SEMINAR I

ADIABATIC QUANTUM COMPUTATION

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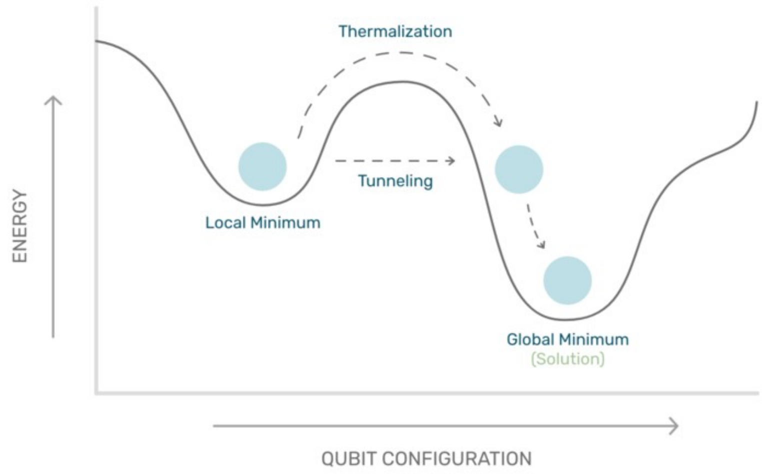
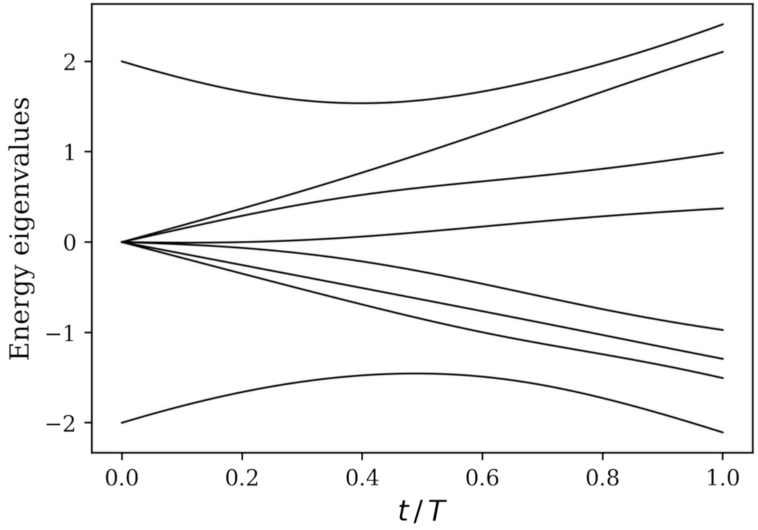
Introduction

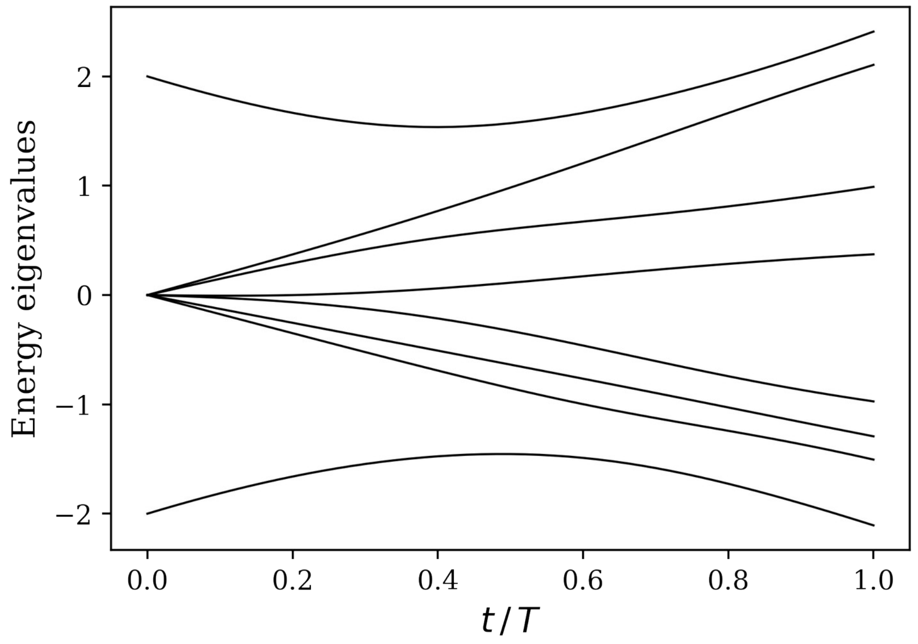
- Optimization algorithm
- Quantum annealing uses quantum fluctuations - tunneling for escaping local minima.
- Adiabatic quantum computer (AQC) uses adiabatic evolution to find the solution.





