Data Reconciliation for NPPs

This report provides a summary of the data reconciliation algorithm of BTB Jansky and its applications in NPPs. It also provides brief background on other related techniques.

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This section outlines three general techniques for handling measurement uncertainty of dynamic processes; including a detailed discussion of the data reconciliation algorithm used by BTB Jansky.

Passive Sensing (Measurement Uncertainty Analysis)

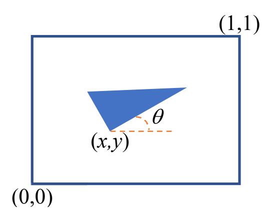
This approach attempts to estimate and reduce measurement uncertainty through local statistical analysis of individual sensors and controls. The measurement uncertainties of the individual process elements are aggregated to estimate the uncertainty of KPI of the process (e.g., RTP). See Uncertainty in Measurement Guide (GUM) and summary in Appendix 1. A step-by-step example from Basu and Bruggeman [8] is reproduced in the companion excel spreadsheet.

Similar techniques include simple Kalman filtering of sensor and controller data (e.g., for steady state and real-time temperature samples).

Analytical Redundancy (Sensorless Data Reconciliation)

One can use process physics (e.g., energy balance) to reduce or eliminate uncertainty about an internal state (or a KPI) of a dynamic process. Under certain conditions this can be achieved without using any measurements at all. Below is a simple real-life example of how to achieve such *sensorless data reconciliation* (where *data* is collected only from a *virtual sensor*, which is the *analytical* model)

Process - Consider the uncertainty associated with the position and orientation of a simple triangular object placed within a cardboard box as shown in the figure below.



The state, $\underline{\boldsymbol{p}}$, of the object within the box can be represented as,

$$\underline{\boldsymbol{p}} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

We can change the state $\underline{\boldsymbol{p}}$ by tilting the box to allow the triangle to slide under gravity (as a way to control the state of the triangle according to the conservation of potential and kinetic energy, and the geometric constraints of the box).

Uncertainty Sources – We'll consider the uncertainty about the position and orientation[1]. Specifically, our knowledge (guess or estimate), $\underline{\boldsymbol{m}}$, about the unmeasured state, $\underline{\boldsymbol{p}}$, can be represented as,

$$\underline{\boldsymbol{m}} = \underline{\boldsymbol{p}} \pm \delta\underline{\boldsymbol{p}}$$

[1] Other possible sources of uncertainty are associated with the forces that affect the object's motion, e.g., gravity force and friction.

$$\delta\underline{\boldsymbol{p}} = \begin{pmatrix} \delta x \\ \delta y \\ \delta \theta \end{pmatrix}$$

Sensorless Reconciliation Task – Assume that no measurements are available about the initial state, $\underline{\boldsymbol{p}}_0$, e.g., the box is covered. Then, the initial state $\underline{\boldsymbol{p}}_0$ of the triangle is completely unknown except that it exists within the box, i.e., $\underline{\boldsymbol{m}}_0 \in [0,1] \times [0,1] \times [0,2\pi)$.

The goal is to find a sequence of box-tilting operations that will completely eliminate the uncertainty. In other words, after executing the last tilting operation of the sequence, we'll know the state of the triangle with 100% level of confidence, i.e., $\underline{\boldsymbol{m}}_{final} = \underline{\boldsymbol{p}}_{final}$ and $\delta\underline{\boldsymbol{p}}_{final} = \underline{\boldsymbol{0}}$.

Note that, since we do not have any measurements for feedback (open-loop), the same sequence of operations must force all possible random initial state $\underline{\boldsymbol{p}}_0$ to the same final state $\underline{\boldsymbol{p}}_{final}$. This is analogous to marbles falling from different random positions into a funnel.

The solution to this reconciliation problem *exists only if* the base and height of the triangle are noticeably different. It consists of a sequence of very few box-tilting operations.

For example, the first titling operation (in any direction) will reduce the orientation uncertainty from the infinite set $[0,2\pi)$ to a set of three possible orientations $\{0, \frac{\pi}{2}, \varphi\}$ where $\varphi = \tan^{-1}(\frac{heigh}{base})$. The next few titling operations should further reduce the uncertainty about orientation to a single point (i.e., eliminating all uncertainty).

We'll leave it to the reader to discover the sequence of titling operations that would eliminate all uncertainty about the position and orientation of the triangle.

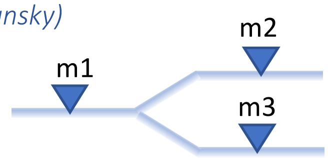
So, <u>under certain conditions</u> one can use analytical redundancy (e.g., conservation laws and process constraints) and <u>active</u> control strategies to reduce or eliminate process uncertainty. (i.e., exchanging control energy for information).

In NPPs, active strategies for reducing uncertainty include optimal sensor/transmitter placement. For example, Basu and Bruggeman [8] actively reduced uncertainty about reactor inlet temperature (at inner zone) by relocating (sliding) its sensor away from stratified flow.

The problem and general techniques of optimal sensor placement are discussed later in this document.

Data Reconciliation (combining measurements and analytical redundancy)

Example[2] - Flow Splitter (BTB Jansky)



The following are the measurement data, $\boldsymbol{m}$, collected from 3 mass flow meters located at a flow splitter:

$$\underline{\boldsymbol{m}} = \begin{bmatrix} \boldsymbol{m_1} \\ \boldsymbol{m_2} \\ \boldsymbol{m_3} \end{bmatrix}$$

The mass flow rate measurement accuracy of the meters are represented using the 95% confidence level of sample-mean distribution, $\underline{\boldsymbol{u}}_{\boldsymbol{m}}$ (which is the initial uncertainty)

$$\underline{\boldsymbol{u}}_{\boldsymbol{m}} = \begin{bmatrix} \boldsymbol{u_1} \\ \boldsymbol{u_2} \\ \boldsymbol{u_3} \end{bmatrix}$$

The mass balance of the splitter (conservation law) is violated due to the measurement errors:

$$\boldsymbol{f}(\underline{\boldsymbol{m}}) = [\mathbf{1} \quad -\mathbf{1} \quad -\mathbf{1}]\,\underline{\boldsymbol{m}} = \underline{\boldsymbol{c}}^{\boldsymbol{T}} . \underline{\boldsymbol{m}} = \boldsymbol{e_{balance}} \neq \mathbf{0}$$

The goal is to reconcile the mass flow rate measurement data with the mass balance. The reconciled values, $\underline{\widehat{\boldsymbol{m}}}$ can be represented as follows (where $\underline{\boldsymbol{v}}$ is the adjustments required for the reconciliation)

$$\underline{\widehat{\boldsymbol{m}}} = \underline{\boldsymbol{m}} + \underline{\boldsymbol{v}}$$

$$\boldsymbol{f}(\underline{\boldsymbol{v}}) = \underline{\boldsymbol{c}}^{\boldsymbol{T}} . \ \underline{\widehat{\boldsymbol{m}}} = \mathbf{0}$$

The adjustments $\underline{\boldsymbol{v}}$, can be calculated using the Gaussian correction principle

$$\underline{\boldsymbol{v}} = \boldsymbol{arg} \ \min_{\underline{\boldsymbol{v}}} \underline{\boldsymbol{v}}^{\boldsymbol{T}} \boldsymbol{S}_{\underline{\boldsymbol{m}}}^{-1} \underline{\boldsymbol{v}}$$
$$\textbf{subject to } \boldsymbol{f}(\underline{\boldsymbol{v}}) = \mathbf{0}$$

where $S_{\underline{m}}$ is the covariance matrix of the measurements data

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[2] **Reference**: J Jansky, "Process data reconciliation in nuclear power plants", 2003.
File: [1.0] Process data reconciliation in nuclear power plants.pdf
Path: "D:\R&D\MW Optimization\Data Reconciliation\CANDU\BTB-Jansky (VALI)"
Worksheet: See *DR Example 1* tab in "D:\R&D\MW Optimization\Bruce\BP_Basu_Bruggeman.xlsx"

$$\boldsymbol{S}_{\underline{\boldsymbol{m}}} = \begin{bmatrix} \boldsymbol{s}_{\boldsymbol{m},\mathbf{11}} & \cdots & \boldsymbol{s}_{\boldsymbol{m},\mathbf{13}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{s}_{\boldsymbol{m},\mathbf{31}} & \cdots & \boldsymbol{s}_{\boldsymbol{m},\mathbf{33}} \end{bmatrix} \textbf{ where } \boldsymbol{s}_{\boldsymbol{m},\boldsymbol{ii}} = (\underline{\boldsymbol{u}}_{\boldsymbol{m}_{\boldsymbol{i}}}.\mathbf{1}/(\mathbf{1.96}))\text{^}\mathbf{2} \textbf{ and } \boldsymbol{s}_{\boldsymbol{m},\boldsymbol{ij}} = \mathbf{0}, \boldsymbol{i} \neq \boldsymbol{j}$$

In this example, let $\underline{\boldsymbol{m}} = \begin{bmatrix} 500.000 \\ 245.000 \\ 250.000 \end{bmatrix}$ and $\underline{\boldsymbol{u}}_{\boldsymbol{m}} = \begin{bmatrix} 25.000 \\ 12.250 \\ 12.500 \end{bmatrix}$ which corresponds to 5% accuracy

Then

$$\boldsymbol{S}_{\underline{\boldsymbol{m}}} = \begin{bmatrix} 162.6926 & 0 & 0 \\ 0 & 39.0625 & 0 \\ 0 & 0 & 40.67316 \end{bmatrix}$$

The solution of the constrained optimization associated with the Gaussian correction principle can be obtained[3] by minimizing the augmented (penalty) objective function $J(\underline{v})$, where

$$J(\underline{v}) = \underline{\boldsymbol{v}}^{\boldsymbol{T}} \boldsymbol{S}_{\underline{\boldsymbol{m}}}^{-\mathbf{1}} \underline{\boldsymbol{v}} + \lambda\, f(\underline{\boldsymbol{v}})$$

