SUPERPREVODNI KVANTNI SPOMIN

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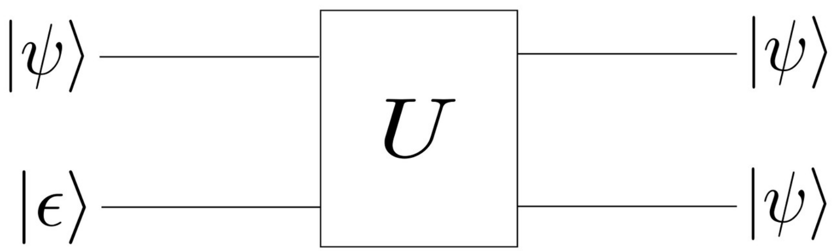
Kratek pregled

- ‘No-cloning’ teorem
- Kvantni bit
- LC resonator
- Josephsonov spoj
- Nabojni kubit
- Manipulacija kubitov

‘No-cloning’ teorem

- Ali obstaja unitaren operator U
  $U\left(|\psi\rangle_A |\epsilon\rangle_B\right) = \mathrm{e}^{i\delta(\psi,\epsilon)} |\psi\rangle_A |\psi\rangle_B$?



- Veljati mora tudi:
  $U\left(|\phi\rangle_A |\epsilon\rangle_B\right) = \mathrm{e}^{i\delta(\phi,\epsilon)} |\phi\rangle_A |\phi\rangle_B$

- Zmnožimo enačbi:

$$\left(\langle\varphi|_A \langle\epsilon|_B\right) U^\dagger U \left(|\psi\rangle_A |\epsilon\rangle_B\right) = \langle\varphi|_A \langle\varphi|_B \,\mathrm{e}^{-i\delta(\varphi,\epsilon)}\mathrm{e}^{i\delta(\psi,\epsilon)} |\psi\rangle_A |\psi\rangle_B$$

$$\langle\varphi|\psi\rangle \underbrace{\langle\epsilon|\epsilon\rangle}_{=1} = \mathrm{e}^{i(\delta(\psi,\epsilon)-\delta(\varphi,\epsilon))} \langle\varphi|\psi\rangle^2 .$$

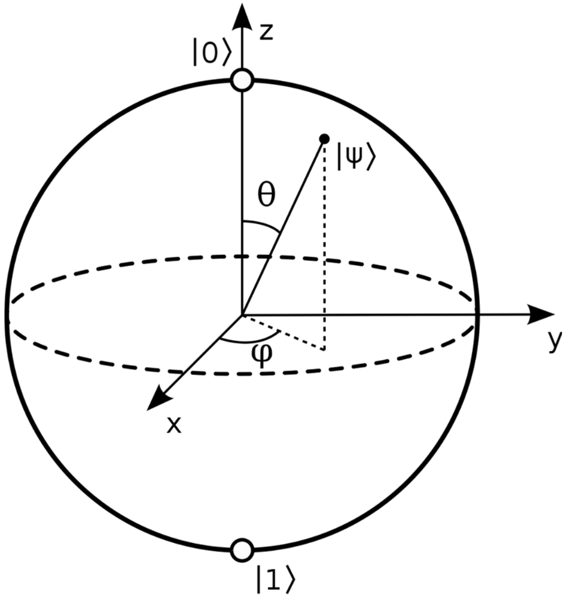
$$\Downarrow$$

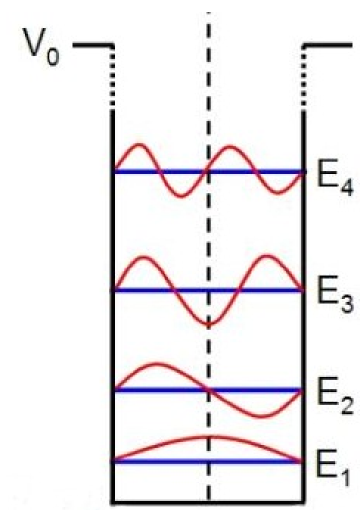
$$|\langle\varphi|\psi\rangle| = 0, 1$$

- Lahko kopiramo ortogonalna stanja (na primer $|0\rangle$ in $|1\rangle$).

Kvantni bit

- Dvovinojski kvanten sistem
  $$|\psi\rangle = a\,|0\rangle + b\,|1\rangle = \cos\tfrac{\theta}{2}\,|0\rangle + \mathrm{e}^{i\varphi}\sin\tfrac{\theta}{2}\,|1\rangle$$
- Rotacija za kot ϑ okoli osi **n**:
  $$\mathrm{e}^{-i\frac{\vartheta}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}}\,|\psi\rangle$$
- Časovni razvoj sistema narekuje hamiltonian $H = \frac{1}{2}\hbar\omega\sigma_z$.
  $$|\psi(t)\rangle = \mathrm{e}^{-i\frac{H}{\hbar}t}\,|\psi(0)\rangle = \mathrm{e}^{-i\frac{\omega t}{2}\sigma_z}\,|\psi(0)\rangle$$
- Večnivojski sistem kot kubit?
  - Potencialne jame ni v naravi
  - Harmonski oscilator?