The algorithm

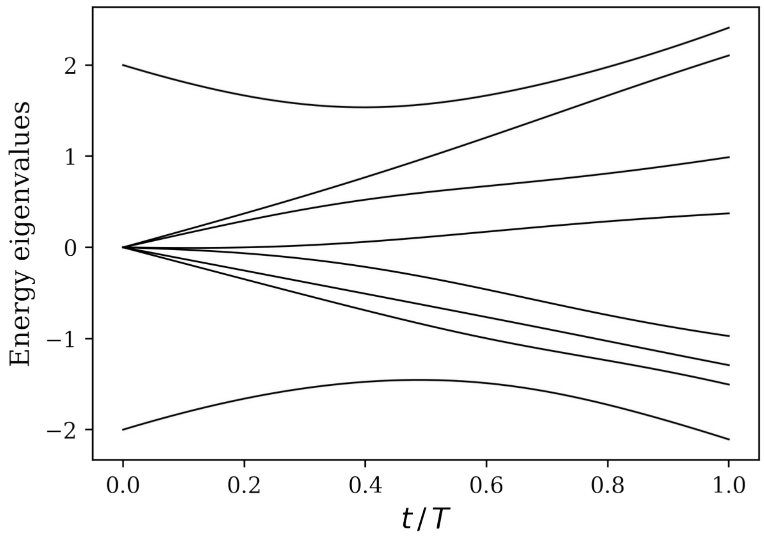
1. Initialize the system in the ground state of the Hamiltonian $\mathrm{H}(0) = \mathrm{H}_0$.
2. Vary the Hamiltonian for a period T. The final Hamiltonian $\mathrm{H}(\mathrm{T}) = \mathrm{H}_p$ should encode the optimization problem.
3. Final state of the system represents the solution of the encoded problem.



Comparison with the gate model

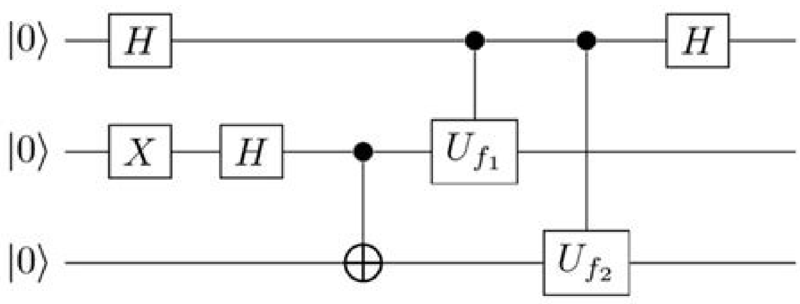
AQC

- Uses only the ground state of a time-dependent Hamiltonian.
- Equivalent to the gate model.



Gate model

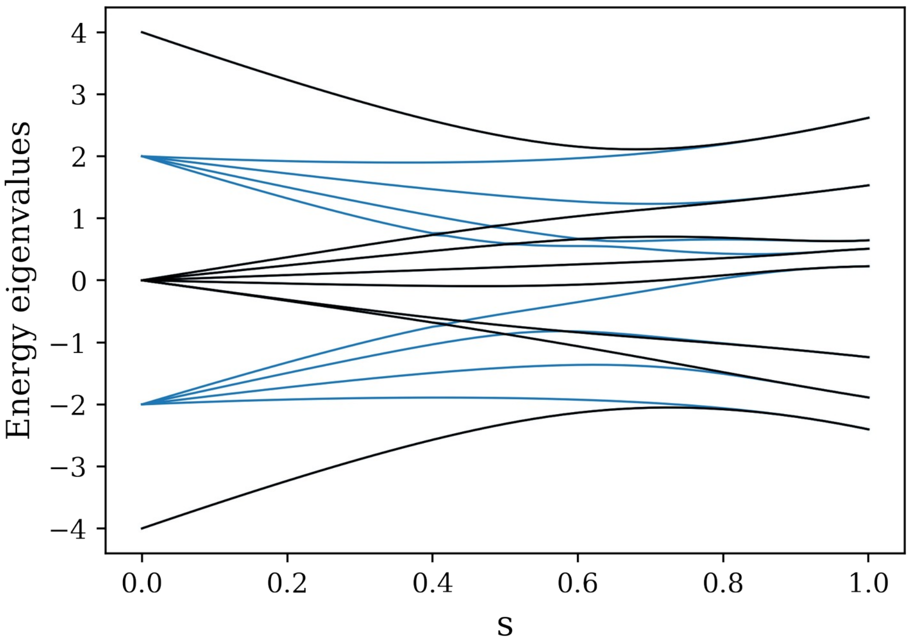
- Series of quantum gates to manipulate qubits.
- Uses full Hilbert space.