The solution to the above problem can be expressed as follows:

$$\underline{v} = -\left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}} \,.\, S_{\underline{m}}\right)^{T} \left(\left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}} \,.\, S_{\underline{m}}\right) . \left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}}\right)^{T} \right)^{-1} .\, f(\underline{\boldsymbol{v}})$$

Where $\frac{\partial f(\underline{v})}{\partial \underline{v}} = \underline{\boldsymbol{c}}^{\boldsymbol{T}} = [1 \quad -1 \quad -1]$ and $f(\underline{\boldsymbol{v}})|_{\underline{v}=\underline{0}} = 5.00$ (with an initial guess $\underline{v} = \underline{0}$)

Substituting the data into the above expression,

$$\underline{v} = \begin{bmatrix} -0.67110 \\ 0.16113 \\ 0.16777 \end{bmatrix} \times 5.00$$

$$\underline{v} = \begin{bmatrix} -3.35548 \\ 0.805651 \\ 0.838870 \end{bmatrix}$$

The reconciled mass flow rate can be calculated using,

$$\underline{\widehat{\boldsymbol{m}}} = \underline{\boldsymbol{m}} + \underline{\boldsymbol{v}}$$

as,

$$\underline{\widehat{\boldsymbol{m}}} = \begin{bmatrix} 496.6445 \\ 245.8057 \\ 250.8389 \end{bmatrix}$$

The corresponding value of the objective function is:

[3] Other direct approaches can be used for general nonlinear constrained optimization formulation of the Gaussian correction principle.

$$J(\underline{v}) = \frac{(-3.35548)^2}{162.6926} + \frac{(0.805651)^2}{39.0625} + \frac{(0.838870)^2}{40.67316} - 2\lambda(496.6445 - 245.8057 - 250.8389)$$
$$J(\underline{v}) = 0.103123 - \ 2\lambda\ (0) = \ \ 0.103123$$

The uncertainty (confidence interval) $\boldsymbol{u_{\hat{\underline{m}}}}$ of the reconciled mass flow rate $\hat{\underline{\boldsymbol{m}}}$, can be calculated from the covariance matrix $S_{\hat{\underline{m}}}$ as follows:

$$S_{\hat{\underline{m}}} = S_{\underline{m}} - S_{\underline{v}}$$

Where $S_{\underline{v}} = \left(\frac{\partial f(\underline{v})}{\partial \underline{v}} \cdot S_{\underline{m}}\right)^T \left(\left(\frac{\partial f(\underline{v})}{\partial \underline{v}} \cdot S_{\underline{m}}\right) \cdot \left(\frac{\partial f(\underline{v})}{\partial \underline{v}}\right)^T\right)^{-1} \cdot \left(\frac{\partial f(\underline{v})}{\partial \underline{v}} \cdot S_{\underline{m}}\right)$

$$S_{\underline{v}} = \begin{bmatrix} 109.1824 & -26.2147 & -27.29559 \\ -26.2147 & 6.294146 & 6.553671 \\ -27.2956 & 6.553671 & 6.823897 \end{bmatrix}$$

$$S_{\hat{\underline{m}}} = \begin{bmatrix} 53.51027 & 26.21468 & 27.29559 \\ 26.2147 & 32.76835 & -6.553671 \\ 27.2956 & -6.553671 & 33.84926 \end{bmatrix}$$

The uncertainty (95% confidence interval) $\boldsymbol{u_{\hat{\underline{m}}}}$ is calculated as follows:

$$\boldsymbol{u_{\hat{\underline{m}}}} = \begin{bmatrix} 1.96\sqrt{\boldsymbol{s_{\hat{m},11}}} \\ 1.96\sqrt{\boldsymbol{s_{\hat{m},22}}} \\ 1.96\sqrt{\boldsymbol{s_{\hat{m},33}}} \end{bmatrix}$$

$$\boldsymbol{u_{\hat{\underline{m}}}} = \begin{bmatrix} 14.33754 \\ 11.21976 \\ 11.40330 \end{bmatrix}$$

$$\boldsymbol{u_{\hat{\underline{m}}}} = \begin{bmatrix} 14.33754/496.6445 \\ 11.21976/245.8057 \\ 11.40330/250.8389 \end{bmatrix} = \begin{bmatrix} 2.89\% \\ 4.56\% \\ 4.55\% \end{bmatrix}$$

Since $J(\underline{v}) = \chi^2_{\hat{\underline{m}}}$ is *a random variable having a* χ^2 *distribution with df=1*, we can use a Chi-Square test to ensure the reconciled values $\hat{\underline{\boldsymbol{m}}}$ does not include ***global*** gross (non-random) measurement errors. A typical approach[4] is to show that the probability $Pr(\chi^2_{\hat{\underline{m}}} \geq \chi^2)$ from a Chi-Square distribution table is very small to ensure goodness of fit[5]. Alternatively, see Jansky (2003), show that $\boldsymbol{J(\hat{\underline{v}}) < \chi^2(0.05, df)}$ for a 95% confidence level. In the above example, $J(\underline{v}) = \chi^2_{\hat{\underline{m}}}$ =0.103123 and $\chi^2(df, 0.05) = 3.8415$ (see χ^2 table[6] entry for df[7,8]=1 and p=0.05) and hence the reconciled estimates of mass flow rate are of good quality.

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[4] See example and rationale in M. Camara, et. al. "Numerical Aspects of Data Reconciliation in Industrial Applications", Processes, October 2017 (pdf at D:\R&D\MW Optimization\Data Reconciliation\Algorithms)

[5] https://www.asc.ohio-state.edu/gan.1/teaching/spring04/Chapter6.pdf (pp 7)

[6] https://www.di-mgt.com.au/chisquare-table.html

[7] The degree of freedom, df is the rank of M which is a matrix generated during a data reconciliation solution process. See S. Guo, P. Liu and Z. Li, "Data reconciliation for the overall thermal system of a steam turbine power plant" (2016)

[8] M. Câmara "Numerical Aspects of Data Reconciliation in Industrial Applications," Processes, 5, 56, October 2017

Finally, verify that ***individual*** measurement corrections $\hat{v}_i$ should be within 2 standard deviations (as obtained from correction, not the measurement or reconciliated, covariance matrix): $\frac{|\hat{v}_i|}{\sqrt{s_{\hat{v},ii}}} \leq 1.96 \quad i = 1,2,3$; otherwise corresponding data and reconciled value should not be accepted.

Notes on Data Reconciliation Algorithm for Isolating Sensor Errors

- The purpose of *Global Tests* (e.g., chi square described above[9]) is to detect the existence of gross errors in the reconciled correction vector. It does not support isolation (identification or labeling) of individual sensor or measurements with gross errors (bias or trends). A separate set of tests, called *Measurement Tests* are employed (e.g., in RECON software[10]) for such task; also illustrated in the example above using $\frac{|\hat{v}_i|}{\sqrt{s_{\hat{v},ii}}} \leq 1.96$. (or $\frac{|\hat{v}_i|}{\sqrt{\max\left(s_{\hat{v},ii}, \frac{s_{m,ii}}{10}\right)}} \leq 1.96$ per VDI 2048[11])
- A workflow for iteratively handling sensor gross errors is described in the following section. It should be noted that the simple test described in the flow split example above works only for the case of linear constraints (e.g., mass flow balance equations), but not in general[12].
- A more detailed discussion of the numerical aspects of the general data reconciliation (combined with parameter estimation, e.g., of **steam generator and preheaters fouling levels**) problem is covered in [4][13]

$$\hat{\underline{z}}, \hat{\underline{\theta}}, = \arg min_{\underline{z},\underline{\theta}}\ J(\underline{z}_m, \underline{z})$$

$$f(\underline{z}, \underline{\theta}) = 0 \ \text{(process models)}$$

$$g(\underline{z}, \underline{\theta}) \leq 0 \ \text{(process constraints)}$$

$$\underline{z}^L \leq \underline{z} \leq \underline{z}^U$$

$$\underline{\theta}^L \leq \underline{\theta} \leq \underline{\theta}^U$$

$$\underline{\theta} \ \text{vector of process parameters}$$

$$\underline{z}_m = \begin{bmatrix} \underline{y}_m \\ \underline{u}_m \end{bmatrix} \text{vector of (control and outputs) measurements}$$

- A modified weighted least square estimator formulation of the objective function is typically as follows,

$$J(\underline{z}_m, \underline{z}) = (\underline{z}_m - \underline{z})^T V_{\underline{z}}^{-1} (\underline{z}_m - \underline{z}) + (\underline{\theta}_m - \underline{\theta})^T V_{\underline{\theta}}^{-1} (\underline{\theta}_m - \underline{\theta})$$

$$\underline{\theta}_m \ \text{vector of process estimated parameters}$$

[9] The chi-square test is valid for the case of linear process models (e.g., linear conservations laws), but not necessarily valid for nonlinear models. See J.W. Hines and R. Seibert "Technical Review of On-line Monitoring Techniques for Performance Assessment,", Volume I: State-of-the-Art, U.S. Nuclear Regulatory Commission, 2005, pp 25.

[10] M. Syed, et.al., "Data Reconciliation and Suspect Measurement Identification for Gas Turbine Cogeneration Systems," at *D:\R&D\MW Optimization\Data Reconciliation\Fundamentals\RECON*

[11] H. Lee and G Heo, "Study on VDI-2048 for Plant Efficiency Calculation," Transactions of the Korean Nuclear Society Spring Meeting, Jeju, Korea, May 29-30, 2014.

[12] J.W. Hines and R. Seibert "Technical Review of On-line Monitoring Techniques for Performance Assessment,", Volume I: State-of-the-Art, U.S. Nuclear Regulatory Commission, 2005, pp 39 (pdf page 55/127).

