)
```

As a result we get a situation where block *c* is on itself! Thus, a better definition of can would not allow a block to be moved to itself, as well as other similar restrictions. Here is such a definition:

```
can( move( Block, From, To),
    [ clear( Block), clear( To), on( Block, From), different( Block, To),
      different( From, To), different( Block, From)] ).
```

Here different(X, Y) means that X and Y do not denote the same object. A condition like different(X, Y) does not depend on the state of the world. So it cannot be made true by an action, but it has to be checked by evaluating the corresponding predicate. One way to handle such quasi goals is to add the following extra clause to the procedure satisfied in our planners:

```
satisfied( State, [Goal | Goals]) :-
  holds( Goal),                     % Goal independent of State
  satisfied( Goals).
```

Accordingly, predicates like different(X, Y) would have to be defined by the procedure holds:

```
holds( different( X, Y))
```

This definition could be along the following lines:

- If X and Y do not match then different(X, Y) holds.
- If X and Y are literally the same (X == Y) then this condition is false, and it will always be false regardless of further actions in the plan. Such conditions could then be handled in a similar way as goals declared in the relation impossible.
- Otherwise (X and Y match, but are not literally the same), we cannot tell at the moment. The decision whether they denote the same object or not should be postponed until X and Y become further instantiated.

As illustrated by this example, the evaluation of conditions like different(X, Y) that are independent of the state sometimes has to be deferred. Therefore, it would be practical to maintain such conditions as an extra argument of the procedure plan and handle them separately from those goals that are achieved by actions.

This is not the only complication introduced by variables. For example, consider the move:

```
move( a, 1, X)
```

Does this move delete the relation clear(b)? Yes, if X = b, and not, if different(X, b). This means that we have two possibilities, and the corresponding two alternatives are associated with the value of X: in one alternative, X is equal to b; in the other, an extra condition different(X, Y) is added.

17.7.2 Partial-order planning

One deficiency of our planners is that they consider all possible orderings of actions even when the actions are completely independent. As an example, consider the planning task of Figure 17.10 where the goal is to build two stacks of blocks from two disjoint sets of blocks that are already well separated. The two stacks can be built independently of each other by two plans, one for each stack:

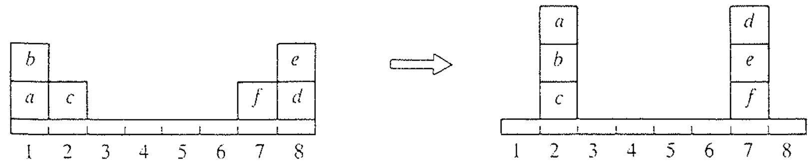

Figure 17.10 A planning task consisting of two independent subtasks.

```
Plan1 = [ move( b, a, c), move( a, 1, b)]
Plan2 = [ move( e, d, f), move( d, 8, e)]
```

The important point is that these two plans do not interact with each other. Therefore, only the order of actions *within* each plan is important; it does not matter in which order they are executed – first P1 and then P2, or first P2 and then P1, or it is even possible to switch between them and execute a bit of one and then a bit of the other. For example, this is an admissible execution sequence:

```
[ move( b, a, c), move( e, d, f), move( d, 8, e), move( a, 1, b)]
```

In spite of this, our planners would, in the planning process, potentially consider all 24 permutations of the four actions, although there are essentially only four alternatives – two permutations for each of the two constituent plans. This problem arises from the fact that our planners strictly insist on *complete* ordering of actions in a plan. An improvement then would be to allow, in the cases that the order does not matter, the precedence between actions to remain unspecified. Thus our plans would be *partially ordered sets* of actions, instead of totally ordered sequences. Planners that allow partial ordering are called *partial-order planners* (sometimes also *non-linear planners*).

Let us look at the main idea of partial-order planning by considering the example of Figure 17.1 again. The following is a sketch of how a non-linear planner may solve this problem. Analyzing the goals on(a, b) and on(b, c), the planner concludes that the following two actions will necessarily have to be included into the plan:

```
M1 = move( a, X, b)
M2 = move( b, Y, c)
```

There is no other way of achieving the two goals. But the order in which these two actions are to be executed is not yet specified. Now consider the preconditions of the two actions. The precondition for move(a, X, b) contains clear(a). This is not satisfied in the initial situation; therefore, we need an action of the form:

```
M3 = move( U, a, V)
```

This has to precede the move M1, so we now have a constraint on the order of the actions:

```
before( M3, M1)
```

Now we see if M2 and M3 can be the same move that achieves the objectives of both. This is not the case, so the plan will have to include three different moves. Now the planner has to answer the question: Is there a permutation of the three moves [M1, M2, M3] such that M3 precedes M1, the permutation is executable in the initial situation and the overall goals are satisfied in the resulting state? Due to the precedence constraint, only three out of altogether six permutations are taken into account:

```
[ M3, M1, M2]
[ M3, M2, M1]
[ M2, M3, M1]
```

The second of these satisfies the execution constraint by the instantiation: U = c, V = 2, X = 1, Y = 3. As can be seen from this example, the combinatorial complexity cannot be completely avoided by partial-order planning, but it can be alleviated.

Projects

Develop a program, using the techniques of this chapter, for planning in a more interesting variation of the simple blocks world used throughout this chapter. Figure 17.11 shows an example task in this new world. The new world contains blocks of different sizes and the stability of structures has to be taken into account. To make the problem easier, assume that blocks can only be placed at whole-numbered positions, and they always have to be properly supported, so that they are safely stable. Also assume that they are never rotated by the robot and the move trajectories are simple: straight up until the block is above any other block, then horizontally, and then straight down. Design specialized heuristics to be used by this planner.

A robot world, more realistic and interesting than the one defined in Figure 17.2, would also comprise perception actions by a camera or touch sensor. For example, the action look(Position, Object) would recognize the object seen by the camera at Position (that is, instantiate variable Object). In such a world it is realistic to assume that the scene is not completely known to the robot, so it will possibly include, in its plan, actions whose only purpose is to acquire information. This can be further complicated by the fact that some measurements of this kind cannot be done

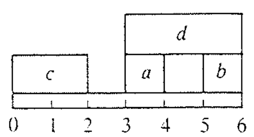

Figure 17.11 A planning task in another blocks world: Achieve on(a, c), on(b, c), on(c, d).

immediately, as some objects cannot be seen (bottom objects cannot be seen by a top-view camera). Introduce other relevant goal relations and modify our planners if necessary.

Modify the goal regression planner of Figure 17.8 so that it will correctly handle variables in goals and actions, according to the discussion in Section 17.7.1.

Write a non-linear planner according to the outline in Section 17.7.2.

Summary

- In planning, available actions are represented in a way that enables explicit reasoning about their effects and their preconditions. This can be done by stating, for each action, its precondition, its add-list (relationships the action establishes) and delete-list (relationships the action destroys).
- Means-ends derivation of plans is based on finding actions that achieve given goals and enabling the preconditions for such actions.
- *Goal protection* is a mechanism that prevents a planner destroying goals already achieved.
- Means-ends planning involves search through the space of relevant actions. The usual search techniques therefore apply to planning as well: depth-first, breadth-first and best-first search.
- To reduce search complexity, domain-specific knowledge can be used at several stages of means-ends planning, such as: which goal in the given goal-list should be attempted next; which action among the alternative actions should be tried first; heuristically estimating the difficulty of a goal-list in best-first search.
- Goal regression is a process that determines what goals have to be true before an action, to ensure that given goals are true after the action. Planning with goal regression typically involves backward chaining of actions.
- Allowing uninstantiated variables in goals and actions can make planning more efficient; however, on the other hand, it significantly complicates the planner.
- Partial-order planning recognizes the fact that actions in plans need not be always totally ordered. Leaving the order unspecified whenever possible allows economical treatment of sets of equivalent permutations of actions.
- Concepts discussed in this chapter are:

  action precondition, add-list, delete-list
  means-ends principle of planning
  goal protection
  goal regression
  partial-order planning

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The early studies of basic ideas of means-ends problem solving and planning in artificial intelligence were done by Newell, Shaw and Simon (1960). These ideas were implemented in the celebrated program GPS (General Problem Solver) whose behaviour is studied in detail in Ernst and Newell (1969). Another historically important planning program is STRIPS (Fikes and Nilsson 1971; 93), which can be viewed as an implementation of GPS. STRIPS introduced the representation of the planning space by the relations **adds**, **deletes** and **can**, which is also used in this chapter. Two other logic-based representations for planning (not discussed in this chapter) are situation calculus (Kowalski 1979) and event calculus (Kowalski and Sergot 1986; Shanahan 1997). The mechanisms of STRIPS, and various related ideas and refinements, are described in Nilsson (1980), where the elegant formulations of planning in logic by Green and Kowalski are also presented. Warren's (1974) program WARPLAN is an early interesting planner written in Prolog. It can be viewed as another implementation of STRIPS, refined in a certain respect. The WARPLAN program appeared in other places in the literature – for example, in Coelho and Cotta (1988). Waldinger (1977) studied phenomena of interaction among conjunctive goals, among them also the principle of goal regression. Goal regression corresponds to determining the *weakest precondition* used in proving program correctness. Difficulties that a planner with goal protection may have with the problem of Figure 17.1 are known in the literature as the Sussman anomaly (see, for example, Waldinger 1977). Early developments of partial-order planning are Sacerdoti (1977) and Tate (1977). Chapman (1987) is an attempt to provide a uniform theoretical framework for describing and investigating various mechanisms in partial-order planning. Weld (1994) gives an overview of partial-order planning. Guaranteed completeness of a planner is theoretically desirable, but its price in terms of combinatorial complexity is high. Therefore, a practically more promising approach is one by Clark (1985) who studied various ways of introducing domain-specific knowledge into a planner to reduce combinatorial complexity. Allen, Hendler and Tate (1990) edited a collection of classical papers on planning. The general books on AI by Russell and Norvig (1995) and by Poole *et al.* (1998) include substantial material on planning. High quality papers on planning appear in the *Artificial Intelligence* journal: see, for example, AIJ volume 76 (1995), a special issue on planning and scheduling.

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chapter 18

Machine Learning

Of the forms of learning, learning concepts from examples is the most common and best understood. Learning algorithms depend on the language in which the learned concepts are represented. In this chapter we develop programs that learn concepts represented by if-then rules and decision trees. We also look at the pruning of decision trees as a method for learning from noisy data, when examples possibly contain errors.

18.1 Introduction

There are several forms of learning, ranging from 'learning by being told' to 'learning by discovery'. In the former case, the learner is explicitly told what is to be learned. In this sense, programming is a kind of learning by being told. The main burden in this type of learning is on the teacher although the learner's task can also be difficult as it may not be easy to understand what the teacher had in mind. So learning by being told may require intelligent communication including a learner's model of the teacher. At the other extreme, in learning by discovery, the learner autonomously discovers new concepts merely from unstructured observations or by planning and performing experiments in the environment. There is no teacher involved here and all the burden is on the learner. The learner's environment plays the role of an oracle.

Between these two extremes lies another form of learning: learning from examples. Here the initiative is distributed between the teacher and the learner. The teacher provides examples for learning and the learner is supposed to make generalizations about the examples – that is, find a kind of theory, explaining the given examples. The teacher can help the learner by selecting good training examples and by describing the examples in a language that permits formulating elegant general rules. In a sense, learning from examples exploits the known empirical observation that experts (that is 'teachers') find it easier to produce good examples than to provide explicit and complete general theories. On the other hand, the task of the learner to generalize from examples can be difficult.

Learning from examples is also called *inductive learning*. Inductive learning is the most researched kind of learning in artificial intelligence and this research has produced many solid results. From examples, several types of task can be learned: one can learn to diagnose a patient or a plant disease, to predict weather, to predict the biological activity of a new chemical compound, to determine the biological degradability of chemicals, to predict mechanical properties of steel on the basis of its chemical characteristics, to make better financial decisions, to control a dynamic system, or to improve efficiency in solving symbolic integration problems.

Machine learning techniques have been applied to all these particular tasks and many others. Practical methods exist that can be effectively used in complex applications. One application scenario is in association with knowledge acquisition for expert systems. Knowledge is acquired automatically from examples, thus helping to alleviate the knowledge-acquisition bottleneck. Another way of applying machine learning methods is *knowledge discovery in databases* (KDD), also called *data mining*. Data in a database are used as examples for inductive learning to discover interesting patterns in large amounts of data. For example, it can be found with data mining, that in a supermarket a customer who buys spaghetti is likely also to buy Parmesan cheese.

In this chapter we will be mostly concerned with learning concepts from examples. We will first define the problem of learning concepts from examples more formally. To illustrate key ideas we will then follow a detailed example of learning concepts represented by semantic networks. Then we will look at induction of rules and decision trees.

18.2 The problem of learning concepts from examples

18.2.1 Concepts as sets

The problem of learning concepts from examples can be formalized as follows. Let U be a universal set of objects – that is, all of the objects that the learner may encounter. There is, in principle, no limitation on the size of U. A concept C can be formalized as a subset of objects in U. To learn concept C means to learn to recognize objects in C. In other words, once C is learned, the system is able, for any object X in U, to recognize whether X is in C.

This definition of concept is sufficiently general to enable the formalization of such diverse concepts as an arch, a certain disease, arithmetic multiplication, or the concept of poisonous:

- The concept of poisonous: For example, in the world U of mushrooms, the concept 'poisonous' is the set of all poisonous mushrooms.
- The concept of an arch in the blocks world: The universal set U is the set of all structures made of blocks in a blocks world. Arch is the subset of U containing all the arch-like structures and nothing else.
- The concept of multiplication: The universal set U is the set of tuples of numbers. *Mult* is the set of all triples of numbers (a, b, c) such that $a * b = c$. More formally:

  $Mult = \{(a, b, c) | a * b = c\}$
- The concept of a certain disease D: U is the set of all possible patient descriptions in terms of some chosen repertoire of features. D is the set of all those descriptions of patients that suffer from the disease in question.

18.2.2 Examples and hypotheses

To introduce some terminology, consider the following, hypothetical problem of learning whether a mushroom is edible or poisonous. We have collected a number of example mushrooms and for each of them we have an expert opinion. Suppose that each mushroom is (unrealistically simply!) described just by its height and its width. We say that each of our example objects has two *attributes*: height and width, in centimetres. In our case both attributes are numerical. In addition, for each example mushroom its *class* is also given: poisonous or edible. From the point of view of the concept 'edible', the two class values are appropriately abbreviated into '+' (edible) and '−' (not edible). Accordingly, the given edible mushrooms are *positive examples* for the concept 'edible'; the given poisonous mushrooms are *negative examples* for the concept 'edible'.

Figure 18.1 shows our learning data. To learn about mushrooms, then, means to be able to classify a new mushroom into one of the two classes '+' or '−'. Suppose now that we have a new mushroom whose attributes are: W = 3, H = 1. Is it edible or poisonous? Looking at the examples in Figure 18.1, most people would say 'edible' without much thought. Of course, there is no guarantee that this mushroom actually is edible, and there may be a surprise. So this classification is still a *hypothesis*. However, this hypothesis seems very likely because the attribute values of this mushroom are similar to those of many known edible mushrooms, and dissimilar to all the poisonous ones. In general, the main assumption in machine learning is that objects that look in some way similar also belong to the same class. Generally, the world appears to be kind in that this similarity assumption is usually true in real life, and this is what makes machine learning from examples possible. However, how one determines that two objects are similar and others are not is another question. What is the measure of similarity, either explicit or implicit? Learning systems differ significantly in this respect.

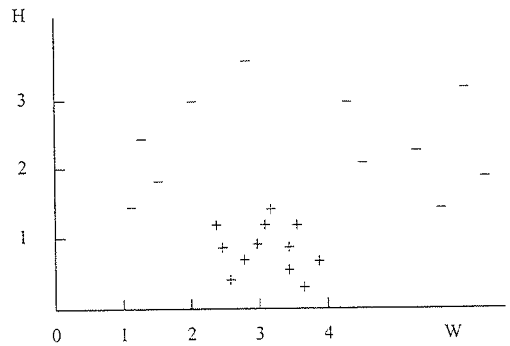


Figure 18.1 Examples for learning about mushrooms. The attributes are W (width) and H (height) of a mushroom. The pluses indicate examples of edible mushrooms, the minuses examples of poisonous ones.

For the same reason of similarity, another mushroom with W = 5 and H = 4 would quite obviously seem to be poisonous. However, a mushroom with W = 2 and H = 2 is hard to decide and any classification seems reasonable and risky.

Usually the result of learning is a *concept description*, or a *classifier* that will classify new objects. Such a classifier can be stated in various ways, using various formalisms. These formalisms are alternatively called *concept description languages* or *hypothesis languages*. The reason for calling them *hypothesis* languages is that they describe the learner's hypotheses, on the basis of the learning data, about the *target concept*. Usually the learner is never sure that a hypothesis, induced from the data, actually does correspond to the target concept.

Here are some possible hypotheses that can be induced from the mushroom data:

Hypothesis 1: If $2 < W$ and $W < 4$ and $H < 2$
then 'edible' else 'poisonous'

Hypothesis 2: If $H > W$ then 'poisonous'
else if $H > 6 - W$ then 'poisonous'
else 'edible'

Hypothesis 3: If $H < 3 - (W - 3)^2$ then 'edible' else 'poisonous'

These hypotheses are illustrated in Figure 18.2. All three hypotheses are stated in the form of if-then rules. Another popular hypothesis language in machine learning

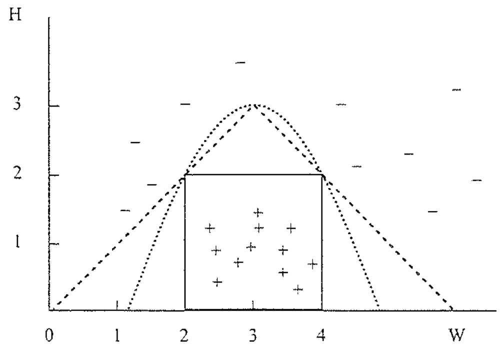


Figure 18.2 Three hypotheses about edible mushrooms: hypothesis 1 – solid line, hypothesis 2 – dashed line, hypothesis 3 – dotted line.

in AI is that of decision trees. Hypothesis 1 is represented as a decision tree in Figure 18.3.

All the three hypotheses are *consistent* with the data: they classify all the learning objects into the same class as given in the examples. However, there are differences between the hypotheses when classifying new objects. According to hypothesis 1, a mushroom with $W = 3$ and $H = 2.5$ is classified as poisonous; according to hypotheses 2 and 3, this mushroom is edible. From the point of view of the concept 'edible', hypothesis 1 is said to be the *most specific* of the three hypotheses. Hypotheses 2 and 3 are said to be more *general* than hypothesis 1. The set of mushrooms that are poisonous according to hypothesis 1 is a subset of those according to hypothesis 2 or 3. On the other hand, hypothesis 2 is neither more general nor more specific than hypothesis 3.

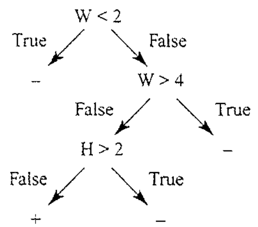


Figure 18.3 Hypothesis 1 represented by a decision tree. Internal nodes of the tree are labelled by attributes, the leaves by classes, and branches correspond to attribute values. For example, the left-most branch corresponds to $W < 2$. The left-most leaf says that a mushroom that falls into this leaf is poisonous (class '–'). An object falls into a particular leaf if the object satisfies all the conditions along the path from the root to the leaf.

18.2.3 Description languages for objects and concepts

For any kind of learning we need a language for describing objects and a language for describing concepts (hypothesis language). In general we distinguish between two major kinds of description:

- *relational descriptions* and
- *attribute-value descriptions*.

In a relational description an object is described in terms of its components and the relations between them. A relational description of an arch may say the following: an arch is a structure consisting of three components (two posts and a lintel); each component is a block; both posts support the lintel; the posts do not touch. Such a description is *relational* because it talks about relations between components. On the other hand, in an attribute-value description we describe an object in terms of its global features. Such a description is a vector of attribute values. An attribute description may, for example, mention the attributes length, height and colour. So an attribute-value description of a particular arch may be: length = 9 m, height = 7 m, colour = yellow.

Attribute-value descriptions are a special case of relational descriptions. Attributes can be formalized as components of an object. So an attribute-value description is always easily translated into a relational description language. On the other hand, transforming a relational description into an attribute-value description is often awkward and sometimes impossible.

Description languages that can be used in machine learning are of course similar to those that can be used for representing knowledge in general. Some formalisms often used in machine learning are:

- attribute-value vectors to represent objects;
- if-then rules to represent concepts;
- decision trees to represent concepts;
- semantic networks;
- predicate logic of various types (for example, Prolog).

Semantic networks, if-then rules and decision trees will be used later in this chapter. Using predicate logic in machine learning is called *inductive logic programming* (ILP). ILP is the topic of Chapter 19.

18.2.4 Accuracy of hypotheses

The problem of learning from examples is usually formulated as follows. There i some target concept C that we want to learn about. There is some hypothesi language L in which we can state hypotheses about C. No definition of C is known and the only source of information for learning about C is a set of classified examples. Usually, examples are pairs (Object, Class) where Class says what concept Object belongs to. The aim of learning is to find a formula H in the hypothesis language L so that H corresponds as well as possible to the target concept C. However, how can we know how well H corresponds to C? The only way to estimate how well H corresponds to C is to use the example set S. We can evaluate how well H performs on the example set S. If H usually classifies the examples in S correctly (that is, into the same classes as given in the examples), then we may hope that H will classify other, new objects correctly as well. So one sensible policy is to choose, among possible hypotheses, one that re-classifies all the example objects into the same class as given in the set S. Such a hypothesis is said to be *consistent* with the data. A consistent hypothesis has 100 percent classification accuracy on the learning data. Of course, we are more interested in a hypothesis' predictive accuracy. How well does the hypothesis predict the class of new objects, those not given in S? The prediction accuracy is the probability of correctly classifying a randomly chosen object in the domain of learning. Possibly surprisingly, it sometimes turns out that hypotheses that achieve the highest accuracy on the learning data S will not stand the best chance to also achieve the highest accuracy on new data, outside S. This observation pertains particularly to the case of learning from noisy data when the learning data contain errors. This will be discussed in Section 18.6.

The most usual criterion of success in inductive machine learning is the predictive accuracy of the induced hypothesis. However, there are other criteria of success, most notably the criterion of *comprehensibility* or 'understandability' of induced hypotheses. How meaningful is an induced hypothesis to a human expert? This will be discussed in more detail in Section 18.7.

18.3 Learning relational descriptions: a detailed example

Learning about structures in the blocks world, and about arches in particular, was introduced as a study domain by Winston (1975) in his early work in machine learning. We will use it here to illustrate some important mechanisms involved in learning. Our treatment will basically, although not entirely, follow Winston's program called ARCHES.

The program ARCHES can be made to learn the concept of an arch from examples as shown in Figure 18.4. The given examples are processed sequentially and the learner gradually updates the current hypothesis about the arch concept. In the case of Figure 18.4, after all four examples have been processed by the learner, the hypothesis (that is, the learner's final understanding of an arch) may informally look like this:

(1) An arch consists of three parts; let us call them **post1**, **post2** and **lintel**.

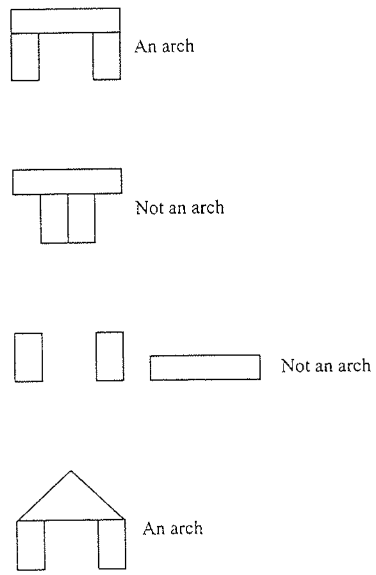


Figure 18.4 A sequence of examples and counter-examples for learning the concept of arch.

(2) **post1** and **post2** are rectangles; **lintel** can be a more general figure, a kind of polygon, for example (this may be concluded from examples 1 and 4 in Figure 18.4).

(3) **post1** and **post2** must not touch (this can be concluded from the negative example 2).

(4) **post1** and **post2** must suport **lintel** (this can be concluded from the negative example 3).

In general, when a concept is learned by sequentially processing the learning examples, the learning process proceeds through a sequence of hypotheses, H_1, H_2, etc., about the concept that is being learned. Each hypothesis in this sequence is an approximation to the target concept and is the result of the examples seen so far. After the next example is processed, the current hypothesis is updated resulting in the next hypothesis. This process can be stated as the following algorithm:

To learn a concept C from a given sequence of examples $E_1, E_2, \ldots, E_n$ (where E_1 must be a positive example of C) do:

(1) Adopt E_1 as the initial hypothesis H_1 about C.

(2) Process all the remaining examples: for each E_i ($i = 2, 3, \ldots$) do:

2.1 Match the current hypothesis H_{i-1} with E_i; let the result of matching be some description D of the differences between H_{i-1} and E_i.

2.2 Act on H_{i-1} according to D and according to whether E_i is a positive or a negative example of C. The result of this is a refined hypothesis H_i about C.

The final result of this procedure is H_n, which represents the system's understanding of the concept C as learned from the given examples. In an actual implementation, steps 2.1 and 2.2 need some refinements. These are complicated and vary between different learning systems. To illustrate some ideas and difficulties, let us consider in more detail the case of learning about the arch from the examples in Figure 18.4.

First, we have to become more specific about the representation. The ARCHES program uses semantic networks to represent both learning examples and concept descriptions. Figure 18.5 shows examples of such semantic networks. These are graphs in which nodes correspond to entities and links indicate relations between entities.

The first example, represented by a semantic network, becomes the current hypothesis of what an arch is (see H_1 in Figure 18.5).

The second example (E_2 in Figure 18.5) is a negative example of an arch. It is easy to match E_2 to H_1. As both networks are very similar it is easy to establish the correspondence between the nodes and links in H_1 and E_2. The result of matching shows the difference D between H_1 and E_2. The difference is that there is an extra relation, **touch**, in E_2. Since this is the only difference, the system concludes that this must be the reason why E_2 is not an arch. The system now updates the current hypothesis H_1 by applying the following general heuristic principle of learning:

if
  example is negative and
  example contains a relation R which is not in the current hypothesis H
then
  forbid R in H (add **must_not_R** in H)

The result of applying this rule on H_1 will be a new hypothesis H_2 (see Figure 18.5). Notice that the new hypothesis has an extra link **must_not_touch**, which imposes an additional constraint on a structure should it be an arch. Therefore we say that this new hypothesis H_2 is *more specific* than H_1.

The next negative example in Figure 18.4 is represented by the semantic network E_3 in Figure 18.5. Matching this to the current hypothesis H_2 reveals two differences: two **support** links, present in H_2, are not present in E_3. Now the learner has to make a guess between three possible explanations:

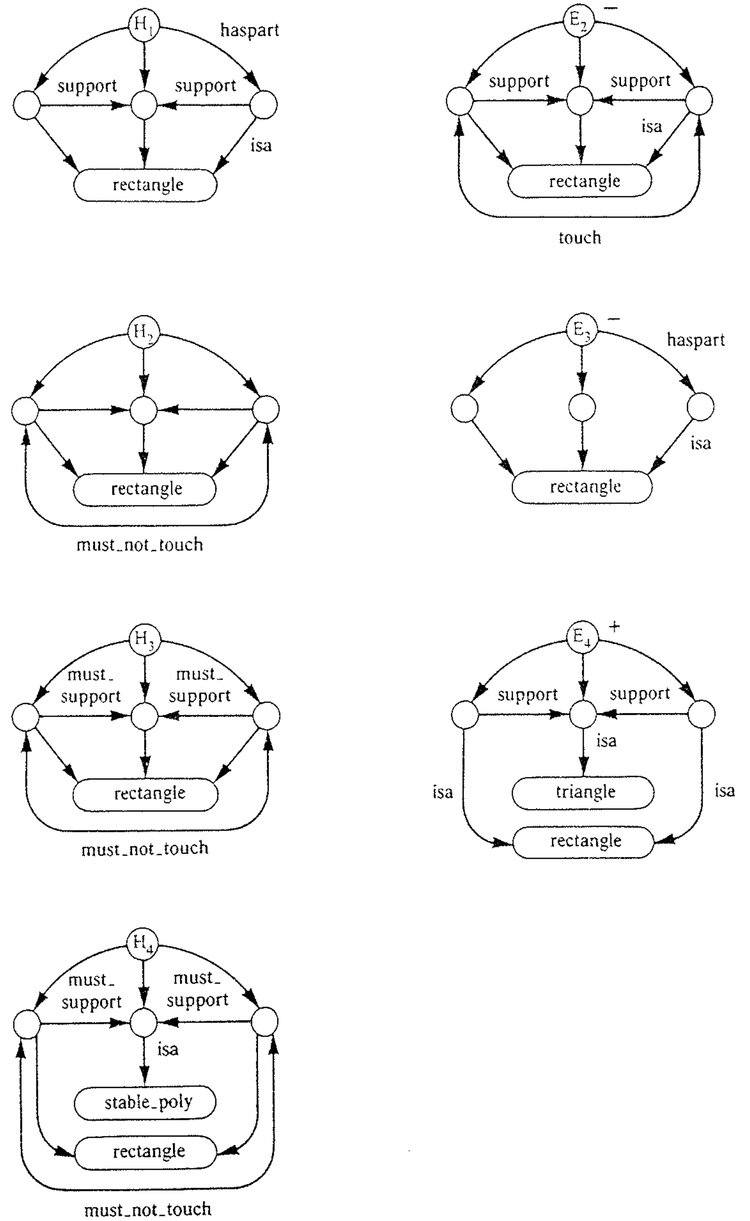


Figure 18.5 Evolving hypotheses about the arch. At each stage the current hypothesis H_i is compared with the next example E_{i+1} and the next, refined hypothesis H_{i+1} is produced.

(1) E_3 is not an arch because the left **support** link is missing, or

(2) E_3 is not an arch because the right **support** link is missing, or

(3) E_3 is not an arch because both **support** links are missing.

Accordingly, the learner has to choose between the three possible ways of updating the current hypothesis. Let us assume that the learner's mentality is more radical than conservative, thus favouring explanation 3. The learner will thus assume that *both* **support** links are necessary and will therefore convert both **support** links in H_2 into **must_support** links in the new hypothesis H_3 (see Figure 18.5). The situation of missing links can be handled by the following condition-action rule, which is another general heuristic about learning:

if
 example is negative and
 example does not contain a relation R which is present in the current hypothesis H
then
 require R in the new hypothesis (add **must_R** in H)

Notice again that, as a result of processing a negative example, the current hypothesis has become still more specific since further necessary conditions are introduced: two **must_support** links. Notice also that the learner could have chosen a more conservative action; namely, to introduce just one **must_support** link instead of two. Obviously, then, the individual learning style can be modelled through the set of condition-action rules the learner uses to update the current hypothesis. By varying these rules the learning style can be varied between conservative and cautious to radical and reckless.

The last example, E_4, in our training sequence is again positive. Matching the corresponding semantic networks E_4 to H_3 shows the difference: the top part is a triangle in E_4 and a rectangle in H_3. The learner might now redirect the corresponding **isa** link in the hypothesis from rectangle to a new object class: **rectangle_or_triangle**. An alternative (and more common) reaction in a learning program is based on a predefined hierarchy of concepts. Suppose that the learner has, as the domain-specific background knowledge, a taxonomy of concepts, as in Figure 18.6. Having found that, according to this taxonomy, rectangle and triangle are both a kind of **stable_poly**, the learner may update the current hypothesis to obtain H_4 (see Figure 18.5).

Notice that this time a positive example has been processed which resulted in a more general new hypothesis (stable polygon instead of rectangle). We say that the current hypothesis was *generalized*. The new hypothesis now allows the top part to be a trapezium although no example of an arch with a trapezium was ever shown to the learner. If the system is now shown the structure in Figure 18.7 and asked to classify it, it would declare it as an arch since it completely satisfies the system's final understanding of an arch – that is, the hypothesis H_4.

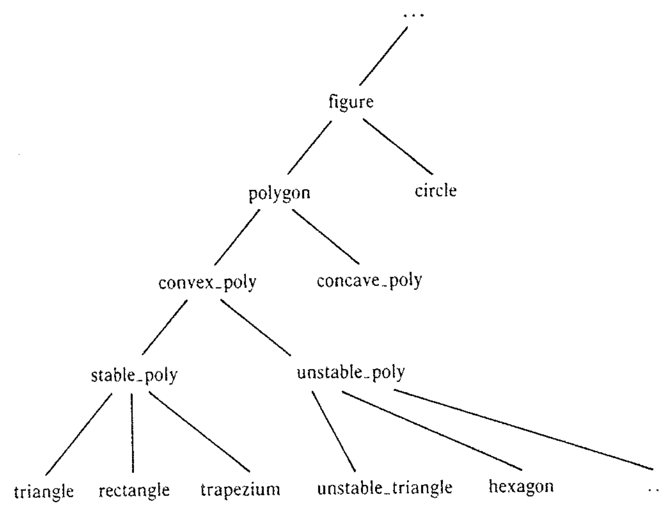


Figure 18.6 A hierarchy of concepts.

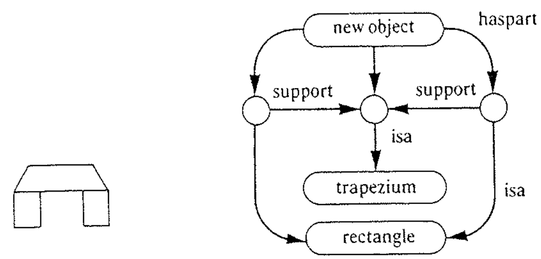


Figure 18.7 Left: a new object; right: its representation. This object matches the concept definition H_4 of the arch in Figure 18.5, using the concept hierarchy of Figure 18.6.

Here are some important points illustrated by the foregoing example:

- The procedure for matching an object to a hypothesis depends on the learning system. Matching is often complicated and can be computationally complex. For example, in the case of semantic networks used as a hypothesis language, we may have to try all possible candidate correspondences between the nodes in the two networks that are being matched to establish the best fit.

- Given two hypotheses, one of them may be more general than the other, or more specific than the other; or they can be incomparable with respect to their generality or specificity.
- Modifications of a hypothesis during the learning process enhance either: the generality of the hypothesis in order to make the hypothesis match a given positive example; or the specificity of the hypothesis in order to prevent the hypothesis from matching a negative example.
- Concept modification principles for a given learning system can be represented as condition-action rules. By means of such rules, the 'mentality' of the learning system can be modelled, ranging from conservative to reckless.

The previous edition of this book (Bratko, *Prolog Programming for Artificial Intelligence*, Addison-Wesley 1990) includes a Prolog implementation of learning relational descriptions as discussed in this section. Here we omit this implementation, although the ARCHES program is a very good illustration of fundamental concepts in machine learning. However, there are learning algorithms that have established themselves as more effective learning tools for practical applications. These include the learning of if-then rules (Section 18.4) and decision trees (Section 18.5). We return to the learning of relational descriptions in the next chapter, using the framework of inductive logic programming.

18.4 Learning simple if-then rules

18.4.1 Describing objects and concepts by attributes

In this section we look at a kind of learning where examples and hypotheses are described in terms of a set of attributes. In principle, attributes can be of various types, depending on their possible values. So an attribute can be numerical or non-numerical. Further, if the attribute is non-numerical, its set of values can be ordered or unordered. We will limit ourselves to non-numerical attributes with unordered value sets. Such a set is typically small, containing a few values only.

An object is described by specifying concrete values of the attributes in the object. Such a description is then a vector of attribute values.

Figure 18.8 shows some objects that will be used as an illustration in this section. These objects belong to five classes: nut, screw, key, pen, scissors. Suppose that these objects have been shown to a vision system. Their silhouettes have been captured by a camera and further processed by a vision-program. This program can then extract some attribute values of each object from the camera image. In our case, the

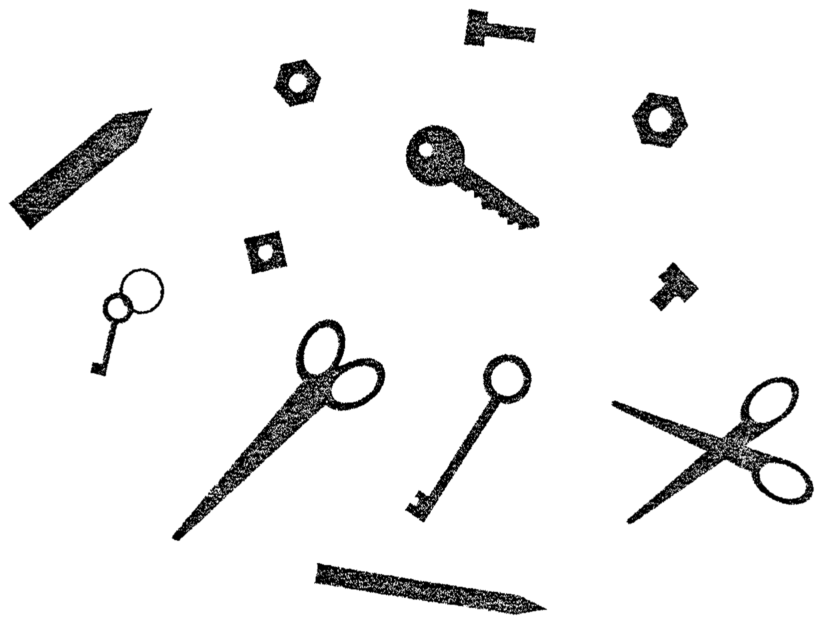


Figure 18.8 Camera image of some objects.

attributes are: the size, the shape and the number of holes in an object. Let the possible values of these attributes be:

size: small, large
shape: long, compact, other
holes: none, 1, 2, 3, many

Assume that the vision system has extracted the three attribute values for each object. Figure 18.9 shows the attribute definitions and these examples represented as a set of Prolog clauses of the form:

```
example( Class, [ Attribute1 = Val1, Attribute2 = Val2, ... ]).
```

Now assume that these examples are communicated to a learning program that is supposed to learn about the five classes. The result of learning will be a description of the classes in the form of rules that can be used for classifying new objects. The format of rules is exemplified by the following possible rules for the classes nut and key:

```
nut <== [[ size = small, holes = 1]]
key <== [[ shape = long, holes = 1], [ shape = other, holes = 2]]
```

```
attribute( size, [ small, large]).
attribute( shape, [ long, compact, other]).
attribute( holes, [ none, 1, 2, 3, many]).

example( nut, [ size = small, shape = compact, holes = 1]).
example( screw, [ size = small, shape = long, holes = none]).
example( key, [ size = small, shape = long, holes = 1]).
example( nut, [ size = small, shape = compact, holes = 1]).
example( key, [ size = large, shape = long, holes = 1]).
example( screw, [ size = small, shape = compact, holes = none]).
example( nut, [ size = small, shape = compact, holes = 1]).
example( pen, [ size = large, shape = long, holes = none]).
example( scissors, [ size = large, shape = long, holes = 2]).
example( pen, [ size = large, shape = long, holes = none]).
example( scissors, [ size = large, shape = other, holes = 2]).
example( key, [ size = small, shape = other, holes = 2]).
```

Figure 18.9 Attribute definitions and examples for learning to recognize objects from their silhouettes (from Figure 18.8).

The meaning of these rules is:

An object is a nut if
its size is small and
it has 1 hole.

An object is a key if
its shape is long and
it has 1 hole
or
its shape is 'other' and
it has 2 holes.

The general form of such rules is:

Class <== [Conj1, Conj2, ...]

where Conj1, Conj2, etc., are lists of attribute values of the form:

[Att1 = Val1, Att2 = Val2, ...]

A class description [Conj1, Conj2, ...] is interpreted as follows:

(1) an object matches the description if the object satisfies at least one of Conj1, Conj2, etc.;

(2) an object satisfies a list of attribute values Conj if all the attribute values in Conj are as in the object.

For example, an object described by:

[size = small, shape = long, holes = 1]

matches the rule for key by satisfying the first attribute-value list in the rule. Thus attribute values in Conj are related conjunctively: none of them may be contradicted by the object. On the other hand, the lists Conj1, Conj2, etc., are related disjunctively: at least one of them has to be satisfied.

The matching between an object and a concept description can be stated in Prolog as:

```
match( Object, Description) :-
  member( Conjunction, Description),
  satisfy( Object, Conjunction).

satisfy( Object, Conjunction) :-
  not (
      member( Att = Val, Conjunction),      % Value in concept
      member( Att = ValX, Object),          % and value in object
      ValX \== Val).                        % are different
```

Notice that this definition allows for partially specified objects when some attribute value may be unspecified – that is, not included in the attribute-value list. In such a case this definition assumes that the unspecified value satisfies the requirement in Conjunction.

18.4.2 Inducing rules from examples

Now we will consider how rules can be constructed from a set of examples. As opposed to the arches example of the previous section, a class description will not be constructed by processing the examples sequentially one-by-one. Instead, all the examples will be processed 'in one shot'. This is also called *batch learning* as opposed to *incremental learning*.

The main requirement here is that the constructed class description matches exactly the examples belonging to the class. That is, the description is satisfied by all the examples of this class and no other example.

When an object matches a description, we say that the description *covers* the object. Thus we have to construct a description for the given class that covers all the examples of this class and no other example. Such a description is said to be *complete* and *sound*: complete because it covers all the positive examples and sound because it covers no negative example.

A widely used approach to constructing a consistent hypothesis as a set of if-then rules is the *covering algorithm* shown in Figure 18.10. It is called the 'covering' algorithm because it gradually covers all the positive examples of the concept learned. The covering algorithm starts with the empty set of rules. It then iteratively induces rule by rule. No rule may cover any negative example, but has to cover some positive examples. Whenever a new rule is induced, it is added to the hypothesis, and the positive examples covered by the rule are removed from the example set.

```
To induce a list of rules RULELIST for a set S of classified examples, do:

RULELIST := empty;
E := S;
while E contains positive examples do
  begin
    RULE := InduceOneRule(E);
    Add RULE to RULELIST;
    Remove from E all the examples covered by RULE
  end
```

Figure 18.10 The covering algorithm. Procedure InduceOneRule(E) generates a rule that covers some positive examples and no negative one.

The next rule is induced with this new, reduced example set, and so on until all the positive examples are covered.

The covering algorithm in Figure 18.10 insists on consistent rules. The induced rules have to cover *all* the positive examples and *no* negative one. In practice, especially when learning from noisy data, relaxed variants of the covering algorithm are used. In such variants, the algorithm may finish before all the positive examples are covered. Also, the procedure InduceOneRule may be allowed to cover some negative examples in addition to positive ones, provided that the covered positive examples have sufficiently convincing majority.

Figure 18.11 gives an implementation of the covering algorithm. The procedure

learn(Examples, Class, Description)

in this program constructs a consistent description for Class and Examples. It works roughly as follows:

To cover all the examples of class Class in Examples do:

if no example in Examples belongs to Class then
  Description = [] (all positive examples covered),
otherwise Description = [Conj | Conjs] where Conj and Conjs
  are obtained as follows:

(1) construct a list Conj of attribute values that covers at least one positive example of Class and no example of any other class;

(2) remove from Examples all the examples covered by Conj and cover the remaining, uncovered objects by description Conjs.

Each attribute-value list is constructed by the procedure:

learn_conj(Examples, Class, Conjunction)


```
% Learning of simple if-then rules

:- op( 300, xfx, <==).

% learn( Class): collect learning examples into a list, construct and
% output a description for Class, and assert the corresponding rule about Class

learn( Class) :-
  bagof( example( ClassX, Obj), example( ClassX, Obj), Examples),   % Collect examples
  learn( Examples, Class, Description),                             % Induce rule
  nl, write( Class), write(' <== '), nl,                            % Output rule
  writelist( Description),
  assert( Class <== Description).                                   % Assert rule

% learn( Examples, Class, Description):
%   Description covers exactly the examples of class Class in list Examples

learn( Examples, Class, [ ]) :-
  not member( example( Class, _ ), Examples).          % No example to cover

learn( Examples, Class, [Conj | Conjs]) :-
  learn_conj( Examples, Class, Conj),
  remove( Examples, Conj, RestExamples),                % Remove examples that match Conj
  learn( RestExamples, Class, Conjs).                   % Cover remaining examples

% learn_conj( Examples, Class, Conj):
%   Conj is a list of attribute values satisfied by some examples of class Class and
%   no other class

learn_conj( Examples, Class, [ ]) :-
  not ( member( example( ClassX, _ ), Examples),       % There is no example
  ClassX \== Class), !.                                 % of different class

learn_conj( Examples, Class, [Cond | Conds]) :-
  choose_cond( Examples, Class, Cond),                  % Choose attribute value
  filter( Examples, [ Cond], Examples1),
  learn_conj( Examples1, Class, Conds).

choose_cond( Examples, Class, AttVal) :-
  findall( AV/Score, score( Examples, Class, AV, Score), AVs),
  best( AVs, AttVal).                                   % Best score attribute value

best( [ AttVal/_], AttVal).

best( [ AV0/S0, AV1/S1 | AVSlist], AttVal) :-
  S1 > S0, !,                                           % AV1 better than AV0
  best( [AV1/S1 | AVSlist], AttVal)
  ;
  best( [AV0/S0 | AVSlist], AttVal).

% filter( Examples, Condition, Examples1):
%   Examples1 contains elements of Examples that satisfy Condition
```

Figure 18.11 A program that induces if-then rules.

Figure 18.11 *contd*

```
filter( Examples, Cond, Examples1) :-
  findall( example( Class, Obj),
          ( member( example( Class, Obj), Examples), satisfy( Obj, Cond)),
          Examples1).

% remove( Examples, Conj, Examples1):
%   removing from Examples those examples that are covered by Conj gives Examples1

remove( [], _, []).

remove( [example( Class, Obj) | Es], Conj, Es1) :-
  satisfy( Obj, Conj), !,                      % First example matches Conj
  remove( Es, Conj, Es1).                      % Remove it

remove( [E | Es], Conj, [E | Es1]) :-          % Retain first example
  remove( Es, Conj, Es1).

satisfy( Object, Conj) :-
  not( member( Att = Val, Conj),
       member( Att = ValX, Object),
       ValX \== Val).

score( Examples, Class, AttVal, Score) :-
  candidate( Examples, Class, AttVal),         % A suitable attribute value
  filter( Examples, [ AttVal], Examples1),     % Examples1 satisfy condition Att = Val
  length( Examples1, N1),                      % Length of list
  count_pos( Examples1, Class, NPos1),         % Number of positive examples
  NPos1 > 0,                                   % At least one positive example
  Score is 2 * NPos1 - N1.                     % matches AttVal

candidate( Examples, Class, Att = Val) :-
  attribute( Att, Values),                     % An attribute
  member( Val, Values),                        % A value
  suitable( Att = Val, Examples, Class).

suitable( AttVal, Examples, Class) :-          % At least one negative example
  member( example( ClassX, ObjX), Examples),   % must not match AttVal
  ClassX \== Class,                            % Negative example
  not satisfy( ObjX, [ AttVal]), !.            % that does not match

% count_pos( Examples, Class, N):
%   N is the number of positive examples of Class

count_pos( [], _, 0).

count_pos( [example( ClassX,_ ) | Examples], Class, N) :-
  count_pos( Examples, Class, N1),
  ( ClassX = Class, !, N is N1 + 1; N = N1).

writelist( []).

writelist( [X | L]) :-
  tab( 2), write( X), nl,
  writelist( L).
```


The attribute-value list Conjunction emerges gradually, starting with the empty list and adding to this list conditions of the form:

Attribute = Value

Notice that, in this way, the attribute-value list becomes more and more specific (it covers fewer objects). The attribute-value list is acceptable when it becomes so specific that it only covers positive examples of Class.

The process of constructing such a conjunction is highly combinatorial. Each time a new attribute-value condition is added, there are almost as many alternative candidates to be added as there are attribute-value pairs. It is not easy to immediately see which of them is the best. In general, we would like to cover all the positive examples with as few rules as possible, and with as short rules as possible. Thus learning can be viewed as a search among possible descriptions with the objective of minimizing the length of the concept description. Because of the high combinatorial complexity of this search, we normally have to resort to some heuristic. The program in Figure 18.11 relies on a heuristic scoring function that is used locally. At each point, only the best-estimated attribute value is added to the list, immediately disregarding all other candidates. The search is thus reduced to a deterministic procedure without any backtracking. This is also called *greedy search* or *hill-climbing*. It is 'greedy' because it always chooses the best-looking alternative. However, in such search there is a risk of missing the shortest concept description.

The heuristic estimate is simple and based on the following intuition: a useful attribute-value condition should discriminate well between positive and negative examples. Thus, it should cover as many positive examples as possible and as few negative examples as possible. Figure 18.12 shows the construction of such a heuristic scoring function. This function is in our program implemented as the procedure:

score(Examples, Class, AttributeValue, Score)

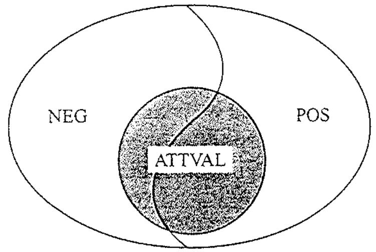


Figure 18.12 Heuristic scoring of an attribute value. POS is the set of positive examples of the class to be learned; NEG is the set of negative examples of this class. The shaded area, ATTVAL, represents the set of objects that satisfy the attribute-value condition. The heuristic score of the attribute value is the number of positive examples in ATTVAL minus the number of negative examples in ATTVAL.

Score is the difference between the number of covered positive and covered negative examples.

The program in Figure 18.11 can be run to construct some class descriptions for the examples in Figure 18.9 with the query:

```
?- learn( nut), learn( key), learn( scissors).
nut <==
  [ shape = compact, holes = 1]
key <==
  [ shape = other, size = small]
  [ holes = 1, shape = long]
scissors <==
  [ holes = 2, size = large]
```

The procedure learn also asserts rules about the corresponding classes in the program. These rules can be used to classify new objects. A corresponding recognition procedure that uses the learned descriptions is:

```
classify( Object, Class) :-
  Class <== Description,          % Learned rule about Class
  member( Conj, Description),     % A conjunctive condition
  satisfy( Object, Conj).         % Object satisfies Conj
```

18.5 Induction of decision trees

18.5.1 Basic tree induction algorithm

Induction of decision trees is probably the most widespread approach to machine learning. In this case, hypotheses are represented by decision trees. Induction of trees is efficient and easy to program.

Figure 18.13 shows a decision tree that can be induced from the examples of Figure 18.9 (that is, the objects in Figure 18.8). Internal nodes in the tree are labelled with attributes. The leaves of the tree are labelled with classes or the symbol 'null'. 'null' indicates that no learning example corresponds to that leaf. Branches in the tree are labelled with attribute values. In classifying an object, a path is traversed in the tree starting at the root node and ending at a leaf. At each internal node, we follow the branch labelled by the attribute value in the object. For example, an object described by:

[size = small, shape = compact, holes = 1]

would be, according to this tree, classified as a nut (following the path: holes = 1, shape = compact). Notice that, in this case, the attribute value size = small is not needed to classify the object.

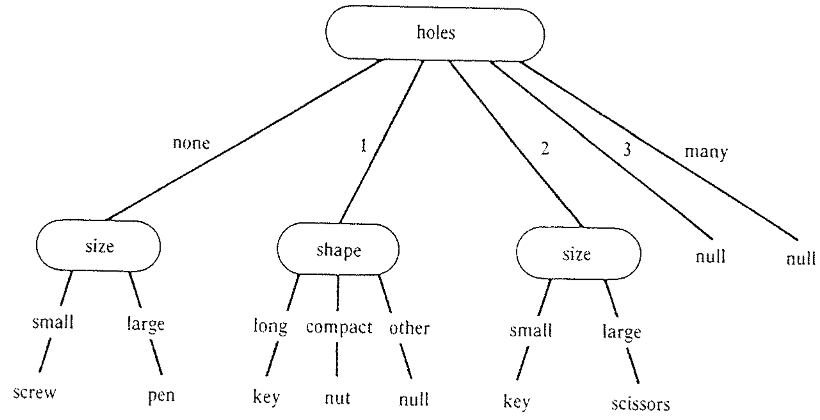


Figure 18.13 A decision tree induced from examples of Figure 18.9 (shown graphically in Figure 18.8).

In comparison with if-then rules to describe classes, discussed in the previous section, trees are a more constrained representation. This has both advantages and disadvantages. Some concepts are more awkward to represent with trees than with rules: although every rule-based description can be translated into a corresponding decision tree, the resulting tree may have to be much lengthier than the rule-based description. This is an obvious disadvantage of trees.

On the other hand, the fact that trees are more constrained reduces the combinatorial complexity of the learning process. This may lead to substantial improvement in the efficiency of learning. Decision tree learning is one of the most efficient forms of learning. It should be noted, however, that computational efficiency of learning is only one criterion of success in learning, as will be discussed later in this chapter.

The basic tree induction algorithm is as shown in Figure 18.14. The algorithm aims at constructing a smallest tree consistent with the learning data. However, search among all such trees is prohibitive because of combinatorial complexity. Therefore the common approach to tree induction is heuristic, without a guarantee of optimality. The algorithm in Figure 18.14 is greedy in the sense that it always chooses 'the most informative' attribute, and it never backtracks. There is no guarantee of finding a smallest tree. On the other hand, the algorithm is fast and has been found to work well in practical applications.

Even in its simplest implementation, this basic algorithm needs some refinements:

(1) If S is empty then the result is a single-node tree labelled 'null'.

(2) Each time a new attribute is selected, only those attributes that have not yet been used in the upper part of the tree are considered.

To construct a decision tree T for a learning set S do:

if all the examples in S belong to the same class, C,
then the result is a single node tree labelled C
otherwise
select the most 'informative' attribute, A, whose values are $v_1, \ldots, v_n$;
partition S into $S_1, \ldots, S_n$ according to the values of A;
construct (recursively) subtrees $T_1, \ldots, T_n$ for $S_1, \ldots, S_n$;
final result is the tree whose root is A and whose subtrees are $T_1, \ldots, T_n$,
and the links between A and the subtrees are labelled by $v_1, \ldots, v_n$;
thus the decision tree looks like this:

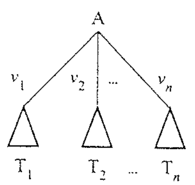


Figure 18.14 The basic tree induction algorithm.

(3) If S is not empty, and not all the objects in S belong to the same class, and there is no attribute left to choose, then the result is a single-node tree. It is useful to label this node with the list of all the classes represented in S, together with their relative frequencies in S. Such a tree is sometimes called a *class probability tree*, instead of a decision tree. In such a case the set of available attributes is not sufficient to distinguish between class values of some objects (objects that belong to different classes have exactly the same attribute values).

(4) We have to specify the criterion for selecting the 'most informative' attribute. This will be discussed in the next section.

18.5.2 Selecting 'the best' attribute

Criteria for selecting 'the best' attribute are a much investigated topic in machine learning. Basically these criteria measure the 'impurity', with respect to class, of a set of examples. A good attribute should split the examples in subsets as pure as possible.

One approach to attribute selection exploits ideas from information theory. Such a criterion can be developed as follows. To classify an object, a certain amount of information is needed. After we have learned the value of some attribute of the object, we only need some remaining amount of information to classify the object. This remaining amount will be called the 'residual information'. Of course, residual information should be smaller than the initial information. The 'most informative' attribute is the one that minimizes the residual information. The amount of information is determined by the well-known *entropy* formula. For a domain exemplified by the learning set S, the average amount of information I needed to classify an object is given by the entropy measure:

$$I = -\sum_c p(c) \log_2 p(c)$$

where c stands for classes and $p(c)$ is the probability that an object in S belongs to class c. This formula nicely corresponds to the intuition about impurity. For a completely pure set, the probability of one of the classes is 1, and 0 for all other classes. For such a set, information $I = 0$. On the other hand, information I is maximal in the case that the probabilities of all the classes are equal.

After applying an attribute A, the set S is partitioned into subsets according to the values of A. The residual information I_{res} is then equal to the weighted sum of the amounts of information for the subsets:

$$I_{res}(A) = -\sum_v p(v) \sum_c p(c \mid v) \log_2 p(c \mid v)$$

where v stands for the values of A, $p(v)$ is the probability of value v in set S, and $p(c \mid v)$ is the conditional probability of class c given that attribute A has value v. The probabilities $p(v)$ and $p(c \mid v)$ are usually approximated by statistics on set S.

Further refinements can be made to the information-theoretic criterion given above. A defect of this criterion is that it tends to favour attributes with many values. Such an attribute will tend to split the set S into many small subsets. If these subsets are very small, with just a few examples, they will tend to be pure anyway, regardless of the genuine correlation between the attribute and the class. The straightforward I_{res} will thus give such an attribute undeserved credit. One way of rectifying this is *information gain ratio*. This ratio takes into account the amount of information $I(A)$ needed to determine the value of an attribute A:

$$I(A) = -\sum_v p(v) \log_2(p(v))$$

$I(A)$ will tend to be higher for attributes with more values. Information gain ratio is defined as:

$$GainRatio(A) = \frac{Gain(A)}{I(A)} = \frac{I - I_{res}(A)}{I(A)}$$

The attribute to choose is the one that has the highest gain ratio.

Another idea of circumventing the problem with many-valued attributes is the *binarization* of attributes. An attribute is *binarized* by splitting the set of its values into two subsets. As a result, the splitting results in a new (binary) attribute whose two values correspond to the two subsets. When such a subset contains more than one value, it can be split further into two subsets, giving rise to another binary attribute, etc. When choosing a good split, the criterion generally is to maximize the information gain of the so obtained binary attribute. After all the attributes have been made binary, the problem of comparing many-valued attributes with few-valued attributes disappears. The straightforward residual information criterion gives a fair comparison of the (binary) attributes.

There exist other sensible measures of impurity, like the Gini index, defined by:

$$Gini = \sum_{i \neq j} p(i)p(j)$$

where i and j are classes. After applying attribute A, the resulting Gini index is:

$$Gini(A) = \sum_{v} p(v) \sum_{i \neq j} p(i \mid v)p(j \mid v)$$

where v stands for values of A and $p(i \mid v)$ denotes the conditional probability of class i given that attribute A has value v.

It should be noted that impurity measures are used here to assess the effect of a single attribute. Therefore, the criterion 'most informative' is local in the sense that it does not reliably predict the combined effect of several attributes applied jointly. The basic tree induction algorithm is based on this local minimization of impurity. As mentioned earlier, global optimization would be computationally much more expensive.

Exercises

18.1 Consider the problem of learning from objects' silhouettes (Figure 18.9). Calculate the entropy of the whole example set with respect to class, the residual information for attributes 'size' and 'holes', the corresponding information gains and gain ratios. Estimate the probabilities needed in the calculations simply by the relative frequencies, e.g. $p(\text{nut}) = 3/12$ or $p(\text{nut} \mid \text{holes}{=}1) = 3/5$.

18.2 Disease D occurs in 25 percent of all the cases. Symptom S is observed in 75 percent of the patients suffering from disease D, and only in one sixth of other cases. Suppose we are building a decision tree for diagnosing disease D, so the classes are just D (person has D) and ~D (does not have D). S is one of the attributes. What are the information gain and gain ratio of attribute S?

18.5.3 Implementing decision tree learning

Let us now sketch a procedure to induce decision trees:

```
induce_tree( Attributes, Examples, Tree)
```

where **Tree** is a decision tree induced from **Examples** using attributes in list **Attributes**. If the examples and attributes are represented as in Figure 18.9, we can collect all the

examples and available attributes into lists, used as arguments for our induction procedure:

```
induce_tree( Tree) :-
  findall( example( Class, Obj), example( Class, Obj), Examples),
  findall( Att, attribute( Att, _ ), Attributes),
  induce_tree( Attributes, Examples, Tree).
```

The form of the tree depends on the following three cases:

(1) Tree = null if the example set is empty.

(2) Tree = leaf(Class) if all of the examples are of the same class Class.

(3) Tree = tree(Attribute, [Val1 : SubTree1, Val2 : Subtree2, ...]) if examples belong to more than one class, Attribute is the root of the tree, Val1, Val2, ... are Attribute's values, and SubTree1, SubTree2, ... are the corresponding decision subtrees.

These three cases are handled by the following three clauses:

```
% induce_tree( Attributes, Examples, Tree)

induce_tree( _, [], null) :- !.

induce_tree( _, [example( Class,_ ) | Examples], leaf( Class)) :-
  not ( member( example( ClassX, _), Examples),    % No other example
        ClassX \== Class), !.                      % of different class

induce_tree( Attributes, Examples, tree( Attribute, SubTrees)) :-
  choose_attribute( Attributes, Examples, Attribute),
  del( Attribute, Attributes, RestAtts),           % Delete Attribute
  attribute( Attribute, Values),
  induce_trees( Attribute, Values, RestAtts, Examples, SubTrees).
```

induce_trees induces decision SubTrees for subsets of Examples according to Values of Attribute:

```
% induce_trees( Att, Vals, RestAtts, Examples, SubTrees)

induce_trees( _, [], _, _, [] ).                   % No attributes, no subtrees

induce_trees( Att, [Val1 | Vals], RestAtts, Exs, [Val1 : Tree1 | Trees]) :-
  attval_subset( Att = Val1, Exs, ExampleSubset),
  induce_tree( RestAtts, ExampleSubset, Tree1),
  induce_trees( Att, Vals, RestAtts, Exs, Trees).
```

attval_subset(Attribute = Value, Examples, Subset) is true if Subset is the subset of examples in Examples that satisfy the condition Attribute = Value:

```
attval_subset( AttributeValue, Examples, ExampleSubset) :-
  findall( example( Class, Obj),
           ( member( example( Class, Obj), Examples),
             satisfy( Obj, [ AttributeValue])),
           ExampleSubset).
```

The predicate satisfy(Object, Description) is defined as in Figure 18.11. The predicate choose_attribute selects the attribute that discriminates well among the classes. This involves the impurity criterion. The following clause minimizes the chosen impurity measure using setof. setof will order the available attributes according to increasing impurity:

```
choose_attribute( Atts, Examples, BestAtt) :-
  setof( Impurity/Att,
        ( member( Att, Atts), impurity1( Examples, Att, Impurity)),
        [ MinImpurity/BestAtt | _ ] ).
```

Procedure

```
impurity1( Examples, Attribute, Impurity)
```

implements a chosen impurity measure. Impurity is the combined impurity of the subsets of examples after dividing the list Examples according to the values of Attribute.

Exercises

18.3 Implement a chosen impurity measure by writing the impurity1 predicate. This measure can be, for example, the residual information content or Gini index as discussed previously in this section. For the examples in Figure 18.9 and attribute size, use of the Gini index as impurity measure gives:

```
?- Examples = ...                        % Examples from Figure 18.9
   impurity1( Examples, size, Impurity).

   Impurity = 0.647619
```

Approximating probabilities by relative frequencies, Impurity is calculated as follows:

$$\begin{aligned}\text{Impurity} &= Gini(\text{ size})\\ &= p(\text{small})*(p(\text{nut}\mid\text{small})*p(\text{screw}\mid\text{small}) + \ldots) + p(\text{large})*(\ldots)\\ &= 7/12 * (3/7 * 2/7 + \ldots) + 5/12 * (\ldots)\\ &= 7/12 * 0.653061 + 5/12 * 0.64\\ &= 0.647619\end{aligned}$$

18.4 Complete the tree induction program of this section and test it on some learning problem, for example the one in Figure 18.9. Note that the procedure choose_attribute, using setof, is very inefficient and can be improved. Also add a procedure

```
show( DecisionTree)
```

for displaying decision trees in a readable form. For the tree in Figure 18.13, a suitable form is:


```
holes
  none
    size
      small ==> screw
      large ==> pen
  1
    shape
      long ==> key
      compact ==> nut
      other ==> null
  2
    size
      small ==> key
      large ==> scissors
  3 ==> null
  many ==> null
```

18.6 Learning from noisy data and tree pruning

In many applications the data for learning are imperfect. One common problem is errors in attribute values and class values. In such cases we say that the data are *noisy*. Noise of course makes the learning task more difficult and requires special mechanisms. In the case of noise, we usually abandon the consistency requirement that induced hypotheses correctly reclassify the learning examples. We allow the learned hypothesis to misclassify some of the learning objects. This concession is sensible because of possible errors in the data. We hope the misclassified learning objects are those that contain errors. Misclassifying such objects only indicates that erroneous data have in fact been successfully ignored.

Inducing decision trees from noisy data with the basic tree induction algorithm has two problems: first, induced trees unreliably classify new objects and, second, induced trees tend to be large and thus hard to understand. It can be shown that some of this tree complexity is just the result of noise in the learning data. The learning algorithm, in addition to discovering genuine regularities in the problem domain, also traces noise in the data.

As an example, consider a situation in which we are to construct a subtree of a decision tree, and the current subset of objects for learning is S. Let there be 100 objects in S, 99 of them belonging to class C1 and one of them to class C2. Knowing that there is noise in the learning data and that all these objects have the same values of the attributes already selected up to this point in the decision tree, it seems plausible that the class C2 object is in S only as a result of an error in the data. If so, it is best to ignore this object and simply return a leaf of the decision tree labelled with class C1. Since the basic tree induction algorithm would in this situation further expand the decision tree, we have, by stopping at this point, in effect *pruned* a subtree of the complete decision tree.

Tree pruning is the key to coping with noise in tree induction programs. A program may effectively prune decision trees by using some criterion that indicates whether to stop expanding the tree or not. The stopping criterion would typically take into account the number of examples in the node, the prevalence of the majority class at the node, to what extent an additional attribute selected at this node would reduce the impurity of the example set, etc.

This kind of pruning, accomplished through stopping the tree expansion, is called *forward pruning* as opposed to another kind of pruning, called *post-pruning*. Post-pruning is done after the learning program has first constructed the *complete* decision tree. Then parts of the tree that seem unreliable are pruned away. This is illustrated in Figure 18.15. After the bottom parts of the tree are removed, the accuracy of the tree on new data may increase. This may appear paradoxical because by pruning we in fact throw away some information. How can accuracy increase after that?

This can be explained by that we are actually pruning the unreliable parts of the tree, those that contribute to the tree's misclassification errors most. These are the parts of the tree that mainly trace noise in the data and not the genuine regularities in the learning domain. Intuitively it is easy to see why the bottom parts of the tree are the least reliable. Our top-down tree induction algorithm takes into account all the learning data when building the top of the tree. When moving down the tree, the learning data gets fragmented among the subtrees. So the lower parts of the tree are induced from less data. The smaller the data set, the greater the danger that it is critically affected by noise. This is why the lower parts of the tree are generally less reliable.

Of the two types of pruning, forward pruning and post-pruning, the latter is considered better because it exploits the information provided by the complete tree. Forward pruning, on the other hand, only exploits the information in the top part of the tree.

The big question remains, however, how to determine exactly which subtrees to prune and which not. If we prune too much then we may throw away also healthy information and then the accuracy will decrease. So, how can we know that we have not pruned too little or too much? This is an intricate question and there are several

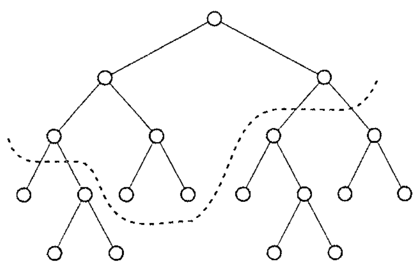


Figure 18.15 Pruning of decision trees. After pruning the accuracy may increase.

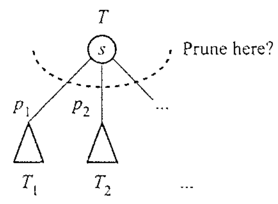


Figure 18.16 Deciding about pruning.

methods that offer different, more or less satisfactory answers. We will now look at one method of post-pruning, known as *minimal error pruning*.

The key decision in post-pruning is whether to prune the subtrees below a given node or not. Figure 18.16 illustrates the situation. T is a decision tree whose root is node s, T_1, T_2, ... are T's subtrees, and p_i are the probabilities of a random object passing from s to subtree T_i. T can be in turn a subtree of a larger decision tree. The question is to decide whether to prune below s (i.e. remove the subtrees $T_1, \ldots$), or not to prune. We here formulate a criterion based on the minimization of the expected classification error. We assume that the subtrees $T_1, \ldots$ have already been optimally pruned using the same criterion on their subtrees.

The classification *accuracy* of a decision tree T is the probability that T will correctly classify a randomly chosen new object. The classification *error* of T is the opposite, i.e. the probability of incorrect classification. Let us analyze the error for the two cases:

(1) If T is pruned just below s then s becomes a leaf. s is then labelled with the most likely class C at s, and everything in this leaf is classified into class C. The error at s is the probability that a random object that falls into s belongs to a class other than C. This is called the *static error* at s:

$$e(s) = p(\text{class} \neq C \mid s)$$

(2) If the tree is not pruned just below s, then its error is the weighted sum of the errors $E(T_1)$, $E(T_2)$, ... of the optimally pruned subtrees $T_1, T_2, \ldots$:

$$p_1 E(T_1) + p_2 E(T_2) + \ldots$$

This is called the *backed-up error*.

The decision rule about pruning below s then is: if the static error is less than or equal to the backed-up error then prune, otherwise do not prune. Accordingly we can define the error of the optimally pruned tree T as:

$$E(T) = \min(e(s), \sum_i p_i E(T_i))$$

Of course, if T has no subtrees then simply $E(T) = e(s)$.

The remaining question is how to estimate the static error $e(s)$, which boils down to the probability of the most likely class C at node s? The evidence we can use for this estimate is the set of examples that fall into node s. Let this set be S, the total number of examples in S be N, and the number of examples of class C be n. Now, the estimate of the probability of C at s is, in fact, an intricate problem. Most people would immediately propose that we just take the proportion n/N (relative frequency) of class C examples at node s. This is reasonable if the number of examples at s is large, but obviously becomes debatable if this number is small. For instance let there be just one example at s. Then the proportion of the most likely class is $1/1 = 100$ percent, and the error estimate is $0/1 = 0$. But given that we only have one example at s, this estimate is statistically completely unreliable. Suppose we were able to get another learning example at s, and that this example was of another class. This single additional example would then drastically change the estimate to $1/2 = 50$ percent!

Another good illustration of the intricacies in estimating probabilities is the outcome of a flip of a particular coin. Suppose that in the first experiment with the coin we get the head. The relative frequency now gives that the probability of the head is 1. This is completely counterintuitive because our *a priori* expectation is that this probability is 0.5. Even if this coin is not quite 'honest', the probability of the head should still be close to 0.5, and the estimate 1 is obviously inappropriate. This example also indicates that the probability estimate should depend not only on the experiments, but also on the prior expectation about this probability.

Obviously we need a more elaborate estimate than relative frequency. We here present one such estimate, called m-estimate, that has a good mathematical justification. According to the m-estimate, the expected probability of event C is:

$$p = \frac{n + p_a m}{N + m}$$

Here p_a is the *a priori* probability of C, and m is a parameter of the estimate. The formula is derived using the Bayesian approach to probability estimation. Roughly, the Bayesian procedure assumes that there is some, possibly very vague, a priori knowledge about the probability of event C. This *a priori* knowledge is stated as the prior probability distribution for event C. Then experiments are performed giving additional information about the probability of C. Prior probability distribution is updated with this new, experimental information, using Bayes' formula for conditional probability. The m-estimate formula above gives the expected value p of this distribution. So the formula allows us to take into account the prior expectation about the probability of C, which is useful if we have some background knowledge about C in addition to the given examples. This prior expectation is in the m-estimate formula expressed by p_a and m, as discussed below.

The m-estimate formula can be rewritten as:

$$p = p_a * \frac{m}{N + m} + \frac{n}{N} * \frac{N}{N + m}$$

This provides a nice interpretation of the m-estimate: probability p is simply equal to the *a priori* probability p_a, modified by the evidence coming from N examples. If there are no examples then $N = 0$ and $p = p_a$. If there are many examples (N very large) then $p \approx n/N$. Otherwise p is between these two values. The strength of the prior probability is varied by the value of parameter m ($m \geqslant 0$): the larger m, the greater the relative weight of the prior probability.

Parameter m has a specially handy interpretation in dealing with noisy data. If the domain expert believes that the data are very noisy and the background knowledge is trustworthy, then he or she will set m high (e.g. $m = 100$, giving much weight to prior probability). If on the other hand the learning data are trustworthy and the prior probability less so then m will be set low (e.g. $m = 0.2$, thus giving much weight to the data). In practice, to avoid the uncertainty with appropriate setting of parameter m, m can be varied. In this way a sequence of differently pruned trees are obtained, each of them being optimal with respect to a different value of m. Such a sequence of trees can then be studied by the domain expert, who may be able to decide which of the trees make more sense.

There is now one remaining and non-trivial question: How to determine the prior probability p_a? If there is expert knowledge available then this should be used in setting p_a. If not, the commonly used technique is to determine the prior probabilities by statistics on the complete learning data (not just the fragment of it at node s), using simply the relative frequency estimate on the complete, large set. An alternative (often inferior to this) is to assume that all the classes are *a priori* equally likely and have uniform prior probability distribution. This assumption leads to a special case of the m-estimate, known as the *Laplace estimate*. If there are k possible classes altogether, then for this special case we have:

$$p_a = 1/k, \qquad m = k$$

So the Laplace probability estimate is:

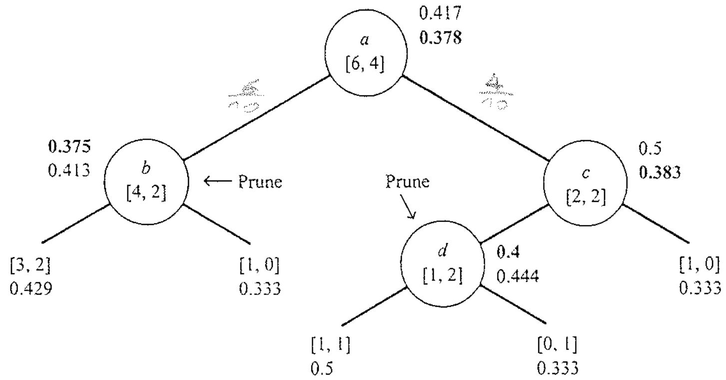
$$p = \frac{n + 1}{N + k}$$

This is handy because it does not require parameters p_a and m. On the other hand, it is based on the usually incorrect assumption of the classes being *a priori* equally likely. Also, it does not allow the user to take into account the estimated degree of noise.

Figure 18.17 shows the pruning of a decision tree by minimum-error pruning, using the Laplace estimate. In this figure, the left-most leaf of the unpruned tree has class frequencies [3, 2], meaning that there are three objects of class C1 and two objects of class C2 falling into this leaf. The static error estimate, using Laplace formula, for these class frequencies is:

$$e(b\_left) = 1 - \frac{n + 1}{N + k} = 1 - \frac{3 + 1}{5 + 2} = 0.429$$

Before pruning:



After pruning:

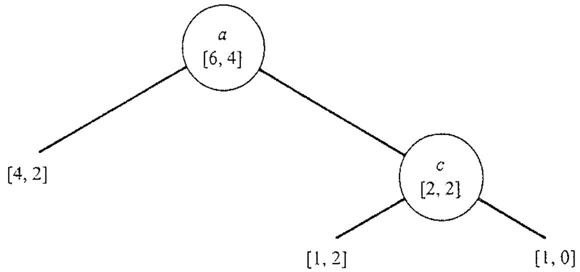


Figure 18.17 Minimal-error pruning of decision trees. Pairs of numbers in square brackets are the numbers of class C1 and class C2 examples that fall into the corresponding nodes. The other numbers attached to the nodes are error estimates. For internal nodes, the first number is a static error estimate and the second number is a backed-up estimate. The lower of the two numbers (in bold face) is propagated upwards.

For the right-hand child of node b, the error estimate is $e(b\_right) = 0.333$. For node b, the static error estimate is:

$$e(b) = 0.375$$

The backed-up error estimate for b is:

$$BackedUpError(b) = 5/6 * 0.429 + 1/6 * 0.333 = 0.413$$

Because the backed-up estimate is greater than the static estimate, the subtrees of b are pruned and the error at b after pruning is:

$$E(b) = 0.375$$

The use of m-estimate for estimating the error at the nodes of a decision tree can be criticized on the following grounds. The m-estimate formula assumes that the available data is a random sample. However, this is not quite true for the data subsets at the nodes of a decision tree. During tree construction, the data subsets at the nodes of a tree have been selected by the attribute selection criterion among possible partitions, according to the attributes, of the learning data. In spite of this theoretical reservation, experience has shown that minimal-error pruning with m-estimate works well in practice. When exploring data with decision trees, experimenting with various values of parameter m for pruning is particularly useful.

The minimum-error pruning strategy can in principle use any approach to estimating errors. In any case, the expected error of the pruned tree will be minimized. Of course, a pruned tree thus obtained is only optimal with respect to the particular error estimates. One alternative approach to estimating errors is to partition the available learning data S into two subsets: 'growing set' and 'pruning set'. The growing set is used to build a complete decision tree consistent with the 'growing' data. The pruning set is used just to measure the error at the nodes of the tree, and then prune so as to minimize this error. Since the pruning set is independent of the growing set, we can simply classify the examples in the pruning set by the decision tree and count the tree's errors. This gives an estimate of predictive accuracy on new data. The approach with a pruning set is sensible when the learning data are ample. However, it is at a disadvantage when the learning data are scarce. In such a case, holding out a pruning set means even less data for growing the tree.

Exercises

18.5 A volcano ejects lava completely randomly. In long-term statistics, the volcano was on average, active one day in ten days and inactive the remaining days. In the most recent 30 days it was active 25 days. This recently increased activity was taken into account by experts in the forecast for the next day. The radio news said that the probability of the volcano being active the next day was at least 30 percent. The prediction on TV quoted the probability between 50 and 60 percent. Both experts, on radio and TV, were known to use the m-estimate method. How can the difference in their estimates be explained?

18.6 Write a procedure

prunetree(Tree, PrunedTree)

that prunes a decision tree Tree according to the minimum-error pruning method discussed in this section. The leaves of Tree contain class frequency information represented as lists of integers. Use the Laplace estimate in the program. Let the number of classes be stored in the program as a fact: number_of_classes(K).

18.7 Success of learning

We started this chapter by considering the ARCHES style of learning relational descriptions. ARCHES is a good illustration of the main principles and ideas in learning. Also, its sequential style of processing examples is perhaps closer to human learning than the one-shot learning in the other two approaches discussed in this chapter. On the other hand, the learning of attribute-value descriptions, in the form of either rules or trees, is simpler and better understood than learning relational descriptions. Therefore, the simpler approaches have until now enjoyed more attention and success in practical applications.

It has been shown that learning is inherently a combinatorial process. It involves search among possible hypotheses. This search can be heuristically guided. It can also be significantly constrained by the 'linguistic bias'; that is, the effect of the hypothesis language only allowing certain forms of hypotheses to be constructed. Both our algorithms for learning if-then rules and decision trees were very strongly heuristically guided – for example, by the impurity criterion. Therefore, these algorithms are efficient, although in our Prolog implementation (Figure 18.11) the evaluation of such a heuristic itself is not efficiently implemented. These heuristics require the computation of certain statistics for which Prolog is not the most suitable.

In general, the success of learning is measured by several criteria. In the following sections we will look at some usual criteria, and discuss the benefits of tree pruning in the view of these criteria.

18.7.1 Criteria for success of learning

Here are some usual criteria for measuring the success of a learning system:

- *Classification accuracy*: This is usually defined as the percentage of objects correctly classified by the induced hypothesis H. We distinguish between two types of classification accuracy:
  (1) Accuracy on new objects; that is, those not contained in the training set S.
  (2) Accuracy on the objects in S. (Of course, this is only interesting when inconsistent hypotheses are allowed.)
- *Comprehensibility* (understandability) of the induced hypothesis H: It is often important for the generated description to be comprehensible in order to tell the user something interesting about the application domain. Such a description can also be used by humans directly, without machine help, as an enhancement to humans' own knowledge. In this sense, machine learning is one approach to computer-aided synthesis of new knowledge. Donald Michie (1986) was an early proponent of this idea. The comprehensibility criterion is also very important when the induced descriptions are used in an expert system whose behaviour has to be easy to understand.
- *Computational complexity*: What are the required computer resources in terms of time and space? Usually, we distinguish between two types of complexity:
  (1) Generation complexity (resources needed to induce a concept description from examples).
  (2) Execution complexity (complexity of classifying an object using the induced description).

18.7.2 Estimating the accuracy of learned hypotheses

The usual question after learning has been done is: How well will the learned hypothesis predict the class in new data? Of course, when new data become available, this accuracy can simply be measured by classifying the new objects and comparing their true class with the class predicted by the hypothesis. The difficulty is that we would like to estimate the accuracy *before* any new data become available.

The usual approach to estimating the accuracy on new data is to randomly split the available data into two sets: *training set* and *test set*. Then we run the learning program on the training set, and test the induced hypothesis on the test set as if this was new, future data. This approach is simple and unproblematic if a lot of data is available. A common situation is, however, shortage of data. Suppose we are learning to diagnose in a particular area of medicine. We are limited to the data about past patients, and the amount of data cannot be increased. When the amount of learning data is small, this may not be sufficient for successful learning. The shortage of data is then aggravated by holding out part of the data as a test set.

When the number of learning examples is small, the results of learning and testing are susceptible to large statistical fluctuations. They much depend on the particular split into a training set and test set. To alleviate this statistical uncertainty, the learning and testing is repeated a number of times (typically, ten times), each time for a different random split. The accuracy results are then averaged and their variance gives an idea of the stability of the estimate.

An elaboration of this type of repetitive testing is *k-fold cross-validation*. Here the complete learning set is randomly split into k subsets. Then the learning and testing is repeated for each of the subsets as follows: the ith subset is removed from the data, the rest of the data is used as a training set, and the ith subset is then used for testing the induced hypothesis. The k accuracy results so obtained are averaged and their variance is computed. There is no particular method to choose k, but the choice $k = 10$ is the most usual in machine learning experiments.

A particular case of cross-validation arises when the subsets only contain one element. In each iteration, the learning is done on all the data but one example, and the induced hypothesis is tested on the remaining example. This form of cross-validation is called 'leave-one-out'. It is sensible when the amount of available data is particularly small.

18.7.3 How pruning affects accuracy and transparency of decision trees

Pruning of decision trees is of fundamental importance because of its beneficial effects when dealing with noisy data. Pruning affects two measures of success of learning: first, the classification accuracy of a decision tree on new objects and, second, the transparency of a decision tree. Let us consider both of these effects of pruning.

The comprehensibility of a description depends on its structure and size. A well-structured decision tree is easier to understand than a completely unstructured tree. On the other hand, if a decision tree is small (consisting of just ten or so nodes) then it is easy to understand regardless of its structure. Since pruning reduces the size of a tree, it contributes to the comprehensibility of a decision tree. As has been shown experimentally in many noisy domains, such as medical diagnosis, the reduction of tree size can be dramatic. The number of nodes is reduced to, say, ten percent of their original number, while retaining at least the same level of classification accuracy.

Tree pruning can also improve the classification accuracy of a tree. This effect of pruning may appear counter-intuitive since, by pruning, we throw away some information, and it would seem that as a result some accuracy should be lost. However, in the case of learning from noisy data, an appropriate amount of pruning normally improves the accuracy. This phenomenon can be explained in statistical terms: statistically, pruning works as a sort of noise suppression mechanism. By pruning, we eliminate errors in learning data that are due to noise, rather than throw away healthy information.

Project

Carry out a typical project in machine learning research. This consists of implementing a learning algorithm, and testing its accuracy on experimental data sets using the 10-fold cross-validation. Investigate how tree pruning affects the accuracy on new data. Investigate the effect of minimal error pruning when varying the value of parameter m in m-estimate. Many real-life learning data sets are electronically

available for such experiments from the well-known UCI Repository for Machine Learning (University of California at Irvine; http://www.ics.uci.edu/~mlearn/MLRepository.html).

Summary

- Forms of learning include: learning by being told, learning from examples, learning by discovery. Learning concepts from examples is also called inductive learning. This form of learning has attained most success in practical applications.
- Learning from examples involves:

  objects and concepts as sets;
  positive and negative examples of concept to be learned;
  hypotheses about target concept;
  hypothesis language.
- The goal of learning from examples is to construct a hypothesis that 'explains' sufficiently well the given examples. Hopefully such a hypothesis will accurately classify future examples as well. A hypothesis is *consistent* with the learning examples if it classifies all the learning data as given in the examples.
- The inductive learning process involves search among possible hypotheses. This is inherently combinatorial. To reduce computational complexity, typically this process is heuristically guided.
- During its construction, a hypothesis may be generalized or specialized. Normally, the final hypothesis is a generalization of the positive examples.
- Programs developed in this chapter are:
  - A program that learns if-then rules from examples defined by attribute-value vectors.
  - A program that learns decision trees from examples defined by attribute-value vectors.
- The pruning of decision trees is a powerful approach to learning from noisy data. Minimal-error pruning method was presented in detail.
- Difficulty of estimating probabilities from small samples was discussed, and the m-estimate was introduced.
- Criteria for assessing the success of a method of learning from examples include:
  - accuracy of induced hypotheses;
  - comprehensibility of learned concept descriptions;
  - computational efficiency both in inducing a hypothesis from data, and in classifying new objects with the induced hypothesis.

- The expected accuracy of learned hypotheses on new data is usually estimated by *cross-validation*. 10-fold cross-validation is the most common. The leave-one-out method is a special form of cross-validation.
- Concepts discussed in this chapter are:

  machine learning
  learning concepts from examples, inductive learning
  hypothesis languages
  relational descriptions
  attribute-value descriptions
  generality and specificity of hypotheses
  generalization and specialization of descriptions
  ARCHES-type learning of relational descriptions
  learning of if-then rules
  top-down induction of decision trees
  learning from noisy data
  tree pruning, post-pruning, minimal-error pruning
  estimating probabilities
  cross validation

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Mitchell's book (1997) is an excellent general introduction to machine learning (ML). In Michalski, Bratko and Kubat (1998), ML applications are presented in a variety of areas, ranging from engineering to medicine, biology and music. Current research in machine learning is published in the AI literature, most notably in the journals *Machine Learning* (Boston: Kluwer) and *Artificial Intelligence* (Amsterdam: North-Holland), and at annual conferences ICML (Int. Conf. on Machine Learning), ECML (European Conf. on Machine Learning) and COLT (Computational Learning Theory). Many classical papers on ML are included in Michalski, Carbonell and Mitchell (1983, 1986) and Kodratoff and Michalski (1990). Gillies (1996) investigates implications of the technical developments in ML and its applications regarding a traditional controversy in the philosophy of science.

Many data sets for experimentation with new ML methods are electronically available from the UCI Repository (University of California at Irvine; http://www.ics.uci.edu/~mlearn/MLRepository.html).

Our example of learning relational descriptions (Section 18.3) follows the ideas of an early learning program ARCHES (Winston 1975). The program in Section 18.4 that constructs attribute-based descriptions is a simple version of the AQ-type learning. The AQ family of learning programs was developed by Michalski and his co-workers (for example, Michalski 1983). CN2 is a well known program for learning if-then rules (Clark and Niblett 1989).

Induction of decision trees is one of the most widely used approaches to learning, also known under the name TDIDT (top-down induction of decision trees, coined by Quinlan, 1986). TDIDT learning was much influenced by Quinlan's early program ID3, Iterative Dichotomizer 3 (Quinlan 1979), and tree induction is therefore often simply referred to as ID3. Tree induction was also studied outside AI by Breiman and his co-workers (1984), who

independently discovered several pertinent mechanisms, including pruning of decision trees. The technique of minimal-error tree pruning was introduced by Niblett and Bratko (1986) and improved with the *m*-estimate in Cestnik and Bratko (1991). The *m*-estimate formula was derived by Cestnik (1990) using the Bayesian procedure for probability estimation. Esposito *et al.* (1997) carried out detailed experimental comparison of various methods of tree pruning. Several other refinements, in addition to pruning, of the basic tree induction algorithm are necessary to make it a practical tool in complex applications with real-life data. Several such refinements, along with early experiments in which machine-learned knowledge outperforms human experts, are presented in Cestnik *et al.* (1987) and Quinlan (1986). Structuring decision trees to improve their transparency was studied by Shapiro (1987).

Let us mention some of the many interesting developments in and approaches to machine learning that could not be covered in this chapter.

An early attempt at machine learning is Samuel's (1959) learning program that played the game of checkers. This program improved its position-evaluation function through the experience it obtained in the games it played.

Mitchell (1982) developed the idea of *version space* as an attempt to handle economically the search among alternative concept descriptions.

In reinforcement learning (Sutton and Barto 1998) the learner explores its environment by performing actions and receiving rewards from the environment. In learning to act so as to maximize the reward, the difficulty is that the reward may be *delayed*, so it is not clear which particular actions are to be blamed for success or failure.

In *instance-based* learning, related to *case-based reasoning* (Kolodner 1993), the learner reasons about a given case by comparing it to similar previous cases.

Neural networks (also introduced in Mitchell (1997)) is a large area of learning which aspires to some extent to resemble biological learning. The learning occurs through adjustment of numerical weights associated with artificial neurons in the network. Neural networks have not been at the centre of AI because they lack explicit symbolic representation of what has been learned. Hypotheses resulting from neural learning can be very good predictors, but are hard to understand and interpret by humans.

An interesting approach to learning is *explanation-based learning* (e.g. Mitchell *et al.* 1986), also known as *analytical learning*. Here the learning system uses background knowledge to 'explain' a given example by a deductive process. As a result, background knowledge gets compiled into a more efficiently executable form. One approach to explanation-based generalization is programmed in Chapter 23 of this book as a kind of meta-programming exercise.

Some learning programs learn by autonomous discovery. They discover new concepts through exploration, making their own experiments. A celebrated example of this kind of program is AM, Automatic Mathematician (Lenat 1982). AM, for example, starting with the concepts of set and 'bag', discovered the concepts of number, addition, multiplication, prime number, etc. Shrager and Langley (1990) edited a collection of papers on ML for scientific discovery.

Logic formalisms are used as a hypothesis language in the approach to learning called inductive logic programming (ILP), which is studied in the next chapter of this book.

The mathematical theory of learning is also known as COLT (computational learning theory; e.g. Kearns and Vazirani 1994). It is concerned with questions like: How many examples are needed so that it is likely to attain some specified accuracy of the induced hypothesis? What are various theoretical complexities of learning for different classes of hypothesis languages?

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chapter 19

Inductive Logic Programming

Inductive logic programming is an approach to machine learning where definitions of relations are induced from examples. In ILP, logic is used as a hypothesis language and the result of learning is usually just a Prolog program. So Prolog programs are automatically constructed from examples. For instance, the user provides some examples of how lists are sorted and how they are not, and a program for sorting lists is constructed automatically. In comparison with other approaches to machine learning, ILP provides a very general way of specifying 'background knowledge', that is knowledge known to the learner before the learning starts. The price for this flexibility in ILP is a generally high computational complexity. In this chapter we develop an ILP program, called HYPER (Hypothesis Refiner), that is capable of solving typical learning exercises in the ILP setting.

19.1 Introduction

Inductive logic programming (ILP) is an approach to machine learning. It is a way of learning relations from examples. In ILP, logic is used as a language for defining hypotheses. Thus the result of learning is a formula in predicate logic, usually just a Prolog program.

In this framework machine learning looks like automatic programming when the user (a 'programmer') does not write programs directly. Instead, the user only specifies by examples what the intended program is supposed to do. Examples indicate what the intended program should do, and counter examples indicate what the program should not do. In addition, the user also specifies some 'background' predicates that may be used in writing the intended, new program.

Here is an example of such automatic programming in Prolog. Suppose we already have predicates parent(X,Y), male(X) and female(X), defining some family relations as in Figure 1.1 (in Chapter 1). Now we want to find a definition of a new predicate has_daughter(X). Let there be some examples about this new predicate. Suppose there are two positive examples:

```
has_daughter(tom)   has_daughter(bob)
```

There are two negative examples:

```
has_daughter(pam),   has_daughter(jim)
```

The learning task is to find a definition of the predicate has_daughter(X) in terms of the predicates parent, male and female, so that it is true for all the given positive examples and not true for any of the given negative examples. An ILP program may come up with the following hypothesis about has_daughter:

```
has_daughter( X) :-
  parent( X, Y),
  female( Y).
```

This hypothesis explains all the given examples.

Notice here the typical feature of relational learning as opposed to attribute-value learning. The property has_daughter of X above is defined not only in terms of X's attributes. Instead, to determine whether an object X has the property has_daughter, look at another object Y related to X, and check properties of Y.

Let us now introduce some technical terminology of ILP. In the foregoing example, the predicate to be learned, has_daughter, is called a *target predicate*. The given predicates parent, male and female are called *background knowledge* (BK), or *background predicates*. This is the knowledge known to the learner before the learning starts. So the background predicates determine the language in which the learner can express hypotheses about the target predicate.

We can now state the usual formulation of the ILP problem:

Given:

(1) A set of positive examples E_+ and a set of negative examples E_-, and
(2) Background knowledge BK, stated as a set of logic formulas, such that the examples E_+ cannot be derived from BK

Find:
A hypothesis H, stated as a set of logic formulas, such that:

(1) All the examples in E_+ can be derived from BK and H, and
(2) No negative example in E_- can be derived from BK and H.

We also say that H, together with BK, has to *cover* all the positive examples, and *must not cover* any of the negative examples. Such a hypothesis H is said to be *complete* (covers all the positive examples) and *consistent* (does not cover any negative example).

Usually both BK and H are simply sets of Prolog clauses, that is Prolog programs. So from the viewpoint of automatic programming in Prolog this corresponds to the following. Suppose our target predicate is p(X) and we have, among others, a positive example p(a) and a negative example p(b). A possible conversation with program BK would be:

```
?- p(a).        % A positive example
no              % Cannot be derived from BK

?- p(b).        % A negative example
no              % Cannot be derived from BK
```

Now suppose an ILP system would be called to automatically induce an additional set of clauses H and add them to the program BK. The conversation with the so extended program BK plus H would now be:

```
?- p(a).        % A positive example
yes             % Can be derived from BK and H

?- p(b).        % A negative example
no              % Cannot be derived from BK and H
```

Well-known exercises in ILP are the automatic construction of Prolog programs to concatenate or sort lists. The programs are constructed from positive examples of how lists are concatenated or sorted, and from negative examples of how they are not. In this chapter we will develop an ILP program called HYPER, and apply it to such exercises.

Let us here consider the motivation for ILP in comparison with other, non-logical approaches to machine learning that are essentially at the level of attribute-value learning. ILP's strength rests in the power of its hypothesis language, and the way of incorporating background knowledge into the learning process. The hypothesis language allows for relational definitions that may even involve recursion. This is usually not possible with other approaches to machine learning.

In ILP, background knowledge can in principle be any Prolog program. This enables the user to provide in a very natural way prior domain-specific knowledge to be used in learning. The use of background knowledge enables the user to develop a good problem representation and to introduce problem-specific constraints into the learning process. By contrast, attribute-value learners can typically accept background knowledge in rather limited forms only, for example in the form of relevant new attributes. In ILP on the other hand, if the problem is to learn about properties of chemical compounds, the molecular structures can be introduced as background knowledge in terms of the atoms and bonds between them. If the learning task is to automatically construct a model of a physical system from its observed behaviours, the complete mathematical apparatus that is considered relevant to the modelling

domain can be included in background knowledge. If this involves the treatment of time and space, axioms of reasoning about time and space may be included as background knowledge. In a typical application of ILP, the emphasis is on the development of a good representation of examples together with relevant background knowledge. A general purpose ILP system is then applied to carry out the induction.

The power of hypothesis language and flexibility of background knowledge in ILP do not come without a price. This flexibility adds to the combinatorial complexity of the learning task. Therefore attribute-value learning, such as decision trees, is much more efficient than ILP. Therefore in learning problems where attribute-value representations are adequate, attribute-value learning is recommended for efficiency reasons.

In this chapter we will develop an ILP program called HYPER (*Hypothesis refiner*), which constructs Prolog programs through gradual refinement of some starting hypotheses. To illustrate the main ideas, we will first develop a simple and inefficient version of it, called MINIHYPER. This will then be elaborated into HYPER.

19.2 Constructing Prolog programs from examples

19.2.1 Representation of a learning problem

Let us consider again the family relations example to see how hypotheses may be constructed automatically from examples. Suppose that our BK and the examples are as in Figure 19.1. The corresponding family graph is similar to the one in Figure 1.1 (in Chapter 1), with the addition that Pat has a daughter, Eve. The program developed in this section will assume the representational conventions illustrated in Figure 19.1. The positive examples are represented by the predicate **ex(Example)**, e.g.:

```
ex( has_daughter(tom)).        % Tom has a daughter
```

The negative examples are represented by the predicate **nex(Example)**, e.g.:

```
nex( has_daughter(pam)).       % Pam does not have a daughter
```

The predicate **backliteral** specifies the form of literals that the ILP program may use as goals in constructing Prolog clauses. For example,

```
backliteral( parent(X,Y), [X,Y]).
```

says that literals of the form **parent(X,Y)**, with the variables X and Y possibly renamed, are part of the hypothesis language. The second argument in **backliteral** is a list of the variables in the literal. Such 'background literals' can be added as goals

```
% Learning from family relations

% Background knowledge

backliteral( parent(X,Y), [X,Y]).          % A background literal with vars. [X, Y]
backliteral( male(X), [X]).
backliteral( female(X), [X]).

prolog_predicate( parent(_,_)).            % Goal parent(_,_) executed directly by Prolog
prolog_predicate( male(_)).
prolog_predicate( female(_)).

parent( pam, bob).
parent( tom, bob).
parent( tom, liz).
parent( bob, ann).
parent( bob, pat).
parent( pat, jim).
parent( pat, eve).

female( pam).
male( tom).
male( bob).
female( liz).
female( ann).
female( pat).
male( jim).
female( eve).

% Positive examples

ex( has_daughter(tom)).                    % Tom has a daughter
ex( has_daughter(bob)).
ex( has_daughter(pat)).

% Negative examples

nex( has_daughter(pam)).                   % Pam doesn't have a daughter
nex( has_daughter(jim)).

start_hyp( [ [has_daughter(X)] / [X] ] ).  % Starting hypothesis
```

Figure 19.1 A definition of the problem of learning predicate **has_daughter**.

to the body of clauses that are being constructed. Background literals are calls to BK predicates that may be specified directly in Prolog. The predicate **prolog_predicate** specifies goals that are to be directly executed by the Prolog interpreter. For example,

```
prolog_predicate( parent(X,Y)).
```

says that goals that match **parent(X,Y)** are evaluated by Prolog directly, executing the BK predicate **parent**. Other goals, such as **has_daughter(tom)**, will be executed by a

Prolog-like interpreter with special properties that we will implement specifically for use in ILP.

19.2.2 Refinement graph

Let us now consider how a complete and consistent hypothesis can be generated for the learning problem of Figure 19.1. We may start with some overly general hypothesis that is complete (covers all the positive examples), but inconsistent (also covers negative examples). Such a hypothesis will have to be specialized in a way to retain its completeness and attain consistency. This can be done by searching a space of possible hypotheses and their refinements. Each refinement takes a hypothesis H1 and produces a more specific hypothesis H2, so that H2 covers a subset of the cases covered by H1.

Such a space of hypotheses and their refinements is called a *refinement graph*. Figure 19.2 shows part of such a refinement graph for the learning problem of Figure 19.1. The nodes of this graph correspond to hypotheses, the arcs between hypotheses correspond to refinements. There is a directed arc between hypotheses H1 and H2 if H2 is a refinement of H1.

Once we have a refinement graph the learning problem is reduced to searching this graph. The start node of search is some over-general hypothesis. A goal node of

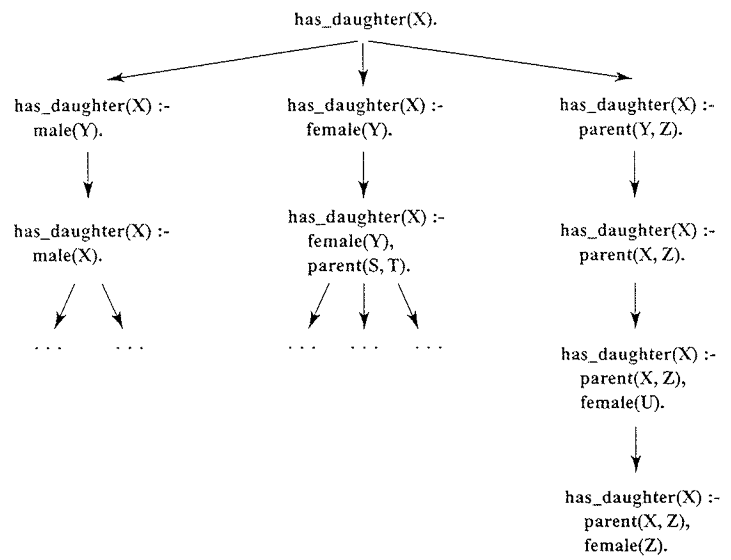

Figure 19.2 Part of the refinement graph for the learning problem of Figure 19.1. Many possible refinements are omitted in this diagram.

search is a hypothesis that is consistent and complete. In our example of Figure 19.2, it is sufficient that all the hypotheses are just single clauses. In general, hypotheses consist of multiple clauses.

To implement this approach we have to design two things:

(1) A *refinement operator* that will generate refinements of hypotheses (such an operator defines a refinement graph).

(2) A search procedure to carry out the search.

In the graph of Figure 19.2, there are two types of refinement. A refinement of a clause is obtained by either:

(1) matching two variables in the clause, or

(2) adding a background literal to the body of the clause.

An example of the first type of refinement is:

```
has_daughter(X) :- parent(Y,Z).
```

is refined into

```
has_daughter(X) :- parent(X,Z).
```

by matching X=Y. An example of the second type of refinement is:

```
has_daughter(X).
```

is refined into

```
has_daughter(X) :- parent(Y,Z).
```

There are a few important points to notice. First, each refinement is a *specialization*. That is, a successor of a hypothesis in the refinement graph only covers a subset of the cases covered by the hypothesis' predecessor. Therefore it suffices that, during search, we only consider complete hypotheses. An incomplete hypothesis can never be refined into a complete one.

A second point about a refinement operator is that it has to produce sufficiently 'small' refinement steps. Otherwise a refinement operator may fail to generate the target hypothesis. Too coarse an operator may jump from a complete and inconsistent hypothesis straight to an incomplete and consistent one, over-jumping a consistent and complete hypothesis in between.

There is another type of refinement, one concerned with refining variables to structured terms. For example, the clause:

```
member( X1, L1) :- member( X1, L3).
```

may be refined into

```
member( X1, [X2 | L2]) :- member( X1, L3).
```


where the variable L1 is refined into the structure [X2 | L2]. For simplicity, the program MINIHYPER that we will develop in this section will not be able to handle problems that require structured terms. We will defer this until the next section when we develop a more sophisticated program HYPER.

Another point obvious from Figure 19.2 is the combinatorial complexity of refinement graphs and the ensuing search complexity. Again, in MINIHYPER we will not worry about this and simply use the uninformed iterative deepening search. This will also later be improved to a best-first search in HYPER.

19.2.3 Program MINIHYPER

We are now ready to start writing our first ILP program. We will choose to represent hypotheses as lists of clauses:

```
Hypothesis = [Clause1, Clause2, ...]
```

Each clause will be represented by a list of literals (the head of the clause followed by the body literals) and the list of variables in the clause:

```
Clause = [Head, BodyLiteral1, BodyLiteral2, ...] / [Var1, Var2, ...]
```

For example, the hypothesis

```
pred(X,Y) :- parent(X,Y).
pred(X,Z) :- parent(X,Y), pred(Y,Z).
```

is in this convention represented as:

```
[ [pred(X1,Y1), parent(X1,Y1)] / [X1,Y1],
  [pred(X2,Z2), parent(X2,Y2), pred(Y2,Z2)] / [X2,Y2,Z2] ]
```

Although it is not strictly necessary explicitly to represent the list of variables in a clause, it is handy for the implementation of hypothesis refinement by matching variables. Notice that we need different names of variables for each clause in a hypothesis because they actually are different variables.

To test whether a hypothesis covers an example we need a Prolog-like interpreter for hypotheses represented as above. To this end we will define the predicate:

```
prove( Goal, Hypothesis, Answer)
```

that for a given **Goal** and **Hypothesis** finds **Answer** indicating whether **Goal** can be logically derived from **Hypothesis**. This predicate will basically try to prove **Goal** from **Hypothesis** in a way similar to Prolog itself. Programming this is an exercise in metaprogramming which is discussed in more detail in Chapter 23. A specific difficulty in our case is the danger of infinite loops. Our refinement operator may easily generate a recursive clause like

```
[ p(X), p(X)]
```

which stands for p(X) :- p(X). This may lead to an infinite loop. We have to make our **prove** predicate immune from such loops. An easy way to do this is by limiting the length of proofs. When proving Goal, if the number of predicate calls reaches this limit, then the procedure **prove** will simply stop without reaching a definitive answer. The argument Answer of **prove** will therefore have the following possible meanings:

Answer = yes:	Goal has been derived from Hypothesis within proof limit
Answer = no:	Goal definitively cannot be derived from Hypothesis even if the limit was relaxed
Answer = maybe:	proof search was terminated because maximum proof length D was reached

The case 'Answer = maybe' means any one of the following three possibilities if Goal was executed by the standard Prolog interpreter:

(1) The standard Prolog interpreter (with no limit on proof length) would get into infinite loop.

(2) The standard Prolog interpreter would eventually find a proof of length greater than limit D.

(3) The standard Prolog interpreter would find, at some length greater than D, that this derivation alternative fails. Therefore it would backtrack to another alternative and there possibly find a proof (of length possibly no greater than D), or fail, or get into an infinite loop.

Figure 19.3 gives the program code for **prove**. The proof-length limit is specified by the predicate:

```
max_proof_length( D)
```

D is by default set to 6, but it can be adjusted depending on the particular learning problem. Calls of background predicates (declared with prolog_predicate) are simply delegated to the standard Prolog interpreter and they do not incur an increase in proof length. So only the calls of the target predicates defined in the hypothesis count.

The learning program will react to 'maybe' cautiously as follows:

(1) When proving a positive example, 'maybe' is treated as 'the example not covered'.

(2) When proving a negative example, 'maybe' is treated as not 'not covered', that is as covered.

A rationale behind this cautious interpretation of 'maybe' is that, among the possible complete hypotheses, computationally efficient ones are preferred. The answer 'maybe' at best indicates that the hypothesis is computationally inefficient, and at worst that it is incomplete.