Adiabatic theorem

- Define $s = t/T \in [0, 1]$.
- Assumptions:
  - lowest two eigenvalues, $E_0(s), E_1(s)$, do not cross.
  - Zero temperature, isolated system
- $g_{min} = \min_{0 \le s \le 1} E_1(s) - E_0(s)$
- System will remain in the instantaneous ground state if

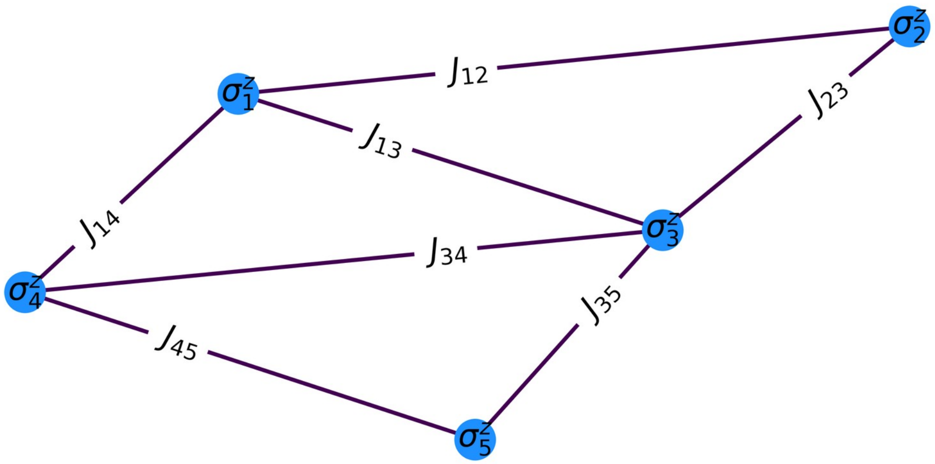
$$T \gg h \max_s \frac{\left|\langle E_1; s \left| \frac{\partial H}{\partial s} \right| E_0; s \rangle\right|}{g_{min}^2}$$



Spectrum of $H(s) = (1-s)\sum_i \sigma_i^x + s\sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z$ which is invariant under $\sigma^z \longrightarrow -\sigma^z$. Only eigenvalues from different sectors cross.

Problem Hamiltonian

- Ising Hamiltonian $H_p = \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z + \sum_i h_i\sigma_i^z$ is used by D-Wave.
  - A lot of optimization problems already translated into this form.
  - Searching for the ground state of H_p is an NP-hard problem for classical computers.

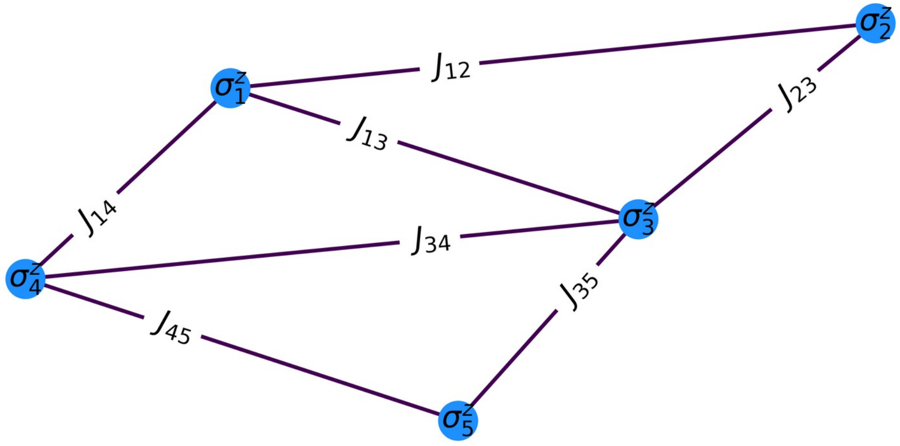


Graph partitioning

- Cut a graph with vertices V and edges E into two partitions of equal size, such that the number of edges between them is minimal.
- We place a spin $\sigma_i^z = \pm 1$ on each vertex. Problem is equivalent to the minimization of