[13] M. Câmara "Numerical Aspects of Data Reconciliation in Industrial Applications," Processes, 5, 56, October 2017

- For the above general problem, the degree of freedom[14] and the global test for the absence of global gross error, can be expressed as follows,

$$df = \dim(\underline{z_m}) - \dim(\underline{u_m}) - \dim(\underline{\theta})$$
$$J(\underline{z_m}, \underline{z}) < \chi^2(0.05, df)$$

- The conservation laws can be represented by a set of nonlinear relations of normally distributed random state variables,

$$\boldsymbol{f}(\underline{\boldsymbol{x}}) = \underline{\boldsymbol{0}};\ \underline{\boldsymbol{x}} \in \mathbb{R}^n, \boldsymbol{f}(.): \mathbb{R}^n \to \mathbb{R}^r \text{ where } 0 < r < n$$

  - The Linearization of process conservation laws is permissible as long as the partial derivates of the function, $\frac{\partial \boldsymbol{f}}{\partial \underline{x}}$, is practically constant within the confidence intervals of the measured variables[15]. This condition can be fulfilled with adequate accuracy of measurement.

$$\frac{\partial \boldsymbol{f}}{\partial x_i} = const \implies \frac{\partial^2 \boldsymbol{f}}{\partial x_i^2} = 0;\ \forall x_i = E(x_i) + 1.96\sigma_{x_i}$$

  - Where the random variable of the approximate linear relation is,

$$\boldsymbol{f}(\underline{\boldsymbol{x}}) \approx \tilde{\boldsymbol{f}}(\underline{\boldsymbol{x}}) = \left(\frac{\partial \boldsymbol{f}}{\partial \underline{\boldsymbol{x}}}\right)^T . \left(\underline{\boldsymbol{x}} - E(\underline{\boldsymbol{x}})\right) + \boldsymbol{f}(E(\underline{\boldsymbol{x}}))$$

  - A linearized approximation of the conservation equations can be used to estimate a reconciled state vector, $\hat{\underline{\boldsymbol{x}}}$, from measurement $\underline{\boldsymbol{x}_{\boldsymbol{m}}}$.

$$\boldsymbol{f}(\hat{\underline{\boldsymbol{x}}}) = \boldsymbol{f}(\underline{\boldsymbol{x}_{\boldsymbol{m}}} + \underline{\boldsymbol{v}}) = \underline{\boldsymbol{0}} \implies \tilde{\boldsymbol{f}}(\hat{\underline{\boldsymbol{x}}}) = \left(\frac{\partial \boldsymbol{f}}{\partial \underline{\boldsymbol{x}}}\right)^T . (\underline{\boldsymbol{v}}) + \boldsymbol{f}(\underline{\boldsymbol{x}_{\boldsymbol{m}}}) = \underline{\boldsymbol{0}}$$

  - The reconciled vector $\hat{\underline{\boldsymbol{x}}} = \underline{\boldsymbol{x}_{\boldsymbol{m}}} + \underline{\boldsymbol{v}}$ can obtained as the solution to the following least squared problem

$$\arg\min_{\underline{v}} J(\underline{v}) = \underline{v}^T S_{\underline{x_m}}^{-1} \underline{v} + \underline{\boldsymbol{\lambda}}^T \left(\boldsymbol{f}(\underline{\boldsymbol{x}}) + \left(\frac{\partial \boldsymbol{f}}{\partial \underline{\boldsymbol{x}}}\right)^T . (\underline{\boldsymbol{v}})\right)$$

  - The solution of the reconciliation problem is illustrated in the n-dimensional vector space of $\aleph(0,1)$ random variables. It is assumed that the Jacobian matrix $\left(\frac{\partial f}{\partial \underline{x}}\right)$ has full rank, i.e., $rank\left(\frac{\partial f}{\partial \underline{x}}\right) = r$. The initial measurement uncertainty vector is represented by the ***n***-dimensional vector $\underline{\boldsymbol{\ell}} \in \aleph^n(0,1)$. The vector $\underline{\boldsymbol{0}} \in \aleph^n(0,1)$ represents the mean value of the process state vector. The initial violation of the balance equations is represented by ***r***-dimensional vector $\underline{\boldsymbol{u}} \in \aleph^r(0,1)$ in the *N subspace*. The measurement uncertainty of reconciled solution is represented by the (***n-r***)-dimensional projection vector $\hat{\underline{\boldsymbol{\ell}}} \in \aleph^{n-r}(0,1)$ in the *M* subspace. The vector $\hat{\underline{\boldsymbol{\ell}}}$ reflects the remaining uncertainty, which might be further reduced by adding more equations.

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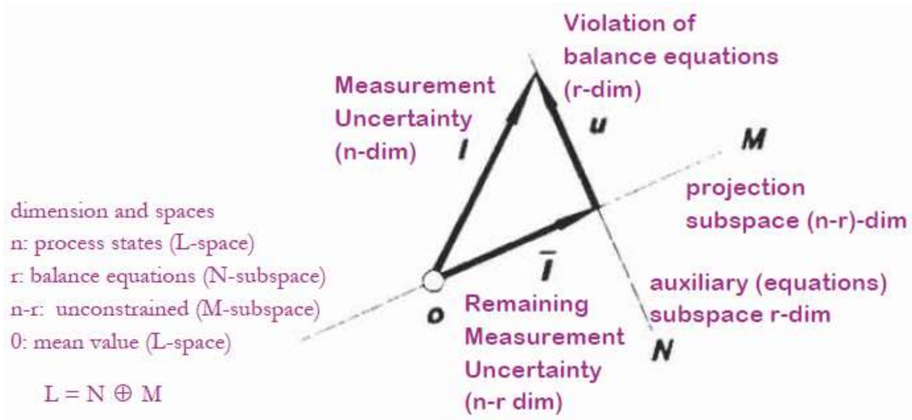
[14] Assumes full-rank matrices (e.g., uncorrelated measurements and linear independence of process equations); otherwise use rank(Sensitivity Matrix) – see S. Guo, P. Liu and Z. Li, “Data reconciliation for the overall thermal system of a steam turbine power plant” (2016). Also, Heyen G., E. Maréchal and B. Kalitventzeff, “Sensitivity Calculations and Variance Analysis in Plant Measurement Reconciliation”, Computers and Chemical Engineering, vol. 20S, 539-544, 1996.

[15] [5] S. Streit “State Estimation in Thermal Power Plants,” IFAC Control of Power Plants and Systems, Munich, 1992.



- The Covariance matrix of the estimated variables can be estimated from the measurement covariance matrix as $S_{\underline{\hat{x}}} = S_{\underline{x}} - S_{\underline{v}}$, where[16]

$$S_{\underline{v}} = \left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}} \cdot S_{\underline{x}}\right)^T \left(\left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}} \cdot S_{\underline{x}}\right) \cdot \left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}}\right)^T\right)^{-1} \cdot \left(\frac{\partial \boldsymbol{f}(\underline{\boldsymbol{v}})}{\partial \underline{v}} \cdot S_{\underline{x}}\right)$$

- When using a Lagrange vector to augment violation of the balance equations, a quadratic term of the imbalance vector $\underline{\boldsymbol{u}} \in \aleph^r(0,1)$ is added into the the objective function. The quadratic term is represented as a single random variable $\underline{\boldsymbol{u}}^2 \in \chi_r^2$ distributed a chi-square with ***r*** degree of freedom, which is the number of effective balance equations $r = rank\left(\frac{\partial f}{\partial \underline{x}}\right)$.
- When the **global** test $\underline{\boldsymbol{u}}^2 \leq \chi_{r,95\%}^2$ is violated, it indicates that the measurement values exceed their assumed uncertainty (confidence intervals), which cause the corrections $\underline{\hat{\boldsymbol{v}}}$ to also exceed their expected uncertainty (correction confidence intervals) at the individual measurement level, hence the **measurement** test $\frac{|\hat{v}_i|}{\sqrt{\max\left(s_{\hat{v},ii}, \frac{s_{m,ii}}{10}\right)}} \leq 1.96,\ i = 1, \dots, n$ must be performed.
- When solution of the reconciliation problem is obtained using the least-square formulation (including the balance equations), the variances $\boldsymbol{s}_{\hat{x},ii}$ of the reconciled variables will be the minimum variances (since least-square formulation is an efficient estimator). Hence, the variances $\boldsymbol{s}_{\hat{x},ii}$ of the reconciled variables are less than the initial variances $\boldsymbol{s}_{x,ii}$ of the measured variables, i.e., $\boldsymbol{s}_{\hat{x},ii} \leq \boldsymbol{s}_{x,ii}$.
- When the balance equations are augmented into the least-square formulation, a stochastic dependency is introduced between the random variables of the reconciled state variables, $\underline{\hat{x}}$, which can be used to isolate the *subset* of measurements[17] $\left\{\hat{v}_k \mid \hat{\rho}_{i,j} \cong 1,\ k \neq i\right\}$ which are *strongly correlated* to a failed measurement test $\frac{|\hat{v}_i|}{\sqrt{\max\left(s_{\hat{v},ii}, \frac{s_{m,ii}}{10}\right)}} \leq 1.96$. The correlations of the reconciliation variables $\underline{\hat{v}}$ can be obtained from the reconciliation covariance matrix, $S_{\underline{\hat{v}}}$, as, [6]

$$\hat{\rho}_{i,j} = \frac{cov(\hat{v}_i, \hat{v}_j)}{\sigma_{\hat{v}_i} \cdot \sigma_{\hat{v}_i}} = \frac{cov(\hat{v}_i, \hat{v}_j)}{\boldsymbol{s}_{\hat{v},ii} \cdot \boldsymbol{s}_{\hat{v},jj}} = \frac{\boldsymbol{s}_{\hat{v},ij}}{\sqrt{\boldsymbol{s}_{\hat{v},ii}} \cdot \sqrt{\boldsymbol{s}_{\hat{v},jj}}}$$

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[16] Review this expression for all state-estimation algorithms (e.g., correlated, non-smooth, mixed-integer, nonlinear constrained optimization). Find the optimality necessary and sufficient conditions and required statistical properties.

[17] [6] S. Streit, M. Langenstein, B. Laipple and H. Eitschberger, "A new method for evaluation and correction of thermal reactor power and present operational applications," ICONE13, Beijing, 2005.