```
% Interpreter for hypotheses
% prove( Goal, Hypo, Answ):
%    Answ = yes, if Goal derivable from Hypo in at most D steps
%    Answ = no, if Goal not derivable
%    Answ = maybe, if search terminated after D steps inconclusively

prove( Goal, Hypo, Answer) :-
  max_proof_length( D),
  prove( Goal, Hypo, D, RestD),
  (RestD >= 0, Answer = yes                    % Proved
   ;
   RestD < 0, !, Answer = maybe                % Maybe, but it looks like inf. loop
  ).

prove( Goal, _, no).                           % Otherwise Goal definitely cannot be proved

% prove( Goal, Hyp, MaxD, RestD):
%    MaxD allowed proof length, RestD 'remaining length' after proof;
%    Count only proof steps using Hyp

prove( G, H, D, D) :-
  D < 0, !.                                    % Proof length overstepped

prove( [], _, D, D) :- !.

prove( [G1 | Gs], Hypo, D0, D) :- !,
  prove( G1, Hypo, D0, D1),
  prove( Gs, Hypo, D1, D).

prove( G, _, D, D) :-
  prolog_predicate( G),                        % Background predicate in Prolog?
  call( G).                                    % Call of background predicate

prove( G, Hyp, D0, D) :-
  D0 =< 0, !, D is D0-1                        % Proof too long
  ;
  D1 is D0-1,                                  % Remaining proof length
  member( Clause/Vars, Hyp),                   % A clause in Hyp
  copy_term( Clause, [Head | Body] ),          % Rename variables in clause
  G = Head,                                    % Match clause's head with goal
  prove( Body, Hyp, D1, D).                    % Prove G using Clause
```

Figure 19.3 A loop-avoiding interpreter for hypotheses.

The rest of the MINIHYPER program is given in Figure 19.4. The predicates in this program are as follows.

refine(Clause, Vars, NewClause, NewVars)
Refines a given clause Clause with variables Vars and produces a refined clause NewClause with new variables NewVars. The refined clause is obtained by matching two variables in Vars or by adding a new background literal to Clause. New literals are

```
% Program MINIHYPER

% induce( Hyp):
%   induce a consistent and complete hypothesis Hyp by gradually
%   refining start hypotheses

induce( Hyp) :-
  iter_deep( Hyp, 0).                  % Iterative deepening starting with max. depth 0

iter_deep( Hyp, MaxD) :-
  write( 'MaxD = '), write( MaxD), nl,
  start_hyp( Hyp0),
  complete( Hyp0),                     % Hyp0 covers all positive examples
  depth_first( Hyp0, Hyp, MaxD)        % Depth-limited depth-first search
  ;
  NewMaxD is MaxD + 1,
  iter_deep( Hyp, NewMaxD).

% depth_first( Hyp0, Hyp, MaxD):
%   refine Hyp0 into consistent and complete Hyp in at most MaxD steps

depth_first( Hyp, Hyp, _) :-
  consistent( Hyp).

depth_first( Hyp0, Hyp, MaxD0) :-
  MaxD0 > 0,
  MaxD1 is MaxD0 - 1,
  refine_hyp( Hyp0, Hyp1),
  complete( Hyp1),                     % Hyp1 covers all positive examples
  depth_first( Hyp1, Hyp, MaxD1).

complete( Hyp) :-                      % Hyp covers all positive examples
  not (ex( E),                         % A positive example
       once( prove( E, Hyp, Answer)),  % Prove it with Hyp
       Answer \== yes).                % Possibly not proved

consistent( Hyp) :-          % Hypothesis does not possibly cover any negative example
  not (nex( E),                        % A negative example
       once( prove( E, Hyp, Answer)),  % Prove it with Hyp
       Answer \== no).                 % Possibly provable

% refine_hyp( Hyp0, Hyp):
%   refine hypothesis Hyp0 into Hyp

refine_hyp( Hyp0, Hyp) :-
  conc( Clauses1, [Clause0/Vars0 | Clauses2], Hyp0),  % Choose Clause0 from Hyp0
  conc( Clauses1, [Clause/Vars | Clauses2], Hyp),     % New hypothesis
  refine( Clause0, Vars0, Clause, Vars).              % Refine the Clause

% refine( Clause, Args, NewClause, NewArgs):
%   refine Clause with arguments Args giving NewClause with NewArgs
```



```
% Refine by unifying arguments

refine( Clause, Args, Clause, NewArgs) :-
  conc( Args1, [A | Args2], Args),     % Select a variable A
  member( A, Args2),                   % Match it with another one
  conc( Args1, Args2, NewArgs).

% Refine by adding a literal

refine( Clause, Args, NewClause, NewArgs) :-
  length( Clause, L),
  max_clause_length( MaxL),
  L < MaxL,
  backliteral( Lit, Vars),             % Background knowledge literal
  conc( Clause, [Lit], NewClause),     % Add literal to body of clause
  conc( Args, Vars, NewArgs).          % Add literal's variables

% Default parameter settings

max_proof_length( 6).        % Max. proof length, counting calls to 'non-Prolog' pred.

max_clause_length( 3).       % Max. number of literals in a clause
```

Figure 19.4 MINIHYPER – a simple ILP program.

only added until user-defined maximal clause length is reached (defined by predicate max_clause_length(MaxL)).

refine_hyp(Hyp, NewHyp)
Refines a hypothesis Hyp producing NewHyp by non-deterministically choosing a clause in Hyp and refining that clause.

induce(Hyp)
Induces a consistent and complete hypothesis Hyp for the given learning problem, by iterative deepening search of the refinement graph starting with a start hypothesis (given by predicate start_hyp(StartHyp)).

iter_deep(Hyp, MaxD)
Finds a complete and consistent hypothesis Hyp by iterative deepening search, starting with depth-limit MaxD and increasing the limit until Hyp is found. Complete iterative-deepening search is carried out by iter_deep(Hyp,0).

depth_first(Hyp0, Hyp, MaxD)
Performs depth-first search limited to depth MaxD, starting with a hypothesis Hyp0. If search succeeds within MaxD then Hyp is a complete and consistent hypothesis obtained in no more than MaxD successive refinements of Hyp0.

complete(Hyp)
True if Hyp covers all the given positive examples (under the 'cautious' interpretation of Answer in prove/3).

consistent(Hyp)
True if Hyp does not cover any of the given negative examples (under the cautious interpretation of Answer in prove/3).

To use MINIHYPER, the code in Figures 19.3 and 19.4 is to be loaded into Prolog, together with the frequently used predicates member/2, conc/3, not/1, once/1 and copy_term/2, and a definition of the learning problem (such as the one in Figure 19.1). The parameters max_proof_length and max_clause_length can of course be redefined as appropriate for the particular learning problem.

For example, learning the predicate has_daughter(X) of Figure 19.1 looks as follows:

```
?- induce(H).

MaxD = 0
MaxD = 1
MaxD = 2
MaxD = 3
MaxD = 4

H = [[has_daughter(A),parent(A,B),female(B)]/[A,B]]
```

The increasing depth limits in iterative deepening search are displayed as MaxD = Limit. A consistent and complete hypothesis is found at refinement depth 4 (cf. Figure 19.2). If we add counters of hypotheses to the program, they would show the number of all generated hypotheses was 105, and 25 of these were refined. The resulting hypothesis H above, translated into the usual Prolog syntax is:

```
has_daughter(A) :- parent(A,B), female(B).
```

This corresponds to our target predicate.

Exercise

19.1 Experiment with MINIHYPER with modified sets of examples about has_daughter. How do these modifications affect the results?

Let us now consider a slightly more complicated learning exercise: learning the predecessor relation, using the same background knowledge as in Figure 19.1. This is more difficult because it requires a recursive definition, but this is exactly what ILP enables. We may define some positive and negative examples as follows:


```
ex( predecessor( pam, bob)).
ex( predecessor( pam, jim)).
ex( predecessor( tom, ann)).
ex( predecessor( tom, jim)).
ex( predecessor( tom, liz)).

nex( predecessor( liz, bob)).
nex( predecessor( pat, bob)).
nex( predecessor( pam, liz)).
nex( predecessor( liz, jim)).
nex( predecessor( liz, liz)).
```

'Guessing' that our target hypothesis comprises two clauses, we may define a start hypothesis as:

```
start_hyp( [ [predecessor(X1,Y1)] / [X1,Y1],
             [predecessor(X2,Y2)] / [X2,Y2] ] ).
```

The relevant background predicates are:

```
backliteral( parent(X,Y), [X,Y]).
backliteral( predecessor(X,Y), [X,Y]).
prolog_predicate( parent(X,Y)).
```

Exercise

19.2 Work out how many refinement steps are needed to obtain the target hypothesis from the start hypothesis above.

We may now try to run MINIHYPER with this problem specification, but it will turn out to be too inefficient. The search space up to the required refinement depth is large. In addition, our search procedure repeatedly generates equal hypotheses that are reachable by different refinement paths. This leads to repeated generation of large subspaces.

We will upgrade MINIHYPER to HYPER in the next section. However, we may just make the predecessor exercise possible for MINIHYPER with the following trick. To control the search complexity (not quite without 'guessing' the result of learning) we may constrain the background literals parent(X,Y) and predecessor(X,Y) to be called with the first argument X instantiated. This can be done by requiring that X be an atom by the following modified definition of background knowledge:

```
backliteral( [atom(X), parent(X,Y)], [X,Y]).
backliteral( [atom(X), predecessor(X,Y)], [X,Y]).
prolog_predicate( parent(X,Y)).
prolog_predicate( atom(X)).
```

This means that in a clause refinement a *pair* of literals is added to the clause, the first literal being atom(X). In testing the completeness of such a hypothesis, the goals in the body will fail unless the argument X is instantiated (tested by atom(X)). This will render many useless hypotheses incomplete and they will immediately be discarded from search. The search task now becomes easier, although it may still take considerable time, that is minutes or tens of minutes, depending on the computer and implementation of Prolog. Eventually the goal induce(H) will result in:

```
H = [[predecessor(A,B),[atom(A),parent(A,C)],[atom(C),predecessor(C,B)]] / [A,C,B],
    [predecessor(D,E),[atom(D),parent(D,E)]]/[D,E]]
```

This is an expected definition of predecessor.

It is clear that MINIHYPER will soon run out of steam when faced with slightly more difficult learning problems. In the next section we will therefore make several improvements.

Exercise

19.3 Run MINIHYPER with the predecessor exercise and measure the execution time. How many hypotheses are generated, how many are refined? The 'guard' literal atom(X) in the background literals above is helpful, but there is a problem. At the moment that a literal pair like [atom(X),parent(X,Y)] is added, all the goals to it will fail (because X is a new variable not yet matched to anything else). Only after a further refinement, when X is matched to an existing variable, may such a literal succeed. So after a refinement with matching, such a hypothesis may become more *general*. This is different from the usual case when refinements produce more specific hypotheses. This anomaly in the case of atom(X) makes the refinement process non-monotonic: the hypotheses along a refinement path are not necessarily increasingly more specific. In this case an incomplete hypothesis may become complete after a refinement which violates the basic assumption on which the search of the hypothesis space is based! However, in this particular case the search still works. Find a sequence of refinements of *complete* hypotheses in the refinement graph that, despite this anomaly, leads to the target hypothesis.

19.2.4 Generalization, specialization and θ-subsumption

As usual in machine learning, the space of candidate hypotheses in ILP is partially ordered by the relations 'more general than' or 'more specific than'. A hypothesis H1 is more general than H2 if H1 covers at least all the cases covered by H2. Our refinement operator corresponds to such a generality relation between hypotheses. This generality relation between hypotheses can be determined syntactically – refinements are just syntactic operations on hypotheses.

There is another generality relation frequently used in ILP, called θ-subsumption. Although we will not be directly using θ-subsumption in the programs in this chapter, we will introduce it here for completeness.

First let us define the notion of substitution. A *substitution* θ = {Var1/Term1, Var2/Term2, ...} is a mapping of variables Var1, Var2, etc. into terms Term1, Term2, etc. A substitution θ is applied to a clause C by substituting the clause's variables by terms as specified in θ. The application of substitution θ to clause C is written as $C\theta$. For example:

C = has_daughter(X) :- parent(X,Y), female(Y).
θ = { X/tom, Y/liz}
$C\theta$ = has_daughter(tom) :- parent(tom,liz), female(liz).

Now we can define θ-subsumption. It is a generality relation between *clauses*. Clause C_1 θ-subsumes clause C_2 if there is a substitution θ such that every literal in $C_1\theta$ occurs in C_2. For example, the clause

parent(X,Y).

θ-subsumes the clause

parent(X,liz).

where θ = { Y/liz }. The clause

has_daughter(X) :- parent(X,Y).

θ-subsumes the clause

has_daughter(X) :- parent(X,Y), female(Y).

where $\theta = \{\}$. The notion of θ-subsumption provides a way of syntactically checking the generality between clauses. If clause C_1 θ-subsumes clause C_2 then C_2 logically follows from C_1, so C_1 is more general than C_2. C_1, together with the rest of a hypothesis, enables the coverage of at least all the examples covered by C_2 and the rest of the hypothesis. There is a simple relation between our refinement operator and θ-subsumption. The refinement operator takes a clause C_1 and produces clause C_2 so that C_1 θ-subsumes C_2.

Exercise

19.4 Let there be the fact C_0:

num(0).

Let clause C_1 be:

num(s(X)) :- num(X).

and clause C_2 be:

```
num( s(s(X))) :- num( X).
```

Is the hypothesis $\{C_0, C_1\}$ more general than hypothesis $\{C_0, C_2\}$? Does clause C_1 θ-subsume clause C_2?

19.3 Program HYPER

We will now introduce the following improvements into the ILP program of Figure 19.4:

(1) To prevent useless refinements by inappropriate matches, the variables in clauses will be typed. Only the variables of the same type will be allowed to match in clause refinements.

(2) Handling structured terms: the refinement operator will also refine variables into terms.

(3) Distinction between input and output arguments of literals: the refinement operator will immediately take care of instantiating the input arguments.

(4) Best-first search of refinement graph instead of iterative deepening.

Let us discuss the details of these improvements.

19.3.1 Refinement operator

The refinement operator will be as in MINIHYPER, but enhanced with the types of arguments and distinction between input and output arguments. To introduce some syntactic conventions consider the problem definition for learning about the predicate member(X,L) (Figure 19.5). The clause

```
backliteral( member(X,L), [L:list], [X:item] ).
```

says that member(X,L) is a background literal (enabling recursive calls) where L is an input variable of type list, and X is an output variable of type item. The general form of defining background literals is:

```
backliteral( Literal, InArgs, OutArgs)
```

InArgs and OutArgs are the lists of input and output arguments respectively:

```
InArgs = [In1:TypeI1, In2:TypeI2, ...]
OutArgs = [Out1:TypeO1, Out2: TypeO2, ...]
```

where In1, In2, ... are the names of input variables, and TypeI1, TypeI2, ... are their types. Similarly the list OutArgs specifies the output variables and their types.


```
% Problem definition for learning about member(X,L)

backliteral( member(X,L), [L:list], [X:item] ).          % Background literal

% Refinement of terms

term( list, [X|L], [ X:item, L:list]).
term( list, [ ], [ ]).

prolog_predicate( fail).                                  % No background predicate in Prolog

start_clause( [ member(X,L) ] / [ X:item, L:list] ).

% Positive and negative examples

ex( member( a, [a])).
ex( member( b, [a,b])).
ex( member( d, [a,b,c,d,e])).

nex( member( b, [a])).
nex( member( d, [a,b])).
nex( member( f, [a,b,c,d,e])).
```

Figure 19.5 Problem definition for learning list membership.

The meaning of input and output is as follows. If an argument of a literal is *input* then it is supposed to be instantiated whenever the literal is executed. In refining a clause, this means that when such a literal is added to the body of the clause, each of its input variables has to be immediately matched with some existing variable in the clause. In the example above, whenever the literal member(X,L) is added to a clause, the variable L has to be immediately matched with an existing variable (also of type 'list'). Such a matching should take care that the input argument is instantiated at the time the literal is executed. Output arguments *may* be matched with other variables later. So in clause refinement, the treatment of output variables is the same as the treatment of all the variables in MINIHYPER. However, the type restrictions prevent the matching of variables of different types. So X of type item cannot be matched with L of type list.

Possible refinements of variables into structured terms are defined by the predicate:

```
term( Type, Term, Vars)
```

This says that a variable of type Type in a clause can be replaced by a term Term. Vars is the list of variables and their types that occur in Term. So in Figure 19.5, the clauses

```
term( list, [X|L], [ X:item, L:list]).
term( list, [ ], [ ]).
```

say that a variable of type list can be refined into [X|L] where X is of type item and L is of type list, or this variable is replaced by the constant [] (with no variables). We assume throughout that ':' has been introduced as an infix operator.

In Figure 19.5, the clause

```
start_clause( [ member(X,L) ] / [ X:item, L:list] ).
```

declares the form of clauses in the start hypotheses in the refinement graph. Each start hypothesis is a list of up to some maximum number of (copies of) start clauses. The list of start hypotheses will be generated automatically. The maximum number of clauses in a hypothesis is defined by the predicate max_clauses. This can be set appropriately by the user according to specifics of the learning problem.

Let us now state the refinement operator in HYPER in accordance with the foregoing discussion. To refine a clause, perform one of the following:

(1) Match two variables in the clause, e.g. X1 = X2. Only variables of the same type can be matched.

(2) Refine a variable in the clause into a background term. Only terms defined by the predicate term/3 may be used and the type of the variable and the type of the term have to match.

(3) Add a background literal to the clause. All of the literal's input arguments have to be matched (non-deterministically) with the existing variables (of the same type) in the clause.

As in MINIHYPER, to refine a hypothesis H_0, choose one of the clauses C_0 in H_0, refine clause C_0 into C, and obtain a new hypothesis H by replacing C_0 in H_0 with C. Figure 19.6 shows a sequence of refinements when learning about member.

In the HYPER program, we will add to this some useful heuristics that often save complexity. First, if a clause is found in H_0 that alone covers a negative example, then only refinements arising from this clause are generated. The reason is that such a clause necessarily has to be refined before a consistent hypothesis is obtained. The second heuristic is that 'redundant' clauses (which contain several copies of the same literal) are discarded. And third, 'unsatisfiable clauses' are discarded. A clause is unsatisfiable if its body cannot be derived by predicate prove from the current hypothesis.

This refinement operator aims at producing *least specific specializations* (LSS). A specialization H of a hypothesis H_0 is said to be *least specific* if there is no other specialization of H_0 more general than H. However, our refinement operator really only approximates LSS. This refinement operator does LSS under the constraint that the number of clauses in a hypothesis after the refinement stays the same. Without this restriction, an LSS operator would have to increase the number of clauses in a hypothesis. This would lead to a rather impractical refinement operator due to complexity. The number of clauses in a refined hypothesis could become very large. The limitation in our program to preserve the number of clauses in the hypothesis


```
member(X1,L1).
member(X2,L2).
     |  Refine term L1 = [X3|L3]
     v
member(X1, [X3|L3]).
member(X2,L2).
     |  Match X1 = X3
     v
member(X1, [X1|L3]).
member(X2,L2).
     |  Refine term L2 = [X4|L4]
     v
member(X1, [X1|L3]).
member(X2, [X4|L4]).
     |  Add literal member (X5,L5) and match input L5 = L4
     v
member(X1, [X1|L3]).
member(X2, [X4|L4]) :- member (X5,L4).
     |  Match X2 = X5
     v
member(X1, [X1|L3]).
member(X2, [X4|L4]) :- member (X2,L4).
```

Figure 19.6 The sequence of refinements from a start hypothesis to a target hypothesis. Notice the fourth step when a literal is added and its input argument immediately matched with an existing variable of the same type.

after its refinement is, however, not overly restrictive. If a solution requires a hypothesis with more clauses, then such a hypothesis can be generated from another start hypothesis that has a sufficient number of clauses.

19.3.2 Search

Search starts with a set of start hypotheses. This is the set of all possible bags of user-defined start clauses, up to some maximal number of clauses in a hypothesis. Multiple copies of a start clause typically appear in a start hypothesis. A typical start clause is something rather general and neutral, such as: conc(L1, L2, L3). In search, the refinement graph is treated as a tree (if several paths lead to the same hypothesis, several copies of this hypothesis appear in the tree). The search starts with multiple start hypotheses that become the roots of disjoint search trees. Therefore strictly speaking the search space is a forest.

HYPER performs a best-first search using an evaluation function Cost(Hypothesis) that takes into account the size of a hypothesis as well as its accuracy with respect to the given examples. The cost of a hypothesis H is defined simply as:

$$\text{Cost}(H) = w_1 * \text{Size}(H) + w_2 * \text{NegCover}(H)$$

where NegCover(H) is the number of negative examples covered by H. The definition of 'H covers example E' is understood under the cautious interpretation of Answer in predicate prove. w_1 and w_2 are weights. The size of a hypothesis is defined as a weighted sum of the number of literals and number of variables in the hypothesis:

$$\text{Size}(H) = k_1 * \#\text{literals}(H) + k_2 * \#\text{variables}(H)$$

The actual code of HYPER in this chapter uses the following settings of the weights: $w_1 = 1$, $w_2 = 10$, $k_1 = 10$, $k_2 = 1$, which corresponds to:

$$\text{Cost}(H) = \#\text{variables}(H) + 10 * \#\text{literals}(H) + 10 * \text{NegCover}(H)$$

These settings are *ad hoc*, but their relative magnitudes are intuitively justified as follows. Variables in a hypothesis increase its complexity, so they should be taken into account. However, the literals increase the complexity more, hence they contribute to the cost with a greater weight. A covered negative example contributes to a hypothesis' cost as much as a literal. This corresponds to the intuition that an extra literal should at least prevent one negative example from being covered. Experiments show that, somewhat surprisingly, these weights can be varied considerably without critically affecting the search performance (see reference (Bratko 1999) at the end of this chapter).

19.3.3 Program HYPER

Figure 19.7 shows an implementation of the foregoing design as program HYPER. The main predicates in this program are:

refine_hyp(Hyp0, Hyp)
Refines hypothesis Hyp0 into Hyp by refining one of the clauses in Hyp0.

refine(Clause, Vars, NewClause, NewVars)
Refines a given clause Clause with variables Vars and produces a refined clause NewClause with variables NewVars. The refined clause is obtained by matching two variables of the same type in Vars, or by refining a variable in Vars into a term, or by adding a new background literal to Clause.

induce_hyp(Hyp)
Induces a consistent and complete hypothesis Hyp for the given learning problem. It does best-first search by calling predicate best_search/2.


```
% Program HYPER (Hypothesis Refiner) for learning in logic

:- op( 500, xfx, :).

% induce( Hyp):
%   induce a consistent and complete hypothesis Hyp by gradually
%   refining start hypotheses

induce( Hyp) :-
  init_counts, !,                        % Initialize counters of hypotheses
  start_hyps( Hyps),                     % Get starting hypotheses
  best_search( Hyps, _:Hyp).             % Specialized best-first search

% best_search( CandidateHyps, FinalHypothesis)

best_search( [Hyp | Hyps], Hyp) :-
  show_counts,                           % Display counters of hypotheses
  Hyp = 0:H,                             % cost = 0: H doesn't cover any neg. example
  complete(H).                           % H covers all positive examples

best_search( [C0:H0 | Hyps0], H) :-
  write('Refining hypo with cost '), write( C0),
  write(:), nl, show_hyp(H0), nl,
  all_refinements( H0, NewHs),           % All refinements of H0
  add_hyps( NewHs, Hyps0, Hyps), !,
  add1( refined),                        % Count refined hypos
  best_search( Hyps, H).

all_refinements( H0, Hyps) :-
  findall( C:H,
        ( refine_hyp(H0,H),              % H new hypothesis
          once(( add1( generated),       % Count generated hypos
                 complete(H),            % H covers all pos. exampl.
                 add1( complete),        % Count complete hypos
                 eval(H,C)               % C is cost of H
        )) ),
        Hyps).

% add_hyps( Hyps1, Hyps2, Hyps):
%   merge Hyps1 and Hyps2 in order of costs, giving Hyps

add_hyps( Hyps1, Hyps2, Hyps) :-
  mergesort( Hyps1, OrderedHyps1),
  merge( Hyps2, OrderedHyps1, Hyps).

complete( Hyp) :-                        % Hyp covers all positive examples
  not ( ex( P),                          % A positive example
        once( prove( P, Hyp, Answ)),     % Prove it with Hyp
        Answ \== yes).                   % Possibly not proved
```

Figure 19.7 The HYPER program. The procedure prove/3 is as in Figure 19.3.

Figure 19.7 *contd*

```
% eval( Hypothesis, Cost):
%    Cost of Hypothesis = Size + 10 * # covered negative examples

eval( Hyp, Cost) :-
  size( Hyp, S),                          % Size of hypothesis
  covers_neg( Hyp, N),                    % Number of covered neg. examples
  ( N = 0, !, Cost is 0;                  % No covered neg. examples
    Cost is S + 10*N).

% size( Hyp, Size):
%    Size = k1*#literals + k2*#variables in hypothesis;
%    Settings of parameters: k1=10, k2=1

size( [], 0).

size( [Cs0/Vs0 | RestHyp], Size) :-
  length(Cs0, L0),
  length( Vs0, N0),
  size( RestHyp, SizeRest),
  Size is 10*L0 + N0 + SizeRest.

% covers_neg( H, N):
%    N is number of neg. examples possibly covered by H
%    Example possibly covered if prove/3 returns 'yes' or 'maybe'

covers_neg( Hyp, N) :-                    % Hyp covers N negative examples
  findall( 1, (nex(E), once(prove(E,Hyp,Answ)), Answ \== no), L),
  length( L, N).

% unsatisfiable( Clause, Hyp):
%    Clause can never be used in any proof, that is:
%    Clause's body cannot be proved from Hyp

unsatisfiable( [Head | Body], Hyp) :-
  once( prove( Body, Hyp, Answ)), Answ = no.

start_hyps( Hyps) :-                      % Set of starting hypotheses
  max_clauses( M),
   setof( C:H,
   (start_hyp(H,M), add1(generated),
    complete(H), add1(complete), eval(H,C)),
   Hyps).

% start_hyp( Hyp, MaxClauses):
%    A starting hypothesis with no more than MaxClauses

start_hyp( [], _).

start_hyp( [C | Cs], M) :-
  M > 0, M1 is M - 1,
  start_clause( C),                       % A user-defined start clause
  start_hyp( Cs, M1).
```



```
% refine_hyp( Hyp0, Hyp):
%    refine hypothesis Hyp0 into Hyp

refine_hyp( Hyp0, Hyp) :-
  choose_clause( Hyp0, Clause0/Vars0, Clauses1, Clauses2),  % Choose a clause
  conc( Clauses1, [Clause/Vars | Clauses2], Hyp),           % New hypothesis
  refine( Clause0, Vars0, Clause, Vars),                    % Refine chosen clause
  non_redundant( Clause),                                   % No redundancy in Clause
  not unsatisfiable( Clause, Hyp).                          % Clause not unsatisfiable

choose_clause( Hyp, Clause, Clauses1, Clauses2) :-          % Choose Clause from Hyp
  conc( Clauses1, [Clause | Clauses2], Hyp),                % Choose a clause
  nex(E),                                                   % A negative example E
  prove( E, [Clause], yes),                                 % Clause itself covers E
  !                                                         % Clause must be refined
  ;
  conc( Clauses1, [Clause | Clauses2], Hyp).          % Otherwise choose any clause

% refine( Clause, Args, NewClause, NewArgs):
%    refine Clause with variables Args giving NewClause with NewArgs

% Refine by unifying arguments

refine( Clause, Args, Clause, NewArgs) :-
  conc( Args1, [A | Args2], Args),                    % Select a variable A
  member( A, Args2),                                  % Match it with another one
  conc( Args1, Args2, NewArgs).

% Refine a variable to a term

refine( Clause, Args0, Clause, Args) :-
  del( Var:Type, Args0, Args1),                       % Delete Var:Type from Args0
  term( Type, Var, Vars),                             % Var becomes term of type Type
  conc( Args1, Vars, Args).                           % Add variables in the new term

% Refine by adding a literal

refine( Clause, Args, NewClause, NewArgs) :-
  length( Clause, L),
  max_clause_length( MaxL),
  L < MaxL,
  backliteral( Lit, InArgs, RestArgs),                % Background knowledge literal
  conc( Clause, [Lit], NewClause),                    % Add literal to body of clause
  connect_inputs( Args, InArgs),                      % Connect literal's inputs to other args.
  conc( Args, RestArgs, NewArgs).                     % Add rest of literal's arguments

% non_redundant( Clause): Clause has no obviously redundant literals

non_redundant( [_]).                                  % Single literal clause

non_redundant( [Lit1 | Lits]) :-
  not literal_member( Lit1, Lits),
  non_redundant( Lits).
```

Figure 19.7 *contd*

```
literal_member( X, [X1 | Xs]) :-             % X literally equal to member of list
  X == X1, !
  ;
  literal_member( X, Xs).

% show_hyp( Hypothesis):
%   Write out Hypothesis in readable form with variables names A, B, ...

show_hyp( [ ]) :- nl.

show_hyp( [C/Vars | Cs]) :- nl,
  copy_term( C/Vars, C1/Vars1),
  name_vars( Vars1, ['A','B','C','D','E','F','G','H','I','J','K','L','M','N']),
  show_clause( C1),
  show_hyp( Cs), !.

show_clause( [Head | Body]) :-
  write( Head),
  ( Body = [ ]; write( ' :- ' ), nl ),
  write_body( Body).

write_body( [ ]) :-
  write('.'), !.

write_body( [G | Gs]) :- !,
  tab( 2), write( G),
  ( Gs = [ ], !, write('.'), nl
   ;
   write(','), nl,
   write_body( Gs)
  ).

name_vars( [ ], _).

name_vars( [Name:Type | Xs], [Name | Names]) :-
  name_vars( Xs, Names).

% connect_inputs( Vars, Inputs):
%   Match each variable in list Inputs with a variable in list Vars

connect_inputs( _, [ ]).

connect_inputs( S, [X | Xs]) :-
  member( X, S),
  connect_inputs( S, Xs).

% merge( L1, L2, L3), all lists sorted

merge( [ ], L, L) :- !.

merge( L, [ ], L) :- !.
```



```
merge( [X1|L1], [X2|L2], [X1|L3]) :-
  X1 @=< X2, !,                     % X1 'lexicographically precedes' X2 (built-in predicate)
  merge( L1, [X2|L2], L3).

merge( L1, [X2|L2], [X2|L3]) :-
  merge( L1, L2, L3).

% mergesort( L1, L2): sort L1 giving L2

mergesort( [ ], [ ]) :- !.

mergesort( [X], [X]) :- !.

mergesort( L, S) :-
  split( L, L1, L2),
  mergesort( L1, S1),
  mergesort( L2, S2),
  merge( S1, S2, S).

% split( L, L1, L2): split L into lists of approx. equal length

split( [ ], [ ], [ ]).

split( [X], [X], [ ]).

split( [X1,X2 | L], [X1|L1], [X2|L2]) :-
  split( L, L1, L2).

% Counters of generated, complete and refined hypotheses

init_counts :-
  retract( counter(_,_)), fail          % Delete old counters
  ;
  assert( counter( generated, 0)),      % Initialize counter 'generated'
  assert( counter( complete, 0)),       % Initialize counter 'complete'
  assert( counter( refined, 0)).        % Initialize counter 'refined'

add1( Counter) :-
  retract( counter( Counter, N)), !, N1 is N+1,
  assert( counter( Counter, N1)).

show_counts :-
  counter(generated, NG), counter( refined, NR), counter( complete, NC),
  nl, write( 'Hypotheses generated: '), write(NG),
  nl, write( 'Hypotheses refined: '), write(NR),
  ToBeRefined is NC - NR,
  nl, write( 'To be refined: '), write( ToBeRefined), nl.

% Parameter settings

max_proof_length( 6).       % Max. proof length, counting calls to preds. in hypothesis
max_clauses( 4).            % Max. number of clauses in hypothesis
max_clause_length( 5).      % Max. number of literals in a clause
```

best_search(Hyps, Hyp)
Starts with a set of start hypotheses Hyps, generated by predicate start_hyps/1, and performs best-first search of the refinement forest until a consistent and complete hypothesis Hyp is found. It uses the cost of hypotheses as the evaluation function to guide the search. Each candidate hypothesis is combined with its cost into a term of the form Cost:Hypothesis. When the list of such terms is sorted (by merge sort), the hypotheses are sorted according to their increasing costs.

prove(Goal, Hyp, Answer)
Proof-length limited interpreter defined in Figure 19.3.

eval(Hyp, Cost)
Evaluation function for hypotheses. Cost takes into account both the size of Hyp and the number of negative examples covered by Hyp. If Hyp does not cover any negative example then Cost = 0.

start_hyps(Hyps)
Generates a set Hyps of start hypotheses for search. Each start hypothesis is a list of up to MaxClauses start clauses. MaxClauses is defined by the user with the predicate max_clauses. Start clauses are defined by the user with the predicate start_clause.

show_hyp(Hyp)
Displays hypothesis Hyp in the usual Prolog format.

init_counts, show_counts, add1(Counter)
Initializes, displays and updates counters of hypotheses. Three types of hypotheses are counted separately: generated (the number of all generated hypotheses), complete (the number of generated hypotheses that cover all the positive examples), and refined (the number of all refined hypotheses).

start_clause(Clause)
User-defined start clauses, normally something very general like:

```
start_clause( [ member(X,L) ] / [ X:item, L:list] ).
```

max_proof_length(D), max_clauses(MaxClauses), max_clause_length(MaxLength)
Predicates defining the parameters: maximum proof length, maximum number of clauses in a hypothesis, and maximum number of literals in a clause. For example, for learning member/2 or conc/3, MaxClauses=2 suffices. Default settings in the program of Figure 19.7 are:

```
max_proof_length(6).   max_clauses(4).   max_clause_length(5).
```

The program in Figure 19.7 also needs the frequently used predicates: not/1, once/1, member/2, conc/3, del/3, length/2, copy_term/2.

As an illustration let us execute HYPER on the problem of learning about the predicate member(X,L). The problem definition in Figure 19.5 has to be loaded into Prolog in addition to HYPER. The question is:

```
?- induce(H), show_hyp(H).
```

During the execution HYPER keeps displaying the current counts of hypotheses (generated, refined and waiting-to-be-refined), and the hypothesis currently being refined. The final results are:

```
Hypotheses generated:  105
Hypotheses refined:    26
To be refined:         15

member(A,[A|B]).
member(C,[A|B]) :-
  member(C,B).
```

The induced hypothesis is as expected. Before this hypothesis was found, 105 hypotheses were generated all together, 26 of them were refined, and 15 of them were still in the list of candidates to be refined. The difference 105 – 26 – 15 = 51 hypotheses were found to be incomplete and were therefore discarded immediately. The needed refinement depth for this learning problem is 5 (Figure 19.6). The total number of possible hypotheses in the refinement space defined by the (restricted) refinement operator in HYPER is several thousands. HYPER only searched a fraction of this (less than 10 percent). Experiments show that in more complex learning problems (list concatenation, path finding) this fraction is much smaller.

Exercise

19.5 Define the learning problem, according to the conventions in Figure 19.5, for learning predicate conc(L1, L2,L3) (list concatenation) and run HYPER with this definition. Work out the refinement depth of the target hypothesis and estimate the size of the refinement tree to this depth for a two-clause start hypothesis. Compare this size with the number of hypotheses generated and refined by HYPER.

19.3.4 Experiments with HYPER

HYPER with its refinement restrictions and heuristic search is much more effective than MINIHYPER. However, HYPER too is faced with the generally high complexity of learning in logic. It is interesting to explore where a boundary between feasible and infeasible learning problems lies for HYPER. The boundary can be considerably extended by cleverly designing the learning problem (background knowledge, input and output variables in background literals, set of examples). We will here look at some illustrative learning exercises with HYPER.

Simultaneously learning two predicates odd(L) and even(L)

HYPER can be applied without modification to multi-predicate learning, that is learning several predicates simultaneously where one predicate may be defined in terms of another one. This may even involve mutual recursion when the predicates learned call each other. We will here illustrate this by learning the predicates odd(List) and even(List) (true for lists of odd or even length respectively). Figure 19.8 shows a definition of this learning problem. The result of learning is:

```
Hypotheses generated: 85
Hypotheses refined:   16
To be refined:        29

even([ ]).
even([A,B|C]) :-
  even(C).
odd([A|B]) :-
  even(B).
```

This corresponds to the target concept. However, HYPER found a definition that is not mutually recursive. By just requesting another solution (by typing a semicolon as usual), HYPER continues the search and next finds a mutually recursive definition:

```
% Inducing odd and even length for lists

backliteral( even( L), [ L:list], [ ]).
backliteral( odd( L), [ L:list], [ ]).

term( list, [X|L], [ X:item, L:list]).
term( list, [ ], [ ]).

prolog_predicate( fail).

start_clause([ odd( L) ] / [ L:list]).
start_clause([ even( L) ] / [ L:list]).

ex( even( [ ])).
ex( even( [a,b])).
ex( odd( [a])).
ex( odd( [b,c,d])).
ex( odd( [a,b,c,d,e])).
ex( even( [a,b,c,d])).

nex( even( [a])).
nex( even( [a,b,c])).
nex( odd( [ ])).
nex( odd( [a,b])).
nex( odd( [a,b,c,d])).
```

Figure 19.8 Learning about odd-length and even-length lists simultaneously.


```
Hypotheses generated: 115
Hypotheses refined:   26
To be refined:        32

even([ ]).
odd([A|B]) :-
  even(B).
even([A|B]) :-
  odd(B).
```

The first, non mutually recursive definition can be prevented by a more restrictive definition of term refinement. Such a more restrictive definition would allow lists to be refined to depth 1 only. This can be achieved by replacing type list with type list(D), and changing the first clause about list refinement into:

```
term( list(D), [X|L], [ X:item, L:list(1)]) :- var(D).
```

This definition prevents a variable of type list(1) from being refined further. So terms like [X,Y|L] cannot be generated. Of course, the other clause about term and the start_clause predicate would have to be modified accordingly.

Learning predicate path(StartNode, GoalNode, Path)

Figure 19.9 shows a domain definition for learning the predicate path in a directed graph (specified by the background predicate link/2). The learning is accomplished smoothly, resulting in:

```
Hypotheses generated: 401
Hypotheses refined:   35
To be refined:        109

path(A,A,[A]).
path(A,C,[A,B|E]) :-
  link(A,B),
  path(B,C,[D|E]).
```

The last line of the induced definition may appear surprising, but it is in this context, in fact, equivalent to the expected path(B,C,[B|E]) and requires one refinement step less. The fact that only 35 hypotheses were refined in this search may appear rather surprising in the view of the following facts. The refinement depth of the path hypothesis found above is 12. An estimate shows that the size of the refinement tree to this depth exceeds 10^{17} hypotheses! Only a tiny fraction of this is actually searched. This can be explained by the fact that the hypothesis completeness requirement constrains the search in this case particularly effectively.

Learning insertion sort

Figure 19.10 gives a definition for this learning problem. This definition invites debate because background knowledge is very specifically targeted at inducing

```
% Learning about path: path(StartNode,GoalNode,Path)

% A directed graph

link(a,b).
link(a,c).
link(b,c).
link(b,d).
link(d,e).

backliteral( link(X,Y), [ X:item], [ Y:item]).

backliteral( path(X,Y,L), [ X:item], [ Y:item, L:list]).

term( list, [X|L], [ X:item, L:list]).
term( list, [ ], [ ]).

prolog_predicate( link(X,Y)).

start_clause( [ path(X,Y,L)] / [X:item,Y:item,L:list] ).

% Examples

ex( path( a, a, [a])).
ex( path( b, b, [b])).
ex( path( e, e, [e])).
ex( path( f, f, [f])).
ex( path( a, c, [a,c])).
ex( path( b, e, [b,d,e])).
ex( path( a, e, [a,b,d,e])).

nex( path( a, a, [ ])).
nex( path( a, a, [b])).
nex( path( a, a, [b,b])).
nex( path( e, d, [e,d])).
nex( path( a, d, [a,b,c])).
nex( path( a, c, [a])).
nex( path( a, c, [a,c,a,c])).
nex( path( a, d, [a,d])).
```

Figure 19.9 Learning about a path in a graph.

insertion sort. The obvious comment is that, when defining such background knowledge, we almost have to know the solution already. This, however, illustrates a typical problem with machine learning. To make the learning most effective, we have to present the learning program with as good a representation as possible, including background knowledge. This inevitably requires speculation by the user about possible solutions. In our case of sorting, the learning problem would be very hard without such a helpful definition of background knowledge. Even so this turns out to be the hardest problem among our experiments so far. The code of Figure 19.10 requires extra comments which, is due to the fact that in sort(L1,L2),


```
% Learning sort

backliteral( sort( L, S), [L:list], [S:list]).
backliteral( insert_sorted( X, L1, L2), [X:item, L1:list], [L2:list]).

term( list, [X | L], [X:item, L:list]).
term( list, [ ], [ ]).

prolog_predicate( insert_sorted( X, L0, L)).
prolog_predicate( X=Y).

start_clause( [sort(L1,L2)] / [L1:list, L2:list] ).

ex( sort( [ ], [ ])).
ex( sort( [a], [a])).
ex( [ sort( [c,a,b], L), L = [a,b,c] ] ).      % Uninstantiated 2nd arg. of sort!
ex( sort( [b,a,c], [a,b,c])).
ex( sort( [c,d,b,e,a], [a,b,c,d,e])).
ex( sort( [a,d,c,b], [a,b,c,d])).

nex( sort( [ ], [a])).
nex( sort( [a,b], [a])).
nex( sort( [a,c], [b,c])).
nex( sort( [b,a,d,c], [b,a,d,c])).
nex( sort( [a,c,b], [a,c,b])).
nex( sort( [ ], [b,c,d])).

insert_sorted( X, L, _) :-                   % Guarding clause: test instantiation of args.
  var(X), !, fail
  ;
  var( L), !, fail
  ;
  L = [Y|_], var(Y), !, fail.

insert_sorted( X, [ ], [X]) :- !.

insert_sorted( X, [Y | L], [X,Y | L]) :-
  X @< Y, !.                                 % Term X lexicographically precedes term Y (built-in)

insert_sorted( X, [Y | L], [Y | L1]) :-
  insert_sorted( X, L, L1).
```

Figure 19.10 Learning insertion sort.

L1 is expected to be instantiated, whereas L2 is an output argument that gets instantiated after the execution of sort. To make sure that the induced sort will work with L2 uninstantiated, one of the examples is specified as:

```
ex( [ sort( [c,a,b], L), L = [a,b,c] ] ).
```

This ensures that sort is called with L uninstantiated, so that sort has to *construct* (not only recognize!) the result L, and only then is this checked for correctness. Care is also needed when defining the background predicate insert_sorted(X,L1,L2), to make sure that the arguments are instantiated as expected (e.g. X is not a variable). The results are:

```
Hypotheses generated: 3708
Hypotheses refined:   284
To be refined:        448

sort([ ],[ ]).
sort([A|B],D) :-
  sort(B,C),
  insert_sorted(A,C,D).
```

Learning about the arch

This is similar to the relational learning example in Chapter 18, where examples are structures made of blocks (Figure 19.11), and there is a hierarchy among the types of blocks (ako(X,Y): Y is a kind of X; similar to Figure 18.6). The examples are triples of blocks. The first two blocks in a triple are the sides of an arch, the third block is the top. So one positive example is arch(a1,b1,c1) where a1, b1 and c1 are all rectangles, both a1 and b1 support c1 and there is no touch relation between a1 and b1. The blocks a2, b2 and c2 form a negative example (nex(a2,b2,c2)) because block c2 is not supported by a2 and b2. The blocks a5, b5 and c5 form another negative example because c5 is not a 'stable polygon'. The arch learning problem is defined in Figure 19.12. The background predicates in this figure also include negation (not) applied to touch/2 and support/2. This background predicate is executed directly by Prolog as negation as failure, as usual. In comparison with Figure 18.4, there are a few additional negative examples to constrain the choice of consistent hypotheses. The result of learning is:

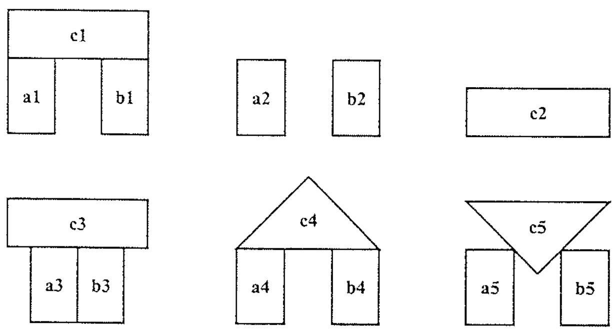

Figure 19.11 A blocks world with two examples of an arch and three counter examples. The blocks **a1**, **b1** and **c1** form one example, the blocks **a4**, **b4** and **c4** form the other example. The blocks **a2**, **b2** and **c2** form one of the counter examples.


```
Hypotheses generated: 368
Hypotheses refined:   10
To be refined:        251

arch(A,B,C) :-             % Arch consists of posts A and B, and top C
  support(B,C),            % B supports C
  not touch(A,B),          % A and B do not touch
  support(A,C),            % A supports C
  isa(C,stable_poly).      % C is a stable polygon
```

```
% Learning about arch

backliteral( isa(X,Y), [X:object], [ ]) :-
  member( Y, [polygon, convex_poly, stable_poly, unstable_poly, triangle, rectangle,
           trapezium, unstable_triangle, hexagon]).      % Any figure
backliteral( support(X,Y), [X:object, Y:object], [ ]).
backliteral( touch(X,Y), [X:object, Y:object], [ ]).
backliteral( not G, [X:object,Y:object], [ ]) :-
  G = touch(X,Y); G = support(X,Y).

prolog_predicate( isa(X,Y)).
prolog_predicate( support(X,Y)).
prolog_predicate( touch(X,Y)).
prolog_predicate( not G).

ako( polygon, convex_poly).                 % Convex polygon is a kind of polygon
ako( convex_poly, stable_poly).             % Stable polygon is a kind of convex polygon
ako( convex_poly, unstable_poly).           % Unstable polygon is a kind of convex poly.
ako( stable_poly, triangle).                % Triangle is a kind of stable polygon
ako( stable_poly, rectangle).               % Rectangle is a kind of stable polygon
ako( stable_poly, trapezium).               % Trapezium is a kind of stable polygon
ako( unstable_poly, unstable_triangle).     % Unstable triangle is a.k.o. unstable poly.
ako( unstable_poly, hexagon).               % Hexagon is a kind of unstable polygon

ako( rectangle, X) :-
  member( X, [a1,a2,a3,a4,a5,b1,b2,b3,b4,b5,c1,c2,c3]).    % All rectangles

ako( triangle, c4).                         % Stable triangle
ako( unstable_triangle, c5).                % Triangle upside down

isa( Figure1, Figure2) :-                   % Figure1 is a Figure2
  ako( Figure2, Figure1).

isa( Fig0, Fig) :-
  ako( Fig1, Fig0),
  isa( Fig1, Fig).

support(a1,c1). support(b1,c1).
support(a3,c3). support(b3,c3). touch(a3,b3).
support(a4,c4). support(b4,c4).
support(a5,c5). support(b5,c5).
```

Figure 19.12 Learning the concept of arch.

Figure 19.12 *contd*

```
start_clause( [ arch(X,Y,Z)] / [X:object,Y:object,Z:object]).
ex( arch(a1,b1,c1)).
ex( arch(a4,b4,c4)).
nex( arch(a2,b2,c2)).
nex( arch(a3,b3,c3)).
nex( arch(a5,b5,c5)).
nex( arch(a1,b2,c1)).
nex( arch(a2,b1,c1)).
```

Summary

- Inductive logic programming (ILP) combines logic programming and machine learning.
- ILP is inductive learning using logic as the hypothesis language. ILP is also an approach to automatic programming from examples.
- In comparison with other approaches to machine learning: (1) ILP uses a more expressive hypothesis language that allows recursive definitions of hypotheses, (2) ILP allows more general form of background knowledge, (3) ILP generally has greater combinatorial complexity than attribute-value learning.
- In a *refinement graph over clauses*, nodes correspond to logic clauses, and arcs correspond to refinements between clauses.
- In a *refinement graph over hypotheses*, nodes correspond to sets of logic clauses (Prolog programs), and arcs to refinements between hypotheses.
- A refinement of a clause (a hypothesis) results in a more specific clause (hypothesis).
- A clause can be refined by: (1) matching two variables in the clause, or (2) substituting a variable with a term, or (3) adding a literal to the body of the clause.
- θ-subsumption is a generality relation between clauses that can be determined syntactically based on substitution of variables.
- Program HYPER developed in this chapter induces Prolog programs from examples by searching a refinement graph over hypotheses.
- Concepts discussed in this chapter are:
  inductive logic programming
  clause refinement

  hypothesis refinement
  refinement graphs over clauses or hypotheses
  θ-subsumption
  automatic programming from examples

References

The term inductive logic programming was introduced by Stephen Muggleton (1991). The early work in this area, before the term was actually introduced, includes Plotkin (1969), Shapiro (1983) and Sammut and Banerji (1986). The HYPER program of this chapter is based on Bratko (1999). The book by Lavrač and Džeroski (1994) gives a good introduction to ILP. Muggleton (1992) and De Raedt (1996) edited collections of papers on ILP. FOIL (Quinlan 1990) and Progol (Muggleton 1995) are among the best-known ILP systems. Bratko *et al.* (1998) review a number of applications of ILP.

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chapter 20

Qualitative Reasoning

Traditional, quantitative modelling and simulation give precise numerical answers. For everyday use, such answers are often overly elaborate. When filling a bath tub, it is sufficient to know that the level of water will reach the top of the bath tub if the inflow of water is not stopped in time. We do not have to know the level precisely at any point in time. A common sense description of the tub-filling process is *qualitative*: 'The level of water will keep increasing and will eventually reach the top, which will cause overflow . . .'. This gives just a useful summary of a possibly large amount of quantitative information. Qualitative reasoning is an area in artificial intelligence concerned with the formalization of and algorithms for qualitative modelling and simulation of the physical world.

20.1 Common sense, qualitative reasoning and naive physics

20.1.1 Quantitative vs qualitative reasoning

Consider the bath tub in Figure 20.1. Assume the tub is initially empty, there is a constant flow from the tap, and that the drain is closed, so there is no outflow. What will happen?

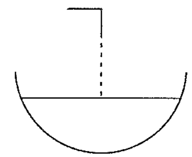

Figure 20.1 Bath tub with some input flow and closed drain.

To answer this question, the physicist's solution would be to write down a differential equation model of this system, and run this model by numerical simulation. The numerical simulation would produce a table with, say, 1,593 rows, giving the exact values of the level of water at consecutive tabulated time points. The table would show, for example, that the level will reach the top of the tub at 62.53 cm in 159.3 seconds. For everyday use, such an elaborate answer is an overkill. A common sense answer that suffices for everyday purposes is rather something like this: 'The water level will keep increasing and will eventually reach the top. After this, water will overflow and cause a flood in the bathroom.' The physicist's answer was *quantitative*, giving precise numerical information. The common sense answer was *qualitative*, just giving a useful summary of the large amount of quantitative information.

The area of qualitative reasoning in artificial intelligence is concerned with the formalization of and algorithms for qualitative reasoning about the world, producing qualitative, non-numerical answers to questions that are typically answered numerically by 'proper' physics. To emphasize the contrast between the 'proper' physics as taught in schools, and the qualitative, common sense reasoning about the physical world, the qualitative physics is sometimes also called *naive physics*.

20.1.2 Qualitative abstraction of quantitative information

Qualitative reasoning is often viewed as an abstraction of quantitative reasoning. Accordingly, in qualitative reasoning some numerical details are discarded; instead, a rather simpler qualitative summary of these numerical details is retained. There are many ways of abstracting away from detailed numerical information. Table 20.1 gives some examples of quantitative statements and their qualitative abstractions, typical of qualitative reasoning in AI. The abstraction principles in these examples are discussed in the following paragraphs.

Abstraction of numbers into symbolic values and intervals

Quantitative statement: *Level* at 3.2 s is 2.6 cm, formally written as:

Level(3.2 s) = 2.6 cm

Table 20.1 Examples of quantitative statements and their qualitative abstractions.

<table>
<tr><th>Quantitative statement</th><th>Qualitative statement</th></tr>
<tr><td>Level(3.2 s) = 2.6 cm</td><td>Level(t1) = zero .. top</td></tr>
<tr><td>Level(3.2 s) = 2.6 cm</td><td>Level(t1) = pos</td></tr>
<tr><td>d/dt Level(3.2 s) = 0.12 m/s</td><td>Level(t1) increasing</td></tr>
<tr><td>Amount = Level * (Level + 5.7)</td><td>M⁺(Amount, Level)</td></tr>
<tr><td><table><tr><th>Time</th><th>Amount</th></tr><tr><td>0.0</td><td>0.00</td></tr><tr><td>0.1</td><td>0.02</td></tr><tr><td>...</td><td>...</td></tr><tr><td>159.3</td><td>62.53</td></tr></table></td><td>Amount(start .. end) = zero .. top/inc</td></tr>
</table>

Qualitative abstraction: *Level* at time *t*1 is between the bottom and the top of the bath tub. This may be written formally as:

Level(*t*1) = *zero* .. *top*

Notice here that 3.2 s has been replaced by a symbolic time point *t*1. So instead of giving exact time, this just says that there is a time point, referred to as *t*1, at which *Level* has the given qualitative value. Regarding this qualitative value, the whole set of numbers between 0 and 62.53 has been collapsed into a symbolic interval *zero* .. *top*. A further abstraction would be to disregard the top of the tub as an important value, and simply state: *Level* at time *t*1 is positive, written as:

Level(*t*1) = *pos*

Abstraction of time derivatives into directions of change

Quantitative statement about the time derivative of *Level*:

$$\frac{d}{dt} Level(3.2\text{ s}) = 0.12\text{ m/s}$$

Qualitative abstraction: *Level* at time *t*1 is increasing.

Abstraction of functions into monotonic relations

Quantitative statement: *Amount* = *Level* * (*Level* + 5.7)
A qualitative abstraction: For *Level*$\geqslant$0, *Amount* is a monotonically increasing function of *Level*, written formally as: M^+(*Amount*, *Level*). That is, if *Level* increases then *Amount* increases as well, and vice versa.

Abstraction of increasing time sequences

A whole table giving the values of *Amount* at consecutive time points between time 0 and 159.3 s may be abstracted into a single qualitative statement: The value of *Amount* in the time interval between *start* and *end* is between *zero* and *full*, and is increasing. This can be formally written as:

Amount(*start* .. *end*) = *zero* .. *full*/*inc*

Qualitative reasoning is related to *qualitative modelling*. Numerical models are an abstraction of the real world. Qualitative models are often viewed as a further abstraction of numerical models. In this abstraction some quantitative information is abstracted away. For example, a quantitative model of the water flow in a river may state that the flow *Flow* depends on the level *Level* of water in the river in some complicated way which also takes into account the shape of the river bed. In a qualitative model this may be abstracted into a monotonically increasing relation:

M^+(*Level*, *Flow*)

This just says that the greater the level the greater the flow, without specifying this in any more concrete and detailed way. Obviously, it is much easier to design such coarse qualitative models than precise quantitative models.

20.1.3 Motivation and uses for qualitative modelling and reasoning

This section discusses advantages and disadvantages of qualitative modelling with respect to the traditional, quantitative modelling. Of course, there are many situations where a qualitative model, due to lack of precise numerical information, is not sufficient. However, there are also many situations in which a qualitative model has advantages.

First, qualitative modelling is easier than quantitative modelling. Precise relations among the variables in the system to be modelled may be hard or impossible to determine, but it is usually still possible to state some qualitative relations among the variables. Also, even if a complete quantitative model is known, such a model still requires the knowledge of all the, possibly many, numerical parameters in the model. For example, a numerical physiological model may require the precise electrical conductance of a neuron, its length and width, etc. These parameters may be hard or impossible to measure. Yet, to run such a numerical model, a numerical simulator will require the values of all these parameters to be specified by the user before the simulation can start. Usually the user will then make some guesses at these parameters and hope that they are not too far off their real values. But then the user will not know how far the simulation results are from the truth. The user will typically not know even if the obtained results are *qualitatively* correct. With a qualitative model, much of such guesswork can be avoided, and in the end the user will at least be sure about the qualitative correctness of the simulations. So, paradoxically, quantitative results, although more precise than qualitative results, are in greater danger of being incorrect and completely useless, because the accumulated error may become too gross. For example, in an ecological model, even without knowing the precise parameters of growth and mortality rates, etc. for the species in the model, a qualitative model may answer the question whether certain species will eventually become extinct, or possibly different species will interchange their temporal domination in time cycles. A qualitative simulator may find such an answer by finding all the possible qualitative behaviours that correspond to all possible combinations of the values of the parameters in the model.

Another point is that for many tasks, numerical precision is not required. Often it only obscures the essential properties of the system. Generic tasks in which qualitative modelling is often more appropriate include functional reasoning, diagnosis and structural synthesis. We will look at these tasks in the following paragraphs.

Functional reasoning is concerned with questions like: How does a device or a system work? For example:

How does the thermostat work?
How does a lock work?
How does a clock work?
How does the refrigerator attain its cooling function?
How does the heart achieve its blood-pumping function?

In all these cases we are interested in the (qualitative) mechanism of how the system works. If the numerical values of the parameters of the system change a little, usually the basic functional mechanism is still the same. All hearts are a little different, but the basic functional principle is always the same.

In a *diagnostic task* we are interested in defects that caused the observed abnormal behaviour of the system. Usually, we are only interested in those deviations from the normal state that caused a behaviour that is qualitatively different from normal.

The problem of *structural synthesis* is: Given some basic building blocks, find their combination which achieves a given function. For example, put the available components together to achieve the effect of cooling. In other words, invent the refrigerator from 'first principles'. The basic building blocks can be available technical components, or just the laws of physics, or materials with certain properties. In such design from first principles, the goal is to synthesize a structure capable of achieving some given function through some mechanism. In the early, most innovative stage of design, this mechanism is described qualitatively. Only at a later stage of design, when the structure is already known, does quantitative synthesis also become important.

The use of qualitative models requires qualitative reasoning. In the remainder of this chapter we will discuss and implement some ideas for qualitative modelling and reasoning. First, in Section 20.2, we look at static systems (where the quantities in

the system do not change in time). In Section 20.3 we look at qualitative reasoning about dynamic systems, which also requires reasoning about changes in time. The mathematical basis for the approach in the latter section consists of *qualitative differential equations* (QDE), an abstraction of ordinary differential equations.

20.2 Qualitative reasoning about static systems

Consider simple electric circuits consisting of switches, bulbs and batteries (Figure 20.2). Switches can be open or closed (off or on), bulbs can be light or dark, blown or intact. We are interested in questions related to prediction, diagnosis or control. A diagnostic question about circuit 1 is: If the switch is on and the bulb is dark, what is the state of the bulb? Simple qualitative reasoning suffices to see that the bulb is blown.

A more interesting diagnostic question about circuit 2 is: Seeing that bulb 2 is light and bulb 3 dark, can we reliably conclude that bulb 3 is blown? Qualitative reasoning confirms that bulb 3 is necessarily blown. For bulb 2 to be light, there must be a non-zero current in bulb 2, and switch 2 must be on. If there is a non-zero current in bulb 2, there must be a non-zero voltage on bulb 2. This requires that switch 3 is off. The same non-zero voltage is on bulb 3. So with the same voltage, bulb 2 is light and bulb 3 is dark; therefore, bulb 3 must be blown.

In our qualitative model of these circuits, electric currents and voltages will just have qualitative values 'pos', 'zero' and 'neg'. The abstraction rule for converting a real number X into a qualitative value is:

if $X > 0$ then pos
if $X = 0$ then zero
if $X < 0$ then neg

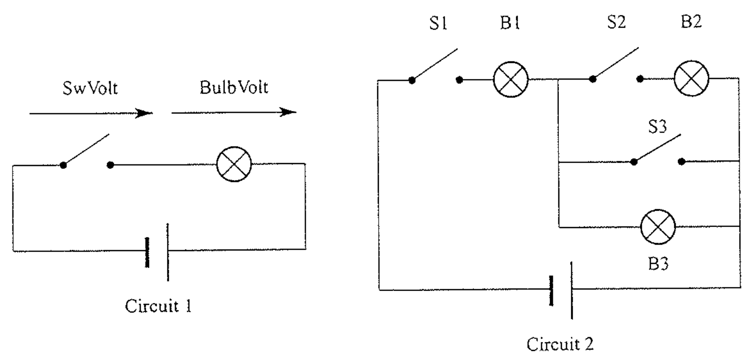

Figure 20.2 Simple circuits made of switches, bulbs and batteries.

In standard, numerical models of electric circuits, we use some basic laws such as Kirchhoff's laws and Ohm's law. Kirchhoff's laws state that (1) the sum of all the voltages along any closed loop in a circuit is 0, and (2) the sum of all the currents into any vertex in a circuit is 0. To apply these laws in a qualitative model, we need a qualitative version of arithmetic summation. In our program, the usual arithmetic summation $X + Y = Z$ will be reduced to a qualitative summation, implemented as the predicate:

```
qsum( X, Y, Z)
```

The qsum relation can be defined simply by a set of facts. These state, for example, that the sum of two positive numbers is a positive number:

```
qsum( pos, pos, pos).
```

The sum of a positive and a negative number can be anything:

```
qsum( pos, neg, pos).
qsum( pos, neg, zero).
qsum( pos, neg, neg).
```

This summation is 'non-deterministic'. Due to lack of precise information, lost in the qualitative abstraction, we sometimes cannot tell what the actual result of summation is. This kind of non-determinism is rather typical of qualitative reasoning.

The program in Figure 20.3 specifies a qualitative model of our circuits, and carries out the qualitative reasoning about this model. A model of a circuit specifies the components of the circuit, and takes into account the connections among the components. The qualitative behaviour of the two types of component, switches and bulbs, is defined by the predicates:

```
switch( SwitchPosition, Voltage, Current)
bulb( BulbState, Lightness, Voltage, Current)
```

Their qualitative behaviours are simple and can be stated by Prolog facts. For example, an open switch has zero current and the voltage can be anything:

```
switch( off, AnyVoltage, zero).
```

```
% Modelling simple electric circuits
% Qualitative values of voltages and currents are: neg, zero, pos

% Definition of switch
% switch( SwitchPosition, Voltage, Current)

switch( on, zero, AnyCurrent).          % Switch on: zero voltage
switch( off, AnyVoltage, zero).         % Switch off: zero current

% Definition of bulb
% bulb( BulbState, Lightness, Voltage, Current)

bulb( blown, dark, AnyVoltage, zero).
bulb( ok, light, pos, pos).
bulb( ok, light, neg, neg).
bulb( ok, dark, zero, zero).