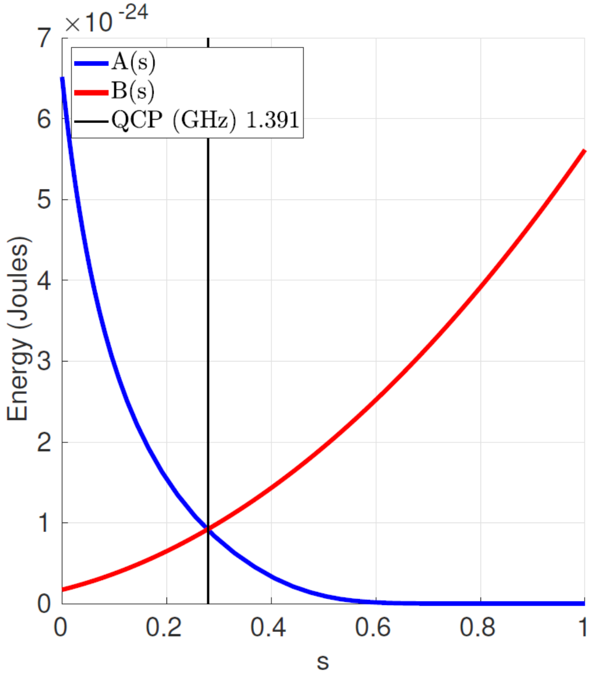
$$H_p = \left(\sum_i \sigma_i^z\right)^2 - \alpha \sum_{(i,j)\in E} \sigma_i^z \sigma_j^z = 2\sum_{i,\, j>i} \sigma_i^z \sigma_j^z - \alpha \sum_{(i,j)\in E} \sigma_i^z \sigma_j^z .$$

- H_p is the Ising Hamiltonian with $J_{ij} = \begin{cases} 2-\alpha; & (i,j)\in E \\ 2; & (i,j)\notin E \end{cases}$



Full Hamiltonian

- Initial Hamiltonian requirements:
  - Easily achievable ground state,
  - Must not include only σ^z operators.
- D-Wave has chosen $H_0 = \sum_i \sigma_i^x$ with the ground state $\Pi_i \frac{|0\rangle_i + |1\rangle_i}{\sqrt{2}}$.



D-Wave Systems, D-Wave Annealing implementations and controls

- Full time-dependent Hamiltonian

$$H(s) = -A(s)\sum_i \sigma_i^x + B(s)\left[\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z\right].$$

Complexity of AQC

- System size – number of qubits: $n^p; \;\; p = 1,2$ for most NP problems.
- Time complexity: scaling of time it takes to run an algorithm with problem size n:

$$T \gg h \max_s \frac{\left|\langle E_1;s\middle|\frac{\partial H}{\partial s}\middle|E_0;s\rangle\right|}{g^2_{min}(n)}$$

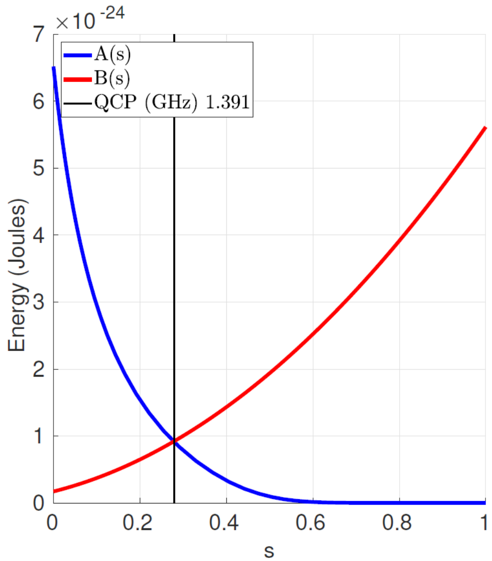
- Scaling of the gap:
  - $J_{ij} = \pm 1 \implies g_{min} \sim 1/n$
  - $\{J_{ij}\}$ random $\implies g_{min} \sim \mathrm{e}^{-n}$
- For general optimization problems, there is no exponential speedup compared to classical algorithms ➡ improved methods: diabatic, reverse adiabatic computation.
- Polynomial speedup (e.g. Grover's problem: $\mathcal{O}(\sqrt{n})$ unordered list search)