Notes on Data Reconciliation Algorithm for Estimating Reactor KPI (e.g., RTP)

- For any arbitrary[18] *KPI*, $h(\boldsymbol{\underline{x}})$, the estimate $\underline{\hat{x}}$ can be used to calculate a reconciled value, $\hat{h}$,of the *KPI* as $\hat{h} = h(\boldsymbol{\underline{\hat{x}}})$
- The covariance matrix (uncertainty) of the *KPI* can be calculated, in terms of the covariance matrix $S_{\underline{\hat{x}}}$ of reconciled states, as follows:
$$S_{\hat{h}} = S_{h(\underline{\hat{x}})} = \left(\frac{\partial h}{\partial \underline{\hat{x}}}\right)^T S_{\underline{\hat{x}}} \left(\frac{\partial h}{\partial \underline{\hat{x}}}\right)$$
- The estimated *KPI* is guaranteed to have the minimum variance (confidence interval). A mathematical proof is available in [6] (and VDI 2048 [7]), which can be used to show that the uncertainty associated with a reconciled estimate of the *KPI*, $\widehat{H}$, is guaranteed to be less than that associated with any other estimate.
- Standard partial differentiation rules can be applied to estimate additional functions of KPI.
- As part of a plant license, the licensee must prove that certain *KPI's* (e.g., reactor thermal power) will not exceed a maximum safety value. Let H be a *KPI* which is calculated from the measurements of the process state variables, $\underline{x}$, as
$$H = h(\underline{x})$$
- Due to the measurement uncertainty of the state variables, the safety *KPI*, H, is treated as a random variable with *unknown* true mean value, μ_H, and a variance σ_H^2.
- The licensee goal is to <u>guarantee</u> (with a specific high degree of confidence) that the *true* mean value[19] of the KPI, μ_H, is guaranteed <u>not to exceed</u> a licensed safety value, μ_H^*. The degree of confidence in the guarantee can be pressed in terms of the probability,
$$p = Pr(\mu_H \leq \mu_H^*)$$
- The approach is to calculate the sample probability $p_{\underline{x}}$ for any measured state values, $\underline{x}$, and a licensed value, μ_H^*,
$$p_{\underline{x}} = Pr\left(\mu_{h(\underline{x})} \leq \mu_H^*\right)$$
- Define a random variable, Δ_H, for the safety margin with a mean value, μ_{Δ_H}, and variance, $\sigma_{\Delta \mathrm{H}}^2$, where,
$$\Delta_H \triangleq \mu_H^* - H$$
$$\mu_{\Delta_H} = \mu_H^* - \mu_H$$
$$\sigma_{\Delta_H} = \sigma_H$$

  Note: Similarly, one can use the sample safety KPI, $h(\underline{x})$, to define a new random variable, $\Delta_h(\underline{x})$, as the sample safety margin:
$$\Delta_h(\underline{x}) \triangleq \mu_H^* - h(\underline{x})$$
$$\mu_{\Delta_h(\underline{x})} = \mu_H^* - \mu_{h(\underline{x})}$$
$$\sigma_{\Delta_h(\underline{x})} = \sigma_{h(\underline{x})}$$

- Using the standardized deviation, the random variable Δ_H can be transformed into a zero mean and unit standard deviation, i.e.,

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[18] For example an estimate of the Reactor Thermal Power (RTP), based on the reconciled heat and flow rate balances.

[19] A *True* mean value, μ_h, is the ensemble mean.

$$\frac{\Delta_H - \mu_{\Delta_H}}{\sigma_{\Delta_H}} \in \aleph(0,1)$$

Let n_p be the *p*-quantile of the standardized normal distribution of the safety margin, then

$$p \triangleq \Pr\left(\frac{\Delta_H - \mu_{\Delta_H}}{\sigma_{\Delta_H}} \le n_p\right) = \Pr\left(\frac{(\mu_H^* - H) - (\mu_H^* - \mu_H)}{\sigma_H} \le n_p\right)$$

Which can be rewritten as,

$$p \triangleq \Pr\left(\mu_H \le H + n_p \, . \sigma_H\right)$$

The value of n_p in the above expression can be arbitrarily set. One choice is to set n_p such that

$$p \triangleq \Pr\left(\mu_H \le H + n_p \, . \sigma_H\right) = \Pr(\mu_H \le \mu_H^*)$$

i.e., select n_p such that

$$H + n_p \, . \sigma_H = \mu_H^* \text{ or}$$

$$n_p = \frac{\mu_H^* - H}{\sigma_H}$$

- Using the *distribution* function of a standardized normal distribution $F(z)$, then the probability

$$p \triangleq \Pr(z \le z^*) = F(z^*) = \frac{1}{\sqrt{2\pi}} . \int_{-\infty}^{z^*} e^{-\frac{t^2}{2}} \, dt$$

- In order to use the standardized normal distribution to calculate $\Pr(\mu_H \le \mu_H^*)$, use the following standardized deviations of H,

$$z = \frac{\mu_H - H}{\sigma_H}$$

$$z^* = \frac{\mu_H^* - H}{\sigma_H} = n_p$$

Then,

$$p \triangleq \Pr(\mu_H \le \mu_H^*) = \Pr(z \le z^*) = \mathrm{F}\left(n_p\right) = F\left(\frac{\mu_H^* - H}{\sigma_H}\right)$$

- To estimate the probability $p = \Pr(\mu_H \le \mu_H^*)$ use the sample (e.g., reconciled) value of H, $g(\underline{x})$, and the sample variance $\sigma_{h(\underline{x})}^2$ as follow:

$$p \triangleq \Pr(\mu_H \le \mu_H^*) = F\left(\frac{\mu_H^* - h(\underline{x})}{\sigma_{h(\underline{x})}}\right)$$

$$\sigma_{h(\underline{x})}^2 = \left(\frac{\partial h(\underline{x})}{\partial \underline{x}}\right)^T S_{\underline{x}} \left(\frac{\partial h(\underline{x})}{\partial \underline{x}}\right)$$

- Since the reconciled state values, $\hat{\underline{x}}$, have minimal variances, compared to variances of measured state variables,

$$\forall_{\underline{x}} \, \sigma_{h(\hat{\underline{x}})} \le \sigma_{h(\underline{x})} \Rightarrow \forall_{\underline{x}} \, \hat{p} = F\left(\frac{\mu_H^* - h(\hat{\underline{x}})}{\sigma_{h(\hat{\underline{x}})}}\right) \ge p = F\left(\frac{\mu_H^* - h(\underline{x})}{\sigma_{h(\underline{x})}}\right)$$

- then it can be shown that:

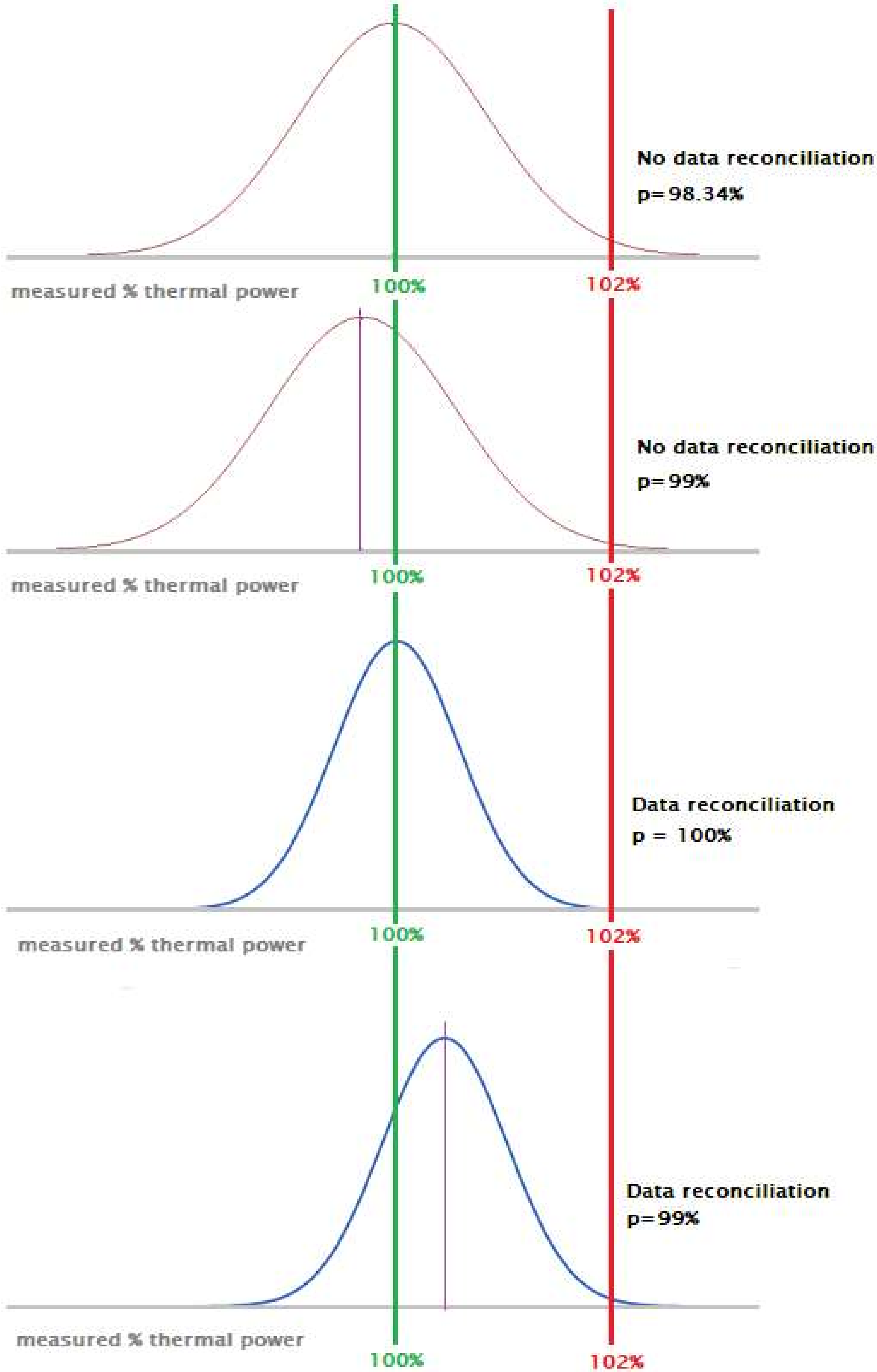
$$\forall_{\underline{x}} \, \sigma_{h(\hat{\underline{x}})} \le \sigma_{h(\underline{x})} \Rightarrow \forall_{\underline{x}} \, \hat{p} = Pr\left(\mu_{h(\hat{\underline{x}})} \le \mu_H^*\right) \ge p = Pr\left(\mu_{h(\underline{x})} \le \mu_H^*\right)$$