% A simple circuit consisting of a bulb, switch and battery

circuit1( SwitchPos, BulbState, Lightness) :-
  switch( SwitchPos, SwVolt, Curr),
  bulb( BulbState, Lightness, BulbVolt, Curr),
  qsum( SwVolt, BulbVolt, pos).                % Battery voltage = pos

% A more interesting circuit made of a battery, three bulbs and
% three switches

circuit2( Sw1, Sw2, Sw3, B1, B2, B3, L1, L2, L3) :-
  switch( Sw1, VSw1, C1),
  bulb( B1, L1, VB1, C1),
  switch( Sw2, VSw2, C2),
  bulb( B2, L2, VB2, C2),
  qsum( VSw2, VB2, V3),
  switch( Sw3, V3, CSw3),
  bulb( B3, L3, V3, CB3),
  qsum( VSw1, VB1, V1),
  qsum( V1, V3, pos),
  qsum( CSw3, CB3, C3),
  qsum( C2, C3, C1).

% qsum( Q1, Q2, Q3):
%   Q3 = Q1 + Q2, qualitative sum over domain [pos,zero,neg]

qsum( pos, pos, pos).
qsum( pos, zero, pos).
qsum( pos, neg, pos).
qsum( pos, neg, zero).
qsum( pos, neg, neg).
qsum( zero, pos, pos).
qsum( zero, zero, zero).
qsum( zero, neg, neg).
qsum( neg, pos, pos).
qsum( neg, pos, zero).
qsum( neg, pos, neg).
qsum( neg, zero, neg).
qsum( neg, neg, neg).
```

Figure 20.3 Qualitative modelling program for simple circuits.

A blown bulb is dark, has no current and any voltage:

```
bulb( blown, dark, AnyVoltage, zero).
```

An intact bulb is light unless both the voltage and current in the bulb are zero. Here we are assuming that any non-zero current is sufficiently large to make a bulb light. The voltage and the current are either both zero, both positive, or both negative. Notice that this is a qualitative abstraction of Ohm's law:

Voltage = Resistance * Current

Since Resistance is positive, Voltage and Current must have the same sign and therefore the same qualitative value.

Once our components have been defined, it is easy to define a whole circuit. A particular circuit is defined by a predicate, such as:

```
circuit1( SwPos, BulbState, Lightness)
```

Here the switch position, the state of the bulb and the lightness have been assumed to be the important properties of the circuit, hence they were made the arguments of circuit1. Other quantities in the circuit, such as the current in the bulb, are not visible from the outside of predicate circuit1. The model of the circuit consists of stating that these arguments have to obey:

(1) The laws of the bulb

(2) The laws of the switch

(3) The (qualitative) Kirchhoff's law: switch voltage + bulb voltage = battery voltage.

The physical connections between the components are also reflected in that the switch current is equal to the bulb current.

The model of circuit 2, although more complex, is constructed in a similar way.

Here are some usual types of questions that the program of Figure 20.3 answers easily.

Prediction-type question

What will be the observable result of some 'input' to the system (switch positions), given some functional state of the system (bulbs OK or blown). For example, what happens if we turn on all the switches, and all the bulbs are OK?

```
?- circuit2(on,on,on,ok,ok,ok,L1,L2,L3).
L1 = light
L2 = dark
L3 = dark
```

Diagnostic-type question

Given the inputs to the system and some observed manifestations, what is the system's functional state (normal or malfunctioning; what is the failure?). For example, if bulb 1 is light, bulb 3 is dark, and switch 3 is off, what are the states of the bulbs?


```
?- circuit2( _, _, off, B1, B2, B3, light, _, dark).
B1 = ok
B2 = ok
B3 = blown
```

Control-type question

What should be the control input to achieve the desired output? For example, what should be the positions of the switches to make bulb 3 light, assuming all the bulbs intact?

```
?- circuit2( SwPos1, SwPos2, SwPos3, ok, ok, ok, _, _, light).
SwPos1 = on
SwPos2 = on
SwPos3 = off;

SwPos1 = on
SwPos2 = off
SwPos3 = off
```

Exercises

20.1 Define the qualitative multiplication relation over signs:

```
qmult( A, B, C)
```

where C = A*B, and A, B and C can be qualitative values pos, zero or neg.

20.2 Define qualitative models of a resistor and a diode:

```
resistor( Voltage, Current)
diode( Voltage, Current)
```

The diode only allows current in one direction. In a resistor, the signs of Voltage and Current are the same. Define qualitative models of some circuits with resistors, diodes and batteries.

20.3 Qualitative reasoning about dynamic systems

In this section we consider an approach to qualitative reasoning about dynamic systems. The approach considered here is based on the so-called qualitative differential equations (QDE). QDEs can be viewed as a qualitative abstraction of ordinary differential equations. To develop the intuition and basic ideas of this approach, let us consider an example of filling the bath tub with an open drain

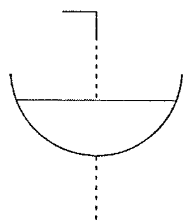

Figure 20.4 Bath tub with open drain and constant input flow.

(Figure 20.4). To begin with we will carry out informally some qualitative reasoning about this system. The variables we observe are: the flow into the tub, the flow out, the amount of water in the tub, and the level of water.

Let the process start with an empty bath tub. The outflow at the drain depends on the level of water: the higher the level, the greater the outflow. The inflow is constant. Net flow is the difference between the inflow and outflow. Initially the level is low, the inflow is greater than the outflow, and therefore the amount of water in the tub is increasing. Therefore the level is also increasing, which causes the outflow to increase. So at some time, the outflow may become equal to the inflow. According to a precise quantitative analysis, this only happens after a 'very long' time (infinite). When this happens, both flows are at equilibrium and the level of water becomes steady. The (quantitative) time behaviour of the water level looks like the one in Figure 20.5.

The *quantitative* behaviour of the level in Figure 20.5 can be simplified into a *qualitative* behaviour as follows. Initially, the level is zero and increasing. We choose to represent this as:

Level = zero/inc

In the subsequent time interval, the level is between zero and top, and it is increasing. We do not qualitatively distinguish between the exact numerical values

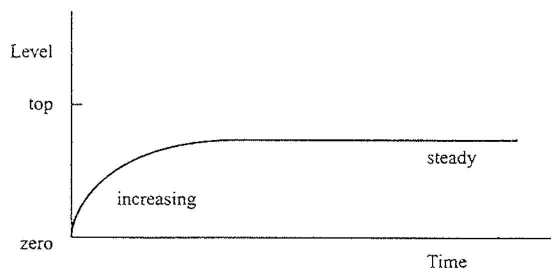

Figure 20.5 The behaviour of water level in time.

between zero and top. We take that these values are all sufficiently similar and therefore qualitatively the same. So we write:

Level = zero . . top/inc

The next qualitative change occurs when the level stops increasing and becomes steady:

Level = zero . . top/std

This is the final qualitative state of the water level.

We will now formalize in more detail the qualitative reasoning indicated above. First we define a *qualitative model* of the bath tub system. The variables in the system are:

Level = level of water
Amount = amount of water
Inflow = input flow
Outflow = output flow
Netflow = net flow (Netflow = Inflow − Outflow)

For each variable we specify its distinguished values, called *landmarks*. Typically we include minus infinity (minf), zero and infinity (inf) among the landmarks. For Level, the top of the bath tub is also an important value, so we choose also to include it among the landmarks. On the other hand, as the level is always non-negative, there is no need for including minf among the landmarks for Level. The landmarks are always ordered. So for Level we have the following ordered set of landmarks:

zero < top < inf

For Amount we may choose these landmarks:

zero < full < inf

Now we define the dependences among the variables in the model. These dependences are called *constraints* because they constrain the values of the variables.

We will use some types of constraints typical of qualitative reasoning. One such constraint states the dependence between Amount and Level: the greater the amount of water, the greater the level. We write:

M_0^+(Amount, Level)

In general the notation $M^+(X,Y)$ means that Y is a monotonically increasing function of X: whenever X increases, Y also increases and vice versa. $M_0^+(X,Y)$ means that Y is a monotonically increasing function of Y such that $Y(0) = 0$. We say that (0,0) is a pair of *corresponding values* for this M^+ relationship. Another pair of corresponding values for this M^+ relationship is (full,top). Notice that $M^+(X,Y)$ is equivalent to $M^+(Y,X)$.

The monotonically increasing constraint is very convenient and often greatly alleviates the definition of models. By stating M_0^+(Amount,Level), we just say that the level will rise whenever the amount increases, and the level will drop whenever the amount decreases. Notice that this is true for every container of *any* shape. If instead we wanted to state the precise *quantitative* functional relation

Amount = f(Level)

this would depend on the shape of the container, illustrated in Figure 20.6. Qualitatively, however, the relation between the level and the amount is always monotonically increasing, regardless of the shape of the container. So to define a qualitative model of the bath tub, we do not have to study the intricacies of the shape. This often greatly simplifies the modelling task. Our simplified, qualitative model still suffices for reliably deriving some important properties of the modelled system. For example, if there is a flow into a container with no outflow, the amount will be increasing and therefore the level will be increasing as well. So there will be

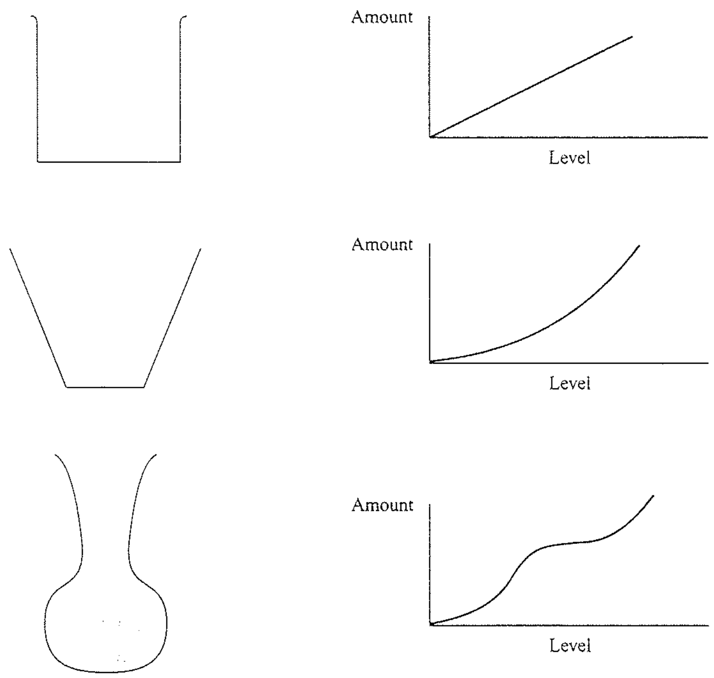

Figure 20.6 The precise relation between the amount and the level depends on the shape of a container. However, the amount is always a monotonically increasing function of the level.

Table 20.2 Types of qualitative constraints.

M^+(X,Y)	Y is a monotonically increasing function of X
M^-(X,Y)	Y is a monotonically decreasing function of X
sum(X,Y,Z)	Z = X+Y
minus(X,Y)	Y = −X
mult(X,Y,Z)	Z = X∗Y
deriv(X,Y)	Y = dX/dt (Y is time derivative of X)

some time point when the level reaches the top and water starts overflowing. All (possibly complicated) containers share this qualitative behaviour.

Similarly, the precise relation between **Outflow** and **Level** may be complicated. Qualitatively, we may simply state that it is monotonically increasing.

The types of constraints we will be using in our qualitative models are shown in Table 20.2. In the bath tub model we have the following constraints:

M_0^+(Amount, Level)
M_0^+(Level, Outflow)
sum(Outflow, Netflow, Inflow)
deriv(Amount, Netflow)
Inflow = constant = inflow/std

As usual, variable names start with capital letters, and constants start with lower case letters.

Sometimes it helps to illustrate the constraints by a graph. The nodes of the graph correspond to the variables in the model; the connections among the nodes correspond to the constraints. Figure 20.7 shows our bath tub model represented by such a graph.

Now let us carry out some qualitative simulation reasoning using the model in Figure 20.7. Without causing ambiguity, we will use a somewhat liberal notation. Writing **Amount=zero** will mean: the qualitative value of **Amount** is zero. Writing

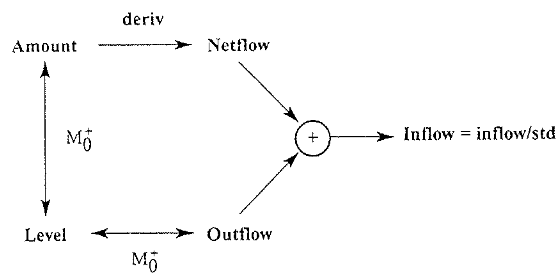

Figure 20.7 A graphical representation of the bath tub model.

Amount=zero/inc will mean: the qualitative value of Amount is zero and it is increasing. We start with the initial condition:

Amount = zero

Due to the constraint M_0^+ between Amount and Level (Figure 20.7), we infer:

Level = zero

This is propagated through the other M_0^+ constraint to infer:

Outflow = zero

Now the constraint Outflow + Netflow = Inflow instantiates to:

zero + Netflow = inflow

This yields: Netflow = inflow

Now consider the deriv constraint between Amount and Netflow, that is Netflow is equal to the time derivative of Amount. Since Netflow = inflow > zero, we infer that Amount must be increasing:

Amount = zero/inc

Propagating this through the M_0^+ constraints we have:

Level = zero/inc
Outflow = zero/inc

Now the constraint Outflow + Netflow = Inflow instantiates to:

zero/inc + Netflow = inflow/std

To satisfy this constraint, Netflow has to be:

Netflow = inflow/dec

Thus we have the complete initial qualitative state of the bath tub:

Amount = zero/inc
Level = zero/inc
Outflow = zero/inc
Netflow = inflow/dec

Now let us consider the possible transitions to a next qualitative state of the system. We assume that all the variables behave smoothly: their values change continuously in time, and their time derivatives are continuous. Consequently, a variable that is negative cannot become positive without first becoming zero. So a negative quantity can in the next time point either stay negative or become zero. Similarly, a variable that is increasing can either stay increasing or become steady. But it cannot instantaneously become decreasing, it has to become steady first. In other words, if a variables direction of change is 'inc' then it can either stay 'inc' or become 'std', but not 'dec'. Another constraint on possible transitions is that a changing variable cannot spend more than an instant at a landmark value. Therefore a transition from 'zero/inc' to 'zero/inc' is not possible.

The smoothness assumption is obviously reasonable in our bath tub system, at least when the level is between zero and top, so there is no water overflow. Given the smoothness constraint, the next qualitative state of Level is:

Level = zero . . top/inc

This value, and the constraints in the model of Figure 20.7 determine the qualitative states of the other variables. So the next qualitative state of the system is:

Level = zero . . top/inc
Amount = zero . . full/inc
Outflow = zero . . inflow/inc
Netflow = zero . . inflow/dec

What are the next possible qualitative states of Level? There are now four possibilities:

$$\text{Level} = \begin{cases} \text{zero .. top/inc} \\ \text{zero .. top/std} \\ \text{top/std} \\ \text{top/inc} \end{cases}$$

In the first case Level's qualitative value is the same as in the previous state. Then our model determines that the other variables also stay unchanged. So the previous qualitative state description still holds, and there is no need to introduce a new qualitative state in this case. Notice that, as in this case, a qualitative state may last for a whole time interval.

The remaining three possible transitions correspond to three alternative behaviours of the system:

(1) Level stops increasing and becomes steady before it has reached the top. The constraints in Figure 20.7 dictate that the other variables also become steady and there is no change from now on. So the final state of the simulation in this case is:

    Level = zero . . top/std
    Amount = zero . . full/std
    Outflow = inflow/std
    Netflow = zero/std

    Strictly, this steady state is only reached after infinite time, but that makes no difference in our qualitative description, because it does not take into account the durations of time intervals.

(2) Level becomes steady exactly at the moment when it reaches the top. Although this is theoretically possible, in reality this is an unlikely coincidence. Then all the other variables become steady, and this is again a final state of the simulation, similar to case 1.

(3) The level reaches the top and is at this moment still increasing. Then water starts flowing over the top and our model of Figure 20.7 no longer holds. At this point there is a discontinuous change into a new 'operating region'. A new model would now be needed for this new operating region. Discontinuous transition between operating regions would also require a special treatment. This will not be done here. Exiting the operating region of the model in Figure 20.7 will therefore be regarded here as another final state.

This example shows that a qualitative model may exhibit several qualitative behaviours. A real bath tub with its concrete physical parameters and a constant inflow will, of course, only behave in one of these three ways. Our qualitative model is a rather gross abstraction of reality. All the numerical information about the bath tub has been abstracted away. Therefore the qualitative simulation could not decide which of the three qualitative behaviours would correspond to an actual tub. Instead, the qualitative simulation in our case quite sensibly states that one of the three alternatives are possible.

This section demonstrated some basic ideas for qualitative modelling and simulation. These ideas are developed into a qualitative simulation program in the next section.

20.4 A qualitative simulation program

Figure 20.8 shows a qualitative simulation program that executes QDE models along the lines of the previous section. The following paragraphs describe details of this implementation.

```
% An interpreter for Qualitative Differential Equations

:- op( 100, xfx, ..).
:- op( 500, xfx, :).

% landmarks( Domain, [ Land1, Land2, ...]
%    Land1, Land2 etc. are landmarks for domain Domain
%    This is part of qualitative model definition, user-defined

% correspond( Constraint):
%    Constraint specifies corresponding values for a type of constr.

correspond( sum( Dom1:zero, Dom2:zero, Dom3:zero)).

correspond( sum( Dom1:L, Dom2:zero, Dom1:L)) :-
  qmag( Dom1:L), L \== zero, not (L = _.._).          % L is nonzero landmark in Dom1
```

Figure 20.8 A simulation program for qualitative differential equations. The program uses the usual list predicates **member/2** and **conc/3**.


```
correspond( sum( Dom1:zero, Dom2:L, Dom2:L)) :-
  qmag( Dom2:L), L \== zero, not (L = _.._).          % L is nonzero landmark in Dom2

correspond( sum( V1, V2, V3)) :-
  correspond( V1, V2, V3).                             % User-defined corr. values

% qmag( Domain:QualMagnitude)

qmag( Domain:Qm) :-
  landmarks( Domain, Lands),
  qmag( Lands, Qm).

qmag( Lands, L) :-
  member( L, Lands),
  L \== minf, L \== inf.                               % A finite landmark

qmag( Lands, L1..L2) :-                                % Interval
  conc( _, [L1,L2 | _], Lands).                        % Two adjacent landmarks

% relative_qmag( Domain1:QM, Domain2:Landmark, Sign):
%    Sign is the sign of the difference between QM and Landmark
%    if QM < Landmark then Sign = neg, etc.

relative_qmag( Domain:Ma..Mb, Domain:Land, Sign) :- !,
  landmarks( Domain, Lands),
  ( compare_lands( Ma, Land, Lands, neg), Sign = neg, !
   ;
   Sign = pos
  ).

relative_qmag( Domain:M1, Domain:M2, Sign) :-
  landmarks( Domain, Lands),
  compare_lands( M1, M2, Lands, Sign), !.

% qdir( Qdir, Sign):
%    Qdir is qualitative direction of change with sign Sign

qdir( dec, neg).
qdir( std, zero).
qdir( inc, pos).

% Laws of qualitative summation
% qsum( Q1, Q2, Q3):
%    Q3 = Q1 + Q2, qualitative sum over domain [pos,zero,neg]

qsum( pos, pos, pos).
qsum( pos, zero, pos).
qsum( pos, neg, pos).
qsum( pos, neg, zero).
qsum( pos, neg, neg).
qsum( zero, pos, pos).
qsum( zero, zero, zero).
qsum( zero, neg, neg).
qsum( neg, pos, pos).
```

Figure 20.8 *contd*

```
qsum( neg, pos, zero).
qsum( neg, pos, neg).
qsum( neg, zero, neg).
qsum( neg, neg, neg).

% qdirsum( D1, D2, D3):
%   qualitative sum over directions of change

qdirsum( D1, D2, D3) :-
  qdir( D1, Q1), qdir( D2, Q2), qdir( D3, Q3),
  qsum( Q1, Q2, Q3).

% sum( QV1, QV2, QV3):
%   QV1 = QV2 + QV3,
%   qualitative sum over qualitative values of form Domain:Qmag/Dir
%   When called, this predicate assumes that the
%   domains of all three arguments are instantiated

sum( D1:QM1/Dir1, D2:QM2/Dir2, D3:QM3/Dir3) :-
  qdirsum( Dir1, Dir2, Dir3),                % Directions of change: Dir1 + Dir2 = Dir3
  qmag( D1:QM1), qmag( D2:QM2), qmag( D3:QM3),
      % QM1+QM2=QM3 must be consistent with all corresponding values:
  not (
    correspond( sum( D1:V1, D2:V2, D3:V3)),      % V1 + V2 = V3
    relative_qmag( D1:QM1, D1:V1, Sign1),
    relative_qmag( D2:QM2, D2:V2, Sign2),
    relative_qmag( D3:QM3, D3:V3, Sign3),
    not qsum( Sign1, Sign2, Sign3) ).

% mplus( X, Y):
%   Y is a monotonically increasing function of X

mplus( D1:QM1/Dir, D2:QM2/Dir) :-            % Equal directions of change
  qmag( D1:QM1), qmag( D2:QM2),
      % QM1, QM2 consistent with all corresponding values between D1, D2:
  not ( correspond( D1:V1, D2:V2),
        relative_qmag( D1:QM1, D1:V1, Sign1),
        relative_qmag( D2:QM2, D2:V2, Sign2),
        Sign1 \== Sign2 ).

% deriv( Var1, Var2):
%   time derivative of Var1 is qualitatively equal Var2

deriv( Dom1:Qmag1/Dir1, Dom2:Qmag2/Dir2) :-
  qdir( Dir1, Sign1),
  qmag( Dom2:Qmag2),
  relative_qmag( Dom2:Qmag2, Dom2:zero, Sign2),    % Sign2 = sign of Qmag2
  Sign1 = Sign2.
```



```
% transition( Domain:Qmag1/Dir1, Domain:Qmag2/Dir2):
%   Variable state transitions between 'close' time points

transition( Dom:L1..L2/std, Dom:L1..L2/Dir2) :-
  qdir( Dir2, AnySign).

transition( Dom:L1..L2/inc, Dom:L1..L2/inc).

transition( Dom:L1..L2/inc, Dom:L1..L2/std).

transition( Dom:L1..L2/inc, Dom:L2/inc) :-
  L2 \== inf.

transition( Dom:L1..L2/inc, Dom:L2/std) :-
  L2 \== inf.

transition( Dom:L1..L2/dec, Dom:L1..L2/dec).

transition( Dom:L1..L2/dec, Dom:L1..L2/std).

transition( Dom:L1..L2/dec, Dom:L1/dec) :-
  L1 \== minf.

transition( Dom:L1..L2/dec, Dom:L1/std) :-
  L1 \== minf.

transition( Dom:L1/std, Dom:L1/std) :-
  L1 \== A..B.                                % L1 not an interval

transition( Dom:L1/std, Dom:L1..L2/inc) :-
  qmag( Dom:L1..L2).

transition( Dom:L1/std, Dom:L0..L1/dec) :-
  qmag( Dom:L0..L1).

transition( Dom:L1/inc, Dom:L1..L2/inc) :-
  qmag( Dom:L1..L2).

transition( Dom:L1/dec, Dom:L0..L1/dec) :-
  qmag( Dom:L0..L1).

% system_trans( State1, State2):
%   System state transition;
%   system state is a list of variable values

system_trans( [], []).

system_trans( [Val1 | Vals1], [Val2 | Vals2]) :-
  transition( Val1, Val2),
  system_trans( Vals1, Vals2).

% legal_trans( State1, State2):
%   possible transition between states according to model

legal_trans( State1, State2) :-
  system_trans( State1, State2),
  State1 \== State2,                          % Qualitatively different next state
  legalstate( State2).                        % Legal according to model
```

Figure 20.8 *contd*

```
% simulate( SystemStates, MaxLength):
%   SystemStates is a sequence of states of simulated system
%   not longer than MaxLength

simulate( [State], MaxLength) :-
  ( MaxLength = 1                           % Max length reached
   ;
   not legal_trans( State, _)               % No legal next state
  ) , !.

simulate( [State1,State2 | Rest], MaxLength) :-
  MaxLength > 1, NewMaxL is MaxLength - 1,
  legal_trans( State1, State2),
  simulate( [State2 | Rest], NewMaxL).

% simulate( InitialState, QualBehaviour, MaxLength)

simulate( InitialState, [InitialState | Rest], MaxLength) :-
  legalstate( InitialState),                % Satisfy system's model
  simulate( [InitialState | Rest], MaxLength).

% compare_lands( X1, X2, List, Sign):
%   if X1 before X2 in List then Sign = neg
%   if X2 before X1 then Sign = pos else Sign = zero

compare_lands( X1, X2, [First | Rest], Sign) :-
  X1 = X2, !, Sign = zero
  ;
  X1 = First, !, Sign = neg
  ;
  X2 = First, !, Sign = pos
  ;
  compare_lands( X1, X2, Rest, Sign).
```

20.4.1 Representation of qualitative states

The variables in the model can take qualitative values from *domains*. For example, Outflow and Netflow can have a value in terms of the landmarks from the domain 'flow', defined by the bath tub model. A domain is defined by its name and its landmarks, for example:

landmarks(flow, [minf, zero, inflow, inf]).

A *qualitative state* of a variable has the form:

Domain: QMag/Dir

QMag is a *qualitative magnitude*, which can be a landmark or the interval between two adjacent landmarks, written as Land1..Land2. **Dir** is a direction of change whose possible values are: inc, std, dec. Two example qualitative states of **Outflow** are:

flow: inflow/dec
flow: zero..inflow/dec

A *qualitative state of a system* is the list of qualitative states of the system's variables. For example, the initial state of the bath tub system consists of the values of the four variables Level, Amount, Outflow and Netflow:

[level:zero/inc, amount:zero/inc, flow:zero/inc, flow:inflow/dec]

A *qualitative behaviour* is the list of consecutive qualitative states.

20.4.2 Constraints

The program of Figure 20.8 implements three types of QDE constraints as the predicates: deriv(X, Y), sum(X, Y, Z), mplus(X, Y). The arguments X, Y and Z are all qualitative states of variables. We look at each of these constraints in turn.

Constraint deriv(X, Y): Y is qualitatively the time derivative of X. This is very simple to check: the direction of change of X has to agree with the sign of Y.

Constraint mplus(X, Y): Y is a monotonically increasing function of X. Here X and Y have the form Dx:QmagX/DirX and Dy:QmagY/DirY. First, the directions of change have to be consistent: DirX = DirY. Second, the given corresponding values have to be respected. The technique of checking this is based on 'relative qualitative magnitudes' of X and Y (relative with respect to the pairs of corresponding values). For example, the relative qualitative magnitude of level:zero..top with respect to top is neg. For each pair of corresponding values, the qualitative magnitudes of X and Y are transformed into the relative qualitative magnitudes. The resulting relative qualitative magnitude of X has to be equal to that of Y.

Constraint sum(X, Y, Z): X + Y = Z, where all X, Y and Z are qualitative states of variables of the form Domain:Qmag/Dir. Both the directions of change and the qualitative magnitudes have to be consistent with the summation constraint. First, the consistency of directions of change is checked. For example,

inc + std = inc

is true, and

inc + std = std

is false. Second, the qualitative magnitudes must be consistent with summation. In particular, they have to be consistent with respect to all the given corresponding values among X, Y and Z. The following are three examples of qualitative magnitudes satisfying the sum constraint:

```
flow:zero + flow:inflow = flow:inflow
flow:zero..inflow + flow:zero..inflow = flow:inflow
flow:zero..inflow + flow:zero..inflow = flow:zero..inflow
```

However, the following is false:

```
flow:zero..inflow + flow:inflow = flow:inflow
```

The technique of checking the consistency of qualitative magnitudes with respect to the corresponding values is as follows. First transform the given qualitative magnitudes into the relative qualitative magnitudes (relative to the corresponding values). Then check the consistency of these relative qualitative values with respect to qualitative summation. As an example consider:

```
sum( flow:zero..inflow/inc, flow:zero..inflow/dec, flow:inflow/std)
```

First, the directions of change satisfy the constraint: inc+dec = std. Second, consider the generally valid triple of corresponding values for summation:

```
correspond( sum( D1:zero, D2:Land, D2:Land))
```

D1 and D2 are domains, and Land is a landmark in D2. Applying this to the flow domain we have:

```
correspond( sum( flow:zero, flow:inflow, flow:inflow))
```

Are the qualitative magnitudes in our sum constraint consistent with these corresponding values? To check this, the three qualitative magnitudes in the sum constraint are transformed into the relative qualitative magnitudes as follows:

flow:zero..inflow is 'pos' with respect to flow:zero
flow:zero..inflow is 'neg' with respect to flow:inflow
flow:inflow is 'zero' with respect to flow:inflow

Now these relative qualitative magnitudes have to satisfy the constraint qsum(pos, neg, zero). This is true.

The mathematical basis for this procedure for checking the sum constraint is as follows. Let the constraint be sum(X,Y,Z) with a triple of corresponding values $(x0, y0, z0)$. We can express X, Y and Z in terms of their differences from the corresponding values:

$$X = x0 + \Delta X, \quad Y = y0 + \Delta Y, \quad Z = z0 + \Delta Z$$

The sum constraint then means:

$$x0 + \Delta X + y0 + \Delta Y = z0 + \Delta Z$$

Since $x0 + y0 = z0$, it follows that $\Delta X + \Delta Y = \Delta Z$. The signs of ΔX, ΔY and ΔZ are the relative qualitative magnitudes of X, Y and Z with respect to $x0$, $y0$ and $z0$. These relative qualitative magnitudes have to satisfy the relation qsum.

20.4.3 Qualitative state transitions

A key relation in the program of Figure 20.8 is:

```
transition( QualState1, QualState2)
```

where QualState1 and QualState2 are consecutive qualitative states of a variable. They both have the form: Domain:Qmag/Dir. The transition relation defines the possible transitions between qualitative states of variables, respecting the assumption about the 'smoothness' of variables. According to the smoothness assumption, the variables in the system may, within the same operating region, only change in time continuously and smoothly. For example, if a variable is approaching a landmark, it may reach the landmark, but may not jump over it. Similarly, if the direction of change of a variable is inc, it may remain inc, or become std, but cannot become dec before becoming std first. The transition predicate also makes sure that a non-steady variable can only stay at a landmark value for an instant, but not longer. Consecutive qualitative states correspond to consecutive time points that are, in principle, infinitesimally close to each other. To avoid generating infinite sequences of indistinguishable qualitative states during simulation, the simulator only generates a next system's state when there is some qualitative change with respect to the previous state. As a consequence of this, a qualitative state of the system may hold for as short as a single time point, or it may hold over a whole time interval. Such a time interval comprises all the consecutive time points in which there is no qualitative change.

Figure 20.9 shows our bath tub model in the format expected by the simulator in Figure 20.8. The query to start the simulation from the empty bath tub, with a maximum length of 10 states, is:

```
?- initial( S), simulate( S, Behaviour, 10).
```

The first of Prolog's answers (slightly edited) to this query is:

```
S = [level:zero/inc,amount:zero/inc,flow:zero/inc,flow:inflow/dec]
Behaviour = [
  [level:zero/inc,amount:zero/inc,flow:zero/inc,flow:inflow/dec],
  [level:zero..top/inc,amount:zero..full/inc, flow:zero..inflow/inc,
   flow:zero..inflow/dec],
  [level:zero..top/std,amount:zero..full/std, flow:inflow/std,flow:zero/std] ]
```

Our simulator of Figure 20.8 can easily be used for running other models. Figure 20.10 shows an electric circuit with two capacitors and a resistor. Figure 20.11 shows a qualitative model of this dynamic circuit and the corresponding initial state. In the initial state, the left capacitor has some initial voltage whereas the right capacitor is empty. The query to start the simulation and the simulator's answer (slightly edited) are:

```
% A bath tub model

landmarks( amount, [ zero, full, inf]).
landmarks( level, [ zero, top, inf]).
landmarks( flow, [ minf, zero, inflow, inf]).

correspond( amount:zero, level:zero).
correspond( amount:full, level:top).

legalstate( [ Level, Amount, Outflow, Netflow]) :-
  mplus( Amount, Level),
  mplus( Level, Outflow),
  Inflow = flow:inflow/std,                    % Constant inflow
  sum( Outflow, Netflow, Inflow),              % Netflow = Inflow - Outflow
  deriv( Amount, Netflow),
  not overflowing( Level).                     % Water not over the top

overflowing( level:top..inf/_).                % Over the top

initial( [ level: zero/inc,                    % Initial level
           amount: zero/inc,                   % Initial amount
           flow: zero/inc,                     % Initial outflow
           flow: inflow/dec ] ).               % Initial inflow
```

Figure 20.9 A qualitative model of bath tub.

```
?- initial( S), simulate( S, Behaviour, 10).
Behaviour =
[ [volt:v0/dec,volt:zero/inc,..],
  [volt:zero..v0/dec,volt:zero..v0/inc, ...],
  [volt:zero..v0/std,volt:zero..v0/std, ...] ]
```

Basically this says that the voltage on capacitor C1 will be decreasing and the voltage on C2 will be increasing until both voltages become equal (the current in and voltage on the resistor become zero).

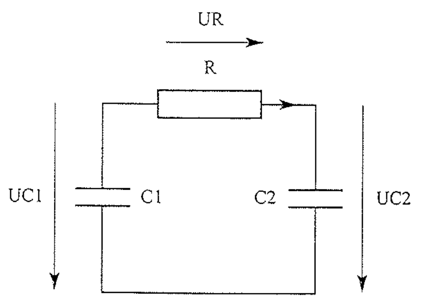

Figure 20.10 An electric circuit with two capacitors and a resistor.


```
% Qualitative model of electric circuit with resistor and capacitors

landmarks( volt, [minf, zero, v0, inf]).              % Voltage on capacitors
landmarks( voltR, [minf, zero, v0, inf]).             % Voltage on resistor
landmarks( current, [minf, zero, inf]).

correspond( voltR:zero, current:zero).

legalstate( [ UC1, UC2, UR, CurrR]) :-
  sum( UR, UC2, UC1),
  mplus( UR, CurrR),                                  % Ohm's law for resistor
  deriv( UC2, CurrR),
  sum( CurrR, current:CurrC1, current:zero/std),      % CurrC1 = - CurrR
  deriv( UC1, current:CurrC1).                        % CurrC1 = d/dt UC1

initial( [ volt:v0/dec, volt:zero/inc, voltR:v0/dec, current:zero..inf/dec]).
```

Figure 20.11 A qualitative model of the circuit in Figure 20.10.

Exercises

20.3 There are two variables in the system: X and Y. Their (quantitative) time behaviours have the form:

$$X(t) = a1*\sin(k1*t), \quad Y(t) = a2*\sin(k2*t)$$

a1, a2, k1 and k2 are constant parameters of the system, all of them greater than 0. The initial time point is t0 = 0, so the initial qualitative state of the system is: X(t0) = Y(t0) = zero/inc. (a) Give all the possible sequences of the first three qualitative states of this system. (b) Now suppose that there is a qualitative constraint between X and Y: $M_0^+(X,Y)$. Give all the possible sequences of the first three qualitative states of the system consistent with this constraint.

20.4 A qualitative model of a system contains variables X, Y and Z, and the constraints:

$M_0^+(X,Y)$
sum(X,Y,Z)

The landmarks for the three variables are:

X, Y: minf, zero, inf
Z: minf, zero, landz, inf

At time t0, the qualitative value of x is x(t0) = zero/inc. What are the qualitative values Y(t0) and Z(t0)? What are the possible qualitative values of X, Y and Z in the next qualitative state of the system which holds over time interval t0..t1, until the next qualitative change? After the next qualitative change, at time t1, what are the possible new qualitative values X(t1), Y(t1) and Z(t1)?

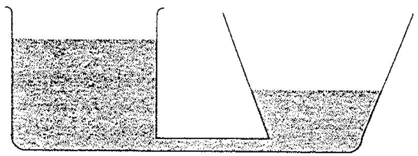

Figure 20.12 U-tube; two containers connected by a thin pipe.

20.5 Define a qualitative model of the 'U-tube' system (Figure 20.12) in the form of a Prolog program expected by the simulator of Figure 20.8. In this system the two containers are connected by a thin pipe (so thin that the inertia of water flow in the pipe can be neglected). Experiment with the model and the initial state as indicated in Figure 20.12.

20.6 Extend the simulator in Figure 20.8 by implementing other qualitative constraints that typically appear in QDEs: minus(X,Y) (X = −Y), m_minus(X,Y) (Y is a monotonically decreasing function of X), mult(X,Y,Z) (Z = X ∗ Y).

20.7 Consider the following qualitative model of accelerated motion:

```
landmarks( x, [ minf, zero, x1, inf]).
landmarks( v, [ minf, zero, v0, inf]).

legalstate( [ X, V]) :-
  V = v:_/inc,                    % Positive acceleration
  deriv( X, V).

initial( [ x:zero..x1/inc, v:v0/inc]).
```

The qualitative simulation program of Figure 20.8 produces the following (Prolog's output slightly edited):

```
?- initial( S), simulate( S, Behav, 3).
Behav = [ [x : zero .. x1 / inc, v : v0 / inc],
          [x : zero .. x1 / inc, v : v0 .. inf / inc],
          [x : x1 / inc, v : v0 .. inf / inc] ];
Behav = [ [x : zero .. x1 / inc, v : v0 / inc],
          [x : x1 / inc, v : v0 .. inf / inc],
          [x : x1 .. inf / inc, v : v0 .. inf / inc] ]
```

The second of the two generated behaviours is, strictly speaking, not correct. The problem is in the transition between the first and the second state of the system. The first state includes v:v0/inc, which can only last for a single point in time (the value of an increasing variable can only be at a landmark for an instant, not longer). The second state includes x:x1/inc, so this is also a time-point state. However, a time-point state cannot be immediately followed by another time-point state. There has to be a time interval in between the two time points (time interval in which X would reach x1 from the interval zero..x1). It does not help to argue that X can be arbitrarily close to x1 in the initial state. No matter how close the initial X is to x1, there is always another real number between the initial X and x1, and X will first have to move to this number before reaching x1. How can the program in Figure 20.8 be modified to fix this deficiency? Hint: modify procedure legal_trans.

20.5 Discussion of the qualitative simulation program

Our qualitative simulator of the previous section is largely based on the QSIM algorithm (Kuipers 1986; 1994). In the interest of simplicity, there are some differences between the program in Figure 20.8 and the original QSIM algorithm: somewhat different treatment of time intervals, which simplifies the qualitative state transition table a little; not all of the constraint types have been implemented; our program does not generate new landmarks during simulation, whereas QSIM does. Regardless of these differences, the following discussion of advantages and drawbacks applies in general to this kind of simulation based on QDEs.

Let us start with some advantages. It has already been emphasized that building qualitative models is generally easier than quantitative models, and that qualitative models are more appropriate for some types of tasks. The QSIM-type qualitative simulation also has an elegant advantage over numerical simulation in that the simulation time step is adaptive, whereas in numerical simulation the time step is normally fixed. This flexibility in qualitative simulation can be particularly advantageous in comparison with fixed step numerical simulation in cases where the modelled system changes its behaviour abruptly. As an illustration of this, consider the bath tub again. When the water level is below the top, the system behaves according to the specified constraints in the model. When the level reaches the top, overflowing may begin and transition to another 'operating region' occurs. So the laws in the model change abruptly. For simulation it is therefore important to detect *exactly* the moment when the level reaches the top. In QSIM, a variable reaching its landmark is defined as a qualitative change, so the very time point when Level=top is automatically generated by the simulator. On the other hand, in a numerical simulator with fixed time step it will be unlikely that the level will exactly equal the top at one of the generated time points. Most likely, the next simulated level value will be either under the top, or, incorrectly, a little over the top.

Let us now discuss some problems of QSIM-type simulation. Qualitative simulation naturally appears to be computationally more economical than numerical simulation. However, paradoxically exactly the opposite may be true in practice. The reason lies in the non-determinism of qualitative simulation. Our simple bath tub model produced three different behaviours. In more complex models, the number of possible behaviours often grows exponentially with their length, which may result in combinatorial explosion. This may render qualitative simulation practically useless.

In the bath tub case, non-determinism was due to the lack of information in the model. All the three generated behaviours were consistent with the model. Real bath tubs are possible whose actual behaviours correspond to the three qualitative behaviours found by the simulator. So the simulator's results quite reasonably branch three ways. However, a more problematic kind of combinatorial branching in QSIM-type simulation is also possible. This kind of simulation may sometimes generate behaviours that do not correspond to any concrete quantitative instance of the qualitative model. Such behaviours are simply incorrect; they are inconsistent with the given qualitative model. Technically they are called *spurious behaviours*.

As an example consider a simple oscillating system consisting of a sliding block and a spring (Figure 20.13). We assume zero friction between the block and the surface. Assume that initially at time $t0$ the spring is at its 'rest length' (X = zero) and the block has some initial velocity $v0$ to the right. Then X will be increasing and the spring will be pulling the block back, causing negative acceleration of the block, until the block stops and starts moving backwards. It will then cross zero with some negative velocity, reach the extreme position on the left, and return to X = zero. As there is no friction, we may expect that the block's velocity will be at that time $v0$ again. Now the whole cycle will be repeated. The resulting behaviour is steady oscillation.

Let us try to model this with our simulator of Figure 20.8. A quantitative model is:

$$\frac{d^2}{dt^2}X = A$$

$$A = -\frac{kX}{m}$$

X is the position of the block, A is its acceleration, m is its mass, and k is the coefficient of the spring. An appropriate qualitative model is given in Figure 20.14.

Let us execute this model from the initial state with:

```
?- initial( S), simulate( S, Beh, 11).
```

The generated behaviour Beh is as expected up to state 8:

```
[ x:minf..zero/inc, v:zero..v0/inc, a:zero..inf/dec]
```

Here the behaviour branches three ways. In the first branch the behaviour continues as follows:

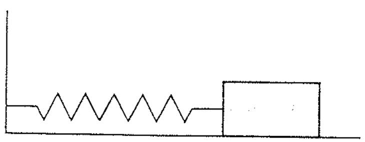

Figure 20.13 Sliding block and spring, no friction between block and support surface.


```
% Model of block on spring

landmarks( x, [ minf, zero, inf]).          % Position of block
landmarks( v, [ minf, zero, v0, inf]).      % Velocity of block
landmarks( a, [ minf, zero, inf]).          % Acceleration of block

correspond( x:zero, a:zero).

legalstate( [ X, V, A]) :-
  deriv( X, V),
  deriv( V, A),
  MinusA = a:_,
  sum( A, MinusA, a:zero/std),              % MinusA = -A
  mplus( X, MinusA).                        % Spring pulling mass back

initial( [ x:zero/inc, v:v0/std, a:zero/dec]).
```

Figure 20.14 A qualitative model of the block and spring system.