Experimental realization

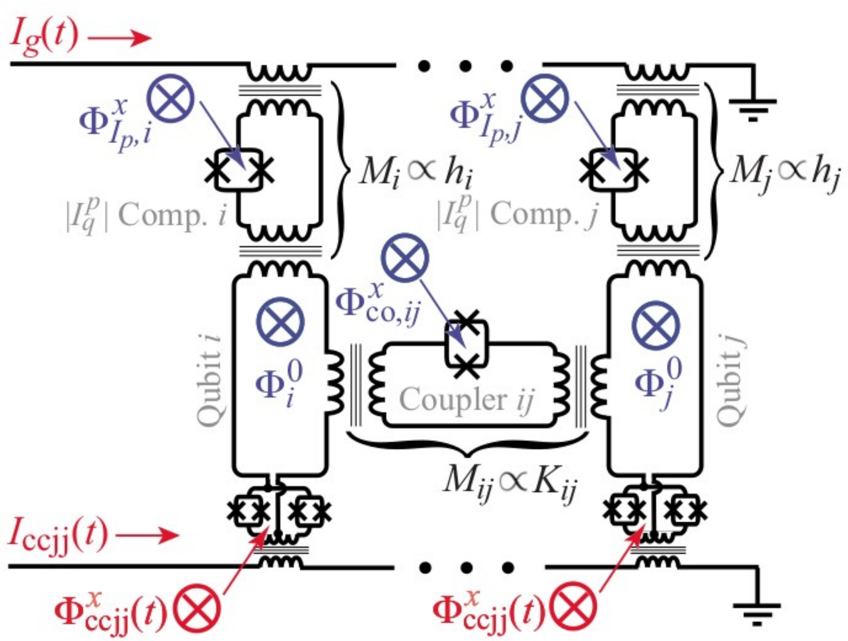
- D-Wave uses radio-frequency SQUIDs (superconducting quantum interference devices) to implement

$$H(s) = -A(s)\sum_i \sigma_i^x + B(s)\left[\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z\right].$$

- 5700 qubits: each connected with 15 others.
- 40000 couplers.
- Finite temperature (15mK), coupling to the environment and imperfect qubits worsen the results.



D-Wave Systems, D-Wave Annealing implementations and controls

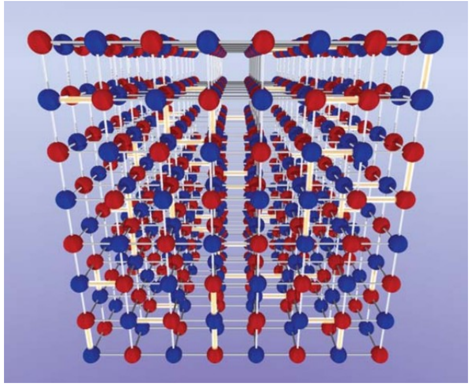


R Harris et al., Physical Review B, vol. 82, no. 2, p. 024511 (2010)

Ising spin glasses

- Simulation of Transverse field Ising model (TFIM) on 8x8x8 cubic lattice:

$$H = -\Gamma \sum_i \sigma_i^x + \mathcal{J} \sum_{\langle i,\, j \rangle} J_{ij} \sigma_i^z \sigma_j^z$$

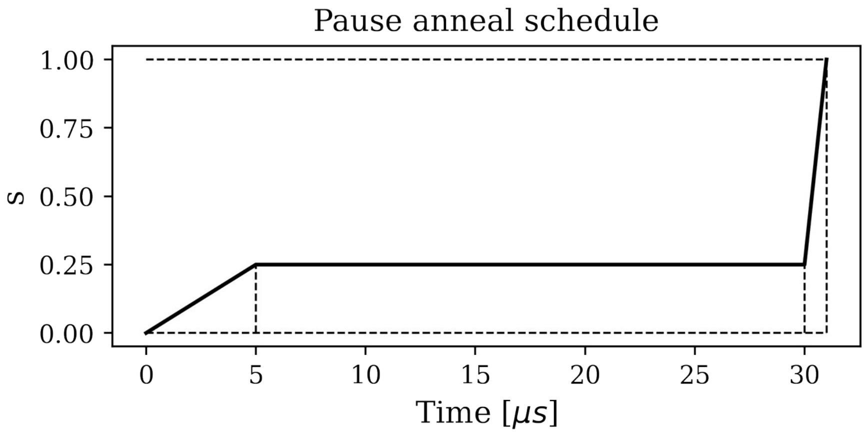


R Harris et al., Science, vol. 361, no. 6398, pp. 162–165 (2018)