- For any given target level of confidence, p°, one can estimate the corresponding estimate of the *KPI*, H°, such that $p^{\circ} = Pr\left(H^{\circ} \le H^*\right)$ where H^* is the safety (maximum) limit of H, as follows:

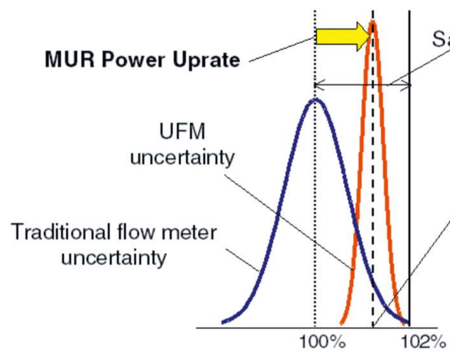
$$H^{\circ} = H^* - \sigma_H \, . \, F^{-1}\left(p^{\circ}\right)$$

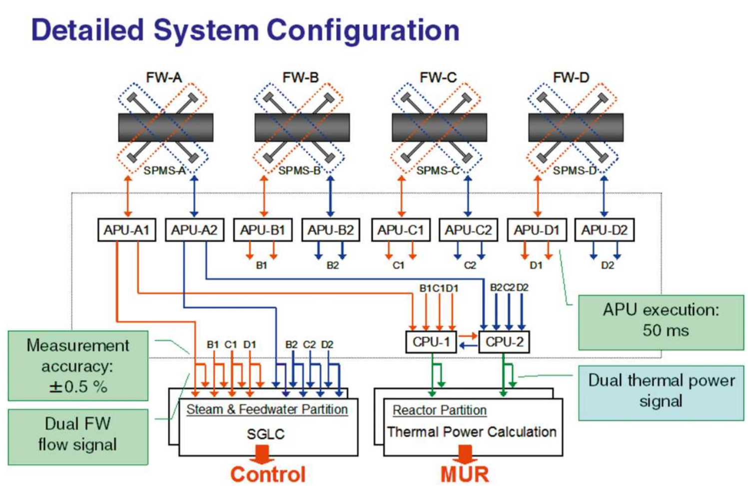
Use $\mu_{h(\hat{\underline{x}})}$ and $\sigma_{h(\hat{\underline{x}})}$ in the above expression to calculate the reconciled value of $\widehat{H}^{\circ}$

- The next figure illustrates the use of data reconciliation for documenting a technical requirement of an MUR-based *uprate* application to a licensing authority.[20]

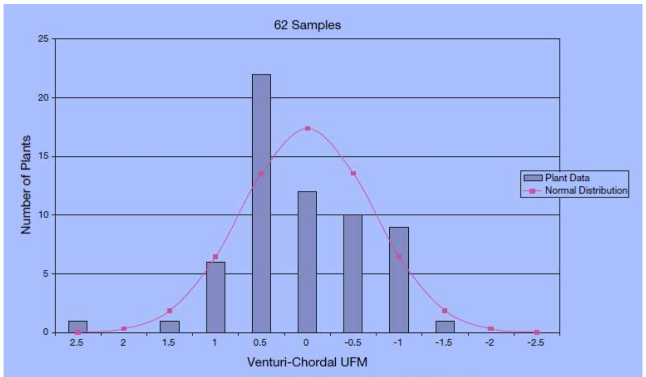


- **Measurement uncertainty recapture (MUR) uprate.** A term applied to the regulatory process of reducing certain emergency core cooling system (ECCS) assumptions regarding reactor power measurement uncertainty from a standard assumption (typically 2–3%) to a specific value based on the use of more accurate feedwater flow measurement devices. The reduction in the uncertainty assumption can result in an increase in reactor licensed thermal power of 1.2–1.7% above currently licensed thermal power.
- Measurement Uncertainty Recapture (MUR) without data reconciliation - CANDU Example[21] – The following diagram illustrate MUR through use of ultrasound flowmeter (UFM) for more accurate measurement of feedwater flow measurements, replacing conventional flow nozzles and venturis.





[20] "Backgrounder - Power Uprates for Nuclear Plants," US NRC. See Measurement Uncertainty Recapture (MUR).
[21] Application Of Ultrasonic Flow Measurement For Power Uprates Based On Measurement Uncertainty Recapture," Appendix IV, in Power Uprate in Nuclear Power Plants: Guidelines and Experience, IAEA Nuclear Energy Series No. NP-T-3.9, 2011.



Distribution of Venturi–chordal differences.

- Measurement Uncertainty Recapture (MUR) application of data reconciliation – Example[22]

VDI 2048 – An Outline

The above analysis is based on VDI 2048 standard [23], which is summarized[24] in this section.

VDI 2048 basic requirements of data reconciliation:

- Corrections to measurement data must be as small as possible (to ensure reconciliation results are not based on gross errors)
- Reconciliation algorithm must reconcile not only measurement values but also measurement uncertainty (i.e., to reconcile covariances) in order to asses confidence in reconciled values.

Open Source Data Reconciliation

Attempts to support data reconciliation in open *Modelica* is outlined in project OPENCPS[25]. The approach is to automatically extract the relevant constraint equations from the *Modelica* models and compute the Jacobian matrix of the constraint equations for the selected state variables. The data reconciliation is based on VDI 2048. The user is expected to specify list of state variables to be reconciled, the covariance matrix of the variables (and flag equations to be ignored by the algorithm). The global and Measurement tests are used for convergence (see workflow section below)

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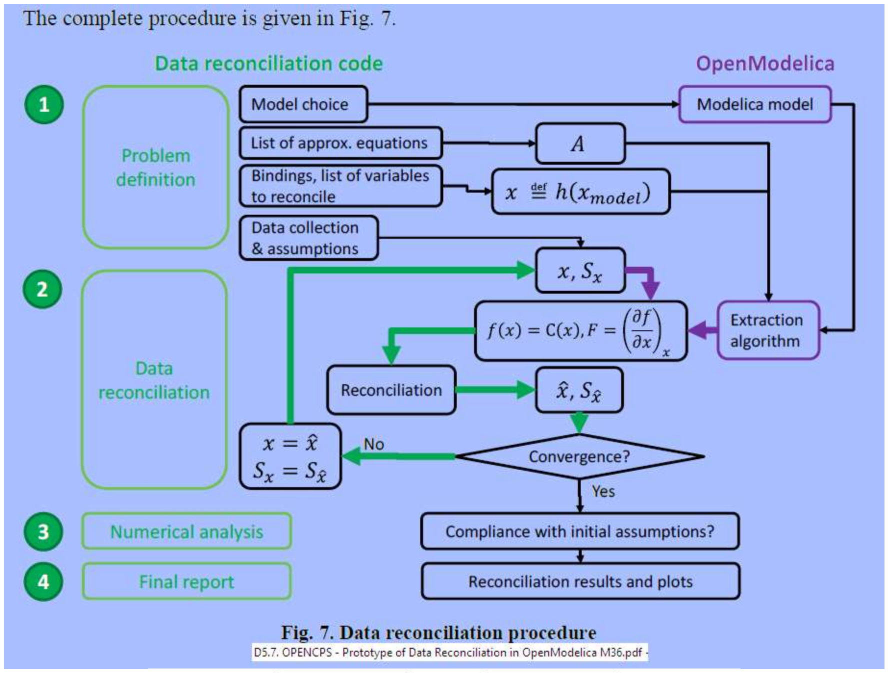
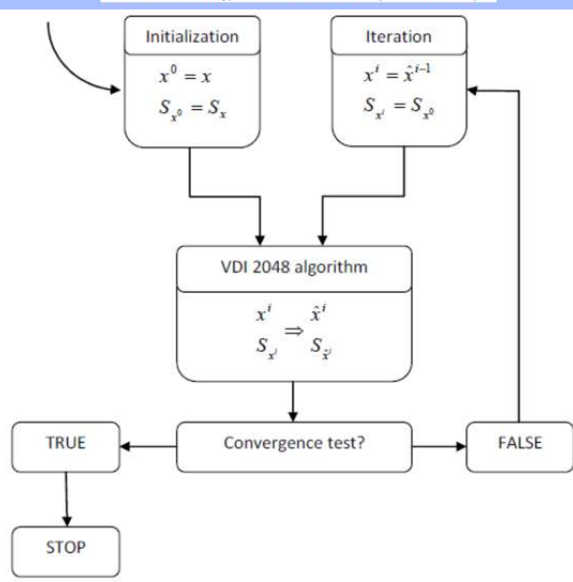
[22] "Use For Measurement Uncertainty Recapture, Power Uprate, Process Monitoring, Component Monitoring And Acceptance Tests In NPPs," Appendix VII, in Power Uprate in Nuclear Power Plants: Guidelines and Experience, IAEA Nuclear Energy Series No. NP-T-3.9, 2011.

[23] Magnus Langenstein and Bernd Laipple (BTB Jansky) "Global Balance of Plant in NPPs Using Process Data Reconciliation According to VDI 2048," 2014 22nd International Conference on Nuclear Engineering, ICONE22-31277 (5 pages), Prague, 2014 (original paper was presented in 2009 IECONE 17). Not available.

[24] H. Lee and G Heo, "Study on VDI-2048 for Plant Efficiency Calculation," Transactions of the Korean Nuclear Society Spring Meeting, Jeju, Korea, May 29-30, 2014.