```
[x:minf..zero/inc, v:v0/inc, a:zero..inf/dec]
[x:minf..zero/inc, v:v0..inf/inc, a:zero..inf/dec]
[x:zero/inc, v:v0..inf/std, a:zero/dec]
```

Here the velocity has reached the initial velocity v0 already before X became equal zero. At the time when X = zero, velocity is already greater than v0. This looks like a physically impossible case: the total energy in the system has increased, and the behaviour looks like an increasing oscillation. In the second branch, state 8 is followed by this:

```
[x:zero/inc, v:zero..v0/std, a:zero/dec]
[x:zero..inf/inc, v:zero..v0/dec, a:minf..zero/dec]
[x:zero..inf/std, v:zero/dec, a:minf..zero/std]
```

Here the block has reached X = zero with velocity lower than v0. The total energy in the system here is less than in the initial state, so this appears to be a decreasing oscillation. In the third branch, state 8 is followed by:

```
[x:zero/inc, v:v0/std, a:zero/dec]
etc.
```

This corresponds to the expected case of steady oscillation.

The question is: Are the two unexpected behaviours only a consequence of lack of information in the qualitative model, or is there a problem in the simulation algorithm? It can be shown that, in fact, the model, although qualitative, contains enough information to allow the steady oscillation only. Therefore the other two behaviours, increasing and decreasing oscillations, are mathematically inconsistent with the model. They are said to be spurious. The weakness is in the simulation algorithm. The immediate question is then: Why not quickly fix the bug in the algorithm? The difficulty is that this is not a simple bug, but a complicated computational problem: How to check qualitative behaviours against *all* the constraints imposed by the model? QSIM-type simulation algorithm checks the consistency of individual states, but not also their sequences as a whole. Although improvements have been found that eliminate many spurious behaviours, a complete solution has not been discovered.

Given this drawback of the QSIM-type simulation, do we have any guarantees regarding the results of simulation? There is a theorem (Kuipers 1986) that QSIM is guaranteed to generate *all* qualitative behaviours consistent with the model. So incorrectness is limited to the opposite cases: QSIM may generate incorrect behaviours, those not consistent with the model (spurious behaviours). Figure 20.15 illustrates the relations between the abstraction levels involved: differential equations and their solutions at the quantitative level, and QDEs and qualitative behaviours generated by QSIM at the qualitative level. Those generated behaviours that are not abstractions of any quantitative solutions are spurious.

The practical significance of the 'one sided' correctness of QSIM is the following. Suppose that a QSIM-type simulator has generated some qualitative behaviours from our model. Now we know that this set is complete in the sense that there exists no other behaviour of our modelled system that is not included among the results. We know that nothing else can happen. However, we have no guarantee that all the generated behaviours are in fact possible.

One way to eliminate spurious behaviours is to add more constraints to the model, whereby the correctness of the model should be preserved. This is not easy in

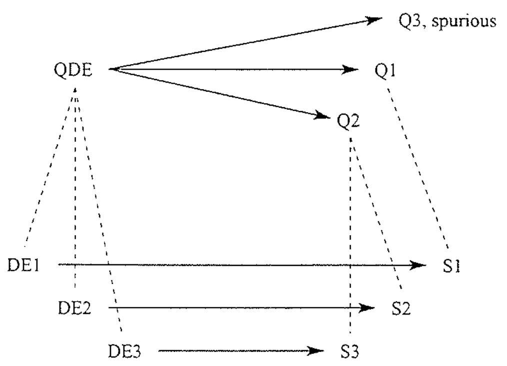

Figure 20.15 Qualitative abstractions of differential equations and their solutions. QDE is the qualitative abstraction of three differential equations DE1, DE2 and DE3. S1, S2 and S3 are respective solutions of the three differential equations. Qualitative behaviours Q1, Q2 and Q3 are generated as solutions of QDE. Q1 is the qualitative abstraction of S1, Q2 is the abstraction of both S2 and S3. Q3 is spurious because it is not the abstraction of the solution of any corresponding differential equation.

general, but it is straightforward in the case of the block and spring system. We know that the energy in this system must be constant, because there is no loss of energy through friction, and there is no input of energy into the system. The total energy is the sum of kinetic and potential energy. So the energy conservation constraint is:

$$\frac{1}{2}mV^2 + \frac{1}{2}kX^2 = \text{const.}$$

m is the mass of the block, and k is the coefficient of the spring. A weaker version of this constraint is sufficient for our purpose. Namely, a consequence of the above energy conservation constraint is: whenever $X = 0$, $V^2 = v_0{}^2$. An equivalent constraint is: if $V = v_0$ then $X = 0$, and if $X = 0$ then $V = v_0$ or $V = -v_0$. The following modification of the block and spring model of Figure 20.14 does not generate any spurious solutions:

```
legalstate( [ X, V, A]) :-
  deriv( X, V),
  deriv( V, A),
  MinusA = a:_,
  sum( A, MinusA, a:zero/std),       % MinusA = -A
  mplus( X, MinusA),                 % Spring pulling block back
  energy( X, V).                     % Weak energy conservation constraint

energy( X, V) :-
  V = v:v0/_, !, X = x:zero/_        % If V=v0, X must be zero
  ;
  X = x:zero/_, !, V = v:minf..zero/_   % Here V must be -v0
  ;
  true.                              % Otherwise X non-zero, no constraint on V
```

Summary

- 'Proper physics' solutions often comprise more numerical details than needed for everyday purposes. Common sense, qualitative reasoning and 'naive physics' are therefore more appropriate in such cases.
- Qualitative modelling and reasoning is usually viewed as an abstraction of quantitative modelling and reasoning. Numbers are reduced to their signs, symbolic values (sometimes called *landmarks*) or intervals. Time may be simplified into symbolic time points and intervals. Time derivatives may be simplified into directions of change (increasing, decreasing or steady). Concrete functional relationships may be simplified into more vague ones, such as monotonic relationships.

- Qualitative models are easier to construct than quantitative models. Due to lack of numerical precision, qualitative models are not always sufficient, but they are generally appropriate for the tasks of diagnosis, functional reasoning, and design from 'first principles'.
- In qualitative reasoning, arithmetic operations are reduced to qualitative arithmetic. An example is qualitative summation over the signs *pos*, *zero* and *neg*. Typically, qualitative arithmetic is non-deterministic.
- Qualitative differential equations (QDE) are an abstraction of differential equations. QSIM is a qualitative simulation algorithm for models defined by QDEs. The main underlying assumption in QSIM-type simulation is that of 'smoothness': within the same 'operating region', the values of variables can only change smoothly.
- The difficulty of ensuring correctness of qualitative simulation is asymmetrical in the following sense. It is relatively easy to ensure that *all* the behaviours consistent with the given QDEs are, in fact, generated by the simulator. On the other hand, it is hard to ensure that *only* the behaviours consistent with the given QDEs are generated. This problem is known as the problem of *spurious behaviours*.
- Concepts discussed in this chapter are:

  qualitative reasoning
  qualitative modelling
  common sense and naive physics
  qualitative abstractions
  landmarks
  qualitative arithmetic
  qualitative summation
  qualitative differential equations (QDEs)
  monotonic functional constraints
  qualitative value, qualitative state of a variable
  qualitative simulation
  the QSIM algorithm
  smoothness assumption in qualitative simulation of dynamic systems
  smooth qualitative transitions
  spurious behaviours

References

Bobrow (1984) edited a special issue of the *Artificial Intelligence* journal entitled Qualitative Reasoning about Physical Systems. Some papers in that volume became classical references in the qualitative reasoning field, such as the papers by de Kleer and Brown (1984) on *confluences*, and by Forbus on *Qualitative Physics Theory*. Although these papers are more interested in descriptions at higher levels than qualitative differential equations (QDE), they are strongly related to QDEs. QDEs can be viewed as the underlying low-level formalism to which these higher-level descriptions can be compiled. de Kleer and Williams (1991) edited another special issue on qualitative reasoning of the *Artificial Intelligence* journal. A useful selection of important papers published prior to 1990 was edited by Weld and de Kleer (1990). Faltings and Struss (1992) is another collection of papers on qualitative reasoning. A specialized forum for rapid publication of ongoing research in qualitative reasoning is the annual Workshop on Qualitative Reasoning.

The qualitative simulation program for dynamic systems in this chapter is based on the QSIM algorithm (Kuipers 1986) for QDE models. In the interest of simplicity, our program does not implement the full repertoire of qualitative constraints in QSIM, it does not introduce new landmarks during simulation, and it does not make explicit difference between time points and time intervals. Makarovič (1991), and Sacks and Doyle (1991) analyze the difficulties of QSIM-style simulation. Improvements that alleviate the problem of spurious behaviours, and some other developments of QSIM, are described in Kuipers (1994). Further contributions to the treatment of spurious behaviours are Say (1998a, 1998b) and Say and Kuru (1993). Exercise 20.7 was suggested by Cem Say (personal communication).

Of course, qualitative reasoning about physical systems does not have to be based on differential equations or their direct abstraction. An approach that does not assume any underlying connection to differential equations was applied by Bratko, Mozetič and Lavrač (1989) in the modelling of a complex physiological system. A model of the heart explaining the relations between cardiac arrhythmias and ECG signals was defined in terms of logic-based qualitative descriptions.

Forbus and Falkenhainer (1992) explored an interesting idea of combining qualitative and numerical simulation.

An important practical question is how to construct qualitative models, and can this be automated. Automated construction of QDE models of dynamic systems from observed behaviours of the modelled systems was studied by Coiera (1989), Bratko, Muggleton and Varšek (1991), Kraan, Richards and Kuipers (1991), Varšek (1991), Say and Kuru (1996), and Hau and Coiera (1997). Small scale QDE-type models were synthesized from given behaviours in all these works. Mozetič (1987a; 1987b; also described in Bratko *et al.* 1989) synthesized by means of machine learning a substantial non-QDE-type qualitative model of the electrical behaviour of the heart.

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chapter 21

Language Processing with Grammar Rules

Many Prolog implementations provide a notational extension called DCG (definite clause grammars). This makes it very easy to implement formal grammars in Prolog. A grammar stated in DCG is directly executable by Prolog as a syntax analyzer. DCG also facilitates the handling of the semantics of a language so that the meaning of a sentence can be interleaved with the syntax. This chapter shows how DCG enables elegant definitions of the syntax and meaning of non-trivial natural language sentences, such as: 'Every woman that admires a man that paints likes Monet'.

21.1 Grammar rules in Prolog

Grammar is a formal device for defining sets of sequences of symbols. Such a sequence of symbols can be abstract, without any practical meaning, or, more interestingly, it can be a statement in a programming language, or a whole program; it can be a sentence in a natural language such as English.

One popular grammar notation is BNF (Backus-Naur form), which is commonly used in the definition of programming languages. We will start our discussion by considering BNF. A grammar comprises *production rules*. Here is a simple BNF grammar of two rules:

$$\langle s \rangle ::= a\ b$$
$$\langle s \rangle ::= a\ \langle s \rangle\ b$$

The first rule says: whenever the symbol *s* appears in a string, it can be rewritten with the sequence *ab*. The second rule says that *s* can be rewritten with the sequence *a*, followed by *s*, followed by *b*. In this grammar, *s* is always enclosed by brackets '$\langle\,\rangle$'. This indicates that *s* is a *non-terminal* symbol of the grammar. On the other hand, *a* and *b* are *terminal* symbols. Terminal symbols can never be rewritten. In BNF, the two production rules above are normally written together as one rule:

$$\langle s \rangle ::= a\ b \mid a\ \langle s \rangle\ b$$

But for the purpose of this chapter we will be using the expanded, longer form.

A grammar can be used to *generate* a string of symbols, called a *sentence*. The generation process always starts with some starting non-terminal symbol, *s* in our example. Then symbols in the current sequence are replaced by other strings according to the grammar rules. The generation process terminates when the current sequence does not contain any non-terminal symbol. In our example grammar, the generation process can proceed as follows. Start with:

s

Now by the second rule, *s* is replaced by:

a s b

The second rule can be used again giving:

a a s b b

Applying the first rule, a sentence is finally produced:

a a a b b b

Obviously, our grammar can generate other sentences – for example, *ab*, *aabb*, etc. In general, this grammar generates strings of the form $a^n b^n$ for $n = 1, 2, 3, \ldots$. The set of sentences, generated by a grammar, is called the *language* defined by the grammar.

Our example grammar is simple and very abstract. However, we can use grammars to define much more interesting languages. Formal grammars are used for defining programming languages and also subsets of natural languages.

Our next grammar is still very simple, but slightly less abstract. Suppose a robot arm can be sent sequences of commands:

up: move one step upwards, and
down: move one step downwards

Here are two examples of command sequences that such a robot would accept:

up
up up down up down

Such a sequence triggers the corresponding sequence of steps performed by the robot. We will call a sequence of steps a 'move'. A move, then, consists of one step, or a step followed by a move. This is captured by the following grammar:

$$\langle move \rangle ::= \langle step \rangle$$
$$\langle move \rangle ::= \langle step \rangle\ \langle move \rangle$$
$$\langle step \rangle ::= up$$
$$\langle step \rangle ::= down$$

We will use this grammar later to illustrate how the meaning can be handled within Prolog's grammar notation.

As shown earlier, a grammar generates sentences. In the opposite direction, a grammar can be used to *recognize* a given sentence. A recognizer decides whether a given sentence belongs to some language; that is, it recognizes whether the sentence can be generated by the corresponding grammar. The recognition process is essentially the inverse of generation. In recognition, the process starts with the given string of symbols, to which grammar rules are applied in the opposite direction to generation: if the current string contains a substring, equal to the right-hand side of some rule in the grammar, then this substring is rewritten with the left-hand side of this rule. The recognition process terminates successfully when the complete given sentence has been reduced to the starting non-terminal symbol of the grammar. If there is no way of reducing the given sentence to the starting non-terminal symbol, then the recognizer rejects the sentence.

In such a recognition process, the given sentence is effectively disassembled into its constituents; therefore, this process is often also called *parsing*. To implement a grammar, normally means to write a parsing program for the grammar. We will see that in Prolog such parsing programs can be written very easily. What makes this particularly elegant in Prolog is a special grammar rule notation, called DCG (definite clause grammar). Many Prolog implementations support this special notation for grammars. A grammar written in DCG is already a parsing program for this grammar. To transform a BNF grammar into DCG we only have to change some notational conventions. Our example BNF grammars can be written in DCG as follows:

```
s --> [ a], [ b].
s --> [ a], s, [ b].

move --> step.
move --> step, move.

step --> [ up].
step --> [ down].
```

Notice the differences between the BNF and DCG notations. '::=' is replaced by '-->'. Non-terminals are not in brackets any more, but terminals are in square brackets, thereby making them Prolog lists. In addition, symbols are now separated by commas, and each rule is terminated by a full stop as every Prolog clause.

In Prolog implementations that accept the DCG notation, our transformed grammars can be immediately used as recognizers of sentences. Such a recognizer expects sentences to be essentially represented as difference lists of terminal symbols. (Difference-list representation was introduced in Chapter 8.) So each sentence is represented by two lists: the sentence represented is the difference between both lists. The two lists are not unique, for example:

aabb can be represented by lists [a, a, b, b] and [ ]
or by lists [a, a, b, b, c] and [c]
or by lists [a, a, b, b, 1, 0, 1] and [1, 0, 1]
...

Taking into account this representation of sentences, our example DCG can be asked to recognize some sentences by questions:

```
?- s( [ a, a, b, b], [ ]).            % Recognize string aabb
yes
?- s( [ a, a, b], [ ]).
no
?- move( [ up, up, down], [ ]).
yes
?- move( [ up, up, left], [ ]).
no
?- move( [ up, X, up], [ ]).
X = up;
X = down;
no
```

Let us now explain *how* Prolog uses the given DCG to answer such questions. When Prolog consults grammar rules, it automatically converts them into normal Prolog clauses. In this way, Prolog converts the given grammar rules into a program for recognizing sentences generated by the grammar. The following example illustrates this conversion. Our four DCG rules about robot moves are converted into four clauses:

```
move( List, Rest) :-
  step( List, Rest).

move( List1, Rest) :-
  step( List1, List2),
  move( List2, Rest).

step( [up | Rest], Rest).

step( [down | Rest], Rest).
```


What is actually achieved by this conversion? Let us look at the move procedure. The relation move has two arguments – two lists:

move(List, Rest)

is true if the difference of the lists List and Rest is an acceptable move. Example relationships are

move([up, down, up], [ ]).

or

move([up, down, up, a, b, c], [a, b, c])

or:

move([up, down, up], [down, up])

Figure 21.1 illustrates what is meant by the clause:

```
move( List1, Rest) :-
  step( List1, List2),
  move( List2, Rest).
```

The clause can be read as:

The difference of lists List1 and Rest is a move if
the difference between List1 and List2 is a step and
the difference between List2 and Rest is a move.

This also explains why difference-list representation is used: the pair (List1, Rest) represents the concatenation of the lists represented by the pairs (List1, List2) and (List2, Rest). As shown in Chapter 8, concatenating lists in this way is much more efficient than with conc.

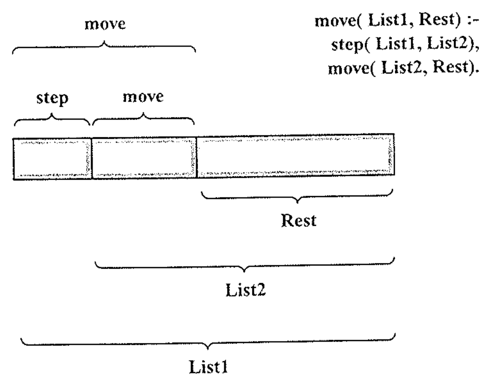


Figure 21.1 Relations between sequences of symbols.

Now we are ready to formulate more generally the translation between DCG and standard Prolog. Each DCG rule is translated into a Prolog clause according to the following basic scheme: let the DCG rule be:

```
n --> n1, n2, ..., nn.
```

If all n1, n2, ..., nn are non-terminals then the rule is translated into the clause:

```
n( List1, Rest) :-
  n1( List1, List2),
  n2( List2, List3),
  ...
  nn( Listn, Rest).
```

If any of n1, n2, ..., nn is a non-terminal (in square brackets in the DCG rule) then it is handled differently. It does not appear as a goal in the clause, but is directly inserted into the corresponding list. As an example, consider the DCG rule:

```
n --> n1, [ t2], n3, [ t4].
```

where n1 and n3 are non-terminals, and t2 and t4 are terminals. This is translated into the clause:

```
n( List1, Rest) :-
  n1( List1, [t2 | List3]),
  n3( List3, [t4 | Rest]).
```

More interesting examples of grammars come from programming languages and natural languages. In both cases they can be elegantly implemented using DCG. Here is an example grammar for a simple subset of English:

```
sentence --> noun_phrase, verb_phrase.

verb_phrase --> verb, noun_phrase.

noun_phrase --> determiner, noun.

determiner --> [ a].
determiner --> [ the].

noun --> [ cat].
noun --> [ mouse].

verb --> [ scares].
verb --> [ hates].
```

Example sentences generated by this grammar are:

```
[ the, cat, scares, a, mouse]
[ the, mouse, hates, the, cat]
[ the, mouse, scares, the, mouse]
```

Let us add nouns and verbs in plural to enable the generation of sentences like [the, mice, hate, the, cats]:


```
noun --> [ cats].
noun --> [ mice].

verb --> [ scare].
verb --> [ hate].
```

The grammar thus extended will generate the intended sentence. However, in addition, it will unfortunately also generate some unintended, incorrect English sentences, such as:

```
[ the, mouse, hate, the cat]
```

The problem lies in the rule:

```
sentence --> noun_phrase, verb_phrase.
```

This states that *any* noun phrase and verb phrase can be put together to form a sentence. But in English and many other languages, the noun phrase and verb phrase in a sentence are not independent: they have to agree in number. Both have to be either singular or plural. This phenomenon is called *context dependence*. A phrase depends on the context in which it occurs. Context dependencies cannot be directly handled by BNF grammars, but they can be easily handled by DCG grammars, using an extension that DCG provides with respect to BNF – namely, *arguments* that can be added to non-terminal symbols of the grammar. For example, we may add 'number' as an argument of noun phrase and verb phrase:

```
noun_phrase( Number)

verb_phrase( Number)
```

With this argument added we can easily modify our example grammar to force number agreement between the noun phrase and verb phrase:

```
sentence( Number) --> noun_phrase( Number), verb_phrase( Number).

verb_phrase( Number) --> verb( Number), noun_phrase( Number1).

noun_phrase( Number) --> determiner( Number), noun( Number).

noun( singular) --> [ mouse].

noun( plural) --> [ mice].

...
```

When DCG rules are read by Prolog and converted into Prolog clauses, the arguments of non-terminals are simply added to the usual two list arguments, with the convention that the two lists come last. Thus:

```
sentence( Number) --> noun_phrase( Number), verb_phrase( Number).
```

is converted into:

```
sentence( Number, List1, Rest) :-
  noun_phrase( Number, List1, List2),
  verb_phrase( Number, List2, Rest).
```

Questions to Prolog now have to be modified accordingly so that the extra arguments are included:

```
?- sentence( plural, [ the, mice, hate, the, cats], [ ]).

yes

?- sentence( plural, [ the, mice, hates, the, cats], [ ]).

no

?- sentence( plural, [ the, mouse, hates, the, cat], [ ]).

no

?- sentence( Number, [ the, mouse, hates, the, cat], [ ]).

Number = singular

?- sentence( singular, [ the, What, hates, the, cat], [ ]).

What = cat;
What = mouse;
no
```

Exercises

21.1 Translate into standard Prolog the DCG rule:

```
s --> [ a], s, [ b].
```

21.2 Write a Prolog procedure

```
translate( DCGrule, PrologClause)
```

that translates a given DCG rule into the corresponding Prolog clause.

21.3 One DCG rule in our grammar about robot moves is:

```
move --> step, move.
```

If this rule is replaced by

```
move --> move, step.
```

the language, defined by the so-modified grammar, is the same. However, the corresponding recognition procedure in Prolog is different. Analyze the difference. What is the advantage of the original grammar? How would the two grammars handle the question:

```
?- move( [ up, left], [ ]).
```


21.2 Handling meaning

21.2.1 Constructing parse trees

Let us first illustrate by an example the concept of a *parse tree*. According to our example grammar, the sentence

```
[ the, cat, scares, the, mice]
```

is parsed as shown by the parse tree of Figure 21.2. Some parts of the sentence are called *phrases* – those parts that correspond to non-terminals in the parse tree. In our example, [the, mice] is a phrase corresponding to the non-terminal **noun_phrase**; [scares, the, mice] is a phrase corresponding to **verb_phrase**. Figure 21.2 shows how the parse tree of a sentence contains, as subtrees, parse trees of phrases.

Here now is a definition of parse tree. The parse tree of a phrase is a tree with the following properties:

(1) All the leaves of the tree are labelled by terminal symbols of the grammar.

(2) All the internal nodes of the tree are labelled by non-terminal symbols; the root of the tree is labelled by the non-terminal that corresponds to the phrase.

(3) The parent-children relation in the tree is as specified by the rules of the grammar. For example, if the grammar contains the rule

```
s --> p, q, r.
```

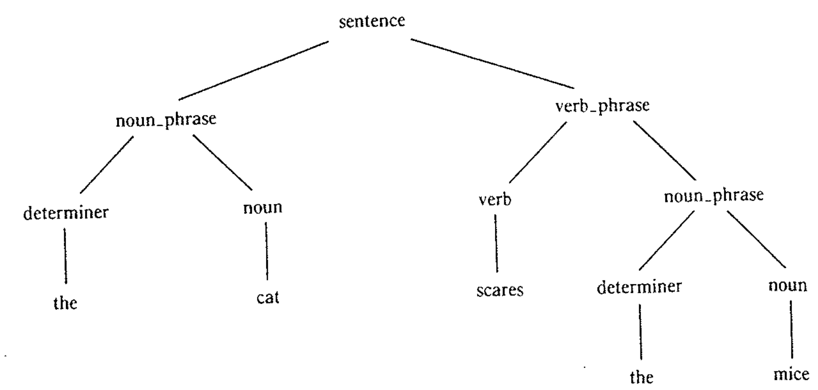
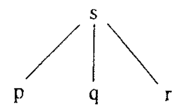


Figure 21.2 The parse tree of the sentence 'The cat scares the mice'.

then the tree may contain the node s whose children are p, q and r:



Sometimes it is useful to have the parse tree explicitly represented in the program to perform some computation on it – for example, to extract the meaning of a sentence. The parse tree can be easily constructed using arguments of non-terminals in the DCG notation. We can conveniently represent a parse tree by a Prolog term whose functor is the root of the tree and whose arguments are the subtrees of the tree. For example, the parse tree for the noun phrase 'the cat' would be represented as:

```
noun_phrase( determiner( the), noun( cat))
```

To generate a parse tree, a DCG grammar can be modified by adding to each non-terminal its parse tree as an argument. For example, the parse tree of a noun phrase in our grammar has the form:

```
noun_phrase( DetTree, NounTree)
```

Here DetTree and NounTree are the parse trees of a determiner and a noun respectively. Adding these parse trees as arguments into our noun phrase grammar rule results in the modified rule:

```
noun_phrase( noun_phrase( DetTree, NounTree)) -->
  determiner( DetTree), noun( NounTree).
```

This rule can be read as:

A noun phrase whose parse tree is noun_phrase(DetTree, NounTree) consists of:

- a determiner whose parse tree is DetTree, and
- a noun whose parse tree is NounTree.

We can now modify our whole grammar accordingly. To ensure number agreement, we can retain the number as the first argument and add the parse tree as the second argument. Here is part of the modified grammar:

```
sentence( Number, sentence( NP, VP)) -->
  noun_phrase( Number, NP),
  verb_phrase( Number, VP).

verb_phrase( Number, verb_phrase( Verb, NP)) -->
  verb( Number, Verb),
  noun_phrase( Number1, NP).

noun_phrase( Number, noun_phrase( Det, Noun)) -->
  determiner( Det),
  noun( Number, Noun).
```



```
determiner( determiner( the)) --> [ the].

noun( singular, noun( cat)) --> [ cat].

noun( plural, noun( cats)) --> [ cats].
```

When this grammar is read by Prolog it is automatically translated into a standard Prolog program. The first grammar rule above is translated into the clause:

```
sentence( Number, sentence( NP, VP), List, Rest) :-
  noun_phrase( Number, NP, List, Rest0),
  verb_phrase( Number, VP, Rest0, Rest).
```

Accordingly, a question to Prolog to parse a sentence has to be stated in this format; for example:

```
?- sentence( Number, ParseTree, [ the, mice, hate, the, cat], [ ]).

Number = plural
ParseTree = sentence( noun_phrase( determiner( the), noun( mice)),
            verb_phrase( verb( hate), noun_phrase( determiner( the),
            noun( cat))))
```

21.2.2 From the parse tree to the meaning

Prolog grammars are particularly well suited for the treatment of the meaning of a language, in particular in natural languages. Arguments that are attached to non-terminal symbols of a grammar can be used to handle the meaning of sentences. One approach to extract the meaning involves two stages:

(1) Generate the parse tree of the given sentence.

(2) Process the parse tree to compute the meaning.

Of course this is only practical if the syntactic structure, represented by the parse tree, also reflects the semantic structure; that is, both the syntactic and semantic decomposition have similar structures. In such a case, the meaning of a sentence can be composed from the meanings of the syntactic phrases into which the sentence has been parsed.

A simple example will illustrate this two-stage method. For simplicity we will consider robot moves again. A grammar about robot moves that generates the parse tree is:

```
move( move( Step)) --> step( Step).
move( move( Step, Move)) --> step( Step), move( Move).

step( step( up)) --> [ up].
step( step( down)) --> [ down].
```

Let us define the meaning of a move as the distance between the robot's position before the move and after it. Let each step be 1 mm in either the positive or negative

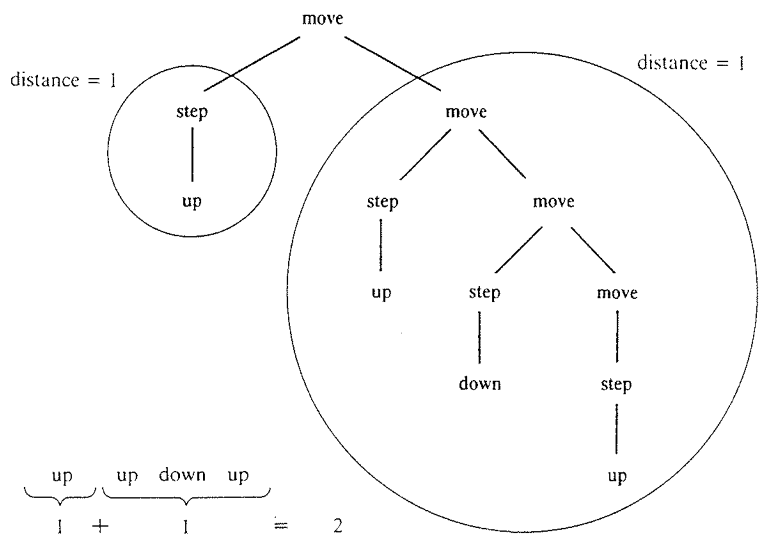


Figure 21.3 Extracting the meaning of a move as its distance.

direction. So the meaning of the move *'up up down up'* is 1 + 1 − 1 + 1 = 2. The distance of a move can be computed from the move's parse tree as illustrated in Figure 21.3. The corresponding arithmetic is:

distance(*'up up down up'*) =
distance(*'up'*) + distance(*'up down up'*) = 1 + 1 = 2

The following procedure computes the meaning of a move (as the corresponding distance) from the move's parse tree:

```
% meaning( ParseTree, Value)
meaning( move( Step, Move), Dist) :-
  meaning( Step, D1),
  meaning( Move, D2),
  Dist is D1 + D2.

meaning( move( Step), Dist) :-
  meaning( Step, Dist).

meaning( step( up), 1).

meaning( step( down), -1).
```

This can be used to compute the meaning of the move *'up up down up'* by:

```
?- move( Tree, [ up, up, down, up], [ ]), meaning( Tree, Dist).

Dist = 2
Tree = move( step( up), move( step( up), move( step( down), move( step( up)))) )
```


Exercise

21.4 Is it possible to use this grammar and the **meaning** procedure for the inverse task; that is, to find a move with the given distance – for example:

```
?- move( Tree, Move, [ ]), meaning( Tree, 5).
```

Discuss this application.

21.2.3 Interleaving syntax and semantics in DCG

The DCG notation actually makes it possible to incorporate the meaning directly into a grammar, thereby avoiding the intermediate construction of the parse tree. There is another notational extension, provided by DCG, that is useful in this respect. This extension allows us to insert normal Prolog goals into grammar rules. Such goals have to be enclosed in curly brackets to make them distinguishable from other symbols of the grammar. Thus everything enclosed in curly brackets is, when encountered, executed as normal Prolog goals. This execution may, for example, involve arithmetic computation.

This feature can be used to rewrite our robot move grammar so that the meaning extraction is interleaved directly with the syntax. To do that, we have to add the meaning; that is, the move's distance as an argument of the non-terminals **move** and **step**. For example

```
move( Dist)
```

now denotes a **move** phrase whose meaning is **Dist**. A corresponding grammar is:

```
move( D) --> step( D).
move( D) --> step( D1), move( D2), {D is D1 + D2}.

step( 1) --> [ up].
step( -1) --> [ down].
```

The second rule here exemplifies the curly bracket notation. This rule can be read as follows:

A move whose meaning is D consists of:

- a step whose meaning is D1, and
- a move whose meaning is D2, where the relation D is D1 + D2 must also be satisfied.

In fact, handling semantics by incorporating the meaning formation rules directly into a DCG grammar is so convenient that the intermediate stage of constructing the parse tree is often avoided altogether. Avoiding such an intermediate stage results in a 'collapsed' program. The usual advantages of this are a shorter and more efficient program, but there are also disadvantages: the collapsed program may be less transparent, less flexible and harder to modify.

As a further illustration of this technique of interleaving the syntax and meaning, let us make our robot example a little more interesting. Suppose that the robot can be in one of two gears: *g1* or *g2*. When a step command is received in gear *g1*, the robot will move by 1 mm up or down; in gear *g2* it will move by 2 mm. Let the whole program for the robot consist of the gear commands (to switch to gear *g1* or *g2*) and step commands, finally ending with *stop*. Example programs are:

stop
g1 up up stop
g1 up up g2 down up stop
g1 g1 g2 up up g1 up down up g2 stop

The meaning (that is, the distance) of the last program is:

Dist = 2 * (1 + 1) + 1 * (1 − 1 + 1) = 5

To handle such robot programs, our existing robot move grammar has to be extended with the following rules:

```
prog( 0) --> [ stop].
prog( Dist) --> gear( _ ), prog( Dist).
prog( Dist) --> gear( G), move( D), prog( Dist1), {Dist is G * D + Dist1}.

gear( 1) --> [ g1].
gear( 2) --> [ g2].
```

21.3 Defining the meaning of natural language

21.3.1 Meaning of simple sentences in logic

Defining the meaning of natural language is an extremely difficult problem that is the subject of on-going research. An ultimate solution to the problem of formalizing the complete syntax and meaning of a language like English is far away. But (relatively) simple subsets of natural languages have been successfully formalized and consequently implemented as working programs.

In defining the meaning of a language, the first question is: How will the meaning be represented? There are of course many alternatives, and good choice will depend on the particular application. The important question therefore is: What will the meaning extracted from natural language text be used for? A typical application is natural language access to a database. This involves answering natural language questions regarding information in the database and updating the database by new information extracted from natural language input. In such a case, the target representation of the meaning extraction process would be a language for querying and updating the database.

Logic has been accepted as a good candidate for representing the meaning of natural language sentences. In comparison with database formalisms, logic is in general more powerful and, while essentially subsuming database formalisms, also allows more subtle semantic issues to be dealt with. In this section we will show how interpretations of simple natural language sentences in logic can be constructed using the DCG notation. The logical interpretations will be encoded as Prolog terms. We will only look at some interesting ideas, so many details necessary for a more general coverage will be omitted. A more complete treatment would be far beyond the scope of this book.

To start with, it is best to look at some natural language sentences and phrases and try to express in logic what they mean. Let us consider first the sentence 'John paints'. The natural way to express the meaning of this sentence in logic, as a Prolog term, is:

```
paints( john)
```

Notice that 'paints' here is an intransitive verb and therefore the corresponding predicate **paints** only has one argument.

Our next example sentence is 'John likes Annie'. The formalized meaning of this can be:

```
likes( john, annie)
```

The verb 'likes' is transitive and accordingly the predicate **likes** has two arguments.

Let us now try to define, by DCG rules, the meaning of such simple sentences. We will start with the bare syntax and then gradually incorporate the meaning into these rules. Here is a grammar that comfortably covers the syntax of our example sentences:

```
sentence --> noun_phrase, verb_phrase.

noun_phrase --> proper_noun.

verb_phrase --> intrans_verb.             % Intransitive verb
verb_phrase --> trans_verb, noun_phrase.  % Transitive verb

intrans_verb --> [ paints].
trans_verb --> [ likes].

proper_noun --> [ john].
proper_noun --> [ annie].
```

Now let us incorporate meaning into these rules. We will start with the simpler categories – noun and verb – and then proceed to the more complicated ones. Our foregoing examples suggest the following definitions. The meaning of proper noun **john** is simply **john**:

```
proper_noun( john) --> [ john].
```

The meaning of an intransitive verb like 'paints' is slightly more complicated. It can be stated as

```
paints( X)
```

where X is a variable whose value only becomes known from the context; that is, from the noun phrase. Correspondingly, the DCG rule for **paints** is:

```
intrans_verb( paints( X)) --> [ paints].
```

Let us now look at the question: How can we construct from the two meanings, **john** and **paints(X)**, the intended meaning of the whole sentence: **paints(john)**? We have to force the argument X of **paints** to become equal to **john**.

It may be helpful at this point to consider Figure 21.4. This shows how the meanings of phrases accumulate into the meaning of the whole sentence. To achieve the effects of the propagation of the meanings of phrases, we can first simply define that **noun_phrase** and **verb_phrase** receive their meanings from **proper_noun** and **intrans_verb** respectively:

```
noun_phrase( NP) --> proper_noun( NP).
verb_phrase( VP) --> intrans_verb( VP).
```

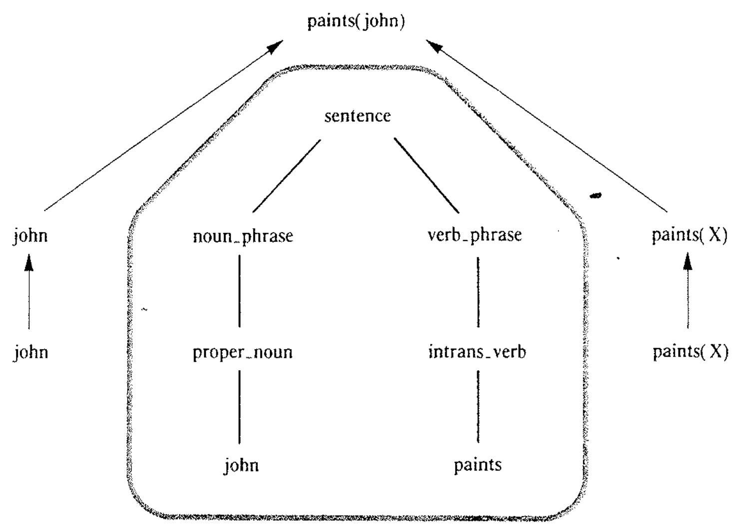


Figure 21.4 The parse tree of the sentence 'John paints' with meaning attached to the nodes. The logical meaning of each phrase is attached to the corresponding non-terminal in the tree. The arrows indicate how the meanings of phrases accumulate.

It remains to define the meaning of the whole sentence. Here is a first attempt:

```
sentence( S) --> noun_phrase( NP), verb_phrase( VP), {compose( NP, VP, S)}.
```

The goal **compose(NP, VP, S)** has to assemble the meanings of the noun phrase **john** and the verb phrase **paints(X)**. Let us say that X is the *actor* in **paints(X)**. Let us define the relation

```
actor( VP, Actor)
```

so that **Actor** is the **actor** in the meaning **VP** of a verb phrase. One clause of the procedure **actor** then is:

```
actor( paints( X), X).
```

Once the **actor** relation is available, the composition of the noun phrase's and verb phrase's meanings can be defined as:

```
compose( NP, VP, VP) :-          % Meaning of sentence is VP where
  actor( VP, NP).                % actor in VP is NP
```

This works, but there is a shorter method. We can avoid the extra predicates **compose** and **actor** if we make the argument X in term **paints(X)** 'visible' from the outside of the term, so that it becomes accessible for instantiation. This can be achieved by redefining the meaning of **verb_phrase** and **intrans_verb** so that X becomes an extra argument:

```
intrans_verb( Actor, paints( Actor)) --> [ paints].
verb_phrase( Actor, VP) --> intrans_verb( Actor, VP).
```

This makes the argument **Actor** easily accessible and facilitates a simpler definition of the meaning of a sentence:

```
sentence( VP) --> noun_phrase( Actor), verb_phrase( Actor, VP).
```

This forces the 'actor' argument of the verb's meaning to become equal to the meaning of the noun phrase.

This technique of making parts of meanings visible is a rather common trick in incorporating meaning into DCG rules. The technique essentially works as follows. The meaning of a phrase is defined in a 'skeleton' form – for example, **paints(SomeActor)**. This defines the general form of the meaning, but leaves some of it uninstantiated (here, variable **SomeActor**). Such an uninstantiated variable serves as a slot that can be filled later depending on the meaning of other phrases in the context. This filling of slots can be accomplished by Prolog matching. To facilitate this, however, slots are made visible by being added as extra arguments to non-terminals. We will adopt the following convention regarding the order of these arguments: first will come the 'visible slots' of the phrase's meaning, followed by the meaning itself – for example, **verb_phrase(Actor, VPMeaning)**.

The meaning of an intransitive verb like 'paints' is slightly more complicated. It can be stated as

```
paints( X)
```

where X is a variable whose value only becomes known from the context; that is, from the noun phrase. Correspondingly, the DCG rule for **paints** is:

```
intrans_verb( paints( X)) --> [ paints].
```

Let us now look at the question: How can we construct from the two meanings, **john** and **paints(X)**, the intended meaning of the whole sentence: **paints(john)**? We have to force the argument X of **paints** to become equal to **john**.

It may be helpful at this point to consider Figure 21.4. This shows how the meanings of phrases accumulate into the meaning of the whole sentence. To achieve the effects of the propagation of the meanings of phrases, we can first simply define that **noun_phrase** and **verb_phrase** receive their meanings from **proper_noun** and **intrans_verb** respectively:

```
noun_phrase( NP) --> proper_noun( NP).
verb_phrase( VP) --> intrans_verb( VP).
```

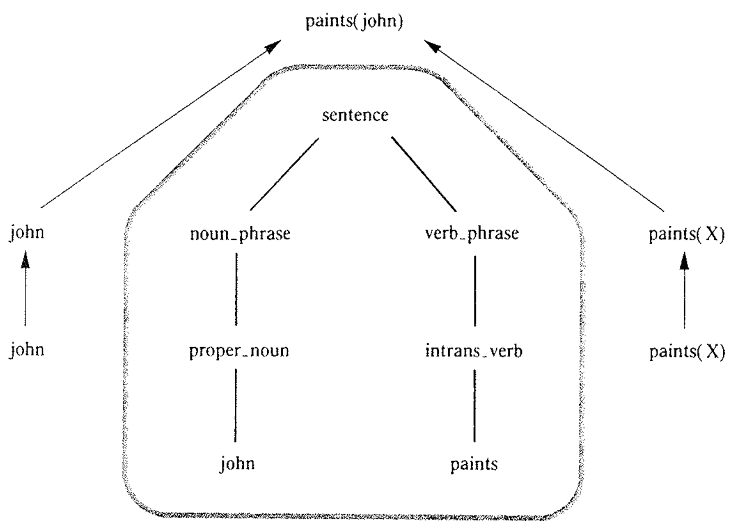


Figure 21.4 The parse tree of the sentence 'John paints' with meaning attached to the nodes. The logical meaning of each phrase is attached to the corresponding non-terminal in the tree. The arrows indicate how the meanings of phrases accumulate.

It remains to define the meaning of the whole sentence. Here is a first attempt:

```
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```

The goal **compose(NP, VP, S)** has to assemble the meanings of the noun phrase **john** and the verb phrase **paints(X)**. Let us say that X is the *actor* in **paints(X)**. Let us define the relation

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actor( VP, Actor)
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so that **Actor** is the **actor** in the meaning **VP** of a verb phrase. One clause of the procedure **actor** then is:

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actor( paints( X), X).
```

Once the **actor** relation is available, the composition of the noun phrase's and verb phrase's meanings can be defined as:

```
compose( NP, VP, VP) :-          % Meaning of sentence is VP where
  actor( VP, NP).                % actor in VP is NP
```

This works, but there is a shorter method. We can avoid the extra predicates **compose** and **actor** if we make the argument X in term **paints(X)** 'visible' from the outside of the term, so that it becomes accessible for instantiation. This can be achieved by redefining the meaning of **verb_phrase** and **intrans_verb** so that X becomes an extra argument:

```
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```

This makes the argument **Actor** easily accessible and facilitates a simpler definition of the meaning of a sentence:

```
sentence( VP) --> noun_phrase( Actor), verb_phrase( Actor, VP).
```

This forces the 'actor' argument of the verb's meaning to become equal to the meaning of the noun phrase.

This technique of making parts of meanings visible is a rather common trick in incorporating meaning into DCG rules. The technique essentially works as follows. The meaning of a phrase is defined in a 'skeleton' form – for example, **paints(SomeActor)**. This defines the general form of the meaning, but leaves some of it uninstantiated (here, variable **SomeActor**). Such an uninstantiated variable serves as a slot that can be filled later depending on the meaning of other phrases in the context. This filling of slots can be accomplished by Prolog matching. To facilitate this, however, slots are made visible by being added as extra arguments to non-terminals. We will adopt the following convention regarding the order of these arguments: first will come the 'visible slots' of the phrase's meaning, followed by the meaning itself – for example, **verb_phrase(Actor, VPMeaning)**.

This technique can be applied to transitive verbs as follows. The meaning of the verb 'likes' is **likes(Somebody, Something)** (somebody likes something) where **Somebody** and **Something** are slots that should be visible from outside. Thus:

```
trans_verb( Somebody, Something, likes( Somebody, Something)) --> [ likes].
```

A verb phrase with a transitive verb contains a noun phrase that provides the value for **Something**. Therefore:

```
verb_phrase( Somebody, VP) -->
  trans_verb( Somebody, Something, VP), noun_phrase( Something).
```

The foregoing discussion has introduced some basic ideas; the DCG rules given handle but the simplest sentences. When noun phrases contain determiners like 'a' and 'every', the meaning expressions become more complicated. We will look at this in the next section.

21.3.2 Meaning of determiners 'a' and 'every'

Sentences that contain determiners such as 'a' are much more difficult to handle than those in the previous section. Let us consider an example sentence: 'A man paints'. It would now be a gross mistake to think that the meaning of this sentence is **paints(man)**. The sentence really says: There exists some man that paints. In logic this is phrased as:

There exists an X such that
  X is a man and X paints.

In logic, the variable X here is said to be *existentially* quantified ('there exists'). We will choose to represent this by the Prolog term:

```
exists( X, man( X) and paints( X))
```

The first argument in this term is a variable, X – the variable that is meant to be existentially quantified. **and** is assumed to be declared as an infix operator:

```
:- op( 100, xfy, and).
```

The syntactic entity that dictates this logic interpretation is, possibly surprisingly, the determiner 'a'. So 'a' in a way dominates the whole sentence. To better understand how the meaning is constructed, let us look at the noun phrase 'a man'. Its meaning is:

There exists some X such that
  X is a man.

However, in sentences where the phrase 'a man' appears, such as 'a man paints', we always want to say something else about this man (not only that he exists, but also that he paints). So a suitable form for the meaning of the noun phrase 'a man' is:

```
exists( X, man( X) and Assertion)
```

where **Assertion** is some statement about X. This statement about X depends on the context; that is, on the verb phrase that follows the noun phrase 'a man'. The variable **Assertion** will only be instantiated when the context in which it appears becomes known.

A similar line of thought leads us to find a proper formulation of the meaning of the determiner 'a'. This determiner indicates that:

There exists some X such that
  X has some property (for example, **man(X)**) and
  some further assertion about X holds (for example, **paints(X)**).

As a Prolog term this can be represented as:

```
exists( X, Property and Assertion)
```

Both variables **Property** and **Assertion** are slots for meanings from the context to be plugged in. To facilitate the importation of the meanings from other phrases in context, we can, as explained in the previous section, make parts of the meaning of determiner 'a' visible. A suitable DCG rule for determiner 'a' is:

```
determiner( X, Prop, Assn, exists( X, Prop and Assn)) --> [ a].
```

The logical meaning of the tiny determiner 'a' may seem surprisingly complicated.

Another determiner, 'every', can be handled in a similar way. Consider the sentence: 'Every woman dances'. The logic interpretation of this is:

For all X,
  if X is a woman then X dances.

We will represent this by the following Prolog term:

```
all( X, woman( X) => dances( X))
```

where '=>' is an infix operator denoting logical implication. Determiner 'every' thus indicates a meaning whose skeleton structure is:

```
all( X, Property => Assertion)
```

A DCG rule that defines the meaning of determiner 'every' and makes the slots in the skeleton visible is:

```
determiner( X, Prop, Assn, all( X, Prop => Assn)) --> [ every].
```

Having defined the meaning of determiners, we shall now concentrate on how their meaning integrates with the meanings of other phrases in the context, leading finally to the meaning of the whole sentence. We can get a first idea by looking again at the sentence 'A man paints' whose meaning is:

```
exists( X, man( X) and paints( X))
```

We have already defined the meaning of 'a' as:

```
exists( X, Prop and Assn)
```

Comparing these two meanings it is immediately obvious that the main structure of the meaning of the sentence is dictated by the determiner. The meaning of the sentence can be assembled as illustrated by Figure 21.5: start with the skeleton meaning enforced by 'a':

```
exists( X, Prop and Assn)
```

Prop then becomes instantiated by the noun and Assn by the verb phrase. The main structure of the meaning of the sentence is received from the noun phrase. Notice that this is different from the simpler grammar in the previous section where it was the verb phrase that provided the sentence's meaning structure. Again, applying the technique of making some parts of the meaning visible, the relations between meanings indicated in Figure 21.5 can be stated by the following DCG rules:

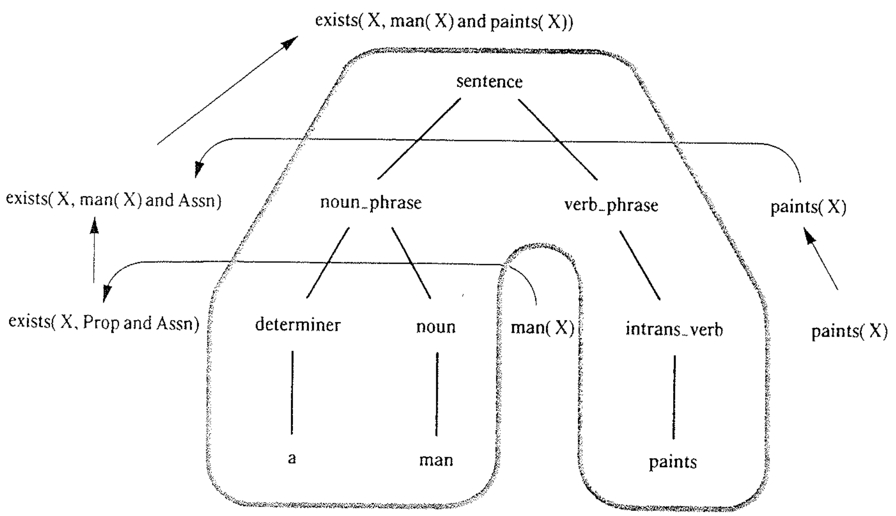

Figure 21.5 Accumulation of meaning for the sentence 'A man paints'. Determiner 'a' dictates the overall form of the meaning of the sentence. Arrows indicate the importation of meaning between phrases.


```
sentence( S) --> noun_phrase( X, Assn, S), verb_phrase( X, Assn).
noun_phrase( X, Assn, S) --> determiner( X, Prop, Assn, S), noun( X, Prop).
verb_phrase( X, Assn) --> intrans_verb( X, Assn).
intrans_verb( X, paints( X)) --> [ paints].
determiner( X, Prop, Assn, exists( X, Prop and Assn)) --> [ a].
noun( X, man( X)) --> [ man].
```

This grammar can be asked to construct the meaning of 'A man paints':

```
?- sentence( S, [ a, man, paints], [ ]).
S = exists( X, man( X) and paints( X))
```

Prolog's answer was polished by replacing a Prolog-generated variable name like _123 with X.

The grammar of the previous section handles sentences like 'John paints'. Now that we have modified our grammar we need to ensure that this new grammar that can handle 'A man paints' can also handle the simpler 'John paints'. To do this, the meaning of proper nouns has to be incorporated into the new noun phrase. The following rules accomplish this:

```
proper_noun( john) --> [ john].
noun_phrase( X, Assn, Assn) --> proper_noun( X).
```

This last rule simply says that the whole meaning of this kind of noun phrase is the same as the value in the second 'visible slot' – that is, Assn. This slot's value is obtained from the context (from the verb phrase) as in the question:

```
?- sentence( S, [ john, paints], [ ]).
S = paints( john)
```

Exercise

21.5 Study how our modified grammar constructs the meaning of 'John paints'. Essentially the following should happen: the meaning received from the verb phrase becomes the noun phrase's meaning, which eventually becomes the sentence's meaning.

21.3.3 Handling relative clauses

Nouns can be qualified by relative clauses – for example, 'Every man that paints admires Monet'. The phrase 'every man that paints' is a noun phrase in which 'that paints' is a relative clause. To cover this syntactically, our grammar rules about noun phrase can be redefined as follows:

```
noun_phrase --> determiner, noun, rel_clause.

rel_clause --> [ that], verb_phrase.
rel_clause --> [ ].                 % Empty relative clause
```

Let us now consider the meaning of such noun phrases. The sentence 'Every man that paints admires Monet' logically means:

```
For all X,
  if X is a man and X paints
  then X admires Monet.
```

This can be represented as the Prolog term:

```
all( X, man( X) and paints( X) => admires( X, monet))
```

where it is assumed that the operator 'and' binds stronger than '=>'. Thus a suitable form for the logical meaning of the noun phrase 'every man that paints' is:

```
all( X, man( X) and paints( X) => Assn)
```

In general, this form is:

```
all( X, Prop1 and Prop2 => Assn)
```

Prop1 is determined by the noun, Prop2 by the verb phrase of the relative clause and Assn by the verb phrase of the sentence. DCG rules for the noun phrase that ensure this are:

```
rel_clause( X, Prop1, Prop1 and Prop2) --> [ that], verb_phrase( X, Prop2).

noun_phrase( X, Assn, S) -->
  determiner( X, Prop12, Assn, S), noun( X, Prop1),
  rel_clause( X, Prop1, Prop12).
```

To cover the case of the empty relative clause we have to add:

```
rel_clause( X, Prop1, Prop1) --> [ ].
```

Figure 21.6 gives a complete DCG with the features developed in this section, including determiners 'a' and 'every', and relative clauses. This grammar is capable of extracting the logical meaning of sentences, such as:

John paints.
Every man that paints admires Monet.
Annie admires every man that paints.
Every woman that admires a man that paints likes Monet.


```
:- op( 100, xfy, and).
:- op( 150, xfy, =>).

sentence( S) -->
  noun_phrase( X, Assn, S), verb_phrase( X, Assn).

noun_phrase( X, Assn, S) -->
  determiner( X, Prop12, Assn, S), noun( X, Prop1), rel_clause( X, Prop1, Prop12).

noun_phrase( X, Assn, Assn) -->
  proper_noun( X).

verb_phrase( X, Assn) -->
  trans_verb( X, Y, Assn1), noun_phrase( Y, Assn1, Assn).

verb_phrase( X, Assn) -->
  intrans_verb( X, Assn).

rel_clause( X, Prop1, Prop1 and Prop2) -->
  [ that], verb_phrase( X, Prop2).

rel_clause( X, Prop1, Prop1) --> [ ].

determiner( X, Prop, Assn, all( X, Prop => Assn)) --> [ every].
determiner( X, Prop, Assn, exists( X, Prop and Assn)) --> [ a].

noun( X, man( X)) --> [ man].
noun( X, woman( X)) --> [ woman].

proper_noun( john) --> [ john].
proper_noun( annie) --> [ annie].
proper_noun( monet) --> [ monet].

trans_verb( X, Y, likes( X, Y)) --> [ likes].
trans_verb( X, Y, admires( X, Y)) --> [ admires].

intrans_verb( X, paints( X)) --> [ paints].

% Some tests

test1( Meaning) :-
  sentence( Meaning, [ john, paints], [ ]).

test2( Meaning) :-
  sentence( Meaning, [ a, man, paints], [ ]).

test3( Meaning) :-
  sentence( Meaning, [ every, man, that, paints, admires, monet], [ ]).

test4( Meaning) :-
  sentence( Meaning, [ annie, admires, every, man, that, paints], [ ]).

test5( Meaning) :-
  sentence( Meaning, [ every, woman, that, admires, a, man, that, paints, likes, monet], [ ]).
```

Figure 21.6 A DCG handling the syntax and meaning of a small subset of natural language.

For example, these sentences can be submitted to our grammar as the questions:

```
?- sentence( Meaning1, [ every, man, that, paints, admires, monet], [ ]).
Meaning1 = all( X, man( X) and paints( X) => admires( X, monet))

?- sentence( Meaning2, [ annie, admires, every, man, that, paints], [ ]).
Meaning2 = all( X, man( X) and paints( X) => admires( annie, X))

?- sentence( Meaning3, [ every, woman, that, admires, a, man, that, paints, likes,
                        monet], [ ]).
Meaning3 = all( X, woman( X) and exists( Y, ( man( Y) and paints( Y))
                and admires( X, Y)) => likes( X, monet))
```

A further interesting problem concerns the use of the meanings extracted from natural language input to answer questions. For example, how can we modify our program so that, after it has processed the given sentences, it can answer questions like: 'Does Annie admire anybody who admires Monet?' The answer to this logically follows from the sentences above, and we just have to make our program do some necessary reasoning. In general, we need a theorem prover to deduce answers from the meanings represented in logic. Of course, it is most practical to simply use Prolog itself as such a theorem prover. To do that we would have to translate the logical meanings into equivalent Prolog clauses. This exercise in general requires some work, but in some cases such a translation is trivial. Here are some easily translatable meanings written as Prolog clauses:

```
paints( john).

admires( X, monet) :-
  man( X),
  paints( X).

admires( annie, X) :-
  man( X),
  paints( X).
```

The example query 'Does Annie admire anybody who admires Monet?' would have to be translated into the Prolog query:

```
?- admires( annie, X), admires( X, monet).
X = john
```

Exercises

21.6 State in logic the meaning of the sentences:

(a) Mary knows all important artists.