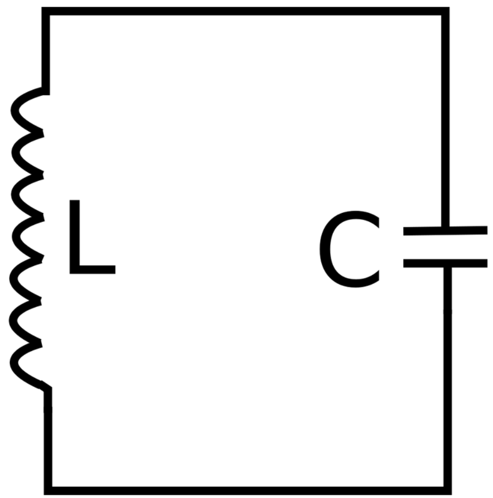
LC resonator

- Hamiltonian sistema $$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

- V primerjavi s harmonskim oscilatorjem

$$\begin{aligned} p &\longleftrightarrow Q, \\ x &\longleftrightarrow \Phi, \\ m &\longleftrightarrow C \end{aligned}$$

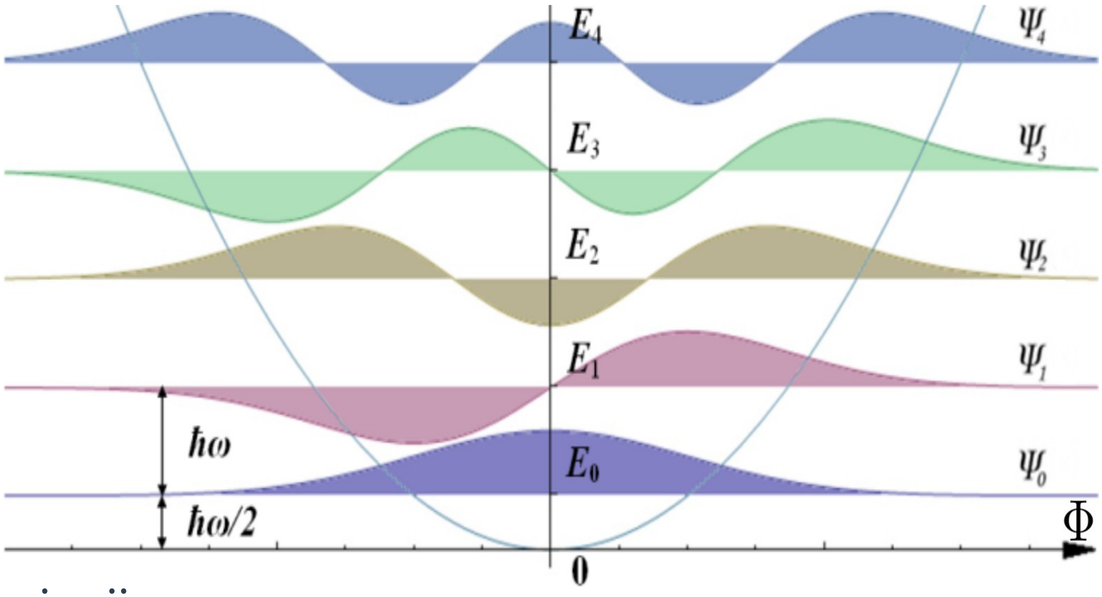
- Lastna frekvenca $$\omega = \frac{1}{\sqrt{LC}}$$



Kvantizacija LC resonatorja

- Velike energijske razlike: $L, C \longrightarrow 0$
- Nizka temperatura: $k_B T \ll \hbar\omega$
- Brez disipacije energije: superprevodni elementi
- Lestvena operatorja

$$a = \frac{1}{\sqrt{2\hbar Z}} \left(\Phi + iZQ\right),$$
$$a^\dagger = \frac{1}{\sqrt{2\hbar Z}} \left(\Phi - iZQ\right).$$
$$\Downarrow$$
$$H = \hbar\omega \left(a^\dagger a + 1/2\right)$$



- Težava: enakomerna razdalja med nivoji.

Superprevodnost

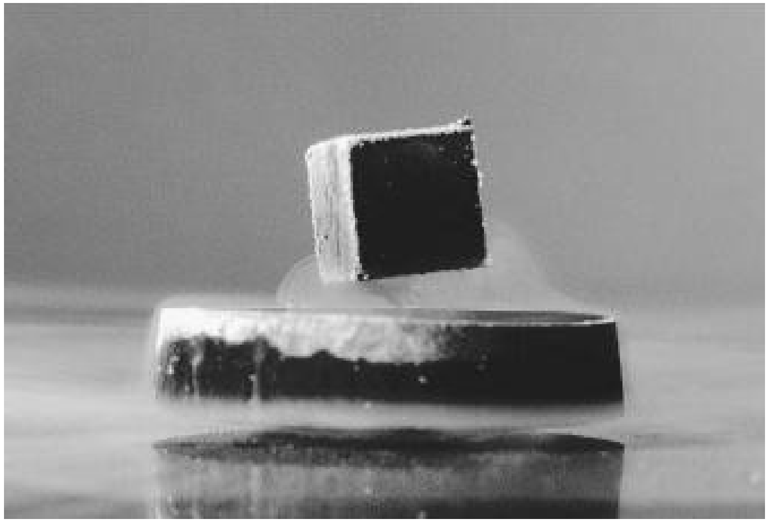
- Cooperjev par: bozon $(s = 1/2, q = -2e)$
- Valovna funkcija osnovnega stanja