[25] Adrian Pop, et. al., "Prototype of data reconciliation in OpenModelica - D5.7", part of OPENCPS - Open Cyber-Physical System Model-Driven Certified Development M36, (2018). An earlier version (OPENPROD), Handling of Uncertainties in OpenModelica: Data Reconciliation and Propagation of Uncertainties, based on OpenTURNS





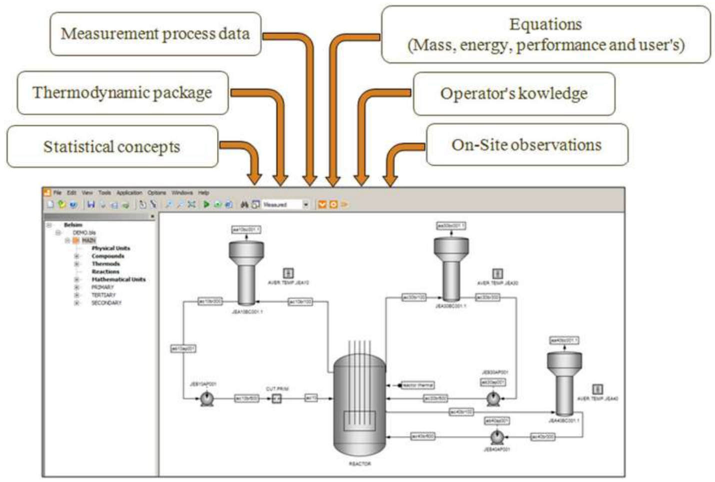
NPPS Data Reconciliation Workflow[26]

Initial Configuration

After installation, an initial configuration of the software is performed to capture existing process knowledge and technical and business requirements. This is accomplished by integrating process

[26] Tran Quang and R. Chares, "Ageing nuclear plants inspection, maintenance and performance monitoring eased by Data Validation and Reconciliation (DVR) approach", Belsim (BTB Jansky)

thermodynamics, list of process state variables and KPI, list of sensors and all available prior knowledge about measurement statistics (e.g., calibration and known drift and biases).



Run-Time Monitoring Tasks

After the initial configuration, the data reconciliation software will continuously monitor the process variables and KIP in order to:

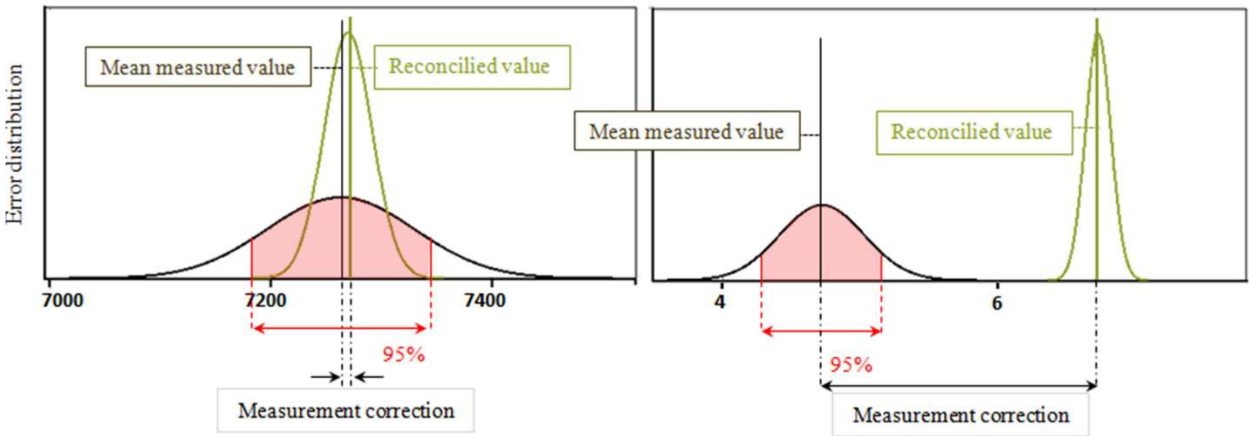
- reduce measurements uncertainty by augmenting analytical redundancy (virtual sensors based on conservation laws and other process and KPI models),
- detect and estimate unexpected gross errors and drifts
- isolate (locate) sources of degradations, including unreliable sensors or process parameters– e.g., heat transfer coefficient due to fouling or leaks
- verify impact of process changes due to maintenance or upgrades, via before-and-after delta analysis
- recommend enhancements/optimizations to KPI to improve performance and schedule maintenance (re-calibration) to reduce gross errors
- integrate new physical/operational relations and constraints into the software configuration
- identify needs for additional sensors, including sensor requirements and optimal placement

Gross Errors Elimination

The identification and elimination of gross errors is essential for the quality assurance of the reconciliation results (all dependent tasks). Typically, this involves an iterative sequence of "identify and eliminate" steps is implemented based on a (global) quality metric, Q, of reconciled data, e.g., Chi-Square test

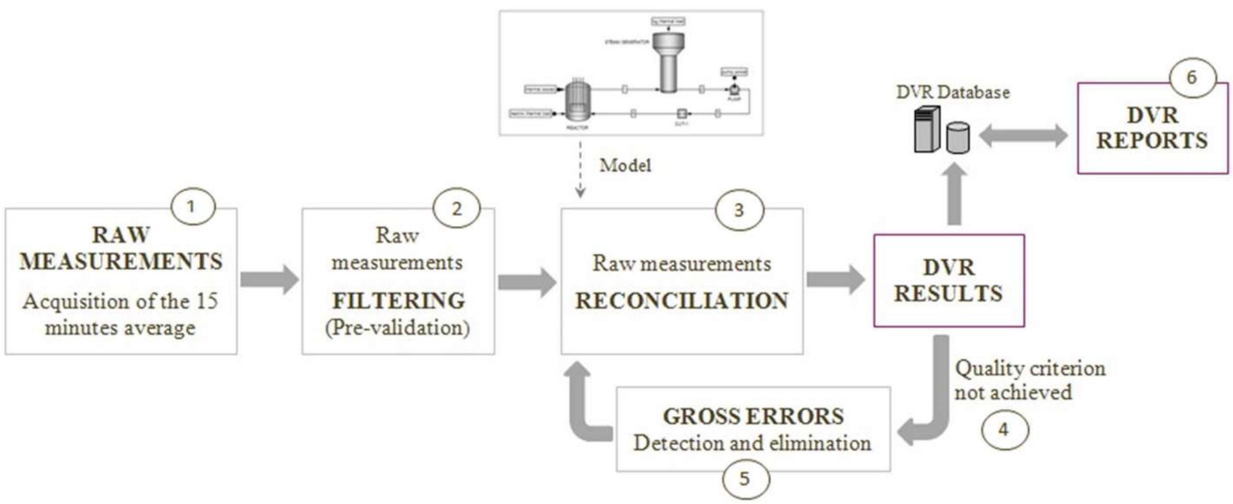
$$Q = \frac{\epsilon_0}{\chi^2_{0.05}} \leq 1, where\ \epsilon_0 = J(\underline{\hat{x}})$$

The following is an example distributions of measured and reconciled sampled mean values showing measurement corrections in two scenarios (a) no gross errors $Q \leq 1$ and (b) gross errors $Q > 1$



Reconciliation Workflow

The workflow associated with the run-time tasks is summarized in the following diagram.



1. **Run-Time data acquisition step** – In this step data is collected within a 15-minute time window. This includes data from all sensors and any other data sources, e.g., other software database. The sample mean of the process measurement $\underline{\hat{Y}}(t_n)$ are then updated using data within the window $\underline{Y}(t_{n-1} + k\Delta t), k = 1, \ldots, \frac{T_{window}}{\Delta t}$ where Δt is the sampling rate within the time window.

2. Filtering step – This step filters or smooths outlier values in the sampled mean values, based on physical constraints within the specified operating mode of the process. It is not clear if missing sensors values are handled in this step (or within the reconciliation step).

3. Reconciliation Step – This is the core step, which updates the values of the process state variables and their associated uncertainty. See the Example on page 1 of this document.

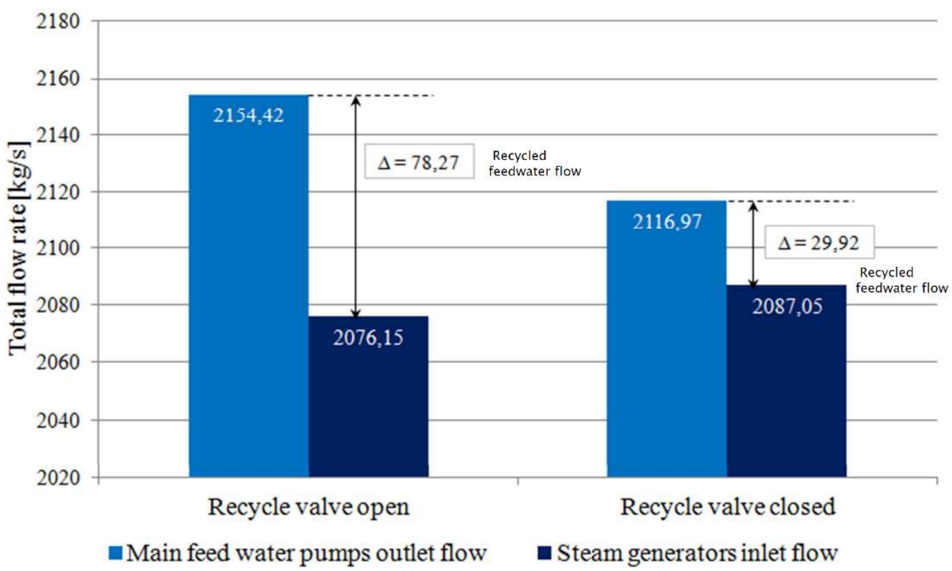
4. Evaluate the quality of the reconciliated data using Chi-Square test outlined in the previous section.

5. Gross error step – In this step the result of the quality test are used to isolate measurements (and process parameter estimates) that include gross errors (bias and drifts). Actions to address the gross errors must be performed before attempts are made to use the reconciled data. Example actions are re-calibration of sensors or updating the configuration of the reconciliation software to take the errors into consideration. Steps 4 and 5 are repeated until all quality tests in step 4 successfully pass.

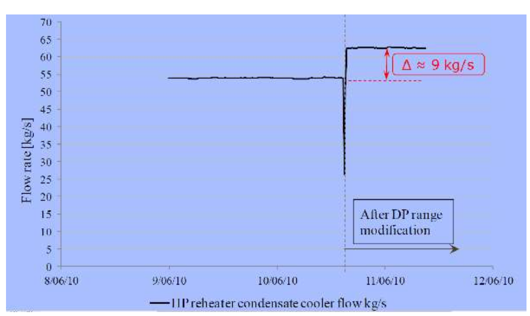
6. Report step. The reconciliation results are stored in a database, along with any recommendation for performance/efficiency improvements or predictive maintenance.