(b) Every teacher who teaches French and studies music understands Chopin.

(c) A charming lady from Florida runs a beauty shop in Sydney.

21.7 The grammar of Figure 21.6 can also be executed in the opposite direction: given a meaning, generate a sentence with this meaning. For example, we may try the opposite of test5 in Figure 21.6 as follows:

```
?- M = all(X,woman(X) and exists(Y,(man(Y) and paints(Y)) and admires(X,Y))
     => likes(X,monet)), sentence( M, S, [ ]).
```

The first Prolog's answer is:

```
S = [every,woman,that,admires,a,man,that,paints,likes,monet]
M = all(_022C,woman(_022C) and exists(_02FC,(man(_02FC) and paints(_02FC))
and admires(_022C,_02FC)) => likes(_022C,monet))
```

This is as expected. However, if we want another solution then we get a surprise:

```
M = all(monet,woman(monet) and exists(_0364,(man(_0364) and paints(_0364))
and admires(monet,_0364)) => likes(monet,monet))
S = [monet,likes,every,woman,that,admires,a,man,that,paints]
```

Explain how this was obtained, and suggest a modification of the grammar to prevent this. Hint: The problem is that the grammar allows a quantified variable (e.g. X in all(X,...)) to match a proper noun (monet); this can easily be prevented.

21.8 Extend the grammar of Figure 21.6 to handle composite sentences with connectives 'if', 'then', 'and', 'or', 'neither', 'nor', etc. For example: John paints and Annie sings. If Annie sings then every teacher listens.

Project

Modify the grammar of Figure 21.6 to represent the meaning of sentences as directly executable Prolog clauses. Write a program that reads natural language sentences in normal text format (not as lists; see a relevant program in Chapter 6) and asserts their meaning as Prolog clauses. Extend the grammar to handle simple questions in natural language which would result in a complete conversation system for a small subset of natural language. You may also consider use of the grammar to generate sentences with the given meaning as natural language answers to the user.

Summary

- Standard grammar notations, such as BNF, can be trivially translated into the DCG notation (definite clause grammars). A grammar in DCG can be read and executed directly by Prolog as a recognizer for the language defined by the grammar.

- The DCG notation allows non-terminal symbols of the grammar to have arguments. This enables the treatment of context dependencies in a grammar and direct incorporation of the semantics of a language into its grammar.
- Interesting DCG grammars have been written that cover the syntax and meaning of non-trivial subsets of natural language.

References

The DCG definitions of the syntax and meaning of natural language in this chapter follow the classical paper by Pereira and Warren (1980). In their excellent book, Pereira and Shieber (1987) give many further developments of this, including an elegant connection between the corresponding mathematical basis for defining meaning and its implementation in Prolog. Markus (1986) gives a DCG grammar to interpret natural language queries to databases. Szpakowicz (1987) gives an example of applying DCG in programming language translation. Exercise 21.8 is borrowed from Kononenko and Lavrač (1988), who also give a solution. The books by Allen (1995), Covington (1994) and Gazdar and Mellish (1989) cover a variety of topics in natural language processing.

Allen, J.F. (1995) *Natural Language Understanding*. Redwood City, CA: Benjamin/Cummings.

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Kononenko, I. and Lavrač, N. (1988) *Prolog through Examples: A Practical Programming Guide*. Wilmslow, UK: Sigma Press.

Markus, C. (1986) *Prolog Programming*. Addison-Wesley.

Pereira, F.C.N. and Shieber, S.M. (1987) *Prolog and Natural-Language Analysis*. Menlo Park, CA: CSLI – Center for the Study of Language and Information.

Pereira, F.C.N. and Warren, D.H.D. (1980) Definite clause grammars for language analysis – a survey of the formalism and comparison with augmented transition networks. *Artificial Intelligence* 13: 231–278.

Szpakowicz, S. (1987) Logic grammars. *BYTE* (August 1987): 185–195.

chapter 22

Game Playing

In this chapter we will consider techniques for playing two-person, perfect-information games, such as chess. For interesting games, trees of possible continuations are far too complex to be searched exhaustively, so other approaches are necessary. One method is based on the minimax principle, efficiently implemented as the alpha-beta algorithm. In addition to this standard technique, we will develop in this chapter a program based on the Advice Language approach for introducing pattern knowledge into a chess-playing program. This rather detailed example further illustrates how well Prolog is suited for the implementation of knowledge-based systems.

22.1 Two-person, perfect-information games

The kind of games that we are going to discuss in this chapter are called two-person, perfect-information games. Examples of games of this kind are chess, checkers and go. In such games there are two players that make moves alternatively, and both players have the complete information of the current situation in the game. Thus this definition excludes most card games. The game is over when a position is reached that qualifies as 'terminal' by the rules of the game – for example, mate in chess. The rules also determine what is the outcome of the game that has ended in this terminal position.

Such a game can be represented by a *game tree*. The nodes in such a tree correspond to situations, and the arcs correspond to moves. The initial situation of the game is the root node; the leaves of the tree correspond to terminal positions.

In most games of this type the outcome of the game can be *win*, *loss* or *draw*. We will now consider games with just two outcomes: *win* and *loss*. Games where a draw is a possible outcome can be reduced to two outcomes: *win*, *not-win*. The two players will be called 'us' and 'them'. 'Us' can win in a non-terminal 'us-to-move' position if there is a legal move that leads to a won position. On the other hand, a non-terminal 'them-to-move' position is won for 'us' if *all* the legal moves from this position lead to won positions. These rules correspond to AND/OR tree representation of problems discussed in Chapter 13. The concepts from AND/OR trees and games correspond as follows:

game positions	problems
terminal won position	goal node, trivially solved problem
terminal lost position	unsolvable problem
won position	solved problem
us-to-move position	OR node
them-to-move position	AND node

Clearly, many concepts from searching AND/OR trees can be adapted for searching game trees.

A simple program that finds whether an us-to-move position is won can be defined as follows:

```
won( Pos) :-
  terminalwon( Pos).                % A terminal won position

won( Pos) :-
  not terminallost( Pos),
  move( Pos, Pos1),                 % A legal move to Pos1
  not ( move( Pos1, Pos2),          % No opponent's move leads to a
        not won( Pos2) ).           % not-won position
```

The rules of the game are built into the predicates **move(Pos, Pos1)** to generate legal moves, and **terminalwon(Pos)** and **terminallost(Pos)** to recognize terminal positions that are won or lost by the rules of the game. The last rule above says, through the double use of **not**: there is no them-move that leads to a not-won position. In other words: *all* them-moves lead to a won position.

As with analogous programs for searching AND/OR graphs, the above program uses the depth-first strategy. In addition, this program does not prevent cycling between positions. This may cause problems as the rules of some games allow repetition of positions. However, this repetition is often only superficial. By rules of chess, for example, after a three-fold repetition the game can be claimed a draw.

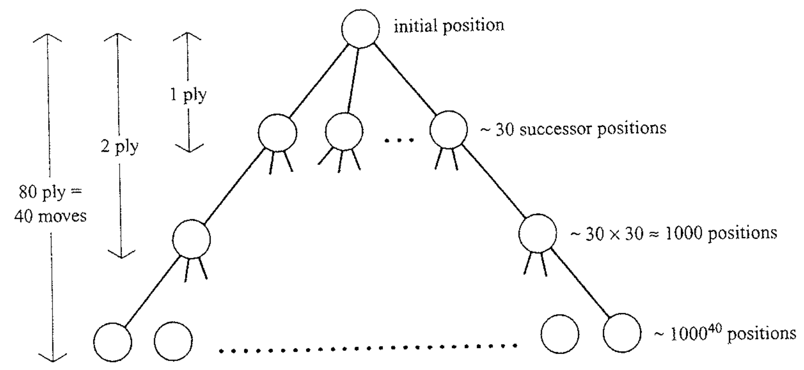

Figure 22.1 The complexity of game trees in chess. The estimates here are based on an approximation that there are about 30 legal moves from each chess position, and that terminal positions occur at a depth of 40 moves. One move is 2 plies (1 half-move by each side).

The foregoing program shows the basic principle. However, much more powerful techniques are necessary for practically dealing with complicated games like chess or go. The combinatorial complexity of these games makes our naive search algorithm, which only stops at terminal positions of the game, completely infeasible. Figure 22.1 illustrates this point with respect to chess. The search space of astronomical proportions includes some 10^{120} positions. It can be argued that equal positions in the tree of Figure 22.1 occur at different places. Still, it has been shown that the number of different positions is far beyond anything manageable by foreseeable computers.

Project

Write a program to play some simple game (like *nim*) using the straightforward AND/OR search approach.

22.2 The minimax principle

As searching game trees exhaustively is not feasible for interesting games, other methods that rely on searching only part of the game tree have been developed. Among these, a standard technique used in computer game playing (chess) is based on the *minimax* principle. A game tree is only searched up to a certain depth, typically a few moves, and then the tip nodes of the search tree are evaluated by some evaluation function. The idea is to assess these terminal search positions without searching beyond them, thus saving time. These terminal position estimates then propagate up the search tree according to the minimax principle. This yields position values for all the positions in the search tree. The move that leads from the initial, root position to its most promising successor (according to these values) is then actually played in the game.

Notice that we distinguish between a 'game tree' and a 'search tree'. A search tree is normally a part of the game tree (upper part) – that is, the part that is explicitly generated by the search process. Thus, terminal search positions do not have to be terminal positions of the game.

Much depends on the evaluation function, which, in most games of interest, has to be a heuristic estimator that estimates the winning chances from the point of view of one of the players. The higher the value the higher the player's chances are to win, and the lower the value the higher the opponent's chances are to win. As one of the players will tend to achieve a high position value, and the other a low value, the two players will be called MAX and MIN respectively. Whenever MAX is to move, he or she will choose a move that maximizes the value; on the contrary, MIN will choose a move that minimizes the value. Given the values of the bottom-level positions in a search tree, this principle (called *minimax*) will determine the values of all the other positions in the search tree. Figure 22.2 illustrates. In the figure, levels of positions with MAX to move alternate with those with MIN to move. The bottom-level position values are determined by the evaluation function. The values of the internal nodes can be computed in a bottom-up fashion, level by level, until the root

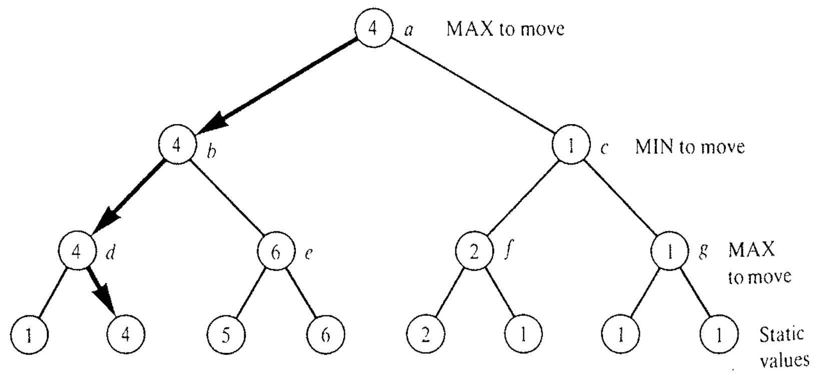

Figure 22.2 Static values (bottom level) and minimax backed-up values in a search tree. The indicated moves constitute the *main variation* – that is, the minimax optimal play for both sides.

node is reached. The resulting root value in Figure 22.2 is 4, and accordingly the best move for MAX in position *a* is *a-b*. The best MIN's reply is *b-d*, etc. This sequence of play is also called the *main variation*. The main variation defines the 'minimax-optimal' play for both sides. Notice that the value of the positions along the main variation does not vary. Accordingly, correct moves are those that preserve the value of the game.

We distinguish between the bottom-level values and the backed-up values. The former values are called 'static' since they are obtained by a 'static' evaluation function, as opposed to backed-up values that are obtained 'dynamically' by propagation of static values up the tree.

The value propagation rules can be formalized as follows. Let us denote the static value of a position P by:

$v(P)$

and the backed-up value by:

$V(P)$

Let $P_1, \ldots, P_n$ be legal successor positions of P. Then the relation between static values and backed-up values can be defined as:

$V(P) = v(P)$ if P is a terminal position in a search tree ($n = 0$)

$V(P) = \max_i V(P_i)$ if P is a MAX-to-move position

$V(P) = \min_i V(P_i)$ if P is a MIN-to-move position

A Prolog program that computes the minimax backed-up value for a given position is shown in Figure 22.3. The main relation in this program is

```
minimax( Pos, BestSucc, Val)
```

where **Val** is the minimax value of a position **Pos**, and **BestSucc** is the best successor position of **Pos** (the move to be played to achieve **Val**). The relation

```
moves( Pos, PosList)
```

corresponds to the legal-move rules of the game: **PosList** is the list of legal successor positions of **Pos**. The predicate **moves** is assumed to fail if **Pos** is a terminal search position (a leaf of the search tree). The relation

```
best( PosList, BestPos, BestVal)
```

selects the 'best' position **BestPos** from a list of candidate positions **PosList**. **BestVal** is the value of **BestPos**, and hence also of **Pos**. 'Best' is here either maximum or minimum, depending on the side to move.

```
% minimax( Pos, BestSucc, Val):
%   Pos is a position, Val is its minimax value;
%   best move from Pos leads to position BestSucc

minimax( Pos, BestSucc, Val) :-
  moves( Pos, PosList), !,                 % Legal moves in Pos produce PosList
  best( PosList, BestSucc, Val)
  ;
  staticval( Pos, Val).                    % Pos has no successors: evaluate statically

best( [ Pos], Pos, Val) :-
  minimax( Pos, _, Val), !.

best( [Pos1 | PosList], BestPos, BestVal) :-
  minimax( Pos1, _, Val1),
  best( PosList, Pos2, Val2),
  betterof( Pos1, Val1, Pos2, Val2, BestPos, BestVal).

betterof( Pos0, Val0, Pos1, Val1, Pos0, Val0) :-   % Pos0 better than Pos1
  min_to_move( Pos0),                             % MIN to move in Pos0
  Val0 > Val1, !                                  % MAX prefers the greater value
  ;
  max_to_move( Pos0),                             % MAX to move in Pos0
  Val0 < Val1, !.                                 % MIN prefers the lesser value

betterof( Pos0, Val0, Pos1, Val1, Pos1, Val1).     % Otherwise Pos1 better than
```

Figure 22.3 A straightforward implementation of the minimax principle.

22.3 The alpha-beta algorithm: an efficient implementation of minimax

The program in Figure 22.3 systematically visits *all* the positions in the search tree, up to its terminal positions in a depth-first fashion, and statically evaluates *all* the terminal positions of this tree. Usually not all this work is necessary in order to correctly compute the minimax value of the root position. Accordingly, the search algorithm can be economized. The improvement can be based on the following idea: Suppose that there are two alternative moves; once one of them has been shown to be clearly inferior to the other, it is not necessary to know *exactly* how much inferior it is for making the correct decision. For example, we can use this principle to reduce the search in the tree of Figure 22.2. The search process here proceeds as follows:

(1) Start with position *a*.

(2) Move down to *b*.

(3) Move down to *d*.

(4) Take the maximum of *d*'s successors yielding $V(d) = 4$.

(5) Backtrack to *b* and move down to *e*.

(6) Consider the first successor of *e* whose value is 5. At this point MAX (who is to move in *e*) is guaranteed at least the value of 5 in position *e* regardless of other (possibly better) alternatives from *e*. This is sufficient for MIN to realize that, at node *b*, the alternative *e* is inferior to *d*, even without knowing the exact value of *e*.

On these grounds we can neglect the second successor of *e* and simply assign to *e* an *approximate* value 5. This approximation will, however, have no effect on the computed value of *b* and, hence, of *a*.

The celebrated *alpha-beta algorithm* for efficient minimaxing is based on this idea. Figure 22.4 illustrates the action of the alpha-beta algorithm on our example tree of Figure 22.2. As Figure 22.4 shows, some of the backed-up values are approximate. However, these approximations are sufficient to determine the root value precisely. In the example of Figure 22.4, the alpha-beta principle reduces the search complexity from eight static evaluations (as originally in Figure 22.2) to five static evaluations.

As said before, the key idea of the alpha-beta pruning is to find a 'good enough' move, not necessarily the best, that is sufficiently good to make the correct decision. This idea can be formalized by introducing two bounds, usually denoted *Alpha* and *Beta*, on the backed-up value of a position. The meaning of these bounds is: *Alpha* is the minimal value that MAX is already guaranteed to achieve, and *Beta* is the maximal value that MAX can hope to achieve. From MIN's point of view, *Beta* is

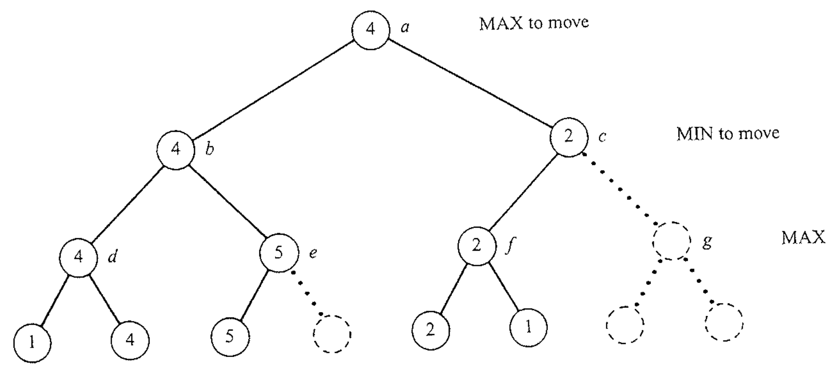

Figure 22.4 The tree of Figure 22.2 searched by the alpha-beta algorithm. The alpha-beta search prunes the nodes shown by dotted lines, thus economizing the search. As a result, some of the backed-up values are inexact (nodes c, e, f; compare with Figure 22.2). However, these approximations suffice for determining the root value and the main variation correctly.

the worst value for MIN that MIN is guaranteed to achieve. Thus, the actual value (that is to be found) lies between *Alpha* and *Beta*. If a position has been shown to have a value that lies outside the *Alpha-Beta* interval then this is sufficient to know that this position is not in the main variation, without knowing the exact value of this position. We only have to know the exact value of this position if this value is between *Alpha* and *Beta*. Formally, we can define a 'good enough' backed-up value *V(P, Alpha, Beta)* of a position *P*, with respect to *Alpha* and *Beta*, as any value that satisfies the following requirements:

$$\begin{array}{lll} V(P, Alpha, Beta) < Alpha & \text{if} & V(P) < Alpha \\ V(P, Alpha, Beta) = V(P) & \text{if} & Alpha \leq V(P) \leq Beta \\ V(P, Alpha, Beta) > Beta & \text{if} & V(P) > Beta \end{array}$$

Obviously we can always compute the exact value $V(P)$ of a root position P by setting the bounds as follows:

$$V(P, -infinity, +infinity) = V(P)$$

Figure 22.5 shows a Prolog implementation of the alpha-beta algorithm. The main relation is

alphabeta(Pos, Alpha, Beta, GoodPos, Val)

where **GoodPos** is a 'good enough' successor of **Pos**, so that its value **Val** satisfies the requirements stated above:

$$Val = V(Pos, Alpha, Beta)$$

The procedure

boundedbest(PosList, Alpha, Beta, GoodPos, Val)

finds a good enough position **GoodPos** in the list **PosList** so that the backed-up value **Val** of **GoodPos** is a good enough approximation with respect to **Alpha** and **Beta**.

The alpha-beta interval may get narrower (but never wider!) at deeper recursive calls of the alpha-beta procedure. The relation

newbounds(Alpha, Beta, Pos, Val, NewAlpha, NewBeta)

defines the new interval (**NewAlpha, NewBeta**). This is always narrower than or equal to the old interval (**Alpha, Beta**). So at deeper levels in the search tree, the *Alpha-Beta* bounds tend to shrink, and positions at deeper levels are evaluated with tighter bounds. Narrower intervals allow for grosser approximations, and thus more tree pruning. An interesting question is now: How much effort does the alpha-beta algorithm save compared with the exhaustive minimax search program of Figure 22.3?

The efficiency of the alpha-beta search depends on the order in which positions are searched. It is advantageous to consider strong moves for each side first. It is easy to demonstrate by examples that if the order is unfortunate then the alpha-beta procedure will have to visit *all* the positions visited by the exhaustive minimax


```
% The alpha-beta algorithm

alphabeta( Pos, Alpha, Beta, GoodPos, Val) :-
  moves( Pos, PosList), !,
  boundedbest( PosList, Alpha, Beta, GoodPos, Val);
  staticval( Pos, Val).                              % Static value of Pos

boundedbest( [Pos | PosList], Alpha, Beta, GoodPos, GoodVal) :-
  alphabeta( Pos, Alpha, Beta, _, Val),
  goodenough( PosList, Alpha, Beta, Pos, Val, GoodPos, GoodVal).

goodenough( [], _, _, Pos, Val, Pos, Val) :- !.     % No other candidate

goodenough( _, Alpha, Beta, Pos, Val, Pos, Val) :-
  min_to_move( Pos), Val > Beta, !;                  % Maximizer attained upper bound
  max_to_move( Pos), Val < Alpha, !.                 % Minimizer attained lower bound

goodenough( PosList, Alpha, Beta, Pos, Val, GoodPos, GoodVal) :-
  newbounds( Alpha, Beta, Pos, Val, NewAlpha, NewBeta),      % Refine bounds
  boundedbest( PosList, NewAlpha, NewBeta, Pos1, Val1),
  betterof( Pos, Val, Pos1, Val1, GoodPos, GoodVal).

newbounds( Alpha, Beta, Pos, Val, Val, Beta) :-
  min_to_move( Pos), Val > Alpha, !.                 % Maximizer increased lower bound

newbounds( Alpha, Beta, Pos, Val, Alpha, Val) :-
  max_to_move( Pos), Val < Beta, !.                  % Minimizer decreased upper bound

newbounds( Alpha, Beta, _, _, Alpha, Beta).          % Otherwise bounds unchanged

betterof( Pos, Val, Pos1, Val1, Pos, Val) :-         % Pos better than Pos1
  min_to_move( Pos), Val > Val1, !;
  max_to_move( Pos), Val < Val1, !.

betterof( _, _, Pos1, Val1, Pos1, Val1).             % Otherwise Pos1 better
```

Figure 22.5 An implementation of the alpha-beta algorithm.

search. That means that in the worst case alpha-beta will have no advantage over the exhaustive minimax search. If the order is favourable, however, savings can be significant. Let N be the number of terminal search positions statically evaluated by the exhaustive minimax algorithm. It has been proved that in the best case, when the strongest move is always considered first, the alpha-beta algorithm will only have to statically evaluate $\sqrt{N}$ positions.

On a similar note, this same result is relevant in a practical aspect in tournament play. In a tournament, a chess-playing program is usually given a certain amount of time for computing the next move in the game, and the depth to which the program can search will depend on this amount of time. The alpha-beta algorithm will be able, in the best case, to search *twice as deep* as the exhaustive minimax search. Experience shows that the same evaluation function applied at a greater depth in the tree will usually produce stronger play.

The economization effect of the alpha-beta algorithm can also be expressed in terms of the effective branching factor (number of branches stemming from each internal node) of the search tree. Assume that the game tree has a uniform branching factor b. Due to the pruning effect, alpha-beta will only search some of the branches, thus effectively reducing the branching factor. The reduction is, in the best case, from b to $\sqrt{b}$. In chess-playing programs the effective branching factor due to the alpha-beta pruning becomes about 6 compared to the total of about 30 legal moves. A less optimistic view on this result is that in chess, even with alpha-beta, deepening the search by 1 ply (one half-move) increases the number of terminal search positions by a factor of about 6.

Project

Consider a two-person game (for example, some non-trivial version of tic-tac-toe). Write game-definition relations (legal moves and terminal game positions) and propose a static evaluation function to be used for playing the game with the alpha-beta procedure.

22.4 Minimax-based programs: refinements and limitations

The minimax principle, together with the alpha-beta algorithm, is the basis of many successful game-playing programs, most notably chess programs. The general scheme of such a program is: perform the alpha-beta search on the current position in the game, up to some fixed depth limit (dictated by the time constraints imposed by tournament rules), using a game-specific evaluation function for evaluating the terminal positions of the search. Then execute the best move (according to alpha-beta) on the play board, accept the opponent's reply, and start the same cycle again.

The two basic ingredients, then, are the alpha-beta algorithm and a heuristic evaluation function. To build a good program for a complicated game like chess many refinements to this basic scheme are needed. We will briefly review some standard techniques.

Much depends on the evaluation function. If we had a perfect evaluation function we would only have to consider the immediate successors of the current position, thus practically eliminating search. But for games like chess, any evaluation function of practically acceptable computational complexity will necessarily be just a heuristic estimate. This estimate is based on 'static' features of the position (for example, the number of pieces on the board) and will therefore be more reliable in some positions than in others. Consider for example such a material-based evaluation function for chess and imagine a position in which White is a knight up. This function will, of course, assess the position in White's favour. This is fine if the position is quiescent, Black having no violent threat at his disposal. On the other hand, if Black can capture the White's queen on the next move, such an evaluation can result in a disastrous blunder, as it will not be able to perceive the position *dynamically*. Clearly, we can better trust the static evaluation in quiescent positions than in turbulent positions in which each side has direct threats of capturing the opponent's pieces. Obviously, we should use the static evaluation only in quiescent positions. Therefore a standard trick is to extend the search in turbulent positions beyond the depth limit until a quiescent position is reached. In particular, this extension includes sequences of piece captures in chess.

Another refinement is *heuristic pruning*. This aims at achieving a greater depth limit by disregarding some less promising continuations. This technique will prune branches in addition to those that are pruned by the alpha-beta technique itself. Therefore this entails the risk of overlooking some good continuation and incorrectly computing the minimax value.

Yet another technique is *progressive deepening*. The program repeatedly executes the alpha-beta search, first to some shallow depth, and then increases the depth limit on each iteration. The process stops when the time limit has been reached. The best move according to the deepest search is then played. This technique has the following advantages:

- enables the time control; when the time limit is reached there will always be some best move found so far;
- the minimax values of the previous iteration can be used for preliminary ordering of positions on the next iteration, thus helping the alpha-beta algorithm to search strong moves first.

Progressive deepening entails some overhead (researching upper parts of the game tree), but this is relatively small compared with the total effort.

A known problem with programs that belong to this general scheme is the 'horizon effect'. Imagine a chess position in which the program's side inevitably loses a knight. But the loss of the knight can be delayed at the cost of a lesser sacrifice, a pawn say. This intermediate sacrifice may push the actual loss of the knight beyond the search limit (beyond the program's 'horizon'). Not seeing the eventual loss of the knight, the program will then prefer this variation to the quick death of the knight. So the program will eventually lose *both* the pawn (unnecessarily) and the knight. The extension of search up to a quiescent position can alleviate the horizon effect.

There is, however, a more fundamental limitation of the minimax-based programs, which lies in the limited form of the domain-specific knowledge they use. This becomes very conspicuous when we compare the best chess programs with human chess masters. Strong programs often search millions (and more) of positions before deciding on the move to play. It is known from psychological studies that human masters typically search just a few tens of positions, at most a few hundred. Despite this apparent inferiority, a chess master may still beat a program. The masters' advantage lies in their knowledge, which far exceeds that contained in the programs. Games between machines and strong human players show that the enormous advantage in the calculating power cannot always completely compensate for the lack of knowledge.

Knowledge in minimax-based programs takes three main forms:

- evaluation function,
- tree-pruning heuristics,
- quiescence heuristics.

The evaluation function reduces many aspects of a game situation into a single number, and this reduction can have a detrimental effect. A good player's understanding of a game position, on the contrary, spans over many dimensions. Let us consider an example from chess: an evaluation function will evaluate a position as equal simply by stating that its value is 0. A master's assessment of the same position can be much more informative and indicative of a further course of the game. For example, Black is a pawn up, but White has a good attacking initiative that compensates the material, so chances are equal.

In chess, minimax-based programs usually play best in sharp tactical struggles when precise calculation of forced variations is decisive. Their weakness is more likely to show in quiet positions where their play falls short of long-range plans that prevail in such slow, strategic games. Lack of a plan makes an impression that the program keeps wandering during the game from one idea to another.

In the rest of this chapter we will consider another approach to game playing, based on introducing pattern knowledge into a program by means of 'advice'. This enables the programming of goal-oriented, plan-based behaviour of a game-playing program.

22.5 Pattern knowledge and the mechanism of 'advice'

22.5.1 Goals and move-constraints

The method of representing game-specific knowledge that we consider in this section belongs to the family of Advice Languages. In Advice Languages the user specifies, in a declarative way, what ideas should be tried in certain types of situations. Ideas are formulated in terms of goals and means of achieving the goals. An Advice Language interpreter then finds out, through search, which idea actually works in a given situation.

The fundamental concept in Advice Languages is a 'piece-of-advice'. A piece-of-advice suggests what to do (or to *try* to do) next in a certain type of position.

Generally speaking, advice is expressed in terms of *goals* to be achieved, and *means* of achieving these goals. The two sides are called 'us' and 'them'; advice always refers to the 'us' point of view. Each piece-of advice has four ingredients:

- *better-goal*: a goal to be achieved;
- *holding-goal*: a goal to be maintained during play toward the better-goal;
- *us-move-constraints*: a predicate on moves that selects a subset of all legal us-moves (moves that should be considered of interest with respect to the goals specified);
- *them-move-constraints*: a predicate to select moves to be considered by 'them' (moves that may undermine the goals specified).

As a simple example from the chess endgame king and pawn vs king, consider the straightforward idea of queening the pawn by simply pushing the pawn forward. This can be expressed in the form of advice as:

- *better-goal*: pawn queened;
- *holding-goal*: pawn is not lost;
- *us-move-constraints*: pawn move;
- *them-move-constraints*: approach the pawn with the king.

22.5.2 Satisfiability of advice

We say that a given piece-of-advice is *satisfiable* in a given position if 'us' can force the achievement of the better-goal specified in the advice under the conditions that:

(1) the holding-goal is never violated,
(2) all the moves played by 'us' satisfy us-move-constraints,
(3) 'them' is only allowed to make moves that satisfy them-move-constraints.

The concept of a *forcing-tree* is associated with the satisfiability of a piece-of-advice. A forcing-tree is a detailed strategy that guarantees the achievement of the better-goal under the constraints specified by the piece-of-advice. A forcing-tree thus specifies exactly what moves 'us' has to play on any 'them' reply. More precisely, a forcing-tree T for a given position P and a piece-of-advice A is a subtree of the game tree such that:

- the root node of T is P;
- all the positions in T satisfy the holding-goal;
- all the terminal nodes in T satisfy the better-goal, and no internal node in T satisfies the better-goal;

- there is exactly one us-move from each internal us-to-move position in T; and that move must satisfy the us-move-constraints;
- T contains all them-moves (that satisfy the them-move-constraints) from each non-terminal them-to-move position in T.

Each piece-of-advice can be viewed as a definition of a small special game with the following rules. Each opponent is allowed to make moves that satisfy his or her move-constraints; a position that does not satisfy the holding-goal is won for 'them'; a position that satisfies the holding-goal and the better-goal is won for 'us'. A non-terminal position is won for 'us' if the piece-of-advice is satisfiable in this position. Then 'us' will win by executing a corresponding forcing-tree in the play.

22.5.3 Integrating pieces-of-advice into rules and advice-tables

In Advice Languages, individual pieces-of-advice are integrated in the complete knowledge representation schema through the following hierarchy. A piece-of-advice is part of an if-then rule. A collection of if-then rules is an *advice-table*. A set of advice-tables is structured into a hierarchical network. Each advice-table has the role of a specialized expert to deal with some specific subproblem of the whole domain. An example of such a specialized expert is an advice-table that knows how to mate in the king and rook vs king ending in chess. This table is summoned when such an ending occurs in a game.

For simplicity, we will consider a simplified version of an Advice Language in which we will only allow for one advice-table. We shall call this version Advice Language 0, or AL0 for short. Here is the structure of AL0 already syntactically tailored toward an easy implementation in Prolog.

A program in AL0 is called an *advice-table*. An advice-table is an *ordered* collection of if-then rules. Each rule has the form:

```
RuleName :: if Condition then AdviceList
```

Condition is a logical expression that consists of predicate names connected by logical connectives **and**, **or**, **not**. **AdviceList** is a list of names of pieces-of-advice. An example of a rule called 'edge_rule', from the king and rook vs king ending, can be:

```
edge_rule ::
  if their_king_on_edge and our_king_close
  then [ mate_in_2, squeeze, approach, keeproom, divide].
```

This rule says: if in the current position their king is on the edge and our king is close to their king (or more precisely, kings are less than four squares apart), then try to satisfy, in the order of preference as stated, the pieces-of-advice: 'mate_in_2', 'squeeze', 'approach', 'keeproom', 'divide'. This advice-list specifies pieces-of-advice in the decreasing order of ambition: first try to mate in two moves, if that is not possible then try to 'squeeze' the opponent's king toward a corner, etc. Notice that with an appropriate definition of operators, the rule above is a syntactically correct Prolog clause.

Each piece-of-advice will be specified by a Prolog clause of the form:

```
advice( AdviceName,
        BetterGoal :
        HoldingGoal :
        Us_Move_Constraints :
        Them_Move_Constraints).
```

The goals are expressions that consist of predicate names and logical connectives **and**, **or**, **not**. Move-constraints are, again, expressions that consist of predicate names and the connectives **and** and **then**: **and** has the usual logical meaning, **then** prescribes the ordering. For example, a move-constraint of the form

```
MC1 then MC2
```

says: first consider those moves that satisfy **MC1**, and then those that satisfy **MC2**.

For example, a piece-of-advice to mate in 2 moves in the king and rook vs king ending, written in this syntax, is:

```
advice( mate_in_2,
        mate :
        not rooklost :
        (depth = 0) and legal then (depth = 2) and checkmove :
        (depth = 1) and legal).
```

Here the better-goal is **mate**, the holding-goal is **not rooklost** (rook is not lost). The us-move-constraints say: at depth 0 (the current board position) try any legal move, then at depth 2 (our second move) try checking moves only. The depth is measured in plies. Them-move-constraints are: any legal move at depth 1.

In playing, an advice-table is then used by repeating, until the end of the game, the following main cycle: build a forcing-tree, then play according to this tree until the play exits the tree; build another forcing-tree, etc. A forcing-tree is generated each time as follows: take the current board position **Pos** and scan the rules in the advice-table one by one; for each rule, match **Pos** with the precondition of the rule, and stop when a rule is found such that **Pos** satisfies its precondition. Now consider the advice-list of this rule: process pieces-of-advice in this list one by one until a piece-of-advice is found that is satisfiable in **Pos**. This results in a forcing-tree that is the detailed strategy to be executed across the board.

Notice the importance of the ordering of rules and pieces-of-advice. The rule used is the first rule whose precondition matches the current position. There must be for any possible position at least one rule in the advice-table whose precondition will match the position. Thus an advice-list is selected. The first satisfiable piece-of-advice in this list is applied.

An advice-table is thus largely a non-procedural program. An AL0 interpreter accepts a position and by executing an advice-table produces a forcing-tree which determines the play in that position.

22.6 A chess endgame program in Advice Language 0

Implementation of an AL0-based game-playing program can be conveniently divided into three modules:

(1) an AL0 interpreter,

(2) an advice-table in AL0,

(3) a library of predicates (including rules of the game) used in the advice-table.

This structure corresponds to the usual structure of knowledge-based systems as follows:

- The AL0 interpreter is an inference engine.
- The advice-table and the predicate library constitute a knowledge base.

22.6.1 A miniature AL0 interpreter

A miniature, game-independent AL0 interpreter is implemented in Prolog in Figure 22.6. This program also performs the user interaction during play. The central function of the program is the use of knowledge in an AL0 advice-table; that is, interpreting an AL0 advice-program for the generation of forcing-trees and their execution in a game. The basic forcing-tree generation algorithm is similar to the depth-first search in AND/OR graphs of Chapter 13; a forcing-tree corresponds to an AND/OR solution tree. On the other hand, it also resembles the generation of a proof tree in an expert system (Chapter 15).

For simplicity, in the program of Figure 22.6 'us' is supposed to be White, and 'them' is Black. The program is started through the procedure

```
playgame( Pos)
```

where Pos is a chosen initial position of a game to be played. If it is 'them' to move in Pos then the program reads a move from the user, otherwise the program consults the advice-table that is attached to the program, generates a forcing-tree and plays its move according to the tree. This continues until the end of the game is reached as specified by the predicate **end_of_game** (mate, for example).

A forcing-tree is a tree of moves, represented in the program by the following structure

```
Move .. [ Reply1 .. Ftree1, Reply2 .. Ftree2, ... ]
```



```
% A miniature implementation of Advice Language 0
%
% This program plays a game from a given starting position using knowledge
% represented in Advice Language 0

:- op( 200, xfy, [:, ::]).
:- op( 220, xfy, ..).
:- op( 185, fx, if).
:- op( 190, xfx, then).
:- op( 180, xfy, or).
:- op( 160, xfy, and).
:- op( 140, fx, not).

playgame( Pos) :-                          % Play a game starting in Pos
  playgame( Pos, nil).                     % Start with empty forcing-tree

playgame( Pos, ForcingTree) :-
  show( Pos),
  ( end_of_game( Pos),                     % End of game?
    write( 'End of game'), nl, !
    ;
    playmove( Pos, ForcingTree, Pos1, ForcingTree1), !,
    playgame( Pos1, ForcingTree1)
  ).

% Play 'us' move according to forcing-tree

playmove( Pos, Move .. FTree1, Pos1, FTree1) :-
  side( Pos, w),                           % White = 'us'
  legalmove( Pos, Move, Pos1),
  showmove( Move).

% Read 'them' move

playmove( Pos, FTree, Pos1, FTree1) :-
  side( Pos, b),
  write( 'Your move: '),
  read( Move),
  ( legalmove( Pos, Move, Pos1),
    subtree( FTree, Move, FTree1), !       % Move down forcing-tree
    ;
    write( 'Illegal move'), nl,
    playmove( Pos, FTree, Pos1, FTree1)
  ).

% If current forcing-tree is empty generate a new one

playmove( Pos, nil, Pos1, FTree1) :-
  side( Pos, w),
  resetdepth( Pos, Pos0),                  % Pos0 = Pos with depth 0
  strategy( Pos0, FTree), !,               % Generate new forcing-tree
  playmove( Pos0, FTree, Pos1, FTree1).
```

Figure 22.6 A miniature implementation of Advice Language 0.

Figure 22.6 *contd*

```
% Select a forcing-subtree corresponding to Move

subtree( FTrees, Move, FTree) :-
  member( Move .. FTree, FTrees), !.

subtree( _, _, nil).

strategy( Pos, ForcingTree) :-                    % Find forcing-tree for Pos
  Rule :: if Condition then AdviceList,           % Consult advice-table
  holds( Condition, Pos, _), !,                   % Match Pos against precondition
  member( AdviceName, AdviceList),                % Try pieces-of-advice in turn
  nl, write( 'Trying'), write( AdviceName),
  satisfiable( AdviceName, Pos, ForcingTree), !.  % Satisfy AdviceName in Pos

satisfiable( AdviceName, Pos, FTree) :-
  advice( AdviceName, Advice),                    % Retrieve piece-of-advice
  sat( Advice, Pos, Pos, FTree).                  % 'sat' needs two positions
                                                  % for comparison predicates

sat( Advice, Pos, RootPos, FTree) :-
  holdinggoal( Advice, HG),
  holds( HG, Pos, RootPos),                       % Holding-goal satisfied
  sat1( Advice, Pos, RootPos, FTree).

sat1( Advice, Pos, RootPos, nil) :-
  bettergoal( Advice, BG),
  holds( BG, Pos, RootPos), !.                    % Better-goal satisfied

sat1( Advice, Pos, RootPos, Move .. FTrees) :-
  side( Pos, w), !,                               % White = 'us'
  usmoveconstr( Advice, UMC),
  move( UMC, Pos, Move, Pos1),                    % A move satisfying move-constr.
  sat( Advice, Pos1, RootPos, FTrees).

sat1( Advice, Pos, RootPos, FTrees) :-
  side( Pos, b), !,                               % Black = 'them'
  themmoveconstr( Advice, TMC),
  bagof( Move .. Pos1, move( TMC, Pos, Move, Pos1), MPlist),
  satall( Advice, MPlist, RootPos, FTrees).       % Satisfiable in all successors

satall( _, [], _, [] ).

satall( Advice, [Move .. Pos | MPlist], RootPos, [Move .. FT | MFTs] ) :-
  sat( Advice, Pos, RootPos, FT),
  satall( Advice, MPlist, RootPos, MFTs).

% Interpreting holding and better-goals:
% A goal is an AND/OR/NOT combination of predicate names

holds( Goal1 and Goal2, Pos, RootPos) :- !,
  holds( Goal1, Pos, RootPos),
  holds( Goal2, Pos, RootPos).
```



```
holds( Goal1 or Goal2, Pos, RootPos) :- !,
  ( holds( Goal1, Pos, RootPos)
   ;
   holds( Goal2, Pos, RootPos)
  ).

holds( not Goal, Pos, RootPos) :- !,
  not holds( Goal, Pos, RootPos).

holds( Pred, Pos, RootPos) :-
  ( Cond =.. [ Pred, Pos]                         % Most predicates do not depend on RootPos
   ;
   Cond =.. [ Pred, Pos, RootPos] ),
  call( Cond).

% Interpreting move-constraints

move( MC1 and MC2, Pos, Move, Pos1) :- !,
  move( MC1, Pos, Move, Pos1),
  move( MC2, Pos, Move, Pos1).

move( MC1 then MC2, Pos, Move, Pos1) :- !,
  ( move( MC1, Pos, Move, Pos1)
   ;
   move( MC2, Pos, Move, Pos1)
  ).

% Selectors for components of piece-of-advice

bettergoal( BG : _, BG).

holdinggoal( BG : HG : _, HG).

usmoveconstr( BG : HG : UMC : _, UMC).

themmoveconstr( BG : HG : UMC : TMC, TMC).

member( X, [X | L] ).

member( X, [Y | L] ) :-
  member( X, L).
```

where '..' is an infix operator; Move is the first move for 'us'; **Reply1**, **Reply2**, etc. are the possible 'them' replies; and Ftree1, Ftree2, etc. are forcing-subtrees that correspond to each of the 'them' replies respectively.

22.6.2 An advice-program for the king and rook vs king ending

A broad strategy for winning with the king and rook against the sole opponent's king is to force the king to the edge, or into a corner if necessary, and then deliver mate in a few moves. An elaboration of this broad principle is:

While making sure that stalemate is never created or the rook left undefended under attack, repeat until mate:

(1) Look for a way to mate the opponent's king in two moves.
(2) If the above is not possible, then look for a way to constrain further the area on the chessboard to which the opponent's king is confined by our rook.
(3) If the above is not possible, then look for a way to move our king closer to the opponent's king.
(4) If none of the above pieces-of-advice 1, 2 or 3 works, then look for a way of maintaining the present achievements in the sense of 2 and 3 (that is, make a waiting move).
(5) If none of 1, 2, 3 or 4 is attainable, then look for a way of obtaining a position in which our rook divides the two kings either vertically or horizontally.

These principles are implemented in detail as an AL0 advice-table in Figure 22.7. This table can be run by the AL0 interpreter of Figure 22.6. Figure 22.8 illustrates the meaning of some of the predicates used in the table and the way the table works.

The predicates used in the table are:

Goal predicates

mate	their king mated
stalemate	their king stalemated
rooklost	their king can capture our rook
rookexposed	their king can attack our rook before our king can get to defend the rook
newroomsmaller	area to which their king is restricted by our rook has shrunk
rookdivides	rook divides both kings either vertically or horizontally
okapproachedcsquare	our king approached 'critical square', see Figure 22.9; here this means that the Manhattan distance has decreased
lpatt	'L-pattern' (Figure 22.9)
roomgt2	the 'room' for their king is greater than two squares

Move-constraints predicates

depth = N	move occurring at depth = N in the search tree
legal	any legal move
checkmove	checking move
rookmove	a rook move
nomove	fails for any move
kingdiagfirst	a king move, with preference for diagonal king moves


```
% King and rook vs king in Advice Language 0

% Rules

edge_rule :: if    their_king_edge and kings_close
              then [ mate_in_2, squeeze, approach, keeproom,
                     divide_in_2, divide_in_3 ].

else_rule :: if    true
              then [ squeeze, approach, keeproom, divide_in_2, divide_in_3 ].

% Pieces-of-advice

advice( mate_in_2,
        mate :
        not rooklost and their_king_edge :
        (depth = 0) and legal then (depth = 2) and checkmove :
        (depth = 1) and legal).

advice( squeeze,
        newroomsmaller and not rookexposed and
        rookdivides and not stalemate :
        not rooklost :
        (depth = 0) and rookmove :
        nomove).

advice( approach,
        okapproachedcsquare and not rookexposed and not stalemate and
        (rookdivides or lpatt) and (roomgt2 or not our_king_edge) :
        not rooklost :
        (depth = 0) and kingdiagfirst :
        nomove).

advice( keeproom,
        themtomove and not rookexposed and rookdivides and okorndle and
        (roomgt2 or not okedge) :
        not rooklost :
        (depth = 0) and kingdiagfirst :
        nomove).

advice( divide_in_2,
        themtomove and rookdivides and not rookexposed :
        not rooklost :
        (depth < 3) and legal :
        (depth < 2) and legal).

advice( divide_in_3,
        themtomove and rookdivides and not rookexposed :
        not rooklost :
        (depth < 5) and legal :
        (depth < 4) and legal).
```

Figure 22.7 An AL0 advice-table for king and rook vs king. The table consists of two rules and six pieces-of-advice.

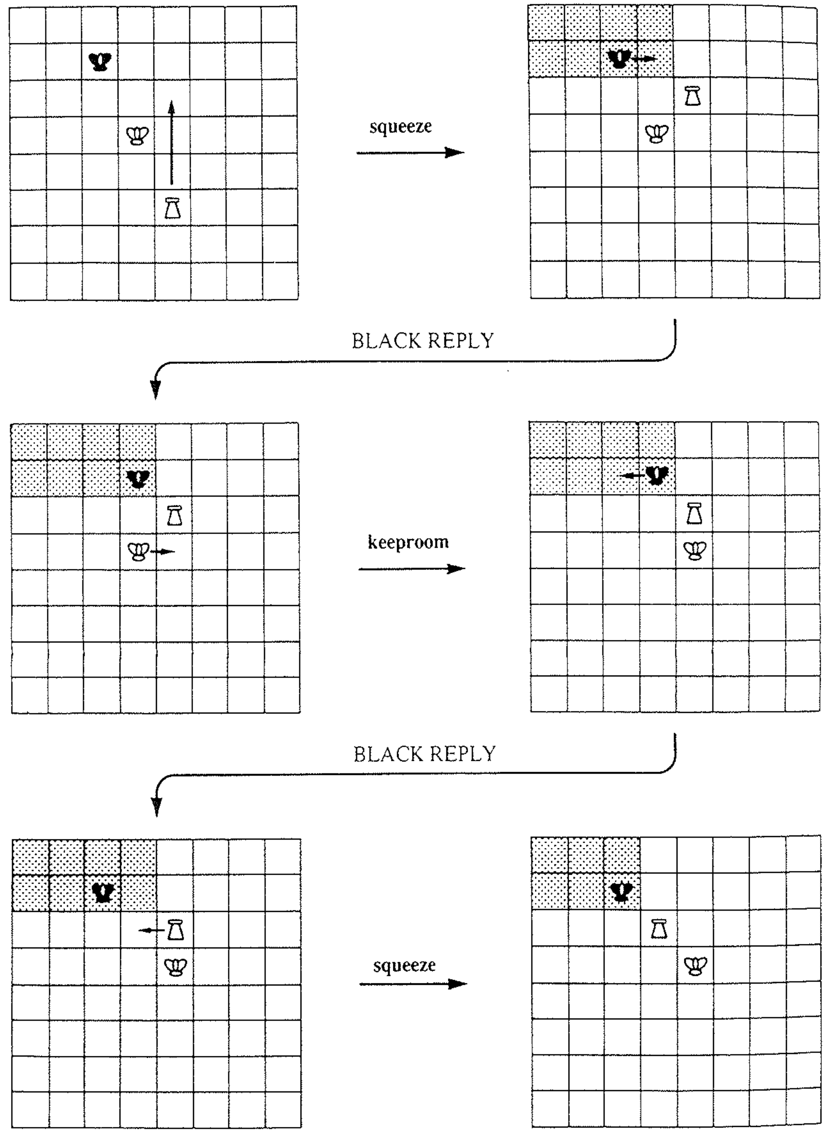

Figure 22.8 A game fragment played by the advice-table of Figure 22.7, illustrating the method of squeezing their king toward a corner. Pieces-of-advice used in this sequence are **keeproom** (waiting move preserving 'room') and **squeeze** ('room' has shrunk). The area to which their king is confined by our rook ('room') is shadowed. After the last **squeeze**, 'room' shrinks from eight to six squares.

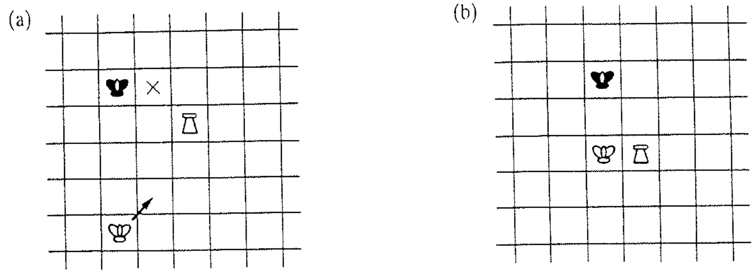

Figure 22.9 (a) Illustration of the 'critical square' (a crucial square in the squeezing manoeuvres, indicated by a cross); the White king approaches the critical square by moving as indicated. (b) The three pieces form an L-shaped pattern.

The arguments of these predicates are either positions (goal predicates) or moves (move-constraints predicates). Goal predicates can have one or two arguments. One argument is always the current search node; the second argument (if it exists) is the root node of the search tree. The second argument is needed in the so-called comparison predicates, which compare in some respect the root position and the current search position. An example is the predicate **newroomsmaller**, which tests whether the 'room' for their king has shrunk (Figure 22.8). These predicates, together with chess rules for king and rook vs king, and a board displaying procedure (show(Pos)), are programmed in Figure 22.10.

An example of how this advice-program plays is shown in Figure 22.8. The game would continue from the last position of Figure 22.8 as in the following variation (assuming 'them' moves as given in the variation). The algebraic chess notation is used where the files of the chessboard are numbered 'a', 'b', 'c', etc, and ranks are numbered 1, 2, 3, etc. For example, the move 'BK b7' means: move the Black king to the square in file 'b' and rank 7.