$$\psi(\mathbf{r}) = \sqrt{n(\mathbf{r})}\mathrm{e}^{i\phi(\mathbf{r})}$$

- Gostota toka v superprevodniku

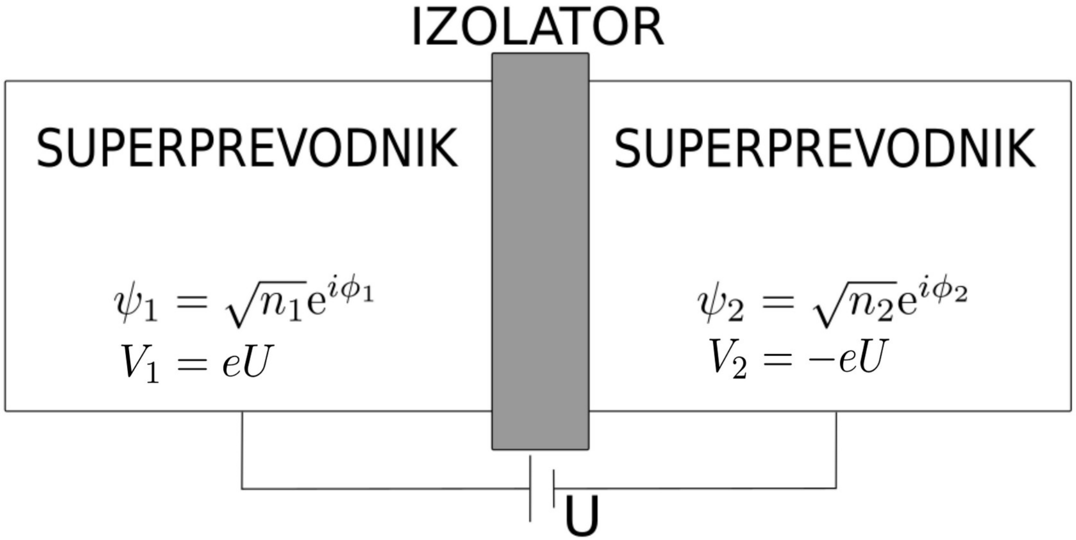
$$\mathbf{j} = \frac{q\hbar}{2im}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = \frac{q\hbar n}{m}\nabla\phi$$

- Ravnovesje: $\nabla\phi = 0 \implies \phi(\mathbf{r}) = konst$



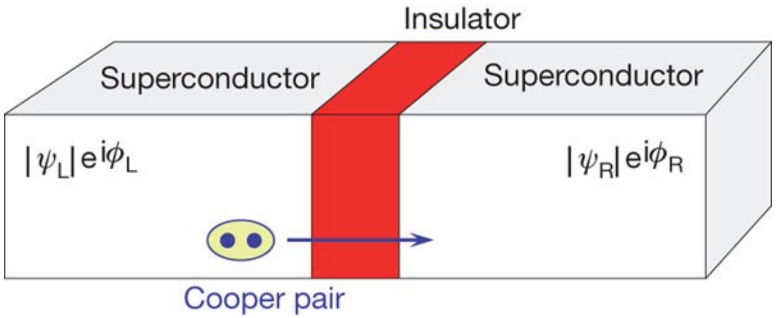
Josephsonov spoj

- Brian Josephson 1962.



- Pari tunelirajo skozi plast izolatorja.
- Dinamiko podajata Schrödingerjevi enačbi

$$i\hbar\frac{\partial\psi_1}{\partial t} = V_1\psi_1 + K\psi_2,$$
$$i\hbar\frac{\partial\psi_2}{\partial t} = V_2\psi_2 + K\psi_1.$$