Example Applications

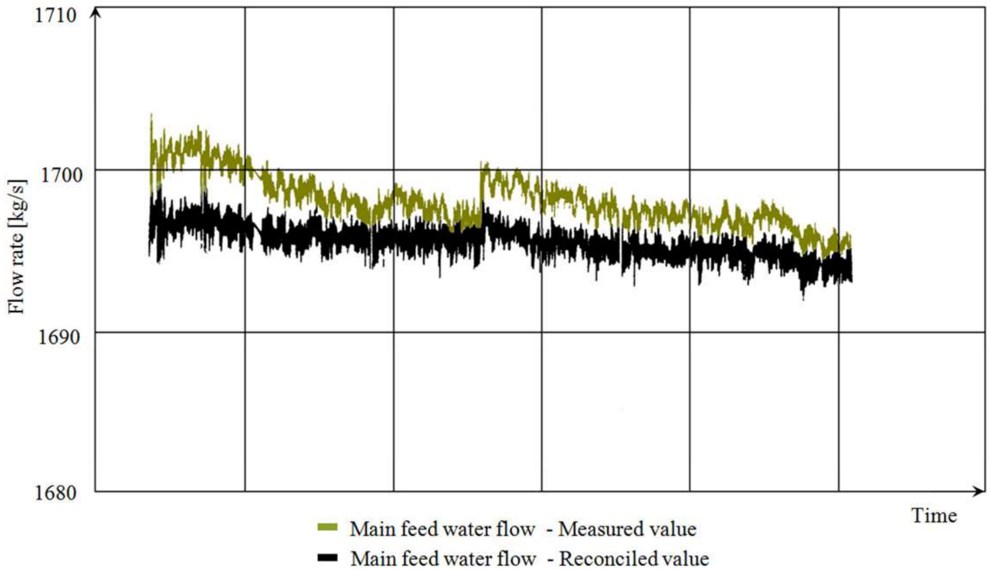
- Detecting abnormal mass imbalance causing inefficiency – detected a high flow rate of recycled water to main feed water tank, which was reducing water flow to the steam generator and hence reducing secondary-side efficiency.



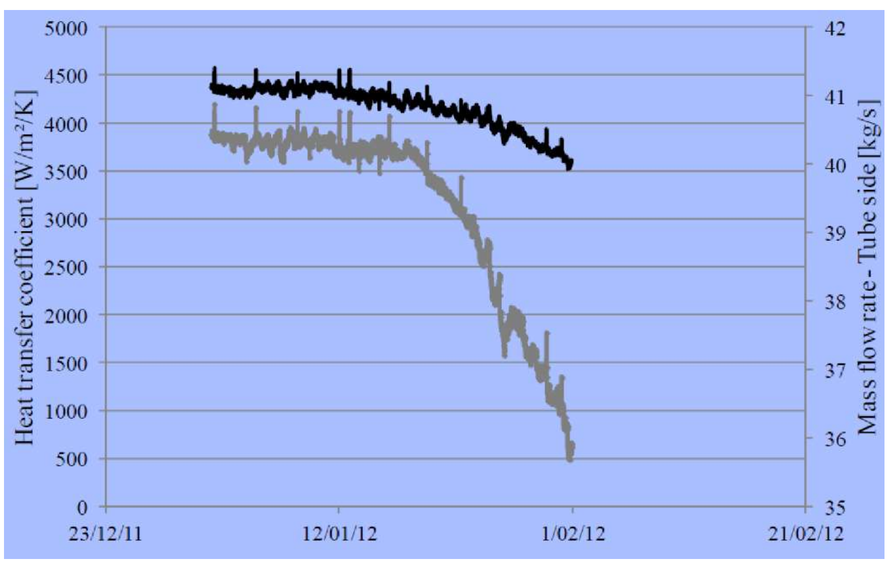
- Detecting sensor bias after maintenance (measurement gross errors) – detected a flow rate sensor bias after inaccurate calibration using delta analysis (before-and-after analysis).



- Detecting sensor drift in the feed water flow – detected drift as a quality violation indicating gross error during data reconciliation quality checks.



- Accurate calculation of KPIs and internal parameters – An example of calculation of KPIs, includes accurate calculations of reactor thermal power (with less than 0.5% uncertainty). Also, detection of deterioration trend in internal heat transfer coefficient of a heat exchanger.



Redundancy Analysis

Sensor Redundancy Analysis

Incidence Matrix Structural Analysis

The incidence matrix[27] of sensor measurement set can be used to analyze the degree of redundancy, with the goal of identifying missing measurements, as well as difficult-to-validate measurements[28].

Analytical Redundancy Analysis

Sensor Fault Isolation (Parity Analysis)

Sensor Placement Optimization

The original BelSim and other software were used to design redundant sensor networks29,30 . See other references in . Of particular interest is the work of Bagajewicz and colleagues , for example "Software Accuracy-Based Sensor Network Design and Upgrade in Process Plants,"[31]

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[27] Heyen G., E. Maréchal and B. Kalitventzeff, "[Sensitivity Calculations and Variance Analysis in Plant Measurement Reconciliation]", Computers and Chemical Engineering, vol. 20S, 539-544, 1996. Available

[28] D. Brown, et. al., "Application of data reconciliation to the simulation of system closure in a paper deinking process," discusses the sensor placement and expert knowledge fusion based on Belsim software.

[29] G. Heyen, et.al., "Computer-Aided Design of Redundant Sensor Networks," Computer-aided chemical engineering, 2002.

[30] D. Nguyen and M. Bagajewicz "Design of Nonlinear Sensor Networks for Process Plants," Ind. Eng. Chem Res., 2008, 47, 5529-5542

[31] See other related and more recent references at D:\R&D\MW Optimization\Data Reconciliation\Sensor Redundancy and Fault Isolation

Observer-Based Data Reconciliation and FDI[32]

The general form of an observer-based process observation is:

$$Z = g(X)$$
$$Y = g(X) + e$$

where
X is the vector of state variables,
Z the measured process variables,
Y the measurement value of Z.

Note: In the trivial case of no state variables, we can still introduce a fictitious once to simplify the analysis

$$Z = C^T X$$
$$Y = C^T X + e$$

A general formulation of the observer-based data reconciliation problem is as follows:

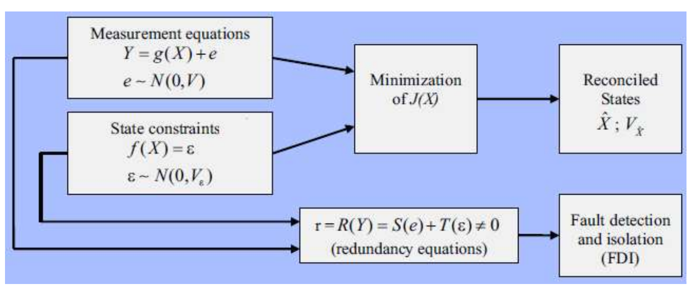
Find the reconciliation vector $\hat{X}$ and variance matrix $V_{\hat{X}}$, that minimize the following objective function $J(\hat{X})$, where

$$J(\hat{X}) = (Y - Z)^T V^{-1} (Y - Z) + \varepsilon^T V_\varepsilon^{-1} \varepsilon$$

Subject to the constraints,

$$Z = g(x)$$
$$Y = Z + e; \;\; e \sim N(0, V)$$
$$\varepsilon = f(X); \;\; \varepsilon \sim N(0, V_\varepsilon)$$
$$X_{min} \le X \le X_{max}$$

X are the plant states
Y the measured values of Z
e the measurement errors assumed to have zero mean values and known variance matrix V,
ε the constraint uncertainty values with zero mean and known variance matrix V_ε
f(.) the State equations of mass and energy conservation constraints and any additional constraints
g(.) observation equations that maps measured variables to state variables



In addition to data reconciliation, the above formulation can support *Fault Detection and Isolation* (FDI), in order to use ***redundancy*** and residuals to detect measurement biases or abnormal deviations to mass and energy conservation laws.

The state variables, X, are said to be observable if they can be estimated from the measurements, Y.

[32] Daniel Hodouin, "Process Observers and Data Reconciliation Using Mass and Energy Balance Equations"

Historical Perspective on Measurement Uncertainty and Data Reconciliation in NPPs

Basic Instrument Calibration Monitoring[33]

The referenced section summarizes generic recommendations that were introduced by EPRI and US NRC in 2000. The measurement uncertainty technique **at specific channel** is generic and hence should not be used for implementations, since it does not take the global process context (models) into consideration (in contrast to data reconciliation).

However, the section provides a historical perspective on how the industry approached uncertainty before introducing DR. It also provides an insight into how measurement uncertainty was used to setup trip thresholds and how to extend the calibration intervals of safety related nuclear instrumentation.

Redundant sensor calibration monitoring[34]

Still single channel but multiple sensors

Non-Redundant Sensor Monitoring[35]

Attempt to add analytical redundancy. The section describes some ad-hoc techniques (including ANN combined with Monte Carlo/Bootstrapping simulation to estimate the uncertainty – similar to what we done in case of structural analysis).

The only technique of interest here is the data reconciliation. A brief historical perspective is given on pp54; including earlier applications of Kalman Filter to linear cases; along with an overview of commercial software, including VALI (Belsim and BTB Jansky) and VDI 2048.

The section also replicates the Flow Splitter example from VALI 2003 paper. With a key note on the importance of gross error detection for the validation of the data reconciliation analysis. Specifically, the note emphasizes that the simple test described in the flow split example above works only for the case of linear constraints (e.g., mass flow balance equations), but not in case of general (nonlinear) constraints.