...	BK b7
WK d5	BK c7
WK c5	BK b7
WR c6	BK a7
WR b6	BK a8
WK b5	BK a7
WK c6	BK a8
WK c7	BK a7
WR c6	BK a8
WR a6	mate

Some questions can now be asked. First, is this advice-program *correct* in the sense that it mates against any defence if the game starts from any king and rook vs king

```
% Predicate library for king and rook vs king

% Position is represented by: Side..Wx : Wy..Rx : Ry..Bx : By..Depth
% Side is side to move ('w' or 'b')
% Wx, Wy are X and Y-coordinates of White king
% Rx, Ry are X and Y-coordinates of White rook
% Bx, By are coordinates of Black king
% Depth is depth of position in search tree

% Selector relations

side( Side.._, Side).                      % Side to move in position
wk( _..WK.._, WK).                         % White king coordinates
wr( _.._..WR.._, WR).                      % White rook coordinates
bk( _.._.._..BK.._, BK).                   % Black king coordinates
depth( _.._.._.._..Depth, Depth).          % Depth of position in search tree

resetdepth( S..W..R..B..D, S..W..R..B..0). % Copy of position with depth 0

% Some relations between squares

n( N, N1) :-                               % Neighbour integers 'within board'
  ( N1 is N + 1;
    N1 is N - 1),
  in( N1).

in( N) :-
  N > 0, N < 9.

diagngb( X : Y, X1 : Y1) :-                % Diagonal neighbour squares
  n( X, X1), n( Y, Y1).

verngb( X : Y, X : Y1) :-                  % Vertical neighbour squares
  n( Y, Y1).

horngb( X : Y, X1 : Y) :-                  % Horizontal neighbour squares
  n( X, X1).

ngb( S, S1) :-                             % Neighbour squares, first diagonal
  diagngb( S, S1);
  horngb( S, S1);
  verngb( S, S1).

end_of_game( Pos) :-
  mate( Pos).

% Move-constraints predicates
% These are specialized move generators:
% move( MoveConstr, Pos, Move, NewPos)

move( depth < Max, Pos, Move, Pos1) :-
  depth( Pos, D),
  D < Max, !.
```

Figure 22.10 Predicate library for king and rook vs king.


```
move( depth = D, Pos, Move, Pos1) :-
  depth( Pos, D), !.

move( kingdiagfirst, w..W..R..B..D, W-W1, b..W1..R..B..D1) :-
  D1 is D + 1,
  ngb( W, W1),                 % 'ngb' generates diagonal moves first
  not ngb( W1, B),             % Must not move into check
  W1 \== R.                    % Must not collide with rook

move( rookmove, w..W..Rx : Ry..B..D, Rx : Ry-R, b..W..R..B..D1) :-
  D1 is D + 1,
  coord( I),                   % Integer between 1 and 8
  ( R = Rx : I; R = I : Ry),   % Move vertically or horizontally
  R \== Rx : Ry,               % Must have moved
  not inway( Rx : Ry, W, R).   % White king not in way

move( checkmove, Pos, R-Rx : Ry, Pos1) :-
  wr( Pos, R),
  bk( Pos, Bx : By),
  (Rx = Bx; Ry = By),          % Rook and Black king in line
  move( rookmove, Pos, R-Rx : Ry, Pos1).

move( legal, w..P, M, P1) :-
  ( MC = kingdiagfirst; MC = rookmove),
  move( MC, w..P, M, P1).

move( legal, b..W..R..B..D, B-B1, w..W..R..B1..D1) :-
  D1 is D + 1,
  ngb( B, B1),
  not check( w..W..R..B1..D1).

legalmove( Pos, Move, Pos1) :-
  move( legal, Pos, Move, Pos1).

check( _..W..Rx : Ry..Bx : By.._) :-
  ngb( W, Bx : By);            % King's too close
  ( Rx = Bx; Ry = By),
  Rx : Ry \== Bx : By,         % Not rook captured
  not inway( Rx : Ry, W, Bx : By).

inway( S, S1, S1) :- !.

inway( X1 : Y, X2 : Y, X3 : Y) :-
  ordered( X1, X2, X3), !.

inway( X : Y1, X : Y2, X : Y3) :-
  ordered( Y1, Y2, Y3).

ordered( N1, N2, N3) :-
  N1 < N2, N2 < N3;
  N3 < N2, N2 < N1.

coord(1). coord(2). coord(3). coord(4).
coord(5). coord(6). coord(7). coord(8).
```

Figure 22.10 *contd*

```
% Goal predicates

true( Pos).

themtomove( b.._ ).                        % Black = 'them' to move

mate( Pos) :-
  side( Pos, b),
  check( Pos),
  not legalmove( Pos, _, _ ).

stalemate( Pos) :-
  side( Pos, b),
  not check( Pos),
  not legalmove( Pos, _, _ ).

newroomsmaller( Pos, RootPos) :-
  room( Pos, Room),
  room( RootPos, RootRoom),
  Room < RootRoom.

rookexposed( Side..W..R..B.._ ) :-
  dist( W, R, D1),
  dist( B, R, D2),
  (Side = w, !, D1 > D2 + 1
   ;
   Side = b, !, D1 > D2).

okapproachedcsquare( Pos, RootPos) :-
  okcsquaremdist( Pos, D1),
  okcsquaremdist( RootPos, D2),
  D1 < D2.

okcsquaremdist( Pos, Mdist) :-    % Manhattan distance between WK and critical square
  wk( Pos, WK),
  cs( Pos, CS),                            % Critical square
  manhdist( WK, CS, Mdist).

rookdivides( _..Wx : Wy..Rx : Ry..Bx : By.._ ) :-
  ordered( Wx, Rx, Bx), !;
  ordered( Wy, Ry, By).

lpatt( _..W..R..B.._ ) :-                 % L-pattern
  manhdist( W, B, 2),
  manhdist( R, B, 3).

okorndle( _..W..R.._, _..W1..R1.._ ) :-
  dist( W, R, D),
  dist( W1, R1, D1),
  D =< D1.

roomgt2( Pos) :-
  room( Pos, Room),
  Room > 2.
```



```
our_king_edge( _..X : Y.._ ) :-                 % White king on edge
  ( X = 1, !; X = 8, !; Y = 1, !; Y = 8).

their_king_edge( _..W..R..X : Y.._ ) :-        % Black king on edge
  ( X = 1, !; X = 8, !; Y = 1, !; Y = 8).

kings_close( Pos) :-                            % Distance between kings < 4
  wk( Pos, WK), bk( Pos, BK),
  dist( WK, BK, D),
  D < 4.

rooklost( _..W..B..B.._ ).                      % Rook has been captured

rooklost( b..W..R..B.._ ) :-
  ngb( B, R),                                   % Black king attacks rook
  not ngb( W, R).                               % White king does not defend

dist( X : Y, X1 : Y1, D) :-                     % Distance in king moves
  absdiff( X, X1, Dx),
  absdiff( Y, Y1, Dy),
  max( Dx, Dy, D).

absdiff( A, B, D) :-
  A > B, !, D is A - B;
  D is B - A.

max( A, B, M) :-
  A >= B, !, M = A;
  M = B.

manhdist( X : Y, X1 : Y1, D) :-                 % Manhattan distance
  absdiff( X, X1, Dx),
  absdiff( Y, Y1, Dy),
  D is Dx + Dy.

room( Pos, Room) :-                             % Area to which B. king is confined
  wr( Pos, Rx : Ry),
  bk( Pos, Bx : By),
  ( Bx < Rx, SideX is Rx - 1; Bx > Rx, SideX is 8 - Rx),
  ( By < Ry, SideY is Ry - 1; By > Ry, SideY is 8 - Ry),
  Room is SideX * SideY, !
  ;
  Room is 64.                                   % Rook in line with Black king

cs( _..W..Rx : Ry..Bx : By.._, Cx : Cy) :-     % 'Critical square'
  ( Bx < Rx, !, Cx is Rx - 1; Cx is Rx + 1),
  ( By < Ry, !, Cy is Ry - 1; Cy is Ry + 1).

% Display procedures

show( Pos) :-
  nl,
  coord( Y), nl,
  coord( X),
  writepiece( X : Y, Pos),
  fail.
```

Figure 22.10 *contd*

```
show( Pos) :-
  side( Pos, S), depth( Pos, D),
  nl, write( 'Side= '), write( S),
  write( 'Depth= '), write( D), nl.

writepiece( Square, Pos) :-
  wk( Pos, Square), !, write( 'W');
  wr( Pos, Square), !, write( 'R');
  bk( Pos, Square), !, write( 'B');
  write( '.').

showmove( Move) :-
  nl, write( Move), nl.
```

position? It is shown in Bratko (1978) by means of a formal proof that an advice-table, effectively the same as the one in Figure 22.7, is correct in this sense.

Another question can be: Is this advice program *optimal* in the sense that it always delivers mate in the smallest number of moves? It can easily be shown by examples that the program's play is not optimal in this sense. It is known that optimal variations (optimally played by both sides) in this ending are at most 16 moves long. Although our advice-table can be rather far from this optimum, it was shown that the number of moves needed by this advice-table is still very safely under 50. This is important because of the 50-moves rule in chess: in endgames such as king and rook vs king the stronger side has to mate within 50 moves; if not, a draw can be claimed.

Project

Consider some other simple chess endgame, such as king and pawn vs king, and write an AL0 program (together with the corresponding predicate definitions) to play this endgame.

Summary

- Two-person games fit the formalism of AND/OR graphs. AND/OR search procedures can be therefore used to search game trees.
- The straightforward depth-first search of game trees is easy to program, but is too inefficient for playing interesting games. In such cases, the minimax principle, in association with an evaluation function and depth-limited search, offers a more feasible approach.

- The alpha-beta algorithm is an efficient implementation of the minimax principle. The efficiency of alpha-beta depends on the order in which alternatives are searched. In the best case, alpha-beta in effect reduces the branching factor of a game tree to its square root.
- Some refinements to the basic alpha-beta algorithm include: extending the search until a quiescent position is reached, progressive deepening and heuristic pruning.
- Numerical evaluation is a very restrictive form of applying game-specific knowledge. A more knowledge-intensive approach to game playing should provide for pattern-based knowledge. Advice Languages realize such an approach, where knowledge is represented in terms of goals and means of achieving goals.
- Programs written in this chapter are: an implementation of minimax and alpha-beta, an interpreter for Advice Language 0, and an advice-table for playing the king and rook vs king chess endgame.
- Concepts discussed in this chapter are:

  two-person, perfect-information games
  game trees
  evaluation function, minimax principle
  static values, backed-up values
  alpha-beta algorithm
  progressive deepening, heuristic pruning, quiescence heuristics
  horizon effect
  Advice Languages
  goals, constraints, piece-of-advice, advice-table

References

The minimax principle, implemented as the alpha-beta algorithm, is the most popularly used approach to game-playing programs, and in particular to chess programs. The minimax principle was introduced by Shannon (1950). The development of the alpha-beta technique had a rather complicated history when several researchers independently discovered or implemented the method or at least part of it. This interesting history is described by Knuth and Moore (1975), who also present a more compact formulation of the alpha-beta algorithm using the 'neg-max' principle instead of minimax, and give a mathematical analysis of its performance. A comprehensive treatment of several minimax-based algorithms and their analyses is Pearl (1984). Kaindl (1990) also reviews search algorithms. Platt *et al.* (1996) introduce a more recent variation of alpha-beta search. There is another interesting question regarding the minimax principle: Knowing that the static evaluation is only reliable to some degree, will the minimax backed-up values be more reliable than the static values themselves? Pearl (1984) has also collected results of mathematical analyses that pertain to this question.

Results on error propagation in minimax trees explain when and why the minimax look-ahead is beneficial.

Bramer (1983), Frey (1983), and Marsland and Schaeffer (1990) edited collections of papers on computer game playing, and chess in particular. On-going research on computer chess is published in the Advances in Computer Chess series and in the ICCA journal.

The Advice Language approach to using pattern knowledge in chess was introduced by Michie, and further developed in Bratko and Michie (1980), and Bratko (1982, 1984, 1985). The king and rook vs king advice-program of this chapter is a slight modification of the advice-table that was mathematically proved correct in Bratko (1978).

Other interesting experiments in knowledge-intensive approach to chess (as opposed to search-intensive approaches) include Berliner (1977), Pitrat (1977) and Wilkins (1980). Interest in knowledge-intensive chess programming seems to have declined over the time, probably due to competitive success of the search-intensive, 'brute force' approach to chess programming. Due to increasing power of computer hardware, including special-purpose chess hardware, up to hundreds of millions of positions can be searched per second, so the sheer power more than compensates for the lack of more subtle knowledge. The brute-force approach culminated in 1997 in the eventual defeat of the world leading chess player Gary Kasparov by the program Deep Blue (Hsu et al. 1990), an extreme example of brute-force. However, this competitive success does not remove the known shortcoming of brute-force programs: they cannot explain their play in conceptual terms. So from the points of view of explanation, commentary and teaching, the knowledge-intensive approach remains necessary. The game of go, on the other hand, seems to require a knowledge-based approach also for the mere competitive reasons. Brute-force has not worked so well in go as in chess because go has a much greater combinatorial complexity.

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chapter 23

Meta-Programming

Due to its symbol-manipulation capabilities, Prolog is a powerful language for implementing other languages and programming paradigms. In this chapter we discuss the writing of Prolog meta-interpreters – that is, interpreters for Prolog in Prolog. We look at a special program-compilation technique, called explanation-based generalization, that was invented as an approach to machine learning. We also develop simple interpreters for two other approaches to programming: object-oriented programming and pattern-directed programming.

23.1 Meta-programs and meta-interpreters

A *meta-program* is a program that takes other programs as data. Interpreters and compilers are examples of meta-programs. *Meta-interpreter* is a particular kind of meta-program: an interpreter for a language written in that same language. So a Prolog meta-interpreter is an interpreter for Prolog, itself written in Prolog.

Due to its symbol-manipulation capabilities, Prolog is a powerful language for meta-programming. Therefore, it is often used as an implementation language for other languages. It has often been observed that Prolog is particularly suitable as a language for rapid prototyping where we are interested in implementing new ideas quickly. This is important when we want to develop a new language or a new

programming paradigm or program architecture. New ideas are rapidly implemented and experimented with. In prototyping the emphasis is on bringing new ideas to life quickly and cheaply, so that they can be immediately tested. On the other hand, there is not much emphasis on efficiency of implementation. Once the ideas are developed, a prototype may have to be re-implemented, possibly in another, more efficient programming language. Even if this is necessary, the prototype is useful because it usually helps to speed up the creative development stage.

Several meta-programs can be found in previous chapters of this book; for example, if-then rule interpreters of Chapters 15 and 16. They process the language of if-then rules, which is, in fact, a programming language, although programs written in it are usually called knowledge bases because of their specific contents. Another example is the interpreter for hypotheses in ILP in Chapter 19. In this chapter we show some further examples to illustrate how easily meta-programs can be written in Prolog:

- writing Prolog meta-interpreters;
- explanation-based generalization;
- implementing other programming paradigms in Prolog, in particular object-oriented programming and pattern-directed programming.

23.2 Prolog meta-interpreters

23.2.1 Basic Prolog meta-interpreter

A Prolog meta-interpreter takes a Prolog program and a Prolog goal, and executes the goal with respect to the program; that is, the meta-interpreter attempts to prove that the goal logically follows from the program. However, to be of any practical interest, the meta-interpreter must not behave exactly as the original Prolog interpreter; it has to offer some additional functionality, such as generating a proof tree or tracing the execution of programs.

We will, for simplicity, assume that the program has already been consulted by the Prolog system that executes the meta-interpreter. So our meta-interpreter can be stated as a procedure **prove** with one argument, the goal to be satisfied:

```
prove( Goal)
```

The simplest Prolog meta-interpreter is trivial:

```
prove( Goal) :-
  call( Goal).
```

Here all the work has been delegated (by **call**) to the original Prolog interpreter, and our meta-interpreter behaves exactly like the original Prolog interpreter. This is of course of no practical value because it does not provide any additional feature. To enable features such as the generation of proof trees, we first have to reduce the 'grain size' of the interpreter. This reduction in granularity of the meta-interpreter is made possible by a built-in predicate, provided in many Prolog implementations:

```
clause( Head, Body)
```

This 'retrieves' a clause from the consulted program. **Head** is the head of the retrieved clause and **Body** is its body. For a unit clause (a fact), **Body = true**. In a non-unit clause (a rule), the body can contain one or several goals. If it contains one goal, then **Body** is this goal. If the body contains several goals then they are retrieved as a pair:

```
Body = ( FirstGoal , OtherGoals)
```

The comma in this term is a built-in infix operator. In the standard Prolog notation, this pair is equivalently written as:

```
,( FirstGoal, OtherGoals)
```

Here **OtherGoals** may again be a pair consisting of another goal and remaining goals. In a call **clause(Head, Body)**, the first argument **Head** must not be a variable. Suppose the consulted program contains the usual **member** procedure. Then the clauses of **member** can be retrieved by:

```
?- clause( member( X, L), Body).

X = _14
L = [_14 | _15]
Body = true;

X = _14
Y = [_15 | _16]
Body = member( _14, _16)
```

Figure 23.1 shows a basic meta-interpreter for Prolog at the level of granularity that has proved to be useful for most purposes. It should be noted, however, that this is a

```
% The basic Prolog meta-interpreter

prove( true).

prove( ( Goal1, Goal2)) :-
  prove( Goal1),
  prove( Goal2).

prove( Goal) :-
  clause( Goal, Body),
  prove( Body).
```

Figure 23.1 The basic Prolog meta-interpreter.

meta-interpreter for pure Prolog only. It does not handle built-in predicates, in particular the cut. The usefulness of this basic meta-interpreter lies in the fact that it provides a scheme that can be easily modified to obtain interesting effects. One such well-known extension results in a trace facility for Prolog. Another possibility is to prevent the Prolog interpreter from getting into infinite loops, by limiting the depth of subgoal calls (Exercise 23.2).

Exercises

23.1 What happens if we try to execute the meta-interpreter of Figure 23.1 with itself – for example, by:

```
?- prove( prove( member( X, [ a, b, c]))).
```

There is a problem because our meta-interpreter cannot execute built-in predicates such as **clause**. How could the meta-interpreter be easily modified to be able to execute itself, as in the query above?

23.2 Modify the meta-interpreter of Figure 23.1 by limiting the depth of Prolog's search for proof. Let the modified meta-interpreter be the predicate **prove(Goal, DepthLimit)**, which only succeeds if **DepthLimit** $\geqq 0$. Each recursive call reduces the limit.

23.2.2 A tracing meta-interpreter

A first attempt to extend the basic meta-interpreter of Figure 23.1 to a tracing interpreter is:

```
prove( true) :- !.

prove( ( Goal1, Goal2)) :- !,
  prove( Goal1),
  prove( Goal2).

prove( Goal) :-
  write( 'Call: '), write( Goal), nl,
  clause( Goal, Body),
  prove( Body),
  write( 'Exit: '), write( Goal), nl.
```

The cuts are needed here to prevent the display of 'true' and composite goals of the form **(Goal1, Goal2)**. This tracer has several defects: there is no trace of failed goals and no indication of backtracking when the same goal is re-done. The tracer in Figure 23.2 is an improvement in these respects. To aid readability, it also indents the displayed goals proportionally to the depth of inference at which they are called. It is, however, still restricted to pure Prolog only. An example call of this tracer is:

```
?- trace( ( member( X, [ a, b]), member( X, [ b, c]))).

Call: member( _0085, [ a, b])
Exit: member( a, [ a, b])
Call: member( a, [ b, c])
  Call: member( a, [ c])
    Call: member( a, [ ])
    Fail: member( a, [ ])
  Fail: member( a, [ c])
Fail: member( a, [ b, c])
Redo: member( a, [ a, b])
  Call: member( _0085, [ b])
  Exit: member( b, [ b])
Exit: member( b, [ a, b])
Call: member( b, [ b, c])
Exit: member( b, [ b, c])
```

```
% trace( Goal): execute Prolog goal Goal displaying trace information

trace( Goal) :-
  trace( Goal, 0).

trace( true, Depth) :- !.                       % Red cut; Depth = depth of call

trace( ( Goal1, Goal2), Depth) :- !,           % Red cut
  trace( Goal1, Depth),
  trace( Goal2, Depth).

trace( Goal, Depth) :-
  display( 'Call: ', Goal, Depth),
  clause( Goal, Body),
  Depth1 is Depth + 1,
  trace( Body, Depth1),
  display( 'Exit: ', Goal, Depth),
  display_redo( Goal, Depth).

trace( Goal, Depth) :-                          % All alternatives exhausted
  display( 'Fail: ', Goal, Depth),
  fail.

display( Message, Goal, Depth) :-
  tab( Depth), write( Message),
  write( Goal), nl.

display_redo( Goal, Depth) :-
  true                                          % First succeed simply
  ;
  display( 'Redo: ', Goal, Depth),              % Then announce backtracking
  fail.                                         % Force backtracking
```

Figure 23.2 A Prolog meta-interpreter for tracing programs in pure Prolog.

This tracer outputs the following information for each goal executed:

(1) The goal to be executed (**Call: Goal**).

(2) Trace of the subgoals (indented).

(3) If the goal is satisfied then its final instantiation is displayed (**Exit: InstantiatedGoal**); if the goal is not satisfied then **Fail: Goal** is displayed.

(4) In the case of backtracking to a previously satisfied goal, the message is: **Redo: InstantiatedGoal** (instantiation in the previous solution of this goal).

Of course, it is possible to further shape the tracing interpreter according to specific users' requirements.

23.2.3 Generating proof trees

Another well-known extension of the basic interpreter of Figure 23.1 is the generation of proof trees. So after a goal is satisfied, its proof tree is available for further processing. In Chapters 15 and 16, the generation of proof trees was implemented for rule-based expert systems. Although the syntax of rules there was different from Prolog, the principles of generating a proof tree are the same. These principles are easily introduced into the meta-interpreter of Figure 23.1. For example, we may choose to represent a proof tree depending on the case as follows:

(1) For a goal **true**, the proof tree is **true**.

(2) For a pair of goals (**Goal1, Goal2**), the proof tree is the pair (**Proof1, Proof2**) of the proof trees of the two goals.

(3) For a goal **Goal** that matches the head of a clause whose body is **Body**, the proof tree is **Goal <== Proof**, where **Proof** is the proof tree of **Body**.

This can be incorporated into the basic meta-interpreter of Figure 23.1 as follows:

```
:- op( 500, xfy, <==).

prove( true, true).

prove( ( Goal1, Goal2), ( Proof1, Proof2)) :-
  prove( Goal1, Proof1),
  prove( Goal2, Proof2).

prove( Goal, Goal <== Proof) :-
  clause( Goal, Body),
  prove( Body, Proof).
```

Such a proof tree can be used in various ways. In Chapters 15 and 16 it was used as the basis for generating the 'how' explanation in an expert system. In the next section we will look at another interesting use of a proof tree, explanation-based generalization.

23.3 Explanation-based generalization

The idea of explanation-based generalization comes from machine learning, where the objective is to generalize given examples into general descriptions of concepts. *Explanation-based generalization* (EBG) is a way of building such descriptions from typically one example only. The lack of examples is compensated for by the system's 'background knowledge', usually called *domain theory*.

EBG rests on the following idea for building generalized descriptions: given an instance of the target concept, use the domain theory to *explain how* this instance in fact satisfies the concept. Then analyze the explanation and try to generalize it so that it applies not only to the given instance, but also to a set of 'similar' instances. This generalized explanation then becomes part of the concept description and can be subsequently used in recognizing instances of this concept. It is also required that the constructed concept description must be 'operational'; that is, it must be stated in terms of concepts declared by the user as operational. Intuitively, a concept description is operational if it is (relatively) easy to use. It is entirely up to the user to specify what is operational.

In an implementation of EBG, these abstract ideas have to be made more concrete. One way of realizing them in the logic of Prolog is:

- A concept is realized as a predicate.
- A concept description is a predicate definition.
- An explanation is a proof tree that demonstrates how the given instance satisfies the target concept.
- A domain theory is represented as a set of available predicates defined as a Prolog program.

The task of explanation-based generalization can then be stated as:

Given:

A domain theory: A set of predicates available to the explanation-based generalizer, including the target predicate whose operational definition is to be constructed.

Operationality criteria: These specify the predicates that may be used in the target predicate definition.

Training example: A set of facts describing a particular situation and an instance of the target concept, so that this instance can be derived from the given set of facts and the domain theory.

Find:

A generalization of the training instance and an operational definition of the target concept; this definition consists of a sufficient condition (in terms of the operational predicates) for this generalized instance to satisfy the target concept.

Thus stated, explanation-based generalization can be viewed as a kind of program compilation from one form into another. The original program defines the target concept in terms of domain theory predicates. The compiled program defines the same target concept (or subconcept) in terms of the 'target language' – that is, operational predicates only. The compilation mechanism provided by EBG is rather unusual. Execute the original program on the given example, which results in a proof tree. Then generalize this proof tree so that the structure of the proof tree is retained, but the constants are replaced by variables whenever possible. In the generalized proof tree thus obtained, some nodes mention operational predicates. The tree is then reduced so that only these 'operational nodes' and the root are retained. The result constitutes an operational definition of the target concept.

All this is best understood by an example of EBG at work. Figure 23.3 defines two domains for EBG. The first domain theory is about giving a gift, while the second is about lift movements. Let us consider the first domain. Let the training instance be:

```
gives( john, john, chocolate)
```

Our proof-generating meta-interpreter finds this proof:

```
gives( john, john, chocolate) <==
  ( feels_sorry_for( john, john) <== sad( john) ,
    would_comfort( chocolate, john) <== likes( john, chocolate)
  )
```

```
% A domain theory: about gifts

gives( Person1, Person2, Gift) :-
  likes( Person1, Person2),
  would_please( Gift, Person2).

gives( Person1, Person2, Gift) :-
  feels_sorry_for( Person1, Person2),
  would_comfort( Gift, Person2).

would_please( Gift, Person) :-
  needs( Person, Gift).

would_comfort( Gift, Person) :-
  likes( Person, Gift).

feels_sorry_for( Person1, Person2) :-
  likes( Person1, Person2),
  sad( Person2).

feels_sorry_for( Person, Person) :-
  sad( Person).
```

Figure 23.3 Two problem definitions for explanation-based generalization.

Figure 23.3 *contd*

```
% Operational predicates

operational( likes( _, _ )).
operational( needs( _, _ )).
operational( sad( _ )).

% An example situation

likes( john, annie).
likes( annie, john).
likes( john, chocolate).
needs( annie, tennis_racket).
sad( john).

% Another domain theory: about lift movement

% go( Level, GoalLevel, Moves) if
%    list of moves Moves brings lift from Level to GoalLevel

go( Level, GoalLevel, Moves) :-
  move_list( Moves, Distance),              % A move list and distance travelled
  Distance =:= GoalLevel - Level.

move_list( [ ], 0).

move_list( [Move1 | Moves], Distance + Distance1) :-
  move_list( Moves, Distance),
  move( Move1, Distance1).

move( up, 1).
move( down, -1).

operational( A =:= B).
```

This proof can be generalized by replacing constants **john** and **chocolate** by variables:

```
gives( Person, Person, Thing) <==
  ( feels_sorry_for( Person, Person) <== sad( Person) ,
    would_comfort( Thing, Person) <== likes( Person, Thing)
  )
```

Predicates **sad** and **likes** are specified as operational in Figure 23.3. An operational definition of the predicate **gives** is now obtained by eliminating all the nodes from the proof tree, apart from the 'operational' ones and the root. This results in:

```
gives( Person, Person, Thing) <==
  ( sad( Person) ,
    likes( Person, Thing)
  )
```


Thus a sufficient condition **Condition** for **gives(Person, Person, Thing)** is:

```
Condition = ( sad( Person), likes( Person, Thing))
```

This new definition can now be added to our original program with:

```
asserta( ( gives( Person, Person, Thing) :- Condition))
```

As a result, we have the following new clause about **gives** that only requires the evaluation of operational predicates:

```
gives( Person, Person, Thing) :-
  sad( Person),
  likes( Person, Thing).
```

Through the generalization of the given instance

```
gives( john, john, chocolate)
```

a definition (in operational terms) of giving as self-consolation was derived as one general case of **gives**. Another case of this concept would result from the example:

```
gives( john, annie, tennis_racket).
```

The explanation-based generalization would in this case produce the clause:

```
gives( Person1, Person2, Thing) :-
  likes( Person1, Person2),
  needs( Person2, Thing).
```

The lift domain in Figure 23.3 is slightly more complicated and we will experiment with it when we have EBG implemented in Prolog.

EBG can be programmed as a two-stage process: first, generate a proof tree for the given example, and, second, generalize this proof tree and extract the 'operational nodes' from it. Our proof-generating meta-interpreter could be used for this. The two stages are, however, not necessary. A more direct way is to modify the basic meta-interpreter of Figure 23.1 so that the generalization is intertwined with the process of proving the given instance. The so-modified meta-interpreter, which carries out EBG, will be called **ebg** and will now have three arguments:

```
ebg( Goal, GenGoal, Condition)
```

where **Goal** is the given example to be proved, **GenGoal** is the generalized goal and **Condition** is the derived sufficient condition for **GenGoal**, stated in terms of operational predicates. Figure 23.4 shows such a generalizing meta-interpreter. For our gift domain of Figure 23.3, **ebg** can be called as:

```
?- ebg( gives( john, john, chocolate), gives( X, Y, Z), Condition).

X = Y
Condition = ( sad( X), likes( X, Z))
```

```
% ebg( Goal, GeneralizedGoal, SufficientCondition):
%   SufficientCondition in terms of operational predicates guarantees that
%   generalization of Goal, GeneralizedGoal, is true.
%   GeneralizedGoal must not be a variable

ebg( true, true, true).

ebg( Goal, GenGoal, GenGoal) :-
  operational( GenGoal),
  call( Goal).

ebg( ( Goal1, Goal2), ( Gen1, Gen2), Cond) :- !,
  ebg( Goal1, Gen1, Cond1),
  ebg( Goal2, Gen2, Cond2),
  and( Cond1, Cond2, Cond).                % Cond = ( Cond1, Cond2) simplified

ebg( Goal, GenGoal, Cond) :-
  not operational( Goal),
  clause( GenGoal, GenBody),
  copy_term( ( GenGoal, GenBody), ( Goal, Body)),  % Fresh copy of ( GenGoal, GenBody)
  ebg( Body, GenBody, Cond).

% and( Cond1, Cond2, Cond):
%   Cond is (possibly simplified) conjunction of Cond1 and Cond2

and( true, Cond, Cond) :- !.               % (true and Cond) <==> Cond

and( Cond, true, Cond) :- !.               % (Cond and true) <==> Cond

and( Cond1, Cond2, ( Cond1, Cond2)).
```

Figure 23.4 Explanation-based generalization.

Let us now try the lift domain. Let the goal to be solved and generalized be to find a move sequence Moves that brings the lift from level 3 to level 6:

```
go( 3, 6, Moves)
```

We can invoke ebg and cash the resulting generalized goal and its condition by:

```
?- Goal = go( 3, 6, Moves),
   GenGoal = go( Level1, Level2, GenMoves),
   ebg( Goal, GenGoal, Condition),
   asserta( ( GenGoal :- Condition) ).

   Goal = go( 3, 6, [ up, up, up])
   GenGoal = go( Level1, Level2, [ up, up, up])
   Condition = ( 0 + 1 + 1 + 1 =:= Level2 - Level1)
   Moves = [ up, up, up]
   ...
```


The resulting new clause about go is:

```
go( Level1, Level2, [ up, up, up]) :-
  0 + 1 + 1 + 1 =:= Level2 - Level1.
```

By means of EBG, the straight three-step upward movement has thus been generalized to any two levels at distance three. To solve the goal go(3, 6, Moves), the original program performs a search among sequences of up/down actions. Using the new derived clause, moving up between any two levels at distance three (for example, go(7, 10, Moves)) is solved immediately, without search.

The ebg meta-interpreter in Figure 23.4 is, again, a derivation of the basic meta-interpreter of Figure 23.1. This new meta-interpreter calls the built-in procedure copy_term. The call

```
copy_term( Term, Copy)
```

constructs, for a given term Term, a copy Copy of this term with variables renamed. This is useful when we want to preserve a term as it is, while at the same time processing the term so that its variables may become instantiated. We can use the copy in this processing, so that the variables in the original term remain unaffected.

The last clause of the ebg procedure deserves special explanation. The call

```
clause( GenGoal, GenBody)
```

retrieves a clause that can be used to prove the generalized goal. This means that the meta-interpreter is actually trying to prove the generalized goal. The next line, however, imposes a constraint on this:

```
copy_term( ( GenGoal, GenBody), ( Goal, Body))
```

This requires that GenGoal and Goal match. The matching is done on the copy of GenGoal, so that variables in the general goal remain intact. The point of requiring this match is to limit the execution of the generalized goal to those alternative branches (clauses) that are applicable to the given example (Goal). In this way the example guides the execution of the generalized goal. This guidance is the essence of program compilation in the EBG style. Without this guidance, a proof for GenGoal could possibly be found that does not work for Goal. In such a case the generalization would not correspond to the example at all.

Once an example has been generalized and stated in terms of operational predicates, it can be added as a new clause to the program to answer future similar questions by evaluating operational predicates only. In this way the EBG compilation technique transforms a program into an 'operational' language. The translated program can be executed by an interpreter that only 'knows' the operational language. One possible advantage of this can be a more efficient program. The efficiency may be improved in two ways: first, operational predicates may be easier to evaluate than other predicates; second, the sequence of predicate evaluation indicated by the example may be more suitable than the one in the original program, so that failed branches do not appear in the compiled definition at all. When a program is compiled by the EBG technique, care is needed during compilation so that new clauses do not interfere in an uncontrolled way with the original clauses.

Exercise

23.3 It may appear that the same compilation effect achieved in EBG can be obtained without an example, simply by substituting goals in the original concept definition by their subgoals, taken from corresponding clauses of the domain theory, until all the goals are reduced to operational subgoals. This procedure is called *unfolding* (a goal 'unfolds' into subgoals). Discuss this idea with respect to EBG and show that the guidance by an example in EBG is essential. Also show that, on the other hand, EBG-generated new concept definitions are only a generalization of given examples and are therefore not necessarily equivalent to the original program (new concept definitions may be incomplete).

23.4 Object-oriented programming

It is easy to adopt the object-oriented style of programming in Prolog. We will demonstrate this in this section by a simple interpreter for object-oriented programs. A program in the object-oriented paradigm is stated in terms of *objects* that *send messages* to each other. Computation occurs as a result of objects responding to these messages. Each object has some private memory and some procedures called *methods*. An object can also *inherit* a method from another object. This is similar to the inheritance mechanism in frame-based representation discussed in Chapter 15. An object responds to a message by executing one of the object's methods. The message determines which method is to be executed. In fact, the act of sending a message to an object is a kind of procedure call. Implementing an object-oriented program amounts to simulating the sending of messages between objects and objects responding to the messages.

To translate this paradigm into Prolog terms, let us consider as a first example an object-oriented program about geometric figures. One object in such a program is a rectangle whose methods can be: a procedure to describe itself, and a procedure to compute its area. If a rectangle is sent the message **area(A)** it responds by calculating its area and the variable A becomes instantiated to the area. In our implementation, objects will be represented as the relation:

```
object( Object, Methods)
```

where **Object** is a term that names the object (and possibly specifies its parameters) and **Methods** is a list of Prolog terms specifying the methods. The terms in this list

have the form of Prolog clauses – that is, Prolog facts and rules (except that they do not end with a period). In our Prolog implementation, an object definition will possibly specify a whole class of objects, such as the class of all rectangles referred to as **rectangle(Length, Width)**. A particular rectangle with sides 4 and 3 is then referred to as **rectangle(4, 3)**. In general, then, the object **rectangle(Length, Width)** with two methods **area** and **describe** can be defined by:

```
object( rectangle( Length, Width),
  [ ( area( A) :-
       A is Length * Width),
    ( describe :-
       write( 'Rectangle of size '),
       write( Length * Width)) ] ).
```

In our implementation, the sending of a message to an object will be simulated by the procedure:

```
send( Object, Message)
```

We can make the rectangle with sides 4 and 3 decribe itself and compute its area by sending it the corresponding messages:

```
?- Rec1 = rectangle( 4, 3),
   send( Rec1, describe),
   send( Rec1, area( Area)).

Rectangle of size 4 * 3
Area = 12
```

The procedure **send(Object, Message)** is easy to program. We first have to retrieve the methods of **Object**. These methods in fact specify a Prolog program that is local to **Object**. We have to execute **Message** as a goal with this program. The program, represented as a list, consists of items of the form **Head :- Body**, or just **Head** in the case of empty body. To execute **Message** we have to find some head that matches **Message** and then execute the corresponding body with Prolog's own interpreter.

Before we program this, we have to consider the other essential mechanism of object-oriented programming: *inheritance* of methods according to the 'is a' relation between objects. For example, a square is a rectangle. For calculating its area, a square can use the same formula as the rectangles. So a square does not need its own **area** method; it may inherit this method from the class of rectangles. To make this happen, we have to define the object **square** and also state that it is a rectangle.

```
object( square( Side),
  [ ( describe :-
       write( 'Square with side '),
       write( Side)) ] ).

isa( square( Side), rectangle( Side, Side)).
```

Now:

```
?- send( square( 5), area( Area)).

Area = 25
```

The message **area(Area)** is processed as follows: first, the object **square(5)** searches for **area(Area)** among its methods and cannot find it. Then through the **isa** relation it finds its 'super-object' **rectangle(5, 5)**. The super-object has the relevant method **area** which is executed.

An interpreter for object-oriented programs along the lines discussed here is given in Figure 23.5. Figure 23.6 provides a completed object-oriented program about geometric figures.

Until now we have not mentioned the problem of *multiple inheritance*. This arises when the **isa** relation defines a *lattice* so that an object has more than one parent object, as is the case for **square** in Figure 23.6. So more than one parent object may potentially supply a method to be inherited by the object. In such a case the question is: Which one of the several potentially inherited methods should be used? The program of Figure 23.5 searches for an applicable method among the objects in the graph defined by the **isa** relation. The search strategy in Figure 23.5 is simply depth first although some other strategy may be more appropriate. Breadth first would, for example, ensure that the 'closest inheritable' method is used.

```
% An interpreter for object-oriented programs

% send( Message, Object):
%    find Object's methods and execute the method that corresponds to Message

send( Object, Message) :-
  get_methods( Object, Methods),               % Find Object's methods
  process( Message, Methods).                  % Execute corresponding method

get_methods( Object, Methods) :-
  object( Object, Methods).                    % Private methods

get_methods( Object, Methods) :-
  isa( Object, SuperObject),
  get_methods( SuperObject, Methods).          % Inherited methods

process( Message, [Message | _ ]).             % Use a fact

process( Message, [ (Message :- Body) | _ ]) :-   % Use a rule
  call( Body).

process( Message, [ _ | Methods]) :-
  process( Message, Methods).
```

Figure 23.5 A simple interpreter for object-oriented programs.


```
/*
                    polygon( [ Side1, Side2, ...])
                       /                  \
     rectangle( Length, Width)        reg_polygon( Side, N)
                       \                /        \
                      square( Side)        pentagon( Side)
*/

object( polygon( Sides),                          % Polygon with sides Sides
  [ ( perimeter( P) :-
       sum( Sides, P))]).                         % Perimeter is the sum of sides

object( reg_polygon( Side, N),                    % Regular polygon with N sides
  [ ( perimeter( P) :-
       P is Side * N),
    ( describe :- write( 'Regular polygon'))]).

object( square( Side),
  [ ( describe :-
       write( 'Square with side '),
       write( Side))]).

object( rectangle( Length, Width),
  [ ( area( A) :-
       A is Length * Width),
    ( describe :-
       write( 'Rectangle of size '),
       write( Length * Width))]).

object( pentagon( Side),
       [ ( describe :- write( 'Pentagon'))]).

isa( square( Side), rectangle( Side, Side)).

isa( square( Side), reg_polygon( Side, 4)).

isa( rectangle( Length, Width), polygon( [ Length, Width, Length, Width])).

isa( pentagon( Side), reg_polygon( Side, 5)).

isa( reg_polygon( Side, N), polygon( L)) :-
  makelist( Side, N, L).

% makelist( Item, N, List):
%    List is the list in which Item appears N times

makelist( _, 0, []).

makelist( Item, N, [Item | List]) :-
  N > 0, N1 is N - 1,
  makelist( Item, N1, List).
```

Figure 23.6 An object-oriented program about geometric figures.

Figure 23.6 *contd*

```
% sum( ListOfNumbers, Sum):
%    Sum is the sum of numbers in ListOfNumbers

sum( [], 0).

sum( [Number | Numbers], Sum) :-
  sum( Numbers, Sum1),
  Sum is Sum1 + Number.
```

To further illustrate the style of object-oriented programming enabled by our interpreter in Figure 23.5, let us consider the situation in Figure 23.7. The figure shows a robot world: a number of blocks arranged in stacks on the table. There is a camera on the ceiling that can capture the top view of the objects on the table. For simplicity we assume that all of the blocks are cubes with side equal 1. The blocks have names *a*, *b*, ..., and the camera can recognize by name those blocks that are not obstructed from the top, and locate them with respect to the *x* and *y* coordinates. Suppose that we are interested in computing the locations of the objects – that is, their *x*, *y* and *z* coordinates. Each block has some local information, so it knows what block (if any) is immediately above it or underneath it. Thus the *xy* coordinates of a block B can be obtained in two ways:

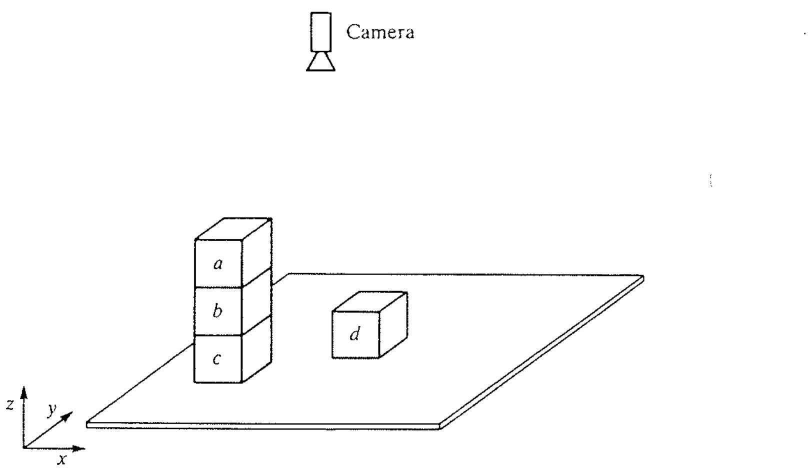

Figure 23.7 A robot world.

(1) If the block has a clear top then it can be seen by the camera, which will determine the *xy* coordinates of the block. This is accomplished by sending the message look(B, X, Y) to the camera.

(2) If block B is underneath some block B1 then it is helpful to observe that all of the blocks in the same stack share the same *xy* coordinates. So to determine the *xy* coordinates of block B, send the message xy_coord(X, Y) to block B1.

```
% A robot world: table, blocks and camera

object( camera,
  [ look( a, 1, 1),                    % Find xy coord. of a visible block
    look( d, 3, 2),
    xy_coord( 2, 2),                   % xy coordinates of camera
    z_coord( 20)]).                    % z coordinate of camera

object( block( Block),
  [ ( xy_coord( X, Y) :-
        send( camera, look( Block, X, Y))),
    ( xy_coord( X, Y) :-
        send( Block, under( Block1)),
        send( Block1, xy_coord( X, Y))),
    ( z_coord( 0) :-
        send( Block, on( table))),
    ( z_coord( Z) :-
        send( Block, on( Block1)),
        send( Block1, z_coord( Z1)),
        Z is Z1 + 1)]).

object( physical_object( Name),
  [ ( coord( X, Y, Z) :-
        send( Name, xy_coord( X, Y)),
        send( Name, z_coord( Z)))]).

object( a, [ on( b)]).
object( b, [ under( a), on( c)]).
object( c, [ under( b), on( table)]).
object( d, [ on( table)]).

isa( a, block( a)).
isa( b, block( b)).
isa( c, block( c)).
isa( d, block( d)).
isa( block( Name), physical_object( Name)).
isa( camera, physical_object( camera)).
```

Figure 23.8 An object-oriented program about a robot world.

Similar considerations lead to determining the z coordinate of a block. If block B is on the table, its z coordinate is 0. If B is on another block B1 then send the message **z_coord(Z1)** to B1 and add the height of B1 to Z1, thereby obtaining the z coordinate of B. This is programmed in Figure 23.8, where blocks a, b, etc., are defined as instances of the block class **block(BlockName)** from which they inherit the methods **xy_coord** and **z_coord**.

Let us conclude this section with a general comment regarding the benefits of object-oriented programming. Our example object-oriented programs can be implemented (in fact more compactly) in the usual Prolog style without reference to objects and messages. These examples illustrate the principles, but are too small to convincingly indicate the possible advantages of object-oriented programming. The object-oriented model is obviously advantageous in writing programs for simulation systems in which physical components in fact exchange a kind of message – for example, in a manufacturing process there are flows of material between machines. Another benefit of the object-oriented model is that it provides a useful framework for organizing large programs. Both memory and procedures (methods) are clustered around objects, and the inheritance mechanism provides a way of structuring the program.

Exercises

23.4 Study what happens when the message-sending question

```
?- send( square( 6), perimeter( P)).
```

is interpreted by the programs in Figures 23.5 and 23.6. Are there alternatives for backtracking? How can the program of Figure 23.5 be modified to prevent undesired alternatives?

23.5 The interpreter in Figure 23.5 is inefficient because it spends much time looking for appropriate methods. Write a compiler for object-oriented programs that compiles object definitions (their methods in particular) into a more efficient Prolog program. Such a compiler would, for example, in compiling the program of Figure 23.6, generate clauses like:

```
area( square( Side), Area) :-
  Area is Side * Side.

area( rectangle( Length, Width), Area) :-
  Area is Length * Width.
```

Also, redefine the procedure **send(Object, Message)** so that it would use these new generated clauses.

23.5 Pattern-directed programming

23.5.1 Pattern-directed architecture

By *pattern-directed systems* we here refer to an architecture for program systems. This architecture is better suited for certain types of problems than conventional systems organization. Among problems that naturally fit into the pattern-directed architecture are many artificial intelligence applications – for example, expert systems. The main difference between conventional systems and pattern-directed systems is in the mechanisms of invocation of program modules. In conventional organization, modules of the system call each other according to a fixed, explicitly predefined scheme. Each program module decides which module will be executed next by *explicitly* calling other modules. The corresponding flow of execution is sequential and deterministic.

In contrast to this, in pattern-directed organization the modules of the system are not directly called by other modules. Instead, they are 'called' by *patterns* that occur in their 'data environment'. Therefore such modules are called *pattern-directed modules*. A *pattern-directed program* is a collection of pattern-directed modules. Each module is defined by:

(1) a precondition pattern, and

(2) an action to be executed if the data environment matches the pattern.

The execution of program modules is triggered by patterns that occur in the system's environment. The data environment is usually called the *database*. We can imagine such a system as shown in Figure 23.9.

There are some notable observations about Figure 23.9. There is no hierarchy among modules, and there is no explicit indication about which module can invoke which other module. Modules communicate with the database rather than with other modules directly. The structure itself, in principle, permits execution of several modules in parallel, since the state of the database may simultaneously satisfy several preconditions and thus, in principle, fire several modules at the same time. Consequently such an organization can also serve as a natural model of parallel computation in which each module would be physically implemented by its own processor.

Pattern-directed architecture has certain advantages. One major advantage is that the design of the system does not require all the connections between modules to be carefully planned and defined in advance. Consequently, each module can be designed and implemented relatively autonomously. This renders a high degree of modularity. The modularity is manifested, for example, in that the removal of some module from the system is not necessarily fatal. After the removal, the system would often still be able to solve problems, only *the way* of solving problems might change.

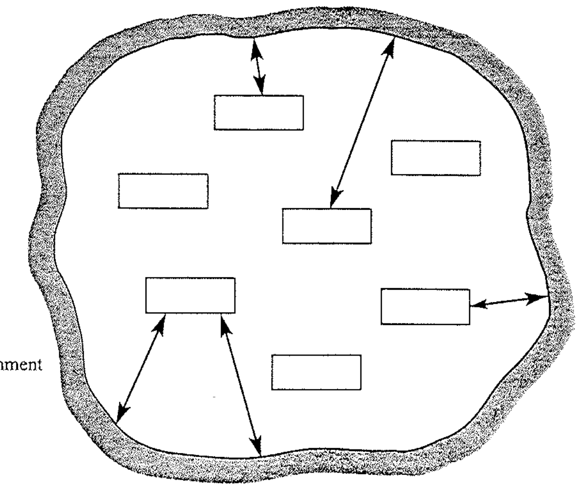

Figure 23.9 A pattern-directed system. Rectangles represent pattern-directed modules. Arrows indicate modules' triggering patterns occurring in data.

The same is true for the addition of new modules and for modifications of the existing modules. If similar modifications are carried out in systems with conventional organization, at least the calls between modules have to be properly modified.

The high degree of modularity is especially desirable in systems with complex knowledge bases because it is difficult to predict in advance all the interactions between individual pieces of knowledge in the base. The pattern-directed architecture offers a natural solution to this: each piece of knowledge, represented by an if-then rule, can be regarded as a pattern-directed module.

Let us further elaborate the basic scheme of pattern-directed systems with the view on an implementation. Figure 23.9 suggests that the parallel implementation would be most natural. However, let us assume the system is to be implemented on a traditional sequential processor. Then in a case that the triggering patterns of several modules simultaneously occur in the database there is a conflict: which of all these potentially active modules will actually be executed? The set of potentially active modules is called a *conflict set*. In an actual implementation of the scheme of Figure 23.9 on a sequential processor, we need an additional program module, called the *control module*. The control module resolves the conflict by choosing and activating one of the modules in the conflict set. One simple rule of resolving conflicts can be based on a predefined, fixed ordering of modules.

The basic life cycle of pattern-directed systems, then, consists of three steps:

(1) *Pattern matching*: find in the database all the occurrences of the condition patterns of the program modules. This results in a conflict set.

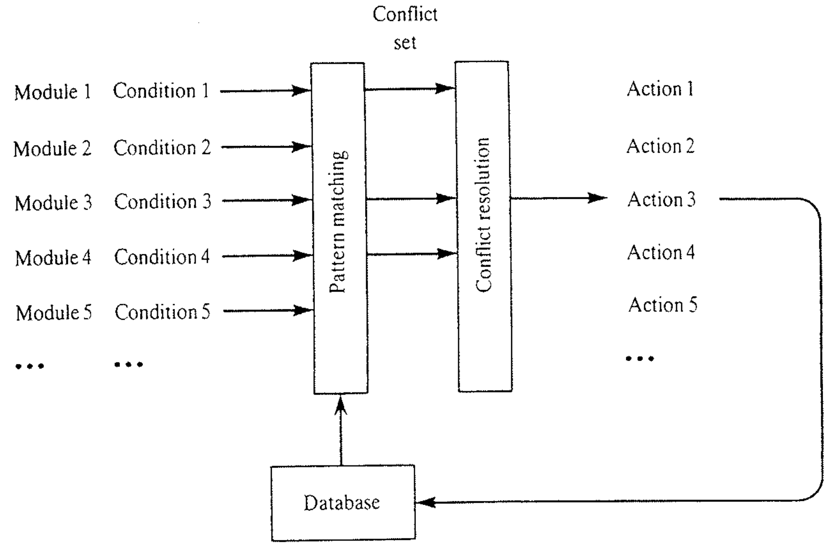

Figure 23.10 The basic life cycle of pattern-directed systems. In this example the database satisfies the condition pattern of modules 1, 3 and 4; module 3 is chosen for execution.

(2) *Conflict resolution*: choose one of the modules in the conflict set.

(3) *Execution*: execute the module that was chosen in step 2.

This implementational scheme is illustrated in Figure 23.10.

23.5.2 Prolog programs as pattern-directed systems

Prolog programs themselves can be viewed as pattern-directed systems. Without much elaboration, the correspondence between Prolog and pattern-directed systems is along the following lines:

- Each Prolog clause in the program can be viewed as a pattern-directed module. The module's condition part is the head of the clause, the action part is specified by the clause's body.
- The system's database is the current list of goals that Prolog is trying to satisfy.
- A clause is fired if its head matches the first goal in the database.
- To execute a module's action (body of a clause) means: replace the first goal in the database with the list of goals in the body of the clause (with the proper instantiation of variables).

- The process of module invocation is non-deterministic in the sense that several clauses' heads may match the first goal in the database, and any one of them can, in principle, be executed. This non-determinism is actually implemented in Prolog through backtracking.

23.5.3 Writing pattern-directed programs: an example

Pattern-directed systems can also be viewed as a particular style of writing programs and thinking about problems, called *pattern-directed programming*.

To illustrate this, consider an elementary programming exercise: computing the greatest common divisor D of two integer numbers A and B. The classical Euclid's algorithm can be written as follows:

To compute the greatest common divisor, D, of A and B:

While A and B are not equal, repeat the following:

if $A > B$ then replace A with $A - B$
else replace B with $B - A$.

When this loop is over, A and B are equal; now the greatest common divisor D is A (or B).

We can define the same process by two pattern-directed modules:

Module 1
Condition There are two numbers X and Y in the database such that $X > Y$.
Action Replace X in the database with the difference $X - Y$.

Module 2
Condition There is a number X in the database.
Action Output X and stop.

Whenever the condition of Module 1 is satisfied, so is the condition of Module 2 and we have a conflict. This will be resolved by a simple control rule: Module 1 is always preferred to Module 2. Initially the database contains the two numbers A and B.

As a pleasant surprise, our pattern-directed program in fact solves a more general problem: computing the greatest common divisor of any number of integers. If several integers are stored in the database the system will output the greatest common divisor of all of them. Figure 23.11 shows a possible sequence of changes in the database before the result is obtained, when the initial database contains four numbers: 25, 10, 15, 30. Notice that a module's precondition can be satisfied at several places in the database.

In this chapter we will implement an interpreter for a simple language for specifying pattern-directed systems, and illustrate the flavour of pattern-directed programming by programming exercises.

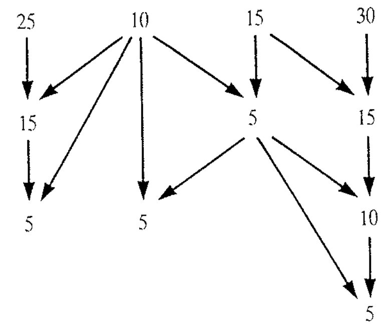

Figure 23.11 A possible execution of the pattern-directed program for computing the greatest common divisor of a set of numbers. In this example tha database initially contains the numbers 25, 10, 15 and 30. Vertical arrows connect numbers with their replacements. The final state of the database is: 5, 5, 5, 5.

23.5.4 A simple interpreter for pattern-directed programs

Let us choose the following syntax for specifying pattern-directed modules:

ConditionPart ---> ActionPart

The condition part is a list of conditions

[Cond1, Cond2, Cond3, ...]

where **Cond1**, **Cond2**, etc. are simply Prolog goals. The precondition is satisfied if all the goals in the list are satisfied. The action part is a list of actions:

[Action1, Action2, ...]

Each action is, again, simply a Prolog goal. To execute an action list, all the actions in the list have to be executed. That is, all the corresponding goals have to be satisfied. Among available actions there will be actions that manipulate the database: *add*, *delete* or *replace* objects in the database. The action 'stop' stops further execution.

Figure 23.12 shows our pattern-directed program for computing the greatest common divisor written in this syntax.

The simplest way to implement this pattern-directed language is to use Prolog's own built-in database mechanism. Adding an object into the database and deleting an object can be accomplished simply by the built-in procedures:

assertz(Object) **retract(Object)**

```
% Production rules for finding greatest common divisor (Euclid algorithm)

:- op( 300, fx, number).

[ number X, number Y, X > Y ] --->
[ NewX is X - Y, replace( number X, number NewX) ].

[ number X] ---> [ write( X), stop ].

% An initial database

number 25.

number 10.

number 15.

number 30.
```

Figure 23.12 A pattern-directed program to find the greatest common divisor of a set of numbers.

Replacing an object with another object is also easy:

```
replace( Object1, Object2) :-
  retract( Object1), !,
  assertz( Object2).
```

The cut in this clause is used to prevent **retract** from deleting (through backtracking) more than one object from the database.

A small interpreter for pattern-directed programs along these lines is shown in Figure 23.13. This interpreter is perhaps an oversimplification in some respects. In particular, the conflict resolution rule in the interpreter is extremely simple and rigid: always execute the *first* potentially active pattern-directed module (in the order as they are written). So the programmer's control is reduced just to the ordering of modules. The initial state of the database for this interpreter has to be asserted as Prolog facts, possibly by consulting a file. Then the execution is triggered by the goal:

```
?- run.
```

23.5.5 Possible improvements

Our simple interpeter for pattern-directed programs is sufficient for illustrating some ideas of pattern-directed programming. For more complex applications it should be elaborated in several respects. Here are some critical comments and indications for improvements.