Josephsonov spoj

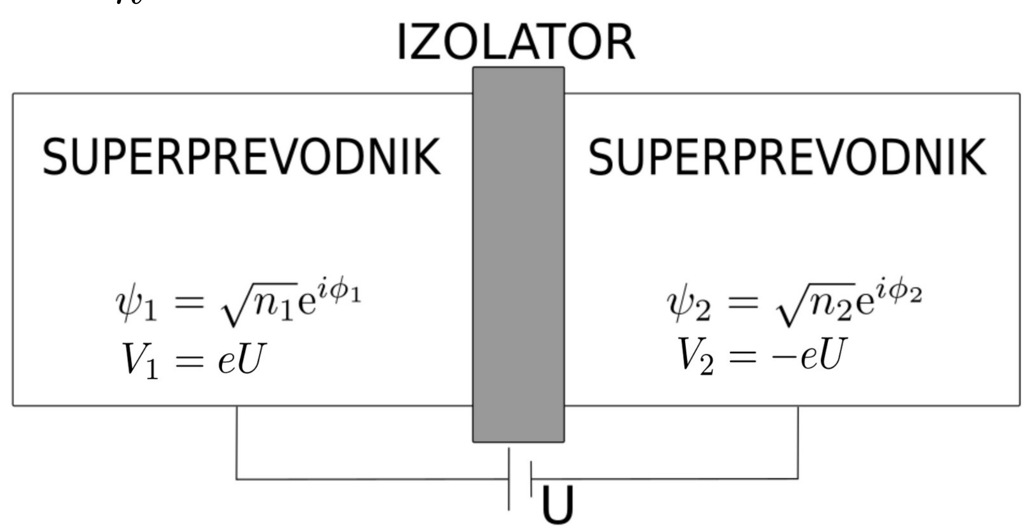
- Nastavke damo v enačbi (označimo $\delta = \phi_2 - \phi_1$):

$$\left.\begin{aligned} \dot{n}_1 &= \frac{2K\sqrt{n_1 n_2}}{\hbar}\sin\delta, \\ \dot{n}_2 &= -\frac{2K\sqrt{n_1 n_2}}{\hbar}\sin\delta, \end{aligned}\right\} \quad I = I_0 \sin\delta$$

$$\left.\begin{aligned} \hbar\dot{\phi}_1 &= -eU - K\sqrt{\frac{n_2}{n_1}}\cos\delta, \\ \hbar\dot{\phi}_2 &= eU - K\sqrt{\frac{n_1}{n_2}}\cos\delta. \end{aligned}\right\} \quad \dot{\delta} = \frac{2eU}{\hbar}$$

Josephsonovi relaciji

- Zakaj velja $n_1 = n_2$?



Josephsonov spoj

$$I = I_0 \sin \delta,$$
$$\dot{\delta} = 2eU/\hbar.$$

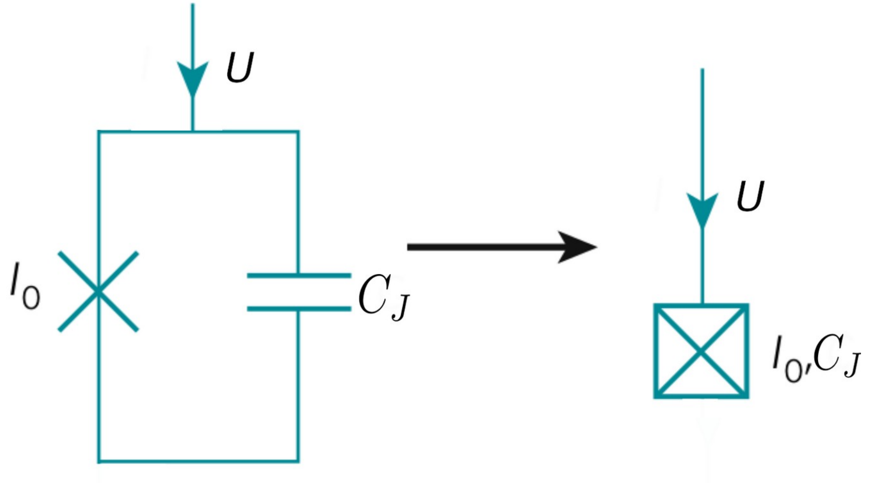
- DC Josephsonov pojav $U \neq U(t) \implies \delta(t) = \delta(0) + 2eUt/\hbar$
  - Definicija volta napetosti prek zveze $I(t) = I_0 \sin\left(\delta(0) + \frac{2eUt}{\hbar}\right).$

- Koliko energije je shranjene na spoju?

$$\Delta E = \int_1^2 IU\mathrm{d}t = \int_1^2 I\mathrm{d}\left(\frac{\Phi_0 \delta}{2\pi}\right) = \frac{\Phi_0}{2\pi}\int_1^2 I_0 \sin(\delta)\,\mathrm{d}\delta = -E_J \Delta \cos\delta$$

- Vpeljemo $E = E_J\left(1 - \cos\delta\right).$
- Spoj ima kapaciteto C_J:

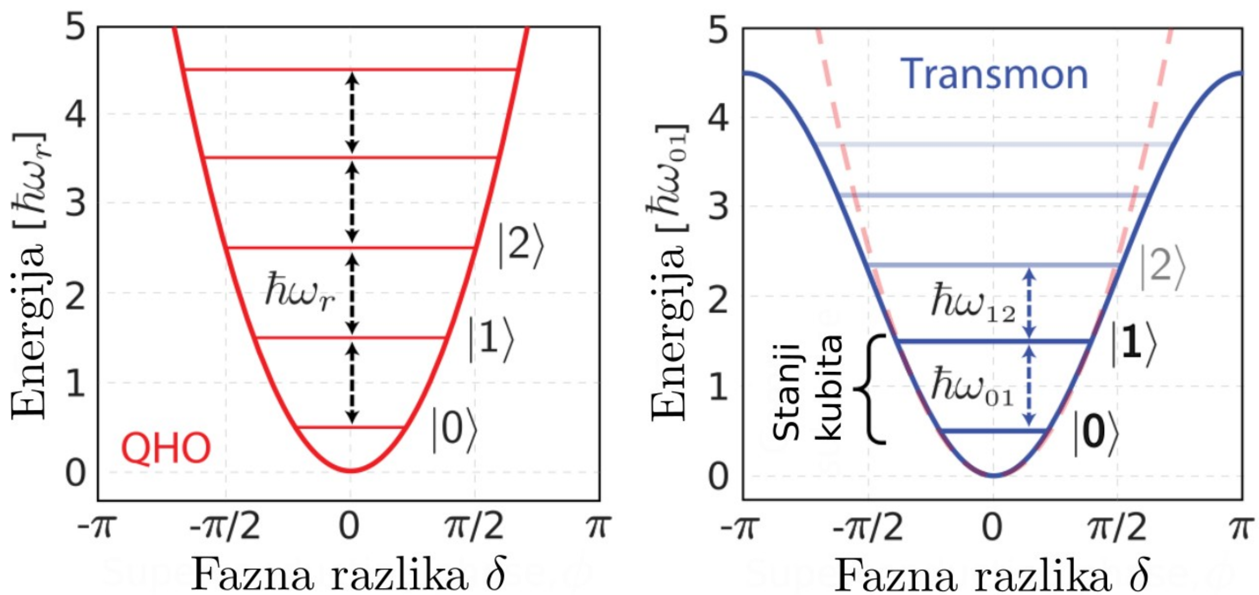
$$E_C = \frac{Q^2}{2C_J}$$



Josephsonov spoj

- Spoj je anharmonski oscilator:

$$H = \frac{Q^2}{2C_J} + E_J\left(1 - \cos\delta\right) \approx \frac{Q^2}{2C_J} + \frac{E_J}{2}\delta^2 - \frac{E_J}{24}\delta^4$$

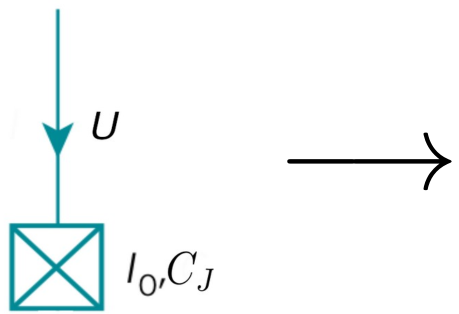


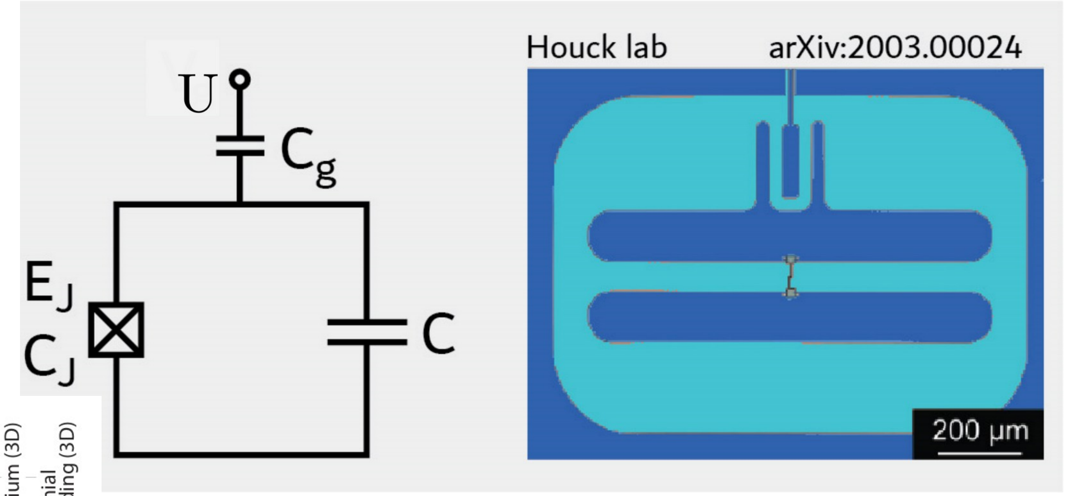
- Najnižji stanji uporabimo za stanji kubita:

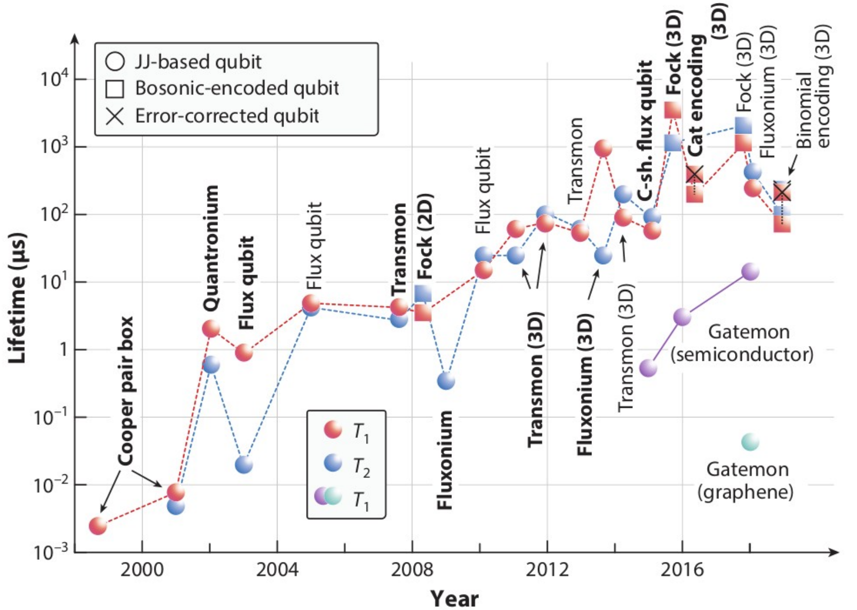
$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle = \cos\tfrac{\theta}{2}\,|0\rangle + \mathrm{e}^{i\varphi}\sin\tfrac{\theta}{2}\,|1\rangle$$

Nabojni kubit

- Dodamo kondenzator za podaljšanje življenske dobe: transmon kubit.



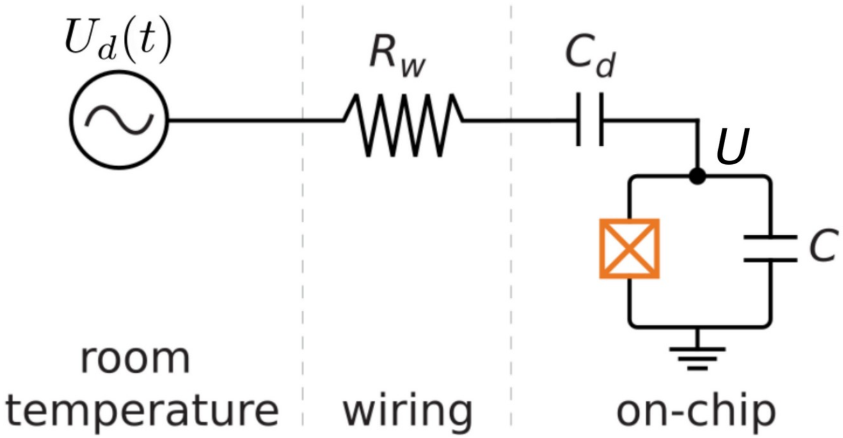




Manipulacija kubitov

- Sklopitev prek kondenzatorja:

$$H_{\mathrm{int}} = C_d U_d(t) U = C_d U_d(t) \frac{Q}{C}$$



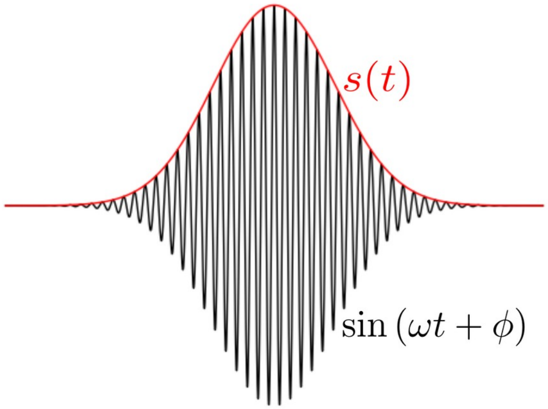
- Spomnimo se: $Q = -iQ_0 \left(a - a^\dagger\right) = Q_0 \sigma_y$

$$a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, a^\dagger = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

- Vzbujamo s pulzi napetosti $U_d(t) = U_0 s(t) \sin\left(\omega t + \phi\right)$

$$\vdots$$

$$H_{\mathrm{int}} = -\frac{1}{2} \Omega U_0 s(t) \left(\sigma_x \cos\phi + \sigma_y \sin\phi\right); \quad \Omega = \frac{Q_0 C_d}{C}$$