This Global Test (chi-square for detecting presence of gross errors) and Measurement Tests (for isolating sensors with gross errors) needs to be assessed in the context of the latest technology (e.g., latest implementation in BTB Jansky software). A key starting point is the research related to RECON software.[36]

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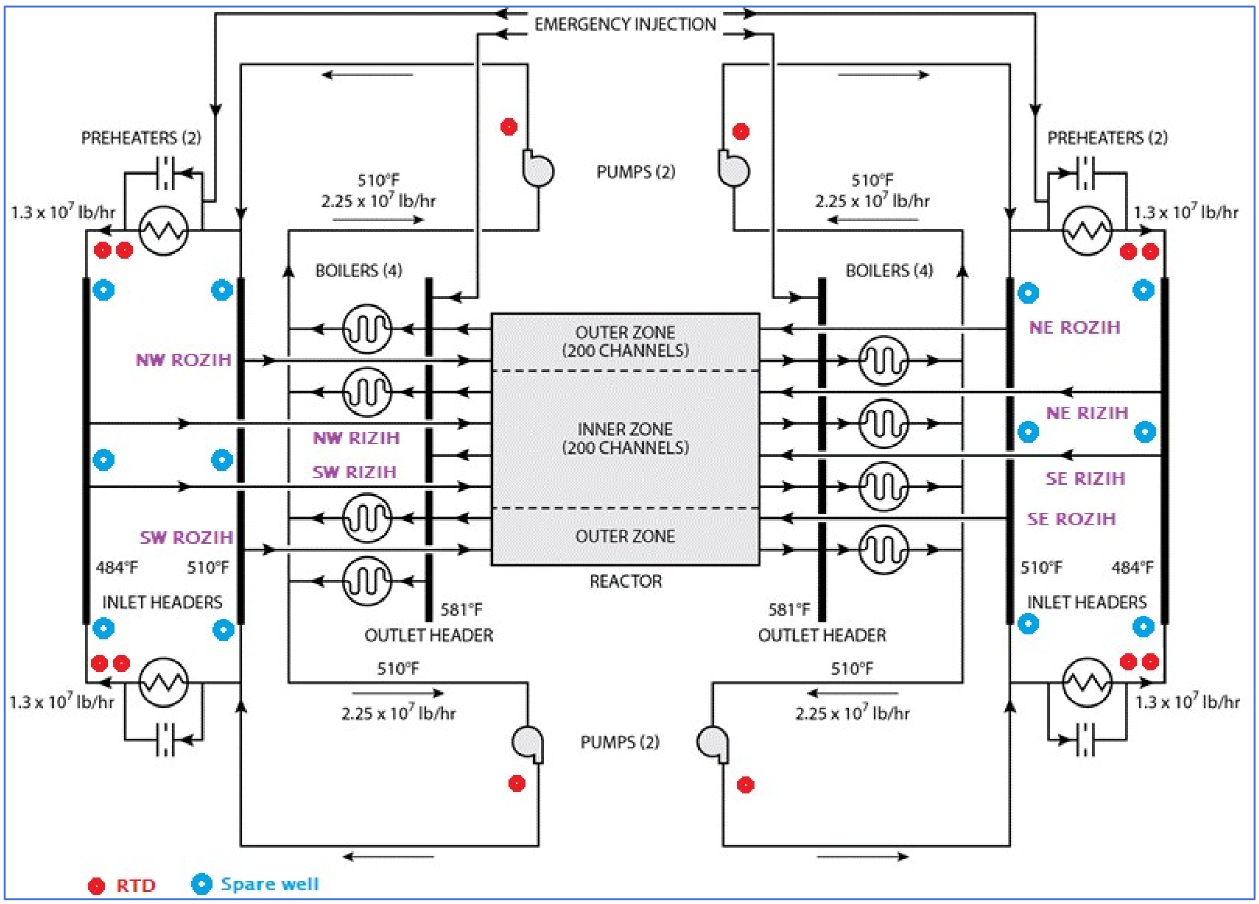
[33] J.W. Hines and R. Seibert "Technical Review of On-line Monitoring Techniques for Performance Assessment,", Volume I: State-of-the-Art, U.S. Nuclear Regulatory Commission, 2005, pp 6 (pdf page 22/157).
[34] J.W. Hines and R. Seibert "Technical Review of On-line Monitoring Techniques for Performance Assessment,", Volume I: State-of-the-Art, U.S. Nuclear Regulatory Commission, 2005, pp 20 (pdf page 36/157).
[35] J.W. Hines and R. Seibert "Technical Review of On-line Monitoring Techniques for Performance Assessment,", Volume I: State-of-the-Art, U.S. Nuclear Regulatory Commission, 2005, pp 25 (pdf page 41/157).
[36] M. Syed, et.al., "Data Reconciliation and Suspect Measurement Identification for Gas Turbine Cogeneration Systems," at *D:\R&D\MW Optimization\Data Reconciliation\Fundamentals\RECON*

CANDU Background

A simplified Heat Transport System (System Context for the Data Reconciliation)

A simplified Diagram of the Primary Heat Transport (PHT) is depicted below. A key challenge is the effect of boilers and feedwater preheaters performance issues (aging/fouling) on the RIH temperature and hence the overall system performance (Reactor Thermal Power RTP). See [3] and [8] for current approaches to handle increased RIH temperatures.



Appendix - Basic Statistics

See links to background material in local YouTrack

- CORTEX-11 Measurement Uncertainty, also see Uncertainty in Measurement Guide (GUM)
-

Central Limit Theorem

Let $\underline{x} \in \mathcal{R}^n$ be a vector of random variables representing measured process states. Let $\boldsymbol{p}(\underline{x})$ be an arbitrary distribution of $\underline{x}$ with a true mean μ and variance σ^2. Let $\mu_{\bar{x}}(m)$ and $\sigma_{\underline{x}}^2(m)$ be the sample mean and variance of the sampling distribution of m observations of $\{\underline{x}_j, j = 1, .., m\}$, i.e.,

$$\mu_{\bar{x}}(m) = \frac{1}{m}\sum_{j=1}^{m}(\underline{x}_j)$$

$$\sigma_{\underline{x}}^2(m) = \frac{1}{m-1}\sum_{j=1}^{m}\left(\underline{x}_j - \mu_{\bar{x}}(m)\right)$$

Then the distribution of the sampling distribution of the sample mean $p(\mu_{\bar{x}}(m))$ will be approximately a *normal* distribution, if sample size is sufficiently large, e.g., $m \geq 30$. See simulation of the sampling distribution

Furthermore, the variance, $\sigma_{\underline{x}}^2(m)$, of the sampling distribution of the sample mean, will inversely proportional to the number of samples, m

$$\sigma_{\underline{x}}^2(m) = \frac{\sigma^2}{m}$$

Hence, for m observations of process states $\underline{x}$ the standard deviation (standard error) of the sampling (normal) distribution of the sample mean is,

$$\sigma_{\underline{x}}(m) = \frac{\sigma}{\sqrt{m}}$$

Measurement Uncertainty

- Frequency Approach

$$U(\underline{x}) \triangleq \sigma_{\underline{x}}(m) = \frac{\sigma}{\sqrt{m}}$$

- Bayesian Approach
  - Non-Pooled measurements case

$$U(\underline{x}) \triangleq \sqrt{\left(\frac{m-1}{m-3}\right)}\frac{\sigma}{\sqrt{m}}, m \geq 4$$

  - Pooled measurements case, where $1 \leq m < 4$

$$U(\underline{x}) \triangleq \sqrt{\left(\frac{m_p-1}{m_p-3}\right)}\frac{\sigma}{\sqrt{m_p}}, m_p \geq 20$$

Covariance

$$cov = \frac{1}{n}\sum \Delta X.\Delta Y$$

Correlation

$$r = \frac{\frac{1}{n}\sum(\Delta X.\Delta Y)}{\sqrt{\frac{1}{n}\sum(\Delta X)^2}\sqrt{\frac{1}{n}\sum(\Delta Y)^2}} = \frac{cov}{std(X).std(Y)}$$

Using Cauchy-Schwarz Inequality, one can show that $-1 \leq r \leq 1$ ($|r| = 1$ when X and Y are strongly correlated, e.g., $\underline{Y} = m.\underline{X}$).

$$\left(\sum \Delta X.\Delta Y\right)^2 = \langle\underline{\Delta X}, \underline{\Delta Y}\rangle^2 \leq \langle\underline{\Delta X}, \underline{\Delta X}\rangle.\langle\underline{\Delta Y}, \underline{\Delta Y}\rangle = \left(\sum(\Delta X)^2\right).\left(\sum(\Delta X)^2\right)$$

$$\Rightarrow r^2 \leq 1 \Rightarrow -1 \leq r \leq 1$$

References

[1] *The Essential CANDU* - Heat Transfer and Fluid Flow Design (Section 7), Thermal-Hydraulic Design (chapter 6).

[2] Chapter 3 - Heat Transport System Thermalhydraulics - 20043705.pdf (D:\R&D\MW Optimization\CANDU Background\Course 2.1 - Reactor Thermalhydraulics Design)

[3] Preston Tang and Akash Bhatia, David Zobin, Khurram Khan, Kurt Gilbride and Jefferson Tse, "Proposed Mitigation of the Reactor Inlet Header Temperature Increase in Bruce Power Units", 11th International Conference on CANDU Maintenance and Nuclear Components, 2017. (Bruce Power and Amec Foster Wheeler). See also "Design Modification to Mitigate the Reactor Inner Zone Inlet Header Temperature in a CANDU Reactor," in Pressure Vessels and Piping Conference, 2017, HI, USA.

[4] M. Câmara "Numerical Aspects of Data Reconciliation in Industrial Applications," Processes, 5, 56, October 2017

[5] S. Streit "State Estimation in Thermal Power Plants," IFAC Control of Power Plants and Systems, Munich, 1992.

[6] S. Streit, M. Langenstein, B. Laipple and H. Eitschberger, "A new method for evaluation and correction of thermal reactor power and present operational applications," ICONE13, Beijing, 2005.

[7] VDI 2048 Part 1, "Uncertainties of measurement during acceptance tests on energy-conversion and power plants – fundamentals", October 2000.

[8] S. Basu and D. Bruggeman - POWER RAISE THROUGH IMPROVED REACTOR INLET HEADER TEMPERATURE MEASUREMENT AT BRUCE A NUCLEAR GENERATING STATION.