In our interpreter, the conflict resolution is reduced to a fixed, predefined order. Much more flexible schemas are often desired. To enable more sophisticated control, all the potentially active modules should be found and fed into a special user-programmable control module.


```
% A small interpreter for pattern-directed programs
% The system's database is manipulated through assert/retract

:- op( 800, xfx, --->).

% run: execute pattern-directed modules until action 'stop' is triggered

run :-
  Condition ---> Action,                 % A production rule
  test( Condition),                      % Precondition satisfied?
  execute( Action).

% test( [ Condition1, Condition2, ...]) if all conditions true

test( []).                               % Empty condition

test( [First | Rest]) :-                 % Test conjunctive condition
  call( First),
  test( Rest).

% execute( [ Action1, Action2, ...]): execute list of actions

execute( [ stop]) :- !.                  % Stop execution

execute( []) :-                          % Empty action (execution cycle completed)
  run.                                   % Continue with next execution cycle

execute( [First | Rest]) :-
  call( First),
  execute( Rest).

replace( A, B) :-                        % Replace A with B in database
  retract( A), !,                        % Retract once only
  assertz( B).
```

Figure 23.13 A small interpreter for pattern-directed programs.

When the database is large and there are many pattern-directed modules in the program then pattern matching can become extremely inefficient. The efficiency in this respect can be improved by a more sophisticated organization of the database. This may involve the indexing of the information in the database, or partitioning of the information into sub-bases, or partitioning of the set of pattern-directed modules into subsets. The idea of partitioning is to make only a *subset* of the database or of the modules accessible at any given time, thus reducing the pattern matching to such a subset only. Of course, in such a case we would need a more sophisticated control mechanism that would control the transitions between these subsets in the sense of activating and de-activating a subset. A kind of meta-rules could be used for that.

Unfortunately our interpreter, as programmed, precludes any backtracking due to the way that the database is manipulated through **assertz** and **retract**. So we cannot study alternative execution paths. This can be improved by using a different implementation of the database, avoiding Prolog's **assertz** and **retract**. One way would be to represent the whole state of the database by a Prolog term passed as an argument to the **run** procedure. The simplest possibility is to organize this term as a list of objects in the database. The interpreter's top level could then look like this:

```
run( State) :-
   Condition ---> Action,
   test( Condition, State),
   execute( Action, State).
```

The **execute** procedure would then compute a new state and call **run** with this new state.

Project

Implement an interpreter for pattern-directed programs that does not maintain its database as Prolog's own internal database (with **assertz** and **retract**), but as a procedure argument according to the foregoing remark. Such a new interpreter would allow for automatic backtracking. Try to design a representation of the database that would facilitate efficient pattern matching.

23.6 A simple theorem prover as a pattern-directed program

Let us implement a simple theorem prover as a pattern-directed system. The prover will be based on the *resolution principle*, a popular method for mechanical theorem proving. We will limit our discussion to proving theorems in the simple *propositional logic* just to illustrate the principle, although our resolution mechanism will be easily extendable to handle the first-order predicate calculus (logic formulas that contain variables). Basic Prolog itself is a special case of a resolution-based theorem prover.

The theorem-proving task can be defined as: given a formula, show that the formula is a theorem; that is, the formula is always true regardless of the interpretation of the symbols that occur in the formula. For example, the formula

$$p \vee \sim p$$

read as 'p or not p', is always true regardless of the meaning of p.

We will be using the following symbols as logic operators:

~	negation, read as 'not'
&	conjunction, read as 'and'
∨	disjunction, read as 'or'
=>	implication, read as 'implies'

The precedence of these operators is such that 'not' binds strongest, then 'and', then 'or', and then 'implies'.

In the resolution method we negate the conjectured theorem and then try to show that this negated formula is a contradiction. If the negated formula is in fact a contradiction then the original formula must be a tautology. Thus the idea is: demonstrating that the negated formula is a contradiction is equivalent to proving that the original formula is a theorem (always holds). The process that aims at detecting the contradiction consists of a sequence of *resolution steps*.

Let us illustrate the principle with a simple example. Suppose we want to prove that the following propositional formula is a theorem:

$$(a => b) \;\&\; (b => c) => (a => c)$$

This formula is read as: if b follows from a, and c follows from b, then c follows from a.

Before the resolution process can start we have to get our negated, conjectured theorem into a form that suits the resolution process. The suitable form is the *conjunctive normal form*, which looks like this:

$$(p_1 \vee p_2 \vee \ldots) \;\&\; (q_1 \vee q_2 \vee \ldots) \;\&\; (r_1 \vee r_2 \vee \ldots) \;\&\; \ldots$$

Here all p's, q's and r's are simple propositions or their negations. This form is also called the *clause form*. Each conjunct is called a *clause*. So $(p_1 \vee p_2 \vee \ldots.)$ is a clause.

We can transform any propositional formula into this form. For our example theorem, this transformation can proceed as follows. The theorem is

$$(a => b) \;\&\; (b => c) => (a => c)$$

The negated theorem is:

$$\sim((a => b) \;\&\; (b => c) => (a => c))$$

The following known equivalence rules will be useful when transforming this formula into the normal conjunctive form:

(1)	$x => y$	is equivalent to	$\sim x \vee y$
(2)	$\sim(x \vee y)$	is equivalent to	$\sim x \;\&\; \sim y$
(3)	$\sim(x \;\&\; y)$	is equivalent to	$\sim x \vee \sim y$
(4)	$\sim(\sim x)$	is equivalent to	x

Applying rule 1 to our formula we get:

$$\sim (\sim ((a => b) \;\&\; (b => c)) \vee (a => c))$$

By rules 2 and 4 we get:

$$(a => b) \;\&\; (b => c) \;\&\; \sim(a => c)$$

Using rule 1 at several places we get:

$(\sim a \vee b) \;\&\; (\sim b \vee c) \;\&\; \sim(\sim a \vee c)$

By rule 2 we finally get the clause form we need:

$(\sim a \vee b) \;\&\; (\sim b \vee c) \;\&\; a \;\&\; \sim c$

This consists of four clauses: $(\sim a \vee b)$, $(\sim b \vee c)$, a, $\sim c$. Now the resolution process can start.

The basic resolution step can occur any time that there are two clauses such that some proposition p occurs in one of them, and $\sim p$ occurs in the other. Let two such clauses be:

$p \vee Y$ and $\sim p \vee Z$

where p is a proposition, and Y and Z are propositional formulas. Then the resolution step on these two clauses produces a third clause:

$Y \vee Z$

It can be shown that this clause logically follows from the two initial clauses. So by adding the expression $(Y \vee Z)$ to our formula we do not alter the validity of the formula. The resolution process thus generates new clauses. If the 'empty clause' (usually denoted by 'nil') occurs then this will signal that a contradiction has been found. The empty clause *nil* is generated from two clauses of the forms:

x and $\sim x$

which is obviously a contradiction.

Figure 23.14 shows the resolution process that starts with our negated conjectured theorem and ends with the empty clause.

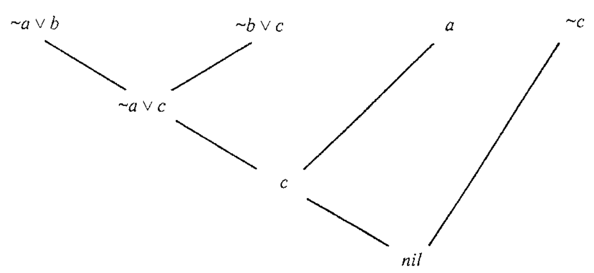

Figure 23.14 Proving the theorem (a => b) & (b => c) => (a => c) by the resolution method. The top line is the negated theorem in the clause form. The empty clause at the bottom signals that the negated theorem is a contradiction.

Figure 23.15 shows how this resolution process can be formulated as a pattern-directed program. This program operates on clauses asserted into the database. The resolution principle can be formulated as a pattern-driven activity:

if

there are two clauses *C1* and *C2*, such that *P* is a (disjunctive) subexpression of *C1*, and $\sim P$ is a subexpression of *C2*

then

remove *P* from *C1* (giving *CA*), remove $\sim P$ from *C2* (giving *CB*), and add into the database a new clause: $CA \vee CB$.

Written in our pattern-directed language this becomes:

```
[ clause( C1), delete( P, C1, CA),
  clause( C2), delete( ~P, C2, CB) ] --->
[ assertz( clause( CA v CB) ) ].
```

This rule needs a little elaboration to prevent repeated actions on the same clauses, which would merely produce new copies of already existing clauses. The program in Figure 23.15 records into the database what has already been done, by asserting:

```
done( C1, C2, P)
```

The condition parts of rules will then recognize and prevent such repeated actions.

The rules in Figure 23.15 also deal with some special cases that would otherwise require the explicit representation of the empty clause. Also, there are two rules that just simplify clauses when possible. One of these rules recognizes true clauses such as

$a \vee b \vee \sim a$

and removes them from the database since they are useless for detecting a contradiction. The other rule removes redundant subexpressions. For example, this rule would simplify the clause

$a \vee b \vee a$

into $a \vee b$.

A remaining question is how to translate a given propositional formula into the clause form. This is not difficult, and the program of Figure 23.16 does it. The procedure

```
translate( Formula)
```

translates a formula into a set of clauses C1, C2, etc., and asserts these clauses into the database as:

```
clause( C1).
clause( C2).
...
```

```
% Production rules for resolution theorem proving

% Contradicting clauses

[ clause( X), clause( ~X) ] --->
[ write('Contradiction found'), stop].

% Remove a true clause

[ clause( C), in( P, C), in( ~P, C) ] --->
[ retract( C) ].

% Simplify a clause

[ clause( C), delete( P, C, C1), in( P, C1) ] --->
[ replace( clause( C), clause( C1)) ].

% Resolution step, a special case

[ clause( P), clause( C), delete( ~P, C, C1), not done( P, C, P) ] --->
[ assertz( clause( C1)), assertz( done( P, C, P)) ].

% Resolution step, a special case

[ clause( ~P), clause( C), delete( P, C, C1), not done( ~P, C, P) ] --->
[ assertz( clause( C1)), assertz( done( ~P, C, P)) ].

% Resolution step, general case

[ clause( C1), delete( P, C1, CA),
  clause( C2), delete( ~P, C2, CB), not done( C1,C2,P) ] --->
[ assertz( clause( CA v CB)), assertz( done( C1, C2, P)) ].

% Last rule: resolution process stuck

[] ---> [ write('Not contradiction'), stop].

% delete( P, E, E1) if deleting a disjunctive subexpression P from E gives E1

delete( X, X v Y, Y).

delete( X, Y v X, Y).

delete( X, Y v Z, Y v Z1) :-
  delete( X, Z, Z1).

delete( X, Y v Z, Y1 v Z) :-
  delete( X, Y, Y1).

% in( P, E) if P is a disjunctive subexpression in E

in( X, X).

in( X, Y) :-
  delete( X, Y, _ ).
```

Figure 23.15 A pattern-directed program for simple resolution theorem proving.


```
% Translating a propositional formula into (asserted) clauses

:- op( 100, fy, ~).                        % Negation
:- op( 110, xfy, &).                       % Conjunction
:- op( 120, xfy, v).                       % Disjunction
:- op( 130, xfy, =>).                      % Implication

% translate( Formula): translate propositional Formula
%   into clauses and assert each resulting clause C as clause( C)

translate( F & G) :-                       % Translate conjunctive formula
  !,                                       % Red cut
  translate( F),
  translate( G).

translate( Formula) :-
  transform( Formula, NewFormula),         % Transformation step on Formula
  !,                                       % Red cut
  translate( NewFormula).

translate( Formula) :-                     % No more transformation possible
  assert( clause( Formula)).

% Transformation rules for propositional formulas

% transform( Formula1, Formula2):
%   Formula2 is equivalent to Formula1, but closer to clause form

transform( ~( ~X), X).                     % Eliminate double negation

transform( X => Y, ~X v Y).                % Eliminate implication

transform( ~ ( X & Y), ~X v ~Y).           % De Morgan's law

transform( ~ ( X v Y), ~X & ~Y).           % De Morgan's law

transform( X & Y v Z, ( X v Z) & ( Y v Z)).   % Distribution

transform( X v Y & Z, ( X v Y) & ( X v Z)).   % Distribution

transform( X v Y, X1 v Y) :-
  transform( X, X1).                       % Transform subexpression

transform( X v Y, X v Y1) :-
  transform( Y, Y1).                       % Transform subexpression

transform( ~ X, ~ X1) :-
  transform( X, X1).                       % Transform subexpression
```

Figure 23.16 Translating a propositional calculus formula into a set of (asserted) clauses.

Now the pattern-directed theorem prover can be triggered by the goal **run**. So, to prove a conjectured theorem using these programs, we translate the negated theorem into the clause form and start the resolution process. For our example theorem, this is done by the question:

```
?- translate( ~( ( a => b) & ( b => c) => ( a => c) ) ), run.
```

The program will respond with 'Contradiction found', meaning that the original formula is a theorem.

Summary

- Meta-programs treat other programs as data. A Prolog meta-interpreter is an interpreter for Prolog in Prolog.
- It is easy to write Prolog meta-interpreters that generate execution traces, proof trees and offer other extra features.
- Explanation-based generalization is a special technique for program compilation. It can be viewed as symbolic execution of a program, guided by a specific example. Explanation-based generalization was invented as an approach to machine learning.
- An object-oriented program consists of objects that send messages between themselves. Objects respond to messages by executing their methods. Methods can also be inherited from other objects.
- A pattern-directed program is a collection of pattern-directed modules whose execution is triggered by patterns in the 'database'.
- Prolog programs themselves can be viewed as pattern-directed systems.
- The parallel implementation of pattern-directed systems would be most natural. The sequential implementation requires conflict resolution among the modules in the conflict set.
- A simple interpreter for pattern-directed programs was implemented in this chapter and applied to resolution-based theorem proving in propositional logic.
- Concepts discussed in this chapter are:

  meta-programs, meta-interpreters
  explanation-based generalization
  object-oriented programming
  objects, methods, messages
  inheritance of methods
  pattern-directed systems, pattern-directed architecture
  pattern-directed programming
  pattern-directed module
  conflict set, conflict resolution
  resolution-based theorem proving, resolution principle

References

Writing Prolog meta-interpreters is part of the traditional Prolog programming culture. Le (1993), Shoham (1994) and Sterling and Shapiro (1994) give some other interesting examples of Prolog meta-interpreters.

The idea of explanation-based generalization was developed in the area of machine learning. The formulation used in this chapter is as in the paper by Mitchell, Keller and Kedar-Cabelli (1986). Our EBG program is similar to that in Kedar-Cabelli and McCarty (1987). This program is a delightful illustration of how elegantly a complicated symbolic method can be implemented in Prolog. With this program, Kedar-Cabelli and McCarty transformed pages of previous vague descriptions of EBG into a succinct, crystal clear and immediately executable description.

Our interpreter for object-oriented programs is similar to the one in Stabler (1986). Moss (1994) describes Prolog++, an efficient implementation of an object-oriented environment on top of Prolog.

Waterman and Hayes-Roth (1978) is the classical book on pattern-directed systems. Several elaborations of the basic pattern-directed architecture can be found under the name *blackboard systems*. A useful collection of papers on blackboard systems is Engelmore and Morgan (1988). Our illustrative application of pattern-directed programming is a basic example of mechanical theorem proving. Fundamentals of mechanical theorem proving in predicate logic are covered in many general books on artificial intelligence, such as those by Genesereth and Nilsson (1987), Ginsberg (1993), Poole *et al.* (1998), and Russell and Norvig (1995).

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appendix A

Some Differences Between Prolog Implementations

The syntax for Prolog in this book follows the tradition of the Edinburgh Prolog, which has been adopted by the majority of Prolog implementations and also the ISO standard for Prolog. Typically implementations of Prolog offer many additional features. Generally, the programs in the book use a subset of what is provided in a typical implementation, and is included in the ISO standard. However, there are still some differences between various Prologs that may require small changes when executing the programs in the book with a particular Prolog. This appendix draws attention to some of such more likely differences.

Dynamic and static predicates

The ISO standard and some Prolog implementations distinguish between *static* and *dynamic* predicates. Static predicates may be compiled into a more efficient code than dynamic ones. Only dynamic predicates can be manipulated by assert(a/z) and **retract**, and can be retrieved by **clause(Head, Body)**. A predicate is assumed static, unless announced in the program by a declaration of the form:

```
:- dynamic PredicateName( PredicateArity).
```

e.g. :- **dynamic member(2).**

Predicates introduced through assert(a/z) only, are automatically assumed as dynamic.

Assert and retract

The standard only includes the predicates **asserta** and **assertz**, but not **assert**. Virtually all the implementations also provide **assert**. If **assert** is not available, **assertz** can simply be used instead. To remove all the clauses about a predicate, built-in predicates **retractall(Clause)** or **abolish(PredicateName/Arity)** may be provided.

Undefined predicates

In some Prologs, a call to a predicate not defined in the program at all, simply fails. Other Prologs in such cases complain with an error message. In such Prologs, undefined predicates can be made to fail (without error messages) by a built-in predicate like: **unknown(_, fail)**.

Negation as failure: *not* and '\+'

In this book we use **not Goal** for negation as failure. Many Prologs (and the standard) use the (somewhat less pretty) notation:

```
\+ Goal
```

to emphasize that this is not the proper logical negation, but negation defined through failure. For compatibility with these Prologs, 'not' should be replaced by '\+', or (with less work), **not** introduced as a user-defined predicate (see Appendix B).

Predicate *name(Atom, CodeList)*

This predicate is provided by most implementations, but not included in the standard (instead: **atom_codes/2**). There are small differences between Prologs in the behaviour of **name** in special cases, e.g. when the first argument is a number.

Loading programs with *consult* and *reconsult*

Loading programs with **consult** and **reconsult** varies between implementations. Differences occur when programs are loaded from multiple files and the same predicate is defined in more than one file (the new clauses about the same predicate may simply be added to the old clauses; alternatively, just the clauses in the most recent file are loaded, abandoning the previous clauses about the same predicate).

Modules

In some Prologs, the program can be divided into *modules* so that predicate names are local to a module unless they are specifically made visible from other modules. This is useful when writing large programs, when predicates with the same name and arity may mean different things in different modules.

appendix B

Some Frequently Used Predicates

Some basic predicates such as **member/2** and **conc/3** are used in many programs throughout the book. To avoid repetition, the definition of such predicates is usually not included in the program's listing. To run a program, these frequently used predicates also have to be loaded into Prolog. This is done most easily by consulting (or compiling) a file, such as one given in this appendix, that defines these predicates. The listing below includes some predicates that may already be included among the built-in predicates, depending on the implementation of Prolog. For example, negation as failure written as **not Goal** is also included below for compatibility with Prologs that use the notation **\+ Goal** instead. When loading into Prolog the definition of a predicate that is already built-in, Prolog will typically just issue a warning message and ignore the new definition.

```
% File frequent.pl: Library of frequently used predicates

% Negation as failure
%   This is normally available as a built-in predicate,
%   often written with the prefix operator '\+', e.g. \+ likes(mary,snakes)
%   The definition below is only given for compatibility among Prolog implementations

:- op( 900, fy, not).

not Goal :-
  Goal, !, fail
  ;
  true.

% once( Goal): produce one solution of Goal only (the first solution only)
%   This may already be provided as a built-in predicate

once( Goal) :-
  Goal, !.

% member( X, List): X is a member of List

member( X, [ X | _]).                 % X is head of list

member( X, [ _ | Rest]) :-
  member( X, Rest).                   % X is in body of list
```

```
% conc( L1, L2, L3): list L3 is the concatenation of lists L1 and L2

conc( [ ], L, L).

conc( [X | L1], L2, [X | L3]) :-
  conc( L1, L2, L3).

% del( X, L0, L): List L is equal to list L0 with X deleted
%   Note: Only one occurrence of X is deleted
%   Fail if X is not in L0

del( X, [X | Rest], Rest).                 % Delete the head

del( X, [Y | Rest0], [Y | Rest]) :-
  del( X, Rest0, Rest).                    % Delete from tail

% subset( Set, Subset): list Set contains all the elements of list Subset
%   Note: The elements of Subset appear in Set in the same order as in Subset

subset( [ ], [ ]).

subset( [First | Rest], [First | Sub]) :-  % Retain First in subset
  subset( Rest, Sub).

subset( [First | Rest], Sub) :-            % Remove First
  subset( Rest, Sub).

% set_difference( Set1, Set2, Set3): Set3 is the list representing
%   the difference of the sets represented by lists Set1 and Set2
%   Normal use: Set1 and Set2 are input arguments, Set3 is output

set_difference( [ ], _, [ ]).

set_difference( [X | S1], S2, S3) :-
  member( X, S2), !,                       % X in set S2
  set_difference( S1, S2, S3).

set_difference( [X | S1], S2, [X | S3]) :- % X not in S2
  set_difference( S1, S2, S3).

% length( List, Length): Length is the length of List
%   Note: length/2 may already be included among built-in predicates
%   The definition below is tail-recursive
%   It can also be used to generate efficiently list of given length

length( L, N) :-
  length( L, 0, N).

length( [ ], N, N).

length( [_ | L], N0, N) :-
  N1 is N0 + 1,
  length( L, N1, N).
```



```
% max( X, Y, Max): Max = max(X,Y)

max( X, Y, Max) :-
  X >= Y, !, Max = X
  ;
  Max = Y.

% min( X, Y, Min): Min = min(X,Y)

min( X, Y, Min) :-
  X =< Y, !, Min = X
  ;
  Min = Y.

% copy_term( T1, T2): term T2 is equal to T1 with variables renamed
%   This may already be available as a built-in predicate
%   Procedure below assumes that copy_term is called so that T2 matches T1

copy_term( Term, Copy) :-
  asserta( term_to_copy( Term)),
  retract( term_to_copy( Copy)), !.
```

Solutions to Selected Exercises

Chapter 1

1.1 (a) no

(b) X = pat

(c) X = bob

(d) X = bob, Y = pat

1.2 (a) ?- parent(X, pat).

(b) ?- parent(liz, X).

(c) ?- parent(Y, pat), parent(X, Y).

1.3 (a)

```
happy( X) :-
    parent( X, Y).
```

(b)

```
hastwochildren( X) :-
    parent( X, Y),
    sister( Z, Y).
```

1.4

```
grandchild( X, Z) :-
  parent( Y, X),
  parent( Z, Y).
```

1.5

```
aunt( X, Y) :-
  parent( Z, Y),
  sister( X, Z).
```

1.6 Yes it is.

1.7 (a) no backtracking

(b) no backtracking

(c) no backtracking

(d) backtracking

Chapter 2

2.1 (a) variable

(b) atom

(c) atom

(d) variable

(e) atom

(f) structure

(g) number

(h) syntactically incorrect

(i) structure

(j) syntactically incorrect

2.3 (a) A = 1, B = 2

(b) no

(c) no

(d) D = 2, E = 2

(e) P1 = point(−1,0)
P2 = point(1,0)
P3 = point(0,Y)

This can represent the family of triangles with two vertices on the x-axis at 1 and -1 respectively, and the third vertex anywhere on the y-axis.

2.4 seg(point(5,Y1), point(5,Y2))

2.5

```
regular( rectangle( point(X1,Y1), point(X2,Y1), point(X2,Y3),
point(X1,Y3) ) ).
% This assumes that the first point is the left bottom vertex.
```

2.6 (a) A = two

(b) no

(c) C = one

(d) D = s(s(1));
D = s(s(s(s(s(1)))))

2.7

```
relatives( X, Y) :-
  predecessor( X, Y)
  ;
  predecessor( Y, X)
  ;
  predecessor( Z, X),
  predecessor( Z, Y)
  ;
  predecessor( X, Z),
  predecessor( Y, Z).
```

2.8

```
translate( 1, one).
translate( 2, two).
translate( 3, three).
```

2.9 In the case of Figure 2.10 Prolog does slightly more work.

2.10 According to the definition of matching of Section 2.2, this succeeds. X becomes a sort of circular structure in which X itself occurs as one of the arguments.

Chapter 3

3.1 (a)
```
conc( L1, [_, _, _], L)
```
(b)
```
conc( [_, _, _ | L2], [_, _, _], L)
```

3.2 (a)
```
last( Item, List) :-
    conc( _, [Item], List).
```
(b)
```
last( Item, [Item] ).

last( Item, [First | Rest] ) :-
    last( Item, Rest).
```

3.3
```
evenlength( [ ] ).

evenlength( [First | Rest] ) :-
  oddlength( Rest).

oddlength( [First | Rest] ) :-
  evenlength( Rest).
```

3.4
```
reverse( [ ], [ ] ).

reverse( [First | Rest], Reversed) :-
  reverse( Rest, ReversedRest),
  conc( ReversedRest, [First], Reversed).
```

3.5
```
% This is easy using reverse

palindrome( List) :-
  reverse( List, List).

% Alternative solution, not using reverse

palindrome1( [ ] ).

palindrome1( [_] ).

palindrome1( List) :-
  conc( [First | Middle], [First], List),
  palindrome1( Middle).
```

3.6
```
shift( [First | Rest], Shifted) :-
  conc( Rest, [First], Shifted).
```

3.7
```
translate( [ ], [ ] ).

translate( [Head | Tail], [Head1 | Tail1] ) :-
  means( Head, Head1),
  translate( Tail, Tail1).
```

3.8
```
% The following assumes the order of elements in Subset as in Set

subset( [ ], [ ] ).

subset( [First | Rest], [First | Sub] ) :-     % Retain First in subset
  subset( Rest, Sub).

subset( [First | Rest], Sub) :-                % Remove First
  subset( Rest, Sub).
```


3.9
```
dividelist( [ ], [ ], [ ] ).                   % Nothing to divide

dividelist( [X], [X], [ ] ).                   % Divide one-element list

dividelist( [X, Y | List], [X | List1], [Y | List2] ) :-
  dividelist( List, List1, List2).
```

3.10
```
canget( state( _, _, _, has), [ ] ).           % Nothing to do

canget( State, [Action | Actions] ) :-
  move( State, Action, NewState),              % First action
  canget( NewState, Actions).                  % Remaining actions
```

3.11
```
flatten( [Head | Tail], FlatList) :-           % Flatten non-empty list
  flatten( Head, FlatHead),
  flatten( Tail, FlatTail),
  conc( FlatHead, FlatTail, FlatList).

flatten( [ ], [ ] ).                           % Flatten empty list

flatten( X, [ X] ).                            % Flatten a non-list

% Note: On backtracking this program produces rubbish
```

3.12
```
Term1 = plays( jimmy, and( football, squash) )
Term2 = plays( susan, and( tennis, and( basketball, volleyball) ) )
```

3.13
```
:- op( 300, xfx, was).
:- op( 200, xfx, of).
:- op( 100, fx, the).
```

3.14 (a) A = 1 + 0

(b) B = 1 + 1 + 0

(c) C = 1 + 1 + 1 + 1 + 0

(d) D = 1 + 1 + 0 + 1;
D = 1 + 0 + 1 + 1;
D = 0 + 1 + 1 + 1 % Further backtracking causes indefinite cycling

3.15
```
:- op( 100, xfx, in).
:- op( 300, fx, concatenating).
:- op( 200, xfx, gives).
:- op( 100, xfx, and).
:- op( 300, fx, deleting).
:- op( 100, xfx, from).

% List membership

Item in [Item | List].

Item in [First | Rest] :-
  Item in Rest.

% List concatenation

concatenating [ ] and List gives List.

concatenating [X | L1] and L2 gives [X | L3] :-
  concatenating L1 and L2 gives L3.
```

```
% Deleting from a list
deleting Item from [Item | Rest] gives Rest.
deleting Item from [First | Rest] gives [First | NewRest] :-
  deleting Item from Rest gives NewRest.
```

3.16
```
max( X, Y, X) :-
  X >= Y.
max( X, Y, Y) :-
  X < Y.
```

3.17
```
maxlist( [X], X).                       % Maximum of single-element list
maxlist( [X, Y | Rest], Max) :-         % At least two elements in list
  maxlist( [Y | Rest], MaxRest),
  max( X, MaxRest, Max).                % Max is the greater of X and MaxRest
```

3.18
```
sumlist( [], 0).
sumlist( [First | Rest], Sum) :-
  sumlist( Rest, SumRest),
  Sum is First + SumRest.
```

3.19
```
ordered( [X] ).                         % Single-element list is ordered
ordered( [X, Y | Rest] ) :-
  X =< Y,
  ordered( [Y | Rest] ).
```

3.20
```
subsum( [], 0, [] ).
subsum( [N | List], Sum, [N | Sub] ) :-   % N is in subset
  Sum1 is Sum - N,
  subsum( List, Sum1, Sub).
subsum( [N | List], Sum, Sub) :-          % N is not in subset
  subsum( List, Sum, Sub).
```

3.21
```
between( N1, N2, N1) :-
  N1 =< N2
between( N1, N2, X) :-
  N1 < N2,
  NewN1 is N1 + 1,
  between( NewN1, N2, X).
```

3.22
```
:- op( 900, fx, if).
:- op( 800, xfx, then).
:- op( 700, xfx, else).
:- op( 600, xfx, :=).
if Val1 > Val2 then Var := Val3 else Anything :-
  Val1 > Val2,
  Var = Val3.
if Val1 > Val2 then Anything else Var := Val4 :-
  Val1 =< Val2,
  Var = Val4.
```


Chapter 4

4.1
```
(a) ?- family( person( _, Name, _, _), _, [] ).
(b) ?- child( person( Name, SecondName, _, works( _, _) ) ).
(c) ?- family( person( _, Name, _, unemployed),
              person( _, _, _, works( _, _) ), _).
(d) ?- family( Husband, Wife, Children),
       dateofbirth( Husband, date( _, _, Year1) ),
       dateofbirth( Wife, date( _, _, Year2) ),
       ( Year1 - Year2 >= 15
         ;
         Year2 - Year1 >= 15
       ),
       member( Child, Children).
```

4.2
```
twins( Child1, Child2) :-
  family( _, _, Children),
  del( Child1, Children, OtherChildren),   % Delete Child1
  member( Child2, OtherChildren),
  dateofbirth( Child1, Date),
  dateofbirth( Child2, Date).
```

4.3
```
nth_member( 1, [X | L], X).               % X is first element of list [X | L]
nth_member( N, [Y | L], X) :-             % X is nth element of [Y | L]
  N1 is N - 1,
  nth_member( N1, L, X).
```

4.4 The input string shrinks on each non-silent cycle, and it cannot shrink indefinitely.

4.5
```
accepts( State, [], _) :-
  final( State).
accepts( State, [X | Rest], MaxMoves) :-
  MaxMoves > 0,
  trans( State, X, State1),
  NewMax is MaxMoves - 1,
  accepts( State1, Rest, NewMax).
accepts( State, String, MaxMoves) :-
  MaxMoves > 0,
  silent( State, State1),
  NewMax is MaxMoves - 1,
  accepts( State1, String, NewMax).
```

4.6 The order is defined in the goal member(Y, [1,2,3,4,5,6,7,8]).

4.7
```
(a) jump( X/Y, X1/Y1) :-              % Knight jump from X/Y to X1/Y1
      (
       dxy( Dx, Dy)                   % Knight distances in x and y directions
       ;
       dxy( Dy, Dx),                  % or the other way round
      ),
```

```
    X1 is X + Dx,
    inboard( X1),            % X1 is within chessboard
    Y1 is Y + Dy,
    inboard( Y1).            % Y1 is within chessboard

  dxy( 2, 1).                % 2 squares to right, 1 forward
  dxy( 2, -1).               % 2 squares to right, 1 backward
  dxy( -2, 1).               % 2 to left, 1 forward
  dxy( -2, -1).              % 2 to left, 1 backward

  inboard( Coord) :-         % Coordinate within chessboard
    0 < Coord,
    Coord < 9.
```

(b)
```
knightpath( [Square] ).      % Knight sitting on Square

knightpath( [S1, S2 | Rest] ) :-
  jump( S1, S2),
  knightpath( [S2 | Rest] ).
```

(c) `?- knightpath( [ 2/1,R,5/4,S,X/8] ).`

Chapter 5

5.1 (a)
```
X = 1;
X = 2;
```
(b)
```
X = 1
Y = 1;

X = 1
Y = 2;

X = 2
Y = 1;

X = 2
Y = 2;
```
(c)
```
X = 1
Y = 1;

X = 1
Y = 2;
```

5.2 Assume that procedure **class** is called with second argument uninstantiated.

```
class( Number, positive) :-
  Number > 0, !.

class( 0, zero) :- !.

class( Number, negative).
```

5.3
```
split( [], [], [] ).

split( [X | L], [X | L1], L2) :-
  X >= 0, !,
  split( L, L1, L2).

split( [X | L], L1, [X | L2] ) :-
  split( L, L1, L2).
```


5.4 `member( Item, Candidates), not member( Item, RuledOut)`

5.5
```
set_difference( [], _, [] ).

set_difference( [X | L1], L2, L) :-
  member( X, L2), !,
  set_difference( L1, L2, L).

set_difference( [X | L1], L2, [X | L] ) :-
  set_difference( L1, L2, L).
```

5.6
```
unifiable( [], _, [] ).

unifiable( [First | Rest], Term, List) :-
  not( First = Term), !,
  unifiable( Rest, Term, List).

unifiable( [First | Rest], Term, [First | List] ) :-
  unifiable( Rest, Term, List).
```

Chapter 6

6.1
```
findterm( Term) :-               % Assuming current input stream is file f
  read( Term), !,                % Current term in F matches Term?
  write( Term)                   % If yes, display it
  ;
  findterm( Term).               % Otherwise process the rest of file
```

6.2
```
findallterms( Term) :-
  read( CurrentTerm),            % Assuming CurrentTerm not a variable
  process( CurrentTerm, Term).

process( end_of_file, _) :- !.

process( CurrentTerm, Term) :-
  ( not( CurrentTerm = Term), !  % Terms do not match
  ;
  write( CurrentTerm), nl        % Otherwise output current term
  ),
  findallterms( Term).           % Do the rest of file
```

6.4
```
starts( Atom, Character) :-
  name( Character, [Code] ),
  name( Atom, [Code | _] ).
```

6.5
```
plural( Noun, Nouns) :-
  name( Noun, CodeList),
  name( s, CodeS),
  conc( CodeList, CodeS, NewCodeList),
  name( Nouns, NewCodeList).
```

Chapter 7

7.2
```
add_to_tail( Item, List) :-
  var( List), !,                        % List represents empty list
  List = [Item | Tail].

add_to_tail( Item, [_ | Tail] ) :-
  add_to_tail( Item, Tail).

member( X, List) :-
  var( List), !,                        % List represents empty list
  fail.                                 % so X cannot be a member

member( X, [X | Tail] ).

member( X, [_ | Tail] ) :-
  member( X, Tail).
```

7.5
```
% subsumes( Term1, Term2):
%   Term1 subsumes Term2, e.g. subsumes( t(X,a,f(Y)), t(A,a,f(g(B))))
% Assume Term1 and Term2 do not contain the same variable
% In the following procedure, subsuming variables get instantiated
% to terms of the form literally( SubsumedTerm)

subsumes( Atom1, Atom2) :-
  atomic( Atom1), !,
  Atom1 == Atom2.

subsumes( Var, Term) :-
  var( Var), !,                         % Variable subsumes anything
  Var = literally( Term).               % To handle other occurrences of Var

subsumes( literally( Term1), Term2) :- !, % Another occurrence of Term2
  Term1 == Term2.

subsumes( Term1, Term2) :-              % Term1 not a variable
  nonvar( Term2),
  Term1 =.. [Fun | Args1],
  Term2 =.. [Fun | Args2],
  subsumes_list( Args1, Args2).

subsumes_list( [ ], [ ]).

subsumes_list( [First1 | Rest1], [First2 | Rest2]) :-
  subsumes( First1, First2),
  subsumes_list( Rest1, Rest2).
```

7.6
```
(a) ?- retract( product( X, Y, Z)), fail.
(b) ?- retract( product( X, Y, 0)), fail.
```

7.7
```
copy_term( Term, Copy) :-
  asserta( term_to_copy( Term)),
  retract( term_to_copy( Copy)).
```

7.9
```
copy_term( Term, Copy) :-
  bagof( X, X = Term, [Copy]).
```


Chapter 8

8.2
```
add_at_end( L1 - [Item | Z2], Item, L1 - Z2).
```

8.3
```
reverse( A - Z, L - L) :-               % Result is empty list if
  A == Z, !.                            % A - Z represents empty list

reverse( [X | L] - Z, RL - RZ) :-       % Non-empty list
  reverse( L - Z, RL - [X | RZ] ).
```

8.6
```
% Eight queens program

sol( Ylist) :-
  functor( Du, u, 15),                  % Set of upward diagonals
  functor( Dv, v, 15),                  % Set of downward diagonals
  sol( Ylist,
     [1,2,3,4,5,6,7,8],                 % Set of X-coordinates
     [1,2,3,4,5,6,7,8],                 % Set of Y-coordinates
     Du, Dv).

sol( [ ], [ ], [ ], _, _).

sol( [Y | Ys], [X | XL], YL0, Du, Dv) :-
  del( Y, YL0, YL),                     % Choose a Y coordinate
  U is X+Y-1,
  arg( U, Du, X),                       % Upward diagonal free
  V is X-Y+8,
  arg( V, Dv, X),                       % Downward diagonal free
  sol( Ys, XL, YL, Du, Dv).

del( X, [X | L], L).

del( X, [Y | L0], [Y | L]) :-
  del( X, L0, L).
```

Chapter 9

9.4
```
%mergesort( List, SortedList): use the merge-sort algorithm

mergesort( [ ], [ ] ).

mergesort( [X], [X] ).

mergesort( List, SortedList) :-
  divide( List, List1, List2),          % Divide into approx. equal lists
  mergesort( List1, Sorted1),
  mergesort( List2, Sorted2),
  merge( Sorted1, Sorted2, SortedList). % Merge sorted lists

divide( [ ], [ ], [ ] ).

divide( [X], [X], [ ] ).

divide( [X, Y | L], [X | L1], [Y | L2] ) :-  % Put X, Y into separate lists
  divide( L, L1, L2).

% merge( List1, List2, List3): See Section 8.3.1
```

9.5 (a)
```
binarytree( nil).

binarytree( t( Left, Root, Right) ) :-
  binarytree( Left),
  binarytree( Right).
```

9.6
```
height( nil, 0).

height( t( Left, Root, Right), H) :-
  height( Left, LH),
  height( Right, RH),
  max( LH, RH, MH),
  H is 1 + MH.

max( A, B, A) :-
  A >= B, !.

max( A, B, B).
```

9.7
```
linearize( nil, [] ).

linearize( t( Left, Root, Right), List) :-
  linearize( Left, List1),
  linearize( Right, List2),
  conc( List1, [Root | List2], List).
```

9.8
```
maxelement( t( _, Root, nil), Root) :- !.        % Root is right-most element

maxelement( t( _, _, Right), Max) :-             % Right subtree non-empty
  maxelement( Right, Max).
```

9.9
```
in( Item, t( _, Item, _), [Item] ).

in( Item, t( Left, Root, _), [Root | Path] ) :-
  gt( Root, Item),
  in( Item, Left, Path).

in( Item, t( _, Root, Right), [Root | Path] ) :-
  gt( Item, Root),
  in( Item, Right, Path).
```

9.10
```
% Display a binary tree from top to bottom
% This program assumes that each node is just one character

show( Tree) :-
  dolevels( Tree, 0, more).                      % Do all levels from top

dolevels( Tree, Level, alldone) :- !.            % No more nodes beyond Level

dolevels( Tree, Level, more) :-                  % Do all levels from Level
  traverse( Tree, Level, 0, Continue), nl,       % Output nodes at Level
  NextLevel is Level + 1,
  dolevels( Tree, NextLevel, Continue).          % Do lower levels

traverse( nil, _, _, _).

traverse( t( Left, X, Right), Level, Xdepth, Continue) :-
  NextDepth is Xdepth + 1,
  traverse( Left, Level, NextDepth, Continue),   % Traverse left subtree
  ( Level = Xdepth, !,                           % Node X at Level?
    write( X), Continue = more                   % Output node, more to do
    ;
    write(' ')                                   % Otherwise leave space
  ),
  traverse( Right, Level, NextDepth, Continue).  % Traverse right subtree
```


Chapter 10

10.1
```
in( Item, l( Item) ).                            % Item found in leaf

in( Item, n2( T1, M, T2) ) :-                    % Node has two subtrees
  gt( M, Item), !,                               % Item not in second subtree
  in( Item, T1)                                  % Search first subtree
  ;
  in( Item, T2).                                 % Otherwise search the second

in( Item, n3( T1, M2, T2, M3, T3) ) :-           % Node has three subtrees
  gt( M2, Item), !,                              % Item not in second or third
  in( Item, T1)                                  % Search first subtree
  ;
  gt( M3, Item), !,                              % Item not in third subtree
  in( Item, T2)                                  % Search second subtree
  ;
  in( Item, T3).                                 % Search third subtree
```

10.3
```
avl( Tree) :-
  avl( Tree, Height).                            % Tree is AVL-tree with height Height

avl( nil, 0).                                    % Empty tree is AVL and has height 0

avl( t( Left, Root, Right), H) :-
  avl( Left, HL),
  avl( Right, HR),
  ( HL is HR; HL is HR + 1; HL is HR - 1),       % Subtrees heights almost equal
    max1( HL, HR, H).

max1( U, V, M) :-                                % M is 1 + max of U and V
  U > V, !, M is U + 1
  ;
  M is V + 1.
```

10.4 The item at the root is initially 5, then 8, and finally 5 again.

Chapter 11

11.1
```
depthfirst1( [Node | Path], [Node | Path] ) :-
  goal( Node).

depthfirst1( [Node | Path], Solution) :-
  s( Node, Node1),
  not member( Node1, Path),
  depthfirst1( [Node1, Node | Path], Solution).
```

11.3
```
% Iterative deepening search that stops increasing depth
% when there is no path to current depth

iterative_deepening( Start, Solution) :-
  id_path( Start, Node, [ ], Solution),
  goal( Node).

% path( First, Last, Path): Path is a list of nodes between First and Last

path( First, First, [First]).

path( First, Last, [First, Second | Rest]) :-
  s( First, Second),
  path( Second, Last, [Second | Rest]).

% Iterative deepening path generator
% id_path( First, Last, Template, Path): Path is a path between First and
%   Last not longer than template list Template. Alternative paths are
%   generated in the order of increasing length

id_path( First, Last, Template, Path) :-
  Path = Template,
  path( First, Last, Path)
  ;
  copy_term( Template, P),
  path( First, _, P), !,                      % At least one path of Template length
  id_path( First, Last, [_ | Template], Path).  % Longer template
```

11.6 Breadth-first search: 15 nodes; iterative deepening: 26 nodes

$N(b, 0) = 1$
$N(b, d) = N(b, d-1) + (b^{d+1} - 1)/(b-1)$ for $d > 0$

11.8
```
solve( StartSet, Solution) :-                 % StartSet is list of start nodes
  bagof( [Node], member( Node, StartSet), CandidatePaths),
  breadthfirst( CandidatePaths, Solution).
```

11.9 Backward search is advantageous if the branching in the backward direction is lower than in the forward direction. Backward search is only applicable if a goal node is explicitly known.

```
% Let 'origs( Node1, Node2)' be the original state-space relation
% Define new s relation:

s( Node1, Node2) :-
  origs( Node2, Node1).
```

11.10
```
% States for bidirectional search are pairs of nodes StartNode-EndNode
% of the original state space, denoting start and goal of search

s( Start - End, NewStart - NewEnd) :-
  origs( Start, NewStart),                    % A step forward
  origs( NewEnd, End).                        % A step backward

% goal( Start - End) for bidirectional search

goal( Start - Start).                         % Start equal end

goal( Start - End) :-
  origs( Start, End).                         % Single step to end
```


11.11 **find1**: depth-first search; **find2**: iterative deepening (**conc(Path,_,_)** generates list templates of increasing length forcing **find1** into iterative deepening regime); **find3**: backward search.

11.12 Bidirectional search with iterative deepening from both ends.

Chapter 12

12.2 Not correct. $h \leqslant h^*$ is sufficient for admissibility, but not necessary.

12.3 $h(n) = \max\{h_1(n), h_2(n), h_3(n)\}$

12.7
```
% Specification of eight puzzle for IDA*

s( Depth:State, NewDepth:NewState) :-
  s( State, NewState, _),
  NewDepth is Depth + 1.

f( Depth:[Empty | Tiles], F) :-
  goal( [Empty0 | Tiles0]),
  totdist( Tiles, Tiles0, Dist),
  F is Depth + Dist.                % Use total dist. as heuristic function

goal( _:[2/2,1/3,2/3,3/3,3/2,3/1,2/1,1/1,1/2]).

% Initial states for IDA* are of the form 0:State, where State
% is a start state for A* (Figure 12.6)

showpos( Depth:State) :-
  showpos( State).
```

For initial state [1/2,3/3,3/1,1/3,3/2,1/1,2/3,2/1,2/2]: A* (using non-admissible heuristic) quickly finds a solution of length 38; IDA* (using total distance heuristic) needs much more time, but finds optimal solution of length 24.

12.9 Order: $a, b, c, f, g, j, k, d, e, f, g, j, l, k, m, j, l$
Updates of $F(b)$ and $F(c)$: $F(b) = 2$, $F(c) = 1$, $F(c) = 3$, $F(b) = 5$

Chapter 13

13.4 There are three solution trees whose costs are: 8, 10 and 11. Intervals: $0 \leqslant h(c) < 9$, $0 \leqslant h(f) < 5$.

Chapter 14

14.1 We get the same result regardless of the order.

14.6 A diode in series with R5, in direction from left to right, should not affect the voltage at T51. A diode in the opposite direction affects this voltage.

Chapter 15

15.2 No error when $p(A|B) = 1$ or $p(B|A) = 1$. Greatest error (0.5) when $p(A) = p(B) = 0.5$ and $p(A|B) = 0$.

15.4 A = 6, B = 5, C = 25.

Chapter 17

17.6 No. The reason is that one action may achieve more than one goal simultaneously.

Chapter 18

18.1 Whole example set: I = 2.2925
Ires(size) = 1.5422, Gain(size) = 0.7503
Info(size) = 0.9799, GainRatio(size) = 0.7503/0.9799 = 0.7657
Ires(holes) = 0.9675, Gain(holes) = 1.324
Info(holes) = 1.5546, GainRatio(holes) = 0.8517

18.2 Gain = I – Ires
I = – (p(D) log p(D) + p(~D) log p(~D))
= – (0.25 log 0.25 + 0.75 log 0.75) = 0.8113
To compute Ires we need p(S), p(~S), p(D|S), p(D|~S)
p(S) = p(S|D) p(D) + p(S|~D) p(~D) = 0.75 * 0.25 + 1/6 * 0.75 = 0.3125
Using Bayes formula:
p(D|S) = p(D) p(S|D) / p(S) = 0.25 * 0.75 / 0.3125 = 0.6
p(~D|S) = 0.4
p(D|~S) = p(D)*p(~S|D) / p(~S) = 0.25 * 0.25 / (1–0.3125) = 0.09090
Ires = p(S)*I(D|S) + p(~S)*I(D|~S) = 0.6056
Notation I(D|S) means information of disease given symptom present.
Gain = 0.2057
GainRatio = Gain(S) / I(S) = 0.2057/0.8960 = 0.2296

18.5
```
% prunetree( Tree, PrunedTree): PrunedTree is optimally pruned Tree
%   with respect to estimated classification error
%   Assume trees are binary:
%   Tree = leaf( Node, ClassFrequencyList), or
%   Tree = tree( Root, LeftSubtree, RightSubtree)

prunetree( Tree, PrunedTree) :-
  prune( Tree, PrunedTree, Error, FrequencyList).

% prune( Tree, PrunedTree, Error, FrequencyList):
%   PrunedTree is optimally pruned Tree with classification Error,
%   FrequencyList is the list of frequencies of classes at root of Tree

prune( leaf( Node, FreqList), leaf( Node, FreqList), Error, FreqList) :-
  static_error( FreqList, Error).

prune( tree( Root, Left, Right), PrunedT, Error, FreqList) :-
  prune( Left, Left1, LeftError, LeftFreq),
  prune( Right, Right1, RightError, RightFreq),
  sumlists( LeftFreq, RightFreq, FreqList),        % Add corresponding elements
  static_error( FreqList, StaticErr),
  sum( LeftFreq, N1),
  sum( RightFreq, N2),
  BackedErr is ( N1 * LeftError + N2 * RightError) / ( N1 + N2),
  decide( StaticErr, BackedErr, Root, FreqList, Left1, Right1, Error, PrunedT).

% Decide to prune or not:

decide( StatErr, BackErr, Root, FreqL, _, _, StatErr, leaf( Root, FreqL)) :-
  StatErr =< BackErr, !.                     % Static error smaller: prune subtrees

% Otherwise do not prune:

decide( _, BackErr, Root, _, Left, Right, BackErr, tree( Root, Left, Right)).

% static_error( ClassFrequencyList, Error): estimated classification error

static_error( FreqList, Error) :-
  max( FreqList, Max),                        % Maximum number in FreqList
  sum( FreqList, All),                        % Sum of numbers in FreqList
  number_of_classes( NumClasses),
  Error is ( All – Max + NumClasses – 1) / ( All + NumClasses).

sum( [], 0).

sum( [Number | Numbers], Sum) :-
  sum( Numbers, Sum1),
  Sum is Sum1 + Number.

max( [X], X).

max( [X,Y | List], Max) :-
  X > Y, !, max( [X | List], Max)
  ;
  max( [Y | List], Max).

sumlists( [], [], []).

sumlists( [X1 | L1], [X2 | L2], [X3 | L3]) :-
  X3 is X1 + X2,
  sumlists( L1, L2, L3).

% A tree

tree1( tree( a,                                                       % Root
           tree( b, leaf( e, [3,2]), leaf( f, [1,0])),                % Left subtree
           tree( c, tree( d, leaf( g, [1,1]), leaf( h, [0,1])), leaf( i, [1,0])))).

number_of_classes( 2).

% Test query: ?- tree1( Tree), prunetree( Tree, PrunedTree).
```

Chapter 19

19.2 Seven steps.

19.3 The number of hypotheses generated is 373179, the number of hypotheses refined is 66518.

19.4 $\{C_0, C_1\}$ is more general than $\{C_0, C_2\}$. C_1 does not θ-subsume C_2.

Chapter 20

20.1
```
qmult( pos, pos, pos).
qmult( pos, zero, zero).
qmult( pos, neg, neg).
...
```

20.2
```
resistor( pos, pos).
resistor( zero, zero).
resistor( neg, neg).

diode( zero, pos).
diode( zero, zero).
diode( neg, zero).
```

20.3 (a) First state: X = zero/inc, y = zero/inc
Second state: X = zero . . inf/inc, Y = zero . . inf/inc
Third state: X = zero . . inf/std, Y = zero . . inf/inc, or
X = zero . . inf/std, Y = zero . . inf/std, or
X = zero . . inf/inc, Y = zero . . inf/std

(b) The same as answer (a) except that the third state can only be:
X = zero . . inf/std, Y = zero . . inf/std

20.4 At T=t0: X = zero/inc, Y &equ zero/inc, Z = zero/inc
At T=t1: X = zero . . inf/inc, Y = zero . . inf/inc, Z = zero . . landz/inc
At T=t2: X = zero . . inf/inc, Y = zero . . inf/inc, Z = landz/inc, or
X = zero . . inf/std, Y = zero . . inf/std, Z = zero . . landz/std, or
X = zero . . inf/std, Y = zero . . inf/std, Z = landz/std

Note: These results can also be obtained by the simulator of Figure 20.8 and the following model that corresponds to this exercise:

```
landmarks( x, [minf,zero,inf]).
landmarks( y, [minf,zero,inf]).
landmarks( z, [minf,zero,landz,inf]).

correspond( x:zero, y:zero).

legalstate( [X,Y,Z]) :-
  mplus( X, Y),
  sum( X, Y, Z).

initial( [x:zero/inc, y:Y0, z:Z0]).
```

The query is:

```
?- initial(S), simulate(S,Beh,3).
```

20.5
```
% Model of U-tube
% levA0, levB0 are initial levels in containers A and B
% fAB0 is the initial flow from A to B, fBA0 is the initial flow from B to A

landmarks( level, [ zero, levB0, levA0, inf]).
landmarks( leveldiff, [minf, zero, inf]).
landmarks( flow, [minf, fBA0, zero, fAB0, inf]).

correspond( leveldiff:zero, flow:zero).
correspond( flow:fAB0, flow:fBA0, flow:zero).

legalstate( [ LevA, LevB, FlowAB, FlowBA]) :-
  deriv( LevA, FlowBA),
  deriv( LevB, FlowAB),
  sum( FlowAB, FlowBA, flow:zero/std),          % FlowBA = -FlowAB
  DiffAB = leveldiff:_,
```



```
  sum( LevB, DiffAB, LevA),                      % DiffAB = LevA - LevB
  mplus( DiffAB, FlowAB).

initial( [ level:levA0/dec, level:levB0/inc, flow:fAB0/dec, flow:fBA0/inc]).
```

20.7
```
legal_trans( State1, State2) :-
  system_trans( State1, State2),
  State1 \== State2,                 % Qualitatively different next state
  not (point_state( State1),         % Not State1 a time-point state
       point_state( State2)),        % and State2 also a time-point state
  legalstate( State2).               % Legal according to model

point_state( State) :-
  member( _:Qmag/Dir, State),
  not (Qmag = _.._),                 % Qmag a landmark, not interval
  Dir \== std.                       % Not steady
```

Chapter 21

21.1
```
s( [a | List], Rest) :-
  s( List, [b | Rest]).
```

21.3 The modified definition is less efficient and may lead to indefinite cycling.

Chapter 23

23.1 Add this clause to the meta-interpreter:

```
prove( clause( Head, Body)) :-
  clause( Head, Body).
```

23.3 There are alternatives because **square(6)** inherits the **perimeter** method from several objects. This multiple inheritance can be prevented by a cut in the **send** procedure.

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