- Spin glass: TFIM with $J_{ij} = \begin{cases} -1 \text{ with propability } p \\ 1 \text{ with propability } 1-p \end{cases}.$
- Pause anneal schedule enables fixing parameters on the machine

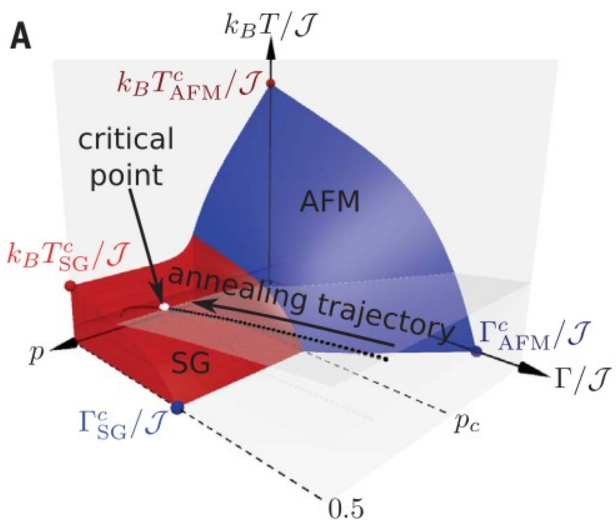
$$H(s) = -A(s) \sum_i \sigma_i^x + B(s) \sum_{\langle i,\, j \rangle} J_{ij} \sigma_i^z \sigma_j^z.$$

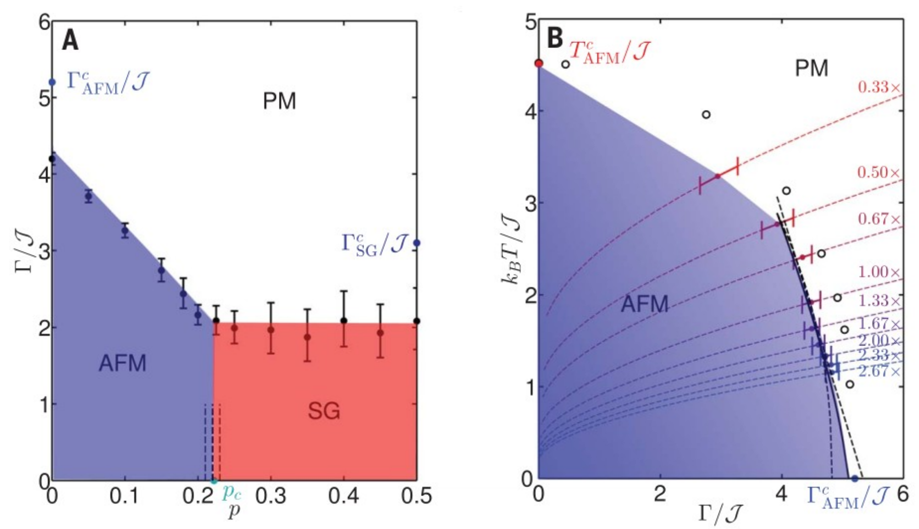
- Measurement problem:
  - Duration: $1\,\mu$s
  - Energy scale: $h \cdot 10\, GHz$



Ising spin glasses

- Paramagnetic (PM), antiferromagnetic (AFM) and spin glass (SG) phases.
- SG: every instance $\{J_{ij}\}$ has many different approximate ground states. We determine the phase of the system by averaging results (e.g. magnetization) over many instances.
- Critical values $\Gamma_c,\ T_c,\ p_c$ agree with numerical data up to a few percent (less than 1% for $p_c = 0.22$)





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Conclusion

- Optimization algorithm, can be used to simulate physics
- Theoretically equivalent to the gate model
- Usable in practice
- Ising Hamiltonian as the problem Hamiltonian
- $T = \mathcal{O}(g_{min}^{-2})$:
  - Only polynomial speedup compared to classical computation.