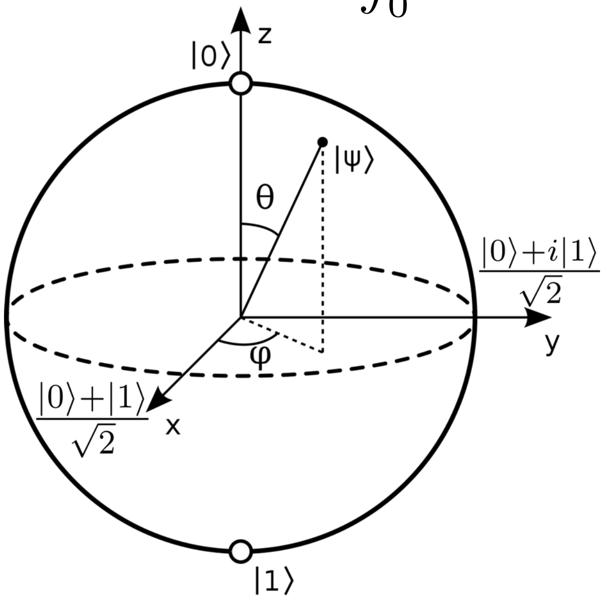
Manipulacija kubitov

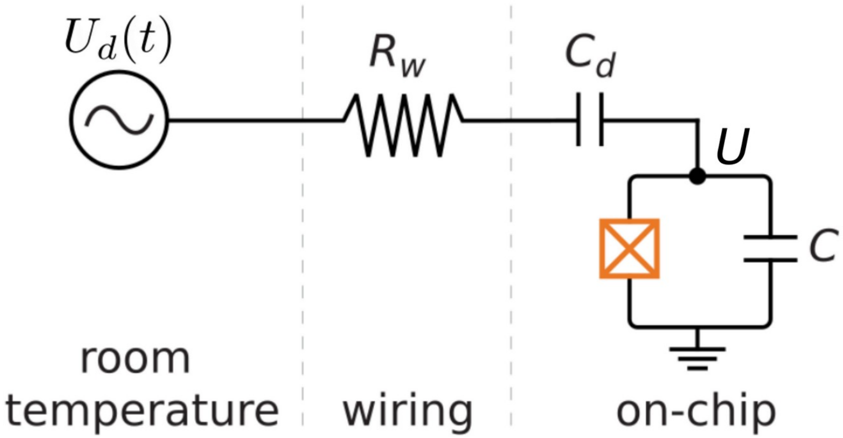
- $$H_{\mathrm{int}} = -\frac{1}{2}\Omega U_0 s(t)\left(\sigma_x \cos\phi + \sigma_y \sin\phi\right)$$

$$\Downarrow$$

- Faza pulza nadzoruje os rotacije, amplituda in oblika pa kot:

$$\Theta(t) = \Omega U_0 \int_0^t s(t')\mathrm{d}t'$$

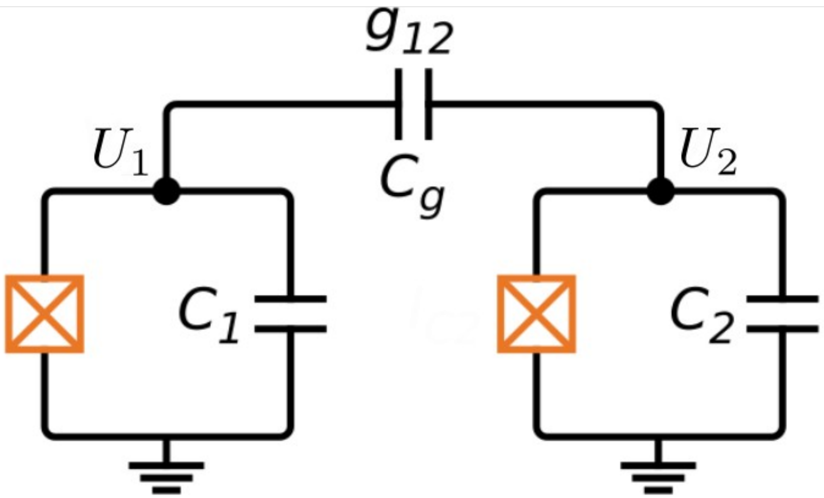




Sklapljanje kubitov

- Sklopitev prek kondenzatorja:

$$H_{\text{int}} = C_g U_1 U_2$$



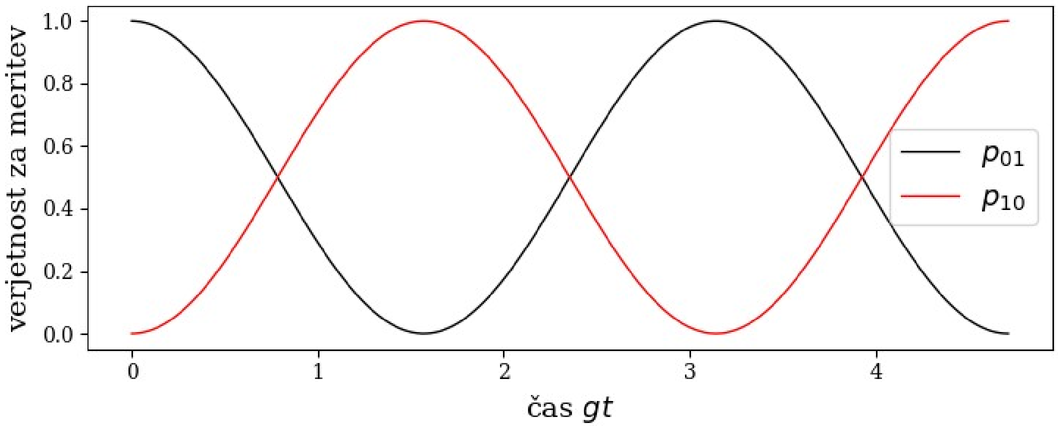
- Splošno stanje sistema:

$$|\psi\rangle = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle$$

- Primer: $|\psi(0)\rangle = |01\rangle$

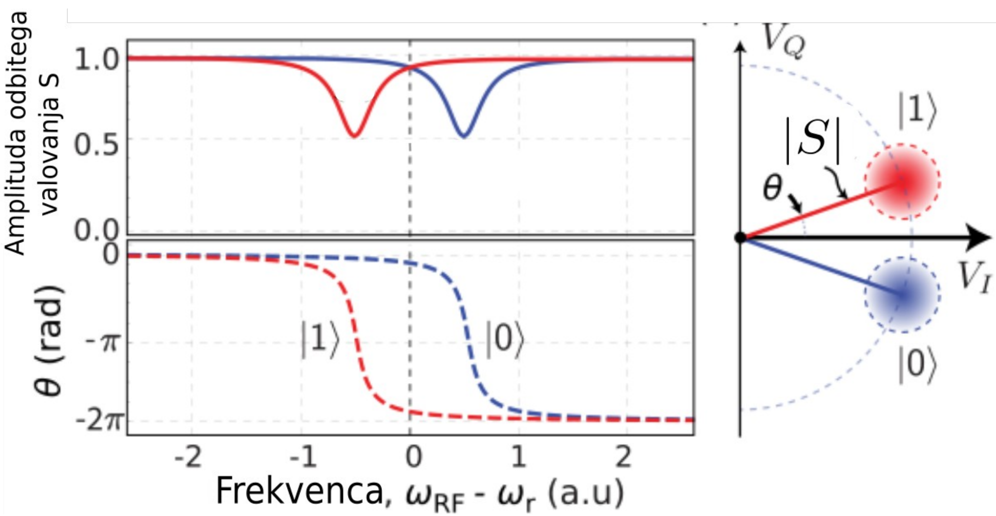
$$|\psi(t)\rangle = \cos(gt)\,|01\rangle - i\sin(gt)\,|10\rangle$$

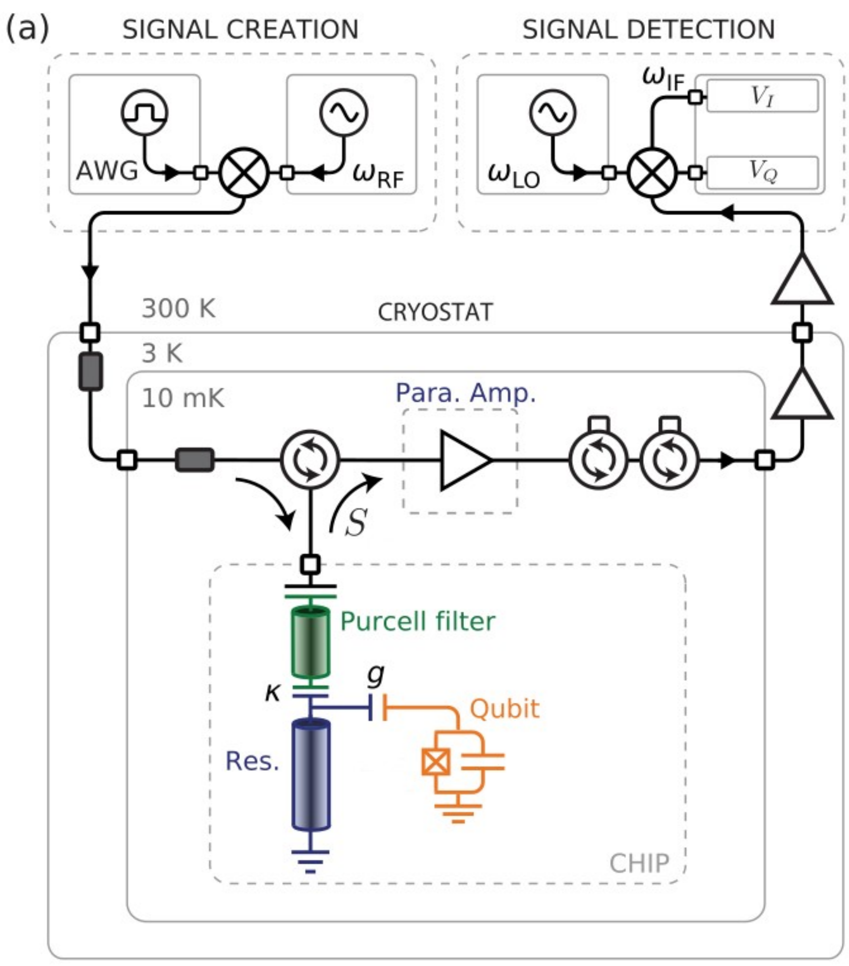
  - Lahko ustvarimo prepletena stanja



Branje kubitov

- Meritve prek sklopitve z resonatorjem
- Lastna frekvenca resonatorja se spremeni glede na stanje kubita. Nekaj valovanja, ki ga pošljemo proti resonatorju se odbije, nekaj pa ga resonančno vzbudi.





Povzetek

- ‘No-cloning’ teorem
- LC resonator $\longrightarrow$ Josephsonov spoj

$\downarrow$

Manipulacija kubitov $\longleftarrow$ Nabojni kubit

$\downarrow$

2-kubitni superprevodni procesor